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The kinked demand curve revisited

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Abstract

In a Stackelberg oligopoly with cost asymmetry and possibility of entry, the Stackelberg leader faces a kinked demand curve. For a robust interval of cost of the leader, the equilibrium price is rigid with respect to small changes in demand and costs of active firms.

Keywords: Kinked demand curve; Price rigidity; Stackelberg oligopoly

JEL classification: D21; D43; L11; L13

1. Introduction

The kinked demand curve theory of oligopoly has a distinguished lineage. Put forward independently by Hall and Hitch (1939) and Sweezy (1939), this theory sought to explain the rigidity of prices under oligopoly. It was argued that given an existing price in an oligopoly, if a single firm raises its price, its rivals will not respond, while if it cuts its price, other firms will cut their prices too. Thus, the demand curve faced by an individual firm will have a kink at the existing level of price and as a consequence, this price will not change for small changes in cost and demand. While empirical evidence remains mixed, the model of kinked demand has been criticized on theoretical ground mainly because of its arbitrariness—both in regard to the existing price as well as the response of the firms.[1] Relatively recent works of Bhaskar (1988) and Maskin and Tirole

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[1] The literature of kinked demand theory, both theoretical and empirical, is large (Stigler, 1947, 1978; Peck, 1961; Bhaskar et al., 1991; Rothschild, 1992, to name only a few) and we do not attempt to summarize it here. We refer to Reid (1981) for a comprehensive survey.

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(1988) have addressed this criticism by providing equilibrium foundation to this theory. Considering price competition under duopoly, they have shown that in equilibrium, both firms charge a sufficiently high common price; this collusive outcome is sustained by the use of the kinked demand strategy on off-the-equilibrium-path.[2] Neither of these theories, however, predicts price rigidity—a phenomenon that the original theory of kinked demand sought to explain. The aim of the present paper is thus two-fold: first, to derive the kinked demand curve on the basis of strategic interaction among firms and second, to obtain equilibrium price that is rigid with respect to small changes in cost and demand. Departing from the existing literature, which has mainly focused on price competition, we consider a simple model of Stackelberg oligopoly with quantity-setting firms. We first show that under entry possibilities and asymmetry of costs, the Stackelberg leader will face a kinked demand curve in any subgame-perfect equilibrium. Then it is shown that the equilibrium price is attained at a kink of the demand curve of the leader, implying rigidity of price.[3] The intuition underlying our result is simple. When the possibility of entry is taken into account, the quantity set by the Stackelberg leader effectively determines the market structure: a high level of quantity drives down the price and prevents relatively inefficient potential entrants from entering while a low level of quantity has the opposite effect. Under asymmetry of costs among the followers, the leader will have one or more "threshold" levels of quantity—each corresponding to a change in the market structure. This in turn gives rise to a kinked demand curve for the leader, with kinks at the threshold levels of quantity. For a robust interval of cost, the Stackelberg leader finds it optimal to set the quantity at one of these threshold levels, thus maximizing her profit while maintaining the existing market structure. The equilibrium results in a price that is rigid for small changes in demand and costs of the active firms in the industry.

2. The model

We consider a Stackelberg oligopoly with three quantity-setting firms: L, 1 and 2.[4] Firm L is the Stackelberg leader while firms 1 and 2 are followers. For $i \in \{L, 1, 2\}$, let q_i be the quantity produced by firm i and let $Q = q_L + q_1 + q_2$. The demand function of the industry is linear and is given by $Q = a - p$, for $p \leq a$ and $Q = 0$, otherwise. Each firm produces under constant marginal cost. For $i \in \{L, 1, 2\}$, c_i is the cost of firm i. The following assumptions will be maintained throughout the paper.

A1. $0 < c_1 < c_2 < a$.
A2. $2c_2 < a + c_1 < 5c_2$.

[2] Considering an extensive-form duopoly where a firm can undercut the price of its rival, Bhaskar (1988) has shown that in the unique subgame-perfect equilibrium, both firms charge the "minimum optimal" price. Maskin and Tirole (1988) have considered a Bertrand duopoly under dynamic setting and have shown that under certain reasonable refinement criteria, in the unique equilibrium, both firms charge the monopoly price and share the market.

[3] See Rothschild (1992) for an alternative explanation of price rigidity based on relative substitutability of products of different firms.

[4] For clarity of presentation, we consider a simple model that captures cost asymmetry and possibility of entry. It will be evident to the reader that our conclusions will continue to hold qualitatively for larger oligopolies as well as the monopoly and for more general demand and cost functions.

2.1. The Stackelberg game G

The strategic interaction among the firms is modeled as a three-stage extensive-form game: the Stackelberg game G. In the first stage, the leader L sets her quantity q_L. In the second stage, the followers, firms 1 and 2, observe q_L and simultaneously set their respective quantities: q_1 and q_2. In the third and final stage, profits are realized and the game terminates. We employ the standard backward induction method to find the (unique) subgame-perfect equilibrium of the game G.

Lemma 1. *Suppose A1–A2 hold. Denote $q_D \equiv a + c_1 - 2c_2$ and $q_M \equiv a - c_1$. In any subgame-perfect equilibrium of G, when L produces q_L, the quantity produced by firm i is given by $f_i(q_L)$ for $i \in \{1, 2\}$, where*

$$(f_1(q_L), f_2(q_L)) = \begin{cases} \left(\dfrac{a - 2c_1 + c_2 - q_L}{3}, \dfrac{a + c_1 - 2c_2 - q_L}{3}\right) & \textit{for } q_L \in [0, q_D], \\ \left(\dfrac{a - c_1 - q_L}{2}, 0\right) & \textit{for } q_L \in [q_D, q_M], \\ (0, 0) & \textit{for } q_L \geq q_M. \end{cases} \tag{1}$$

Proof. See the Appendix A. □

Proposition 1. *Suppose A1–A2 hold. Then in any subgame-perfect equilibrium of G, the demand curve faced by the leader L is a kinked demand curve. Specifically, it is given by $D(q_L)$, where*

$$D(q_L) = \begin{cases} D_1(q_L) \equiv \dfrac{a + c_1 + c_2 - q_L}{3} & \textit{for } q_L \in [0, q_D], \\ D_2(q_L) \equiv \dfrac{a + c_1 - q_L}{2} & \textit{for } q_L \in [q_D, q_M], \\ D_3(q_L) \equiv a - q_L & \textit{for } q_L \geq q_M. \end{cases} \tag{2}$$

Proof. Note that in any subgame-perfect equilibrium, the demand curve faced by L when she sets the quantity q_L is given by $D(q_L) = a - q_L - f_1(q_L) - f_2(q_L)$. Then the result follows from Lemma 1. □

The demand curve of L, $D(q_L)$, is given by $A_1K_1K_2B$ in Fig. 1. The lines A_1K_1, $A_2K_1K_2$ and A_3K_2B correspond to $D_1(q_L)$, $D_2(q_L)$ and $D_3(q_L)$, respectively. The demand curve has two kinks: K_1 and K_2.[5] The kink K_1 corresponds to the quantity q_D, where the price is c_2 (the cost of firm 2). When $q_L < q_D$, both

[5] Observe that $|\text{slope of } D_1(q_L)| < |\text{slope of } D_2(q_L)| < |\text{slope of } D_3(q_L)|$. Thus, the demand curve faced by L has *obtuse* kinks. See Reid (1981), pp. 15–16, for details on different types of kinks. Diagrams similar to Fig. 1 have been used to illustrate kinked demand in the literature (e.g. Sweezy, 1939, Stigler, 1947).

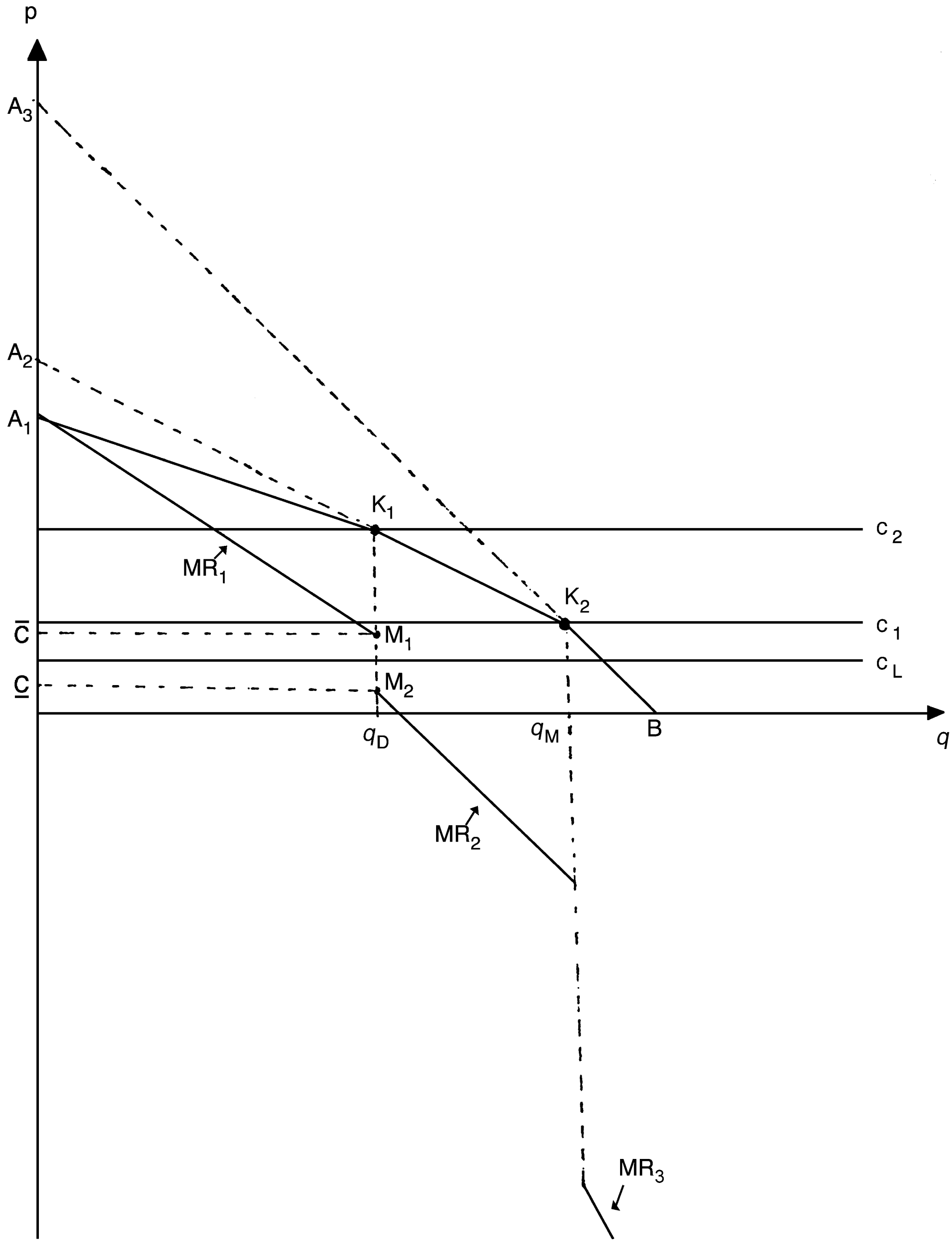

Fig. 1. The demand curve of the Stackelberg leader.

firms 1 and 2 produce positive quantity, while for $q_L \geq q_D$, firm 2 produces zero. The kink K_2 corresponds to the quantity q_M, where the price is c_1 (the cost of firm 1). Firm 1 produces positive quantity when $q_D \leq q_L < q_M$ and it produces zero when $q_L \geq q_M$. Thus, each kink corresponds to a change in the market structure.

Proposition 2. *Suppose A1–A2 hold. Then there exist constants $0 \le \underline{c} < \bar{c} < c_2$ such that when $c_L \in [\underline{c}, \bar{c}]$, in the unique subgame-perfect equilibrium of G, firm L produces the quantity q_D that corresponds to the kink K_1 of the demand curve. The equilibrium price is given by c_2 that results in a Stackelberg duopoly with firms L and 1.*

Proof. From Proposition 1, it follows that the marginal revenue of L, $MR(q_L)$, is not defined at $q_L = q_D$ and $q_L = q_M$. For all other values of q_L, it is given by the following (see Fig. 1).

$$MR(q_L) = \begin{cases} MR_1(q_L) \equiv \dfrac{a + c_1 + c_2 - 2q_L}{3} & \text{for } q_L \in [0, q_D), \\ MR_2(q_L) \equiv \dfrac{a + c_1 - 2q_L}{2} & \text{for } q_L \in (q_D, q_M), \\ MR_3(q_L) \equiv a - 2q_L & \text{for } q_L > q_M. \end{cases} \tag{3}$$

Note further that $MR_1(q_L = q_D) = (5c_2 - a - c_1)/3$ and $MR_2(q_L = q_D) = (4c_2 - a - c_1)/2$. Denote $\bar{c} \equiv (5c_2 - a - c_1)/3$ and $\underline{c} \equiv \max\{(4c_2 - a - c_1)/2, 0\}$. Then $0 \le \underline{c} < \bar{c} < c_2$ due to A2. Now suppose that $c_L \in [\underline{c}, \bar{c}]$ (in Fig. 1, $[\underline{c}, \bar{c}]$ has been identified and c_L has been chosen inside this interval). For profit to be maximized at some q_L, it is required that there is an interval $[q_L - \Delta q_L, q_L + \Delta q_L]$, $\Delta q_L > 0$, such that the following holds.[6]

$$MR(q_L + \Delta q_L) < c_L \text{ and } MR(q_L - \Delta q_L) > c_L. \tag{4}$$

First, noting that $MR_1(q_L = q_D) = \bar{c} \ge c_L$ and $MR_2(q_L = q_D) \le \underline{c} \le c_L$, since both MR_1 and MR_2 are downward sloping, we have $MR_1(q_L) > c_L$ for $q_L < q_D$ and $MR_2(q_L) < c_L$ for $q_D < q_L < q_M$. Next, observing that $MR_3(q_L = q_M) = 2c_1 - a < \underline{c} \le c_L$, using the fact that MR_3 is also downward sloping, we have $MR_3(q_L) < c_L$ for $q_L > q_M$. All these facts imply that condition (4) holds only when $q_L = q_D$. Thus, when $c_L \in [\underline{c}, \bar{c}]$, in the unique subgame-perfect equilibrium, the leader L sets the quantity q_D. Evaluating $D(q_L)$ at $q_L = q_D$ from (2), we conclude that the equilibrium price is c_2. Then firm 2 does not produce and the equilibrium results in a Stackelberg duopoly with firms L and 1. This completes the proof.□

The essence of Propositions 1 and 2 can be described as follows. Under asymmetry of costs among the followers, the Stackelberg leader determines the market structure through her choice of quantity. By setting a quantity at a level that corresponds to a change in the market structure, she maximizes her profit in the existing market and at the same time maintains the structure by effectively deterring entry from potential entrants. The equilibrium market structure will of course depend on the cost of the leader. Proposition 2 shows that for a robust interval of cost, the equilibrium market structure is a duopoly with firms L and 1. Since the equilibrium price depends only on the cost of the potential entrant, it is clearly rigid with respect to small changes in either demand or costs of the operating firms in the industry.

[6] For the derivation and an excellent discussion on optimality conditions on different forms of kinked demand curves, see Reid (1981), pp. 20–24.

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Appendix A

Proof of Lemma 1. When firm L chooses the quantity q_L, the profit functions of firms 1 and 2 are given by the following.

$$\Pi_1 = (a - q_L - q_1 - q_2)q_1 - c_1 q_1, \quad \Pi_2 = (a - q_L - q_1 - q_2)q_2 - c_2 q_2.$$

The first-order conditions yield

$$q_1 = \frac{a - q_L - 2c_1 + c_2}{3}, \quad q_2 = \frac{a - q_L + c_1 - 2c_2}{3}. \tag{5}$$

We consider the following cases.

Case 1. $a - q_L + c_1 - 2c_2 \geq 0$. For this case, the quantities produced by firms 1 and 2 are given by (5).

Case 2. $a - q_L + c_1 - 2c_2 \leq 0$. For this case, firm 2 produces zero and the profit function of firm 1 is given by $\Pi_1 = (a - q_L - q_1)q_1 - c_1 q_1$. The first-order condition yields $q_1 = (a - q_L - c_1)/2$. We consider the following subcases.

Subcase 2. (a). $a - q_L - c_1 \geq 0$. For this case, from the first-order condition of firm 1, we have $f_1(q_L) = (a - q_L - c_1)/2$, while $f_2(q_L) = 0$.

Subcase 2. (b). $a - q_L - c_1 \leq 0$. For this case, firm 1 also produces zero, so that $f_1(q_L) = f_2(q_L) = 0$.

The lemma follows from Case 1 and Subcases 2(a) and (b). □

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