# Section 6: Maneuvering in Space

To maneuver, a satellite in orbit must use rocket engines (thrusters) to change the magnitude or direction of its velocity. Because the orbital speed of satellites is so large, the velocity changes required for maneuvering may also be large, requiring the thrusters to use large amounts of propellant.

How much and how quickly a satellite can maneuver depends on the amount and type of propellant it carries. There are practical limits to the amount of propellant a satellite can carry since it increases the total mass that must be launched into orbit. These constraints on maneuvering in space have important consequences for satellite operations.

This section discusses the different types of satellite maneuvers and the changes in satellite velocity required for each. Section 7 outlines the amount of propellant required for these maneuvers.

## BASIC SATELLITE MANEUVERS

When a satellite maneuvers, it changes orbit. Since the speed of a satellite is related to its orbit, maneuvering can be complicated.

Three basic maneuvers are used to change orbits: (1) changing the shape or size of an orbit within the orbital plane; (2) changing the orbital plane by changing the inclination of the orbit; and (3) changing the orbital plane by rotating the plane around the Earth's axis at constant inclination. (Recall that all satellite orbits lie in a plane that passes through the center of the Earth.)

We discuss each of these in more detail below, as well as several common orbital changes that use these basic maneuvers. Maneuvers within the orbital plane allow the user to change the altitude of a satellite in a circular orbit, change the shape of the orbit, change the orbital period, change the relative location of two satellites in the same orbit, and de-orbit a satellite to allow it to return to Earth. To indicate the scale of velocity changes required for some common orbital maneuvers, Table 6.1 lists such maneuvers along with a characteristic value of the velocity change needed in each case (see the Appendix to Section 6 for more details).[1]

A velocity change is typically referred to as *delta-V*, or $\Delta V$, since the term "delta" is commonly used in technical discussions to indicate a change in some quantity. To get a feel for what these numbers mean, it is helpful to keep in mind that a speed of 1 km/s is roughly four times faster than a passenger jet. In addition, as Section 7 shows, generating a velocity change of 2 km/s with conventional propulsion technologies would require a satellite to carry its own mass in propellant—thus doubling the mass of the satellite.

1. A general maneuver will be combination of these basic maneuvers. Designing a maneuver that changes the altitude and orbital plane at the same time, rather than through sequential maneuvers, can reduce the velocity change required.

Maneuvers that change the orbital plane of a satellite can require very large changes in the satellite's velocity, especially for satellites in low earth orbit (see Table 6.1). This has important implications for the feasibility and utility of space-based systems that require such maneuvers.

**Table 6.1.** This table shows the change in satellite velocity ($\Delta V$) required for various types of maneuvers and activities in space, where $\Delta\theta$ is the change in inclination.

| Type of Satellite Maneuver | Required $\Delta V$ (km/s) |
|---|---|
| Changing orbital altitude within LEO (from 400 to 1,000 km) | 0.3 |
| Stationkeeping in GEO over 10 years | 0.5–1 |
| De-orbiting from LEO to Earth | 0.5–2 |
| Changing inclination of orbital plane in GEO | |
| by $\Delta\theta$ = 30° | 2 |
| by $\Delta\theta$ = 90° | 4 |
| Changing orbital altitude from LEO to GEO (from 400 to 36,000 km) | 4 |
| Changing inclination of orbital plane in LEO | |
| by $\Delta\theta$ = 30° | 4 |
| by $\Delta\theta$ = 90° | 11 |

These numbers are calculated in the Appendix to Section 6. (LEO = low earth orbit, GEO = geosynchronous orbit)

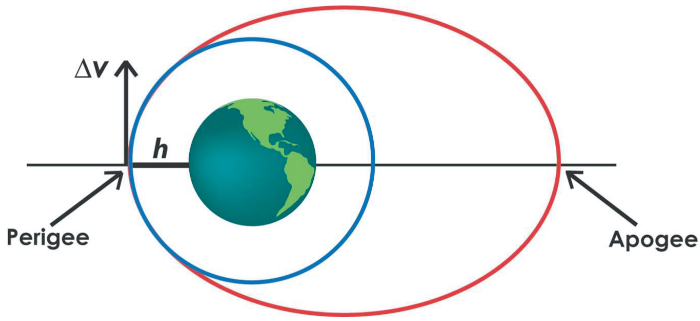
## MANEUVERS WITHIN THE ORBITAL PLANE

Maneuvers that change the shape or size of a satellite's orbit without changing its orbital plane can be made by changing the magnitude but not the direction of the velocity. These kinds of maneuvers can require significantly less $\Delta V$ than maneuvers that change the direction of the velocity.

### *Changing the Shape of the Orbit*

Consider a satellite that is initially in a circular orbit with altitude $h$. As discussed in Section 4, the laws of physics require it to have a particular speed for that altitude, which is given by Figure 4.1 and Equation 4.2. If the speed of the satellite is suddenly increased by $\Delta V$ at some point on the orbit (without changing the direction of the velocity), the satellite does not go faster around the same orbit; instead, the orbit becomes an ellipse in the same orbital plane (see Figure 6.1). The perigee of the new orbit (where the satellite is closest to Earth) lies at the point where the speed was increased, and this point will remain at an altitude $h$. As is always the case for elliptical orbits, the major axis passes through the center of the Earth, with the perigee and apogee of the new orbit at opposite ends. The orbital altitude at apogee is greater than $h$ and depends on the value of $\Delta V$, as discussed in the Appendix to Section 6.

**Figure 6.1.** The speed of a satellite in a circular orbit with altitude $h$ is increased by $\Delta V$ at the point shown. This point becomes the perigee of the new elliptical orbit that the satellite will follow.




If the speed of a satellite on a circular orbit is reduced at some point on the orbit by thrusting in the direction opposite to the satellite motion, that point becomes the apogee of an elliptical orbit, with an altitude of $h$ at apogee. The perigee then lies at an altitude less than $h$.

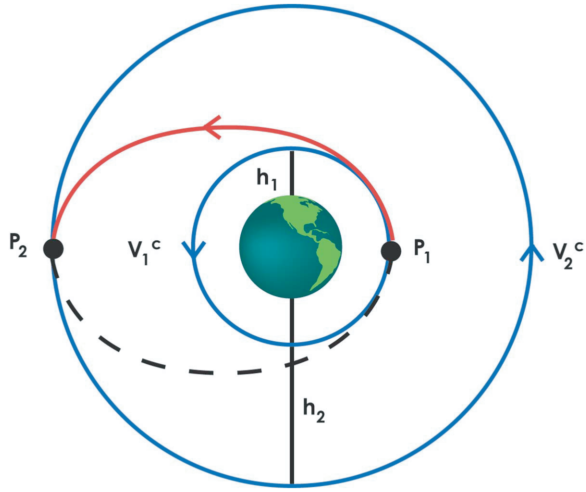
As shown in the Appendix to Section 6, a relatively small value of $\Delta V$ results in a significant change in altitude at apogee. As an example, for a satellite in orbit at an altitude of 400 km, a $\Delta V$ of 0.1 km/s would lead to a change in altitude at apogee of 350 km, so that apogee lies at an altitude of 750 km.

In the more general case of an elliptical orbit, changing the speed but not the direction of the velocity of the satellite results in another elliptical orbit, but of a different shape and orientation within the plane. The resulting orbit depends on both the value of $\Delta V$ and the point at which the velocity changed. However, in two specific cases, an elliptical orbit can be changed into a circular orbit, with one of two altitudes. Increasing the speed at apogee by the required amount results in a circular orbit with an altitude equal to that at the apogee of the ellipse. Decreasing the speed at perigee by a specific amount results in a circular orbit with an altitude equal to that at the perigee of the ellipse.

### *Changing the Altitude of a Satellite in a Circular Orbit*

The strategy described above to change the shape of the orbit, can also be used to increase the altitude of a circular orbit from $h_1$ to $h_2$, through a two-step process (see Figure 6.2). The first step is to increase the speed of the satellite by $\Delta V_1$ so that the resulting elliptical orbit has an altitude at apogee of $h_2$. Recall that the perigee of the new orbit lies at the point where the velocity increase ($\Delta V_1$) is applied and has an altitude of $h_1$. Once this is done, the speed of the satellite at apogee is less than its speed would be if it were on a circular orbit with altitude $h_2$. The second step is to change the elliptical orbit the satellite is on to a circular one at altitude $h_2$ by increasing the speed at apogee by the appropriate amount ($\Delta V_2$). By choosing $\Delta V_1$ to make the apogee of the elliptical orbit at $h_2$, the satellite's velocity will be tangent to the larger circular orbit (at point $P_2$ in Figure 6.2), and $\Delta V_2$ needs to change only the satellite's speed and not its direction.

**Figure 6.2.** This figure shows the elliptical Hohmann transfer orbit between two circular orbits, the initial one with altitude $h_1$ and the final one with altitude $h_2$. $\Delta V_1$ is applied at point $P_1$ and $\Delta V_2$ at $P_1$. The satellite travels only half an orbit on the transfer orbit; it does not travel the half of the ellipse indicated by the dashed line.




The total $\Delta V$ required to make the orbital change described above is the sum of the velocity changes applied in each of these two steps: $\Delta V = \Delta V_1 + \Delta V_2$. These velocity changes are calculated in the Appendix to Section 6.

The elliptical orbit used to move between these two circular orbits, which is tangent to both orbits, is called a *Hohmann transfer orbit* (see Figure 6.2). This method is fuel-efficient since it requires the minimum $\Delta V$ needed to transfer between two orbits. The time required for such a transfer is half the period of the elliptical transfer orbit.

This time can be shortened and the transfer done more quickly by applying a larger $\Delta V_1$ in the first step of the process than that described above. In this case, the velocity of the satellite will not be tangent to the larger circular orbit when it reaches that orbit, so $\Delta V_2$ will need to adjust the speed of the satellite as well as rotate its direction to put it on the circular orbit. Since both $\Delta V_1$ and $\Delta V_2$ will be larger in this case, it is clear that using the Hohmann transfer orbit requires the minimum energy for this transfer.

Satellites placed in geostationary orbits are frequently placed in a low earth orbit initially, and then moved to geostationary orbit using a Hohmann transfer orbit.

The calculations in the Appendix to Section 6 show that for a satellite in low earth orbit, a significant change in altitude requires a relatively small $\Delta V$. For example, maneuvering from a circular orbit at 400 km to a circular orbit at 1,000 km requires a total $\Delta V$ of only 0.32 km/s. On the other hand, if the satellite were transferred from a 400 km orbit to a geosynchronous orbit at 36,000 km altitude, this maneuver would require a total $\Delta V$ of 3.9 km/s.

Not surprisingly, the $\Delta V$ required to change from one circular orbit to another is related to the difference in orbital speeds of the two orbits. Since the orbital speed of circular orbits changes relatively slowly with altitude, orbital changes do not require large values of $\Delta V$ unless the change in altitude is very large. This is because the orbital speed is related not to the altitude of the satellite (its distance above the Earth), but to the satellite's distance from the center of the Earth. A relatively large fractional change in altitude, say from 500 to 1,000 km (a 100% change), represents only a small fractional change in the distance to the center of the Earth, in this case from 6,870 to 7,370 km, a 7% change; as a result, the orbital speed changes by less than 4%.

### *Changing the Orbital Period*

Since the orbital period of a satellite depends on the altitude and shape of the orbit, maneuvers to change the shape and altitude of the orbit can be used to change the period. Such maneuvers may be useful, for example, to vary the revisit time of a reconnaissance satellite, making it less predictable.

The equation for the change in period produced by a change in velocity is given in the Appendix to Section 6. As an example, a satellite in a circular orbit with an altitude of 400 km has an orbital speed of 7.67 km/s and a period of 92.2 minutes. Increasing the orbital speed by 0.1 km/sec would increase the period by about 3.6 minutes, while an increase of 0.3 km/sec would increase the period by 10.8 minutes. As discussed above, these velocity changes would cause the orbit to become elliptical: the resulting apogees would have altitudes of 750 km and 1,460 km, respectively, while the perigee would remain at 400 km.

### *Changing the Relative Location of Satellites in the Same Orbit*

Changing the period of one satellite can change its position relative to other satellites in the same orbit through a multi-step process. Consider, for example, two satellites in the same circular orbit. Since they must have the same speed, the distance between them will stay the same as they move around the orbit. To change the distance between them, simply increasing the speed of one of the satellites will not work, since that would change its orbit.

Instead, one satellite can be moved relative to the other by putting it temporarily into a higher or lower orbit to change its period, and then moving it back into the original orbit after enough time has passed to put the satellites in the desired relative positions. The amount of propellant required for this process depends on how quickly the change must be made: a small $\Delta V$ leads to a small change in period, and the satellites require a long time to reach the desired relative position.

For example, consider two satellites that are near one another in a circular orbit at an altitude of 400 km. Giving one satellite a $\Delta V$ of 0.1 km/s to place it on an elliptical orbit changes its period by 3.6 minutes, requiring about 13 orbits, or 20 hours, to move it halfway around the orbit relative to the second satellite, which remains on the original orbit. Moving the first satellite back onto the original circular orbit requires another $\Delta V$ of 0.1 km/s, for a total $\Delta V$ of 0.2 km/s. Doubling the amount of $\Delta V$ cuts the transition time roughly in half since it changes the period of the satellite by twice as much (7.2 minutes) as in the previous example.

This type of maneuvering can be used to rendezvous one satellite with another. It can also be used to position multiple satellites around an orbit, as discussed below, to increase the ground coverage of a satellite constellation. These satellites can be placed in the same orbit by a single launcher, then shifted around the orbit by this kind of maneuver.

## MANEUVERS THAT CHANGE THE ORBITAL PLANE

Maneuvers that change the plane of the orbit require changing the direction of the velocity of the satellite. Since the orbital velocity of a satellite is very large (it varies from roughly 3 to 8 km/sec for typical orbits—see Table 4.1), changing its direction by a significant amount requires adding a large velocity component perpendicular to the orbital velocity. Such large changes in velocity require large amounts of propellant.

Figure 6.3 shows an example for a satellite in a 500 km-altitude orbit, with an orbital velocity of 7.6 km/sec. The figure illustrates that a $\Delta V$ of 2 km/s rotates the orbital velocity by only 15 degrees.

**Figure 6.3.** This figure shows a $\Delta V$ of 2 km/s being added to a speed of 7.6 km/s, which is the orbital speed of a satellite in a 500 km-altitude orbit. Even this large $\Delta V$ will lead to only a relatively small change, rotating the orbital plane by an angle of only 15°.

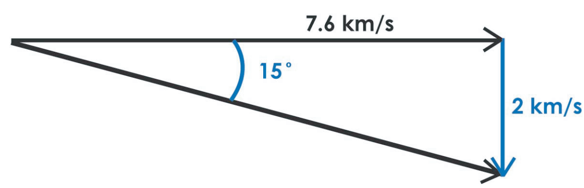



Figure 6.3 shows that the larger the satellite's velocity, the larger the value of $\Delta V$ required to rotate the velocity by a given angle. As a result, changing the plane of a satellite in a low altitude circular orbit will require more $\Delta V$ than the same change at higher altitudes, because satellites travel at a slower velocity at higher altitudes.

It is convenient to look at two different types of plane-changing maneuvers: those that change the inclination of the plane, and those that rotate the plane at constant inclination. Recall that the orbital plane is partly described by its inclination angle $\theta$, which is measured with respect to the Earth's equatorial plane (see Fig 4.3).

### *Maneuvers to Change Inclination*

The simplest type of plane change to conceptualize is one that changes the inclination of the orbital plane by an angle $\Delta\theta$. Such a maneuver requires rotating the velocity vector of the satellite by the same angle $\Delta\theta$ (see Figure 6.4).[2]

2. This can be thought of as rotating the plane about the line formed by the intersection of the orbital plane and the equatorial plane.

**Figure 6.4.** This figure shows two orbits with different inclinations. The velocity vector for a satellite in each orbit is denoted by the arrows labeled $V_1$ and $V_2$. For the satellite to change its orbit from one plane to the other, the satellite's thrusters must produce a $\Delta V$ large enough to rotate its velocity from $V_1$ to $V_2$.

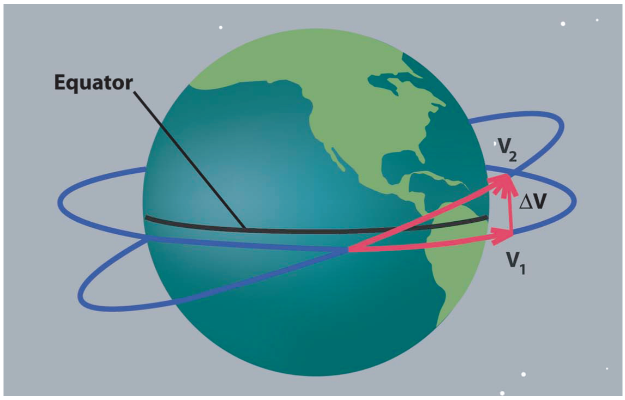



Table 6.2 shows the $\Delta V$ required for several values of $\Delta\theta$ for a satellite at an altitude of 500 km; these values are calculated using Equation 6.13 in the Appendix to Section 6.

**Table 6.2.** This table shows values of $\Delta V$ required to change the inclination angle $\theta$ by an amount $\Delta\theta$ for a satellite at an altitude of 500 km.

| $\Delta\theta$ (degrees) | $\Delta V$ (km/s) |
|---|---|
| 15 | 2.0 |
| 30 | 3.9 |
| 45 | 5.8 |
| 90 | 11 |

Since the orbital speed decreases with altitude, the $\Delta V$ required for a given change of $\Delta\theta$ also decreases with orbital altitude, but the decrease is relatively slow. For example, for orbits at 1,000-km altitude, the required $\Delta V$ is only 3% lower than for orbits at 500 km (Table 6.2). On the other hand, the required $\Delta V$ at geosynchronous altitude (36,000 km) is about 40% of the value of $\Delta V$ at 500 km.

For this reason, rotations are made at high altitudes when possible. For, example, consider a satellite that is intended for an equatorial orbit (zero inclination) at geosynchronous altitude, but is launched into a plane with a nonzero inclination due to the location of the launch site. The satellite is placed in an orbit at geosynchronous altitude with nonzero inclination before the orbit is rotated to have zero inclination.

Since the $\Delta V$ required for a given $\Delta\theta$ decreases when the satellite's speed decreases, large rotations of the orbital plane can be made somewhat more economically using a three-step process. First the satellite is given a $\Delta V$ to increase its altitude at apogee. Since the satellite's speed is slower at apogee, it is rotated at that altitude, then given a final $\Delta V$ to reduce the altitude at apogee to its original value. As discussed above, maneuvers that change the altitude require relatively small values of $\Delta V$; consequently, this three-step procedure can, in some cases, require a lower overall $\Delta V$ than simply rotating the plane of the original orbit. However, this procedure can take much longer than a simple inclination change because it takes time for the satellite to move into a higher orbit and then return.

As an example, consider a satellite in a circular orbit at an altitude of 500 km. For inclination changes of $\Delta\theta$ less than about 40°, changing altitudes before rotating requires more $\Delta V$ than rotating at the original altitude. However, for rotations through larger angles, changing altitude first requires less energy. For example, if $\Delta\theta$ is 90°, performing the rotation at an altitude of 10,000 km reduces the total required $\Delta V$ to 8.2 km/s, or 76% of the 10.8 km/s required for such a rotation at the original 500 km altitude. In this case, the total transit time to and from the higher altitude is about 3.5 hours. Rotating instead at an altitude of 100,000 km reduces the required $\Delta V$ by nearly 40% to 6.6 km/s and increases the transit time to 37 hours. Going to even higher altitudes reduces the required $\Delta V$ only marginally while further increasing transit time.

### *Rotating the Orbital Plane at Constant Inclination*

Another maneuver that can require a large velocity change is rotating the orbital plane around the Earth's axis while keeping the inclination fixed.[3] Such a maneuver might be used if multiple satellites were put into orbit by a single launch vehicle and then moved into different orbital planes—all with the same inclination—to increase the ground coverage of the constellation. A set of three satellites, for example, might be maneuvered to place each in a plane rotated 120° with respect to the others. The energy requirements of such maneuvers are an important consideration when planning to orbit a constellation of satellites.

The $\Delta V$ required for this maneuver depends on the angle $\Delta\Omega$ through which the orbital plane is rotated around the Earth's axis, as well as the inclination angle $\theta$ of the orbit and the altitude (and therefore the speed) of the satellite when the maneuver is carried out.

Table 6.3 shows the $\Delta V$ required for a satellite in a circular orbit at an altitude of 500 km for several rotation angles $\Delta\Omega$ and two inclination angles $\theta$. For practical applications the rotation angle can be large, resulting in very large values of $\Delta V$. As above, the required $\Delta V$ decreases slowly with the altitude of the orbit; values for a 1,000 km-altitude orbit are about 3% lower than those for a 500 km orbit.

3. This can be thought of as rotating the line formed by the intersection of the orbital plane and the equatorial plane about the Earth's axis, while keeping the inclination fixed.

**Table 6.3.** This table shows the $\Delta V$ required for rotations of $\Delta\Omega$ degrees of an orbital plane around the Earth's axis, for inclinations $\theta$ of 45 and 90 degrees. These values assume the satellite is in a circular orbit at an altitude of 500 km.

| | $\theta = 45°$ | $\theta = 90°$ |
|---|---|---|
| $\Delta\Omega$ (degrees) | $\Delta V$ (km/s) | $\Delta V$ (km/s) |
| 45 | 4.1 | 5.8 |
| 90 | 7.6 | 10.8 |
| 120 | 9.3 | 13.2 |

## DE-ORBITING MANEUVERS

For some missions, an object in orbit will use its thrusters to accelerate out of orbit and back toward the Earth. The Space Shuttle must do this to return to Earth; similarly, an orbiting weapon intended to strike the Earth would need to carry propellant to kick it out of orbit. The $\Delta V$ required for this maneuver will depend on how fast the return to Earth must be. The dynamics of the de-orbiting are complicated because once the satellite moves to low enough altitudes, the increasing density of the atmosphere affects its trajectory.[4]

Figure 6.5 illustrates the de-orbiting process for three values of $\Delta V$. This example assumes a relatively high circular orbit—3,000 km—to show the de-orbiting trajectories more clearly. At this altitude, the satellite has an orbital velocity of 6.5 km/s. In this illustration, a thrust is applied instantaneously at point *P* in a direction opposite to the satellite's velocity, so that it reduces the velocity by $\Delta V$. This reduction in speed causes the satellite to follow an elliptical orbit with a perigee below its original altitude. If the perigee is low enough, the orbit will intersect the Earth.

Making the satellite fall vertically to Earth under the influence of gravity requires reducing its orbital speed to zero—a $\Delta V$ of 6.5 km/s. In this case, it would take the satellite 19 minutes to fall to Earth and it would strike the Earth at point *O* in Figure 6.5, directly below the point at which the velocity change occurred (point *P*).[5]

4. These effects include drag forces, which slow the object, and lift forces, which are sideways forces and pull the object off its trajectory. At high speeds, both effects can be important.

5. Of course, due to the rotation of the Earth, the point on the Earth that was under the satellite when the $\Delta V$ was applied would in general move during the time it took the satellite to reach the Earth; the motion would range from zero at the poles to 500 km at the equator.

**Figure 6.5.** This figure shows a satellite in an initial orbit at an altitude of 3,000 km, and the paths the satellite would follow if its speed were decreased at the point $P$ by the values of $\Delta V$ shown. Applying $\Delta V$ = 6.5 km/s gives the satellite zero speed, and it falls vertically to the point $O$ on Earth in a time $t$ of 19 minutes. For smaller values of $\Delta V$, the de-orbiting time $t$ is longer. In each case, the range $r$ along the Earth's surface is given for the impact point relative to the point $O$.

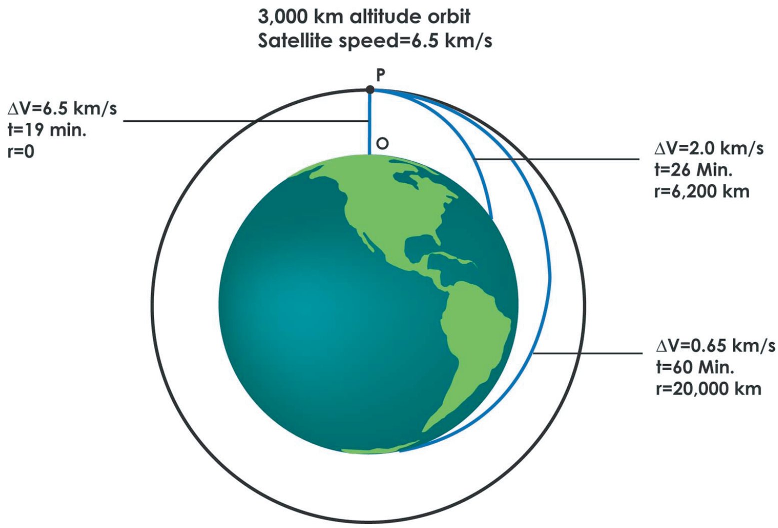



Figure 6.5 also shows the reentry trajectory if the satellite's orbital speed were reduced by 2 km/s. In this case, it would take 26 minutes for the satellite to fall to Earth, and it would hit the Earth at a point 6,200 km along the Earth's surface from point $O$. If the orbital speed were reduced by only 0.65 km/s, so that the satellite takes 60 minutes to de-orbit, it would hit the Earth halfway around the world from point $O$—at a ground range of roughly 20,000 km.

If $\Delta V$ were much less than 0.65 km/s, the satellite would not hit the Earth, but would pass by the Earth at low altitude and follow an elliptical orbit to return to point $P$. However, the drag of passing so low through the atmosphere on its near encounter with the Earth would reduce the satellite's speed, so that it would reach an altitude somewhat less than 3,000 km when it returns to $P$ and would slowly spiral downward on subsequent orbits until it hit the Earth.[6]

A case more relevant to space security issues is a satellite in an orbit with an altitude of 500 to 1,000 km, since this is where missile defense or ground-attack satellites might be stationed. In calculating the de-orbit time and $\Delta V$ required in this case, assume that the thrust given to the satellite is oriented

6. The object may also be able to use lift forces to assist in de-orbiting, so that the trajectory need not simply be determined by the object's speed.

vertically downward toward the Earth. Applying thrust in this direction results in somewhat shorter de-orbit times than simply reducing the orbital speed as done for the cases illustrated in Figure 6.5.

For a satellite in a circular orbit at an altitude of 500 km (with an orbital speed of 7.6 km/s), a $\Delta V$ of 0.7 km/s results in a de-orbit time of about 15 minutes, and 1 km/s in a de-orbit time of 10 minutes (see the Appendix to Section 6 for calculations). (The precise time required for the satellite to de-orbit depends in part on its drag coefficient, which is partially determined by its shape.)

For a satellite in a circular orbit at an altitude of 1,000 km (with an orbital speed of 7.4 km/s), a $\Delta V$ of 1.4 km/sec results in a de-orbit time of roughly 15 minutes, and a $\Delta V$ of 2 km/sec gives a time of 9 to 10 minutes.

Higher values of $\Delta V$ can lead to shorter de-orbit times. Though the satellite would need to carry a large amount of propellant, high $\Delta V$s have been discussed for kinetic energy weapons intended to attack ground targets, which must hit their targets at high speeds. A $\Delta V$ of 4 km/s gives de-orbit times of 2 to 3 minutes from an altitude of 500 km, 4 to 5 minutes from 1,000 km, and 14 to 15 minutes from 3,000 km. A $\Delta V$ of 6 km/s results in de-orbit times of 1.5 to 2 minutes from an altitude of 500 km, 3 to 3.5 minutes from 1,000 km, and 8.5 to 9.5 minutes from 3,000 km. Section 7 discusses the amount of propellant required for producing these values of $\Delta V$.

### *Reentry Heating*

An important issue in de-orbiting is that as the atmosphere slows the satellite large amounts of heat build up in the layers of air around the satellite. (This occurs as the kinetic energy of the satellite is converted to thermal energy of the air, largely through compression of the air in front of the satellite.)

If the object is not to burn up during re-entry, it must carry a heat shield to withstand this intense heat. The heating rate increases rapidly with the speed of the object moving through it and with the density of the atmosphere. If de-orbiting occurs too fast, the satellite will be moving at high speeds low in the atmosphere where the atmospheric density is high, and this can lead to extreme heating.

Atmospheric heating is important when considering the possibility of delivering kinetic energy weapons either from space or by ballistic missile. The motivation for such weapons is that their destructive power would come from the kinetic energy resulting from their high speed rather than from an explosive charge. To be effective, such weapons must hit the ground with very high speed. For example, a mass must be moving at about 3 km/s for its kinetic energy to be equal to the energy released in the explosion of an equal mass of high explosive.[7] The heat load on an object traveling faster than 3 km/s at atmospheric densities near the ground is very large. For comparison, a modern U.S. nuclear reentry vehicle, which is designed to pass through the atmosphere quickly to improve its accuracy, has a speed of about 2.5 km/s

7. The energy released by TNT is roughly 1,000 calories per gram, which equals $4.2 \times 10^6$ J/kg. The kinetic energy of a one kilogram mass moving at 3 km/s is $V^2/2 = 4.5 \times 10^6$ J/kg.

when it reaches the ground: designing the warhead to travel faster is limited by its ability to withstand the heating. A penetrator made of a tungsten rod would be more heat tolerant than a nuclear warhead, but the intense heating at the tip of the rod could reduce its structural strength. Since an object traveling at 5 km/s would have a heating rate eight times as high as an object traveling at 2.5 km/s, a kinetic energy weapon traveling at 5 km/s would have to withstand eight times the heating rate that a modern U.S. nuclear warhead is designed to tolerate.

Not only do atmospheric forces cause drag, which leads to heating, they can also produce strong lateral forces—called *lift forces*—that change the object's trajectory. The reentering body can be designed to use the significant lift forces resulting from its high speed in the atmosphere to maneuver in directions perpendicular to its trajectory. Documents describing the goals for ground-attack weapons state that these weapons should be able to travel thousands of kilometers in these directions using only lift forces.

## STATIONKEEPING

A number of forces act on a satellite to change its orbit over time. These include the slight asymmetries in the Earth's gravitational field due to the fact that the Earth is not completely spherically symmetric; the gravitational pull of the Sun and Moon; solar radiation pressure; and, for satellites in low earth orbit, atmospheric drag.

As a result, the satellite must periodically maneuver to maintain its prescribed orbit. Thus, it must carry sufficient propellant for this task. While satellite lifetimes used to be limited by the lifetime of the electronics in the satellite, the quality of electronics has improved to the point that lifetimes are now typically limited by the amount of propellant carried for stationkeeping.[8]

How much propellant is needed for stationkeeping depends on several factors. First, satellites that travel for all or part of their orbit at low altitudes (up to several hundred kilometers) must compensate for more atmospheric drag than those at high altitudes. This is especially necessary during high solar activity when the outer parts of the Earth's atmosphere expand, resulting in increased drag at a given altitude. Second, the orbits of some satellites must be strictly maintained, either to fulfill their missions or because their orbital locations are governed by international agreements. For example, the locations of satellites in geosynchronous orbits are tightly controlled by international rules to prevent satellites from interfering with one another. Third, the propellant required depends on the type of thrusters used for stationkeeping, and their efficiency. Until recently, conventional chemical thrusters were used for stationkeeping, but other options that reduce propellant requirements are now available. For example, ion thrusters, which provide lower thrust over longer times, are discussed in Section 7.

8. Bruno Pattan, *Satellite Systems: Principles and Technologies* (New York: Van Nostrand Reinhold, 1993), 36.

To get a rough sense of how much maneuvering is required for stationkeeping in geostationary orbits, consider the Intelsat communication satellites. Each year, these use an amount of propellant equal to roughly 2 to 2.5% of their total initial mass (when placed in orbit) for stationkeeping. Thus, for a ten-year satellite lifespan, a propellant mass of 20% to 25% of the satellite's initial mass is required for stationkeeping, which corresponds to a total $\Delta V$ over ten years of roughly 0.5–1.0 km/s (see the Appendix to Section 6).

# Section 6 Appendix: Technical Details of Maneuvering

## CHANGING THE SHAPE OF THE ORBIT

A satellite in a circular orbit at altitude $h$ will have a velocity $V_h^c = \sqrt{GM_e/(h+R_e)}$, where $G$ is the gravitational constant, $M_e$ is the mass of the Earth ($GM_e$ = 3.99 × $10^{14}$ $m^3/s^2$), and $R_e$ is the average radius of the Earth (6,370 km) (see the Appendix to Section 4). If the speed of the satellite is suddenly increased by $\Delta V$ at some point on the orbit (without changing the direction of the velocity), the orbit becomes an ellipse. The perigee of the new orbit remains at altitude $h$. The altitude at apogee depends on the value of $\Delta V$. For small $\Delta V$ (*i.e.*, $\Delta V/V \ll 1$), the change in altitude $h$ at apogee is given approximately by[9]

$$\Delta h \approx 4(h+R_e)\frac{\Delta V}{V} \tag{6.1}$$

This equation can be rewritten using $r \equiv (h + R_e)$ as

$$\frac{\Delta r}{r} \approx 4\frac{\Delta V}{V} \tag{6.2}$$

which shows that the fractional change in $r$ at apogee is just four times the fractional change in the velocity at perigee.

Similarly, if the speed of a satellite on a circular orbit is reduced at some point on the orbit, that point becomes the apogee of an elliptical orbit, and the altitude at perigee is less than the altitude of the original orbit by an amount given by Equations 6.1 and 6.2.

Equation 6.1 shows why maneuvers that change altitudes take relatively little $\Delta V$: since the change in velocity is multiplied by the radius of the Earth, even a relatively small change in velocity will lead to a significant change in $h$. This is especially true for satellites maneuvering between low earth orbits, since the altitude band of interest—about 1,000 km—is small compared to $R_e$. For a satellite orbiting at an altitude of 400 km, a $\Delta V$ of 0.1 km/s would lead to a change in altitude at apogee of 350 km.

If the original orbit is not circular, but elliptical with eccentricity $e$, the approximate equations for the change in the altitude of the orbit at apogee ($\Delta h_a$) and perigee ($\Delta h_p$) that result from a velocity change applied at perigee ($\Delta V_p$) and at apogee ($\Delta V_a$) are, respectively[10]

9. R. Bate, D. Mueller, and J. White, *Fundamentals of Astrodynamics* (New York: Dover, 1971), 163.

10. Bate et al., 163.

$$\Delta h_a \approx \frac{4a^2}{GM_e} V_p \Delta V_p = 4 \frac{h_a + R_e}{1-e} \frac{\Delta V_p}{V_p} \quad \text{or} \quad \frac{\Delta r_a}{r_a} \approx \frac{4}{1-e} \frac{\Delta V_p}{V_p} \tag{6.3}$$

and

$$\Delta h_p \approx \frac{4a^2}{GM_e} V_a \Delta V_a = 4 \frac{h_p + R_e}{1+e} \frac{\Delta V_a}{V_a} \quad \text{or} \quad \frac{\Delta r_p}{r_p} \approx \frac{4}{1+e} \frac{\Delta V_a}{V_a} \tag{6.4}$$

Note that these equations are only valid for $\Delta V/V \ll 1$. For larger values of $\Delta V$, the exact equations given below are required.

## MANEUVERING BETWEEN CIRCULAR ORBITS

Here we calculate the minimum $\Delta V$ required to increase the altitude of a circular orbit from $h_1$ to $h_2$, through a two-step process using a Hohmann transfer orbit. The transfer orbit is an ellipse with its perigee at $h_1$ and apogee at $h_2$ and eccentricity $e = (r_2 - r_1)/(r_2 + r_1)$, where $r_i \equiv h_i + R_e$.

The first step is to move the satellite from the initial circular orbit onto the transfer orbit by increasing the speed of the satellite from its initial circular velocity $V_1^c = \sqrt{GM_e/r_1}$ to $V_p = V_1^c\sqrt{1+e}$, where $e$ is the eccentricity of the transfer ellipse. This gives

$$\Delta V_p \equiv V_p - V_1^c = V_1^c\left(\sqrt{1+e} - 1\right) \tag{6.5}$$

The speed of the satellite at apogee of the transfer orbit is $V_a = V_2^c\sqrt{1-e}$, where $V_2^c = \sqrt{GM_e/r_2}$ is the velocity of a circular orbit at altitude $h_2$. The second step is to make the satellite's orbit circular by increasing the speed at apogee to $V_2^c$. This gives

$$\Delta V_a \equiv V_2^c - V_a = V_2^c\left(1 - \sqrt{1-e}\right) \tag{6.6}$$

The total $\Delta V$ required for this orbit change is just the sum of these two:

$$\Delta V_{tot} = \Delta V_p + \Delta V_a \tag{6.7}$$

For relatively small altitude changes, so that $e \ll 1$, this becomes

$$\Delta V_{tot} \approx e\frac{(V_1^c + V_2^c)}{2} \tag{6.8}$$

Equation 6.8 shows that maneuvering from a circular orbit at 400 km to a circular orbit at 1,000 km requires $\Delta V_{tot}$ = 0.32 km/s (in this case, $e$ = 0.041 for the transfer orbit). Moving the satellite from a 400 km orbit to a geosynchronous orbit at 36,000 km altitude requires using a transfer orbit with $e$ = 0.71, so Equation 6.8 cannot be used; Equation 6.7 gives $\Delta V_{tot}$ = 3.9 km/s.

Two other useful approximate expressions are those for the speed of a satellite at perigee and apogee after a small change of a circular orbit with radius $r$ to an elliptical orbit with semi-major axis of length $r + \Delta r$:

$$V_p \approx V^c\left(1+\frac{1}{4}\frac{\Delta r}{r}\right) \quad \text{and} \quad V_a \approx V^c\left(1-\frac{3}{4}\frac{\Delta r}{r}\right) \tag{6.9}$$

where $V_c$ is the speed of the satellite on the original circular orbit.[11]

## CHANGING THE PERIOD OF A SATELLITE

From Equation 4.5 for the period of an elliptical orbit with major axis $a$

$$\frac{\partial P}{\partial a} = \frac{3}{2}\frac{P}{a} \tag{6.10}$$

and from Equation 4.4 for the speed of a satellite on an elliptical orbit

$$\frac{\partial a}{\partial V} = \frac{2a^2 V}{GM_e} \tag{6.11}$$

Combining these expressions, the change in period $\Delta P$, for small eccentricities, is given approximately by

$$\frac{\Delta P}{P} \approx 3\frac{\Delta V}{V} \tag{6.12}$$

for $\Delta V/V << 1$.

## CHANGING THE INCLINATION OF THE ORBIT

Changing the inclination angle of an orbit by an angle $\Delta\theta$ requires rotating the velocity vector of the satellite by $\Delta\theta$. Vector addition shows that the required $\Delta V$ is

$$\Delta V = 2V\sin\frac{\Delta\theta}{2} \tag{6.13}$$

where $V$ is the speed of the satellite when the maneuver occurs.

For circular orbits, the required $\Delta V$ decreases with orbital altitude, since the orbital speed decreases with altitude; in this case, $\Delta V$ is proportional to $1/\sqrt{h+R_e} \equiv 1/\sqrt{r}$.

## ROTATING THE ORBITAL PLANE AT CONSTANT INCLINATION

For circular orbits, the $\Delta V$ required to rotate an orbital plane with an inclination angle $\theta$ by an angle $\Delta\Omega$ around the Earth's axis is

$$\Delta V = 2V\sin\theta\sin\frac{\Delta\Omega}{2} \tag{6.14}$$

11. Oliver Montenbruck and Eberhard Gill, *Satellite Orbits* (New York: Springer-Verlag, 2000), 47.

where $V$ is the speed of the satellite when the maneuver occurs.[12] As with the previous maneuver, the required $\Delta V$ decreases with the altitude of the orbit, since $V$ does.

This process is also known as changing the right ascension of the ascending node.

### GENERAL ROTATIONS

For circular orbits, the $\Delta V$ required for a maneuver that both changes the inclination by $\Delta\theta$ and rotates the orbital plane by an angle $\Delta\Omega$ around the Earth's axis is given by[13]

$$\Delta V = 2V\sqrt{\sin^2\frac{\Delta\theta}{2} + \sin\theta_1 \sin\theta_2 \sin^2\frac{\Delta\Omega}{2}} \tag{6.15}$$

where $V$ is the speed of the satellite when the maneuver occurs, $\theta_1$ and $\theta_2$ are the initial and final values of the inclination, and $\Delta\theta \equiv \theta_1 - \theta_2$. Notice that this equation reduces to Equations 6.13 and 6.14 for $\Delta\Omega = 0$ and $\Delta\theta = 0$, respectively. As with the previous maneuvers, the required $\Delta V$ decreases with the altitude of the orbit.

### DE-ORBITING

De-orbiting times and trajectories were calculated using a computer program that integrates the equations of motion for an object, assuming a round Earth with an atmosphere. We assumed the satellite was initially in a circular orbit at altitude $h$. A velocity change vector of magnitude $\Delta V$ was added to the orbital velocity vector, with the change pointing either opposite to the velocity vector or in a vertical direction pointing toward the Earth. We repeated the calculation using a range of drag coefficients for the object, but assumed no lift forces. The drag coefficient enters the calculations through the combination $mg/(C_dA)$ called the *ballistic coefficient*, where $m$ is the mass of the object, $g$ is the acceleration of gravity at the altitude of the object, $C_d$ is the drag coefficient, and $A$ is the cross-sectional area of the object perpendicular to its motion.

In particular, we varied the ballistic coefficient by a factor of 10 from a value comparable to a modern strategic warhead (150,000 Newtons/m$^2$ (N/m$^2$), or 3,000 lb/ft$^2$), to a value for an object with much higher drag (15,000 N/m$^2$, or 300 lb/ft$^2$). As an illustration, consider the case in which the velocity change vector is oriented in the vertical direction. Results are given in Table 6.4.

12. Vallado, 333. Using trigonometric identities, the equation given in this book can be put in the simpler form given here.

13. Vallado, 335.

**Table 6.4.** This table lists the de-orbiting time for a satellite in a circular orbit at the given altitude when a velocity change $\Delta V$ is applied in the vertical direction. The results are given for two different values of the ballistic coefficient, which is inversely proportional to the drag coefficient of the object; the larger value is comparable to that of a modern strategic ballistic missile warhead.

| Altitude (km) | $\Delta V$ (km/s) | De-orbiting Time (min) | |
|---|---|---|---|
| | | Ballistic coefficient 150,000 N/m$^2$ (3,000 lb/ft$^2$) | Ballistic coefficient 15,000 N/m$^2$ (300 lb/ft$^2$) |
| 500 | 0.7 | 14.6 | 15.2 |
| | 1 | 9.4 | 10.3 |
| | 2 | 4.4 | 5.5 |
| | 4 | 2.1 | 2.9 |
| 1,000 | 1.4 | 14.4 | 15.3 |
| | 2 | 9.1 | 10.2 |
| | 4 | 4.3 | 5.1 |
| | 6 | 2.8 | 3.4 |
| 3,000 | 4 | 14.0 | 15.1 |
| | 6 | 8.7 | 9.4 |

The heating rate for an object moving through the atmosphere is roughly proportional to $\rho V^3$, where $\rho$ is the atmospheric density.[14] This expression shows that the heating rate increases rapidly with velocity and with decreasing altitude, since the atmospheric density increases roughly exponentially with decreasing altitude.

## STATIONKEEPING

Data from the Intelsat communication satellites suggest the scale of the $\Delta V$ required for stationkeeping in geosynchronous orbit using conventional thrusters.[15] The Intelsat V satellite has a mass of 1,005 kg when placed in orbit, of which 175 kg is propellant (with a specific impulse of 290 to 300 s), intended for a lifetime of 7 years. The propellant mass is 17.4% of this initial mass; assuming all the propellant is used for stationkeeping, this corresponds to 2.5% of the initial mass used per year. The Intelsat VII has a mass of 2,100 kg when placed in orbit, of which 650 kg is propellant (with a specific impulse of 235 s) and a planned lifetime of 17 years. The propellant is 31% of

14. For a more detailed discussion of heating at hypersonic speeds, see John Anderson, *Hypersonic and High Temperature Gas Dynamics* (New York: McGraw-Hill, 1989), 291.

15. Robert A. Nelson, "Rocket Science: Technology Trends in Propulsion," *Via Satellite*, June 1999, http://www.aticourses.com/rocket_tutorial.htm, accessed January 20, 2005.

the initial mass, and 1.8% is used each year. This indicates that these satellites use roughly 2 to 2.5% of their initial mass per year for stationkeeping. Over a 10-year lifespan, this would require that 20 to 25% of the initial mass be propellant reserved for stationkeeping. Using the rocket equation (see Section 7), these masses can be shown to correspond to a total $\Delta V$ over 10 years of roughly 0.5–1.0 km/s.