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On Money as a Medium of Exchange

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We analyze economies in which individuals specialize in consumption and production and meet randomly over time in a way that implies that trade must be bilateral and quid pro quo. Nash equilibria in trading strategies are characterized. Certain goods emerge endogenously as media of exchange, or commodity money, depending both on their intrinsic properties and on extrinsic beliefs. There are also equilibria with genuine fiat currency circulating as the general medium of exchange. We find that equilibria are not generally Pareto optimal and that introducing fiat currency into a commodity money economy may unambiguously improve welfare. Velocity, acceptability, and liquidity are discussed.

> After the thunderstorms of recent years, it is with particular diffidence and even apprehension that one ventures to open one's mouth on the subject of money. [Hicks 1935, p. 1]

Many friends, colleagues, and participants in seminars have provided valuable assistance in this project—unfortunately, too many to mention by name. However, we would like to thank those who acted as discussants at conferences or in similar capacities, including Martin Eichenbaum, Robert Jones, Robert Townsend, and Neil Wallace. The usual disclaimer applies. Kiyotaki acknowledges the hospitality of the Department of Economics at the University of Western Ontario, as well as a grant from the University of Wisconsin Graduate School; Wright acknowledges a grant from the University of Pennsylvania Research Foundation.

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I. Introduction

The basic goal of this project is to analyze a simple general equilibrium matching model, in which the objects that become media of exchange will be determined endogenously as part of the noncooperative equilibrium. This medium of exchange function is the essential feature of money and the one emphasized by the classical and early neoclassical economists; yet formal modeling has been illusive.[1] In the model presented here, we demonstrate how trade using media of exchange can emerge in equilibrium, with different commodities potentially playing this role depending both on their intrinsic properties and on extrinsic beliefs. We also demonstrate how genuine fiat currency may or may not circulate in the economy, depending on extrinsic beliefs, or social custom, as well as preferences and technology. We also investigate the welfare properties of fiat and commodity money equilibria and study the equilibrium behavior of variables such as the velocity, the acceptability, and the liquidity of various assets.

From ideas going back at least to Adam Smith (1776, chap. 4), the driving force behind the use of money is specialization, which implies that agents do not necessarily consume what they produce. Here, they will also meet randomly over time in a way that implies that trade must be bilateral and quid pro quo. This leads to Jevons's (1875) "double coincidence of wants" problem with direct barter—when we meet, you not only have to have what I want but also have to want what I have—which is behind the genesis of indirect trade and the use of media of exchange. As was also stressed by the early monetary economists, we assume that different objects have different intrinsic properties (here it will be storability), making them more or less natural candidates for the role of money. These properties notwithstanding, a critical factor in determining if an object can serve as a medium of exchange is whether or not agents believe that it will. In other words, the use of money necessarily involves strategic elements and certain aspects of social custom.

[1] The dominant paradigms in monetary economics today are the overlapping generations models (e.g., Wallace 1980) and cash-in-advance models (e.g., Lucas 1980*a*). Overlapping generations models basically ignore the medium of exchange role, concentrating on money's store of value function. Cash-in-advance models simply impose the medium of exchange role by an ad hoc restriction that goods can be acquired only using money. These approaches are useful when we are interested only in getting money into the system so that we can proceed to analyze some substantive economic issues, but they have no hope of explaining endogenously either the nature of money or the development of monetary exchange. The spatial separation models of Townsend (1980) are closer in spirit to our general approach but are very different in structure. Some other attempts at modeling the medium of exchange role of money are discussed at the end of this section.

In Section II we describe our economy and the equilibrium concept in detail. Basically, agents have to choose trading strategies, which (given that our assumptions imply that they always store one unit of one good at a time) amount to decision rules determining whether or not they should trade when they have good i and meet another agent with good j. These choices also determine endogenously the distribution of goods held as inventories by agents. We look for strategies that maximize individuals' expected utility, given the strategies of others and the distribution of inventories, and also imply this distribution, as an equilibrium. Since there is only a finite number of possible strategy profiles in our model, by exhaustively checking all possibilities, we are able to completely characterize the set of such equilibria. When a commodity is accepted in trade not to be consumed or used in production, but to be used to facilitate further trade, it becomes a medium of exchange and is called commodity money. If an object with no intrinsic value becomes a medium of exchange, it is called fiat money. We will analyze the properties of each of our equilibria in terms of the numbers and types of media of exchange they display.

For one specification, discussed in Section III, there is never more than a single equilibrium for any given parameter values, although for an alternative specification in Section IV, multiple equilibria sometimes coexist with different goods acting as media of exchange. In Section V, we construct equilibria with fiat money taking on value, essentially as a self-fulfilling prophecy. We then characterize the velocity, acceptability, and liquidity of various assets as a function of the stock of real balances in circulation. Welfare implications are discussed in Section VI. We show that equilibria are not generally Pareto optimal and that when multiple equilibria coexist they are not generally Pareto comparable. We also show that introducing fiat currency into a commodity money economy (in some but not all cases) can unambiguously improve welfare, although too much money is always welfare reducing. Concluding remarks are contained in Section VII.

To close this introductory section, we briefly review some recent attempts to model money in related ways. Jones (1976) provides an interesting framework in which one can examine which of many commodities will circulate as media of exchange, although his traders are somewhat naive concerning both the equilibrium matching distribution and their choice of strategies. Oh (in press) updates this model so that traders follow optimal sequential strategies. Iwai (1988) analyzes a similar framework, in which expectations are fully rational but agents are able to choose simple trading patterns and not sequential strategies because they visit deterministic trading zones rather than matching randomly. Our agents use individually optimal sequential strategies, based on rational expectations of others' strategies and the stochastic matching distribution, which we make tractable (at the cost of some generality) by keeping the number of goods and agents small. An extensive survey of much other work in this area is provided by Ostroy and Starr (1988). Formally, our structure is similar to some recent advances in search and sequential bargaining theory, such as Mortensen (1982), Rubinstein and Wolinsky (1985), and Gale (1986*a*), although these papers do not discuss monetary issues. Diamond (1984) and Gale (1986*b*) do present monetary versions of the matching framework but impose a medium of exchange exogenously via a cash-in-advance constraint (as game-theoretic models do such as those discussed by Shubik [1986]). To be perfectly clear, the goal of the present paper is to use the sequential matching model to derive commodity and/or fiat money endogenously.

II. The Economy

First, we describe the basic physical environment. Time is discrete and continues forever, and at each date there are three indivisible commodities called goods 1, 2, and 3. There is a continuum of infinitely lived agents with unit mass, with equal proportions of types 1, 2, and 3, that specialize in both consumption and production: type i agents derive utility only from the consumption of good i and are able to produce only good $i^* \neq i$.[2] All goods are storable at a cost, but agents can store only one unit at a time and, since goods are indivisible, therefore only one good at a time. Storage costs may be type and good specific, and we let c_{ij} denote the cost (in terms of instantaneous disutility) to type i of storing good j. We assume that $c_{i3} > c_{i2} > c_{i1} > 0$ for all i.

For type i, let U_i denote the instantaneous utility from consuming good i, D_i the instantaneous disutility from producing good i^*, and $\beta \in (0, 1)$ the discount factor (common across types). Then i's expected discounted lifetime utility is given by

$$E \sum_{t=0}^{\infty} \beta^t [I_i^U(t)U_i - I_{i*}^D(t)D_i - I_{ij}^c(t)c_{ij}],$$

where $I_i^U(t)$ is a (random) indicator function that equals one if the agent eats his consumption good i, zero otherwise; $I_{i*}^D(t)$ equals one if

[2] There are two interesting ways to combine consumption and production specialties in what follows. In model A, type 1 agents produce good 2, type 2 agents produce good 3, and type 3 agents produce good 1. In model B, type 1 agents produce good 3, type 2 agents produce good 1, and type 3 agents produce good 2. Since these goods will have different intrinsic properties, these are not simply relabelings of the same economy, and both cases will be analyzed below.

he produces his production good i^*, zero otherwise; and $I^c_{ij}(t)$ equals one if he stores any good j, zero otherwise, at the end of period t. We assume that the net utility of consuming plus producing, $u_i = U_i - D_i$, is large enough that agents will not want to drop out of the economy, a sufficient condition for which is the following assumption (see lemma 1).

ASSUMPTION A. For all i, $u_i > (c_{ii^*} - c_{ik})/(1 - \beta)$, for all k.

At date t, if type i is lucky enough to acquire his consumption good i, he will consume it and produce a new unit of i^*. Thus each type i always has an inventory of exactly one unit of one good other than good i (since when he gets good i he will eat it). If there was a centralized market in which all agents came together at each date, then everyone would produce, exchange, and consume every period, which could be supported as a Walrasian equilibrium. But that is not the way this economy operates. Rather, each period, agents are matched randomly in pairs and must decide whether or not to trade bilaterally, without the benefit of an auctioneer or some other outside authority to impose any arrangement. Trade always entails a one-for-one swap of inventories, given the physical environment, and occurs if and only if mutually agreeable (there is no credit since a given pair will meet again with probability zero). The distribution of potential matches can be characterized by the time path of $\mathbf{p}(t) = [\ldots p_{ij}(t) \ldots]$, where $p_{ij}(t)$ is the proportion of type i agents holding good j in inventory at date t.

This completes the description of the physical environment. We now proceed to consider behavior. Each individual chooses a trading strategy to maximize his expected discounted utility from consumption net of production and storage costs, taking as given the strategies of other agents and $\mathbf{p}(t)$. A trading strategy is a rule determining the circumstances under which i is willing to trade, most generally as a function of the date and everything that has happened to him up until that point. However, since the environment here is time-invariant, the planning horizon is infinite, and we consider only steady-state equilibria in what follows—that is, $\mathbf{p}(t) = \mathbf{p}$ for all t—we restrict attention to strategies for i that depend only on the good j he has in inventory and the good k in the inventory of the agent with whom he is currently matched.[3] Let $\tau_i(j, k) = 1$ if i wants to trade j for

[3] Typically in sequential bargaining theory, given a match, one agent proposes a trade and the other either accepts or rejects and makes a counteroffer. Counteroffers are not necessary here since there is really nothing to bargain over: either you trade or you do not. Also, as this is assumed to be an anonymous game (Jovanovic and Rosenthal 1988), a strategy does not depend on the type with which you are currently matched. Finally, note that we always assume that all agents of the same type play the same strategy, and we ignore randomized play.

k, and zero otherwise. Then when type i with good j meets type h with good k, they trade if and only if $\tau_i(j, k) \cdot \tau_h(k, j) = 1$.

Given the underlying physical setup and our notion of strategies, we define equilibrium as follows.

DEFINITION. A steady-state Nash equilibrium is a set of trading strategies $\{\tau_i\}$, one for each type i, together with a steady-state distribution of inventories $\mathbf{p}$, that satisfies (a) maximization: each individual i chooses τ_i to maximize expected utility given the strategies of others and the distribution $\mathbf{p}$; and (b) rational expectations: given $\{\tau_i\}$, $\mathbf{p}$ is the resulting steady-state distribution.

Our goal is to characterize equilibrium for different particular specifications of the production and consumption specialties. Before doing so, however, we describe some general properties of the model that will prove useful.[4]

Let $V_i(j)$ be the expected discounted utility for type i when he exits a trading opportunity with good j, given that he follows a maximizing strategy; that is, $V_i(j)$ is the indirect utility of leaving with good j. When i exits with his own consumption good i, he consumes it and immediately produces a new unit of i^*, which yields the instantaneous utility $u_i = U_i - D_i$ plus the indirect utility of storing i^*.[5] Therefore, $V_i(i) = u_i + V_i(i^*)$. The indirect utility for i of storing good $j \neq i$ is described by Bellman's equation of dynamic programming (see, e.g., Bertsekas 1976):

$$V_i(j) = -c_{ij} + \max \beta E[V_i(j')|j],$$

where $E[V_i(j')|j]$ is the expectation of V_i at next period's random state j', conditional on j, and the maximization is over strategies. Standard techniques guarantee that the "value function" $V_i(\cdot)$ is well defined (again, see Bertsekas 1976).

Now with the shorthand notation $V_{ij} \equiv V_i(j)$, for $j \neq k$, an optimal strategy clearly satisfies

$$\tau_i(j, k) = 1 \quad \text{iff } V_{ik} > V_{ij},$$

which says that i is willing to trade j for $k \neq j$ iff k provides more indirect utility than j (we assume that i will not trade if $V_{ij} = V_{ik}$). For $j = k$, trade is irrelevant, and $\tau_i(j, j) = 0$. Observe that for $j \neq k$, $\tau_i(j, k) = 1$ iff $\tau_i(k, j) = 0$; therefore, in equilibrium, agents of the same type

[4] There will always be a trivial equilibrium in which everyone believes that all others will play "never trade" strategies ($\tau \equiv 0$), and so they may as well play $\tau \equiv 0$ themselves. We ignore this degenerate outcome for the rest of the analysis.

[5] That i always accepts and eats good i is verified explicitly in lemma 1 below. For simplicity, we provisionally do not allow agents to consume without producing or to dispose of goods except by eating them, but these assumptions can be relaxed as indicated following theorem 1.

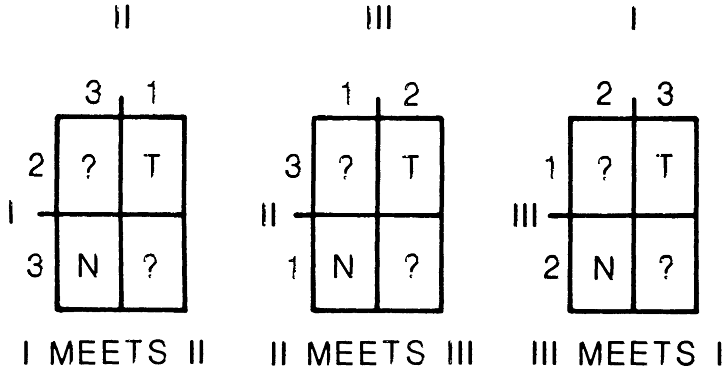

FIG. 1.—Trading matrices

never trade since both cannot prefer what the other has. We now verify that agents are always willing to trade for their own consumption good, and when they get it they immediately eat it and produce a new unit of their production good (as long as u_i is sufficiently large).

LEMMA 1. Under assumption A, each type i will accept good i, eat it, and produce a new unit of good i^* whenever he has the opportunity. That is, for all i, $\max_j V_{ij} = V_{ii} = u_i + V_{ii^*}$.

Proof. Suppose that some i prefers $k \neq i$ to all other goods; that is, $V_{ik} = \max_j V_{ij}$ for $k \neq i$. Then if i acquires good k, he keeps it forever, so

$$V_{ik} = -\frac{c_{ik}}{1-\beta} \geq V_{ii} \geq u_i - \frac{c_{ii^*}}{1-\beta}.$$

The first inequality follows from $V_{ik} = \max_j V_{ij}$ and the second from the fact that V_{ii} can be no less than the value of eating i and storing i^* forever. Rearranging yields $u_i \leq (c_{ii^*} - c_{ik})/(1 - \beta)$, contradicting assumption A, and therefore we conclude that $V_{ii} = \max_j V_{ij}$. This means that i always accepts good i. If he does not consume it,

$$V_{ii} = -\frac{c_{ii}}{1-\beta} \geq u_i + V_{ii^*} \geq u_i - \frac{c_{ii^*}}{1-\beta},$$

again contradicting assumption A. Q.E.D.

Since type i always wants to consume good i and produce i^* (even in the worst-case situation, in which he expects to have to store i^* for the rest of time), we know that $\tau_i(j, i) = 1$.[6] Therefore, trade always occurs when a double coincidence emerges, that is, whenever type i with good j meets type j with good i. Figure 1 displays all possible

[6] Assumption A can actually be weakened substantially since i will never really have to store i^* for the rest of time (e.g., he can always trade it to a type i^* agent); see the discussion following theorem 1.

matches, except for the uninteresting cases in which two individuals of the same type meet, using roman numerals for agent types to avoid confusion with the numbers used for commodity types. For example, the "trading matrix" on the left depicts the case in which type I meets type II: the rows represent the possibility of I holding either 2 or 3, while the columns represent the possibility of II holding 3 or 1 (recall that agent i never stores good i). The T in each box representing a double coincidence indicates that trade will definitely occur, while the N in each box in which both agents have the same inventory indicates that no trade will occur. The ? in the remaining boxes indicates that trade may or may not occur, depending on the strategies chosen by the agents.

We now describe an algorithm for finding equilibria. Begin by conjecturing an arbitrary set of trading strategies, $\{\tau_i\}$; in fact, this will be equivalent to ranking the value functions V_{ij}, $j = 1, 2, 3$, for each i since, as we argued above, $\tau_i(j, k) = 1$ iff $V_{ij} < V_{ik}$. This determines exactly when trade occurs, and so we can replace each ? by either T or N in the trading matrices and determine how any initial distribution of inventories evolves over time. It is then a routine matter to calculate the implied steady-state distribution, $\mathbf{p}$. Finally, check to see if the conjectured strategies are in fact maximizing for each individual, given $\mathbf{p}$ and the strategies of others. If so, we have an equilibrium; if not, we do not. Since there is only a finite number of possible $\{\tau_i\}$ here (i.e., only a finite number of ways to rank the V_{ij}'s), we can exhaustively search over strategy profiles to completely characterize the set of equilibria. In the next two sections, we describe the outcome of this characterization for two slightly different versions of the model and interpret the results.

III. Equilibrium: Model A

Here we consider a case we call model A, in which type I produces good 2, II produces good 3, and III produces good 1 (i.e., $1^* = 2$, $2^* = 3$, and $3^* = 1$). Before stating the results precisely, we describe things roughly as follows. For certain parameter values there will exist one equilibrium, for other parameter values there exists another equilibrium with different qualitative properties, while for the remaining parameter values satisfying the maintained assumptions there will exist no equilibrium. For no parameter values will multiple equilibria coexist. One equilibrium is referred to as a *fundamental* equilibrium since agents always prefer a lower-storage-cost commodity to a higher-storage-cost commodity unless the latter is their own consumption good (so agents need look only at "fundamentals"—storage costs and utility values—when deciding whether or not to

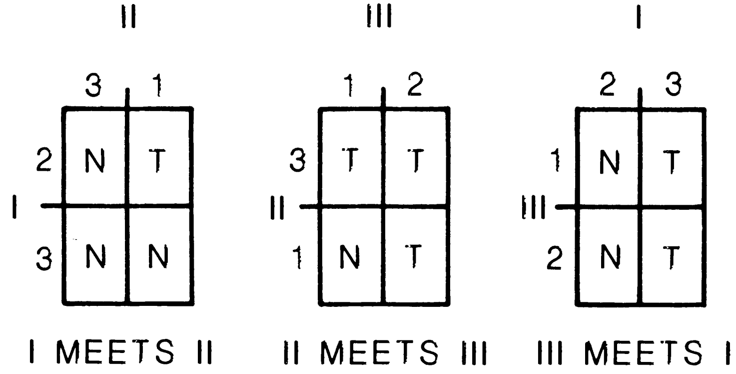

FIG. 2.—Fundamental equilibrium (model A)

trade). We refer to the other as a *speculative* equilibrium since sometimes agents trade a lower- for a higher-storage-cost commodity not because they wish to consume it, but because they rationally expect that this is the best way to ultimately trade for another good that they do want to consume, that is, because it is more marketable.[7]

The fundamental strategies are described by $V_{ii} = \max_j V_{ij}$ for all i (agents always prefer their consumption good) and the inequalities $V_{12} > V_{13}$, $V_{21} > V_{23}$, and $V_{31} > V_{32}$ (otherwise they prefer lower-storage-cost goods). The completed trading matrices are shown in figure 2. To check that these actually constitute an equilibrium, we must demonstrate that they are maximal for each type i when others use these strategies, given the implied inventory distribution. We shall not actually compute the **p** distribution until after we check to see if these strategies are maximal since this will allow us to determine conditions under which they are that depend on **p** in an intuitively reasonable way. Since $V_{ii} = \max_j V_{ij}$ has already been established in lemma 1, it remains only to verify the inequalities above. This is slightly complicated, but it is also worthwhile since it demonstrates explicitly how the matching and exchange processes work in this economy.

Consider first a typical type I agent. When he exits a match with good 2, he immediately pays c_{12} in storage costs, and next period he meets an individual of type I, II, or III, each with probability $\frac{1}{3}$. If he meets another type I, he cannot trade, so he keeps good 2 and leaves with indirect utility V_{12}. If he meets type II, with probability p_{21} there

[7] Marketability is closely related to Menger's (1892, p. 248) notion of saleability: "when any one has brought goods not highly saleable to market, the idea uppermost in his mind is to exchange them, not only for such as he happens to be in need of, but, if this cannot be effected directly, for other goods also, which, while he did not want them himself, were nevertheless more saleable than his own."

is double coincidence and he leaves with utility $u_1 + V_{12}$, while with probability p_{23} he has the option of leaving with either V_{12} or V_{13} (given that II always wants to trade good 3 for good 2). If he meets III, he cannot trade (given that III never accepts good 2) and he leaves with V_{12}. If $b = \beta/3$, this implies

$$V_{12} = -c_{12} + b[V_{12} + p_{21}(u_1 + V_{12}) + p_{23}\max(V_{12}, V_{13}) + V_{12}].$$

A similar story when I exits a match with good 3 implies

$$V_{13} = -c_{13} + b[V_{13} + V_{13} + p_{31}(u_1 + V_{12}) + p_{32}\max(V_{12}, V_{13})].$$

It is easy to show $V_{12} > V_{13}$ iff $c_{13} - c_{12} > (p_{31} - p_{21})bu_1$, which determines the parameters and values for the p_{ij} for which fundamental play by type I is the best response to fundamental play by others.[8]

Consider next a typical type II agent. By analyzing his options in each possible match as we did for type I, we have

$$V_{21} = -c_{21} + b[p_{12}(u_2 + V_{23}) + p_{13}\max(V_{21}, V_{23}) + V_{21} + p_{31}V_{21} + p_{32}(u_2 + V_{23})],$$

$$V_{23} = -c_{23} + b[V_{23} + V_{23} + p_{31}\max(V_{21}, V_{23}) + p_{32}(u_2 + V_{23})].$$

It is easy to show $V_{21} > V_{23}$ for *all* parameter values and p_{ij}. The same sort of argument for type III implies $V_{31} > V_{32}$ for all parameter values and p_{ij}. Hence, to sum up, fundamental strategies are always the best response to fundamental strategies for types II and III and best for type I iff $c_{13} - c_{12} > (p_{31} - p_{21})bu_1$. This inequality says that the cost of storing good 3 rather than 2 exceeds the discounted utility benefit conveyed by the relative marketability of good 3 compared with good 2, which is positive iff there is a greater probability of III holding good 1 than of II holding good 1 in equilibrium.

The steady-state inventory distribution $\mathbf{p}$ may be summarized here by three numbers (since $p_{ii} = 0$ and $\Sigma_j p_{ij} = 1$ for all i). For these fundamental strategies, this is given by $(p_{12}, p_{23}, p_{31}) = (1, .5, 1)$, and therefore these strategies constitute equilibrium iff $c_{13} - c_{12} > .5bu_1$. We have thus constructed one equilibrium for a region of parameter space. Since $p_{12} = p_{31} = 1$ in this equilibrium, types I and III always keep their production goods until they can trade directly for their consumption goods, never using indirect trade. Type II agents trade their production good 3 for good 1 whenever possible, however, and end up holding each exactly half the time. They thereby act as *middle-*

[8] Parameter space is the set of β, u_i, and c_{ij} satisfying the explicit assumptions in the text; the p_{ij} are not parameters, but endogenous variables to be determined exactly below. Nevertheless, it is desirable to have the p_{ij} (rather than their equilibrium values) in the condition that determines whether $V_{12} > V_{13}$ or $V_{12} < V_{13}$ since this condition will then have a meaningful economic interpretation.

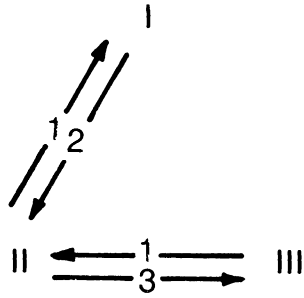

FIG. 3.—Fundamental equilibrium exchange pattern (model A)

men, transferring good 1 from type III to type I (see fig. 3). Good 1 is the unique medium of exchange, or commodity money, in this equilibrium since by definition a *medium of exchange* is "an object which is taken in exchange, not for its own account, i.e. not to be consumed by the receiver or to be employed in technical production, but to be exchanged for something else within a longer or shorter period of time" (Wicksell [1911] 1967, p. 15).

If $c_{13} - c_{12} < (p_{31} - p_{21})bu_1$, on the other hand, fundamental play by all agents does not constitute an equilibrium. The best response by type I to fundamental play in this case is to speculate by attempting to trade good 2 for good 3, which has a higher storage cost but is also more marketable. We will show that fundamental play is still the best response by II and III, and therefore the strategies corresponding to $V_{ii} = \max_j V_{ij}$ and the inequalities $V_{12} < V_{13}$, $V_{21} > V_{23}$, and $V_{31} > V_{32}$ also constitute an equilibrium in some other region of parameter space. By analyzing II's options in each possible match, given that type I speculates in this manner and type III plays fundamental, we find

$$V_{21} = -c_{21} + b[p_{12}(u_2 + V_{23}) + p_{13}\max(V_{21}, V_{23}) + V_{21} + p_{31}V_{21} + p_{32}(u_2 + V_{23})],$$

$$V_{23} = -c_{23} + b[p_{12}(u_2 + V_{23}) + p_{13}V_{23} + V_{23} + p_{31}\max(V_{21}, V_{23}) + p_{32}(u_2 + V_{23})].$$

It is easy to show $V_{21} > V_{23}$, and a similar argument for III also implies $V_{31} > V_{32}$, for any parameter values and p_{ij}.

Hence, to sum up, types II and III should indeed use fundamental strategies when I speculates, and I should speculate iff $c_{13} - c_{12} < (p_{31} - p_{21})bu_1$. The inventory distribution implied by these strategies is given by $(p_{12}, p_{23}, p_{31}) = (.5\sqrt{2}, \sqrt{2} - 1, 1)$, and so speculative equilibrium obtains iff $c_{13} - c_{12} < (\sqrt{2} - 1)bu_1$. By engaging in speculation, type I agents now also play the role of middlemen in some

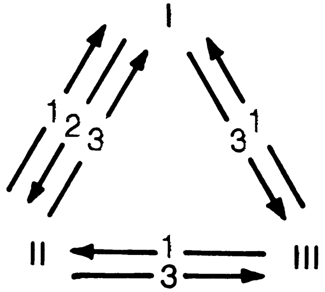

FIG. 4.—Speculative equilibrium exchange pattern (model A)

trades, transferring good 3 from type II to type III. Type II agents are still middlemen in other trades, and they continue to use good 1 as a medium of exchange, while type I uses good 3 as a medium of exchange (see fig. 4). In this equilibrium, we therefore have dual commodity monies, with both the most storable and the least storable objects (i.e., goods 1 and 3) used to achieve indirect trade in different instances by different individuals. It is perhaps particularly interesting that type I trades the lower-storage-cost good 2 for the high-storage-cost good 3. If we think of storage costs as (negative) instantaneous rates of return to holding an asset, then since there are no capital gains here, good 3 has a lower objective rate of return than good 2. Yet because good 3 has superior marketability, for type I, $V_{13} > V_{12}$, and therefore I prefers holding good 3. This is an example of an object being used as a medium of exchange in spite of the fact that it is dominated in rate of return by another object (see Hicks 1935).

There are six cases left to consider as possible equilibria, with lemma 1 and the fact that there are exactly eight ways to choose $\max(V_{12}, V_{13})$, $\max(V_{21}, V_{23})$, and $\max(V_{31}, V_{32})$. It may be shown that no other set of strategies is consistent with equilibrium.[9] Hence we conclude that in the intermediate region, where $(\sqrt{2} - 1)bu_1 < c_{13} - c_{12} < .5bu_1$, no equilibrium will exist. That is, no pure strategy, steady-state equilibria exist in which all agents of the same type play the same strategy. It is possible to show that in this region there are mixed strategy equilibria, or equilibria in which agents of the same type use different strategies (details are available on request). However, in

[9] For example, suppose $V_{12} < V_{13}$, $V_{21} > V_{23}$, and $V_{31} < V_{32}$ (I and III speculate while II plays fundamental). For I, given others' strategies,

$$V_{12} = -c_{12} + b[V_{12} + p_{21}(u_1 + V_{12}) + p_{23}\max(V_{12}, V_{13}) + p_{31}(u_1 + V_{12}) + p_{32}V_{12}],$$

$$V_{13} = -c_{13} + b[V_{13} + V_{13} + p_{31}(u_1 + V_{12}) + p_{32}\max(V_{12}, V_{13})].$$

It is easy to show $V_{12} > V_{13}$, contradicting the supposition and ruling out these strategies as an equilibrium. The remaining cases are similar.

terms of our symmetric, pure strategy definition of equilibrium, we summarize the results in the following theorem.

THEOREM 1. In model A, under the maintained assumptions, (*a*) if $c_{13} - c_{12} > .5bu_1$, then there is a unique equilibrium in which all agents use fundamental strategies and good 1 serves as the unique commodity money; (*b*) if $c_{13} - c_{12} < (\sqrt{2} - 1)bu_1$, then there is a unique equilibrium in which type II and type III agents use fundamental strategies while type I agents speculate, and both goods 1 and 3 serve as commodity monies; (*c*) these are the only equilibria.

To close this section, recall that so far we have been assuming that agents cannot consume without producing and cannot freely dispose of goods. We now argue that these assumptions are not binding as long as the parameter values satisfy certain restrictions. Thus consider the fundamental equilibrium. The value to type i of searching for a trading partner with his production good i^* in hand is given by V_{ii^*} evaluated at equilibrium values for the endogenous variables, which may be simplified to yield

$$\bar{V}_{12} = (1 - \beta)^{-1}(.5bu_1 - c_{12}),$$

$$\bar{V}_{23} = (1 - \beta)^{-1}(1 - b)^{-1}[b^2u_2 - bc_{21} - (1 - 2b)c_{23}],$$

$$\bar{V}_{31} = (1 - \beta)^{-1}(.5bu_3 - c_{31}).$$

Clearly, as long as we assume $\bar{V}_{ii^*} > D_i$, which we can guarantee by making u_i large, i would rather produce i^* than drop out of the economy. Furthermore, free disposal is irrelevant since under this assumption no one will ever want to throw anything away. Finally, it may be shown that $\bar{V}_{ii^*} > D_i$ is also sufficient for the conclusion of lemma 1 to hold, and so assumption A becomes redundant when $\bar{V}_{ii^*} > D_i$ is imposed.

IV. Equilibrium: Model B

In this section we briefly consider what we call model B, where $1^* = 3$, $2^* = 1$, and $3^* = 2$. Explicit derivations will be omitted since the arguments mimic closely those in the previous section. It turns out that for any parameter values there exists an equilibrium with all agents playing fundamental strategies, preferring a low-storage-cost good to a high-storage-cost good unless the latter is their consumption good (i.e., $V_{12} > V_{13}$, $V_{21} > V_{23}$, and $V_{31} > V_{32}$). Additionally, for parameter values satisfying $c_{23} - c_{21} < (p_{32} - p_{12})bu_2$ and $c_{32} - c_{31} < p_{23}bu_2$ (where the p_{ij} are given below), there exists an equilibrium with types II and III speculating while type I plays fundamental (i.e., $V_{12} > V_{13}$, $V_{21} < V_{23}$, and $V_{31} < V_{32}$). The conditions above are interpretable in terms of the relative cost (in terms of storage) and benefit

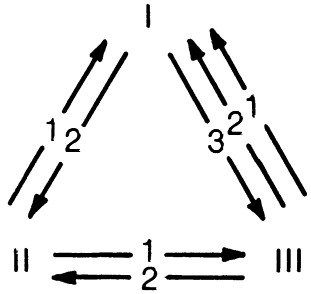

FIG. 5.—Fundamental equilibrium exchange pattern (model B)

(in terms of marketability) to speculation by types II and III. There are no other equilibria.

The distribution in fundamental equilibrium is given by $(p_{13}, p_{21}, p_{32}) = (.5\sqrt{2}, 1, \sqrt{2} - 1)$. Type II agents store their production good until they can buy their consumption good directly, while types I and III trade their production goods for more storable commodities whenever possible. Both goods 1 and 2 serve as media of exchange (see fig. 5). In speculative equilibrium, $(p_{13}, p_{21}, p_{32}) = (\sqrt{2} - 1, .5\sqrt{2}, 1)$. In this case, type III agents speculate by not trading their higher-storage-cost good 2 when offered good 1. Knowing this, type II agents speculate by acquiring the costly good 3 from type I to facilitate trade with III. Type I buys good 2 from type III to reduce his storage cost and also to facilitate trade with type II. Thus both goods 2 and 3 serve as commodity money while, perhaps surprisingly, the most storable good 1 does not (see fig. 6). Note that in some nonempty region of parameter space these two equilibria coexist. Either goods 1 and 2 or goods 2 and 3 may end up as commodity monies, depending solely on extrinsic beliefs and in spite of the fact that fundamentals suggest that good 1 is a much better medium of exchange than good

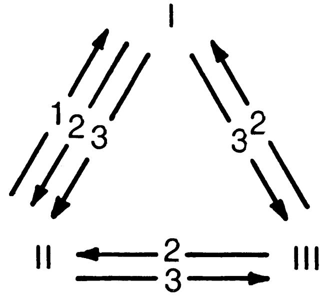

FIG. 6.—Speculative equilibrium exchange pattern (model B)

3. This is by no means a novel suggestion.[10] We have simply shown that such self-fulfilling prophecies are consistent with our notion of equilibrium.

Summarizing this discussion to facilitate comparison with the previous section, we have the following theorem.

THEOREM 2. In model B, under the maintained assumptions, (*a*) there always exists an equilibrium in which all agents play fundamental, with goods 1 and 2 serving as commodity money; (*b*) for parameter values implying $c_{23} - c_{21} < (\sqrt{2} - 1)bu_2$ and $c_{32} - c_{31} < (1 - .5\sqrt{2})bu_3$, there also exists an equilibrium in which types II and III speculate while type I agents play fundamental, with goods 2 and 3 serving as commodity money; (*c*) these are the only equilibria.

This completes our description of model B. Furthermore, since all other consumption-production specializations (which do not have anyone producing his own consumption good) are simply relabelings of either model A or model B, this completes our characterization of commodity money equilibria.

V. Fiat Money

So far we have been discussing commodity money, an important topic in its own right. But one of the more interesting challenges in economics is to explain how *fiat* money takes on value in equilibrium.[11] Niehans (1978, p. 14) describes the objective as follows:

> The problem was to explain precisely why money stocks are useful. It is clear that, except perhaps for irrational misers, cash balances are not one of the genuine consumer goods appearing in consumer theory. . . . It is also clear that money is not one of the genuine producer goods, appearing in an ordinary production function. . . . Rather than from direct utility and production, the services of money arise from exchange, being derived from the utility of money spent. The challenge was to make explicit how the utility of cash balances *held* is derived from the utility of cash balances *spent* in the exchange process. This requires a theory of exchange with frictions, which neoclassical theory failed to develop.

[10] As Hahn (1982, p. 28) puts it, "there may be a perfectly good monetary equilibrium with gold or cowry shells as well as with pound notes. Theory will help only marginally deciding which it will be."

[11] Fiat money is by definition an object that is intrinsically worthless (does not appear in any utility or production function) and inconvertible (is not a redeemable claim to something that does, such as a stock certificate); see Wallace (1980). An early advocate of a threefold classification of consumption goods, production goods, and media of exchange was von Mises (1912). Many objects can play more than one role, of course, such as commodity money or any object that is both a consumption good and a production good; but fiat money, by definition, can be *only* a medium of exchange.

In this section, we ask if the frictions in our model can lead to valued fiat money. We concentrate exclusively on model A (B is similar).

Suppose that the economy is endowed with a fixed quantity M of a new object called good 0. No one ever has or ever will derive utility from good 0, and it is of no help in production. It is, *by definition,* fiat money. To be explicit about its storage properties, assume $c_{i0} = 0$ for all i. However, good 0 does take a strictly positive, although perhaps small, amount of space, and so agents could never hold both fiat currency and real commodities at the same time since they have only a single unit of storage capacity available (whether or not good 0 is divisible is of little consequence when the other goods are not). This implies that, as in the previous sections, the inventory of each individual can contain no more than one object at any date. This is convenient, for although the state space expands beyond the earlier version of the model once we introduce good 0, it remains finite, and our technique for characterizing equilibrium remains manageable.

Could there exist steady-state equilibria in which good 0 circulates as the medium of exchange? We will demonstrate that the answer is yes. If P units of good 0 are required to buy one unit of each of the real commodities, then $S = M/P$ will be the quantity of real balances in circulation. Given that each agent holding fiat money will have exactly P units of the stuff in inventory, S will also equal the proportion of all agents holding good 0; if we let p_{i0} be the proportion of type i holding fiat money, $S = \Sigma_i p_{i0}/3$. We will construct equilibria in which individuals voluntarily accept good 0 in exchange for their real commodities and use it to acquire different real commodities in the future. Others willingly take the fiat money from them, and it continues to circulate as a medium of exchange—in fact, as the *general* medium of exchange, which is by definition an object "which is habitually, and without hesitation, taken by anybody in exchange for any commodity" (Wicksell 1967, p. 17).[12]

First we demonstrate that there exist equilibria in which fiat money does not circulate. Assume that in commodity money equilibrium $V_{ii*} > D_i > 0$, the condition that means that no one wants to drop out of the system. Then $V_{ij} > 0$ for all j such that $p_{ij} > 0$ (if $V_{ij} < 0$, then i will never acquire good j). Now if i believes that no one will accept fiat money from him in the future, then if he takes it now he will be stuck

[12] Although we are mainly interested in steady states and not the initial introduction of fiat money, we might ask how good 0 gets into circulation in the first place. One can imagine a "government" or a "monetary authority" trying to buy real commodities with intrinsically worthless and unbacked paper. If private agents do not believe in the paper, this outside agent will have no chance of success. But if private agents do believe in it, they will accept fiat money from him in exchange for their goods and use it to buy other goods in the future, while the outside agent collects $S = M/P$ as seigniorage.

with it forever, which implies utility $V_{i0} = 0 < V_{ij}$. Hence he will not take good 0. If no one believes in fiat money, then it cannot get off the ground (like the Susan B. Anthony dollar). This "tenuousness" of fiat currency is shared by the overlapping generations model, although not by the cash-in-advance model, and we think that it is a property that a good theory of money ought to have. The value of any medium of exchange, and especially fiat money, ultimately depends at least partially on faith. The next step is to show that, when such faith is present, good 0 can indeed take on value in our economy.

To this end, we now suppose that everyone believes that others *will* accept fiat money and ask if this could be an equilibrium. We begin by arguing that good 0 is preferred by agent i to all goods other than his own consumption good. Clearly, i could not prefer good 0 to good i (since then he would want to get good 0 and keep it forever, equivalent to dropping out of the economy), and so we need only to show that good 0 is preferred to the other goods. But this is also obvious because both fundamentals and marketability work in the same direction (good 0 has the lowest storage cost by assumption, and everybody always accepts it by construction). We still must rank the remaining goods; for example, for type I we know that good 1 is best and good 0 is second-best, but what about goods 2 and 3? This is slightly complicated because things depend on the distribution $\mathbf{p}$, which in turn depends on the quantity of real balances in circulation. We will parameterize this dependence here by a variable $\pi = \pi(S)$, which is a decreasing function of real balances S, and give formulae for computing $\mathbf{p}$ in terms of π in the Appendix.

We now state a result indicating that, for certain values of S, there are equilibria in which fiat money circulates.

THEOREM 3. Choose S, determining $\pi = \pi(S)$. Then if the following two conditions are satisfied,

$$\text{(i)} \qquad [1 - 2b + b\pi^3(1 + \pi)^{-1}(1 + \pi - \pi^2)^{-1}](c_{13} - c_{12})$$

$$> b\pi(1 + \pi)^{-1}(1 + \pi - \pi^2)^{-1}\left[\pi(1 - \pi)c_{12} + u_1\left(1 - 2b + \frac{b\pi^2}{1 + \pi}\right)\right],$$

$$\text{(ii)} \qquad \left(1 - 2b - \frac{b\pi^2}{1 + \pi}\right)(c_{32} - c_{31}) > b\pi^3(1 - \pi)(1 + \pi)^{-1} \cdot (1 + \pi - \pi^2)^{-1}c_{31},$$

there exists an equilibrium in which all agents play fundamental strategies; that is, $V_{ii} = \max_j V_{ij}$ for all i, and $V_{10} > V_{12} > V_{13}$, $V_{20} > V_{21} > V_{23}$, and $V_{30} > V_{31} > V_{32}$ (they always prefer their own consumption goods; otherwise they rank goods or money according to their storage costs).

I \ II	3	1	0
2	N	T	T
3	N	N	N
0	N	T	N

I MEETS II

II \ III	1	2	0
3	T	T	T
1	N	T	N
0	N	T	N

II MEETS III

III \ I	2	3	0
1	N	T	T
2	N	T	N
0	N	T	N

III MEETS I

FIG. 7.—Fiat money equilibrium

The proof is nothing more than a construction along the lines of those in previous sections, but it involves a lot of algebra, which is available on request. This equilibrium corresponds to introducing fiat currency into the fundamental commodity money equilibrium described in Section III (see the trading matrices in fig. 7), and condition i is a generalization of the condition there that rules out speculation by type I. Condition ii is less easy to interpret. However, we note that it is redundant for small S since, with formulae in the Appendix, $S = 0$ implies $\pi = 1$ and condition ii reduces to $(1 - \beta)(c_{32} - c_{31}) > 0$, which is automatically satisfied.[13]

Notice that conditions i and ii hold for *any* value of S in [0, 1] if c_{13} and c_{32} are sufficiently large. In the extreme case of $S = 0$, we are back to commodity money equilibrium. In the other extreme $S = 1$, there is nothing but fiat money in circulation. The more interesting outcomes involve $0 < S < 1$, where there are both real commodities and fiat money in circulation. In such cases, all agents take fiat money sometimes: type III accepts it from I for good 1, type I accepts it from II for good 2, and type II accepts it both from I for good 1 and from III for good 3 (see fig. 8). An interesting example is when type I buys good 1 from II using fiat currency, and then II buys good 2 from type I using the same money (see the arrow pointing both ways in fig. 8). Type II also accepts good 1 from III for future trade with I, so good 1 is a medium of exchange in some trades, too, and commodity money exists alongside fiat money. However, fiat money is the only *general*

[13] For certain other restrictions on the parameter values, there exists a fiat money equilibrium corresponding to the speculative equilibrium in Sec. III (types II and III play fundamental, while type I ranks goods according to $V_{10} > V_{13} > V_{12}$; i.e., he prefers good 3 over good 2). These are the only fiat money equilibrium, given that good 0 is the most preferred object after one's own consumption good.

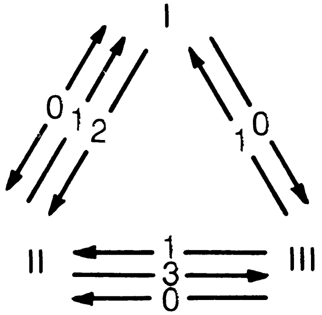

FIG. 8.—Fiat money equilibrium exchange pattern

medium of exchange: no agent ever offers good 0 for good j and gets refused, while, for example, sometimes III offers good 1 to II and gets turned down (when II has good 0).[14]

We close this section with a discussion of some measures of "moneyness." All remarks will concern the fundamental fiat money equilibrium of model A, that is, the one described in theorem 3. In figure 9, we graph the equilibrium values of the stock of good j (x_j), the number of times it gets traded per period (t_j), its velocity of circulation (v_j), and the probability it gets accepted when offered (a_j) on the vertical axis, against the stock of real balances (S) on the horizontal axis.[15] The vertical intercepts ($S = 0$) yield values for the commodity money

[14] Good 0 satisfies Clower's (1967, p. 5) criterion: "A commodity is regarded as money for our purposes if and only if it can be traded directly for all other commodities in the economy." Clower went on to advocate imposing a unique medium of exchange exogenously—i.e., assuming a cash-in-advance constraint—on the basis of the apparent observation that "money buys goods and goods buy money; but goods do not buy goods." It is hard to imagine why two agents who meet and happen to have a double coincidence in real commodities (as is the case when type I with good 2 meets type II with good 1 in the equilibrium discussed in the text) should not be allowed to trade without using fiat currency.

[15] The total stock of good j is given by $x_j = \Sigma_i \theta_{ij}$, where $\theta_{ij} = p_{ij}/3$ is the proportion of the entire population that are type i agents holding good j. The number of times j gets offered in trade per period is the number of meetings between type i with good j and type h with good k in which i wants to trade, summed over h, i, and k:

$$o_j = \sum_h \sum_i \sum_k \theta_{hk}\theta_{ij}\tau_i(j, k).$$

The number of times good j gets traded in a period is given by the number of meetings between type i with good j and type h with good k in which both want to trade, summed over h, i, and k:

$$t_j = \sum_h \sum_i \sum_k \theta_{hk}\theta_{ij}\tau_h(k, j)\tau_i(j, k).$$

Now velocity is $v_j \equiv t_j/x_j$ and the acceptance probability is $a_j \equiv t_j/o_j$. Figure 9 was constructed by computer, using formulae for the p_{ij} in the Appendix.

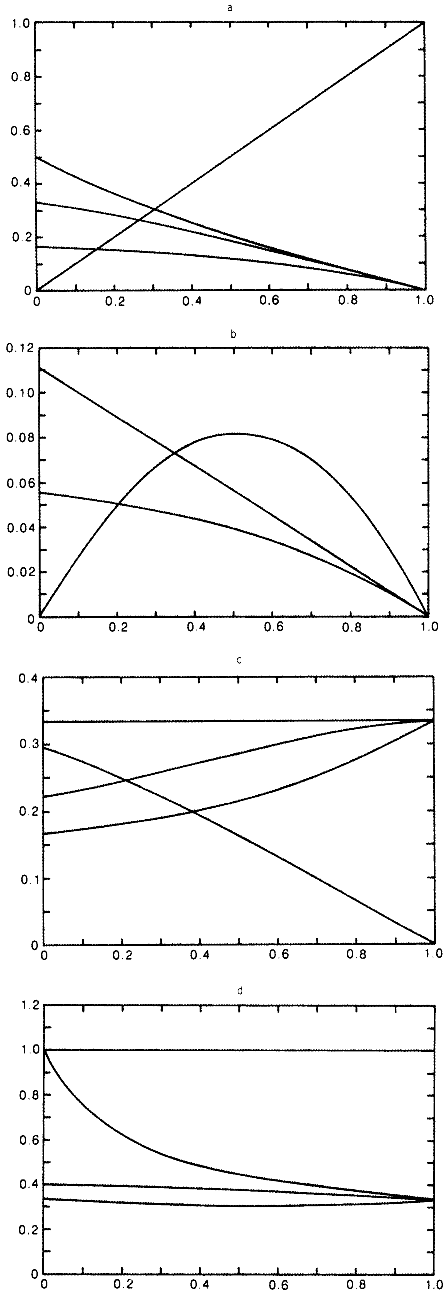

FIG. 9.—*a*, Stocks. *b*, Transactions. *c*, Velocities. *d*, Acceptabilities

equilibrium. As S increases, the stock of each real commodity falls, as does the number of transactions in real commodities. The number of transactions in fiat money first rises and then falls since when there are very many people holding money there are very few holding goods with whom to trade. As a net result the velocity of money is globally decreasing in S. Notice that velocity (fig. 9*c*) is a very poor indicator of moneyness. Good 3 has the highest velocity, surpassing both goods 0 and 1 even though they are media of exchange, simply because the stock of good 3 is so low in equilibrium.[16] Acceptability is a much better measure of moneyness (fig. 9*d*). Fiat currency has an acceptability of unity (which is what makes it a general medium of exchange). Good 1 has a high acceptability, too, since it sometimes also serves as a medium of exchange; but as S increases, its role as money diminishes and a_1 falls.

Another measure of moneyness is liquidity. One way to think about the liquidity of good j for agent i is the following: starting with good j, how long, on average, will it take i to trade for his consumption good?[17] He need not trade j directly for good i but might trade indirectly via some other good k, of course. It is a straightforward although somewhat tedious matter to compute these average durations, call them d_{ij}; for agent i, we say that good j is more liquid when d_{ij} is smaller. We find that in fiat money equilibrium, liquidity depends on the quantity of real balances. In particular, the greater is S, the larger are all d_{ij} since there are fewer real commodities in circulation (a clear case of "too much money chasing too few goods"). We also find that, for any S, $d_{12} > d_{13} > d_{10}$, $d_{23} > d_{21} > d_{20}$, and $d_{32} = d_{31} = d_{30}$. This says that for type III, all objects are equally liquid, which is why III can always play fundamental. For type II, lower-storage-cost goods are more liquid, which is why II can also always play fundamental. For type I, however, good 3 is more liquid than good 2, which is why he may sometimes speculate, depending on relative storage costs. This is merely another way of saying that, for type I, good 3 is more marketable.

VI. Welfare

In this section we discuss some welfare implications. We focus on steady-state utility levels, given by $W_i = (1 - \beta)\Sigma_j p_{ij} V_{ij}$ (up to a con-

[16] It initially seemed reasonable to rank objects by their equilibrium velocity and call those with the highest v_j money, those with slightly lower v_j near money, and so on. We did not realize that goods that are not used as media of exchange can have a very high v_j until after we calculated velocity explicitly. With the benefit of hindsight, Robert Hall told us that we should have had more foresight since associating velocity with moneyness leads, e.g., to the conclusion that electricity is a medium of exchange.

[17] See Lippman and McCall (1986), and the sources they reference, for some other, related, notions of liquidity.

stant of proportionality).[18] Computing these for the fundamental commodity money equilibrium in model A, we find

$$W_1 = \frac{\beta u_1}{6} - c_{12},\ W_2 = \frac{\beta u_2}{6} - \tfrac{1}{2}(c_{21} + c_{23}),\ W_3 = \frac{\beta u_3}{6} - c_{31}$$

(agents can expect to eat next period with probability 1/6, type I always stores good 2, type II stores goods 1 and 3 each half of the time, and type III always stores good 1). Is there any reason to expect this outcome to be efficient? Clearly, it is not interesting to ask if a social planner could Pareto-dominate the equilibrium by an arbitrary reallocation since he must abide by the same rules of spatial and temporal separation that impinge on private agents (e.g., he cannot simply impose the Walrasian allocation). Instead, we ask if the equilibrium outcome is optimal relative to other, not necessarily equilibrium, sets of trading strategies.

The answer is not generally yes. Suppose, for example, that agents were to follow the (nonequilibrium) strategy of always trading, regardless of the match: $\tau_i(j, k) = 1$ for all i, j, k. Letting W_i^* denote i's steady-state welfare when everyone uses these strategies, we calculate the difference $\Delta_i = W_i^* - W_i$ between this and equilibrium utility:

$$\Delta_1 = \frac{\beta u_1}{6} - \tfrac{1}{3}(c_{13} - c_{12}),$$

$$\Delta_2 = \frac{\beta u_2}{6} - \tfrac{1}{6}(c_{23} - c_{21}),$$

$$\Delta_3 = \frac{\beta u_3}{6} - \tfrac{1}{3}(c_{32} - c_{31}).$$

If u_i is large, $\Delta_i > 0$ for all i since individuals eat more frequently when all use the $\tau \equiv 1$ strategy, and we conclude that equilibria are not generally optimal. Unfortunately these strategies are not implementable; in a given match, trade may not be in an individual's self-interest, and he has incentive to reject offers of high-storage-cost goods even though when everyone behaves so "selfishly" they will all be worse off in the long run.

We next compare the relative welfare of fundamental and speculative equilibria. Consider first model B, where they coexist for some parameter values. Letting superscripts F and S denote utility levels in

[18] Using steady-state utilities W_i ignores the effect of initial conditions (the current inventory of agent i) on V_{ij}. Since we are basically interested in examples in which the outcome is not efficient, our use of W_i is sufficient because, for β near one, V_{ij} is arbitrarily close to W_i.

fundamental and speculative equilibria, respectively, one can compute

$$W_1^F - W_1^S = (.5\sqrt{2} - 1)(c_{13} - c_{12}) < 0,$$
$$W_2^F - W_2^S = (1 - .5\sqrt{2})(c_{23} - c_{21}) > 0,$$
$$W_3^F - W_3^S = (2 - \sqrt{2})(c_{32} - c_{31}) > 0.$$

The equilibria are noncomparable: types II and III are better off in the fundamental while I is better off in the speculative equilibrium. Type I gains because his output (good 3 in model B) serves as one of the commodity monies in speculative equilibrium. In model A the two equilibria never coexist, so they cannot be directly compared. However, it is possible to show that whenever speculative equilibrium exists, it Pareto-dominates the allocation that results if the (nonequilibrium) fundamental strategies are imposed. If a balanced budget intervention could be introduced to prevent speculation by type I (e.g., tax c_{13} and subsidize c_{12}), this would serve only to reduce welfare; speculation is not inefficient here and is not something to be discouraged.

Finally, we consider the possibility of improving steady-state welfare in a commodity money economy by the introduction of fiat money. Since the fundamental commodity money equilibrium for model A is actually a special case of the fiat money equilibrium with $S = 0$, we compute W_i in fiat money equilibrium as a function of real balances and consider $\partial W_i/\partial S$ at $S = 0$. In the Appendix we show $\partial W_i/\partial S > 0$ for all i as long as the u_i are not too large. This results because using fiat money reduces the inefficient storage of real commodities in the same manner that fiat money can improve welfare in some versions of overlapping generations models. However, since the only way to get good 0 into the system here is to reduce the amount of real goods and therefore the frequency of consumption, it is essential that the u_i not be too large relative to the c_{ij} if we are to enjoy a net welfare increase.[19]

There is also a sense in which fiat money is neutral here. Welfare depends solely on real balances, $S = M/P$, not nominal balances, M. The quantity equation holds exactly, and since it does not matter whether a unit of good j sells for P or λP units of fiat money, for any $\lambda > 0$, the nominal money supply can be of any size. What matters is the amount of valuable resources taken out of the system, S, not the amount of intrinsically worthless stuff put in. Similarly, the stock of outside money need not be homogeneous, and all our results are valid

[19] It is *not* correct to say that we have improved welfare by introducing a new technology for storing or freely disposing of commodities that agents previously lacked. The critical factor is that we introduced good 0, which in addition to having efficient storage properties could also be very marketable.

if good 0 is interpreted as a composite of two other objects—say, two bills of different denomination or maybe fiat money and bonds—as long as the private agents regard them as perfect substitutes. Of course, they might not regard these different assets as perfect substitutes, even if they happen to have the same fundamental properties, but we will not pursue the possible implications for open market operations in this paper.

VII. Conclusion

We have presented a model with specialization and explicit frictions that lead to indirect exchange. We have characterized steady-state equilibria in terms of existence and uniqueness and discussed how different objects endogenously come to play the role of commodity monies. Sometimes there was one and sometimes more than one medium of exchange. In some cases there were multiple outcomes with different media of exchange. We also constructed equilibria with intrinsically worthless stuff, fiat money, circulating as the general medium of exchange, and we discussed the tenuousness of this result. We looked at the way some variables such as velocity, acceptability, and liquidity depend on real balances, and we examined how well they capture the notion of moneyness. We analyzed welfare implications and found that decentralized outcomes are not necessarily Pareto efficient, that multiple equilibria cannot be Pareto-ranked, and that introducing fiat currency into a commodity money economy can improve welfare.

This is a simple model with a special explicit structure.[20] Are our conclusions robust? Extension to more types or goods should not change the message but would make things much messier (three is the minimum number leading to indirect exchange). The assumption of indivisible consumption goods makes the state space finite and the model tractable. Relaxing this would be interesting because then relative prices would have to be determined, for example, as the solution to bargaining problems. One implication of this assumption, plus limited storage capacity, is that even if good 0 is perfectly divisible, agents cannot hold both it and real commodities at the same time. Some people might consider this unrealistic. We are not sure if it is more or less unrealistic than other aspects of the model, but it does not seem

[20] Simplicity is a virtue but gives the impression that generality has been sacrificed. The methodology advocated by Lucas (1980*b*) requires that we construct a "fully articulated artificial economy," and this means specific assumptions about technology, preferences, matching, etc. Shubik's (1986) criteria for models in this class also require that the game played by the agents in the model should have a simple enough structure so that, at least in principle, it could be played by real people in an actual experiment.

particularly important. The defining characteristic of fiat money is *not* that you can carry it and other stuff at the same time; the defining characteristic of fiat money is that it is intrinsically useless. Many other features that money possesses have been ignored, and we concentrated on storability exclusively because it seemed the easiest to handle technically. In principle, similar analyses might examine the implications of features such as recognizability in models with informational frictions, for example.

How does the theory presented here compare with existing models? The cash-in-advance model has proved useful, but, ceteris paribus, it is better to derive money endogenously, and sometimes it is essential. For example, as Kareken and Wallace (1980, pp. 6–7) put it, "no Clower constraint model determines what is used as a means of payment. That is given exogenously. So, independent of what happens to the physical environment, or, more specifically, to government policy, there is never any switching from one thing to another. . . . The foregoing objection would be without force if no one had ever observed any variation in the means of payment. It is, however, a matter of record that different things have been used at different times and in different places." Similar remarks apply to theories that give up the postulate of intrinsic uselessness by putting money in utility or production functions. Although little time was allocated here to phenomena such as switching from one medium of exchange to another, the potential clearly does exist in models of this class.

The overlapping generations model has proved useful and enlightening also, but its implications have been challenged because it fails to capture the medium of exchange role (e.g., Tobin 1980; McCallum 1983). Some of this criticism is misguided, and it is also apparent that a given model need not capture every function or nuance of money to provide us with insights. Yet protagonists of the model themselves recognize certain drawbacks. According to Kareken and Wallace again, "There is a clearly discernible real world pattern of transaction velocities, a pattern displayed by nearly all real world economies, past and present. Some one thing has a large transactions velocity, or a few things do, and all other things have small transactions velocities. . . . [A] model of a monetary economy, to be successful, must explain not only valued fiat money but also the real-world pattern of transactions velocities" (1980, pp. 8–9). Our model has several implications for velocity that standard overlapping generations (or cash-in-advance) models do not: for one thing, it demonstrates that velocity is not a very good indicator of moneyness.

This model can also generate rate-of-return dominance, a fundamental issue in monetary theory since Hicks (1935) that requires auxiliary assumptions such as "legal restrictions" in overlapping generations models. Objects with a low rate of return (i.e., a high storage cost in the existing setup) may become equilibrium media of exchange without such assumptions here, as demonstrated explicitly by our speculative commodity money equilibria. A potentially valuable extension would be to see if fiat money equilibrium can exist when the storage cost of good 0 is not lower than that of real goods.[21] Finally, as Wallace (1980, p. 78) says, another "limitation of the overlapping generations model is that money, in a way, works too well in that model. Money completely overcomes the friction. Given a real friction, there is no reason why it should be feasible to overcome it completely." A genuine role for media of exchange arises from real frictions in our model, which money may help lubricate but not dissolve. We are optimistic that versions of this or related models may be used to study these issues and, it is hoped, will generate insights into important substantive questions in the future.

Appendix

Here we verify that introducing fiat money into a commodity money economy may improve welfare. We will need explicit formulae for the steady-state **p** distribution, which are rather complicated because the p_{ij} depend on $S = M/P$. If we parameterize the system by a variable $\pi = \pi(S)$, a little algebra results in the following solution:

$$p_{12} = \pi, \qquad p_{13} = 0, \qquad p_{10} = 1 - \pi,$$

$$p_{21} = \frac{\pi^2}{(1 + \pi - \pi^2)(1 + \pi)}, \quad p_{23} = \frac{\pi}{1 + \pi}, \quad p_{20} = \frac{1 + \pi - 2\pi^2}{(1 + \pi - \pi^2)(1 + \pi)},$$

$$p_{31} = \frac{\pi}{1 + \pi - \pi^2}, \qquad p_{32} = 0, \qquad p_{30} = \frac{1 - \pi^2}{1 + \pi - \pi^2}$$

(and, as always, $p_{ii} = 0$ for all i), where π satisfies the identity $\Sigma_i p_{i0}/3 = S$. With the formulae above, this identity can now be written as

$$\pi^4 + (3S - 2)\pi^3 - 5\pi^2 + 3(1 - 2S)\pi + 3(1 - S) = 0.$$

The extreme case $S = 0$ (yielding $p_{i0} = 0$ for all i) gives the values for p_{ij} in commodity money equilibrium, while the other extreme $S = 1$ (yielding $p_{i0} = 1$ for all i) is the case in which everyone holds fiat money and there are no real commodities left in circulation.

Let V_i be the steady-state utility of type i. We wish to show $\partial V_i/\partial S > 0$ when $S = 0$. First observe that $S = 0$ iff $\pi = 1$. Then notice that $\pi = \pi(S)$ is a decreasing function because $\partial p_{i0}/\partial \pi < 0$ for all i, and so differentiating the

[21] We are currently working on a related framework that does not impose the severe asymmetries (i.e., differential storage costs, as well as consumption and production specializations) that are present in this paper (see Kiyotaki and Wright 1988). That model is not as natural for studying commodity money but turns out to be much more tractable for the analysis of fiat currency, and it is therefore probably better for pursuing the rate-of-return dominance issue.

identity $\Sigma_i p_{i0} = 3S$ implies $\pi'(S) < 0$. Hence, $\partial V_i/\partial S > 0$ at $S = 0$ iff $\partial V_i/\partial \pi < 0$ at $\pi = 1$. Calculating each V_i as a function of the p_{ij} and then inserting the expressions for p_{ij} in terms of π and simplifying, we have

$$V_1 = \frac{\pi}{1+\pi}\frac{\beta u_1}{3} - \pi c_{12},$$

$$V_2 = \frac{\pi}{1+\pi}\frac{\beta u_2}{3} - \frac{\pi}{1+\pi}\left(c_{23} + \frac{\pi}{1+\pi-\pi^2}c_{21}\right),$$

$$V_3 = \frac{\pi}{1+\pi}\frac{\beta u_3}{3} - \frac{\pi}{1+\pi-\pi^2}c_{31}.$$

Differentiating with respect to π and evaluating at $\pi = 1$, we get

$$\frac{\partial V_1}{\partial \pi} = \frac{\beta u_1}{12} - c_{12}, \frac{\partial V_2}{\partial \pi} = \frac{\beta u_2}{\partial \pi} - \tfrac{5}{4}c_{21} - \tfrac{1}{4}c_{23}, \frac{\partial V_3}{\partial \pi} = \frac{\beta u_3}{12} - 2c_{31}.$$

As long as u_i is not too large, V_i is decreasing in π at $\pi = 1$, and therefore V_i is increasing in S at $S = 0$. This completes the proof.

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