Unit One
# The Difference of Squares
Organizing Data

Unit 1 Objectives
Difference of Squares: ORGANIZING DATA

In this unit we continue to develop good work habits and the ability to work as a productive team member. Many of the problems in this course can be solved by using a variety of strategies and tools. In this unit you will look at ways to **organize and analyze data** in tables, graphs and charts. You will use models to review integer operations and combine like terms. You will also develop the problem solving strategy of Guess and Check and explore the use of your calculator when dealing with large numbers.
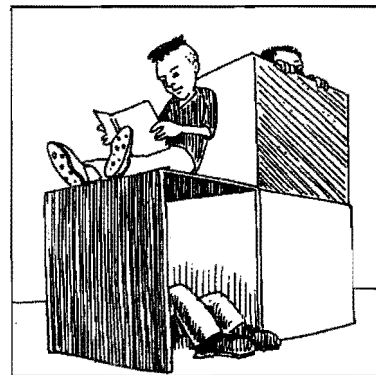
In this unit you will have the opportunity to:

- understand the rules for doing arithmetic with integers.

- combine algebraic terms using addition and subtraction.

- interpret graphs.

- organize data using a variety of methods, including systematic lists and tables.

- investigate patterns and use them to predict outcomes.

- solve word problems using the "Guess and Check" strategy as preparation to write and solve equations.

Read the problem below. Over the next few days you will gain the skills to solve it. **Do not try to complete it now.**

SQ-0.   DIFFERENCE OF TWO SQUARES

Many numbers can be written as a difference of two squares, such as the ones below. You will find ways to write numbers as a difference of two squares, organize your data and look for patterns. The difference of two squares will occur again in a later unit.

NUMBER	SOLUTIONS (Difference of Squares)
8	$3^2 - 1^2$
15	$8^2 - 7^2; 4^2 - 1^2$

Problem Solving
Graphing
Writing and Solving Equations
Ratios
Geometry
Symbol Manipulation

Unit 1
Difference of Squares: ORGANIZING DATA

PZL-5 WHAT TO DO IN THIS COURSE

What is your primary responsibility as a student in this algebra class ? I am going to answer that question with one simple phrase: **to think!** Throughout this course, the number one requirement will be to think about mathematics. This requires you to be active in your learning. The point of CPM and its style of teaching and learning is to help you think. **You will need to read the book and do your homework every night to practice thinking.** Your years of college prep mathematics and even beyond will require you to be an active thinker.

I will not kid you by saying this is easy, but it will get easier as you do it on a regular basis. You also have other responsibilities, most of which were mentioned in the welcome note in the "Getting Started" unit (PZL-1). Make sure you read this note again. Also, be sure you know all the requirements for this course. If you are still not sure what they are, ask your teacher as soon as possible.

MISSION POSSIBLE (something to do)

Re-read the second to last paragraph in the welcome note.

a)   Do you agree with the statements about what it takes to learn how to play a musical instrument? Why or why not?

b)   Read the box below, then begin today's lesson with SQ-1. By the way, what is your primary responsibility in this course?
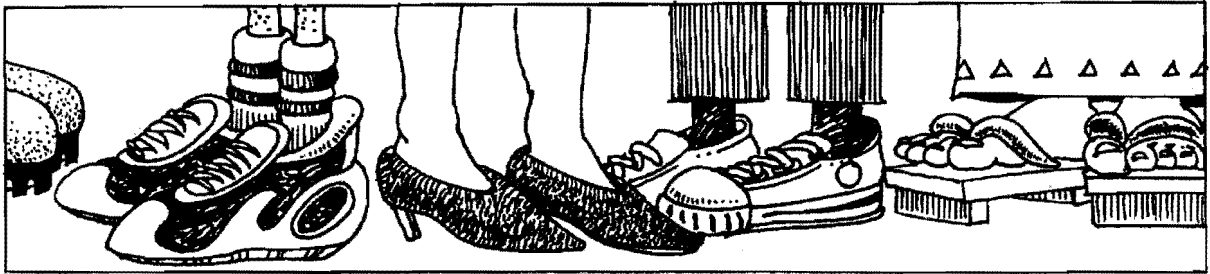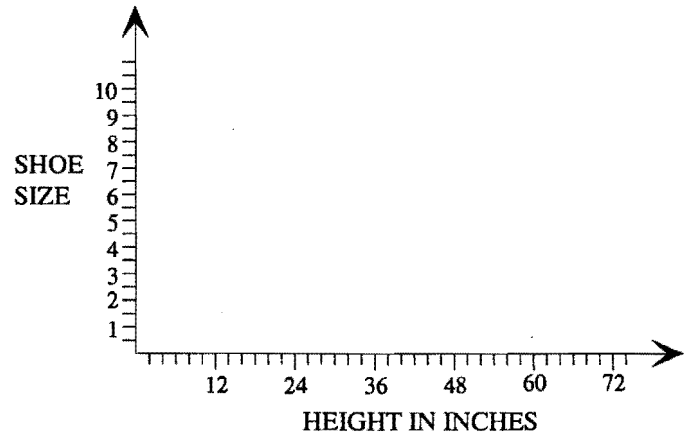

REMINDER: Guidelines for Study Teams

1)   Each member of the team is responsible for his or her own behavior.

2)   Each member of the team must be willing to help any other team member who asks for help.

3)   You should only ask the teacher for help when all team members have the same question.

4)   Use your team voice.

SQ-1.   On the poster paper on the wall, place a sticky dot above your height to indicate your shoe size.

Do the same for your family member's shoe size.

SHOE SIZE — (vertical axis labeled 1 through 10)

HEIGHT IN INCHES — (horizontal axis labeled 12, 24, 36, 48, 60, 72)

SQ-2.   THE BEEBOPPER SHOE STORE

As the manager of the Beebopper Shoe Store, you are responsible for ordering the shoes to stock a brand new line of athletic shoes. You have no idea how many shoes of each size to order, but you have heard from a wise old woman (who does not live in a shoe) that you can probably figure out a person's shoe size by estimating his or her height. The woman believes that taller people wear larger shoe sizes and shorter people wear smaller sizes. You are not sure about this wisdom, so you have decided to conduct an investigation by asking the local algebra students to provide heights and shoe sizes.

Make a table that represents the data from the Beebopper Shoe Store graph on the wall. You should make your own data table and verify the entries with your team members before writing them down.

SQ-3.   Use the table you made in problem SQ-2 to make your own graph of the class's height and shoe size data on graph paper. On the horizontal (height) axis, use a scale where one unit represents 2 inches of height. On the vertical (shoe size) axis, let each unit represent a half shoe size.

SQ-4. Use your own Beebopper Shoe Store graph (or the graph on the wall) to answer the following questions in complete sentences:

a) What are the smallest and largest shoe sizes represented?

b) What are the shortest and tallest heights represented?

c) Which shoe size would you stock the most of if you were the manager of the Beebopper Shoe Store? Which size would you stock the least? Would it be profitable to carry every possible size? Why or why not?

d) What difference, if any, do you notice between the data for males and the data for females?

e) Was the old woman correct? Can you predict a person's shoe size if you know his or her height? Why or why not? Do you think this relationship would be valid for adults? Why or why not?

f) Explain how you would use the graph to predict the shoe size for someone 5'6" tall. Explain how you would use the graph to predict the shoe size for someone 7' tall.

g) Look at the graph. Find the height represented most frequently. Calculate the average shoe size for classmates of this height. Does this shoe size make sense? Why or why not?

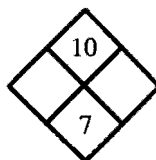h) Is there any data point that does not seem to follow the trend?

SQ-5. Use the pattern you found in the Getting Started Unit to complete these Diamond Problems. Create two new Diamond Problems of your own.
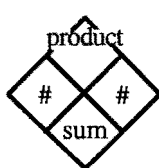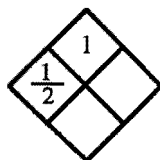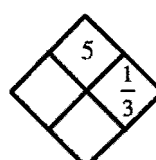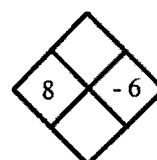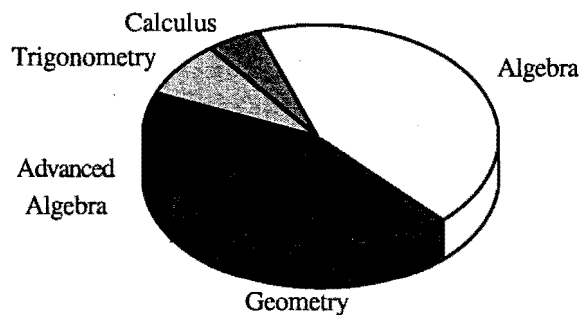

SQ-6. Enrollment in math courses at a particular high school is shown in the pie chart at right. If there are 1,000 students enrolled in math courses, approximately how many students are enrolled in Algebra? In Geometry? In Calculus?

SQ-7.

A local radio station is giving away prizes to the ninth caller. After a listener successfully dials in, the radio station then spins two spinners. The first determines if you win a cassette or CD, while the second selects from five artists: two of whom you like.

a) Assuming that you are the ninth caller, what is the probability that you win a CD?

b) What is the probability of getting one of your favored artists on tape or CD?

c) What is the probability that you win a CD of one of your favored artists?

d) What is the relationship between the fractions in (a) and (b) to your answer in (c)?

ADDING AND SUBTRACTING INTEGERS

The number 3 can be represented by

+++ or **+++++++** **————** or **+++++** **——**

The number 0 can be represented by the same number of positive and negative numbers. An equal number of positive and negative tiles is called a **NEUTRAL FIELD**.

++ **——** or **+++++** **—————** or **+++** **———**

The number - 1 can be represented by

— or **+++** **————** or **++++** **—————**

SQ-8. Use a minimum number of tiles to represent the following integers. Build each model and then sketch it on your paper.

a) 2   b) - 7   c) 0

Use at least 12 tiles to represent each of the integers below. Sketch each model after you build it.

d) 5   e) - 1   f) 0

SQ-9.   Represent each of the statements below using tiles. Sketch each model and write the problem with its answer below your drawing.

   a)   -7 + (-2)   c)   3 + (-4)   e)   -2 - 5

   b)   -1 + 5   d)   4 - (-2)   f)   -7 - (-8)

Problems that are especially important have a box around the problem number. They help you develop understanding and consolidate ideas. Pay careful attention to these problems, and be sure to revise your work if necessary. Complete, correct solutions should be in your notebook to serve as examples of why a concept or procedure works and as an example of how to do it.

SQ-10.   Khalifa has never worked with integers before. Explain to her your rules for doing integer problems, including a diagram. Answer in complete sentences.

   a) -4 + (-2)   b) 3 + (-7)   c) 2 - 5   d) 3 - (-2)

SQ-11.   Lindsay thinks the problems 2 + 4 and -2 + (-4) are similar while Juan disagrees. Explain your position on this discussion.

SQ-12.   Compute <u>without</u> a calculator:

   a)   427 - (-3)   b)   -50 + (-150)

SQ-13.   Sam and Samantha put the problem 3 + 2 · 4 in their calculators. Sam got 20 for the answer while Samantha got 11. Ms. Speedi then knew Samantha had a scientific calculator and Sam did not. Sam had to return his and buy a scientific calculator for his algebra class.

   a)   How did Ms. Speedi know Samantha's answer was correct and Sam's was wrong? What did Samantha's calculator do that was different from Sam's?

   b)   Check your calculator to see if it is a scientific calculator by doing the problem. What kind of calculator do you have? If it is not scientific you will need to make a trip to the store and buy one.
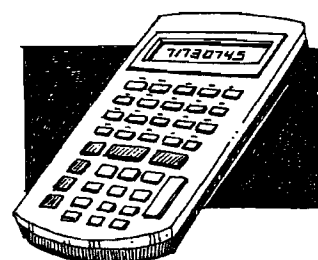
SQ-14.   Deondre was tired of playing Roll and Win. Instead of subtracting the two dice, he decided to add them. He called his new game Roll and Add. Use the strategy you used in problems GS-20 through GS-22 to decide which number Deondre should choose to win. Show all your work.
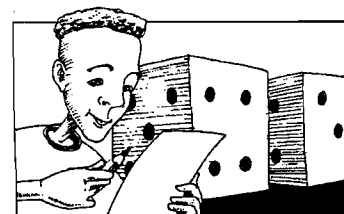
SQ-15.   Sketch at least 12 tiles that represent each of the integers below.

   a)   4   b)   -2   c)   -5

SQ-16.   Represent each of the statements below using tiles. Sketch each model and write the problem with its answer below your drawing.

   a)   2 + (-4)   b)   -3 - (-5)

SQ-17.   Compute without a calculator.

   a)   -15 + 7   e)   -50 - 30   i)   9 + (-14)

   b)   8 - (-21)   f)   3 - (-9)   j)   28 - (-2)

   c)   -12 - (-4)   g)   -75 - (-75)   k)   -3 + (-2) + (-5)

   d)   -9 + (-13)   h)   (-3) + 6   l)   3 + 2 + 5

INTERPRETING GRAPHS AND COMPARING AREAS

SQ-18.   Today you received a sample page for your **ALGEBRA TOOL KIT**, essentially a summary of what has been covered so far in this course. YOU will be responsible for the creation of subsequent entries in your tool kit.
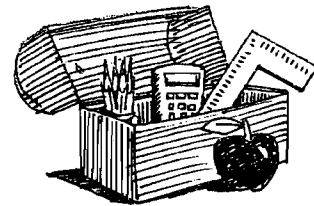
We suggest that you summarize each new idea, with an appropriate diagram or example, as it is presented. It is also a good idea to label each entry with the textbook page number or problem number where the idea appeared.

Your tool kit is a handy reference guide you will create during the year. As you identify and organize the important ideas, you will have a useful tool to make algebra more manageable.

Add to your tool kit what you know about adding and subtracting integers.

INTEGER ADDITION AND SUBTRACTION

In class we have been using integer tiles to model the operations of addition and subtraction. Here is an example that explains the process with words and a sketch of what you record.
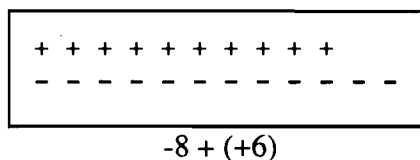
Addition Example: -8 + 6

Start with a neutral field, that is, an equal number of positive and negative tiles, that has a value of zero. You can review the definition of "neutral field" in the box after problem SQ-7.

```
+ + + +
- - - -
```
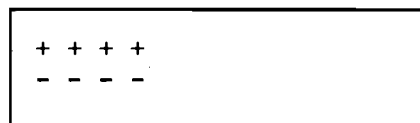
Display the two numbers using tiles.

```
-8                          +6
- - - - - -      + + + + + +

+ + + +
- - - -
```
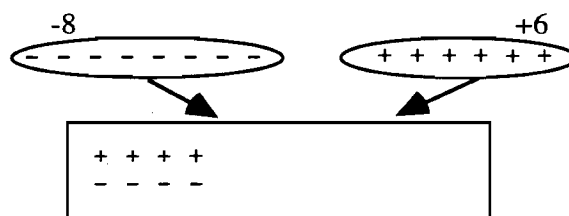
Combine the two numbers with the neutral field.

```
+ + + + + + + + + +
- - - - - - - - - - - -
```
-8 + (+6)

Circle the "zeros".

Record the sum.

```
(+)(+)(+)(+)(+)(+)(+)(+)(+) - -
(-)(-)(-)(-)(-)(-)(-)(-)(-)
```
-8 + 6 = -2
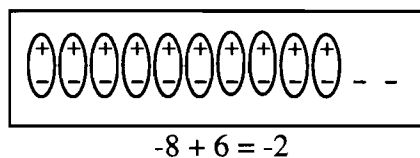
Subtraction Example: -2 - (-4)

Start with the first number displayed with a neutral field.

```
+ + + + +
- - - - - - -
```
-2
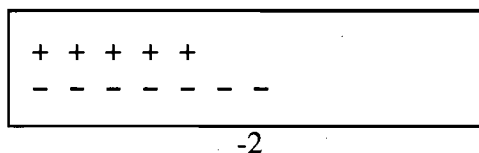
Circle the second number in your sketch and show with an arrow that it will be removed.

```
+ + + + +
- - -( - - - )
```
-2 - (-4)
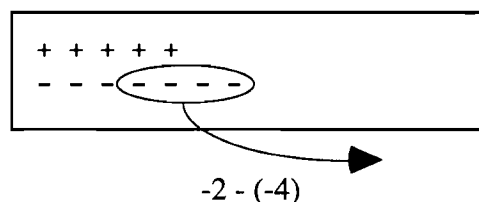
Remove the second number.

Circle the "zeros"
Record the difference.

```
(+)(+)(+) + +
(-)(-)(-)
```
-2 - (-4) = 2
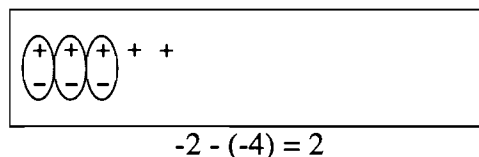
SQ-19.   This problem must be done with a scientific calculator. Use your study team to make sense of the calculator results.
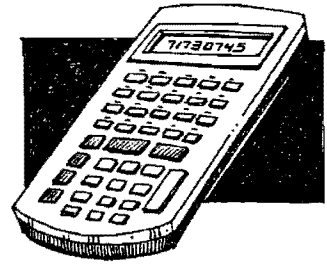
a)   Fill your calculator display with 9's. How many 9's does your calculator display? Copy the display on your paper with commas in the appropriate places.

b)   Without using your calculator, add one to the number you just wrote down. Compare your answer with others. Count your zeros, put in commas and write that answer in words.

c)   Now use your calculator to add one to your original number. Write down exactly what your calculator displays. This is the calculator's way to display those numbers that do not fit in its screen.

d)   Compare your answers for parts (b) and (c).

e)   Emil's calculator displays $\boxed{5.286^{12}}$. What does the "12" represent?

f)   Extension: Can you get your calculator to display $\boxed{5.286^{12}}$? Write down your steps.

SQ-20.   Latisha is determined to do well in school this year. Her goal is to maintain an at least 85% average in all her courses. You will need to help her keep track of her average each night. So far she has two scores in Algebra, 72 and 89. Find her average now. Show your work.
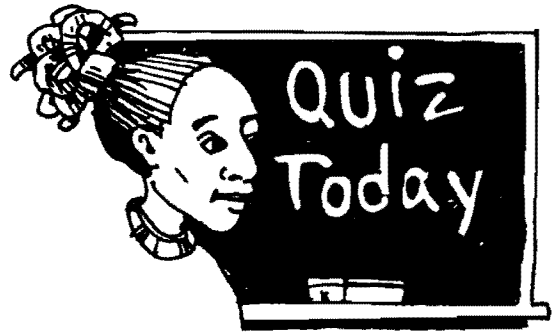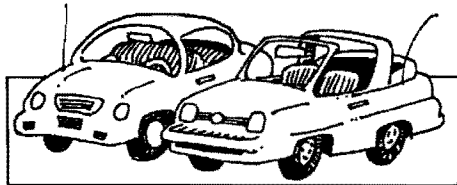
SQ-21. CAR COMPARISON

The following three graphs describe two cars, A and B.
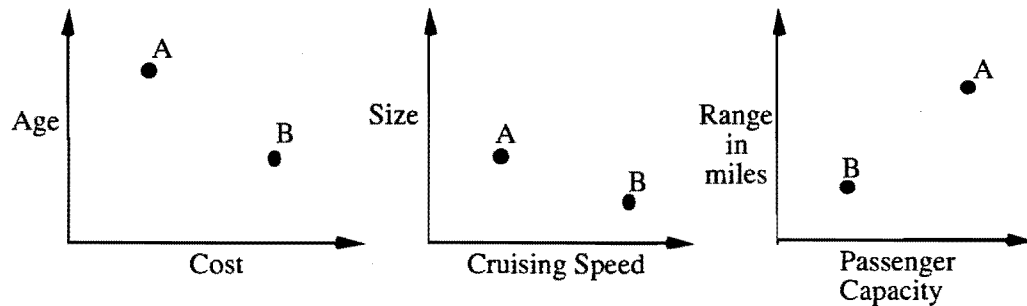
Decide whether each of the following statements is true or false. Explain your reasoning.

a) The newer car is more expensive.

b) The slower car is larger.

c) The larger car is newer.

d) The less expensive car carries more passengers.

SQ-22. Suppose we discover the island shown in the drawing at right and want to know its area. If we cover the island with a grid we can approximate the area.
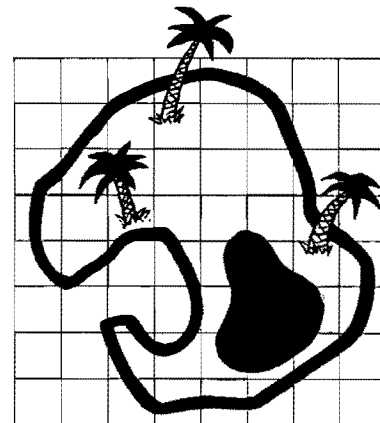
a) What is the area of the grid?

b) The island's area is less than the area of the grid. Why?

c) Using the grid, approximate the area of the island, including the lake.

d) Does the coastline represent the area or the perimeter of the island? Explain the difference between area and perimeter.

SQ-23. a) Copy the two graphs at right. On each graph label two points that would represent cars A and B as described in the Car Comparison Problem, SQ-21.
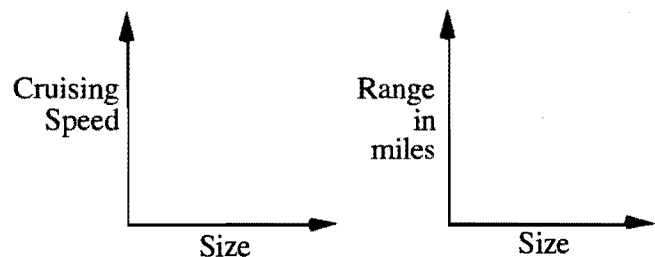
b) Which car would you buy? Why? Explain your answer in complete sentences.

SQ-24.   First try these problems without a calculator. Then check your answers on your calculator.

a) $32 + (-7)$

b) $-5 + (-10)$

c) $-13 + (-12)$

d) $(-8) + (-8) - (-8)$

e) $\left(-1\frac{1}{2}\right) + \left(-2\frac{1}{3}\right)$

f) $\left(-4\frac{1}{5}\right) + (10)$

g) $\left(\frac{3}{4}\right) - \left(-2\frac{1}{3}\right)$

h) $\left(16\frac{1}{8}\right) + \left(-\frac{3}{4}\right)$

SQ-25.   Copy and complete each of these Diamond Problems:
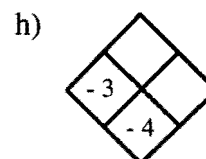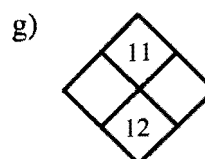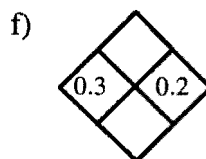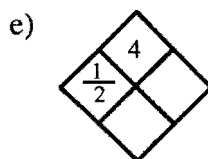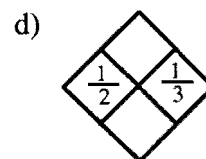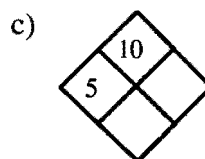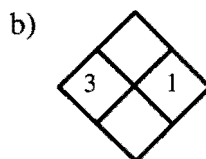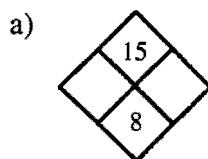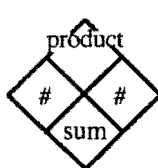
SQ-26.   Use the work from Deondre's Roll and Add game to answer the following questions. Refer to your work from SQ-14.

a)   Which sums have the same probability of showing up? Explain why.

b)   If Deondre rolled the dice 72 times, how many times would you expect the sum of 12 to come up? the sum of 7? the sum of 10?
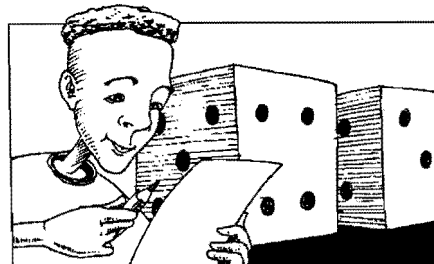
SQ-27.   Max, who is in the third grade, asked his older sister, Maxine, to explain what $3 \cdot 4$ means.

a)   In complete sentences, explain how Maxine could help Max understand what "3 times 4" means. Draw a picture that Maxine could use to show Max what "3 times 4" means.

b)   Give a real life example of when you might use $3 \cdot 4$.
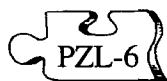
PZL-6   UNDERSTANDING HOW TO USE THIS TEXTBOOK

By now you have noticed that this Algebra 1 course expects you to be an active learner. You work with other students in study teams, the problems guide you to learning the mathematical ideas, you are asked questions about what your work means, and your teacher works with you in several ways during class time.
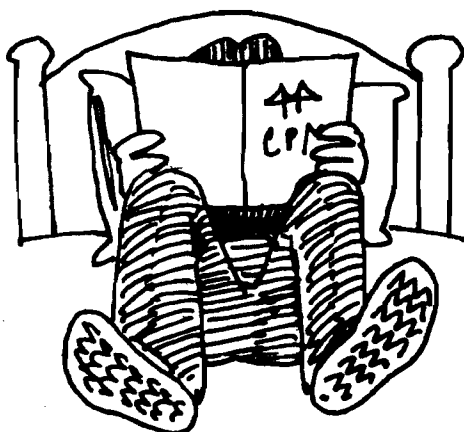
Another key step to get the most out of this course is to understand the textbook. **Understanding the textbook** is quite a bit different from understanding the problems within it.

Quickly <u>glance</u> at the welcome note (PZL-1 in Unit 0) and see if you can find the three big ideas it contains about this course.

How did you find them? If you noticed that they are in **bold print**, you have just discovered **one way for the authors to relate big ideas to you**. Flip through a few pages and find a few other examples of using bold print for emphasis. Bold print is one of the most common ways to point out information. **Sentences in bold print contain valuable information.** Always make sure you <u>add information in bold print to your tool kit and know how the surrounding paragraphs relate to the idea(s)</u>.

There are some other signs to help you find key information in the text. Every unit of this text contains several main ideas. These ideas are the fundamental concepts of algebra. **You will absolutely need to know these ideas to be successful in this course.**

- The mathematical ideas and definitions are in **single** and **double line boxes.** These boxes sometimes contain **examples** of how to use an idea or do a procedure.

- Important problems--usually ones that develop an idea or help pull it together--have a **box around the problem number.**

The information you find in boxes and important problems should be in your tool kit.

There are a few more ways to locate key information in this course. In the front of your book there is a **table of contents** that lists what you will study unit by unit. In the back there is an **index** that helps you locate where a term or concept is introduced. At the beginning of each unit there is a **list of what you will learn** in that unit. At the end of each unit, everything you should have learned and added to your algebra tool kit is listed in a **"Tool Kit Check-up."** In addition to your textbook, there is a **"Parent Guide with Review"** that will help your parent(s) understand how to help you with this course. It has additional examples and lots more practice problems. Ask you teacher how to obtain these resources or use the ordering information in the "Note to the Parents" at the beginning of the book.

MISSION POSSIBLE (something to do)

Make a list of all the locations in the text that will help you find the important information it contains.

SQ-28. Padraig has never multiplied integers before. Explain to him your rules for doing integer problems. Include a diagram. Then compute the following products:

a) 3(2)

c) -3(5)

b) 5(-3)

d) -4(-2)

SQ-29. Compute:

a) -12 ÷ 3

b) $\frac{6}{-2}$

c) $\frac{-8}{-4}$

SQ-30. What do the rules you use for multiplying and dividing integers have in common? Be specific enough that a student who transfers into class tomorrow would easily understand your response.

SQ-31. Try these problems without a calculator first. Then use a calculator to check your answer.

a) -16 + 7

e) -49 - 36

i) (-6) · 9

b) 10 - (-24)

f) -56 ÷ (-7)

j) 27 - (-3) - 12

c) (3)(-9)

g) 15 ÷ -3

k) (-5)(-5)

d) -9 + (-11)

h) -7 ÷ 7

l) -6 · (43)

SQ-32. Show all your work to prove that your answer is correct.

a) If you spin both spinners and add the two numbers, what is the probability that the answer will be positive? Make a list of the possible outcomes.

b) If you multiply the results of the spins, what is the probability that the answer will be positive?

Spinner #1   Spinner #2
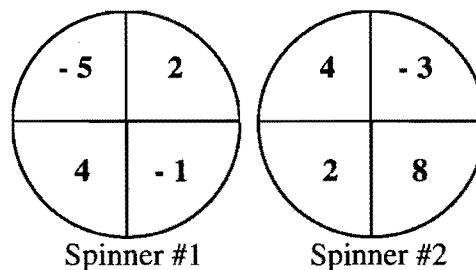
SQ-33. Latisha's newest score is a 90. Use her scores from SQ-20 to figure out her average now. Be sure to show your work.

SQ-34. Draw a diagram to represent $\frac{2}{3}$. Label your diagram with the fraction. Would 0.66 or 0.67 be a more appropriate decimal equivalent?

SQ-35. Write what you know about how to round decimals. Include several examples.

SQ-36.*   The diagram below shows a map surrounding the local Community Center. Five of Tara's friends are at the locations shown and are all traveling to meet her there to play basketball. Kelly and Ricardo are each riding their bikes. Elizabeth is walking. One of the remaining friends walked while the other drove. Assume that all the same modes of transportation travel at the same rate. Analyze the map and determine each person's placement on the graph below.
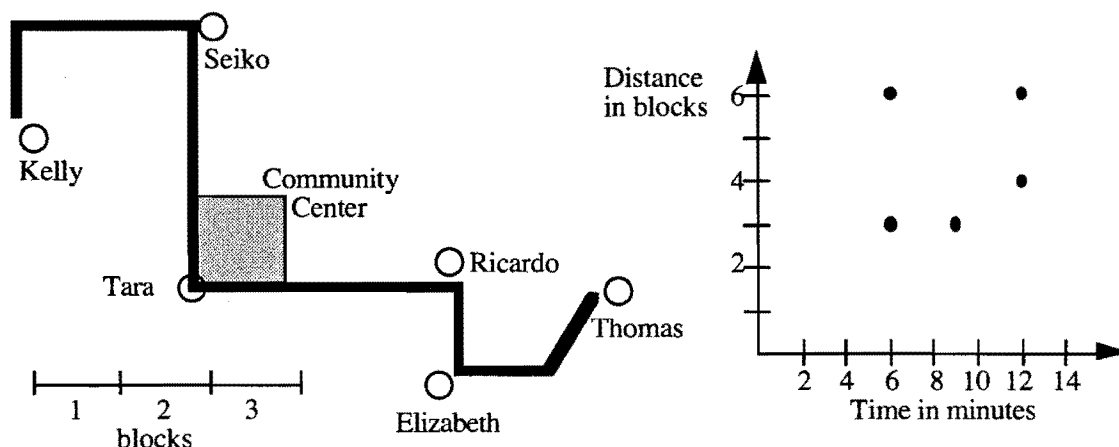
a)   Trace the graph above on your paper. Label each point with the name of its corresponding person.

b)   How long did it take Ricardo to get to the Community Center?

c)   How did Seiko get to the Community Center? d) How did Thomas get to the Community Center?
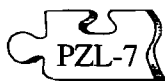
PZL-7   HUMAN RESOURCES

In PZL-6, I listed all the printed resources that are available for help. I forgot to mention another kind of resource -- the most important one -- YOU and your study team members! Parts of the textbook are written for you to work with other students. The authors <u>assume</u> that you will have other students with you so that you can talk about what you are doing, ask questions, and get immediate help from your partner or teammates. They also believe (and have read research that agrees) that talking about the math you are doing will help you learn it--even if you are the so-called "smartest" person in the class.

GET THE PHONE NUMBERS OF STUDENTS IN YOUR TEAM so that you have someone to talk to when you are doing work out of class. You may want to form study pairs with a friend who is also taking this course or study teams to work after school, in the evening, and/or on the weekend. The point is that there is plenty of help available if you take advantage of it. Caution! Be careful not to fall into any of the traps that some students think is the way to use their study team:

- letting the smart one do all the work and then copying his or her answers.

- doing the work on their own because they don't like working with others.

- visiting with their friends and then asking the teacher for help.

If you think like one of these students, you will have to change how you study in a team. Teams are there to use as a resource. Let your friends help you understand things that you do not, and in turn you can help them with things they need. EVERYONE SHOULD CONTRIBUTE TO THE WORK OF THE TEAM.

MISSION POSSIBLE (something to do)

Discuss with your partner or teammates several ways for everyone to contribute to the daily classwork. Write a brief summary of your conclusions. When you are finished, continue with today's lesson.

SQ-37.   Suppose you toss three coins, a penny, a nickel and a dime. They might come up "heads, heads, heads" (H, H, H).

     a)   What other possible outcomes might occur? List all the possibilities you can find.

     b)   How can you be sure that you listed all possibilities in part (a)? Describe a strategy for checking your list.

     c)   Find the probability that <u>exactly</u> (only) one coin comes up "heads."

     d)   Find the probability that <u>at least</u> one (or more) coin(s) comes up "heads."

SQ-38.   You roll a die and it comes up a "6" three times in a row. What is the probability of rolling a "6" on the next toss?

SQ-39.   Add to your tool kit what you know about how to multiply and divide integers.

INTEGER MULTIPLICATION

Multiplication is repeated addition or subtraction in a problem with two factors: The first factor tells us how many groups we are adding (+) or subtracting (-). The second factor tells us how many are in each group and whether they are positive (+) or negative (-).

For example: (2)(3) means add 2 groups of 3 positive tiles.
                    (2)(-3) means add 2 groups of 3 negative tiles.
                    (-2)(3) means remove 2 groups of 3 positive tiles.
                    (-2)(-3) means remove 2 groups of 3 negative tiles.

Below are examples of the diagrams used to record multiplication.

Example: (2)(-5)

Start with a neutral field. Since the first factor is positive groups will be added to the neutral field. Build 2 groups of 5 negative tiles.
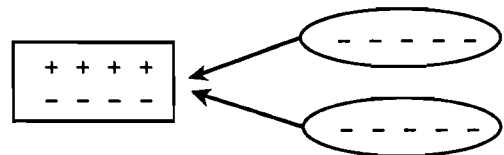
Physically push the groups into the neutral field lining up the positive and negative tiles.

(2)(-5)

Circle the "zeros".

Record the product.

(2)(-5) = -10

Example: (-2)(-5)

Start with a neutral field.

Since the first factor is negative, circle the two groups of negatives that will be removed. Use arrows to indicate removal.

Remove the groups.
Circle the "zeros" and record the product.
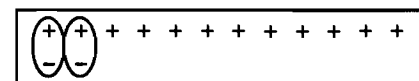
(-2)(-5) = 10

SQ-40.   Try these first, then check your answer with a calculator.

a) -50 - 30   e) $-\frac{1}{2} - 2\frac{3}{4}$

b) 4(-15)   f) $-\frac{1}{2} + (-\frac{1}{6})$

c) (-13)(-2)   g) (2.35)(-4.01)

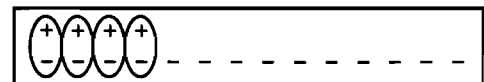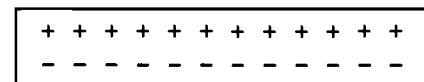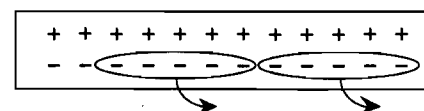d) -178 - (-3)   h) (0.005) ÷ (-0.021)

SQ-41.   Today, Latisha earned a score of 67. Figure out her
         average now. How much did her grade drop? Show
         your work and use your previous work from SQ-20
         and SQ-33 to help you.

SQ-42.   Write another fraction that is equivalent to $\frac{4}{5}$. Draw
         diagrams to show that they are equal.

   a)   Find the equivalent decimal for both fractions. Was rounding your answer
        necessary?

   b)   Find the equivalent percent for both fractions.

SQ-43.   Trace the number line below on your paper. Locate the following numbers by placing
         the lower case letters a through e on the number line corresponding to the values
         given below. Part (a) is done for you.

$$\overset{\textstyle \mathbf{a}}{\underset{\displaystyle \text{-5 -4 -3 -2 -1 \ 0 \ 1 \ 2 \ 3 \ 4 \ 5}}{\longleftarrow\!\!+\!+\!+\!+\!+\!+\!+\!+\!\bullet\!+\!+\!+\!\longrightarrow}}$$
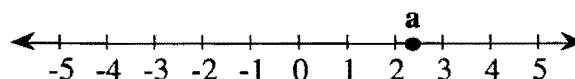
   a) $2\frac{1}{3}$ c) $-1\frac{1}{2}$ e) $33\frac{1}{3}$ % of 1

   b)   2.7   d)   -0.2

SQ-44.   Estimate the area of Montana
         and California using the grid.
         Which state is larger?
         Compare the area of Montana
         to the area of California.

SQ-45.   In the past, you have guessed at the answer to a problem and then figured out if you were right or wrong. In this algebra course, a method called "Guess and Check" is a first step to mastering the power of algebra and solving problems. The problems you do in class should be well organized and readable. THE ORGANIZATION IS IMPORTANT. It tells you how you checked your guess. Be neat and clear in your work, like the examples you will see in class. Otherwise, your work may not be acceptable, and it will certainly be less useful when we connect the process to algebraic methods. Show four guesses and checks for each problem, even if you get the answer sooner. Copy this example, starting with Step 5, in your tool kit.
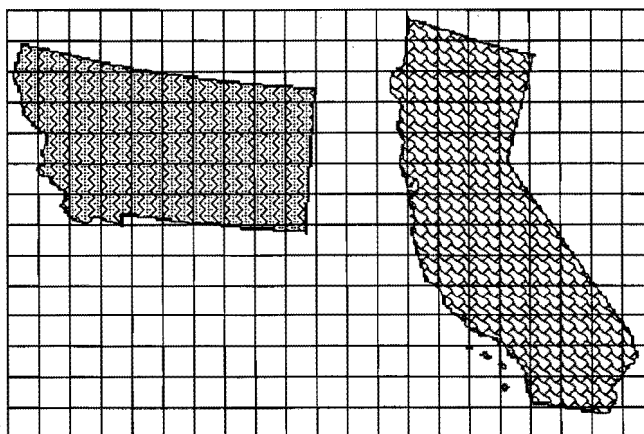

GUESS AND CHECK TABLES: AN EXAMPLE

The base of a rectangle is three centimeters more than twice the height. The perimeter is 60 centimeters. Use a Guess and Check table to find the base and height of the rectangle.

height

base

STEP 1   Draw a diagram and start a table. Why is the height a good choice to guess?

Guess Height

STEP 2   Make a Guess.

Guess Height	
10	

STEP 3   Calculate the base.

Guess Height	2(height) + 3 Base	
10		

STEP 4   Find the perimeter. Remember, rectangles have two "base" sides and two "height" sides.

Guess Height	2(height) + 3 Base	Perimeter = 2(Height) +2(Base)
10	2(10) + 3 = 23	2·10 + 2·23 = 66

STEP 5   Check the perimeter against 60 and identify it as correct, too high, or too low.

Guess Height	2(height) + 3 Base	Perimeter = 2(Height) +2(Base)	Check 60?
10	2(10) + 3 = 23	2·10 + 2·23 = 66	too high

STEP 6   Go back to Step 2 and make a new Guess. Use the previous guess to make a more accurate guess. Repeat this process until you find the solution, using a minimum of four guesses.

STEP 7   Write your answer to the problem in a complete sentence.

Solve each of the following problems by making a Guess and Check table with at least four guesses as modeled in the example above. The table for SQ-46 is started for you.

SQ-46. Find two consecutive odd numbers whose sum is 376.

First Odd Number	Second Odd Number	Sum	Check 376?
101	103	204	too low

SQ-47. One number is five more than a second number. The product of the numbers is 3,300. Find the two numbers.

SQ-48. The perimeter of a triangle is 76 centimeters. The second side is twice as long as the first side. The third side is four centimeters shorter than the second side. How long is each side?

SQ-49. Find four consecutive odd integers such that the sum of the second integer and twice the fourth integer is 65

SQ-50. Copy the axes at right and put a dot for:

a) Student A, who studies hard but gets only average grades.

b) Student B, who studies little but gets good grades.

Time Spent Studying

Grade as a Percentage
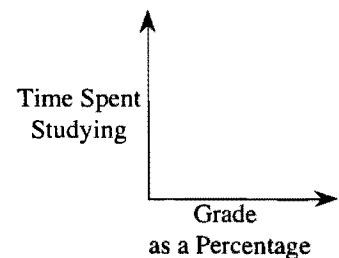
SQ-51. Copy and complete the drawing at right to show -3 + (-1).

+ + +
– – – – –
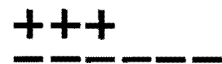
SQ-52. Latisha got a score of 95 today. Calculate her new average. How much did her average improve? Show your work. You may wish to refer back to SQ-41.

SQ-53. Compute:

a) -15 + 7

b) -50 - 30

c) 5.8 + (-6.37)

d) (-2)(-25)

e) (-6 + 17) - 20

f) 0.0001 ÷ 10

g) [-12+ (-18)] - 15

h) (4982)(-556)(0)

i) 0.0001 · 10

SQ-54. Draw a diagram that represents the quantity $\frac{1}{2} \cdot 3 \cdot 5$.

SQ-55.   Copy and complete the table.

10^1 10   ten
10^2 100   one hundred
10^3
10^4
10^6
10^9

SQ-56.   A scientific calculator is necessary for this problem. Do not attempt it with a graphing calculator.
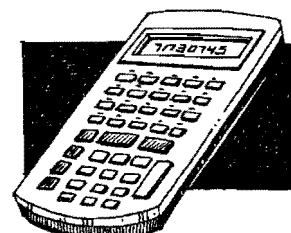
a)   Using a scientific calculator, enter a nine followed by all zeros. Write that number carefully on your paper, with commas. This is called **standard notation**. How many zeros does your number contain?

b)   Write its name. For example, 1,000 would be called "one thousand".

c)   On paper, multiply this number by two.

d)   On the calculator, multiply the original number by two. Copy the display on your paper. Why do you think your calculator displayed the answer that way?

e)   Compare the number of digits to the right of the one (1) on your paper with the calculator display. Write down your observations. The table in SQ-55 may help with your conclusion.

f)   What is the biggest number your calculator can hold? Compare your result with your team members.

SQ-57.   In the pattern at right, each figure is composed of tiles.

figure 1   figure 2   figure 3

a)   Copy the first three figures in the pattern and draw the fourth.
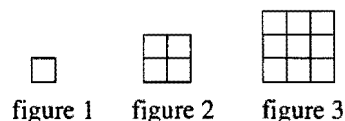
b)   Copy and complete the table:

figure #	1	2	3	4	5	6	10	23
# of tiles	1	4						

c)   Describe with words what pattern you used to fill in the table. Describe any other patterns you see.

d)   The sequence you have developed is a famous pattern called "Square Numbers". Why do you think the pattern has that name?

SQ-58.   In the pattern at right, each figure is composed of tiles.

figure 1   figure 2   figure 3

a)   Copy the first three figures in the pattern and draw the fourth.

b)   Copy and complete the table:

figure #	1	2	3	4	5	6	10	23
# of tiles	2	6						

c)   Describe with words what pattern you used to fill in the table. Describe any other patterns you see.

d)   The sequence you have developed is a famous pattern called "Rectangular Numbers". Why do you think the pattern has that name?

SQ-59.   In the pattern at right, each figure is composed of tiles.

figure 1   figure 2   figure 3

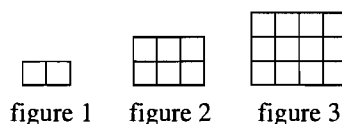a)   Copy the first three figures in the pattern and draw the fourth.
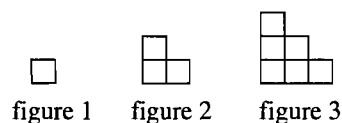
b)   Copy and complete the table:

figure #	1	2	3	4	5	6	10	23
# of tiles	1	3						

c)   Describe with words what pattern you used to fill in the table. Describe any other patterns you see.

d)   The sequence you have developed is a famous pattern called "Triangular Numbers." Why do you think the pattern has that name?

SQ-60.   Copy the tables below into your tool kit, listing at least ten numbers in each pattern.

FAMOUS PATTERNS

	1	2	3	4	5	6	7	8	9	10
The Square Numbers:	1	4	9							
The Rectangular Numbers:	2	6	12							
The Triangular Numbers:	1	3	6							

SQ-61.   Now examine your tool kit entry with the three famous number patterns. Do you see any relationships between them? Describe what you have found.

SQ-62.   Latisha earned an 85 today. Her previous scores were 72, 89, 90, 67, and 95. Calculate her new average.

SQ-63.   Use a Guess and Check table to find the solution and state it in a sentence.

Jabari is thinking of three numbers. The greatest is twice the least. The middle is three more than the least. The numbers total 75. Find the numbers.

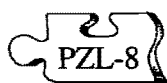SQ-64.   Use a Guess and Check table to find the solution and state it in a sentence.

The total cost for a chair, a desk, and a lamp is $562. The desk costs four times as much as the lamp and the chair costs $23 less than the desk. Find the cost of the chair and the desk.

INTRODUCTION TO VARIABLES AND COMBINING LIKE TERMS

PZL-8   UNDERSTANDING THE ROLE OF THE TEACHER

Earlier I mentioned that your teacher would use several methods to help you learn algebra. The idea is to use several approaches to give you more opportunities to understand algebra. In the past, you may have had math classes where the teacher almost always stood in front of the class and gave you instructions about how to complete different math problems.

In other words, the teacher **told you** exactly what to write, what to say, and what to think. This method of teaching leaves little room for you to think and understand. Being told what to do before understanding the reason(s) behind it makes concepts easy to forget. Past experience by students who learned through reasoning and discovery showed that it helped eliminate the tendency to merely memorize information and then see it quickly slip away.

Your teacher will usually give step by step instructions at key points in the course, frequently when tying together several days of class exploration.

In most cases, your teacher will ask questions designed to help YOU develop the steps yourself. The teacher will not stand up in front of the class and lecture you very often. Rather, you will usually work with a partner or in study teams and help each other. Therefore, try not to get mad at your teacher when you do not get a step by step explanation to your question. **Your instructor wants you to become a strong thinker, not a tape recorder.** You may get frustrated with this style at first, but over time you will see your confidence and success with math grow. The teacher will make sure your frustration does not become overwhelming. Open your mind and relax. Start thinking about the advantages this approach may offer you.

MISSION POSSIBLE (something to do)

a)   Summarize what you can expect from your teacher in this algebra course.

b)   Compare and contrast the above description (list similarities and differences) to the way class time has been used so far this year.

SQ-65. You have made or have been provided with sets of tiles of three sizes. We will call these "algebra tiles". Suppose the big square has a side length of x and the small square has a side length of 1. What is the area of:

a) the big square?   b) the rectangle?   c) the small square?

d) Trace one of each of the tiles in your tool kit. Mark the dimensions along the sides, then write the area of each tile in the center of the tile and circle it.

From now on we will name each tile by its area.
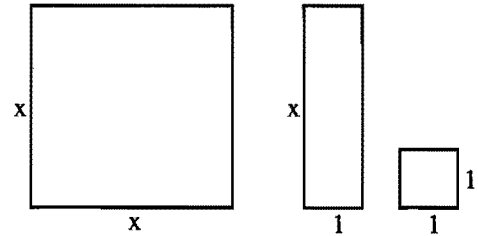
SQ-66. Check your results with your team members.

a) Find the areas of each tile in SQ-65 if x = 4. Find the areas of each tile if x = 6.

b) Why do you think x is called a variable?

SQ-67. Summarize the idea of Combining Like Terms in your tool kit. Then represent the following situations with an algebraic expression.

Combining tiles that have the same area to write a simpler expression is called **COMBINING LIKE TERMS.**

Example:
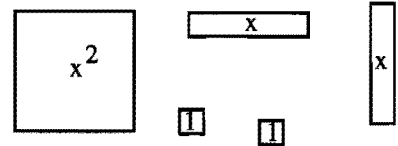
$x^2 + 2x + 2$

We write 2x to show 2(x) or 2 · x.

Represent each of the following situations with an algebraic expression.

a)   b)   c) 38 small squares
                                                            20 rectangles
                                                            5 large squares

d)

SQ-68.   You put your rectangle and two small squares with another pile of three rectangles and five small squares. What is in this new pile?

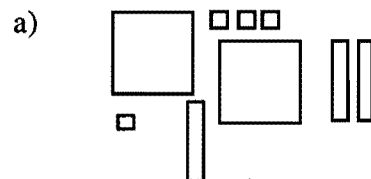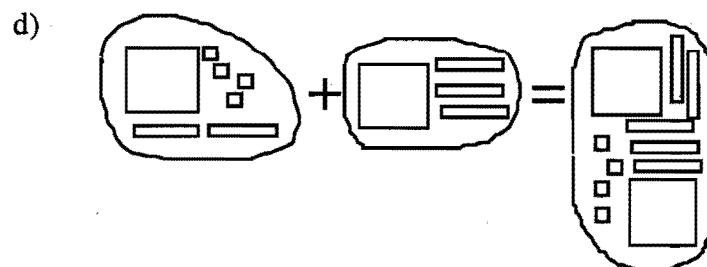SQ-69.   Suppose one person in your team has two big squares, three rectangles, and one small square on his desk and another person has one big square, five rectangles, and eight small squares on her desk. You decide to put all the tiles together on one desk. Write an algebraic equation that represents this situation. Your equation should look like

$($ ____ $) + ($ ____ $) =$ _____ .

SQ-70.   You are busy working on a problem with algebra tiles. You have three big squares, five rectangles, and 10 small squares on your desk when your friend leans over and borrows two big squares, two rectangles, and four small squares. Write an algebraic equation that represents the tiles you had, what your friend took, and the tiles you have left.

SQ-71.

Example: To show that $2x$ does not usually equal x^2, you need two rectangles and one big square.
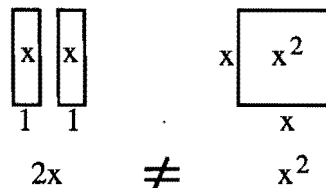
$$2x \neq x^2$$

a)   Show that $3x + x \neq 3x^2$.

b)   Show that $2x - x \neq 2$.

SQ-72.   Bob, Kris, Janelle, and Pat are in a study team. Bob, Kris, and Janelle have algebra tiles on their desks. Bob has two big squares, four rectangles, and seven small squares; Kris has one big square and five small squares; and Janelle has 10 rectangles and three small squares. Pat's desk is empty. The team decides to put all of the tiles from the three desks onto Pat's desk. Write an algebraic equation that represents this situation.

SQ-73.   Another time you were working with one big square, seven rectangles, and six small squares when the teacher walked by and knocked the big square on the floor and the wind blew through and scattered five of the small squares all over the room. Write an equation to represent the tiles you had, what you lost, and the tiles that remained.

SQ-74. Use a Guess and Check table to solve the problem below. Then state your solution in a sentence.

Mairé is thinking of two numbers. When she adds them, she gets 40. When she multiplies them, she gets 351. Help her younger sister, Enya, figure out the numbers.

SQ-75. Latisha is really motivated. She earned a 92 on her test this time.

a) Calculate her grade now. (Recall her past scores were 72, 89, 90, 67, 95, 85.)

b) She has one more grade before the progress report grades are calculated. Figure out what grade she needs to earn to have at least an 85% average. Show your work.

c) Add what you know about **averaging numbers** to your tool kit.

GUESS AND CHECK TABLES: MORE PRACTICE

Make a Guess and Check table to solve problems SQ-76 through SQ-79. Then state your solution in a sentence.

SQ-76. Duke cleared the cash register of nickels and dimes. There were twenty coins in all, and the total value of the coins was $1.35. How many of each type of coin were in the cash register?

The table has been created for you. Copy it and add guesses of your own to solve the problem.

Number of Nickels	Number of Dimes	Value of Nickels	Value of Dimes	Number of Coins	Value of Coins	Check $1.35?
10	10	$0.50	$1.00	20	$1.50	too high

SQ-77. The drama department at Galileo High is having a production. Tickets cost $3 for members of the student body and $5 for anyone else. A total of 515 tickets were sold, bringing in $1785. How many student body members attended?

SQ-78. A cable 84 meters long is cut into two pieces so that one piece is 18 meters longer than the other. Find the length of each piece of cable.

SQ-79. Janelle has only quarters, dimes and pennies in her pocket. There are eight coins in all and their total value is 83 cents.

    a)   How many of each kind of coin does Janelle have?

    b)   If she takes one coin out of her pocket, what is the probability that it is a dime?

SQ-80. Latisha's friend Brandee forgot to make up a test and had these scores: 70, 92, 91, 71, 89, 84, 0, 85.

    a)   Calculate Brandee's average. Does this average score really represent her abilities? Why or why not?

    b)   Brandee persuaded Ms. Speedi to allow her to make up the missed test. However, it had been awhile since the original test date. Brandee received a 60. Calculate her new average.

    c)   What difference did the 0 score make? Does this new average better represent Brandee's ability?

SQ-81. Caitlin was minding her own business using her algebra tiles -- one big square, four rectangles, and five small squares -- when Sean came in and added four rectangles and two of the small squares. Write an equation to represent the tiles Caitlin started with, what Sean added, and the tiles Caitlin had after Sean left.

SQ-82. Copy and solve these Diamond Problems:
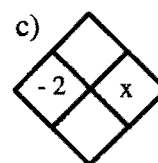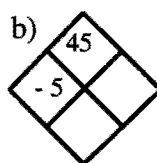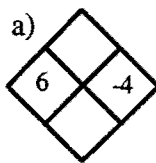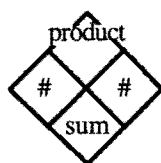
SQ-83. Use a Guess and Check table to find the solution and state it in a sentence.

The sum of two numbers is 111. The greater exceeds the lesser by 17. What are the two numbers?

SQ-84. Latisha earned an 88 as her last grade before progress reports were calculated. Calculate her average. You may wish to refer to SQ-75. Explain what percent grade Latisha has now.

DIFFERENCE OF SQUARES

SQ-85.   DIFFERENCE OF TWO SQUARES

In this unit you have organized data, described patterns with diagrams and words, and talked and worked together in a study team. You now have a problem to investigate that will require all of those skills. This is a big problem that should need the entire team's effort in order to complete it successfully.

a)   Study the examples in the table.

NUMBER	SOLUTIONS(Difference of Squares)
8	$3^2 - 1^2$
15	$8^2 - 7^2; 4^2 - 1^2$

b)   Express the numbers 1 through 25 as a difference of two squares. Some numbers have more than one solution as shown with the number 15. Some numbers might have no solution. Find as many solutions for each number as your team can and organize them neatly and clearly on a chart.

c)   Describe as many patterns as you can find.

d)   Write an explanation of how your study team went about solving this problem. How did you start? Did someone find a technique that helped you find more solutions? Did you use patterns or guess and check to help? How did you decide what your chart would look like?

e)   Prepare a presentation to share your results with the class. Everyone in your study team must be part of the presentation. Your presentation should include an introduction, your chart(s), explanations of your patterns, and an explanation of how your team solved the problem. (Hint: it is important for the class to be able to read your results and see your patterns as you speak. Therefore be aware that your chart needs to be written large enough to be seen easily by everyone.)

Make sure each person in your study team understands what he or she needs to complete before coming to the next class. Your team will need to present your results then.

f)   Extension: Can you describe a way to find solutions for any given number? For example, can you write 43, 99, or 60 as a difference of two squares? If so, explain how you found this general solution and include it in your presentation.

SQ-86.   Compute:

a)   -3 + 8

b)   -12 - 8

c)   17 + (-19)

d)   (-2)(-12)

e) $\frac{1}{3}(-6)$

f)   (-6)(14)

g)   - 4 + 9 - 11

h)   (-1)(-1)(-1)

i)   -15 + 15

SQ-87.   You were minding your own business using your algebra tiles: two big squares, four rectangles, and three small squares. Tom came in and took the four rectangles and two of the small squares. Write an equation to represent the tiles you started with, what Tom took, and the tiles you had after Tom left.

SQ-88.   Copy and complete the drawing at right to show -3 + (-4).   **+ + +**
                                                                      — — — — — —
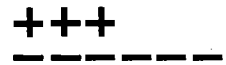
SQ-89.   Make a Guess and Check table to solve and state your solution in a sentence.

          Todd is 10 years older than Jamal. The sum of their ages is 64. How old are Todd and Jamal?

SQ-90.   Write a team and learning reflection for Unit One. Use complete sentences and write as honestly as you can. Use the questions below as a guide. Include any other information you like.

          a)   **Study Team Reflection:** Who was in your study team and how did your team work? Be as detailed as possible. What exactly did you do to help someone? Who helped you and how did they do so? What talents did you find you had? Describe a different talent someone else had. What was disappointing about your team? What are important team skills that help a team work effectively? What team skills do you personally want to work on in your next team?

          b)   **Learning Reflection:** What part of your learning do you feel good about? What is still difficult for you? What learning are you proudest of? Did you do all your assignments? Did you participate in class discussions? What study skill do you want to improve in the next unit? Describe your plan to do so.

PZL-9   **REMINDERS:**   WHERE TO GET INFORMATION and
                             WHAT TO DO IF YOU FALL BEHIND

          Before you start the next problem, go back and review PZL-6. Use the list you made to help assemble a COMPLETE took kit for Unit 1 as directed in the next problem.

          By the way, some of you may have had a hard time getting started with this course and are beginning to fall behind. I have good news for you! The information contained in PZL-6 tells you where to look to catch up quickly. **Now I have to warn you: the number one rule for studying CPM math is DO NOT GET BEHIND!** You will have to study every day. In particular, homework is an essential part of your studies. This means that if you are behind, **use your time wisely.** Do not spend hours doing all the problems when you can choose a select few to get the main ideas. After you understand all the main ideas you can go back for more practice if you have time. Be honest about your level of understanding. If you understand, move on. If you do not, go back and practice.

SQ-91.   TOOL KIT CHECK-UP

Your tool kit contains reference tools for algebra. Return to your tool kit entries. You may need to revise or add entries.

Be sure that your tool kit contains entries for all of the items listed below. Add any topics that are missing to your tool kit NOW, as well as any other items that will help you in your study of algebra.

- Definition of Probability
- Diamond Problems
- Integer Multiplication and Division
- Famous Patterns
- Averaging Numbers

- Area and Perimeter of a Rectangle
- Addition and Subtraction of Integers
- Guess and Check
- Algebra Tiles with Labels
- Combining Like Terms

To The Students

*Gold Medal problems present you with an opportunity to investigate complex, interesting problems over several days. The purpose is to focus on the process of solving complex problems. **You will be evaluated on your ability to show, explain, and justify your work and thoughts.** Save all your work, including what does not work in order to write about the processes you used to reach your answer.*

Completion of a Gold Medal Problem includes four parts:

- ***Problem Statement:*** *State the problem clearly in your own words so that anyone reading your paper will understand the problem you intend to solve.*

- ***Process and Solutions:*** *Describe in detail your thinking and reasoning as you worked from start to finish. Explain your solution and how you know it is correct. Add diagrams when it helps your explanation. Include things that did not work and changes you made along the way. If you did not complete this problem, describe what you do know and where and why you are stuck.*

- ***Reflection:*** *Reflect on your learning and your reaction to the problem. What mathematics did you learn from it? What did you learn about your math problem solving strategies? Is this problem similar to any other problems you have done before? If yes, how?*

- ***Attached work:*** *Include all your work and notes. Your scratch work is important because it is a record of your thinking. Do not throw anything away.*

GM-1.   GOING BANANAS!

Cleopatra ("Cleo") the Camel works for
the owner of a small, remote banana
plantation. This year's harvest consists of
three thousand bananas. Cleo can carry up
to one thousand bananas at a time. The
market place where the bananas are sold is
one thousand miles away. Unfortunately,
Cleo eats one banana each and every mile
she walks.

Your Task:

Of the three thousand bananas harvested,
what is the largest number of bananas Cleo
can get to market?

P.S. This problem is not impossible!

GM-2.   RUMORS, RUMORS, RUMORS!

Part 1

Burton High School has 1,500 students.
During first period, a rumor is started
when Susan tells three friends a secret.
Each of Susan's three friends tell three
of their friends during second period,
who in turn tell three different friends
during third period. Assume each
person tells the rumor to only three
others.

Part 2

In an attempt to discredit the President, a member of the opposite political
party wishes to start a scandalous rumor about him. He plans to tell people
this rumor one week before election day and wants the entire population of
the United States (that's 250,000,000 people!) to know the rumor. He needs
to figure out how many people he should tell (and they each will tell the same
number of people) so that everyone will have heard the rumor by the seventh
day.

Your Task:

• Find the number of students who have heard the rumor at the end of the day.
  (Burton has 6 periods.)

• Find the number of people the evil politician needs to tell so everyone in the
  United States will have heard the rumor by the seventh day. Remember: each
  person needs to tell the same number of people.