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Solar Chimney for Enhanced Stack Ventilation

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A steady state mathematical model has been developed for a solar chimney which is used to enhance the effect of thermally induced ventilation in buildings. The model takes into consideration different sizes of the openings of a solar chimney with varying values of the discharge coefficients. Numerical calculations performed for different values of ambient temperature and solar radiation show that a solar collector area of 2.25 (m^2) is able to induce an air flow between 140 (m^3/hr) to 330 (m^3/hr) for solar radiation of 200 (W/m^2) and 1000 (W/m^2) respectively.

NOMENCLATURE

A = Area (m^2)
C = Coefficient of
g = Gravitational acceleration (m/s^2)
H = Height (m)
h = Heat transfer coefficient ($W/m^2/K$)
m = Mass flow rate (Kg/s)
P = Pressure (N/m^2)
Q = Volume flow rate (m^3/s)
$S(t)$ = Average solar radiation (W/m^2)
T = Temperature (deg. C)
U = Heat loss coefficient ($W/m^2/K$)
W = Width (m)

Greek
α = Absorptance
β = Slope (degree)
τ = Transmittance
σ = Density (kg/m^3)

Subscripts
a = Ambient
b = Bottom
D = Discharge
f = Fluid
i = Inlet
o = Outlet
p = Plate
r = Ratio
R = Room

1. INTRODUCTION

VENTILATION is generally defined as supply of outside air to the interior for air motion and replacement of stale air by fresh outside air for healthy and comfortable interior environment [1].

The two systems of ventilation, namely, natural ventilation and mechanical ventilation can be met by properly placed openings in the former case and by ceiling fans or exhaust fans in the latter. For natural ventilation, there are two causes of air motion through the building, namely (i) aeromotive or wind force, (ii) thermal or temperature forces or stack effect.

When wind strikes a building, a region of higher pressure is created on windward side while the leeward wall and roof are all subjected to reduced pressure. A pressure gradient is thereby created across the building in the direction of the incident wind. This pressure gradient causes the air to flow through the building from openings in the region of higher pressure to openings located in regions of lower pressure. In the simple case of an isolated enclosure in which openings are provided in each of two opposite walls, the rate of air flow can be calculated by the equation

$$Q = K \cdot A \cdot V$$

where Q is rate of air flow (m^3/hr), A is area of smaller opening (m^2), V is outdoor wind speed (m/hr), and K is coefficient of effectiveness.

Thermal ventilation due to stack effect operates when a temperature difference exists between the outside and inside air of a building. A difference is created between their densities and a pressure gradient developed along the vertical direction over the walls of the building. If the temperature inside is higher than that outside, the upper parts of the building will have higher pressure while the lower parts will have lower pressure. When openings are provided in these regions, air enters through the lower openings and escapes through the upper. In case the indoor air temperature is lower than outside, the air flow will be reversed.

The patterns of pressure changes can be described as given below [2].

(i) When a single opening exists at a certain level in the building, the air pressure on either side of the opening equalizes, after which no air flow is induced through the opening inspite of the temperature difference.

The air pressure above and below the opening varies with height, temperature difference between inside and outside and it is proportional to the density of air. If the

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indoor air is warmer and therefore less dense, the indoor vertical pressure gradient is less than that outdoors. This means that inside is at excess pressure at any level above the opening and a depression below it, and these differences increase with vertical distance from the opening. In case no opening is available the air cannot flow out though the pressure difference exists.

(ii) When two openings are provided at different heights and the indoor temperature is higher than outside, the pressure difference is formed in such a way that excess indoor pressure builds-up at the upper opening, where air flows outwards while a depression is created at the lower level, inducing an inward flow of air. When the indoor temperature is lower, the positions are interchanged and the flow direction reversed.

The air flow induced by the thermal force is proportional to the square root of the pressure head and the free area of the opening.

It is given by equation

$$Q = K \cdot A \cdot \{h \cdot (t_i - t_o)\}^{1/2}$$

where Q is volume rate of air flow (m^3/hr), A is free area of inlet opening (m^2), K is a constant depending upon resistance given by the opening, h is vertical distance between inlet and outlet (m), t_i is average temperature of indoor air at height (h) in (°C) and t_o is the temperature of outdoor air (°C) [1].

As the thermal force of ventilation depends on the product of indoor–outdoor temperature difference and the height of the ventilation path, i.e. vertical distance between the openings, it is important that one of these factors is of sufficient magnitude.

In residential buildings, the effective height of ventilation path is very small, less than 2 m in an average single storeyed apartment. So for an air flow of any practical use to be induced by thermal force, there must be an appreciable difference between the indoor and outdoor temperatures. Such differences exist only in winter and mostly in cold regions. Thus in summer the thermal force is usually too small to have any practical application. Ventilation due to temperature forces is therefore considered to be insignificant and usually neglected, as when both aeromotive and thermal forces are acting. The aeromotive forces are assumed to be predominant. Ventilation due to aeromotive forces is well understood but no significant work has so far been done on natural ventilation by thermal forces.

Use of solar energy to create large temperature differences and therefore an appreciable air movement has been an idea given to operate either a turbine for power generation or lately it is in use in buildings with the name of a solar chimney [4, 5, 6]. A solar chimney has been designed and tested for aiding ventilation in Africa [8]. It was shown that if air cools from 30°C to $(30-\theta)$ °C in our concrete, a room of 30 m^3, one can obtain $4(\theta)^{1/2}$ air changes per hour, so that a fall in temperature by 1, 2 or 3 degrees would give 4, 5.5 and 7 air changes per hour. The air changes were also found to be dependent upon the width of the air collector in the chimney. A demonstration project in Alicante, Spain uses the concept of solar chimney for inducing ventilation [9]. No. measurements or theory are however available for quantifying solar chimney ventilation. In this paper a rigorous analytical model has been developed for a solar chimney to see whether the concept can in fact produce desirable movement of air flow.

2. SYSTEM DESCRIPTION

A solar chimney is essentially divided into two parts, one—the solar air heater (collector) and second—the chimney (Fig. 1). The system is desired to be designed to maximize solar gain and thereby maximize the ventilation effect. The critical design parameters being the height, cross-section area and the difference in temperature at the inlet and outlet of the solar air heating system.

Air in a solar collector gets heated during the day, the air inside heats, expands and rises, in turn pulling interior air up and out. One advantage of the system is its ability to self balance; the hotter the day, the hotter the solar air heat collector and faster the air movement.

3. MATHEMATICAL MODELLING

3.1. *Air flow rate equation*

An equation for volume flow rate can be derived with help of Bernoulli's Equation and Continuity Equation from Principle of Conservation of Mass. Referring to Fig. 2 one gets [3]:

$$Q_o = C_D \cdot A_o[2(P_{i1} - P_{o1})/\sigma_o]^{1/2} \quad (1)$$

and

$$Q_i = C_D \cdot A_i[2(P_{o2} - P_{i2})/\sigma_i]^{1/2}. \quad (2)$$

Where Q_o is outflow rate, Q_i is inflow rate, C_D is coefficient of discharge, A_o is outlet area, A_i is inlet area, σ_o is density of air at outlet, σ_i is density of air at inlet. P_{o1}, P_{i1}, P_{o2} and P_{i2} denotes pressures at relative points as shown in Fig. 2.

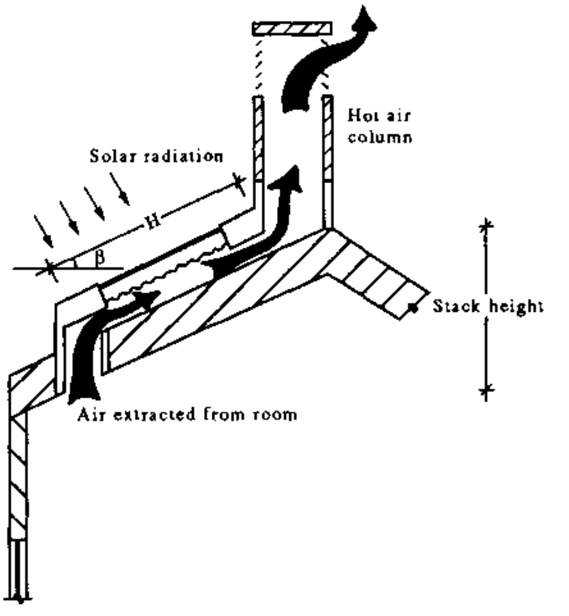

Fig. 1. System description.

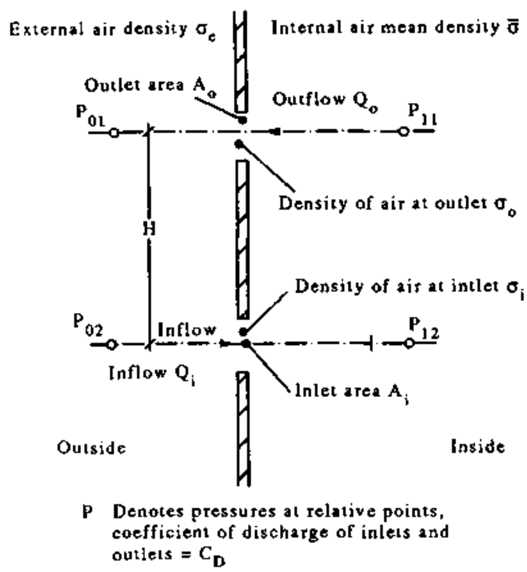

Fig. 2. Pressure and density distribution in a solar chimney system.

Also

$$P_{o2} = P_{o1} + \sigma_e \cdot g \cdot H$$

and

$$P_{12} = P_{11} + \bar{\sigma} \cdot g \cdot H$$

Hence

$$(P_{o2} - P_{12}) + (P_{11} - P_{o1}) = g \cdot H(\sigma_e - \bar{\sigma}). \quad (3)$$

Substituting equations (1), (2) in (3) and using the conservation of mass flow, i.e. $m = \sigma_i \cdot Q_i = \sigma_o \cdot Q_o$, then

$$m = C_D \cdot [2 \cdot g \cdot H(\sigma_e - \bar{\sigma})]^{1/2} \cdot [\{1/(\sigma_i \cdot A_i^2)\} + \{1/(\sigma_o \cdot A_o^2)\}]^{-1/2}. \quad (4)$$

The normal ranges of external and inernal conditions and the differences of density are of much greater importance than the absolute values, thus

$$(\sigma_e - \bar{\sigma}) = \Delta\sigma$$

and

$$\sigma_o = \sigma_i = \sigma$$

also

$$m = \sigma \cdot Q$$

$$m = C_D \cdot A_o[2 \cdot g \cdot H \cdot \sigma \cdot \Delta\sigma]^{1/2} / [(1 + A_r^2)]^{1/2}$$

where

$$A_r = (A_o/A_i)$$

and using the ideal gas law

$$P = \sigma \cdot R \cdot T$$

$$\sigma_i = P/(R \cdot T_i); \quad \sigma_o = P/(R \cdot T_o)$$

and

$$\Delta\sigma = (\sigma_o - \sigma_i)$$

on substituting these we get

$$Q_i = C_D \cdot A_o[2 \cdot (\Delta T/T_o) \cdot g \cdot H]^{1/2} \cdot [(1 + A_r^2)]^{-1/2}. \quad (5)$$

Similarly

$$Q_o = C_D \cdot A_o[2 \cdot (\Delta T/T_i) \cdot g \cdot H]^{1/2} \cdot [(1 + A_r^2)]^{-1/2}. \quad (6)$$

The equations (5) and (6) have been derived considering a solar air heat collector in vertical position. If it is inclined making an angle β with the horizontal, these equations in modified form can be expressed as:

$$Q_i = C_D \cdot A_o[2 \cdot (\Delta T/T_o) \cdot g \cdot H \cdot \sin\beta]^{1/2} \cdot [(1 + A_r^2)]^{-1/2} \quad (7)$$

and

$$Q_o = C_D \cdot A_o[2 \cdot (\Delta T/T_i) \cdot g \cdot H \cdot \sin\beta]^{1/2} \cdot [(1 + A_r^2)]^{-1/2}. \quad (8)$$

Equations (7) and (8) give volume flow rate of air at the inlet and outlet respectively in terms of measurable parameters (temperature). Number of air changes can be calculated from these equations and by varying the size of solar air heat collector, ventilation requirements can be satisfied. These two equations are dimensionally homogeneous.

3.2. *Temperature equation*

The energy balance equations for the absorber plate and the flowing fluid (air in present case) of the solar air heat collector can be written and solved for inlet and outlet air temperatures of the whole system [7].

Energy balance for absorber plate:

$$(\alpha \cdot \tau) \cdot \bar{S}(t) = h_f(T_p - \bar{T}_f) + U_t(T_p - T_a) + U_b(T_p - T_R) \quad (9)$$

where α, τ, $\bar{S}_i$, h_f, T_p, $\bar{T}_f$, U_t, T_a, U_b and T_R are absorptance of the collector plate (dimensionless), transmittance of glazing (dimensionless), average solar radiation (W/m^2), heat transfer coefficient (W/m^2/K), absorber plate temperature (°C), average fluid tempeature (°C), top loss coefficients (W/m^2/K), ambient temperature (°C), bottom loss coefficient (W/m^2/K) and room temperature (°C), respectively.

Energy balance for flowing fluid (i.e. air):

$$m \cdot C_p \cdot \frac{dT_f}{dx} x = h_f \cdot w \cdot \Delta x \cdot (T_p - T_f) \quad (10)$$

where m, C_p, Δx and w are mass flow rate (Kg/s), specific heat (J/Kg/K), elemental width, width of collector (m), respectively. Solving equation (10), with the initial condition that

$$\text{at } x = 0: \quad T_f = T_R$$

therefore

$$T_f(x) = T_p - (T_p - T_R) \cdot \exp(-k \cdot x)$$

$$\bar{T}_f = \int_0^L T_f \cdot dx \Big/ \int_0^L dx \qquad (11)$$

or

$$\bar{T}_f = T_p - [(T_p - T_R) \cdot \{1 - \exp(-k \cdot L)\}/(k \cdot L)] \qquad (12)$$

from equations (9) and (12) we get

$$T_p = [(\alpha \cdot \tau) \cdot \bar{S}(t) + U_t \cdot T_a + (U_b + U_f) \cdot T_R] / [U_t + U_b + U_f] \qquad (13)$$

where

$$U_f = h_f \cdot [\{1 - \exp(k \cdot L)\}/(k \cdot L)]$$

from equations (11) and (13) we get

$$T_f(x) = A(t) + B(t) \cdot \exp(-kx). \qquad (14)$$

At

$$x = L; \quad T_f = T_{fo}$$

therefore

$$T_{fo} = A(t) + B(t) \cdot \exp(-k \cdot L) \qquad (15)$$

where

$$A(t) = [(\alpha \cdot \tau) \cdot \bar{S}(t) + U_t \cdot T_a + U_b \cdot T_R]/[U_t + U_b + U_f]$$

and

$$B(t) = -[(\alpha \cdot \tau) \cdot \bar{S}(t) = U_t \cdot (T_R - T_a)]/[(U_t + U_b + U_f)].$$

3.3. *Example calculation*

Using equations (8) and (15), volume flow rate of air is calculated by iterative method and therefrom number of air changes per hour is calculated. These calculations are done by assuming a room size of 4 m (length) × 4 m (width) × 4 m (ceiling height) giving a volume of 64 m^3. Different sizes of solar air heat collectors were tried and finally optimized size of 1.5 m × 1.5 m × 0.15 m is arrived at to give number of air changes per hour between 3 and 6 (given in Table 1) [1]. Other parameters taken in calculations are as given in Table 2. The results of these calculations are presented in a graphical form in Fig. 3a–3d.

Table 1. Air change schedule [1]

S. No.	Space to be ventilated	Air changes per hour
1	* Assembly hall/auditoria	3 6
2	* Bedrooms, living rooms	3 6
3	Classrooms	3 6
4	* Factories (medium metal works)	3 6
5	* Hospital wards	3 6
6	* Kitchen (domestic)	3 6
7	Laboratories	3 6
8	* Offices	3 6

* Smoking

Table 2. Input parameters considered for computation

1.	Absorptance-transmittance of the collector plate and glazing	0.8
2.	Bottom loss coefficient	0.5 (W/m^2 C)
3.	Top loss coefficient	5.0 (W/m^2 C)
4.	Specific heat	1000 (J/kg C)
5.	Heat transfer coefficient	15 (W/m^2 C)
6.	Length of collector	1.5 (m)
7.	Width of collector	1.5 (m)
8.	Acceleration due to gravity	9.8 (m/sec/sec)
9.	Distance between inlet and outlet	1.5 (m)
10.	Coefficient of discharge of inlet and outlet	0.5, 0.6, 0.7 and 0.8
11.	Inclination of collector with horizontal	30

4. RESULTS AND DISCUSSIONS

The required size of the solar collector area of a solar chimney will depend on the required air changes in a building. As per Bureau of Indian Standards [1], the standard of required ventilation rates in different types of a room are given in Table 1. It is therefore evident that the required collector area will depend on the size of the room also. For the sake of illustration, we assume a room size of 4 m × 4 m × 4 m enclosing an air volume of 64 m^3.

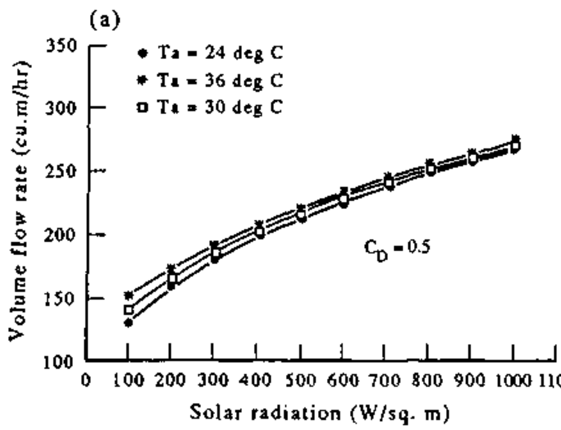

Fig. 3a. Variation of volume flow rate with solar radiation for different ambient temperatures.

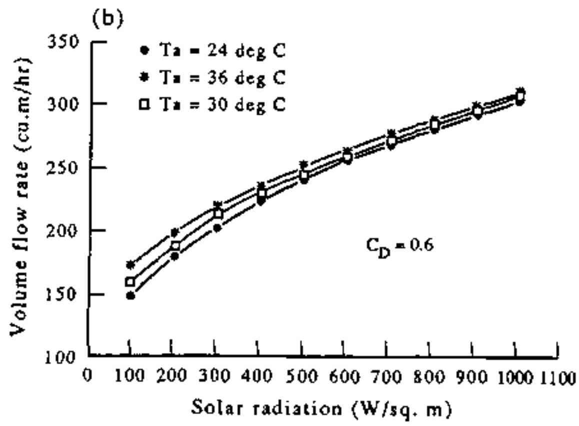

Fig. 3b. Variation of volume flow rate with solar radiation for different ambient temperatures.

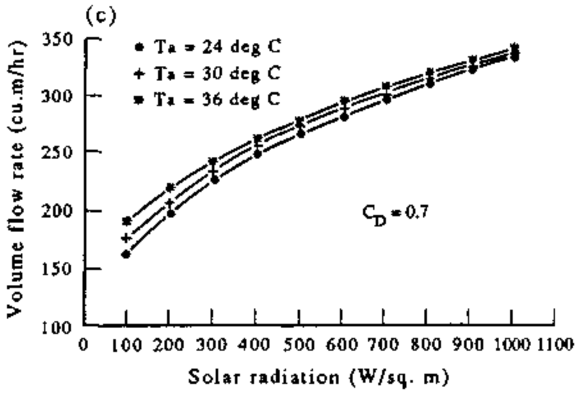

Fig. 3c. Variation of volume flow rate with solar radiation for different ambient temperatures.

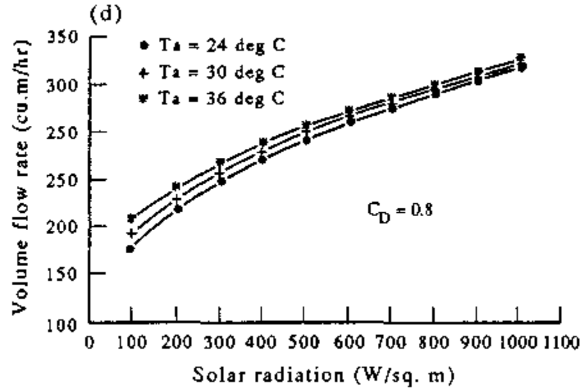

Fig. 3d. Variation of volume flow rate with solar radiation for different ambient temperatures.

Calculations were performed by using equations (8) and (15) and by using an iterative procedure for different values of solar insolation between 200 (W/m^2) and 1000 (W/m^2). It is found that a collector area of 2.25 m^2 and for a duct size of 15 cm (air gap between the absorber plate and upper surface of the rear insulation) leads to air exchange values between 3 and 6 by using various other values of the parameters given in Table 2.

The discharge coefficient (C_D) is a function of the geometry of the solar chimney and the volumetric flow rate. Its value is chosen to be 0.6 [1]. However, the discharge coefficient can take values between 0.5 and 0.8, therefore, a parametric study was performed corresponding to an ambient temperature of 24 to 36 C. The results are illustrated in Figs 3a and 3d which gives the volumetric flow rate and the air changes for different values of the ambient temperature and for an optimized solar collector area of 2.25 m^2.

5. CONCLUSIONS

Considerable air ventilation can be generated by solar induced temperature difference if the system is properly designed. This has become evident by the fact that there is a potential of generating 100–350 (m^3/hr) ventilation rates for a collector area of 2.25 m^2 and for solar radiation values of 100–1000 W/m^2 on the horizontal surface. The values of these induced air flows also depend on the geometry of the air collector, cross-section of the duct and the performance parameters of the air heating solar collector.

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