FRAGILITY ASSESSMENT
THEORY AND TEST PROCEDURE

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A. INTRODUCTION

Packaging for the purpose of limiting shipping damage has long been an engineering problem of economic importance. Beginning with the work of Mindlin[1*] in 1945, much progress has been made. The shipping environment has been studied in detail[2,3,4]. Testing techniques for assessing the dynamic behavior of cushion materials have been standardized[5]. Based upon these extensive efforts, a procedure for designing and testing protective packaging has been developed[5,6].

B. DESIGN PROCEDURE

This report examines one phase of the procedure for designing to prevent shock damage, namely, the assessment of the "fragility" of the item to be packaged. For the purpose it is pertinent to review, in outline, the standard procedure for design. The fundamental questions are:

1. What is the severest shock likely to be encountered in shipping?
2. What is the maximum shock that the item to be packaged can tolerate?
3. What cushioning is required?

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* Numerical superscripts refer to entries in the Bibliography Section F.

1. Shipping Shocks.

It is generally agreed that, regardless of the transportation mode, the severest shocks likely to be encountered in shipping result from handling operations[4,7]. These shocks result from dropping the package onto a floor, dock, or platform. The height and kind (e.g., flat drop or edge drop) of drop likely to be encountered has been found to vary with the weight, size and shape of the package. Despite the complexities of actual shipping environments, it is assumed for purposes of package design, that the severest shock to be expected is that resulting when the package is dropped from a known height to land flat on a non-resilient horizontal surface.

2. Item Fragility.

The nature of damage clearly depends upon the item to be shipped. For an egg, it is breakage. For electronic equipment, excessive deformation might induce an electrical "short". In any event, damage results from excessive internal stress which is induced by inertia forces. Since inertia forces are directly proportional to acceleration, fragility is characterized by the maximum tolerable acceleration. The package designer must know what this acceleration level is[8].

3. Cushion Performance.

Extensive test data have been accumulated to predict the dynamic performance of commonly used cushion materials in drop tests[5, 9, 10, 11, 12]. These data are available in the form of curves showing maximum accelerations (in g's) versus static stress (item weight divided by cushion area). There is a separate curve for each cushion thickness. For a given material a separate set of curves is needed for each drop height.

The designer, equipped with such cushion performance data, may readily select a number of combinations which will keep the transmitted acceleration below the fragility limit. The design may be optimized by selecting the acceptable combination having the lowest overall cost[5].

4. Shortcomings.

The outlined procedure has many shortcomings, as those who have devised it and those who use it are well aware. For example, handling may be expected to involve a number of drops of varying kind and severity. Both the package and the item itself may suffer cumulative damage in the process. A rotational edge drop will subject different parts of a large packaged item to accelerations which differ markedly in both magnitude and direction. Cushion tests performed under standard conditions of temperature and humidity may not predict adequately the performance under the environmental extremes encountered in shipping. Also, cushion test data do not include the effects of cushion shape or the performance changes that will result from confinement which restricts or prevents lateral expansion[13].

Although it is relatively easy to discover such shortcomings as those just described, practical remedies are not readily found. For this reason further consideration is limited to the problem of fragility assessment.

C. FRAGILITY ASSESSMENT - THEORY

1. Shock Transmission.

Consider first the process by which the effect of the abrupt deceleration of the outer package at the termination of a drop is communicated to the packaged item. The nearly instantaneous velocity change which takes place at the outer surface of the package upon striking the floor is accompanied by local accelerations of many thousands of g's. The compliance of the outer package, the cushioning material, and the inner package (if any) transforms the pulse delivered to the packaged item so that the maximum acceleration is greatly reduced and the time required to attain this maximum is many times as long. The situation is represented qualitatively in Figure 1. The maximum cushion deformation ia assumed to occur at B. The corresponding ordinate BM is generally close to the maximum for the packaged item. The shaded areas under the two curves must be substantially equal, since each of these areas corresponds to the striking velocity*. Because the cushioning material exhibits some elastic recovery, upward acceleration of the packaged item continues until point C is reached.

Some additional oscillation of the packaged item will generally occur, but the accompanying accelerations are generally quite small compared with the first maximum.

A useful simplification for analysis and testing results from assuming that the damaging effects result solely from that portion of the curve between A and C. Thus the input motion is simplified as a single acceleration pulse. The shaded area, A to B, is equal to the striking velocity. The unshaded area, B to C, is equal to the rebound velocity. The ratio of rebound velocity to striking velocity is called the coefficient of restitution e. Energy considerations establish that e must lie between 0 (fully plastic impact, no rebound) and 1 (fully elastic impact).

2. Factors Determining Damage.

Usual practice in package design assumes that the maximum acceleration A_p alone measures the severity of shock. If

* There is an area difference, usually negligible, because the packaged item continues to accelerate downward until the cushion force exceeds the item weight.

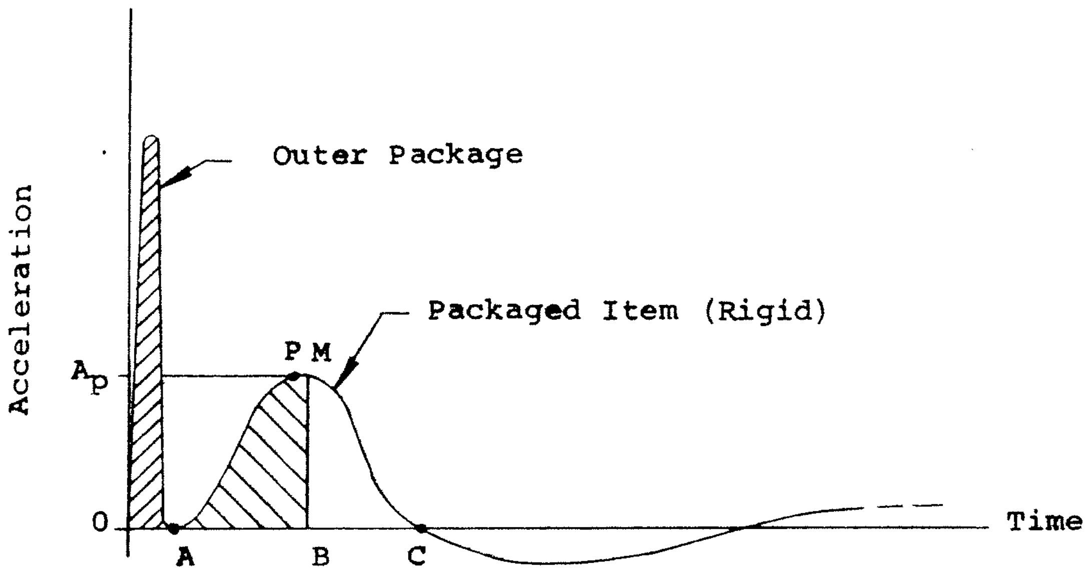

Figure 1. Drop Test Accelerations

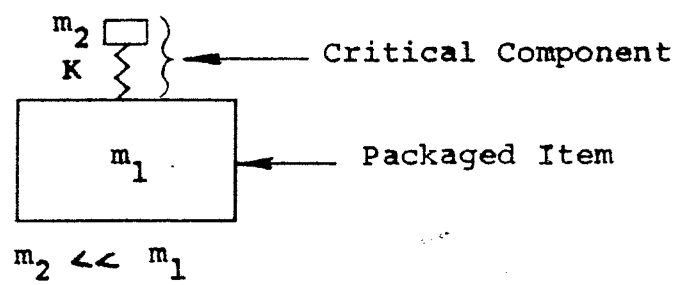

Figure 2. Mathematical Model for Packaged Item

this is less than the rated fragility of the item, a margin of safety against damage is assumed to exist. This method neglects the influence of item flexibility upon the damaging effects of the shock. Because any real item has distributed mass and flexibility, it will undergo elastic (and possibly inelastic) deformations during a shock. Correspondingly, maximum accelerations will not be the same throughout the packaged item.

Itis readily apparent that a fully rational analysis of the response motion of an actual packaged item is not practical. There is available, however, a simplified mathematical model which affords a substantial improvement over the rigid model without introducing unmanageable complexity[1]. This model is the basis for the widely-used shock spectrum. Briefly, the model for the packaged item consists of a rigid mass m_1 to which a second mass m_2 is attached by a spring of stiffness K (see Figure 2). The rigid mass m_1 is assumed to represent the bulk of the item. The small mass m_2 and spring K represent a critical component and its stiffness (flexibility). The fragility of this model is characterized by the maximum allowable acceleration of the critical component (acceleration of m_2). Because m_2 is much smaller than m_1, its effect on the motion of m_1 may be neglected. Accordingly, the system is analyzed by assuming that m_1 undergoes a specified acceleration vs. time history and the resulting maximum acceleration of m_2 is determined.

3. Shock Spectrum.

Results of the foregoing analysis are usually presented as a shock spectrum[14]. For a pulse of specified shape (e.g., a half-cycle sine wave) the ratio of the peak acceleration of m_2 to the maximum acceleration of the input pulse is plotted versus the product of pulse duration by the natural frequency of the critical component. Such a curve is shown in

Figure 3. Symbols used are defined as follows:

A_p = peak acceleration of input pulse

A_c = peak acceleration of critical component

τ_e = effective pulse duration = $\frac{\text{velocity change}}{A_p}$

f_c = natural frequency of critical component

$$= \frac{1}{2\pi} \sqrt{\frac{K}{m_2}}$$

Examination of Figure 3 discloses that, for a given shock (specified τ_e and A_p), the peak acceleration of the critical component depends strongly upon component frequency. In particular, if $f_c \tau_e < \frac{1}{6}$ the component peak acceleration depends only on the velocity change $V = A_p \tau_e$ of the pulse. Such a low frequency component is, in a sense, its own shock isolator and it benefits little, if at all, from the cushioning. On the other hand, for components with $f_c \tau_e > \frac{1}{6}$, the component peak acceleration exceeds that of the pulse. The ratio may vary from 1 to about 1.8, depending on component frequency.

To make rational use of the shock spectrum the designer needs much information not usually available. For the equipment he needs to know:

A_{cs} = maximum safe peak acceleration of critical component;

f_c = natural frequency of critical component.

It should be observed that there may be many fragile components, each of which might be critical for a particular pulse. To verify the adequacy of a particular design the following are needed:

A_p = peak acceleration transmitted to item;

τ_e = effective duration of acceleration pulse.

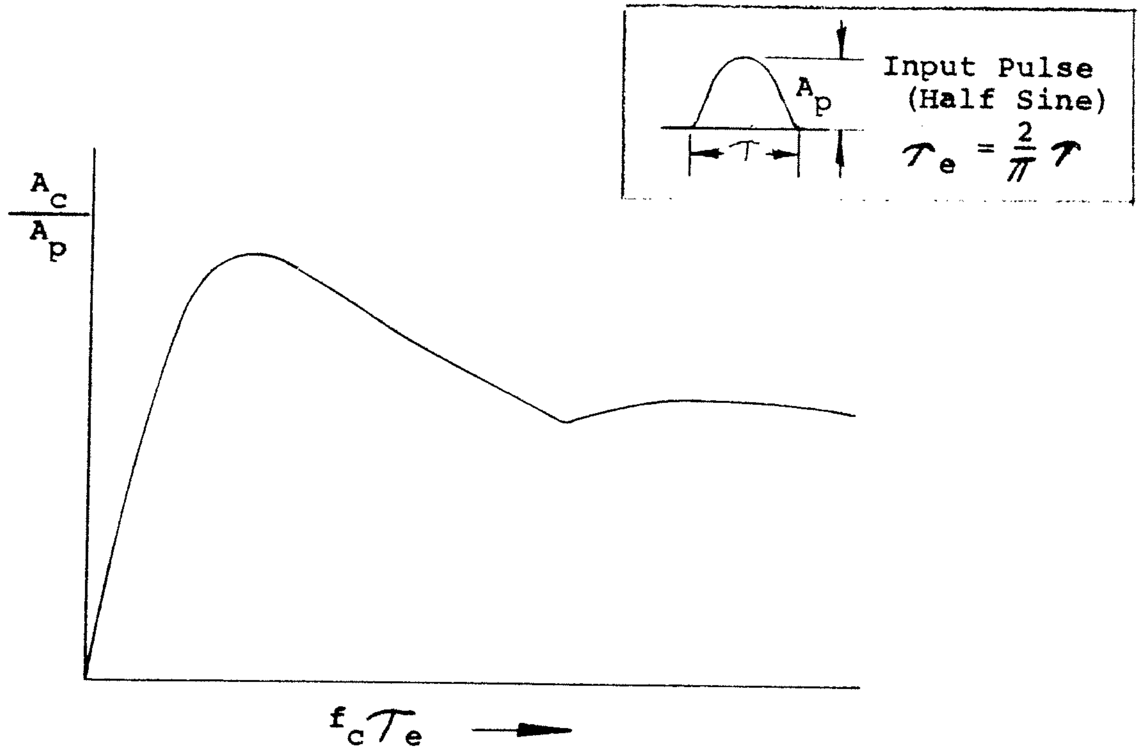

Figure 3. Shock Spectrum, Half Sine Pulse

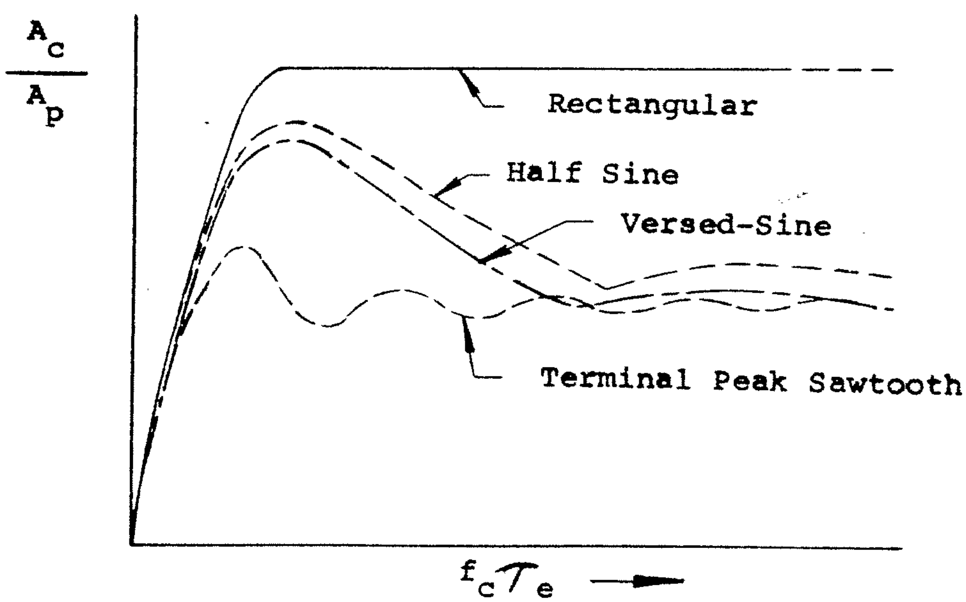

Figure 4. Shock Spectra, Various Pulses

If the acceleration pulse were always a half-sine, the above information would suffice. Unfortunately, this is not the case and the shock spectrum is sensitive to pulse shape. In Figure 4 the shock spectrum curves for four different acceleration pulses are shown. Examination discloses that the maximum component acceleration may range from about 0.9 A_p to 2.0 A_p for shocks of different shapes, even though τ_e and A_p are fixed.

A realistic appraisal of the data requirements makes evident the impracticality of using a fully rational version of shock spectrum analysis for routine package design. A particular item, to be shipped in large numbers in individual packages, might easily warrant the tests needed to exploit fully the shock spectrum approach. For routine use a simpler, and somewhat less accurate, procedure is needed. Specifically, it is proposed that fragility tests be based on shock spectra, but that cushion selection may continue to be based on drop height, static stress, and peak acceleration. Details are given in the following section.

D. FRAGILITY ASSESSMENT - TESTING METHOD

Comparison of the shock spectrum curves for various pulse shapes (Figure 4) reveals that the rectangular pulse curve provides an upper bound. Thus, if an item is subjected to a shock pulse of given peak acceleration A_p and effective duration τ_e the peak acceleration of any component will not exceed that which would result from a rectangular pulse having the same A_p and τ_e. Accordingly, it is proposed that item fragility tests be performed using rectangular pulses. The only complication in this procedure is that the effective duration τ_e to be used is not initially known.

In order to determine an appropriate pulse duration for tests it is pertinent to recall that

$$V = A_p \tau_e, \qquad (1)$$

where V is the velocity change (pulse area). Thus $\tau_e = V/A_p$ and it is uniquely determined by these two parameters. The velocity change in a drop test may be expressed as

$$V = (1 + e)\sqrt{2gH} \qquad (2)$$

where e is the coefficient of restitution, g is the acceleration of gravity, and H is the drop height. Now, as has been observed earlier, e must be between 0 and 1. Accordingly, the V for a given drop height is known within the limits of uncertainty on e and

$$\sqrt{2gH} \lesssim V \lesssim 2\sqrt{2gH} \qquad (2')$$

Ignoring this uncertainty in V for an actual drop, a test procedure may be ~~used~~ based upon controlled values of V and A_p.

Using a square wave (rectangular pulse) programmer, A_p is determined by selection of pre-charge pressure and V is determined by drop height H. (Since V may be measured in such a test, the uncertainty indicated by Equation 2' does not enter here.) For a chosen V, drops are made for successively increasing peak pulse accelerations A_p. The item is inspected for damage following each drop. The maximum peak pulse acceleration A_{ps} that can be sustained without damage is taken to be the fragility measure of the item at that velocity change V.

Customary practice ignores the indicated dependence on V. It is instructive to explore the nature of this dependence. Details of the supporting analysis are given in Appendix A. Consider an item having a critical component with natural frequency f_c = 100 hz. It is assumed that the component can sustain a maximum peak acceleration A_{cs} = 100 g without damage. Figure 5 shows the manner in which A_{ps} depends upon V. For this item, a conventional fragility rating would be simply A_{ps} = 50 g, corresponding to the horizontal portion of the damage boundary for V > 96 in/sec. The

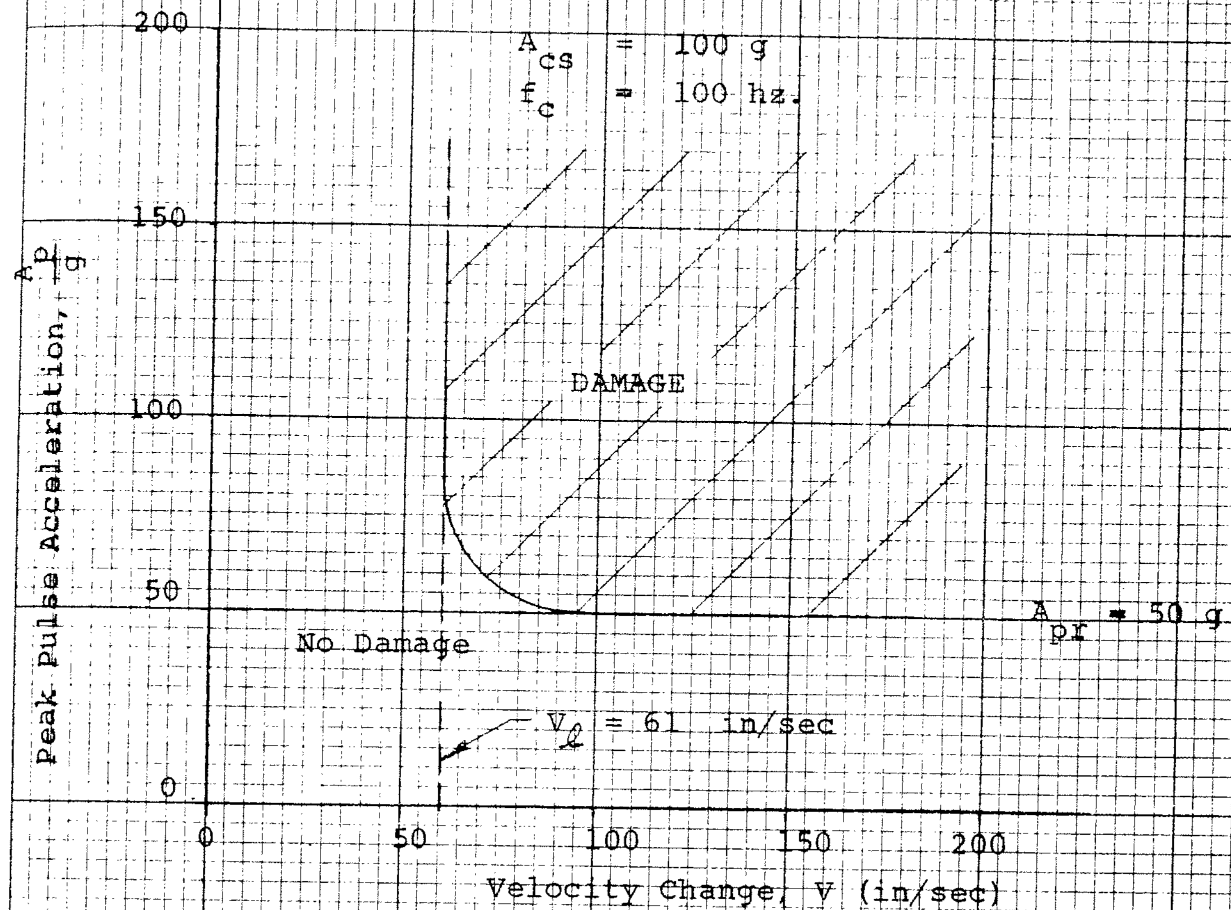

Figure 5. DAMAGE BOUNDARY

behavior below $V = V_\ell = 61$ in/sec., not considered in normal design practice, results from the fact that the pulses in this region are very short and, regardless of pulse shape, the maximum component acceleration depends solely on velocity change V.

The practical importance of this portion of the damage boundary depends upon the drop height against which protection is to be afforded. In the example of Figure 5, a velocity change of 61 in/sec corresponds to a drop height of 1.2 inches for e = 1 (see Equation 2). For e = 0 the height is 4.8 inches. If the item needs to be protected against 24 inch drops (V = 136 in/sec for e = 0), the region in question would not enter into cushion design. On the other hand, for an item having sufficiently high V_ℓ, it may occur that no cushioning is needed to prevent damage during drops from the design height.

To establish the complete damage boundary by test is straight-forward. As already described, tests at constant V and progressively increasing A_p will establish the ordinate A_{pr} for the right hand portion (bottom line) of the damage boundary. A second sequence of tests at constant A_p (= 3 A_{pr}, say) with successively increasing V will establish V_ℓ, the velocity change defining the left-hand portion of the damage boundary. It is shown in Appendix A that the transition curve is tangent to the vertical line $V = V_\ell$ at $A_p = 2 A_{pr}$ and is tangent to the horizontal line $A_p = A_{pr}$ at $V = 1.57 V_\ell$. The transition curve may be plotted from calculated points or a square corner may be substituted.

It should be noted that only a limited range of velocity changes V is of practical interest. The lower limit is found from Equation 2 using the minimum H of interest with e = 0. The upper limit is given by the maximum H and e = 1. Clearly there is no reason to conduct fragility tests outside this range of velocity change V.

Discussion in this section and Appendix A has thus far been based upon the shock spectrum of a rectangular pulse. A square wave programmer produces an actual pulse with finite rise and decay times. Such a pulse may be approximated by a symmetric trapezoid. In Figure 6 the shock spectra of a rectangular pulse and a symmetric trapezoidal pulse are compared[14]. For very large values of $f_c \tau_e$ the curve for the trapezoid approaches $A_c/A_p = 1$. The corresponding effect on the damage boundary is shown in Figure 7. Even though this modified curve lies above the rectangular pulse curve, the resulting boundary is still safe as long as the shock spectrum curve for the actual drop lies below that for the trapezoid. In practice it is reasonable to expect that experimentally derived damage boundaries will provide conservative results.

One other possible complication of the experimentally determined damage boundary should be mentioned. In Figure 8, the ideal (rectangular pulse) damage curves for two vulnerable components are superimposed. Component 1, having the lower V_ℓ, defines the extreme left-hand portion of the combined damage region. Component 2, having the lower A_{pr}, defines the lowest portion of the boundary. It is believed that such a situation would rarely occur in the velocity change range of interest. If it should occur, it can be recognized because the damage resulting from failure of one component would be distinguishable from that resulting from failure of a different component. Details of the corresponding program of fragility tests are omitted here, but it is evident that, by conducting a larger number of tests, the damage boundary may still be adequately defined.

E. CONCLUSIONS

The rationale and customary technique of a package design have been examined. On the basis of shock spectra, a test procedure for determining item fragility has been proposed. It is believed that this procedure, employing machine drop tests with a square wave programmer, affords a rational and economical basis for fragility assessment.

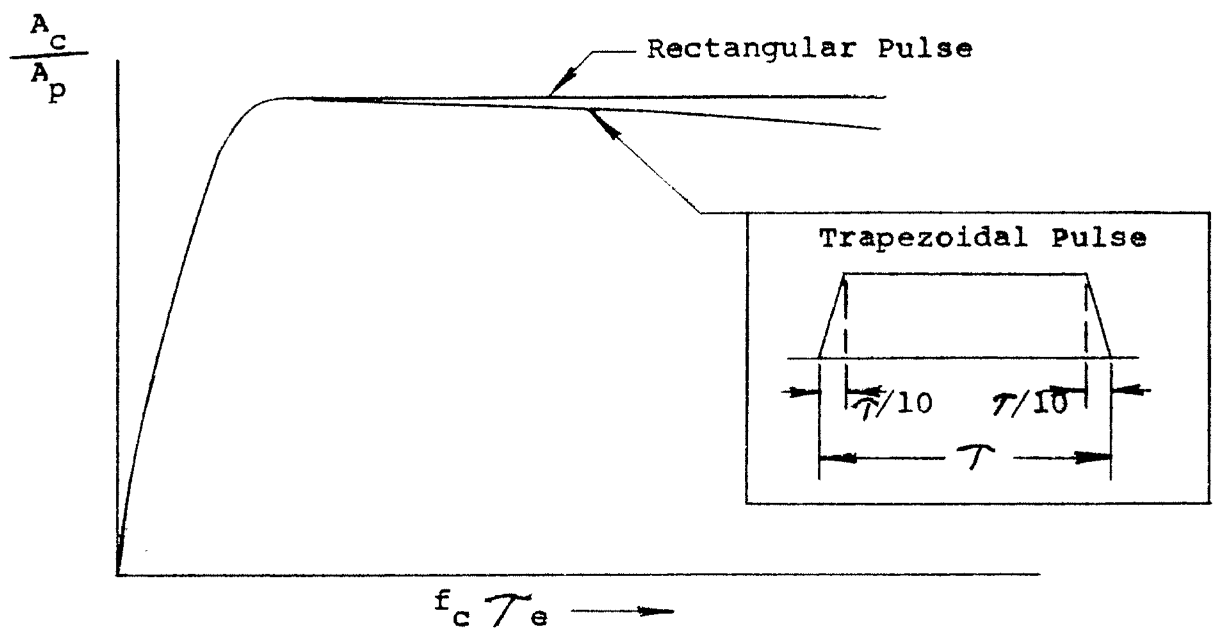

Figure 6. Shock Spectra, Rectangular and Trapezoidal Pulses

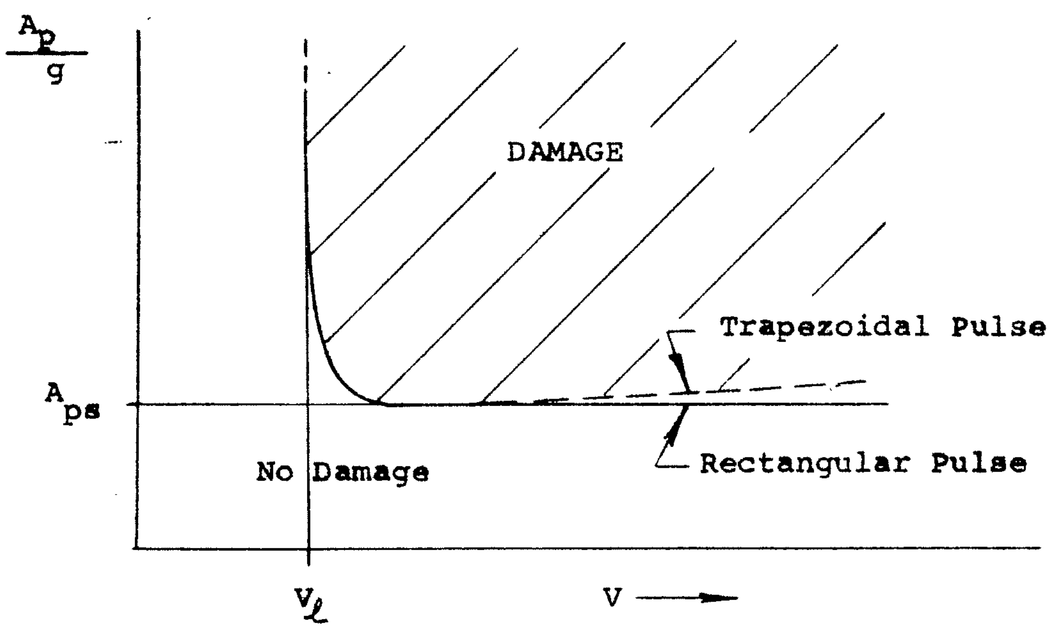

Figure 7. Effect of Pulse Shape on Damage Boundary

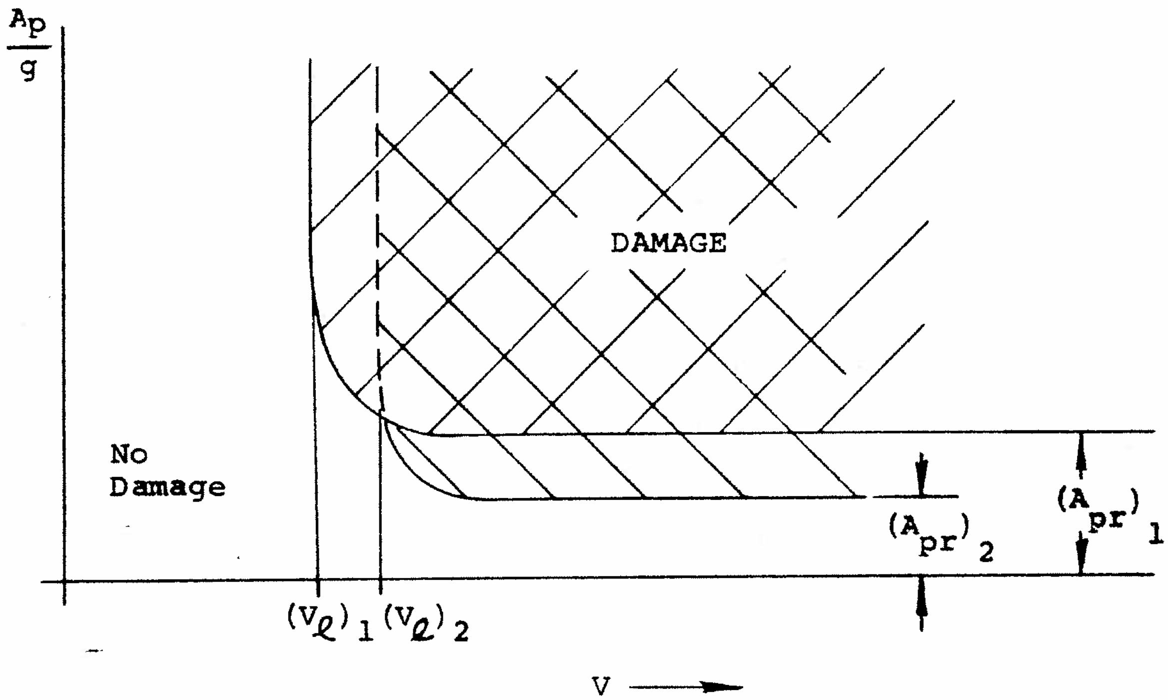

Figure 8. Damage Boundary - Two Critical Components

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APPENDIX A

DERIVATION OF DAMAGE BOUNDARY

A rectangular pulse is completely defined by peak acceleration A_p and effective (also actual) duration τ_e. Since, from Equation 1, $V = A_p \tau_e$, an alternate description is provided by specifying A_p and the velocity change V. The present analysis seeks to define the portion of the A_p, V plane corresponding to shocks which will not damage the critical components of an item subjected to such shocks.

Consider the shock spectrum for a rectangular pulse shown in Figure A-1.

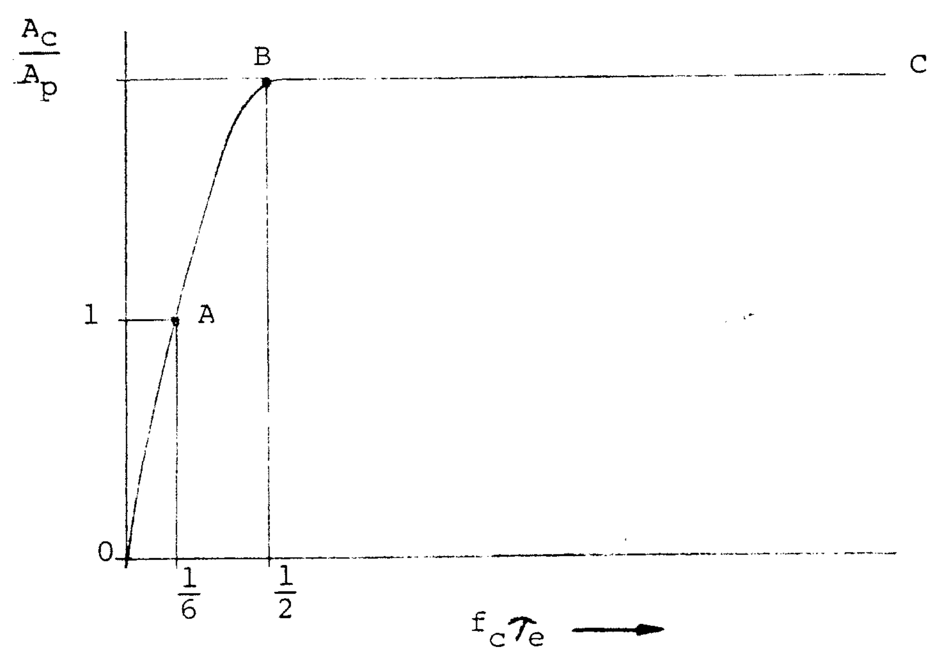

Figure A-1. Shock Spectrum, Rectangular Pulse

It is easy to show that the segments of OA, AB, and BC are represented with adequate accuracy as follows:

$$\text{OA:} \quad \frac{A_c}{A_p} = 2\pi f_c \tau_e, \quad 0 \lesssim f_c \tau_e < \frac{1}{6} \; ; \qquad \text{(A-1)}$$

$$\text{AB:} \quad \frac{A_c}{A_p} = 2 \sin (\pi f_c \tau_e), \quad \frac{1}{6} \lesssim f_c \tau_e \lesssim \frac{1}{2} \; ; \qquad \text{(A-2)}$$

$$\frac{\omega_c T_e}{2}$$

$$\text{BC:} \quad \frac{A_c}{A_p} = 2, \quad \frac{1}{2} \lesssim f_c \tau_e . \qquad \text{(A-3)}$$

Assume now that the critical component has a specified natural frequency f_c and that it may safely sustain a peak acceleration A_{cs} without damage. Substituting A_{cs} for A_c in Equation A-1, replacing $A_p \tau_e$ by V (Equation 1) and solving for V gives

$$V_\ell = \frac{1}{2\pi} \frac{A_{cs}}{f_c}, \qquad \text{(A-4)}$$

where a subscript ℓ has been appended to V. It is easy to show that a pulse for which $V \leq V_\ell$ will result in a peak component acceleration $A_c \leq A_{cs}$. It is evident from inspection of Equation A-3 that any shock for which $A_p \leq A_{pr}$ will likewise give $A_c \leq A_{cs}$, where

$$A_{pr} = \frac{1}{2} A_{cs} \qquad \text{(A-5)}$$

Note also that

$$\text{Point A:} \quad V = V_\ell , \quad A_p = A_{cs}; \qquad \text{(A-6)}$$

$$\text{Point B:} \quad V = \frac{\pi}{2} V_\ell, \quad A_p = \frac{1}{2} A_{cs} \qquad \text{(A-7)}$$

Finally, substitution in Equation A-2 gives the relation

$$\frac{A_{cs}}{A_p} = 2 \sin \frac{\pi f_c V}{A_p}, \qquad \text{(A-8)}$$

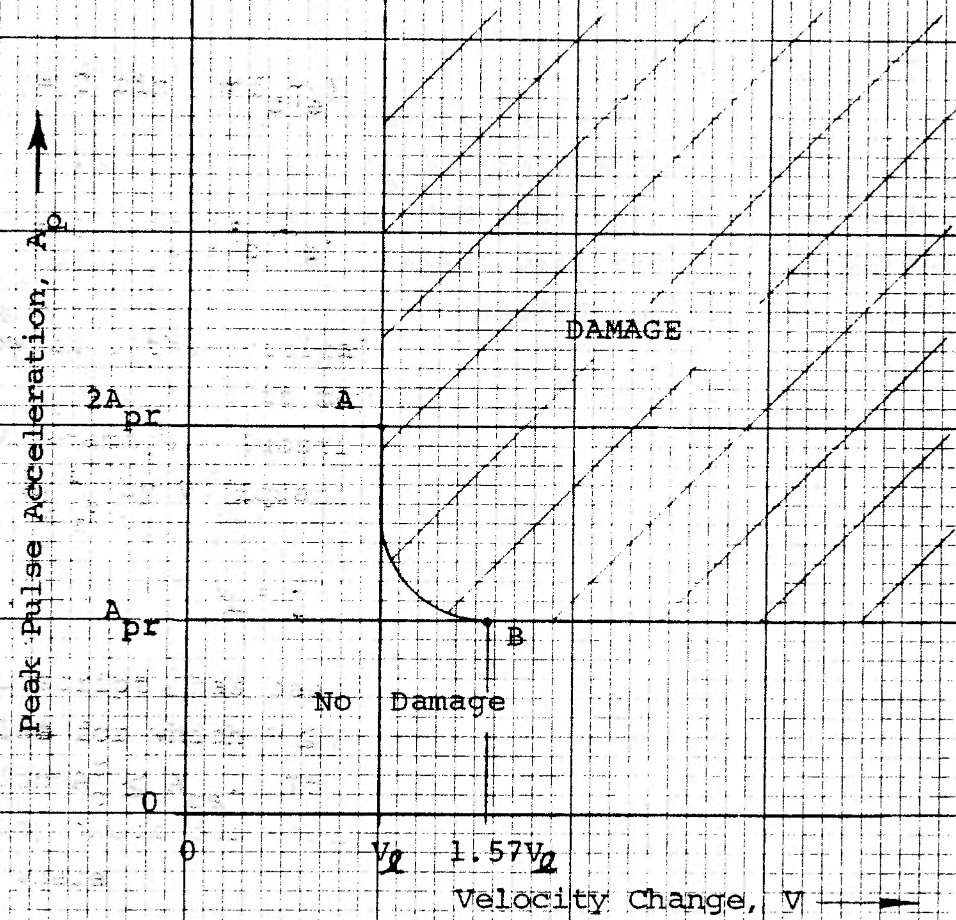

Figure A-2. Ideal Damage Boundary

which defines the relation between A_p and V in the segment between A and B.

Using the results of Equations A-4, 5, 6, 7, 8, Figure A-2 is plotted.

In Figure A-2, the curve O A B C is the image in the V, A_p plane of the shock spectrum curve with $A_c = A_{cs}$ and f_c = constant. (Point O lies at $V = V_\ell$, $A_p = \infty$ in the V, A_p plane.) For shocks on the curve or in the unshaded region to the left and below, $A_c \leq A_{cs}$ and no component damage will result. Conversely, shocks corresponding to points in the shaded region give $A_c > A_{cs}$ and damage will result.

APPENDIX B

TRANSFORMATION OF SHOCK SPECTRUM TO DAMAGE BOUNDARY

The shock spectrum may conveniently be presented as a graphical relation between A_c/A_p and $f_c\tau_e$ for pulses of specified shape. Symbols used are:

A_c = maximum component acceleration

A_p = peak pulse acceleration

f_c = component natural frequency

τ_e = effective pulse duration

The effective duration τ_e is expressible as

$$\tau_e = \frac{V}{A_p}$$

where

V = velocity change

The damage boundary is a transformation of the shock spectrum to a new set of coordinates so that $\frac{A_p}{A_c}$ is plotted vs. Vf_c / A_c. (This is the dimensionless form. If f_c and the maximum safe value of A_c are known, a specific plot of A_p vs. V is possible for that component.) The procedure for transformation is as follows:

1. For a chosen value of $f_c\tau_e$, read A_c / A_p from the shock spectrum curve.
2. Calculate $\frac{Vf_c}{A_c} = (f_c\tau_e)/(A_c / A_p)$ and $A_p / A_c = 1/(A_c / A_p)$.

3. Below $f_c T_e = \frac{1}{6}$ the shock spectra for all single pulses are represented with adequate accuracy as

$$\frac{A_c}{A_p} = 2\pi f_c T_e$$

Accordingly this gives

$$\frac{V f_c}{A_c} = \frac{1}{2\pi}$$

This portion of the damage boundary extends from $\frac{A_p}{A_c} = 1$ to $\frac{A_p}{A_c} \longrightarrow \infty$.

An example is given on the following page for a terminal peak sawtooth.

EXAMPLE - TERMINAL PEAK SAWTOOTH

(1)	(2)	(3)	(4)
$f_c T_e$	A_c / A_p	A_p / A_c	Vf_c / A_c
.34	1.27	.79	.27
.50	1.00	1.00	.50
.62	.89	1.12	.70
.76	1.00	1.00	.76
.90	1.07	.93	.84
1.00	1.00	1.00	1.00
1.12	.94	1.06	1.19
1.25	1.00	1.00	1.25
1.38	1.05	.95	1.31
1.50	1.00	1.00	1.50
1.62	.96	1.04	1.69
1.75	1.00	1.00	1.75
1.88	1.04	.96	1.81
2.00	1.00	1.00	2.00

(1), (2) - From Figure 8.24 ($\sigma = 1$), p. 8-34, Vol. I of Shock and Vibration Handbook.

$$(T_e = \frac{1}{2} T)$$

(3) = 1/(2)

(4) = (1) · (3)

DAMAGE BOUNDARY