The origins and development of mathematical notation

(A historical outline)

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In the vast part of mathematical community including circles with highest education there is many time rather vagous imagination about the time of appearance, origin and development of mathematical notation. It contributes hardly to the spread of the enlightenment in this domain also the system of mathematical education in which these topics are often laid on the background of educational program or they absent in it at all. Sometimes fragmentary and simplified notes on them (as e. g. an assertion that François Viète is author of mathematical notation) aid very little to the creation of a real image on the lengthy and intricated historical process by which the notation has reached current fashion. The knowledge of historical background of these topics also has a strong didactic aspect: the knowledge of obstacles in the historical process of the formation of mathematical notation stimulates to the concentrated and premeditated effort in the tuition and acquisition of this matter.

The most conspicuous parts of mathematical symbolism in every culture are numerals and the mode of number record. However this theme is so extensive and widespread that it deserves a separate treatise and in the frame of a complete information on all mathematical notation would take too great area and could shift aside the parts of the same importance. Therefore the investigation in this paper will be concentrated on the notation of important relations, operations and objects in the arithmetic, algebra, analysis and geometry. The layout of observation will be chronological principally, only in individual historical periods the subclassification will be arranged by main domains of mathematics.

An informative paper of a basic character cannot exhaust the theme which are a thousand and one pages in monographies and journal articles devoted to. A skilled reader finds hardly new information in it, however the paper can contribute to the precision of chronological sequence of appearance of some symbols.

I. Mathematical notation in the ancient mathematics

1. Sporadic appearance of symbols

There was no stable system of signs for arithmetic relations and operations in the mathematics of ancient Egypt and Mesopotamia. Sporadically the signs occurred that were either an ideogram of an action or words of natural language expressing an operation. In the hieroglyphic transcription of the Ahmes' papyrus from the 18th century B. C. (called also Rhind's or London papyrus) e. g. the operations of addition and subtraction of natural numbers are illustrated in a unique case by a pair of legs walking forward and away respectively (Figure 1a, 1b). In Mesopotanian mathematical texts on the cuneiform clay tables from the period of Old Babylonian empire the operation of addition is represented by the wording "tab" and the subtraction by the wording "lal" with the graphic record in cuneiform script illustrated in Figure 1c. The signs had not always the character of an obligatory standard; this holds particularly for Egypt where in the Moscow papyrus (from the 18th century B. C., called by the place of deposit) the sign of addition is used in the sense of the second power.

Fig. 1

2. The notation in the antique Greek mathematics

The ancient Greek mathematics has elaborated no systematic and standardized arithmetic-algebraic symbolism before Diophantus. The single cases of abbreviation of the full-meaning word connecting the addition of two terms with the result of the operation have been not developed into a generally used norm. Also the system of writing numbers and their placing in arithmetic operations had no positive influence on the creating of the notations; full-meaning words of the natural language were usually used for to denote an action.

Diophantus' notation (*Diophantus* was living in Alexandria about 250 A. D.) is based prevailingly on the syncopate principle. The denotations of the unknown, of its powers and of the unique standardly denoted operation of the subtraction - or more exactly - of the terms with negative coefficients in an expression - are created by the choice of a characteristic letter or syllable of the word denoting the relevant mathematical object, relation or operation in natural or slightly terminologically adapted language.

Diophantus' system of creating symbols is obvious from the following overview table containing symbols of the constant term and all powers of the unknown in equations occurred at Diophantus.

Sign	**Wording**	**Meaning today**
M^o	Μόνας (monas) = unity	constant term (in an equation)
ς	'Αριθμος (arithmos) = number	x (unknown)
Δ^Y ($\delta^{\bar{\upsilon}}$)	Δύναμις (dynamis) = power	x^2
K^Y ($\kappa^{\bar{\upsilon}}$)	Κύβος (kybos) = cube	x^3
$\Delta\Delta^Y$ ($\delta\delta^{\bar{\upsilon}}$)	Δύναμοδύναμις (dynamodynamis)	x^4
ΔK^Y ($\delta\kappa^{\bar{\upsilon}}$)	Δύναμοκύβος (dynamokybos)	x^5
KK^Y ($\kappa\kappa^{\bar{\upsilon}}$)	Κύβοκύβος (kybokybos)	x^6
$ς^{\bar{\kappa}}$	'Αριθμοστον (arithmoston)	$\frac{1}{x}$
Δ^{YK} ($\delta^{\bar{\upsilon}\bar{\kappa}}$)	Δύναμοστον (dynamoston)	$\frac{1}{x^2}$

The set of all terms with negative (integer) coefficients in an equation Diophantus introduces by the sign ⋔ what is likely the stylized inverted letter ψ with the top shortened (the stylization of the inverted form is ⋒, further ⋔) from the word λειψις with the meaning "to be lacking in".

For equality the sign in the archetypal manuscripts seems to have been ι^{σ} deduced from the word ισος (isos = equal); but this sign has been deformed by copyists to a form which was sometimes confused with the sign ч. Similarly, various speculative conjectures on the sign of division are based more on the deformations caused by copyists than on original sources.

Excessive connection of the Diophantus' notation with his algebra of equations in his work *Arithmetica* and the fact that no scientific school was established which would develop systematically Diophantus' work, caused that no general standard of arithmetic and algebraic notation had been developed from Diophantus' symbolism. European mathematics proceeded hard to the Diophantus' level in the notation in the 15th and 16th century but after reaching this level it has achieved relatively quickly a generalization and higher principles of creating on which are based foundations of modern notation.

Special chapters of the ancient Greek mathematical symbolism are represented by Aristotle's notation of general number quantities by capital letters and Euclid's notation of segments (in the sense of their length) by small letters as well as by the symbolism of geometric algebra based on this principle. Both Aristotle's and Euclid's principles were totally unknown in the primitive mathematics of early feudalism in Europe and the European mathematics had to overwork himself to them difficultly and unskillfully in later centuries (e. g. Fibonacci and Iordanus Nemorarius in the 13th century).

II. Mathematical notation until 17th century

In various world regions in the 6th till 15th century the principles were slowly formed at which foundations of a modern mathematical notation have been worked out, that are used with certain modifications also in modern today mathematics. A decay of the European science, culture and education in the second half of the first millennium have caused that the European mathematics played in that process for many centuries an inferior and even neglectable role and it rose to new success especially under the influence of the higher level of mathematics in other cultures of the world.

1. India

a) Anonymous manuscript Bakhshālī (about 200 A.D. or 6th - 7th century)

The manuscript was excavated near the village Bakhshālī in the northwestern India in 1881. Its symbolism is strikingly based on the syncopate principle but it differs - especially phonetically - from the later works of medieval Indian mathematics which were constructed continually by various authors in a historical sequence. The kernel of the symbolism in this manuscript is the denotation of basic arithmetic and algebraic relations and operations by abbreviations but the consequent systematic use is missing.

The following table contains an overview of symbols.

Sign	**Denotation**	**Meaning**
yu	yuta (=added)	addition
xa	xaya (=subtracted)	subtraction
gu	guna (=multiplied)	multiplication

bhā	bhāga (=divided)	division
mū	mūla (=root)	square root
pha	phalah (=equal)	equality

The notation of unknown quantities appears only in one case and it is expressed by ordinal numerals.

b) The notation from the 6th till 12th century

In the works of Bhāskara I (6th century), Brahmagupta (598-about 660; the work Brahma-sphuta-siddhānta - Brahma correct system, 628), Bhāskara II (1114 - after 1178; the work Siddhānta Śīrōmani - Head jewel of accuracy, about 1150), Nārāyana (the 14th century) and other Indian mathematicians a little attention is paid to the notation of operations and relations but a greater stress is put on the notation of unknown quantities and their powers. It followed from the needs of algebra that was a superior science to arithmetic and was in a dominant position in the Indian medieval mathematics.

The syncopate principle is all the time of basic importance; however it already has the character of the stable standard and authors of later periods respect the usage in the notation of their predecessors.

A selection of this notation brings the following table.

Sign	**Wording**	**Meaning**
rū	rūpa (=shape; appearance)	constant term (in an equation)
yā	yāvat-tāvat (=so much-how much)	the first unknown
cā	cālaca (=black)	the second unknown
nī	nīlaca (=blue)	the third unknown
pī	pītaca (=yellow)	the fourth unknown
lo	lohitaca (=red)	the fifth unknown
ha	harītaca (=green)	the sixth unknown
va	varga (=square)	the square, the second power
gha	ghana (=solid, cube)	the cube, the third power
va-va	varga - varga	the fourth power
va-gha-ghāta	varga - ghana - ghāta	the fifth power
va-gha	varga - ghana	the sixth power
va-va-gha-ghāta	varga - varga - ghana - ghāta	the seventh power
va-va-va-	varga - varga - varga	the eighth power
gha-gha	ghana - ghana	the ninth power
ca	caranī (=root)	the square root
	varga - mūla (mūla = root)	the square root
	ghana - mūla	the third root

Although this symbolism could seem primitive and cumbersome, it represented in the time of its origin a top level of the abstraction. It is also evident from the fact that the outstanding mathematicians in Arab and Central Asia countries have not comprehended its importance and didn't adopt it and used further in their works a verbal (rhetorical) algebraic terminology.

2. China

The Chinese medieval mathematics has achieved no systematic notation. Its expressive algorithmic character with the stress on numerical computation led to a stable record in schemas in which the position and arranging determined the kind and order of operations. In such schemas special signs for operations were not necessary. (E. g. also in the contemporary elementary school the settlement of two numbers into two rows setting the digits of the same decimal order into the same column denotes the addition without any additional sign.)

E. g. *Chu Shih-chieh* represents the expression $a+b+c+d$ and its square in the book *The precious mirror of the four elements* (1303) by the schemas

The rigidity of the writing system of numbers and operations has become an obstacle of the progress in later centuries of the Middle Ages.

3. Islamic countries

There is no arithmetic-algebraic notation in the works of mathematicians of the East Arabic Empire from al-Khowārizmī till al-Kāshī. Among the mathematicians of the West Arabic empire al-Hassār has contributed to the mathematical notation by introducing of fractional line in the 12th century and al-Qalasādī (died in 1486) has created a developed mathematical symbolism (see Fig. 2) in the last quarter of the 15th century. In the same time started an intensive process of the creation and development of the mathematical notation in the arithmetic and algebra also in the European mathematics. The sophisticated character of the al-Qalasādī's notation justifies a well-founded conjecture that it is a continuation and completion of the contributions of several preceding authors. Their works are unknown and from significant authorities only al-Bannā (the 13th - 14th century) is mentioned to whom Ibn Khaldūn (14th century) has accredited the use of algebraic symbols serving to "abstract considerations" as well as to "expressive shaping".

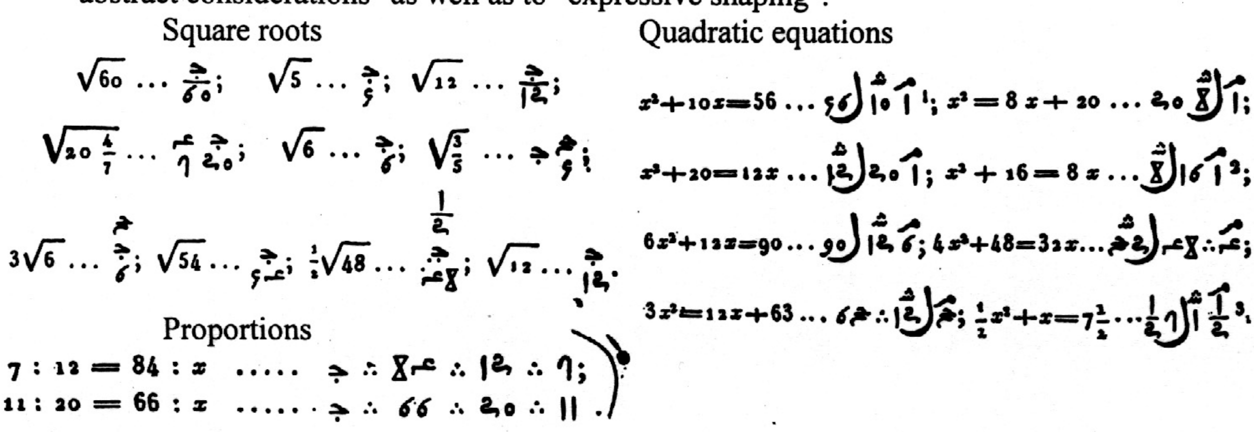


Fig. 2

4. Europe

The early feudal utilitarianly oriented European mathematics, very distant from the scientific level of antiquity, had no chance to develop systematically the mathematical notation e. g. from the Diophantus' level. A very tiny improvement of Diophantus, more in the method as in the content, was the introduction of "the first undescribed" power x^5 and of " the second undescribed" power x^7 at the Byzantine intellectual *Michael Psellus* (the 11th century) as an expression of the fact that the prime number 5 and 7 are not a product of two proper divisors as well as a methodological modification of Diophantus' procedure of solving equations by the writing of single steps of solving into separate rows at *Maximus Planudes* (the 14th century).

In the European mathematics of the 13th century two personalities appeared which have influenced further development of algebra and indicated some future tendencies in the sphere of mathematical notation. *Iordanus Nemorarius* (died 1237) was representative of university course among mathematical authors and the aim of his numerous works was didactic most of all. His arithmetic writings *Arithmetica in decem libris demonstrata* (The arithmetic demonstrated in ten books) and the algebraic treatise in four volumes *De numeris datis* (On given numbers) present systematical use of letters in the sense of numbers. In contrast to the accidental use of such notation at some preceding authors the Nemorarius' use is regular and stable, i. e. the same letter denotes in a problem or example the same number. Symbolic notations of operations and relations absent at Nemorarius what requires denoting every intermediate result of the arithmetic-algebraic process by a new letter. An example of the use is illustrated by the following fragment from Nemorarius' text: "Let . *a*. be a square, . *b*. its root, let . *b*. be multiplied by . *c*. *d*. , . *c* . and . *d* . being its halves; further let . *b*. time . *c*. *d* . equals to . *e*. and let . *a* . *e* . be given . Etc."

The second personality whose importance has overstepped the bounds of the 13th century was *Leonardo of Pisa - Fibonacci* (about 1170 -past 1240). In his work *Liber abaci* (The book on abacus, 1202) which is a detailed manual of contemporary arithmetic and algebra at the top level as well as in other arithmetic-algebraic works besides the mediating of mathematical knowledge of another cultures he also takes on some methods of its notation or revives previous European traditions occasionally. According to al-Hassār he uses the fractional line and in a stable way he also writes mixed numbers, only in the reverse order to our mode of writing, i. e. he writes $\frac{1}{3}14$ instead of our writing $14\frac{1}{3}$. (It shows evidently the influence of Arabic sources.) He applies letters to the denotation of segments - in the sense of their length - in a double mode: either *.a.* denotes the length of the segment denoted by *a* (Fig. 3a) or *.b.g.* denotes the length of the segment with boundary points *b* and *g* (Fig. 3b)

a/ b/

Fig.3

In the spirit of Arab mathematics from which Fibonacci drew a considerable part of his algebraic topics, he replaced the names of terms in equations by its Latin equivalents:

Notation	**Meaning**
numerus, denarius	the number, the absolute term
res, radix	the unknown, the root
census, quadratus	the square of the unknown
cubus	the cube (the third power) of the unknown
census census	the fourth power of the unknown
cubus cubus, census census census	the sixth power of the unknown
census census census census	the eighth power of the unknown

Otherwise, Leonardo of Pisa's mathematical writings are almost wholly rhetorical in mode of exposition. Some germs of syncopate method as e. g. R for radix (the square root), .R. ^{x}. p^{i}. Bino^{ij}. for *primi binomii radix* (the square root of the first binomial) etc. appeared in the works *Practica geometriae* and *Flos* but no concise syncopate system was created.

Nicole Oresme (1323 - 1382) was the first to conceive the notion of powers with rational exponents in the treatise *Algorismus proportionum* (The algorithm of proportions), and was introducing an original notation of these objects. The meaning of the signs is illustrated by the following examples:

$$\boxed{\frac{p\;1}{1\;2}} = 1\tfrac{1}{2}, \quad \boxed{\frac{p\;1}{1\;3}} = 1\tfrac{1}{3}, \quad \boxed{\frac{1\cdot p}{2\cdot 2}} = 2^{\frac{1}{2}}, \quad \boxed{\frac{1\cdot p\cdot 1}{4\cdot 2\cdot 2}} = \left(2\tfrac{1}{2}\right)^{\frac{1}{4}}$$

Oresme's discovery evoked no adequate reception and his notation has found no direct successors.

Nicolas Chuquet (died 1500) has presented relatively compact and systematic notation in the work *Le triparty en la science des numbers* (The science of numbers in three volumes, 1484). The work was written in French and although it was not printed and published its essential ideas were mediated for the general public in the book of *Estienne de la Roche L'arismethique* in 1520. Chuquet appropriated many signs from the works of his predecessors especially from the Italian milieu but he enriched the formation of the symbolism by some original ideas which modified were included into the foundations of later modern notations. The following table illustrated some standard signs of Chuquet's notations.

Sign	**Wording**	**Meaning**
$\tilde{p}$	plus	+
$\tilde{m}$	moins (minus)	-
a,e.i. 7, 12	racine première	number (the first root), the absolute term
℞^{2}	racine seconde	the square root

℞^{3}	racine troisième	the cube root
etc.		etc.
a^n, e. g. 8^3		$a.x^n$, e.g. $8x^3$
$\underline{a}$		*(a)*

E. g. the expression ℞^{2}14p̃℞^{2}180 means $\sqrt{14} + \sqrt{180}$, the expression $R^2\underline{4^2\tilde{p}6^1}\tilde{m}3^1\tilde{p}1$ égaux à 100 means $\sqrt{4x^2+6x} - 3x + 1 = 100$. It is obvious from this example that the underlining indicates the aggregation.

The employ and further development of mathematical relations in the region of Italy and Middle Europe in the 16th century *Luca Pacioli* (about 1445 - about 1514) influenced in an essential way. In his collecting work *Summa de arithmetica, geometria, proportioni et proportionalita* (Summa of arithmetic, geometry, ratios and proportions, published in Venice in 1494) which served as an encyclopaedia of contemporary mathematical knowledge for several decades he created an ample mathematical symbolism based on the syncopate principle, generally acknowledged, accepted and employed by several generations of successors (Francesco Ghaligai, Hieronimo (Girolamo) Cardano, Niccoló Tartaglia, Rafaele Bombelli etc.). The following table contains an overview of the main Pacioli's symbols.

Sign	**Wording**	**Meaning**
p̃	più (plus)	+
m̃	meno (minus)	-
co.	cosa	the unknown (thing), x
ce.	censo	x^2
cu.	cubo	x^3
ce. ce.	censo de censo	x^4
p^o. r^o.	primo relato	x^5
ce. cu.	censo de cubo	x^6
2^o. r^o.	secondo relato	x^7
ce. ce. ce.	censo de censo de censo	x^8
cu. cu.	cubo de cubo	x^9
3^o. r^o.	terzo relato	x^{11}
etc.		
q p^a	quantita, cosa seconda	the second unknown, y
ce. de qp^a	censo de quantita	y^2
n^o.	numero	the number, the absolute term
℞, ℞2	radice (radix) seconda	the square root
℞ cuba, ℞3	radice terza	the cube root, the cubic root
℞ ℞ , ℞4	radice de radice	the fourth root
℞ relato	radice relato	the fifth root
V	universale	(...) (the parenthesis)

The following example illustrates the using of notations: ℞ V60m̃℞250 means $\sqrt{60-\sqrt{250}}$.

Slight modifications and simplification of the Pacioli's notation were brought in the work of *R. Bombelli Algebra* (Venice 1572, Bologna 1576). The following table gives an information about their form and extent.

Sign	Wording	Meaning
p	più (plus)	+
m	meno (minus)	-
R. q.	radice quadrata	the square root
R. c.	radice cubica	the cube root, the cubic root
R. R. q.	radice radice quadrata	the fourth root
R. p. r.	radice prima relata	the fifth root
R. q. c.	radice quadrata cubica	the sixth root
R. s. r.	radice seconda relata	the seventh root
R. ⌊...⌋	radice legata	the taking of the root of the polynomial in the bracket ⌊...⌋

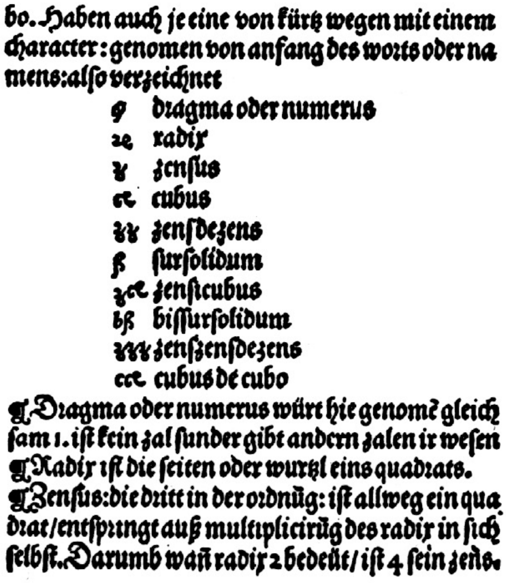
In the Bombelli's manuscript also the sign = appears in the meaning of the notation of equality. It indicates that Italians have reached this sign independently of Recorde and likely before him in view of the fact that Bombelli's manuscript was finished twenty years before its publication.

The signs + and - to denote operations of addition and subtraction appeared for the first time in the book of *Johann Widmann Behennde unnd hübsche Rechnung auff allen Kauffmanschaften* (Nimble and nice computation for all merchants, Leipzig 1489). The earliest manuscript appearance is noted in Dressden in 1486.

An unambiguous connection with the Pacioli's notations is documented by the German cossic symbolism presented in the book of *Christoff Rudolff Behend unnd Hubsch Rechnung durch die kunstreichen regeln Algebre so gemcinicklich die Coss genennt werden* (Nimble and nice computation by clever rules of algebra which usually are called Coss, Strassburg 1525). The following table exhibits basic signs of Rudolff's book.

Sign	Wording	Meaning
φ	dragma, numerus	the number, the absolute term
ꝛ	radix	the root, the unknown, x
ʒ	zensus (census)	the square, x^2
ce	cubus	the cube, x^3
ʒʒ	zensdezens	the fourth power, x^4
β	sursolidum	the fifth power, x^5
ʒce	zensicubus	the sixth power, x^6
bβ	bissursolidum	the seventh power, x^7
ʒʒʒ	zenszensdezens	the eighth power, x^8
cce	cubus de cubo	the ninth power, x^9
q	quantita	the second unknown, y
√	(the stylized letter ꝛ = r - radix)	the square root
w√		the cubic root
√√		the fourth root

The original shape of Rudolff's signs is exhibited on the copies of pages in his book (Fig. 4 and 5).

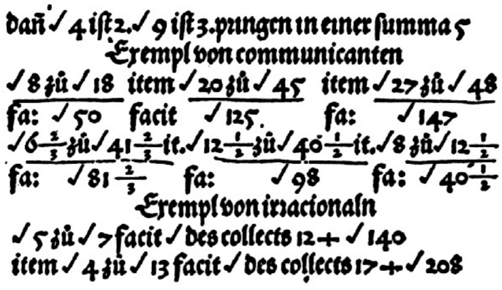


bo. Haben auch je eine von kürtz wegen mit einem character: genomen von anfang des worts oder namens: also verzeichnet

dragma oder numerus
radix
zensus
cubus
zensdezens
sursolidum
zensicubus
bissursolidum
zenszensdezens
cubus de cubo

Dragma oder numerus würt hie genomē gleichsam 1. ist kein zal sunder gibt andern zalen ir wesen
Radix ist die seiten oder wurtzl eins quadrats.
Zensus: die dritt in der ordnūg: ist allweg ein quadrat/entspringt auß multiplicirūg des radix in sich selbst. Darumb wañ radix 2 bedeüt/ ist 4 sein zens.

Fig.4



dañ $\sqrt{4}$ ist 2. $\sqrt{9}$ ist 3. pringen in einer summa 5
Exempl von communicanten
$\sqrt{8}$ zü $\sqrt{18}$ item $\sqrt{20}$ zü $\sqrt{45}$ item $\sqrt{27}$ zü $\sqrt{48}$
fa: $\sqrt{50}$ facit $\sqrt{125}$. fa: $\sqrt{147}$
$\sqrt{6\frac{2}{3}}$ zü $\sqrt{41\frac{2}{3}}$ it. $\sqrt{12\frac{1}{2}}$ zü $\sqrt{40\frac{1}{2}}$ it. $\sqrt{8}$ zü $\sqrt{12\frac{1}{2}}$
fa: $\sqrt{81\frac{2}{3}}$ fa: $\sqrt{98}$ fa: $\sqrt{40\frac{1}{2}}$
Exempl von irracionaln
$\sqrt{5}$ zü $\sqrt{7}$ facit $\sqrt{}$ des collects 12 + $\sqrt{140}$
item $\sqrt{4}$ zü $\sqrt{13}$ facit $\sqrt{}$ des collects 17 + $\sqrt{208}$

Fig.5

The slight modifications of the cossic symbolism appeared in the German milieu in the publications by *Michael Stifel* (the 16th century), *Johannes Scheubel* (the 16th century) and *Christophorus Clavius* (the 17th century). E. g. Stifel suggested the sign $\sqrt{\mathfrak{z}}$ for the square root and $\sqrt{\mathfrak{c}}$ for the cubic root, $\overset{0}{1}$, $1\overset{1}{A}$, $1\overset{2}{A}A$, $1A\overset{3}{A}A$, $1A\overset{4}{A}AA$, ... for the 0th, 1st, 2nd, 3rd, 4th, ... power of the number A respectively. Clavius wrote consequently the signs + and - and used amply parentheses.

The 16th and 17th centuries were in Europe a period of an intensive search for specific expressive means of mathematics especially of arithmetic and algebra which were intended from the rhetorical and syncopate mode of exposition evermore to the symbolic notation. To these tendencies it also belongs the simplified writing of the fractional part of a decimal number due *Simon Stevin* who suggested in the brochure *De Thiende* (The one-tenth, Leiden 1585) writing negative order of the digit in the fractional part with omitting of the sign - under the relevant digit; e. g. the number 2913, 7122 is written in the Stevin's mode in the form

2913 7 1 2 2
⓪①②③④

In the French translation of the Stevin's treatise in 1634 the order is analogously written into the row behind the relevant digit; the mentioned number is written in this mode in the form

2 9 1 3⓪7①1②2③2④

The decimal point (in the form of a dot or comma) came into use step by step in the 17th century.

The expansion of printing has supported the increase of the number of published mathematical works. Many authors attempted with great effort to create and standardized mathematical notations. However, only 2 - 3 decades of them were successful. E. g. the attempts of *Marco Aurel* (the 16th century), *Pedro Nuñez* (the 16th century) and *Albert Girard* (the 17th century) had sunk into oblivion without any trace. From the algebraic work of *Robert Recorde The Whetstone of Witte* (London

1557) only the sign of equality = has found its place in the mathematical notation. It was about five times longer than today sign.

From a plenty of European algebraists in 16th century such as *John Dee, Leonard* and *Thomas Digges, Thomas Masterson, Jacques Peletier, Jean Buteon, Guillaume Gosselin* etc., *François Viète* (1540 - 1603) has at the most contributed to the development and standardization of the mathematical notation. In his at most important work *In artem analyticem isagoge* (Introduction into analytical art, Tours 1511) he has formulated several new principles of the algebraic notation which modified has become stable standard. In the field of notation of arithmetic relations and operations has brought no revolutionary changes: the signs of addition and subtraction are + and - respectively, the multiplication is denoted by the preposition *in* (A in B meant A.B), the division by the fractional line, the powers denoted by words: C quadratus meant C^2, D cubus D^3. He wrote the equality by words *aequatur* (= is equal), *aequantur* (= are equal). The personal Viète's contribution was the introduction of the standard notation of unknowns by the capital vowels (A, E, I, O, U, Y) and of general numbers (constants, coefficients, values of quantities) by the capital consonants (B, C, D, F, G, H. J, K, L, M, N, P, ...). The further development involved these signs in various modifications but the fundamental ideas stayed valid. The Viète's mode to denote the arithmetic difference by the sign = was not successful. (The arithmetical difference A = B meant the result of subtraction despite of the sign of quantities A, B, i. e. the absolute value of the difference A - B.)

It was *William Oughtred* (1573/5 - 1660) who highly contributed to the spread and propagation of the mathematical notation in the Viète's spirit. In his works *Clavis mathematicae* (The key of mathematics, London 1631, 1648, 1652 and many other editions including English translations), *The Circles of Proportion* (London 1632, 1633) and *Trigonometria* (The Trigonometry, London 1657; in Latin; also English translation) he has introduced over 150 mathematical signs mostly appropriated from other authors. This plurality of sources has also found its expression in a nonstability and variability of signs from which he used many ones in several variants. The stable standard signs in our today meaning were = , + , - , × (the sign of multiplication), the fractional line as the sign of the division. The dot and comma varied in the position of a decimal point at decimals, as well as the forms Aq, AA for A^2, AAA or A[3] for A^3, *V*, *Vq* for the square root, *V[n]*, $V\boxed{n}$ ($n \geq 3$) for the *n*th root. The swinging of use also held for the denotation of trigonometric functions, e. g. *s, sin* for the function sine, *t, tan* for the function tangent, *se, sec* for the function secant. The syncopate symbols *s co, t co, se co* belonged to the functions cosine, cotangent and cosecant. From the other important symbols is to mention log, Lo for the logarithm, Gr for degree, || for the parallelism, < and > for inequalities, *Tri, tri* for a triangle etc. The signs for inequality came from the book of *Thomas Harriot* (17th century) *Artis analyticae praxis* (The practice of the analytical art, London 1631) where also the Recorde's sign for equality was contained.

Many symbols of the arithmetic, algebra, geometry and logic were contained in the work of *Pierre Hérigon Cursus mathematicus* (Mathematical course, Paris 1634, 1644). At Hérigon the sign ~ was a symbol for the subtraction (instead of our -), .~: for the difference, 5 < for the pentagon, a2, a3, a4 for the powers a^2, a^3, a^4, == for the parallelism, ⊥ for the orthogonality, 2/2 for the equality, 3/2 for the inequality >, 2/3 for <, < and ∠ for the angle, ⌋ for the right angle, ⊙ for the circle or circular disc, Δ for the triangle, □ for the square, ▭ for the rectangle etc. Only a few Hérigon's symbols are developed into standard ones, many of them are lost without any trace and some of them survive up to our days as substandard (working) denotations.

René Descartes (1596 - 1650) contributed to the simplification of several symbols and to the convergence of the usage to today norm by the writing *La géométrie* (The geometry, 1637). He has introduced the notation of unknowns and variables by the small letters *x, y, z, w* from the end of the alphabet and of known constants by the small letters *a, b, c, d, e* ... from the beginning of the alphabet, the writing of powers in accord with today mode (a^2, a^3, a^4 etc.), though by persisting in the tradition he further wrote a a for the square a^2. Descartes' sign ∝ for the equality was not accepted. *Johann Heinrich Rahn* used for the first time in 1659 the sign * for the operation of multiplication and ÷ for the operation of division; it is interesting that these signs appear in today computer software in an unchanged form.

The sign ∞ for the infinity was used for the first time by *John Wallis* in 1665 in the book *Arithmetica infinitorum* (The arithmetic of infinites) for to denote the "first" member of the reverse sequence to the sequence 0, 1, 2, ...

Mathematical journals contributed significantly to the unification of the mathematical notation and to its standardization. They strained by program to simplify the notation and to clear its structure from various grounds, especially typographical ones. It is conceivable that the influence of chief editor's and publisher's personalities was enforced. E. g. the proclamation of the editorial board of the journal *Acta eruditorum* announces in Leipzig in 1708 the acceptation and obligatory character of the Leibniz' symbolism which introduces normatively among the hints the use of parentheses instead of the horizontal line over the expression (the writing (aa + bb) instead of $\overline{aa + bb}$, the sign : or the fractional line for the operation of division (a : b = $\frac{a}{b}$), a new mode of denoting the proportion (a: b = c : d instead of a.b :: c.d), a new mode of writing of powers and roots of polynomials ($(aa + bb)^n$ instead of $\overline{aa+bb}^{\,n}$, $(aa + bb)^{n:m}$ instead of $\sqrt[m]{\overline{aa+bb}^{\,n}}$).

Similarly, the journal *Miscellanea Berolinensia* codifies Leibniz' notation essentially accordant with the symbols today in recommendations to authors in 1710. It introduces new signs ~ and ≃ for similitude and congruence, respectively; these signs acquitted themselves well in the mathematical community and they were accepted relatively without delay because of their simplicity and easy remembering. On the other side the authors of recommendations were not able to escape fully from the capture of the tradition by letting some speculative symbols; e. g. the sign :: for so called continued proportions (:: a, b, c, d meant a:b:c:d).

III. Mathematical notation in the 18th and 19th century

1. Symbols of important constants

The number π considered as the ratio $\frac{O}{d}$ where O denotes the perimeter and d the length of the diameter of a circle, had no stable denotation a long time. *J. Ch.*

Sturm has denoted it by the letter *e* in 1689. The denotation $\frac{\pi}{\delta}$ for $\frac{O}{d}$ has at W. Oughtred in 1652 obviously a syncopate character: π and δ are initial letters of the Greek words to the denotation of the perimeter and diameter (περιμετρος and διαμετρος respectively). *Wiliam Jones* has used for this ratio the denotation π in 1706. In the same meaning was used the symbol π by *L. Euler* (1736, 1737 and further years), *Johann Bernoulli* (1739 and further), *Nicolas Bernoulli* (1742 and further). It was definitively anchored in the Euler's book *Introductio in analysin infinitorum* (Introduction to the analysis of infinitives, 1748).

Leibniz has denoted the number *e* by the letter *b*. The Euler's symbol *e* used by Euler in 1727 - 8 and 1736 won a general acceptance and in later years it got a character of the standard.

2. The symbolism of the theoretical arithmetic, algebra and number theory

Most symbols in these domains have their origin at pioneers of these subjects and at their significant successors as were Leibniz, Cramer, Vandermonde and others in the early period and during the 18th century as well as Euler, Legendre, Gauss, Jacobi, Dirichlet, Cayley, Kronecker, Weierstrass, Cantor, Peano etc. The original symbol of a leading personality was replaced not once by a more convenient, lifelike and acceptable one, which was suggested by a less important successor.

From generally known and spread symbols e. g. the sign ≡ for the congruence of integers was used for the first time by Gauss in 1801, the sign Σ of the sum by Euler in 1755, the today symbol $\binom{m}{n}$ of the binomial number by Ettingshausen in 1827, the sign *m!* of the factorial by Ch. Kramp in 1808, the symbol of permutation by Cauchy in 1815, the denotation $|\ldots|$ of the determinant by Cayley in 1841, the denotation $\|\ldots\|$ for matrices by Cayley in 1843 and 1845, the denotation (...) for matrices was used by Bôcher (1919), Kowalewski (1909), the denotation [...] by Cullis (1913); the symbols $\aleph_0$ and $\aleph_1$ were introduced by Cantor in 1895, the symbols ω, Ω were used by Cantor and Schoenflies.

Some other symbols of the elementary character

Logarithm: Napier used no abbreviations for logarithm. The abbreviations Log, log., l, lg soon appeared in the works of his contemporaries. E. g. Kepler used the abbreviation Log in 1624 as well as Briggs and Oughtred which also used the denotations log., Lo. According to the Leibniz' symbolism in Acta eruditorum the sign *l* was used for the natural logarithm and *L* for the logarithm to any base $b > 1$. It was also variable the writing of the symbol for the base in the complete symbol of the logarithm. There were problems, especially at the tabulating, with the writing of number values of logarithms (and not only of mantissas) in the form of integers and with the writing of negative values of logarithms.
Logarithm of a complex number:
In 1829 M. Ohm denoted any value of the logarithm of a complex number by the symbol *log* and the principal value by *L*. In 1842 De Morgan denoted the principal

(tabular) value of the logarithm by the abbreviation *log* and reserved the sign λx for any solution of the equation $e^{\lambda x} = x$. At Peano in 1903 the symbol log x denoted the principal value of the logarithm of the complex number x and the symbol $\log^* x$ denoted a class of solutions of the equation $e^y = x$.

The symbol ~ : It has a multifarious meaning:

- Historically it denoted the absolute value of the difference of two numbers.
- It denotes the similitude in the geometry.
- It is one of the sign denoting an approximate equality in numerical disciplines.
- It denotes the equivalence in some theories (e. g. at Kronecker and the members of his school.)

Absolute value: Weierstrass used the sign | | in 1841.

Imaginary unit:

It appeared for the first time at Cardano under the expression R̸m̃.1. Bombelli wrote it in the form R⌊ 0 m. 1 ⌋, Girard in the form V - 1. Euler used the writing $\sqrt{-1}$, since 1777 the symbol i . Gauss is gone in 1801 from the expression $\sqrt{-1}$ to the symbol i. Cauchy used this symbol since 1847 when it already was generally known and spread.

Vector: The symbolism was created gradually from the beginning of the 19th century and was swinging through all forms used up to our days.

The attempts on the unification

The origin and development of new mathematical subjects and the scattering of the symbolism in them in the time of an unprecedented increase of the communication means by the accessibility of printed monographs, text-books and journals at the and of the 19th century have led several prominent scientists and organizers of the mathematical life to the effort to unify the mathematical notation at least in the chief domains of the mathematics. Besides individual attempts to unify the notation in the whole mathematics on the base of the symbolism of the mathematical logic (Peano, Russel, Whitehead) there existed programmed institutionalized steps directed to the solving of these problems. The first attempt in 1895 in the frame of the International Association for Promoting the Study of Quaternions and Allied Systems of Mathematics has been ended after a hopeful start of the discussion without concrete results. The similar fate met also other private and institutionalized initiatives from the years 1903 (L. Prandtl in Gottingen) and 1908 appointed even by a decision of the International Congress of Mathematicians to prepare the suggestions to the next congress in Cambridge in 1912. Sometimes stormy and pointed discussions were not able to overcome irreconcilable positions of the representatives of several national schools and despite the historically objective benefit for the advance of the science from the potential unification no agreement on the unification was closed.

3. Notation in the trigonometry

This domain has origins of its notation in the Old Ages. Ptolemy (the 2nd century) called the degree μοιραι (moirai; plural - μοιρον - moiron) and denoted it by the sign μ^o. The smaller units of the angle measure were the minute with the sign ′ and the second with the sign ".

The European mathematics in the Middle Ages appropriated trigonometric terms from the Arab mathematics through a mechanical translation into Latin of Arab terms which were mostly translated into Arab language (partially incorrectly) from the ancient Greek and Indian astronomy. The units of the angle measure were *signa* (the abbreviation Sig.), *gradus* (Gr.), *minuta* (Min.) and *secunda* (Sec.) etc.; signa occurred sporadically until 18th century. Regiomontanus (15th century) used for the degree the symbol $\overline{\ }$, for the minute ' and for the second the symbol ". The symbol ° for the degree was used by J. Peletier for the first time in a manuscript in 1558 and in a published book in 1569. The unit *radian* for the arc measure of the angle was introduced by James Thomson (lord Kelvin) in 1873 and is denoted by the signs ρ, R, r, $^{(r)}$.

The denominations of the trigonometric functions came into European in Latin written mathematics from the Arab mathematics where some of them were appropriated from the Indian mathematics and astronomy. So the words *sinus, cosinus* and *sinus versus* came in the middle period of the Middle Ages. Th. Finck accepted the denominations of the functions tangent and secant in a translation from the Arab originals in 1583. Tycho de Brahe has taken them from him in 1591. Finck used the abbreviations of the trigonometric functions in a form near to the contemporary notation.

The using of the symbols for the trigonometric functions was stabilized somehow in the 18th century, e. g. at Euler about 1750. It is not fully unified up to our days, there are slight national differences in the notation.

4. Notation of the analysis

In the creation of the system of symbols in the analysis *Gottfried Wilhelm Leibniz* (1646 - 1716) played a key role. His multisided education, the status of an acknowledged scientist and his generalizing philosophic-logical approach enabled him to judge the new formed branch of mathematics from the higher point of view, to estimate its tendencies and possibilities and from this viewpoint also to approach to the search of principles of the creation of its notation. The foundations of this notation were created at Leibniz likely already in 1675.

The development of the Leibniz symbolism was continued by the classics of the analysis in the 18th and 19th century.

Some Leibniz' symbols:

d*x* - differential of the variable *x*; $\frac{d\overline{y}}{d\overline{x}}, \frac{dy}{dx}$, d*y*:d*x* - derivative of the function *y* (dependent on *x*) by *x*

dd*x* - second differential (differential of the second order); ddd*y*, d^3y- third differential (differential of the third order)

d^my - *m*-th differential (differential of the *m* -th order)

$\int$ - integral

We can see that the symbols are no speculative products but they are created by rule on the base of the syncopate principle respecting the progress in the notation made in 16th and 17th century. The sign for the integral is the capital letter S written in a temporal usage as the initial letter of the word *summa* (=sum) from which the integral was deduced. Leibniz suggested a denomination *calculus summatorius* for the integral calculus. The Johann Bernoulli's denomination *calculus integralis* was adopted. Nevertheless his suggestion to denote the integral by the sign *I* or *J* has fallen into oblivion. Just so his original denotation of partial derivatives of the function *m (x, y)* by symbols δm, ϑm, respectively, has been changed historically into $\frac{\partial m}{\partial x}, \frac{\partial m}{\partial y}$, respectively.

Notation of the *Newton' s theory of fluxions*:
$\overset{\shortmid\shortmid}{x}, \overset{\shortmid}{x}, x, \dot{x}, \ddot{x}, \dddot{x}, \ldots$ is a denotation of a sequence of *fluents* (=of functions of one variable), from which every following one is a *fluxia* of the predeceding one.

The dominance of the Leibniz ' symbolism over the Newton's notation on the Continent was supported sizeable by the publishing policy where the deciding editorial positions in mathematical journals and publishing houses were occupied by the Leibniz' followers so that journals, monographs and especially text-books preferred Leibniz' notation.

In the domain of the symbolism for the functions of several variables the 18th century was a period of the search. A. M. Legendre used the effective symbols $\frac{\partial v}{\partial x}, \frac{\partial v}{\partial y}$ for the partial derivatives of the function *v(x, y)* for the first time in 1786. C.G.J. Jacobi contributed importantly to their spread in 19th century.

Conclusions

1. The progress in the mathematical notation especially in the domain of the arithmetic and algebra played a progressive role in the history of mathematics not only in methods but also in the content. A long-time latent or apparent struggle with the charge of the rhetorical description preceded the crucial qualitative breaks in the content and methods of those domains. The creation and application of the syncopate and later sophisticated notation brought an arrangement into the structure of objects and of relations among them, gave an opportunity to formulate problems clearly and to deprive them of the vagueness of the verbal expression and so to form a preparatory grade of the effective solution. The lengthiness of the historical way to this stage indicates an objectively complicated process of the creation and perfection of the symbolism and gives an undoubted witness on its gradually abstract character. A didactic aspect of this feature is particularly important: in the process of tuition it is already to keep in mind that mathematical symbols - and it concerns especially those ones at which is lost the consciousness of a connection between the form and any concrete content - were historically achieved in mathematics by a concentrated effort of its most significant representatives by stubborn struggles and often many failures. It is to keep in view that also at pupils the process of the adopt of the notation is no problemless and simple phenomenon.

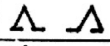
2. The secondary school usually comprehends mathematics as a completed and closed science. A look beyond this horizon opens a fascinating panorama of a

dynamic and proceeding science. In its new branches also new notation arises. Also in classical domains a revaluation and reconstruction of the foundations is no rare phenomenon. This affects also the symbolism in them. New demands for the mathematical notation are also brought from the side of technical means of which role has become steadily more important not only in communication but also in a direct entrance into mathematical activities. This all, together with the qualitative changes in the logical foundations of mathematics which happened in two first decades of the 20th century as a consequence of the turbulence at the turn of the centuries, puts the mathematical logic into a position of a full-value partner of the scientific content of mathematics. The creation of the mathematical notation stops to be an irrelevant unrestrained phenomenon and becomes a directed and sophisticated process. It is a task of the mathematics to defend the inner consistency of its notation as a form of the mathematical content against the compulsion of requirements of the technical character.

3. The compulsions to unify the mathematical notation come from several directions. The most important reason for the unification is one of communication: a stable unified standard of symbols facilitates past the appropriation and automation the position of participants of the communication - both active and passive. The second aspect is technical-technological: in the era of the communication explosion the simplicity and uniformity of the notation is an unneglectable factor of reducing economic charges. Perhaps it is one of the main motives for the effort to codify the frequent mathematical notation as an obligatory technological norm: International norm Quantities and units - Part 11: Mathematical signs for use in physical sciences and technology 1992, and derived national norms. Maybe it is reasonable to have an understanding for such an attitude and to respect justified requirements. But it will be hardly created a norm at some other time that could span the whole richness and variability of the mathematical notation.

Appendix

Chronological table

Date	Locus Author	Sign	Meaning	Note
Period of rhetorical notation				
about –1800	Egypt Ahmes		+, -	(only one example)
about -2300 till -1600	Babylon	tab  (lal)	+ -	

-5th century	Greece	AB	segment (length)	
2nd century	Alexandria Ptolemy	μ^o, ' , "	degree, minute, second	angle measure
12th century	West Arab Empire al-Hassār	–	fractional line	
1202 1220 about 1225	Italy Leonardo of Pisa	– .a. , .b.c. .R^x. (abbr.radix)	fractional line segment (length) square root	germs of the syncopate and symbolic notation
before 1237	Germany, France Iordanus Nemorarius	.a. , .b.c. etc.	natural numbers	
about 1350	France N. Oresme	1·p·1 / 4·2·2	$(2\frac{1}{2})^{\frac{1}{4}}$	powers with rational exponents

Period of syncopate notation

about 250	Alexandria Diophantus	ς , Δ^Y , K^Y , etc. ⋔	x, x^2 , x^3 , etc. (powers of the unknown) –	germs of systematic syncopate notation relation
about 200 or 6th-7th century	Bakhshālī India	yu, xa, etc.	arithmetical operations, square root and equality	
6th-12th century	India	rū, jā, kā, etc,	absolute term, powers of the unknown	standard syncopate notation
1303	China Chu Shi-Chieh	schemas	arithmetical operations with polynomials	
15th century (before 1486)	West Arab Empire al-Qualasādī		syncopate symbolism of arithmetic and algebra	
1484	France N. Chuquet	p̄, m̄, R^2 , R^3 , a ^{n} etc.	+, -, $\sqrt{\ }$, $\sqrt[3]{\ }$, a.x^n etc.	
1487	Italy L. Pacioli	p̄, m̄, co.,ce.,cu. etc. R 2, R 3, R relato etc.	+, -, x, x^2, x^3 , etc.	
1489	Germany J. Widmann	+, -	+, -	the first signs of symbolic notation
1525	Germany Ch. Rudolff	[illegible] etc.	a, x, x ^{2} etc.	continued abbreviation of syncopate symbols, germ of symbolic notation
1544	Germany M. Stifel	1, 1A, 1AA, 1AAA etc.	*n*-th power of the number A	
1572	Italy R. Bombelli	p, m, R.q. etc.	+, -, $\sqrt{\ }$ etc.	

Period of symbolic notation

1557	England R. Recorde	=	=	
1569	France J. Peletier	°	degree	
1585	Holland S. Stevin	2 9 1 3 7 1 2 2 ⓪①②③④ or 2913⓪7①1②2③2④	2913.7122	writing of decimal numbers
1591	France F. Viète	A, E, I, O, U, Y B,C,D,G,F,H,…X, Y,Z	unknowns constants	
1617	England J. Napier	, and .	decimal separatrix	

1624	Germany J. Kepler	Log.	logarithm	
1631	England W. Oughtred	× s, sin; t, tan; se s co; t co; se co Log. , log	multiplication sine, tangent, secant cosine, cotangent, cosecant logarithm	
1631	England Th. Harriot	< , >	inequalities	
1637	France R. Descartes	a^2 , a^3 , a^4 ,... x, y, z a, b, c, d, etc.	*n*-th power of the number a unknowns constants	
1657	England J. Wallis	∞	infinity	
about 1665 - 1687	England I. Newton	$\dot{x}$, $\ddot{x}$, $\dddot{x}$ etc.	derivatives	
about 1680	Germany G. W. Leibniz	$\frac{dy}{dx}$ dx, ddx, d^2x, d^3x,... $\int$	derivative differentials integral	
1706	England W. Jones	π	number π	
1708	Germany Acta eruditorum	( ) a : b = c : d	parentheses proportion	
1710	Germany Miscellanea Berolinensia	~ $\simeq$	similitude congruence (in geometry)	
1727-8	Switzerland L. Euler	*e*	number *e*	
1755	Germany L. Euler	$\sum$	sum	
1777	Russia L. Euler	i	imaginary unit	
1786	France A. M. Legendre	$\frac{\partial v}{\partial x}, \frac{\partial v}{\partial y}$	partial derivatives	
1801	Germany C. F. Gauss	$\equiv$	congruence (in arithmetic)	
1808	Germany, France Ch. Kramp	m!	factorial	
1815	France A. L. Cauchy	$\begin{pmatrix} 1 & 2 & \ldots \\ a_1 & a_2 & \end{pmatrix}$	permutation	
1827	Austria A. v.Ettingshausen	$\binom{m}{n}$	binomial number	
1841 1843-5	England A. Cayley	| ... | || ||	determinant matrix	( ) Kowalewski 1909 Bôcher 1919 [ ] Cullis 1913
1882	Germany G. Cantor	ω	first ordinal number of the second class	
1895	Germany G. Cantor	$\aleph_0$ $\aleph_1$	the smallest transfinite cardinal number the next superior cardinal number	

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Summary

The paper sketches the development of the mathematical notation from the origins of mathematics till the contemporary stage in the elementary domains of the mathematics. The contribution of various world regions to the creation of the mathematical symbolism in the periods of the rhetorical, syncopate and symbolic notation is described. The most part is devoted to the decisive role of the European mathematics in the elaboration of the modern notation in main branches of the elementary and higher mathematics. The appendix exhibits the chronological succession of the first appearance of symbols.

Résumé

Cet article présente une description courte de la développement de la notation mathématique dépuis des origines des mathématiques à l´état contemporain dans les domaines élémentaires des mathématiques. Il décrit la contribution des régions variées du monde à la création du symbolisme mathématique à la période rhétorique, syncopique et symbolique. La plupart d´article est consacrée à la rôle décisive des mathématiques européens en l´élaboration du symbolisme moderne dans les domaines principales des mathématiques élémentaires et des mathématiques plus hautes. On présente dans l´annexe la succession chronologique de l´apparence première des symbols.

Riassunto

L´articolo offre un compendio dello sviluppo dello simbolismo matematico nelle matematiche elementari dalle origine della matematica ad oggi. Si presentano i contributi dei vari regioni mondiali alla creazione del linguaggio simbolico matematico in tre periodi storici: fase retorica, fase sincopata e fase simbolica. Il lavoro è dedicato per lo più allo ruolo decisivo della matematica europea nel elaborazione dello simbolismo contemporeano nei rami fondamentali della matematica elementare e matematica superiore. L´appendice presenta una successione cronologica delle prime apparizioni dei simboli.

Zhrnutie

V článku sa načrtáva vývoj matematickej symboliky od vzniku matematiky po súčasný stav v elementárnych oblastiach matematiky. Opisuje sa prínos rôznych svetových regiónov k tvorbe matematickej symboliky v období rétorického, synkopického a symbolického označovania. Prevažná časť sa venuje rozhodujúcej úlohe európskej matematiky pri vypracúvaní modernej symboliky v hlavných odvetviach elementárnej a vyššej matematiky. V dodatku sa prezentuje chronologická postupnosť prvého výskytu symbolov.