

Removing the Mystique of Glass Selection

Robert E. Fischer & Alastair J. Grant, OPTICS 1, Inc.,

Dr. Ulrich Fotheringham, Dr. Peter Hartmann, Dr. Steffen Reichel, SCHOTT AG

Abstract

Glass selection tends to be both a science and an art. It is the intent of this paper to remove the "mystique" surrounding glass selection, primarily based on the chromatic properties of the glass, and to show via careful parametric analyses how we can optimally select glasses for lenses of different f/numbers, spectral bands, and performance requirements. The important roles of refractive index and Abbe number as well as partial dispersion will be considered. Using the SCHOTT glass map, six separate and identifiable regions along with glasses within each region will be discussed. The goal for this paper is to make glass selection easier to understand.

Basic glass characteristics

The most basic characteristics of optical glass that influence the glass selection process are refractive index which refracts light and V number or Abbe number which is used in quantifying the variation in refraction with wavelength. **Figure 1** shows a plot of the above information on what is called an Abbe diagram (sometimes known as a glass map). The abscissa shows Abbe number increasing from right to left. This places lower dispersion glasses to the left of the map and higher dispersion glasses to the right. The ordinate axis is refractive index. This map therefore allows easy selection of glasses with large or small refractive index, or dispersion values.

The focal length of a lens at the helium yellow d wavelength of $0.5876\mu\text{m}$ can generally be thought of as the primary focal length of a lens. Variation in the foci from red (typically the red hydrogen C lines at $0.6563\mu\text{m}$ is used) to the blue (typically the blue hydrogen F line at $0.4861\mu\text{m}$ is used) gives a definition of primary axial color. As the Abbe number increases the amount of shift between red and blue focus decreases. In theory, if the Abbe number were exceedingly high, such as 500 or 1000, there would be virtually no primary axial color. As the Abbe number decreases, the amount of color shift in focus between red and blue increases. In order to create a lens whose red and blue come to a common focus we need to use at least two different materials, one with a low Abbe number and one with a high Abbe number.

Table 1 below lists the major terminology describing glass characteristics :

Parameter	Definition
Main dispersion = Principal dispersion	$(n_F - n_C)$
Abbe number = V_d number	$(n_d - 1) / (n_F - n_C)$
Relative Partial Dispersion = $P_{x,y}$	$(n_x - n_y) / (n_F - n_C)$
Refractivity	$(n_d - 1)$
Primary Axial Color	Amount of defocus between the best red focus position & the best blue focus position
Secondary Axial Color	Amount of defocus between the best yellow focus position and the best red focus position.
Primary Lateral Color	The lateral displacement (in the image plane) between the best red image and the best blue image, commonly known as color fringing.
Secondary Lateral Color	The lateral displacement (in the image plane) between the best red image and the best yellow image at the d wavelength.
Wavelengths usually referenced	$g = 435.834\text{nm}$, $C = 656.273\text{nm}$, $d = 587.562\text{nm}$, $F = 486.133\text{nm}$

Table 1 : Glass Terminology

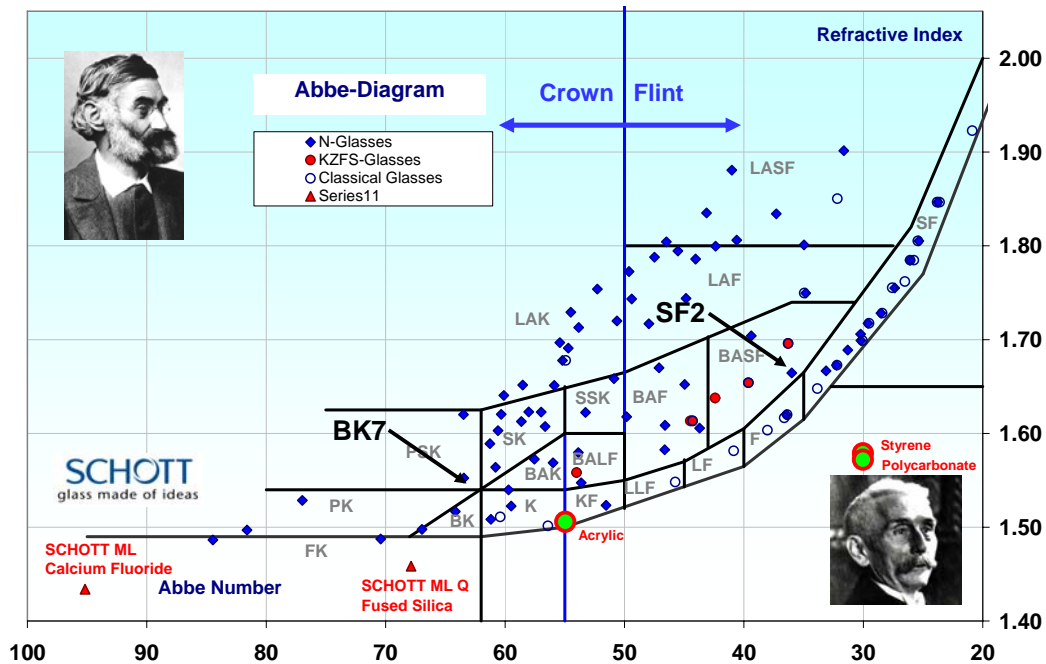


Figure 1 shows a typical Abbe diagram provided by SCHOTT AG. The location BK7 and SF2, two of the more popular and commonly used glass materials, have been noted. Some common plastic lenses have been included for reference.

Primary Axial Color

Primary axial color is defined above as the focal length change between the red and blue wavelength of $0.6563\mu\text{m}$ and $0.4861\mu\text{m}$ (C and F). For a very distant object point, the primary axial color can be expressed as follows [3] :

$$f'_C - f'_F = f'_d \{n_F - n_C\} / (n_d - 1) = f'_d / \text{Abbe Number (or V number)} \dots 1$$

If in **Equation 1**, the Abbe number is 2, then the change in focal length between red and blue will be *one half* of the basic focal length in the yellow at $0.5876\mu\text{m}$. The effect of this can be seen in **Figure 2**, which shows the dispersive properties of a positive singlet element using the exaggerated Abbe value of 2.

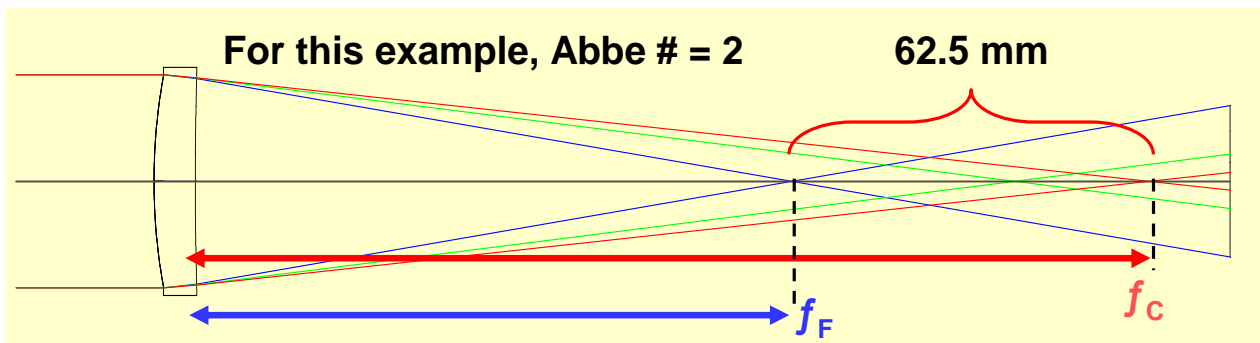


Figure 2 This is a real ray trace showing the red and blue focus positions for a positive singlet with an artificially high dispersion, $V=2$.

For two lenses in contact we find that the optical power for the group of 2 elements is equal to the sum of the optical power of the individual components. It can be shown that a lens can be designed such that focal length in the red and blue are equal to one another, and this is achieved by using two different materials of different Abbe numbers [3].

Specifically, $f_1 = f \{V_1 - V_2\} / V_1$ and $f_2 = -f \{V_2 - V_1\} / V_2 \dots \dots \dots 2$

Equations 2 show how we can determine the focal lengths of the positive powered component and the negative powered component of a doublet, such that the red and blue focuses are at the same position.

Considering an Achromatic Doublet

For an achromatic doublet the focal lengths are equal in the red and the blue i.e. the primary color is zero. The glass in the positively powered element is a crown and that in the negatively powered element is a flint.

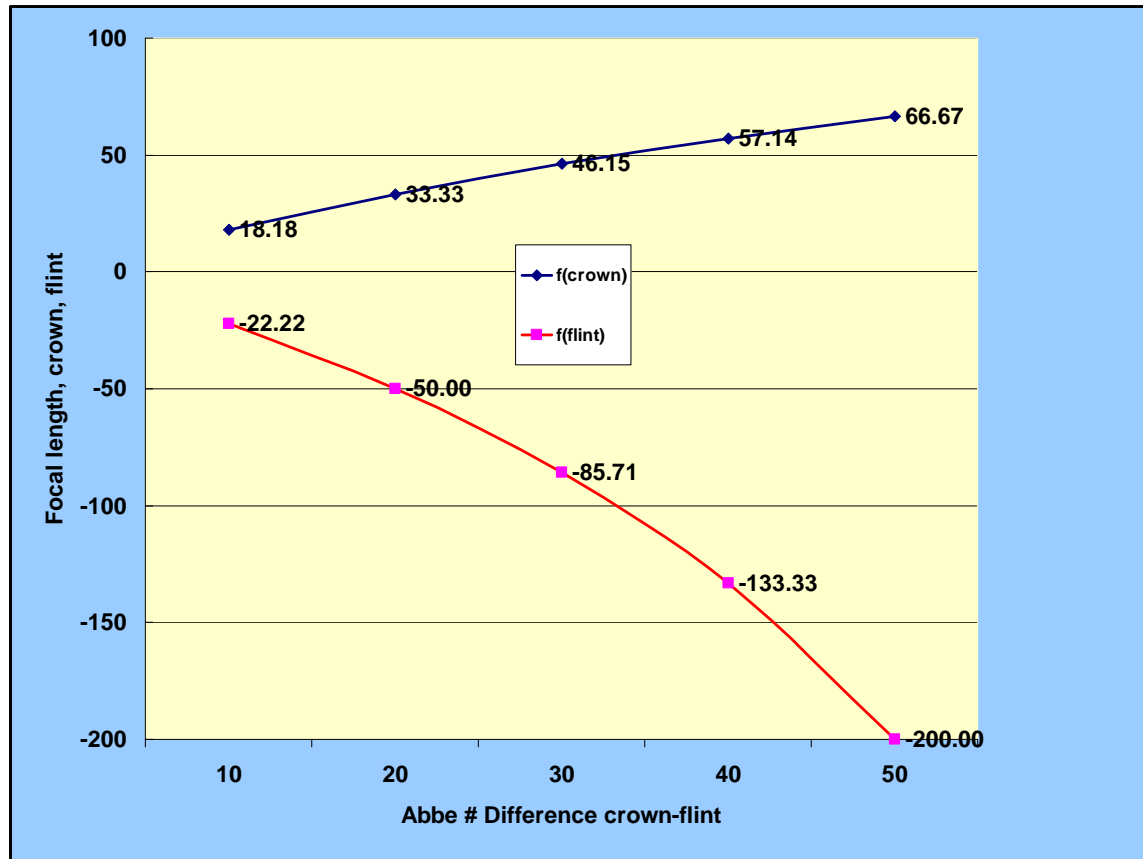


Figure 3 : Achromatic doublet lens focal length variation with Abbe number difference

We show in **Figure 3** the focal lengths of the crown and flint elements for the situation where the focal length is equal in the red and blue wavelengths. This is shown for Abbe numbers that range from 10-50. As the Abbe number difference increases for the two materials from 10 up to 50, the focal length of the crown or positive element increases from 18.18 mm to 66.67 mm, and the focal lengths of the negative or flint element decreases from -22.22 mm to -133.33 mm. Increased Abbe number separation therefore leads to reduced optical power in both elements. The combination of the two optical powers provides a constant focal length at the primary wavelength for the doublet. In this example, the focal length of the achromatic lens is 100mm. It is also the case that the powers of the individual elements comprising the achromat become less if a greater Abbe separation exists between the two materials.

Thus if we consider how we apply this in the real design world, our guideline would be to use glasses that have at least an Abbe number difference of approximately 20 between the crown element and the flint element. If our Abbe number difference gets too small, as shown above example, the individual powers of the positive and negative elements get exceedingly strong. This in turn leads to steep angles of incidence, and this leads to higher orders of spherical aberration and tighter manufacturing tolerances. Increasing the Abbe number difference beyond 20 further alleviates the situation.

Secondary Axial Color

Secondary Axial Color is the separation in focal lengths between the red C wavelength at $0.6563\mu\text{m}$ and the yellow d wavelength at $0.5876\mu\text{m}$. Where the secondary axial color is zero, the focal lengths in the red and the yellow are identical. Where primary and secondary color are zero, the red, green and blue foci are all equal.

We will assume for the following discussion that the reader is familiar with transverse ray aberration curves [1].

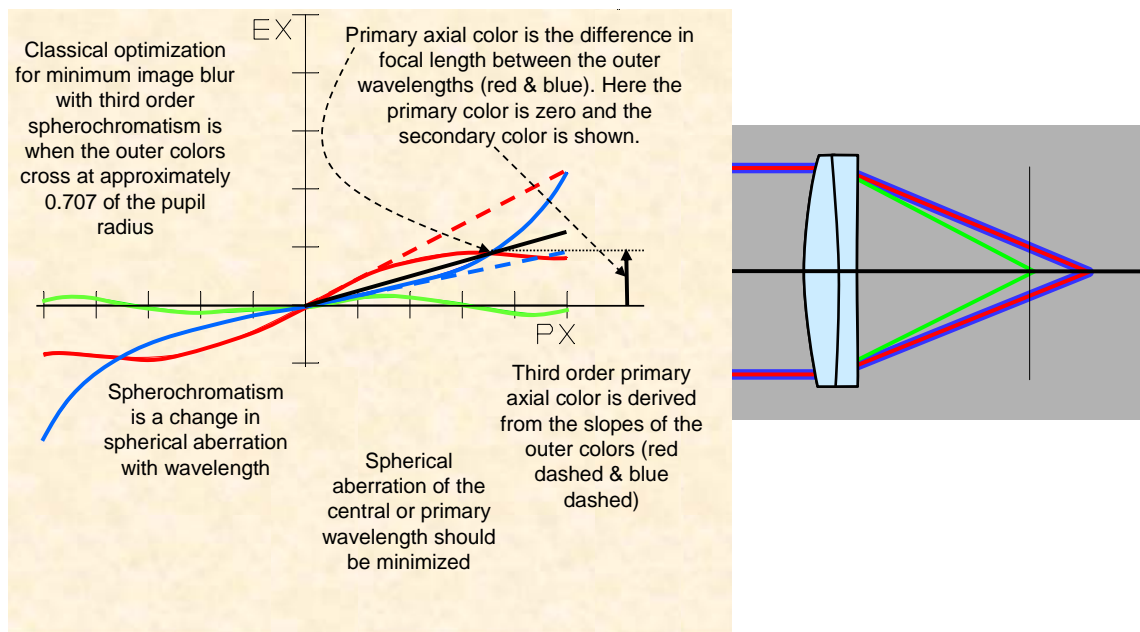


Figure 4 : Hypothetical lens corrected for primary color with transverse ray aberration plot. The transverse ray aberration plot is shown in a classical form, with red and blue being tied at the 0.707 pupil radius point.

Figure 4 shows a hypothetical lens with its set of transverse ray aberration curves. We show the primary axial color as well as the secondary axial color for the lens. Furthermore, we show and define the primary and secondary axial color on the transverse ray fan plots. The transverse ray fan plots have been provided in a classical form where the red and the blue intersect at the 0.707 point across the pupil. This form of presentation helps to understand the chromatic effects which are influencing the overall performance of the lens.

Also seen in **figure 4** is the residual of spherochromatism, which is spherical aberration that changes as a function of wavelength. In this particular case there is undercorrected spherical aberration in the red and overcorrected spherical aberration in the blue.

In order to correct or minimize secondary color, we need to utilize optical glass materials that have a common relative partial dispersion. Relative partial dispersion defined in **Table 1** above. A plot of relative partial dispersion ($P_{g,F}$) versus Abbe number, can be found in **figure 5** below.

To illustrate the effect of relative partial dispersion on the transverse ray aberration plots obtained for a doublet, an f/10 achromatic doublet was set up, wherein the materials were notionally taken to be BK7 and KZFSN11. The value of the BK7 error from the partial dispersion normal line was changed from its nominal value of -0.0009, to 0.024 and to +0.048 shown in the 3 cases in **figure 5** below.

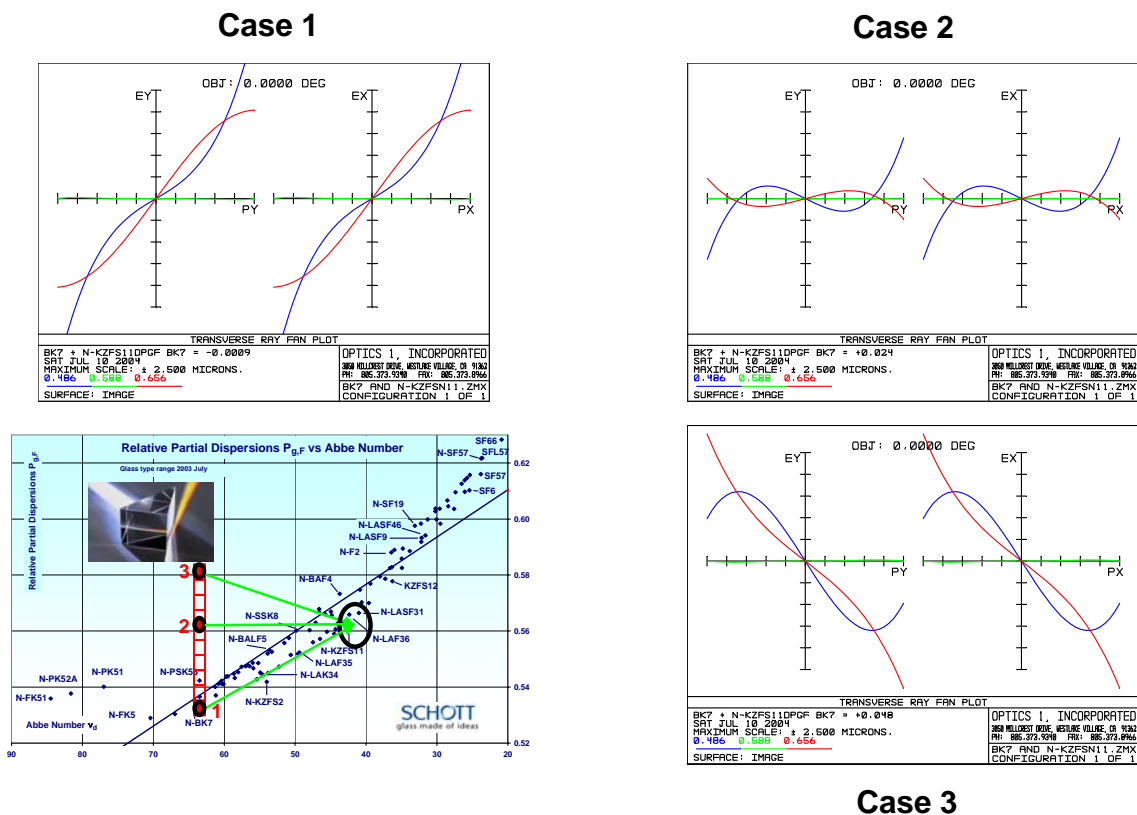


Figure 5 : Examination of secondary color by modeling a difference in relative partial dispersion.

Figure 5 above shows that as the partial dispersion is changed from a value below, to a value similar and to a value greater than the partial dispersion of the second glass (N-KZFSN11) so, the transverse ray fan plot tilts in the red and blue, from a positive slope, to a near zero slope, to a negative slope. This indicates that where the partial dispersions are in agreement, so the secondary color can be removed.

It should be noted that typically optical design programmes, such as Zemax which was used in the **figure 5** example, will use $dP_{g,F}$ values. These values do not fully describe the relative partial dispersion. The $dP_{g,F}$ values are errors from the partial dispersion normal line. This normal line is generated from the K7 and F2 values plotted as $P_{g,F}$ against Abbe number. The $dP_{g,F}$ values are the difference from this 'normal line'. One will therefore find that these K7 and F2 glasses will have a zero $dP_{g,F}$ value.

Care must therefore be taken by the optical designer trying to minimize secondary color, not to choose glasses with a zero $dP_{g,F}$ or even equal $dP_{g,F}$ values; a zero partial dispersion difference ($P_{g,F}$) must exist to eliminate secondary color.

To return to real (as opposed to model) glasses a Hammer optimization was run on Zemax. The glasses initially selected for the acromatic doublet were BK7 and SF2. These glasses can be found in regions 1 and 2 respectively of the glass map shown in **Figure 9**. The BK7 element changed to PSK51A (region 6 of the glass map) and the SF2 element became KZFSN4 (region 2 of the glass map). This is quite rational as it represents a balance of large Abbe number difference and low partial dispersion difference.

Primary and Secondary Lateral Color

Lateral color is observed when different colored rays from an off axis object point focus to different heights on the image plane. In effect this is a chromatic variation in magnification. The difference between the blue and red is the primary lateral color, and the difference between yellow and red (or blue) is the secondary lateral color. Definitions of primary and secondary lateral color can be found in **Table 1** above. Lateral color is responsible for color fringing in images. Lateral color does not exist on axis; a color halo on axis is due to axial color. Lateral color becomes evident with increased field angle and as such is often a significant design challenge in wide field of view optical systems.

It should be noted that lateral color is independent of $f/\#$; it is dependent on the height of the chief rays at the focal plane, for different wavelengths.

Spherochromatism

Spherochromatism is the change of spherical aberration as a function of wavelength. Typically, an achromatic doublet will have under-corrected or negative spherical aberration in the blue and over-corrected or positive spherical aberration in the red. **Figure 6a** shows a plot of the transverse ray aberrations for the following series of different glass types including pairs :

BK7/SF2 ($f/4$ & $f/10$), PSK51A/KZFSN4, FK54/KZFSN4

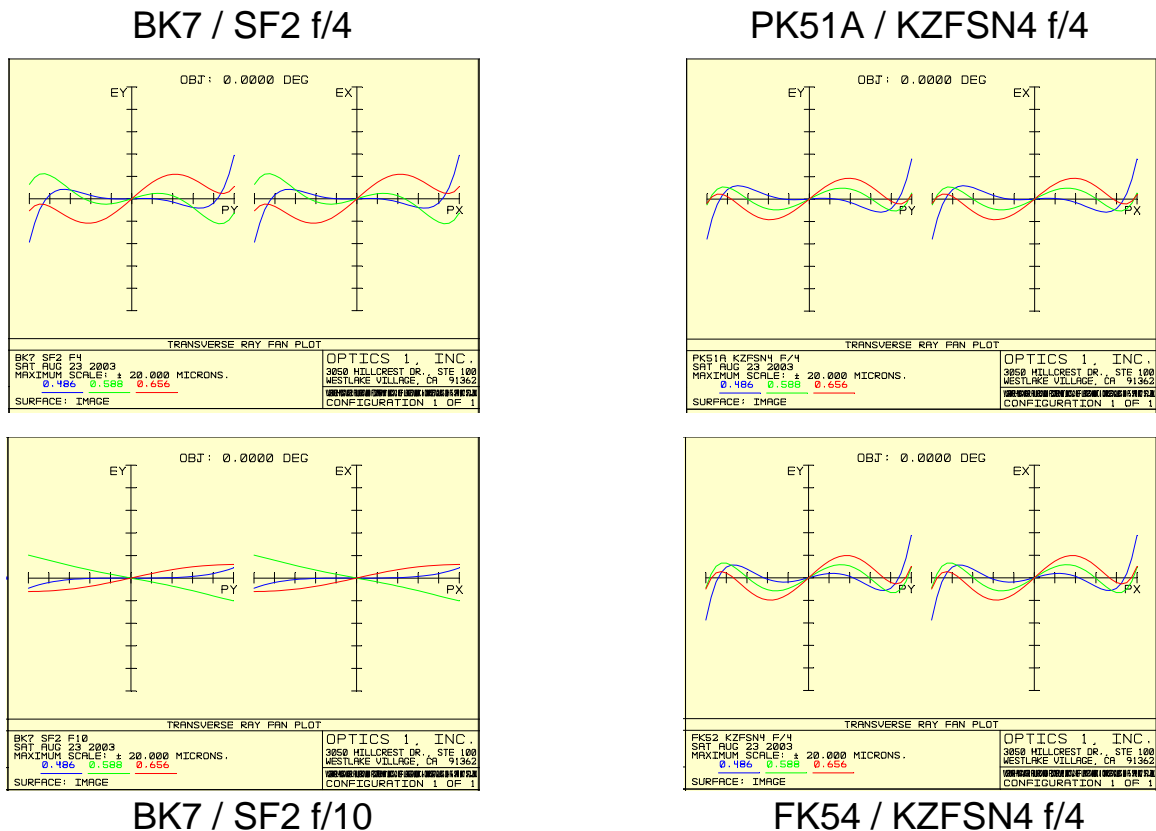


Figure 6a : Transverse ray aberration plots for a series of achromatic doublets exhibiting varying amounts of spherochromatism.

The doublets, which are at the relatively low f /number of $f/4$ exhibit slightly differing amounts of spherical aberration in the primary wavelength and this of course will carry through to the red and the blue wavelengths. In an effort to show only the spherochromatism residual, an aspheric was placed on the first surface of the doublet which served the purpose of eliminating all of the spherical aberration at the central (d) wavelength of $0.5876\mu\text{m}$.

In these examples, using an achromatic doublet, the spherochromatism is similar in each of the three $f/4$ lenses shown. The spherochromatism is improved in the $f/10$ lens. If one considers the center of the pupil of the $f/4$ lens of the same BK7 / SF2 materials, it is seen that the aberration profile is significantly more linear and exhibits lower spherical and spherochromatism. Spherochromatism will therefore become more of a design issue at faster f /numbers.

Glass choice can be shown to influence spherochromatism however, but in a doublet on axis, model glasses need to be used to exhibit a variation in spherochromatism. In the following results plotted in **Figure 6b**, model glasses were used in an f/5 cemented doublet. To allow only spherochromatism to be observed the first surface of the doublet was aspheric to eliminate spherical aberration at the primary wavelength, and secondary color was eliminated by matching partial dispersions. The Abbe numbers for both model glasses in the doublet were $V1 = 64.167$ and $V2 = 33.848$ (these are the Abbe values of BK7 and SF2 respectively).

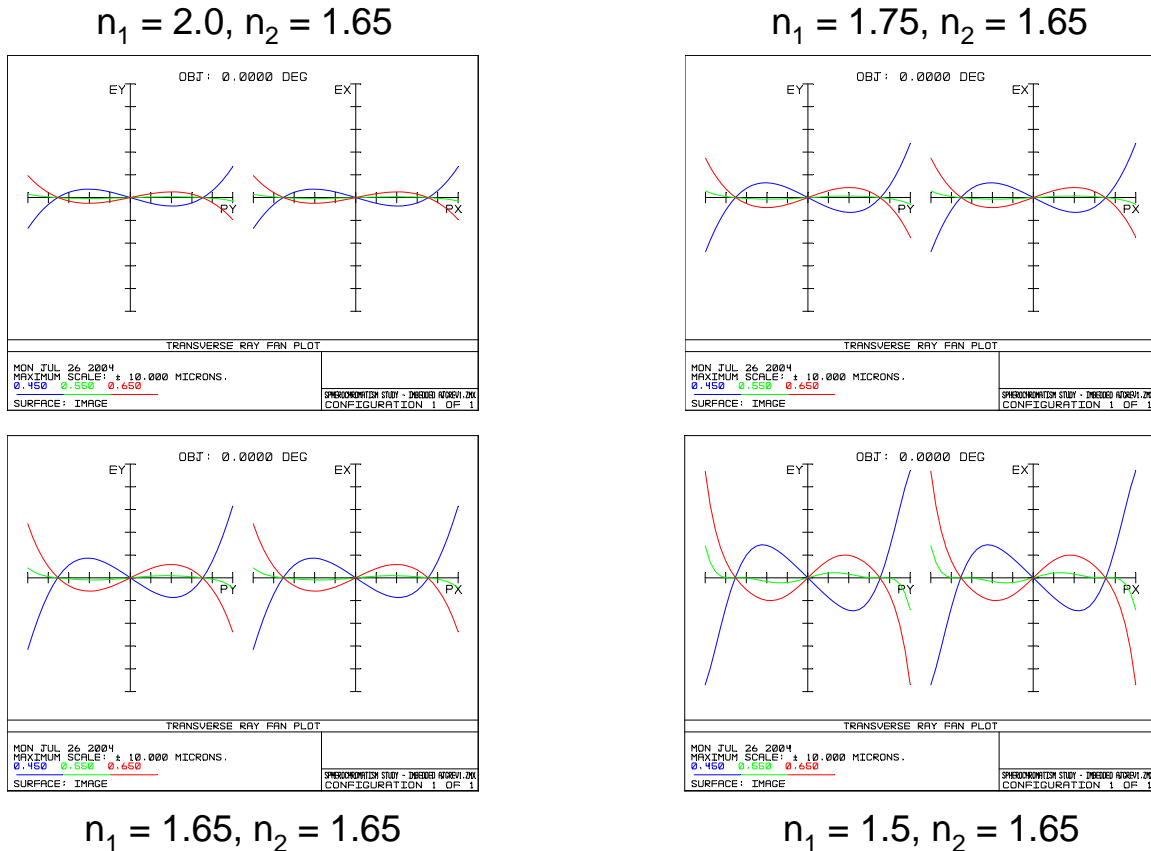
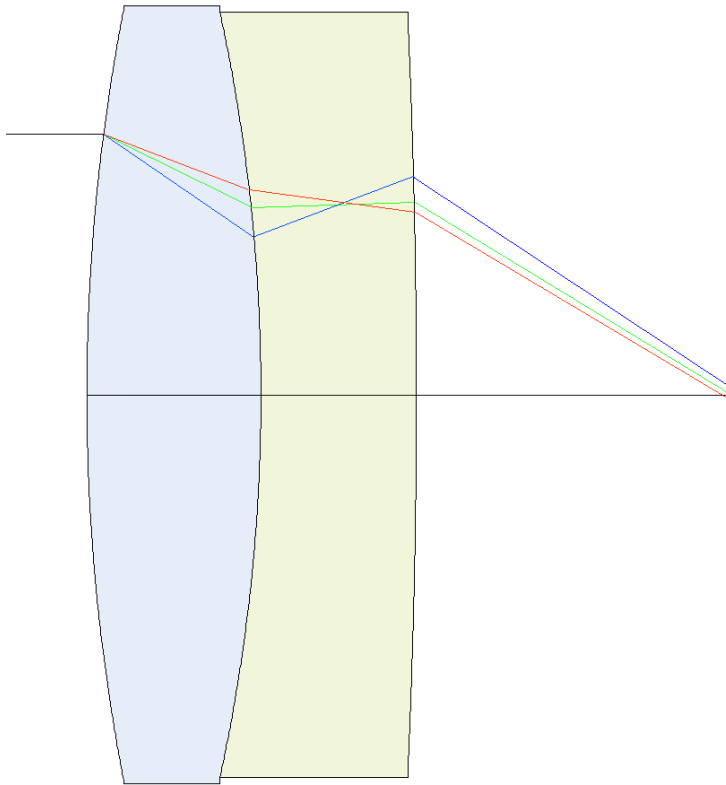


Figure 6b : Model glasses used to exhibit spherochromatism.

Figure 6b shows that for a the model glass f/5 cemented doublet, decreasing refractive index on the first, higher dispersion element lead to increasing spherochromatism. Further modeling also showed that increasing refractive index on the second, lower dispersion element resulted in increasing spherochromatism.

Figure 7 below gives some inclination as to why spherochromatism occurs. In an achromatic doublet the first and last surfaces are responsible for undercorrected spherical aberration. The inner surface is responsible for over corrected spherical aberration. Whilst the primary wavelength may be well corrected for spherical aberration, the chromatic differences do not allow spherical to be corrected at other wavelengths.



Fundamentally, if differences in the paths of the red and blue rays can be minimized, then spherochromatism will reduce. Approaches which could be adopted are to reduce the power on the first and last surfaces, thereby minimizing ray bending (although this may not be possible for other reasons, such as introducing 3rd order spherical aberration). Reducing the thickness of optical elements will further help to a some extent.

Fig 7 : Exaggerated ray paths leading to spherochromatism. Under-correction can be seen at the 1st and 3rd surfaces and over-correction occurs at the cemented surface [2]. At each surface the amount of ray bending of the blue is greater than the green, which is in turn greater than the red. At the focus, the green has no 3rd order spherical aberration, the blue is over-corrected and the red is under-corrected.

In **Figure 8** we show a unique 3 axis plot including, in effect, a traditional glass map as well as the error in partial dispersion from the normal line. The glass types include BK7, PSK53, KZFSN4, FK54, and SF6. This map allows us to see simultaneously the index, and Abbe separations, whilst simultaneously considering partial dispersion; this allows for more rapid and comprehensive selection of an appropriate glass.

By selecting glasses which fall on the same partial dispersion plane, two glasses of similar partial dispersion can be selected thereby minimizing secondary color, whilst simultaneously considering the Abbe separation and the glass refractive indices, all of which are required for appropriate glass selection. Note that because this plot is of delta in partial dispersion from the normal line, the line of equal partial dispersion will be tilted, with the tilt being defined by the gradient of the normal partial dispersion line of glasses K7 and F2.

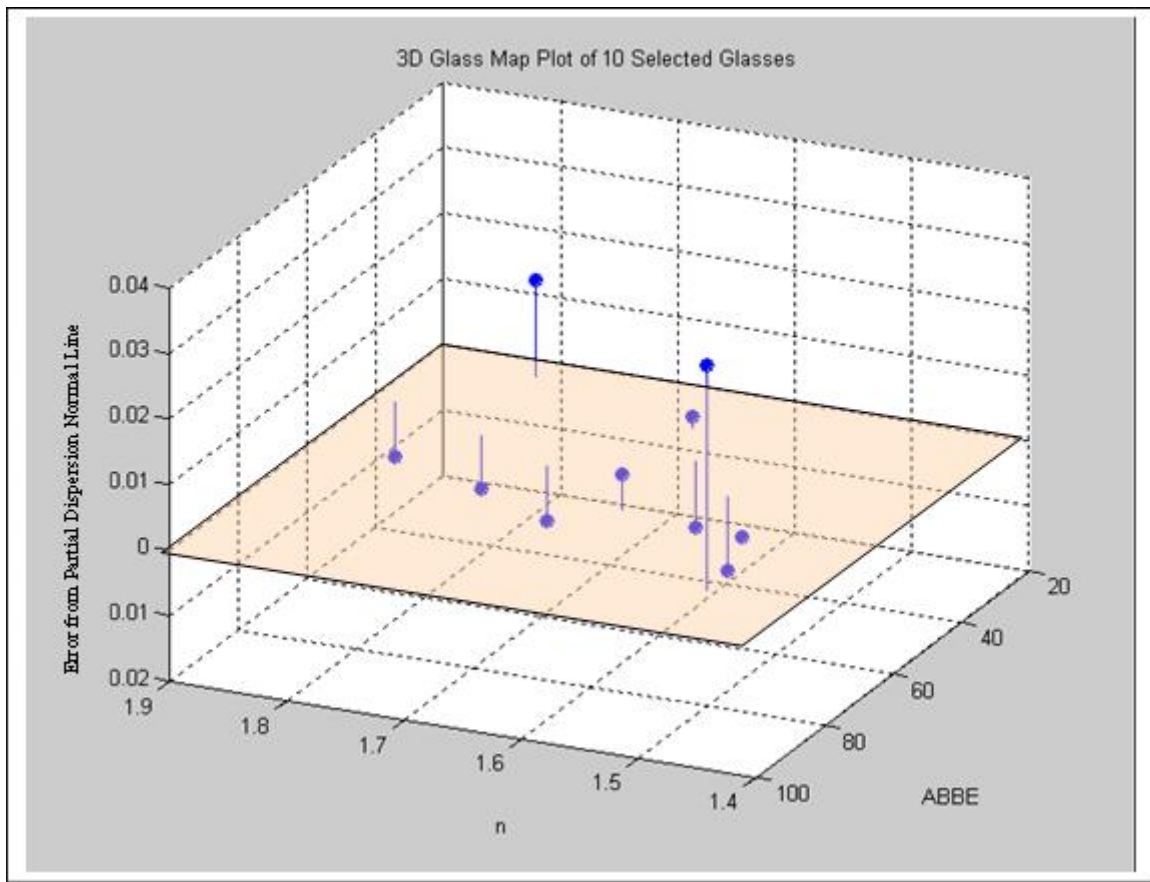


Figure 8 : 3D glass map depicting Refractive Index, Partial Dispersion and Partial Dispersion Delta from the Normal Partial Dispersion Line.

To assist with glass selection, and to get a better feeling for the traditional Abbe glass format, the map has been grouped into 6 different regions. These are shown in **figure 9** below. The groupings allow the general region of the map to be understood. This is useful because typically during an optical design optimization it will be possible to allow the optical parameters of a glass to vary. Whilst this technique may lead to an glass with unavailable parameters being desired, it will allow the designer to determine the direction in which the glass wants to head across the glass map. When this direction is considered in conjunction with the optimization merit function, OPD and transverse ray aberration plots, it may then be possible to select a ‘real’ glass which will benefit the overall design.

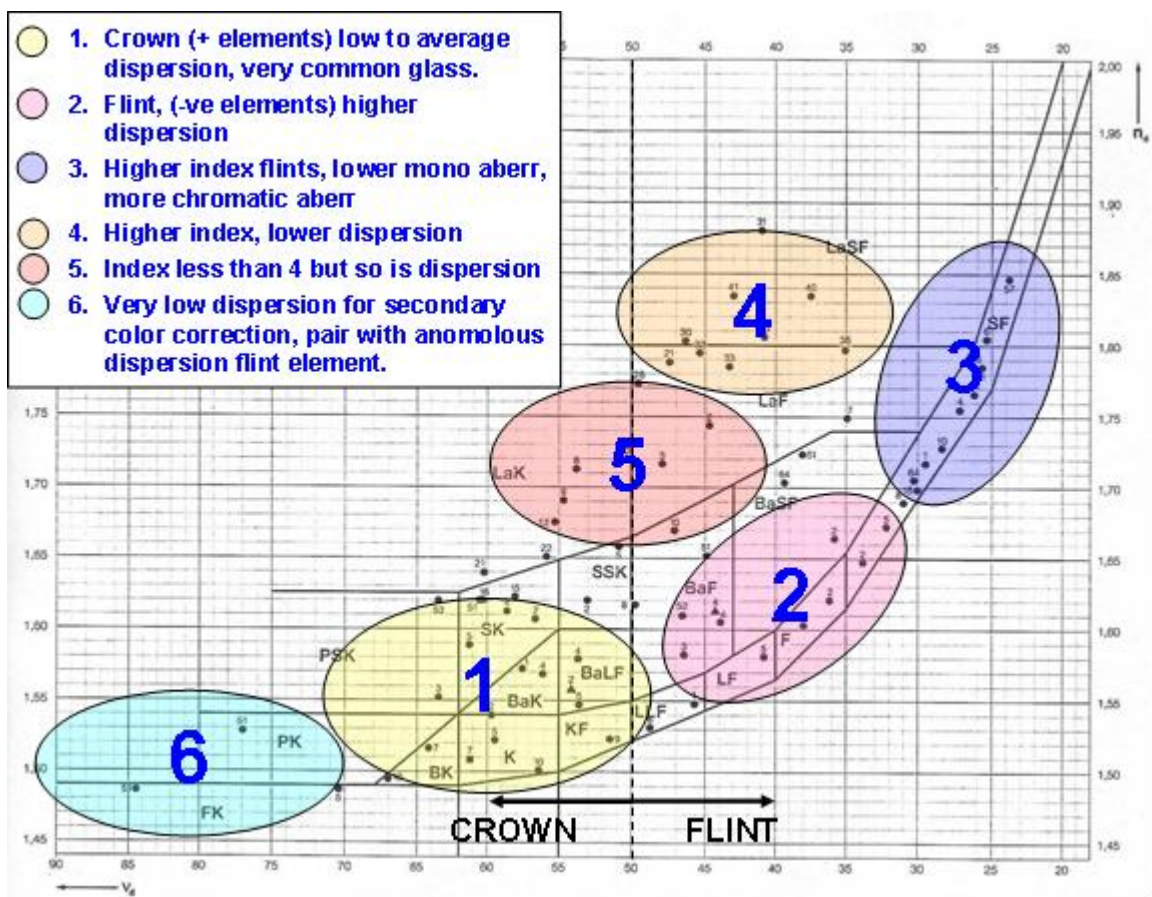


Figure 9 : Balloon map of typical SCHOTT Abbe vs Index glass map allowing a generalized glass selection approach to be adopted.

Summary and Conclusions

In this paper the factors influencing chromatic aberrations in an optical system have been reviewed. Each of the classical chromatic aberrations have been addressed separately. Where the aberrations come from and perhaps more importantly, how to get rid of them, have been discussed. Often in modern optical design the selection of glasses may occur “automatically” through an optimization algorithm. However, there may be times where intervention is required. The optical designer must therefore be aware of the detailed glass properties whilst optimizing an optical design. For example, if the glass for optimal achromatic performance has undesirable environmental, manufacturing or transmission characteristics, or is simply too expensive, then it will be necessary to select an alternative glass. By understanding why the glass was selected in the first place, or why at least an optimization algorithm selected a glass, the designer may effectively choose another with equivalent performance. Work conducted on achromatic doublets has shown that moving towards higher refractive index glasses, with an Abbe separation of greater than 20 and a near equal partial dispersion, will lead to a good solution being achieved.

References

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