Biham-Middleton-Levine Traffic

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Consider two types of cars, red (east-bound) and blue (north-bound) which populate a two-dimensional $n \times n$ square lattice with periodic boundary conditions. Each lattice site is in one of three states: empty, occupied by a red car, or occupied by a blue car. The cars are initially distributed independently and uniformly at random over the lattice sites with spatial density p, implying that at each site,

$$\mathrm{P}\,(\text{red car}) = p/2, \qquad \mathrm{P}\,(\text{blue car}) = p/2, \qquad \mathrm{P}\,(\text{empty}) = 1 - p.$$

This is the only indeterminate step within the traffic model [1].

Time is integer-valued. At each time point, two steps occur, one immediately following the other. First, all red cars simultaneously attempt to move one lattice site to the east. If the site east of a red car is currently empty, it advances; otherwise it is blocked (even if the east site is becoming empty). Second, all blue cars simultaneously attempt to move one lattice site to the north. If the site north of a blue car is currently empty, it advances; otherwise it is blocked (even if the north site is becoming empty).

The velocity v of the system at each time t is the ratio between the number of cars that successfully moved and the total number of cars. If $v = 0$, then no car has moved at t; if $v = 1$, then all the cars have moved. The dependence of v for large t on both p and n is exceedingly interesting – see Figure 1 – depicted is an average of v over many realizations and over a large time interval [2].

Early in the study of this particular traffic model, it was thought that the phase transition exhibited by v would be comparable to other famous systems in statistical mechanics (for example, percolation). Such a belief seems, however, not to be supported by computer simulation. Intermediate stable phases, where regions of gridlock coexist with bands of unrestricted movement, seem to form effortlessly for $32 \leq n \leq 512$ [3, 4]. No one knows what truly happens as $n \to \infty$. Do such critical intervals slowly cascade to $p = 0$ in the limit? Or do they remain intact and disjoint from $p = 0$?

Additional references [5, 6, 7, 8, 9, 10, 11, 12] cover both theoretical and experimental aspects of the subject.
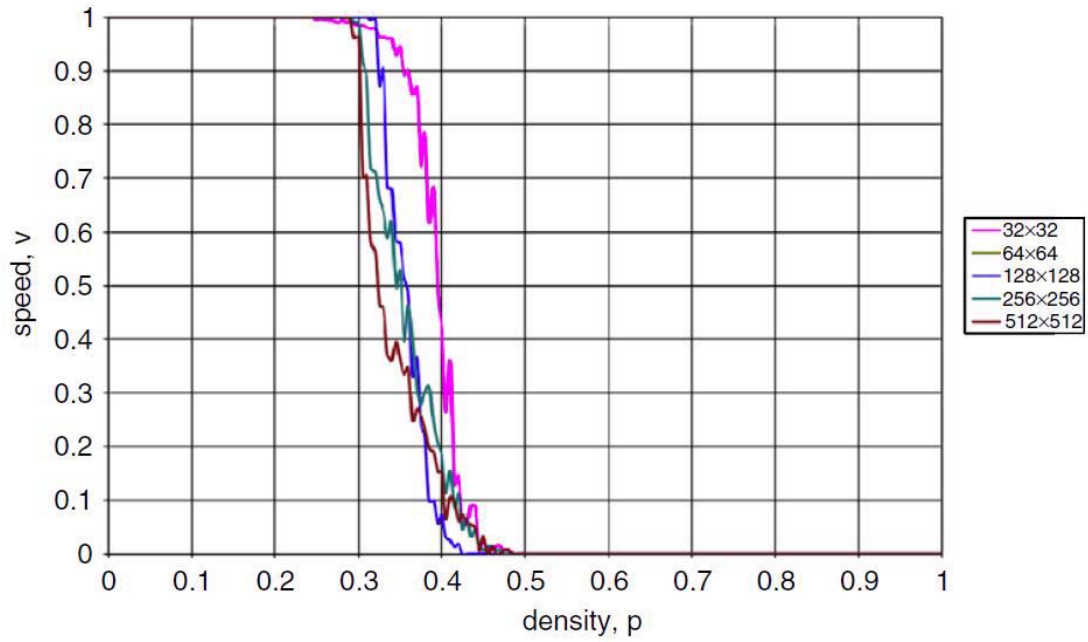

Figure 1: The phase transition from freely-flowing to fully-jammed is not sharp, even for large n. We desire more precise estimates of both upper and lower critical densities, as functions of p and n. Another plot in [2] gives not the mean of v over realizations, but the standard deviation (with well-defined peak).

References

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