LETTERS TO THE EDITOR

PRACTICAL NUMBERS

THE subdivisions of money, weights and measures involve numbers like 4, 12, 16, 20 and 28 which are usually supposed to be so inconvenient as to deserve replacement by powers of 10. It was thought that these numbers can have no important feature to justify their existence except, perhaps, a fairly high composite character. In this note we proceed to show that they have a very remarkable property which ought to have been perceived by the ancients but either forgotten or ignored by the moderns. The revelation of the structure of these numbers is bound to open some good research in the theory of numbers. A preliminary examination is attempted here.

A number N may be called a 'practical number (on account of the association aforesaid) if every number less than N, other than a factor of N, admits of partition into unequal parts all of which are factors of N. Thus the numbers less than 12, which are not factors of 12, are 5(=1+4=2+3), 7(=1+6=3+4), 8 (= 2 + 6), 9 (= 3 + f), 10 (= 4 + 6), and 11 (=1+4+6), where 1, 2, 3, 4 and 6 are factors of 12. 12 is therefore a 'practical' number.

It is easily seen that every 'practical' number greater than 2 must be a multiple of 4 or 6.

It cannot be a deficient number, that is one of which the sum of all divisors is less than twice the number, unless the deficiency is one.

Every perfect number is evidently a 'practical' number.

If N is a 'practical' number, then $2N$ and therefore $2^t N$ is also 'practical'.

Further, the highly composite numbers of Ramanujan can be shown to be 'practical' numbers.

Three special types of 'practical' numbers are noticed :

(1) The α-type :—$N_\alpha = 2^{a_0} p_1^{a_1} p_2^{a_2} \ldots p_n^{a_n}$ where

$$2 < p_1 < p_2 < \ldots\ldots < p_n$$
$$2^{a_0} < p_1 < 2^{a_0+1},$$
$$\text{and } p_i^{a_i} < p_{i+1} < p_i^{a_i+1} \quad (1 \leqslant i \leqslant n-1).$$

In this type a_n *is* unrestricted. It includes even *perfect numbers.*

The product of the first n primes also belongs to this type. *E.g.*: 2.3.5.7.11=2310 is a 'practical' number.

The existence of the numbers of the α-type is manifest from the well-known Bertrand's *postulate.*

(2) The β-type:—$N_\beta = 2^{\beta_0} p_1^{\beta_1} p_2^{\beta_2} \ldots\ldots p_n^{\beta_n}$

$$2 < p_1 < p_2 < \ldots\ldots < p_n,$$
$$p_1 < 2^{\beta_0+1},$$

$$\text{and } p_{i+1} < 2^{\beta_0+1} p_1^{\beta_1} p_2^{\beta_2} \ldots\ldots p_i^{\beta_i} \quad (1 \leqslant i \leqslant n-1),$$

Obviously, $2^{t_0} p_1^{t_1} \ldots p_n^{t_n}$ is also a "practical" number of the β-type when $t_i \geqslant \beta_i$ ($i = 0, 1, 2, \ldots, n$).

This type is of wider scope than the α-type and includes it as a subclass. Ramanujan's highly composite numbers belong to the β-type but not to the α-type *E g.*: 36, 48; 180.

(3) The γ-type ;—$2^{\gamma_0} p_1 \; (2^{\gamma_0+1} p_1 + 1)$ where $2^{\gamma_0} p_1$ is a perfect number and $2^{\gamma_0+1} \; p_1+1$ a prime

E.g. : 2.3.13 = 78.

All 'practical' numbers less than 201 belong *to one or other of the types given above as may* be easily verified from the table given below:—

2	20	42	72	100	132	168
4	24	48	78	104	140	176
6	28	54	80	108	144	180
8	30	56	84	112	150	192
12	32	60	88	120	156	196
16	36	64	90	126	160	198
18	40	66	96	128	162	200

The three types envisaged here do not exhaust *probably all possible cases. The general* structure is, however, unknown. If the tables are enlarged, *up to at least 1000, we may meet* with other types. Our table shows that about 25 per cent. of the first 200 natural numbers are 'practical'. It is a matter for investigation what percentage of the natural numbers will be 'practical' in the long run.

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THE CANONICAL CO-ORDINATE SYSTEM IN GENERAL RELATIVITY

THE canonical co-ordinate system[1] for which, at the origin, all the first order partial derivatives of $g_{\mu\nu}$ vanish and the second order derivatives are given by a set of hundred equations is well known in the literature of general relativity. It is particularly useful for exploring the neighbourhood of an event in the space-time continuum. We have not seen anywhere the Taylor expansions of $g_{\mu\nu}$ defining the canonical co-ordinate system. The expansions *contain explicitly the twenty* independent components of the Riemann-Christoffel[2] tensor R_{hijk}. As the metric tensor defines not only the co-ordinate system but the gravitational field itself, we have found the expansions of special interest and service in discussing the purely geometrical, as well as gravitational properties of the relativity metric. A full report is being prepared for communication elsewhere. We have thought it worthwhile to place only the expansions here on record.

We define the twenty independent components of R_{hijk} at (0, 0, 0, 0) by the following equations :

$$R_{1212} = a, \quad R_{1313} = b, \quad R_{1414} = c,$$
$$R_{2323} = d, \quad R_{2424} = e, \quad R_{3434} = f,$$
$$R_{2113} = g, \quad R_{2114} = h, \quad R_{3114} = i,$$
$$R_{1223} = j, \quad R_{1224} = k, \quad R_{3224} = l,$$
$$R_{1332} = m, \quad R_{1324} = n, \quad R_{2334} = o,$$
$$R_{1442} = p, \quad R_{1443} = q, \quad R_{2443} = r,$$
$$R_{1234} = s, \quad R_{1423} = t.$$

If the powers above the second of the coordinates of an event in the neighbourhood of the origin are ignored we have

$$g_{11} = -1 - \tfrac{1}{3}(ay^2 + bz^2 + c\tau^2 - 2gyz - 2hy\tau - 2iz\tau),$$
$$g_{22} = -1 - \tfrac{1}{3}(ax^2 + dz^2 + e\tau^2 - 2jxz - 2kx\tau - 2lz\tau),$$
$$g_{33} = -1 - \tfrac{1}{3}(bx^2 + dy^2 + f\tau^2 - 2mxy - 2nx\tau - 2oy\tau),$$
$$g_{44} = 1 - \tfrac{1}{3}(cx^2 + ey^2 + fz^2 - 2pxy - 2qxy - 2ryz)$$
$$g_{12} = \tfrac{1}{3}\{mz^2 + p\tau^2 + axy - gxz - hx\tau - jyz - ky\tau - (2t + s)\, z\tau\},$$
$$g_{13} = \tfrac{1}{3}\{jy^2 + q\tau^2 - gxy + bxz - ix\tau - myz - nz\tau + (t - s\}\, y\tau,$$
$$g_{14} = \tfrac{1}{3}\{ky^2 + nz^2 - hxy - ixz + cx\tau - py\tau - qz\tau + (t + 2s\} yz,$$
$$g_{23} = \tfrac{1}{3}\{gx^2 + r\tau^2 - jxy - mxz + dyz - ly\tau - ozt + (t + 2s)\, x\tau\},$$
$$g_{24} = \tfrac{1}{3}\{hx^2 + oz^2 - kxy - px\tau - lyz + ey\tau - iz\tau + (t - s)\, xz\},$$
$$g_{34} = \tfrac{1}{3}\{ix^2 + ly^2 - nxz - qx\tau - oyz - ry\tau + fz\tau - (2t + s)\, xy\}.$$

In the above x, y, z, τ stand for the usual x^1, x^2, x^3, x^4. The algebraic work involved in the above calculation is quite tedious, but the symmetry of the various terms at each stage provides a useful check on the details and simplifies the calculation.

Benares Hindu University, V. V. NARLIKAR.
May 28, 1948. AYODHYA PRASAD.

1. Eddington, A. S., *The Mathematical Theory of Relativity*, 1924, 79. 2. Eisenhart, L. P., *Riemannian Geometry*, 1926. 20.

THE VANISHING OF RAMANUJAN'S FUNCTION (N)

MAKING use of certain congruence properties of Ramanujan's function $\tau(n)$ defined by the relation

$$\prod_{r=1}^{\infty} (1 - x^r)^{24} = \sum_{n=1}^{\infty} \tau(n)\, x^{n-1}, \; | x | < 1,$$

Lehmer has recently shown that

$\tau(n) \neq 0$ for $n < 3316799$.

More recently Chowla and Bambah have proved that

(1) $\tau(n) \equiv \sigma_{11}(n) \pmod{256}$ if n is odd ;

(2) $\tau(n) \equiv 5n^2\, \sigma_7(n) - 4n\, \sigma_9(n) \pmod{125}$ if $(n, 5) = 1$;

(3) $\tau(n) \equiv (n^2 + k)\, \sigma_7(n) \pmod{81}$,
where $k = 9$ if $n \equiv 2 \pmod 3$
and $= 0$ otherwise.

In view of these results, *it is now possible to* state that

$\tau(n) \neq 0$ for $n < 1791071999$.

In fact, the only possible solutions of $\tau(n) = 0$ below 26866079999 are $n = 1791071999$ and 8955359999. Since $\tau(n)$ cannot vanish except when n is a prime, it remains to be seen if any of these numbers is a prime. This has to be verified from a table of primes, which is not accessible to me at present.

Govt. College, Hoshiarpur, HANSRAJ GUPTA.
June 4, 1948.

1. Lehmer, D.H., *Duke Math. Jour.*, 1947, **14**, 429-33
2. Bambah, R. P., and Chowla, S., *Bull. American Math. Soc*, 1947, **53**, 950-55.