PROBLEM SET 4 Solutions
14.02 Principles of Macroeconomics
March 30, 2005
Due April 6, 2005

I. Answer each as True, False, or Uncertain, providing some explanation for your choice.

1. If the expectations formed by workers about the future responded to economic policy, then the cost of a disinflation policy could possibly be lower.

TRUE. If the policy maker embarks on a disinflation program, and the program is credible, then workers can be expected to factor in the policy goal of lower inflation into their wage contracts (essentially this means that the manner in which they set expectations would itself change), allowing for a disinflation associated with a smaller increase in unemployment than would have occurred had expectations not changed (see the discussion in Chapter 9, Section 4).

2. Sacrifice ratios are smaller in countries that have shorter wage contracts.
TRUE.

$$\pi_t - \pi_{t-1} = -\alpha\,(u_t - u_n)$$

Shorter wage contracts imply that nominal wages are less rigid. Nominal wages can change more often in response to any changes in inflation rates, when old contracts are renewed and new contracts are signed. A higher degree of wage flexibility means that unemployment rates need not respond very much to changes in inflation rates (ie. monetary shocks.) From the expectations augmented Phillips Curve above, a small $(u_t - u_n)$ needed means a big α for a given $(\pi_t - \pi_{t-1})$ – thus a small sacrifice ratio $\frac{1}{\alpha}$.

3. A permanent increase in money growth rate leads to a higher real interest rate in the medium run when the growth rate of output is 0.

FALSE. Money is neutral in the medium run. Output is back to the natural level of output, and unemployment rate is back to the NAIRU. The real interest rate is pinned down by the IS relation as follows:

$$Y_n = C(Y_n - \overline{T}) + I(Y_n, r) + \overline{G}$$

where $\overline{G}$ and $\overline{T}$ are constants.
However, the nominal interest rate increases, as it is pinned down by

$$i' = r_n + \pi' = r_n + g'_m > r_n + g_m = i$$

where $\pi' = g'_m > g_m$, if $g_y = 0$ in the medium run.

4. When the yield curve for bonds is downward sloping, it implies that the financial markets expect the short-term real interest rate to decrease in the future.

UNCERTAIN. A downward sloping yield curve tells us that the financial markets expect short-term nominal interest rates to be lower in the future. Recall that a downward sloping yield curve illustrates lower interest rates for long-term bonds than those for short-term bonds. (A slightly downward sloping yield curve in the second half of 2000, as depicted on p.312 in the textbook, is an example.) Whether the short-term real interest rates decrease depends on the expected changes in inflation rates. If the inflation rates are expected to decrease proportionally more than the decrease in the short-term nominal interest rates, the expected real interests rates will be higher.

5. In theory, if the stock market fully anticipated a monetary expansion next period, stock prices will not change next period when the money growth rate actually increases.

TRUE. If a monetary expansion *was* fully anticipated, the financial markets already expect increases in profits and dividends for some time in the short-run. Since stock prices are the present discounted values of all expected future dividends (in theory), the financial markets adjusted their expectations on future dividends when they received the news of a credible monetary expansion, and thus, the stock prices changed also. Once the new information is already reflected in the stock prices, the actual occurrence of the events do not affect the prices anymore.

6. Stock prices increase in response to an unexpected increase in consumer spending, since output will increase in the short run.

UNCERTAIN. It is true that an unexpected increase in consumer spending means higher profits and higher dividends for some time. However, a shift of the IS curve along the LM curve also leads to increases in nominal interest rates in the short run. On one hand higher nominal dividends means higher stock prices, keeping the expected nominal interest rates constant; on the other hand, higher (expected) nominal interest rates mean a higher discounting, which decreases the present discounted value of dividends. The net impact on stock prices is uncertain. In general, if the LM curve is flatter, stock prices tend to increase since the increase in the nominal interest rate is small and the effect of the increases in future profits/ dividends dominates. (Think about the intuition of a flat LM curve and its implication on the stock prices.)

II. Output, Unemployment and Inflation

Consider an economy summed up by the 3 equations described in Chapter 9-2.

$$u_t - u_{t-1} = -\beta(g_{yt} - \overline{g_y}) \quad (1)$$
$$\pi_t - \pi_{t-1} = -\alpha(u_t - u_n) \quad (2)$$
$$g_{yt} = g_{mt} - \pi_t \quad (3)$$

Assume the following parameter values : $\alpha = \beta = 1$, $u_n = 5\%$, $\overline{g_y} = 0\%$

At time $t = 0$, $g_{mt} = 4\%$, $\pi_{t-1} = 4\%$, $u_{t-1} = u_n$

At time $t = 1$, the Central Bank lowers the growth of rate of money to $g_{mt} = 2\%$ (this policy shock was unanticipated by workers and firms at $t = 0$)

(a) Calculate the initial and final medium run equilibrium values, i.e., (π_0, u_0, g_{y0}) and (π_T, u_T, g_{yT}) (where T denotes the time period when the final medium run equilibrium is reached). Comparing the two, provide some reasons for why the differences, if any, arise.

Ans:

According to the definition of a medium run equilibrium, the unemployment rate is equal to the natural rate, and growth rate of output is the normal growth rate. This immediately pins down : $u_0 = u_T = 5\%$, and $g_{y0} = g_{yT} = 0\%$. Only π will differ across the two equilibria, since in order for $g_{y0} = g_{yT}$, we need $g_{mt} = \pi_t$ from the final equation for $t = 0, T$, and so the exogenous money growth rate pins down inflation as $\pi_0 = 4\%$, $\pi_T = 2\%$. The difference between the two equilibria also points to the super-neutrality of money. Note all of the decrease in the money growth rate is manifested in a decrease in the inflation rate, nothing real (for example, u or g_y) changes.

(b) Now focus on the dynamics :

(i) Calculate (π_1, u_1, g_{y1}). Does the Phillips curve shift at time $t = 1$? Why or why not?

Ans:

Solving the equations, $(\pi_1, u_1, g_{y1}) = (3\%, 6\%, -1\%)$

The Phillips curve is given by $\pi_1 = \pi_0 - (u_1 - 5\%) = 9 - u_1$, which is the same as for $t = 0$, since the expected inflation remains the same $\pi_{-1} = \pi_0 = 4\%$, so the Phillips curve does not shift at time $t = 1$.

(ii) Calculate (π_2, u_2, g_{y2})

Ans:

The equations to be solved are :

$u_2 = u_1 - g_{y2} = 6 - g_{y2}$

$\pi_2 = \pi_1 - (u_2 - 5\%) = 8 - u_2$

$g_{y2} = g_{m2} - \pi_2 = 2 - \pi_2$

Solving the equations, $(\pi_2, u_2, g_{y2}) = (2\%, 6\%, 0\%)$

Note that the equilibrium (π_2, u_2) are on a new Phillips curve : $\pi = \pi_{-1} - (u - 5\%) = 8 - u$

(iii) Denote the Phillips curve at time t (also called the short run Phillips curve) by PC_t. In a diagram, draw PC_0, PC_1, PC_2, and PC_T

Ans:

The equations are

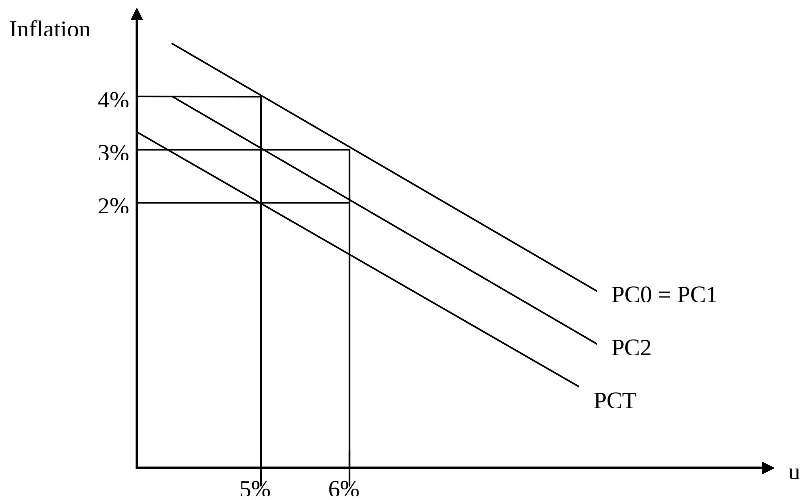
$PC_0 = PC_1 : \pi = 9 - u$

$PC_2 : \pi = 8 - u$

$PC_T : \pi = 7 - u$

(see figure on next page)

(c) Based on the given conditions, use a software of your choice (eg. *Excel*®) to plot π_t against u_t for $t = -1, 0, 1, 2, ... 30$. Describe the evolution of the



inflation rate and the employment rate (ie. the shape of the curve connecting (π_t, u_t)). Why is it like this?

Ans:

See the attached *Excel*® spreadsheet. The curve connection (π_t, u_t) is a spiral converging to the medium run equilibrium (π_T, u_T). With (π_T, u_T) as the center, if we divide the plane into 4 quadrants (I, II, III, IV) from the top right hand clockwise, we have

I : $\frac{\partial u}{\partial t} \geq 0$ and $\frac{\partial \pi}{\partial t} \leq 0$

II : $\frac{\partial u}{\partial t} \leq 0$ and $\frac{\partial \pi}{\partial t} \leq 0$

III : $\frac{\partial u}{\partial t} \leq 0$ and $\frac{\partial \pi}{\partial t} \geq 0$

IV : $\frac{\partial u}{\partial t} \geq 0$ and $\frac{\partial \pi}{\partial t} \geq 0$

Since $g_{yT} = 0$ in the medium run, $\pi_T = g_{mt} = 2\%$ from the AD equation (equation (3) above). Any points above $\pi_T = 2\%$ on the plane (ie. quadrants I and IV) have $\pi_t \geq g_{mt} = 2\%$, which means that $g_{yt} < 0$ according to the AD equation. From Okun's Law (equation (1) above), a negative growth rate of output implies an increase in the unemployment rate. ($u_t - u_{t-1} \geq 0$ or $\frac{\partial u}{\partial t} \geq 0$). With the same logic, any points below $\pi_T = 2\%$ (quadrants II and III) have $u_t - u_{t-1} \leq 0$. In sum, we have a west-east run in u_t in the 2 northern quadrants (I & IV) and an east-west run in u_t in the 2 southern quadrants (II & III).

Since $u_T = u_n = 5\%$ in the medium run. Any points to the left of $u_T = 5\%$ (ie. quadrants III and IV) on the plane have $u_t < u_n$. From the Phillips Curve (equation (2)) above, an unemployment rate below the natural rate of unemployment leads to an increase in the inflation rate. ($\pi_t - \pi_{t-1} \geq 0$ or $\frac{\partial \pi}{\partial t} \geq 0$). With the same logic, any points to the right of $u_T = 5\%$ (quadrants

I and II) have $\pi_t - \pi_{t-1} \leq 0$. Thus, we have a north-south run in π_t in the 2 eastern quadrants (I & II) and the south-north run in π_t in the 2 western quadrants (III & IV).

Both u_t and π_t oscillate for awhile until they are back to the medium-run equilibrium. Keep in mind that convergence is not monotonic!

III. Asset Pricing

(i) Consider a 1-year bond that promises to pay $\$X$ next year. The price of this bond this year is given by $\$P_{1t}$. Express $\$P_{1t}$ in terms of $\$X$ and i_{1t} (the 1-year interest rate from t to $t+1$).

Ans:

$$\$P_{1t} = \frac{\$X}{(1+i_{1t})}$$

(ii) Suppose that at the beginning of year t, people expect the interest rate from year $t+1$ to year $t+2$ to be the same as the current 1-year interest rate. Express the price of the 2-year bond ($\$P_{2t}$), which promises $\$Y$ in year $t+2$, in terms of $\$Y$ and i_{1t}.

Ans:

$$\$P_{2t} = \frac{\$Y}{(1+i_{1t})^2}$$

(iii) Suppose that at the beginning of year t, people expect that the price of the 1-year bond (with the same promise) in year $t+1$ will be $\$P^e_{1\,t+1}$. What is $\$P^e_{1\,t+1}$? Suppose that the agent has $\$Z$ to invest, show that the agent is indifferent between investing all $\$Z$ in either of the two bonds, independent of $\$X$ and $\$Y$.

Ans:

Without any changes in the face value and the 1-year interest rate, $\$P^e_{1\,t+1} = \$P_{1t}$.

At the end of year $t+1$, the amount of of payment that she will receives = number of 1-year bonds*promised payment : $\frac{\$Z X}{\$P_{1t}} = \$Z(1+i_{1t})$

The agent invests $\$Z(1+i_{1t})$ to purchase 1-year bonds. Since the price of the 1-year bond in year $t+1$ is the same, amount of payments that she will receive at the end of year $t+2$ is $\frac{\$Z(1+i_{1t})X}{\$P_{1t}} = \$Z(1+i_{1t})^2$.

Consider the agent who purchases the 2-year bond in year t, at the end of year $t+2$, she has $\frac{\$ZY}{\$P_{2t}} = \$Z(1+i_{1t})^2$.

The bottom line is that the initial prices of the bonds adjust one-for-one to any changes in promised payments. Both investments have a gross rate of return at the end of year $t+2$ of $(1+i_{1t})^2$.

(iv) Suppose that at the beginning of year t, people expect the interest rate from year $t+1$ to year $t+2$ to be θi_{1t}, where $\theta > 1$. Is the price of the 2-year bonds in year t, $\$P_{2t}$, higher now? Why? Show your steps.

Ans:

Now $\$P^e_{1\,t+1} = \frac{\$X}{1+\theta i_{1t}}$. The arbitrage relation (in year $t+2$) requires that

$$\begin{aligned} \frac{\$ZY}{\$P_{2t}} &= \frac{\$ZX}{\$P_{1t}} \frac{\$X}{\$P^e_{1\,t+1}} \\ \frac{\$Y}{\$P_{2t}} &= (1+\theta i_{1t})(1+i_{1t}) \\ \$P_{2t} &= \frac{\$Y}{(1+\theta i_{1t})(1+i_{1t})} \end{aligned}$$

So $\$P_{2t}$ is lower now. Because investors who invest in the 2-year bonds in year t will lose the opportunity to invest in the 1-year bonds in $t+1$, which are expected to yield a higher rate of return at the end of year $t+2$. Competition in the financial markets will drive the the price of the 2-year bonds down to make investments in the bonds of different maturities indifferent.

(v) Consider now a share, which is expected to pay a constant dividend $\$D^e_t = \$D$ every year as long as the investor holds it. Assume that dividends are paid at the end of each year. Also, the expected 1-year interest rate is a constant, $i^e_{1t} = i$, for all t. Express the ex-dividend price $\$Q_t$ in terms of i and $\$D$?

Ans:

$$\begin{aligned} \$Q_t &= \frac{\$D}{1+i} + \frac{\$D}{(1+i)^2} + \frac{\$D}{(1+i)^3} + ... \\ &= \left(\frac{\$D}{1+i}\right)\left(1 + \frac{1}{1+i} + \left(\frac{1}{1+i}\right)^2 + ...\right) \\ &= \frac{\$D}{1+i}\frac{1}{1-\frac{1}{1+i}} \\ &= \frac{\$D}{1+i}\frac{1+i}{i} \\ &= \frac{\$D}{i} \end{aligned}$$

Suppose that the expected dividend payment in year t is proportional to the expected aggregate output in that year.

$$\$D^e_t = \alpha Y^e_t$$

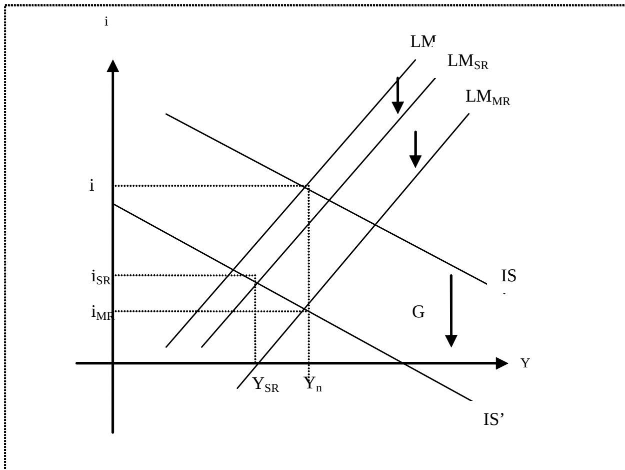


Figure 1: IS-LM (both short run & medium run)

$$\$Q_t = \frac{\alpha \$Y^e_{t+1}}{1+i_{1t}} + \frac{\alpha \$Y^e_{t+2}}{(1+i_{1t})\left(1+i^e_{1t+1}\right)} + \frac{\alpha \$Y^e_{t+3}}{(1+i_{1t})\left(1+i^e_{1t+1}\right)\left(1+i^e_{1t+2}\right)} + ... \quad (4)$$

Consider the case that at the end of year $t = \tau$, the government announces to cut spending permanently, starting from $t = \tau + 1$. What happens to the price level P, the interest rate i and the aggregate output Y in the medium run?

(vi) Draw the relevant AS-AD *and* IS-LM diagrams to show your results. Denote the short-run price level, interest rate and output as P_{SR}, i_{SR} and Y_{SR} respectively; and medium-run price level, interest rate, output as P_{MR}, i_{MR} and Y_n respectively.

Ans: See Figure 1 and 2. In the medium run, output returns to the natural level of output Y_n, but both the price level and the interest rate decrease.

(vii) Assume that the economy is still under short-run adjustment in year $\tau + 1$ ($Y_{\tau+1} = Y_{SR}$), but back to the medium run equilibrium at the beginning of year $\tau + 2$. Everyone expects this to happen! Write down the ex-dividend price $\$Q_\tau$ after the announcement in terms of P_{SR}, P_{MR}, Y_{SR},Y_{MR}, i_{SR} and i_{MR} (as defined in (vi)) and α. Is $\$Q_\tau$ higher or lower than $\$Q_{\tau-1}$? (For simplicity, ignore all within-year changes.)

Ans:

i_τ is pre-determined and does not change in response to the announcement.

In the AS-AD diagram, in the short run, the AD curve shifts to the left, $P_{\tau+1}$ and $Y_{\tau+1}$ both decrease. So $\$Y^e_{\tau+1} = \$Y_{\tau+1} = P_{SR}Y_{SR} < P_\tau Y_\tau = \$Y_\tau$.

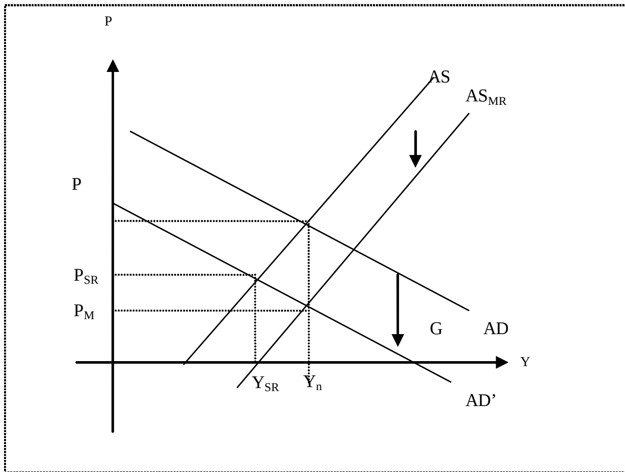


Figure 2: AS-AD

In the IS-LM diagram, the IS curve shifts to the left in response to a decrease in G. LM curve shifts to the right immediately in response to a price drop in $\tau+1$. In $\tau+1$, $i^e_{\tau+1} = i_{SR} < i_\tau$.

In $\tau+2$, the economy is back to the medium equilibrium. $\$Y^e_{\tau+2} = \$Y_{\tau+2} = P_{MR}Y_{MR} < P_\tau Y_\tau$, since $P_{MR} < P_\tau$. And $i^e_{\tau+2} = i_{MR} < i_{SR} < i_\tau$.

Since the economy is back to the medium equilibrium and stays there, $\$Y^e_{\tau+k} = P_{MR}Y_{MR}$ and $i^e_{\tau+k} = i_{MR}\ \forall\ k \geq 2$.

$$
\begin{aligned}
\$Q_\tau &= \frac{\alpha \$Y^e_{\tau+1}}{1+i_{1\tau}} + \frac{\alpha \$Y^e_{\tau+2}}{(1+i_{1\tau})\left(1+i^e_{1\tau+1}\right)} + \frac{\alpha \$Y^e_{\tau+3}}{(1+i_{1\tau})\left(1+i^e_{1\tau+1}\right)\left(1+i^e_{1\tau+2}\right)} + \ldots \\
&= \frac{\alpha P_{SR}Y_{SR}}{1+i} + \frac{\alpha P_{MR}Y_{MR}}{(1+i)(1+i_{SR})} + \frac{\alpha P_{MR}Y_{MR}}{(1+i)(1+i_{SR})(1+i_{MR})} + \ldots \\
&= \frac{\alpha P_{SR}Y_{SR}}{1+i} + \frac{\alpha P_{MR}Y_{MR}}{(1+i)(1+i_{SR})}\left[1 + \frac{1}{1+i_{MR}} + \frac{1}{(1+i_{MR})^2} + \ldots\right] \\
&= \frac{\alpha P_{SR}Y_{SR}}{1+i} + \frac{\alpha P_{MR}Y_{MR}}{(1+i)(1+i_{SR})}\frac{1}{1-\frac{1}{1+i_{MR}}} \\
&= \frac{\alpha P_{SR}Y_{SR}}{1+i} + \frac{\alpha P_{MR}Y_{MR}}{(1+i)(1+i_{SR})}\left(1 + \frac{1}{i_{MR}}\right)
\end{aligned}
$$

The first term on the RHS $\frac{\alpha P_{SR}Y_{SR}}{1+i} < \frac{\alpha PY_n}{1+i}$ depends on whether $\frac{P_{SR}Y_{SR}}{PY_n} < \frac{1+i_{SR}}{1+i}$, since $P_{SR}Y_{SR}$ decreases. The second term $\frac{\alpha P_{MR}Y_{MR}}{(1+i)(1+i_{SR})}\left(1+\frac{1}{i_{MR}}\right) \gtreqless \frac{\alpha PY_n}{(1+i)^2}\left(1+\frac{1}{i}\right)$ depends on whether $\frac{P_{MR}Y_{MR}}{PY_n} = \frac{P_{MR}}{P} \gtreqless \frac{(1+i_{SR})\left(1+\frac{1}{i}\right)}{(1+i)\left(1+\frac{1}{i_{MR}}\right)}$, since i_{SR},

P_{MR} and i_{MR} decrease.

In sum, $\$Q_\tau$ can be higher or lower than $\$Q_{\tau-1}$.