

# The GEOSAT Orbit Adjust

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## Abstract

GEOSAT is a U.S. Navy gravity gradient stabilized spacecraft which aids geodetic and oceanographic researchers by mapping the Earth's shape with a radar altimeter. The transition from the geodetic phase of the mission to the oceanographic phase required modifications of the spacecraft's orbit. The orbital change, implemented as a series of 239 maneuvers comprising over 170,000 seconds of thrusting, was performed during a 37 day period starting 1 October 1986. The subsequent Exact Repeat Mission (ERM) has, to date, maintained a repeat ground track ( $\pm 1$  km) for over a year with  $\sigma = 490$  meters.

## The GEOSAT Mission

GEOSAT was launched from the Western Test Range on March 12, 1985 [1]. The first eighteen months of the mission were dedicated to obtaining a high resolution marine geoid, a dense global grid of geodetic data which could be used to improve models of the Earth's gravitational field [2]. A single instrument, the radar altimeter (RA), was employed for this purpose. The RA is essentially a short pulse radar which measures the distance between the satellite orbit and the subsatellite point on the ocean surface to a precision of a few centimeters. The precise shape of the ocean's surface along the satellite ground track can be determined by combining RA measurements with tracking data. The geodetic mission was successfully completed on October 1, 1986.

GEOSAT's secondary, or "exact repeat," mission (ERM) is the detection of time-dependent mesoscale oceanographic features. These are medium scale (50-300 km) features, such as rings, eddies, and currents, that cause the ocean surface to depart from the geoid. The time variation of these oceanographic features is obtained by performing radar altimetry from a repeat ground track orbit. Transitioning to the ERM required modifying the spacecraft orbit.

GEOSAT is a gravity gradient stabilized spacecraft. The Attitude Control System was designed to point the RA to within one degree of nadir 98% of the time. The principal components [3] are: a twenty foot scissors boom with a 100 pound end

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mass; redundant momentum wheels for roll and yaw stiffness; pitch and roll attitude control thrusters; and a passive, magnetically anchored, libration damper. Attitude sensing is provided by three digital sun sensors and a three axis vector magnetometer. All attitude determination is performed on the ground in real-time or near real-time. On-board attitude control is entirely open-loop. The momentum wheels are used only to provide stiffness and not to perform attitude maneuvers. Spacecraft parameters are shown in table 1.

A Velocity Control Subsystem (VCS) provides attitude and orbit control. The VCS is a cold gas system utilizing Freon-14 with a specific impulse of 40 seconds. The gas was initially pressurized to 2700 psi, which is reduced by pressure regulators to 15 psi at each of six 0.01 lb-f thrusters. One of the thrusters is forward and one aft pointing, thereby providing prograde and retrograde velocity control. The remaining four thrusters are positioned to produce roll and pitch torques. Thrusting is commanded directly in real-time or by loading delayed command sequences to the spacecraft command subsystem. Prior to the orbit adjust, the velocity control thrusters had not been used on orbit. The attitude thrusters had been used only for initial attitude capture subsequent to launch.

An on-board delayed command subsystem allows sequences of commands with associated time tags to be stored for execution at a later, pre-specified time. The resolution of the command system is approximately eight seconds. GEOSAT contains two independent command systems, each of which can be loaded with up to five sequences of 30 commands. Both command systems can be operating simultaneously, but only one sequence in each system can be active at a time. During normal operations, only one command system is used. This provides for autonomous control of spacecraft housekeeping, which includes transmitter and tape recorder control commands. Approximately thirty commands per day are normally executed via delayed commands. Seven commands are required for an orbit adjust maneuver.

TABLE 1. Typical GEOSAT Orbital Parameters

Parameter	Geodetic Mission	Oceanographic Mission
Epoch, Day of 1986	82/17:42:33	312/10:50:55
$a$ , Earth radii	1.1236003	1.1233336
$e$	0.0041672	0.0007971
$i$ , degrees	108.04696	108.04397
$\omega$ , degrees	34.040904	91.493632
$\Omega$ , degrees	348.84433	49.788597
$\Omega'$ , degrees/day	2.0501875	2.0517209
$P_{nodal}$ , seconds	6039.6993	6037.5601
$M$ , degrees	0.0	0.0
Mass, pounds	1408	1363
Fuel, pounds	84	39
Prograde Thrust, lbf	0.0126	0.0126
Retrograde Thrust, lbf	0.0136	0.0136
Inertia, $I_{xx}$ , kg-m	2785	2785
Inertia, $I_{yy}$ , kg-m	2783	2783
Inertia, $I_{zz}$ , kg-m	255	255
Pitch Libration Period, sec	3656	3656

All communication with the spacecraft is performed via the Satellite Tracking Facility (STF) at APL. There are two clusters of spacecraft contacts per day, each composed of two or three passes. Orbit determination and tracking is provided by the Naval Astronautics Group (NAVASTROGRU).

The control center utilizes a Gould SEL 32/77 minicomputer and four HP-9836 microcomputers for real-time operations. A second SEL and several IBM-PC's are used for engineering analysis. Orbit and attitude determination is performed on an IBM mainframe in the APL Computing Center. Software tools on the mainframe and mini-computers include:

1. GRASS (GEOSAT Real Time Attitude Support System)—processes sensor data to determine spacecraft attitude to within  $\pm 0.5^\circ$  accuracy.
2. GEOSIM (GEOSAT attitude SIMulator)—extrapolates an attitude state vector for one to two orbits into the future.
3. ODP (Orbit Determination Program)—a generalized orbit integrator incorporating both analytic and numerical modes.
4. GOATS (GEOSAT Orbit Adjust and Transfer Software)—determines an optimum thrust pattern for orbit adjusts.
5. GMOSS (GEOSAT Mission Operations and Scheduling Software)—performs all spacecraft communications, command and control activities.

To support the orbit adjust, additional engineering analysis tools were implemented on the PC's for purposes such as eclipse prediction; determination of the remaining on-board fuel supply from pressure and temperature measurements; ground track predictions; atmospheric density and drag predictions; orbit adjust scheduling; orbit integration; pass prediction; and attitude simulation. Some of these PC based tools duplicated the functions of the mainframe tools for purposes of validation, CPU time cost reduction, or expanded functionality. In addition, we found that a good general purpose spreadsheet/graphics package, such as SYMPHONY, was one of the most useful support tools for spacecraft engineering analysis.

### Orbit Requirements

The GEOSAT oceanographic mission, in part, continues the geodetic data collection which began with SEASAT. In order to duplicate the conditions under which the SEASAT data was taken, while providing useful data for mesoscale oceanography, the GEOSAT orbit is required to [4] have inclination within 0.0017 radians of that of SEASAT, have node crossings within 10.8 km of those of SEASAT, provide a repeat ground track, maintain ground tracks within a  $\pm 1.0$  km window, and minimize eccentricity.

A candidate orbit [4] was adopted which met these requirements and which could be reached within the on-board fuel budget. This orbit has mean inclination  $i = 108.044^\circ$ , eccentricity  $e = 0.000805$ , and argument of perigee  $\omega = 90$  degrees. The repeat ground track is every 244 orbits, and the Earth-fixed reference nodes are at longitudes  $1.004^\circ + n 1.4754^\circ$  East,  $n = 0, 1, \dots, 243$ . Sets of typical GEOSAT orbital elements at the end of the geodetic mission and the beginning of the oceanographic mission are shown in Table 1.

## Mission Planning Constraints

The primary constraints upon the design of the orbit adjust and subsequent ERM involved mission feasibility, efficiency, attitude stability, time to completion, impact upon data acquisition, and command verification. These are summarized in the following paragraphs.

*Mission Feasibility* — The ERM orbit had to be accessible and maintainable for several years.

*Data acquisition* and spacecraft operations could not be compromised by the maneuver. It was decided that a data loss of not greater than sixty days was acceptable for ERM implementation. No data loss is acceptable during ERM maintenance.

*Attitude stability* — the center-of-mass to center-of-thrust alignment was only known to within 3 centimeters. A large misalignment would force maneuver design to be governed by spacecraft attitude stability rather than fuel efficiency and speed.

*Command Scheduling* — all thruster commands were to be executed via on-board delayed command sequences. These sequences had to be loaded into the two command systems so that the spacecraft would remain in a safe mode should either command system fail.

*Command Uplink* — all loads had to be uplinked via the two daily clusters of passes available to STF.

*Thrust Uncertainty* — The actual thrust level was only measured to one significant figure prior to launch.

*Antenna Pointing* — The along-track pointing error subsequent to a single 1000 second burn would accumulate to the half cone beamwidth of STF's primary antenna ( $0.25^\circ$ ) after 2.7 orbits. Thruster uncertainty and the inaccuracies inherent in modeling all of the orbital perturbations would therefore make it necessary to obtain updated orbital elements frequently.

*Tracking Constraints* — NAVASTROGRU requires approximately 48 hours of tracking data to determine the orbit, during which no thrusting can take place. The 60 day time limit precluded scheduling a tracking period after each of 200 17-minute burns.

## Determination of Burn Duration

Meirovich [5] (equation (12.8)) gives the change of the orbital element due to an impulsive velocity change. The perturbations in the elements are expressed in terms of the components of the velocity change,  $\Delta v$ , and the spacecraft's true anomaly,  $\theta$ . For short duration maneuvers in low eccentricity orbits, Meirovich's orbit adjust equations can be expanded in terms of the eccentricity  $e$ , as shown in Table 2.

When thrusting occurs during a significant fraction of the orbit, the variation in true anomaly  $\theta$  cannot be ignored. If the mass of fuel expelled over a single burn is much smaller than the spacecraft mass and the thruster efficiency is relatively constant, the maneuver can be approximated as providing a constant thrust. In this case

TABLE 2. Effect of Impulsive Velocity Change  $\Delta v$  at True Anomaly  $\theta$ 

Effect of an impulsive velocity change  $\Delta v$  at true anomaly  $\theta$  upon each of the orbital elements, to lowest order in the eccentricity. A constant thrust maneuver extending from  $\theta - \Delta\theta$  to  $\theta + \Delta\theta$  reduces the efficiency by an additional factor of  $(\sin \Delta\theta)/\Delta\theta$ .

element	radial	Along Track	Out of Plane
$\Delta a$	$2ea(\Delta v/v) \sin \theta$	$2a\Delta v/v$	0
$\Delta p$	$3ep(\Delta v/v) \sin \theta$	$3p\Delta v/v$	0
$\Delta e$	$(\Delta v/v) \sin \theta$	$2(\Delta v/v) \cos \theta$	0
$\Delta \Omega$	0	0	$[\sin(\omega + \theta)](\Delta v/v)/\sin i$
$\Delta \omega$	$-(\cos \theta)(\Delta v/v)/e$	$2(\sin \theta)(\Delta v/v)/e$	$-(\Delta v/v)(\cot i) \sin(\omega + \theta)$
$\Delta i$	0	0	$(\Delta v/v) \cos(\omega + \theta)$

the impulsive equations of Table 2 may be averaged over the thrust interval  $\theta - \Delta\theta$  to  $\theta + \Delta\theta$ . The net effect is to reduce the efficiency of each burn by a factor of  $(\sin \Delta\theta)/\Delta\theta$ .

Optimal methods for changing each of the orbital elements with an impulsive thrust are discussed below.

*Semimajor Axis,  $a$ ,* can be changed most efficiently by a thrust along-track. The location of the burn does not affect the magnitude of  $\Delta a$ . The axis can be changed independently of the other elements by performing two burns of thrust level  $\Delta v/2$  at true anomaly  $\theta$  and  $\theta \pm 180^\circ$ , where  $\theta$  is any arbitrarily selected true anomaly.

*Eccentricity,  $e$ ,* can be most efficiently modified by thrusting along track. A pair of  $\pm \Delta v/2$  burns at  $\theta = 0$  and  $\theta = 180^\circ$  will change  $e$  independently of all other elements.

*Argument of Perigee,  $\omega$ ,* is also most efficiently modified by burning along-track. A pair of  $\pm \Delta v/2$  burns at  $\theta = \pm 90^\circ$  will change  $\omega$  independently of the other elements.

*Right Ascension of Ascending Node,  $\Omega$ ,* and *inclination,  $i$ ,* can only be changed by an out of plane maneuver. The optimum change in  $\Omega$  will occur at the poles, i.e., when the argument of latitude  $u = 90^\circ$  or  $u = 270^\circ$ , while for  $i$ , it occurs at the nodes,  $u = 0^\circ$  or  $u = 180^\circ$ . Maneuvers to change  $i$  or  $\Omega$  are not possible on GEOSAT due to thruster orientation.

*Perigee and Eccentricity* can both be modified simultaneously without changing the semi-major axis by performing a pair of translational burns of equal magnitude and opposite direction. A simple geometrical procedure is described in [6,7] to determine the optimum placement of the thrust. This procedure defines orbital state vectors in polar coordinates  $(e, \omega)$  where  $e$  is the magnitude and  $\omega$  the polar angle of the vector. The angle between the  $x$  axis and the difference between the desired and actual state vectors gives the argument of latitude of the burn. In the case where a sequence of maneuvers is required, this angle must be recalculated after each burn. This method is physically justified by the fact that the spacecraft is moving faster at

perigee. Consequently, a translational prograde thrust will always pull perigee towards the point at which the thrust is applied. Each burn must be applied at true anomaly [6]

$$\theta = \{\tan^{-1}[(e \sin \omega - e_0)/(e \cos \omega)]\} - \omega \quad (1)$$

and provide a thrust

$$\Delta v/v = (1/4) [e^2 + e_0^2 - 2ee_0 \sin \omega]^{1/2} \quad (2)$$

To avoid affecting any other elements, burns of equal magnitude and opposite direction should occur at  $\theta$  and  $\theta + 180^\circ$ .

*Maintaining the Frozen Orbit*—A frozen orbit is one in which both the eccentricity and argument of perigee are "fixed" at some values  $e_0$  and  $\omega_0$ . Small perturbations of  $e$  or  $\omega$  away from  $e_0$  or  $\omega_0$  will result in bounded oscillatory motion in the  $(e, \omega)$  plane. A physical explanation of the frozen orbit is given by [6] for SEASAT and [8] for GEOSAT. It has also been used successfully on LANDSAT [9]. In this configuration, the higher order perturbations on  $\omega$  approximately cancel out the secular variation for certain combinations of the elements, while the lowest order perturbation in  $e$ , which depends upon  $\cos \omega$ , goes to zero at  $\omega = 90^\circ$ . For GEOSAT numerical calculations by Heyler [10] and Born [8] indicate a frozen orbit at  $e = 0.000805$  for an inclination of  $i = 108.044^\circ$ .

### Attitude Simulations

The relative alignment of the velocity control thrusters with respect to the spacecraft center of mass was not known with any confidence. Measurements made during integration and testing indicated that the offset was no more than 0.75 cm in roll and yaw and no more than 3.5 centimeters in pitch. Firing a misaligned thruster would produce undesired rotational torques with the offset acting as the moment arm. Due to the magnitude of thrusting required this could pose considerable difficulties. We calculated that at least 216 constant thrust 1000 second burns would be necessary to reach the ERM orbit. Repeatedly burning twice per orbit would undoubtedly perturb the attitude. There should be a steady state characteristic to the attitude, but numerous concerns arose:

1. What was the relationship between burn duration and maximum libration angle?
2. How was efficiency reduced by the libration?
3. What was the maximum safe burn duration?
4. What was the steady state behavior of the attitude?
5. Would we ever reach steady state?
6. What was the effect of the tracking periods upon the attitude?
7. Would attitude damping maneuvers be necessary?
8. What was the relationship between the thrusting frequency and the spacecraft resonance frequency?

GEOSIM was designed to perform short term attitude simulations, on the order of several orbits, but could not simulate the steady state behavior. In order to aid our analyses of these concerns, Mark Holdridge developed the GEOSAT attitude simula-

tion program (GSIM). This package, implemented in FORTRAN on an IBM PC, simulates three-axis attitude dynamics by integrating Euler's equations of motion for the spacecraft.

GSIM solves these equations with a Runge-Kutta integrator in six variables (attitude angles and attitude rates). The equations for a gravity gradient stabilized spacecraft with a spinning momentum wheel are given by [11]. The eddy-current damper is modeled with a simple harmonic torque proportional to the difference ( $\omega_2 - 2\omega_0$ ), where  $\omega_2$  and  $\omega_0$  are the pitch rate and spacecraft orbital rate, respectively. Geomagnetic and solar radiation follow the models given by [12]. Thruster-induced torques are integrated throughout a maneuver.

Figure 1 shows the steady-state pitch libration predicted by GSIM, assuming 100% efficiency and a center-of-mass/center-of-thrust offset of 2 cm. The steady-state thrust pattern used for this simulation, composed of alternating prograde and retrograde thrusts of 1000 seconds duration each applied one half an orbit apart, is superimposed. Similar simulations were run to explore many of the parametric variations to the orbit adjust scenario. As seen from Fig. 1, under certain conditions, we were able to determine that the spacecraft attitude would, in fact, exhibit a steady state behavior. Table 3 shows the maximum pitch libration amplitude for various values of burn duration and thruster offset. The characteristic shape of the curve turned out to be similar for each of these cases.

### Maneuver Implementation

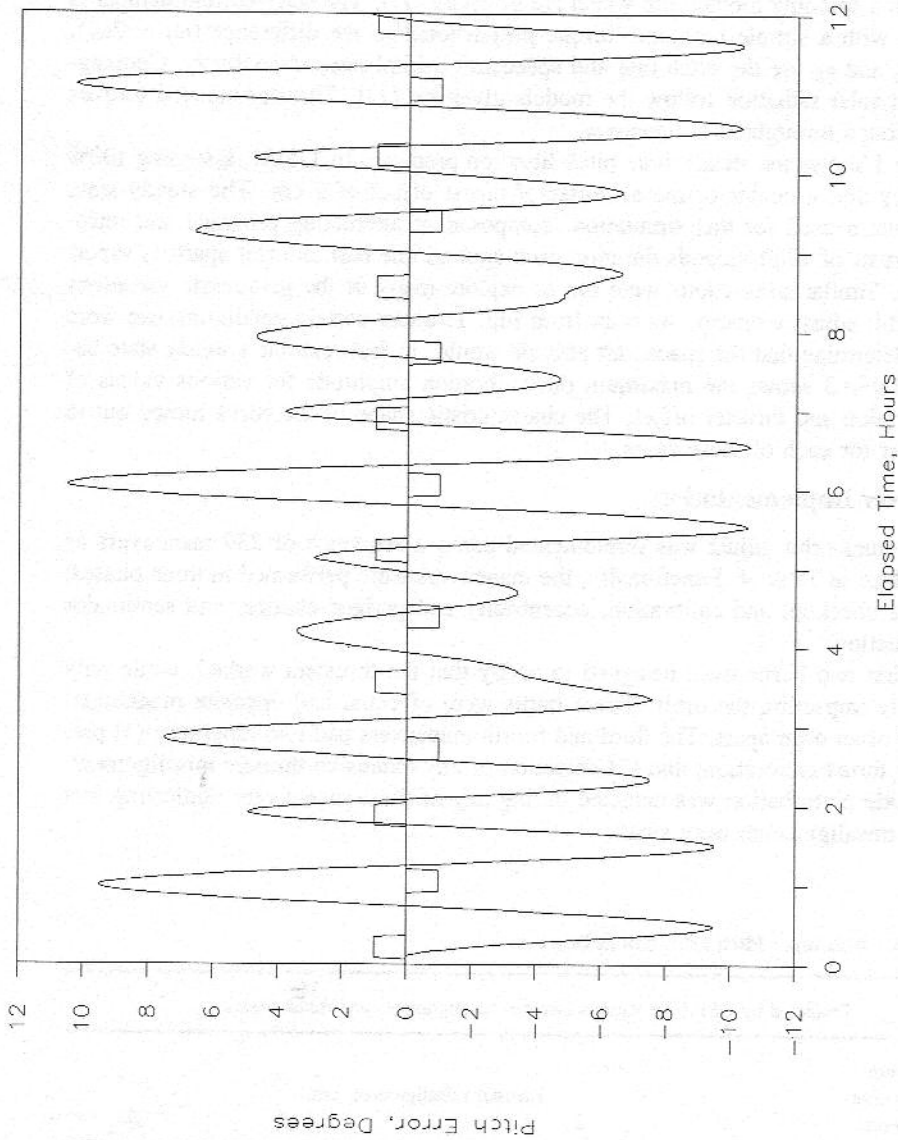
The actual orbit adjust was implemented using a sequence of 239 maneuvers as summarized in Table 4. Functionally, the maneuvers were performed in three phases: hardware checkout and calibration, eccentricity and perigee change, and semimajor axis reduction.

The first two burns were designed to verify that the thrusters worked, while only minimally impacting the orbit. These burns were of equal and opposite magnitude, one half of an orbit apart. The third and fourth maneuvers had two functions: (1) preliminary thrust calibration; and (2) detection of any extensive thruster misalignment. No attitude perturbation was detected during any of these maneuvers, indicating that thruster misalignments were small.

TABLE 3. Maximum Pitch Libration in Degrees

Burn Duration minutes	Thruster misalignment, cm.				
	1	2	3	4	5
3	1.5	3.0	4.0	5.5	6.5
5	2.0	4.0	6.0	8.0	10.0
10	4.0	8.0	11.5	16.0	20.0
17	6.0	12.0	18.0	26.0	35.0

Predicted by GSIM for various thruster misalignments and thrust durations.



**FIG. 1.** Pitch libration amplitude predicted by GSIM assuming a 2 cm thruster offset and 100% thruster efficiency. The thrust pattern, consisting of alternating prograde and retrograde thrusts of 1000 seconds duration spaced one half an orbit apart, is superimposed.



TABLE 4. Summary of the GEOSAT Orbit Adjust

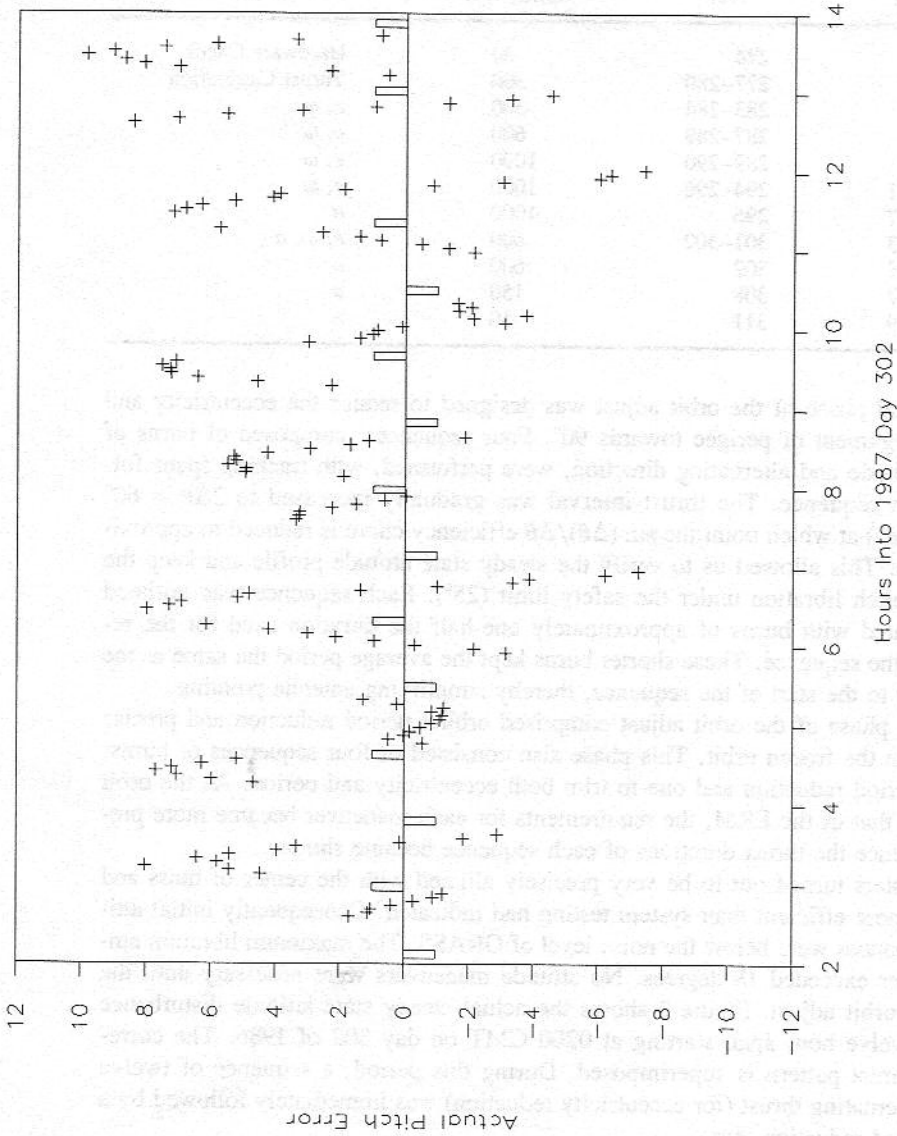
The aggregate thrust duration was 172474 seconds.			
Burn Number	Day of 1986	Typical Burn, secs	Purpose
1-2	274	30	Hardware Check
3-4	277-280	300	Thrust Calibration
5-34	283-284	300	$e, \omega$
35-91	287-289	600	$e, \omega$
92-122	289-290	1000	$e, \omega$
123-191	294-296	1000	$e, \omega$
192-197	296	1000	$a$
198-233	301-302	600	$e, \omega, a$
234-235	302	600	$a$
235-237	308	150	$a$
238-239	311	50	$a$

The second phase of the orbit adjust was designed to reduce the eccentricity and move the argument of perigee towards  $90^\circ$ . Four sequences, composed of burns of equal magnitude and alternating direction, were performed, with tracking spans following each sequence. The thrust interval was gradually increased to  $2\Delta\theta = 60^\circ$  (1000 seconds) at which point the  $\sin(\Delta\theta)/\Delta\theta$  efficiency curve is reduced to approximately 95%. This allowed us to verify the steady state attitude profile and keep the maximum pitch libration under the safety limit ( $25^\circ$ ). Each sequence was initiated and terminated with burns of approximately one-half the duration used for the remainder of the sequence. These shorter burns kept the average period the same as the period prior to the start of the sequence, thereby simplifying antenna pointing.

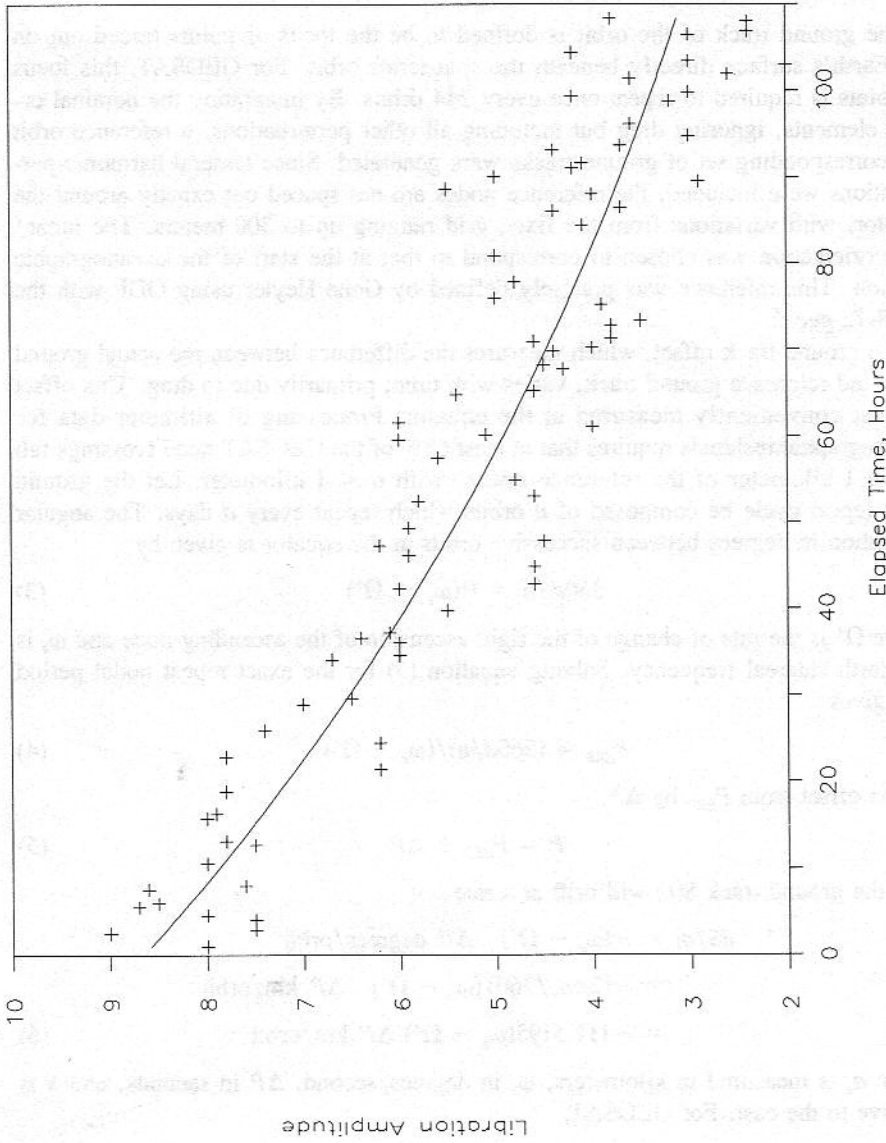
The final phase of the orbit adjust comprised orbital period reduction and precise placement in the frozen orbit. This phase also consisted of four sequences of burns, three for period reduction and one to trim both eccentricity and period. As the orbit approached that of the ERM, the requirements for each maneuver became more precise, and hence the thrust durations of each sequence became shorter.

The thrusters turned out to be very precisely aligned with the center of mass and over 25% more efficient than system testing had indicated. Consequently initial attitude disturbances were below the noise level of GRASS. The maximum libration amplitude never exceeded 18 degrees. No attitude maneuvers were necessary until the end of the orbit adjust. Figure 2 shows the actual steady state attitude disturbance during a twelve hour span starting at 0200 GMT on day 302 of 1986. The corresponding thrust pattern is superimposed. During this period, a sequence of twelve burns of alternating thrust (for eccentricity reduction) was immediately followed by a pair of period reduction burns.

At the completion of the orbit adjust, a single attitude damping maneuver was performed, after which the attitude was allowed to damp itself out naturally from approximately nine degrees. The maximum pitch libration amplitude for each cycle is plotted in Fig. 3. The least squares fit exponential has a natural decay time of 105.9 hours.



**FIG. 2.** Actual GEOSAT pitch libration for a 12 hour period on October 29, 1986. The thrust pattern is superimposed upon the plot. During this period, both eccentricity reduction (the first twelve burns) and period reduction (the last two burns) were performed. In the thrust pattern, retrograde thrusts are indicated by a positive value, and prograde thrusts by a negative value. All of the burns during this period were approximately 600 seconds long.



**FIG. 3.** Natural attitude damping after the orbit adjust. The maximum libration amplitude for each orbit is plotted. The least squares fit exponential has a decay constant of 105.9 hours.

Operationally, a total of 1673 special delayed commands were uplinked and executed to perform 239 maneuvers in 37 days. The entire maneuver required 172,474 seconds of thrust, with an overall thruster efficiency of better than 125%. Approximately 45 pounds of fuel was used.

### Orbit Maintenance During the ERM

The ground track of the orbit is defined to be the locus of points traced out on the Earth's surface directly beneath the spacecraft orbit. For GEOSAT, this locus of points is required to repeat once every 244 orbits. By integrating the nominal orbital elements, ignoring drag but including all other perturbations, a reference orbit and corresponding set of ground tracks were generated. Since tesseral harmonic perturbations were included, the reference nodes are not spaced out exactly around the equator, with variations from the fixed grid ranging up to 300 meters. The lunar/solar orientation was chosen to correspond to that at the start of the oceanographic mission. This reference was precisely defined by Gene Heyler using ODP with the WGS-72 geoid.

The ground track offset, which measures the difference between the actual ground track and reference ground track, varies with time, primarily due to drag. This offset is most conveniently measured at the equator. Processing of altimeter data for oceanographic residuals requires that at least 68% of the GEOSAT node crossings fall within 1 kilometer of the reference nodes, with  $\sigma < 1$  kilometer. Let the ground track repeat cycle be composed of  $n$  orbits which repeat every  $d$  days. The angular separation in degrees between successive orbits at the equator is given by

$$360d/n = P(\omega_e - \Omega') \quad (3)$$

where  $\Omega'$  is the rate of change of the right ascension of the ascending node and  $\omega_e$  is the Earth sidereal frequency. Solving equation (3) for the exact repeat nodal period  $P_{ERP}$  gives

$$P_{ERP} = (360d/n)/(\omega_e - \Omega') \quad (4)$$

If  $P$  is offset from  $P_{ERP}$  by  $\Delta P$ ,

$$P = P_{ERP} + \Delta P \quad (5)$$

then the ground track  $S(t)$  will drift at a rate

$$\begin{aligned} dS/dt &= -(\omega_e - \Omega') \Delta P \text{ degrees/orbit} \\ &= -(2\pi a_e/360)(\omega_e - \Omega') \Delta P \text{ km/orbit} \\ &= -111.3195(\omega_e - \Omega') \Delta P \text{ km/orbit} \end{aligned} \quad (6)$$

where  $a_e$  is measured in kilometers,  $\omega_e$  in degrees/second,  $\Delta P$  in seconds, and  $S$  is positive to the east. For GEOSAT,

$$ds/dt = -0.46246\Delta P \text{ km/orbit} = -39956(\Delta P/P) \text{ km/day} \quad (7)$$

Ideally,  $ds/dt = 0$  and  $P = P_{ERP}$ . The ground track offset will drift westward when  $P > P_{ERP}$  (since  $dS/dt < 0$ ), and eastward when  $P < P_{ERP}$  ( $dS/dt > 0$ ). Suppose the ground track must be maintained within a swath of width  $D$ . By setting  $P = P_{ERP} +$

$\Delta P$  (where  $\Delta P > 0$ ) when  $S = +D/2$ , the offset will drift westward. The westward drift will continue, but at a decreasing rate (since drag causes  $P$  to decrease), until  $P = P_{ERP}$ . Ideally, this will occur when  $S = -D/2$ . As the period continues to decrease,  $dS/dt$  becomes positive (since  $P < P_{ERP}$ ), and hence the ground track offset moves to the east. An orbit adjust is required when  $S = +D/2$ .

A constant drag,  $k = -dP/dt$ , gives a linear period change,

$$P(t) = P_0 - kt \quad (8)$$

Letting  $A$  be the constant in equation (7), and integrating with boundary condition  $S = S_0$  at  $t = 0$ ,

$$\begin{aligned} S(t) &= S_0 - At - (AP_{ERP}/k) \ln(1 - kt/P_0) \\ &= S_0 - At(1 - P_{ERP}/P_0) + [AkP_{ERP}t^2/(2P_0^2)]\text{km} \end{aligned} \quad (9)$$

to order  $|kt/P_0|^2$ . The period which will maximize the time between burns is

$$P_0 = [2kDP_{ERP}/A]^{1/2} + P_{ERP} \quad (10)$$

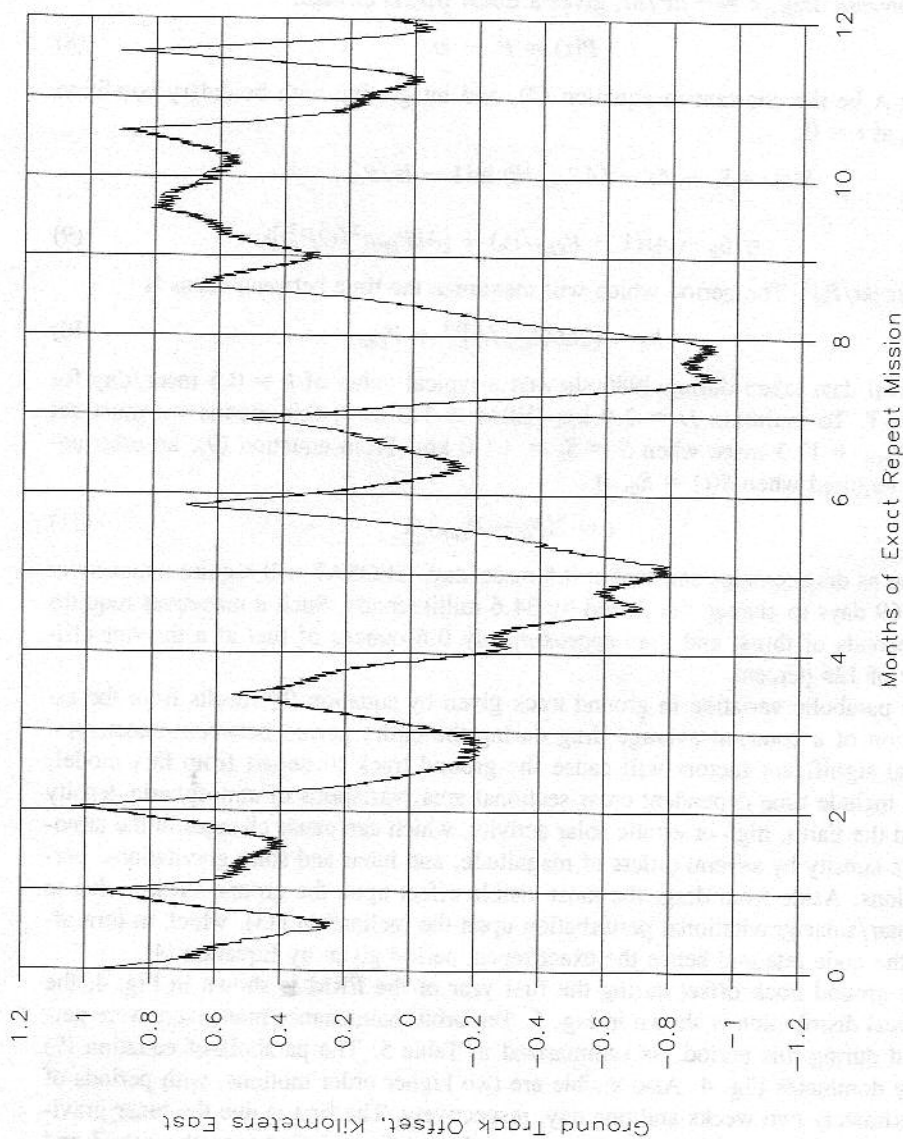
Empirical data taken during 1986 showed a typical value of  $k \approx 0.5$  msec/day for GEOSAT. To maintain  $D = 2.0$  km ( $ERM \pm 1.0$  km), this means we must set  $P_0 = P_{ERM} + 17.3$  msec when  $S = S_0 = +1.0$  km. From equation (9), an orbit adjust is required when  $S(t) = S_0$ , at

$$t = 2(P_0 - P_{ERP})/k \quad (11)$$

So long as drag remains constant at 0.5 msec/day, GEOSAT will require a maneuver every 69 days to change the period by 34.6 milliseconds. Such a maneuver requires 163 seconds of thrust and uses approximately 0.6 ounces of fuel at a thruster efficiency of 125 percent.

The parabolic variation in ground track given by equation (9) results from the assumption of a constant average drag during the entire period between maneuvers. Several significant factors will cause the ground track to depart from this model. These include time dependent cross sectional area, variations of atmospheric density around the Earth, high or erratic solar activity, which can cause changes in the atmospheric density by several orders of magnitude, and lunar and solar gravitational perturbations. Aside from drag, the most visible effect upon the ground track is due to the lunar/solar gravitational perturbation upon the inclination [13], which in turn affects the node rate and hence the exact repeat period given by Equation (4).

The ground track offset during the first year of the ERM is shown in Fig. 4; the statistical distribution is shown in Fig. 5. Ten orbit maintenance maneuvers were performed during this period, as summarized in Table 5. The parabola of equation (9) clearly dominates Fig. 4. Also visible are two higher order motions, with periods of approximately two weeks and one day, respectively. The first is due the lunar gravitational perturbation. The second is due to the relationship between the actual and reference lunar/solar orientation and the gravitational harmonics not modelled by WGS-72. The solar perturbation, though of comparable magnitude to the lunar perturbation, is not clearly visible since its period exceeds that of the typical span between orbit adjusts.



**FIG. 4.** Ground track offset for the first year of the ERM. East is positive, and west negative. Ten orbit adjusters were performed, as summarized in Table 5.

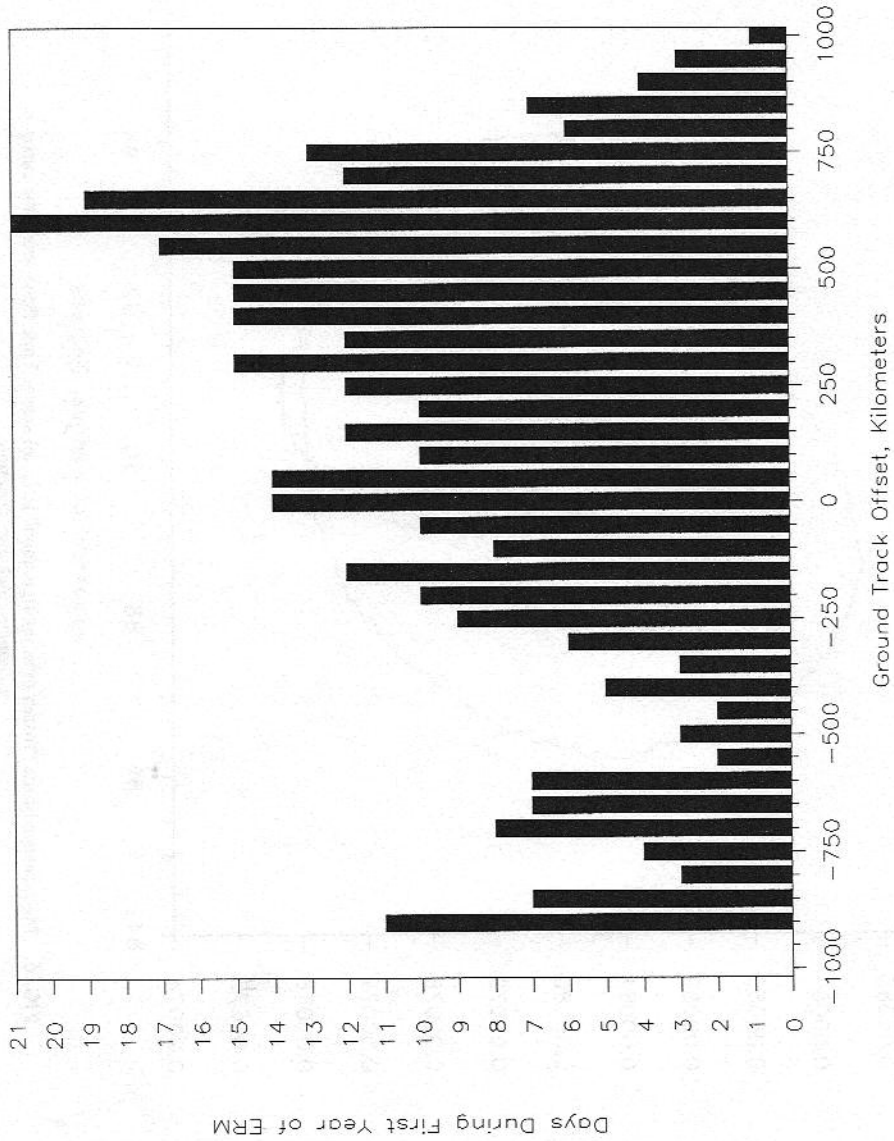
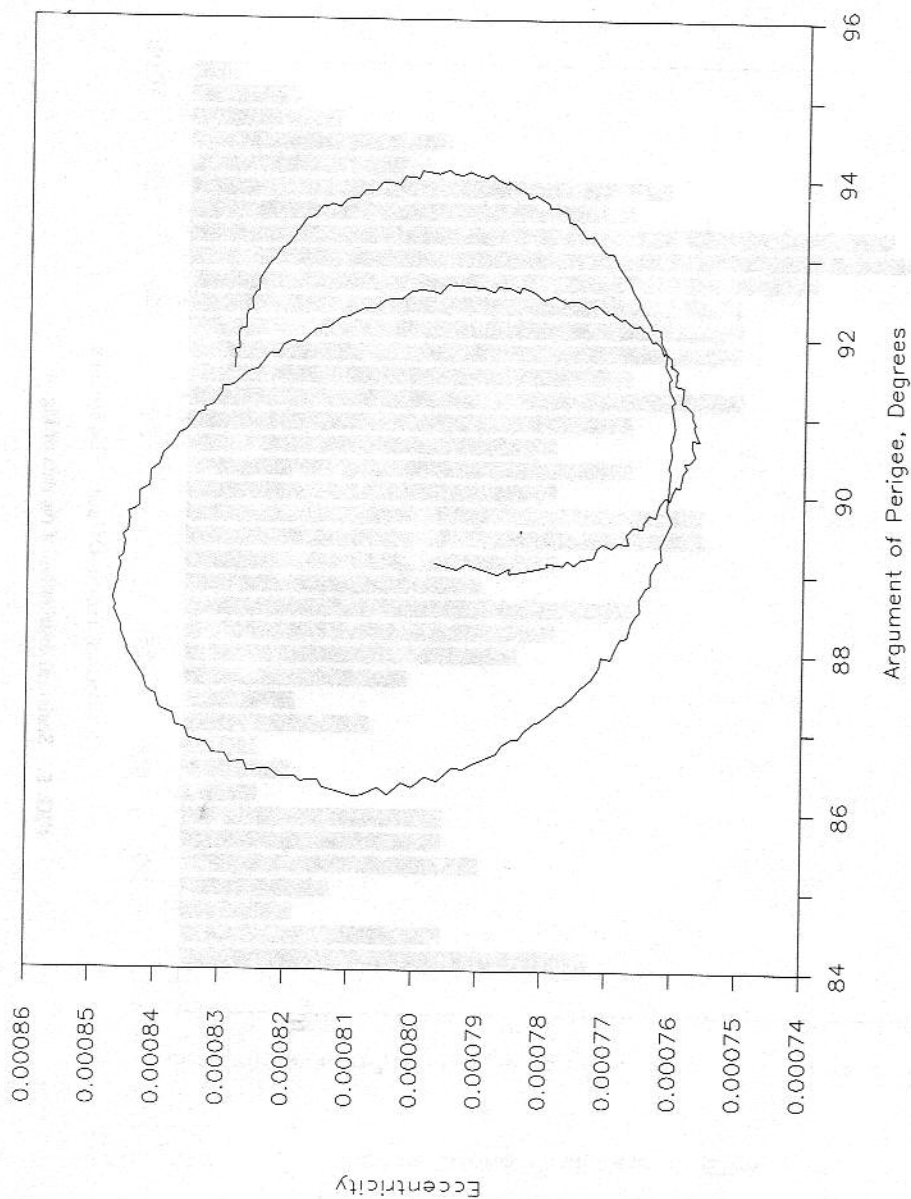


FIG. 5. Statistical distribution of the data of Fig. 4.



**FIG. 6.** Maintenance of the "frozen orbit configuration" in  $(e, \omega)$  space. Time flows along the curve in a counter-clockwise direction.



TABLE 5. GEOSAT Orbit Maintenance Maneuvers

Day of ERM	$\Delta P$ , milliseconds	Days Between Burns	Drag, ms/day
29	22.2	22.9	
60	25.1	31.1	-0.55
104	20.1	43.5	-0.52
176	30.6	72.3	-0.55
209	31.0	33.1	-0.66
226	-5.8	17.0	-0.53
260	23.3	33.7	-0.46
290	9.7	30.1	-0.53
320	17.1	30.0	-0.57
350	35.9	30.1	-0.94

The maintenance of the frozen orbit is shown in Fig. 6. The stability of both eccentricity and argument of perigee is clearly visible. Time progresses counterclockwise along the curve. As predicted by [10] and [14] the orbit is following a flower-shaped path through  $(e, \omega)$  space. No maneuvers have been necessary to further trim the eccentricity or move the perigee.

### Conclusion

The GEOSAT orbit adjust was completed in only 37 days. Maneuvering from the geodetic orbit to the oceanographic orbit, without compromising any of the constraints imposed by fuel budget, attitude stability, data acquisition, and spacecraft operations, was an unqualified success. To our knowledge, the GEOSAT orbit adjust was the most significant maneuver which had been designed principally using PC's to date. There is no question that the personal computer can be used on future missions to provide the bulk of orbital and attitude analysis support required for control center operations, especially for maneuver design and analysis.

The ERM has been particularly successful. The exact repeat ground track has been maintained for over a year with maintenance maneuvers required approximately monthly. Drag has ranged from 0.5 msec/day to 2.0 msec/day due to variations in solar activity. Maneuvers have had no impact upon data acquisition. Statistically, the ground track has been maintained within a  $\pm 1$ km swath with standard deviation  $\sigma = 490$  meters, mean  $\mu = 156$  meters, and 99.8% of the coverage within  $2\sigma$ . The frozen orbit has been virtually maintenance free.

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