

Non-Cooperative Game Theory

Having Fun with Strategic Games

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TEDy

Outline

- 1 What Is A Non-Cooperative Game?
 - Motivating Examples
 - Formal and Informal Definitions of Non-Cooperative Games
- 2 Nash Equilibrium as the Prediction of a Game
 - Formal and Informal Definitions of Nash Equilibrium
 - Examples of Nash Equilibrium
- 3 Interactive Games
 - Divergence from Nash Equilibrium
 - Convergence to Nash Equilibrium?
 - Backward Induction

Real World Examples of Non-Cooperative Games

- Games assigning property rights: rock-paper-scissor; various drinking games; *etc.*
- Games in sports: penalty kick in soccer; batter-pitcher duel; *etc.*
- Games in media: *The Dark Knight*, ferry scene; *Friends*, Season 5, Episode "The One Where Everybody Finds Out";¹ *etc.*
- Games at school: moral hazard in team for group assignments; cleaning up after a party; *etc.*
- In short, game theory is very much a part of our lives, and all of us have been introduced to many of its underlying intuitions. Much of the theoretical work in game theory is formalizing these intuitions. Let's put your intuition to work.

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Formal Definition of a Game

Definition

A game is specified by the following collection:

$$\Gamma \equiv \{ \mathcal{N}, \mathcal{A}, I, p(\cdot), \alpha(\cdot), \mathcal{H}, H(\cdot), \iota(\cdot), \rho(\cdot), \{u_i(\cdot)\}_{i \in I} \}$$

- $p: \mathcal{N} \rightarrow \{ \mathcal{N} \cup \emptyset \}$
- $\alpha: \mathcal{N} \cup \{x_0\} \rightarrow \mathcal{A}$
- $H: \mathcal{N} \rightarrow \mathcal{H}$
- $\iota: \mathcal{H} \rightarrow I$
- $\rho: \mathcal{H}_0 \times \mathcal{A} \rightarrow [0, 1]$
- $u_i: T \rightarrow \mathbb{R}$, where $T \equiv \{x \in \mathcal{N} : p^{-1}(x) = \emptyset\}$

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Informal Definition of a Game

- Players
- Actions
- Strategies: “[A] complete contingent plan, or decision rule, that specifies how the player will act in every possible distinguishable circumstance in which she might be called upon to move...”
- Outcomes/payoffs: The key to game theory is that my payoff is contingent on the action/strategy of the other players playing the game: $u_i(\sigma_i, \vec{\sigma}_{-i})$, where σ_i is player i 's strategy and $\vec{\sigma}_{-i} = [\sigma_1 \dots \sigma_{i-1} \sigma_{i+1} \dots \sigma_I]$ is the other players' strategies

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What Is a Nash Equilibrium?

- Once you define the players, actions, possible strategies, and the payoffs of a game, you have set up the rules of the game. Now what?
- We could ask: which player within the set I will win the game? For example, if the game you have defined is a penalty kick, you could ask who will win the penalty kick by virtue of getting the higher payoff? Will it be the goal-keeper or the shooter?
- Another interesting question might be what is the optimal strategy for a player $i \in I$ given that all of the other players $j \neq i$ and $j \in I \setminus \{i\}$ are also playing their optimal strategies.
- This is the question that underlies the most fundamental concept in Game Theory: the Nash Equilibrium.

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Formal Definition of Nash Equilibrium

Definition

In game where the n player set is I , the strategy profile $(\sigma_1^*, \dots, \sigma_I^*)$ is a Nash Equilibrium if, for each player i , her strategy σ_i^* is a best response to the strategies specified for the other $n - 1$ players, or for $\vec{\sigma}_{-i} = [\sigma_1 \dots \sigma_{i-1} \sigma_{i+1} \dots \sigma_I]$. Formally,

$$u_i(\sigma_i^*, \vec{\sigma}_{-i}) \geq u_i(\sigma_i', \vec{\sigma}_{-i}), \forall i, \forall \sigma_i' \in \Sigma_i,$$

or

$$\sigma_i^* = \arg \max_{\sigma_i} u_i(\sigma_i, \vec{\sigma}_{-i}) \forall i \in I$$

$$\vec{\sigma}_{-i} = (\sigma_1^*, \dots, \sigma_{i-1}^*, \sigma_{i+1}^*, \dots, \sigma_I^*)$$

Nash's Existence Theorem

Theorem

Kakutani's Fixed Point Theorem: Suppose that $\mathbb{A} \subset \mathbb{R}^n$ is a nonempty compact, convex set, and that $f : \mathbb{A} \rightarrow \mathbb{A}$ is an upper hemi-continuous correspondence from \mathbb{A} unto itself with the property that the set $f(\vec{x}) \subset \mathbb{A}$ is nonempty and convex for every $x \in \mathbb{A}$. Then $f(\cdot)$ has a fixed point, or there exists an $\vec{x}^ \in \mathbb{A}$ such that $f(\vec{x}^*) = \vec{x}^*$.*

Based on Kakutani's Fixed Point Theorem, follows Nash's theorem:

Theorem

There exists a mixed strategy Nash Equilibrium for all games with a finite number of players and a finite number of strategies

Informal Definition of Nash Equilibrium

- Huh??
- *Given what others are doing*, there is no profitable deviation for any player.
- Given what all of the players are doing, *check* to see whether there are any changes that would be profitable for any of the players *assuming that the strategies of the other players do not change*.

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Prisoner's Dilemma

The story behind the Prisoner's Dilemma is of two suspects who are caught by the police and are now being interrogated. However, we can apply this to a number of settings. Consider a group project for class. Assume that the payoffs below are in terms of utilities.

Table: Prisoner's Dilemma

	Cooperate	Defect
Cooperate	2,2	0,3
Defect	3,0	1,1

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Prisoner's Dilemma Continued

We solve for the Nash Equilibrium by looking at the “best response” of each player *given* a strategy of the other player. For example, given Player 2’s strategy of “Cooperate”, Player 1’s best response is “Defect.” Let’s underline the payoffs corresponding to each players’ best response.

Table: Prisoner's Dilemma

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Defect	<u>3</u> ,0	<u>1</u> , <u>1</u>

Prisoner's Dilemma Continued

A Nash Equilibrium corresponds to the cell where both payoffs are underlined. Let's check to see whether any of the players can do better by changing her strategy.

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If either player 1 or 2 deviates to "Cooperate," their payoffs fall to 0 from 1. Therefore, "Defect" and "Defect" is the Nash Equilibrium and this is the predicted outcome of this game. However, is this truly what we expect to happen given the group project story?

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Battle of the Sexes

The story behind the Battle of the Sexes is that a man and a woman are trying to decide where to go on their date. They both want to spend time together, but they have preferences over where to go. Assume that player 1 is “woman” and player 2 is “man.” Again assume that the payoffs below are in terms of utilities.

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The Nash prediction of this game is that the man and woman will end up *together* at either the Musical or the Football match. The key is that the two players stay together, despite differing preferences.

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A Beautiful Mind

- Let's watch a short clip from "A Beautiful Mind," where John Nash "figures out" the Nash Equilibrium. Note that there are four players (the four men); five possible strategies (choice of four of the brunettes and one blonde); blonde gives the highest payoff; and the key assumption that "no one wants to be the second choice."
- While watching the clip, considering the following two questions:
 - What is the equilibrium solution of the game identified by John Nash?
 - Is this solution a Nash Equilibrium?

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Ultimatum Game

- Rules of the game:
 - The Offeror offers a split of 1 dollar with the Offeree, for example, x cents for Offeree and $1 - x$ cents for herself, the Offeror;
 - The Offeree can “Accept” or “Reject” the Offeror’s offer. If Offeree “Accepts,” the two get the money. If the Offeree “Rejects,” then both parties get nothing.

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Ultimatum Game Continued

- What is the Nash Equilibrium of this game?
 - There are multiple Nash Equilibria of this game based on empty threats. We won't have time to consider this.
 - There is a unique subgame perfect Nash Equilibrium of the game: Offeror offers 1 cent to the Offeree and the Offeree "Accepts."
 - Check that there are no changes that would increase the payoffs to either player.
- A key assumption of game theory is that all players are perfectly rational. However, as the ultimatum game shows, people are also motivated by a sense of fairness. In the lab, Offerors usually offer 60-40 splits.

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Generalized Beauty Contest

- Rules of the game:
 - Write down your name;
 - Pick a non negative integer between, and including, 0 and 100;
 - I will collect the cards and the number(s) closest to $\frac{1}{2}$ the average of the 10 numbers will win a prize
- Play again with a different group of 10 players

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Generalized Beauty Contest Continued

- What is the Nash Equilibrium of this game?
 - The unique Nash Equilibrium is where all 10 players choose 0. Check to see that no player can win by deviating from 0. Consider one player deviating to $x \neq 0$. Then half of the average is $\frac{x}{20}$. For all $x \in [0, 100]$, 0 is closer to $\frac{x}{20}$ than x .
 - Iterated reasoning: Because the average can never be greater than 100, the numbers from 51-100 are dominated for all the players. If no player will submit numbers greater than 51, then the average can never exceed 50. Since the average cannot exceed 50, the numbers 26-50 are dominated for all players....
 - The result is that all players should play 0 at the equilibrium.

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Generalized Beauty Contest Continued

- Predicted outcome of the game:
 - With the first group of players, the winning number was not equal to 0. In lab run experiments, researchers find that the number is usually around 25.
 - With the second group, notice (hopefully) that the winning number decreased when compared to the first group's winning number.
- Although the Nash Equilibrium was not a great predictor of the first outcome, we might expect to see convergence to the Nash Equilibrium as we continue the learning process.

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Drinking Games with Two Players

- Rules of the game:
 - Starting from 21 pennies, the first player removes 1, 2, or 3 pennies;
 - Thereafter, the second player then also removes 1, 2, or 3 pennies;
 - The process is repeated until only 1 penny remains and the player who removes the last penny wins and the opponent drinks shot of beer.
- Play again, but change the sequence of moves.

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Drinking Games with Two Players Continued

- What is the Nash Equilibrium of this game?
 - The unique Nash Equilibrium is where the first player removes 1 penny at her first move so that 20 pennies remain.
 - Thereafter, player 1 must make sure that 16, 12, 8, and 4 pennies remain.
 - Given player 1's strategy, player 2 is indifferent because he will always lose! Player 2 randomizes.

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Drinking Games with Two Players Continued

- Illustrates the importance of the first mover advantage in some types of games.
- Illustrates the idea of moving backwards from the conclusion of the game to the beginning of the game to find Nash Equilibria.

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Conclusion and Questions

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