Render the Possibilities

SIGGRAPH2016

THE **43RD** INTERNATIONAL
CONFERENCE AND **EXHIBITION** ON

& Computer Graphics
Interactive Techniques

24-28 JULY

ANAHEIM, CALIFORNIA

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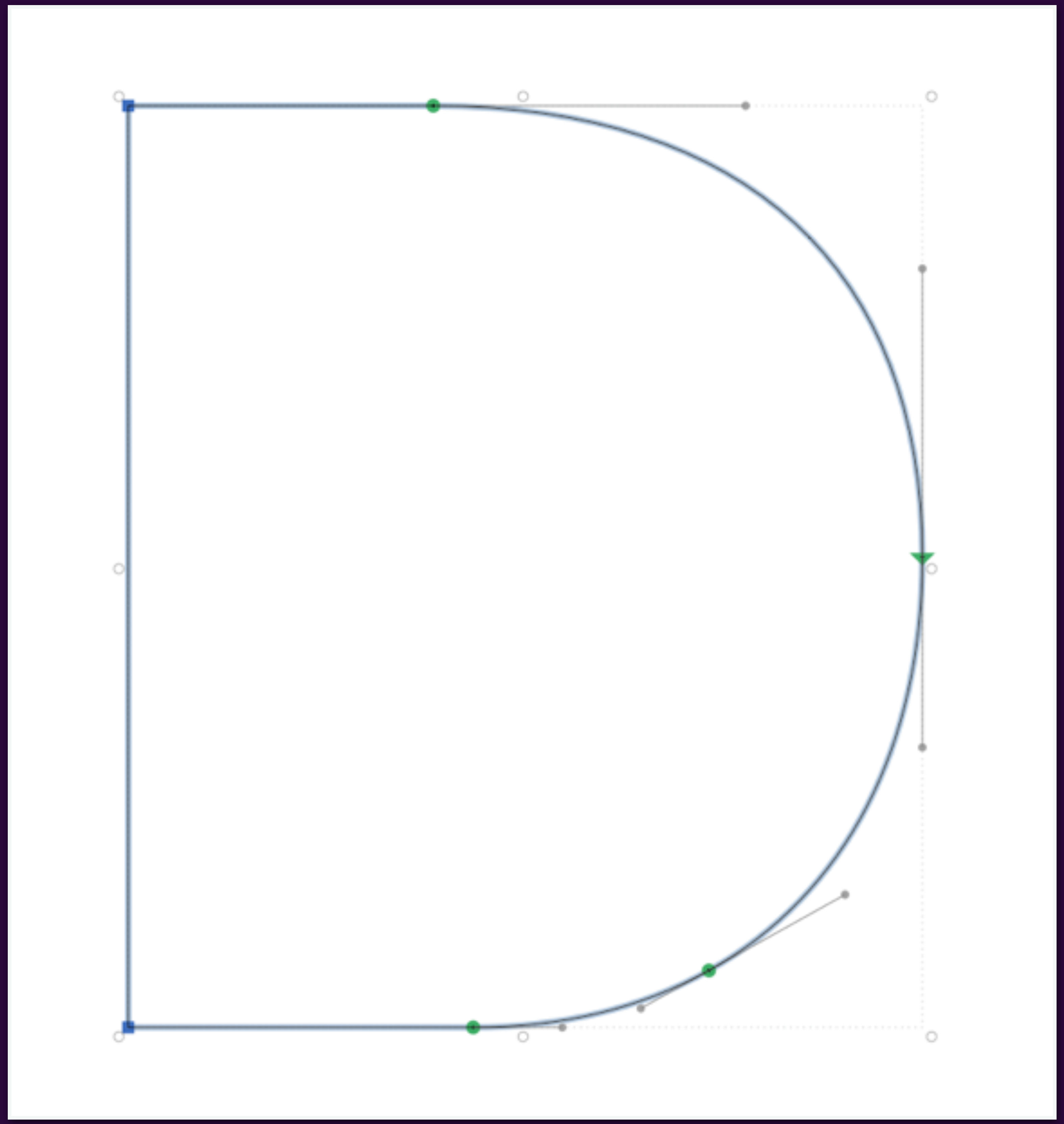
SIGGRAPH2016
THE 43RD INTERNATIONAL CONFERENCE AND EXHIBITION ON
Computer Graphics & Interactive Techniques

ARM

Practical Analytic 2D Signed Distance Field Generation

Wasim Abbas
ARM - Staff Engineer

Path



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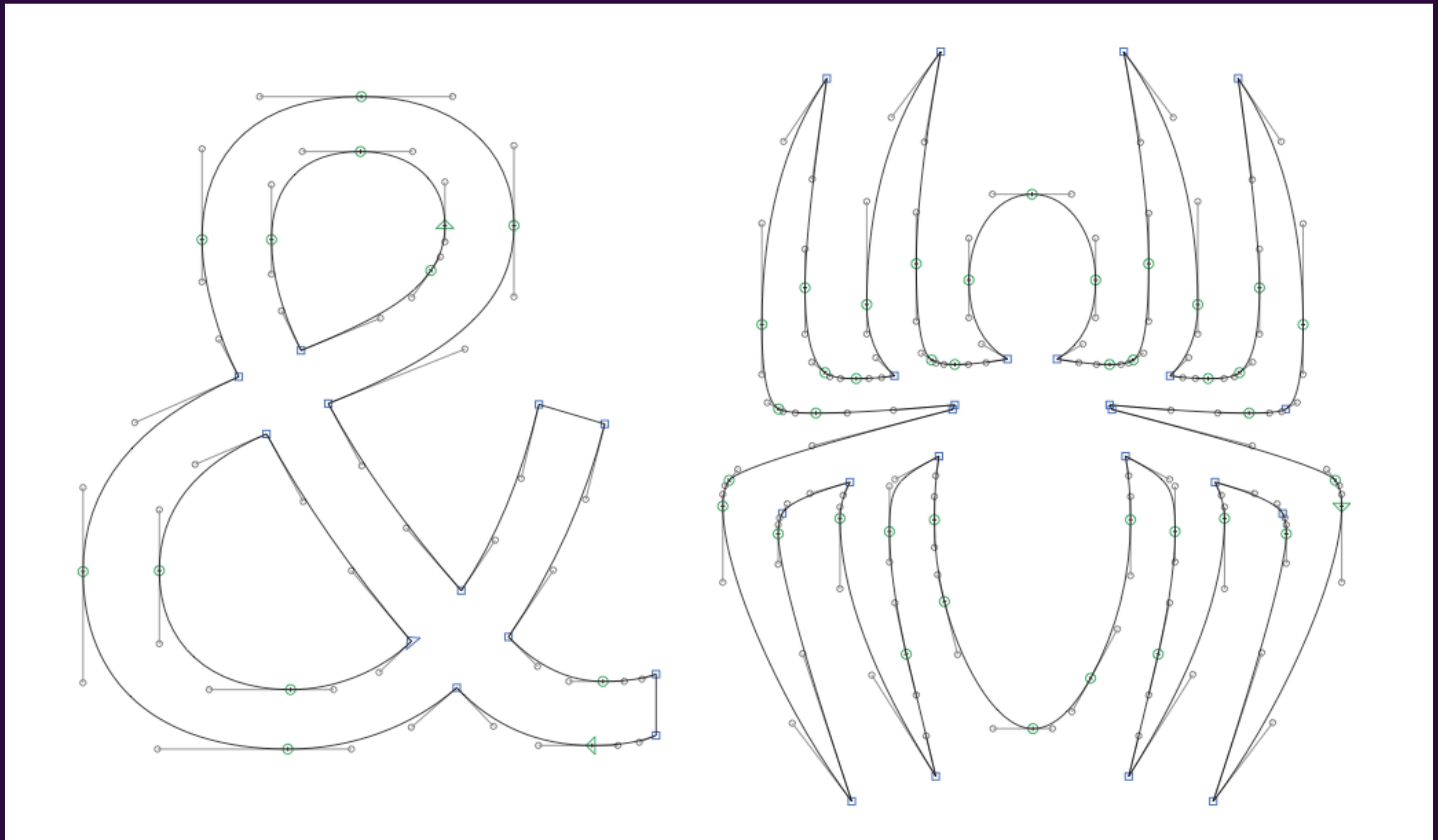
Path



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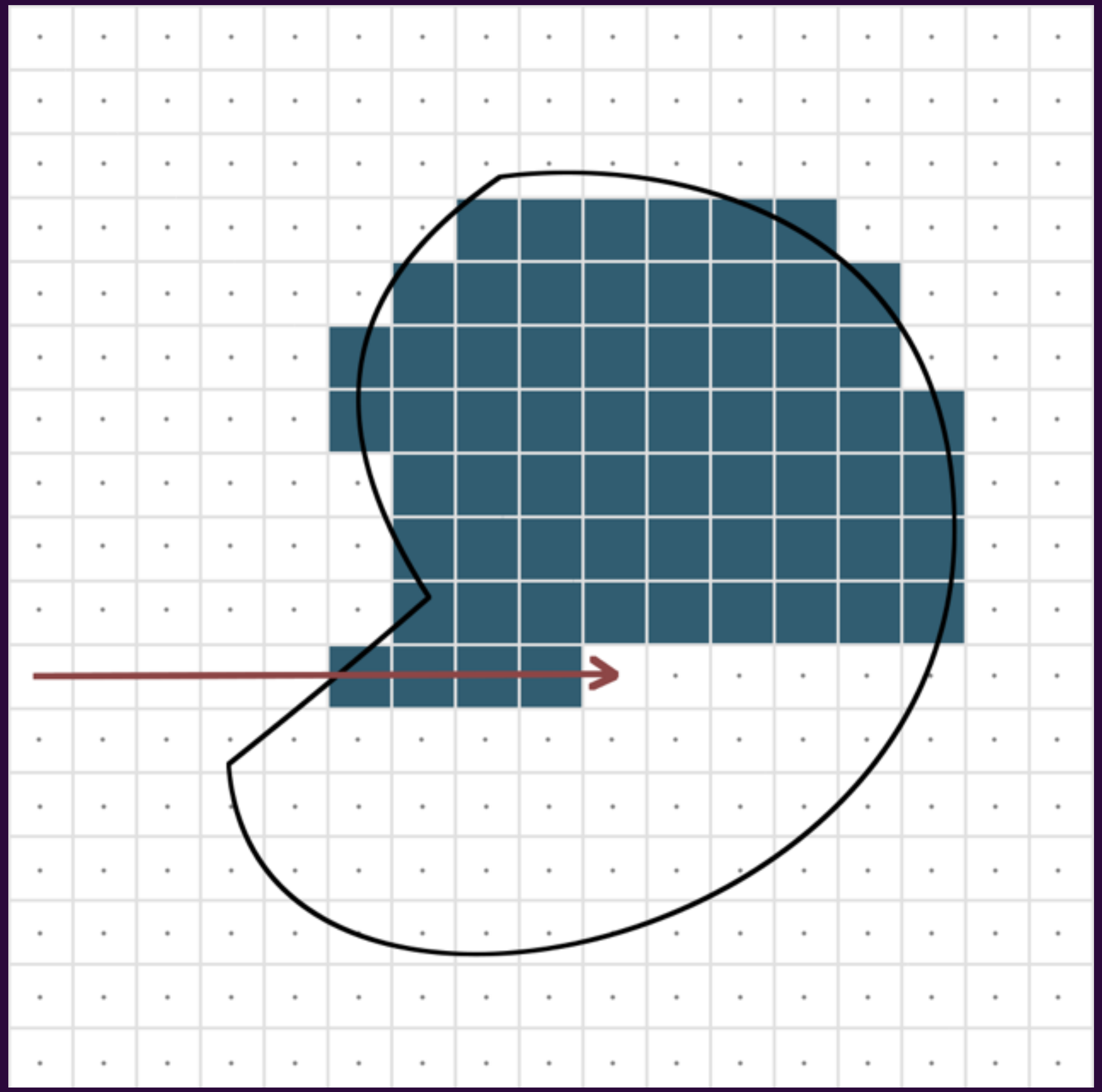
Path

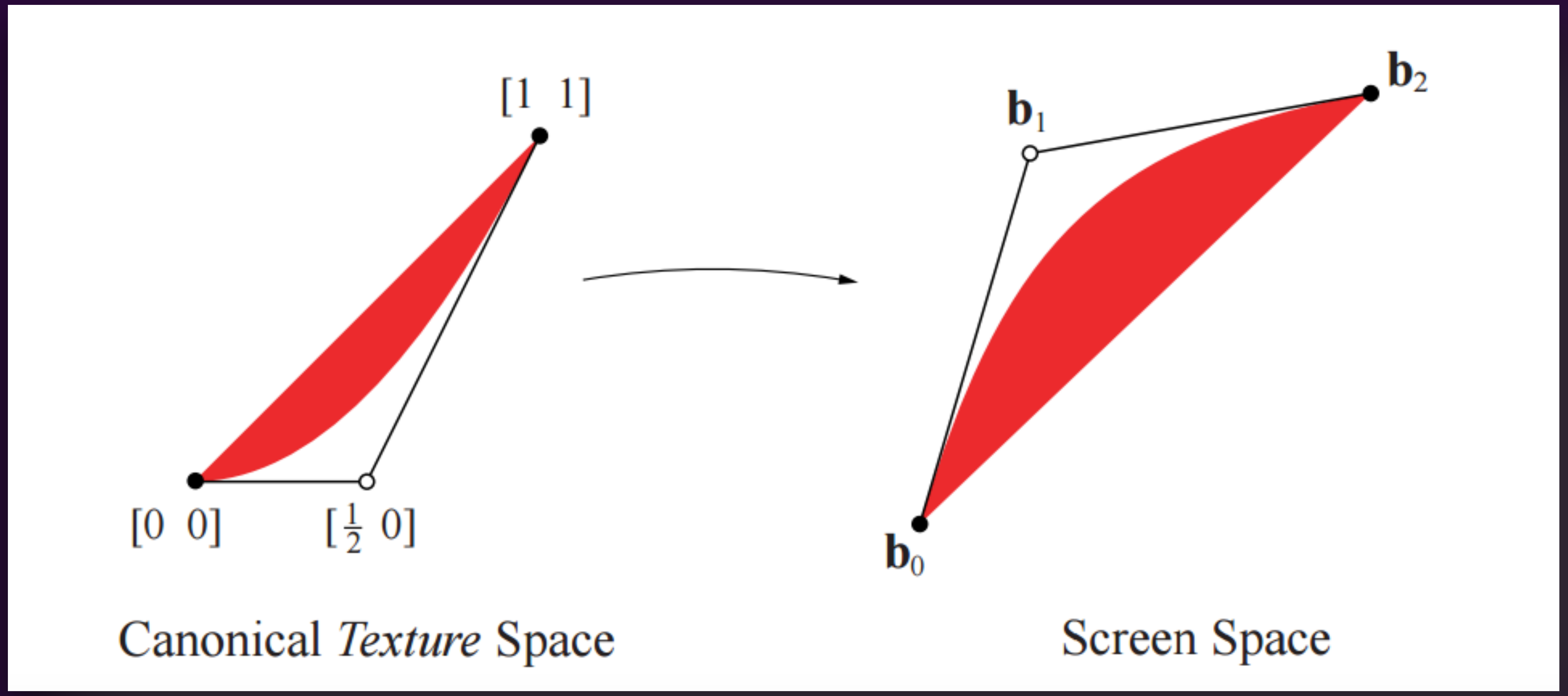
Outline

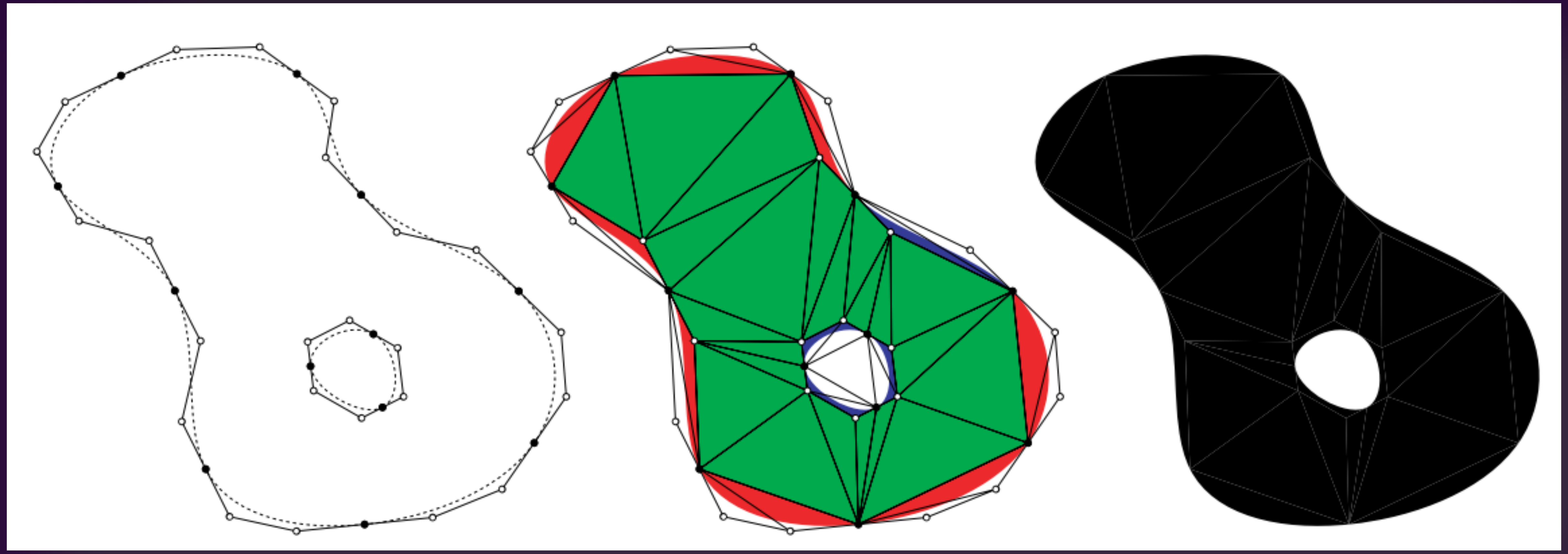
- What is the problem?
- What is our solution?
- Results and conclusion

Rendering a Path



Rendering a Path





C. Loop, J. Blinn
Microsoft Research 2005

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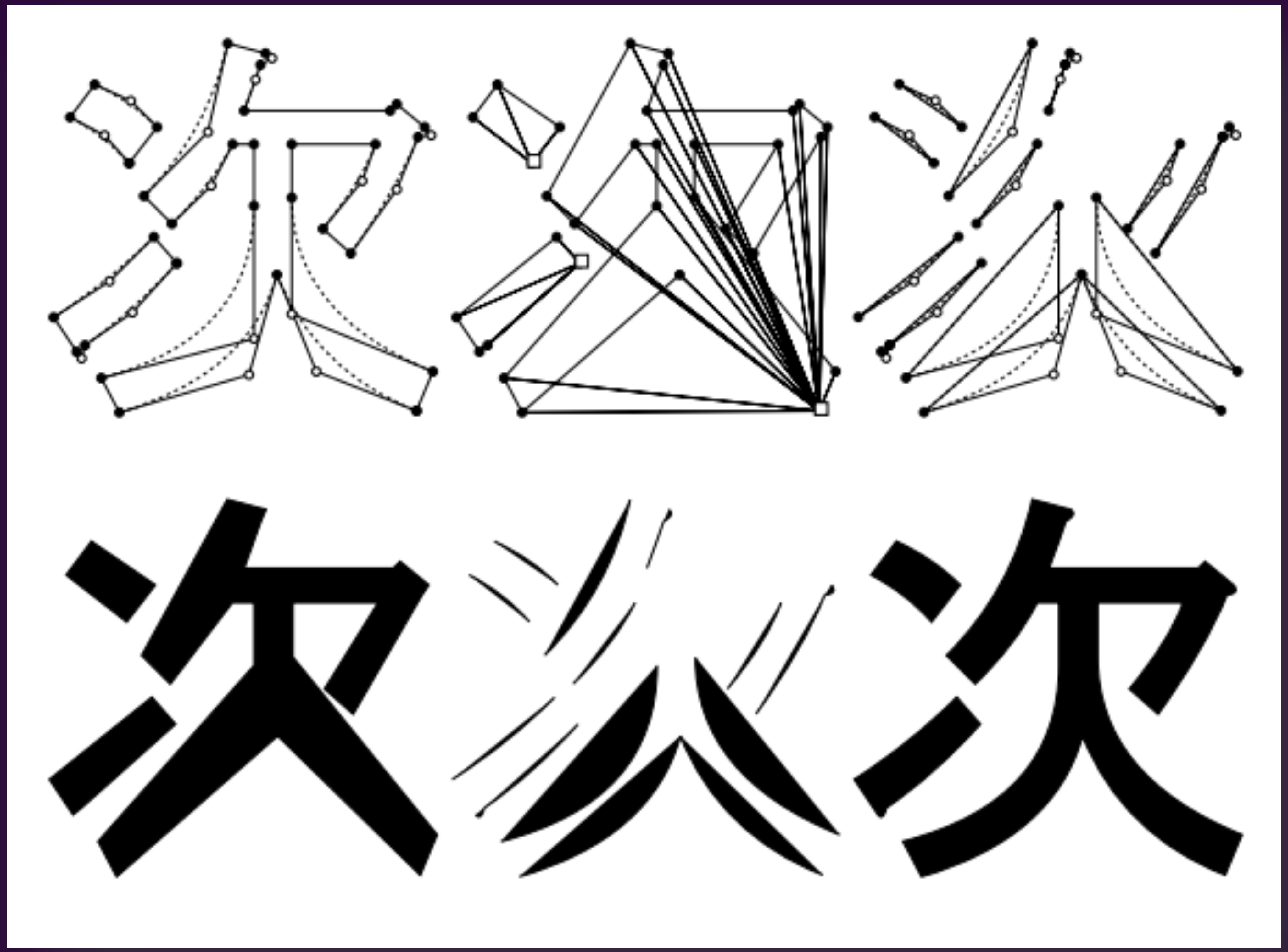
Rendering a Path



Y. Kokojima, K. Sugita, T. Saito, T. Takemoto
Toshiba 2006

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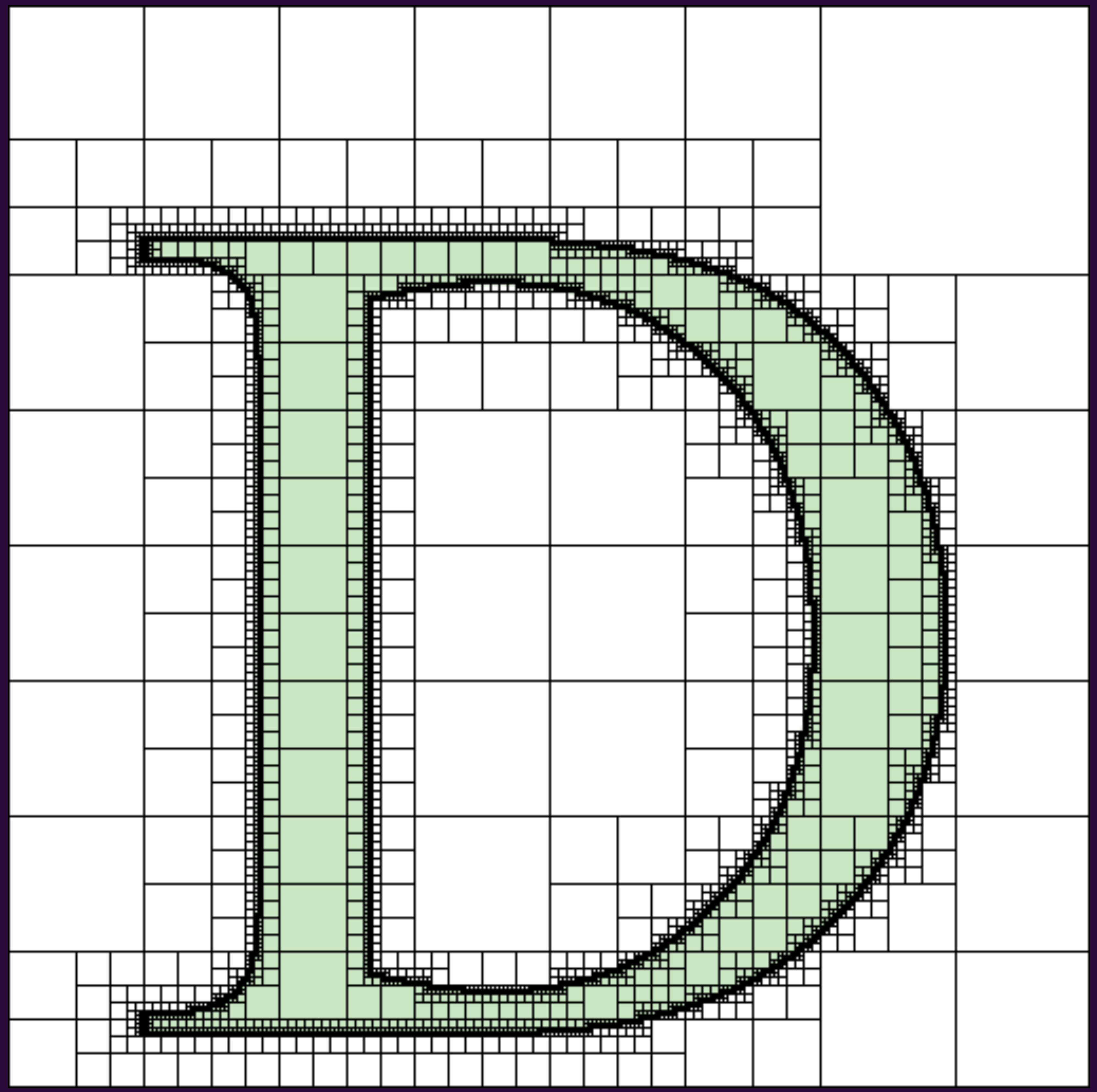
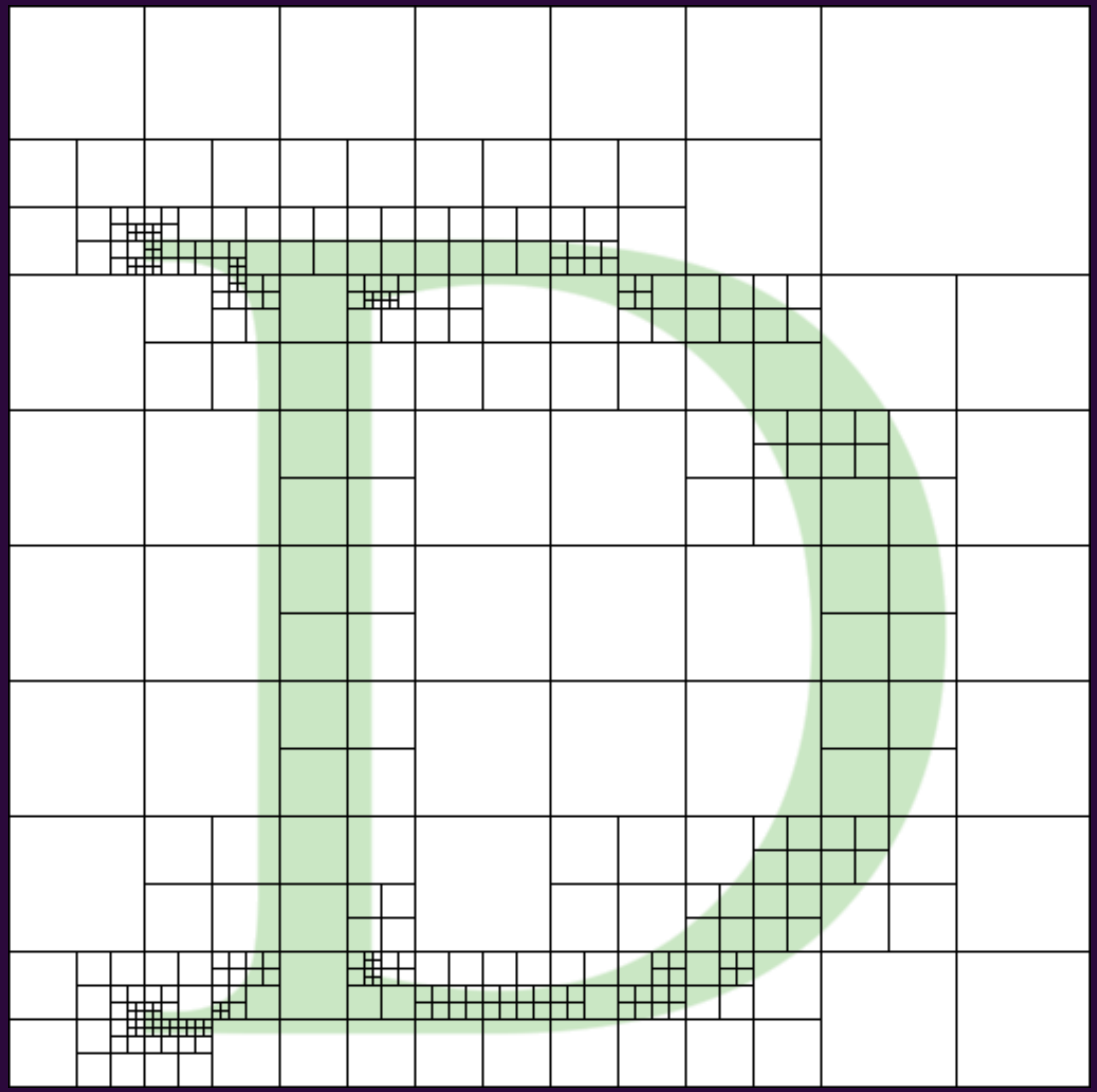
Rendering a Path





S. Frisken, R. Perry, A. Rockwood, T. Jones
Mitsubishi Electric Research Laboratory 2000

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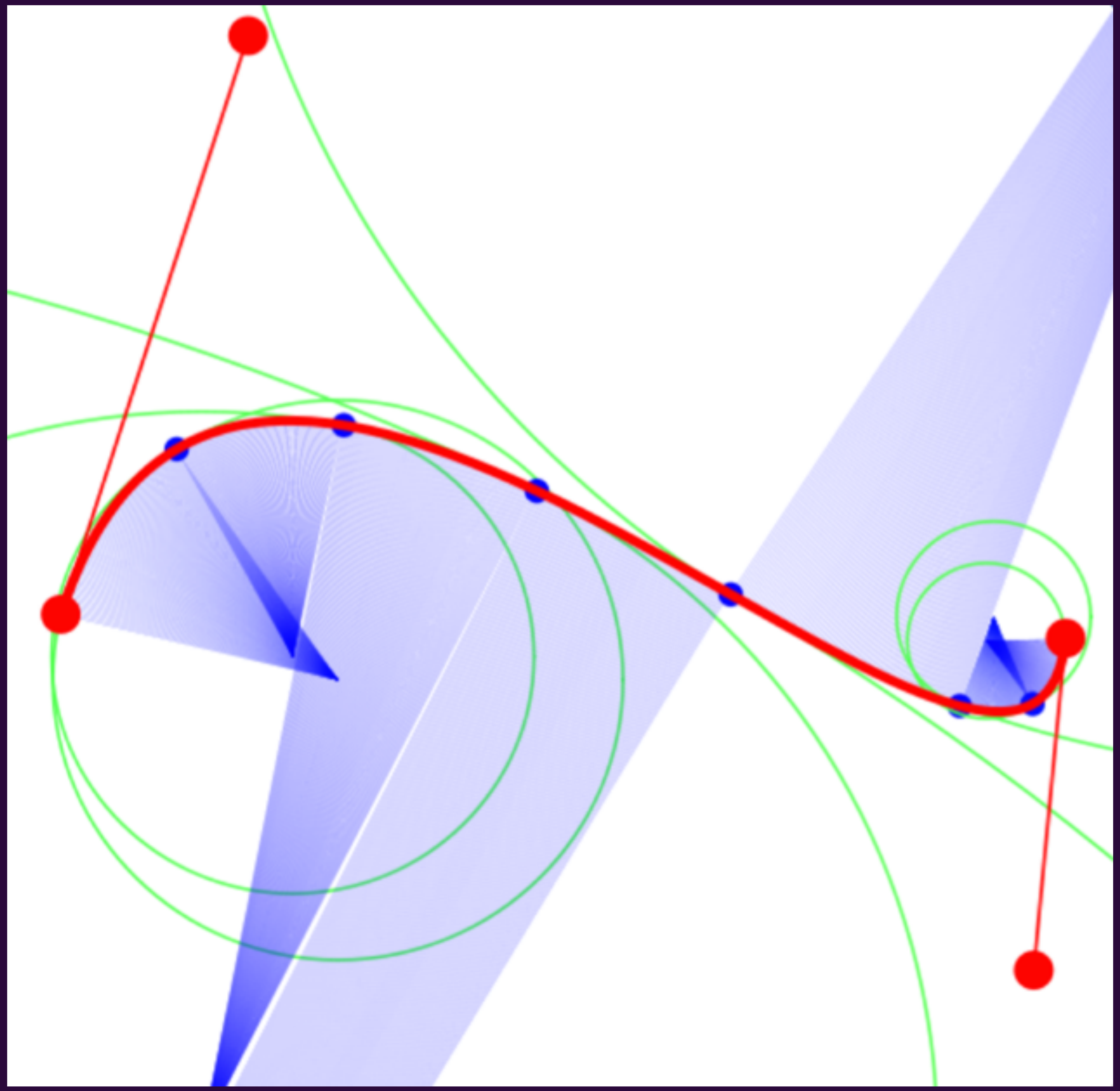
Rendering a Path



B. Esfahbod
Google 2014

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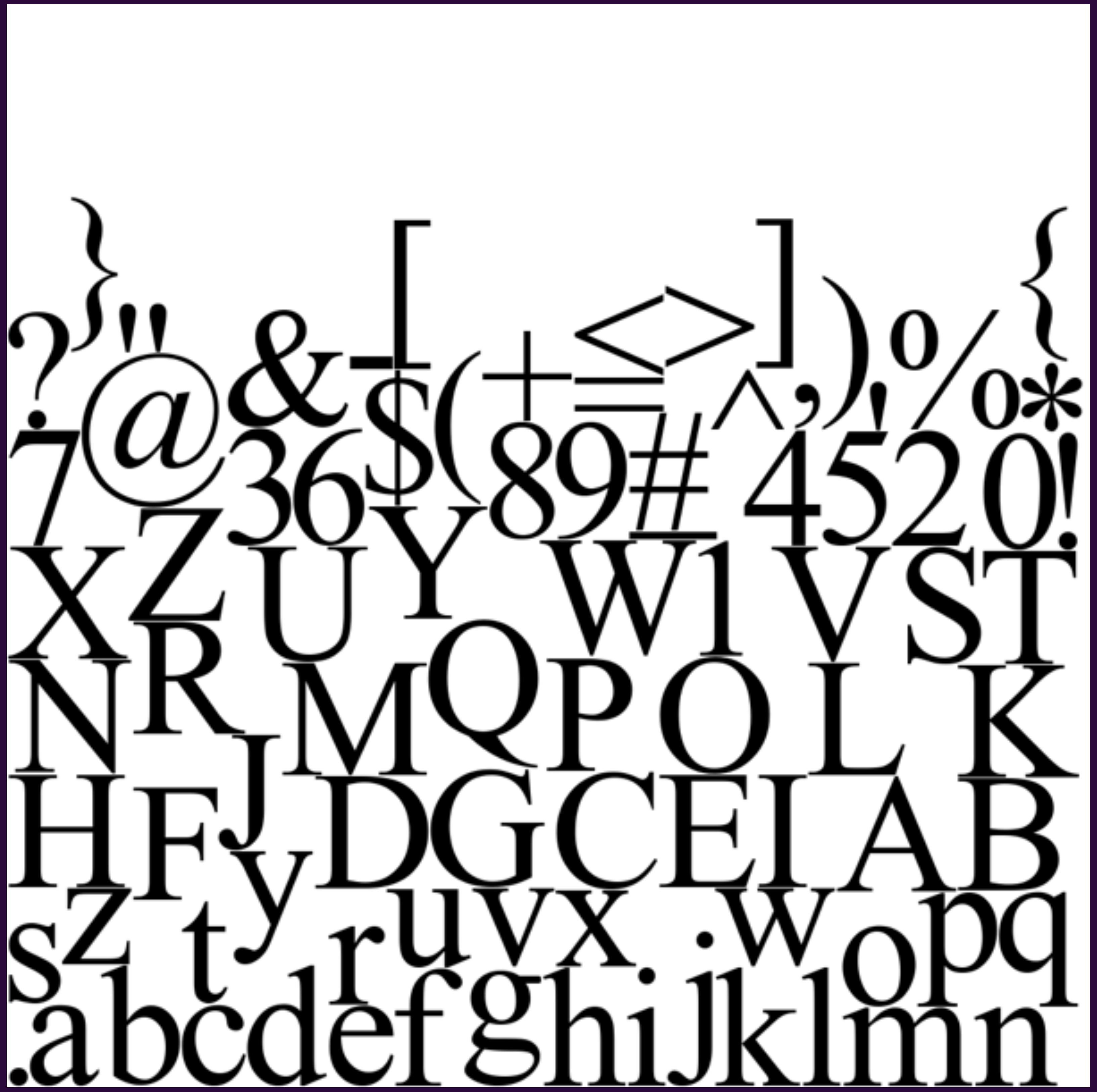
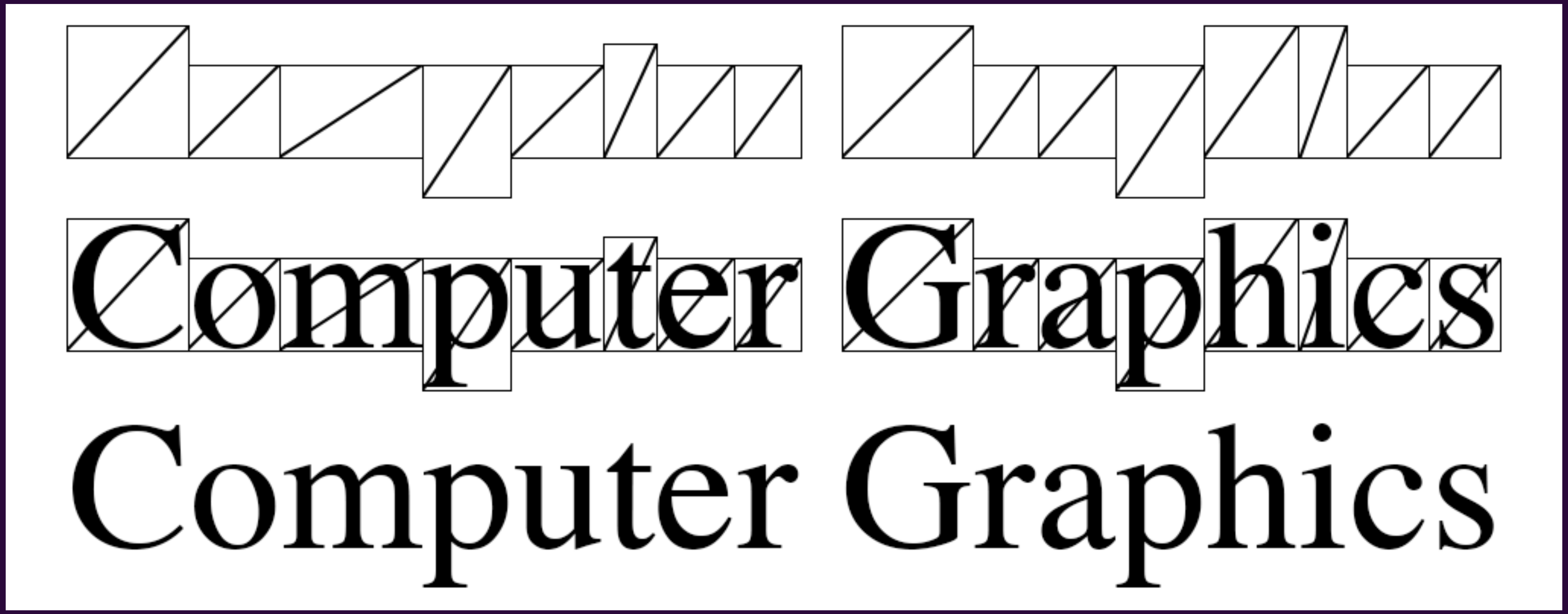
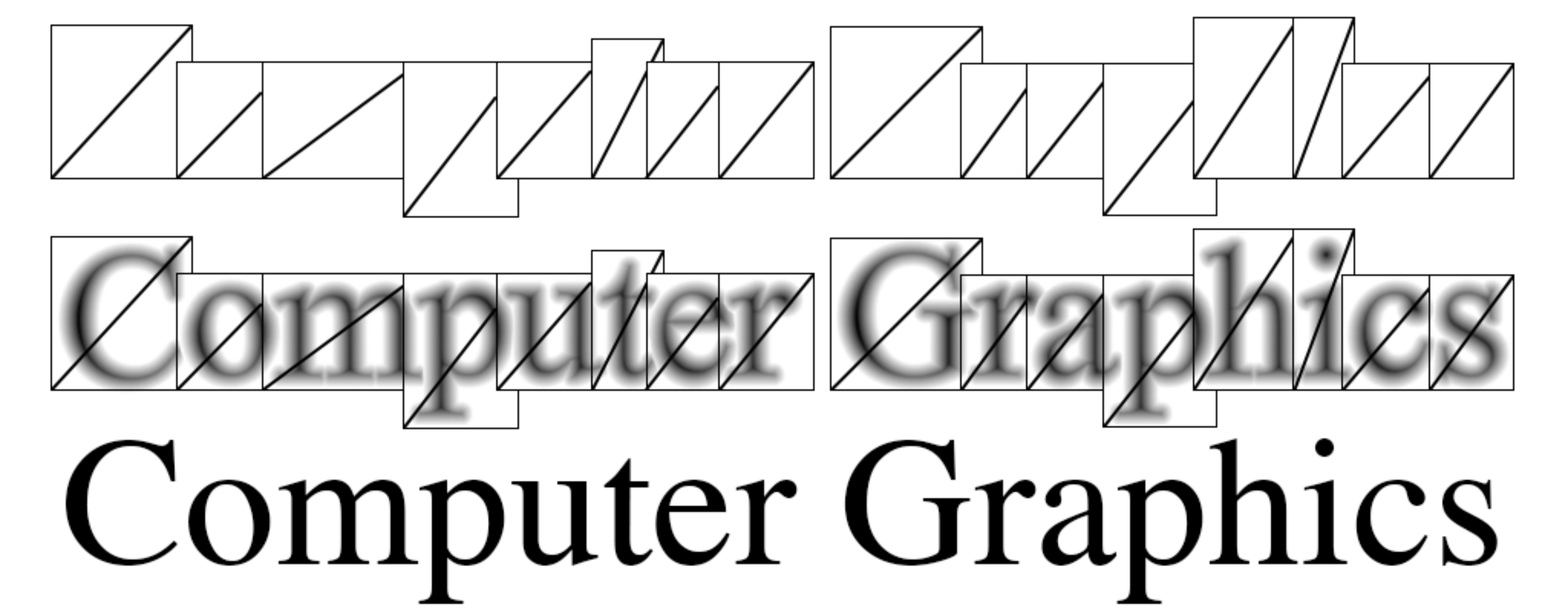
Rendering a Path





Rendering a Path



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Rendering a Path

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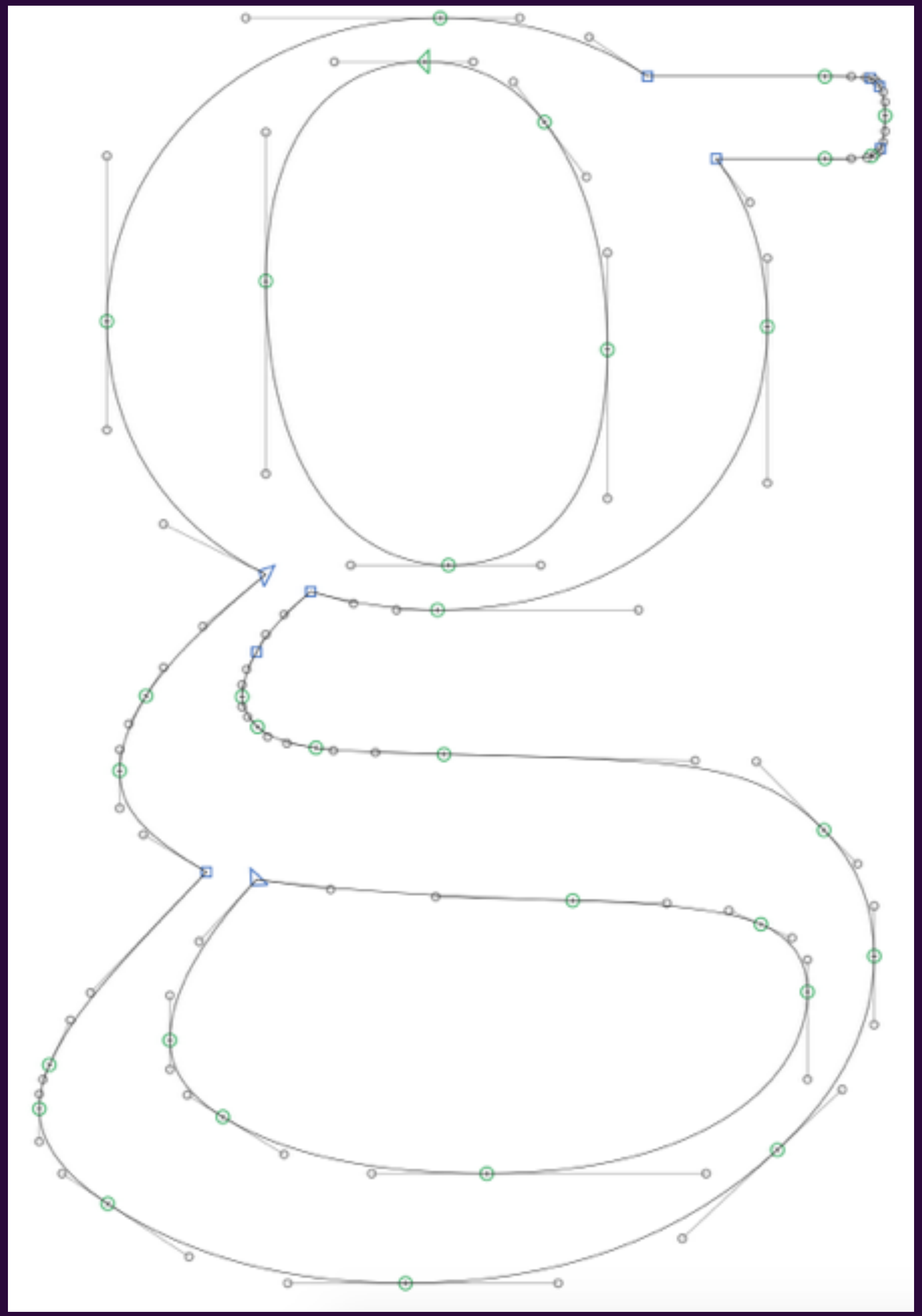
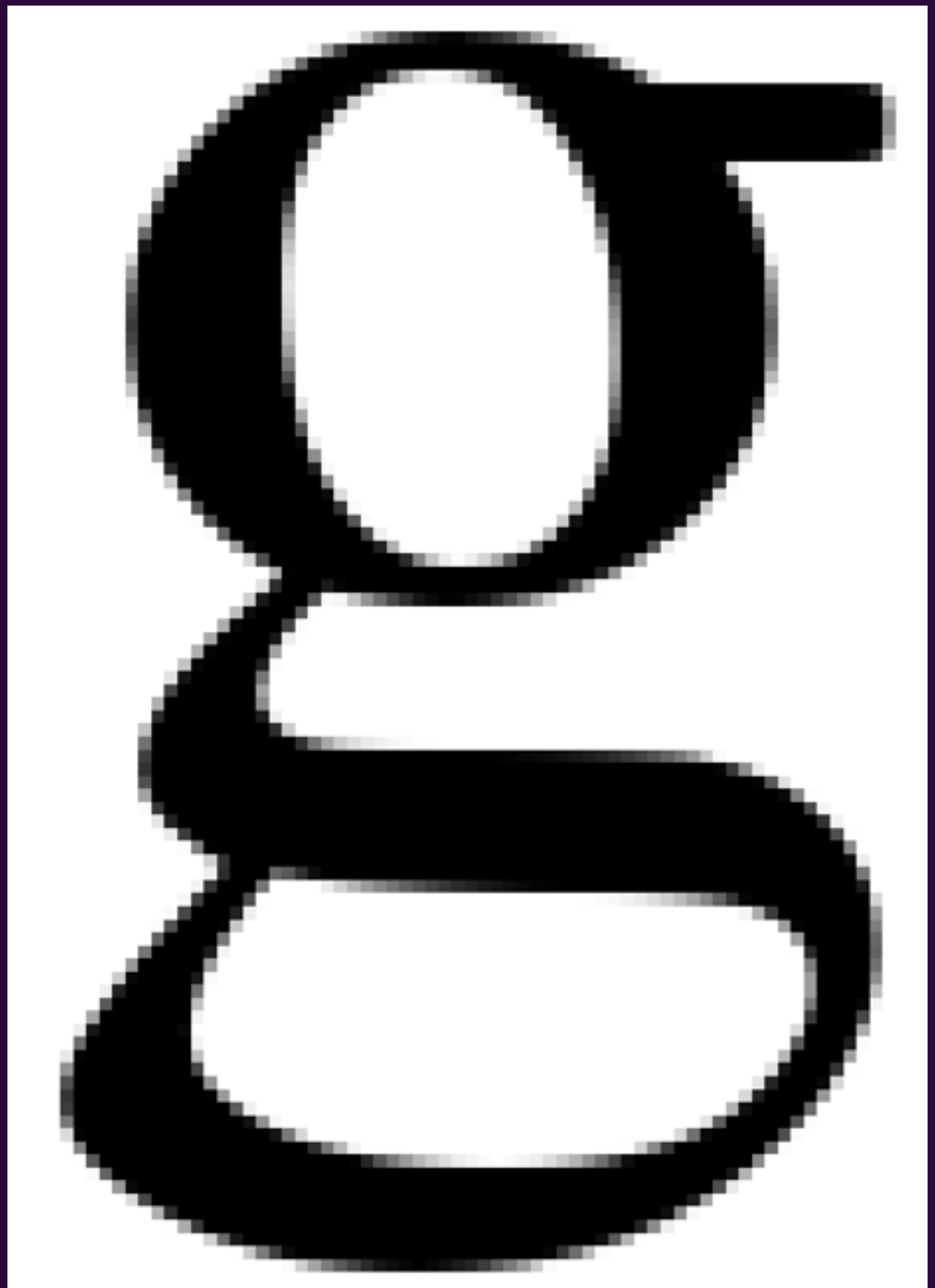
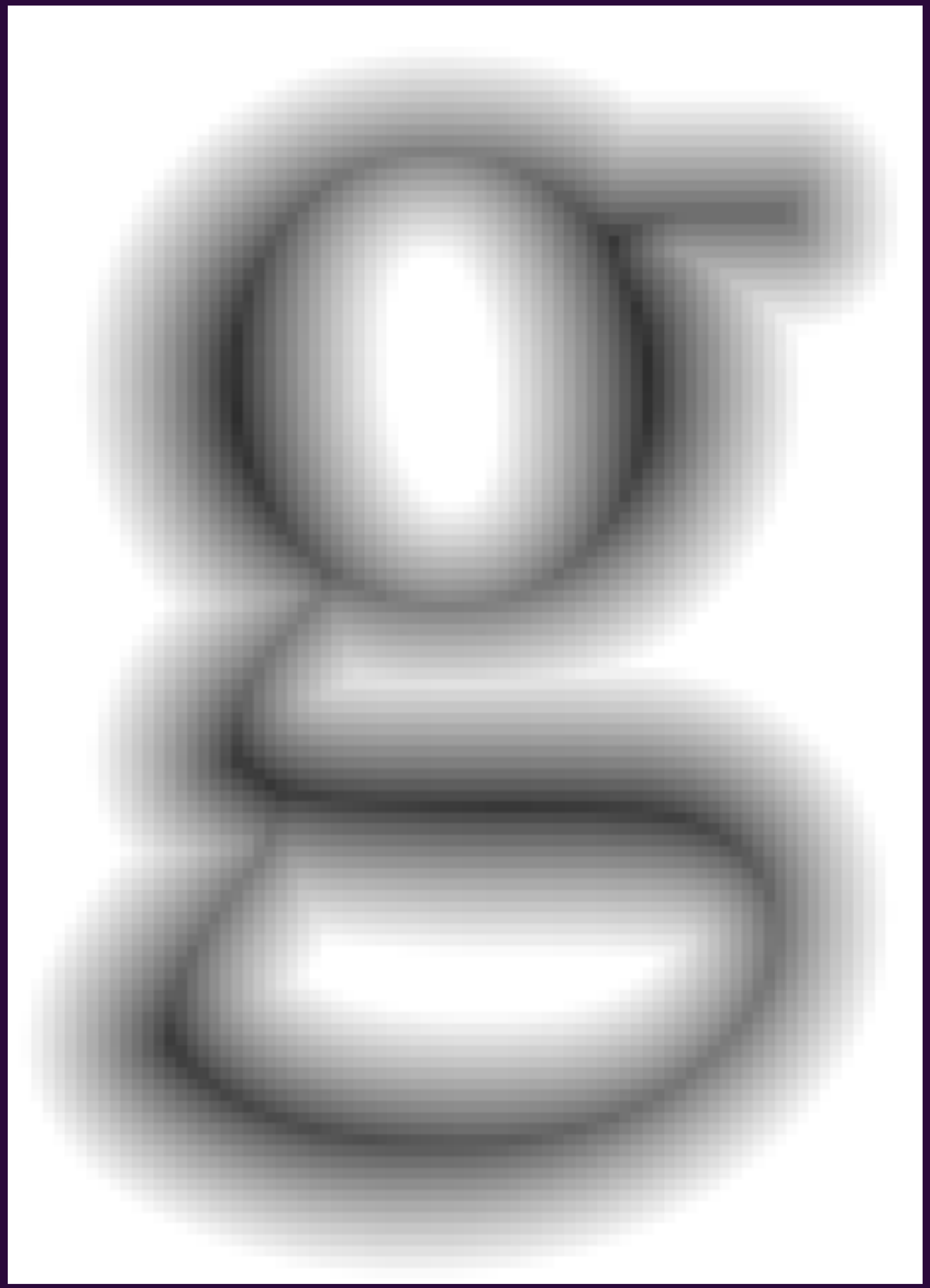
SDFs generation 8SSEDT







Gustavson 2011 and Danielsson 1980

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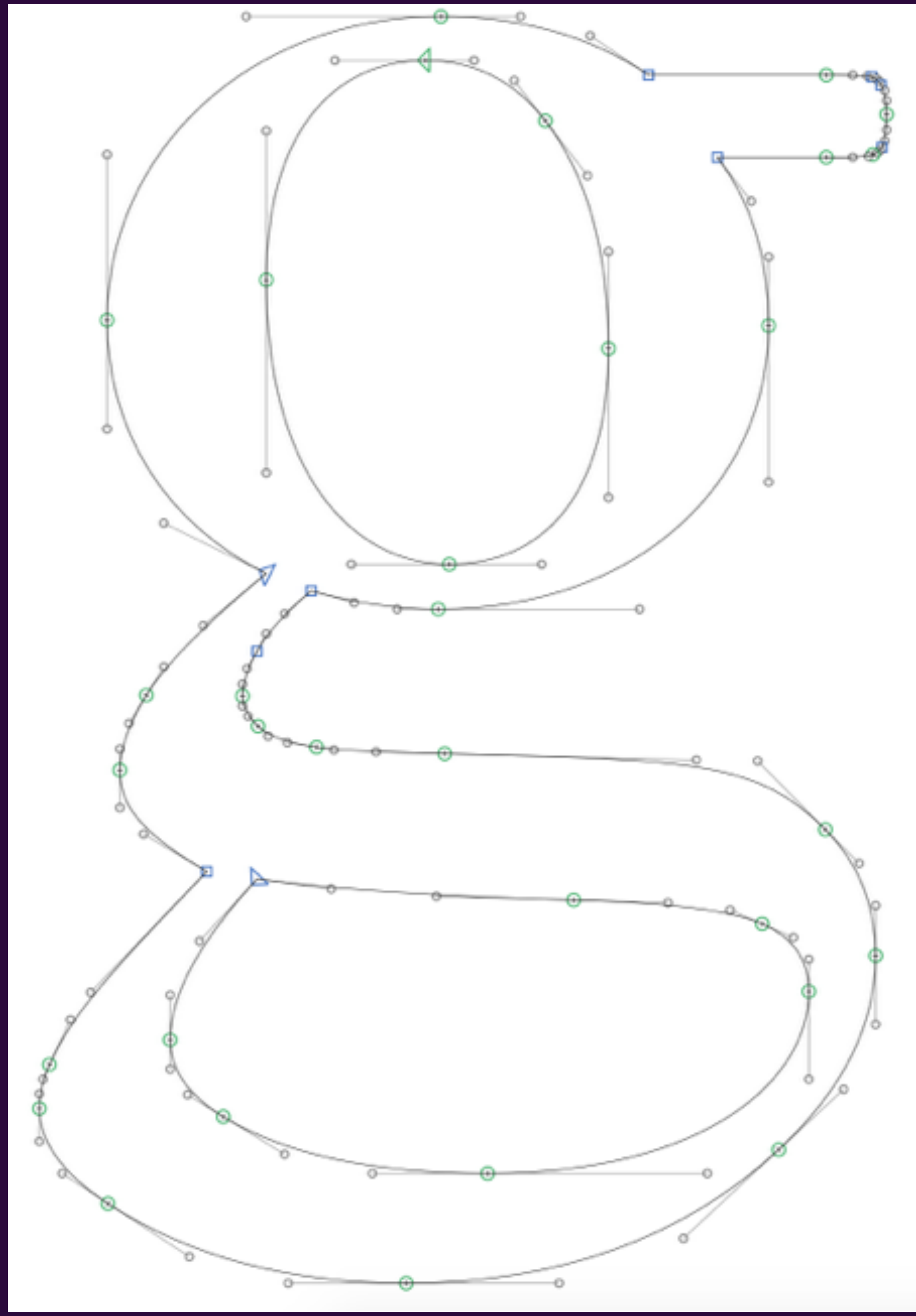
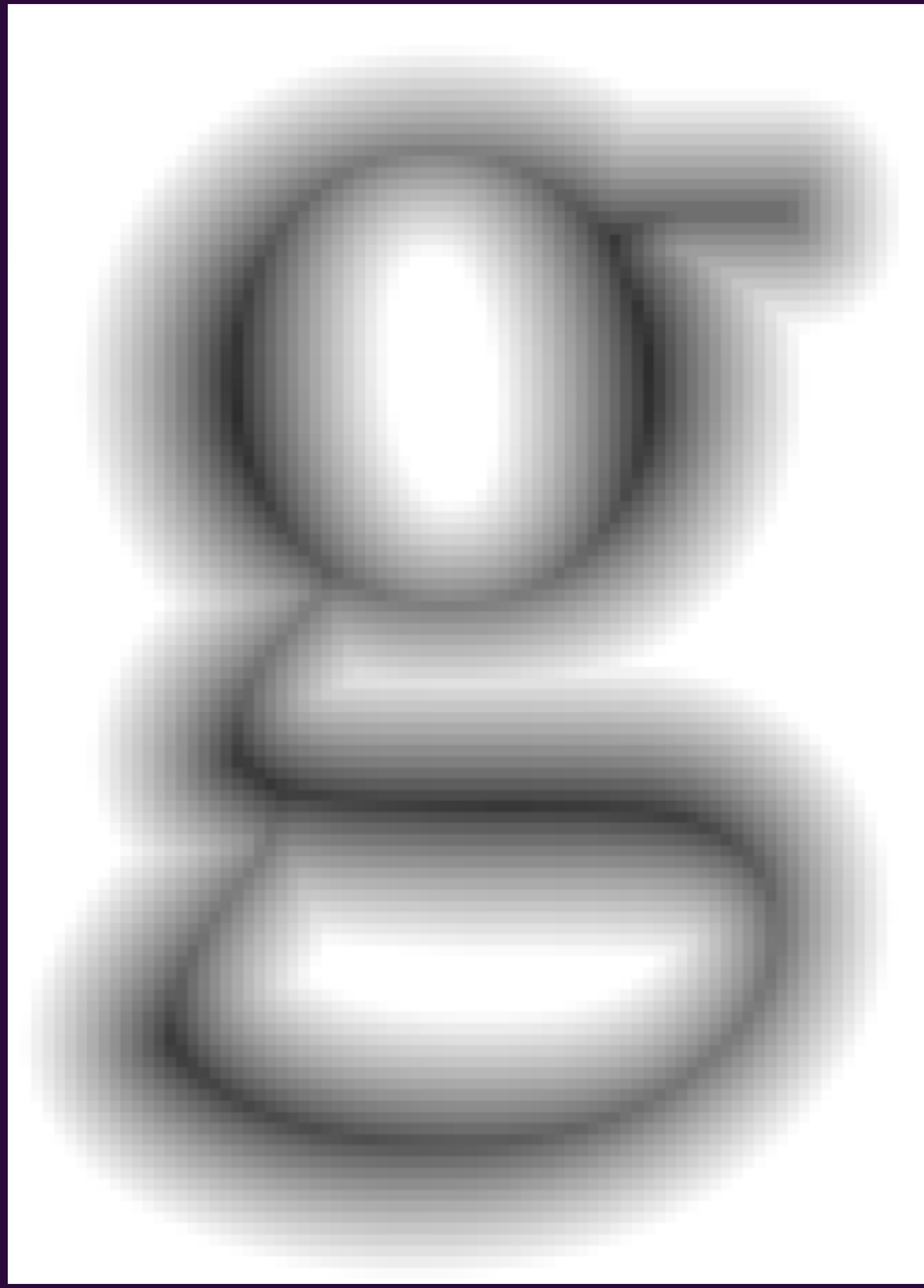
SDFs generation ARM technique





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ARM Technique

- Any Line or Quadratic Bézier curve can be transformed into a fixed, known form with a series of transformations (Translation, Rotation, Scaling)

- All lines can map to a simple horizontal line section of

$$y = 0$$

- All Quadratic Bézier curve can map onto a segment of the curve

$$y = x^2$$

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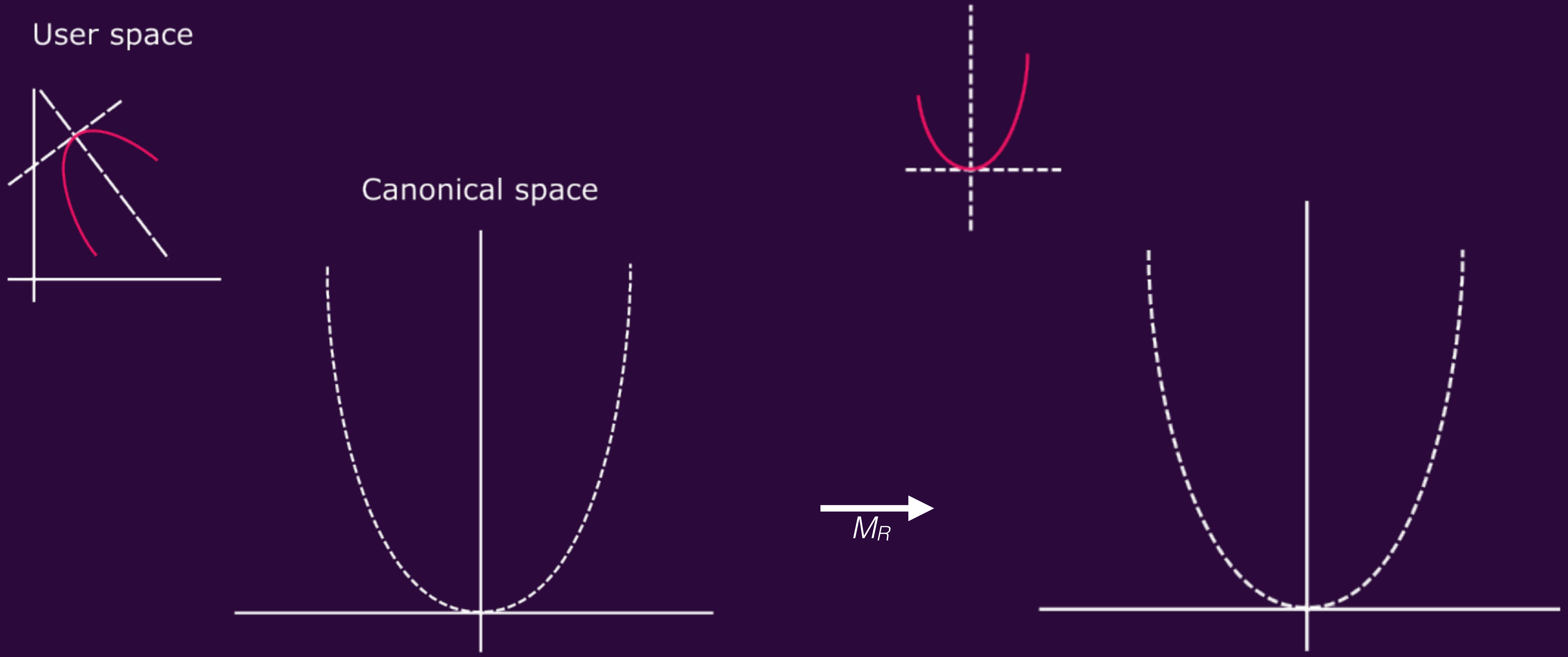
Fundamentals - Quadratic Beziers 1



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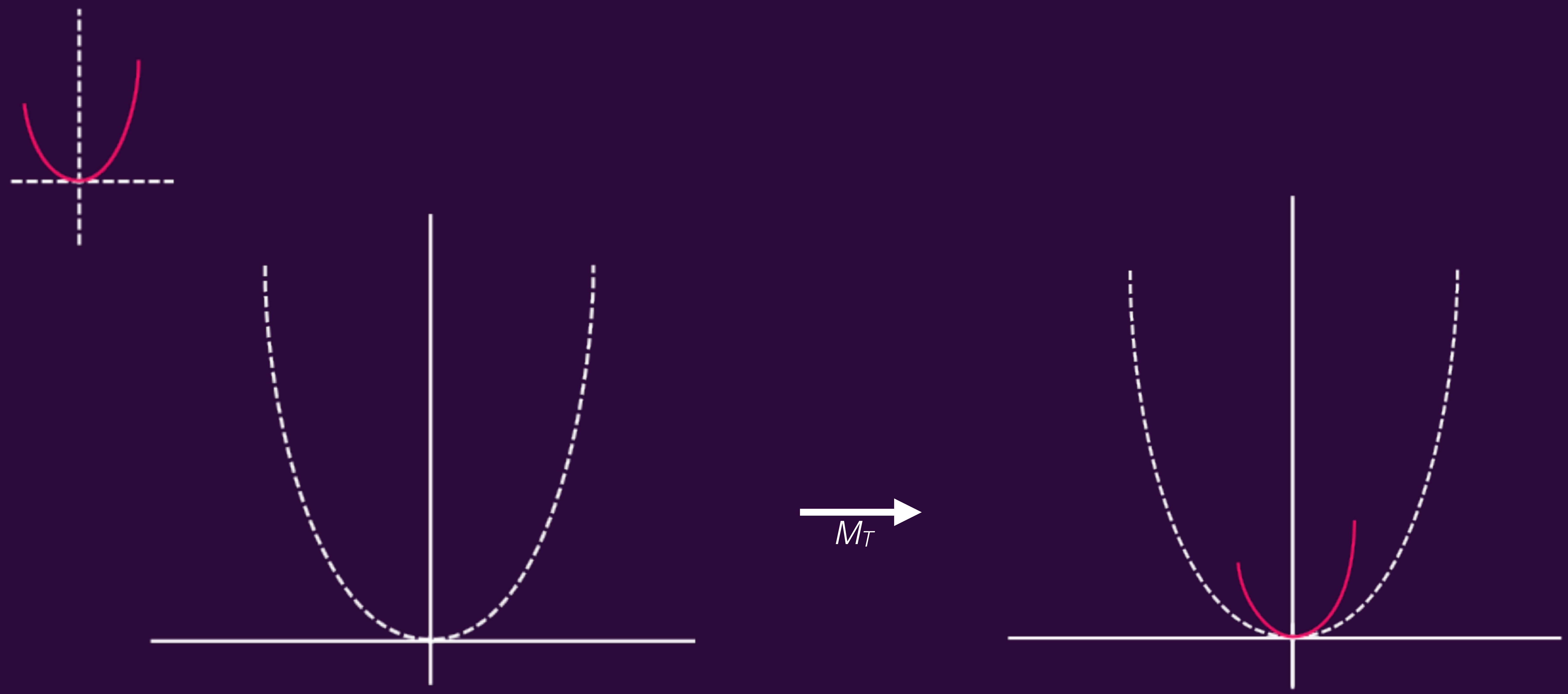
Fundamentals - Quadratic Beziers 2



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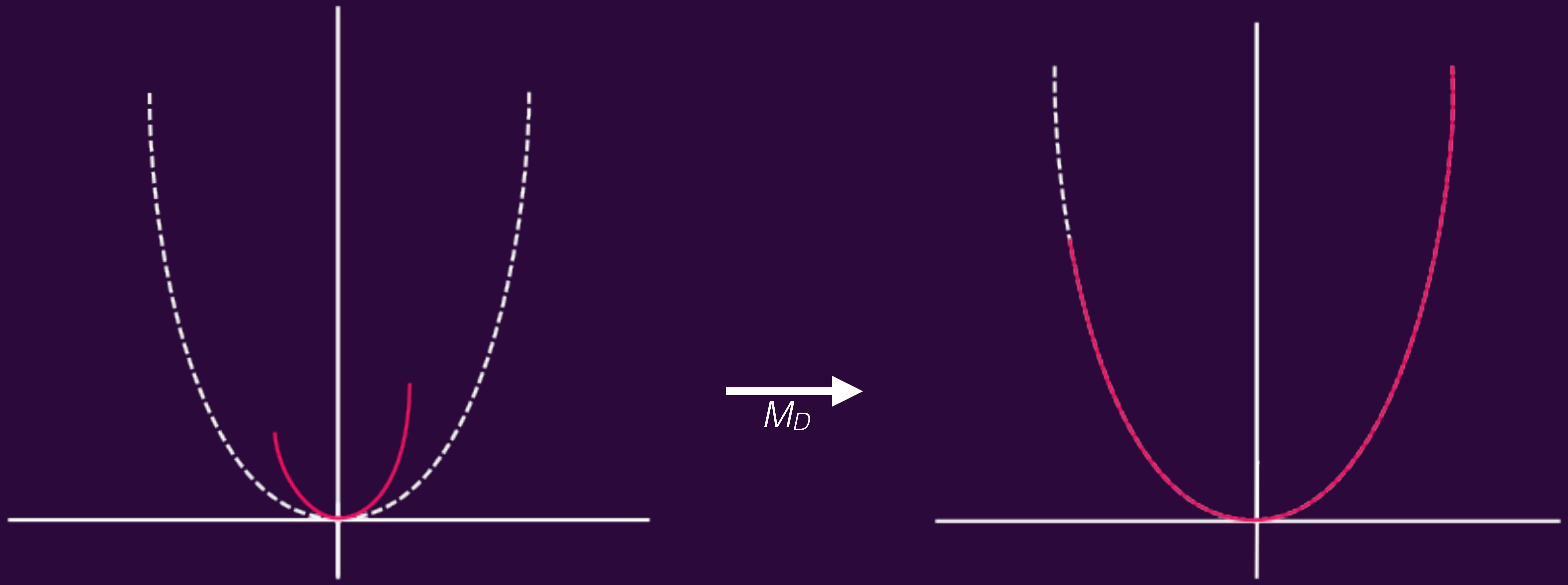
Fundamentals - Quadratic Beziers 3



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Final Transformation Matrix

$$R = DTR_\theta$$

Distance calculation from (u, v)

$$D = \left((x-u)^2 + (x^2-v)^2\right)^{1/2}$$

$$4x^3 + (1-2v)2x - 2u = 0$$

$$x^3 + ax + b = 0$$

where

$$a = \frac{1}{2} - v$$

$$b = -\frac{u}{2}$$

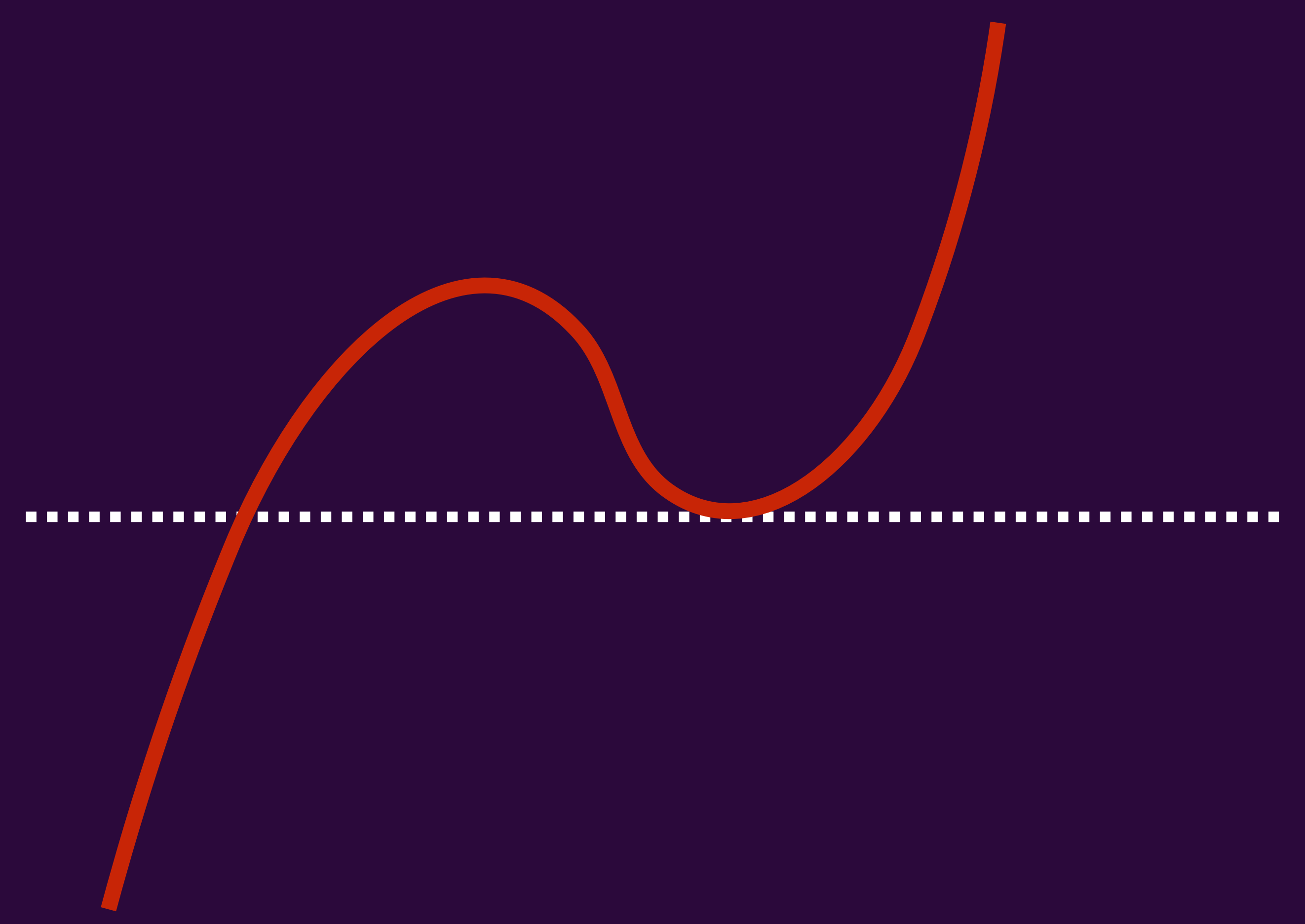
Roots case 1

$$b^2/4 + a^3/27 > 0$$

$$x_1 = \left(-\frac{b}{2} + \sqrt{\frac{b^2}{4} + \frac{a^3}{27}}\right)^{1/3} + \left(-\frac{b}{2} - \sqrt{\frac{b^2}{4} + \frac{a^3}{27}}\right)^{1/3}$$

Roots case 2

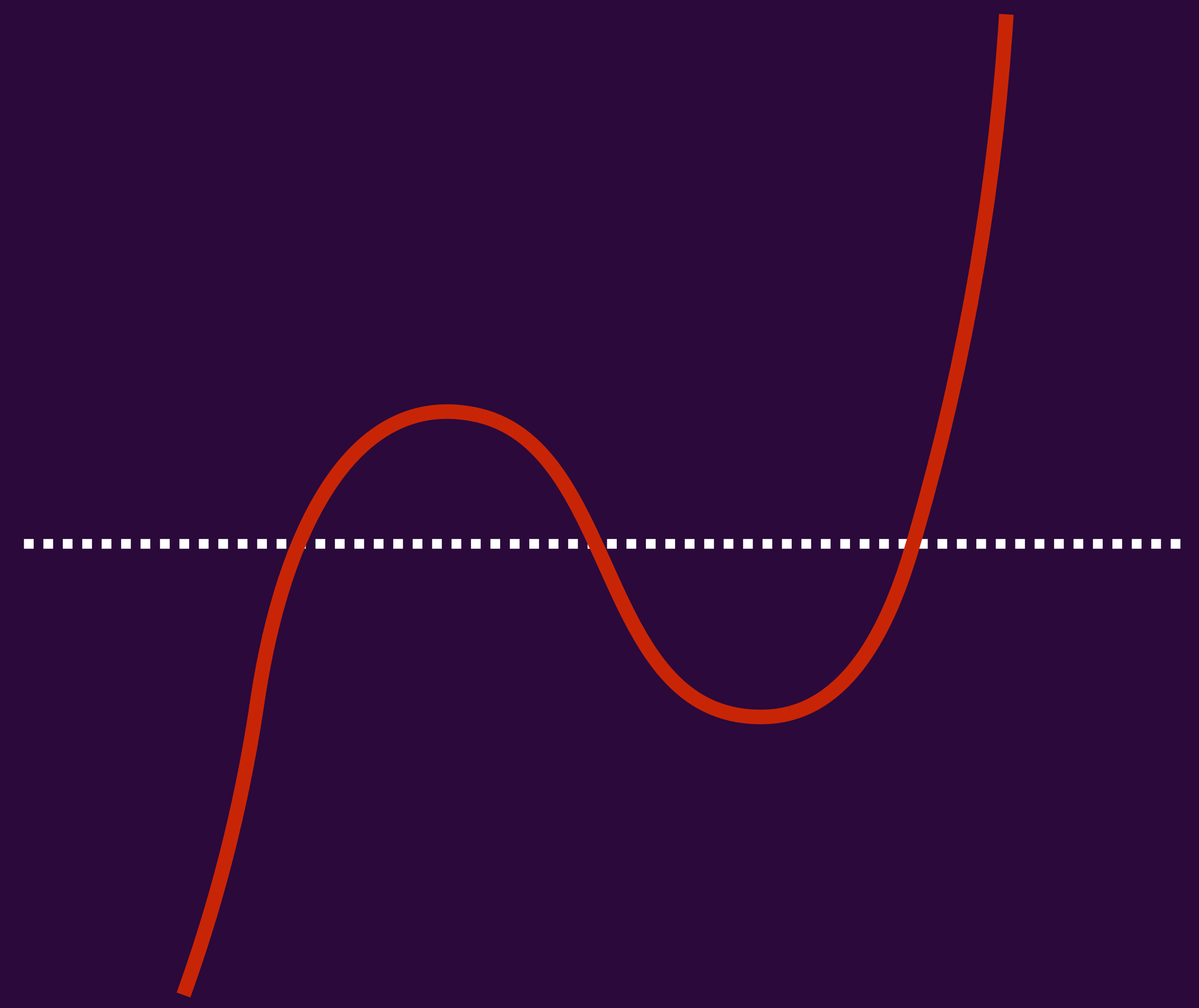
$$b^2/4 + a^3/27 = 0$$



$$x_1 = \left(-\frac{b}{2} + \sqrt{\frac{b^2}{4} + \frac{a^3}{27}}\right)^{1/3} + \left(-\frac{b}{2} - \sqrt{\frac{b^2}{4} + \frac{a^3}{27}}\right)^{1/3}$$

Roots case 3

$$b^2/4 + a^3/27 < 0$$



$$x_k = 2\sqrt{\frac{-a}{3}}\cos\left(\frac{\phi}{3} + \frac{2k\pi}{3}\right) \qquad k = 0, 1, 2 \qquad \cos\phi = \begin{cases} -\sqrt{\dfrac{b^2/4}{-a^3/27}} & \text{if } b > 0 \\ \sqrt{\dfrac{b^2/4}{-a^3/27}} & \text{if } b < 0 \end{cases}$$

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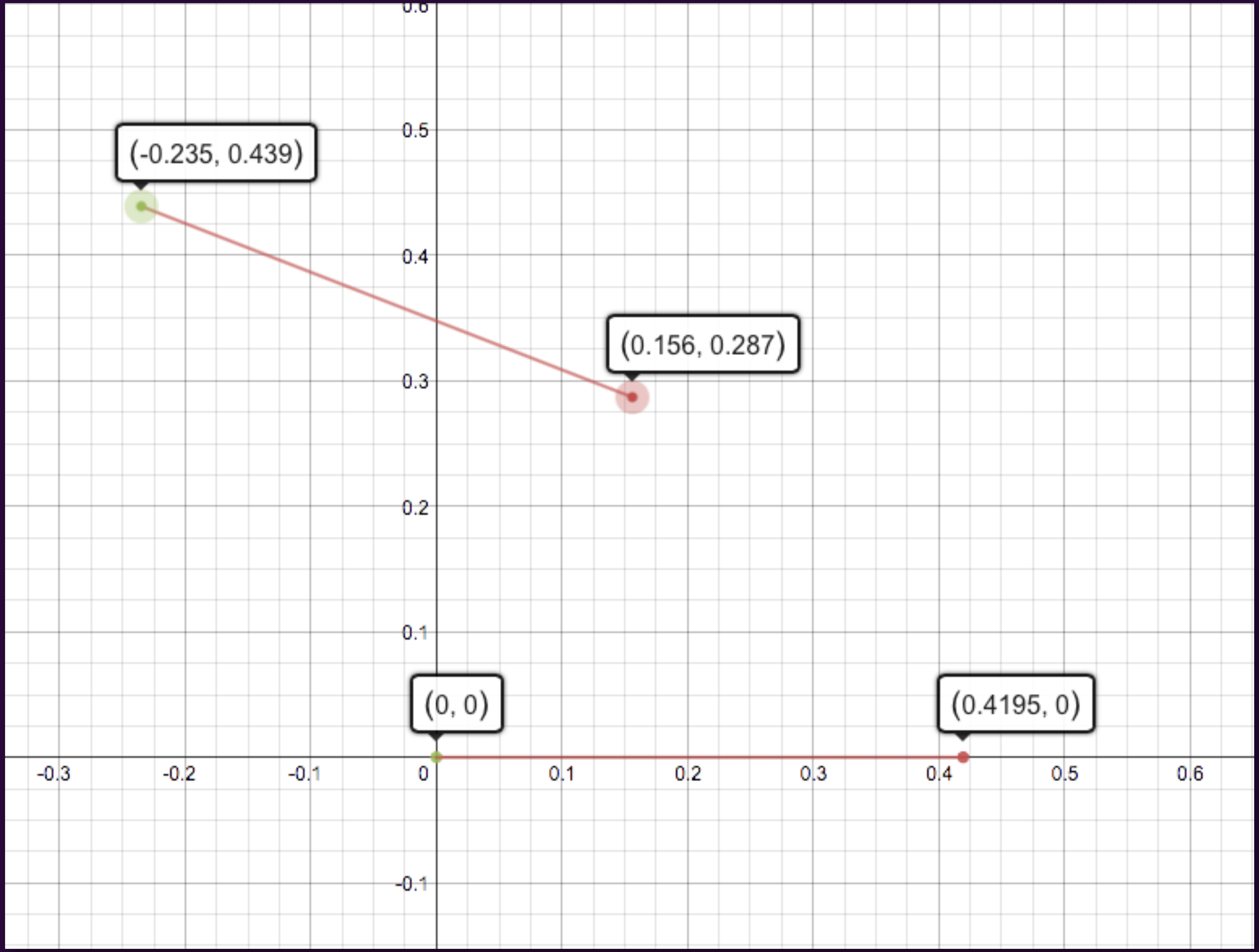
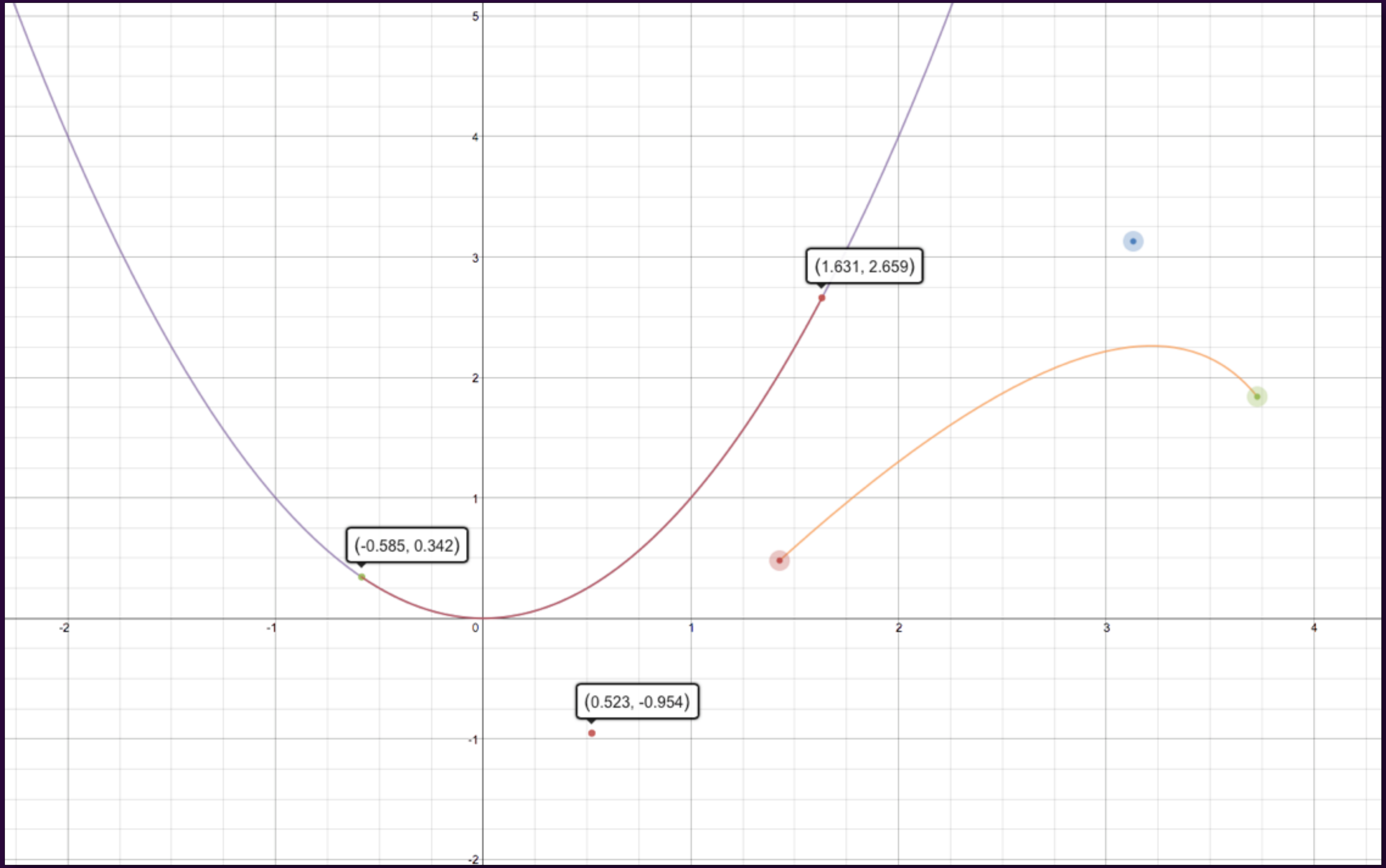
Live Demo: Lines



https://www.desmos.com/calculator/lvwv689bx0

Live Demo: Curves



https://www.desmos.com/calculator/l9ssbp9xqh

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Implementation Theory

- For any given segment
  - We can take an arbitrary point in space around it
    - Using the transformed point, calculate distance to canonical form
    - Scale back to give real distance
    - We can tell if the point is inside or outside the segment
      - Lines: sign of y
      - Quadratic Bézier's side of tangent from nearest point on curve
- Sample around a segment in a pixel grid and we have signed distance values!

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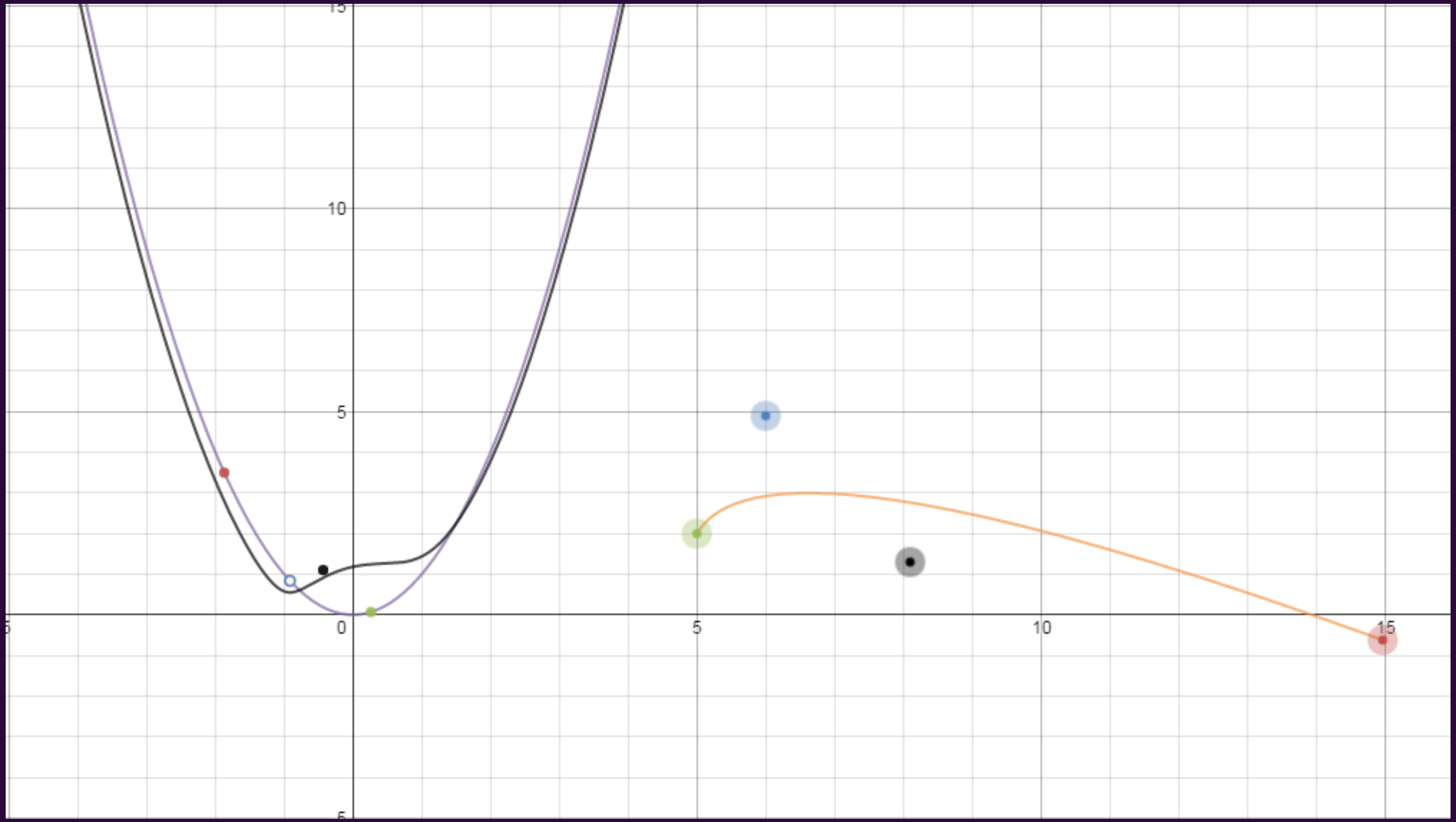
Live Demo! complete



https://www.desmos.com/calculator/wxqfhychxu

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Implementation – Signed Distance Fields

```
// for each segment initialise object and calculate transformation matrices
// calculate bounding box this becomes the sampling grid around segment
// for each sampling point in its bounding box
for each segment
    for each bounds_row
        for each bounds_column
            calculate_distance_and_winding_score();
// for each sampling point in its bounding box
for each row
    for each column
        resolve_to_distance_map();
```

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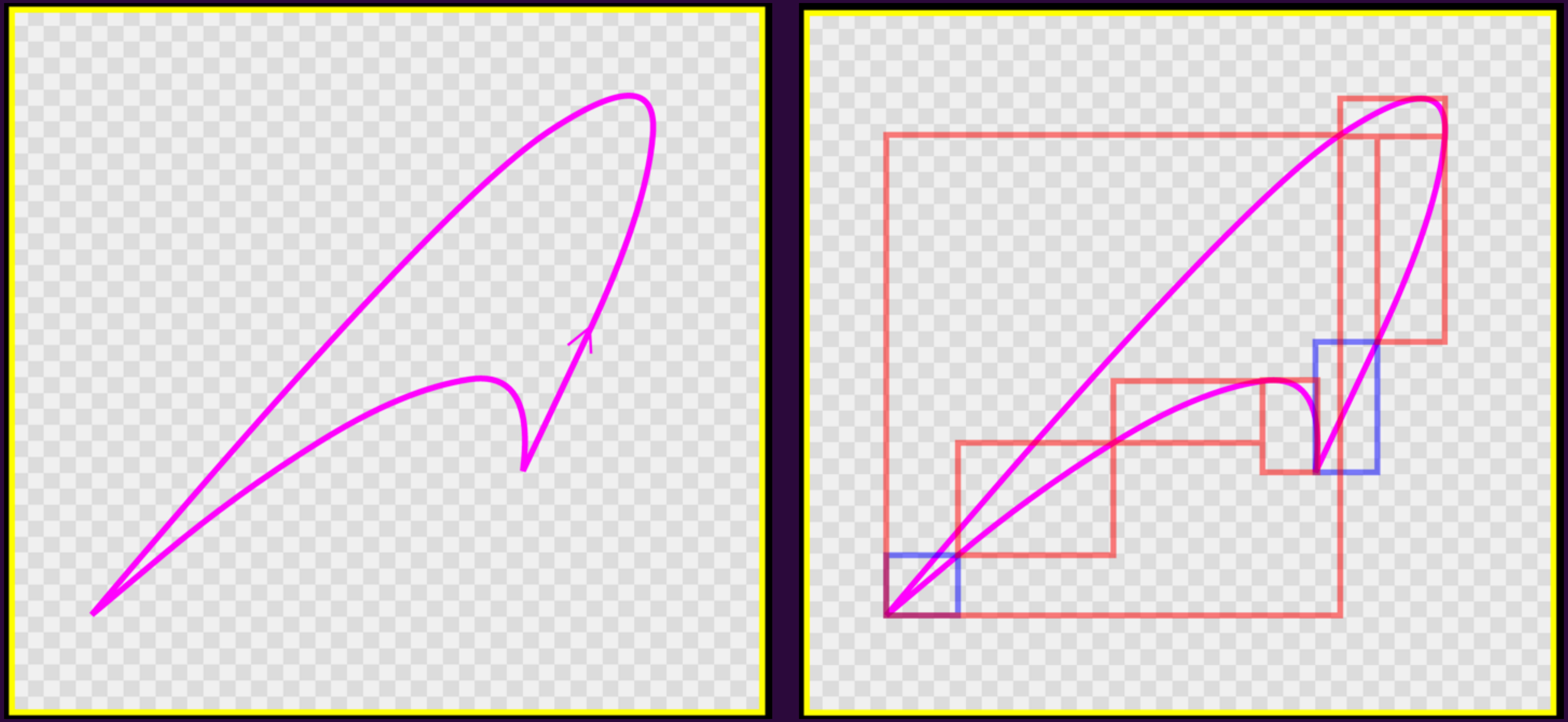
Winding Number



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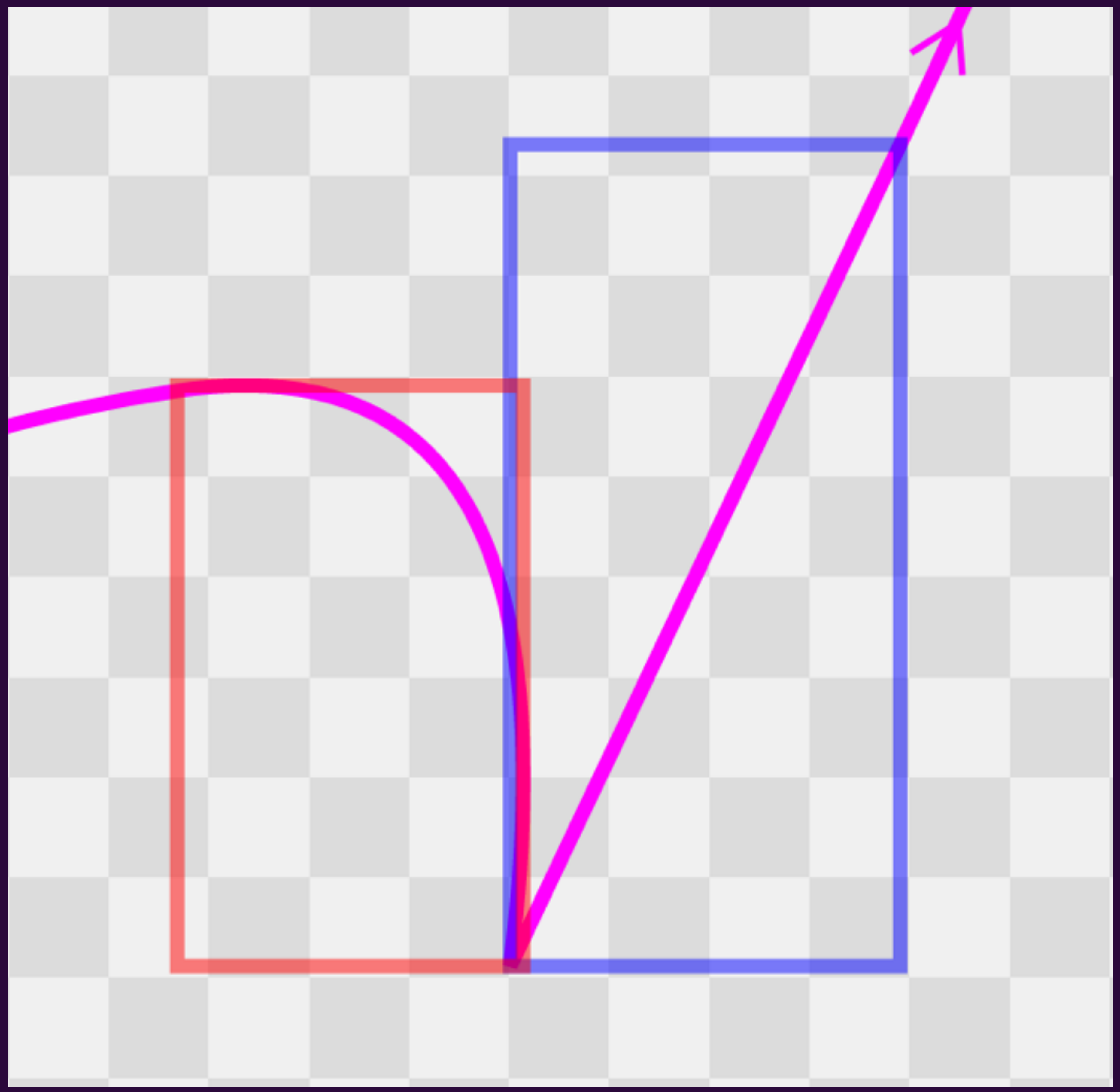
Zooming in on a Section



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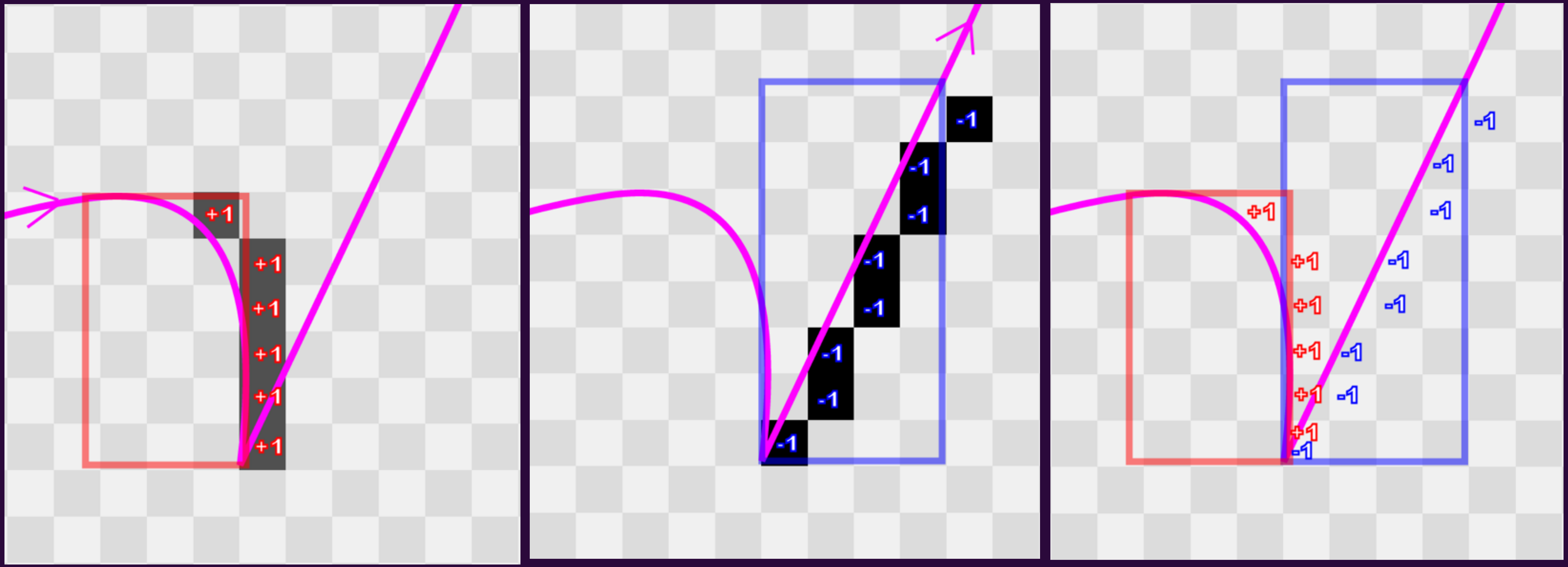
Calculate Delta Winding Score



Right to Left **+1,** Left to Right **-1**

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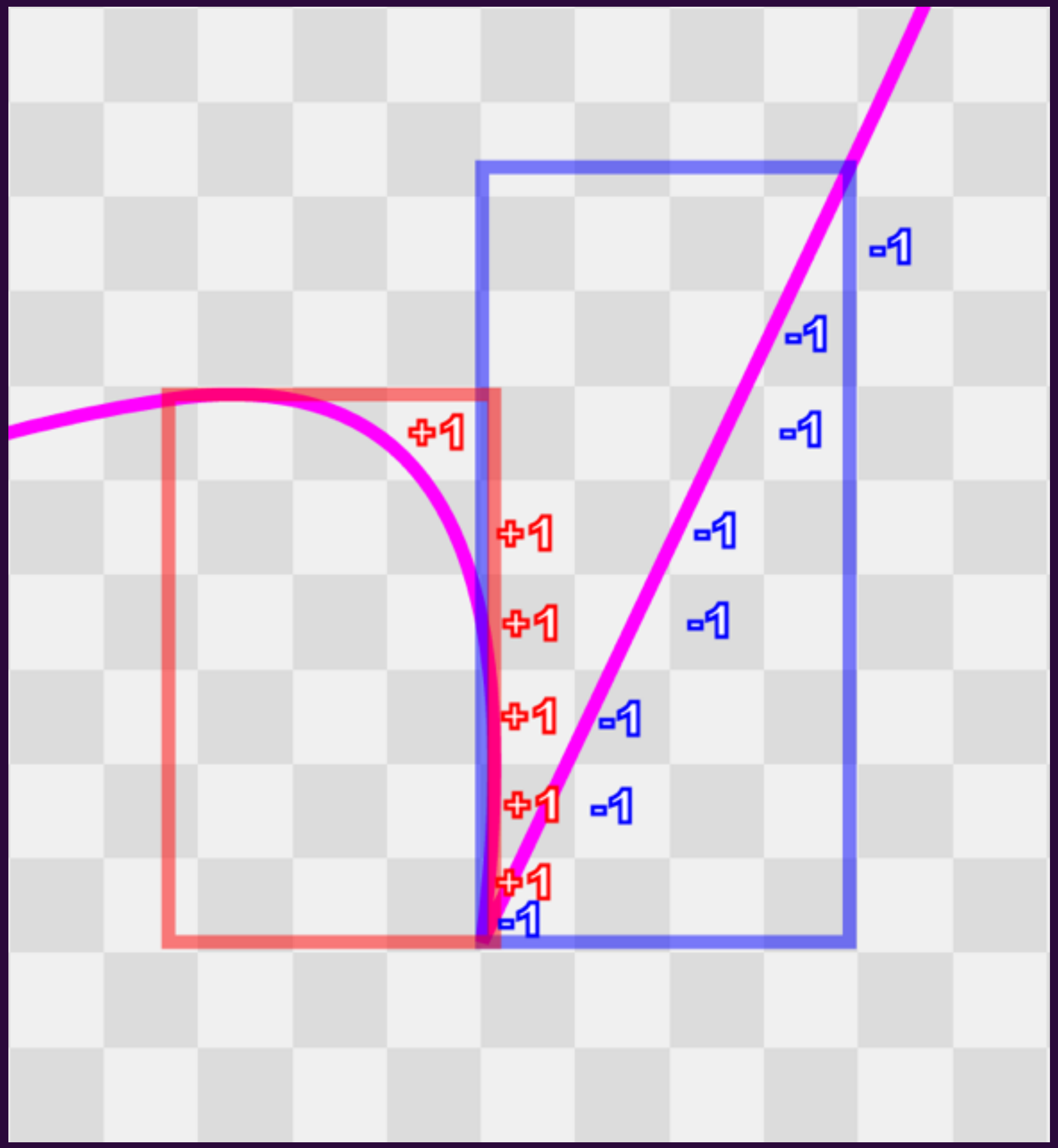
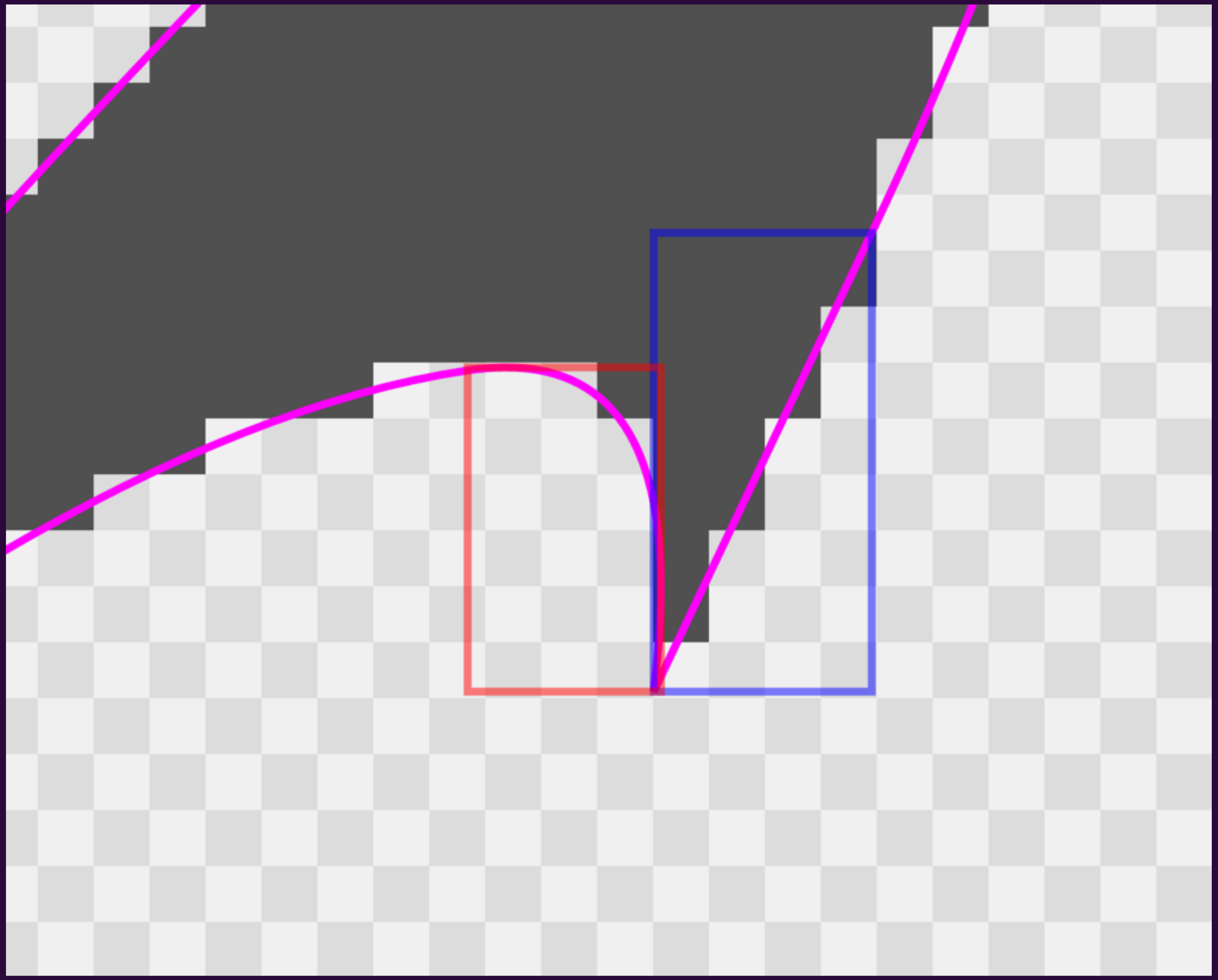
Winding number





Summary the delta winding score from left to right

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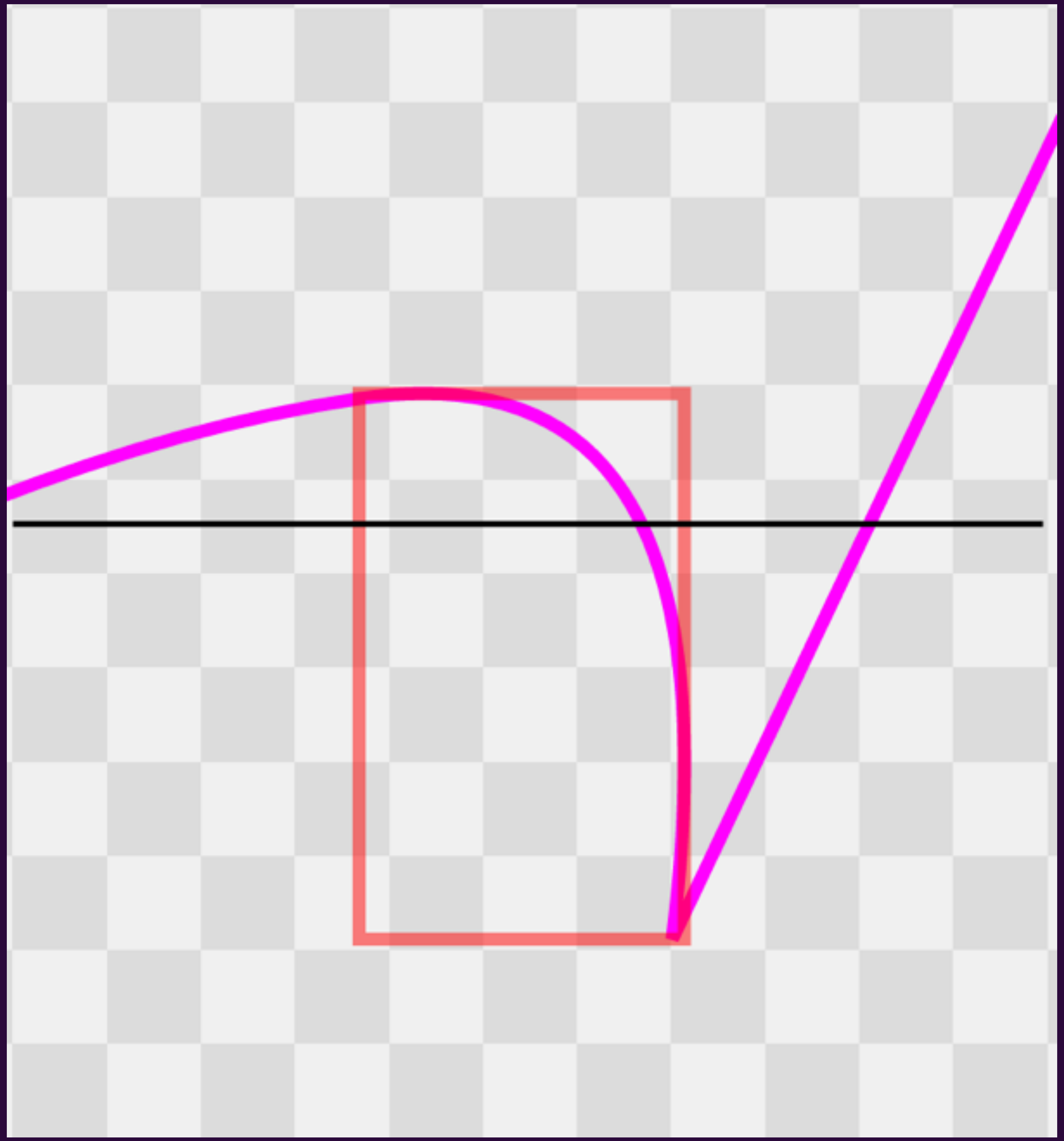
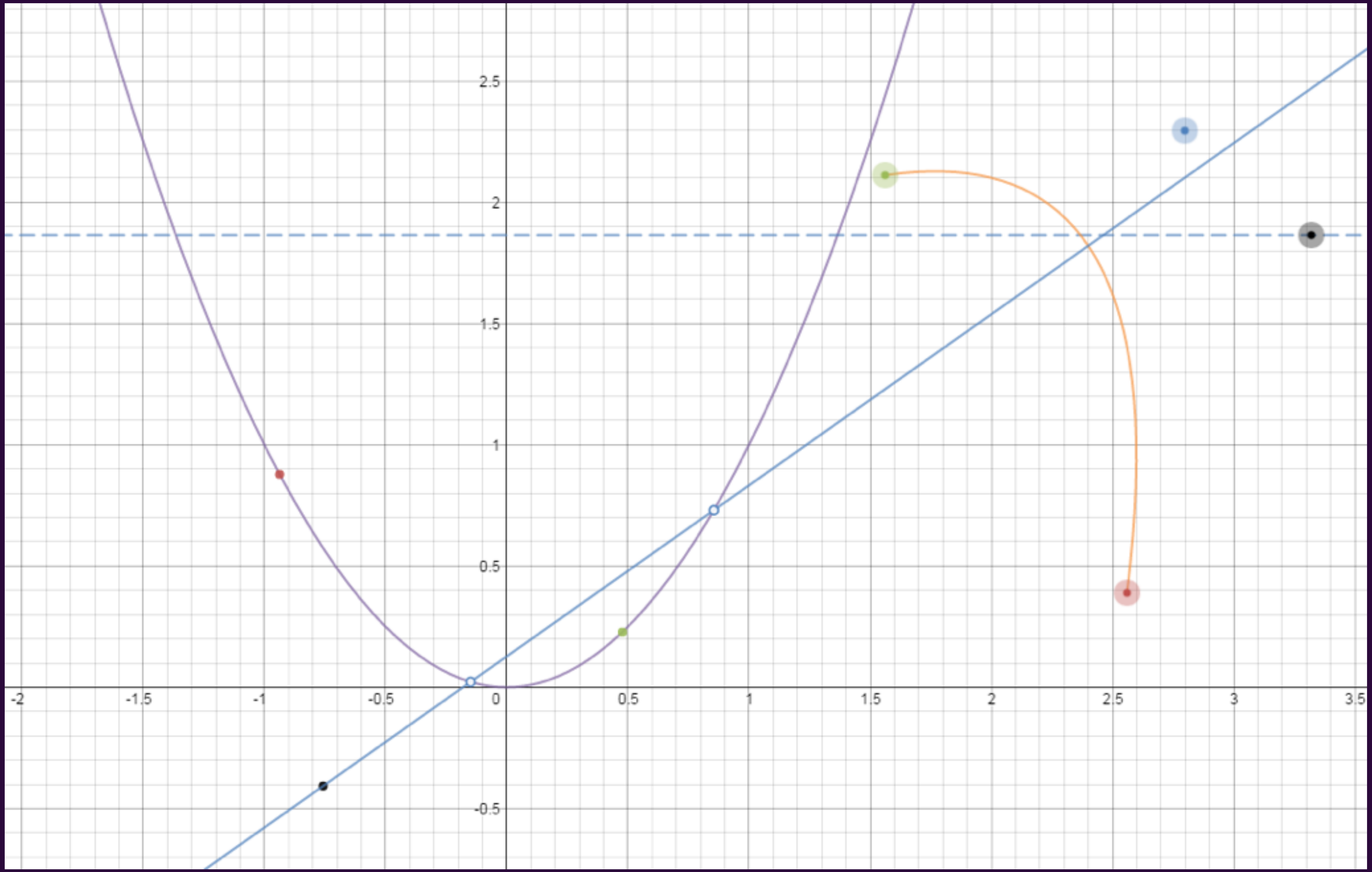
Side of quadratic curve





https://www.desmos.com/calculator/wuugaixiwh

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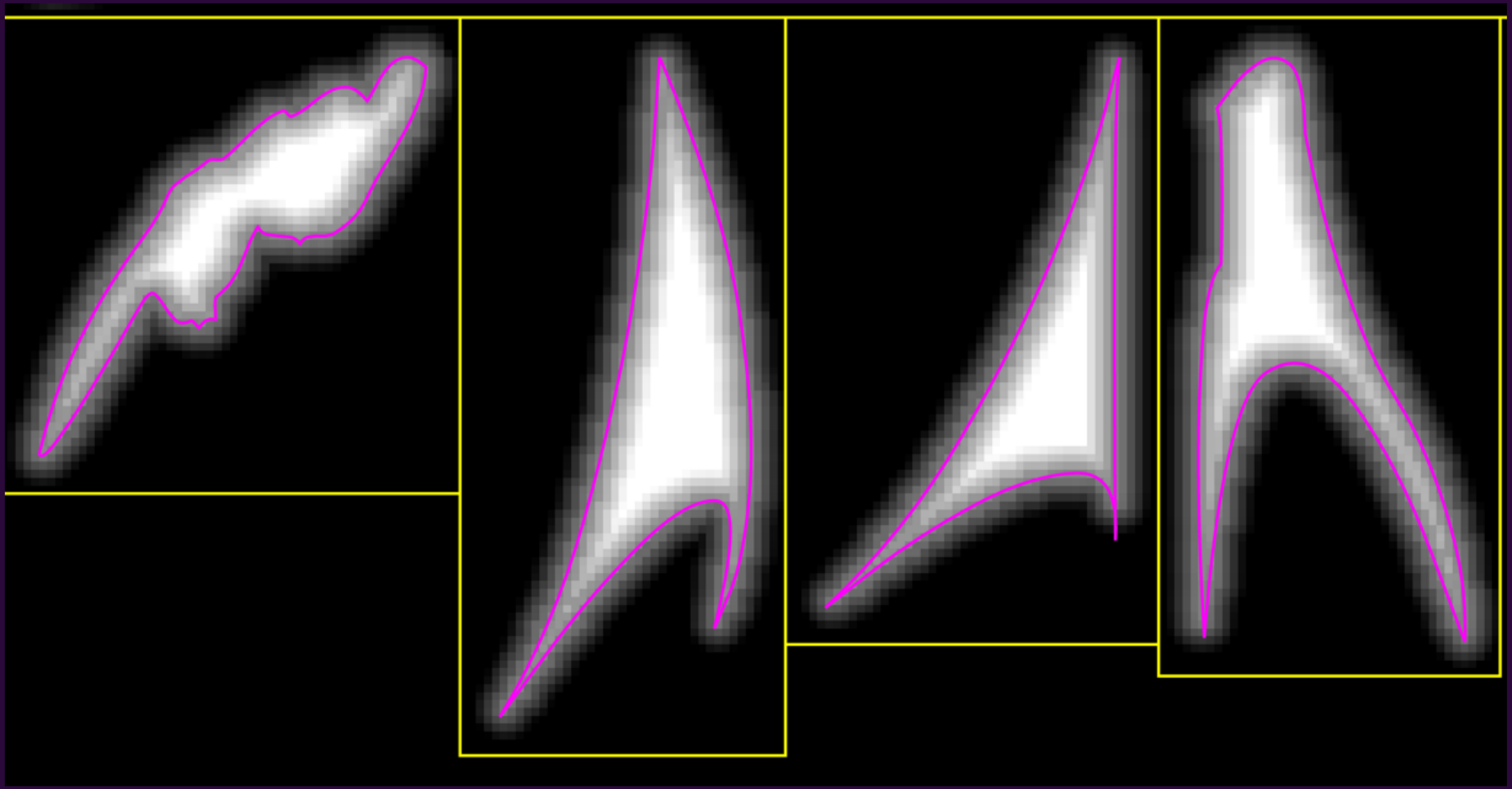
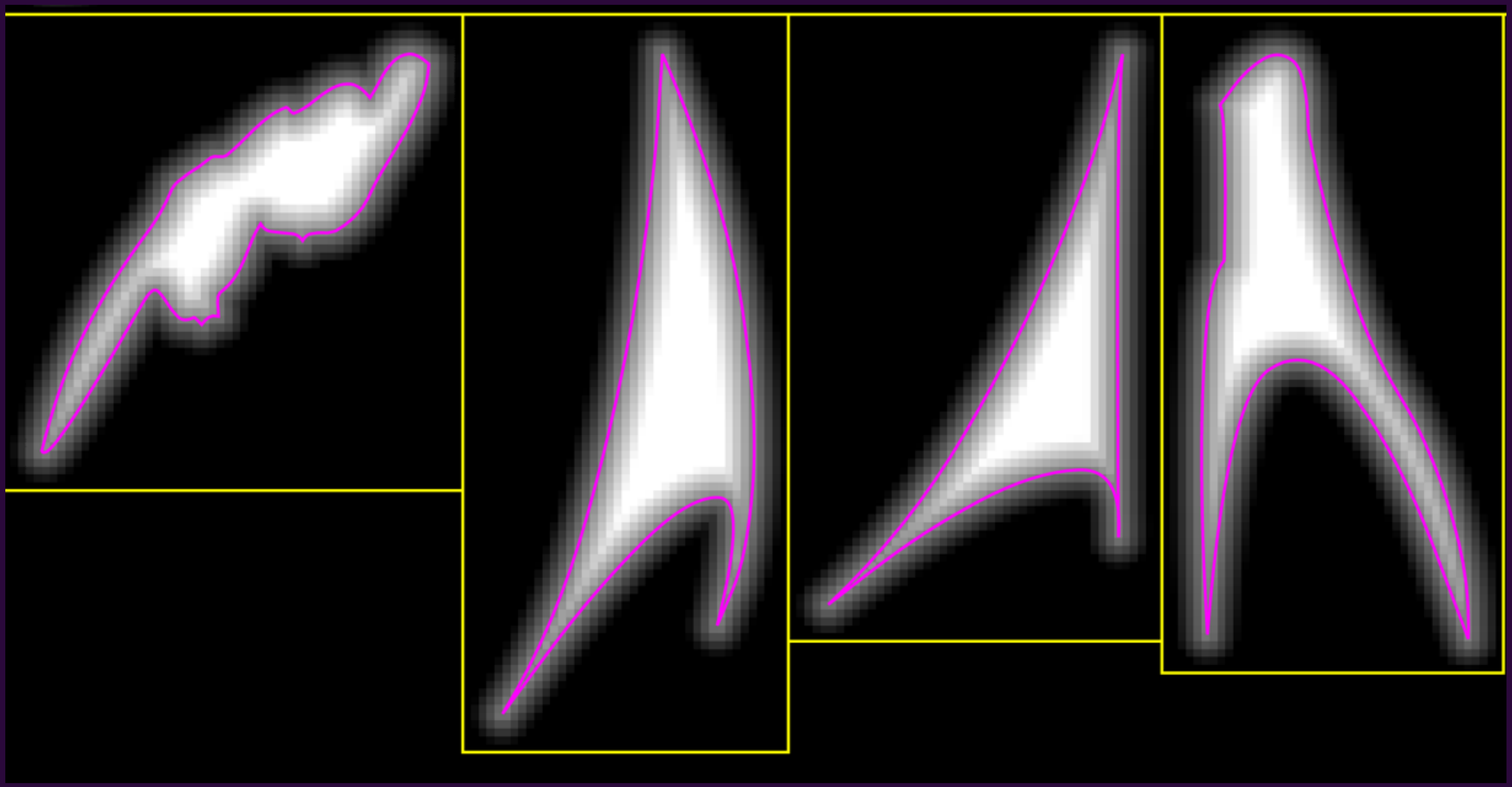
Quality



Skia

Quality



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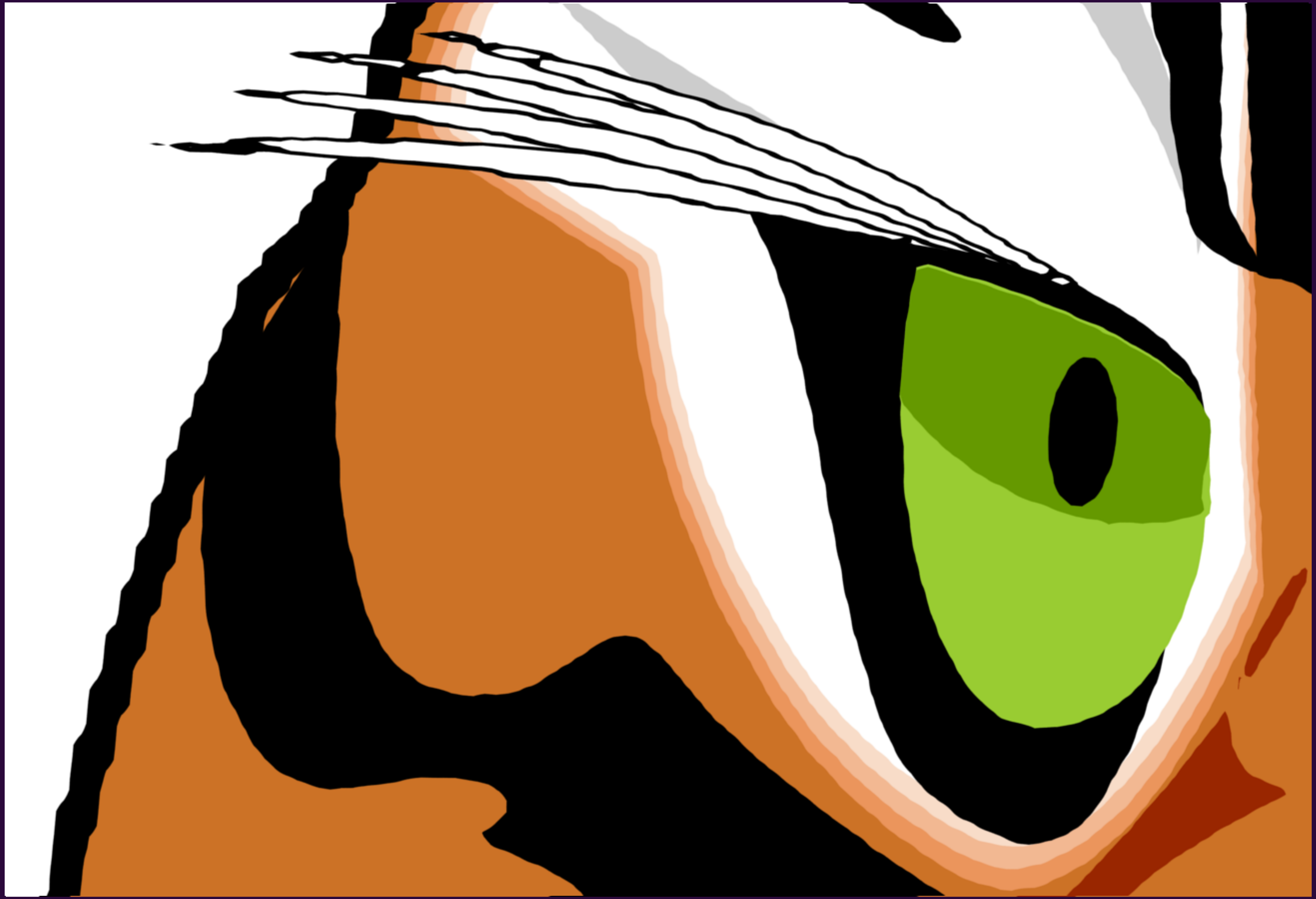
Skia, Reference



Skia

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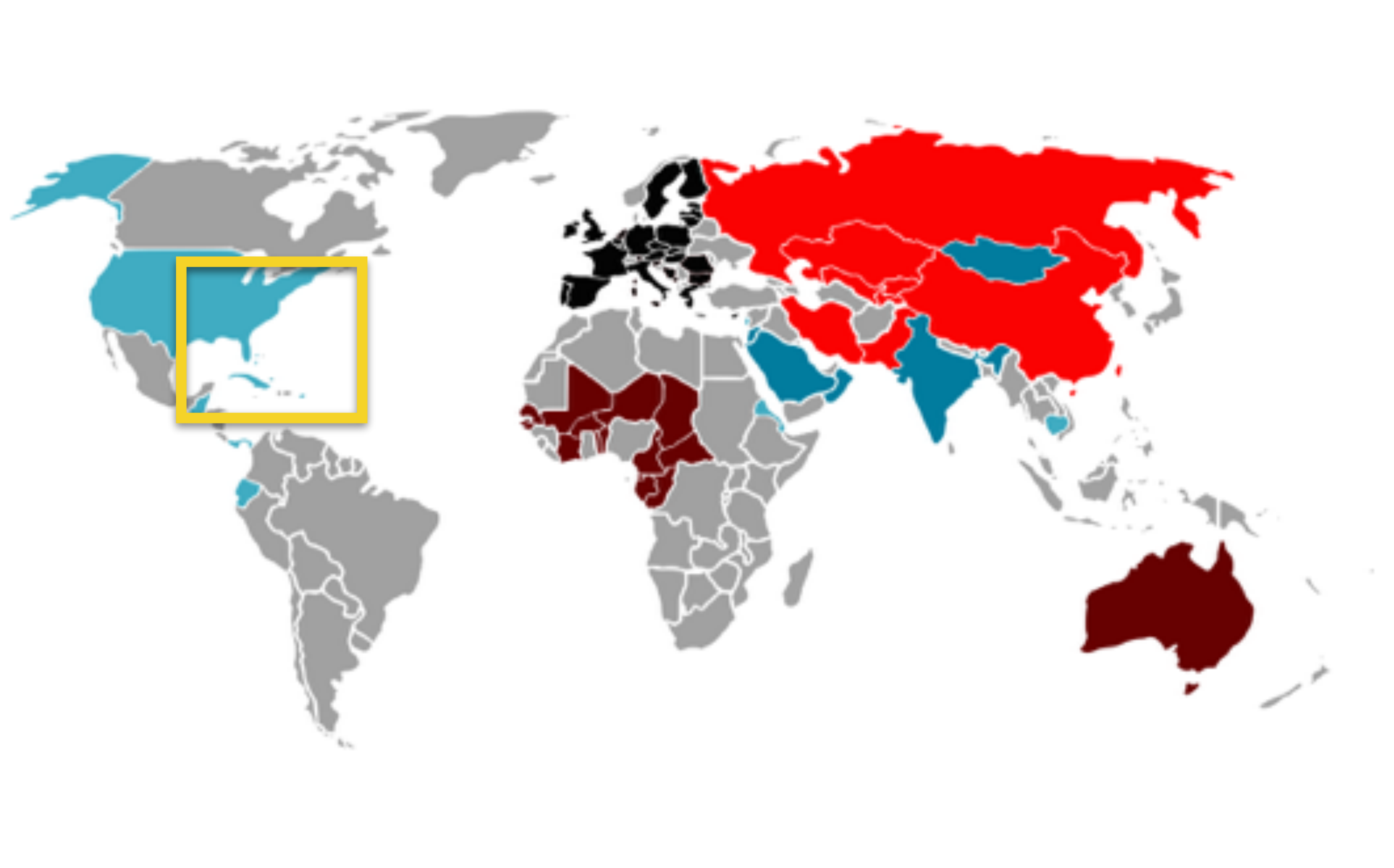
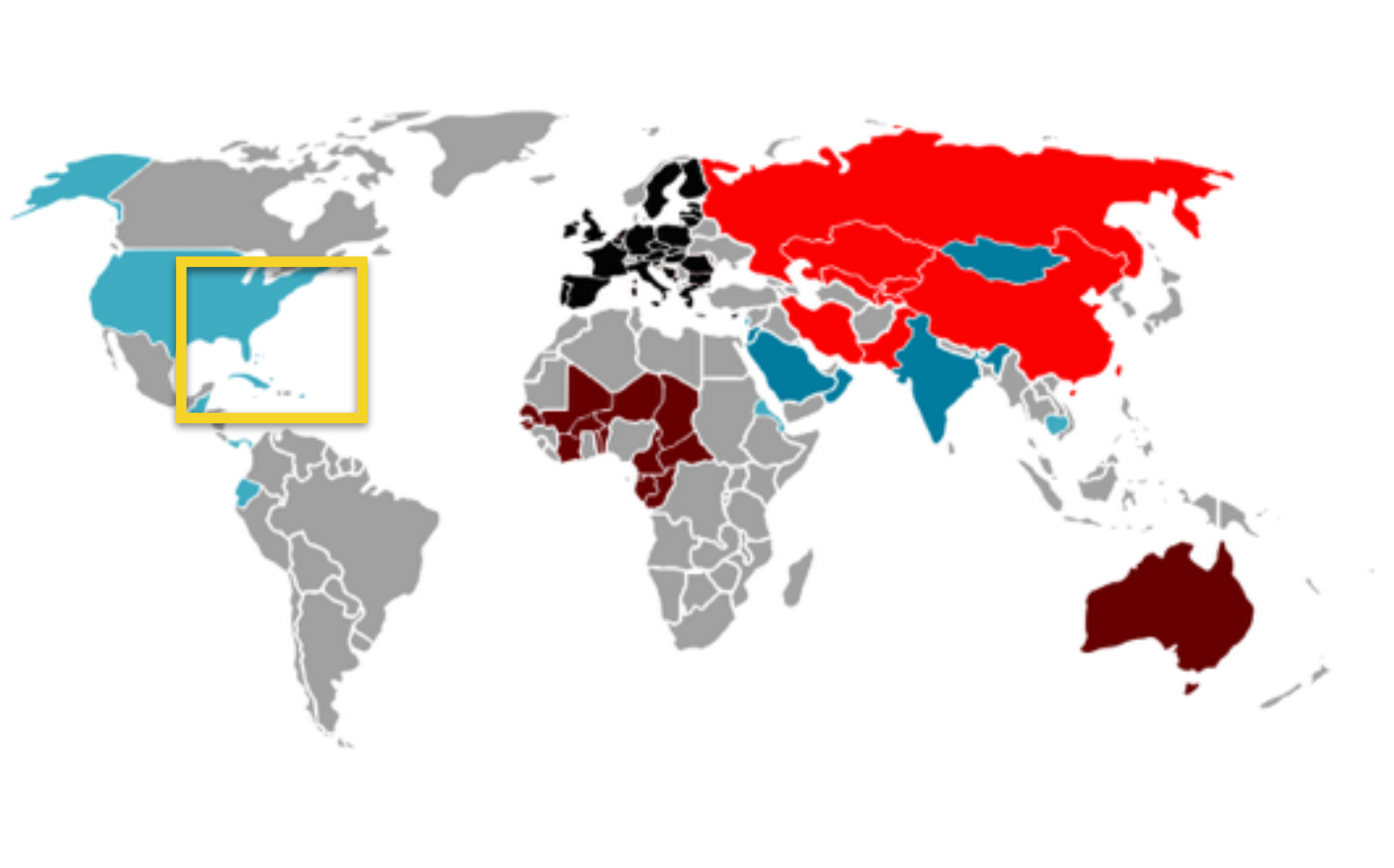


Skia



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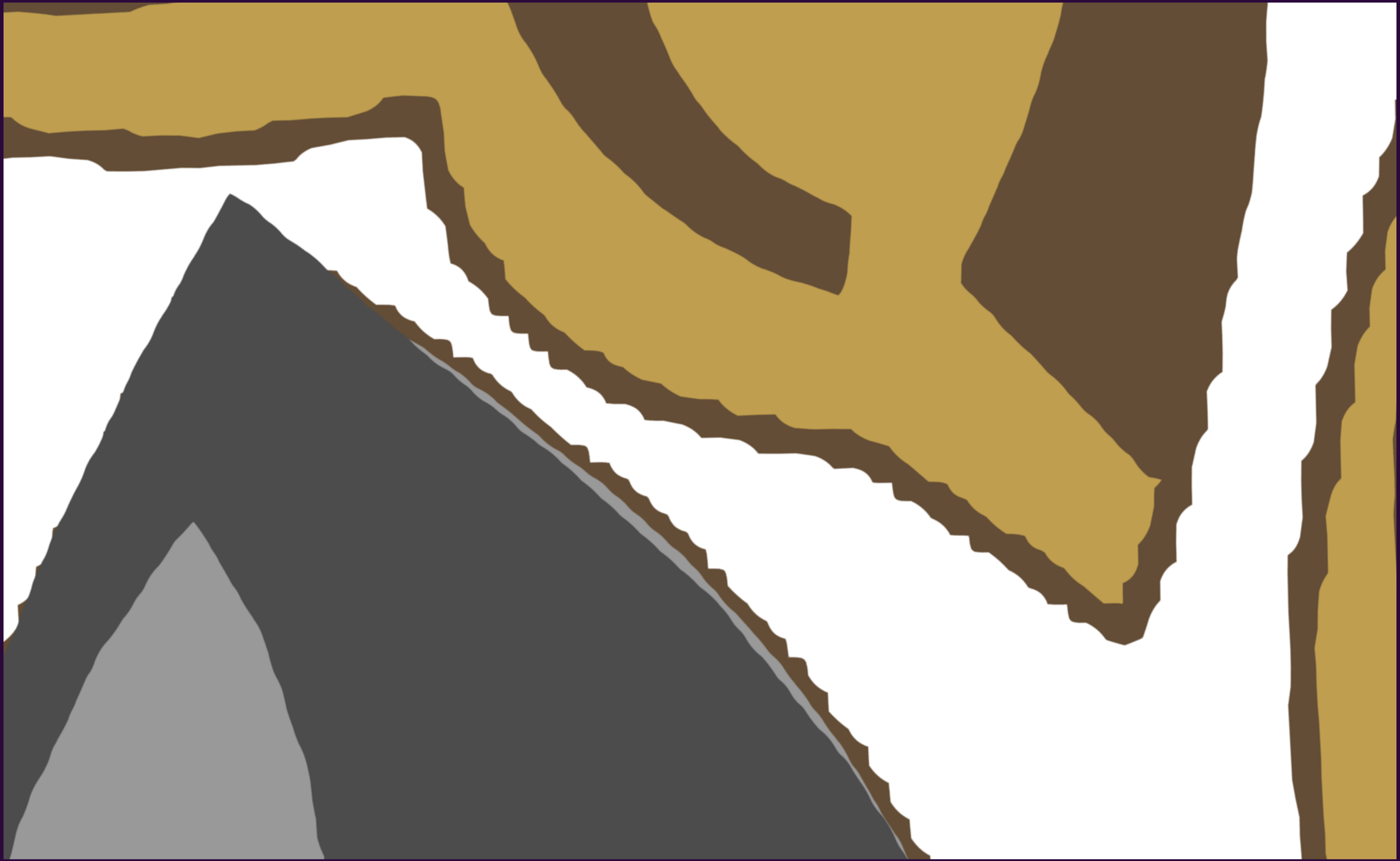
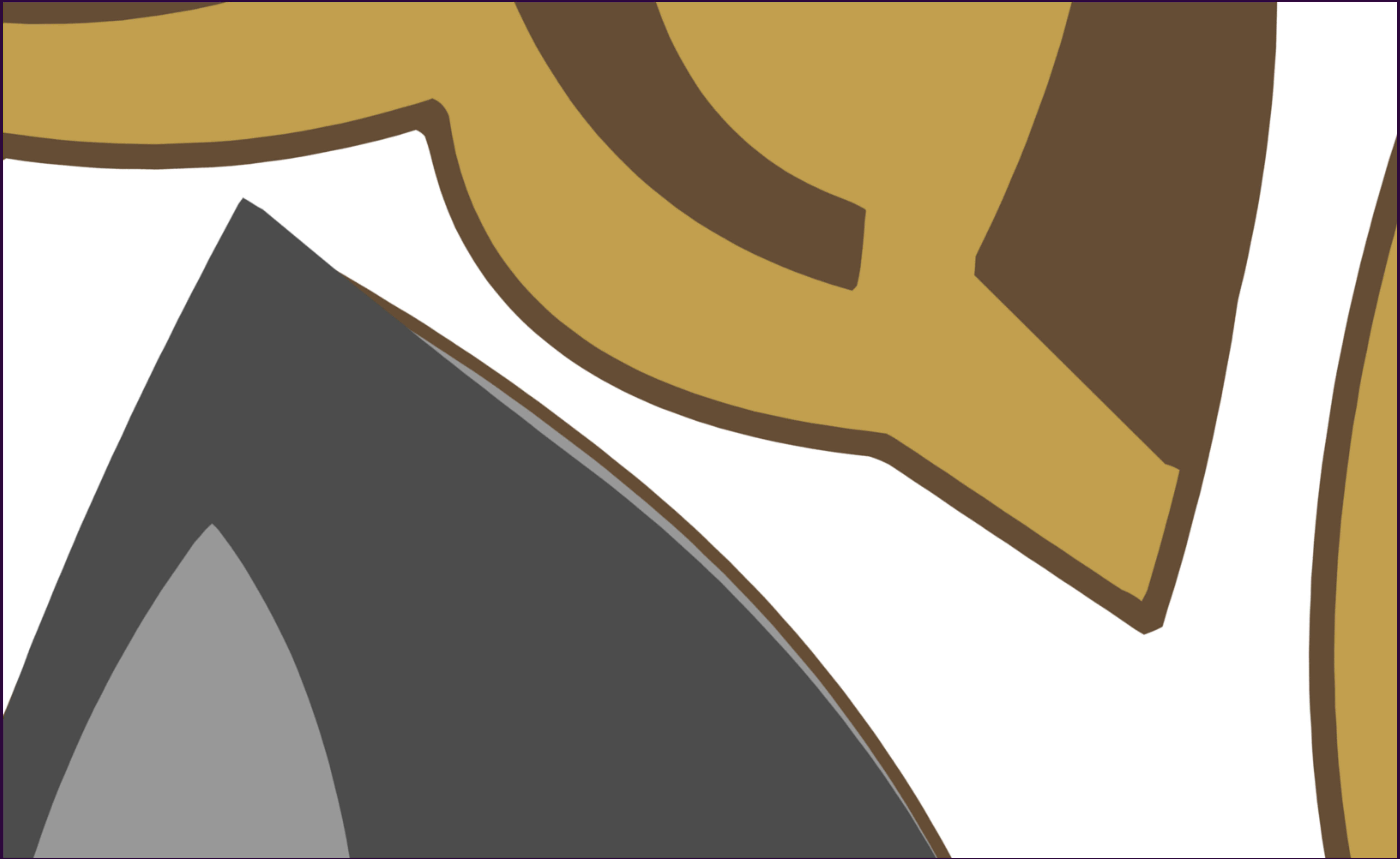
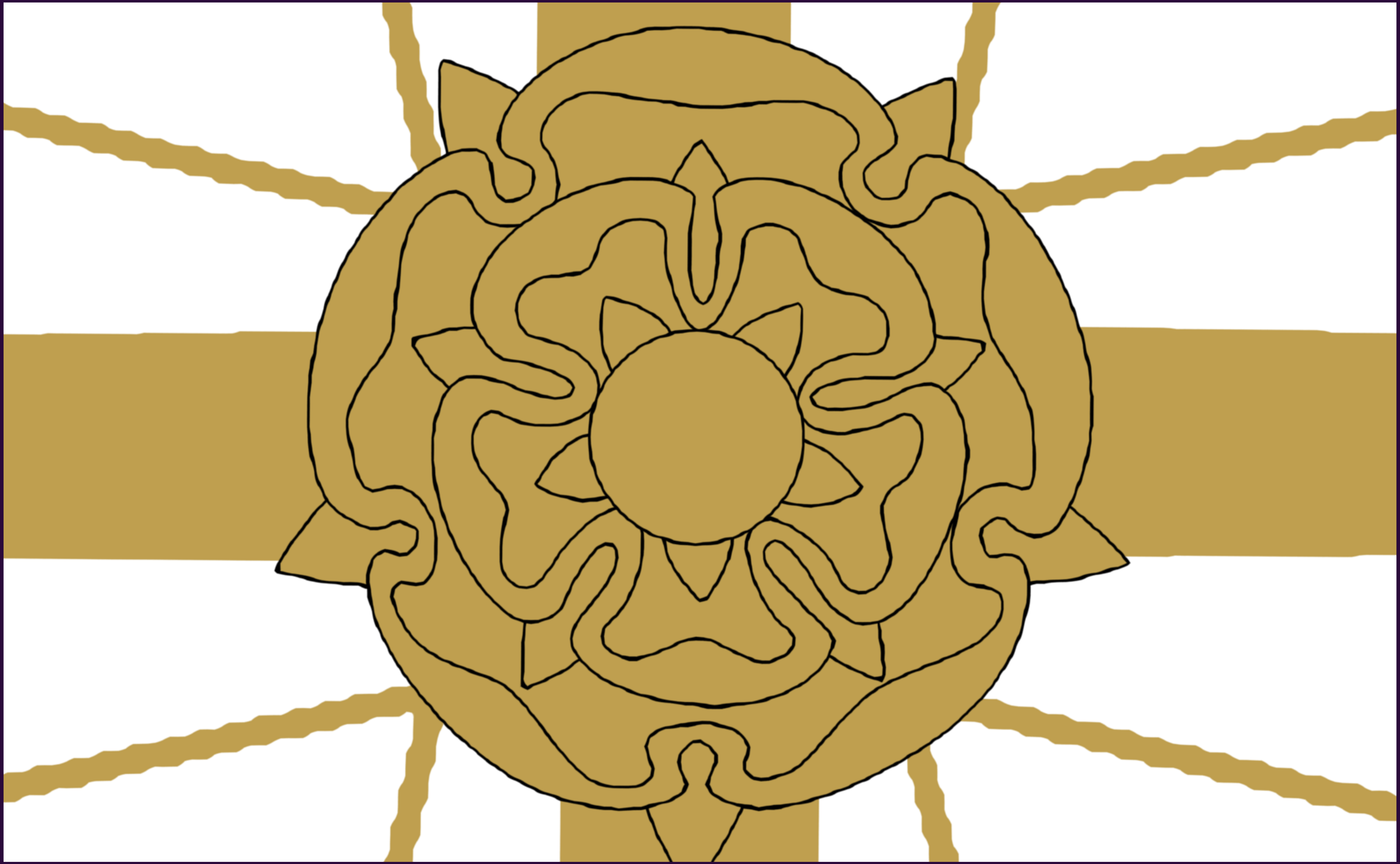
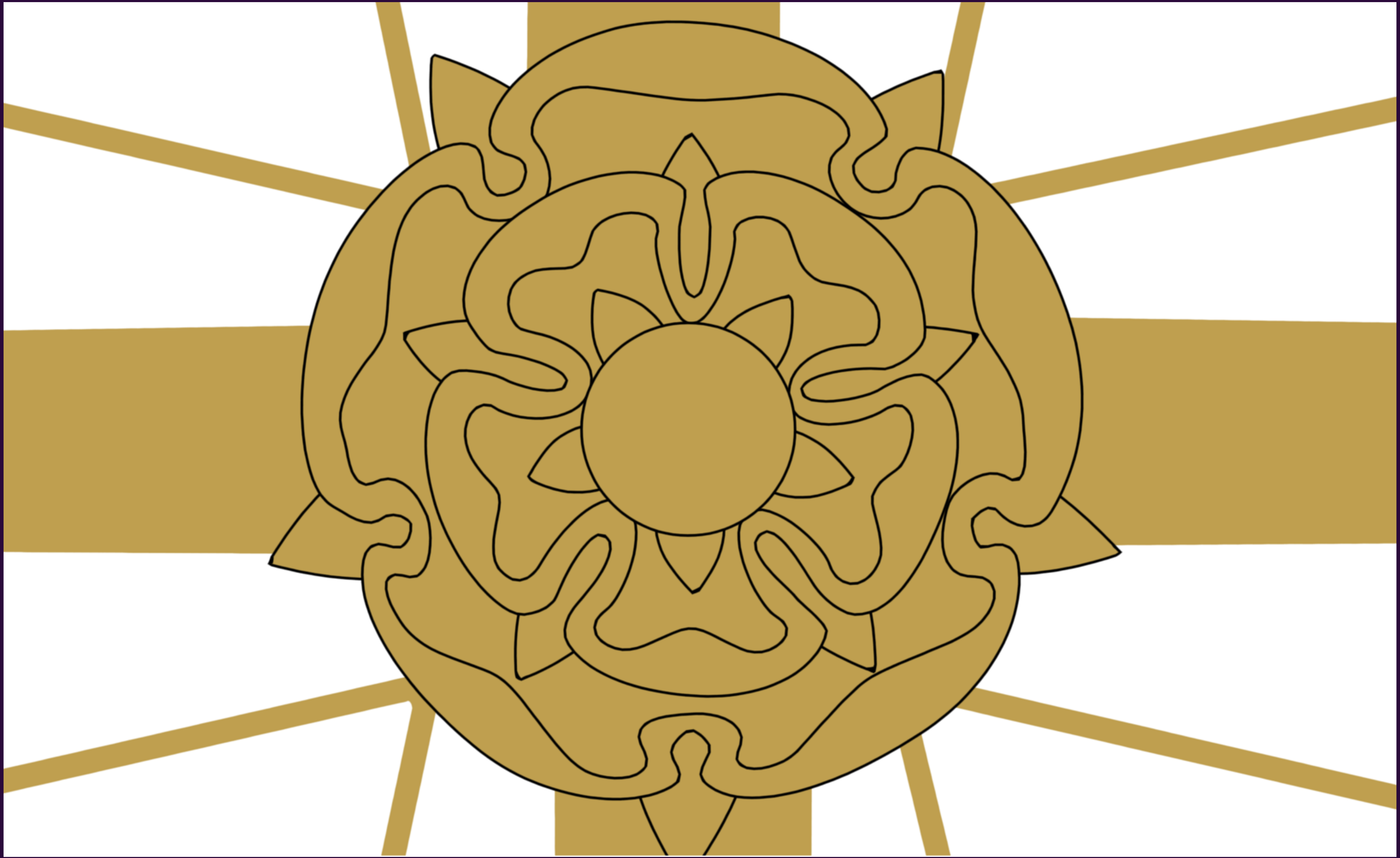
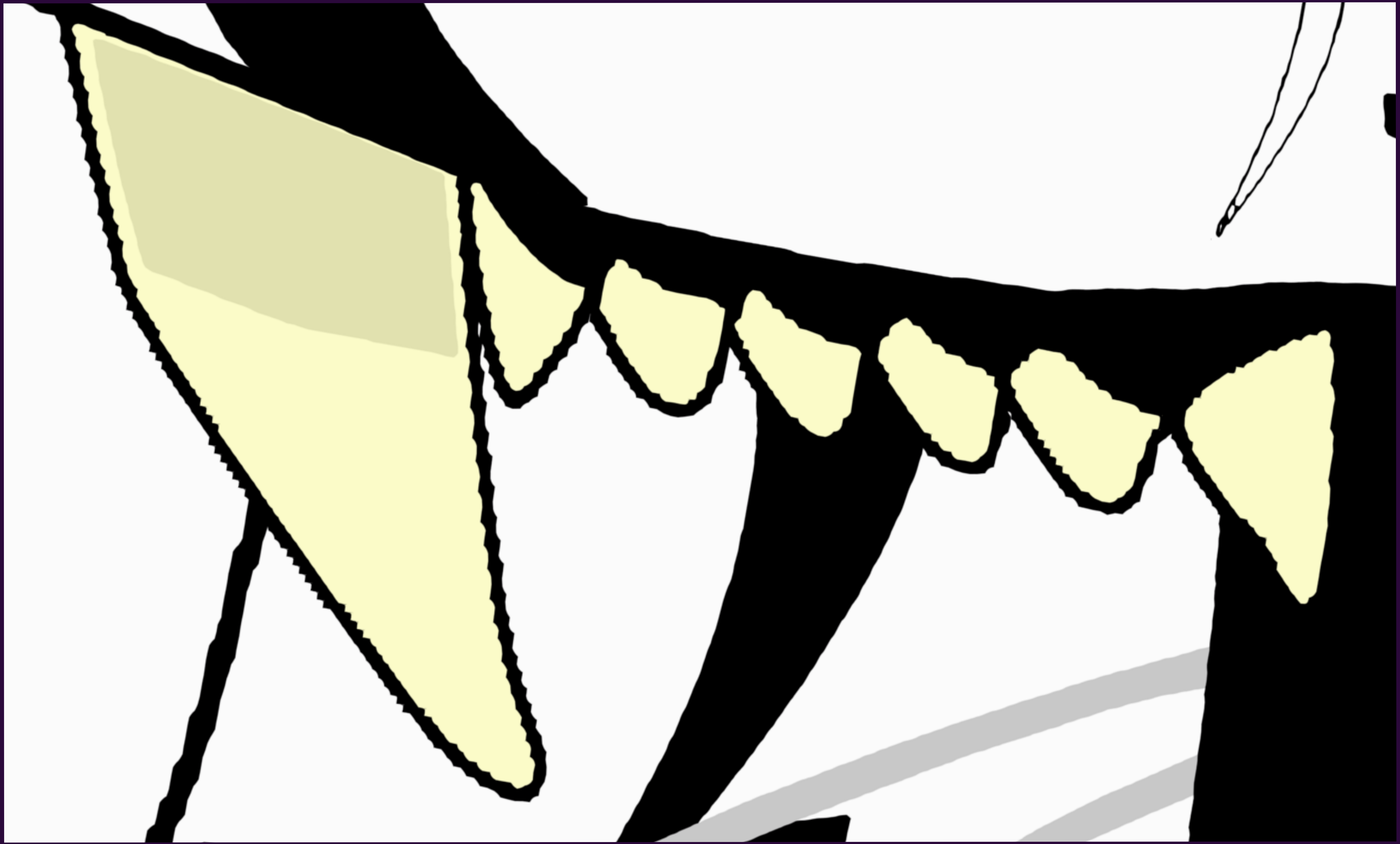


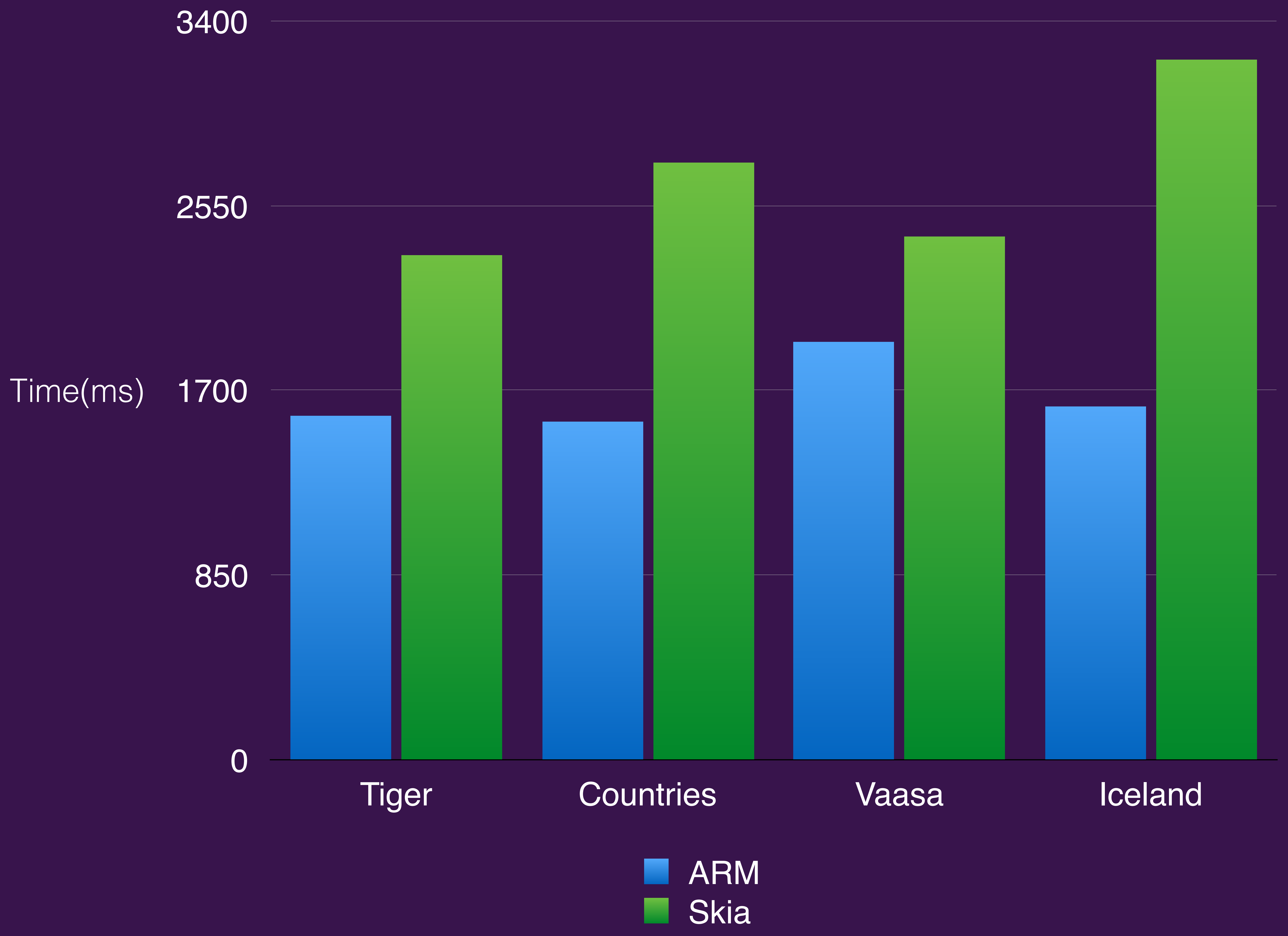




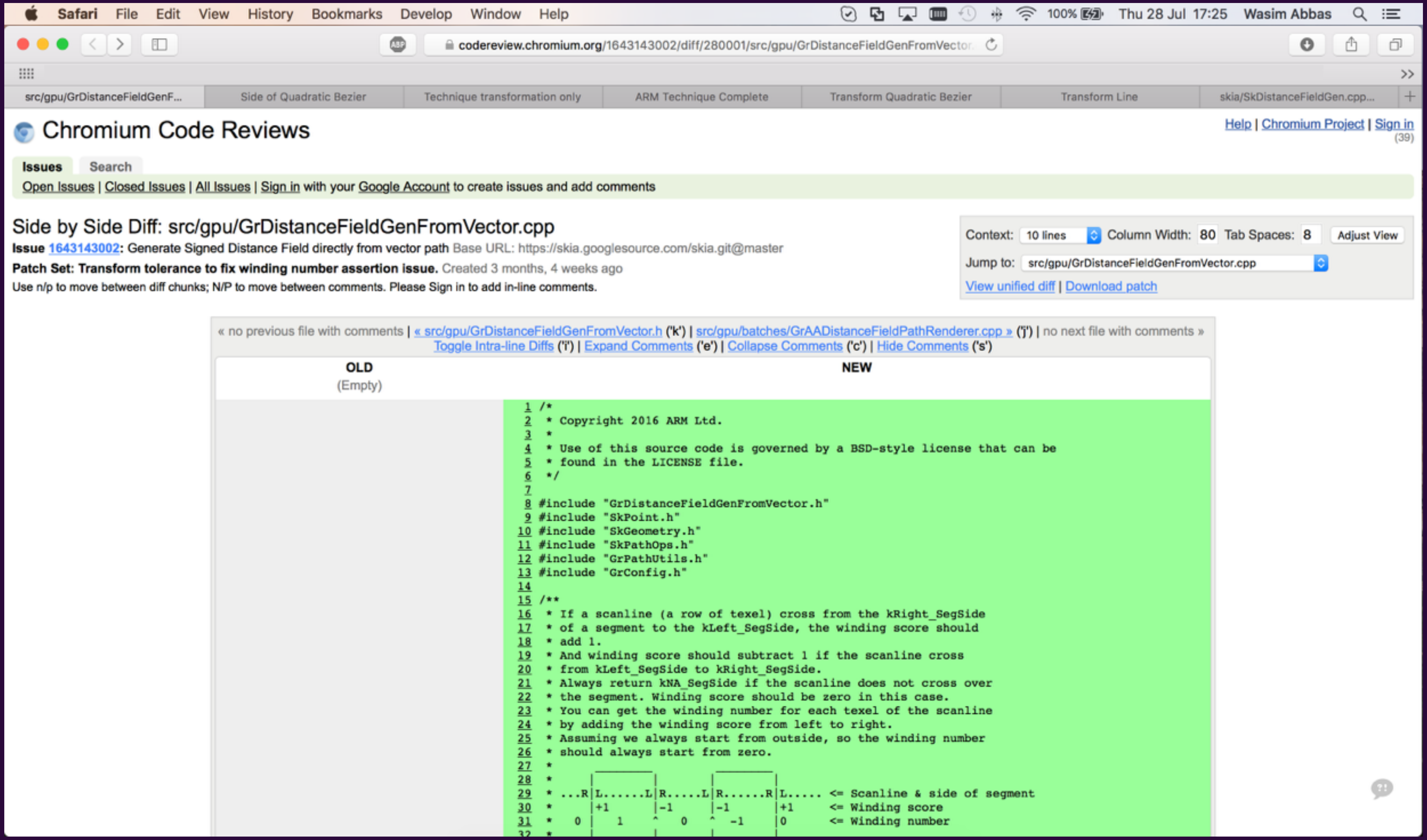




Results in Numbers



Where is the Code?



https://codereview.chromium.org/1643143002

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Summary

- Much better quality
- Higher performance. In its current state we are above 75% faster.
- Highly parallel algorithm (currently single threaded but can be easily multi-threaded)
- Lots of scope for improvements and optimisations (still in beta)
- Only CPU version at the moment but the whole algorithm is based around GPU architecture so it will be much faster with GPU
- Not limited to Font glyphs but any path can be converted to SDF

GPU version

https://www.desmos.com/calculator/lqn6g1tpty

Acknowledgements

- Chris Doran chris.doran@arm.com
- Roberto Lopez Mendez roberto.lopezmendez@arm.com
- Rich Evans rcb.evans@outlook.com
- Joel Liang joel.liang@arm.com

Bonus content

Standard form theory

A general second-degree curve is defined by an equation of the form

$$X^t C X = (x, y, 1) \begin{pmatrix} a & h & g \\ h & b & f \\ g & f & c \end{pmatrix} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix} = 0$$

This is equivalent to the component equation

$$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$$

In order to define a parabola (a second-degree Bezier) the coefficients must also satisfy

$$ab - h^2 = 0$$

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Computing the standard form

We assume the input consists of three points in 2D, b0, b1 and b2 where b1 is control point.
Find an equation which takes 2D vector and returns a 6D vector

$$F(x, y) = (x^2, xy, y^2, x, 1, y)$$

Now we turn to the Bezier control points (b0, b1, b2) and from these define 3 new points along the curve:

$$c_1 = \frac{1}{16}(9b_0 + 6b_1 + b_2)$$
$$c_2 = \frac{1}{4}(b_0 + 2b_1 + b_2)$$
$$c_3 = \frac{1}{16}(b_0 + 6b_1 + 9b_2)$$

Computing the standard form

We now define the 5 × 6 matrix A

$$A = \begin{pmatrix} F(b_0) \\ F(c_1) \\ F(c_2) \\ F(c_3) \\ F(b_2) \end{pmatrix}$$

$$B[i] = \det A_i$$

$$[a, h, b, g, c, f] = [B_0, -1/2B_1, B_2, -1/2B_3, B_4, -1/2B_5]$$

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Standard form Equation Accelerated version

$$a = (y_0 - 2y_1 + y_2)^2$$

$$b = (x_0 - 2x_1 + x_2)^2$$

$$c = x_0^2 y_2^2 - 4x_0x_1y_1y_2 - 2x_0x_2y_0y_2 + 4x_0x_2y_1^2 + 4x_1^2y_0y_2 - 4x_1x_2y_0y_1 + x_2^2y_0^2$$

$$h = -(y_0 - 2y_1 + y_2)(x_0 - 2x_1 + x_2)$$

$$\begin{aligned} g = {} & x_0y_0y_2 - 2x_0y_1^2 + 2x_0y_1y_2 - x_0y_2^2 + 2x_1y_0y_1 - 4x_1y_0y_2 + 2x_1y_1y_2 \\ & - x_2y_0^2 + 2x_2y_0y_1 + x_2y_0y_2 - 2x_2y_1^2 \end{aligned}$$

$$\begin{aligned} f = {} & -(x_0^2y_2 - 2x_0x_1y_1 - 2x_0x_1y_2 - x_0x_2y_0 + 4x_0x_2y_1 - x_0x_2y_2 + 2x_1^2y_0 \\ & + 2x_1^2y_2 - 2x_1x_2y_0 - 2x_1x_2y_1 + x_2^2y_0) \end{aligned}$$

Transformation matrices (Rotation)

$$R_\theta = \begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\cos\theta = \sqrt{\frac{a}{a+b}}$$

$$\sin\theta = -\mathrm{signum}((a+b)h)\sqrt{\frac{b}{a+b}}$$

$$\mathrm{signum}(x) = \begin{cases} -1 & \text{if} \quad x < 0 \\ +1 & \text{otherwise} \end{cases}$$

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Transformation matrices (Translation)

$$T = \begin{pmatrix} 1 & 0 & x_0 \\ 0 & 1 & y_0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$x_0 = \frac{g'}{a+b}$$

$$y_0 = \frac{1}{2f'}\left(c - \frac{g'^2}{a+b}\right)$$

$$\begin{pmatrix} g' \\ f' \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} g \\ f \end{pmatrix}$$

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Transformation matrices (Dilation/Scaling)

$$D = \begin{pmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\lambda = -\frac{a+b}{2f'}$$

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More numbers

Test cases	SDF Generation time(ms)					avg.(ms)	Performance
Tiger (ARM)	1591.877103	1570.867658	1585.281014	1592.140317	1574.57006	1582.94723	1.47x
Tiger (Skia)	2323.360443	2324.164629	2330.242872	2332.234621	2301.774502	2322.355413	
Countries(ARM)	1543.180108	1552.965522	1568.936348	1563.890934	1554.955125	1556.785607	1.76x
Countries(Skia)	2783.946037	2717.692852	2723.564148	2740.092278	2770.129919	2747.085047	
Vaasa(ARM)	1940.449476	1916.500211	1915.157199	1922.863245	1907.956362	1920.585299	1.25x
Vaasa(Skia)	2417.225361	2453.290224	2302.508593	2434.435368	2424.861908	2406.464291	
Iceland(ARM)	1641.598582	1633.31306	1623.172998	1633.633852	1586.660147	1623.675728	1.98x
Iceland(Skia)	3170.907497	3288.432121	3247.562647	3211.12895	3196.832895	3222.972822	

questions?