

8. NDI AND THE EET:

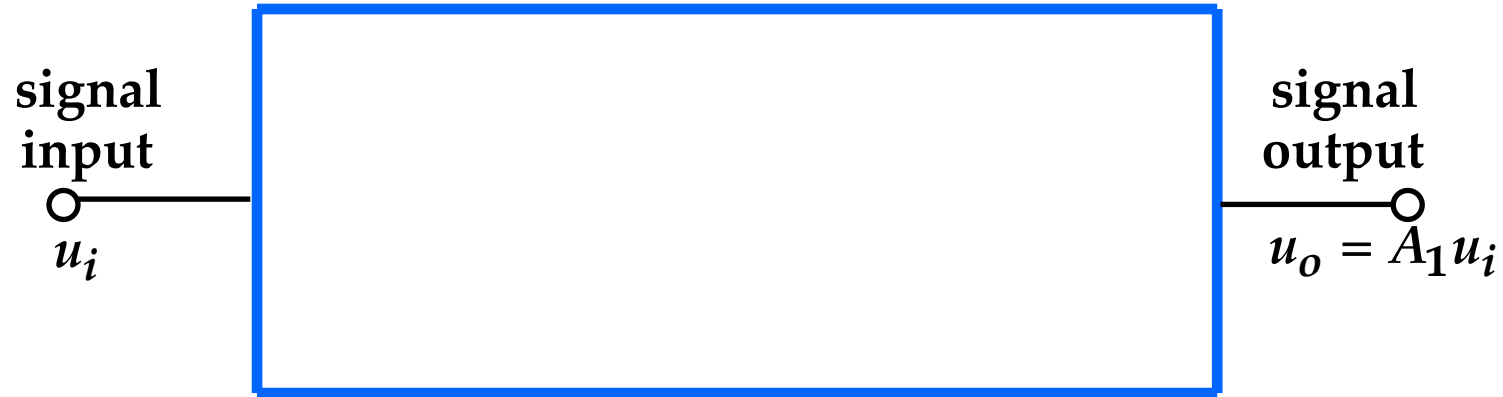
Null Double Injection and the Extra Element Theorem

How to find the contribution of a particular element to the transfer function

Null Double Injection (ndi)

Usually, a transfer function (TF) is calculated as a response to a single independent excitation.

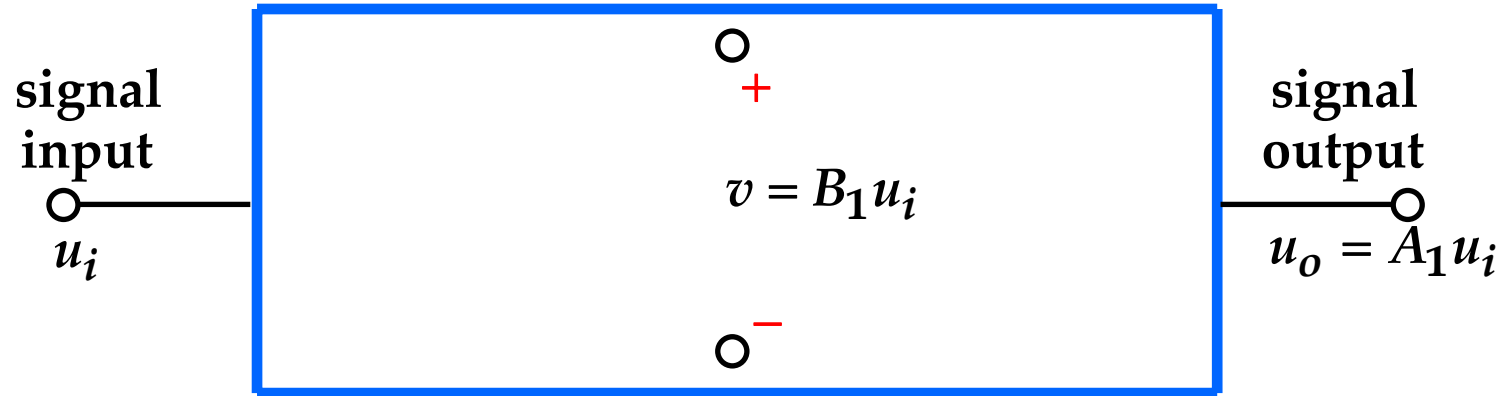
However, large analysis benefits accrue when certain constraints are imposed on several excitations present simultaneously.



The input is an independent signal, the output is a dependent signal.

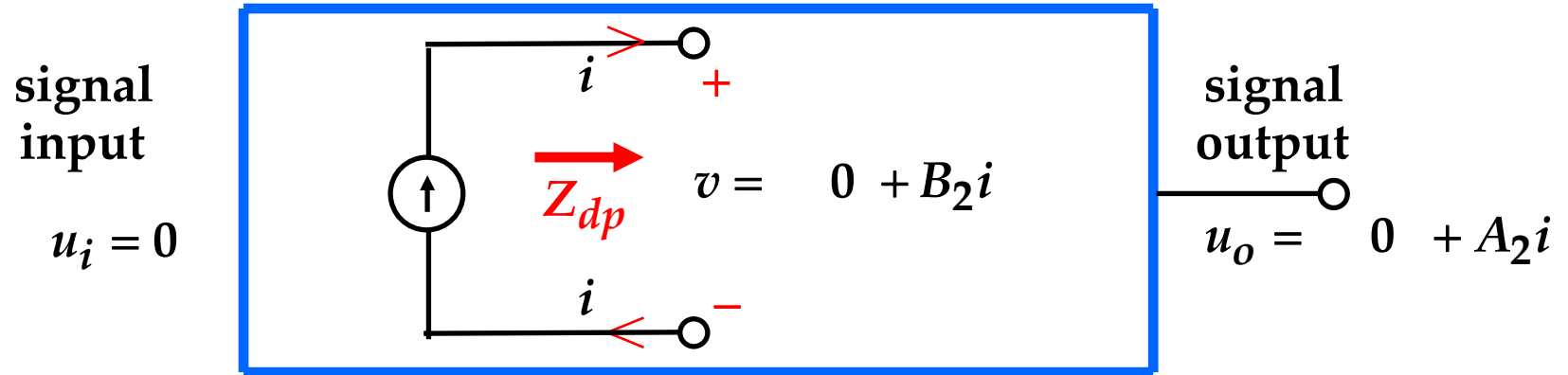
The gain is A_1 .

Consider a second dependent signal, a voltage v at some internal port :



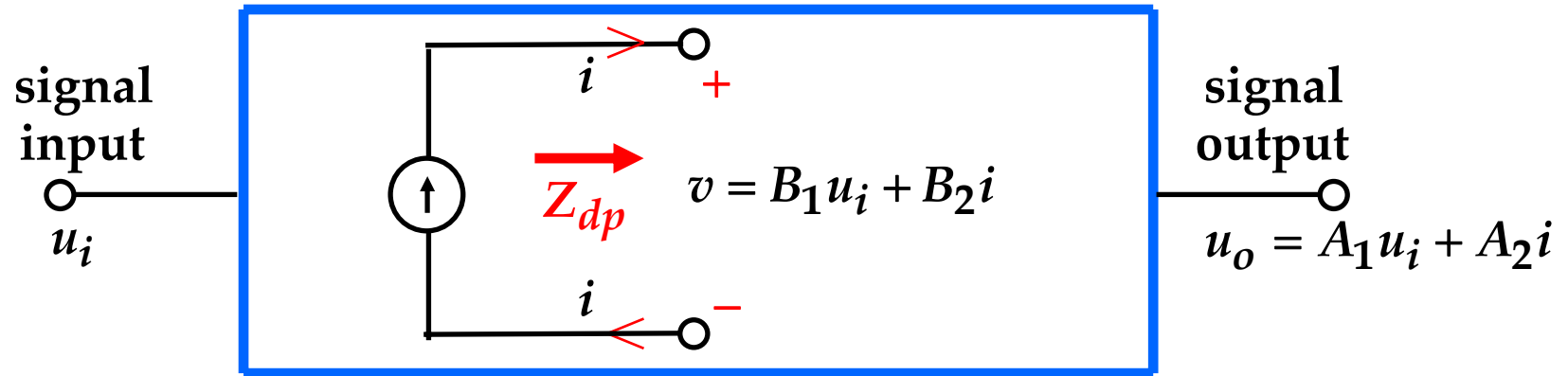
The gain from u_i to v is B_1 .

Apply a second independent signal, a current i at the same internal port :



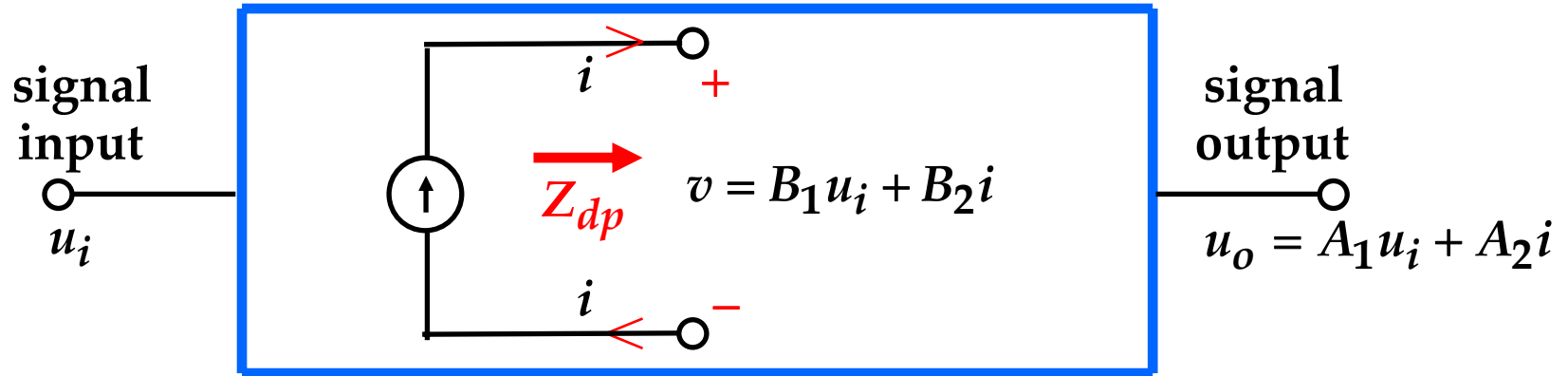
The "gain" from i to v is a driving point impedance B_2 .

Apply both independent signals simultaneously:



For a linear system model, the two dependent signals are the superposition of the values they would have for each independent signal separately.

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For a linear system model, the two dependent signals are the superposition of the values they would have for each independent signal separately.

By adjustment of u_i and i , u_o can be made to have any value we like.

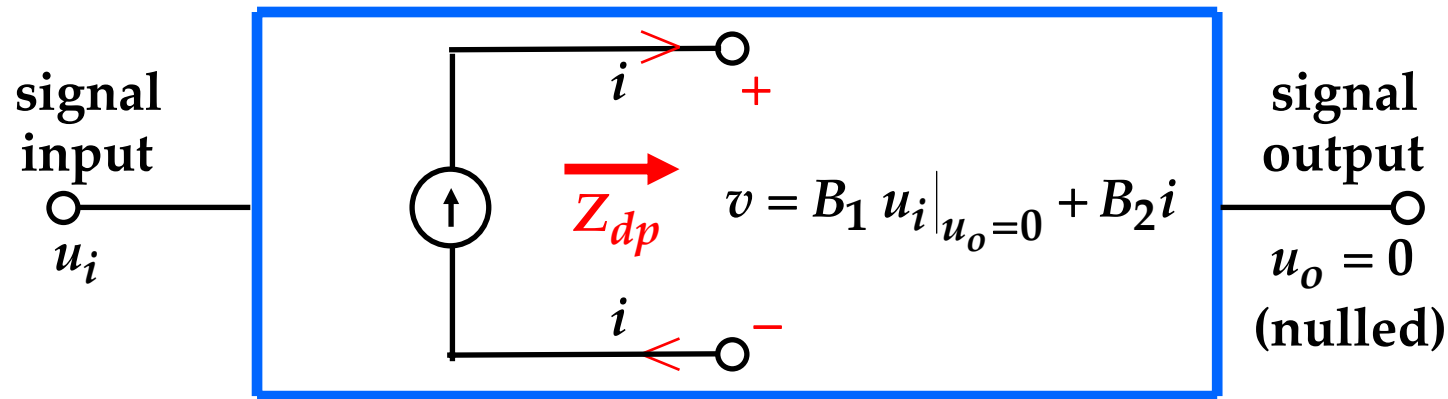
In particular:

u_o can be made zero by adjustment of the relative values of u_i and i , namely

$$u_i|_{u_o=0} = -\frac{A_2}{A_1} i$$

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There is now a null double injection (ndi) condition:



The voltage at the internal port is

$$v = \left(B_2 - B_1 \frac{A_2}{A_1} \right) i$$

so the driving point impedance is

$$Z_{dp} |_{u_o=0} = \left(B_2 - B_1 \frac{A_2}{A_1} \right)$$

Recap:

The driving point impedance (dpi) Z_{dp} at the internal port can have two different values, one when the input is zero, and another when the input is not zero, but is adjusted to null the output:

$$Z_{dp}|_{u_i=0} = B_2$$

↑
from i

$$Z_{dp}|_{u_o=0} = B_2 - B_1 \frac{A_2}{A_1}$$

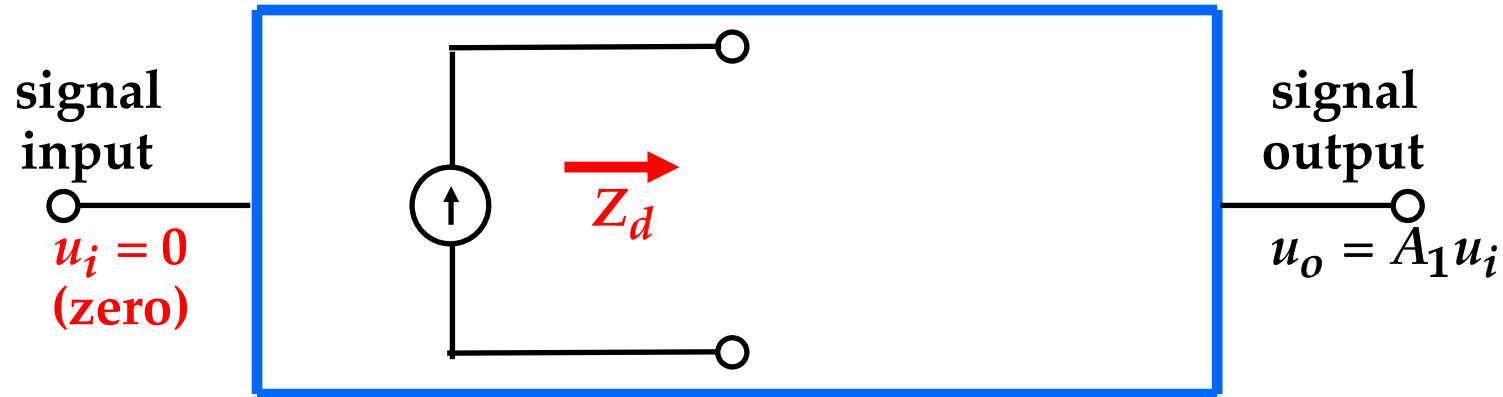
↑
from u_i

Recap:

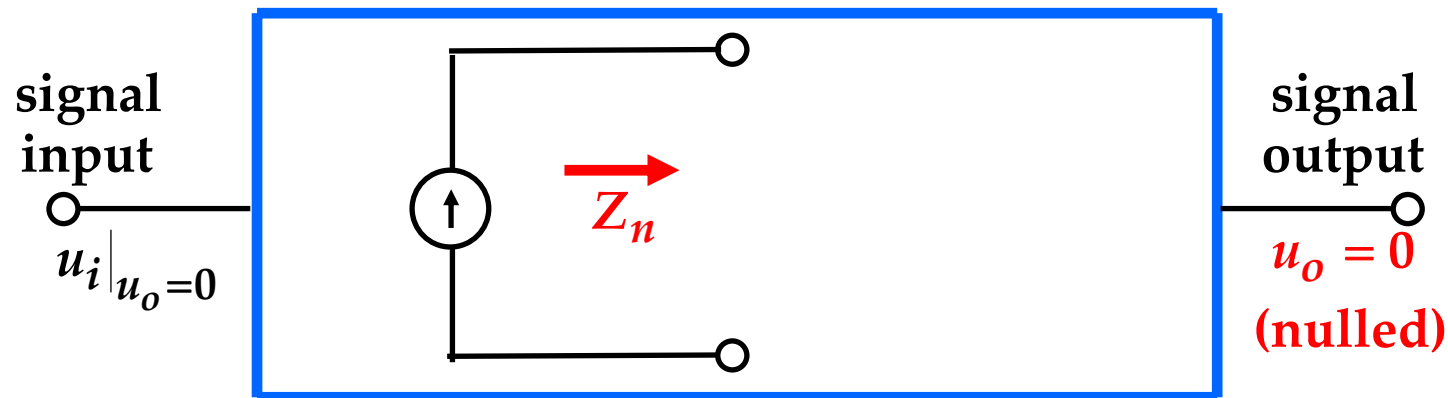
The driving point impedance (dpi) Z_{dp} at the internal port can have two different values, one when the input is zero, and another when the input is not zero, but is adjusted to null the output:

$$\begin{array}{ccc} Z_{dp}|_{u_i=0} = B_2 & \equiv Z_d & \\ \uparrow & & \\ \text{from } i & & \\ \downarrow & & \\ Z_{dp}|_{u_o=0} = B_2 - B_1 \frac{A_2}{A_1} & \equiv Z_n & \\ \uparrow & & \\ \text{from } u_i & & \end{array}$$

The dpi Z_d is calculated under single injection (si) conditions:

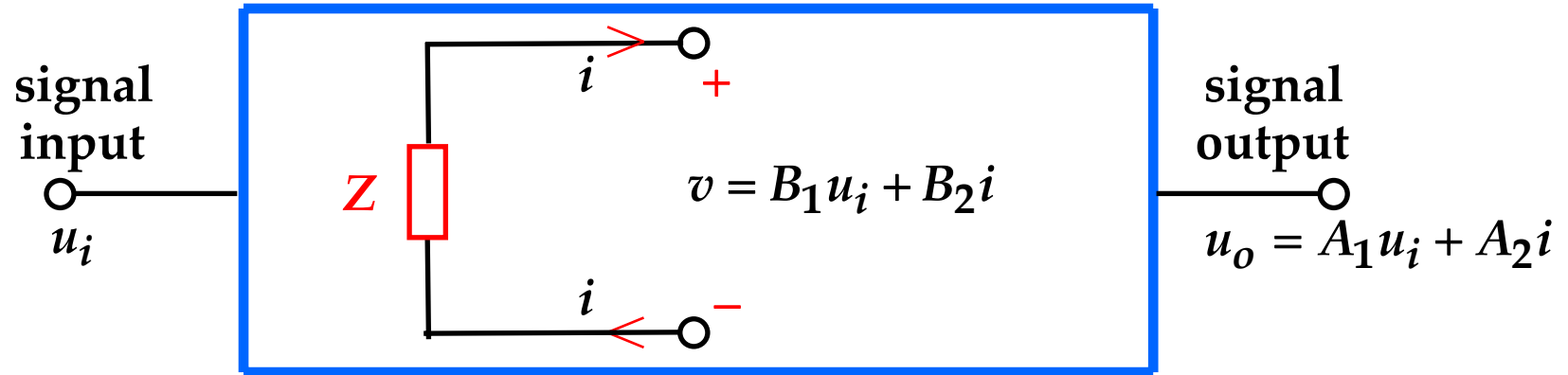


The dpi Z_n is calculated under null double injection (ndi) conditions:



The Extra Element Theorem (EET)

Replace the current source by an impedance Z :



The same linear superposition equations still apply to the rest of the circuit.

However, a relation between v and i is now enforced by Z , namely

$$v = -Zi$$

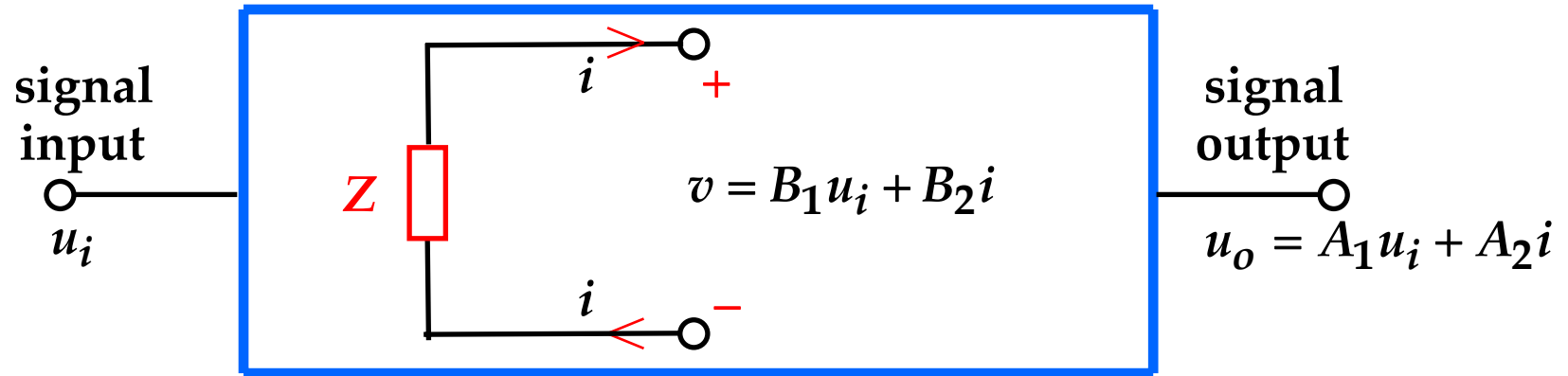
Substitute for v to find i in terms of u_i :

$$v = B_1 u_i + B_2 i = -Zi$$

so

$$i = -\frac{B_1}{B_2 + Z} u_i$$

The Extra Element Theorem (EET)

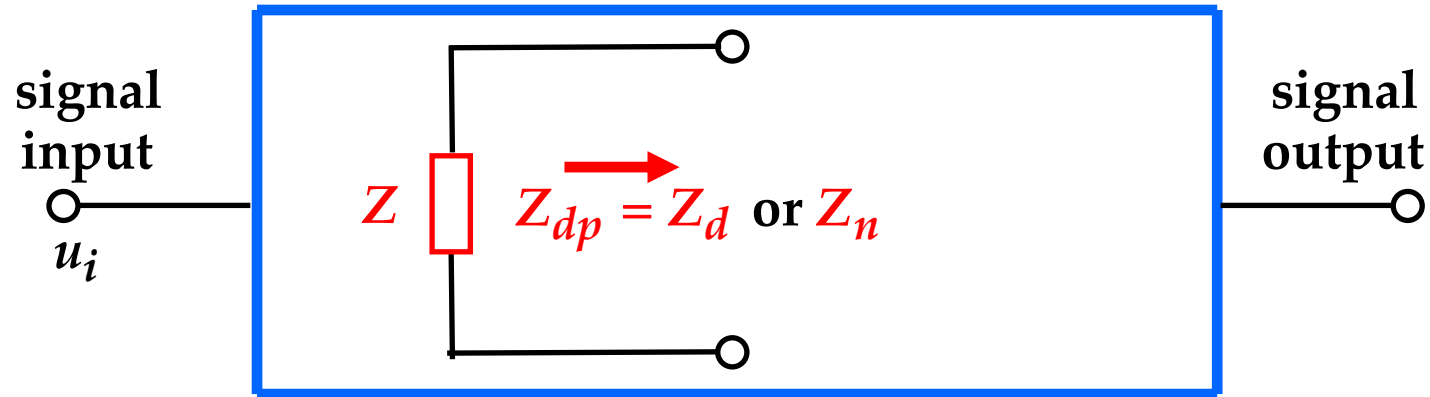


$$i = -\frac{B_1}{B_2 + Z} u_i$$

Now substitute for i in the equation for u_o to find u_o in terms of u_i :

$$\begin{aligned} u_o &= A_1 u_i + A_2 i = A_1 u_i - A_2 \frac{B_1}{B_2 + Z} u_i \\ &= A_1 \left(\frac{B_2 - B_1 \frac{A_2}{A_1} + Z}{B_2 + Z} \right) u_i = A_1 \left(\frac{1 + \frac{B_2 - B_1 \frac{A_2}{A_1}}{Z}}{1 + \frac{B_2}{Z}} \right) u_i \end{aligned}$$

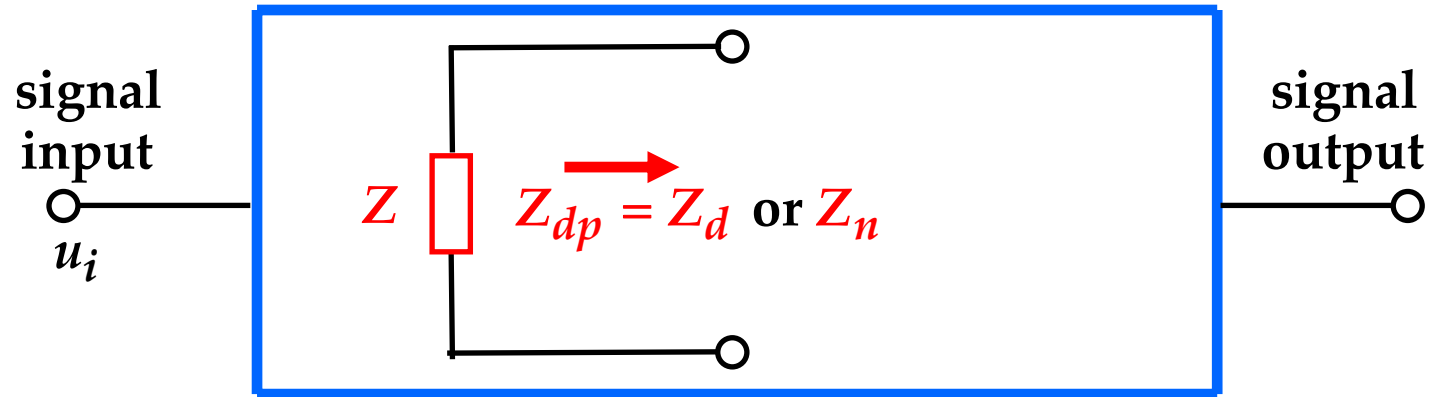
The Extra Element Theorem (EET)



The two combinations of the linear circuit parameters are precisely what have just been defined as Z_d and Z_n , so

$$u_o = A_1 \left(\frac{1 + \frac{Z_n}{Z}}{1 + \frac{Z_d}{Z}} \right) u_i$$

The Extra Element Theorem (EET)



The two combinations of the linear circuit parameters are precisely what have just been defined as Z_d and Z_n , so

$$u_o = A_1 \left(\frac{1 + \frac{Z_n}{Z}}{1 + \frac{Z_d}{Z}} \right) u_i$$

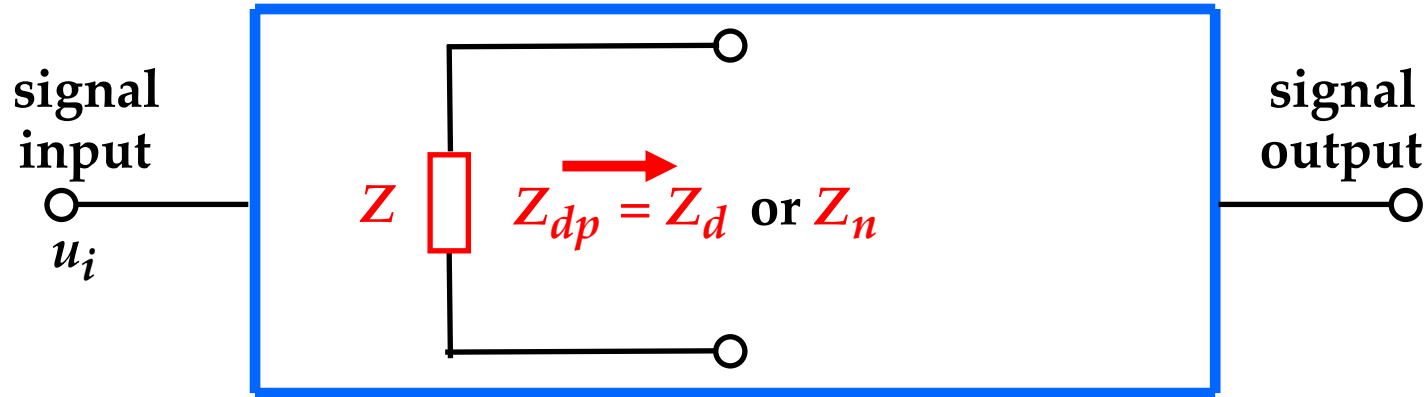
However,

$$\frac{u_o}{u_i} = \text{gain in presence of } Z$$

$$A_1 = \text{gain when } Z = \infty$$

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The Extra Element Theorem (EET)



The two combinations of the linear circuit parameters are precisely what have just been defined as Z_d and Z_n , so

$$u_o = A_1 \left(\frac{1 + \frac{Z_n}{Z}}{1 + \frac{Z_d}{Z}} \right) u_i$$

However,

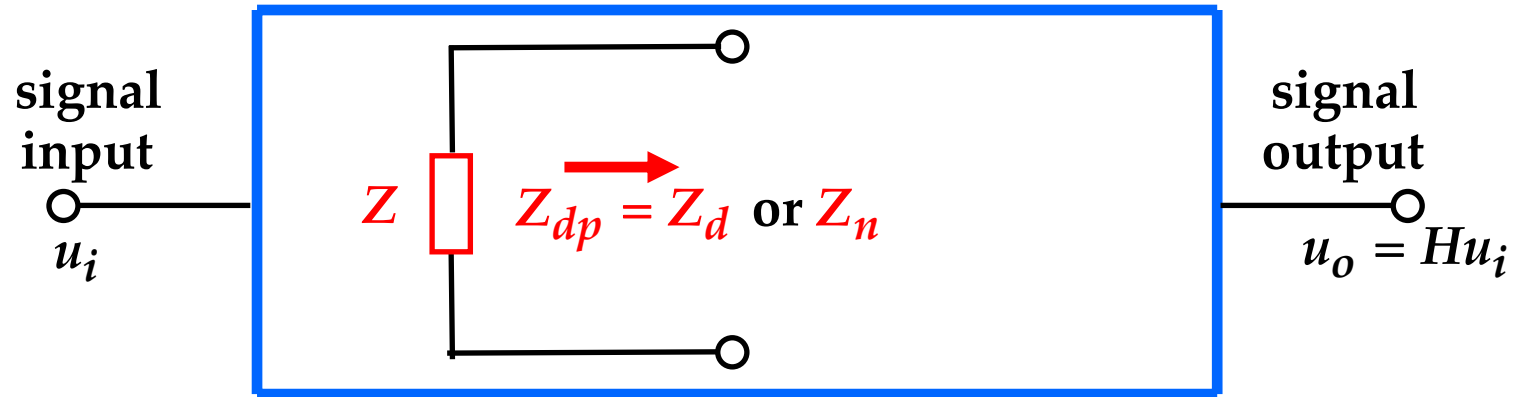
$$\frac{u_o}{u_i} = \text{gain in presence of } Z \equiv H$$

$$A_1 = \text{gain when } Z = \infty$$

$$\equiv H_o$$

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The Extra Element Theorem (EET)



Hence, the Extra Element Theorem (EET) is:

$$H = H_{\infty} \left(\frac{1 + \frac{Z_n}{Z}}{1 + \frac{Z_d}{Z}} \right)$$

H = gain in presence of Z

H_{∞} = gain when $Z = \infty$

$$Z_d = Z_{dp} \Big|_{u_i=0}$$

$u_i = \text{zero}$

$$Z_n = Z_{dp} \Big|_{u_o=0}$$

$u_i \neq \text{zero}, u_o$ nulled

Hence

$$u_{o1} = A_1 \frac{1 + \frac{z_n}{z}}{1 + \frac{z_d}{z}} u_{i1}$$

or

$$\text{gain}|_z = \text{gain}|_{z=\infty} \frac{1 + \frac{z_n}{z}}{1 + \frac{z_d}{z}}$$

This is the Extra Element Theorem: how to calculate the gain, after an extra element is added, by a correction factor instead of starting from scratch.

Hence

$$u_{o1} = A_1 \frac{1 + \frac{z_n}{z}}{1 + \frac{z_d}{z}} u_{i1}$$

or

$$\text{gain} \Big|_z = \text{gain} \Big|_{z=\infty} \frac{1 + \frac{z_n}{z}}{1 + \frac{z_d}{z}}$$

This is the Extra Element Theorem: how to calculate the gain, after an extra element is added, by a correction factor instead of starting from scratch.

The Theorem also proves that any transfer function (e.g. gain) of a linear system is a bilinear function of any single element (e.g. z).

forming Δ_{11} the terms by which z is multiplied must be the minor Δ_{11jj} obtained by omitting both the first and j 'th rows and columns. If we let Δ^0 and Δ_{11}^0 represent, respectively, Δ and Δ_{11} when $z = 0$, therefore, we have

$$Z = \frac{\Delta^0 + z\Delta_{jj}}{\Delta_{11}^0 + z\Delta_{11jj}} \quad (1-11)$$

Since Δ_{ij} and Δ_{11jj} are evidently independent of z they can equally well be written as Δ_{ij}^0 and Δ_{11jj}^0 . This will occasionally be done in later analysis in order to facilitate further transformations.

The relation between Z_T and z can be found in similar fashion. It is given by

$$Z_T = \frac{\Delta^0 + z\Delta_{jj}}{\Delta_{22}^0 + z\Delta_{22jj}} \quad (1-12)$$

If z represents a unilateral coupling term, instead of a bilateral element, the expansion is essentially the same. Thus, if we suppose that z is a part of Z_{ij} in the original determinant, we readily find

$$Z = \frac{\Delta^0 + z\Delta_{ij}}{\Delta_{ii}^0 + z\Delta_{11ij}} \quad (1-13)$$

and

$$Z_T = \frac{\Delta^0 + z\Delta_{ij}}{\Delta_{ii}^0 + z\Delta_{11ij}} \quad (1-14)$$

The "brute-force" method: loop analysis

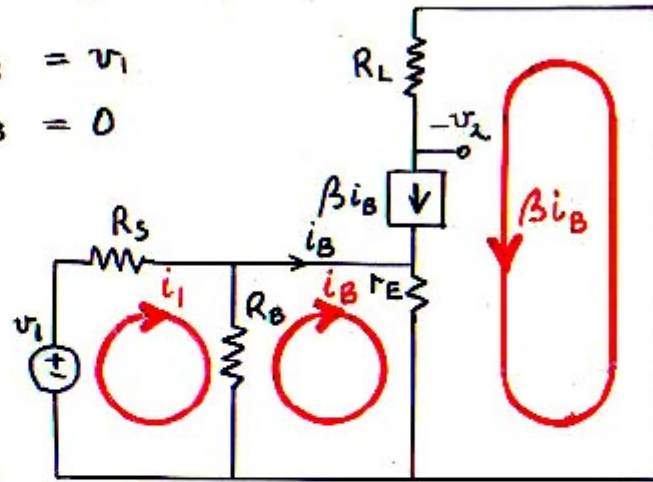
$$\begin{aligned} (R_s + R_B) i_1 - R_B i_B &= v_1 \\ -R_B i_1 + [R_B + (1+\beta)r_E] i_B &= 0 \end{aligned}$$

$$i_B = \frac{\begin{vmatrix} R_s + R_B & v_1 \\ -R_B & 0 \end{vmatrix}}{\begin{vmatrix} R_s + R_B & -R_B \\ -R_B & R_B + (1+\beta)r_E \end{vmatrix}} = \frac{R_B v_1}{(R_s + R_B)[R_B + (1+\beta)r_E] - R_B^2}$$

$$\begin{aligned} &= \frac{R_B v_1}{R_s R_B + (1+\beta)r_E R_s + R_B^2 + (1+\beta)r_E R_B - R_B^2} \end{aligned}$$

Finally, $v_2 = R_L \beta i_B$
which leads to:

$$A_m \equiv \frac{v_2}{v_1} = \frac{\beta R_B R_L}{(1+\beta)r_E R_s + (1+\beta)r_E R_B + R_s R_B}$$



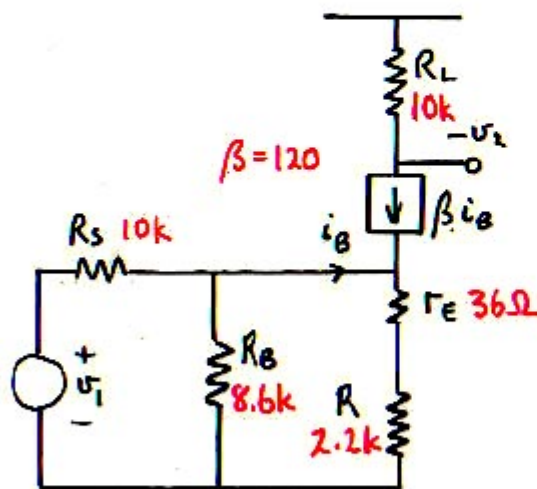
Implementation:

All that is needed is to calculate the driving point impedance across the terminals to which the extra element is to be added, under two conditions:

$$Z_d \equiv Z_{dp} \Big|_{u_{ii}=0} \quad (\text{original input zero})$$

$$Z_n \equiv Z_{dp} \Big|_{u_{oi}=0} \quad (\text{original output nulled})$$

Example: The previously designed CE amplifier
 Suppose the gain has been calculated without the emitter bypass capacitance, and the correction factor resulting from addition of the extra element $Z \rightarrow 1/sC_2$ is desired.

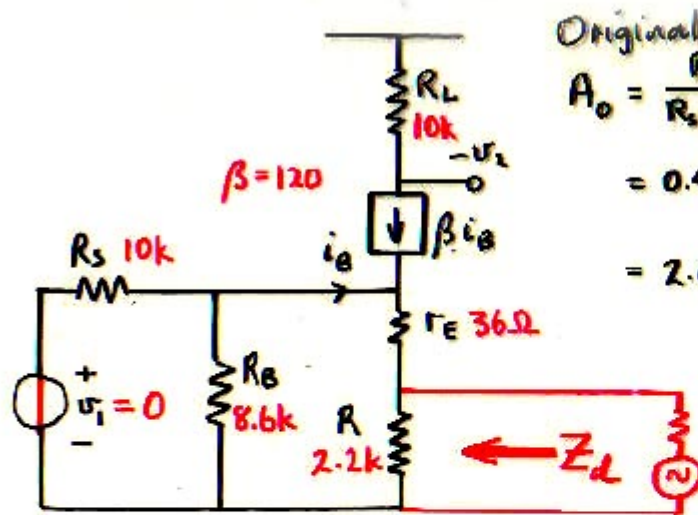


Original gain:

$$A_0 = \frac{R_B}{R_S + R_B} \frac{\alpha R_L}{R + r_e + (R_S \parallel R_B) / (1 + \beta)}$$

$$= 0.46 \frac{10}{2.2 + 0.036 + 0.039}$$

$$= 2.0 \Rightarrow 6 \text{ dB}$$



Original gain:

$$A_o = \frac{R_o}{R_s + R_o} \frac{\alpha R_L}{R + r_e + (R_s \parallel R_o) / (1 + \beta)}$$

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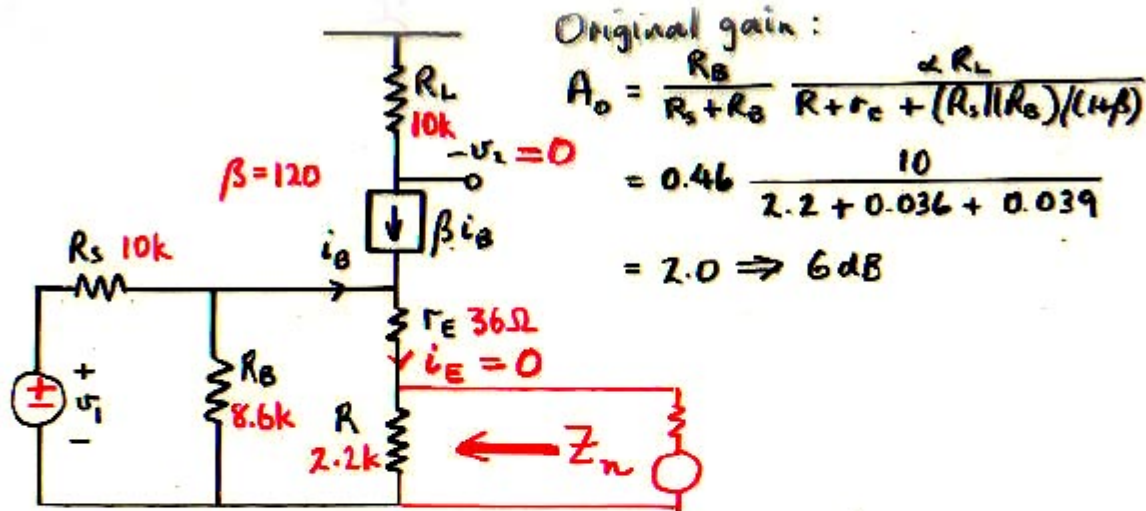
$$= 2.0 \Rightarrow 6 \text{ dB}$$

Step 1. Calculate Z_d by shorting $v_{i1} = v_i$, and applying a second injected signal across R :

$$Z_d = R_d = R \parallel [r_E + (R_s \parallel R_B) / (1 + \beta)]$$

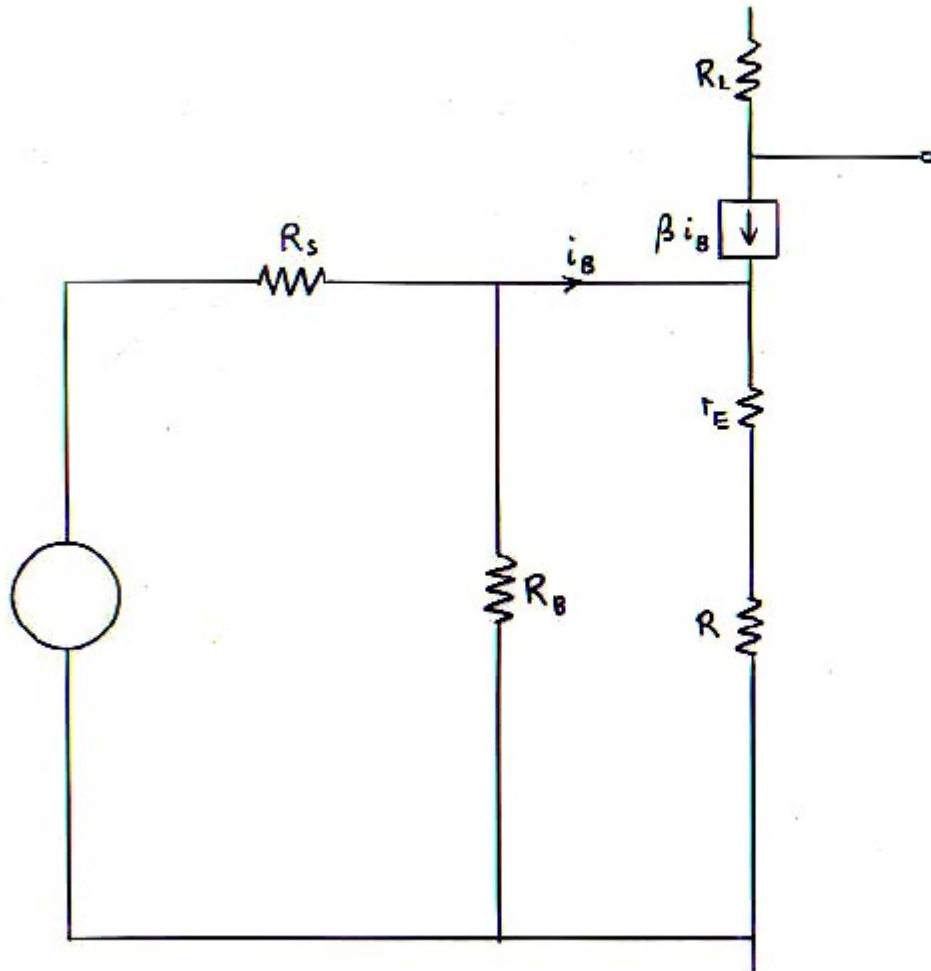
$$= 2.2 \parallel [0.036 + \underbrace{(10 \parallel 8.6) / 120}_{0.039}]$$

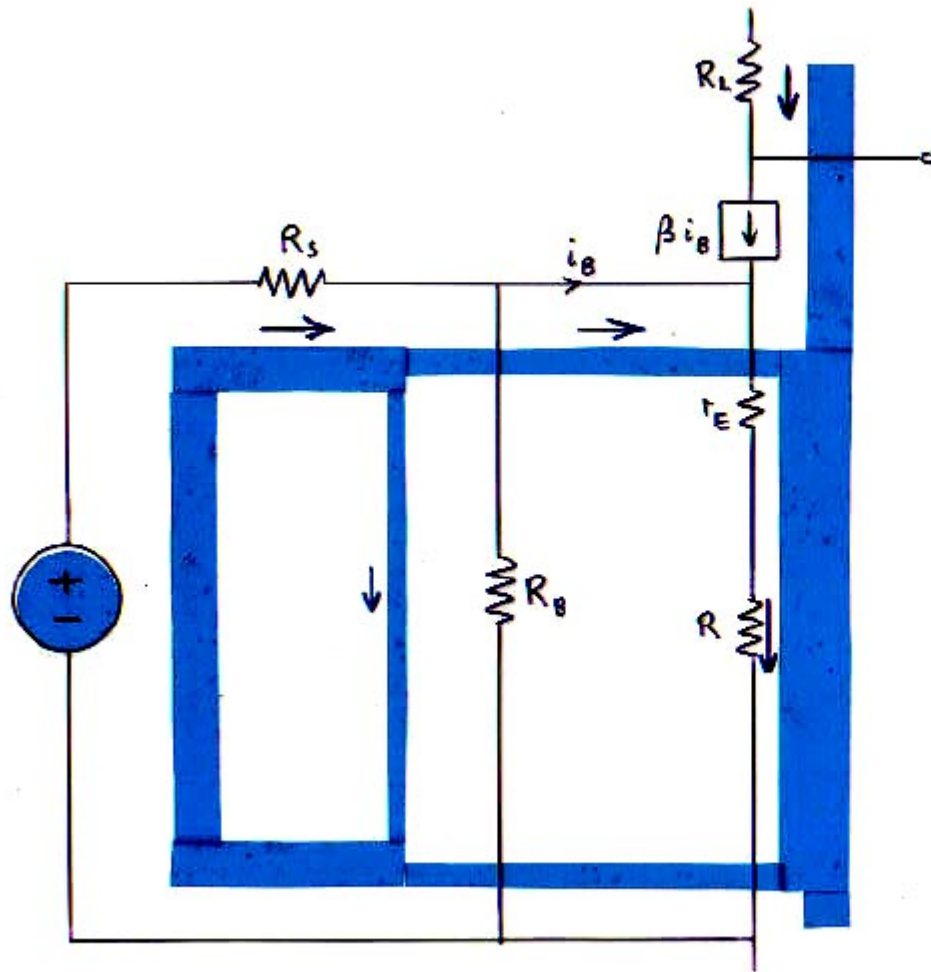
$$= 75 \Omega$$

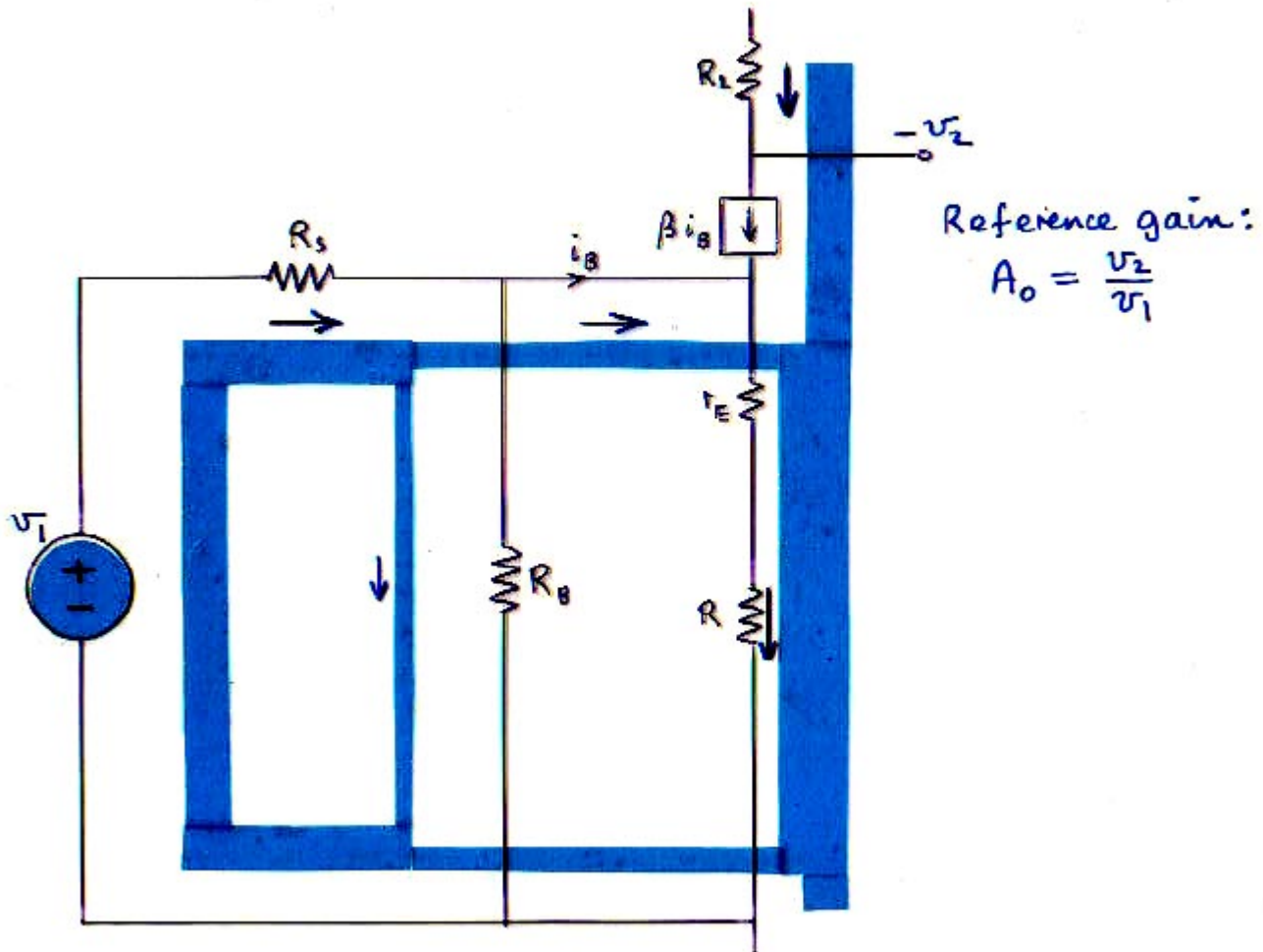


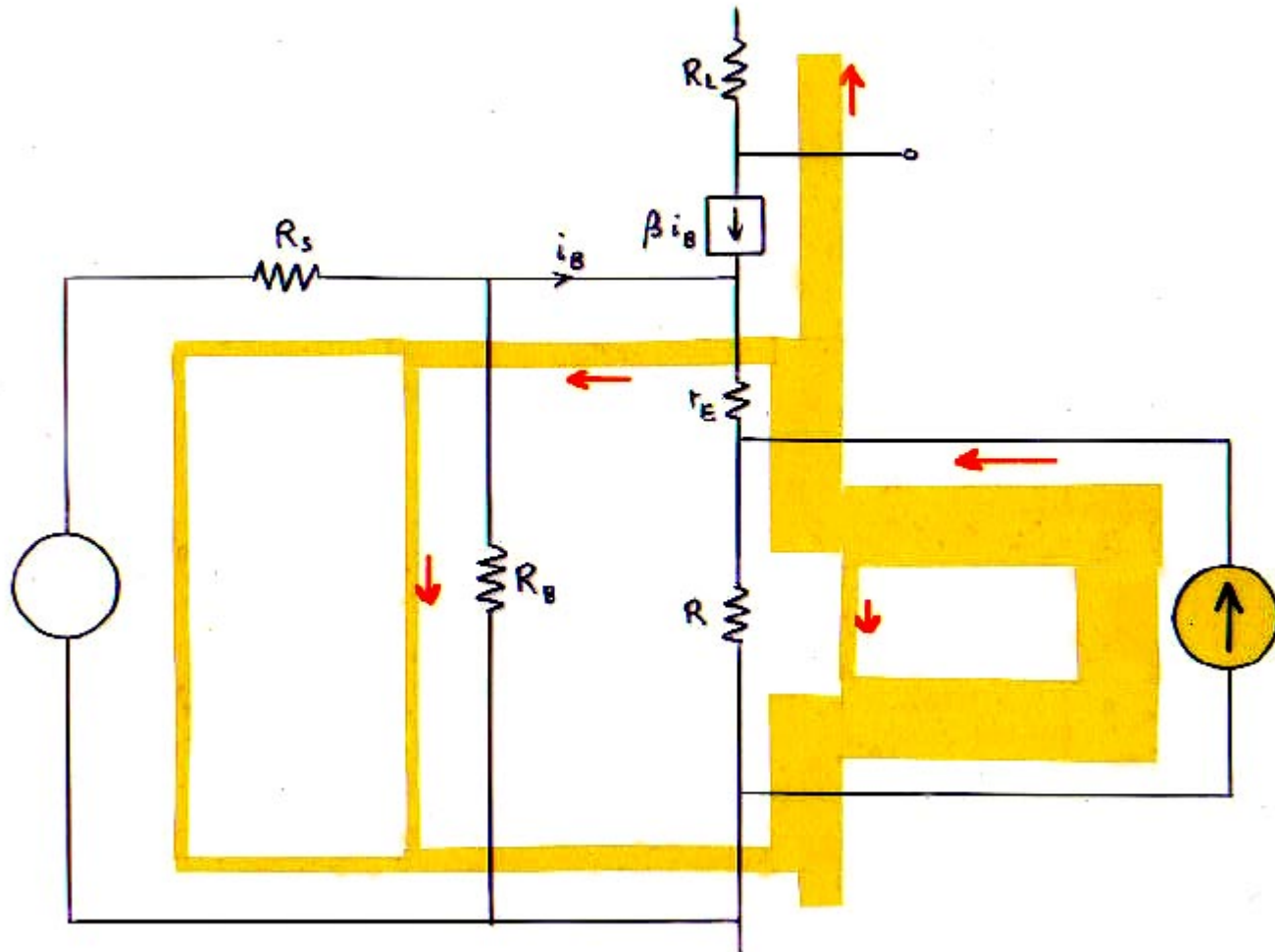
Step 2. Calculate Z_n by applying a second injected signal across R , and adjusting it with respect to v_1 , to null $v_{o1} = v_2 = 0$. Then, since $v_2 = 0$, $i_E = 0$, hence:

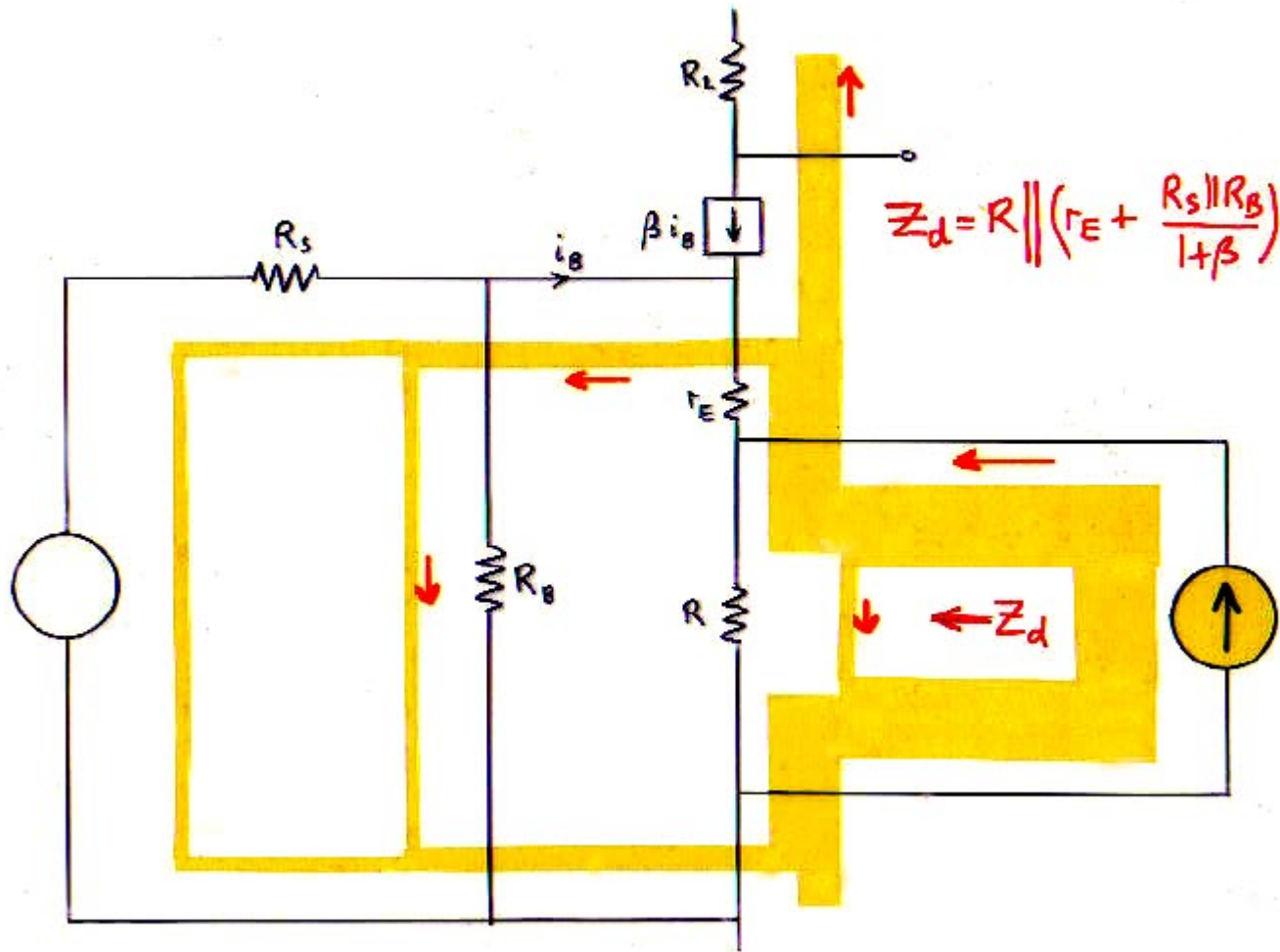
$$Z_n = R_n = R = 2.2 \text{ k}$$

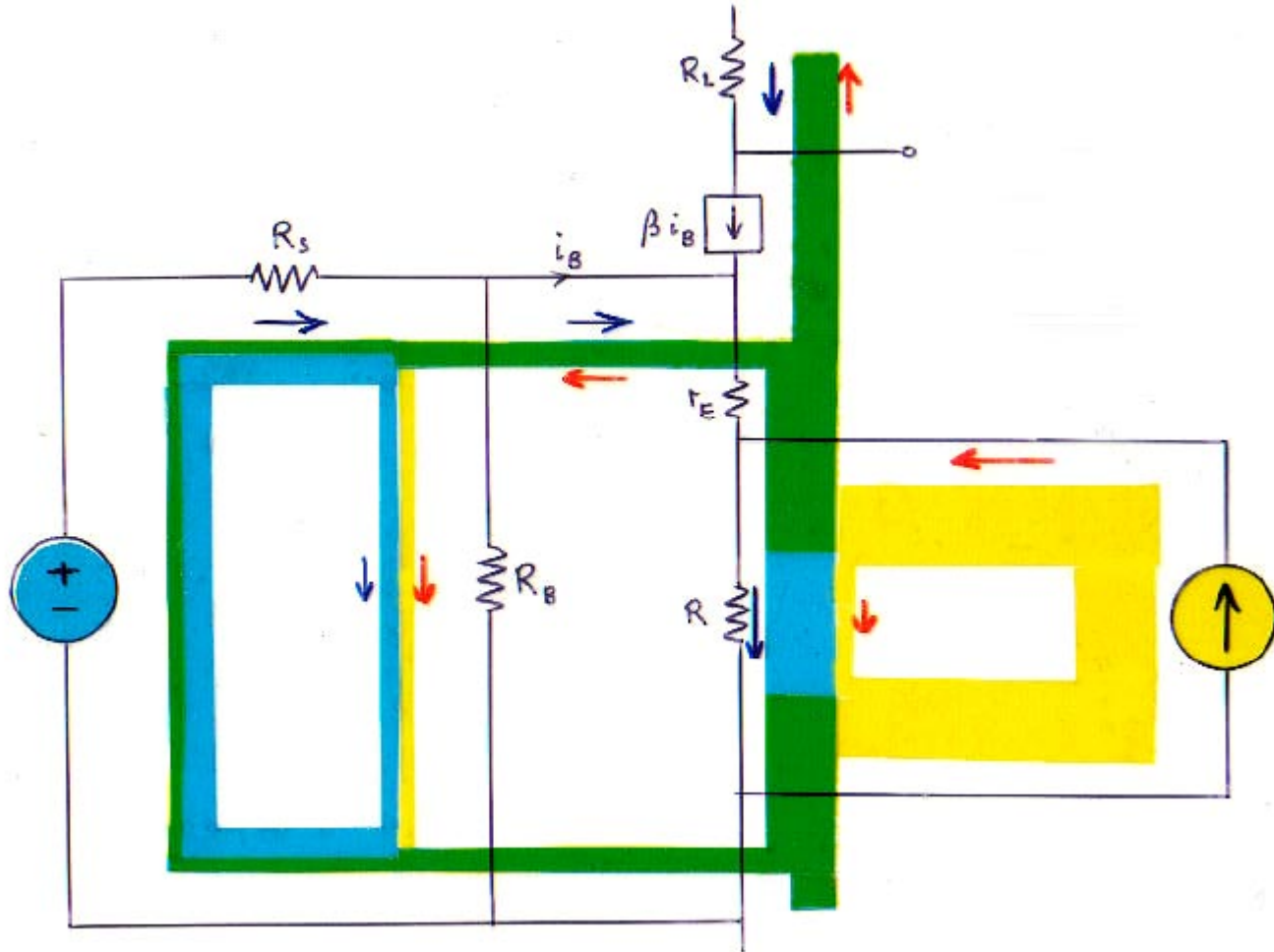


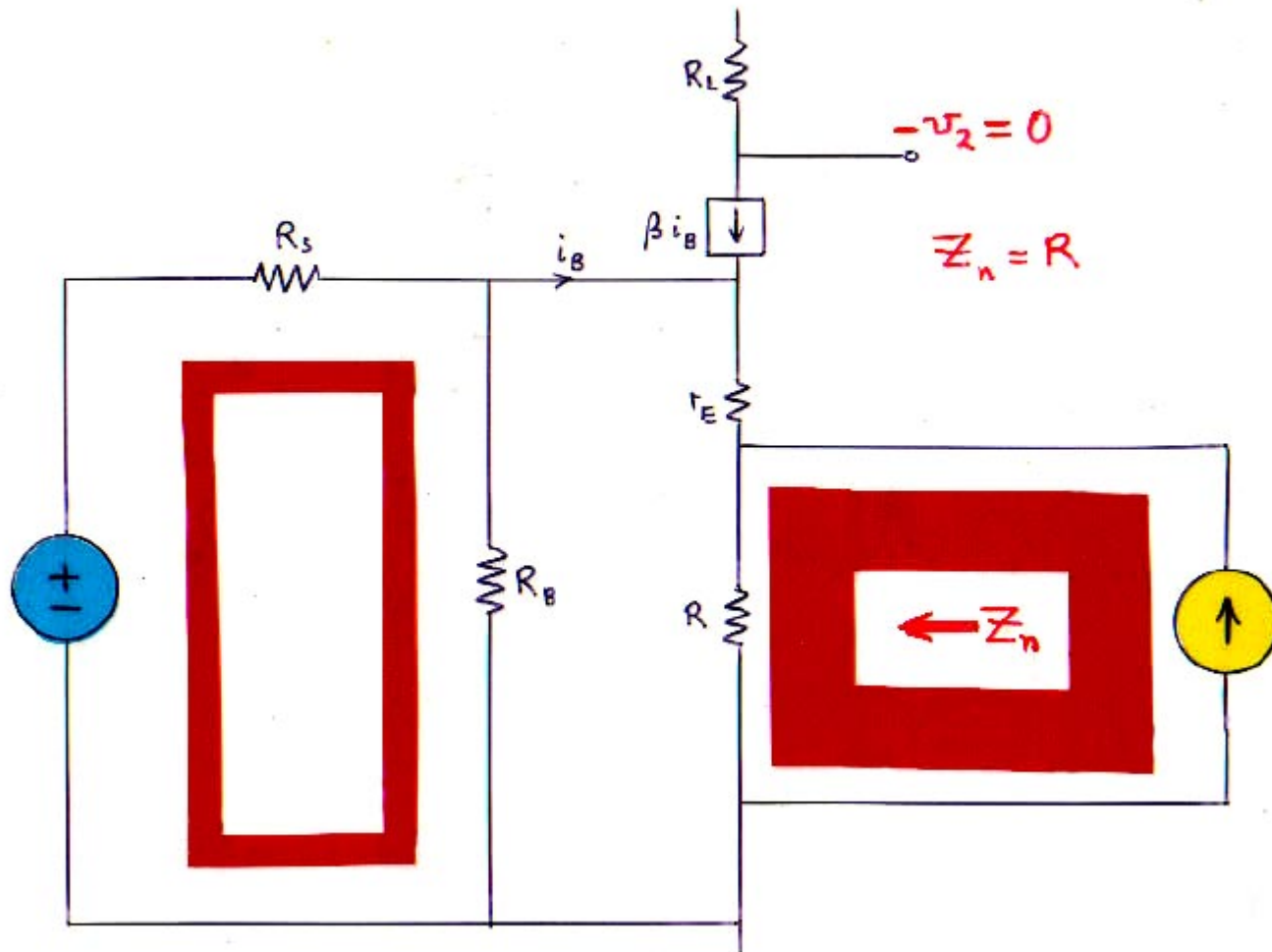


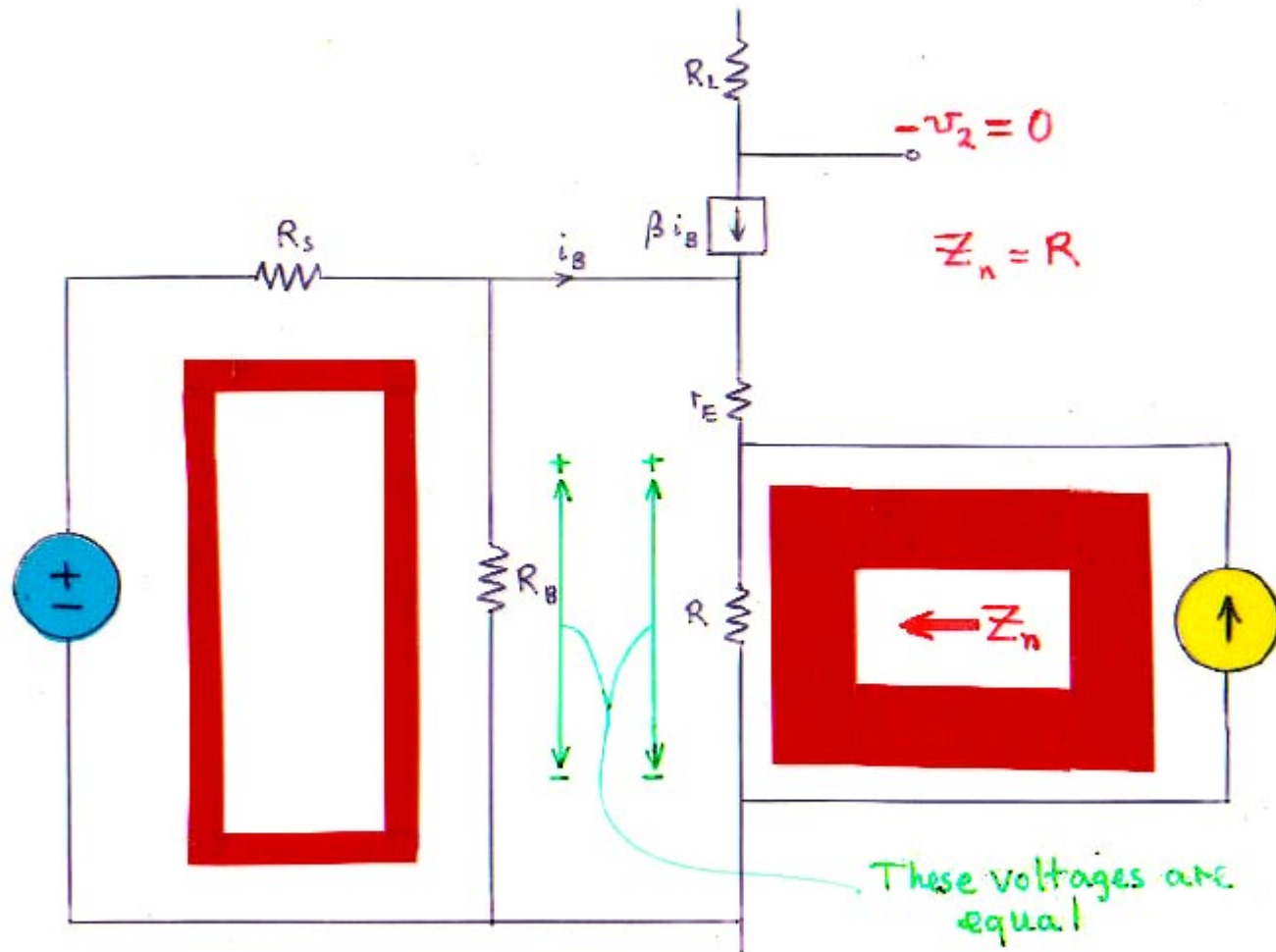




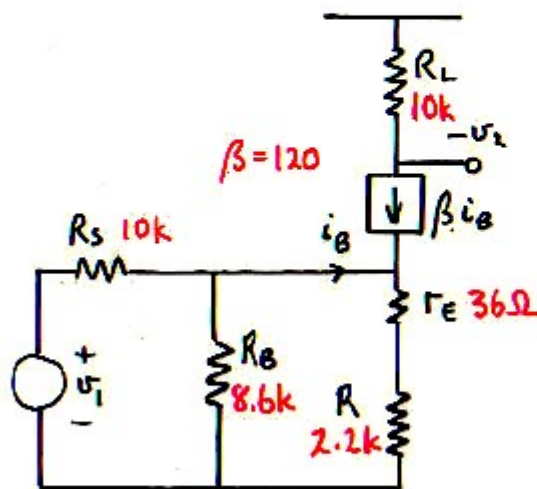








Example: The previously designed CE amplifier
 Suppose the gain has been calculated without the emitter bypass capacitance, and the correction factor resulting from addition of the extra element $Z \rightarrow 1/sC_2$ is desired.

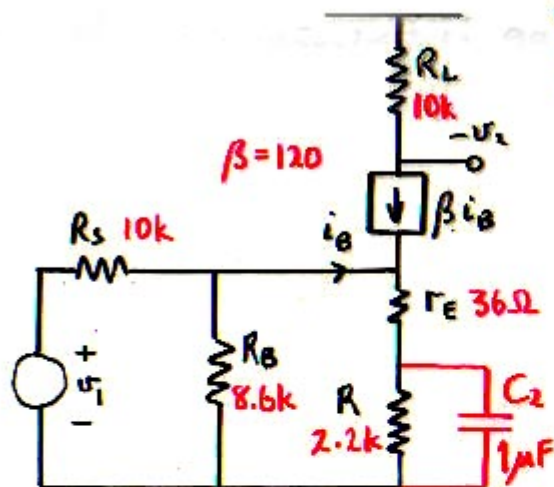


Original gain:

$$A_0 = \frac{R_B}{R_S + R_B} \frac{\alpha R_L}{R + r_e + (R_S \parallel R_B) / (1 + \beta)}$$

$$= 0.46 \frac{10}{2.2 + 0.036 + 0.039}$$

$$= 2.0 \Rightarrow 6 \text{ dB}$$



Original gain:

$$A_0 = \frac{R_B}{R_s + R_B} \frac{\alpha R_L}{R + r_e + (R_s \parallel R_B) / (1 + \beta)}$$

$$= 0.46 \frac{10}{2.2 + 0.036 + 0.039}$$

$$= 2.0 \Rightarrow 6 \text{ dB}$$

Hence, corrected gain in presence of $C_2 = 1\mu\text{F}$ bypass capacitance is:

$$A = A_0 \frac{1 + \frac{R_n}{Z}}{1 + \frac{R_d}{Z}} = A_0 \frac{1 + sC_2 R_n}{1 + sC_2 R_d} = A_0 \frac{1 + \frac{s}{\omega_1}}{1 + \frac{s}{\omega_2}} = A_m \frac{1 + \frac{\omega_1}{s}}{1 + \frac{\omega_2}{s}}$$

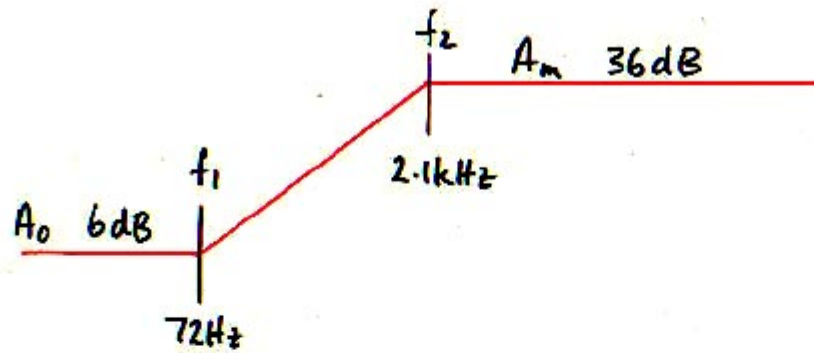
where

$$\omega_1 \equiv \frac{1}{C_2 R_n} \quad f_1 = \frac{159}{1 \times 2.2} = 72 \text{ Hz} \quad \omega_2 \equiv \frac{1}{C_2 R_d} = \frac{159}{1 \times 0.075} = 2.1 \text{ kHz}$$

$$A_m = A_0 \frac{\omega_2}{\omega_1} = A_0 \frac{R_n}{R_d}$$

$$= \frac{R_B}{R_s + R_B} \frac{\alpha R_L}{R + r_E + (R_s \parallel R_B) / (1 + \beta)} \frac{R [R + r_E + (R_s \parallel R_B) / (1 + \beta)]}{R [r_E + (R_s \parallel R_B) / (1 + \beta)]}$$

$$= \frac{R_B}{R_s + R_B} \frac{\alpha R_L}{r_E + (R_s \parallel R_B) / (1 + \beta)} = 62 \Rightarrow 36 \text{ dB}$$



NOTE: Nulling a voltage is not the same as shorting it!

NOTE: the null double injection calculation is easier than the single injection calculation!

Dual forms of the EET

The Extra Element Theorem as derived applies to the correction factor resulting from an extra shunt element.

There is a corresponding form to find the correction factor resulting from an extra series element:

$$\begin{aligned}
 \text{gain}|_z &= \overset{\text{reference gain}}{\downarrow} \text{gain}|_{z=\infty} \frac{1 + \frac{z_n}{z}}{1 + \frac{z_d}{z}} \\
 &= \text{gain}|_{z=\infty} \frac{\frac{z_n}{z}}{\frac{z_d}{z}} \frac{z + 1}{z_d + 1} \\
 &= \frac{z_n}{z_d} \text{gain}|_{z=\infty} \frac{1 + \frac{z}{z_n}}{1 + \frac{z}{z_d}}
 \end{aligned}$$

The Extra Element Theorem as derived applies to the correction factor resulting from an extra shunt element.

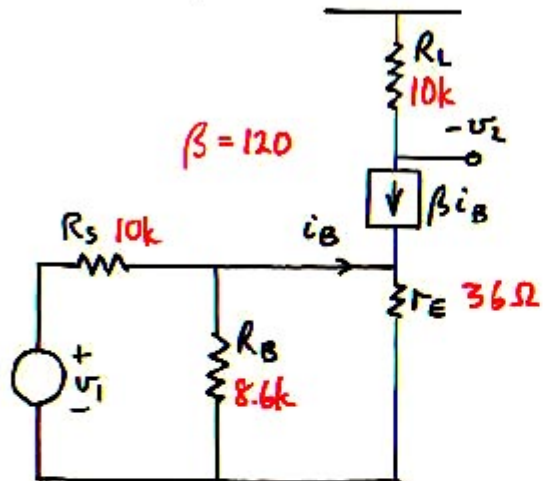
There is a corresponding form to find the correction factor resulting from an extra series element:

$$\begin{aligned} & \text{reference gain} \\ & \downarrow \\ \text{gain}|_z &= \text{gain}|_{z=\infty} \frac{1 + \frac{z_n}{z}}{1 + \frac{z_d}{z}} = \text{gain}|_{z=\infty} \frac{\frac{z_n}{z} + 1}{\frac{z_d}{z} + 1} \end{aligned}$$

$$\begin{aligned} & \text{reference gain} \\ & \downarrow \\ &= \text{gain}|_{z=0} \frac{1 + \frac{z}{z_n}}{1 + \frac{z}{z_d}} = \frac{z_n}{z_d} \text{gain}|_{z=\infty} \frac{1 + \frac{z}{z_n}}{1 + \frac{z}{z_d}} \end{aligned}$$

This must be the gain when $z=0$.

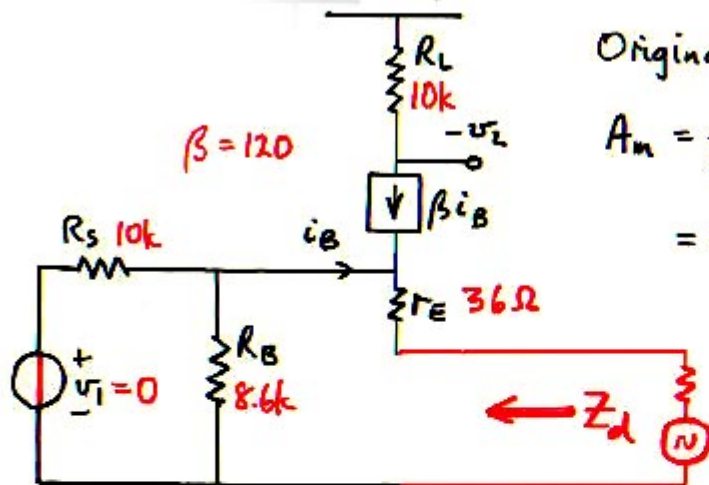
Example: An alternative to the method of the previous example is to find the correction factor to the midband gain A_m resulting from addition of the series "extra element"
 $Z \rightarrow R \parallel 1/sC.$



Original gain:

$$A_m = \frac{R_B}{R_s + R_B} \frac{\alpha R_L}{r_E + (R_s \parallel R_B) / (1 + \beta)}$$

$$= 62 \Rightarrow 36dB$$



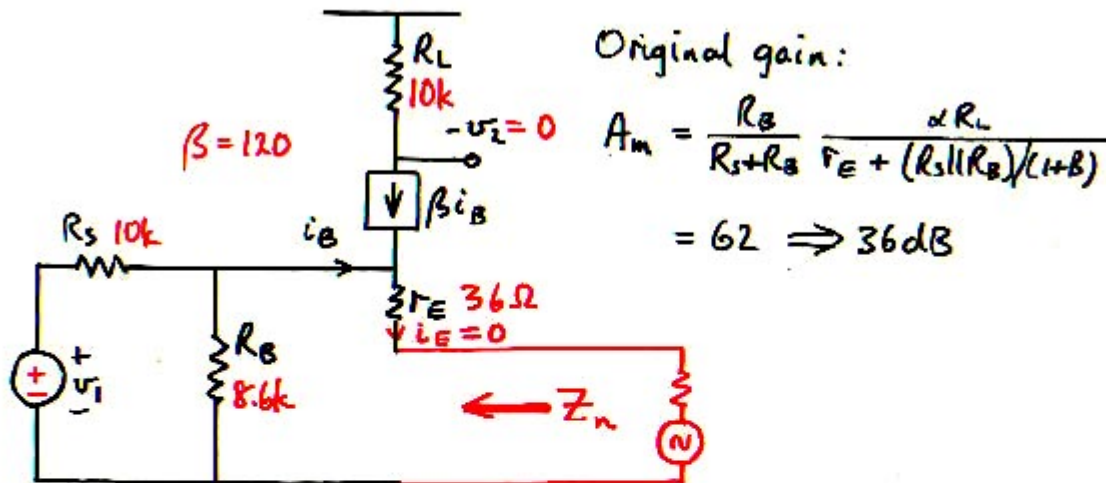
Original gain:

$$A_m = \frac{R_B}{R_s + R_B} \frac{\alpha R_L}{r_E + (R_s || R_B) / (1 + \beta)}$$

$$= 62 \Rightarrow 36 \text{ dB}$$

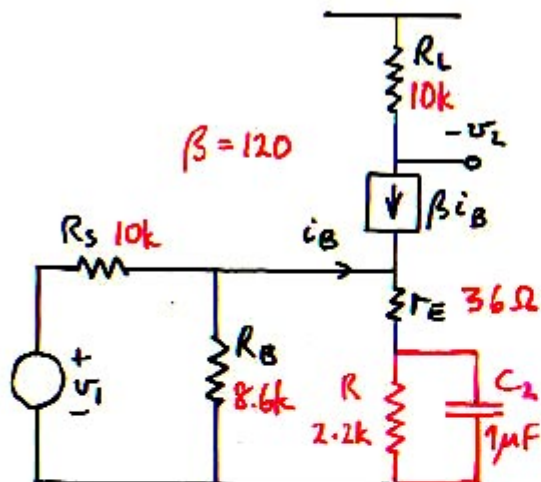
Step 1. Calculate Z_d by shorting $v_{i1} = v_i$ and applying a second injected signal in series with r_E :

$$Z_d \equiv R_d' = r_E + (R_s || R_B) / (1 + \beta)$$



Step 2. Calculate Z_n by applying a second injected signal in series with r_E , and adjusting it with respect to v_1 to null $v_{o1} = v_2 = 0$. Then, since $v_2 = 0$, $i_E = 0$, hence

$$Z_n \equiv R_n' = \infty$$



Original gain:

$$A_m = \frac{R_B}{R_s + R_B} \frac{\alpha R_L}{r_E + (R_s \parallel R_B) / (\beta + 1)}$$

$$= 62 \Rightarrow 36 \text{ dB}$$

Hence, corrected gain in presence of $R \parallel 1/sC_2$ is:

$$A = A_m \frac{1 + \frac{Z_1}{R_d'}}{1 + \frac{Z_2}{R_d'}} = A_m \frac{1}{1 + \frac{R}{R_d'} \frac{1}{1 + sC_2 R}} = A_m \frac{1 + 1/sC_2 R}{1 + 1/sC_2 (R \parallel R_d')}$$

However, $R \parallel R_d' = R_d$, so

$$A = A_m \frac{1 + 1/sC_2 R}{1 + 1/sC_2 R_d} \longrightarrow \text{same result as before}$$

The Parallel and Series forms of the EET

Generalization: Extra Element Theorem - #1

There are two forms of Extra Element Theorem:

1.
$$\text{gain}|_Z = \text{gain}|_{Z=\infty} \frac{1 + \frac{Z_n}{Z}}{1 + \frac{Z_d}{Z}}$$

Provides a correction factor for an extra element added in shunt across a node pair.

2.
$$\text{gain}|_Z = \text{gain}|_{Z=0} \frac{1 + \frac{Z}{Z_n}}{1 + \frac{Z}{Z_d}}$$

Provides a correction factor for an extra element added in series with a branch.

The "extra element" Z can be any two-terminal combination of impedances.

Note that in all cases the null double injection calculation is easier than the single injection calculation.

This results from use of the null condition (which makes several other quantities zero);

and

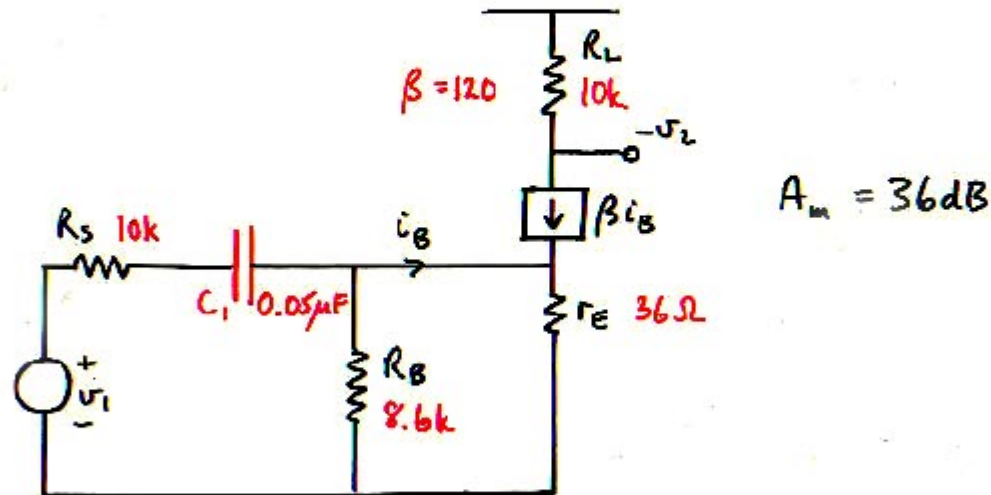
because the relation between u_{i1} and u_{i2} to produce the null is never needed — only the null itself is used.

Exercise 8.1

Insert C_1 by the EET

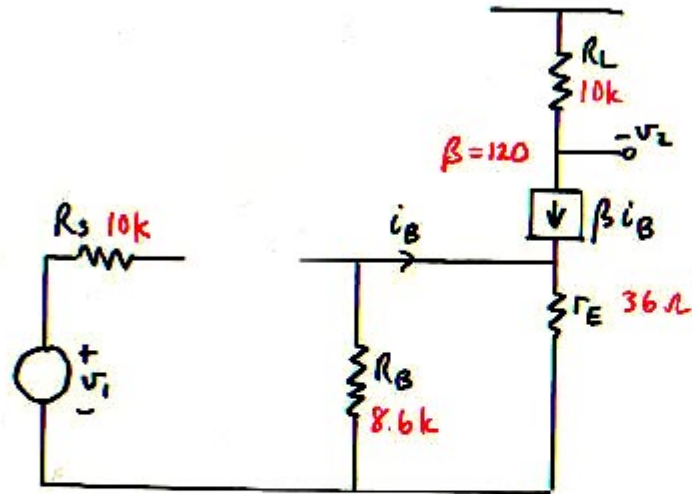
Exercise

In the CE amplifier stage, find the correction factor to the midband gain A_m resulting from inclusion of the coupling capacitance $C_1 = 0.05\mu\text{F}$:



Exercise 8.1 - Solution

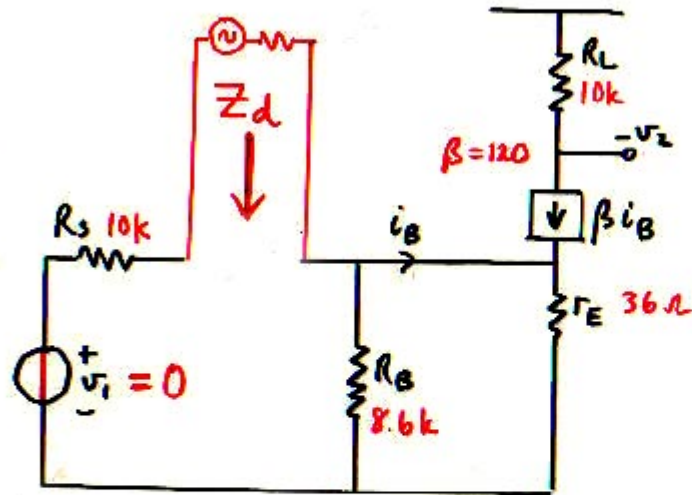
Exercise Solution



$$A_m = 36\text{dB}$$

Exercise 8.1 - Solution

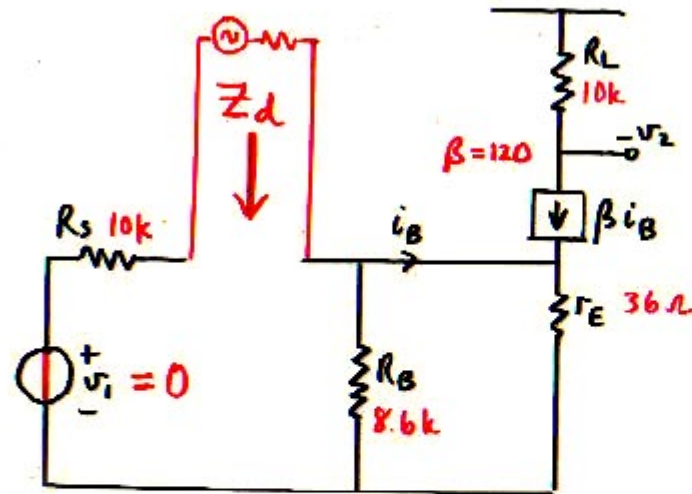
Exercise Solution



$$A_m = 36dB$$

Exercise 8.1 - Solution

Exercise Solution



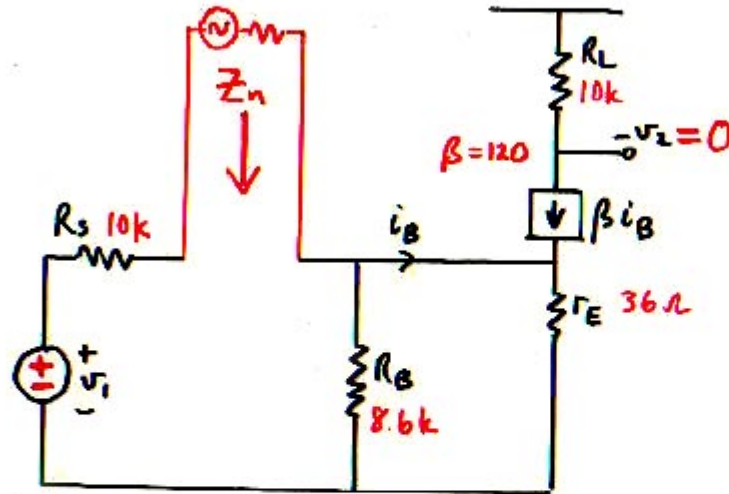
$$A_m = 36\text{dB}$$

Step 1.

$$\begin{aligned} Z_d = R_d &= R_s + R_B \parallel ((1+\beta)r_E) \\ &= 10 + 8.6 \parallel (120 \times 0.036) \\ &= 10 + 8.6 \parallel 4.3 \\ &= 13\text{k} \end{aligned}$$

Exercise 8.1 - Solution

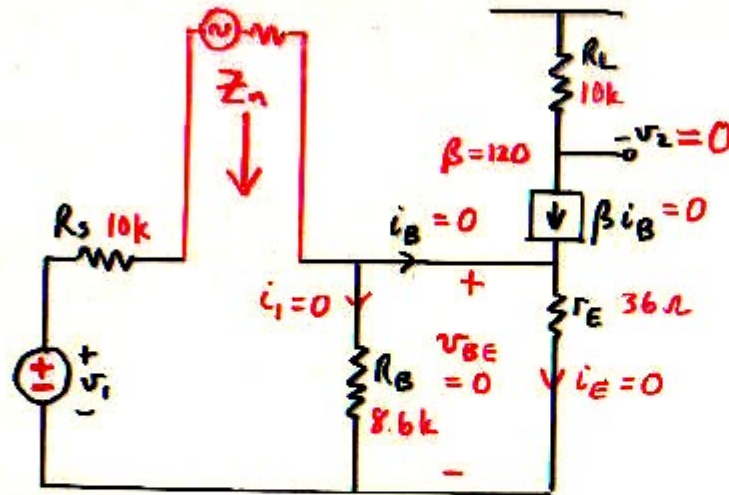
Exercise Solution



$$A_m = 36dB$$

Exercise 8.1 - Solution

Exercise Solution



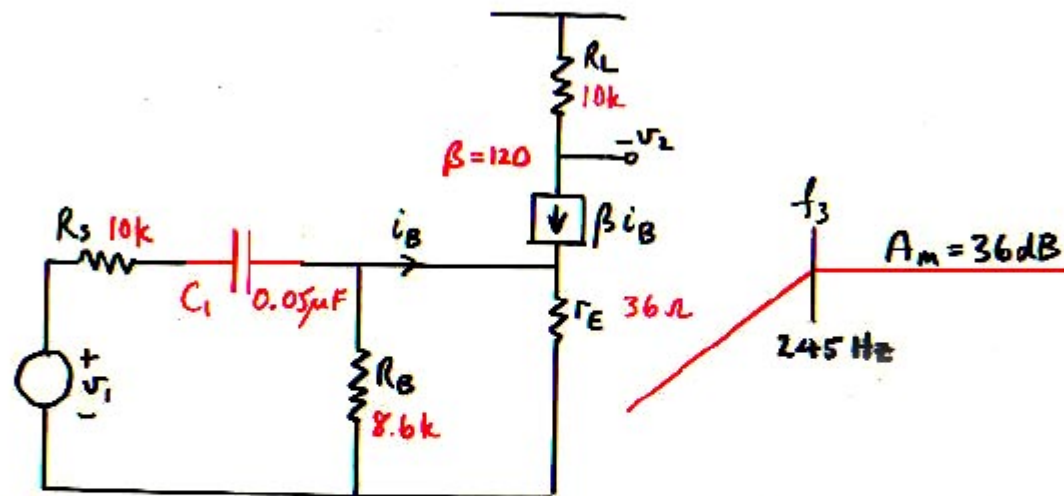
$$A_m = 36dB$$

Step 2.

$$Z_n = R_n = \infty$$

Exercise 8.1 - Solution

Exercise Solution



Hence corrected gain in the presence of $Z \rightarrow 1/sC_1$, is

$$A = A_m \frac{1 + \frac{Z}{Z_d}}{1 + \frac{Z}{Z_d}} = A_m \frac{1}{1 + \frac{1}{sC_1 R_d}} = A_m \frac{1}{1 + \frac{\omega_3}{s}}$$

where

$$\omega_3 \equiv \frac{1}{C_1 R_d} \quad f_3 = \frac{159}{0.05 \times 13} = 245\text{ Hz}$$

Generalization: Extra Element Theorem - #2

If the reference circuit is purely resistive,
 $Z_d = R_d$ and $Z_n = R_n$ are pure resistances.

If, also, the extra element is a pure reactance,
the Extra Element Theorem correction
factor gives the corner frequencies
directly.

Generalization: Extra Element Theorem - #3

The Extra Element Theorem can profitably be used to divide the analysis of a complicated circuit into successive simpler steps:

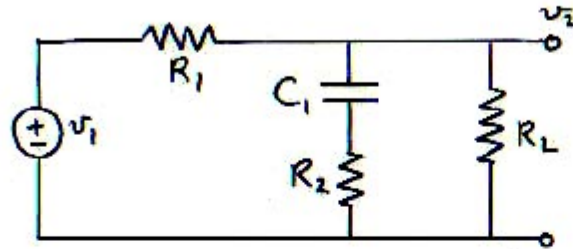
Designate one element as "extra," and the circuit without the element as the "reference circuit." Calculate the gain of the (simpler) reference circuit, then restore the omitted element by the Extra Element Theorem correction factor.

This is a particularly useful approach when the designated "extra" element is a reactance and the reference circuit is purely resistive.

Exercise 8.2

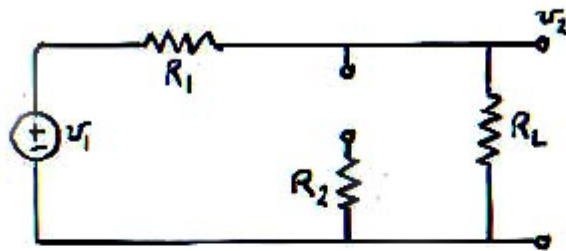
Lag-lead network: Find A by designating C_1 as an extra element.

Exercise: lag-lead network



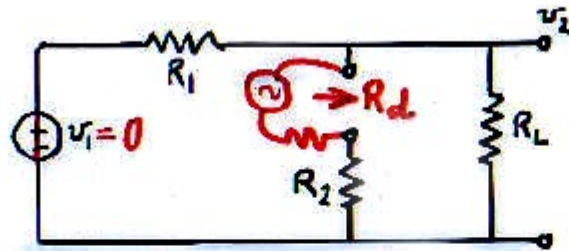
Find the transfer function $A \equiv v_2/v_1$ by designating C_1 as an "extra" element.

Exercise 8.2 - Solution



$$\frac{v_2}{v_1} = \frac{R_L}{R_1 + R_L}$$

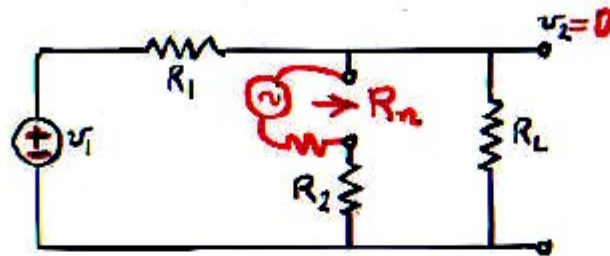
Exercise 8.2 - Solution



$$\frac{v_2}{v_1} = \frac{R_L}{R_1 + R_L}$$

$$R_d = R_2 + R_1 \parallel R_L$$

Exercise 8.2 - Solution

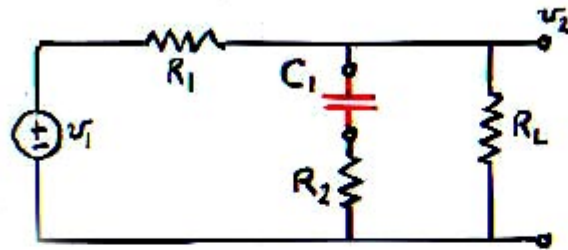


$$\frac{v_2}{v_1} = \frac{R_L}{R_1 + R_L}$$

$$R_d = R_2 + R_1 \parallel R_L$$

$$R_n = R_2$$

Exercise 8.2 - Solution



$$\frac{v_2}{v_1} = \frac{R_L}{R_1 + R_L}$$

$$R_d = R_2 + R_1 \parallel R_L$$

$$R_n = R_2$$

$$\frac{v_2}{v_1} = \frac{R_L}{R_1 + R_L} \frac{1 + sC_1 R_2}{1 + sC_1 (R_2 + R_1 \parallel R_L)}$$

Special case: The EET for a self-impedance

The Extra Element Theorem may be used to find an extra element correction factor for any transfer function of a linear circuit. It is necessary merely to identify the "input" and "output" signals; Z_d and Z_n are then calculated as the driving point impedance seen by the extra element with the "input" zero and with the "output" nulled, respectively.

Examples of transfer functions:

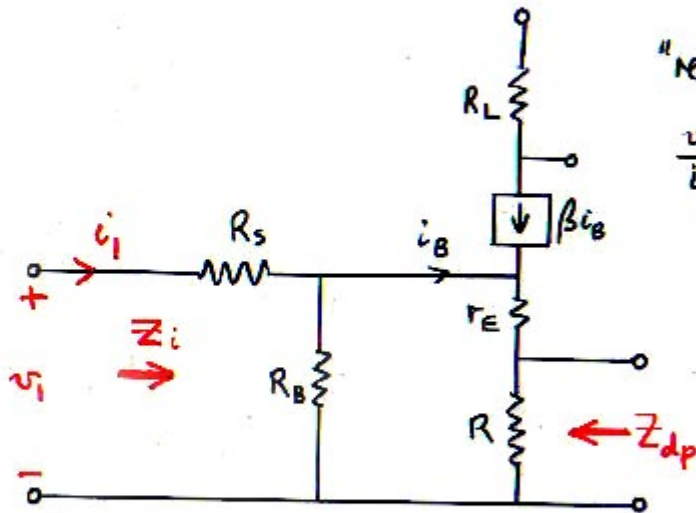
"output" → $\frac{\text{current drawn from power supply}}{\text{"input" → input voltage}}$ (a transadmittance)

"output" → $\frac{\text{output voltage ripple component}}{\text{Power supply ripple voltage}}$ (a voltage gain; audio susceptibility of a power supply)

"output" → $\frac{\text{corresponding driving voltage}}{\text{"input" → any driving current}}$ (a self-impedance, e.g. input or output impedance)

Example: Input impedance Z_i of a CE amplifier stage with emitter bypass capacitance as "extra" element.

"Reference circuit":



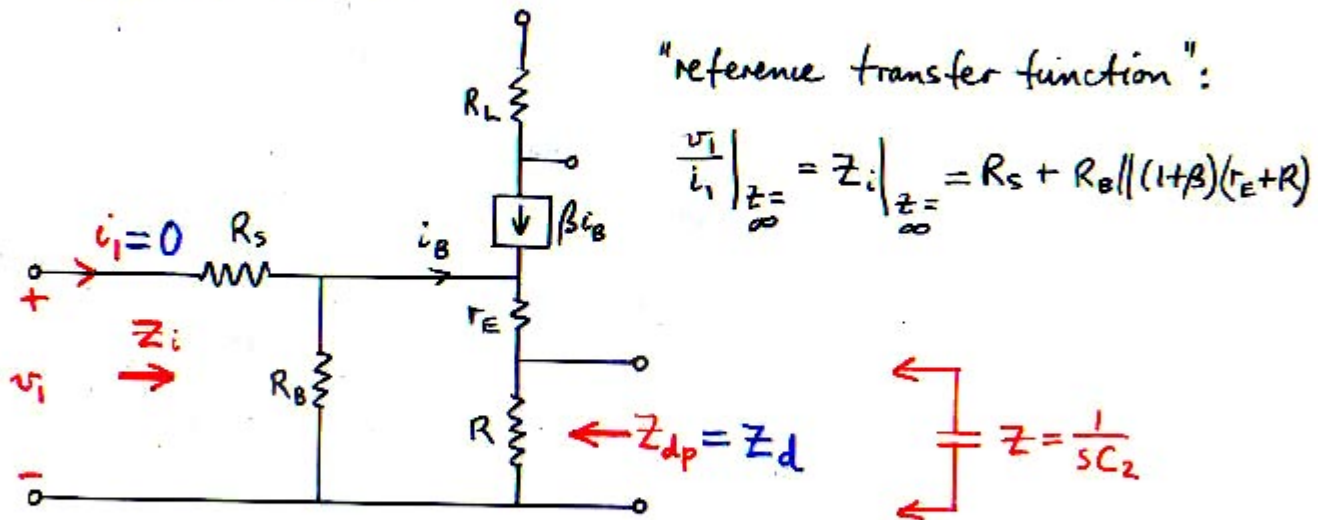
"reference transfer function":

$$\left. \frac{v_1}{i_1} \right|_{z=\infty} = Z_i \Big|_{z=\infty} = R_s + R_B \parallel ((1+\beta)(r_E + R))$$

$$Z = \frac{1}{sC_2}$$

Example: Input impedance Z_i of a CE amplifier stage with emitter bypass capacitance as "extra" element.

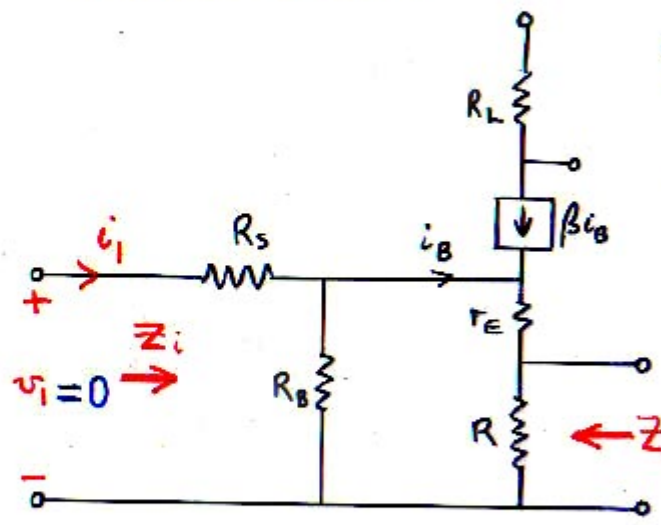
"Reference circuit":



$$Z_d = Z_{dp} \Big|_{\text{"input zero"}} = Z_{dp} \Big|_{i_i=0} = R_d = R \parallel \left(r_E + \frac{R_B}{1+\beta} \right)$$

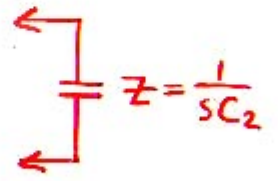
Example: Input impedance Z_i of a CE amplifier stage with emitter bypass capacitance as "extra" element.

"Reference circuit":



"reference transfer function":

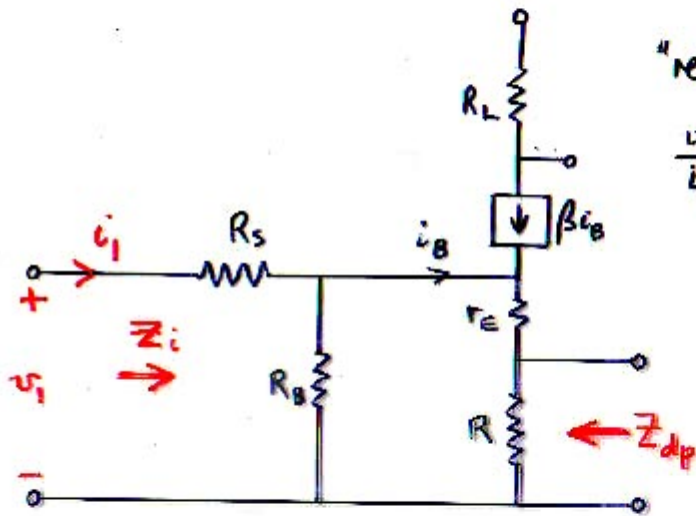
$$\left. \frac{v_1}{i_1} \right|_{Z=\infty} = Z_i \Big|_{Z=\infty} = R_s + R_B \parallel ((1+\beta)(r_E + R))$$



$$Z_n = Z_{dp} \Big|_{\substack{\text{"output"} \\ \text{nullled}}} = Z_{dp} \Big|_{v_1=0} = R_n = R \parallel \left(r_E + \frac{R_s \parallel R_B}{1+\beta} \right)$$

Example: Input impedance Z_i of a CE amplifier stage with emitter bypass capacitance as "extra" element.

"Reference circuit":



"reference transfer function":

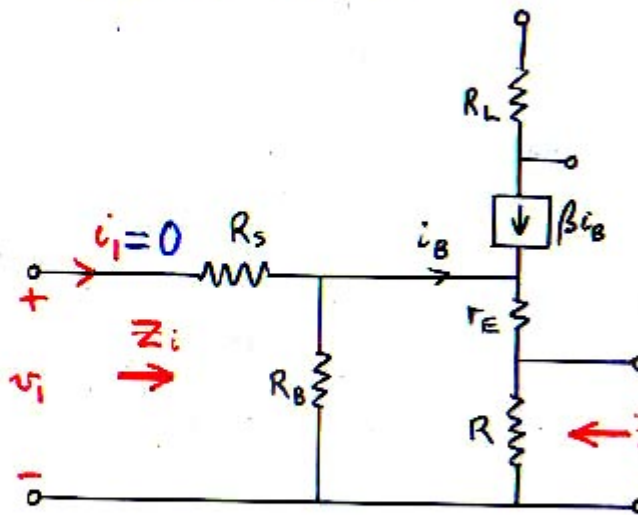
$$\left. \frac{v_1}{i_1} \right|_{z=\infty} = Z_i \Big|_{z=\infty} = R_s + R_B \parallel ((1+\beta)(r_e + R))$$

$$Z = \frac{1}{sC_2}$$

Hence,
$$Z_i = \left[R_s + R_B \parallel ((1+\beta)(r_e + R)) \right] \frac{1 + sC_2 R_n}{1 + sC_2 R_d}$$

Example: Input impedance Z_i of a CE amplifier stage with emitter bypass capacitance as "extra" element.

"Reference circuit":



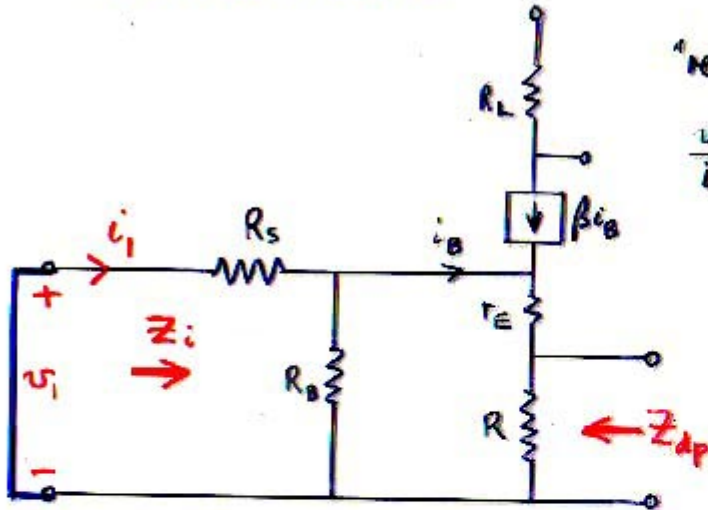
"reference transfer function":

$$\left. \frac{v_1}{i_1} \right|_{z=\infty} = Z_i \Big|_{z=\infty} = R_s + R_B \parallel ((1+\beta)(r_E + R))$$

$$Z = \frac{1}{sC_2}$$

$$Z_d = Z_{dp} \Big|_{\text{"input zero"}} = Z_{dp} \Big|_{i_i=0} = R_d = R \parallel \left(r_E + \frac{R_B}{1+\beta} \right)$$

"Reference circuit":



"reference transfer function":

$$\left. \frac{v_i}{i_i} \right|_{z=\infty} = Z_i \Big|_{z=\infty} = R_s + R_B \parallel (1+\beta)(r_E+R)$$

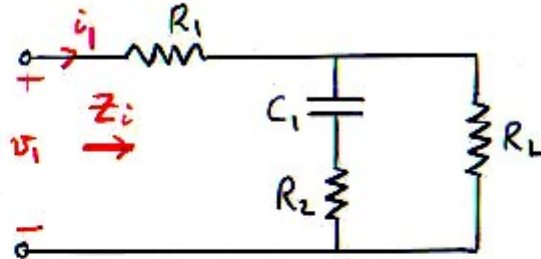
$$Z = \frac{1}{sC_2}$$

NOTE: In the special case of a self-impedance, nulling the "output" voltage is the same as shorting the "input" current, because the "output" and "input" are at the same node pair.

Exercise 8.3

Lag-lead network: Find Z_i by designating C_1 as an extra element.

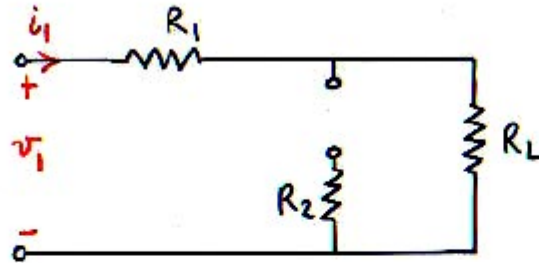
Example: Lag-lead network



Find the input impedance $Z_i = v_i / i_i$ by designating C_1 as an "extra" element.

Exercise 8.3 - Solution

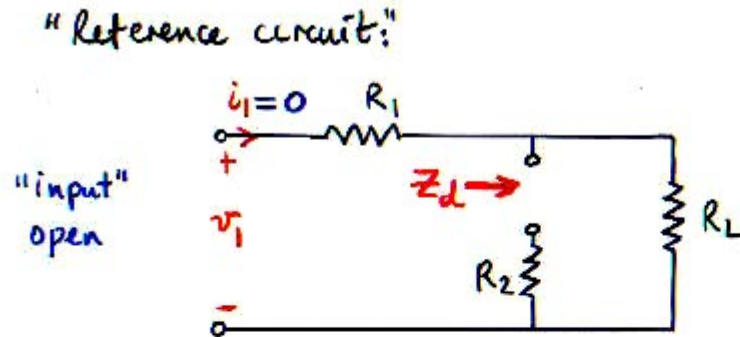
"Reference circuit:"



"Reference" input impedance:

$$Z_i \Big|_{z_o = \infty} = \frac{v_1}{i_1} \Big|_{z_o = \infty} = R_1 + R_L$$

Exercise 8.3 - Solution

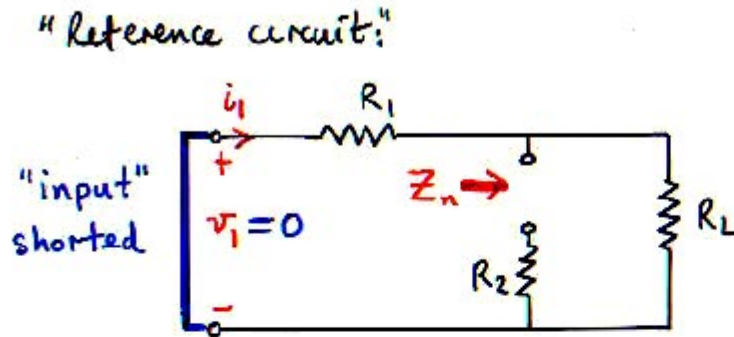


"Reference" input impedance:

$$Z_i \Big|_{z=\infty} = \frac{v_1}{i_1} \Big|_{z=\infty} = R_1 + R_L$$

$$Z_d = R_d = R_2 + R_L$$

Exercise 8.3 - Solution



"Reference" input impedance:

$$Z_i \Big|_{z=0} = \frac{v_1}{i_1} \Big|_{z=0} = R_1 + R_L$$

$$Z_n = R_n = R_2 + R_1 \parallel R_L$$

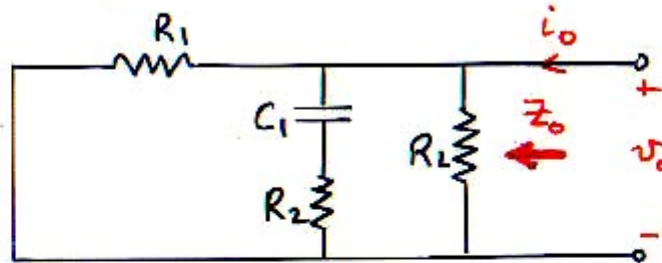
Hence:

$$Z_i = (R_1 + R_L) \frac{1 + sC_1 R_n}{1 + sC_1 R_d}$$

Exercise 8.4

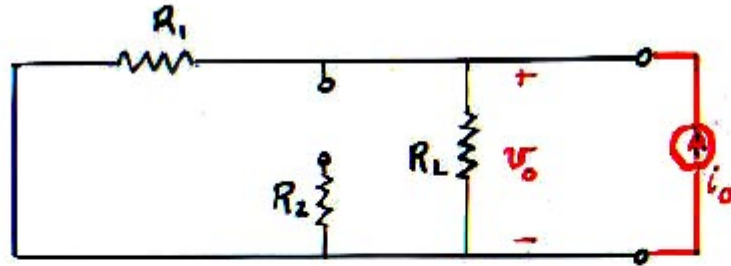
Lag-lead network: Find Z_o by designating C_1 as an extra element.

Exercise: Lag-lead network



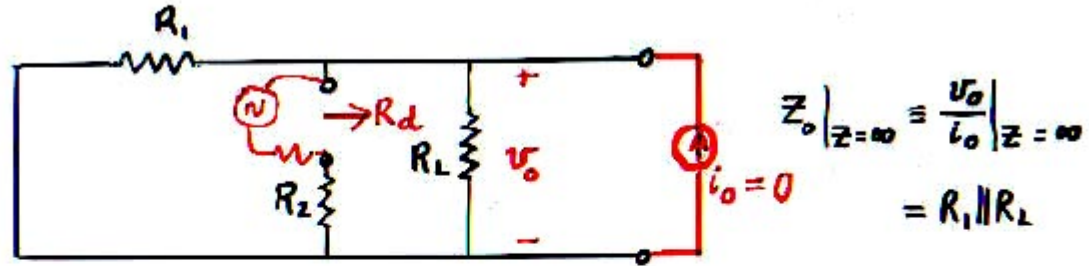
Find the output impedance $Z_o = v_o / i_o$ by designating C_1 as an "extra" element.

Exercise 8.4 - Solution



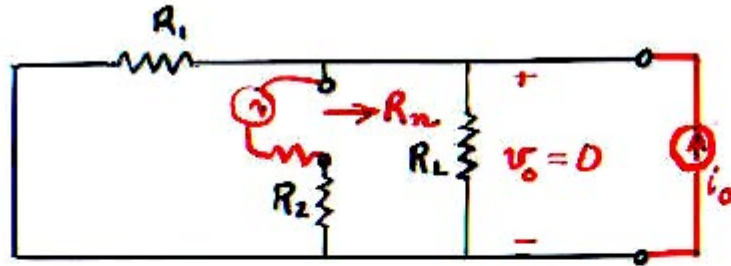
$$\begin{aligned} Z_o \Big|_{z=\infty} &= \frac{V_o}{i_o} \Big|_{z=\infty} \\ &= R_1 \parallel R_L \end{aligned}$$

Exercise 8.4 - Solution



$$R_d = R_2 + R_1 \parallel R_L$$

Exercise 8.4 - Solution



$$\begin{aligned} Z_o \Big|_{z=\infty} &\equiv \frac{v_o}{i_o} \Big|_{z=\infty} \\ &= R_1 \parallel R_L \end{aligned}$$

$$R_d = R_2 + R_1 \parallel R_L$$

$$R_n = R_2$$

With C_1 replaced:

$$Z_o = R_1 \parallel R_L \frac{1 + s C_1 R_2}{1 + s C_1 (R_2 + R_1 \parallel R_2)}$$

Generalization: Extra Element Theorem - #4

The Extra Element Theorem can be used to find an extra element correction factor for any transfer function; Z_d and Z_n are then the driving point impedances seen by the extra element with the "input" zero and with the "output" nulled, respectively.

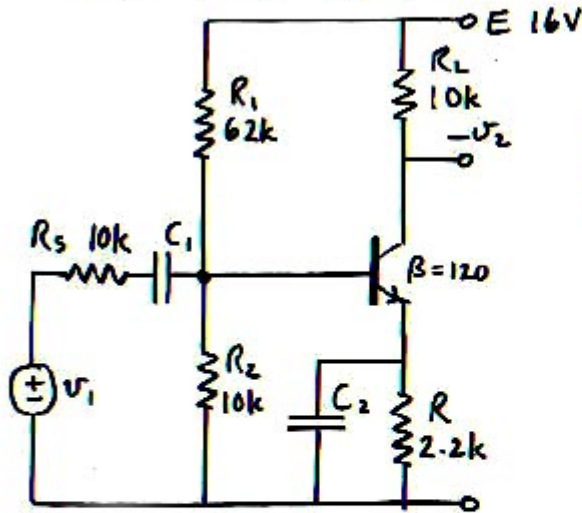
When the transfer function is a self-impedance, such as the input impedance Z_i or the output impedance Z_o , nulling the "output" is the same as shorting the "input," hence

$$Z_d = Z_{dp} \Big|_{\substack{\text{"input"} \\ \text{zero}}} = Z_{dp} \Big|_{\substack{\text{"input"} \\ \text{open}}}$$

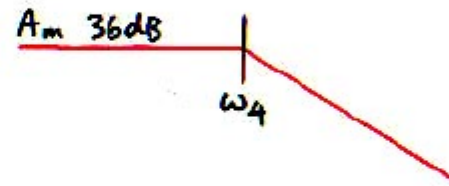
$$Z_n = Z_{dp} \Big|_{\substack{\text{"output"} \\ \text{nulled}}} = Z_{dp} \Big|_{\substack{\text{"input"} \\ \text{shorted}}}$$

1CE: The basic Common-Emitter amplifier stage

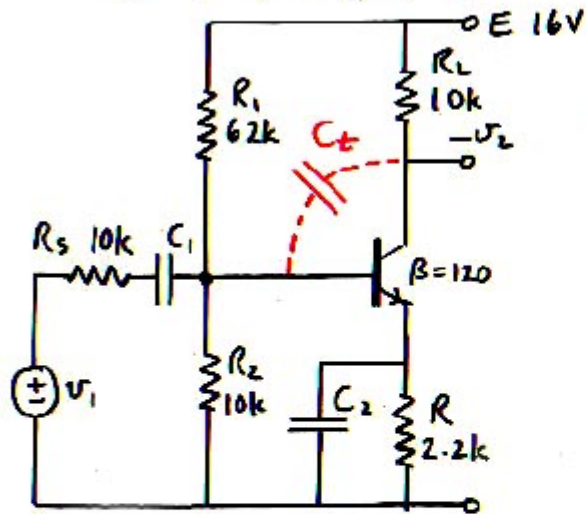
High-frequency properties of CE amplifier



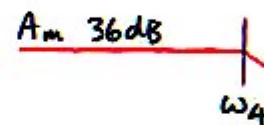
Measurement indicates that there is a high-frequency pole ω_4 :



High-frequency properties of CE amplifier



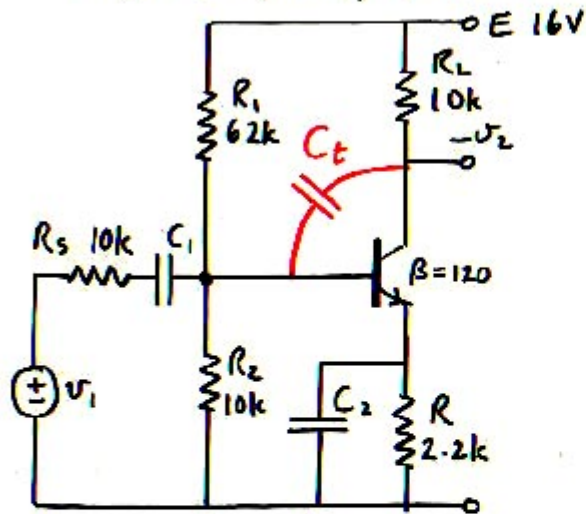
Measurement indicates that there is a high-frequency pole ω_4 :



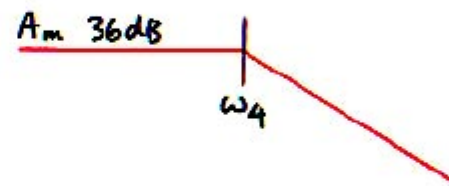
The expectation is that this is caused by the collector-base transition-layer capacitance C_t .

A typical value is $C_t = 5\text{pF}$. The resulting corner frequency with $R_L = 10\text{k}$ is $159/5 \times 10^{-6} \times 10 = 3.2\text{MHz}$. Since the actual corner frequency is much lower, there must be a multiplying effect on C_t resulting from its connection to the transistor base instead of to ground.

High-frequency properties of CE amplifier

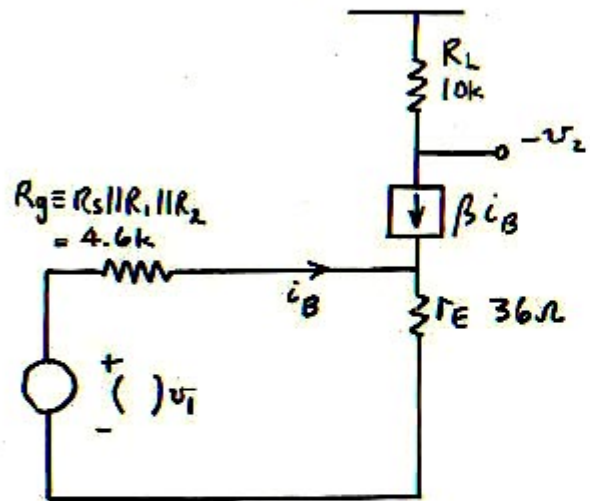


Measurement indicates that there is a high-frequency pole ω_4 :

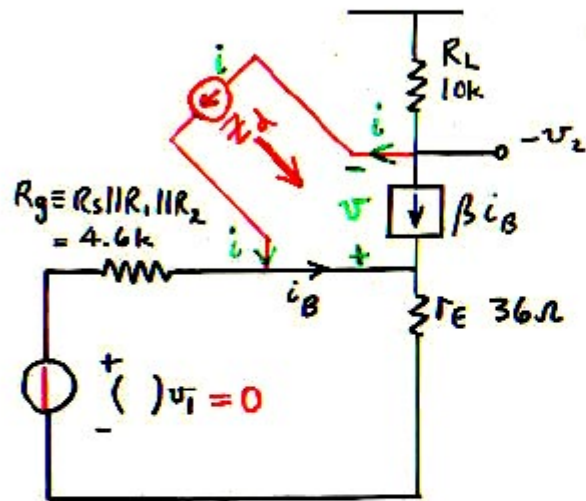


Since the midband gain $A_m = 36\text{dB}$ has already been determined, use the Extra Element Theorem to find the correction factor resulting from inclusion of $Z \rightarrow 1/sC_t$.

Midband model after Thevenin reduction of R_s, R_1, R_2 :

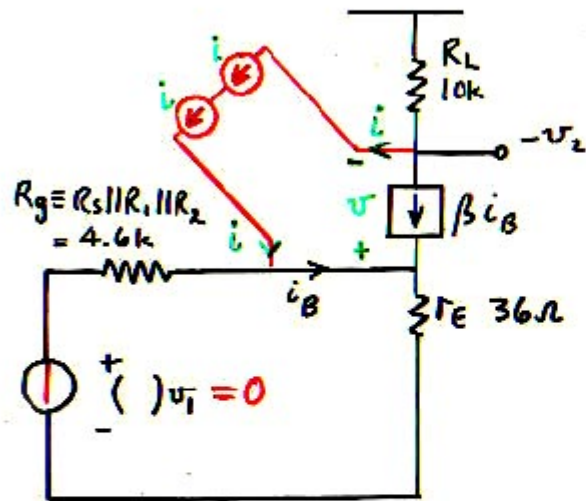


Midband model after Thevenin reduction of R_s, R_1, R_2 :



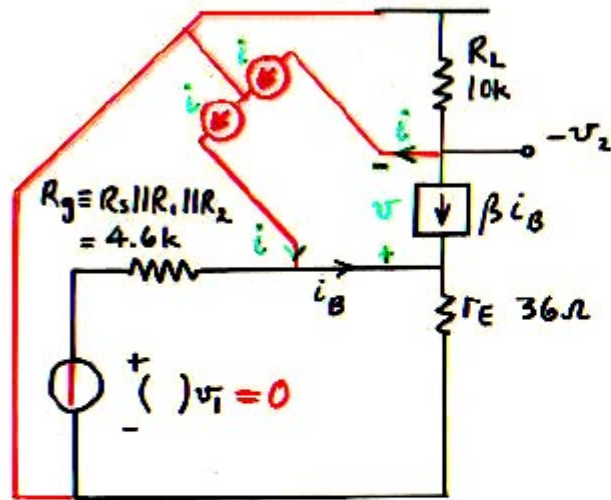
$$Z_d = \frac{v}{i} \Big|_{v_i=0}$$

Midband model after Thevenin reduction of R_s, R_1, R_2 :



The current generator i can be divided into two equal current generators in series.

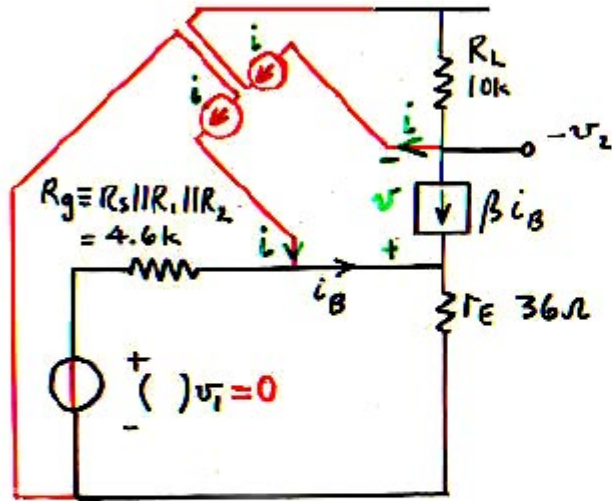
Midband model after Thevenin reduction of R_s, R_1, R_2 :



The current generator i can be divided into two equal current generators in series.

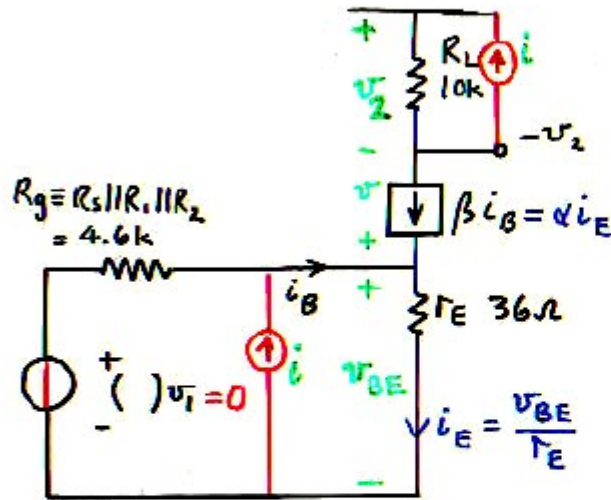
Since the voltage at the junction of the two current generators i is immaterial, the junction can be grounded.

Midband model after Thevenin reduction of R_s, R_1, R_2 :



A separate ground can be identified for each current generator i .

Midband model after Thevenin reduction of R_s, R_1, R_2 :



Rearranged diagram.

$$Z_d = R_d = \frac{v}{i} = \frac{v_{BE}}{i} + \frac{v_2}{i}$$

$$v_{BE} = [R_g \parallel (1+\beta)r_E] i$$

$$v_2 = R_L (\alpha i_E + i) = R_L \left(\frac{\alpha}{r_E} v_{BE} + i \right)$$

$$R_d = \frac{v_{BE}}{i} + R_L \left(\frac{\alpha}{r_E} \frac{v_{BE}}{i} + 1 \right)$$

$$R_d = \left(1 + \frac{\alpha R_L}{r_E} \right) [R_g \parallel (1+\beta)r_E] + R_L = R_L [R_g \parallel (1+\beta)r_E] \left[\frac{1}{R_L} + \frac{\alpha}{r_E} + \frac{1}{R_g \parallel (1+\beta)r_E} \right]$$

$$= R_L [R_g \parallel (1+\beta)r_E] \left[\frac{1}{R_L} + \frac{\beta}{(1+\beta)r_E} + \frac{1}{R_g} + \frac{1}{(1+\beta)r_E} \right]$$

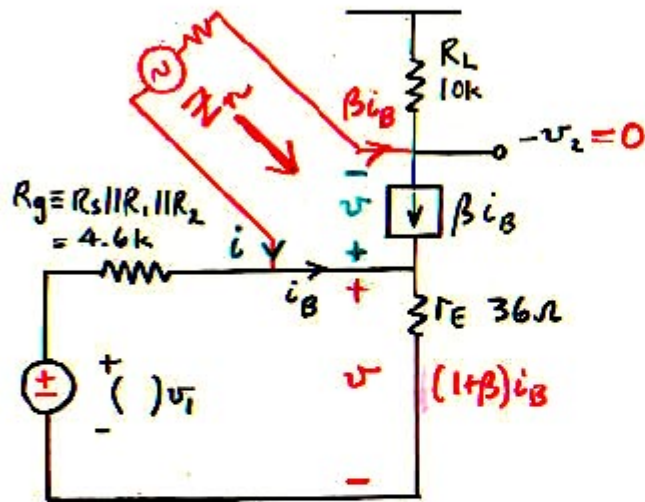
$$= \frac{R_g \parallel (1+\beta)r_E}{R_g \parallel r_E \parallel R_L} R_L = \frac{4.6 \parallel 4.3}{4.6 \parallel 0.036 \parallel 10} R_L = 62 R_L = 620k$$

Generalization: Floating Current Generator

A floating current generator can be replaced by two separate, equal, grounded current generators.

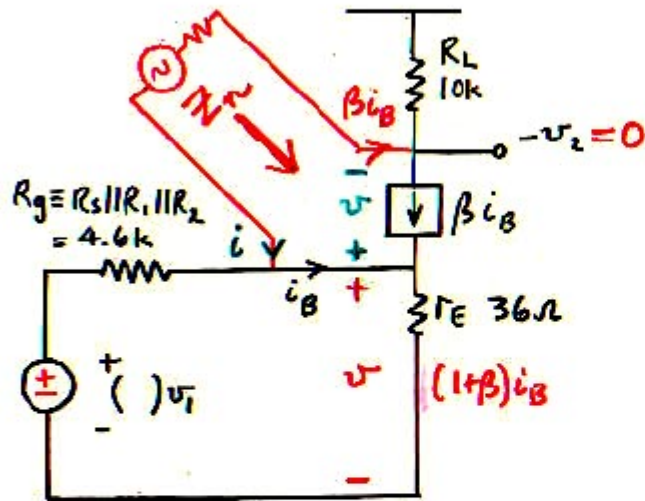
This is a useful technique in "doing the algebra on the circuit diagram."

Midband model after Thevenin reduction of R_s, R_1, R_2 :



$$Z_n = R_n = \frac{v}{i} = \frac{(1+\beta)r_E i_B}{-\beta i_B} = -\frac{r_E}{\alpha} = -36\Omega$$

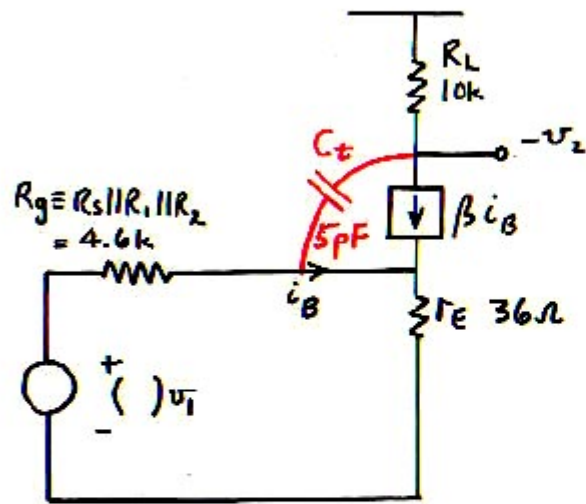
Midband model after Thevenin reduction of R_s, R_1, R_2 :



$$Z_n = R_n = \frac{v}{i} = \frac{(1+\beta)r_E i_B}{-\beta i_B} = -\frac{r_E}{\beta} = -36\Omega$$

Note that the Z_n calculation is much shorter and easier than the Z_d calculation!

Midband model after Thevenin reduction of R_s, R_1, R_2 :



Hence corrected gain after inclusion of C_t is

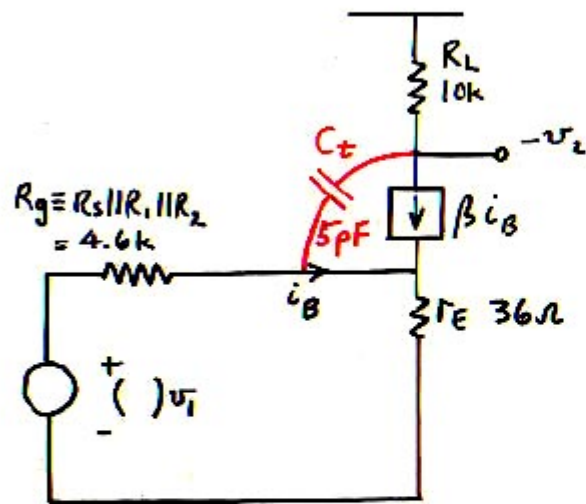
$$A = A_m \frac{1 + \frac{Z_n}{Z}}{1 + \frac{Z_d}{Z}} = A_m \frac{1 + sC_t R_n}{1 + sC_t R_d}$$

$$= A_m \frac{1 - \frac{s}{\omega_5}}{1 + \frac{s}{\omega_4}} \quad \text{where}$$

$$\omega_4 \equiv \frac{1}{C_t R_d} \quad f_4 = \frac{159}{5 \times 10^{-6} \times 620} = 51kHz$$

$$\omega_5 \equiv \frac{1}{C_t R_n} \quad f_5 = \frac{159}{5 \times 10^{-6} \times 0.036} = 880MHz$$

Midband model after Thevenin reduction of R_s, R_1, R_2 :



Hence corrected gain after inclusion of C_c is

$$A = A_m \frac{1 + \frac{z_n}{z}}{1 + \frac{z_d}{z}} = A_m \frac{1 + sC_c R_n}{1 + sC_c R_d}$$

$$= A_m \frac{1 - \frac{s}{\omega_5}}{1 + \frac{s}{\omega_4}} \quad \text{where}$$

$$\omega_4 = \frac{1}{C_c R_d} \quad f_4 = \frac{159}{5 \times 10^{-6} \times 620} = 51 \text{ kHz}$$

$$\omega_5 = \frac{1}{C_c R_n} \quad f_5 = \frac{159}{5 \times 10^{-6} \times 0.036} = 880 \text{ MHz}$$

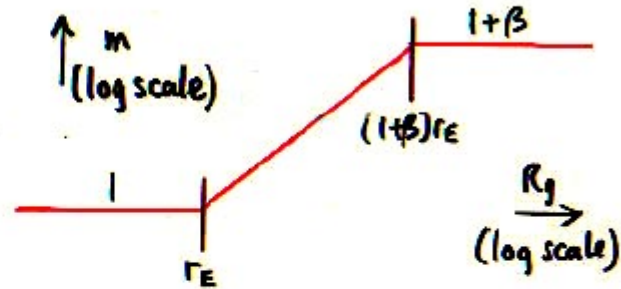
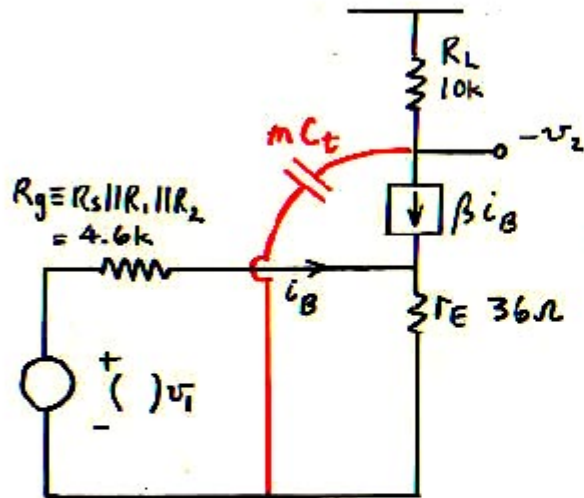
Note that the zero $\omega_5 = \frac{1}{C_c R_n} = \frac{\alpha}{C_c R_E}$ is negative (right half-plane), and is at a very high frequency unless there is substantial external emitter resistance and/or there is substantial external collector-base capacitance (as often exists).

Note that the pole $\omega_4 = \frac{1}{C_t R_d} = \frac{1}{C_t R_L} \frac{R_g \parallel R_e \parallel R_L}{R_g \parallel (1+\beta) r_e}$ is at a much lower frequency than $\omega_b \equiv \frac{1}{C_t R_L}$, and can be ascribed to an effective multiplication of C_t by a factor

$$m \equiv \frac{R_g \parallel (1+\beta) r_e}{R_g \parallel R_e \parallel R_L} = \frac{R_g \parallel (1+\beta) r_e}{R_g \parallel r_e} \left(1 + \frac{R_g \parallel r_e}{R_L} \right)$$

$R_g \gg (1+\beta) r_e \rightarrow 1+\beta$
 $R_g \ll r_e \rightarrow 1$

Midband model after Thevenin reduction of R_s, R_1, R_2 :



This example: $m = 62$

Alternative method for calculation of Z_d

There are two forms of the Extra Element Theorem:

$$A = A|_{z=\infty} \frac{1 + \frac{z_n}{z}}{1 + \frac{z_d}{z}} = A|_{z=0} \frac{1 + \frac{z}{z_n}}{1 + \frac{z}{z_d}}$$

where $A|_{z=0} = \frac{z_n}{z_d} A|_{z=\infty}$

Hence in general

$$\frac{A|_{z=0}}{A|_{z=\infty}} = \frac{z_n}{z_d}$$

It may be easier to find $A|_{z=0}$, $A|_{z=\infty}$, and z_n than to find Z_d directly.

Example: Addition of collector-base capacitance C_t to the CE amplifier stage. $A|_{z=\infty}$ and z_n were easily

found: $A|_{z=\infty} = A_m = \frac{R_B}{R_S + R_B} \cdot \frac{\beta R_L}{R_B + (1 + \beta) r_E}$ $z_n = R_n = -\frac{r_E}{\alpha}$

The Extra Element Theorem as derived applies to the correction factor resulting from an extra shunt element.

There is a corresponding form to find the correction factor resulting from an extra series element:

$$\begin{aligned} \text{reference gain} \\ \downarrow \\ \text{gain}|_z &= \text{gain}|_{z=\infty} \frac{1 + \frac{z_n}{z}}{1 + \frac{z_d}{z}} \\ &= \text{gain}|_{z=\infty} \frac{\frac{z_n}{z} + 1}{\frac{z_d}{z} + 1} \end{aligned}$$

$$\begin{aligned} \text{reference gain} \\ \downarrow \\ &= \text{gain}|_{z=0} \frac{1 + \frac{z}{z_n}}{1 + \frac{z}{z_d}} \\ &= \frac{z_n}{z_d} \text{gain}|_{z=\infty} \frac{1 + \frac{z}{z_n}}{1 + \frac{z}{z_d}} \end{aligned}$$

(This must be the gain when $z=0$.)

Alternative method for calculation of Z_d

There are two forms of the Extra Element Theorem:

$$A = A|_{z=\infty} \frac{1 + \frac{z_n}{z}}{1 + \frac{z_d}{z}} = A|_{z=0} \frac{1 + \frac{z}{z_n}}{1 + \frac{z}{z_d}}$$

where $A|_{z=0} = \frac{z_n}{z_d} A|_{z=\infty}$

Hence in general

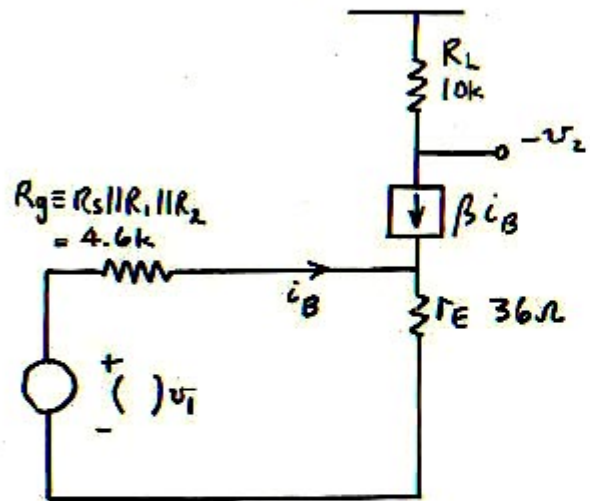
$$\frac{A|_{z=0}}{A|_{z=\infty}} = \frac{z_n}{z_d}$$

It may be easier to find $A|_{z=0}$, $A|_{z=\infty}$, and z_n than to find Z_d directly.

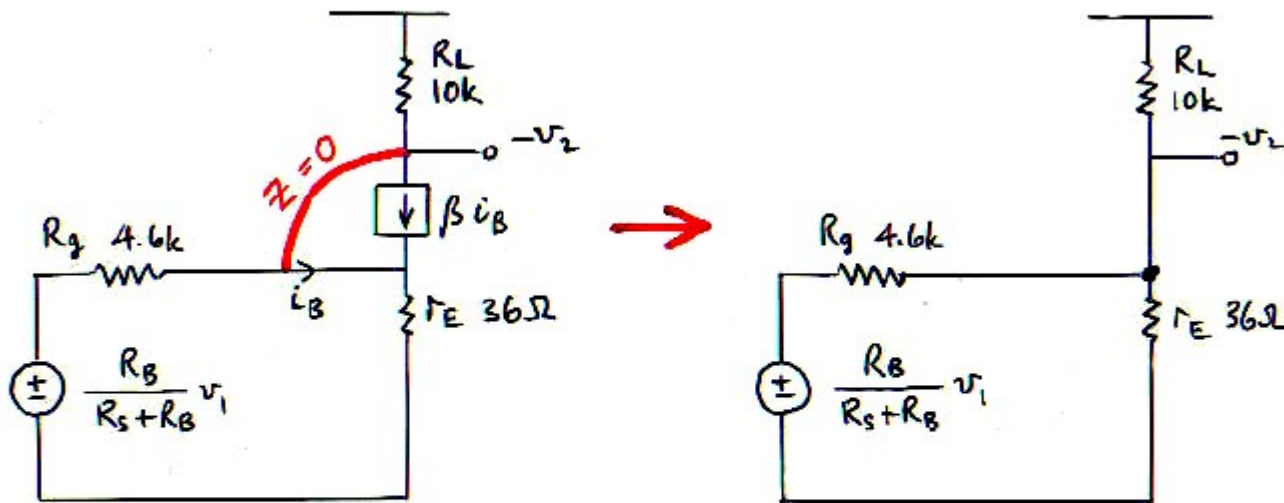
Example: Addition of collector-base capacitance C_t to the CE amplifier stage. $A|_{z=\infty}$ and z_n were easily found:

$$A|_{z=\infty} = A_m = \frac{R_B}{R_s + R_B} \cdot \frac{\beta R_L}{R_B + (1 + \beta) r_E} \quad z_n = R_n = -\frac{r_E}{\alpha}$$

Midband model after Thevenin reduction of R_s, R_1, R_2 :



Model for calculation of $A|_{z=0}$



$$A|_{z=0} = -\frac{R_B}{R_S + R_B} \frac{r_E \parallel R_L}{R_g + r_E \parallel R_L} = -\frac{R_B}{R_S + R_B} \frac{R_g \parallel r_E \parallel R_L}{R_g}$$

Hence: $Z_d = R_d = R_n \frac{A|_{z=0}}{A|_{z=0}} = \frac{r_E}{\alpha} \frac{\beta R_L}{R_g + (1+\beta)r_E} \frac{R_g}{R_g \parallel r_E \parallel R_L} = \frac{R_g \parallel (1+\beta)r_E}{R_g \parallel r_E \parallel R_L} R_L$

This is much easier than was the direct calculation of Z_d !

Generalization: Extra Element theorem - #5

The two reference gains and the two driving point impedances are related by:

$$\frac{A|_{z=0}}{A|_{z=\infty}} = \frac{z_n}{z_d}$$

One reference gain is always known or is easily found.

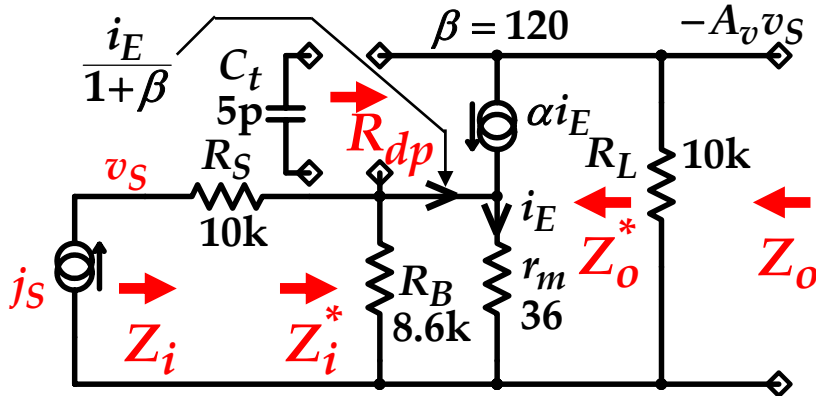
z_n is always easier to find than z_d .

Therefore:

It is often easier to find the other reference gain and to use the above ratio relation for z_d , than to find z_d directly.

Common-emitter (1CE) amplifier stage

Use the EET to find the outer and inner input impedances Z_i and Z_i^*



Previous results:

$$A_v = A_{vm} \frac{1 - s/\omega_z}{1 + s/\omega_p} = 36dB \frac{1 - \frac{s/2\pi}{880MHz}}{1 + \frac{s/2\pi}{51kHz}}$$

$$A_{vm} \equiv \frac{R_B}{R_S + R_B} \frac{\alpha R_L}{r_m + \frac{R_S \parallel R_B}{1 + \beta}} = 62 \Rightarrow 36dB$$

$$R_n = r_E / \alpha = 36\Omega$$

$$R_d = m R_L = 620k$$

$$\omega_z \equiv \frac{1}{C_t R_n} \quad \omega_p \equiv \frac{1}{C_t R_d} \quad m \equiv \frac{R_S \parallel R_B \parallel (1 + \beta) r_m}{R_S \parallel R_B \parallel r_m \parallel R_L} = 62$$

Outer input impedance Z_i :

Use the parallel EET with reference value $Z \equiv 1 / sC_t$ infinite:

$$Z_i \equiv \frac{v_S}{j_S} = R_{im} \frac{1 + sC_t R_{ni}}{1 + sC_t R_{di}}$$

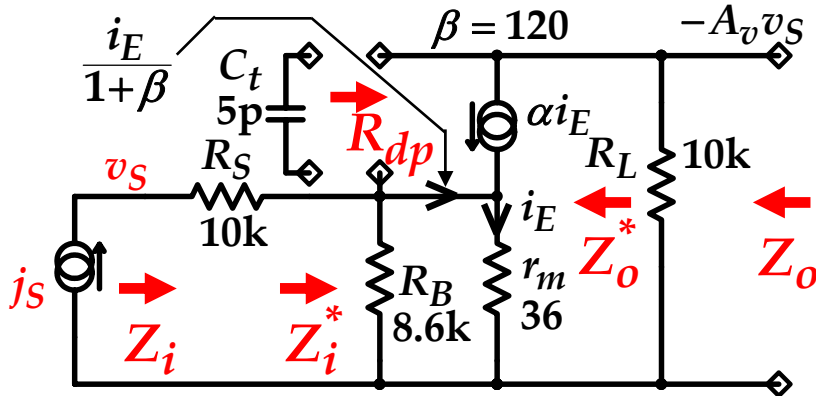
$$R_{im} \equiv R_S + R_B \parallel (1 + \beta) r_m = 13k \Rightarrow 82dB \text{ ref } 1\Omega$$

$$R_{ni} = R_{dp} \text{ with output } v_S \text{ nulled}$$

$$= R_{dp} \text{ with input } j_S \text{ shorted}$$

$$= R_{dp} \text{ for the voltage gain } A$$

Use the EET to find the outer and inner input impedances Z_i and Z_i^*



Previous results:

$$A_v = A_{vm} \frac{1 - s/\omega_z}{1 + s/\omega_p} = 36dB \frac{1 - \frac{s/2\pi}{880MHz}}{1 + \frac{s/2\pi}{51kHz}}$$

$$A_{vm} \equiv \frac{R_B}{R_S + R_B} \frac{\alpha R_L}{r_m + \frac{R_S \parallel R_B}{1 + \beta}} = 62 \Rightarrow 36dB$$

$$R_n = r_E / \alpha = 36\Omega$$

$$R_d = mR_L = 620k$$

$$\omega_z \equiv \frac{1}{C_t R_n} \quad \omega_p \equiv \frac{1}{C_t R_d} \quad m \equiv \frac{R_S \parallel R_B \parallel (1 + \beta)r_m}{R_S \parallel R_B \parallel r_m \parallel R_L} = 62$$

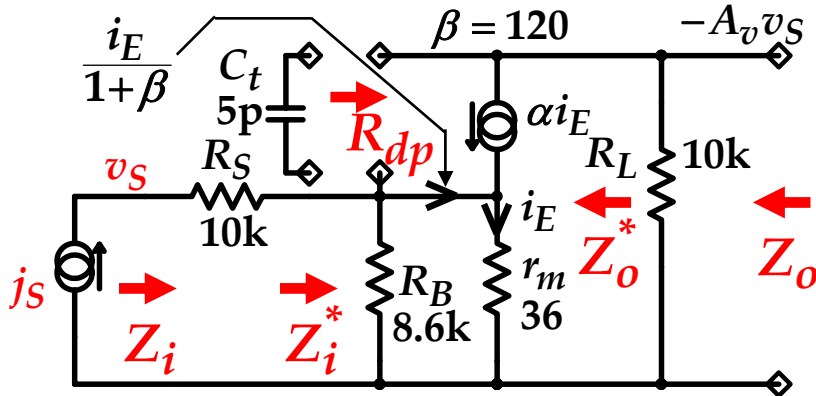
Outer input impedance Z_i :

$$Z_i \equiv \frac{v_S}{j_S} = R_{im} \frac{1 + sC_t R_{ni}}{1 + sC_t R_{di}}$$

$$R_{im} \equiv R_S + R_B \parallel (1 + \beta)r_m = 13k \Rightarrow 82dB \text{ ref } 1\Omega$$

$$R_{ni} = mR_L \equiv \frac{R_S \parallel R_B \parallel (1 + \beta)r_m}{R_S \parallel R_B \parallel r_m \parallel R_L} R_L = 620k$$

Use the EET to find the outer and inner input impedances Z_i and Z_i^*



Previous results:

$$A_v = A_{vm} \frac{1 - s/\omega_z}{1 + s/\omega_p} = 36dB \frac{1 - \frac{s/2\pi}{880MHz}}{1 + \frac{s/2\pi}{51kHz}}$$

$$A_{vm} \equiv \frac{R_B}{R_S + R_B} \frac{\alpha R_L}{r_m + \frac{R_S \parallel R_B}{1 + \beta}} = 62 \Rightarrow 36dB$$

$$R_n = r_E / \alpha = 36\Omega$$

$$R_d = mR_L = 620k$$

$$\omega_z \equiv \frac{1}{C_t R_n} \quad \omega_p \equiv \frac{1}{C_t R_d} \quad m \equiv \frac{R_S \parallel R_B \parallel (1 + \beta)r_m}{R_S \parallel R_B \parallel r_m \parallel R_L} = 62$$

Outer input impedance Z_i :

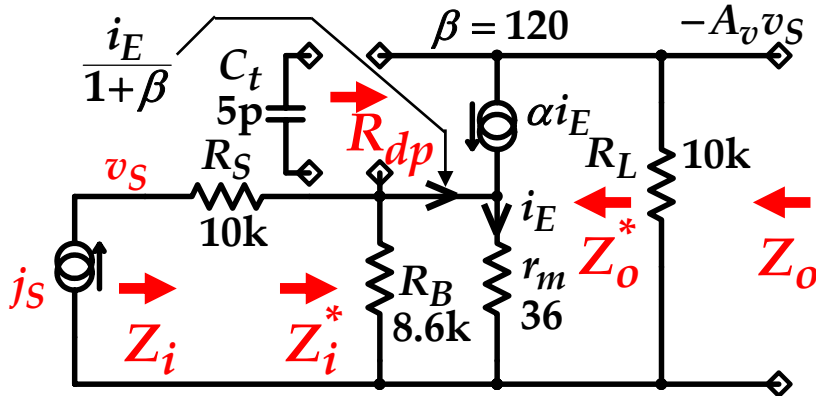
$$Z_i \equiv \frac{v_S}{j_S} = R_{im} \frac{1 + sC_t R_{ni}}{1 + sC_t R_{di}}$$

$$R_{im} \equiv R_S + R_B \parallel (1 + \beta)r_m = 13k \Rightarrow 82dB \text{ ref } 1\Omega$$

$$R_{ni} = mR_L \equiv \frac{R_S \parallel R_B \parallel (1 + \beta)r_m}{R_S \parallel R_B \parallel r_m \parallel R_L} R_L = 620k$$

$$R_{di} = R_{dp} \text{ with input } j_S \text{ open}$$

Use the EET to find the outer and inner input impedances Z_i and Z_i^*



Previous results:

$$A_v = A_{vm} \frac{1 - s/\omega_z}{1 + s/\omega_p} = 36dB \frac{1 - \frac{s/2\pi}{880MHz}}{1 + \frac{s/2\pi}{51kHz}}$$

$$A_{vm} \equiv \frac{R_B}{R_S + R_B} \frac{\alpha R_L}{r_m + \frac{R_S \parallel R_B}{1 + \beta}} = 62 \Rightarrow 36dB$$

$$R_n = r_E / \alpha = 36\Omega$$

$$R_d = mR_L = 620k$$

$$\omega_z \equiv \frac{1}{C_t R_n} \quad \omega_p \equiv \frac{1}{C_t R_d} \quad m \equiv \frac{R_S \parallel R_B \parallel (1 + \beta)r_m}{R_S \parallel R_B \parallel r_m \parallel R_L} = 62$$

Outer input impedance Z_i :

$$Z_i \equiv \frac{v_S}{j_S} = R_{im} \frac{1 + sC_t R_{ni}}{1 + sC_t R_{di}}$$

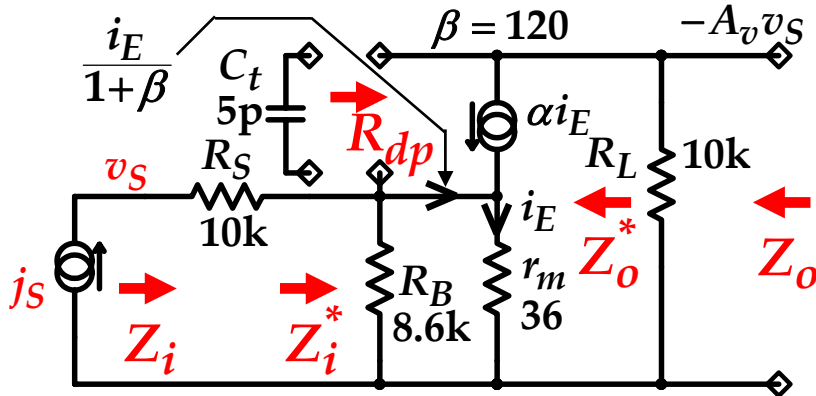
$$R_{im} \equiv R_S + R_B \parallel (1 + \beta)r_m = 13k \Rightarrow 82dB \text{ ref } 1\Omega$$

$$R_{ni} = mR_L \equiv \frac{R_S \parallel R_B \parallel (1 + \beta)r_m}{R_S \parallel R_B \parallel r_m \parallel R_L} R_L = 620k$$

$R_{di} = R_{dp}$ with input j_S open

$$= R_{ni} \Big|_{R_S \rightarrow \infty} = \frac{R_B \parallel (1 + \beta)r_m}{R_B \parallel r_m \parallel R_L} R_L = 820k$$

Use the EET to find the outer and inner input impedances Z_i and Z_i^*



Previous results:

$$A_v = A_{vm} \frac{1 - s/\omega_z}{1 + s/\omega_p} = 36dB \frac{1 - \frac{s/2\pi}{880MHz}}{1 + \frac{s/2\pi}{51kHz}}$$

$$A_{vm} \equiv \frac{R_B}{R_S + R_B} \frac{\alpha R_L}{r_m + \frac{R_S \parallel R_B}{1 + \beta}} = 62 \Rightarrow 36dB$$

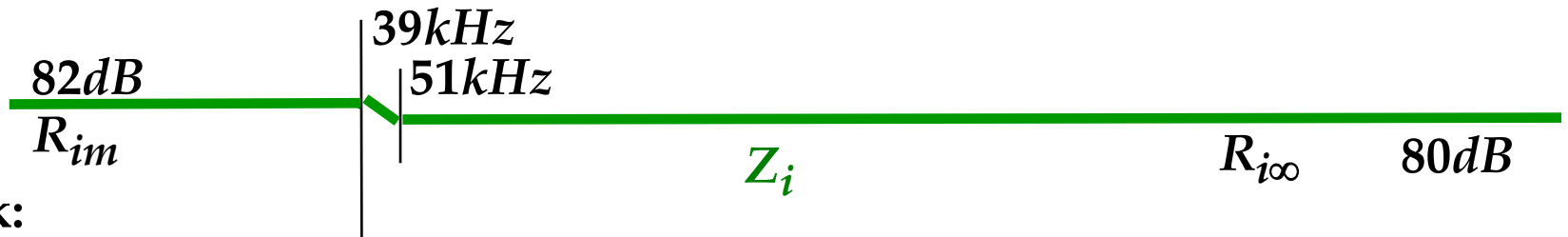
$$R_n = r_E / \alpha = 36\Omega$$

$$R_d = m R_L = 620k$$

$$\omega_z \equiv \frac{1}{C_t R_n} \quad \omega_p \equiv \frac{1}{C_t R_d} \quad m \equiv \frac{R_S \parallel R_B \parallel (1 + \beta) r_m}{R_S \parallel R_B \parallel r_m \parallel R_L} = 62$$

Outer input impedance Z_i :

$$Z_i \equiv \frac{v_S}{j s} = R_{im} \frac{1 + s C_t R_{ni}}{1 + s C_t R_{di}} = 82dB \frac{1 + \frac{s/2\pi}{51kHz}}{1 + \frac{s/2\pi}{39kHz}}$$

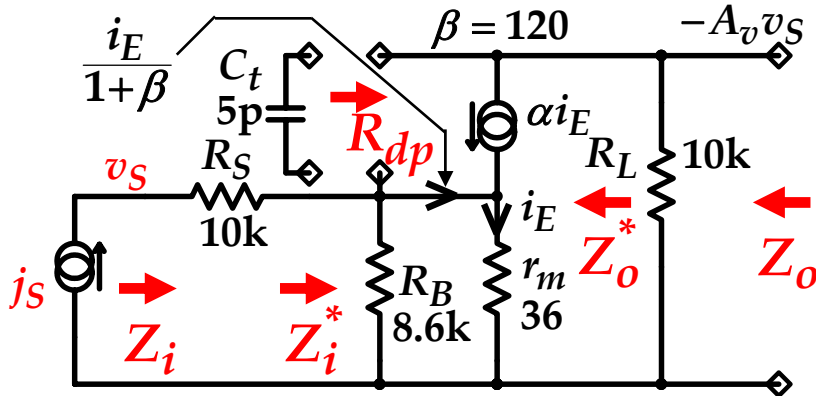


Check:

By GB trade-off: $R_{i\infty} = R_{im} \frac{R_n}{R_d} = 13k \frac{620k}{820k} = 10k \Rightarrow 80dB$

By inspection: $R_{i\infty} = R_S \parallel R_B \parallel r_m \parallel R_L = 10k \Rightarrow 80dB$

Use the EET to find the outer and inner input impedances Z_i and Z_i^*



Previous results:

$$A_v = A_{vm} \frac{1 - s/\omega_z}{1 + s/\omega_p} = 36dB \frac{1 - \frac{s/2\pi}{880MHz}}{1 + \frac{s/2\pi}{51kHz}}$$

$$A_{vm} \equiv \frac{R_B}{R_S + R_B} \frac{\alpha R_L}{r_m + \frac{R_S \parallel R_B}{1 + \beta}} = 62 \Rightarrow 36dB$$

$$R_n = r_E / \alpha = 36\Omega \quad R_d = m R_L = 620k$$

$$\omega_z \equiv \frac{1}{C_t R_n} \quad \omega_p \equiv \frac{1}{C_t R_d} \quad m \equiv \frac{R_S \parallel R_B \parallel (1 + \beta) r_m}{R_S \parallel R_B \parallel r_m \parallel R_L} = 62$$

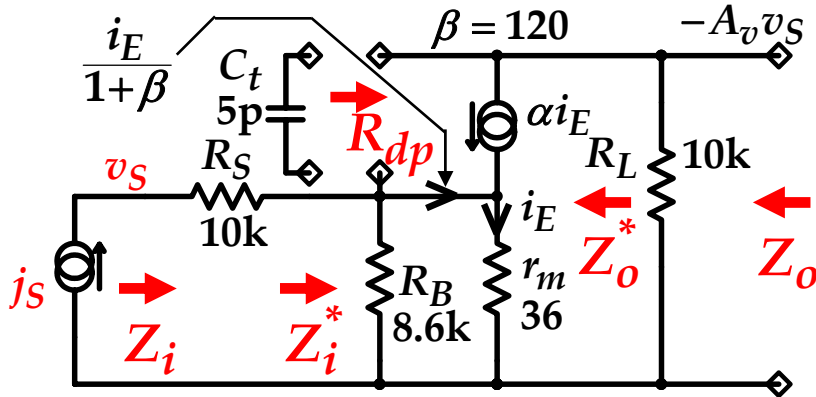
Inner input impedance Z_i^* :

$$Z_i^* = Z_i \Big|_{R_S \rightarrow 0} = R_{im} \frac{1 + sC_t R_{ni}}{1 + sC_t R_{di}} \Big|_{R_S \rightarrow 0} = R_{im}^* \frac{1 + sC_t R_{ni}^*}{1 + sC_t R_{di}}$$

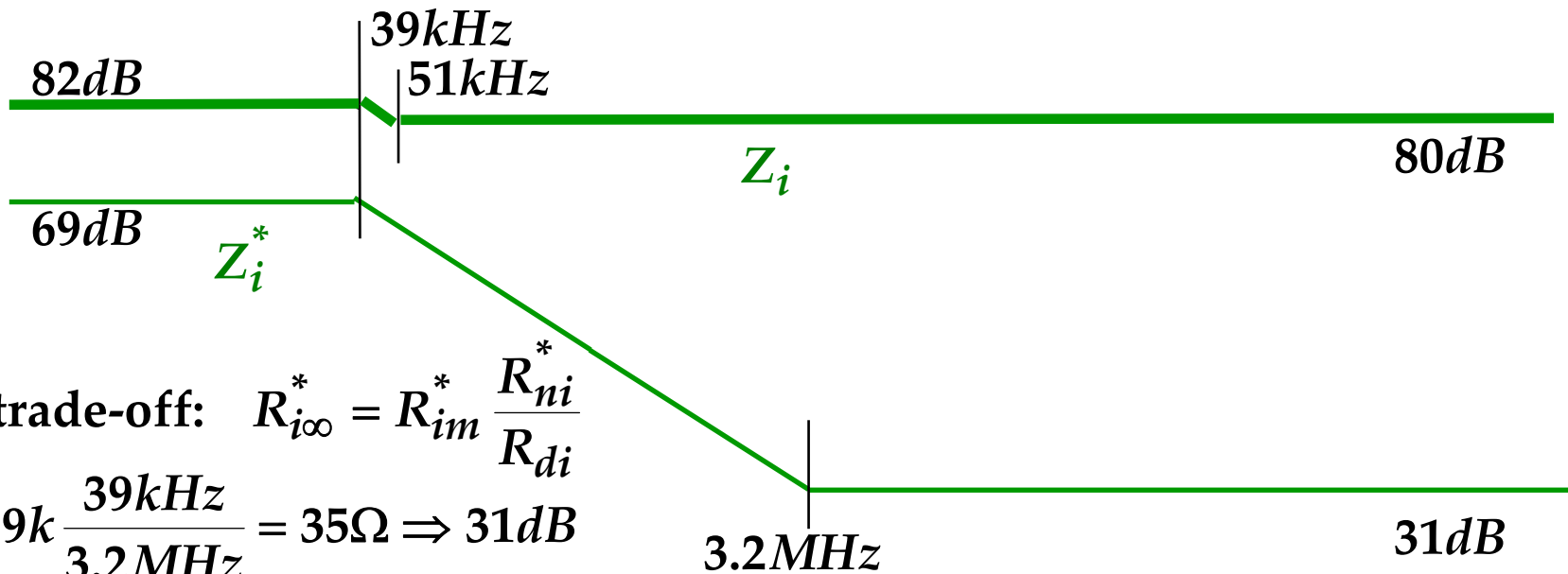
$$R_{im}^* = R_{im} \Big|_{R_S \rightarrow 0} = R_S + R_B \parallel (1 + \beta) r_m \Big|_{R_S \rightarrow 0} = R_B \parallel (1 + \beta) r_m = 2.9k \Rightarrow 69dB$$

$$R_{ni}^* = R_{ni} \Big|_{R_S \rightarrow 0} = \frac{R_S \parallel R_B \parallel (1 + \beta) r_m}{R_S \parallel R_B \parallel r_m \parallel R_L} R_L \Big|_{R_S \rightarrow 0} = R_L = 10k \Rightarrow 80dB$$

Use the EET to find the outer and inner input impedances Z_i and Z_i^*



$$Z_i^* = 69dB \frac{1 + \frac{s/2\pi}{3.2MHz}}{1 + \frac{s/2\pi}{39kHz}}$$



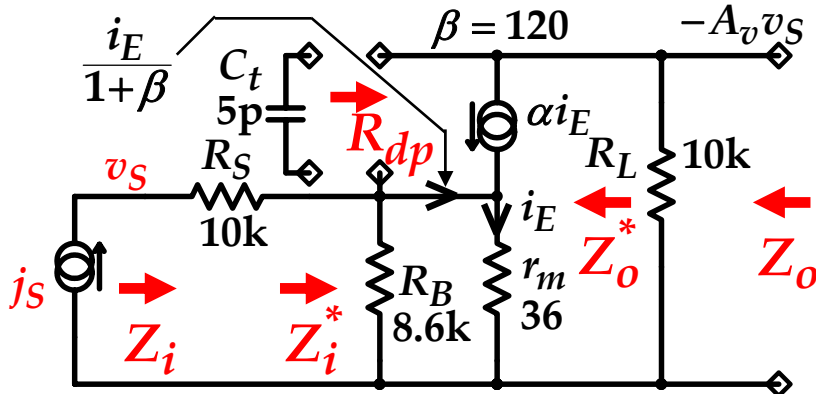
Check:

By GB trade-off: $R_{i\infty}^* = R_{im}^* \frac{R_{ni}^*}{R_{di}}$
 $= 2.9k \frac{39kHz}{3.2MHz} = 35\Omega \Rightarrow 31dB$

By inspection: $R_{i\infty}^* = R_B \parallel r_m \parallel R_L = 35\Omega \Rightarrow 31dB$

Exercise 8.5

Use the EET to find Z_o and Z_o^* for the 1CE amplifier stage



Previous results:

$$A_v = A_{vm} \frac{1 - s/\omega_z}{1 + s/\omega_p} = 36dB \frac{1 - \frac{s/2\pi}{880MHz}}{1 + \frac{s/2\pi}{51kHz}}$$

$$A_{vm} \equiv \frac{R_B}{R_S + R_B} \frac{\alpha R_L}{r_m + \frac{R_S \parallel R_B}{1 + \beta}} = 62 \Rightarrow 36dB$$

$$R_n = r_E / \alpha = 36\Omega$$

$$R_d = m R_L = 620k$$

$$\omega_z \equiv \frac{1}{C_t R_n} \quad \omega_p \equiv \frac{1}{C_t R_d} \quad m \equiv \frac{R_S \parallel R_B \parallel (1 + \beta) r_m}{R_S \parallel R_B \parallel r_m \parallel R_L} = 62$$

Outer output impedance Z_o :

Use the parallel EET with reference value $Z \equiv 1/sC_t$ infinite:

$$Z_o \equiv \frac{v_o}{j_s} = R_{om} \frac{1 + sC_t R_{no}}{1 + sC_t R_{do}}$$

$$R_{om} \equiv R_L = 10k \Rightarrow 80dB \text{ ref } 1\Omega$$

$$R_{no} = R_{dp} \text{ with output } v_o \text{ nulled} = R_{dp} \text{ with input } j_s \text{ shorted}$$

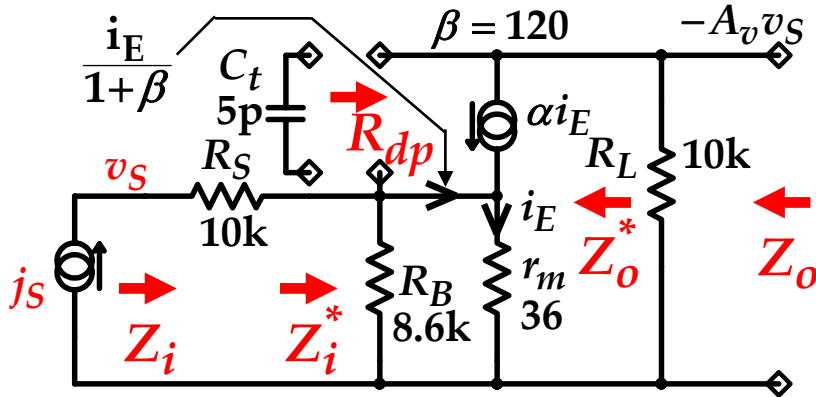
$$= R_S \parallel R_B \parallel (1 + \beta) r_m = 2.2k$$

$$R_{do} = R_{dp} \text{ with input } j_s \text{ open} = R_d \text{ for the voltage gain } A$$

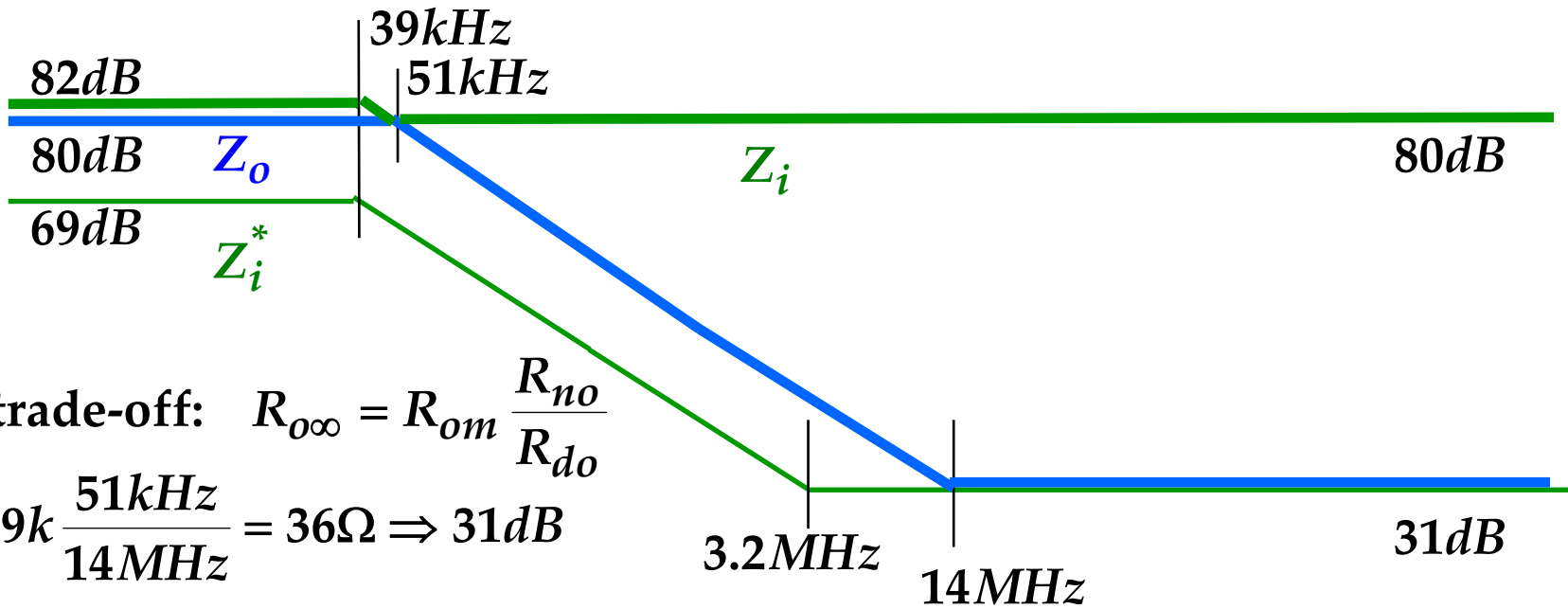
$$v.0.1 \Rightarrow 620k$$

Exercise 8.5: - Solution

Use the EET to find Z_o and Z_o^* for the 1CE amplifier stage



$$Z_o = 80dB \frac{1 + \frac{s/2\pi}{14MHz}}{1 + \frac{s/2\pi}{51kHz}}$$



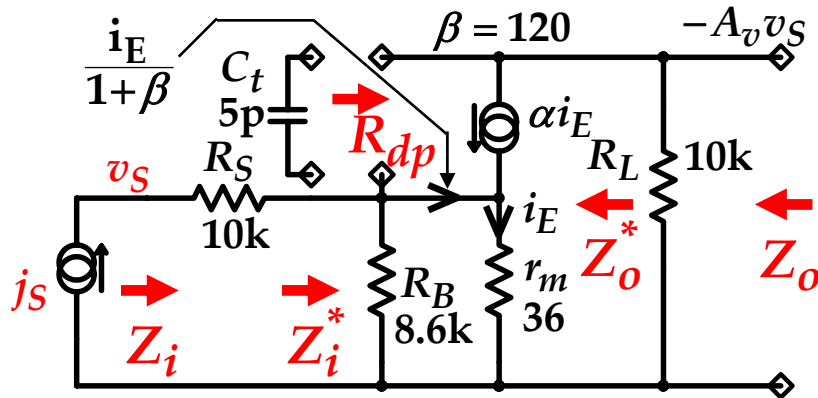
Check:

By GB trade-off: $R_{o\infty} = R_{om} \frac{R_{no}}{R_{do}}$
 $= 2.9k \frac{51kHz}{14MHz} = 36\Omega \Rightarrow 31dB$

By inspection: $R_{o\infty} = R_S \parallel R_B \parallel r_m \parallel R_L = 36\Omega \Rightarrow 31dB$

Exercise 8.5: - Solution

Use the EET to find Z_o and Z_o^* for the 1CE amplifier stage



$$R_{om} \equiv R_L = 10k \Rightarrow 80dB \text{ ref } 1\Omega$$

$$R_{no} = R_S \parallel R_B \parallel (1 + \beta)r_m = 2.2k$$

$$R_{do} \equiv \frac{R_S \parallel R_B \parallel (1 + \beta)r_m}{R_S \parallel R_B \parallel r_m \parallel R_L} R_L = 620k$$

Inner output impedance Z_o^* :

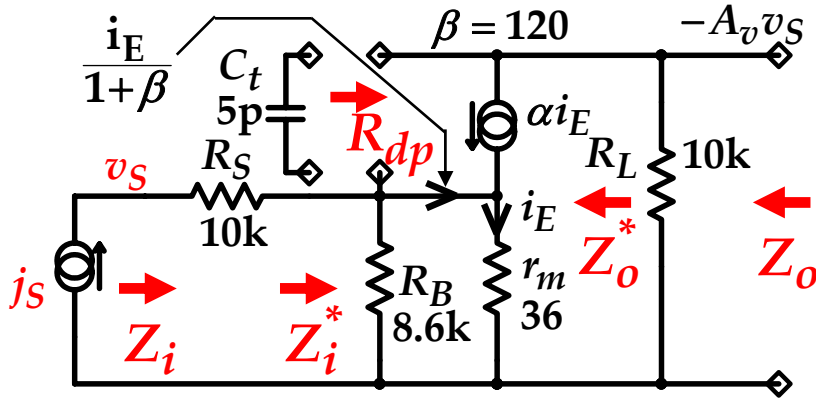
$$Z_o^* = Z_o \Big|_{R_L \rightarrow \infty} = R_{om} \frac{1 + sC_t R_{no}}{1 + sC_t R_{do}} \Big|_{R_L \rightarrow \infty} = R_{om}^* \frac{1 + sC_t R_{no}}{1 + sC_t R_{do}^*}$$

$$R_{om}^* = R_{om} \Big|_{R_L \rightarrow \infty} = R_L \Big|_{R_L \rightarrow \infty} = \infty$$

$$R_{do}^* = R_{do} \Big|_{R_L \rightarrow \infty} = \frac{R_S \parallel R_B \parallel (1 + \beta)r_m}{R_S \parallel R_B \parallel r_m \parallel R_L} R_L \Big|_{R_L \rightarrow \infty} = \infty$$

Exercise 8.5: - Solution

Use the EET to find Z_o and Z_o^* for the 1CE amplifier stage



$$R_{om} \equiv R_L = 10k \Rightarrow 80dB \text{ ref } 1\Omega$$

$$R_{no} = R_S \parallel R_B \parallel (1 + \beta)r_m = 2.2k$$

$$R_{do} \equiv \frac{R_S \parallel R_B \parallel (1 + \beta)r_m}{R_S \parallel R_B \parallel r_m \parallel R_L} R_L = 620k$$

Inner output impedance Z_o^* :

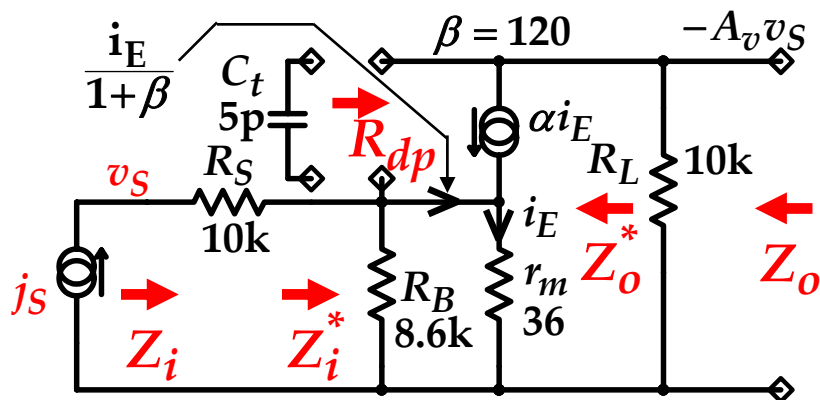
Because R_{om}^* and R_{do}^* both are infinite, change the Z_o reference value from R_{om} to $R_{o\infty}$. Then:

$$\begin{aligned} Z_o^* &= Z_o \Big|_{R_L \rightarrow \infty} = R_{om} \frac{1 + sC_t R_{no}}{1 + sC_t R_{do}} \Big|_{R_L \rightarrow \infty} = R_{o\infty} \frac{1 + 1/sC_t R_{no}}{1 + 1/sC_t R_{do}} \Big|_{R_L \rightarrow \infty} \\ &= R_{o\infty}^* (1 + 1/sC_t R_{no}) \end{aligned}$$

$$R_{o\infty}^* = R_{om} \frac{R_{no}}{R_{do}} \Big|_{R_L \rightarrow \infty} = R_S \parallel R_B \parallel r_m = 36\Omega \Rightarrow 31dB$$

Exercise 8.5: - Solution

Use the EET to find Z_o and Z_o^* for the 1CE amplifier stage



$$Z_o^* = 31dB \left(1 + \frac{14MHz}{s/2\pi} \right)$$

