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Book Review

Burden of Proof: A Review of *Math on Trial*

Reviewed by Paul H. Edelman

Math on Trial
Leila Schneps and Coralie Colmez
Basic Books, 2013
US$26.99, 272 pages
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In *Math on Trial*, Leila Schneps and Coralie Colmez write about the abuse of mathematical arguments in criminal trials and how these flawed arguments "have sent innocent people to prison" (p. ix). Indeed, people "saw their lives ripped apart by simple mathematical errors." The purpose of focusing on these errors, despite mathematics' "relatively rare use in trials" (p. x), is "that many of the common mathematical fallacies that pervade the public sphere are perfectly represented by these trials. Thus they serve as ideal illustrations of these errors and of the drastic consequences that faulty reasoning has on real lives" (p. x). The authors' strategy is to identify common mathematical errors and then illustrate how those errors arose in trials. They seek to accomplish two goals: first, to impress upon the general public the importance of being able to "distinguish whether the numbers brandished in our faces are legitimately providing information or being misused for dangerous ends"; second, "to identify the most important errors that have actually occurred" so that such mistakes can be eliminated in the future.

These are worthy if anodyne goals, and I would not dare argue against them. But the claims that Schneps and Colmez make are strong ones and prompt many questions. Do they adequately support their contention that mathematics has a "disastrous record of causing judicial error?" How influential are mathematical arguments, anyway? Are mathematical arguments more problematic

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than other expert testimony? What role should mathematics play in the judicial system?

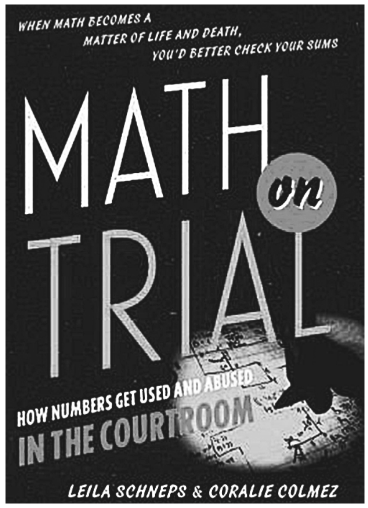
I will get to these questions shortly but, first, a brief description of the book. It consists of ten chapters. Each begins with a short introduction to a particular faulty mathematical argument and then illustrates the error with a discussion of a criminal case in which that argument was advanced. For instance, the first chapter, titled "Math Error Number 1, Multiplying Nonindependent Probabilities", discusses The Case of Sally Clark: Motherhood under Attack. In this case, Ms. Clark, whose first child died in the crib, was charged with the murder of her second child, who also died in her care. The error appears in the guise of testimony by an expert witness that the likelihood of two children dying innocently in her care could be computed by taking the square of the likelihood that a random baby dies innocently while in the care of the family. But that computation relies on the independence of the probabilities, which, if there is some underlying medical issue that caused the death, may well not be the case.

The range of cases presented, both geographically and historically, is in many ways the best feature of the book. Six of the ten cases arise in the United States, while the remainder are from Europe. There are three quite old cases, pre-World War I, and three from the twenty-first century, including one still in litigation. This breadth makes for a very good read, but it also leads to some questions. Do we really think that mathematical errors from the 1860s are as salient as ones from last year? Might advances in knowledge in the intervening one hundred fifty years mediate our concern about such errors? Moreover, continental Europe's legal regime is rather different than that of Britain and the United States. Will those differences have any effect on how mathematics is used? None of these questions is addressed by the book.

There is a lot to like about *Math on Trial*. It is an easy and fun read. The cases, like so many criminal cases, are fascinating in their details. The older cases, in particular, are entertaining, and the mathematical hooks bring a different perspective to the Dreyfus affair (Ch. 10) and the story of Charles Ponzi (Ch. 8). The writing tends toward the breathless, as is common in the true crime genre, but rarely goes over the top. The mathematics is well presented and well integrated into the narrative. Some of the explications are excellent: the discussion of the probabilistic issues in searching DNA databases (Ch. 5) and how Simpson's paradox manifests itself in sex discrimination cases (Ch. 6) are especially noteworthy in this regard.

As entertaining and informative as *Math on Trial* is, have Schneps and Colmez mustered sufficient evidence to justify their claim that mathematics has a "disastrous record of causing judicial error," let alone the claim that "the misuse of mathematics can be deadly?" I think not. To see why, we have to read the presented cases with a more critical eye.

Some of the cases actually do not exhibit any mathematical errors at all. The supposed mathematical issue arising in the case of Charles Ponzi, whose eponymous scheme bilked thousands of people, is that people "got fooled because they did not realize the implications of the incredible rapidity of exponential growth" (p. 149). While that may be a shortcoming of individuals, it is hardly a misuse of mathematics by the legal system. It is also difficult to understand exactly how this led to people being defrauded by Ponzi. After all, investments are all about exponential growth—often in the 5%–10% range but exponential nevertheless.



The chapter about sex discrimination, which describes Simpson's paradox in a very accessible way, actually illustrates a triumph for mathematics in the legal context. The University of California was able to show that what first appeared to be discrimination against women in graduate admissions was, in fact, anything but. Why this chapter is included is a bit of a mystery to me actually, since much of it describes the well-known sex discrimination allegations made by Jenny Harrison against the UC Berkeley mathematics department, a case which never went to trial and in which statistics played little role, if any.

But what of the cases that do, in fact, exhibit material mathematical mistakes? It is one thing for there to be evidence in the record exhibiting a faulty mathematical argument; it is quite another to assume that such an argument was decisive in the outcome of the case. Consider the Dreyfus Affair, which gets quite a nice discussion in the book. Alphonse Bertillon, a handwriting expert, was called to testify on whether the critical memo was written by Dreyfus. He "built up an extraordinary, well-argued theory that Dreyfus had purposely forged an imitation of his own handwriting so that if he were caught, he could attempt to explain away any evidence against him by claiming he had been framed" (p. 196). The authors focus on this theory, which is quite elaborate and quite clearly daft, and conclude, "Bertillon's testimony contributed to Dreyfus' conviction." But they give no reason to believe this, and given the machinations of the French military, it is difficult to imagine that the outcome would have been different if no such "evidence" had been admitted.

Schneps and Colmez make similar jumps throughout. Just because evidence is admitted to the record does not mean that it played a role in the outcome. In the 1865 case of Hetty Green (another fabulously interesting case), who was trying to enforce a contested will, the Harvard mathematician Benjamin Peirce was called in to decide if a certain signature was a forgery. He presented an elaborate, but seriously flawed, model purporting to demonstrate that fact. Perhaps this would be disconcerting but for the fact that the court decided against Ms. Green on purely legal grounds having nothing to do with the signature itself.[1] (I can't help but also note that Ms. Green hardly falls into the category of people whose lives have been ripped apart by a mathematical mistake. She was already worth several million dollars or so in the 1860s.)

Even the most important case in the book, People v. Collins, is subject to this criticism. People v. Collins is the first case in the United States to explicitly consider the role of probability in evidence and is a staple in every evidence course in every law school in the country. The basic facts are the following: On June 18, 1964, Juanita Brooks had her purse snatched while walking home from the grocery store in Los Angeles. She and another witness reported that the assailant was a woman with a blond ponytail who was subsequently picked up by a bearded African American man in a yellow car. A couple meeting that description, Janet and Malcolm Collins, were soon located in the vicinity and were eventually charged and tried.

One thing the district attorney, Ray Sinetar, had going for him was his intuition that there was unlikely to be more than one couple who fit this very uncommon description. In order to push this insight he managed to adduce through the testimony of Daniel Martinez, a professor of mathematics at California State Long Beach, the following table expressing the likelihood of various observations:[2]

	Characteristic	Individual Probability
A.	Partly yellow automobile	1/10
B.	Man with mustache	1/4
C.	Girl with ponytail	1/10
D.	Girl with blond hair	1/3
E.	Negro man with beard	1/10

Sinetar then concluded that the likelihood of such a couple existing is the product of these probabilities and hence about 1 in 12 million. Ultimately the Collinses were convicted, but an appeal was soon filed to challenge, among other things, this probabilistic approach to evidence.

There are, of course, so many things wrong with this argument it is hard to keep count. First of all, the numbers themselves were produced by surveying Sinetar's secretaries (I guess we would call this crowd-sourcing now) and had no factual basis. The probabilities themselves are clearly not independent, so multiplication is obviously inappropriate. And even if both of these problems can be overcome, what exactly it all proves is quite problematic. The argument exhibited what is known as the prosecutor's fallacy: at best he computed the likelihood that a random couple matched the characteristics of the Collinses, not the likelihood that the Collinses were guilty of the crime. Schneps and Colmez do a fine job of explaining the plethora of confusions in the argument.

All that is well and good, and all of it became recognized after the California Supreme Court overturned the verdict. This was just bad mathematics, and it certainly deserves to be excoriated. But did it make a difference in the outcome of the case? One juror is quoted as saying, "I don't remember our discussing the professor much when we deliberated. Maybe we were overwhelmed by the numbers." And a reporter who covered the trial wrote, "Jurors said they disregarded Martinez's testimony,…, and found the couple guilty on evidence given by other witnesses."[3]

Obviously, I cannot go through chapter by chapter, but I think the point is made that many things happen at trial and that to focus on only one aspect of the evidence as the "but-for" cause of the outcome is a mistake. Nevertheless, having bogus mathematical arguments entered into the record is disturbing and the authors ask a legitimate question as to how such arguments can be effectively prevented or countered. They argue that "it is probably going to be necessary to educate the public, from which juries are drawn, to recognize some of the most common mathematical principles that forensic analysis cannot do without" (p. 224). While promoting education is always good, raising the numeracy of the general public is not easy. Fortunately, I do not think it is really necessary in order for the legal system to work adequately.

Mathematical arguments appear in a wide range of legal disputes. They appear in the analysis of race and sex discrimination, anti-trust, stock fraud, and torts, to name a just a few. The vast majority of these applications are not terribly controversial, although any particular model will be subject to criticism and interpretation. *Math on Trial* focuses on the introduction of probabilistic evidence in criminal trials, a very narrow, although important,

[1] *What eventually undid Hetty Green was an interpretation of the parol evidence rule which would not permit her testimony to confirm a contract with her deceased aunt. See* The Howland Will Case, *4 Amer. L. Rev. 625 (1869).*

[2] *Details of how this table was produced are somewhat murky. The reader should read Chapter 2 of the book being reviewed and George Fisher, "The Green Felt Jungle: The Story of People v. Collins", in* Evidence Stories *(Richard Lempert, ed.), Foundation Press, 7 (2006).*

[3] *These quotes are reported in Fisher on page 16.*

area.[4] The argument put forward in People v. Collins is one such example; testimony about the likelihood of a DNA sample coming from a particular individual is another.

Even within this narrow area, the significance of an error in mathematics can easily be overstated. A recent study looked at eighty-six cases in which people were convicted of serious crimes but were later exonerated on the basis of DNA evidence. In 71% of those cases, there were erroneous eyewitness identifications, 63% had forensic science testing errors, 44% had police misconduct, and 28% had prosecutorial misconduct.[5] So there would seem to be many more important problems in the criminal justice system than bad mathematics. Indeed, these more mundane problems arise in most of the cases discussed in *Math on Trial*.

Before we worry about remedying the problem of bad mathematics in criminal trials, we should probably consider what mechanisms are already in place to prevent bad evidence from being introduced. Judges and juries routinely have to cope with evidence of a very technical nature. One cannot hope for them to be adequately educated in all of the areas of knowledge that will be put before them. That is why the legal system provides various procedural safeguards to control what information is put into the legal record. Rules of evidence, standards for the admittance of expert testimony, and other procedural devices all provide means for blocking or refuting bad evidence. It is not a coincidence that one basis for the California Supreme Court overturning the Collinses' conviction, not mentioned in *Math on Trial*, was the inadmissibility of the mathematical testimony, since no empirical support was presented for any of the claims. On reading the cases in *Math on Trial,* one is struck by how ineffectual these devices were, either because of inadequate counsel (another common thread in faulty convictions, as mentioned above) or the failure of the judge to enforce the appropriate rules.

Lawyers have another way to deal with bad testimony, particularly expert testimony—they can provide their own experts to dispute the bad information.[6] In the adversarial system in the United States and Britain we would expect each side to challenge the other by providing their own experts on questions such as the likelihood of some event. This is not a perfect system, since poor defendants may not have the resources to hire such an expert, and that might be the reason there is little evidence of this practice in the cases discussed in *Math on Trial*. Another problem with this approach is that the trial can then become a battle of the experts, the result being that juries (and judges) throw up their hands and ignore the expert testimony altogether. This very reaction was noted in the Hetty Green case, where a contemporaneous account noted that "[T]he result of so much labor of experts, their skill, their ingenuity, their patience, their anxiety, simply demonstrates to the profession their inutility as witnesses in a court of justice."[7]

Should we think that mathematical error is any more prevalent than any other kind of error? One might argue that because they are innumerate, lawyers are worse at coping with mathematical issues than with others. This possibility has been raised before,[8] but I am not sure that I am persuaded by this argument. I know of no studies that indicate that lawyers are less competent at dealing with elementary probability theory than with, say, sophisticated economic modeling questions in anti-trust. And if they are no more prone to error in mathematics than in any other technical area, would it not make more sense to address the issue at the broader level than at the discipline-specific level?

Despite these criticisms, I would agree with the authors that education can certainly play an important role. Indeed, in the case of DNA evidence, it already has. "Immediately after DNA's first courtroom appearance in the 1980s, scientists from disciplines as varied as statistics, psychology, and evolutionary biology debated the strengths and limitations of forensic DNA evidence. Blue-ribbon panels were convened, conferences were held, unscientific practices were identified, data were collected, critical papers were written, and standards were developed and implemented.…Most exaggerated claims and counterclaims about DNA evidence have been replaced by scientifically defensible propositions. Although some disagreement remains, the scientific process worked."[9]

[4] *Three of the ten cases presented are not of this type, but they are also the least persuasive of the chapters: the aforementioned chapters on Ponzi, Hetty Green, and Jenny Harrison.*

[5] *Data reported in Michael J. Saks & Jonathan J. Koehler,* The Coming Paradigm Shift in Forensic Identification Science*, 309 Science 892 (2005). The remainder of issues on the list are false/misleading testimony by forensic scientists (27%), dishonest informants (19%), incompetent defense representation (19%), false testimony by lay witnesses (17%), and false confessions (17%).*

[6] *At least this is true in the United States and Britain. The situation is somewhat different in civil law regimes of continental Europe.*

[7] The Howland Will Case *on page 643.*

[8] *Most recently in Lisa Milot*, Illuminating Innumeracy, *63 Case Western L. Rev. 1 (2013)*

[9] *Saks & Koehler on page 893. I would note here that Colmez and Schneps base two chapters on what they claim to be faulty use of DNA evidence. One is a case involving database trawling and the other is on the methods of DNA testing. While the first case plausibly represents a mathematical error, the latter discussing the Meredith Kercher case (more commonly referred to as the Amanda Knox case in the U.S.) seems to me to be better described as a dispute over testing protocol rather than over mathematics.*

The moral of this tale is that education can be successful, but it is a result of experts working among themselves and coming to a consensus on these highly technical issues. The results are then promulgated through the legal system via these experts. The authors of *Math on Trial* themselves are part of such a project, the "Bayes and the Law" Research Consortium, to develop "a set of criteria and a set of analytic tools that should ensure that probability will henceforth be used correctly" (p. 224). I wish them luck. It takes time (thirty years in the case of DNA) for best practices to be adopted, both because the scientific process is slow and because the legal system is a distributed one and so information disperses slowly through it. Mistakes are going to happen—it is unfortunate but inevitable.

Finally, it is worth thinking about what the role of mathematics in the law should be in a perfect world of sophisticated jurors, judges, and lawyers. There is considerable debate within the legal academy as to whether it is possible to put formal probabilistic foundations under the theory of evidence.[10] And for those who think such a theory can be laid, there are a number of different candidates for how it should be developed.[11] The fact of the matter is that rigorous mathematical thinking is sometimes not in accord with the workings of the judicial system.

The Conjunction Paradox is an example of the kind of problem that arises when trying to establish a probabilistic theory of the burden of proof. In civil actions, such as tort, the plaintiff typically has to establish his case by a preponderance of the evidence, which is usually interpreted to mean that the probability of the offense exceeds 0.5. But sometimes the offense consists of two or more elements. For example, in a common negligence claim, the plaintiff might have to show both that the defendant was negligent and that the plaintiff's injuries resulted from the defendant's actions. Suppose that the plaintiff can establish both claims, the first with probability 0.7 and the second with a probability 0.6. By doing so he has met the burden required to demonstrate his claim, and he should recover his damages. This is the way most courts would analyze the case.

On the other hand, traditional probability would argue that the likelihood of both elements being true is closer to $0.6 \times 0.7 = 0.42$ (assuming the independence of these two events, which seems plausible in this situation), which would not meet the threshold required. Since the conjunction of the events does not exceed the 0.5 probability threshold, the plaintiff should lose. Mathematically this argument seems unexceptionable, but it is not recognized by the legal system.

So where does this leave us with *Math on Trial*? I think it is unconvincing in its claim that the misuse of mathematics in evidence is either particularly significant or novel. It seems much like the other technical expert testimony and is subject to similar costs and benefits. Their proposed solution of educating jurors sounds unpromising to me, but educating lawyers and judges, not so much in the mathematics itself but rather in how to be an educated consumer of the information, is a very reasonable approach. Part of that education is the development of techniques and analyses that gain acceptance within the scientific community. An even more productive response would be to better train the lawyers and judges in law and provide greater access to legal counsel. This would address errors in using evidence across the whole spectrum of disciplines.

Its analysis and prescriptions notwithstanding, *Math on Trial* is an entertaining and informative read for those interested in true crime with a mathematical hook. Perhaps it will impress upon the general public the importance of numeracy and inspire them to look beyond and behind the numbers that are trumpeted around us. If so, that would be all to the good.

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[10] *See, e.g., Ronald J. Allen,* A Reconceptualization of Civil Trials*, 66 B. U. L. Rev. 401 (1986) and Richard Lempert,* The New Evidence Scholarship: Analyzing the Process of Proof*, 66 B. U. L. Rev. 439 (1986).*

[11] *Recent work includes Edward K. Cheng,* Reconceptualizing the Burden of Proof*, 122 Yale L. J. 1254 (2013) and Kevin Clermont,* Death of Paradox: The Killer Logic Beneath the Standards of Proof*, 88 Notre Dame L. Rev. (2012).*