PROBLEM SET 4
14.02 Macroeconomics
April 5, 2006
Due April 12, 2006

I. Answer each as True, False, or Uncertain, and explain your choice.

1. Even in the absence of technological progress, growth could go on forever if capital never depreciated.

   Ans:

   True. A constant saving rate would produce a positive but declining rate of growth.

2. The more you save the more your country will keep growing in the long run.

   Ans:

   False. Saving rate will not affect the rate of growth in the long run.

3. The U.S. capital stock is far below the golden rule level, therefore, individuals should increase their saving.

   Ans:

   Uncertain. The U.S. capital stock is below the golden rule, but that does not necessarily imply that it is desirable for individuals to increase their saving. The increase in future consumption would come at the cost of current consumption.

4. The convergence hypothesis states that, everything else being equal, the absolute increase of output per capita in small countries will be larger than in large countries.

   Ans:

   False. The convergence hypothesis states that the rate of growth of output per capita will be higher in poor countries than in rich countries.

5. Output per capita in the United States is roughly equal to 60% of output per worker.

   Ans:

   True. Output per capita equals output per worker times the participation rate, which is about 66% for the United States today. So, roughly 60% is true.

II. Short Questions

1. The Effects of a Permanent Decrease in the Rate of Nominal Money Growth

   Suppose that the economy can be described by the following three equations:

$$
\begin{aligned}
u_t - u_{t-1} &= -0.4\,(g_{yt} - 3\%) && \text{Okun's law} \\
\pi_t - \pi_{t-1} &= -\,(u_t - 5\%) && \text{Phillips curve} \\
g_{yt} &= g_{mt} - \pi_t && \text{Aggregate demand}
\end{aligned}
$$

a. Reduce the three equations to two by substituting g_{yt} from the aggregate demand equation into Okun's law. Write down the dynamic system of two equations, one of which shows u_t as a function of u_{t-1}, π_{t-1}, g_{mt} and a constant, and the other shows π_t as a function of u_{t-1}, π_{t-1}, g_{mt} and a constant. The current inflation and unemployment rate (π_t, u_t) are determined by the past (π_{t-1}, u_{t-1}) and the exogenously given monetary policy (g_{mt}).
Ans:
Substitute g_{yt} from the aggregate demand equation into Okun's law,

$$\begin{aligned} u_t - u_{t-1} &= -0.4\,(g_{mt} - \pi_t - 3\%) \\ \pi_t - \pi_{t-1} &= -(u_t - 5\%) \end{aligned}$$

Derive (π_t, u_t) in terms of (π_{t-1}, u_{t-1}) and g_{mt},

$$\begin{aligned} \pi_t &= \frac{5}{7}\left[-u_{t-1} + 0.4\,(g_{mt} - 3\%) + \pi_{t-1} + 5\%\right] \\ &= (g_{mt} - 3\%) - \frac{5}{7}\left[u_{t-1} - 5\%\right] + \frac{5}{7}\left[\pi_{t-1} - (g_{mt} - 3\%)\right] \\ u_t &= \frac{5}{7}\left[u_{t-1} - 0.4\,(g_{mt} - 3\%)\right] + \frac{2}{7}\left[\pi_{t-1} + 5\%\right] \\ &= 5\% + \frac{5}{7}\left[u_{t-1} - 5\%\right] + \frac{2}{7}\left[\pi_{t-1} - (g_{mt} - 3\%)\right] \end{aligned}$$

Assume initially that $u_t = u_{t-1} = 5\%$, $g_{mt} = 13\%$, and $\pi_t = 10\%$. Now suppose that money growth is permanently reduced from 13 to 3%, starting in year t.

b. Calculate the final values of the unemployment rate and the inflation rate in the medium-run equilibrium, i.e., (π_T, u_T) where T denotes the time when the economy has reached the medium-run equilibrium with $g_{mt} = 3\%$. Compare your results with the initial (π, u).
Ans:
In the medium run, the unemployment rate equals the natural rate, $u_T = 5\%$, and the growth rate of output equals the normal growth rate, $g_{yT} = 3\%$. Given g_{yT}, the aggregate demand equation thus pins down the medium-run inflation rate, $\pi_T = g_{mT} - g_{yT} = 0$. The reduction of the money growth rate decreases the inflation rate and leaves the real variables unchanged.

c. Now focus on the dynamics. Based on the system of equations you obtained in part (a), compute (using a software of your choice, e.g. Excel®) the unemployment and the inflation rate in year $t, t+1, ..., t+10$.
Instructions:
   i. In a spreadsheet, label the first row as follows:

s	u_t	π_t	g_{mt}	$\bar{g}_y$	u_n

   ii. Fill up row 2 by the given values for time $s = t$.
   iii. Fill up the next 10 rows of the columns corresponding to g_{mt}, $\bar{g}_y$, u_n. Note: $g_{mt} = 3\%$ for all $s \geq t$, $\bar{g}_y = 3\%$, $u_n = 5\%$.
   iv. Fill up $B3$ and $C3$ (corresponding to u_t and π_t) by using the equations you derived in part (a). Type '=' in an empty cell to start entering an

equation. Excel will calculate the numerical value automatically after you finish typing your equation and hit the 'enter' key. Note: while typing an equation, you should refer to the cell that contains the value instead of writing down the value directly. For example, if your equation says $u_{t+1} = g_{mt+1} + (u_t - u_n)$, type "$= D3 + (B2 - F3)$" instead of "$= 3\% + (5\% - 5\%)$" in cell $B3$ (the cell representing for u_{t+1}).

v. "Copy" $B3$ and $C3$ simultaneously, then "paste" them to $B4$ and $C4$, and all the way to row 12 (for $s = t + 10$). Label the time column s.

vi. Print your spreadsheet in one page and include it in your answer key.

Ans:
See the attached Excel® spreadsheet.

d. Does inflation decline smoothly from 10% to 3%? Why or why not?
Ans:
The inflation declines initially, overshoots its medium-run equilibrium level, then climbs back up, and eventually approaches to 3% in oscillation. First, the transition to the new medium-run equilibrium is not monotonic, because unemployment needs to rise first in order for inflation to fall and then inflation needs to rise in order for unemployment to return to its natural level. Secondly, it takes a perpetual oscillation for the economy to converge to the new medium-run equilibrium, because this model introduces an artificial inertia by specifying that $E\pi_t = \pi_{t-1}$ in the Phillips curve relation.

2. The Facts of Growth

In this problem, we compare Russian and U.S. GDP per capita using current exchange rates and the PPP method.

a. According to the International Financial Statistics (IFS), in 2000, Russian GDP was 7,305.65 billion rubles, and the Russian population was 146.56 million. Compute Russian GDP per capita in rubles. The IFS gives the average exchange rate for 2000 as 28.129 rubles per dollar. Divide Russian GDP per capita by the exchange rate to convert the number to dollars.
Ans:
Russian GDP per capita in rubles:

$$\frac{7,305.65}{0.14656} \sim 49,848$$

Russian GDP per capita in dollars:

$$\frac{49,848}{28.129} \sim 1,772$$

b. The IFS gives U.S. GDP as $9,816.97 billion in 2000 and the U.S. population as 284.15 million. Compute U.S. GDP per capita in dollars.
Ans:
U.S. GDP per capita in dollars:

$$\frac{9,816.97}{0.28415} \sim 34,548$$

c. Using the exchange rate method, what was Russian GDP per capita in 2000 as a percentage of U.S. GDP per capita. [Divide your answer in part (*a*) by your answer in part (*b*).]
Ans:

$$\frac{1,772}{34,548} \sim 5.1\%$$

d. In the Penn World tables (*pwt.econ.upenn.edu*), retrieve Russian GDP per capita in 2000 as a percentage of U.S. GDP per capita in PPP terms. This data item is "CGDP Relative to the United States."
Ans:
28.06%

e. Why do the numbers in part (*c*) and (*d*) differ?
Ans:
The Penn numbers are in PPP terms which corrects for the lower general price level of goods and services in Russia.

3. Saving, Capital Accumulation, and Output
(This problem is based on the material in the appendix.) Suppose that the economy's production is given by

$$Y = K^{\alpha} N^{1-\alpha}.$$

a. Is this production function characterized by constant returns to scale? Explain.
Ans:
Yes.

$$F(xK, xN) = (xK)^{\alpha} (xN)^{1-\alpha} = xK^{\alpha} N^{1-\alpha} = xY$$

A proportional increase (or decrease) of all inputs (both capital and labor) leads to the same proportional increase (or decrease) in output.

b. Are there decreasing returns to capital?
Ans:
Yes.

$$F_K(K, N) = \alpha \left(\frac{N}{K}\right)^{1-\alpha}$$

Since $0 < \alpha < 1$, $F_K(K, N)$ is decreasing in K. Given labor, increases in capital lead to smaller and smaller increases in output.

c. Are there decreasing returns to labor?
Ans:
Yes.

$$F_N(K, N) = (1 - \alpha) \left(\frac{K}{N}\right)^{\alpha}$$

Since $0 < \alpha < 1$, $F_N(K, N)$ is decreasing in N. Given capital, increases in labor lead to smaller and smaller increases in output.

d. Transform the production function into a relation between output per worker and capital per worker.

Ans:

$$\frac{Y}{N} = \frac{K^{\alpha}N^{1-\alpha}}{N} = \left(\frac{K}{N}\right)^{\alpha}$$

Let y denote output per worker and k capital per worker, and rewrite the production function as follows:

$$y = f(k) = k^{\alpha}$$

e. For a given saving rate (s) and a depreciation rate (δ), give an expression for capital per worker in the steady state.
Ans:
In the steady state, capital per worker stays constant, therefore,

$$sf(k) = \delta k$$

Given the production function derived in part (d),

$$sk^{\alpha} = \delta k$$

$\rightarrow$

$$k = \left(\frac{s}{\delta}\right)^{\frac{1}{1-\alpha}}$$

f. Give an expression for output per worker in the steady state.
Ans:
Given the production, the steady-state k derived in part (e) determines the steady-state y,

$$y = k^{\alpha} = \left(\frac{s}{\delta}\right)^{\frac{\alpha}{1-\alpha}}$$

Assume that $\alpha = \frac{1}{3}$.

g. Solve for the steady-state level of output per worker when $\delta = 0.08$ and $s = 0.32$.
Ans:

$$y = \left(\frac{s}{\delta}\right)^{\frac{\alpha}{1-\alpha}} = (4)^{\frac{1}{2}} = 2$$

h. Suppose that the depreciation rate remains constant at $\delta = 0.08$ while the saving rate is reduced by half to $s = 0.16$. What is the new steady-state output per worker?
Ans:

$$y = \left(\frac{s}{\delta}\right)^{\frac{\alpha}{1-\alpha}} = \sqrt{2}$$

A lower s decreases the steady-state y.