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Legacy, Works, and Life of
Sir Robert Stawell Ball
Upon the 100th Anniversary of
A Treatise on the Theory of Screws
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PRELIMINARY STUDIES TO SCREW THEORY IN XVIIIth CENTURY

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ABSTRACT - In this paper preliminary works to the Screw Theory are reviewed with the aim to show the efforts in the XVIIIth century and some developments that can be still of current interest. In particular, the early works on rotation of a body by Jean Baptiste D'Alembert, Jean Bernoulli, Leonhard Euler, Paolo Frisi and Giuseppe Luigi Lagrange have been reviewed. An original contribution of this paper from historical viewpoint consists of having recognised that D'Alembert individuated rigorously the instantaneous axis of rotation in 1749; Frisi proved the theorem of composition of instantaneous rotations in 1759; Euler formulated the motion of rotation in a modern form in 1765; Lagrange treated the motion of rotation with mathematical formulation in 1788. Finally, in 1763 Giulio Mozzi wrote a treatise in which he formulated a first Theory of Screw.

Symposium Topic: Historical and biographical perspectives

1. INTRODUCTION

The study of motion of rigid bodies was recognised of fundamental importance since the beginning of a mechanical Science. Nevertheless mathematical approaches have been formulated only from XVIIth century, because of the evolution of proper mathematical means. In addition, technological requirements for mechanical devices did not require the study of three-dimension motion until Astronomical observations stressed no coplanar motion of the planets. Moreover, the Galilean approach gave the first formulation that was successfully used for machines design and operation for long time without any further improvements.

Only in the XVIIIth century specific and rigorous treatments appeared on the analysis of motion of rigid bodies by using also suitable mathematical models. These studies gave great impulse on the development of a theory for motion of rigid bodies that was completed in the XIXth century with the establishment of the well known Screw Theory in the work by Ball (Ball 1876).

In this paper preliminary works to the Screw Theory are reviewed with the aim to show the first efforts and some developments on the Screw Theory that can be still of current interest. In particular, we have reviewed the early works on rotation of a body by Jean Baptiste D'Alembert, Jean Bernoulli, Leonhard Euler, Giuseppe Luigi Lagrange, and Paolo Frisi.

This paper is a first attempt to analyse the historical evolution of studies on the motion of rotation of rigid bodies.

2. EARLY STUDIES ON THE MOTION OF RIGID BODIES

Helicoidal motion was known since the ancient Greeks after Archimedes (287-212 b.C.) who designed the helicoidal screw for pumps elevating water flow. These pumps were widely used over time and they were illustrated by Romans. The mechanical design was based on the study of the motion both of the device and fluid so that one can find some description of the rotational motion and even helicoidal path. Specifically, this kind of study gave very detailed drawings of the mechanical design of the pumps, as shown in the Fig.1, (Barbaro 1584), which is a drawing obtained by interpreting the original Roman text by Vitruvio (Vitruvio 1511).

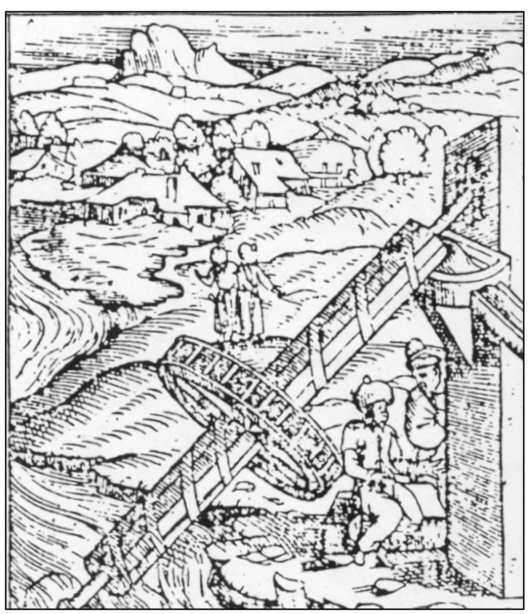


Fig.1 A drawing of Archimedes pump obtained from Vitruvio work reproduced in the Renaissance (Barbaro 1584).

During the Middle Age the classical work were reproduced by rewriting the text but not the drawing. In the Renaissance there was a great interest in the classical works and they were studied in depth and reproduced even with comments about the novelties. Indeed, in the Renaissance these pumps were studied again, although a study of motion was not attempted analytically.
A first description of helicoidal motion can be found due by Leonardo da Vinci (1452-1519) who wrote in the Arundel Codex in folium 140-verso, (Uccelli 1940): "Il moto composto è quello che, oltre al moversi di sito, ancora si move intorno al suo polo" (it can be translated as :"A composed motion is that one which, beside a motion of location, gives a motion about a pole"). Leonardo clarified the text by means of some sketches of different helices that have been reported in Fig.2.

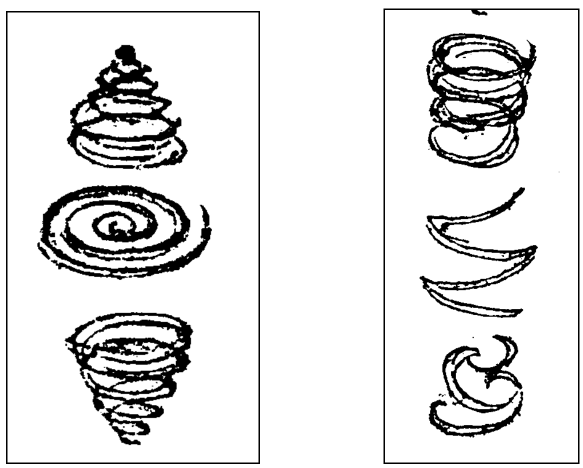
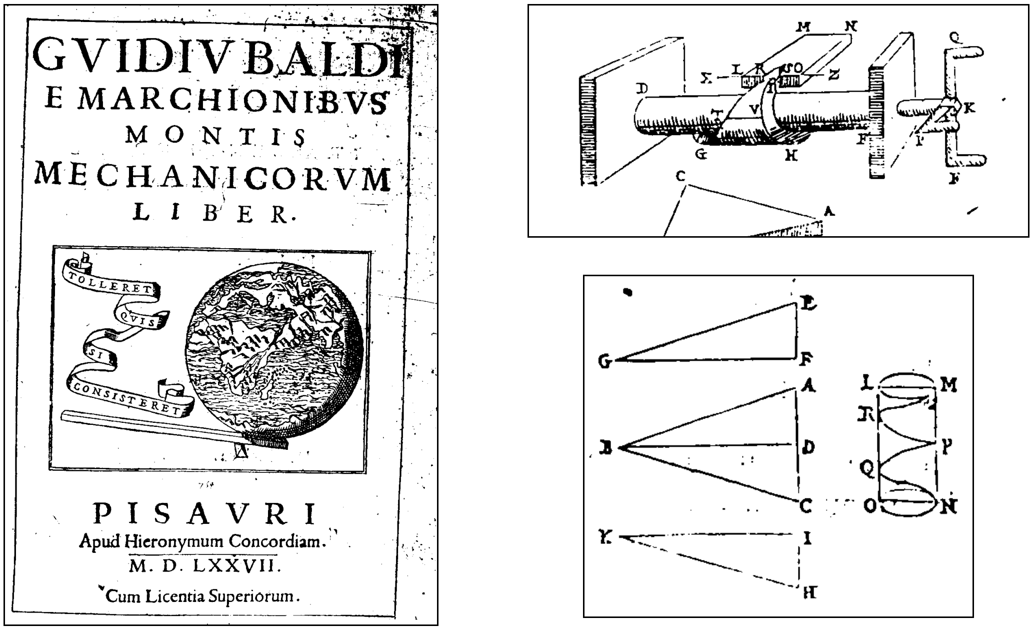


Fig.2 Sketches of helicoidal path by Leonardo da Vinci, (Uccelli 1940).

In the late Renaissance, indeed, there was interest on the motion of rigid bodies, but it was mainly addressed to planar motion and problems regarding composition of actions and kinematics of movements, in the form as reference (Varronis 1584) for example. These subjects were approached over time and specific works were developed like reference (Wallis 1670), which is a typical example after the works of Galilei and Newton. John Wallis (1616-1703) studied mainly Geometry, being professor of Geometry at Oxford for the whole life and he approached the motion of rigid bodies mainly form geometric viewpoint in order to outline several kind of motions in terms of point trajectories.
However mechanical studies on the helicoidal geometry were developed by Guidobaldo Del Monte (1545-1607), (1577), and Galileo Galilei (1564-1642), (1590 and 1600) It is of particular interest the chapter "De Cochlea" by G. Del Monte, (1577), in which the geometry of an helix is analysed and results are used for mechanical design of several devices for which helicoidal motion is clearly described, as in the case shown in Fig. 3b).



a) b)

Fig.3 The work by Guidobaldo Del Monte published in 1577: a) the title page; b) a study for helicoidal path.

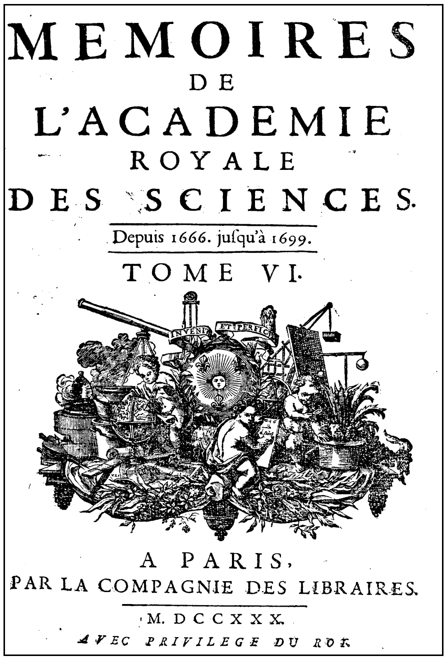
Indeed this kind of description and study of the helicoidal path and movement can be found in many treatises on Mechanics or Machines with the aim to explain both the mechanical design and usefulness of mechanical studies. Typical examples are the fundamental works by Del Monte (1577) and Galilei (1600).

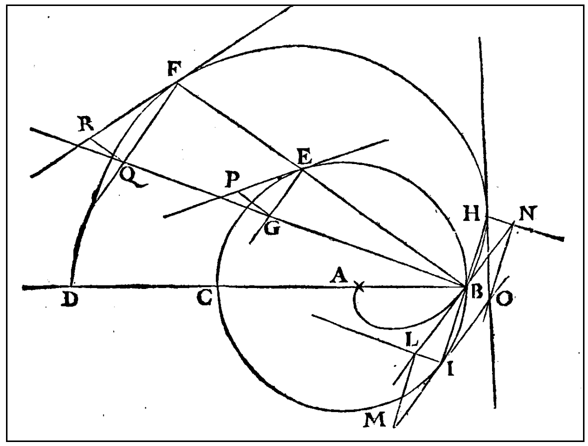
Indeed, the motion of rotation was studied with elementary movements even because machines used simple rotations that could be combined with simple geometrical reasoning, as for example in the cited works by Varronis (1584) and Wallis (1620). It is noteworthy that many authors in Europe were attracted to mechanical studies as applied to practical applications and many others gave specific attention to Mechanics in theoretical and academic studies. Emblematic are the works by Cardano (1570), Stevin (1634) Marci (1639), Torricelli (1644) only to cite some.

This kind of approach was persistent along the XVIIth century as emblematically illustrated in the work by De La Hire (1695) and even in the first part of the XVIIIth century as for example in the works by Ozanam (1720) and Grandi (1739).

Another interesting work was developed by Gilles Personnier De Roberval (1602-1675) who approached the geometry of planar motion by analysing several trajectories, (De Roberval 1730 a), and recognised before the XVIIIth century the need of deep studies on the composition of the motion of rigid bodies in (De Roberval 1730 b).

Figure 4 shows a geometrical study of a rotation of a line BEF with the aim to derive the trajectory of its points.





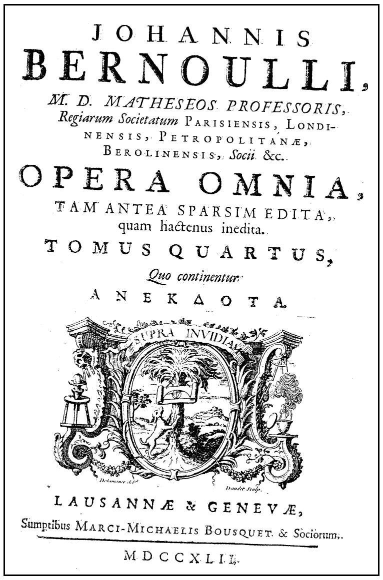
a) b)

Fig.4 The work by Personier De Roberval published in 1730: a) the title page; b) a scheme for a rotation of a line.

Specific attention was addressed to the motion of rigid bodies for astronomical studies giving specific treatises as for example by Kepler (1609). Together with empirical observations theoretical studies on the motion of the planets gave greater and greater impulse to develop better and better knowledge of the Mechanics of rigid bodies.
In this context a specific theoretical interest started on the motion of rigid bodies and several European Academies announced Academy Prizes to solve basically problems rigorously. Thus, a certain community of scientists was attracted specifically to pure Mechanics as part of Mathematics.

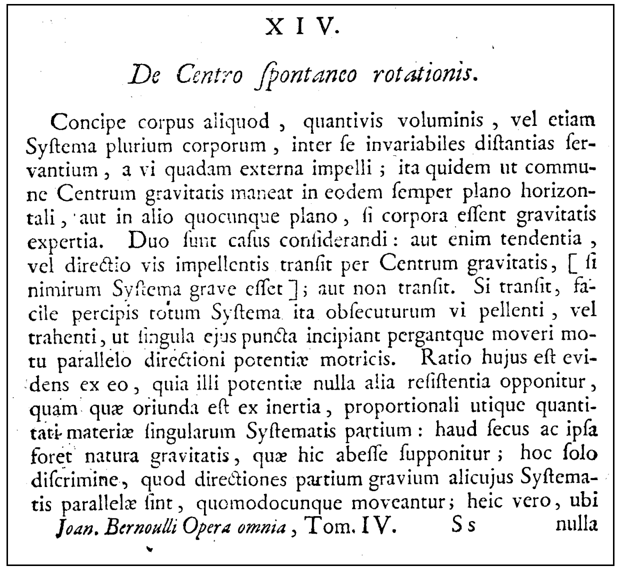
3. PRELIMINARY STUDIES TO SCREW THEORY

Fundamental results of the activity in XVIIIth century on preliminary studies to Screw Theory can be recognised in the work by Leonhard Euler, Jean Baptiste D'Alembert, Giuseppe Luigi Lagrange and Paolo Frisi.
Bernoulli is cited by Mozzi (see next section) since he worked on the subject of the motion of rigid bodies as a matter of application of Mathematics. Nevertheless, in his work (Bernoulli 1742), Fig.5, during this first research we did not find anything else that a description of a rotation and some computation for force evaluation related to a rotation in pages 265 to 271, although the paragraph XIV is titled on the Center of Rotation, as shown in shown in Fig.5b).


JOHANNIS
BERNOULLI,
M. D. MATHESEOS PROFESSORIS,
Regiarum Societatum PARISIENSIS, LONDINENSIS, PETROPOLITANÆ, BEROLINENSIS, Socii &c.
OPERA OMNIA,
TAM ANTEA SPARSIM EDITA,
quam hactenus inedita.
TOMUS QUARTUS,
Quo continentur
ANEKΔOTA.

LAUSANNÆ & GENEVÆ,
Sumptibus MARCI-MICHAELIS BOUSQUET & Sociorum.
MDCCXLII.

a)


XIV.

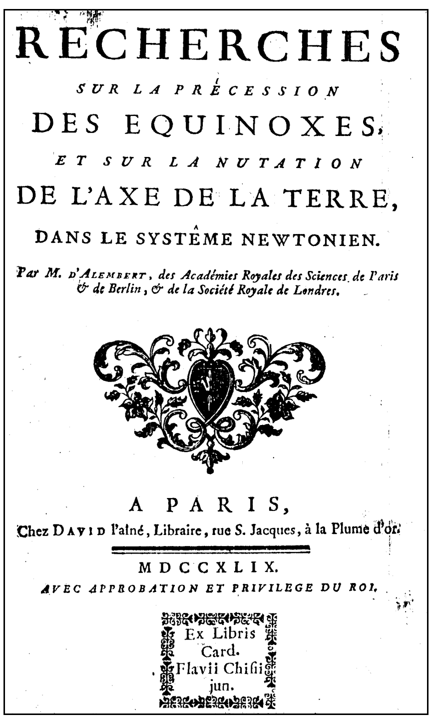
De Centro spontaneo rotationis.

Concipe corpus aliquod, quantivis voluminis, vel etiam Systema plurium corporum, inter se invariabiles distantias servantium, a vi quadam externa impelli; ita quidem ut commune Centrum gravitatis maneat in eodem semper plano horizontali, aut in alio quocunque plano, si corpora essent gravitatis expertia. Duo sunt casus considerandi: aut enim tendentia, vel directio vis impellentis transit per Centrum gravitatis, [si nimirum Systema grave esset]; aut non transit. Si transit, facile percipis totum Systema ita obsecuturum vi pellenti, vel trahenti, ut singula ejus puncta incipiant pergantque moveri motu parallelo directioni potentiæ motricis. Ratio hujus est evidens ex eo, quia illi potentiæ nulla alia resistentia opponitur, quam quæ oriunda est ex inertia, proportionali utique quantitati materiæ singularum Systematis partium: haud secus ac ipsa foret natura gravitatis, quæ hic abesse supponitur; hoc solo discrimine, quod directiones partium gravium alicujus Systematis parallelæ sint, quomodocunque moveantur; heic vero, ubi

Joan. Bernoulli Opera omnia, Tom. IV. Ss nulla

b)

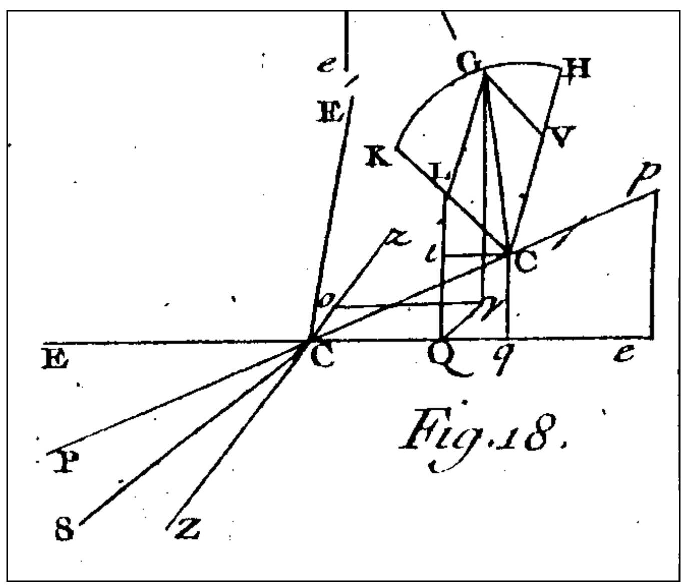
Fig. 5 The work by Jean Bernoulli: a) the title page; b) a paragraph on the motion of rotation.


RECHERCHES
SUR LA PRÉCESSION
DES EQUINOXES,
ET SUR LA NUTATION
DE L'AXE DE LA TERRE,
DANS LE SYSTÊME NEWTONIEN.
Par M. D'ALEMBERT, des Académies Royales des Sciences de Paris & de Berlin, & de la Société Royale de Londres.

A PARIS,
Chez DAVID l'aîné, Libraire, rue S. Jacques, à la Plume d'or.
MDCCXLIX.
AVEC APPROBATION ET PRIVILEGE DU ROI.

Ex Libris
Card.
Flavii Chisii
jun.

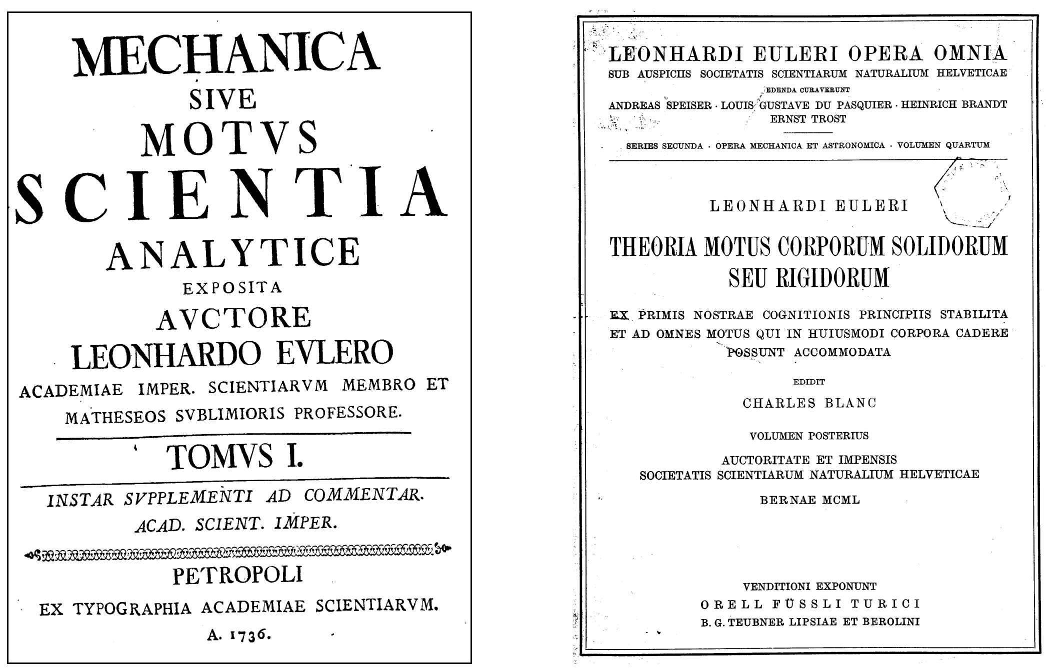
a)



b)

Fig. 6 The work by Jean Baptiste D'Alembert: a) the title page; b) a drawing for the study of rotation.

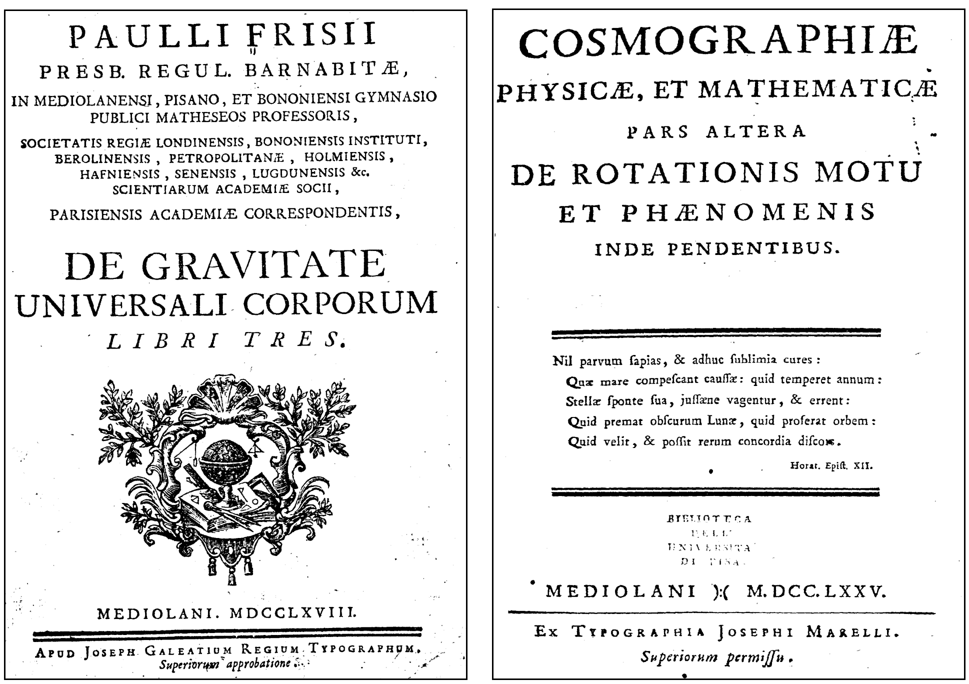
D'Alembert approached the problem of the motion of rotation in several papers and works during many years, but first in the reference (D'Alembert 1749) he defined the instantaneous axis of rotation. The proof is based on a geometric description and calculation. An example of this approach is sketched in Fig. 6b), which is a scheme in (D'Alembert 1749) used to argue on the characteristic of a rotation.
Later D'Alembert dealt with the dynamical aspects of the motion and in his masterpiece (D'Alembert 1796), first published in 1743, he introduced and used the inertia actions as forces acting on a rigid body: he defined the Principle of D'Alembert. But in this case he did not focused on the kinematical aspects of the motion. In fact, in the Traité (1796) D'Alembert approached dynamical problems so that they could be treated as problems of equilibrium and therefore solved likewise problems of Statics.
In (D'Alembert 1749) D'Alembert analysed the motion of the moon and calculated the equinoxes and nutation motions by using his Principle for the first time.
In the same period Euler approached the problem of deriving the equations of motion and made several attempts with several papers before he wrote his masterpiece (Euler 1760), Fig.7b), in which he formulated what now we call the Euler equations of motion for rigid bodies. In this work Euler tried to simplify the equations by referring them to the principal axes of inertia, tried to identify them, but he did not approach the problem of composition of rotations, although his formulation could be easily useful for that. Even in (Euler 1736), Fig.7a), Euler was not interested in this kinematical aspect that would had been very useful for his deduction of an easy formulation of the principal axes of inertia.


MECHANICA
SIVE
MOTVS
SCIENTIA
ANALYTICE
EXPOSITA
AVCTORE
LEONHARDO EVLERO
ACADEMIAE IMPER. SCIENTIARVM MEMBRO ET
MATHESEOS SVBLIMIORIS PROFESSORE.
TOMVS I.
INSTAR SVPPLEMENTI AD COMMENTAR.
ACAD. SCIENT. IMPER.
PETROPOLI
EX TYPOGRAPHIA ACADEMIAE SCIENTIARVM.
A. 1736.

LEONHARDI EULERI OPERA OMNIA
SUB AUSPICIIS SOCIETATIS SCIENTIARUM NATURALIUM HELVETICAE
EDENDA CURAVERUNT
ANDREAS SPEISER · LOUIS GUSTAVE DU PASQUIER · HEINRICH BRANDT
ERNST TROST
SERIES SECUNDA · OPERA MECHANICA ET ASTRONOMICA · VOLUMEN QUARTUM
LEONHARDI EULERI
THEORIA MOTUS CORPORUM SOLIDORUM
SEU RIGIDORUM
EX PRIMIS NOSTRAE COGNITIONIS PRINCIPIIS STABILITA
ET AD OMNES MOTUS QUI IN HUIUSMODI CORPORA CADERE
POSSUNT ACCOMMODATA
EDIDIT
CHARLES BLANC
VOLUMEN POSTERIUS
AUCTORITATE ET IMPENSIS
SOCIETATIS SCIENTIARUM NATURALIUM HELVETICAE
BERNAE MCML
VENDITIONI EXPONUNT
ORELL FÜSSLI TURICI
B. G. TEUBNER LIPSIAE ET BEROLINI

Fig. 7 The title pages of the works by Leonhard Euler.

Indeed, the work by Euler was a great improvement in the direction of a modern approach to Dynamics, since he formulated with mathematical expressions what in the past (and still in his time) was treated by means of geometrical arguments with the Galilean tradition. In (Euler 1736) Euler studied the Dynamics of a point as based on the impulse of a force and the subject is developed by means of the mathematical formulation of the several problems concerning with the motion of a point, considered free or constrained, along a curve or a surface. Successively, in 1760 Euler approached the Dynamics of rigid bodies in his masterpiece (Euler 1760), as already mentioned.
Frisi studied the motion of rigid bodies in several works both to participate to Academy Prizes (that he won in several occasions, although his approaches were discussed with criticism by D'Alembert, Euler and Lagrange yet) and investigate on the motion of the moon and planets since the interest of that time. The main work by Frisi can be considered the reference (Frisi 1775), Fig. 8b), in which he revised and grouped all his previous works concerning with the motion of plants and their effects in a comprehensive form and formulation. Thus, one can find problems and treatments that were already published in (Frisi 1768), Fig.8 a), and even in (Frisi 1759).


PAULLI FRISII
PRESB. REGUL. BARNABITÆ,
IN MEDIOLANENSI, PISANO, ET BONONIENSI GYMNASIO PUBLICI MATHESEOS PROFESSORIS,
SOCIETATIS REGIÆ LONDINENSIS, BONONIENSIS INSTITUTI, BEROLINENSIS, PETROPOLITANÆ, HOLMIENSIS, HAFNIENSIS, SENENSIS, LUGDUNENSIS &c. SCIENTIARUM ACADEMIÆ SOCII,
PARISIENSIS ACADEMIÆ CORRESPONDENTIS,
DE GRAVITATE UNIVERSALI CORPORUM
LIBRI TRES.
MEDIOLANI. MDCCLXVIII.
APUD JOSEPH GALEATIUM REGIUM TYPOGRAPHUM.
Superiorum approbatione.

COSMOGRAPHIÆ
PHYSICÆ, ET MATHEMATICÆ
PARS ALTERA
DE ROTATIONIS MOTU ET PHÆNOMENIS INDE PENDENTIBUS.

Nil parvum sapias, & adhuc sublimia cures:
Quæ mare compescant caussæ: quid temperet annum:
Stellæ sponte sua, jussæne vagentur, & errent:
Quid premat obscurum Lunæ, quid proferat orbem:
Quid velit, & possit rerum concordia discors.
Horat. Epist. XII.

MEDIOLANI):(M.DCC.LXXV.
EX TYPOGRAPHIA JOSEPHI MARELLI.
Superiorum permissu.

a) b)

Fig. 8 The title pages of the works by Paolo Frisi: a) De Gravitate in 1768; b) Cosmographiae in 1775.

Indeed in his work (Frisi 1759) one can find the first original rigorous statement and proof of the composition of rotations, much before the attempts by Euler and Lagrange as Marcolongo recognised in (Marcolongo 1906).
The theorem of he composition of rotations is declared in the form of Corollaries II and III on page 35 in (Frisi 1768) and as Theorem IV with its proof and significant Corollaries on pages 31 and 32 in (Frisi 1775).
However, the original contribution can be found in “Problematum praecessionis aequinoctiorum nutationis terrestris axis, aliarumque vicissitudinum diurni motus geometrica solutio, cius specimen a Regia Berolinensi Scientiarum Academiae anno 1756 praemium obtinuit”, (Fris 1759), where Fris stated: “in quocumque binos motus totations in motum unum componi, eadem prorsus atione, qua duae vires duobus lateribus parallelogrammi alicuius expressae tertiam vim componunt, quae diagonali exprimitur”.
In the following we have outlined the procedure that Frisi used to solve the composition of instantaneous rotations about intersecting axes.
The velocity of a point rotating about an axis is obtained as the sum of the velocities of points, which are the projection points of the given point onto two axes that are orthogonal to each other and the given axis. Consequently (i.e. it is a corollary in Frisi’s work), when a rigid body rotates about two axes, the velocity of a point that does not lye on the plane identified by the two given axes, can be decomposed in four components. Two components are the velocity of the projection points onto the plane; the other two components are orthogonal to the given axes but in the plane. Thus, it is possible to determine an axis in the plane so that the sine functions of the angles between the given axes and the resultant axis are inversely proportional to the corresponding velocity. The points of the resultant axis have no velocity. Thus, this axis is the axis of the resultant rotation. In addition, Frisi proved with geometrical arguments that the velocity of any point of the plane or any point of the body containing the plane is orthogonal to the plane containing the two axes. Moreover, the velocity magnitude of a point is proportional to the distance of the point from the resultant axis. In conclusion, the resultant motion of the two rotations about the two axes can be considered a rotation about the resultant axis of the rotations.
This proof is given in (Frisi 1759) but is repeated more clearly in (Frisi 1768) and is based on the schemes, which have been reported in Fig. 8c).
Finally, it seems that Frisi understood also the generation of the helicoidal motion when in (Frisi 1768) at Propositio XXI he stated : “... corpus duplicem motum concipiet projectionis, et rotationis circa axem per G transeuntem, et eidem plano perpendicularem”.
Frisi wrote several times about the theorem of composition of rotations and his works circulated for long time in Europe. He won also sevral Academy Prizes on subjects regarding Mechanics, and particularly his specific work “Problematun praecessionis ...” (Frisi 1759) got the Prize of the Berlin Academy in 1756 (as decalred in the title as

above-mentioned). Therefore, it is quite curious that the theorem is not recognised to Frisi but to Euler. This is probably due to the fact that Euler achieved greatest results in Dynamics of rigid bodies, particularly concerning with the motion of rotation so that everything developed in his time was ascribed to him by posterity.

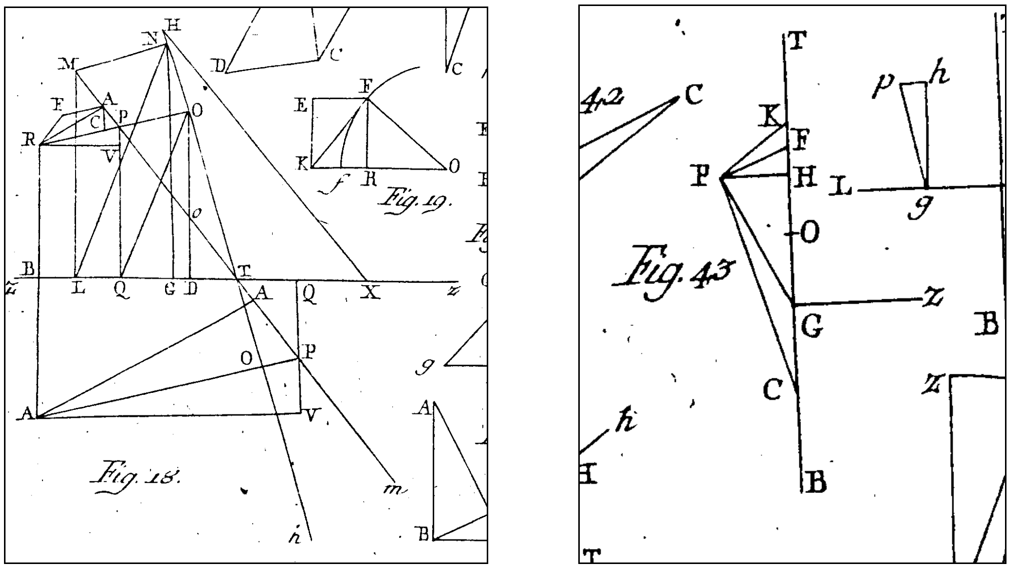
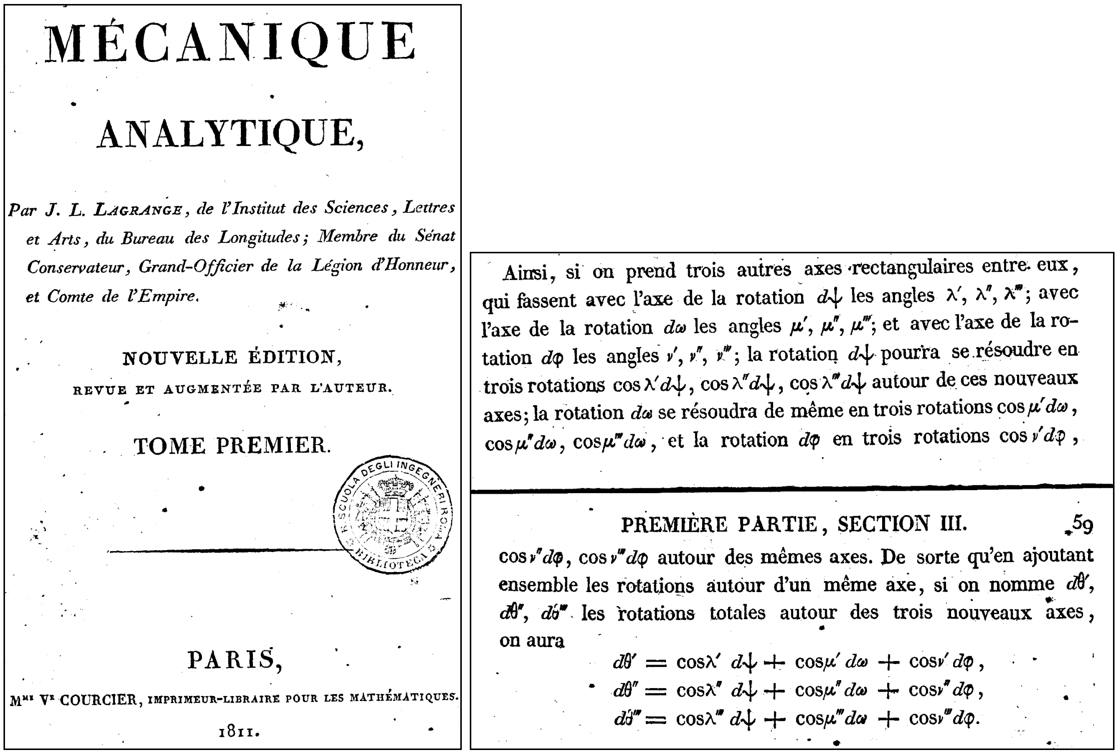


Fig. 8c) Drawings of Fig.18,19,and 20 used by Frisi in (Frisi 1768) to prove the composition of rotations about intersecting axes.

The mathematical evolution of dynamical studies started by Euler was completed by Lagrange in the work (Lagrange 1811), whose first edition was published in 1788. In his masterpiece Lagrange proposed a unified approach, which is based on sevral concepts developed since the time of Galilei, but also gave a general mathemtical procedure useful to solve problems of Mechanics. Thus, in the Preface he declared clearly the aim of the work being directed to formulate and solve equations and he stressed that “no figures have been drawn since the proposed methods are based on algebraic equations only....”. Indeed the work by Lagrange makes valuable fusion of the work by Euler and D’Alembert with the fundamentals of Mechanics, and it is an original contribution with a novel formulation, which is now well-known as Lagrangian formulation.
The contribution of Lagrange to the specific subject of the composition of rotations can be recognised in a clear statement and formulation in the paragraph of his work (Lagrange 1811) from page 57 to 62. Particularly, he solved mathematically the composition and decomposition of rotations in a brilliant and synthetic formulation, which has been reported form the original text in Fig.9b). This formulation can be considered a conclusion of the historical evolution of the solving the composition of rotations of rigid bodies.



MÉCANIQUE

ANALYTIQUE,

Par J. L. Lagrange, de l'Institut des Sciences, Lettres et Arts, du Bureau des Longitudes; Membre du Sénat Conservateur, Grand-Officier de la Légion d'Honneur, et Comte de l'Empire.

NOUVELLE ÉDITION,

REVUE ET AUGMENTÉE PAR L'AUTEUR.

TOME PREMIER.

PARIS,

Mme Ve COURCIER, IMPRIMEUR-LIBRAIRE POUR LES MATHÉMATIQUES.

1811.

Ainsi, si on prend trois autres axes rectangulaires entre eux, qui fassent avec l'axe de la rotation $d\psi$ les angles λ', λ'', λ'''; avec l'axe de la rotation $d\omega$ les angles μ', μ'', μ'''; et avec l'axe de la rotation $d\varphi$ les angles ν', ν'', ν'''; la rotation $d\psi$ pourra se résoudre en trois rotations $\cos\lambda' d\psi$, $\cos\lambda'' d\psi$, $\cos\lambda''' d\psi$ autour de ces nouveaux axes; la rotation $d\omega$ se résoudra de même en trois rotations $\cos\mu' d\omega$, $\cos\mu'' d\omega$, $\cos\mu''' d\omega$, et la rotation $d\varphi$ en trois rotations $\cos\nu' d\varphi$,

PREMIÈRE PARTIE, SECTION III. 59

$\cos\nu'' d\varphi$, $\cos\nu''' d\varphi$ autour des mêmes axes. De sorte qu'en ajoutant ensemble les rotations autour d'un même axe, si on nomme $d\theta'$, $d\theta''$, $d\theta'''$ les rotations totales autour des trois nouveaux axes, on aura

$$d\theta' = \cos\lambda'\, d\psi + \cos\mu'\, d\omega + \cos\nu'\, d\varphi,$$
$$d\theta'' = \cos\lambda''\, d\psi + \cos\mu''\, d\omega + \cos\nu''\, d\varphi,$$
$$d\theta''' = \cos\lambda'''\, d\psi + \cos\mu'''\, d\omega + \cos\nu'''\, d\varphi.$$

a) b)

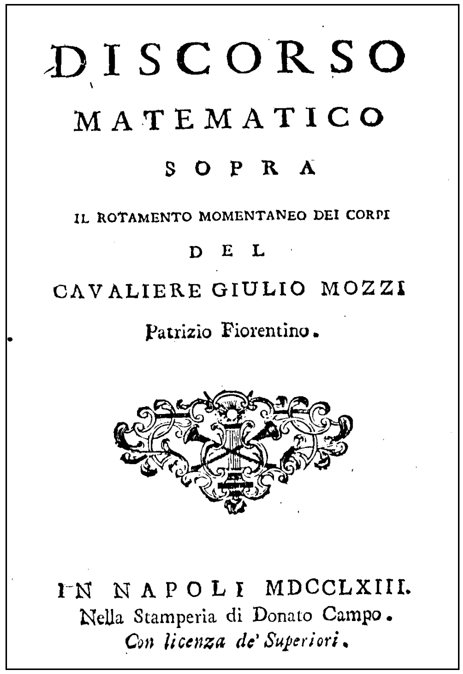
Fig. 9 The work by Lagrange a) the title page; b) the composition of rotations.

4. FIRST FORMULATION OF SCREW THEORY BY GIULIO MOZZI IN 1763

The Treatise by Giulio Mozzi (1763), Fig.10a), can be considered of great importance not only for its fundamental kinematic contribution, but also from the Mechanics viewpoint as discussed in (Ceccarelli 2000). In fact, the Treatise approaches Statics and Dynamics of rigid bodies using impulsive forces to compute consequent motions. In addition, the Treatise deals with several cases of determination of the helicoidal motion for given forces and viceversa.

The screw axis is introduced by Giulio Mozzi in his Treatise on page 5 in Corollario IV, (Mozzi 1763): "Therefore, you can deduce that the above mentioned motions become two others. A first one is linear and common to all the points of a body; it is parallel to the axis of rotation, which crosses the centre of gravity. The latter is a rotational motion whose axis of rotation is parallel to the above-mentioned axis". At the end of the demonstration the screw axis is defined as "asse spontaneo di rotazione" (spontaneous axis of rotation). The demonstration proves the existence of the screw axis by means of a descriptive geometric reasoning based on Fig. 10 b).

The screw axis can be also called Mozzi's axis because of this geometrical definition, following the Italian tradition.



DISCORSO

MATEMATICO

SOPRA

IL ROTAMENTO MOMENTANEO DEI CORPI

DEL

CAVALIERE GIULIO MOZZI

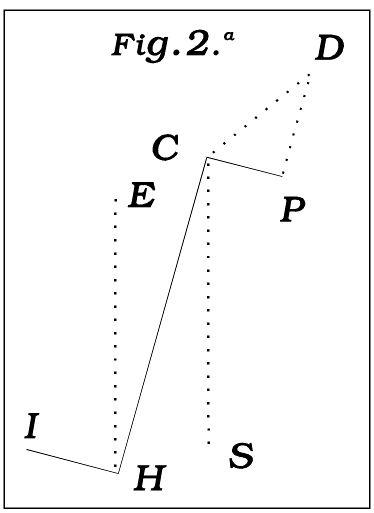
Patrizio Fiorentino.

IN NAPOLI MDCCLXIII.

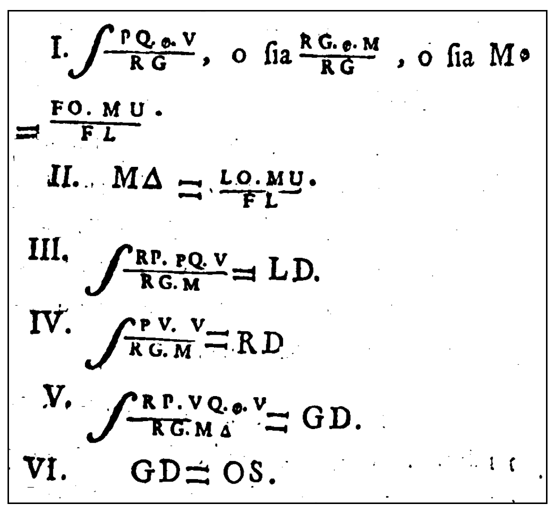
Nella Stamperia di Donato Campo.

Con licenza de' Superiori.

a)



b)



I. $\int \frac{PQ.\varphi.V}{RG}$, o sia $\frac{RG.\varphi.M}{RG}$, o sia $M\varphi = \frac{FO.MU}{FL}$.

II. $M\Delta = \frac{LO.MU}{FL}$.

III. $\int \frac{RP.PQ.V}{RG.M} = LD$.

IV. $\int \frac{PV.V}{RG.M} = RD$

V. $\int \frac{RP.VQ.\varphi.V}{RG.M\Delta} = GD$.

VI. $GD = OS$.

c)

Fig. 10 The work by Giulio Mozzi: a) the title page of the Treatise published in 1763; b) the drawing defining the Screw Axis as the HE line; c) equations of motion for a general problem on page 30.

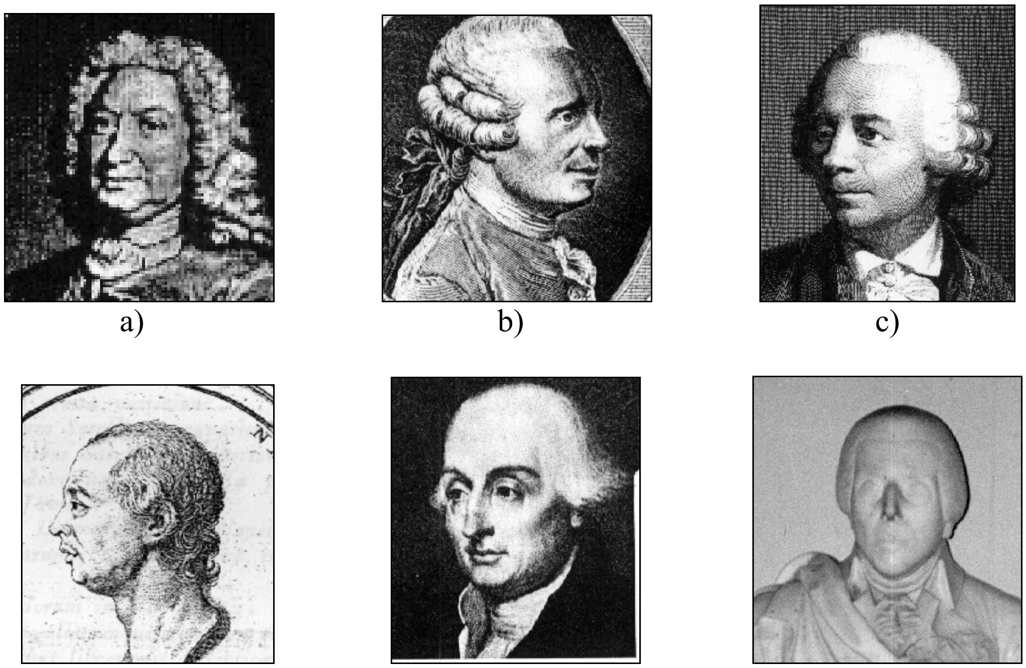
In addition, a very first concept of a force couple was introduced and has been used throughout the Treatise, as it is the case for Lemmas II and III on pages 6 and 12 or Corollario VII on pages 10 and 11.

Moreover, Giulio Mozzi approached the problem to solve a given system of forces as an equivalent set of two forces, whose the first is orthogonal to a given plane and the second is parallel to the plane yet. Then, a general problem is formulated in Problema 1° on page 22 for the determination of the instantaneous helicoidal motion of a rigid body when an impulsive force is given. Thus, dynamic equations are formulated by expressions in page 30 in the form of the Theorem of Impulsive Forces for a given force acting on the body as reported in Fig.10 c). The consequent instantaneous helicoidal motion is determined.
Mozzi discussed the obtained formulas through some examples, also with the aim to illustrate the kinematic and dynamical feasibility of the concept of instantaneous helicoidal motion. He mentioned the works by Bernoulli (1742), Euler (1736) and D'Alembert (1749) to review the recent interest on the topic and acknowledge advances due to P. Frisi (1728-1784) and T. Perelli (1704-1783). Mozzi proposed a generalisation of the results obtained for two forces and proposed an equivalence of a system of applied forces to two suitable forces, which are normal to each other. The result had been verified in the Treatise by computing instantaneous motion for each force and resolving all motions into a unique helicoidal motion. Then, an inverse problem was formulated to compute a force that must act on the rigid body to give a prescribed motion or to make a change from a motion to another one (Problems VI and VIII, from page 50 to 59). Mozzi approached also the problem for the case of constrained rigid bodies. In the last part of the Treatise, the specific cases have been discussed with a fixed point or a fixed axis of the body, or a fixed plane in contact with the body. It is also of interest the very first use of the D'Alembert Principle in terms of instantaneous forces (Lemma VII, page 64). Mozzi used the Principle to determine the instantaneous motion that can be associated with a screw axis in some cases.

5. CONCLUSIONS

An original contribution of this paper from historical viewpoint consists of having recognised that D'Alembert individuated rigorously the instantaneous axis of rotation in 1749; Frisi proved the theorem of composition of instantaneous rotations in 1759; Euler formulated the motion of rotation in a modern form in 1765; Lagrange treated the motion of rotation with mathematical formulation in 1788. These works gave great impulse on the study of general motion of rigid bodies that brought to the fundamental work by Mozzi in 1763 and later the well known contributions on the Screw Theory that has been finally established by Ball in a modern treatment in 1876.

6. BIOGRAPHICAL NOTES



a) b) c) d) e) f)

Fig.11 a) Jean Bernoulli (1667-1748); b) Jean Baptiste D'Alembert (1717-1783); c) Leonhard Euler (1707-1783); d) Paolo Frisi (1728-1784); e) Giuseppe Luigi Lagrange (1736-1813); f) Giulio Mozzi (1730-1813).

6.1 Jean Bernoulli

Jean Bernoulli was born in Basilea on July 27th 1667 and there he died in 1748. His older brother James I (1645-1705) introduced him to Mathematics. However, he studied also Medicine obtaining relevant results on the Mechanics of muscles. Indeed Jean Bernoulli approached several disciplines with great results giving him great repute. In particular Jean Bernoulli studied Mechanics and Mathematics in which he reached highs by applying the infinitesimal calculus introduced by Leibniz. In 1695 he became professor of Mathematics in Groninga and then in 1705 in Basilea, after the death of his brother James I. He was elected member of many European Academies and he was particularly active in the Academie de Sciences of Paris.

6.2 Jean Baptiste D'Alembert

Jean Baptiste D'Alembert was born in Paris on November 16th 1717 and there he died on October 28 th 1783. Just born he was left in front of the door of the St. Jean Le Rond Church, from which he had his name. Nevertheless, the father was always interested of him giving a salary. In 1737 he wrote a paper on the integral calculus, because of which he was accepted as a member in th Academié des Sciences of Paris. Although he was invited everywhere in Europe he never left Paris, where he also collaborated with Denis Diderot on the Encyclopédie.

6.3 Leonhard Euler

Leonhard Euler was born in Basilea on April 15th 1707 and died in St Petersburg on September 7th 1783. His education was due mainly to the Bernoulli family since he was a pupil of Johannis and then Daniel and Nicolas. From the last two he was invited to join them in St Petersburg at the Russian Academy of Sciences because of the interest of Caterina I. There he obtained the chair of professor and in 1733 he succeeded to Daniel Bernoulli. In 1741 he was invited in Berlin and became Director of the Mathematical Section of the Berlin Academy of Sciences. In 1766 Euler came back to St Petersburg where he lived until the death.

6.4 Paolo Frisi

Paolo Frisi was born in Milan, Italy, on April 13th 1728 and there he died on November 12th 1784. He was a Barnabita priest and he started by teaching Philosophy in Lodi, near Milan. He got the position of Professor at the University of Pisa first in "Physics and Etics" and later, from 1755 to 1764, in "Arithmetics and Algebra". Successively he went back to Milan to teach Mathematics. Frisi was quite famous for his works on Astronomy and Mathematics, being a member of several European Academies. According to Marcolongo, he is believed to have first formulated correctly the composition of instantaneous rotations in the works (Cosmografia [A.6, A.7]. Moreover, he was held in great repute also in practical Hydraulic Engineering since he carried out a consultant activity on the design and construction of several channels of rivers.

6.5 Giuseppe Luigi Lagrange

Giuseppe Luigi Lagrange was born in Turin on January 25th 1736 and he died in Paris on April 10th 1813. In 1755 he became professor at the Royal School of Artillery in Turin, where he with others founded a scientific society that later became the Italian Royal Academy of Sciences. His studies were appreciated in several European Academies and in 1766 he was appointed as Director of the Berlin Academy of Sciences. He lived in Berlin having a very fecund scientific activity until 1787 when he moved to Paris as invited members of the Academie des Sciences. His great reputation maintained him away from the turbulence of the French Revolution and even gave him the position of professor at the Ecole Normale and Ecole Polytechnique.

6.6 Giulio Mozzi

Giulio Giuseppe Mozzi was born in Florence on February 23rd 1730 within aristocratic family Mozzi del Garbo and there he died on April 16th 1813. He received a humanistic education but later he was attracted to "mathematical studies" and joined Paolo Frisi at the University of Pisa. In 1754 Giulio Mozzi was elected member of the “Accademia della Crusca” and was so active that in 1784 and again in 1808 he became President of the new “Accademia Fiorentina” that combined together “Accademia della Crusca” and the “Accademia Fiorentina” since 1783. Although during the political changes in 1799 he suffered some misfortune, his personality was recognised when he was nominated senator in 1785 and then in 1801 Minister for Foreign Affairs during the Kingdom of Etruria with Ludovico I di Borbone. Successively Maria Teresa d’Austria confirmed him in this charge until 1807. He was also elected by Napoleon in 1812 as a member of the re-established “Accademia della Crusca” and received the "Gran Croce della Riunione".

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