Cryptography Made Easy

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Why Study Cryptography?

- Secrets are intrinsically interesting
- So much real-life drama:
  - Mary Queen of Scots executed for treason
  - primary evidence was an encoded letter
  - they tricked the conspirators with a forgery
- Students enjoy puzzles
- Real world application of mathematics

Start with an Algorithm

- The Spartans used a scytale in the fifth century BC (transposition cipher)
- Card trick
- Caesar cipher (substitution cipher):

ABCDEFGHIJKLMNOPQRSTUVWXYZ

GHIJKLMNOPQRSTUVWXYZABCDEF

Then add a secret key

- Both parties know that the secret word is "victory":

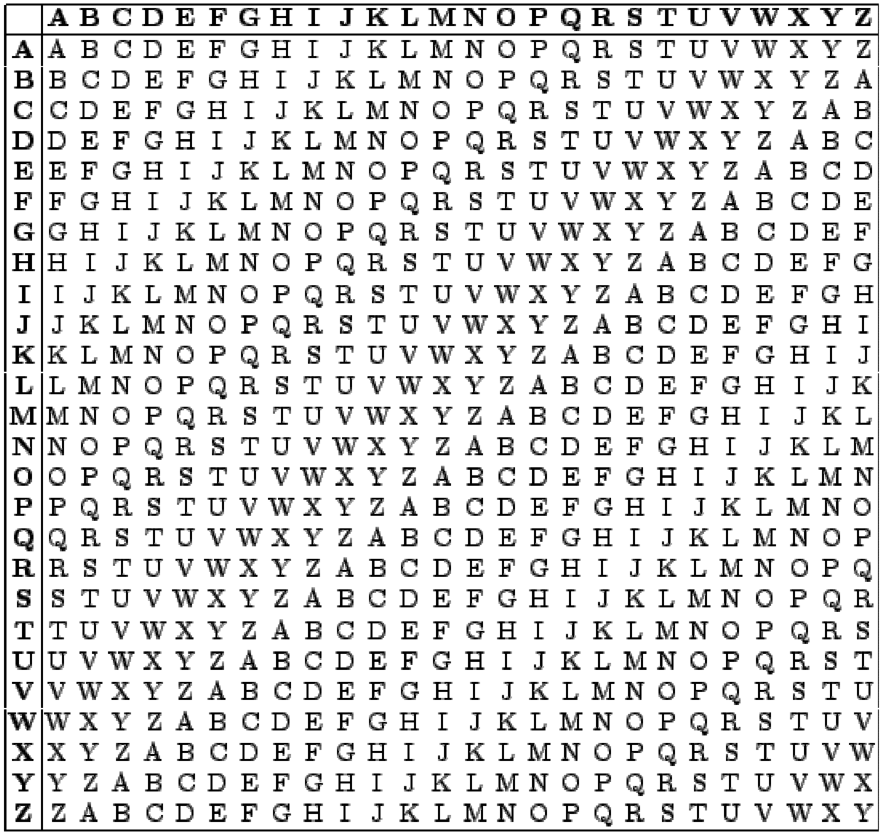
  ```
  ABCDEFGHIJKLMNOPQRSTUVWXYZ
  VICTORYABDEFGHJKLMNPQSUWXZ
  ```

- "state of the art" for hundreds of years
- Gave birth to cryptanalysis first in the Muslim world, later in Europe

Cryptographers vs Cryptanalysts

- A battle that continues today
- Cryptographers try to devise more clever algorithms and keys
- Cryptanalysts search for vulnerabilities
- Early cryptanalysts were linguists:
  - frequency analysis
  - properties of letters

Vigenère Square (polyalphabetic)



	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
A	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
B	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A
C	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B
D	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C
E	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D
F	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E
G	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F
H	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G
I	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H
J	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I
K	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J
L	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K
M	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L
N	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M
O	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N
P	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
Q	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
R	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q
S	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R
T	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S
U	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T
V	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U
W	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V
X	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W
Y	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X
Z	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y

Vigenère Cipher

- More secure than simple substitution
- Confederate cipher disk shown (replica)
- Based on a secret keyword or phrase
- Broken by Charles Babbage

Cipher Machines: Enigma

- Germans thought it was unbreakable
- Highly complex
  - plugboard to swap arbitrary letters
  - multiple scrambler disks
  - reflector for symmetry
- Broken by the British in WW II (Alan Turing)

Public Key Encryption

- Proposed by Diffie, Hellman, Merkle
- First big idea: use a function that cannot be reversed (humpty dumpty)
- Second big idea: use asymmetric keys (sender and receiver use different keys)
- Key benefit: doesn't require the sharing of a secret key

RSA Encryption

- Named for Ron Rivest, Adi Shamir, and Leonard Adleman
- Invented in 1977, still the premier approach
- Based on Fermat's Little Theorem:

  $a^{p-1} \equiv 1 \pmod{p}$ for prime p, gcd(a, p) = 1
- Slight variation:

  $a^{(p-1)(q-1)} \equiv 1 \pmod{pq}$ for distinct primes p and q, gcd(a,pq) = 1
- Requires large primes (100+ digit primes)

Example of RSA

- Pick two primes p and q, compute $n = p \times q$
- Pick two numbers e and d, such that:

  $e \times d = k(p-1)(q-1) + 1$ (for some k)
- Publish n and e (public key), encode with:

  $(\text{original message})^e \bmod n$
- Keep d, p and q secret (private key), decode with:

  $(\text{encoded message})^d \bmod n$

Why does it work?

- Original message is carried to the e power, then to the d power:

  $(msg^{e})^{d} = msg^{e\square d}$

- Remember how we picked e and d:

  $msg^{ed} = msg^{k(p-1)(q-1) + 1}$

- Apply some simple algebra:

  $msg^{ed} = (msg^{(p-1)(q-1)})^{k} \times msg^{1}$

- Applying Fermat's Little Theorem:

  $msg^{ed} = (1)^{k} \times msg^{1} = msg$

Politics of Cryptography

- British actually discovered RSA first but kept it secret
- Phil Zimmerman tried to bring cryptography to the masses with PGP and ended up being investigated as an arms dealer by the FBI and a grand jury
- The NSA hires more mathematicians than any other organization

Exploring further

- Simon Singh, *The Code Book*
- RSA Factoring Challenge (unfortunately the prizes have been withdrawn)
- Shor's algorithm would break RSA if only we had a quantum computer
- Java's BigInteger class has methods for isProbablePrime, nextProbablePrime, modPow

Card Trick Solution

- Given 5 cards, at least 2 will be of the same suit (pigeon hole principle)
- Pick 2 such cards: one will be hidden, the other will be the first card
- First card tells you the suit
- Hide the card that has a rank that is no more than 6 higher than the other (using modular wrap-around of king to ace)
- Arrange other cards to encode 1 through 6

Encoding 1 through 6

- Figure out the low, middle, and high cards
  - rank (ace < 2 < 3 ... < 10 < jack < queen < king)
  - if ranks are the same, use the name of the suit (clubs < diamonds < hearts < spades)
- Some rule for the 6 arrangements, as in:

1: low/mid/hi

2: low/hi/mid

3: mid/low/hi

4: mid/hi/low

5: hi/low/mid

6: hi/mid/low