Kevin Wilson

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Isorhythm and Musical Identities

An analysis of isorhythm as occurs in historical and contemporary composition functions as a viable, enabling focal point for discussions of form and formalism in music. We will consider Guillaume de Machaut's *Messe de Notre Dame* as a noted example of isorhythm in fourteenth century medieval music, evaluating research that attempts to reconstruct Machaut's compositional process. In doing so, we hope to gain first-hand appreciation for the subtlety and complexity of musical phenomena that manifest aurally the relationship between chance and order and between probability and structure. Our analysis of Machaut's Mass, and the understanding of isorhythm that this analysis yields, will establish a basis for comparison with other prepared compositional forms that achieve different forms of perceptible mapping between and across individual parts, lines, or parameters within a piece of music. I am attempting to establish an elucidation of form in music, introducing the concept of isomorphism as a building block. Specifically, we will consider isomorphism between lines within a piece of music, but we will also analyze relationships between and across a range of musical properties and the manipulation of these relationships as compositional intent.

From the onset, I hope to emphasize that one should regard *isorhythm* and *isomorphism,* while relatable in some sense, as very distinct and very different concepts that somewhat incidentally share the same prefix *iso.* Actually, since *iso* in Greek means "same", and since both terms pertain to identity and similarity structures, the linguistic convergence appears less

incidental but still important to establish. Isomorphism is an originally mathematical term that we will use in the context of music to discuss instances in which one maps similarity or identity between sets of notes despite differences. Notably, also, the term *isorhythm* has considerable use outside of music as well.

I would like first to consider and evaluate isorhythm as it occurs historically in one well-known work of Guillaume de Machaut. In attempting to reconstruct Machaut's methods into an ordered series of compositional steps, researchers find that the composition of isomorphic works challenges composers' ability to conceive of music in a nonlinear fashion. Researchers must understand how composers considering matters like ratio, common factors, and otherwise, and the relationship between these abstract musical and numerical properties and the rhythmic and harmonic development of a piece.

Let us treat isorhythm in its most common sense, specifically, that of a fixed rhythm grid (historically known as the *talea*) and an ordered set of note values (the *color*) with which to instantiate this grid. Machaut's first compositional step is to choose a plainchant melody to assigned to the Tenor voice as its *color* (Leech-Wilkinson, 21). The tenor voice of the opening section of *Kyrie,* the first section of *Messe de Notre Dame,* has a 28-note *color* arranged into seven *taleae*. Each *talea,* then, has four notes, such that upon seven instantiations of the *talea* all 28 notes of the *color* have been instantiated.
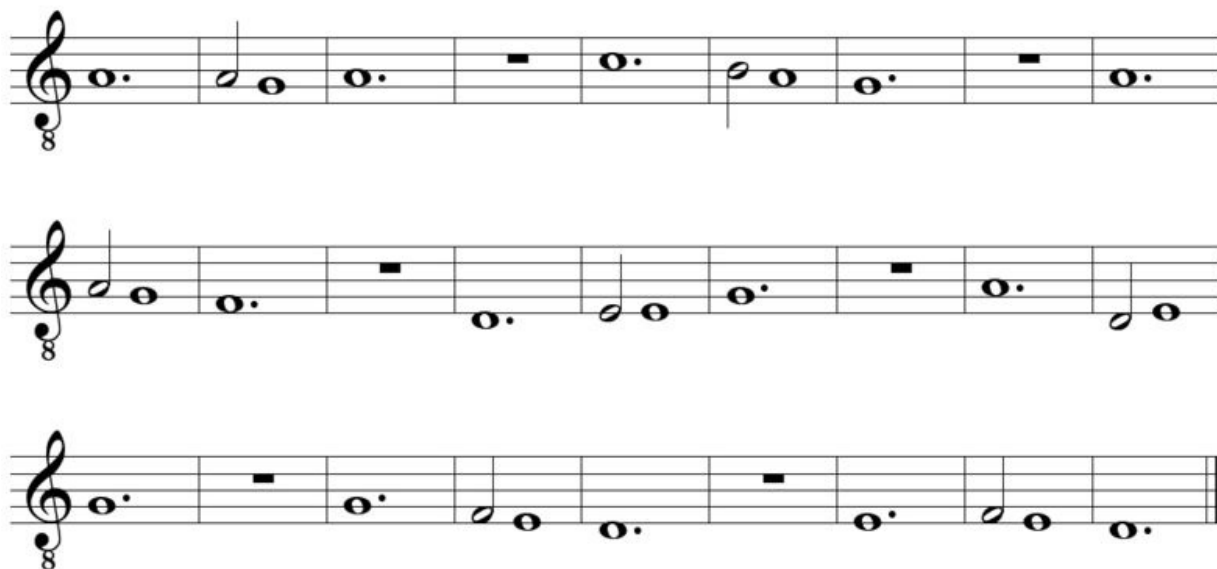
Tenor Part from Kyrie I of Machaut's Messe de Notre Dame (http://en.wikipedia.org/wiki/File:TenMachautKyrie.jpg)

In *Kyrie I,* the Tenor part's *color* is identical to the chosen plainsong. Given that, at least for this period, the composition of Triplum and Motetus parts proceeds that of Tenor and Contratenor parts, Machaut faces the task of simultaneously evaluating the relationship between the Contratenor's *color,* the Tenor's *talea,* and the Contratenor's *talea* (Leech-Wilkinson, 21). While isorhythm is a strictly defined process, an exhaustive enumeration of isorhythmic methods available to composers proves more difficult.

Leech-Wilkinson regards Machaut's *Kyrie* as "exceptional" and "almost unique" for the period in its divergence from strict isorhythm in the Contratenor voice (23), specifically, in terms of its *talea*. The upper voices, which utilize the Contratenor's *talea*, also diverge, but this phenomenon appears more commonly in medieval works. These conditions lead us to conclude that Machaut places high value on strict isorhythm in the Tenor part while allowing some flexibility within other parts in order to adhere to rules of counterpoint.

Leech-Wilkinson argues that Machaut as composer consciously considers the number of notes in the Tenor part's *talea* such that its completion corresponds with that of the 28-note *color.*

Owen Rees in his article *Machaut's Mass and Sounding Number* further contends that Machaut deliberately emphasizes the numbers 7 and 12 as pertains to certain ratios and relationships both within and between sections of the mass, most notably, 12 Tenor notes (i.e., 3 Tenor *taleae*) within each Contratenor *talea* duration in *Kyrie.* On this particular attribute Rees bases his argument against Leech-Wilkinson's characterization of *Kyrie* as "peculiar" due to its incomplete third Contratenor *talea.* To analyze whether Machaut at times consciously invokes a given ratio or factor (in this case, 7 and 12), Rees attempts to establish a significance of deviation between the number of times this factor occurs and number that would be expected at random. For *Kyrie*, Machaut could have chosen *taleae* of length 4 to 14. Rees argues that, statistically significantly, two of the four sections of *Kyrie* contain *taleae* length 7, and three of the four contain *taleae* length 7 or 12. The difficulty in assessing this claim stems from the need to weight the probability of different *taleae* lengths, a somewhat complicated task Rees acknowledges but does not attempt (103).

In the upper voices of Machaut's *Kyrie I,* less strict or "partial" isorhythm occurs, insofar as the rhythmic repetitions occur but with greater deviation, notably, with *taleae* corresponding to those of the Contratenor, not the Tenor part (Leech-Wilkinson, 23) . Contrast between the ranges of respective upper lines and the sizes of steps and leaps within each of these lines demonstrates Machaut's capacity to differentiate function while still achieving multiple part isorhythm. Rhythmic similarity between simultaneous isorhythmic parts potentially contributes to the centralized solemnness and stillness manifest in this piece.

Expanding into a more fundamental discussion of form in music, we can regard isorhythm as one particular type or instantiation of *isomorphism* in music, that is, identity or "sameness" between relatable relative classes of parts of a whole (e.g., degree and direction of

consecutive intervallic change within a part). In this way, we can regard isorhythm present in the Tenor Part of the *Kyrie* as a manifestation of rhythmic identity across time within a piece.

Isorhythm appears less obvious and self-apparent, perhaps, than other compositional forms that create or utilize types of identity between parts, specifically, forms like canon, fugue, or, even more overtly, simple "rounds" (such as "Row Row Row Your Boat"). In each, the listener actively perceives identity between melodic parts that occur rhythmically variably, evidence of the capacity of sequences of notes to function simultaneously in melodic and harmonic roles. In the case of isorhythm, however, the underlying rhythmic identities that occur between parts, perhaps because unsupported by strong melodic identity as in a canon, might result in a more subtle, or, arguably, abstract compositional effect. Note also that any simple repeating figure can be represented formally as specific subset of isorhythm in which the number of notes in a given *talea* is equal to the length of a given *color*.

Considering potential variation in the degree to which listeners can perceive a given isomorphism in a given piece of music, we might question whether perceptibility is in fact necessary for a relationship to exist in a proper sense. Consider inverse, retrograde, and inverse-retrograde manipulations in a canon, for instance. In these cases, one might perceive identity between parts despite polar contrast either in ordering, or direction of relative movement up or down in pitch, or, in the case of inverse-retrogradation, both. Suppose a set of pitches is not inverted, or reversed, but rather its order completely randomized -- does an isomorphic relationship exist between this set and its original form, and is this relationship perceivable? Or, suppose the notes in a sequence are repeated identically, except that random variation within some range of x semitones is introduced? At the very least, I hope to emphasize that perception of identity across different or changing sequences of events functions crucially in one's

experience of music, and that there exists gray area in musical categories and phenomena corresponds with a profound degree of flexibility as well as a potentially endless gradation and range of compositional technique.

The question reverses itself in a complex way if we introduce performance practice and present cultural circumstances into our consideration. Consider retrogradation of a solo line of music, as an example, and its relative perceptibility in three distinct cases. In case one, a performer plays a line and its retrograded form, a therefore palindromic passage. In case two, a performer plays the same line, which is recorded live and replayed in reverse to create a passage parallel to that of case one. Now, consider briefly whether isomorphism across retrogradation is or not more perceptible or salient in case one or two. Then, consider case three, in which a performer plays the same line, and then its retrograded form such that, for its retrogradation, the envelopes of individual notes are also reversed, thus paralleling the reversed recorded sound of case two. At the least, this example demonstrates how distinct characteristics of musical phenomena (e.g., ordered intervallic change, note envelopes) can occur in conjunction and influence the manner and development of form in music. What also comes to bear is the issue of cultural context, such as one in which an entity such as "reversed sound" can in fact exist and cue a listener's perception of potentially isomorphic phenomena.

As we witness, isomorphism between parts or lines within a composition can yield multiple forms of somewhat divergent natures. Furthermore, we see that isomorphism can exist in and relation to multiple factors, including, as we have outlined, pitch, harmony, and rhythm, but also factors like dynamics, timbre, envelope, articulation, and otherwise. Composer, researcher, and audience member alike perceptually organizes musical values into sets, which potentially interrelate in significant ways. To review, then, the specific case of isorhythm and of

Machaut's *Kyrie*, an identity relationship occurs between the ordered sets derivable from the ordered rhythmic durations of the Tenor part's *taleae*.

Twentieth century serial techniques appear to exemplify further logical structural methods by which to organize sets of notes, advancing global operations like retrogradation and inversion already developed for constructing canonic forms, and attempting in new ways to achieve symmetry, balance, and development within music composition. Often presented as a response to tonal development in music, serial composition in its most common form utilizes an ordered set of all twelve values of the chromatic scale (parallel to the *color*), potentially, then, emphasizing different modes and tonal centers equally and thereby achieving an atonal or pan-tonal form. Integral serialism would extend serial practices to other parameters. In addition to dynamic and pitch values, Milton Babbitt serializes rhythmic values in his *Three Compositions for Piano.* Consider the following rows of values that represent sixteen note groupings in the first eight measures of the piece and first iteration of Prime, Inverse, Retrograde, Inverse-Retrograde forms: (Cope 27)

P:	5	1	4	2
I:	1	5	2	4
R:	2	4	1	5
IR:	4	2	5	1

The continued reemergence of the concept of a set and its manipulations evidences the music theorist's imperative to formalize basic units and relationships within music and sound. Iannis Xenakis' book *Formalized Music* stands as one of the twentieth century's more formidable music theoretic attempts to analyze music as a type of formal system (relatable to other types, including art and architecture) whose very nature can be understood and explored as

mathematical and logical phenomena. In attempting to enumerate the underlying logical constructs implicit in the analysis of all types of music, Xenakis provides a model both for understanding present and historical work as well as a conceptual framework in which to envision new forms of composition (155).

Xenakis reasons that sets of salient musical properties describe and instantiate either *structure outside-time* (e.g., pitch, intensity, and duration, which he terms the "ultimate aspects of sonic events") , *temporal structure* (e.g., temporal intervals), and the correspondence between structure outside-time and temporal structure, that is, *structure in-time* (158, 160). These different types of sets demonstrate fundamentally different logical and musical properties, a difference that Xenakis actively addresses as he attempts to delineate fundamental musical comprehension and understanding, axiomatizing the musical construct of an interval and deriving an additive law for conjunct intervals (159).

This difference we already observed in our analysis of isorhythmic forms as relate to the canon and fugue. Using Xenakis' formulation, we can conclude that canonical isomorphism (i.e., isomorphism between ordered sets of intervallic relationships) may often appear more perceptually immediate than isorhythmic isomorphism (i.e., isomorphism between temporal organization) due to significant differences in each form's *structure in-time.* The canon utilizes isomorphic simultaneity, that is, isomorphism as occurs in ordered intervallic relationships of two lines in combination with isomorphism as occurs in the ordered rhythmic relationships of these two lines, thereby organizing *structure outside-time* and *temporal structure* concurrently and creating a form that exists as a structure in-time, the ancient notion of a part or a line or a voice. Isorhythmic forms utilize isomorphism between concurrent temporal structures but not necessarily between structures outside-time. Therefore isorhythm could possess potentially

fewer or at least different perceptual cues or properties than canonical forms due to each process's activation of different sets of musical properties that occur on different levels of music perception. Conceivably, then, Machaut creates an effect of timelessness by utilizing isorhythm and thereby emphasizing temporal structures over structures in time, engaging the piece's underlying plainchant while simultaneously creating an almost mystical effect as the chant is immersed in contrapuntal reflection and self-symmetry.

One of Xenakis' own instructors would use isorhythm in what was to be the first movement of his most famous piece, a prominent twentieth century work whose very subject is the end of time and its connection to ideas of a timeless existence. Liturgie de Cristal, the opening movement of Messiaen's *Quartet for the End of Time*, contains several isorhythmic elements. Considered using the terms we have developed, the cello part has a 15-note *color* which repeats over nearly eight 15-note *taleae.* Notably, the cello part's *talea* is non-retrogradable, that is, palindromic. The piano part has a sequence of 29 chords which repeat over a 17-note *taleae,* Messiaen's use of unrelated prime numbers as *talea* and *color* size markedly contrasting Machaut's use of apparently related multiples. Prevailing interpretation holds that this movement reflects just one surface of the necessarily massive kaleidoscopic structure in-time large enough for these prime sequences ultimately to align.

We witness in Messiaen' s work on the theory of modes of limited transposition a discovery and elucidation of generative principles by which both to organize and enumerate well-established modes (e.g., whole tone, octatonic) as well as lesser-known or, even, potentially unstudied or underutilized modes (e.g., Messiaen's modes 4 - 7) (Pople, 99). We immediately see that, first of all, the modes of limited transposition are the result of an iterative process of construction similar to that of isorhythmic forms, and that, secondly, the relationship between the

size of a given mode's iteration (i.e., number of notes before one complete "cycle" of interval sizes) and this mode's number of transpositions parallels, in the case of isorhythm, the number of notes in a talea and the number of a repetitions of a given color. In a simple example, consider that the octatonic scale is composed of four two-note iterations, and that this scale has three transpositions. Now, as a simple example, consider a talea with two notes, and a color of three notes, a complete cycle (with, as analogy, three transpositions) represented symbolically as follows:

<div align="center">

1 2   3 1   2 3

</div>

Or, similarly, consider a talea with three notes, and a color of two notes, for which a complete cycle, in a sense analogical to the whole-tone scale, has two modes of transposition:

<div align="center">

1 2 1   2 1 2

</div>

An extension of this parallel exists between the process of cycling through ordered pitch values, as occurs with isorhythm, and the process of cycling through ordered intervallic sizes, as occurs when constructing neighboring traditional (by implication, "unlimited") modes. Consider, for example, Aeolian, Dorian and Phrygian modes, constructed respectively as follows (W as whole step, H as half):

<div align="center">

W W **H** W W W **H**

W **H** W W W **H** W

H W W W **H** W W

</div>

Notice, further, that matrices formed from serial tone rows also parallel this construction.

Messiaen's notion of a "charm of impossibilities" directly corresponds to the subject of our analysis, specifically, the relationship between systematic change as occurs across different dimensions of music. Messiaen draws a parallel between the modes of limited transposition and

non-retrogradable rhythm, as both represent identity across structural transformation and change, "the rhythms realizing in the horizontal direction ... what the modes realize in the vertical" (Pople, 26).

A student of Messiaen, Xenakis realizes, possibly as an impetus for his unique methods, that an ancient division between order and disorder respectively valued and devalued presumes and predicts certain historical and contemporary developments in music, and music thereby conceived in a fashion that, as we have witnessed, organizes along lines of symmetry, similarity, and identity over change, hence, isomorphism. Using mathematical and logical entities and constructs inherited from concurrent studies of architecture and mathematics, Xenakis strives to achieve musical phenomena that exist on a new scale and dimension, creating clouds of sound whose resultant sonic effect relates to speed and density of note events, evidencing the probabilistic nature of music perception. Xenakis begins the process as composer of exploring gradations and interrelationships of order and disorder, using methods that balance purpose and incidence (randomness).

Xenakis points to a "linear contradiction" that serialist technique arguably creates, as individual lines are lost in a mass of interweaving parts across registers (8). As we have already observed, different types of isomorphism across events and parameters within music bear different degrees of perceptual apparentness. Xenakis attempts in some ways to de-emphasize the size and scope of serial technique, classifying it as a very specific subset of more general, encompassing, and perceptually apparent stochastic methods, such that probabilistic devices can yield music that parallels that created using serial methods.

Can a composer use random change or variation purposefully? Can one decide *not* to decide, choose *not* to choose, and do so with compositional intent? Our analysis of isorhythm as

a compositional practice has demonstrated the inverse, namely, that a composer can use structural change purposefully, such the very structure of the change has perceivable characteristics. In exploring the subtlety and organization of order, disorder, chance and probability, Xenakis realizes that phenomena like swarms, clouds, masses, crowds, streams and otherwise seem to embody disorder and change as much as symmetry and identity. One could utilize stochastic and markov chain models to create, as analogy, an orchestral or electronic transcription of these phenomena, just as Messiaen transcribes the birdcall and Machaut the plainchant, as just as Xenakis would transcribe the score of his work *Metastasis* into the architectural model for the Philips Pavilion at the 1958 Brussels World's Fair (11). In this way Xenakis expands the isomorphic possibilities of musical and artistic technique.

We witness that the line between purpose and incidence in music appears more dynamic than one originally might expect, and that random processes over time can in fact embody symmetry. Consider Xenakis' example of a coin toss, a process that, while, on a small scale, proves unpredictable, on a large scale, approaches a determinate, probabilistically symmetrical state.

The sogetto cavato technique, first introduced into musical composition during the 15th century, offers a remarkable example of how incidental can interweave with compositional intent, and demonstrates how symbols and forms can possess multiple levels of derivable meaning, just as notes can function harmonically and melodically simultaneously. This technique generally involves mapping words or phrases letter-by-letter onto musical notes. To dismiss the resultant musical line as wholly random and unconnected to the translated words' semantic meaning fails to appreciate the subtle characteristics of the given 'breed' of randomness that this particular isomorphic process generates. Immediately, we see that the pitch classes

assigned to vowels would be activated preferably and with derivable neighbor relations with non-vowel entities. Also, different languages or even different styles of speech could possess different sound and characteristics when transcribed using this method.

Historically, as well as more recently, composers have used techniques like sogetto cavato in conjunction with global-level abstract isomorphic transformations like retrogradation to achieve a synthesis of semantic meaning and formal instantiation. Guillaume Dufay's piece *My Beginning Is My End* serves as a medieval example, one that Milton Babbitt in fact revisits, with this statement transcribed and then retrograded. Similarly, consider writing a piece using the sequence *repetition* as an ostinato pattern. Or, in a weirdly self-reflexive example, consider how, in the context of a large English language passage translated using the sogetto cavato transformation, the sequence *weird* or *exceptional* might perceivably sound weird or exceptional, given the unlikelihood of an *ei* sequence or of the value *x*. As strange as these incidental levels of meaning may seem, there appears evidence that structural incident, such as occurs in written language, fosters a potentially richer and more nuanced probabilistic environment than strictly independent evenly-weighted random integer generation.

Notice, importantly, the parallel between interpreting incidental change and earlier interpretative attempts at reconstructing Machaut's compositional process, determining whether Machaut in fact consciously evaluated matters of ratio and numeral when composing. Rees, in defending his position that Machaut consciously emphasizes numerals seven and twelve when composing *La Messe de Notre Dame*, references the historical and cultural context in which Machaut lived and wrote music, arguing that religious belief would cause Machaut to favor these numbers or religious and symbolic meanings. The question arises, then, of the extent to which this breed of historical or cultural incident affects compositional process. This issue further

exemplifies how symbols -- not only words, but also, in this case, numbers -- can carry

additional, unintentional, or otherwise hidden or strange meaning.

We have seen that different types of isomorphic relationships function distinctively

within a piece of music, and account for unique forms that occur in music. Isorhythmic, canonic,

fugal, and serial techniques collectively represent different methods by which to organize and

envision isomorphic musical phenomena. In recognizing the parallels between these apparently

divergent methods we can begin to understand music in a more fundamental capacity, and begin

potentially to realize new directions in music, including, as we have discussed, probabilistic

methods, clouds of sounds, manipulations of ordered sets of parameters, cross-manipulating

different perceptual categories, and symbolic transformations. We face the challenge of

addressing purpose in the face of randomness or incidence, even, cultural and symbolic

incidence, the context of established technology and customs, as well as an assumed and

underlying mathematical and logical context. In some ways I hope to embrace the obscureness

of the idea of realizing a personal composer's voice as manifested in numbers and ratios as

evidence of music as an unapologetically formalistic phenomenon. This formal underpinning,

while appearing confining or restrictive, actually enables music to possess a richness beyond

enumeration.

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