BritGrav15, University of Birmingham, 2014

The self-force problem: local behaviour of the Detweiler-Whiting singular field

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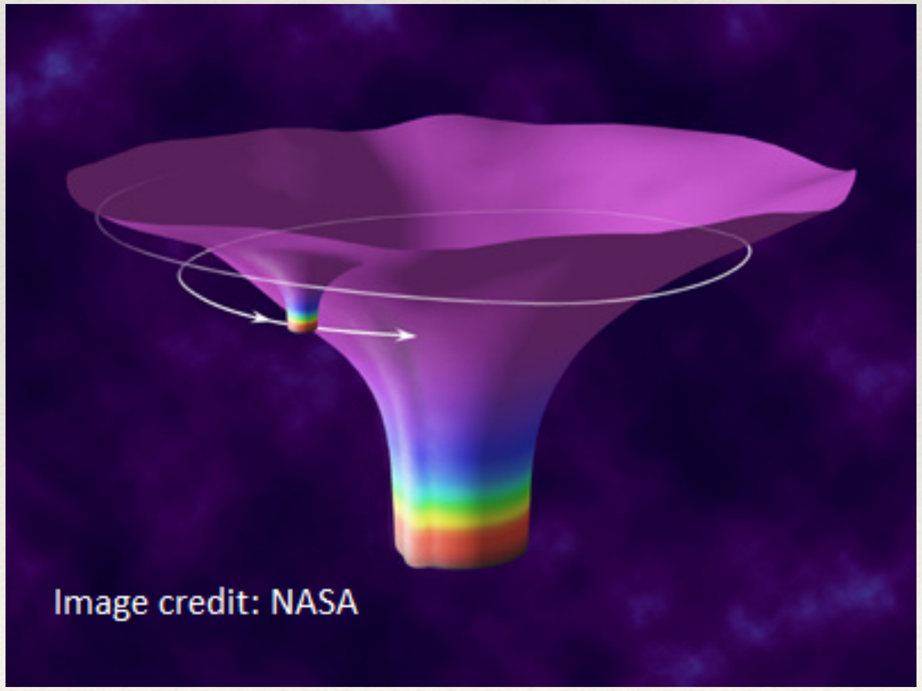
Supervisor: Adrian Ottewill
Collaborator: Barry Wardell

http://arxiv.org/abs/1403.6177

A. Heffernan, A. Ottewill, B. Wardell, Phys. Rev. D 82, 104023 (2012)
A. Heffernan, A. Ottewill, B. Wardell, Phys. Rev. D 89, 024030 (2014)

The self-force



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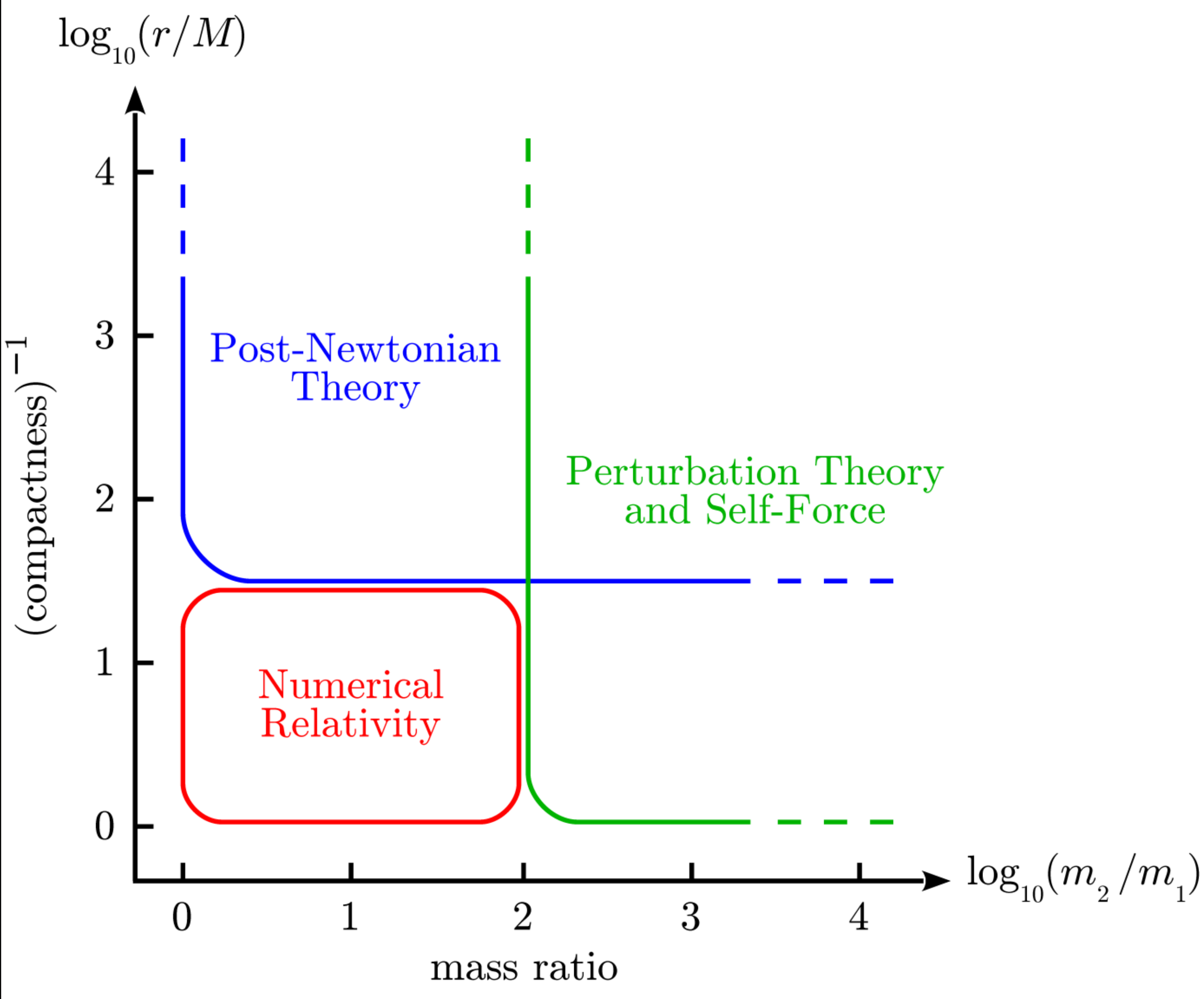
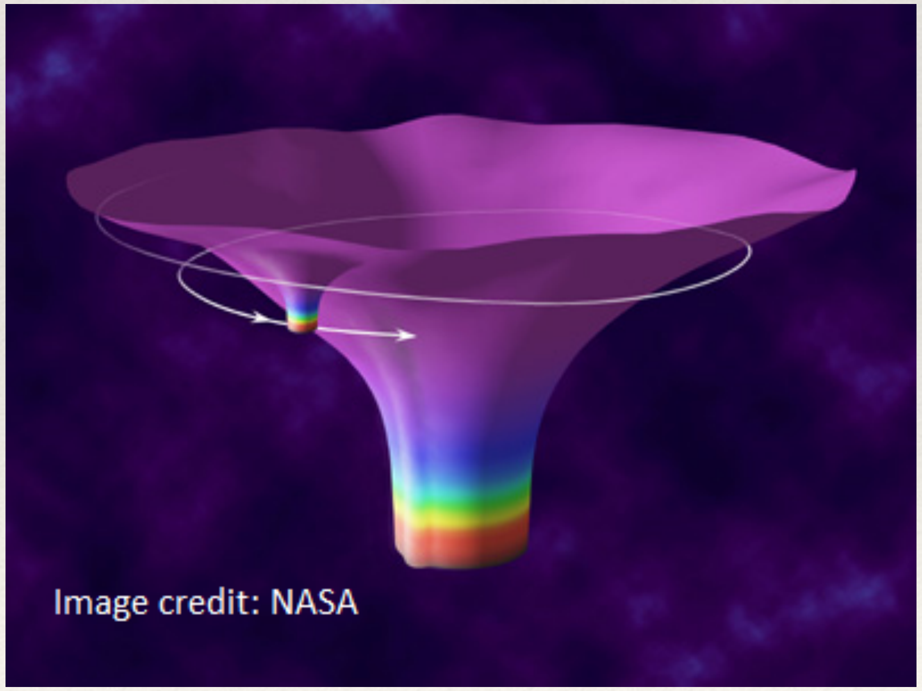


Diagram: Alex Le Tiec, Int. J. Mod. Phys. D. 23:1430022, 2014



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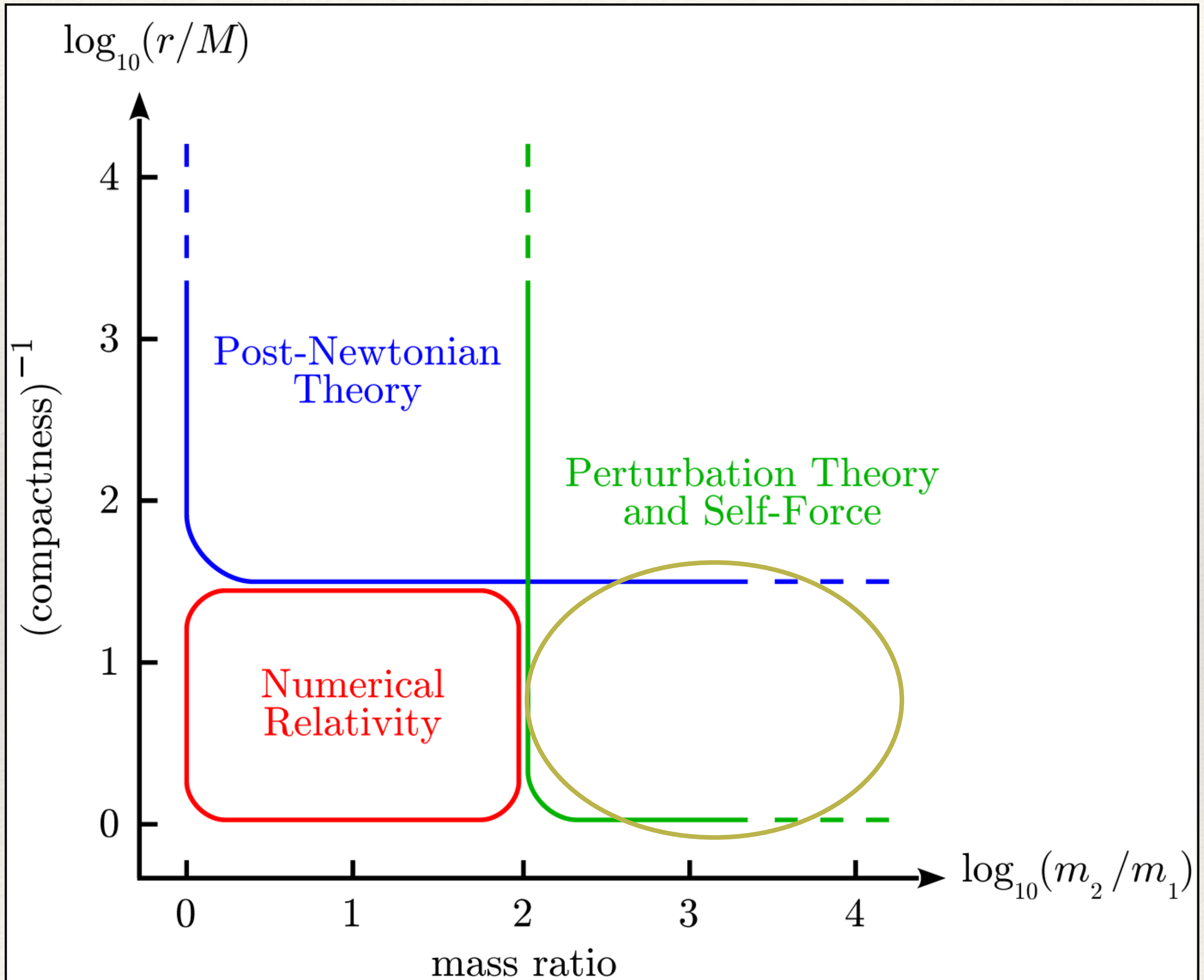

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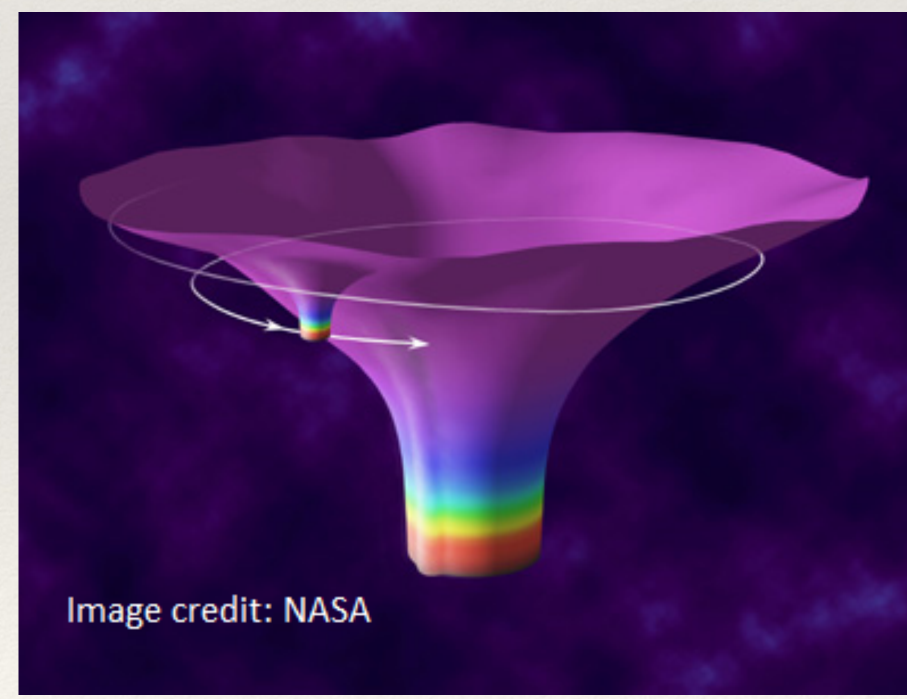

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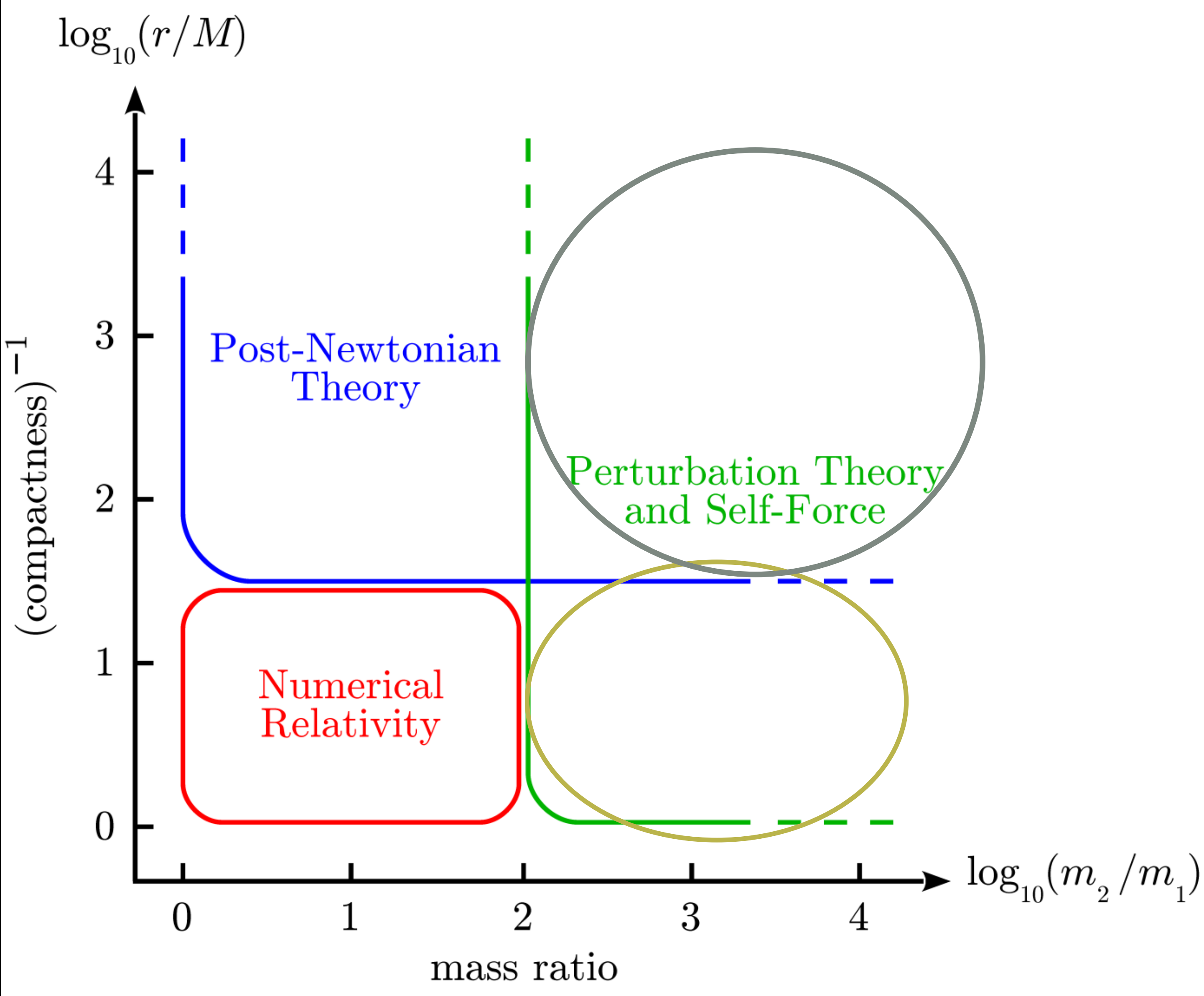

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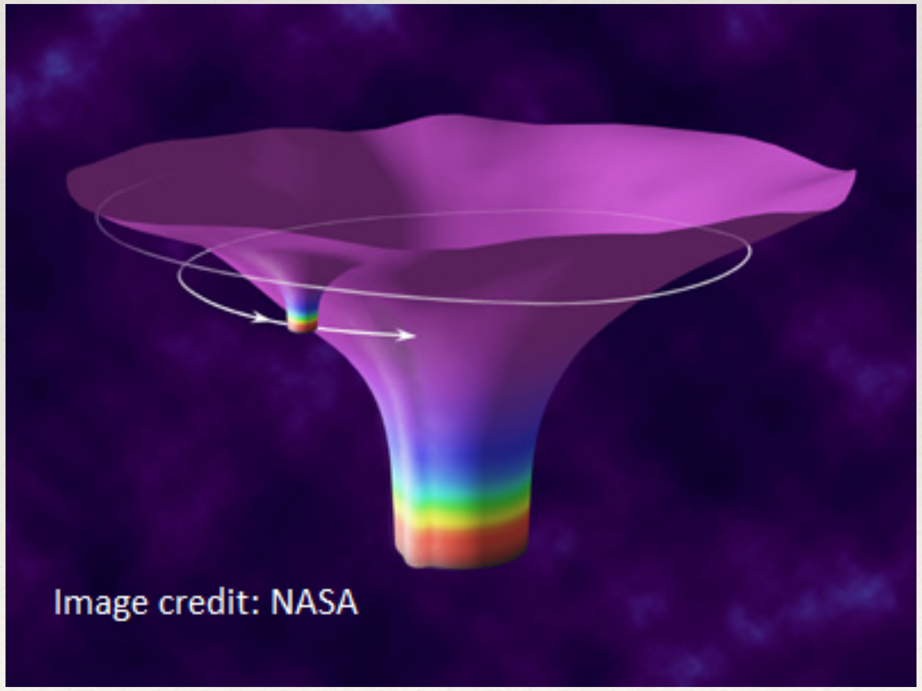

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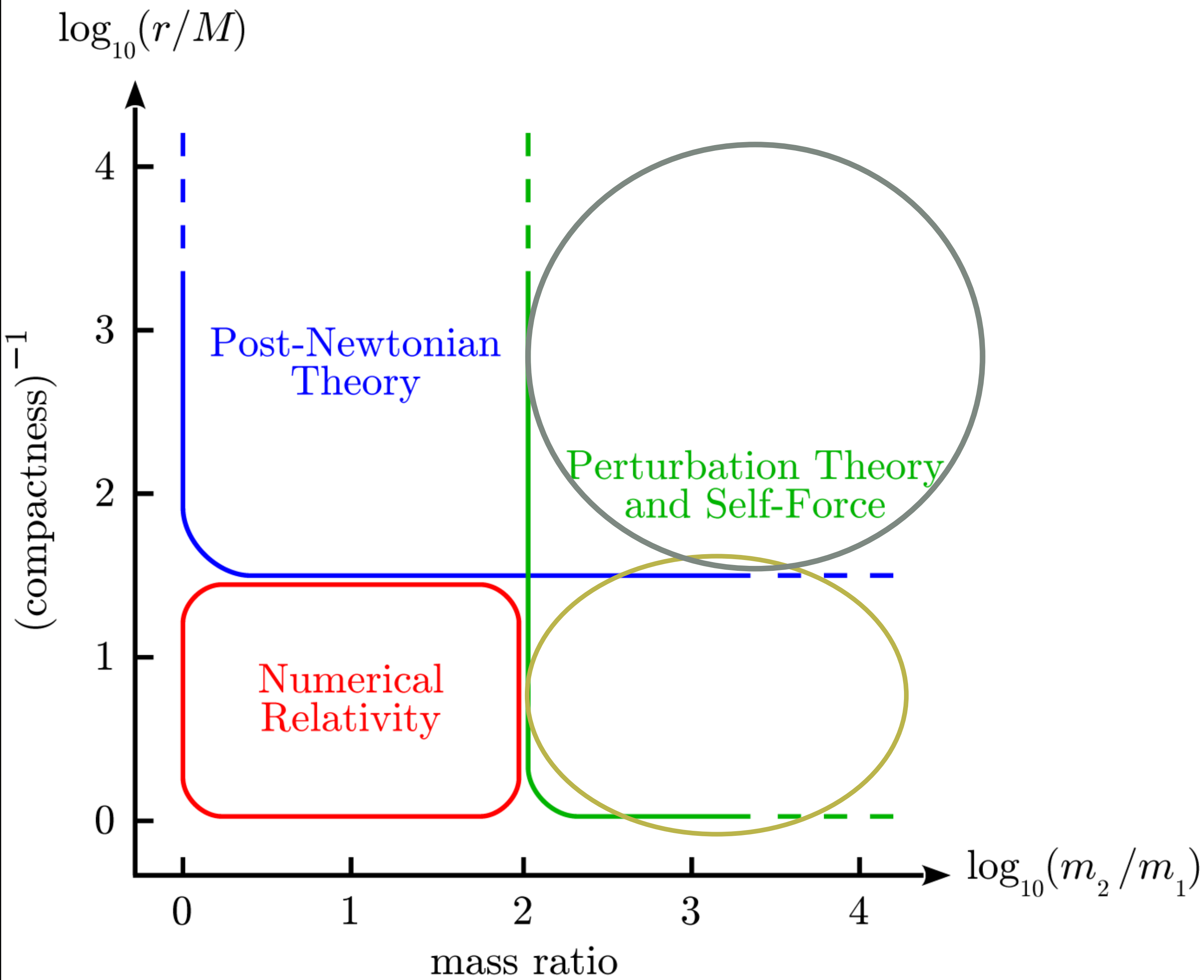

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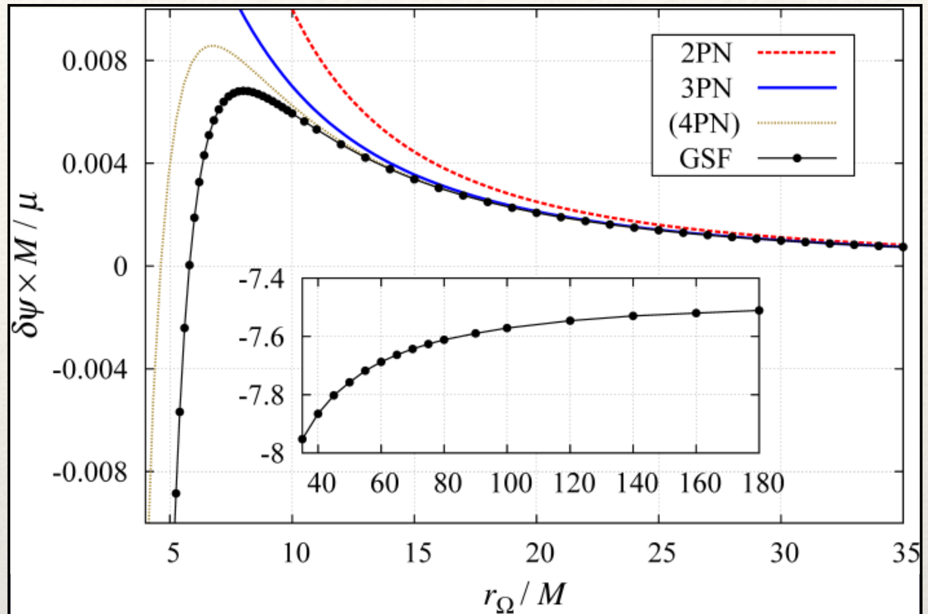
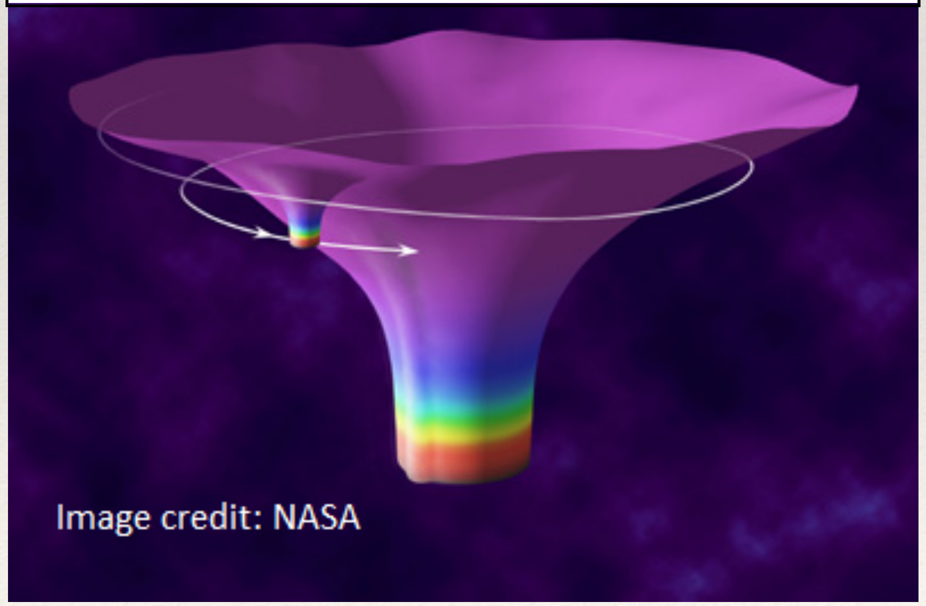
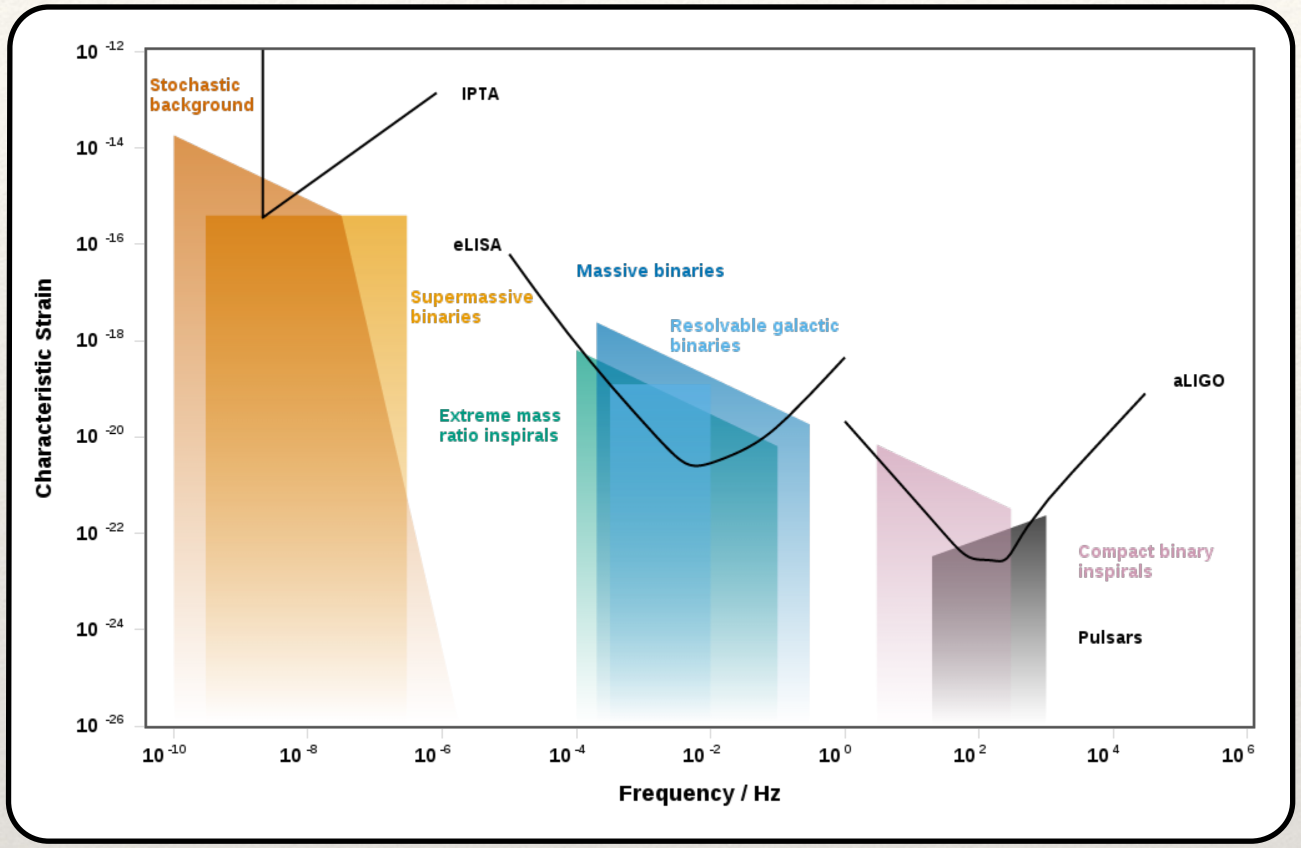


Diagram: S.R. Dolan, N. Warburton, A.I. Harte, A. Le Tiec, B. Wardell, L. Barack, Phys Rev. D 89, 064011 (2014)



Motivation



C.J. Moore, R.H. Cole, C.P.L. Barry, Class. Quantum Grav 32, 015014 (2015)

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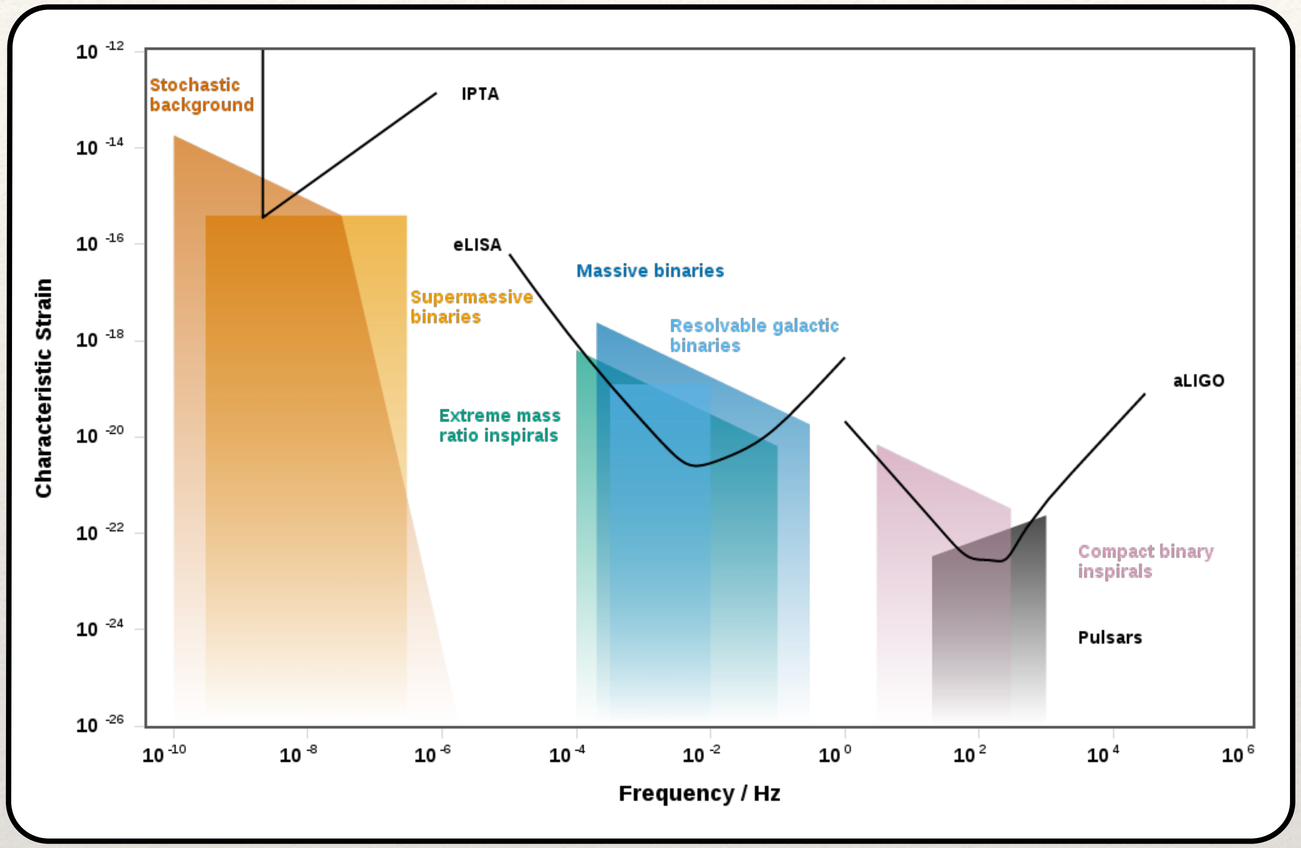

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- Mapping the space-time geometry in strong regime
- Hubble constant

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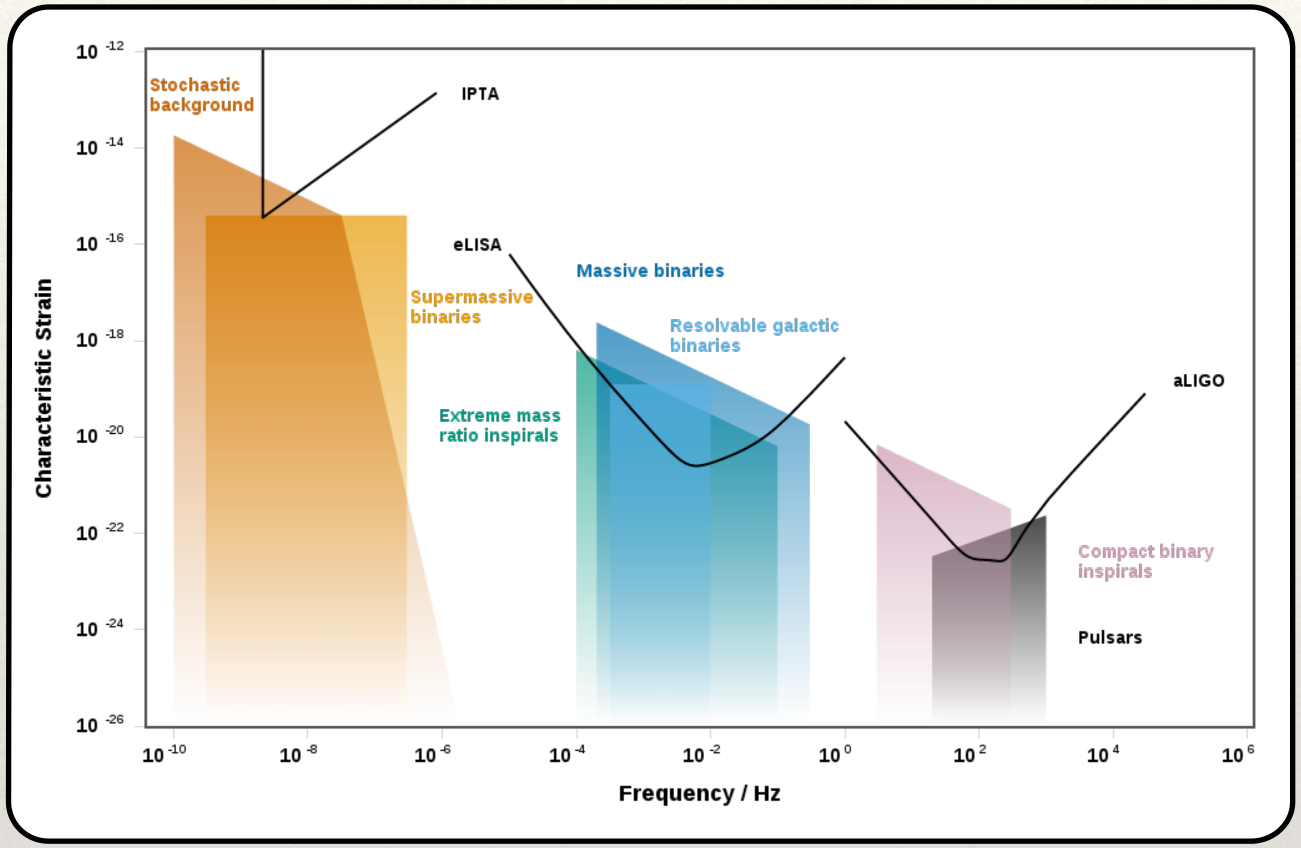

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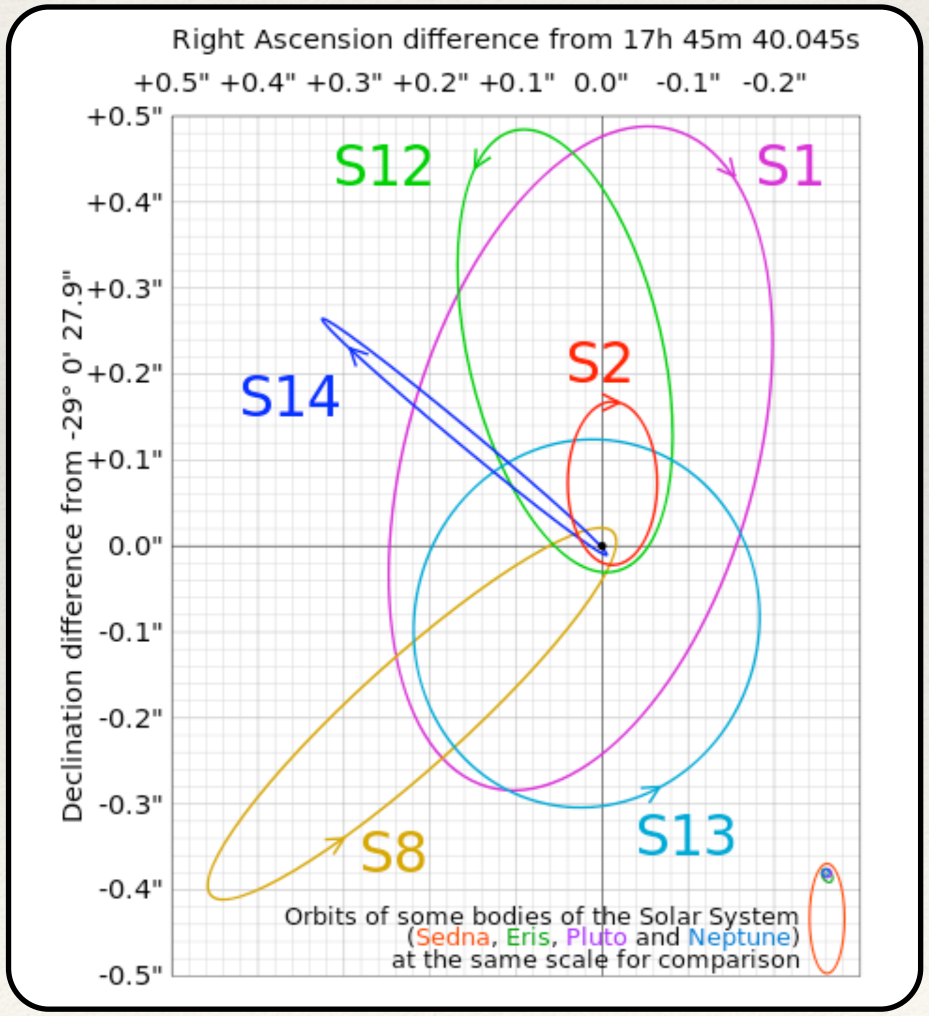
- Galaxy formation
- Galaxy census
- The unknown



F. Eisenheuer et al., Astrophys. J. 628, 246 (2005)

- Mapping the space-time geometry in strong regime
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The singular field

The retarded field satisfies

$$\mathcal{D}^A{}_B \varphi^B = -4\pi \mathcal{Q} \int u^A \delta_4\left(x, z(\tau')\right) d\tau' \quad \text{where} \quad \mathcal{D}^A{}_B = \delta^A{}_B(\Box - m^2) - P^A{}_B$$

The self-force

$$f^a = p^a{}_A \varphi^A_{(\text{ret})}$$

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$$\varphi^A_{(\mathrm{R})} = \varphi^A_{(\mathrm{ret})} - \varphi^A_{(\mathrm{sing})} \qquad\qquad f^a = p^a{}_A \varphi^A_{(R)}$$

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Detweiler-Whiting singular field

$$\varphi^{A}_{(\mathrm{sing})} = \int_{\tau_{(\mathrm{adv})}}^{\tau_{(\mathrm{ret})}} G_{(\mathrm{sing})}{}^{A}{}_{B'}\left(x, z(\tau')\right) u^{B'} d\tau'$$

$$G_{(\mathrm{sing})}{}^{A}{}_{B'}\left(x, x'\right) = \frac{1}{2}\left\{U^{A}{}_{B'}\left(x, x'\right)\delta\left[\sigma\left(x, x'\right)\right] + V^{A}{}_{B'}\left(x, x'\right)\theta\left[\sigma\left(x, x'\right)\right]\right\}$$

The scalar singular field and self-force are

$$\Phi^{(\mathrm{S})}(x) = \left[\frac{U(x,x')}{2\sigma_{a'}u^{a'}}\right]^{x'=x_+}_{x'=x_-} + \int_{\tau_-}^{\tau_+} V(x,z(\tau))d\tau$$

$$f^a = g^{ab}\Phi^{(\mathrm{R})}{}_{,b}.$$

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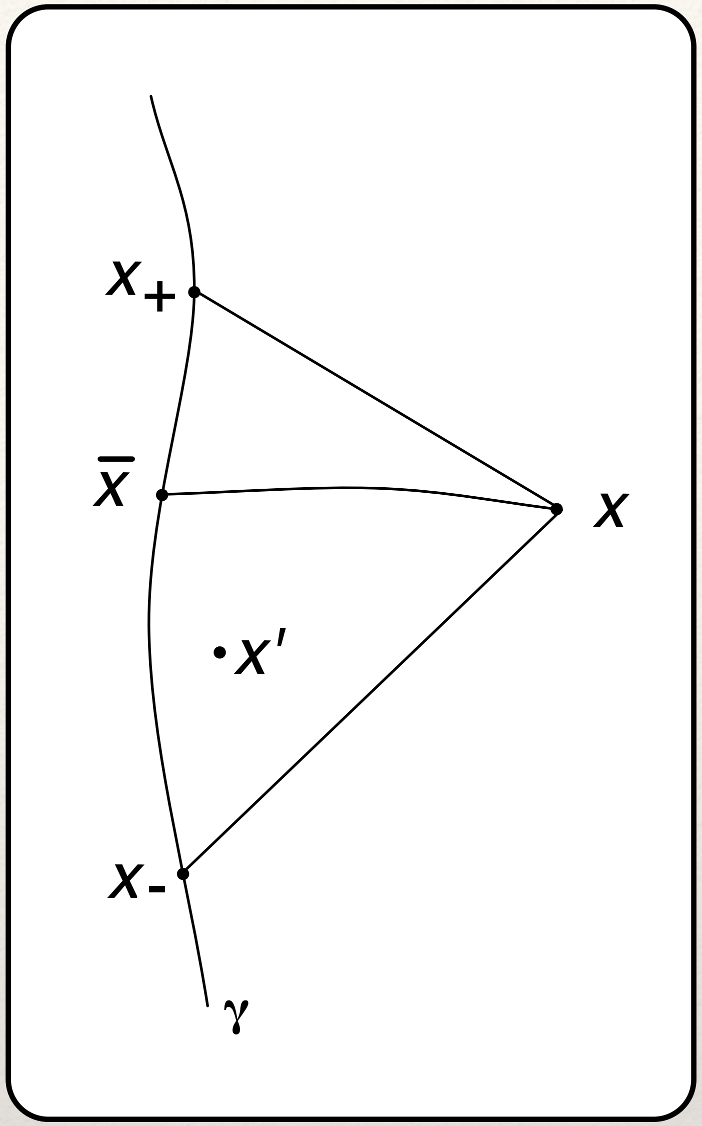

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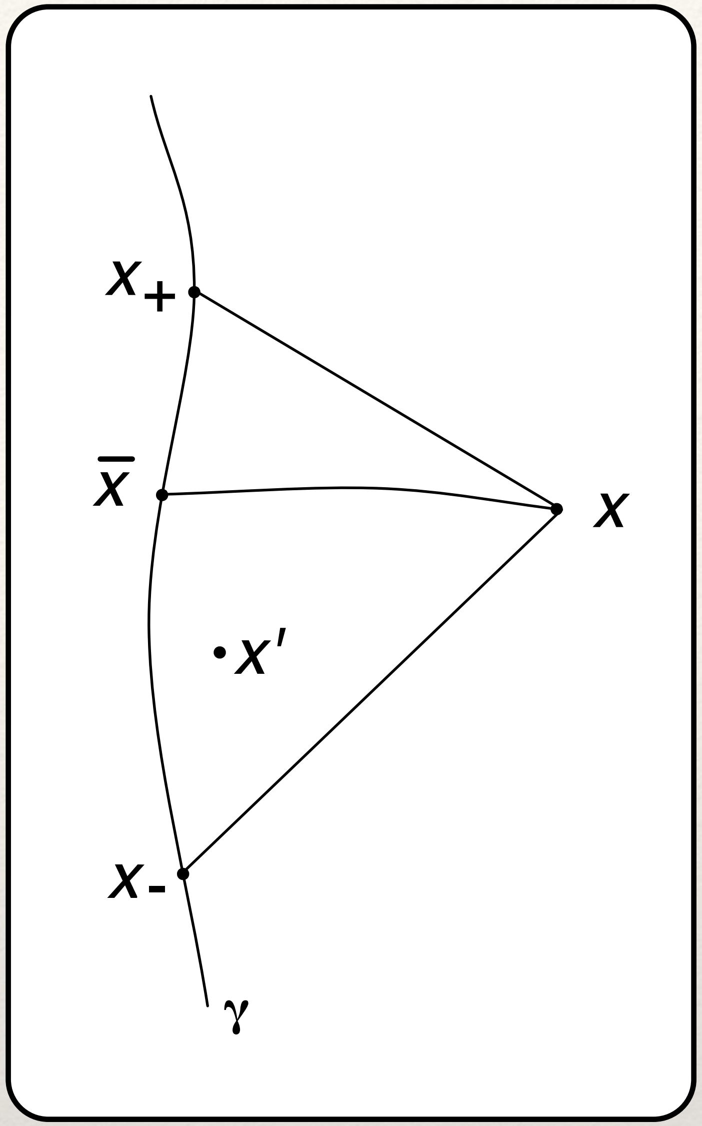

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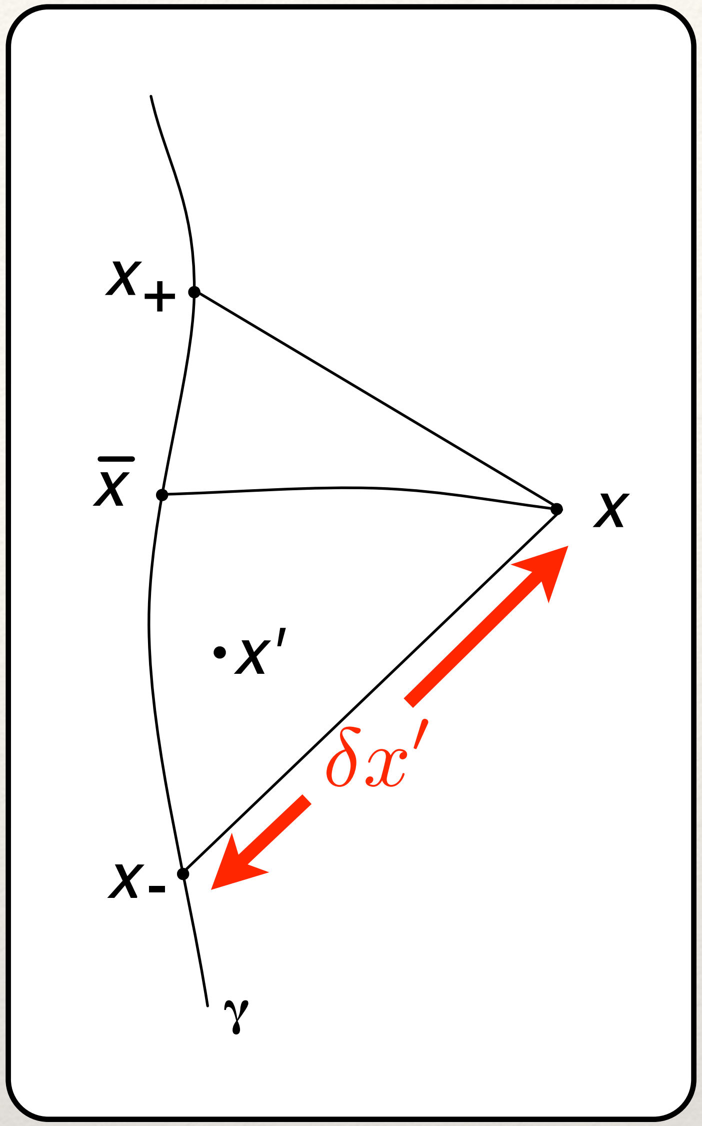

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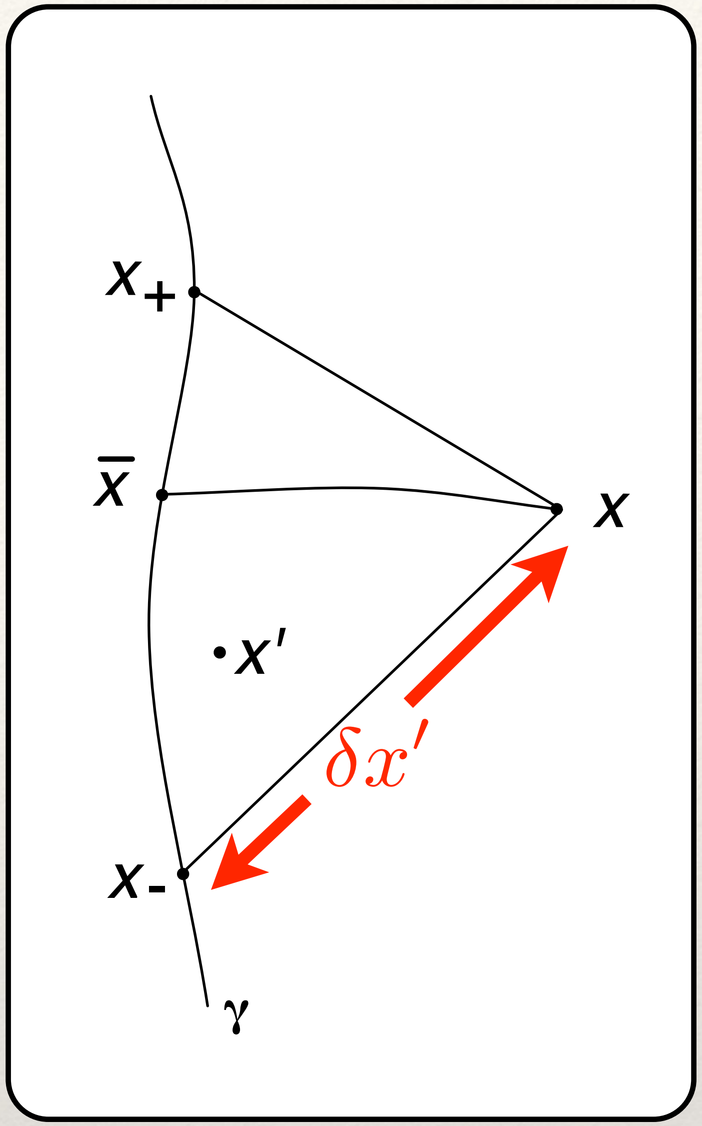
$$2\sigma = \sigma_{a'}\sigma^{a'}$$



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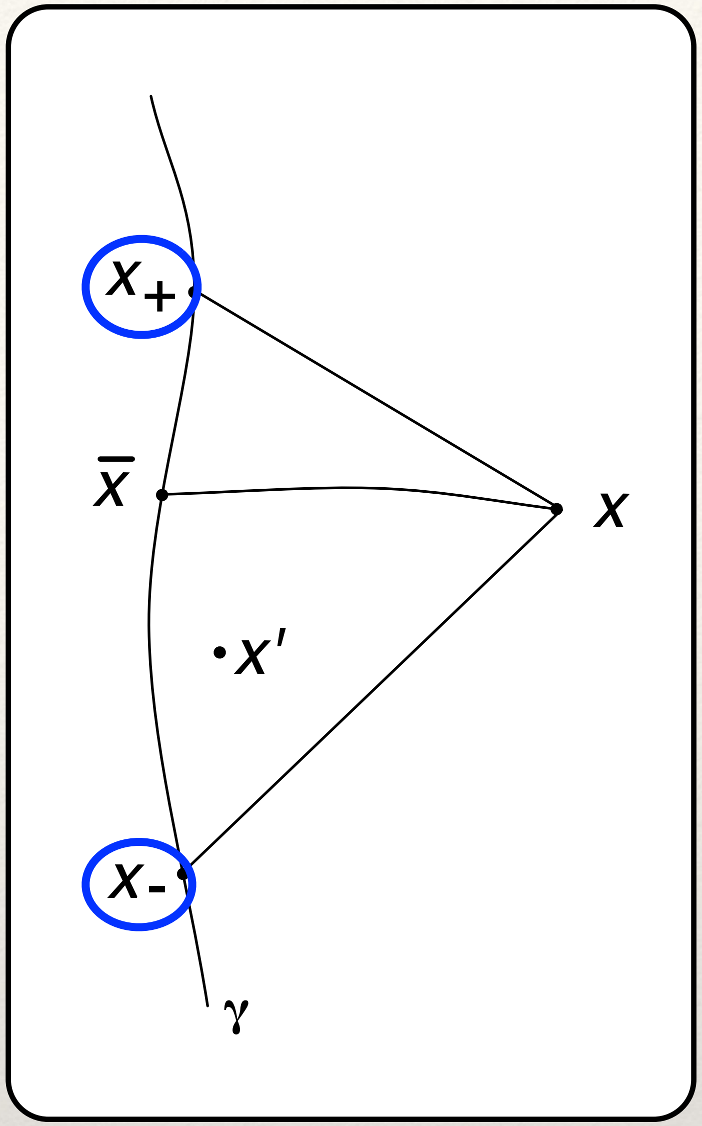

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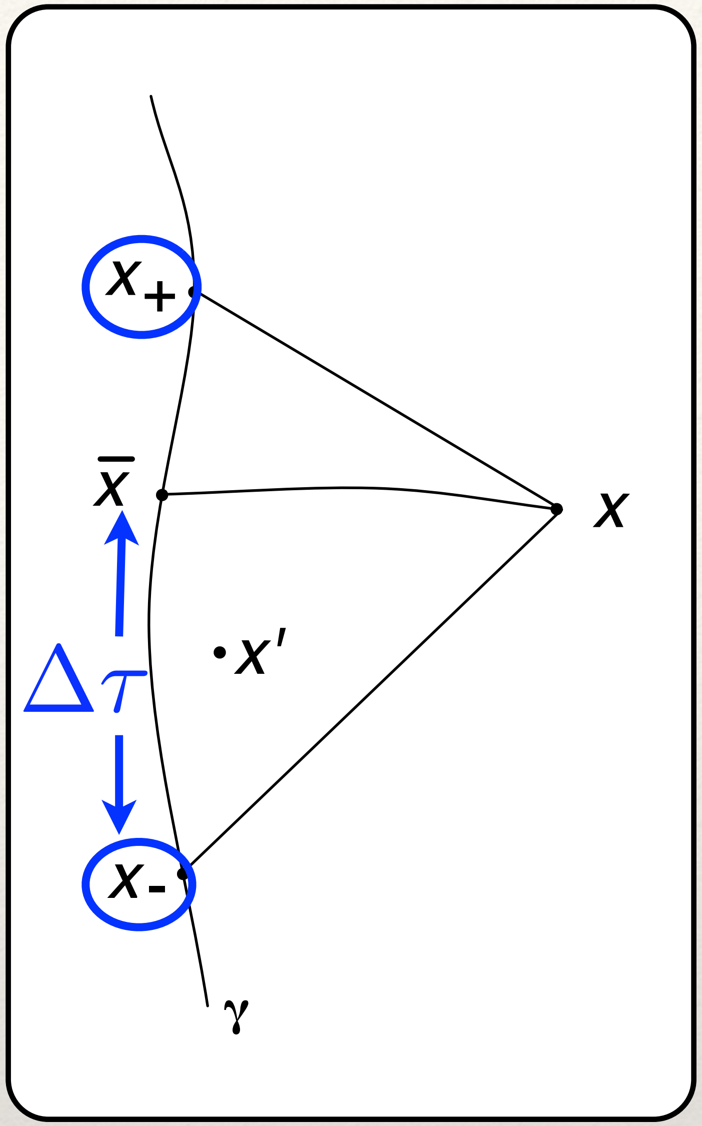

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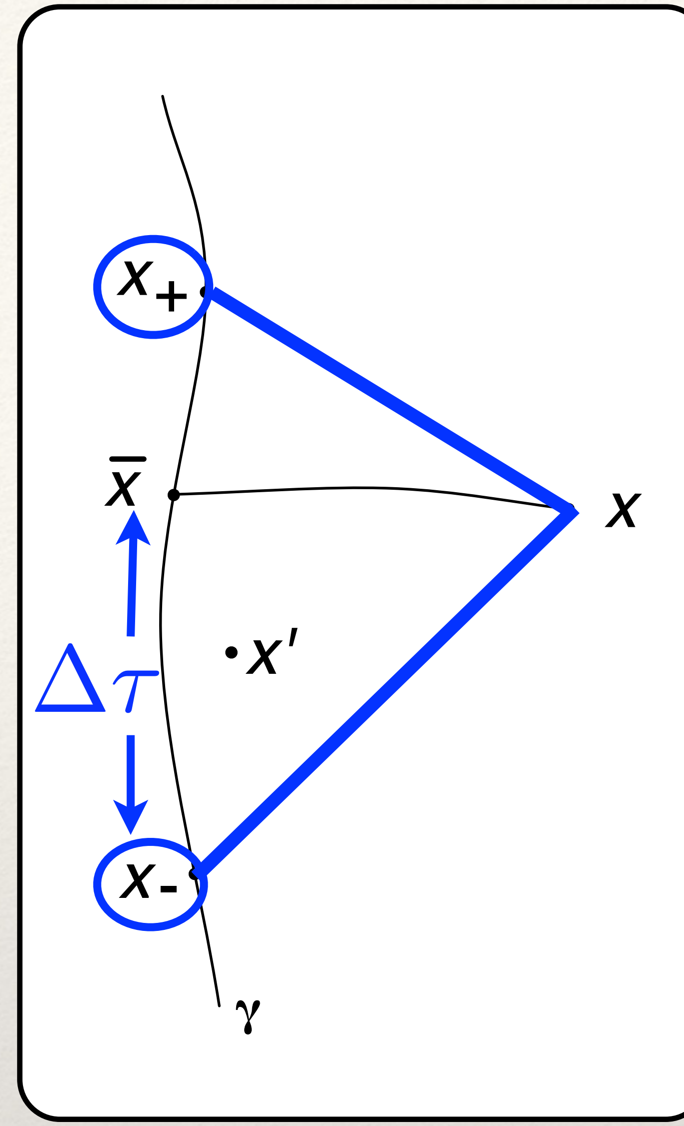

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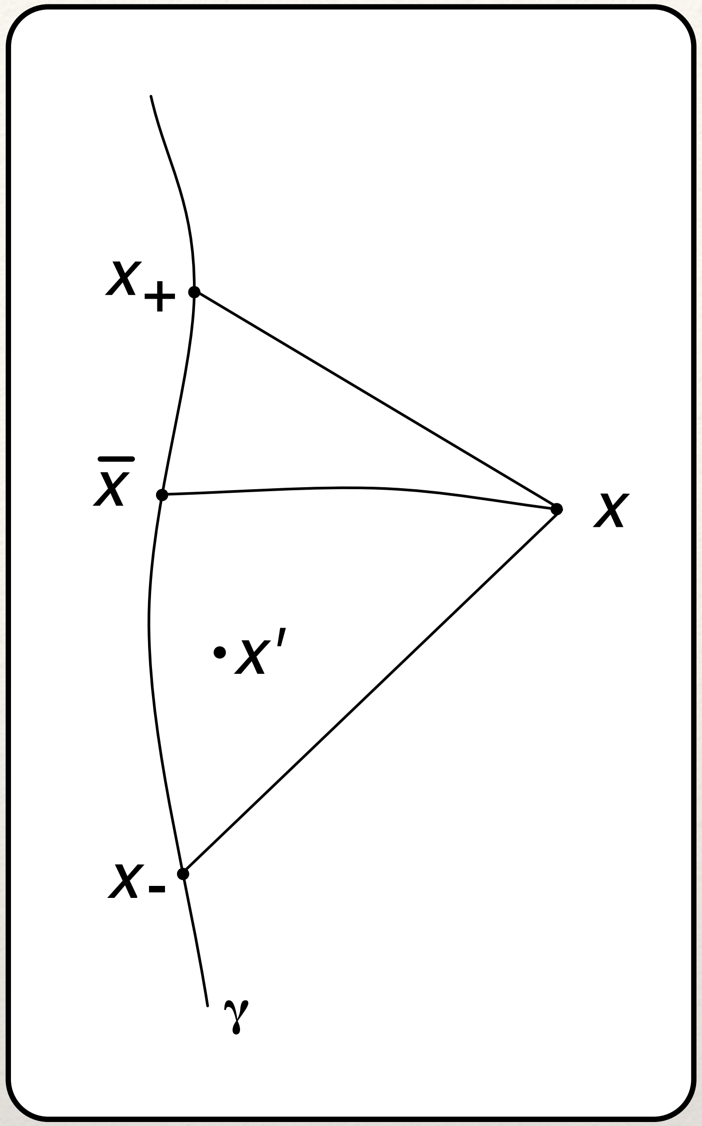

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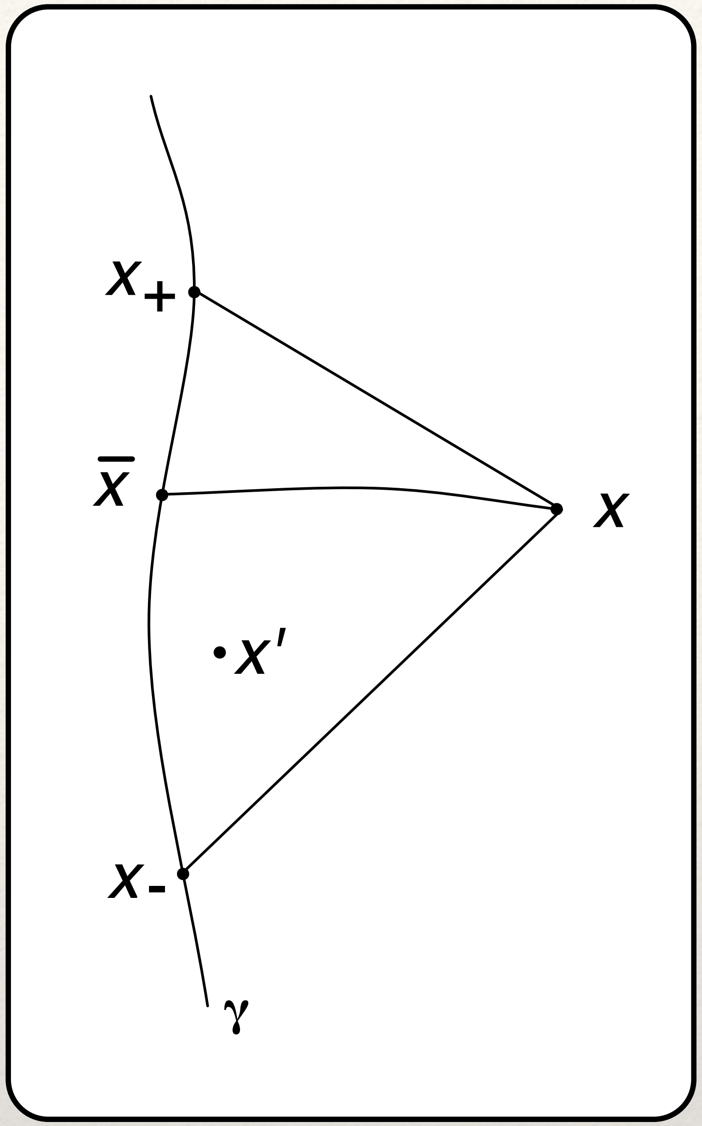

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$$\bar{h}_{ab}^S = \left[\frac{u^{a'}u^{b'}U^{ab}{}_{a'b'}(x,x')}{2\sigma_{a'}u^{a'}}\right]_{x'=x_-}^{x'=x_+} + \int_{\tau_-}^{\tau_+} V^{ab}{}_{a'b'}(x,z(\tau))u^{a'}u^{b'}d\tau \qquad \text{and} \qquad f^a = k^{abcd}\bar{h}^{(R)}_{bc;d},$$

where

$$\Delta^{\frac{1}{2}}(x,x') = \left(-\left[-g(x)\right]^{-\frac{1}{2}}\left|-\sigma_{a'b}(x,x')\right|\left[-g(x')\right]^{-\frac{1}{2}}\right)^{\frac{1}{2}}$$

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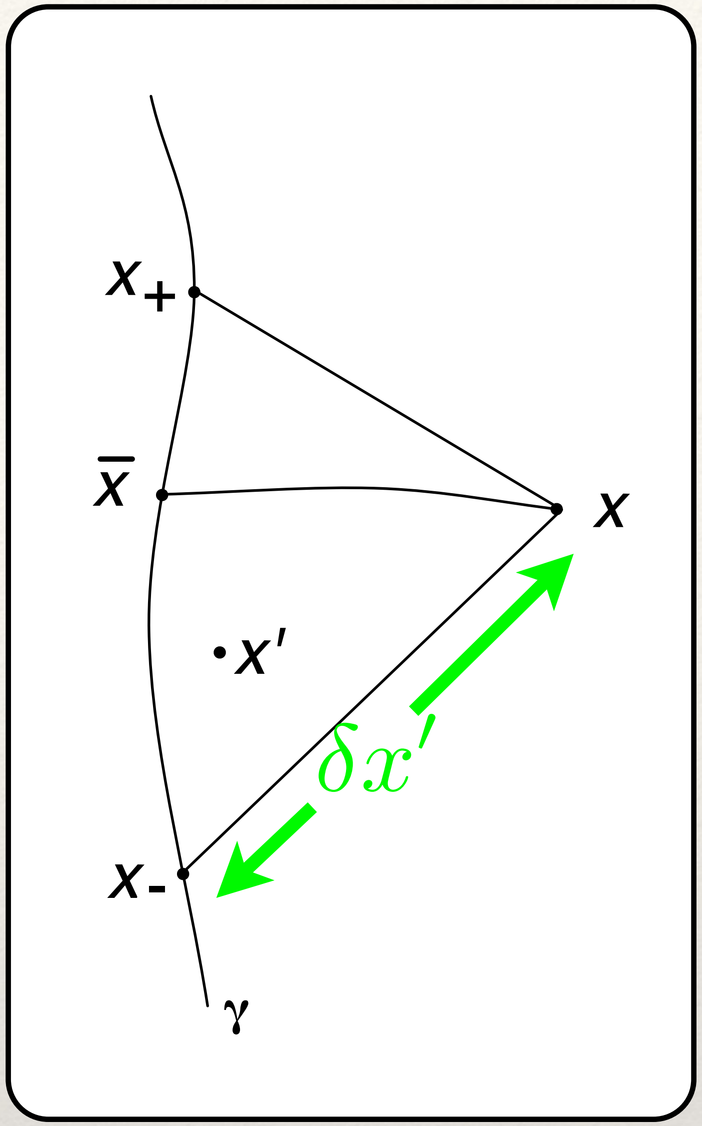

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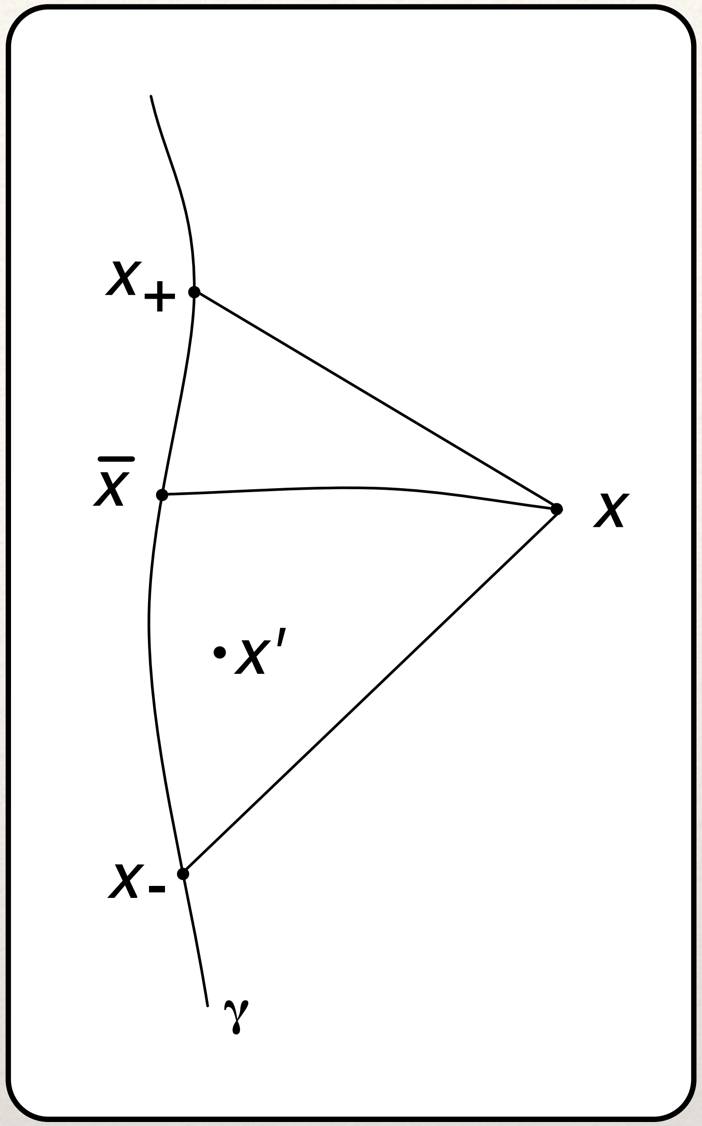

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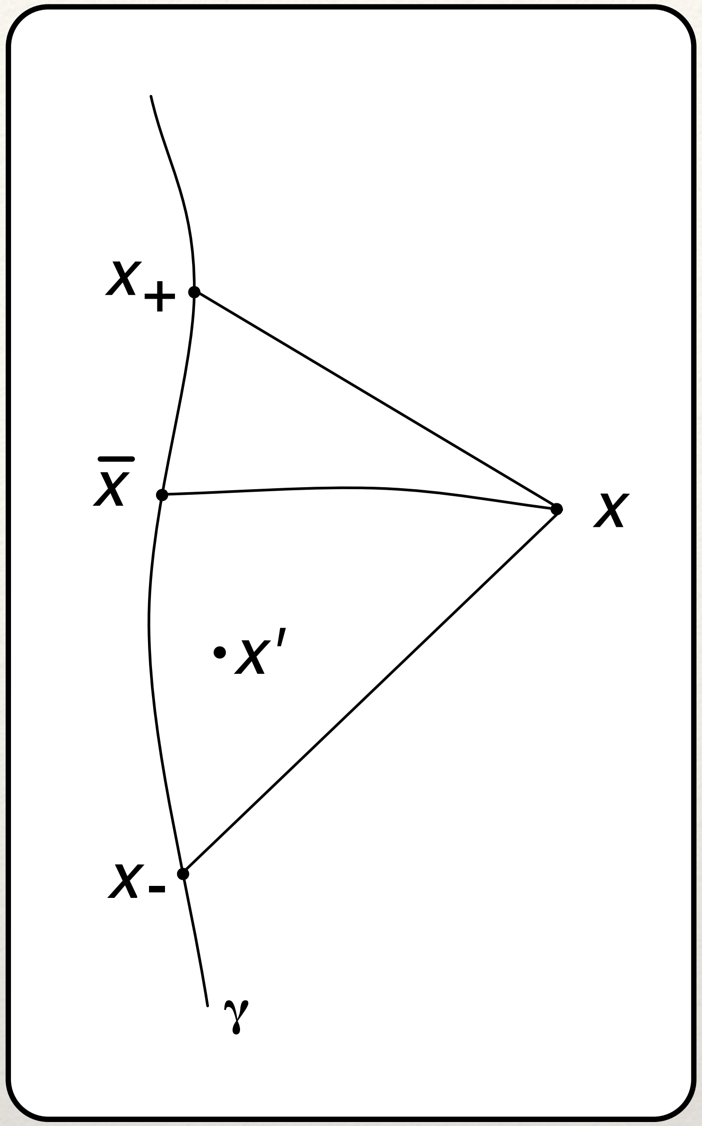

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where $\sigma^{;\alpha'}\left(\Delta^{-1/2}V_n^{AB'}\right)_{;\alpha'} + (n+1)\,\Delta^{-1/2}V_n^{AB'} + \frac{1}{2n}\Delta^{-1/2}\mathcal{D}^{B'}{}_{C'}V_{n-1}^{AC'} = 0$

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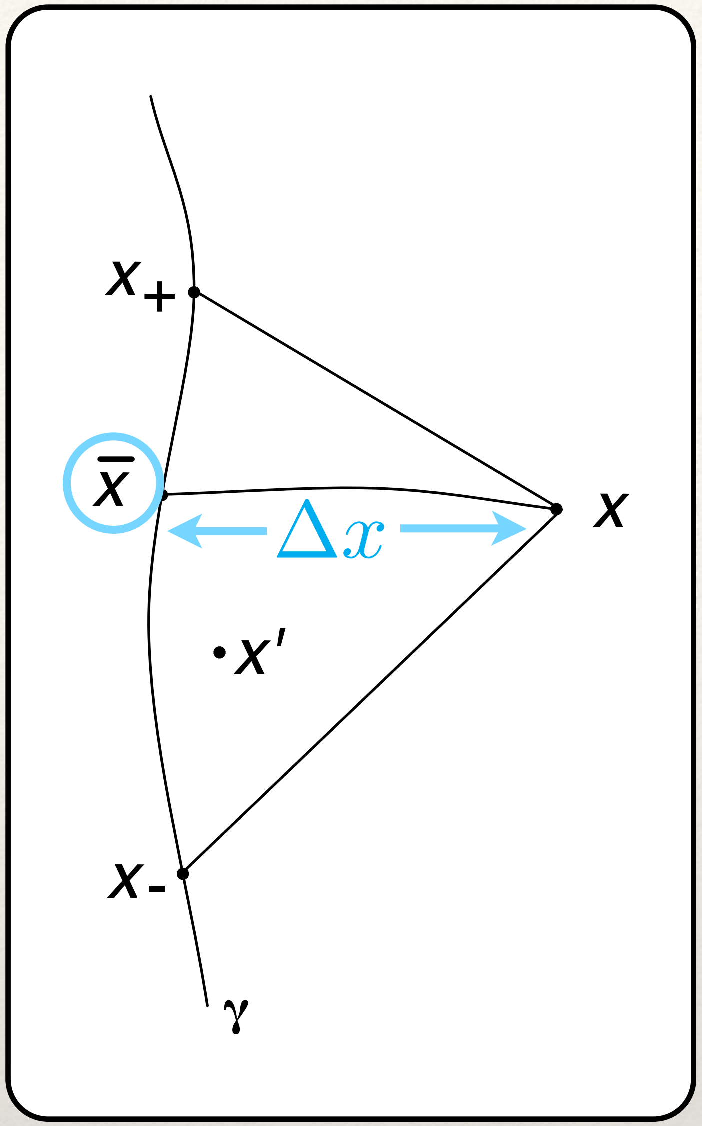

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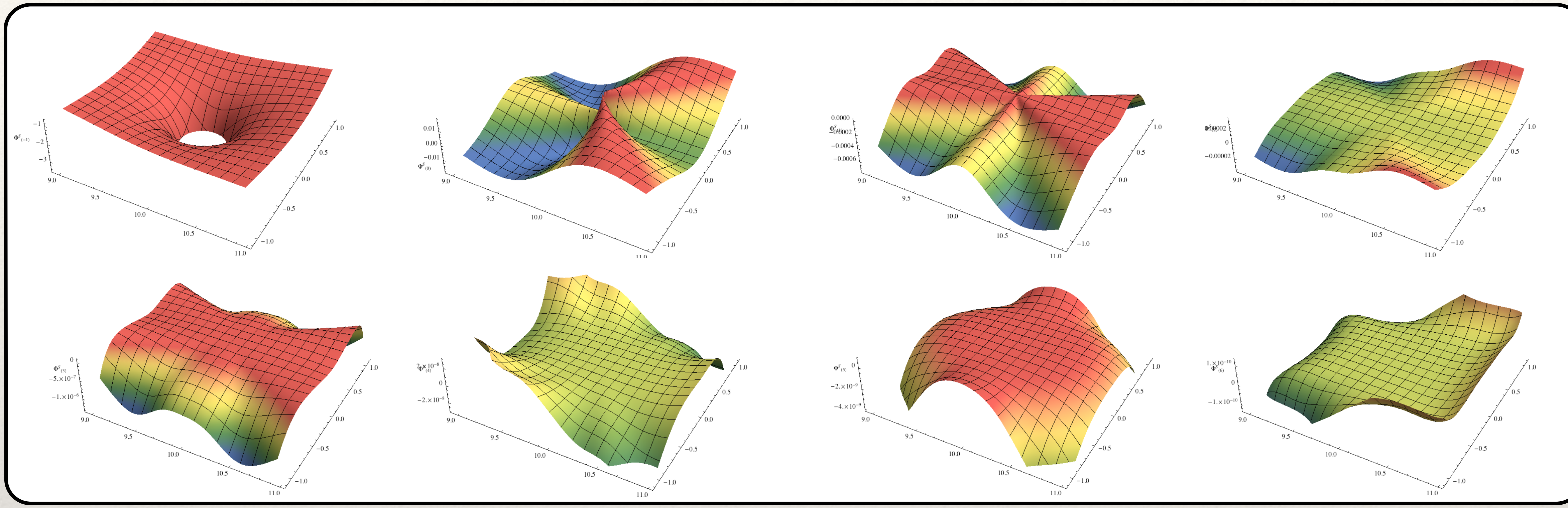
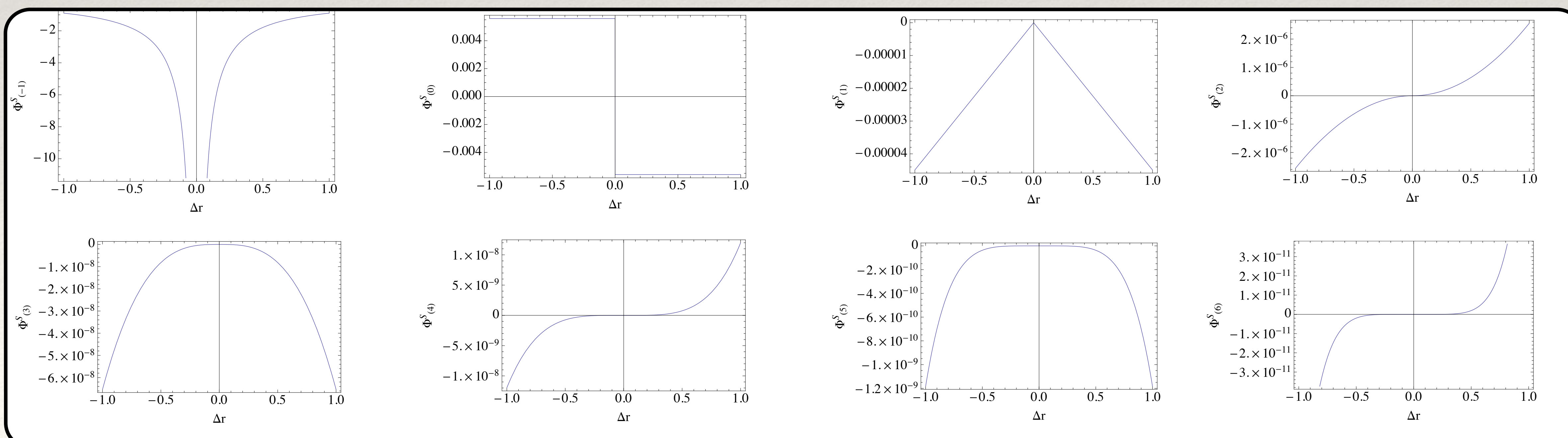
The singular field

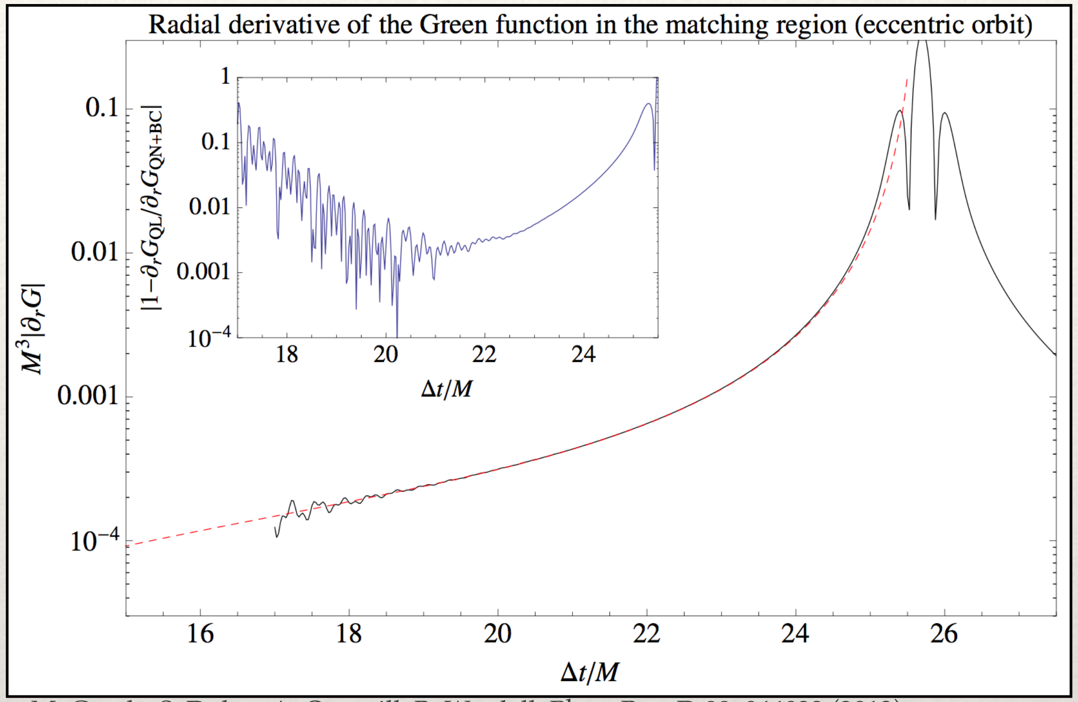




Methods

- Matched Expansions

$$f_\mu = q^2 \nabla_\mu \left[\int_{\tau_m}^{\tau^-} d\tau' V(z(\tau), z(\tau')) + \int_{-\infty}^{\tau_m} d\tau' G_{ret}(z(\tau), z(\tau')) \right]$$



M. Casals, S. Dolan, A. Ottewill, B. Wardell, Phys. Rev. D 88, 044022 (2013)

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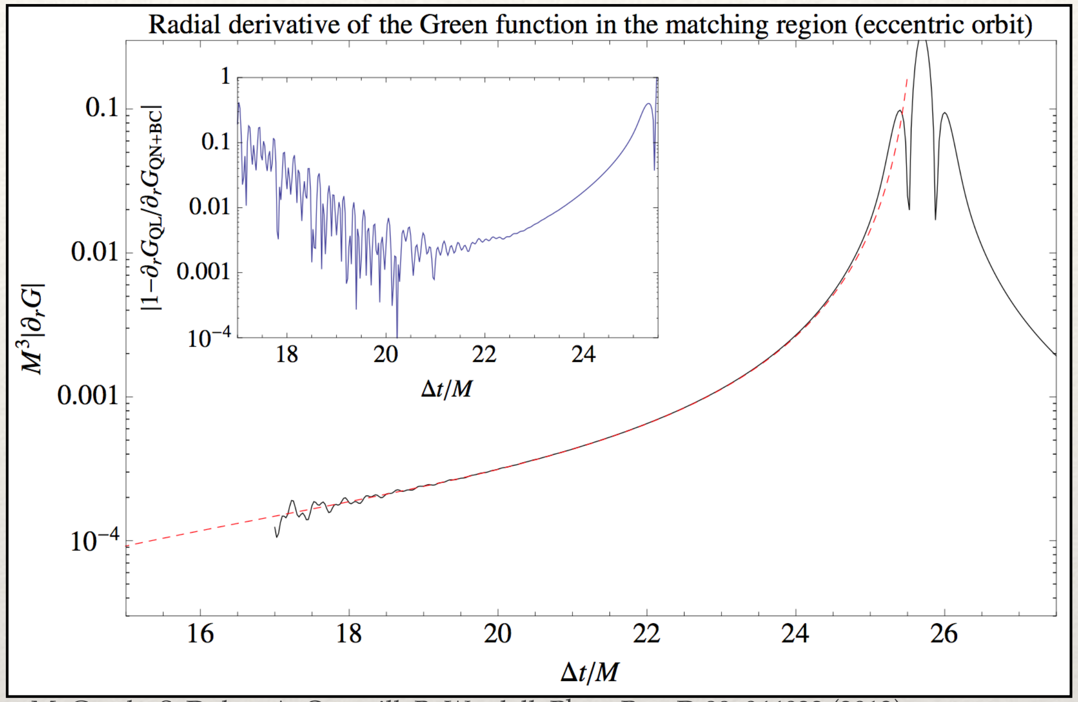

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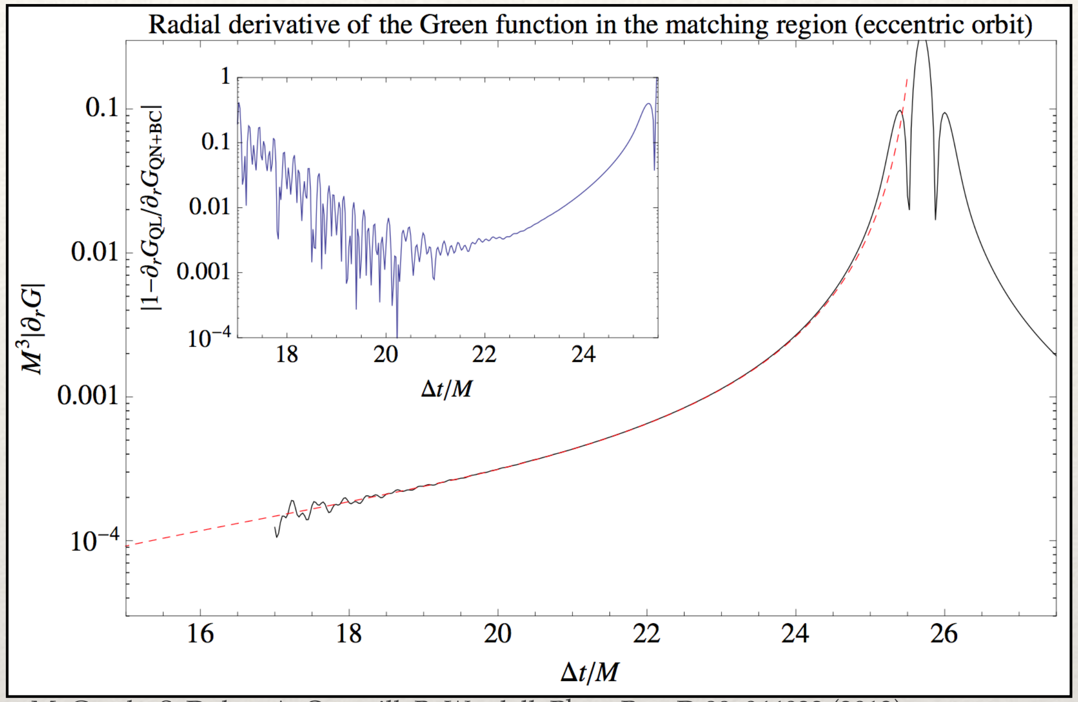

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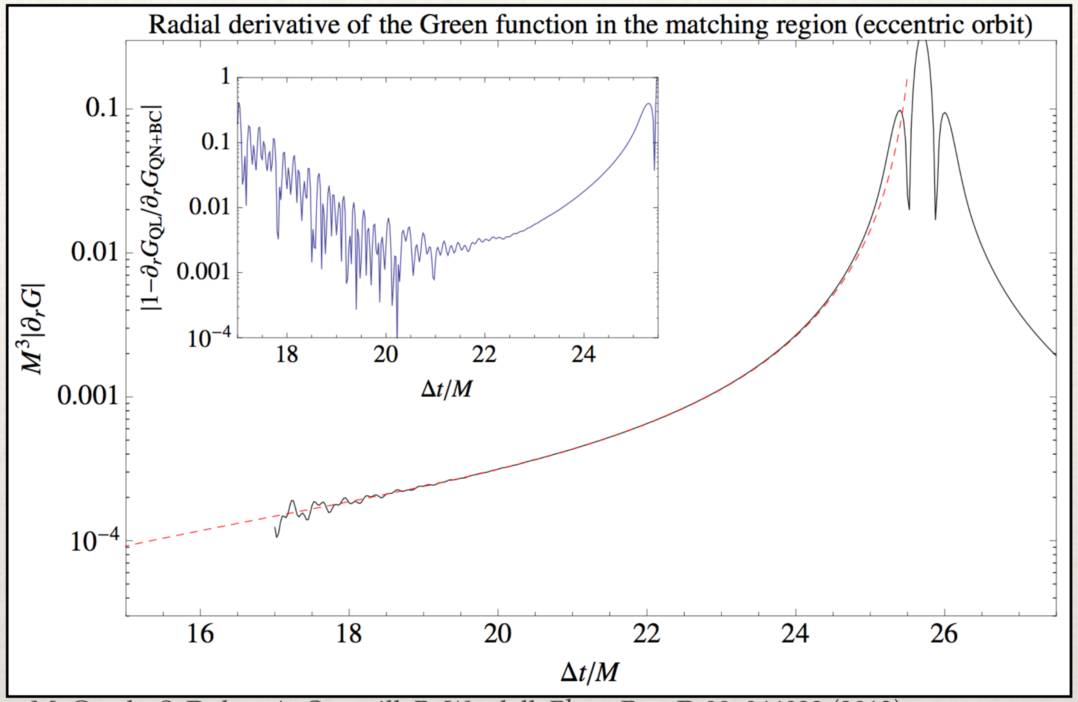

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- Effective Source

$$\varphi^A_{(\mathrm{ret})} = \tilde{\varphi}^A_{(\mathrm{S})} + \tilde{\varphi}^A_{(\mathrm{R})}$$

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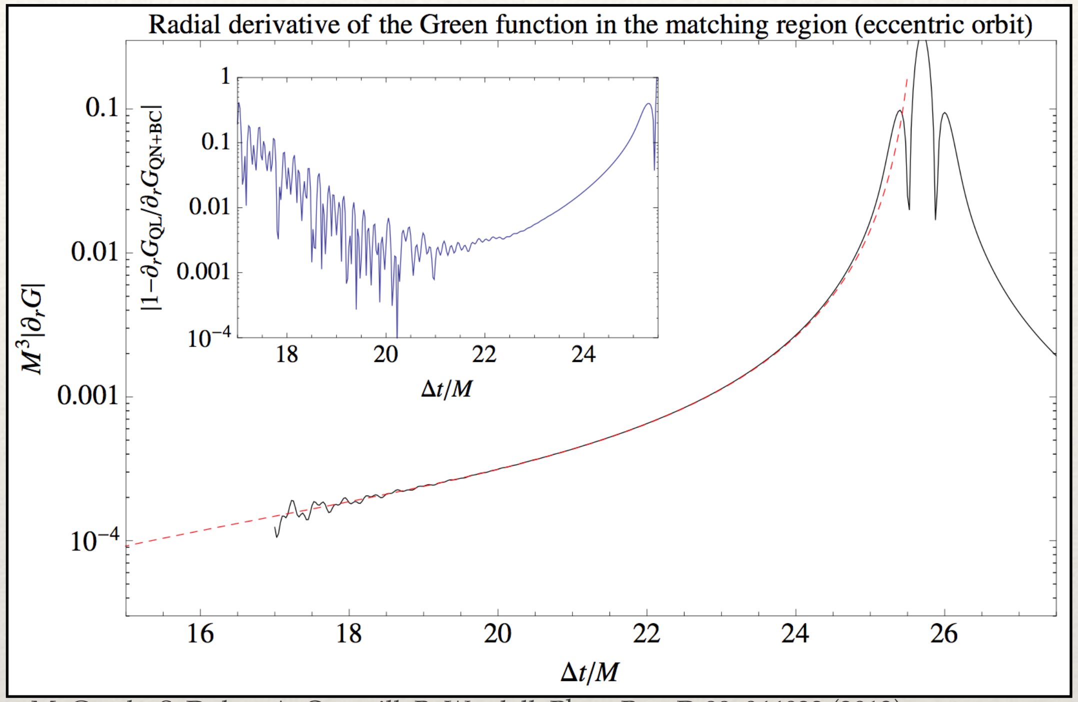

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- Mode-Sum

$$f^l_a \left(r_0, t_0\right) = \lim_{\Delta r \to 0} \sum_m f^{lm}_a \left(r_0 + \Delta r, t_0\right) Y^{lm} \left(\alpha_0, \beta_0\right)$$

$$= \frac{2l+1}{4\pi} \lim_{\Delta r \to 0} \int f_a \left(r_0 + \Delta r, t_0, \alpha, \beta\right) P_l \left(\cos \alpha\right) d\Omega$$

Mode-sum

$$\begin{aligned} f_a^l\left(r_0, t_0\right) &= \lim_{\Delta r \to 0} \sum_m f_a^{lm}\left(r_0 + \Delta r, t_0\right) Y^{lm}\left(\alpha_0, \beta_0\right) \\ &= \frac{2l+1}{4\pi} \lim_{\Delta r \to 0} \int f_a\left(r_0 + \Delta r, t_0, \alpha, \beta\right) P_l\left(\cos\alpha\right) d\Omega \end{aligned}$$

$$\begin{aligned} \sin\theta\cos\phi &= \cos\alpha \\ \sin\theta\sin\phi &= \sin\alpha\sin\beta \\ \cos\theta &= \sin\alpha\cos\beta \end{aligned}$$

Mode-sum

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Singular field contribution:

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$$\rho^2 = (g_{\bar{a}\bar{b}}u^{\bar{a}}\Delta x^b)^2 + g_{\bar{a}\bar{b}}\Delta x^a\Delta x^b$$
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$$F^l_{a[-1]}(r_0,t_0) = (l+1/2)\,\frac{b_{a_r}sgn(\Delta r)}{\zeta\nu r_0}$$

Mode-sum

$$f_a^l(r_0,t_0) = \frac{2l+1}{4\pi}\left[\epsilon^{-2}\lim_{\Delta r\to 0}\int \frac{B_a^{(1)}}{\rho^3}P_l(\cos\alpha)\,d\Omega\right.$$ $F^l_{a[-1]}(r_0,t_0)$

$$\left. + \epsilon^{n-3}\sum_{n=2}\int \rho_0^{n-3}c_{a(n)}(r_0,\beta)P_l(\cos\alpha)\,d\Omega\right]$$

$$w_1 = 2\sin\left(\frac{\alpha}{2}\right)\cos\beta$$

$$w_2 = 2\sin\left(\frac{\alpha}{2}\right)\sin\beta$$

Singular field contribution: $f_a(r,t,\alpha,\beta) = \sum_{n=1}\frac{\mathcal{B}_a^{(3n-2)}}{\rho^{2n+1}}\epsilon^{n-3}$

where $\mathcal{B}_a^{(k)} = b_{a_1a_2\ldots a_k}(\bar{x})\Delta x^{a_1}\Delta x^{a_2}\ldots\Delta x^{a_k}$

$$\rho^2 = (g_{\bar{a}\bar{b}}u^{\bar{a}}\Delta x^b)^2 + g_{\bar{a}\bar{b}}\Delta x^a\Delta x^b$$

$$= \frac{\dot{t}_0^2}{1+r_0^2\dot{\phi}_0^2}\Delta r^2 + \left(r_0^2 + r_0^4\dot{\phi}_0^2\right)\left[\Delta w_1^2 + \frac{\dot{r}_0\dot{\phi}_0}{f(r_0)\left(1+r_0^2\dot{\phi}_0\right)}\Delta r\right]^2 + r_0^2\Delta w_2^2$$

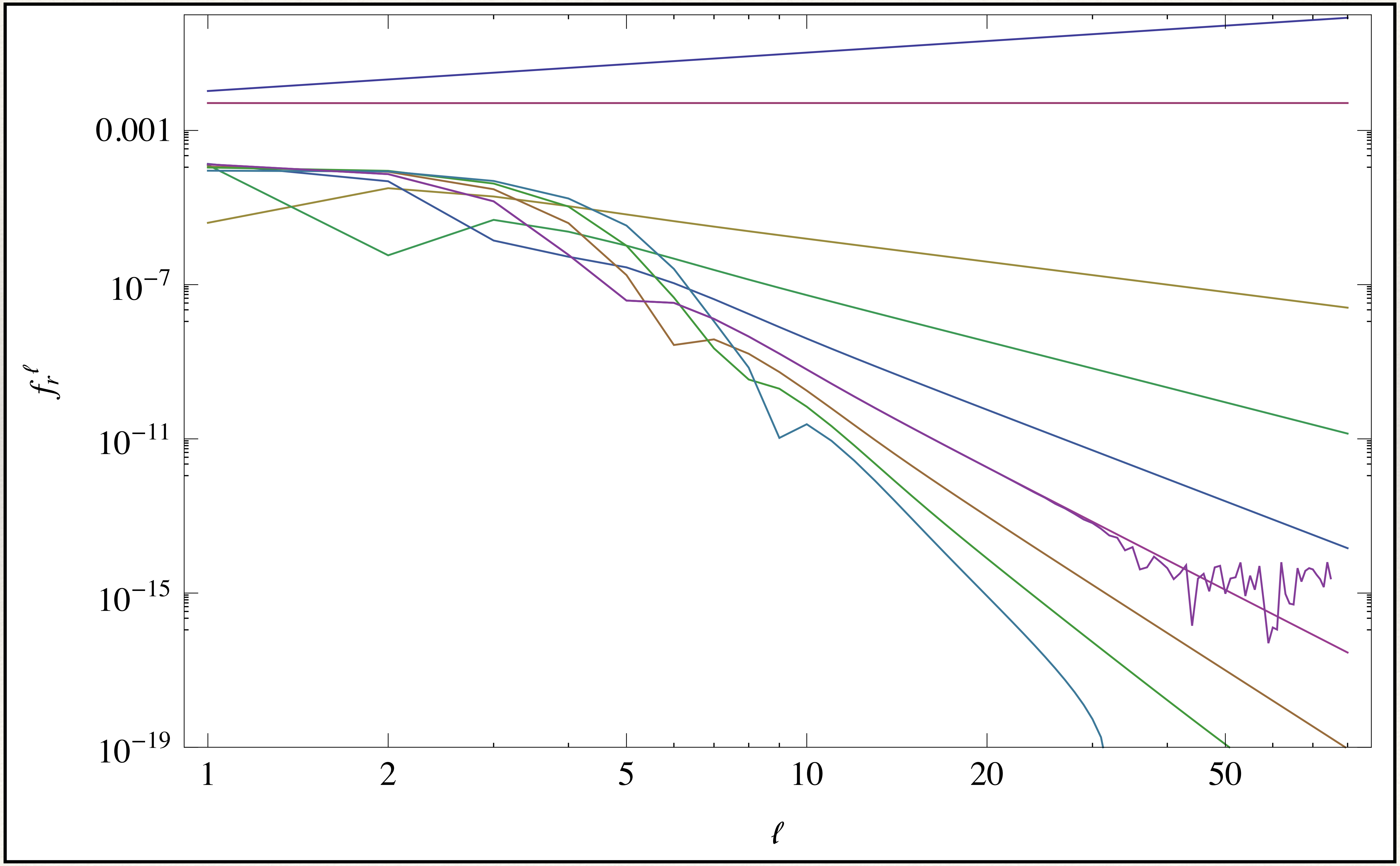
ν^2, ζ^2, $-\mu$

$$\Delta w_1 \to \Delta w_1 + \mu\Delta r \Rightarrow \rho(r,t_0,\alpha,\beta) = \nu^2\Delta r + \zeta^2\Delta w_1^2 + r_0^2\Delta w_2^2$$

$$F^l_{a[-1]}(r_0,t_0) = (l+1/2)\,\frac{b_{a_r}sgn(\Delta r)}{\zeta\nu r_0}$$

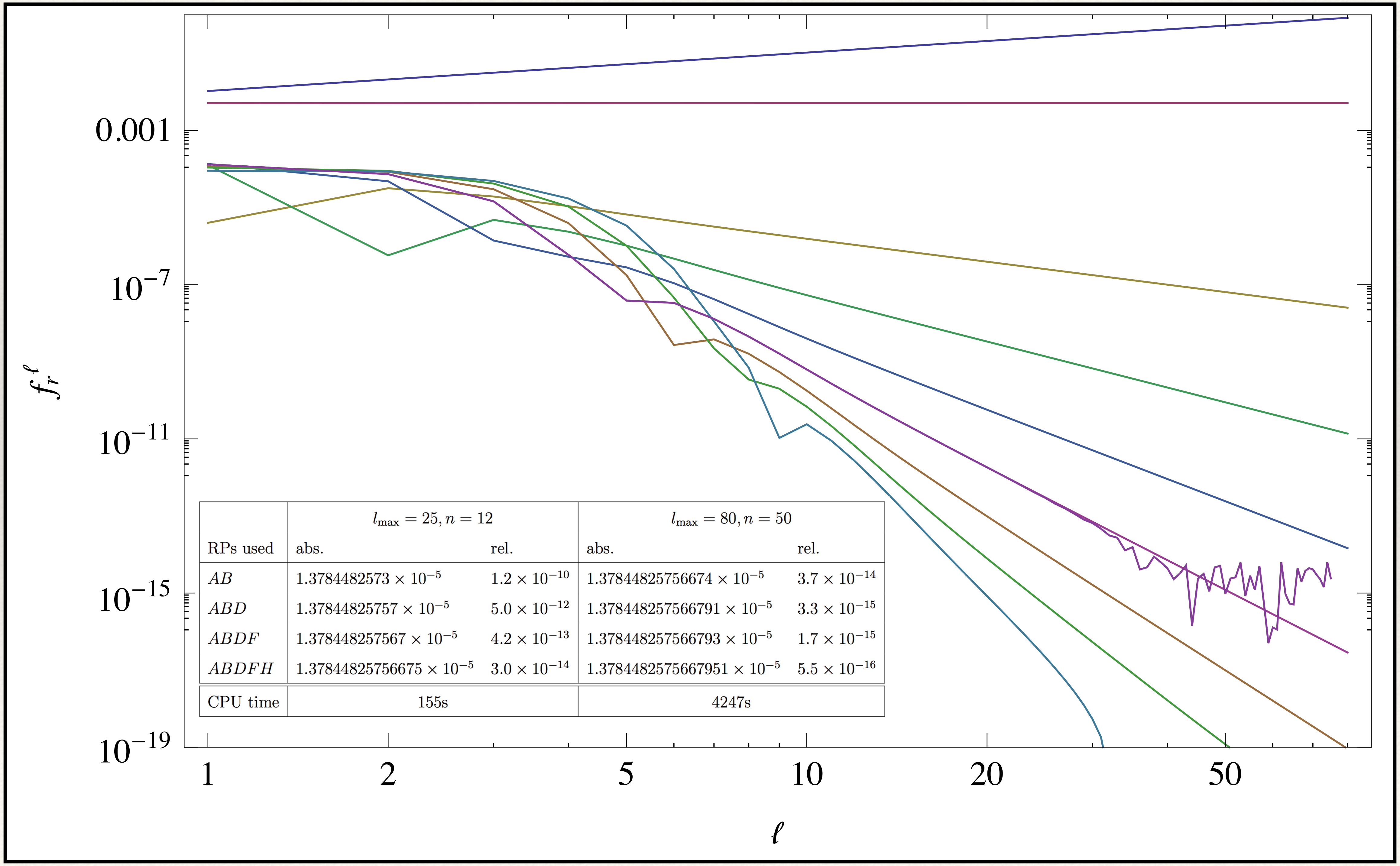
Mode-sum: Schwarzschild scalar

Regularised l-modes for radial scalar self-force in Schwarzschild spacetime



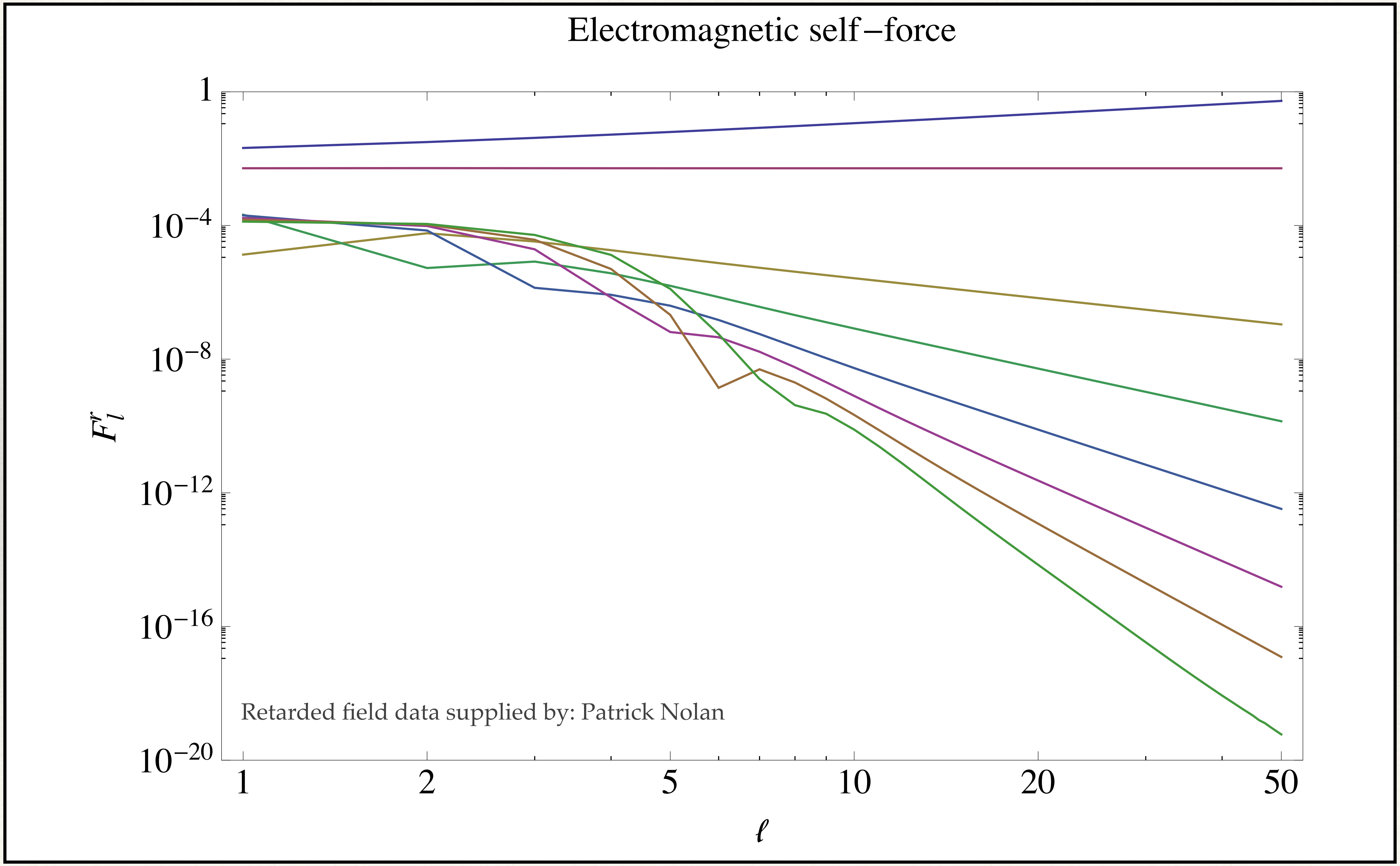
Mode-sum: Schwarzschild scalar

Regularised l-modes for radial scalar self-force in Schwarzschild spacetime

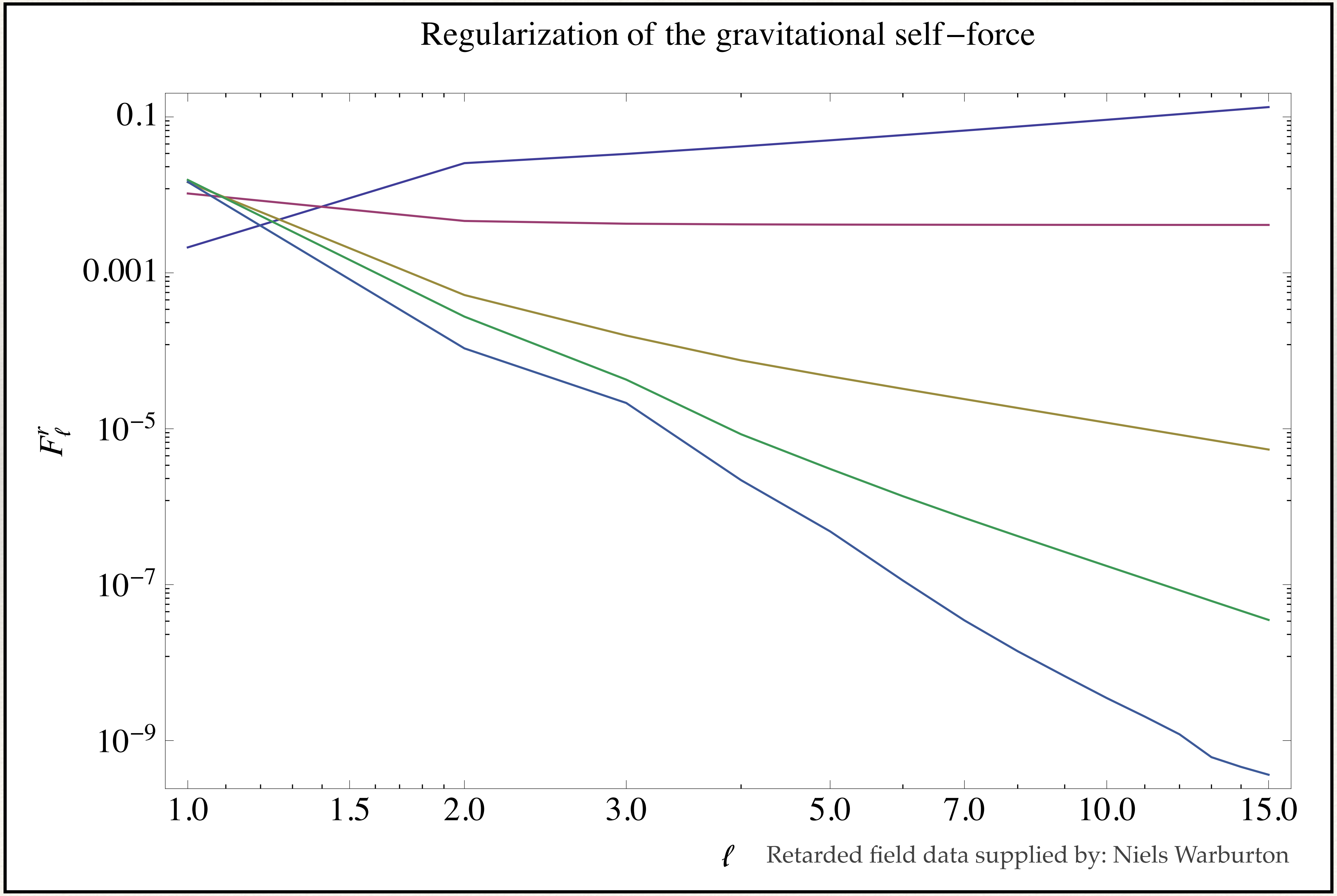


	$l_{max} = 25, n = 12$		$l_{max} = 80, n = 50$	
RPs used	abs.	rel.	abs.	rel.
AB	$1.3784482573 \times 10^{-5}$	1.2×10^{-10}	$1.37844825756674 \times 10^{-5}$	3.7×10^{-14}
ABD	$1.37844825757 \times 10^{-5}$	5.0×10^{-12}	$1.378448257566791 \times 10^{-5}$	3.3×10^{-15}
$ABDF$	$1.378448257567 \times 10^{-5}$	4.2×10^{-13}	$1.378448257566793 \times 10^{-5}$	1.7×10^{-15}
$ABDFH$	$1.37844825756675 \times 10^{-5}$	3.0×10^{-14}	$1.3784482575667951 \times 10^{-5}$	5.5×10^{-16}
CPU time	155s		4247s	

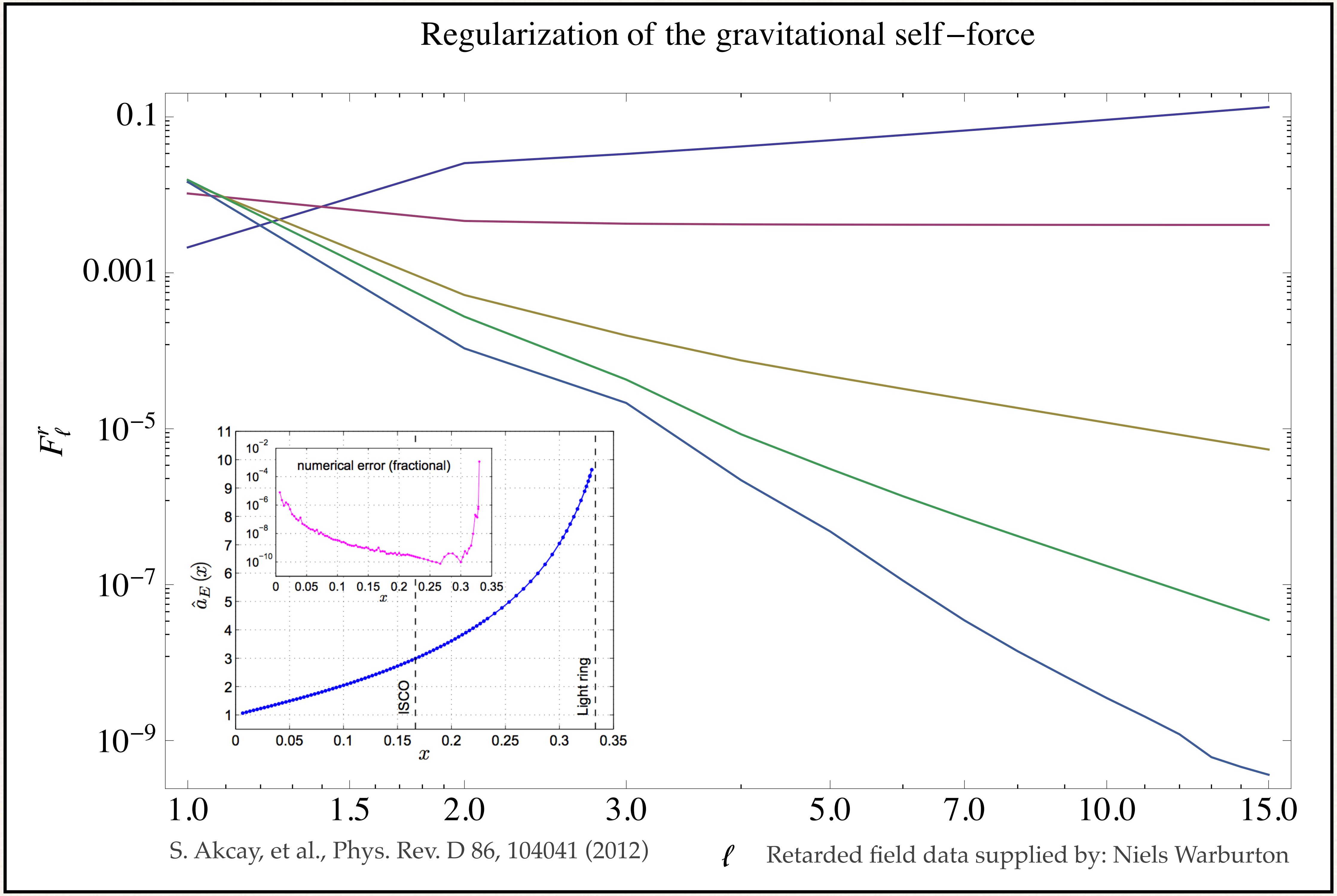
Mode-sum: Schwarzschild EM



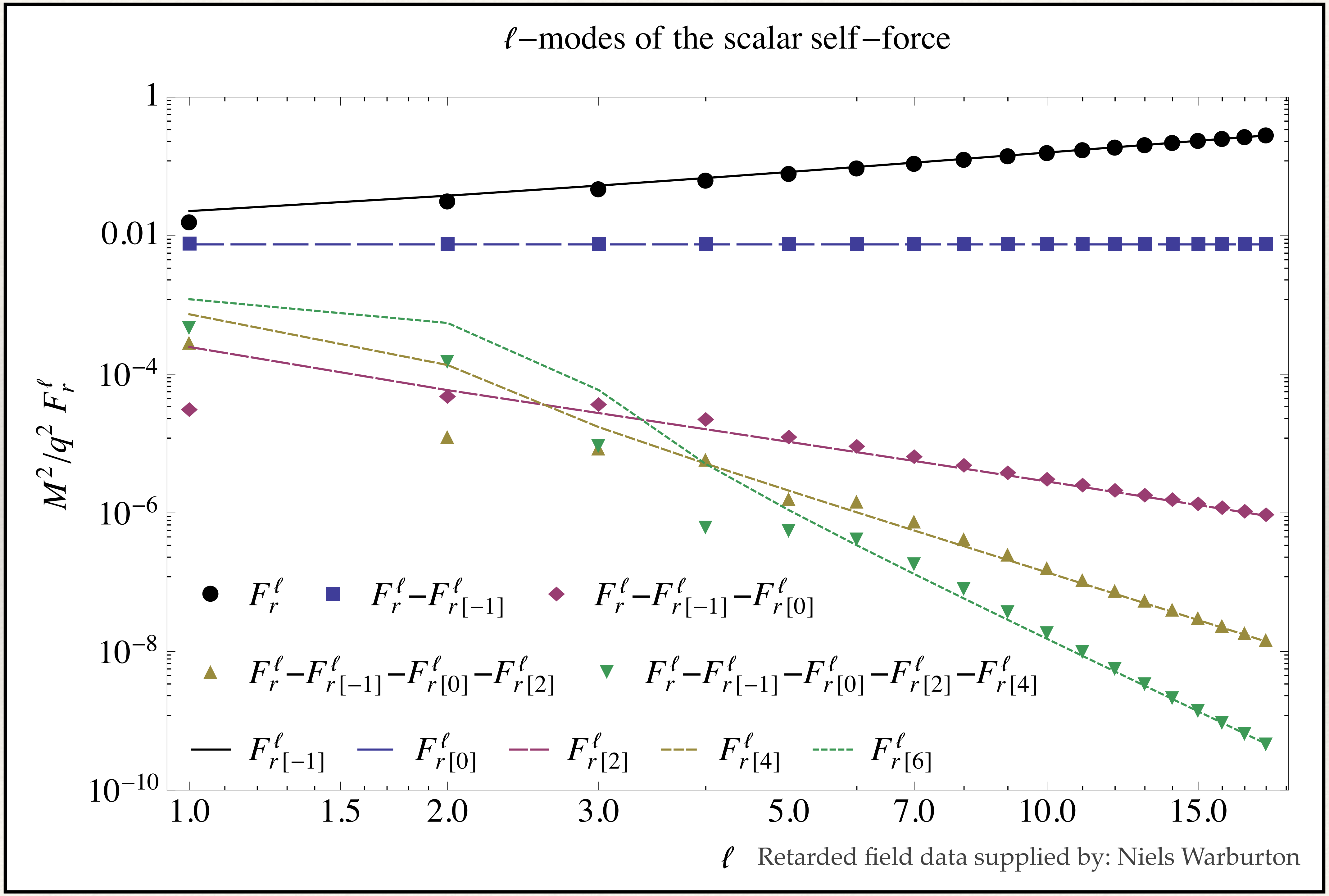
Mode-sum: Schwarzschild gravity



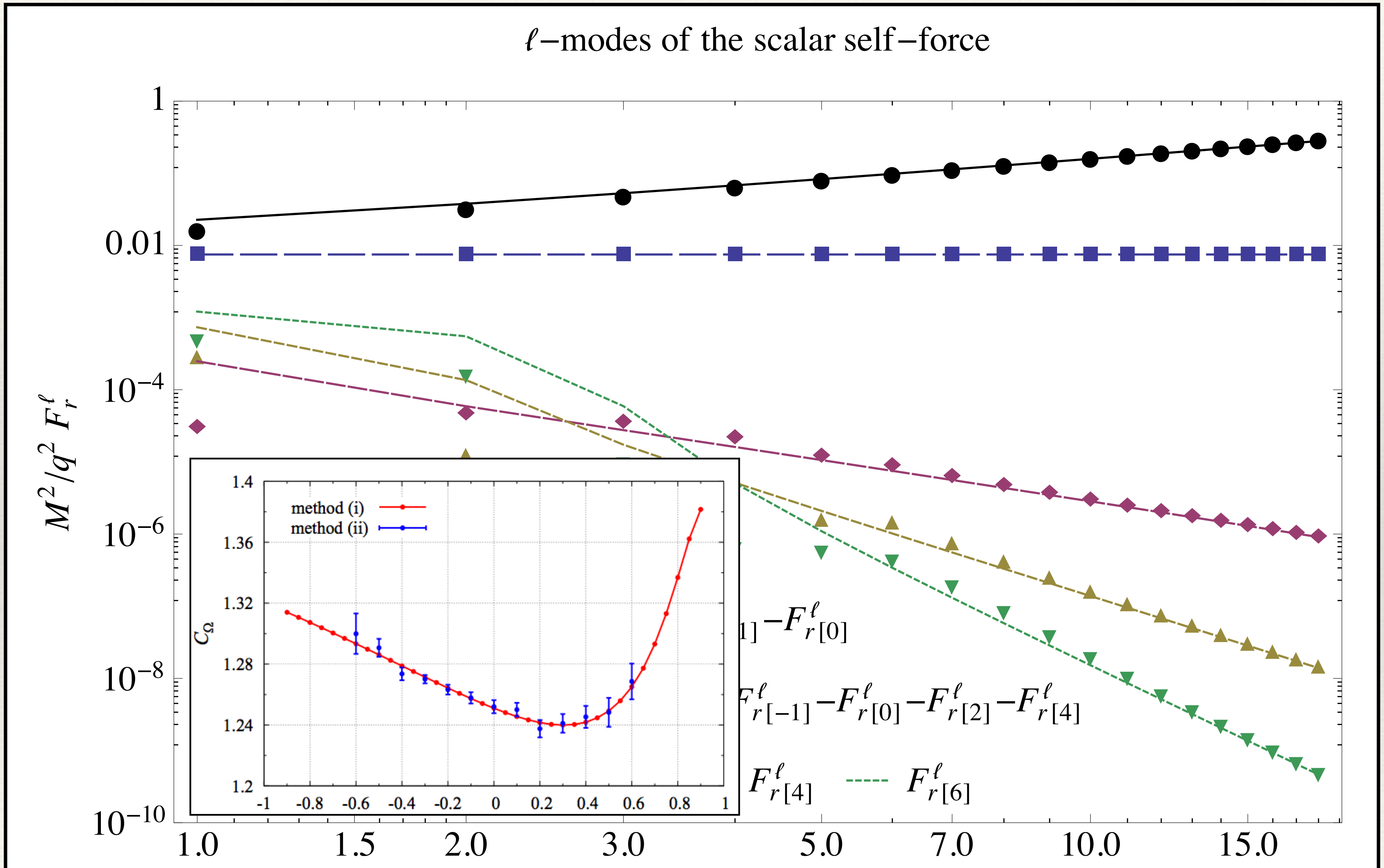
Mode-sum: Schwarzschild gravity



Mode-sum: Kerr scalar



Mode-sum: Kerr scalar

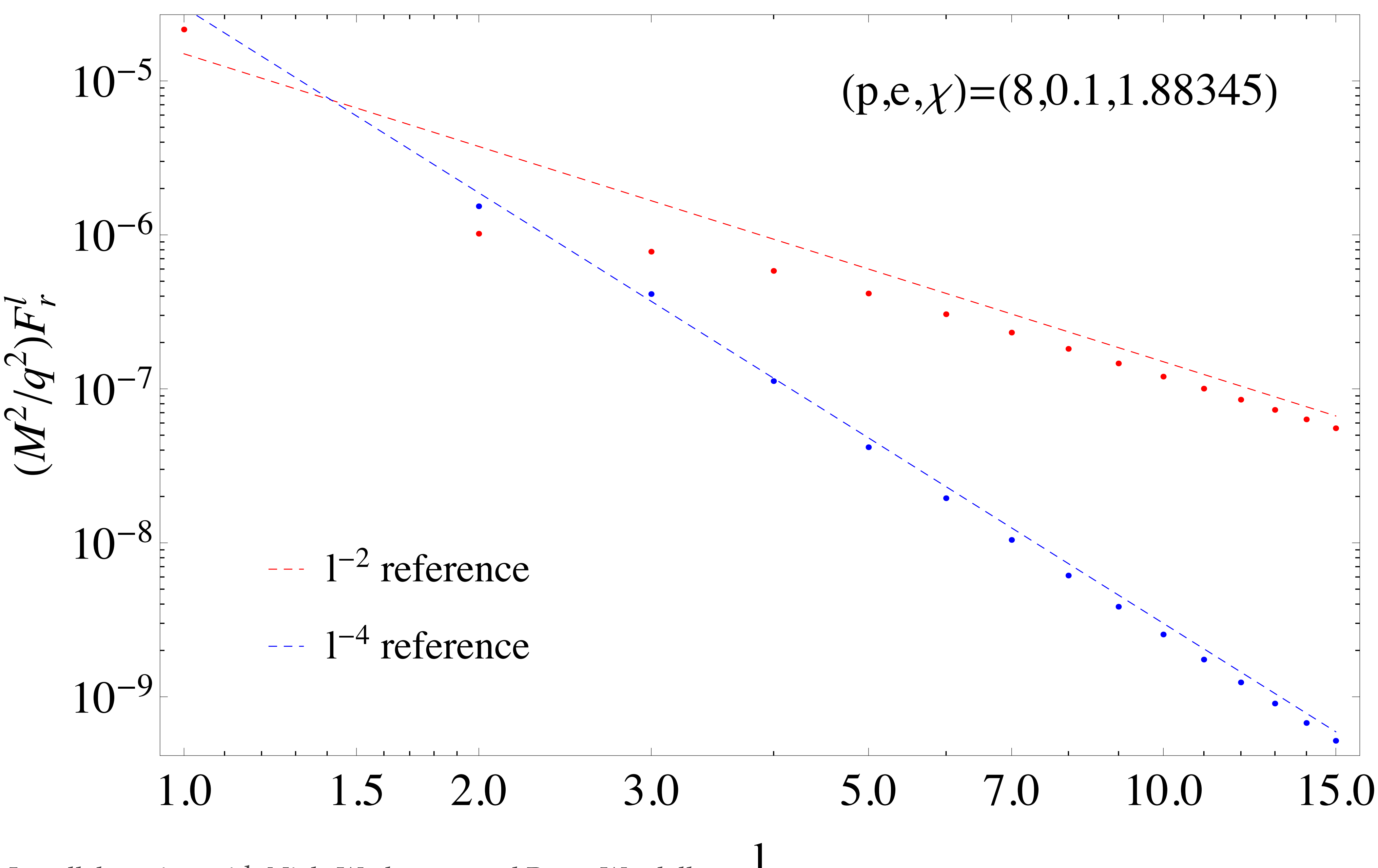


S. Isoyama et al., Phys. Rev. Lett 113, 161101 (2014)

Retarded field data supplied by: Niels Warburton

Mode-sum: Non-geodesic scalar

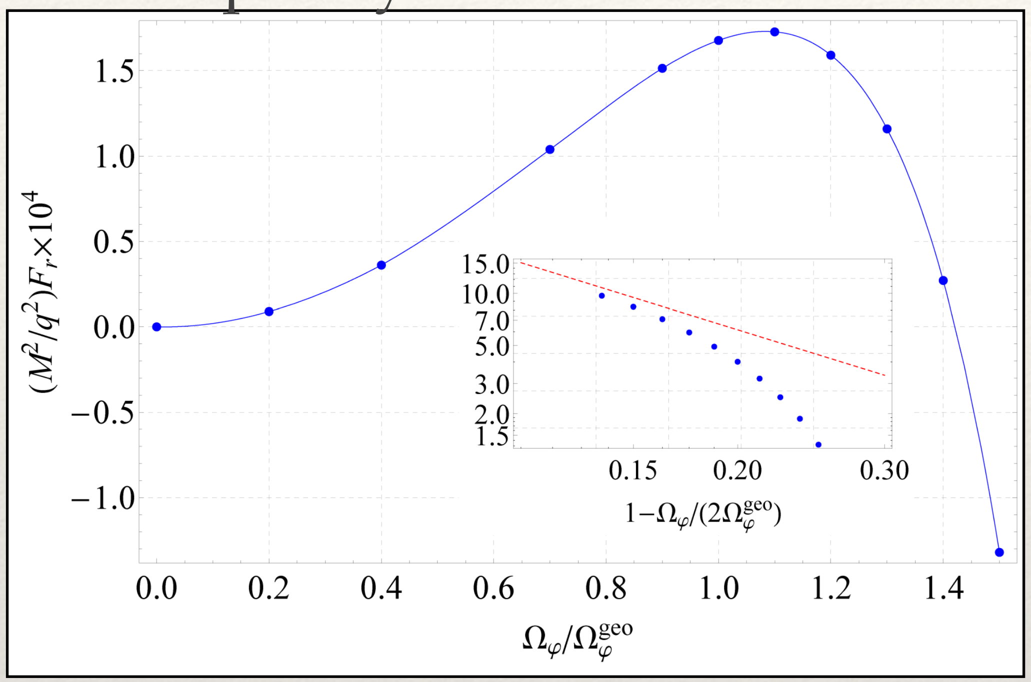
Regularised l-modes for radial scalar self-force for non-geodesic motion



In collaboration with Niels Warburton and Barry Wardell

Non-geodesic scalar in Schwarzschild

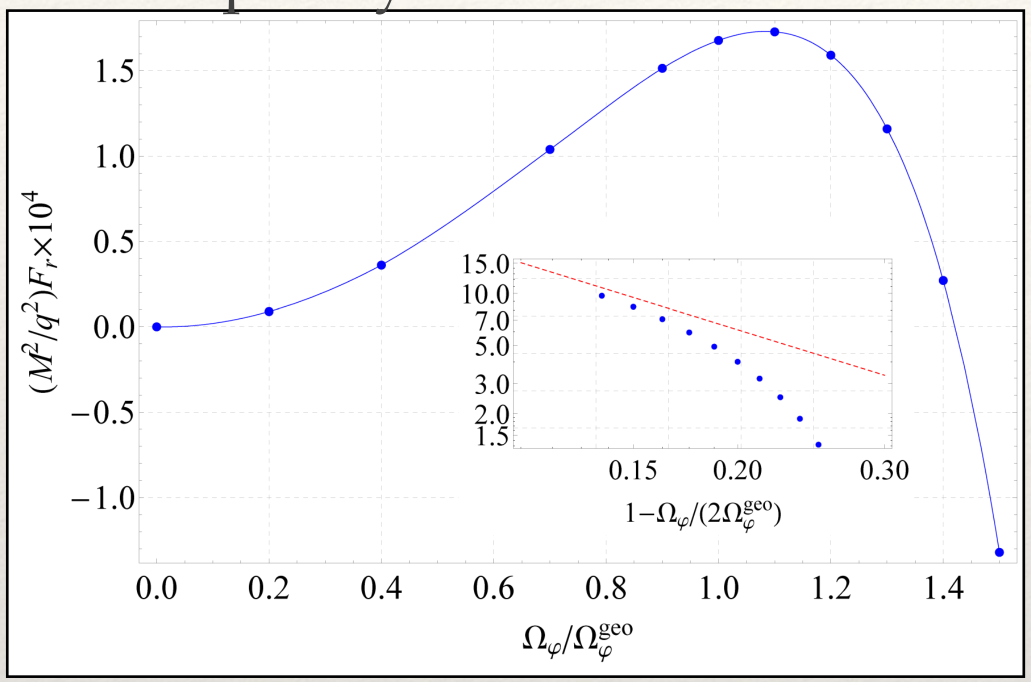
Frequency Domain: Circular



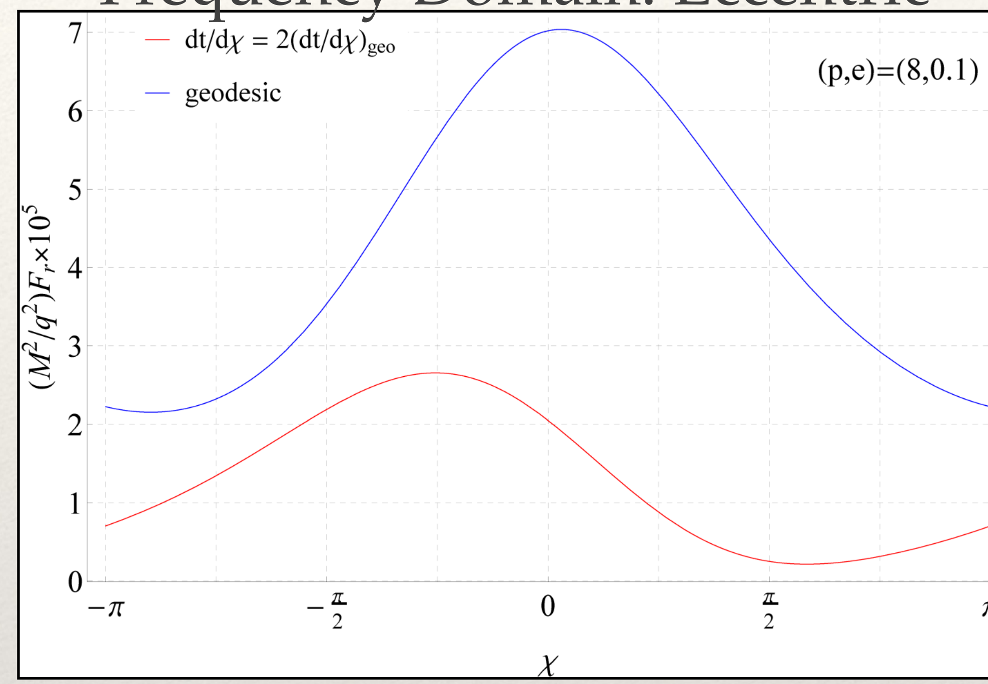
In collaboration with
Niels Warburton
and Barry Wardell

Non-geodesic scalar in Schwarzschild

Frequency Domain: Circular



Frequency Domain: Eccentric



In collaboration with
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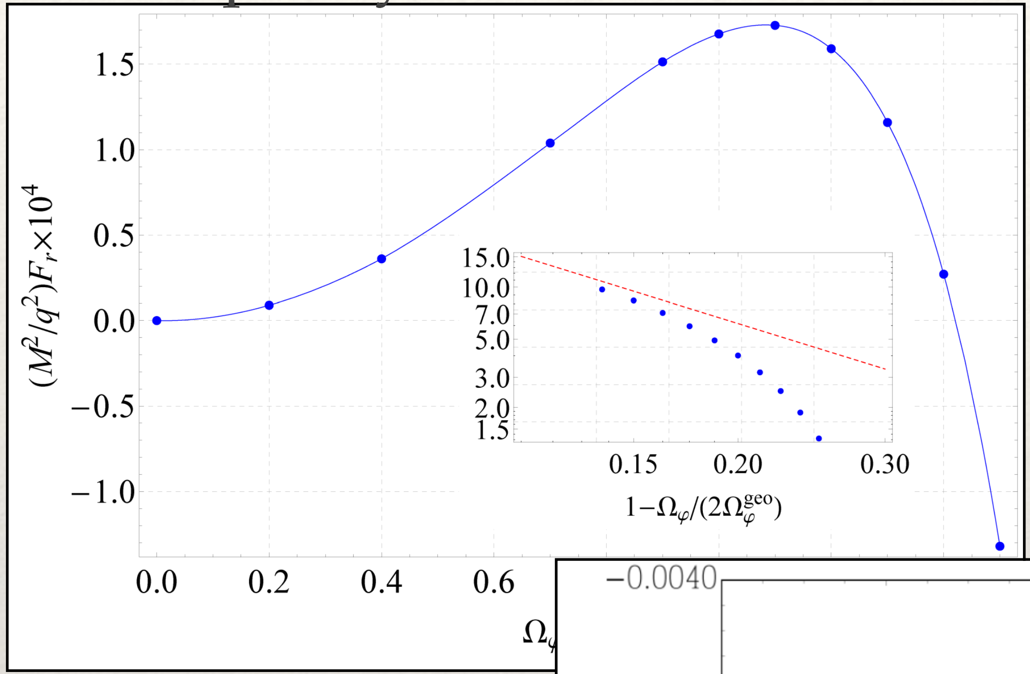
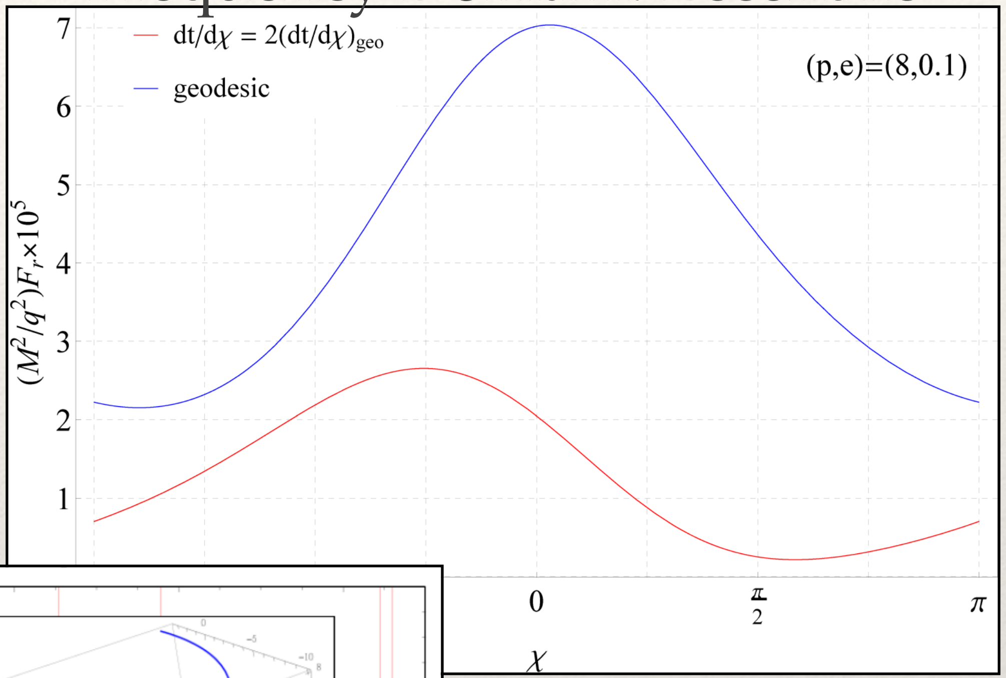
Non-geodesic scalar in Schwarzschild

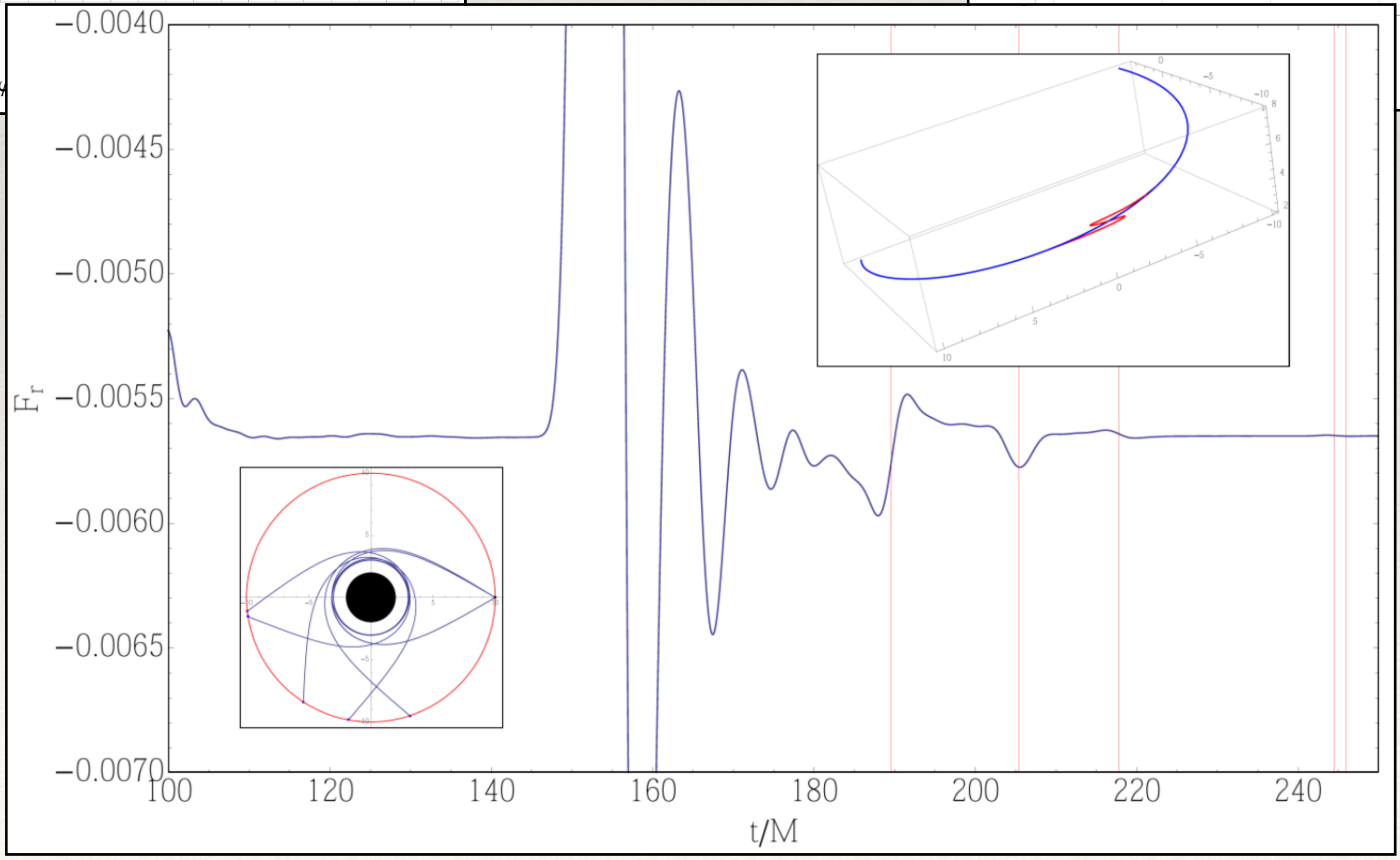
Frequency Domain: Circular



Frequency Domain: Eccentric



Time Domain



In collaboration with
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and Barry Wardell

Effective Source

Approximate regular/singular split $\varphi^{A}_{(\mathrm{ret})} = \tilde{\varphi}^{A}_{(\mathrm{S})} + \tilde{\varphi}^{A}_{(\mathrm{R})}$

$$S_{eff} = \mathcal{D}^{A}{}_{B}\tilde{\varphi}^{B}_{(\mathrm{R})} = -4\pi\mathcal{Q}\int u^{A}\delta_4\left(x, z(\tau')\right)d\tau' - \mathcal{D}\tilde{\varphi}^{A}_{(\mathrm{sing})}$$

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$$S^A_{\mathrm{eff}} = \begin{cases} 0 & \text{(at the particle)} \\ -\mathcal{D}^A{}_B \tilde{\varphi}^B_{(\mathrm{sing})} & \text{(away from the particle)} \end{cases}$$

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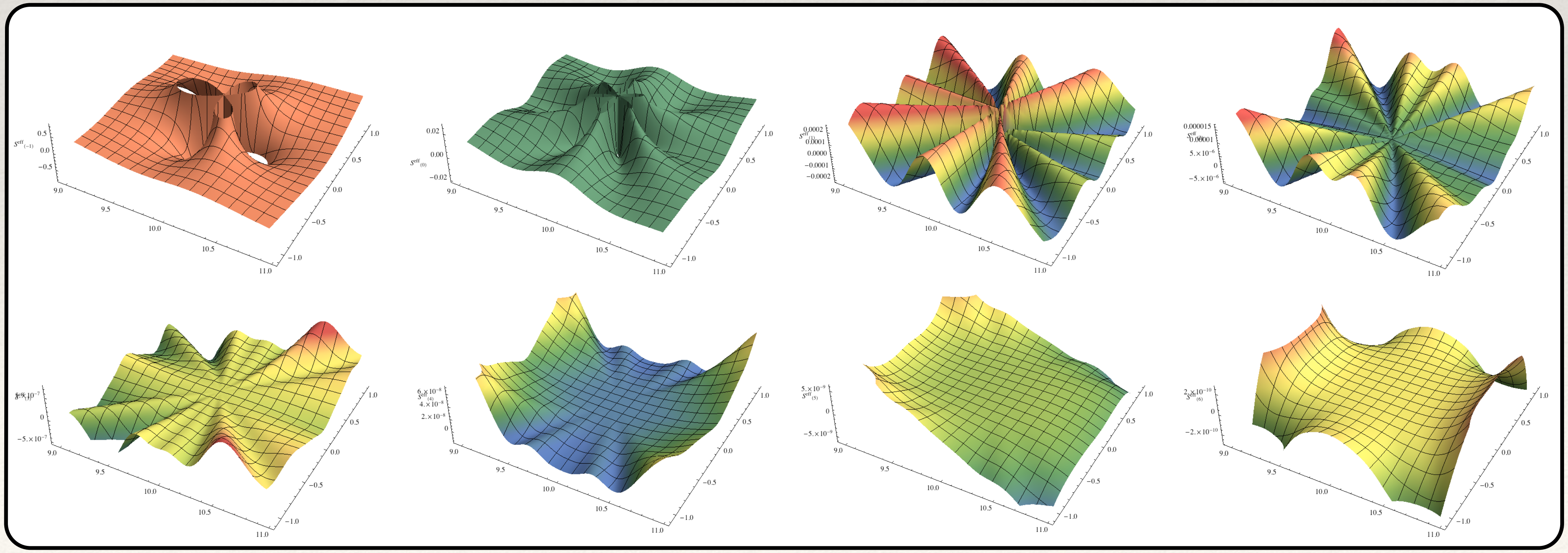

Effective Source: m-modes in Kerr

m-mode decomposition

$$\tilde{\Phi}^{(m)}_{(\mathrm{R})} = \frac{1}{2\pi}\int_{-\pi}^{\pi} \tilde{\Phi}_{(\mathrm{R})} e^{-im\phi} d\phi \quad \Rightarrow \quad \tilde{\Phi}_{(\mathrm{R})} = \sum_{m=-\infty}^{\infty} \tilde{\Phi}^{(m)}_{(\mathrm{R})} e^{im\phi_0}$$

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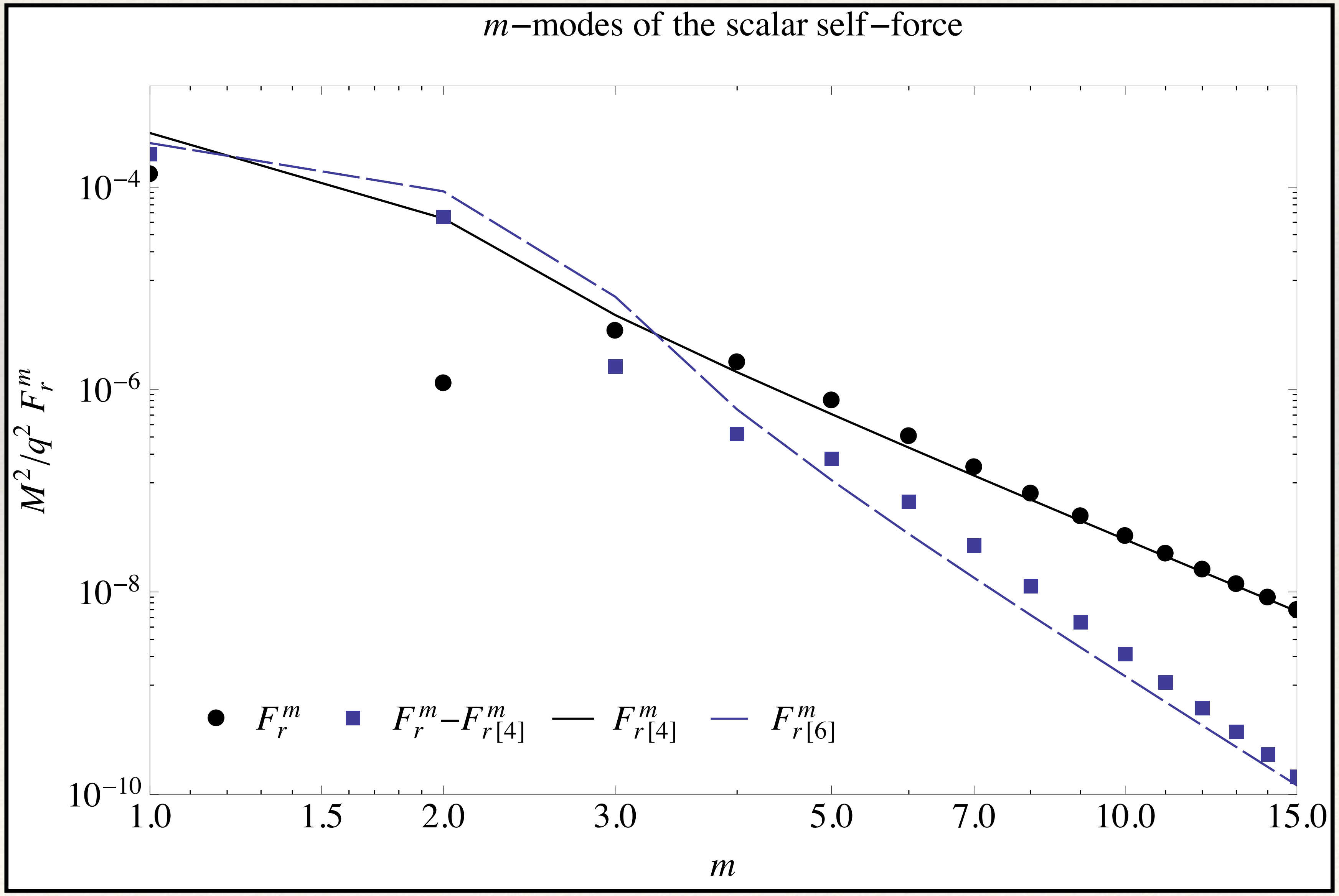

Further work

Self-force waveforms still require:

- More accurate Kerr gravity results required at 1st order
- 2nd order
- Orbit evolution

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Self-force waveforms still require:

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Results of this thesis:

- Highly accurate singular field required for 3 different self-force methods
- Nearly doubling the number of regularisation parameters within mode sum
- Providing the 'puncture' used for the first Kerr gravity calculation
- Introduced a new m-mode post-regularisation method
- Non-geodesic motion -> possible application in orbit evolution
- Singular field is a requirement for 2nd order

Thank you!