# **Algebraic & Differential Topology**

*A Conference in Honor of Wu-chung Hsiang's 70<sup>th</sup> Birthday* 

# Stanford University August 6<sup>th</sup> and 7<sup>th</sup>, 2005

We are pleased to invite you to a conference at Stanford University, during the weekend of August 6 and 7, 2005. The conference is in honor of the 70<sup>th</sup> birthday of Professor Wu-chung Hsiang. There will be lectures during the day on Saturday and Sunday, and a banquet in Wu-chung's honor on Saturday evening.

#### **Speakers include:**

Soren Galatius (Stanford) ~ Thomas Goodwillie (Brown) Allen Hatcher (Cornell) ~ Bob Oliver (Paris-Nord, XIII) Kiyoshi Igusa (Brandeis) ~ Ian Hambleton (McMaster) Lowell Jones (SUNY Stony Brook) ~ Mike Davis (Ohio State)



# **Program of Talks:**

All talks to be held in Room 380-380C, Mathematics Department (Building 380)

Saturday, August 6	
9:00 a.m. – 9:30 a.m.	Continental breakfast outside room 380C
9: 30 a.m. – 10:30 a.m.	Tom Goodwillie: Geometric Language for the Functor/Function Analogy
10:30 a.m. – 11:00 a.m.	Break
11:00 a.m. – 12:00 p.m.	Allen Hatcher: A Topological Proof of Harer's Homology Stability Theorem
12:00 p.m. – 2:30 p.m.	Lunch—You're free to visit any of our local eateries
2:30 p.m. – 3:30 p.m.	Kiyoshi Igusa: Diffeomorphisms of Disks and Spheres Revisited
3:30 p.m. – 4:00 p.m.	Tea/Break
4:00 p.m. – 5:00 p.m.	Lowell Jones: A New Approach to Proving the "Injectivity Half" of the Isomorphism Conjecture
6:30 p.m.	Reception followed by dinner at the Stanford Faculty Club; spouses, family, and friends are welcome (please notify Pat Cahill no later than August 2 if you wish to attend the dinner < <u>cahill@math.stanford.edu</u> >)
Sunday August 7	
Sunday, August 7 9:00 a.m. – 9:30 a.m.	Continental breakfast outside room 380C
9: 30 a.m. – 10:30 a.m.	<b>Soren Galatius:</b> Stable Homology of the Automorphism Group of Free Groups.
10:30 a.m. – 11:00 a.m.	Break
11:00 a.m. – 12:00 p.m.	Ian Hambleton: Free Actions on Products of Spheres
12:00 p.m. – 2:30 p.m.	Lunch—You're free to visit any of our local eateries
2:30 p.m. – 3:30 p.m.	Bob Oliver: p-local Groups
3:30 p.m. – 4:00 p.m.	Tea/Break
4:00 p.m. – 5:00 p.m.	<i>Mike Davis:</i> The Weighted L <sup>2</sup> Cohomology of Coxeter Groups

#### **Titles & Abstracts**

### Saturday, August 6

#### Tom Goodwillie (Brown University)

**Title:** Geometric Language for the Functor/Function Analogy

**Abstract:** This talk is about a point of view. Functors TOP—SPECTRA that preserve weak homotopy equivalences may be considered as global functions on the variety TOP. We sketch a dictionary for this kind of geometry. Functors TOP/B—SPECTRA on the category of spaces over a fixed space B serve as functions in a neighborhood of the point B. The tangent space of TOP at B is the category of parametrized spectra over B. There are cotangent vectors, tensor fields both covariant and contravariant, and differential operators.

There are precisely two tangent connections, both flat, and their difference is the tensor field known as smash product of spectra. I cannot find differential forms (except 0-forms and 1-forms). This has not yet led to nontrivial theorems, but I will at least say something trivial about some nontrivial examples.

### Allen Hatcher (Cornell University)

Title: A Topological Proof of Harer's Homology Stability Theorem

**Abstract:** This is the theorem that the homology groups of mapping class groups of surfaces stabilize as the genus increases. Harer's original proof used some Teichmuller theory. Ivanov later gave a proof using singularity theory instead. Here we describe a proof, worked out with Karen Vogtmann, that uses only topology. The proof can be adapted to other groups. Braid groups are the simplest examples (Arnold's theorem). There are new examples as well, including handlebody mapping class groups.

## Kiyoshi Igusa (Brandeis University)

Title: Diffeomorphisms of disks and spheres revisited

**Abstract:** About 30 years ago Tom Farrell and Wu-Chung Hsiang computed the rational homotopy type of the diffeomorphism groups of disks and spheres in the stable range. We will reexamine this result using axiomatic higher torsion and canceling Hatcher disks.

## Lowell Jones (SUNY Stony Brook)

Title: A New Approach to Proving the "Injectivity Half" of the Isomorphism Conjecture

**Abstract:** *X* denotes a connected CW complex and G(X) denotes the collection of all virtually cyclic subgroups of  $\pi_1(X)$ . For each group  $G \in G(X)$  let  $X_G$  denote the covering space of *X* corresponding to *G*. The covering space projections  $X_G \to X$  induce maps  $P(X_G) \to P(X)$  between the  $\Omega$ -spectra of stable pseudoisotopies for the spaces  $X_G$ , *X*. The Isomorphism Conjecture for stable pseudoisotopy theory states that the collection of these maps can be "assembled" in a simple way to obtain an "assembly map"  $A^* : P(E; \rho) \to P(X)$  which is a homotopy equivalence between  $\Omega$  spectra; here  $\rho : E \to B$  is a stratified fibration whose fibers are the spaces  $X_G$ ,  $G \in G(X)$ , and  $P(E; \rho)$  is the  $\Omega$ -spectrum of stable pseudoisotopies in *E* which are "controlled" under the projection  $\rho$ .

This conjecture has been verified in many special cases. For example, Farrell and Jones have verified it when X is a closed Riemannian manifold having non-positive sectional curvature values everywhere. Their proof is in two parts: (Injectivity) first show that the assembly map A\* induces an injective map on homotopy groups; (Surjectivity) then show that A\* induces a surjective map on homotopy groups. The proof of injectivity uses the canonical "transfer map"  $P(X) \rightarrow P(X_G)$  induced by the path lifting property for covering spaces  $X_G \rightarrow X$ . Their proof for surjectivity, uses another transfer map (transferring from the base of disc bundle to the total space). Unlike the transfer map for covering spaces (where the transfer is unique), there are many transfer maps from the base space to the total space of a disc bundle. Farrell and Jones invented the "focal transfers" (which work for certain disc bundles over certain Riemannian manifolds with non-positive sectional curvatures everywhere) to help prove surjectivity.

In this talk I will explain how to use the "focal transfers" to prove both the injectivity and surjectivity of the Isomorphism Conjecture for closed Riemannian manifolds with non-positive curvature. The advantage of this approach is that not only streamlines the proofs for known results, but also extends to non-compact complete Riemannian manifolds with non-positive sectional curvature values everywhere.

## Sunday, August 7

## Soren Galatius (Stanford University)

Title: Stable homology of the automorphism group of free groups

**Abstract:** I will announce and sketch a proof of the result that the rational homology of the automorphism group of a free group is trivial in the stable range.

### Ian Hambleton (McMaster University)

Title: Free actions on products of spheres

**Abstract:** Which finite groups act freely on a product  $S^n \times S^n$ ? Free actions of finite groups on  $S^n$  have been much studied, and there are strong restrictions on the structure of the groups involved arising from P. A. Smith theory. For free actions of a finite group *G* on the product of two spheres of the same dimension, the structure of the *p*-Sylow subgroups of *G* is not yet known. We will survey some old and new results on this problem.

#### Bob Oliver (University of Paris-Nord, XIII)

Title: *p*-local Groups

**Abstract:** This talk will mostly be a survey of my work with Carles Broto and Ran Levi on *p*-local finite groups, and the properties they share with *p*-completed classifying spaces of finite groups. I will also discuss similar structures which generalize *p*-completed classifying spaces of compact Lie groups, as well as *p*-compact groups.

# Mike Davis (Ohio State University)

**Title:** The weighted  $L^2$ -cohomology of Coxeter groups

**Abstract:** The reduced  $L^2$ -cohomology of the universal cover of a CW complex or of a manifold lies somewhere between its ordinary cohomology and its cohomology with compact supports. One of the most interesting interpretations of the previous sentence comes from the study of the weighted  $L^2$ cohomology of the complex associated to a Coxeter group. It interpolates between ordinary cohomology and cohomology with compact supports. The motivation for studying these weighted cohomology groups is that they compute the ordinary  $L^2$ -cohomology of those buildings with apartments equal to the complex associated to the given Coxeter group. I will discuss work on this topic by Dymara, Januszkiewicz, Okun, and myself.