Classical Aspects of Ring Theory
and
Module Theory

July 14-20, 2013

Będlewo, Poland

Scientific Committee

Alberto Facchini (University of Padova, Italy)
Piotr Grzeszczuk (Bialystok University of Technology, Poland)
Tom Lenagan (University of Edinburgh, Scotland)
Jan Okniński (University of Warsaw, Poland)
Edmund Puczyłowski (University of Warsaw, Poland)
Louis Rowen (Bar-Ilan University, Ramat-Gan, Israel)
Agata Smoktunowicz (University of Edinburgh, Scotland)

Organizing Committee

Czesław Bagiński (Bialystok University of Technology, Poland)
Piotr Grzeszczuk (Bialystok University of Technology, Poland)
Jerzy Matczuk (University of Warsaw, Poland)
Ryszard Mazurek (Bialystok University of Technology, Poland)
Jan Okniński (University of Warsaw, Poland)

Warsaw Center
of Mathematics
and Computer Science

Invited Speakers

Eli Aljadeff (Technion-Israel Institute of Technology, Haifa , Israel)
Pere Ara (Universitat Autónoma de Barcelona, Spain)
Vladimir Bavula (University of Sheffield, England)
Jason Bell (University of Waterloo, Canada)
Ken Brown (University of Glasgow, Scotland)
Ken Goodearl (University of California, Santa Barbara, USA)
Pedro Guil Asensio (University of Murcia, Spain)
Natalia Iyudu (University of Edinburgh, Scotland)
David Jordan (University of Sheffield, England)
Alexei Kanel-Belov (Bar-Ilan University, Ramat-Gan, Israel)
Ulrich Kraehmer (University of Glasgow, Scotland)
Stephane Launois (University of Kent, England)
Pavel Prihoda (Charles University, Prague, Czech Republic)
Lance Small (University of California, San Diego, USA)
Toby Stafford (University of Manchester, England)
Uzi Vishne (Bar-Ilan University, Ramat-Gan, Israel)
Efim Zelmanov (University of California, San Diego, USA)
James Zhang (University of Washington, USA)

Contents

Plenary Talks

Regular gradings on associative algebras and polynomial identities asymptotics

Eli Aljadeff

Department of Mathematics
Technion
Haifa, Israel 36054
aljadeff@tx.technion.ac.il, elialjadeff@gmail.com

Regev and Seeman introduced regular G-gradings on associative algebra where G is a finite abelian group. Roughly, it consists of a decomposition of an algebra A into homogeneous subspaces $A_g,\ g \in G$, which commute up to nonzero scalars $\theta(g,h)$, $g,h \in G$ (for instance the algebra of $n \times n$ matrices admits such a decomposition). Bahturin and Regev conjectured that if the grading is regular and minimal (i.e. the grading on A induced by a proper homomorphic image of G is not longer regular) then the order of the group G is an invariant of the algebra A. Furthermore, given a regular grading on A, one may consider the corresponding "commutation matrix" $\{\theta(g,h)\}_{g,h\in G}$ and here, they conjectured that the commutation matrices which correspond to two minimal gradings on A, have the same determinant. In a joint work with Ofir David we prove these conjectures. In particular we show that the order of the group G (which provides a minimal regular grading on A) coincides with an invariant which arises in asymptotic PI theory.

In the lecture I'll recall the topics involved (namely graded algebras and polynomial identities) and explain how to connect them via graded polynomial identities. These results can be extended to nonabelian groups.

Wild refinement monoids and von Neumann regular rings

Pere Ara

Universitat Autónoma de Barcelona
08193 Bellaterra (Barcelona), Spain
para@mat.uab.cat

There have been some recent advances on the realization problem for von Neumann regular rings, which asks whether any countable conical refinement monoid is isomorphic to the monoid $V(R)$ of isomorphism classes of finitely generated projective modules over some von Neumann regular ring R, see [1] for a survey on this problem. The classes of monoids realized so far are contained in the class of direct limits of finitely generated conical refinement monoids. The regular rings realizing these monoids can be chosen to be K-algebras over an arbitrary field K. We say that a conical refinement monoid M is *tame* in case it can be obtained as a direct limit of finitely generated conical refinement monoids. Otherwise, we say that M is a *wild refinement monoid*. Recent work in [2] and [3] allows to obtain countable wild refinement monoids in a controlled way. We will explore the realization problem for a concrete wild refinement monoid N obtained that way, which can be explicitly described in terms of generators and relations. It turns out that, for any field K, there is an exchange K-algebra R such that $V(R) \cong N$. The K-algebra R is a universal localization of the semigroup algebra over the monogenic free inverse monoid. However, it turns out that N cannot be realized by any von Neumann regular ring which is a K-algebra over an *uncountable* field K. Using a skew version of the construction mentioned before, we are able to realize N as the V-monoid of a von Neumann regular K-algebra over any countable field K.

This is joint work-in-progress with Ken Goodearl.

References

[1] P. Ara, The realization problem for von Neumann regular rings, *Ring Theory 2007. Proceedings of the Fifth China-Japan-Korea Conference*, (eds. H. Marubayashi, K. Masaike, K. Oshiro, M. Sato); World Scientific, 2009, pp. 21–37.

[2] P. Ara, K. R. Goodearl, Leavitt path algebras of separated graphs, *Journal*

für die reine und angewandte Mathematik, **669** (2012), 165–224.

[3] P. Ara, R. Exel, Dynamical systems associated to separated graphs, graph algebras, and paradoxical decompositions, arXiv:1210.6931 [math.OA].

New criteria for a ring to have a semisimple left quotient ring

Vladimir Bavula

Department of Pure Mathematics
University of Sheffield
Hicks Building, Sheffield S3 7RH, UK
v.bavula@sheffield.ac.uk

Goldie's Theorem (1960), which is one of the most important results in Ring Theory, is a criterion for a ring to have a semisimple left quotient ring (i.e. a semisimple left ring of fractions). The aim of my talk is to give four new criteria (using a completely different approach and new ideas). The first one is based on the recent fact that for an arbitrary ring R the set of maximal left denominator sets of R is a non-empty set. The Second Criterion is given via the minimal primes of R and goes further then the First one and Goldie's Theorem in the sense that it describes explicitly the maximal left denominator sets via the minimal primes of R. The Third Criterion is close to Goldie's Criterion but it is easier to check in applications (basically, it reduces Goldie's Theorem to the prime case). The Fourth Criterion is given via certain left denominator sets.

Free subalgebras of division rings

Jason Bell

University of Waterloo
Waterloo, ON, Canada
N2L 3G1
jpbell@uwaterloo.ca

Let A be a countably generated noetherian $\mathbb{C}$-algebra that is a domain and let $Q(A)$ denote its quotient division algebra. We show that either $Q(A)$ is finite-dimensional over its centre or $Q(A)$ contains a copy of the free $\mathbb{C}$-algebra on two generators.

This is joint work with Dan Rogalski and this is a preliminary report.

Construction of finitely presented infinite nil-semigroup

Alexei Ya. Belov*

Mathematics Department Bar-Ilan University
Ramat-Gan, 52900 Israel
kanel@mccme.ru

Ilya A. Ivanov-Pogodaev

Mechmat MGU, Moscow State University
Chair of Algebra
Moscow, Vorobievy Gory, 119899, Russia
ivanov-pogodaev@mail.ru

The talk is devoted to the new construction method for algebraic objects using various aperiodic tilings. We use this method to construct an finitely presented infinite nil-semigroup, answering on the Shevrin problem. The method is based on considering the paths on some tiling as non trivial elements of a semigroup. Also, the structure of tiling induces the relations in the semigroup. The tiling can be presented by the finite number of rules, so the semigroup would have the finite number of defining relations. These facts correspond to Goodman-Strauss theorem about aperiodic hierarchical tilings. There are no periodic paths on the tiling so there are no periodic words in the semigroup. The subject is related to other Burnside type problems in groups and rings. See more detailed abstract [2]

References

[1] Chaim Goodman-Strauss, Matching Rules and Substitution Tilings , Annals of Mathematics, 147 (1998), 181-223

[2] Ivanov-Pogodayev, I.; Kanel-Belov, A. Construction of finitely presented infinite nil-semigroups. (English. Russian original) J. Math. Sci., New York 186, No. 5, 751-752 (2012); translation from Sovrem. Mat. Prilozh. 74 (2011)

Noncommutative Unipotent Groups

Ken Brown

School of Mathematics and Statistics
University of Glasgow
Glasgow G12 8QW, UK
Ken.Brown@glasgow.ac.uk

I will survey recent results of myself and a number of others on the structure and properties of connected Hopf algebras of finite GK-dimension, putting these results in the context of classical work on algebraic groups and on cocommutative Hopf algebras. I will also suggest some directions for future work in this area.

References will be provided at the meeting

Some noteworthy ideals in categories of modules

Alberto Facchini

University of Padova,
Padova, Italy
facchini@math.unipd.it

We will describe some noteworthy ideals in categories of modules. We begin with maximal ideals. Recall that a ring R is *semilocal* if $R/J(R)$ is semisimple artinian. A preadditive category is *semilocal* if it is non-null and the endomorphism ring of every non-zero object is a semilocal ring. There are several natural examples of semilocal categories. Maximal ideals do not exist, in general, in arbitrary preadditive categories, but do exist in semilocal categories. An additive functor $F\colon A \to B$ between preadditive categories A and B is said to be a *local functor* if, for every morphism $f\colon A \to A'$ in A, $F(f)$ isomorphism in B implies f isomorphism in A. If C is a semilocal category, the canonical functor $F\colon C \to \oplus_{M\in(C)} C/M$ is a local functor.

Another ideal we will consider is the Jacobson radical of the category. The kernel of any local functor $F\colon A \to B$ is an ideal of A contained in the Jacobson radical of A. If A is a preadditive category and $I_1, \dots, I_n$ are ideals of A, we will study when the canonical functor $A \to A/I_1 \times \cdots \times A/I_n$ is a local functor. A weak form of the Krull-Schmidt theorem naturally appears in this setting.

We will conclude discussing Birkhoff's Theorem (there exists a subdirect embedding of A into a direct product of subdirectly irreducible preadditive categories) for skeletally small preadditive categories A and for the category $A = R$.

Unique Factorization in Quantum-type Skew Polynomial Rings

Ken Goodearl

University of California,
Santa Barbara, USA
goodearl@math.ucsb.edu

The theme of this talk is noncommutative unique factorization domains and their appearance among quantum algebras, particularly the iterated skew polynomial rings which underlie many quantized coordinate rings. Classical results in algebraic geometry show that many rings of functions on affine varieties are UFDs, and Launois, Lenagan, and Rigal have proved that many quantized coordinate rings are noncommutative UFDs. The latter results are based on showing that the members of a large class of iterated skew polynomial rings, known as torsionfree CGL extensions, are noncommutative UFDs. In recent joint work with Yakimov, we have pinned down the homogeneous irreducible elements in arbitrary CGL extensions – there are only finitely many, up to scalar multiples – and showed that they provide initial clusters for quantum cluster algebra structures.

Ideal Approximation Theory

Pedro A. Guil Asensio

University of Murcia
Murcia, Spain
paguil@um.es

Let $(\mathcal{A};\mathcal{E})$ be an exact category and $\mathcal{F} \subseteq Ext$ a subfunctor. A morphism $\varphi \in \mathcal{A}$ is an $\mathcal{F}$-phantom if the pullback of an $\mathcal{E}$-conflation along φ is a conflation in $\mathcal{F}$. If the exact category $(\mathcal{A};\mathcal{E})$ has enough injective objects and projective morphisms, it is proved that an ideal I of $\mathcal{A}$ is special precovering if and only if there is a subfunctor $\mathcal{F} \subseteq Ext$ with enough injective morphisms such that I is the ideal of $\mathcal{F}$-phantom morphisms. A crucial step in the proof is a generalization of Salce's Lemma for ideal cotorsion pairs: if I is a special precovering ideal, then the ideal cotorsion pair $(\mathcal{I},\mathcal{I}^{\perp})$ generated by $\mathcal{I}$ in $(\mathcal{A};\mathcal{E})$ is complete. This theorem is used to verify: (1) that the ideal cotorsion pair cogenerated by the pure-injective modules of R-Mod is complete; (2) that the ideal cotorsion pair cogenerated by the contractible complexes in the category of complexes $Ch(R-Mod)$ is complete; and, using Auslander and Reiten's theory of almost split sequences, (3) that the ideal cotorsion pair cogenerated by the Jacobson radical $Jac(\Lambda-mod)$ of the category $\Lambda-mod$ of finitely generated representations of an artin algebra is complete.

This is joint work with X.H. Fu, I. Herzog and B. Torrecillas

On the K-theory of graph C^*-algebras

Natalia Iyudu

School of Mathematics
University of Edinburgh
James Clerk Maxwell Building
The King's Buildings, Mayfield Road
Edinburgh, Scotland EH9 3JZ, UK
niyudu@staffmail.ed.ac.uk

We will describe the formulas for the Grothendieck and the Whitehead groups of graph C^*- algebras. By this we answer the question from the paper due to M. Marcolli et. al, 2008, where these groups were calculated via the first Betti number of the graph, in the case of a finite graph. Our formulas express the K-groups in terms of the Betti number of the graph and the branching number. They show, in particular, that torsion part of K_0 vanishes in case of an infinite graph.

These results are published in: N. Iyudu, *K-theory of locally finite graph C^*-algebras*, Journal of Geometry and Physics **71** (2013), $22 - 29$.

Connected quantized Weyl algebras

David Jordan

School of Mathematics and Statistics
University of Sheffield
UK
d.a.jordan@sheffield.ac.uk

Although skew polynomial rings or Ore extensions are established classical objects in noncommutative ring theory, interesting new examples continue to emerge. Here we discuss what appears to be a new and interesting family of examples with a particular emphasis on classifying prime ideals. These examples have arisen from quantization of Poisson algebras coming from periodic quiver mutation.

Let $q \in \mathbb{C}\backslash\{0, 1\}$. We consider $\mathbb{C}$-algebras with n generators such that: (i) each pair of generators generates either the coordinate ring of the quantum plane or the quantized Weyl algebra; (ii) if we draw the graph whose vertices are the generators and where there are edges between those pairs that generate a quantized Weyl algebra then this graph is connected; (iii) the algebra has a PBW basis consisting of the standard monomials in the n generators. These algebras fall into two subfamilies which, because of the nature of their graphs, we call *linear* and *cyclic*. In a naive sense they can be regarded as quantizations of Weyl algebras or polynomial rings in a single indeterminate over a Weyl algebra. In the latter case there is a distinguished central element corresponding to the single indeterminate.

When q is not a root of unity, the prime ideals can be approached using localization and deleting derivations. In the linear case, where the outcome depends on the parity of n, all primes are completely prime but in the cyclic case that is not the case.

This is joint work with Christopher Fish.

Untwisting a Twisted Calabi-Yau Algebra

Ulrich Kraehmer

University of Glasgow
Scotland
ulrich.kraehmer@glasgow.ac.uk

Twisted Calabi-Yau algebras are a generalisation of Ginzburg's notion of Calabi-Yau algebras. Such algebras A come equipped with a modular automorphism $\sigma \in Aut(A)$, the case $\sigma = id$ being precisely the original class of Calabi-Yau algebras. In this joint work with Jake Goodman we prove that the smash product by the modular automorphism is Calabi-Yau.

Efficient recognition of totally nonnegative cells

Stephane Launois

University of Kent
England
S.Launois@kent.ac.uk

A matrix is totally positive if all of its minors are positive, and totally non-negative if all of its minors are nonnegative. The space of mxp totally non-negative matrices admits a cell decomposition whose big cell is the space of mxp totally positive matrices. Efficient criteria to test for total positivity are well-known. In this talk, I will explain how one can use tools developed to study prime ideals in quantum matrices in order to obtain efficient criteria for all totally nonnegative cells.

This talk is based on joint work with Ken Goodearl and Tom Lenagan.

Pure projective modules over chain rings

Pavel Příhoda

Charles University, Faculty of Mathematics and Physics,
Department of Algebra
Prague, Czech Republic
prihoda@karlin.mff.cuni.cz

Recall that a ring is said to be a chain ring if the lattice of its left ideals is a chain and the same is true for the lattice of its right ideals. By the Drozd-Warfield theorem [5, Corollary 3.4], every finitely presented module over a chain ring is a direct sum of cyclic modules. Moreover, all indecomposable decompositions of a finitely presented module over a chain ring are isomorphic. Recall that a module is called pure projective if it is a direct summand of an arbitrary direct sum of finitely presented modules.

What can be said about pure projective modules over chain rings? Puninski [3] proved there exists a chain domain possessing an indecomposable pure projective module which is not finitely generated. What is even worse, he also found an example of a chain ring possessing a superdecomposable pure projective module [4].

In my talk I will give a brief summary of the dimension theory introduced in [2] that can be used to classify pure projective modules over chain rings and explain how could it be applied to give criteria of (non) existence of strange pure projective modules. I will also try to explain what kind of pure projective modules one could expect over classes of chain domains constructed in [1].

References

[1] H. H. Brungs, N. I. Dubrovin: *A classification and examples or rank one chain domains,* Trans. Amer. Math. Soc. 355 (2003), no. 7, 2733-2753.

[2] A. Facchini, P. Příhoda, *Factor categories and infinite direct sums,* Int. Electron. J. Algebra 5 (2009), 135-168.

[3] G. Puninski: *Some model theory over a nearly simple uniserial domain and decompositions of serial modules,* J. Pure Appl. Algebra 163 (2001), no. 3, 319-337.

[4] G. Puninski: *Some model theory over an exceptional uniserial ring and decompositions of serial modules,* J. London Math. Soc. (2) 64 (2001), no.

2, 311-326.

[5] R. B. Warfield, *Serial rings and finitely presented modules,* J. Algebra 37 (1975), no. 2, 187-222.

Polynomials evaluated on nonassociative algebras

Louis Rowen

Department of Mathematics
Bar-Ilan University
Ramat-Gan 52900
Israel
rowen@math.biu.ac.il

Some famous theorems from polynomial identities (PIs), such as Kemer's solution to Specht's problem for algebras over infinite fields, and Belov's solution for affine algebras in characteristic p, have recently been extended to associative algebras with involution and graded algebras. Sviridova has proved an involutory version of Kemer's theorem, and Aljadeff and Belov have a graded version.

In this talk we review the associative results and their proofs, and then gather these notions under the umbrella of universal algebra, and see how certain aspects can be treated for (not necessarily associative) algebras in this setting.

This includes:
Representability of weakly Noetherian algebras;
Nilpotence of nil subalgebras;
Codimensions of T-ideals;
Specht's problem for affine algebras.

Belov, Giambruno, Small, and Vishne are collaboraters in this program. If time permits, evaluations of non-identities will also be discussed, related to joint work with Belov and Malev.

Infinite Dimensional Division Algebras

Lance Small

University of California
San Diego, USA
lwsmall@ucsd.edu

We will discuss some open problems on infinite dimensional division algebras that possess additional properties or that arise as quotient rings. Additionally, we mention some problems remaining on the enveloping algebras of the Witt and Virasoro algebras.

Some results on the Jacobson and nil radicals in graded rings

Agata Smoktunowicz

Institute for Mathematical Sciences, School of Mathematics
University of Edinburgh
James Clerk Maxwell Building, King's Buildings
Mayfield Road, Edinburgh EH9 3JZ, Scotland, UK
a.smoktunowicz@ed.ac.uk

It was shown by Bergman that the Jacobson radical of a $\mathbb{Z}$-graded ring is homogeneous. We will show that the analogous result holds for nil rings, namely, that the nil radical of a $\mathbb{Z}$-graded ring is homogeneous. It is obvious that a subring of a nil ring is nil, but generally a subring of a Jacobson radical ring need not be a Jacobson radical ring. We show that every subring which is generated by homogeneous elements in a graded Jacobson radical ring is always a Jacobson radical ring, and that a ring whose subrings are Jacobson radical rings are nil.

Several important properties of the Brown-McCoy radical were studied by Chebotar, Krempa, Jespers, Lee, Puczylowski, and others. We will show that graded-nil rings, that is graded rings whose all homogeneous elements are nil, are Brown-McCoy radical. We will also look at some strangely-behaving examples of rings which are Brown-McCoy radical but not Jacobson radical.

In addition, we recall some other recent results in this area, some open questions, and pose some new problems.

Classifying orders in the Sklyanin algebra

Toby Stafford

University of Manchester
England
Toby.Stafford@manchester.ac.uk

One of the major open problems in noncommutative algebraic geometry is the classification of noncommutative surfaces and I will describe a significant case of this problem.

Specifically, let S denote the 3-dimensional Sklyanin algebra over an algebraically closed field k and assume that S is not a finite module over its centre. Let A be any connected graded k-algebra that is contained in and with the same quotient ring as some Veronese ring S^{3n}. Then we give a reasonably complete description of the structure of A. The description is most satisfactory when A is a maximal order, in which case we prove that A is finitely generated and noetherian and can be described as a noncommutative blowup of S^{3n} at a (possibly non-effective) divisor in the associated elliptic curve E. It follows that A has surprisingly pleasant properties; for example it satisfies the Artin-Zhang chi conditions and has a balanced dualising complex.

This work is all joint with Dan Rogalski and Sue Sierra.

P-CENTRAL ELEMENTS AND SUBSPACES IN CENTRAL SIMPLE ALGEBRAS, CHAIN LEMMAS AND RELATED NONASSOCIATIVE CONSTRUCTIONS

Uzi Vishne

Bar-Ilan University
Israel
vishne@math.biu.ac.il

The major open question on central simple algebras is the cyclicity problem: are all algebras of prime degree cyclic? Any cyclic algebra of degree p has p-central elements: non-central elements whose p-power is central. The cyclicity problem can thus be studied using subspaces of p-central elements, which will be the main topic of the lecture. We will discuss such subspaces from various points of views: chain lemmas, the symbol length problem, a problem in elementary number theory, and a new construction in nonassociative algebra.

On Leavitt path algebras

Efim Zelmanov

University of California
San Diego, USA
efim.zelmanov@gmail.com

We will discuss: (i) Leavitt path algebras of polynomial growth, their structure, automorphisms and involutions; (ii) Lie algebras and groups associated with Leavitt path algebras. The latters provide new examples of simple infinite finitely presented groups.

This is joint work with A. Alahmedi, H. Alsulami, S. K. Jain.

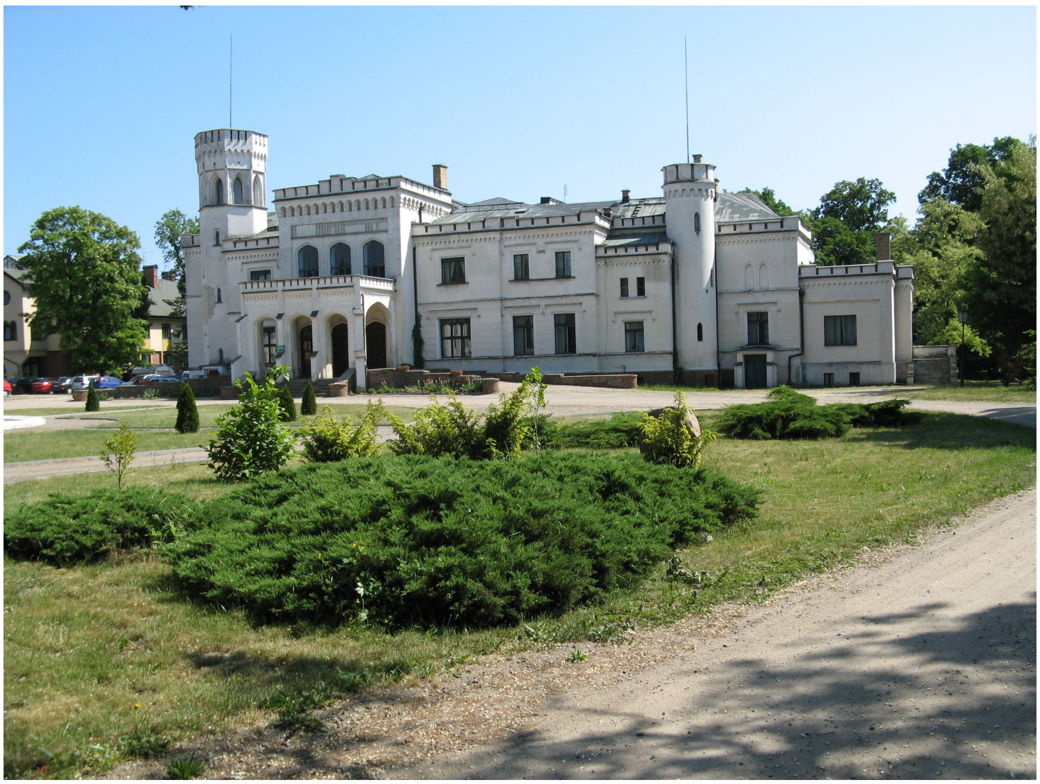
Discriminant controls automorphism group of noncommutative algebras

James Zhang

University of Washington
Washington, USA
zhang@math.washington.edu

We use discriminant to determine the automorphism group of some noncommutative algebras.

This is joint work with Secil Ceken, John Palmieri and Yanhua Wang.



Short Talks

Constructing classes of prime, non-primitive, von Neumann regular algebras

Gene Abrams

University of Colorado
Colorado Springs, USA
abrams@math.uccs.edu

More than forty years ago, Kaplansky posed the question: "Is a regular prime ring necessarily primitive?" A negative answer (via a clever though somewhat ad hoc example) to this question was given by Domanov in 1977.

For any directed graph E and field K, $L_K(E)$ denotes the *Leavitt path K-algebra of E with coefficients in K.* Leavitt path algebras have been defined and subsequently investigated within the past decade. In previous work (done by the author and others), necessary and sufficient conditions on E have been established for which $L_K(E)$ is von Neumann regular; as well, necessary and sufficient conditions on E have been established for which $L_K(E)$ is prime. The current contribution establishes necessary and sufficient conditions on E for which $L_K(E)$ is primitive. The three results together yield algebras of the type about which Kaplansky queried. Various infinite classes of examples (both unital and nonunital) of such algebras will be given.

This is joint work with Jason Bell and K. M. Rangaswamy

Isomorphisms between strongly triangular matrix rings

Phạm Ngọc Ánh

Alfréd Rényi Institute of Mathematics
Hungarian Academy of Sciences
Hungary
anh.pham.ngoc@renyi.mta.hu

This is a joint work with L. van Wyk. In this talk we describe isomorphisms between stongly triangular matrix rings. This description enables us to give the automorphism groups of such rings in terms of ones involved in the matrix decompositions. This shows that in these rings certain idempotents behave like idempotents in semiperfect rings.

Rings whose cyclic modules are direct sums of extending modules

Pınar Aydoğdu*

Department of Mathematics
Hacettepe University
Beytepe 06800 Ankara, Turkey
paydogdu@hacettepe.edu.tr

Noyan Er

Department of Mathematics
University of Rio Grande
Rio Grande, OH 45674, USA
noyaner@yahoo.com

Nil Orhan-Ertaş

Department of Mathematics
Karabük University
78050 Karabük, Turkey
orhannil@yahoo.com

Dedekind domains, Artinian serial rings and right uniserial rings share the following property: Every cyclic right module is a direct sum of uniform modules. We first prove the following improvement of the well-known Osofsky-Smith theorem: A cyclic module with every cyclic subfactor a direct sum of extending modules has finite Goldie dimension. So, rings with the above-mentioned property are precisely rings of the title. Furthermore, a ring R is right q.f.d. (cyclics with finite Goldie dimension) if proper cyclic right R-modules are direct sums of extending modules. R is right serial with all prime ideals maximal and $\cap_{n\in\mathbb{N}} J^n = J^m$ for some m if cyclic right R-modules are direct sums of quasi-injective modules. A right non-singular ring with the latter property is right Artinian. Thus, hereditary Artinian serial rings are precisely one-sided non-singular rings whose right and left cyclic modules are direct sums of quasi-injectives.

References

[1] J. Ahsan, Rings all of whose cyclic modules are quasi-injective, Proc. London Math. Soc., 27(3) (1973), 425-439.

[2] J. Ahsan, On rings with quasi-injective cyclic modules, Proc. Edinb.Math. Soc., II Ser. 19 (1974), 139-145.

[3] N. V. Dung, D. V. Huynh, P. F. Smith and R. Wisbauer, Extending modules, Pitman Res. Notes Math. Ser. 313 (1994).

[4] N. Er, Rings whose modules are direct sums of extending modules, Proc. Amer. Math. Soc. 137(7) (2009), 2265-2271.

[5] S. K. Jain, S. Singh and A. K. Srivastava, On σ-q rings, J. Pure Appl. Algebra. 213 (2009), 969-976.

[6] S. K. Jain, S. Singh and R. G. Symonds, Rings whose proper cyclic modules are quasiinjective, Pacific J. Math. 67 (1976), 461-472.

[7] R. E. Johnson and E. T.Wong, Quasi-injective modules and irreducible rings, J. London Math. Soc. 36 (1961), 260-268.

[8] G. B. Klatt and L. S. Levy, Pre-self injective ringss, Trans. Amer. Math. Soc. 137 (1969), 407-419.

[9] A. Koehler, Rings with quasi-injective cyclic modules, Quarterly J. Math. 25 (1974), 51-55.

[10] B. Osofsky and P. F. Smith, Cyclic modules whose quotients have all complement submodules direct summands, J. Algebra 139 (1991), 342-354.

[11] B. Osofsky, Rings all of whose finitely generated modules are injective, Pacific J. Math. 14 (1964), 645-650.

[12] B. Osofsky, Noncyclic injective modules, Proc. Amer. Math. Soc. 19 (1968), 1383-1384.

[13] S. Singh, Indecomposable modules over Artinian right serial rings in Advances in Ring Theory, (Jain, S. K. and Rizvi, S. T. Editors) (Birkhauser Verlag, Boston, 1997), 295-304.

On the center of a modular group algebra of a finite p-group

Czesław Bagiński

Bialystok University of Technology
Bialystok, Poland
c.baginski@pb.edu.pl

Let G be a finite p-group and F a field of characteristic p and let $\mathcal{Z}_0 = \mathcal{Z}_0(F[G])$ be the subalgebra of $F[G]$ spanned by class sums $\widehat{C}$, where C runs over all conjugacy classes of noncentral elements of G. We show that all finite p-groups are subgroups and homomorphic images of p-groups for which $\widehat{C}^p = 0$. We give also the description of abelian-by-cyclic groups for which $\mathcal{Z}_0$ is an algebra with zero multiplication or is nil of index 2.

This is joint work with J. Kurdics

On Σ-pure injective modules

Simion Breaz

Babeş-Bolyai University
Faculty of Mathematics and Computer Science
400084, Cluj-Napoca, Romania
bodo@math.ubbcluj.ro

It was proved by H. Krause and M. Saorin that $\mathrm{Prod}(M) \subseteq \mathrm{Add}(M)$ if and only if M is Σ-pure injective and product-rigid [2], and this implies $\mathrm{Prod}(M) = \mathrm{Add}(M)$.

We prove, under the set theoretic hypothesis ($V = L$), that a right R-module M is Σ-pure injective if and only if $\mathrm{Add}(M) \subseteq \mathrm{Prod}(M)$. Consequently, if R is a unital ring, the Brown Representability Theorem is valid for the homotopy category $\mathbf{H}(\text{Mod-}R)$ if and only if it is valid for the dual $\mathbf{H}(\text{Mod-}R)^{\circ}$. The main result can be also applied to provide new characterizations for right pure-semisimple rings or to give a partial positive answer to a question of G. Bergman, [1, Question 12].

References

[1] G.M. Bergman: *Two statements about infinite products that are not quite true*, (ed. by Chin, William et al.), Groups, rings and algebras. A conference in honor of Donald S. Passman, Madison, WI, USA, June 10–12, 2005. Providence, RI: American Mathematical Society (AMS). Contemporary Mathematics 420 (2006), 35–58.

[2] H. Krause, M. Saorin: *On minimal approximations of modules*, In: Trends in the representation theory of finite dimensional algebras (ed. by E. L. Green and B. Huisgen-Zimmermann), Contemp. Math. 229 (1998), 227–236.

Representations of the Clifford algebra

Adam Chapman

Department of Mathematics
Bar-Ilan University
Ramat-Gan 52900, Israel
adam1chapman@yahoo.com

A form $f(u_1, \dots, u_n)$ of degree d with n variables over a field F has a finite linearization if for some m there exist matrices $X_1, \dots, X_n \in M_m(F)$ such that for every $u_1, \dots, u_n \in F$, $(u_1X_1 + \cdots + u_nX_n)^d = f(u_1, \dots, u_n)$.

The form f has a finite linearization if and only if its Clifford algebra C_f has a representation of finite rank over F. The question of whether the Clifford algebra always has representations of finite rank has troubled mathematicians over the years, with only partial successes. We prove that the Clifford algebra of any form has representations of finite rank. The talk is based on a joint work with Daniel Krashen and Max Lieblich.

On the involutive Yang-Baxter property in finite groups

Florian Eisele

Vrije Universiteit Brussel
Pleinlaan 2, B-1050 Brussels, Belgium
feisele@vub.ac.be

A finite group G is called involutive Yang-Baxter (IYB for short) if there is some $\mathbb{Z}G$-module M which admits a bijective 1-cocycle $\chi : G \longrightarrow M$. This property can also be characterized in terms of the existence of a particular one-sided ideal contained in the augmentation ideal of the group ring $\mathbb{Z}G$. It is an open problem to characterize those finite groups which are IYB. It has long been known that such a group has to be solvable, and there is to date no known example of a solvable group which isn't IYB. So it might well be that all of them are. In this talk I will report on some results of ongoing research, both theoretical and computational, which provide evidence for the conjecture that all solvable groups are IYB.

Varieties of associative algebras satisfying a reduced semigroup identity

Olga Finogenova

Ural Federal University
Ekaterinburg, Russia
olgafinogenova@gmail.com

A polynomial identity $u = v$ where u, v are distinct semigroup words is called a *semigroup identity*. The semigroup identity $u = v$ is called *reduced* if the first letters of words u, v are different and the last letters of words u, v are different as well. Varieties satisfying a reduced semigroup identity are investigated in the case of associative algebras over an infinite field (see [1] – [3]). We study such varieties in the case of algebras over a finite field and in the case of rings. We describe them in the language of forbidden algebras and prove that every such variety satisfies the identity $x^n y = yx^n$ for some integer $n > 0$.

References

[1] Golubchik, I.Z., Mikhalev, A.V. *On varieties of algebras with semigroup identity*, Mosc. Univ. Math. Bull. 37 (1982), 5–9; translation from Vestn. Mosk. Univ., Ser. I 1982, No.2, 8-11 (1982).

[2] Riley D.M., Wilson M.C. *Associative algebras satisfying a semigroup identity*, Glasgow Math. J. 41 (1999), 453–462.

[3] Samoilov, L.M. *A note on trinomial identities in associative algebras*, Math. Notes 65 (1999), 208–213; translation from Mat. Zametki 65 (1999), 254-260.

Semi-Invariant Subrings, Centralizers and Krull-Schmidt Decompositions

Uriya A. First

Einstein Institute of Mathematics,
Hebrew University of Jerusalem, Israel
uriya.first@gmail.com

A *linearly topologized* (abbrev.: LT) ring is a topological ring admitting a basis of neighborhoods of 0 consisting of two-sided ideals (e.g. any ring with the discrete topology or the p-adic integers). We call a subring R_0 of an LT ring R *semi-invariant* (in R) if there exists an LT ring S containing R and a set of continuous ring endomorphisms $\Sigma \subseteq \mathrm{End}_c(S)$ such that $R_0 = R^\Sigma := \{r \in R \,|\, \sigma(r) = r \,\forall \sigma \in \Sigma\}$. (If we can take $S = R$, then R_0 is just an *invariant subring.*)

While not obvious from the definition, it turns out that all centralizers of subsets of R are semi-invariant subrings. Furthermore, if M is a finitely presented R-module, then $\mathrm{End}_R(M)$ (after suitably topologized) is isomorphic to an epimorphic image of a semi-invariant subring of $\mathrm{M}_n(R) \times \mathrm{M}_m(R)$ for some $n, m \in \mathbb{N}$. Regardless of that, one can show that various properties of LT rings pass to semi-invariant subrings, including: pro-semiprimary (i.e. being an inverse limit of discrete semiprimary rings), pro-semiprimary and semiperfect, pro-right-perfect, pro-right-perfect and semiperfect, semilocal, etc.

Combining all these observations leads to various applications concerning the structure of centralizers and the existence Krull-Schmidt decompositions. In particular, we show that the center and any maximal commutative subring of a semiprimary (right perfect) ring is also semiprimary (right perfect), and generalize results of Bjork ([1]) and Rowen ([3],[4]) by showing that any finitely presented module over a pro-semiprimary semiperfect ring has a Krull-Schmidt decomposition. We also obtain a notion of *Henselianity* for LT rings, which coincides with the standard Henselianity for rank-1 valuation rings.

This contents of this lecture can also be found at [2].

References

[1] J.-E. Björk. Conditions which imply that subrings of semiprimary rings are semiprimary. *J. Algebra*, 19:384–395, 1971.

[2] U.A. First. Semi-invariant subrings. *J. Algebra*, 387:103–132, 2013.

[3] L.H. Rowen. Finitely presented modules over semiperfect rings. *Proc. Amer. Math. Soc.*, 97(1):1–7, 1986.

[4] L.H. Rowen. Finitely presented modules over semiperfect rings satisfying ACC-∞. *J. Algebra*, 107(1):284–291, 1987.

Monomial algebras defined by Lyndon words

Tatiana Gateva-Ivanova

American University in Bulgaria
Blagoevgrad 2700, Bulgaria
tatyana@aubg.bg
and
Institute of Mathematics and Informatics
Bulgarian Academy of Sciences
Sofia 1113, Bulgaria
tatianagateva@yahoo.com

Assume that $X = \{x_1, \cdots, x_g\}$ is a finite alphabet and $\mathbf{k}$ is a field. We study monomial algebras $A = \mathbf{k}\langle X\rangle/(W)$, where W is an antichain of Lyndon words in the alphabet X of arbitrary cardinality. We find a Poincaré-Birkhoff-Witt type basis of A in terms of its *Lyndon atoms* N, but, in general, N may be infinite. We prove that if A has polynomial growth of degree d then A has global dimension d and is standard finitely presented, with $d-1 \leq |W| \leq d(d-1)/2$. Furthermore, A has polynomial growth if and only if the set of Lyndon atoms N is finite. In this case A has a $\mathbf{k}$-basis $\mathfrak{N} = \{l_1^{\alpha_1} l_2^{\alpha_2} \cdots l_d^{\alpha_d} \mid \alpha_i \geq 0, 1 \leq i \leq d\}$, where $N = \{l_1, \cdots, l_d\}$. We give an extremal class of monomial algebras, the Fibonacci-Lyndon algebras, F_n, with global dimension n and polynomial growth, and show that the algebra F_6 of global dimension 6 cannot be deformed, keeping the multigrading, to an Artin-Schelter regular algebra.

References

[1] Tatiana Gateva-Ivanova, Gunnar Fløystad, *Monomial algebras defined by Lyndon words*, arXiv:1207.6256 [math.RA]

[2] Tatiana Gateva-Ivanova, *Quadratic algebras, Yang-Baxter equation, and Artin-Schelter regularity*, Advances in Mathematics **230** (2012), pp. 2152-2175.

H-(co)module algebras and their polynomial H-identities

Alexey S. Gordienko

Vrije Universiteit Brussel,
Belgium
alexey.gordienko@vub.ac.be

When an algebra is endowed with a grading, an action of a group G by automorphisms and anti-automorphisms, an action of a Lie algebra by derivations or a structure of an H-module algebra for some Hopf algebra H, it is natural to consider, respectively, graded, G-, differential or H-identities. During the talk, we will discuss different problems involving numeric characteristics of such identities, as well related problems in the structure theory of H-(co)module associative and Lie algebras.

References

[1] Gordienko, A. S., Structure of H-(co)module Lie algebras. *J. Lie Theory* (to appear).

[2] Gordienko, A. S., Amitsur's conjecture for polynomial H-identities of H-module Lie algebras. *Tran. Amer. Math. Soc.* (to appear)

[3] Gordienko, A. S., Asymptotics of H-identities for associative algebras with an H-invariant radical. `arXiv:1212.1321 [math.RA] 6 Dec 2012`

[4] Gordienko, A. S., Co-stability of radicals and its applications to PI-theory. `arXiv:1301.2446v1 [math.RA] 11 Jan 2013`

Chains of prime ideals

Be'eri Greenfeld

Department of Mathematics
Bar-Ilan University
52900 Ramat-Gan, Israel
beeri.greenfeld@gmail.com

In a commutative ring, the union of an ascending chain of prime ideals is prime. We exhibit several examples of ascending chains of (semi)prime ideals with non-(semi)prime union. We discuss some special cases where the ideals in the chain have other properties, such as primitivity or finite Gel'fand-Kirillov codimension.

On the Jacobson radical of constants of derivations

Piotr Grzeszczuk

Faculty of Computer Science
Białystok University of Technology, Poland
piotrgr@pb.edu.pl

Let R be an associative ring, d its derivation and $R^d = \ker d$ the subring of constants of d. In the talk we will discuss the relationships between the Jacobson radicals of R and R^d. Particular attention will be paid to two special cases, where d is either algebraic or locally nilpotent. Applications to the semiprimitive rings with semilocal rings of constants of algebraic derivations will be presented.

References

[1] J. Bergen, P. Grzeszczuk, Jacobson Radicals of Ring Extensions, Journal of Pure and Applied Algebra, **216** (2012), 2601–2607.

[2] P. Grzeszczuk, On the Jacobson radical of constants of derivations, (2013) preprint.

Hat-functor and its applications

Vsevolod Yu. Gubarev

Sobolev Institute of Mathematics
The Siberian Branch of the Russian Academy of Sciences
4 Acad. Koptyug avenue
630090 Novosibirsk, Russia
vsevolodgu@mail.ru

The classical object of an algebraic study is an algebra over a field, i.e., a linear space with one bilinear operation of multiplication. Among them, there are various classes (classical varieties), like associative, Lie, alternative, Jordan, Malcev algebras et al. To the moment there are enough examples of similar algebraic systems: superalgebras, group algebras, di- and trialgebras, generalised trialgebras, pre- and post-algebras (dendriform di- and trialgebras), Hom-algebras, linearly compatible algebras, totally compatible algebras and other. The algebraic systems from the list are not always varieties in the usual sense but they are all related to the classical varieties.

For some of them there are suggested approaches to determine the varieties, for others definitions are available only for some of the classic varieties.

In this work we study the general approach to the definition of varieties for different types of algebras based on definition of varieties of ordinary algebras. For that purpose, we construct a special functor "hat" and then consider some examples of problems that can be solved by means of this functor.

The work is supported by RFBR grant 12-01-33031 and the Federal Target Grant "Scientific and educational staff of innovation Russia" for 2009–2013 (contract 14.740.11.1510).

Definable classes and Mittag-Leffler conditions

Dolors Herbera

Departament de Matemàtiques Universitat Autònoma de Barcelona
E-08193 Bellaterra (Barcelona), Spain
dolors@mat.uab.cat

Definable classes of modules are a natural object of study in Model Theory. In algebraic terms, a class of modules is said to be definable if it is closed under products, direct limits and pure submodules.

Following Raynaud and Gruson [6] a right R-module M_R is said to be Mittag-Leffler if the natural morphism

$$M \otimes_R \prod_{i \in I} Q_i \to \prod_{i \in I} (M \otimes Q_i) \tag{1}$$

is injective for any family of left R-modules $\{Q_i\}_{i \in I}$. Restricting the choosing of the modules Q_i in (1) we get the relative Mittag-Leffler modules, cf. [7].

Recently the interest in the relative Mittag-Leffler conditions have been strongly renewed. Some reasons for that are its applications to the characterization of the vanishig of the functor Ext [1, 3] and because they provide examples of non-deconstructible and of non precovering classes [2, 4, 8, 9].

In this talk we want to explain how relative Mittag-Leffler conditions have a key role when characterizing definable classes defined either as intersection of kernels of Tor-functors or as intersection of kernels of Ext-functors.

The results we present are contained in [5].

References

[1] L. Angeleri Hügel, S. Bazzoni and D. Herbera, *A solution to the Baer splitting problem*, Trans. Amer. Math. Soc. **360** (2008), 2409-2421.

[2] Silvana Bazzoni and Jan Šťovíček, *Flat Mittag-Leffler modules over countable rings*, Proc. Amer. Math. Soc. **140** (2012), 1527–1533.

[3] I Emmanouil and O. Talelli, *On the flat length of injective modules.* J. London Math. Soc. (2)**84** (2011), 408–432.

[4] Dolors Herbera, Jan Trlifaj, *Almost free modules and Mittag-Leffler conditions*, Adv. Math. **229** (2012), 3436–3467.

[5] Dolors Herbera, *Definable classes and Mittag-Leffler conditions.* Preprint

2012.

[6] M. Raynaud et L. Gruson, *Critères de platitude et de projectivité*, Invent. Math. **13** (1971), 1–89.

[7] P. Rothmaler, Mittag-Leffler modules and positive atomicity. Habilitationsschrift, Kiel, 1994.

[8] J. Šaroch and J. Trlifaj, *Kaplansky classes, finite character and* $\aleph_1$-*projectivity*, Forum Math. **24** (2012), 1091–1109.

[9] A. Slávik and J. Trlifaj, *Approximations and locally free modules*, Preprint (2012).

On lattices of annihilators

Małgorzata Jastrzębska

Institute of Mathematics, University of Warsaw
ul. Banacha 2, 02-097 Warszawa, Poland
mjastrzebska@mimuw.edu.pl

In this talk K will be a field and A an associative K-algebra with $1 \neq 0$.

It is well know that the set $\mathcal{I}_l(A)$ of all left ideals and the set $\mathcal{I}_r(A)$ of all right ideals are complete modular lattices under operations:

$$I \vee J = I + J \quad \text{and} \quad \wedge_{k\in K} I_k = \cap_{k\in K} I_k.$$

However, properties of these lattices can be quite different.

Further, if $X \subseteq A$ is a subset then let $L_A X = LX$ be the left annihilator of X in A and let $R_A X = RX$ be the right annihilator of X in A. Let $\mathcal{A}_l(A)$ be the set of all left annihilators in A and let $\mathcal{A}_r(A)$ be the set of all right annihilators in A. Then $\mathcal{A}_l(A) \subseteq \mathcal{I}_l(A)$ and $\mathcal{A}_r(A) \subseteq \mathcal{I}_r(A)$ are complete lattices under operations for left annihilators given by:

$$I \vee J = LR(I + J) \quad \text{and} \quad \wedge_{k\in K} I_k = \cap_{k\in K} I_k.$$

Analogous formula is for right annihilators. In this case the operators L and R give a Galois correspondence between $\mathcal{A}_l(A)$ and $\mathcal{A}_r(A)$.

In several papers QF-algebras and other algebras with $\mathcal{I}_l(A) = \mathcal{A}_l(A)$ were investigated. On the other hand, some strange properties of the lattice $\mathcal{A}_l(A)$ were exhibited. In this talk we are going to present the following result:

Theorem (with Jan Krempa). *Let L be a lattice. Then there exists an algebra A_L and a natural lattice embedding of L into $\mathcal{A}_l(A_L)$. If L is finite then we can assume that A_L is finite dimensional.*

References

[1] M. Ya. Finkel'stein, *Rings in which annihilators form a sublattice of the lattice of ideals,* Siberian Math. J. 24(6) (1983), 160-167.
[2] M. Jaegermann, J. Krempa, *Rings in which ideals are annihilators*, Fund. Math. 76(1972), 95-107.
[3] J. W. Kerr, *Very long chains of annihilator ideals,* Israel J. Math.

46(1983), 197-204.
[4] J. Krempa, D. Niewieczerzał, *Rings in which annihilators are ideals and their application to semi-group rings,* Bull. Acad. Polon. Sci. 25(1977), 851-856.
[5] D. Niewieczerzał, *Some examples of rings with annihilator conditions,* Bull. Acad. Polon. Sci. 26(1978), 1-5.
[6] C.R. Yohe, *On rings in which every ideal is the annihilator of an element,* Proc. Amer. Math. Soc. 19(1968), 1346-1348.

On commutative finite dimensional Gorenstein algebras and their smoothability

Joachim Jelisiejew

student of
Faculty of Mathematics, Informatics, and Mechanics
University of Warsaw, Poland
jj277546@students.mimuw.edu.pl

We consider finite dimensional commutative algebras over an algebraically closed field k. In this class the reduced algebras are just products of k. It is natural to ask which other algebras are "limits" of reduced ones, i.e. are a member of a flat family of algebras whose general member is reduced. Such algebras are called *smoothable*. In general, the question of smoothability of a given algebra is difficult. Some particular results, for algebras of low length, are obtained in [1].

We investigate a special class of algebras – local *Gorenstein* algebras (studied in e.g. [2]). In our setup a local algebra is Gorenstein if any two nonzero ideals have nonzero intersection. There are known examples of non-smoothable local Gorenstein algebras of length 14 and we work on proving smoothability of all algebras of lower length. Even though the problem is rather specialized, the methods used are general — e.g. the local Gorenstein algebras are classified as quotients of the ring of partial differential operators annihilating a given polynomial, and proving smoothability requires certain criteria for flatness.

References

[1] Dustin A. Cartwright, Daniel Erman, Mauricio Velasco, Bianca Viray, *Hilbert schemes of 8 points.* Algebra Number Theory 3 (2009), no. 7, 763-795.

[2] Gianfranco Casnati, Roberto Notari, *On the Gorenstein locus of some punctual Hilbert schemes.* Journal of Pure and Applied Algebra Volume 213, Issue 11, November 2009, Pages 2055-2074.

Lie regular elements in general linear groups

Pramod Kanwar*

Department of Mathematics,
Ohio University - Zanesville,
Zanesville, OH 43701, USA
kanwar@ohio.edu

R. K. Sharma

Department of Mathematics,
Indian Institute of Technology,
Delhi, India
rksharma@maths.iitd.ernet.in

Pooja Yadav

Department of Mathematics,
Kamla Nehru College,
Delhi, India
iitd.pooja@gmail.com

An element a of a ring R is called *Lie regular* if it can be expressed as a Lie product of an idempotent in R and a unit in R. A unit in R is said to be a *Lie regular unit* if it is Lie regular as an element of R. Among other things it is shown that the linear groups $GL(2, \mathbb{Z}_{2^n})$, $GL(2, \mathbb{Z}_{p^n})$ (where p is an odd prime), $GL(2, \mathbb{Z}_{2p^n})$ (where p is an odd prime), $GL(2, \mathbb{Z}_{3p^n})$ (where p is a prime greater than 3) and $GL(2, \mathbb{Z}_{5p^n})$ (where p is a prime greater than 5) can be generated by Lie regular matrices. Presentations of some linear groups using Lie regular matrices are also given.

References

[1] H. S. M. Coxeter and W. O. J. Mosser, *Generators and Relations for Discrete Groups*, Springer-Verlag, 1980.

[2] J. E. Humphreys, *Introduction to Lie Algebras and Representation Theory*, Springer-Verlag, 1980.

[3] Pramod Kanwar, R. K. Sharma, and Pooja Yadav, Lie Regular Generators of General Linear Groups, Comm. Algebra, **40 (4)**, 1304-1315, 2012.

[4] Pramod Kanwar, R. K. Sharma, and Pooja Yadav, Lie Regular Generators of General Linear Groups II, International Electronic Journal of Algebra, **13**, 91-108, 2013.

[5] G. Karpilovsky, *Unit Group of Group Rings*, Pitmann Monographs, 1989.

[6] A. Karrass, D. Solitar, W. Magnus, *Combinatorial Group Theory*, Dover Publications, INC, 1975.

Characterizations of the strongly prime radical of a module

Algirdas Kaučikas

Ateities str. 20
M. Romeris University, Vilnius, Lithuania
algirdas.kaucikas@mruni.eu

We consider associative rings and unitary left modules over such rings. The notion of a one sided strongly prime ring, introduced in [1] is naturally extended for a module over a ring.
A (left) non-zero module M over a ring R is called *strongly prime* if for any non-zero $x, y \in M$, there exits a finite set of elements $\{a_1, ..., a_n\} \subseteq R$, $n = n(x, y)$, such that $Ann_R\{a_1x, ..., a_nx\} \subseteq Ann_R\{y\}$, i.e., that $ra_1x = ... = ra_nx = 0$, $r \in R$, implies $ry = 0$. Immediately we obtain notions of a one sided strongly prime submodule and, in particular, one sided strongly prime ideal of a ring. Strongly prime modules, submodules and ideals over a commutative ring are called prime.
Let now M be a nonzero left finitely generated R-module. We denote by $M\langle X_1, \ldots, X_n\rangle$ the module of polynomials over M with noncommuting indeterminates $X_1, \ldots, X_n$. Denote by J_n the Jacobson radical of a $R\langle X_1, \ldots, X_n\rangle$-module $M\langle X_1, \ldots, X_n\rangle$.

Let M be a finitely generated R-module. In the talk we discuss the following results on the left strongly prime radical Sp_lM of a module M.
Theorem 1.

$$Sp_lM = \bigcap_{n\in\mathbb{N}} (M \cap J_n).$$

Theorem 2. *If R is commutative, then we may take commuting indeterminates $X_1, \ldots, X_n$ and their number does not exceed the number of generators of a module M.*
Theorem 3. *For any module M over a commutative ring and any element $a \in Sp_lM$ there exists finitely generated submodule $N \subseteq M$ such that $a \in Sp_lN$.*
This result was proved in [4], when R is Noetherian domain of dimension 1. We also give an internal characterization for the prime radical of a module over a commutative ring.

References

[1] A. Amini, B. Amini, On strongly superfluous submodules, Comm. Algebra 40: 2906-2919 (2012)

[2] A. Azizi, Radical formula and prime submodules, J. Algebra, 307(2002), 454-460

[3] D. Handelman, J. Lawrence, Strongly prime rings, T.A.M.S. 211, 209-223, (1975)

[4] J. Jenkins, P. F. Smith, On the prime radical of a module over a commutative ring, Comm. Algebra, 20: 3593-3602 (1992)

[5] A. Kaučikas, R. Wisbauer, Noncommutative Hilbert rings, J. Alg. Appl., Vol.3, No 4, 437-443 (2004)

[6] A. Kaučikas, On the left strongly prime modules, ideals and radicals, in: Analytic and Probabilistic Methods in Number Theory, TEV, Vilnius (2002), pp 119-123.

[7] A.L. Rosenberg, The left spectrum, the Levitzki radical, and noncommutative schemes, Proc. Natl. Acad. Sci.USA, 87, 8583-8588, (1990).

Subexponential estimates in the height theorem and estimates on numbers of periodic parts of small periods

Alexei Belov-Kanel

Bar-Ilan University, Israel
kanelster@gmail.com

Mikhail Kharitonov*

Moscow State University, Russia
mikhailo.kharitonov@gmail.com

The paper is devoted to subexponential estimations in Shirshov's Height theorem. A word W is n-*divisible*, if it can be represented in the following form: $W = W_0W_1 \cdots W_n$ such that $W_1 \prec W_2 \prec \cdots \prec W_n$, where $\prec$ is comparison in lexicographical sense. If an affine algebra A satisfies polynomial identity of degree n then A is spanned by non n-divisible words of generators $a_1 \prec \cdots \prec a_l$. A. I. Shirshov proved ([1]) that the set of non n-divisible words over alphabet of cardinality l has bounded height h over the set Y consisting of all the words of degree $\leqslant n-1$.

We show, that $h < \Phi(n,l)$, where

$$\Phi(n,l) = 2^{96} l \cdot n^{12\log_3 n + 36 \log_3\log_3 n + 91}.$$

Let l, n, $d \geqslant n$ be positive integers. Then all the words over alphabet of cardinality l which length is greater than $\Psi(n,d,l)$ are either n-divisible or contain d-th power of subword, where

$$\Psi(n,d,l) = 2^{27} l (nd)^{3\log_3(nd) + 9\log_3\log_3(nd) + 36}.$$

See also proof of this estimates in [2], [3].

In 1993 E. I. Zelmanov asked the following question in Dniester Notebook [4]:

"Suppose that $F_{2,m}$ is a 2-generated associative ring with the identity $x^m = 0$. Is it true, that the nilpotency degree of $F_{2,m}$ has exponential growth?"

We give the definitive answer to E. I. Zelmanov by this result. We show that the nilpotency degree of l-generated associative algebra with the identity $x^d = 0$ is smaller than $\Psi(d,d,l)$. This imply subexponential estimations on the nilpotency index of nil-algebras of an arbitrary characteristics. Original Shirshov's estimation was just recursive, in 1982 double exponent was obtained ([5]), an exponential estimation was obtained in 1992 ([6], [7]).

Our proof uses Latyshev idea of Dilworth's theorem application ([8]). We think that Shirshov's height theorem is deeply connected to problems of modern combinatorics. In particular this theorem is related to the Ramsey theory.

Above all we obtain lower and upper estimates of the number of periods of length $2, 3, (n-1)$ in some non n-divisible word (see also [9], [10]). These estimates are differ only by a constant.

Keywords: Height theorem, combinatorics on words, n-divisibility, Dilworth's theorem, Burnside type problems, Shirshov's height.

References

[1] A. I. Shirshov. *On rings with identical relations (Russian).* Sb. Math., 1957, Vol. 43, No 2, pages 277–283.

[2] A. Ya. Belov, M. I. Kharitonov. *Subexponential estimates in the height theorem and estimates on numbers of periodic parts of small periods.* Fundamental and Applied Mathematics, Vol. 17 (2011/2012), No 5, pages 21–54.

[3] A. J. Belov, M. I. Kharitonov. *Subexponential estimates in Shirshov theorem on height.* Sb. Math., 2012, No 4, 534–553.

[4] *Dnestr copy-book: a collection of operative information (Russian), issue 4, Novosibirsk: Institute of mathematics, SO AN SSSR, 1993.* page 73.

[5] A. G. Kolotov. *An upper estimate for height in finitely generated algebras with identities (Russian).* Sib. Mat. Zh., 1982, Vol. 23, No 1, pages 187–189.

[6] Belov, A. Ya. *Some estimations for nilpotency of nil-algebras over a field of an arbitrary characteristic and height theorem.* Commun. Algebra. 20 (1992), No. 10, 2919–2922.

[7] Belov, A. Ya., Borisenko, V. V., and Latyshev, V. N. *Monomial algebras.* Algebra 4, J. Math. Sci. (New York) 87 (1997), No 3, 3463–3575.

[8] Latyshev, V. N. *On Regev's theorem on identities in a tensor product of PI-algebras.* Uspehi Mat. Nauk., 27(1972), 213–214.

[9] M. I. Kharitonov. *Two-sided estimates for essential height in Shirshov height theorem (Russian).* Vestnik Moskovskogo Universiteta, Ser. 1, Matematika. Mekhanika, 2012, No 2, pages 20–24.

[10] M. I. Kharitonov. *Estimates for structure of piecewise periodicity in Shirshov height theorem (Russian).* Vestnik Moskovskogo Universiteta, Ser. 1, Matematika. Mekhanika, 2013, No 1, pages 10–16.

Endomorphisms of Quantum Generalized Weyl Algebras

Andrew Kitchin

School of Mathematics, Statistics and Actuarial Science
Cornwallis Building
University of Kent
Canterbury, Kent CT2 7NF, UK
ak562@kent.ac.uk

We study quantum generalized Weyl algebras $A(a, q)$ over a Laurent polynomial ring, where $q \in \mathbb{K}^*$ is not a root of unity and $\sigma(h) = qh$. We give a complete answer to the classification of endomorphisms of $A(a, q)$. In the case where these algebras are simple, our classification shows that every endomorphism is an automorphism.

On left spectrum of rings functions with value in principal ideal V-domain

Mykola Ya. Komarnitskii

Ivan Franko National University of L'viv, Ukraine
mykola_komarnytsky@yahoo.com

Let $\{R_i\}_{i\in I}$ by the family of exemplars of associative ring R with a unit i.e. $\{R_i = R\}$ for all $i \in I$, were I is any infinite set. Denote by $\mathfrak{D}$ a non trivial ultrafilter over the set I. Let $S = \prod\limits_{i\in I} R_i = R^I$ be a ring of functions on I with values from R. For each element $r = (r_i)_{i\in I} \in S$ we define $\Theta(r) = \{i \in \Omega | r_i = 0\}$. The set $\mathfrak{A}_{\mathfrak{D}} = \{r \in S | \Theta(r) \in \mathfrak{D}\}$ is a two sided ideal in S. Factor-ring $S/\mathfrak{A}_{\mathfrak{D}}$ is called an ultrapower of R, relative to the ultrafilter $\mathfrak{D}$ and is specified through $\prod\limits_{i\in I} R/\mathfrak{D}$.

Recall that the ring R is called left (right) V-ring, if every simple left (right) R-module is injective. Further assume that ring R is principal ideal V-domain (i.e. left and right principal ideal V-ring and does not contain zero divisors). Results of talk will be related to the solutions of such problems:

- finding all left maximal (left strongly-prime, left prime ideals) of ring S, where ideals and properties of ring R are known;

- description of topology on the ring S with known topologies, that are determined by A. L. Rosenberg on the ring R, [1];

- description of left torsion-theoretic spectrum of the ring S.

Solutions of such problems are based on the authors results, that are concerned on maximal ideals in determined above ultrapower of domain R, [2].

References

[1] Rosenberg A. Noncommutative algebraic geometry and representations of quantized algebras // Kluwer Academic Publishers, (1995) 316 p.

[2] Komarnitskii, M. Ya. The Cozzens-Faith problem on "countable" ultrapowers of the Koifman-Cozzens V-domain. (Russian), Mat. Stud., 1997, 7 , no. 1, p.3-26,

On rings with finite number of orbits

Jan Krempa

Institute of Mathematics, University of Warsaw
ul. Banacha 2, 02-097 Warszawa, Poland
jkrempa@mimuw.edu.pl

Let R be an associative unital ring with the unit group $U(R) = U$. Then the group $U \times U$ acts on R by left and right multiplication in the following way:

$$(a, b) \rightharpoonup x = axb^{-1}$$

for all $a, b \in U$ and $x \in R$.

If $\mathcal{S} \subseteq 2^R$ denotes: the set of elements of R, of left (right) ideals of R, of twosided ideals of R, or of subgroups of the additive group R^+, then the above action induces in a natural way an action of the group $U \times U$ on $\mathcal{S}$.

In this talk I'm going to discuss some properties of rings R with a finite number of orbits in $\mathcal{S}$ under this action. The results are obtained in cooperation with dr Małgorzata Hryniewicka. Our research was inspired by the papers [1] and [3]. Some results in the same spirit, but for finite dimensional algebras over fields, were recently obtained in [2].

References

[1] Y. Hirano, *Rings with finitely many orbits under the regular action,* Lecture Notes in Pure and Appl. Math. 236, Dekker, New York 2004, 343–347.

[2] A Męcel, J. Okniński, *Conjugacy classes of left ideals of finite dimensional algebra,* Publicationes Mathematique, (to appear).

[3] J Okniński, L.E. Renner, *Algebras with finitely many orbits,* J. Algebra 264 (2003), 479–495.

Remarks on Lie-nilpotent rings

Kamil Kular

Institute of Mathematics
Cracow University of Technology
Kraków, Poland
and
Institute of Mathematics
Jagiellonian University
Kraków, Poland
kamil-kular@wp.pl

Marcin Skrzyński*

Institute of Mathematics
Cracow University of Technology
Kraków, Poland
pfskrzyn@cyf-kr.edu.pl

Let R be an associative ring wih unity. For $a, b \in R$ we define $[a, b] = ab - ba$. Moreover, we define the sequence $\{\ell_n(R)\}_{n=0}^{\infty}$ of subsets of R by

$$\begin{cases} \ell_0(R) = R, \\ \ell_{n+1}(R) = \{[a, b] : \ a \in \ell_n(R), \, b \in R\} . \end{cases}$$

Finally, let $\mathrm{Der}(R)$ be the Lie ring of all the derivations $d : R \longrightarrow R$.

It follows immediately from a known theorem of Gupta and Levin [2] that if $\ell_n(R) = \{0\}$ for some n, then the unit group $U(R)$ is nilpotent with nilpotency class at most n.

If $\ell_n(R) = \{0\}$ for some n, then the ring R is said to be Lie-nilpotent. Notice that the ring $\mathbb{H}(\mathbb{Z})$ of integer quaternions is not Lie-nilpotent, while its unit group is nilpotent of class 2. Catino and Miccoli proved [1] that a local associative ring with unity is Lie-nilpotent if and only if its unit group is nilpotent.

It is easy to see that the ring R is commutative whenever $\mathrm{Der}(R) = \{0\}$, and $\ell_2(R) = \{0\}$ whenever $\mathrm{Der}(R)$ is abelian (i.e., $d_1 \circ d_2 = d_2 \circ d_1$ for all $d_1, d_2 \in \mathrm{Der}(R)$).

In the talk, we will present some examples, new observations and open questions on relationships between Lie-nilpotency of R, nilpotency of $U(R)$, and abelianity of $\mathrm{Der}(R)$.

References

[1] F. Catino & M. M. Miccoli, Local rings whose multiplicative group is nilpotent, *Arch. Math.* **81**, No. 2: 121-125 (2003).

[2] N. Gupta & F. Levin, On the Lie ideals of a ring, *J. Algebra* **81**: 225-231 (1983).

A quadratic Poisson Gel'fand-Kirillov problem in prime characteristic

César Lecoutre

University of Kent
34 Stour Street
CT1 2NZ
Canterbury, Kent, UK
cl335@kent.ac.uk

We study the fields of fractions of certain Poisson polynomial algebras over a field of arbitrary characteristic. Especially we show that such a field of fractions is Poisson isomorphic to a field of rational functions endowed with so-called quadratic Poisson brackets. We get this isomorphism by performing Poisson changes of variables in the Laurent polynomial ring associated to the Poisson polynomial algebra under consideration. The results also extend to homogeneous Poisson prime quotients. Examples include Poisson matrix varieties and Poisson determinantal varieties.

Joint work with Stéphane Launois.

References

[1] K. R. Goodearl and S. Launois, *The Dixmier-Moeglin equivalence and a Gel'fand-Kirillov problem for Poisson polynomial algebras*, Bull. Soc. Math. France **139** (2011), no. 1, 1-39.

[2] S. Launois and C. Lecoutre, *A quadratic Poisson Gel'fand-Kirillov problem in prime characteristic*, to be published.

Singular matrices as products of idempotents

André Leroy*

Université d'Artois, Lens, France
andre.leroy@univ-artois.fr

Adel Alahmadi

King Abdulaziz University, Jeddah, Saudi Arabia
adelnife2@yahoo.com

Surender K. Jain

Ohio University, Athens, USA
and KAU University, Jeddah, Saudi Arabia
jain@ohio.edu

Let R be a ring with unity. A matrix $A \in M_n(R), n \geq 2$ is singular if $lann(A) \neq 0$ and $rann(A) \neq 0$. We say that R has the IP property if any square singular matrix can be written as a product of idempotent matrices. Erdös showed that a field has the IP property and this was extended to the case of a division ring and commutative euclidean domain by Laffey. In the talk we will examine such factorizations over local rings and Bézout domains. We will show that the main step towards proving the IP property for a certain class of ring is to show that the factorization property is true for 2 by 2 singular matrices. More specifically the factorization of matrices of the form $\begin{pmatrix} a & b \\ 0 & 0 \end{pmatrix}$ is already of significant importance. We will emphasize the role of conditions such as stable range one and GE_2.

References

[1] J.A. Erdös, On products of idempotent matrices, Glasgow Math. J. **8** (1967) 118-122.

[2] J.M. Howie, The subsemigroup generated by the idempotents of a full transformation semigroup, J. London Math. Soc. **41** (1) (1966) 707-716.

[3] T.J. Laffey, Products of idempotent matrices, Linear and Multilinear Algebra **14** (4) (1983) 309-314.

Cyclic left and torsion-theoretic spectrums of a module and their relations

Marta O. Maloid-Glebova

Ivan Franko National University of L'viv, Ukraine
martamaloid@gmail.com

Let R be an associative ring with $1 \neq 0$. A left ideal $\mathfrak{p}$ of the ring R is called prime, if for every $x, y \in R$, $xRy \subseteq \mathfrak{p}$ implies either $x \in \mathfrak{p}$ or $y \in \mathfrak{p}$. Clearly, a left prime ideal is two-sided iff it is prime in classical way. The set of all two-sided prime ideals is denoted by $Spec(R)$ and is called (prime) spectrum of ring R. Recall definition of the pre-order $\leq$ on the set of left ideals of a ring R in following way: $\mathfrak{a} \leq \mathfrak{b}$ for left R-ideals $\mathfrak{a}$ and $\mathfrak{b}$ iff there exists a finite subset V of the ring R that $(\mathfrak{a} : V) \subseteq \mathfrak{b}$. A left prime ideal $\mathfrak{p}$ of the ring R is called the left Rosenberg point if $(\mathfrak{p} : x) \leq \mathfrak{p}$ for any $x \in R \backslash \mathfrak{p}$, [5] . The set of all left Rosenberg points of the ring R is called the left Rosenberg spectrum of R and is denoted by $spec(R)$.

The space $spec(R)$ may by defined in another way: it is the set of all strongly prime left ideals. Recall that a left ideal $\mathfrak{p}$ of the ring R is called strongly-prime, if for every $x \in R\backslash\mathfrak{p}$ there exists a finite set V of the ring R, that $(\mathfrak{p} : Vx) = \{r \in R : rVx \subseteq \mathfrak{p}\} \subseteq \mathfrak{p}$. Clearly, every strongly-prime left ideal of a ring R is prime left ideal and every maximal left ideal is strongly-prime. It is known, that if R is noetherian, then $Spec(R) \subseteq spec(R)$.

A nonzero left module M over a ring R is called strongly-prime, if for any nonzero $x, y \in M$ there exists a finite subset $\{a_1, a_2, \ldots, a_n\} \subseteq R$, that $Ann_R\{a_1x, a_2x, \ldots, a_nx\} \subseteq Ann_R\{y\}$, $(ra_1x = ra_2x = \cdots = ra_nx = 0)$, $r \in R$ implies $ry = 0$. [1]

A submodule P of some module M is called strongly-prime, if the quotient module M/P is a strongly-prime R-module. The set of all strongly-prime submodules of a module M is called the left prime spectrum of M and is denoted by $spec(M)$.

The relation of preorder on $spec(R)$ do not have a trivial generalization for modules, but it is possible for cyclic modules. For instance, let M be a cyclic module $Rm = R/Ann(m)$ and L, K are some submodules. We can represent L and K as $\mathfrak{A}/Ann(m)$ and $\mathfrak{B}/Ann(m)$ respectively, where $\mathfrak{A}$ and $\mathfrak{B}$ are some left ideals of ring R. Then we define $K \leq L$ iff $\mathfrak{A} \leq \mathfrak{B}$.

Lemma 1. *A submodule L of the module $Rm = R/Ann(m)$ is prime iff the left ideal $\mathfrak{A}$ is a left prime ideal of R.*

Rosenberg points of $spec(Rm)$ are submodules of Rm, that have the form $\mathfrak{A}/Ann(m)$, were $\mathfrak{A}$ is the Rosenberg point of $spec(R)$. All properties are carried out. So the spectrum of a cyclic module coincides with the set of all it's Rosenberg points.
Then the cyclic spectrum of an arbitrary module M is defined as a union of all spectrums of its cyclic submodules. The cyclic spectrum of a module M is denoted by $Cspec(M)$. Then we can define $L \leq K \iff Cspec(L) \subseteq Cspec(K)$ for all submodules of a module M and obtain preorder on the family of such submodules.
Lemma 2. *Let L and K be left cyclic R-modules. Then $L \leq K$ iff there exists a cyclic left R-module X, a monomorphism $X \leftarrowtail L^n$ and an epimorphism $X \twoheadrightarrow K$. In other words, there exist the diagram $(L)^n \leftarrowtail X \twoheadrightarrow K$.*

This lemma generalizes one statement from [3]. Based on both assertions, most technical results can be transferred on a modular basis. Using Zarisky-like topology on $spec(R)$, we can prove functoriality on $Cspec$, that is defined on morphisms, using scheme from [4]. There are established some properties, that are corresponding to the same from [6].

For torsion-theoretic spectrum of a module M, the concept of $supp(M) = \{\sigma \mid \sigma(M) \neq 0\}$ is given, where $\sigma(M)$ is the set of prime torsion-theories of a module M. Torsion-theoretic spectrum of a module M is defined as $M - sp = R - sp \cap supp(M)$, where $R - sp$ is torsion-theoretic spectrum of a ring R. (see [2])
Lemma 3. *If σ is torsion theory and $\mathfrak{P}$ is a left Rosenberg point of a module M, then $M/\mathfrak{P}$ is either σ-torsion module or σ-torsion free module.*
Proposition 1. *If M is an fully bounded left noetherian module and $\mathfrak{P} \in Cspec(M)$, then torsion theory $\tau_{\mathfrak{P}} = \chi(M/\mathfrak{P})$, cogenerated by module $M/\mathfrak{P}$, is prime.*
Theorem 1. *Mapping $\Phi : Cspec(M) \to M - sp$, where $\Phi(\mathfrak{P}) = \chi(M/\mathfrak{P})$ is continuous and surjective.*

Recall, that in Rosenberg paper, subspaces of left ring spectrum, that are related to left modules, are studied. Such ideals are called support of module and, to some extent, claim to the role of left spectrum of module. But it's construction is less clear than our, and less suitable for calculations. Questions about relationships between such constructions has yet to be explore.

References

[1] Beachy J.A., Some aspects of noncommutative localization, In book: Noncommutative Ring Theory, - LNM. - Vol. 545, Spriger-Verlag, Berlin, 1975, p. 2–31.

[2] Golan J. S. Topologies on the Torsion-Theoretic Spectrum of a Noncommutatie Ring // Pacific Journal of Mathematics, 1974 Vol. 51, No. 2, p 439-450;

[3] Jara P., Verhaeghe P., Verschoren A. On the left spectrum of a ring //Communs. Algebra, - 1994, - 22(8), p. 2983-3002.

[4] Letzter E.S. On continuous and adjoint morphisms between noncommutative prime spectra //Proc. Edinbourgh Math. Soc., 2006, 49, 367-381.

[5] Rosenberg A.L. The left spectrum, the Levitski radical, and noncommutative schemes. Proc. Nat. Acad. Sci. USA, 1990, 87, 8583-8586.

[6] Reyes M.L. Obstructing extensions of the functors Spec to noncommutative rings //Israel J. Math., 192(2012), 667 -698.

Right Gaussian rings

Ryszard Mazurek

Faculty of Computer Science
Bialystok University of Technology
Wiejska 45A, 15-351 Białystok, Poland
r.mazurek@pb.edu.pl

For a ring R and a polynomial $f \in R[x]$, let $c_r(f)$ be the right ideal of R generated by the coefficients of f. A ring R is called

- *right Gaussian* if $c_r(fg) = c_r(f)c_r(g)$ for any $f, g \in R[x]$;
- *Armendariz* if whenever the product of two polynomials over R is zero, then the products of their coefficients are all zero;
- *right distributive* if the lattice of right ideals of R is distributive.

Among commutative domains, Gaussian rings are precisely Prüfer domains. Right Gaussian rings can be characterized as right duo rings all of whose homomorphic images are Armendariz.

In this talk, basic properties of right Gaussian rings and their quotient rings will be presented and connections between right Gaussian rings and right distributive rings will be discussed. Various characterizations of right Gaussian skew power series rings and right Gaussian skew generalized power series rings will be given.

This is joint work with Michał Ziembowski.

References

[1] R. Mazurek, M. Ziembowski, Right Gaussian rings and skew power series rings, J. Algebra 330 (2011), 130-146

[2] R. Mazurek, M. Ziembowski, On a characterization of distributive rings via saturations and its applications to Armendariz rings and Gaussian rings, Rev. Mat. Iberoamericana, to appear

A module theoretic translation of Brown representabilty for homotopy categories

George Ciprian Modoi

Department of Mathematics, Babeş–Bolyai University
1, Mihail Kogălniceanu
400084, Cluj–Napoca, Romania
cmodoi@math.ubbcluj.ro

Brown representability theorem is a property of a triangulated category with coproducts indexed over arbitrary sets, which originates in abstract homotopy theory and provides, in this setting, a useful substitute of the celebrated Freyd adjoint functor theorem.

For a ring R the homotopy category of complexes of R-modules is naturally triangulated and has products and coproducts. We characterize the situation in which this category or its dual satisfies Brown representability in terms of the initial module category. In this way we are lead to some interesting module theoretic problems.

References

[1] G. C. Modoi, J. Šťovíček, *Brown representability often fails for homotopy categories of complexes*, J. K-Theory, **9** (2012), 151–160.

[2] G. C. Modoi, *Brown representability dual for homotopy categories*, preprint arXiv: 1207.2133v2 [math.CT].

Jacobson Pairs Preserve Properties

Pace P. Nielsen

Department of Mathematics
Brigham Young University
Provo, UT 84602, USA
pace@math.byu.edu

"Jacobson's Lemma" says that if $\alpha = 1 - ab$ is a unit then so is $\beta = 1 - ba$. The proof consists in constructing a map from the inverse of α to that of β. Surprisingly, in some cases one can replace the word "unit" with another property and this lemma continues to hold, sometimes even with the same proof. We investigate the implications of this fact for (unit-)regular rings and strongly clean rings, and we are able to make improvements to Jacobson's original proof in these new contexts. In particular, we find an improved "Jacobson map" from the inner inverses of α to those of β (in the case when α is regular). As one application of these ideas, we prove that α and β are always equivalent in a unit-regular ring.

This is joint work with T. Y. Lam.

References

[1] T. Y. Lam and Pace P. Nielsen: *Jacobson's lemma for Drazin inverses.* Denison Conf. Proceedings, Contemp. Math., A.M.S. (2013), 12 pp.

[2] T. Y. Lam and Pace P. Nielsen: *Inner inverses and inner annihilators in rings.* (submitted)

ALGEBRAS DEFINED BY HOMOGENEOUS SEMIGROUP RELATIONS

Jan Okniński

Institute of Mathematics, University of Warsaw
ul. Banacha 2, 02-097 Warszawa, Poland
okninski@mimuw.edu.pl

Recent results on finitely presented algebras defined by homogeneous semigroup relations are discussed. The considered classes of algebras include plactic algebras, Chinese algebras and certain algebras defined by quadratic relations. In particular, we deal with the minimal spectrum, the primitive spectrum and simple modules, and with the Jacobson radical. The talk is based on results obtained in collaboration with F. Cedó, J. Jaszuńska and Ł. Kubat.

References

[1] F. Cedó, J.Okniński, Gröbner bases for quadratic algebras of skew type, Proc. Edinburgh Math. Soc. 55(2012), 387–401.

[2] F. Cedó, J. Okniński, Minimal spectrum and the radical of Chinese algebras, Algebras Repres. Theory, http://link.springer.com/article/10.1007/s10468-012-9339-1#

[3] J. Jaszuńska and J. Okniński, Structure of the Chinese algebras, J. Algebra 346 (2011), 31–81.

[4] Ł. Kubat, J. Okniński, Plactic algebra of rank 3, Semigroup Forum 84 (2012), 241–266.

On Strongly $*$-Clean Rings

A. Çiğdem Özcan

Hacettepe University,
Department of Mathematics,
06800 Beytepe Ankara, Turkey
ozcan@hacettepe.edu.tr

A $*$-ring R is called a strongly $*$-clean ring if every element of R is the sum of a unit and a projection that commute with each other. In this article, we give some characterizations of strongly $*$-clean rings in terms of the ring of real valued functions over the topological space X where X is the set of all maximal ideals, all prime ideals and all prime ideals containing the Jacobson radical, respectively.

This is joint work with H. Chen and A. Harmancı.

About weak factorization properties

Andreas Reinhart

Karl-Franzens-Universität Graz
Heinrichstraße 36
8010 Graz, Austria
andreas.reinhart@uni-graz.at

In this talk we consider various generalizations of factorial domains (and monoids). We will discuss well known classes of integral domains, like Mori domains, Krull domains and FF-domains and their relations with less known concepts, like (quasi) idpf-domains, t-SP-domains and radical factorial domains. In particular, we investigate the gap between Krull domains and t-SP-domains. Moreover, we will show that completely integrally closed, atomic idpf-domains can fail to be Krull domains.

Algebras in Symmetric Monoidal Categories

Dmitriy Rumynin

Mathematics Institute
University of Warwick
Coventry, CV4 7AL
U.K.
D.Rumynin@warwick.ac.uk

We will discuss how to do algebra in a symmetric monoidal category. We will discuss the notion of simple Lie algebra as well as examples of such, including Deligne's conjecture. We will also discuss how one can study identities of algebras in symmetric monoidal categories.

(Semi)primitivity of some classes of semigroup rings

Catarina Santa-Clara

Centro de Algebra da Universidade de Lisboa
Avenida Professor Gama Pinto, 2
1649-003 Lisboa, Portugal
cgomes@fc.ul.pt

In [1,2], W. D. Munn characterized primitivity and semiprimitivity of semigroup rings of inverse semigroups. We look into other classes of semigroups. This is joint work with Gracinda M. S. Gomes and Filipa Soares.

References

[1] W. D. Munn, *A class of contracted inverse semigroup rings*, Proceedings of the Royal Society of Edinburgh, **107A** (1987), 175–196.

[2] W. D. Munn, *On contracted semigroup rings*, Proceedings of the Royal Society of Edinburgh, **115A** (1990), 109–117.

The enveloping algebra of the Witt algebra is not noetherian

Susan J. Sierra

School of Mathematics, University of Edinburgh
King's Buildings, Mayfield Road
Edinburgh, EH9 3JZ, UK
s.sierra@ed.ac.uk

Whether or not it is possible for the enveloping algebra of an infinite-dimensional Lie algebra to be noetherian is a long-standing question. We answer this in the negative for the positive Witt algebra, and as a result for the full Witt algebra and for the Virasoro algebra. It follows that if L is an infinite-dimensional $\mathbb{Z}$-graded simple Lie algebra of polynomial growth, then $U(L)$ is not noetherian.

This is joint work with Chelsea Walton.

References

[1] Susan J. Sierra and Chelsea Walton, *The universal enveloping algebra of the Witt algebra is not noetherian*, arxiv:1304.0114, 2013.

Sets of lengths in maximal orders in central simple algebras

Daniel Smertnig

Institut für Mathematik und Wissenschaftliches Rechnen
Karl-Franzens-Universität Graz
Heinrichstraße 36, Graz, Austria
daniel.smertnig@uni-graz.at

In a noetherian domain every element can be expressed as a product of irreducibles, but in general this factorization is not unique. In the setting of commutative rings and monoids, there is a long tradition of studying this non-uniqueness by means of arithmetical invariants.

In a non-commutative setting we may define similar invariants: Let R be a maximal order in a central simple algebra over a global field, and let $x \in R$ be a non-unit non-zero-divisor. A positive integer l is a length of x if there exists a factorization $x = u_1 \cdot \ldots \cdot u_l$ into irreducibles. The set of lengths of x, $\mathsf{L}(x)$, consists of all such lengths of x and is a finite set. The system of sets of lengths of R is $\mathcal{L}(R) = \{\, \mathsf{L}(x) \mid x \in R,\ x \text{ a non-zero-divisor} \,\}$.

We present some results on $\mathcal{L}(R)$ in this context: If every stably free left R-ideal is free, then sets of lengths behave similarly as in a commutative Krull domain with finite class group. On the other hand, if R is a maximal order in a central simple algebra A over a number field and there exist stably free non-free left R-ideals (in this case A is necessarily a totally definite quaternion algebra), then sets of lengths exhibit distinctively different features.

Centralizers and zero-level centralizers in endomorphism rings

Jenő Szigeti

University of Miskolc, Hungary
matjeno@uni-miskolc.hu

The centralizer $\mathrm{Cen}(\varphi) \subseteq \mathrm{End}_R(M)$ of a nilpotent endomorphism φ of a finitely generated semisimple left R-module ${}_RM$ (over an arbitrary ring R) is the homomorphic image of the opposite of a certain $\mathrm{Z}(R)$-subalgebra of the full $m \times m$ matrix algebra $\mathrm{M}_m(R[z])$, where m is the dimension of $\ker(\varphi)$. If R is a local ring, then we give a complete characterization of $\mathrm{Cen}(\varphi)$ and of the containment $\mathrm{Cen}(\varphi) \subseteq \mathrm{Cen}(\sigma)$, where σ is a not necessarily nilpotent element of $\mathrm{End}_R(M)$. The zero-level centralizer (or two-sided annihilator) $\mathrm{Cen}_0(\varphi)$ of (an arbitrary) endomorphism φ is an ideal of $\mathrm{Cen}(\varphi)$. We also present some results about $\mathrm{Cen}_0(\varphi)$, $\mathrm{Cen}(\varphi)/\mathrm{Cen}_0(\varphi)$ and $\mathrm{Cen}_0(\mathrm{Cen}_0(\varphi))$.

This is joint work with V. Drensky and L. van Wyk

Dimension-friendly rings

Lia Vaš

University of the Sciences
600 S. 43rd St.
Philadelphia, PA 19104 USA
l.vas@usciences.edu

The existence of a well-behaved dimension of a finite von Neumann algebra has lead to the study of such a dimension of some finite Baer $*$-rings. This dimension is closely related to the equivalence relation $\overset{*}{\sim}$ on projections defined by $p \overset{*}{\sim} q$ iff $p = xx^*$ and $q = x^*x$ for some x. However, the equivalence $\overset{a}{\sim}$ on projections (or, in general, idempotents) defined by $p \overset{a}{\sim} q$ iff $p = xy$ and $q = yx$ for some x and y, can also be relevant. There were attempts to unify the two approaches.

We study the assumptions on a ring that guarantee the existence of a well-behaved dimension for any general equivalence relation on projections $\sim$. By interpreting $\sim$ as $\overset{a}{\sim}$, we prove the existence of a well-behaved dimension of strongly semihereditary rings with a positive definite involution. This class is wider than the class of finite Baer $*$-rings with dimension considered in the past: it includes some rings that are not Rickart $*$-rings.

A class of formal matrix rings

Yiqiang Zhou*

Memorial University of Newfoundland
St.John's, Canada
zhou@mun.ca

Gaohua Tang

Guangxi Teacher's Education University
Nanning, China

Given a ring R, our concern is about the formal matrix ring $\mathbb{M}_n(R; s)$ over R defined by a central element s of R. When $s = 1$, $\mathbb{M}_n(R; s)$ is just the matrix ring $\mathbb{M}_n(R)$, but generally $\mathbb{M}_n(R; s)$ can be significantly different from $\mathbb{M}_n(R)$. In this talk, we will present some basic properties of the ring $\mathbb{M}_n(R; s)$, address the isomorphism problem between these rings, and discuss extending some known results from a matrix ring to the formal matrix ring $\mathbb{M}_n(R; s)$.

References

[1] G. Tang, Y. Zhou, A class of formal matrix rings, *Linear Algebra Appl.*, in press.

On classical rings of quotients of duo rings

Michał Ziembowski

Faculty of Mathematics and Information Science,
Warsaw University of Technology, Poland
m.ziembowski@mini.pw.edu.pl

In this talk we will construct a duo ring R such that the classical right ring of quotients $Q^r_{cl}(R)$ of R is neither right nor left duo. This gives a negative answer to [1, Question 1].

References

[1] A.J. Diesl, C.Y. Hong, N.K. Kim, P.P. Nielsen, Properties which do not pass to classical rings of quotients, J. Algebra 379 (2013) 208–222.