Comparison of Plotting Methods for Solar Radiometer Calibration

S. M. ADLER-GOLDEN

Spectral Sciences, Inc., Burlington, Massachusetts

J. R. SLUSSER

UV-B Monitoring and Research Program, Natural Resource Ecology Laboratory, Colorado State University, Fort Collins, Colorado

ABSTRACT

An alternative formulation of the Langley plot relating observed solar irradiance, extraterrestrial solar irradiance, and air mass has been suggested to potentially improve radiometer calibration accuracy. In this study, results from the traditional and alternative plotting methods are compared using both simulated and measured data. The simulations indicate that their relative accuracies depend on the time scale of the atmospheric extinction fluctuations. The two methods are found to be essentially equivalent with the measured data.

Ground-based solar radiometry provides much of our fundamental knowledge about the optical and physical properties of both the sun and the earth's atmosphere (Shaw 1982; Holben et al. 1998; McArthur et al. 2003). Analysis of the data involves combining measurements of the sensor signal $I(\lambda)$ at different wavelengths through a range of airmass values M in order to infer the extraterrestrial solar irradiance signal $I_0(\lambda)$ and the atmospheric extinction coefficient $k(\lambda)$ using the Beer–Lambert–Bouger equation:

$$\ln(I) = \ln(I_0) - kM. \tag{1}$$

In the traditional Langley method [named for the inventor of the bolometer (Langley 1881)], $\ln(I)$ is plotted versus M and a linear regression performed to determine the intercept $\ln(I_0)$ and the slope $-k$. An implicit assumption is that the atmospheric composition, and hence k, can be considered as constant during the measurement period. With this procedure, measurements with calibrated radiometers enabled the derivation of the extraterrestrial solar irradiance spectrum in absolute units. With the solar spectrum thus deter-

mined, it is common to use Langley plots to calibrate radiometers in the field on a frequent basis (Schmid and Wehrli 1995; Slusser et al. 2000). Atmospheric extinction can therefore be accurately measured over time without needing to reference the radiometer to an irradiance standard lamp.

Despite its widespread use, the Langley plot method may not necessarily be the optimal way to derive I_0. In particular, linear regression presumes that error (noise) in the dependent variable [i.e., $\ln(I)$] does not vary systematically with the independent variable (i.e., M). This condition would be satisfied if the major noise contribution were intensity fluctuations in the light source, which is certainly not the case here. Instead, under most weather conditions the bulk of the Langley plot "noise" arises from temporal and/or spatial fluctuations in the atmospheric extinction coefficient. As a result, the error in $\ln(I)$ increases with M, and the linear regression method gives more weight to the high-M data points relative to the low-M points. It is not obvious that this weighting is appropriate. In addition, because the rate of change of M increases with M, the spacing of the plotted data points increases with M, which can make Langley plots a bit tricky to interpret by visual inspection.

In this brief note, we compare the Langley plot with an alternative method that is based on a simple redefi-

Corresponding author address: Dr. Steven Adler-Golden, Spectral Sciences, Inc., 4 Fourth Avenue, Burlington, MA 01803-3304.
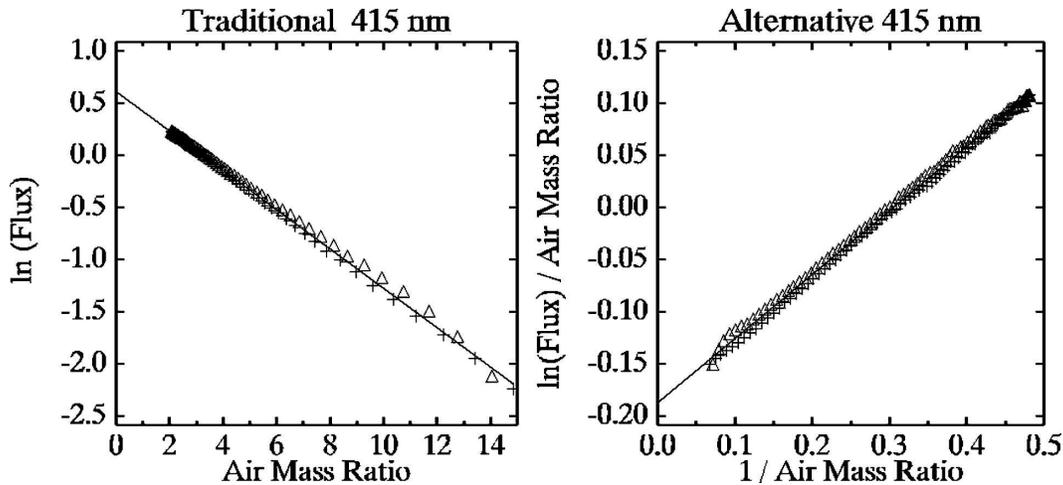E-mail: sag@spectral.com

FIG. 1. Traditional Langley and alternative plots of MFRSR data taken at Davis, CA. Flux units are W m^{-2} nm^{-1}. Plus symbols denote AM data; triangles denote PM data. Solid lines are the linear regression fits.

nition of variables. In the alternative plot, the dependent variable is $\ln(I)/M$, the independent variable is $1/M$, the slope of the plot is $\ln(I_0)$, the intercept is $-k$, and the k noise is uniform across the plot:

$$\ln(I)/M = \ln(I_0)M - k. \qquad (2)$$

This method has been used in previous unpublished work (J. B. Kerr 2004, personal communication) and in a recent study on atmospheric compensation of imagery (Rochford et al. 2005) using data from a multifilter rotating shadow band radiometer (MFRSR) (Harrison et al. 1994). The traditional Langley plot from Eq. (1) and the alternative plot from Eq. (2) are compared in Fig. 1 for MFRSR data on a clear day. Since the linear regressions for the two plotting methods can potentially generate slightly different values of $\ln(I_0)$ as well as slightly different k values, it is desirable to understand and characterize their relative accuracies.

In our initial investigation, some numerical experiments were performed using simulated data. The case presented here assumes equatorial solar equinox, with a baseline k value chosen as 0.4 per atm. The data point spacing is 0.12 h. To mimic atmospheric fluctuations in k, a 20% root-mean-square (RMS) noise waveform was added. The waveform was generated by Gaussian random sampling, followed by taking a running average with a variable width window (1, 3, 7, 15, or 31 points) to introduce a high-frequency cutoff in the fluctuations, and then finally renormalizing to unit RMS. A total of 10 000 test cases were run, corresponding to two different plot lengths for the airmass ratio ($M_{max} = 3$ and 8 air masses), the five different frequency cutoffs, and 1000 different noise waveforms (corresponding to dif-

ferent initializations of the random number generator). For each test case, $\ln(I_0)$ was calculated by linear regression using both the Langley method and the alternative method. The results were compared with the correct value of $\ln(I_0)$ to determine the error. The errors for the 1000 waveforms were combined to yield an RMS average error in $\ln(I_0)$ for each combination of frequency cutoff and plot length.

The results of the simulations are reported in Fig. 2. The errors are seen to decrease with increasing fluctuation frequency, when there is less redundancy among successive measurements and hence more noise averaging. With the short plots ($M_{max} = 3$) the Langley and alternative methods are essentially equivalent. However, with the long plots ($M_{max} = 8$) the differences between the two methods are significant; the alternative method is better with high-frequency fluctuations, and the Langley method slightly better with low-frequency fluctuations, with the two methods being equivalent when the frequency cutoff is around 0.6 h^{-1}. For cutoffs of 1 h^{-1} and greater the alternative method long plot yields errors between 65% and 87% as large as those from the Langley method. The k value errors show exactly the same trends as the $\ln(I_0)$ errors. Although the absolute errors in Fig. 2 are specific to the size of the fluctuations and the average k value chosen, the relative errors of the two methods are not.

A further investigation was performed using measurements from a visible MFRSR instrument located at the U.S. Department of Agriculture UV-B Monitoring and Research Program site at Davis, California. The data are for all cloud-free periods during 2004 for the five water-free narrow wavelength bands of the
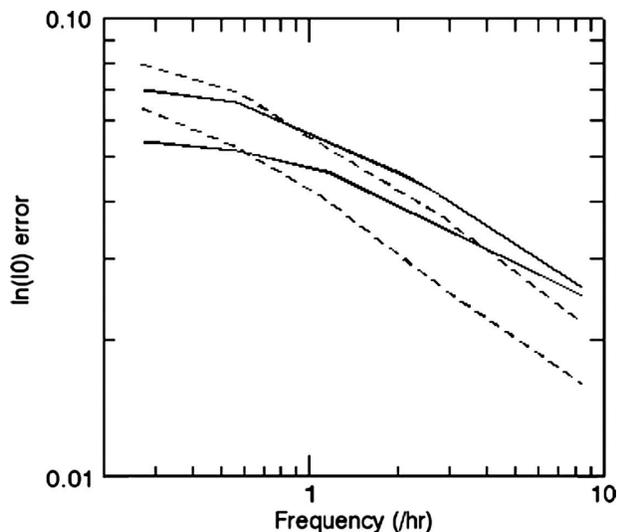
FIG. 2. Dependence of $\ln(I_0)$ error on fluctuation cutoff frequency (per hour). Solid lines denote the Langley plot, and dashed lines denote the alternative plot; the upper curves are for $M_{max} = 3$ and the lower curves are for $M_{max} = 8$.
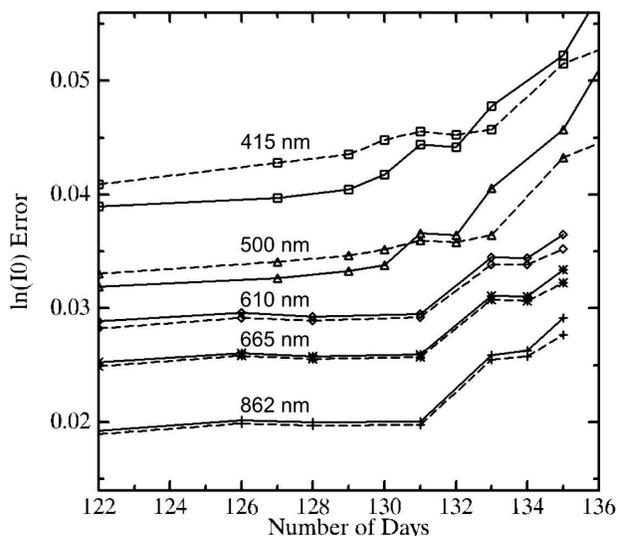

FIG. 3. $\ln(I_0)$ RMS errors from 2004 MFRSR data for Davis, CA. Solid lines denote the traditional Langley method; dashed lines denote the alternative method. Symbols identify individual narrow wavelength bands of the MFRSR. The 415- and 500-nm results are for $M_{max} = 6$; the 610-, 665-, and 862-nm results are for $M_{max} = 3$.

MFRSR; the data from the water-sensing band (940 nm) are omitted since they do not follow the Beer–Lambert–Bouger law. Assuming that the true sensor calibration changes slowly over time, the calibration error metric is taken as the RMS deviation of the daily $\ln(I_0)$ value with respect to a running average of $\ln(I_0)$ over 21 consecutive days. The daily $\ln(I_0)$ values are obtained from plots that combined all data (from both the A.M. and P.M.) for up to $M = M_{max}$.

The results from the measurements turned out to be quite sensitive to the presence of a few relatively poor daily datasets, in which both types of plots showed significant deviation from linearity. To improve the statistics of our plotting method comparison, we weeded out data in which the standard error in the Langley plot slope (k value) exceeded a predetermined threshold. By varying the threshold value, the calibration error was computed as a function of the number of retained days of data.

The results in Fig. 3 are taken from calculations performed for $M_{max} = 3$, 6, and 8. Data for $M > 6$ may be slightly biased by the angular response of the Spectralon diffuser in MFRSR instruments. For each wavelength and plotting method, the error versus number of days curve is shown for the M_{max} value that yields the smallest overall error. As it turned out, at a given wavelength these M_{max} values were the same for both plotting methods; thus, *the two methods are being compared using exactly the same data points*. For the two optically thickest wavelengths (415 and 500 nm), the M_{max} value

of 6 is favored. The alternative plotting method is favored by a small margin when more days of data are retained, while the traditional method is favored by a similarly small margin when fewer days are retained. This indicates that the alternative method may be slightly better than the traditional method with the poorer data. For the longer wavelengths, $M_{max} = 3$ is favored, and virtually identical results are obtained with both plotting methods. In all cases the differences in the mean values of both I_0 and k between the two methods are 1% or less.

Figure 4 shows the individual $\ln(I_0)$ values obtained at the 415-nm wavelength, where the two methods perform a little differently. The alternative method has a tendency to give very slightly higher values, by around 0.01 (~1% in I_0), as indicated by the clustering of data points slightly above the line.

We interpret the near-equivalence of the two plotting methods as indicating that the characteristic time scale of atmospheric extinction fluctuations at Davis, California, is in the intermediate regime of Fig. 2 centered around the 0.6 h^{-1} frequency cutoff. We do not expect this time scale to vary much at different locations around the globe. Therefore, we believe that the results of this study are generally applicable, and conclude that the choice of the traditional Langley plot [Eq. (1)] versus the alternative method [Eq. (2)] is more a matter of taste than accuracy.
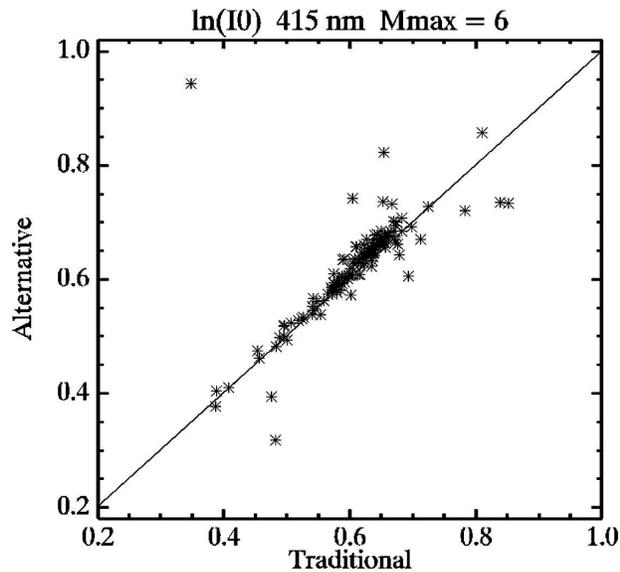
FIG. 4. Comparison of $\ln(I_0)$ values from the traditional and alternative plotting methods for the Davis, CA, dataset. Straight line corresponds to identical values from the two methods.

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