Factor Endowment and Relative Commodity Prices

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1. The purpose of this note is to investigate the effect of an increase in the quantity of a factor of production upon production, consumption and the terms of trade. Although in the model used here the quantity of only one factor is varied, the analysis can easily be extended to variations in the quantities of two factors—the variations in these factors being proportionate or disproportionate.

2. We assume a closed economy with only two factors of production, X and Y, which are perfectly divisible, perfectly mobile, and in some degree substitutable. There are but two industries and each of them is subject to a linear homogeneous production function. They are producing two commodities L and R. The " factor intensity " of each industry is different. By this we mean that technical conditions of production are such that the ratio $\dfrac{\text{marginal physical product of } X}{\text{marginal physical product of } Y}$ is always equalised as between the two industries by using a higher ratio of $\dfrac{X}{Y}$ in one of the industries (the X-intensive industry) and a lower ratio of $\dfrac{X}{Y}$ in the other (the Y-intensive industry).

3. We shall use the box diagram as employed by Stolper and Samuelson[2] to depict production conditions. Thus in Fig. 1 the box $ABCD$ depicts an economy with AB of Factor X and AD of Factor Y. Isoquants for the commodity L are shown by the family of curves convex to the origin A; similarly isoquants for the commodity K are shown by the family of curves convex to the origin C.

If we make commodity K X-intensive and commodity L Y-intensive, the contract curve AC must be convex to D. Equilibrium must lie somewhere on this curve. The position on the curve is determined by the condition that the substitution rate between L and K in production (as indicated by the isoquants) must be equal to the substitution rate in consumption (as indicated by the consumer preference system).

4. Suppose this condition is fulfilled at the point S. An increase in the quantity of Factor X may be shown by extending BA to BA'. The new production box is now $A'BCD'$. The horizontal extension

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[2] See W. F. Stolper and P. A. Samuelson, " Protection and Real Wages ", reprinted in *Readings in the Theory of International Trade*, pp. 347 *et seq.*

of the box leaves the family of the isoquants originating from C unaffected; the family of isoquants originating previously from A must now be shifted to the new origin A'. As the production functions of both commodities are homogeneous and of the first degree, the line AS intersects all members of the family of L-isoquants at points where their slopes are the same as at the point S; similarly CS intersects all members of the family of K-isoquants at points where these curves have this same slope. Therefore, when the line $A'S'$ is drawn parallel to AS it must cut the family of isoquants originating from A' at points where these curves all have the same slope as the curves at S.

FIG 1.
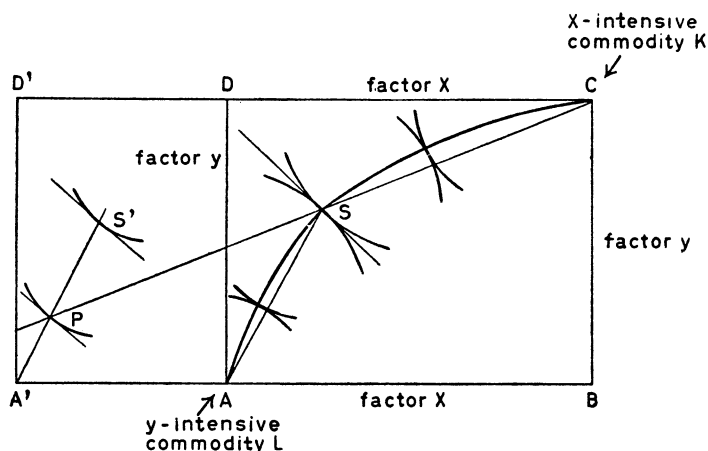
Let the line CS be prolonged to cut $A'S'$ at P. Point P must be on the new " contract " curve because the slopes of the two isoquants passing through that point are by the above reasoning equal, both being the same as the slope of the tangent at S. If the same rates of substitution in production were to remain after Factor X had been increased by AA', P would be the new point of equilibrium.

5. Because of the property of linearity of the two production functions, the amount of a commodity produced may be measured by the distance along any given radial from the origin. Thus since the length $A'P$ is necessarily shorter than AS, production of the Y-intensive commodity must be less at P than at S. The fact that CP is longer than CS shows on the other hand that production on the X-intensive commodity is expanded. This proves *that the maintenance of the same rates of substitution in production after the quantity of one factor has increased must lead to an absolute expansion in production of the commodity using relatively much of that factor, and to an absolute*

curtailment of production of the commodity using relatively little of the same factor.

6. The above proposition may now be carried over into a diagram showing " production possibility " curves. Thus in Fig. 2 the horizontal and vertical co-ordinates measure quantities of commodities K and L respectively. With the given initial quantities of X and Y, the boundary

FIG. 2.
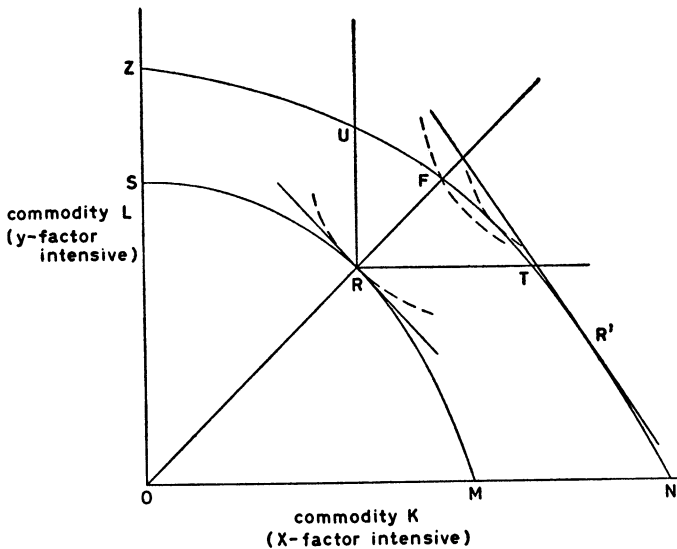
of possible L, K, combinations, or the " production possibility " curve, is SM. The concavity of this curve to the origin connotes conditions of increasing rate of substitution on the production side.

7. The original position of equilibrium is defined by the equality of substitution rates in consumption and production and is represented graphically by the point of tangency between the " production possibility " curve and an indifference curve (point R in Fig. 2). To find the new point of production and consumption equilibrium when the quantity of Factor X is increased, it is necessary to discover the new point of tangency between the new " production possibility " curve and (another) indifference curve. This requires the derivation of the new " production possibility " curve and its analysis in relation to the indifference curve system. Each will be examined below.

8. Now in virtue of our proposition that the maintenance of the same substitution rate in production, after Factor X has increased, requires an absolute increase in the output of the X-intensive commodity, and an absolute reduction in the output of the Y-intensive commodity, it follows that the slope found at R on the production curve SM (Fig. 2) must, on the production curve ZN, lie below RT where less of L is produced. We shall suppose that it lies at R'.

9. So much for the shape of the new " production possibility " curve. What is its relationship to the system of indifference curves ? We assume the absence of any inferior good. This means that as income increases more of both commodities must be consumed. This in turn implies that for any indifference curve above that passing through R, the slope prevailing at R must be found on a sector of the higher indifference curve which lies within the quadrant URT. Let the indifference curve passing through F have at that point the same slope as at R where F is the point of intersection between OR prolonged and ZN. Then since the new " production possibility " curve above R' is flatter than at R and the indifference curve through F has the same slope as at R, it follows that the indifference curve passing through F must cut the new " production possibility " curve from above at that point. But it also follows that, since below F the indifference curves become flatter, there must be a point of tangency between a (higher) indifference curve and the " production possibility " curve below F, but above R'. At such a point the slope of the " production possibility " curve and indifference curve must be flatter than at R. This implies that the terms of trade of the X-intensive commodity have worsened as against the Y-intensive commodity, and proves the proposition that the terms of trade of the commodity using relatively much of the factor whose quantity has increased must deteriorate.

10. No reference has been made so far to the conditions of demand. The fact that the point F was assumed to fall on the straight line OR was equivalent to the assumption that the Marginal Propensity to Consume is equal to the Average Propensity to Consume (the latter falling over the whole range of the change in the position of equilibrium). The implications arising from the possibility of the former being different from the latter must now be examined.

When the Marginal Propensity to Consume the X-intensive good exceeds the Average Propensity to Consume, the income consumption curve will cut the new " production possibility " curve somewhere between T and F. Since it has been proved that the new point of equilibrium must always be below the point where an indifference curve with the same slope as at R (i.e. defined by the income consumption curve) intersects the new " production possibility " curve, it follows that the more the Marginal Propensity to Consume favours the commodity using relatively much of the factor whose quantity has

been increased, the smaller is the worsening of the terms of trade of that commodity.

The reverse holds equally true: when the Marginal Propensity to Consume the good which uses a relatively low proportion of the increased factor exceeds the Average Propensity to Consume, an indifference curve with the same slope as at R will cut the new " production possibility " curve somewhere between U and F, signifying that the terms of trade have turned more against the commodity using relatively much of the increased factor. The greater is the difference between the Marginal and the Average Propensity to Consume, the greater will be the deterioration in the terms of trade for the commodity using relatively much of the increased factor.

11. Our conclusion is that an increase in the quantity of one factor will always lead to a worsening in the terms of trade, or the relative price, of the commodity using relatively much of that factor. The Marginal Propensity to Consume influences the degree of deterioration, but it can never reverse its direction.

12. Despite the change of the relative prices of the two commodities the patterns of production and consumption may remain unaltered, or change in favour of one good or the other. By the unchanged pattern of production and consumption we mean that the quantity of both products has increased proportionately ; a shift in the direction of one commodity signifies a greater than proportionate increase in production and consumption of that good combined with a less than proportionate rise of the other. The new equilibrium is determined again by the difference between the Marginal and the Average Propensity to Consume on the one hand and the shift of the production possibility curve; the latter in turn is determined by the proportion in which one factor has been increased and the technical production functions of the two commodities.

The unequivocal generalisation possible regarding the new production and consumption patterns is that they will change in the direction of the good using relatively much of the factor increased (i.e. the quantity of that good produced and consumed will increase more while that of the other will rise less than proportionately) if the Marginal Propensity to Consume the product using relatively much of the factor increased remains equal to, or is greater than, the Average Propensity to Consume. When the Marginal Propensity to Consume is less than the Average then the new production and consumption patterns may still change in favour of the commodity using much of the factor increased, or may remain unchanged or move in the direction of the other good; the ultimate outcome depends, in this case, on the numerical values assigned to the factors mentioned; it is possible, however, to offer tentative conclusions that the small excess of the Marginal over the Average Propensity to Consume in the direction of the good using little of the factor increased is still likely to shift the production and

consumption patterns in favour of the other commodity; a moderate excess is likely to leave the production and consumption patterns unchanged, while a large difference may be expected to alter both production and consumption in favour of that good. All this is but a corollary of our previous proof.

13. If it is now assumed that the commodity using relatively much of the factor, the quantity of which had been increased, is an item of export, this means that external terms of trade will deteriorate; conversely, should the commodity be an import, the terms of trade must improve.

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