### 14.02 Principles of Macroeconomics

Problem Set \#2
Posted: Wednesday, September 17, 2003
Due Date: Wednesday, September 24, 2003
Solutions revised September 25, 2003
Please remember to write your TA's name and section time on the front page of your problem set.

## Part I. True/False: Decide whether each statement is true or false and justify your answer with a short argument.

1. When nominal income increases the demand for bonds also increases.

True. Recall that an increase in nominal income will shift the money demand curve to the right. At the old interest rate ( $i_{1}$ ), the quantity of money demanded is now higher than the supply of money in the economy (money supply has not changed). Because of this, the interest rate must rise for people to be happy holding the old level of money. When the interest rate rises, the yield on bonds is higher, and the demand for bonds will increase.

2. Expansionary monetary policy decreases the interest rate.

True. $\uparrow M^{s} \Rightarrow \uparrow \$ P_{B} \Rightarrow \downarrow i$
The central bank increases the money supply by buying up bonds. As the central bank buys bonds, reducing the bond supply, the price of bonds will rise. Since $i=(P a y o u t ~ f r o m ~$ bond at maturity- $\left.\$ P_{B}\right) / \$ P_{B}$, the increase in $\$ P_{B}$ will result in a fall in the interest rate. You can show this by graphing the shift in the money supply:

3. The money multiplier is always less than 1 since the reserve ratio $(\theta)$ is less than one. False. The money multiplier is: $\frac{1}{c+\theta(1-c)}$. Since $0<\theta<1$, this is always greater than one if individuals in the economy hold both currency and checkable deposits ( $0<c<1$ ), or if they hold only checkable deposits ( $c=0$ ). If everyone in the economy holds only currency, the money multiplier is equal to 1.
4. Only federal employees can purchase stocks and bonds in the federal funds market.

Fals. The federal funds market is the market for bank reserves in which banks that have excess reserves can lend to banks with insufficient reserves.
5. If nominal income increases and the central bank decreases the money supply, the interest rate will fall.

False. The interest rate will rise. This is easiest to see graphically:


Recall that the increase in nominal income leads to an increase in money demand. Money demand increases and money supply decreases simultaneously - this results in a higher interest rate.

## Part II. Supply and Demand For Money

Assume an economy in which:
$\mathrm{L}(\mathrm{i})=(.5-\mathrm{i})$
$\$ \mathrm{Y}=\mathrm{P} * \mathrm{Y}=\$ 10$ trillion
$\mathrm{M}^{\mathrm{s}}=\$ 3.5$ trillion

1. Graph $\mathrm{M}^{\mathrm{s}}$ and $\mathrm{M}^{\mathrm{d}}$ for this economy.


$$
\begin{aligned}
& M^{d}=10(.5-i) \\
& M^{s}=3.5
\end{aligned}
$$

2. What is the equilibrium rate of interest?

$$
\begin{aligned}
& M^{d}=M^{s} \\
& 10(.5-i)=3.5 \\
& i=.15=15 \%
\end{aligned}
$$

3. Label the equilibrium of the money market at point A on the graph from number 1 .
4. Now, the central bank sells $\$ 1$ trillion worth of bonds via open market operations. Find the new equilibrium interest rate. Label this equilibrium point B on the graph.

$$
\begin{aligned}
& M^{d}=M^{s} \\
& 10(.5-i)=2.5 \\
& 5-10 i=2.5 \\
& i=.25
\end{aligned}
$$

5. At equilibrium A , what is the price of a bond that promises to pay $\$ 100$ at the end of a year?
$i=\left(100-\$ P_{B}\right) / \$ P_{B}$
$\$ P_{B}=100 /(i+1)$
$\$ P_{B}=100 / 1.15=\$ 86.95$
6. At equilibrium B , what is the price of a bond that promises to pay $\$ 100$ at the end of a year?
$i=\left(100-\$ P_{B}\right) / \$ P_{B}$
$\$ P_{B}=100 /(i+1)$
$\$ P_{B}=100 / 1.25=\$ 80$
7. Why did the price of bonds fall? How does this affect the rate of return on a bond that will pay $\$ 100$ at the end of the year?

In order to decrease the money supply, the central bank sells bonds on the open market. This results in more bonds available in the economy. Because the supply of bonds rises, their price must fall. Because the final payout of $\$ 100$ is fixed, the rate of return on the bond is higher when the price is lower ( $i \uparrow$ ).

## Part III. The Money Multiplier

Assume the following:
$D^{\mathrm{d}}=\$ 800$ billion
$\mathrm{M}^{\mathrm{s}}$ (total supply of money) $=\$ 1200$ billion
$\theta=0.1$

1. In equilibrium, what are $\mathrm{CU}^{\mathrm{d}}, \mathrm{R}^{\mathrm{d}}$ and $\mathrm{H}^{\mathrm{d}}$ ?

In equilibrium, $M^{s}=M^{d}$.
$D^{d}=(1-c) M^{d}$, so $800=(1-c) * 1200$
$c=(1 / 3)$
$R^{d}=\theta(1-c) M^{d}=.1(2 / 3)(1200)=\$ 80$ billion
$C U^{d}=c M^{d}=(1 / 3)(1200)=\$ 400$ billion
$H^{d}=C U^{d}+R^{d}=\$ 480$ billion
2. If the Fed purchases $\$ 300$ million worth of T-bills, what is the change in the overall equilibrium money supply?

Money multiplier $=\frac{1}{c+\theta(1-c)}=\frac{1}{(1 / 3)+.1(2 / 3)}=\frac{5}{2}$

The $\$ 300$ million T-bill purchase increases high-powered money by $\$ 300$ million.
Overall money supply will increase by $\frac{5}{2}(300)$ or $\$ 750$ million.

## Part IV. IS-LM Practice Problems - Do Not Need to Be Turned In, But VERY IMPORTANT for Quiz 1

1. Assume the following IS-LM model:
$\mathrm{C}=80+(4 / 5) \mathrm{Y}_{\mathrm{D}}$
$\mathrm{T}=(1 / 4) \mathrm{Y}$
$\mathrm{I}=300-2000 \mathrm{i}$
$\mathrm{G}=120$
$\mathrm{M}^{\mathrm{s}}=700$
$\mathrm{P}=2$
$(M / P)^{d}=(1 / 3) Y+200-1000 i$
a. Derive the equilibrium values of consumption and money demand.
$Z=C+I+G=80+(4 / 5)(Y-.25 Y)+300-2000 i+120=500+(4 / 5)(.75 Y)-2000 i$
Equilibrium in the goods market requires production $=$ demand:
$Y=500+.6 Y-2000 i$
$Y=2.5(500-2000 i)$
$Y=1250-5000 i$

Equilibrium in the money market:
$700 / 2=(1 / 3) Y+200-1000 i$
$350=(1 / 3) Y+200-1000 i$
$Y=450+3000 i$
$I S=L M:$
$450+3000 i=1250-5000 i$
$8000 i=800$
$i=.1$
$Y=750$
$C=80+(4 / 5)(Y-.25 Y)=80+(4 / 5)(562.5)=530$
$(M / P)^{d}=(1 / 3)(750)+200-1000(.1)=450-100=350$
b. Calculate the equilibrium value of investment. How does investment change if the government increases its purchases by 160 ?
$I=300-2000(.1)=100$
Increase $G$ by 160:
$Y=660+.6 Y-2000 i$

$$
\begin{aligned}
& Y=2.5(660-2000 i) \\
& Y=1650-5000 i \\
& I S=L M: \\
& 450+3000 i=1650-5000 i \\
& i=.15 \\
& Y=900 \\
& I=300-2000(.15)=0
\end{aligned}
$$

Investment fell by 100.
c. Keep the new level of government spending from part $\mathrm{b}(\mathrm{G}=280)$. By how much will the equilibrium level of income and the interest rate change if nominal money supply is increased to 1100 ?

Equilibrium in the money market is now:
$1100 / 2=(1 / 3) Y+200-1000 i$
$550=(1 / 3) Y+200-1000 i$
$Y=1050+3000 i$
$I S=L M:$
$1050+3000 i=1650-5000 i$
$i=.075$
$Y=1275$
2. Suppose:
$\mathrm{C}=\mathrm{c}_{0}+\mathrm{c}_{1}(\mathrm{Y}-\mathrm{T})$
$\mathrm{I}=\mathrm{b}_{0}-\mathrm{b}_{1} \mathrm{i}$
$(\mathrm{M} / \mathrm{P})^{\mathrm{d}}=\mathrm{m}_{1} \mathrm{Y}-\mathrm{m}_{2} \mathrm{i}$
G and T are constant
a. How should the parameters $\mathrm{b}_{1}, \mathrm{~m}_{1}$, and $\mathrm{m}_{2}$ be interpreted?
$b_{1}$ is the sensitivity of investment to changes in the interest rate. $m_{1}$ is the sensitivity of real money demand to changes in income, and $m_{2}$ is the sensitivity of real money demand to changes in the interest rate.
b. Use the IS-LM model to graphically determine the effectiveness of fiscal versus monetary policy when investment is very sensitive to changes in $i$, and money demand is very insensitive to changes in i .

IS relation is: $\quad Y=\frac{1}{1-c_{1}}\left[c_{o}+I+G-c_{1} T\right]=\frac{1}{1-c_{1}}\left[c_{o}+b_{0}-b_{1} i+G-c_{1} T\right]$
Notice that when $b_{1}$ is larger (investment more sensitive to changes in $i$ ), the IS curve is flatter.
$L M$ relation is: $\quad \frac{M}{P}=Y L(i)=m_{1} Y-m_{2} i$
When $m_{2}$ is smaller (real money demand less sensitive to changes in $i$ ), the LM curve is steeper.


Output is not very responsive to even a large change in the interest rate when fiscal policy is used; output is very responsive to a small change in the interest rate when monetary policy is used.
c. Now suppose that the government imposes a tax, $t_{1}$, on income, so that the following is true:
$\mathrm{T}=\mathrm{t}_{0}+\mathrm{t}_{1} \mathrm{Y}, 0<\mathrm{t}_{1}<1$
If money demand is very insensitive to changes in the interest rate, is a decrease in the income tax rate an effective way to stimulate the economy?

IS relation is now: $Y=\frac{1}{1-\left(1-t_{1}\right) c_{1}}\left[c_{o}+b_{0}-b_{1} i+G-c_{1} t_{0}\right]$

Lowering the income tax rate increases the multiplier. This results in a flatter IS curve. Since the LM curve is very steep, this is not an effective way to stimulate the economy results in a small change in Y for a large change in $i$.


