

[5N] There should be a greater attempt in this elementary T.M. work, to stick as close as possible to intuitive methods of learning. T.M. should be simply an attempt to explicate one's intuitive concepts.

E.G. ① In "=" we would like T.M. to realize that 1 and 0 are somehow related to "=", but there is no reason for T.M. to pick such a remote ~~unrelated~~ "cause" when 1 and 0 are sufficient.

② When " $\sim$ " is given, again, we would like T.M. to realize 1 and 0 are assoc. with " $\sim$ ". While this is not too improbable, tetragrams are still probably easier (intuitively).

③ ~~When~~ <sup>when</sup>  $\oplus (= \beta \Sigma)$  comes around, T.M. ~~should~~ should be ready to realize that the isolated signs  $=, \sim, \oplus$ , condition the operations occurring near them — or even <sup>just</sup> in some example.

At this point R. foll. epist. rule comes up: "If something happens that was unforeseen, then look for the things near R. unforeseen event that are "unusual". They are good ~~and~~ guesses for the "causes" of the unforeseen events. A good def. of "unusual" has been given before: essentially, an event is "unusual" if it doesn't conform to R. abs. that most examples have conformed to up to R. present time.

It might be well to formulate this in a form for use with ~~the~~ elementary Math T.M. ~~Temporary~~ Suppose one has a bunch of abs. that are very U. in prediction — that one's score is very low. Then one gets a few new examples in which predictions are

However, it may be said that if we have a sufficiently long  $\sim$  or  $=$  problem, it will take less time for T.M. to learn  $\mathcal{R}$ , tetragrams, than to carry on  $\mathcal{R}$ . "contagion" process of learning trigrams like

$V \oplus B \oplus B \oplus 0$

⑥ Say T.M. has learned tetragrams. (The formation of trigrams gone thru on α 61 to α 63 is mainly for illustration of methods of developing ntpsts. by inversion; and the use of "neighboring" strs.)

Next try  $\beta \Sigma$ . ( $\equiv V$ ) ( $\equiv \oplus$ )

$\oplus \begin{matrix} | & 0 & 0 & 0 \\ | & 1 & 0 & 1 \\ | & 1 & 0 & 1 \end{matrix}$  ,  $\oplus \begin{matrix} 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \end{matrix}$  ,  $\oplus \begin{matrix} 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{matrix}$  etc.

The monograms now become of some U, since 1's occur more frequently than 0's.

If  $\square$ 's are put in  $\mathcal{R}$ . bottom line only,

then  $\square$  become of by U, for this particular problem  $\rightarrow$  also  $\leftarrow$  but it is of not much U. in  $\sim$  — so on  $\mathcal{R}$ . average,  $\square$  does much good.

The tetragrams

$\begin{matrix} | & 1 \\ | & \square \end{matrix}$  ,  $\begin{matrix} 0 & 1 \\ 0 & \square \end{matrix}$  ,  $\begin{matrix} 1 & 1 \\ \square & 1 \end{matrix}$  ,  $\begin{matrix} 0 & 0 \\ \square & 0 \end{matrix}$  are of by U. with 100% correctness.

They do not, however, ~~produce~~ <sup>fit</sup> all cases.

The tetragrams  $\begin{matrix} 1 & \square \\ 0 & 1 \end{matrix}$  ,  $\begin{matrix} 0 & \square \\ 0 & 0 \end{matrix}$  etc. are of ~~some~~ lower U.

However, they can be salvaged by adding the space, to obtain pentagrams:

$\begin{matrix} 1 & \square \\ 0 & 1 \end{matrix}$  ,  $\begin{matrix} 0 & \square \\ 0 & 0 \end{matrix}$  etc.

T.M.

of these, only

$\begin{matrix} 2 \\ \square \end{matrix} \begin{matrix} \square \\ \square \end{matrix} \begin{matrix} \square \\ \square \end{matrix}$ ,  $\begin{matrix} \square \\ \square \end{matrix} \begin{matrix} \square \\ \square \end{matrix} \begin{matrix} \square \\ \square \end{matrix}$ ,  $\begin{matrix} \square \\ \square \end{matrix} \begin{matrix} \square \\ \square \end{matrix} \begin{matrix} \square \\ \square \end{matrix}$  are of any U, so

we retain them: they produce

$\neq \sim \begin{matrix} \square \\ \square \end{matrix} \begin{matrix} \square \\ \square \end{matrix} \begin{matrix} \square \\ \square \end{matrix}$ ,  $= \begin{matrix} \square \\ \square \end{matrix} \begin{matrix} \square \\ \square \end{matrix} \begin{matrix} \square \\ \square \end{matrix}$ , etc.

By the way: this fact strengthens the a priori of M, X D, as well as its U.

Similarly, we try neighboring str. to

$\begin{matrix} \square \\ \square \end{matrix} \begin{matrix} \square \\ \square \end{matrix} \begin{matrix} \square \\ \square \end{matrix}$  at a1, and obtain

$\sim \begin{matrix} \square \\ \square \end{matrix} \begin{matrix} \square \\ \square \end{matrix} \begin{matrix} \square \\ \square \end{matrix}$ , etc.

we get  $\sim \begin{matrix} \square \\ \square \end{matrix} \begin{matrix} \square \\ \square \end{matrix} \begin{matrix} \square \\ \square \end{matrix}$  and  $= \begin{matrix} \square \\ \square \end{matrix} \begin{matrix} \square \\ \square \end{matrix} \begin{matrix} \square \\ \square \end{matrix}$

by ~~the~~ similar operations each time.

5N At this point, we would like T.M. to learn that if we ~~can go from~~ it is desirable to go from

$\sim \begin{matrix} \square \\ \square \end{matrix}$  to  $\sim \begin{matrix} \square \\ \square \end{matrix}$  or  $\sim \begin{matrix} \square \\ \square \end{matrix}$  or  $\sim \begin{matrix} \square \\ \square \end{matrix}$

it should be much more immediately desirable to try  $\begin{matrix} \square \\ \square \end{matrix} \begin{matrix} \square \\ \square \end{matrix} \begin{matrix} \square \\ \square \end{matrix}$  and  $\begin{matrix} \square \\ \square \end{matrix} \begin{matrix} \square \\ \square \end{matrix} \begin{matrix} \square \\ \square \end{matrix}$

This seems to make ~~the~~ expedient, T.M.'s ability to use sets of sets of sets....

5 ~~the~~ whether T.M. uses tetragrams or trigrams to predict  $= \sim \sim \sim$ , will depend on the relative a priori attenuations assigned to the various operations used in forming them. 264

$$\boxed{12} \text{ Str.} \times \left[ \begin{array}{c} (=, \sim) \text{ \&S} \\ \text{monogram} \\ \text{set} \end{array} \text{ \&S} \begin{array}{c} (\square, \square, \square, \square) \\ \text{cart.} \\ \text{product} \end{array} \text{ \&S} \begin{array}{c} (\square, \square, \square, \square) \\ \text{digm set:} \end{array} \right]$$

$$S_1 \times [M_1 \text{ \&S} D_1]$$

$S_1$  is of hy a prip. - as initial str. that T.M. starts with.

The trigram set  $M_1 \text{ \&S} D_1$  was obtained in R. foll. way!

$\boxed{12}$  was mult. by R. cart product of

$$\left( \begin{array}{c} \square, \square, \square, \square \\ \square, \square, \square, \square \end{array} \text{ \&S} (=, \sim, \square, \square) \right) \text{ with itself.}$$

are digms that have had by U in R. past

are monogms, and are always of hy U.

The result was a set of 128 trigrams. of these,

only 32 ever occur, only 24 contain  $\square$ . of these, 24,

only 8 are of hy U. They are

except for extensions ambiguous trigrams

Note:  $\boxed{12} \times \left( (=, \square) \right) \rightarrow \left( (= \square, = \square) \right)$

In general, if in multiplying a str. times an ntpl, R. result is ambiguous, then all of the ambiguous ~~elements~~ resultants should be retained.

since  $S_1 \times (M_1 \text{ \&S} D_1) =$  a ngrmst of hy U,

and  $S_1$  is of hy a prip, then  $M_1 \text{ \&S} D_1$  is of hy a prip.

We try str. near  $S_1$  and mult. them by

$M_1 \text{ \&S} D_1$  to obtain trial trigrams.

Some str "near"  $\boxed{12} = S_1$



The inability is due to the fact that e.g.  $\begin{matrix} 1 \\ \square \end{matrix}$  and  $\begin{matrix} 0 \\ \square \end{matrix}$  cannot be, in this method, members of the same categorization group. In the present case, it would be nice to have

$\begin{matrix} 1 \\ \square \end{matrix}$  and  $\begin{matrix} 0 \\ \square \end{matrix}$  members of the same cat. group. — since this would give something closer to the idea of "equality" but this is not essential yet.

It should be looked into soon however. We might make  $\square$  part of the specification of a str. — then

$$\begin{matrix} 1 \\ \square \end{matrix}, \begin{matrix} 0 \\ \square \end{matrix} = \begin{matrix} 1 \\ \square \\ \square \end{matrix} \times \begin{matrix} 0, 1 \\ \text{noogram set.} \\ \text{= 1 tuple set.} \end{matrix}$$

⊕  $\sim \begin{matrix} 1001 \\ 0\square 10 \end{matrix} ; \sim \begin{matrix} 0\square 11 \\ 1100 \end{matrix} ; \sim \begin{matrix} 101\square \\ 0101 \end{matrix}$  is given.

with associated problems. This makes all digits useless. Trigrams are tried.

$$= \begin{matrix} 1 \\ \square \end{matrix}, = \begin{matrix} 0 \\ \square \end{matrix}, \sim \begin{matrix} \square \\ 1 \end{matrix} \text{ etc. are } \cup.$$

However, most cases are not predictable by compact

Trigrams.

ⓐ Tetragrams work in every case!

e.g.  $\begin{matrix} 10 \\ 1\square \end{matrix}, \begin{matrix} 0\square \\ 11 \end{matrix}, \begin{matrix} 11 \\ \square 0 \end{matrix}$  etc.

ⓑ It is poss. to develop ~~trigrams~~ trigrams like

$$= \begin{matrix} 1 \\ \square \end{matrix}, = \begin{matrix} \square \\ \square \end{matrix} \text{ etc. in the foll. way:}$$

This method ~~is~~ involves more operations, but the result seems, intuitively, more desirable.

The ~~trigram~~ ngram set

$$= \begin{matrix} 1 \\ \square \end{matrix}, \sim \begin{matrix} \square \\ 0 \end{matrix} \text{ etc. can be factored into}$$

from 258.40:

Math

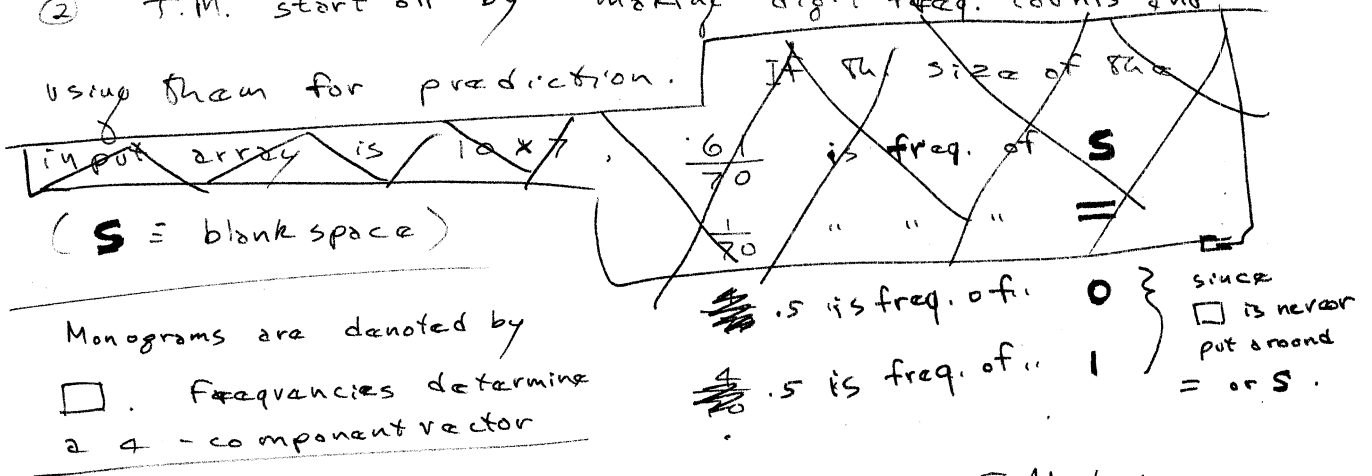
More detailed descriptions of T.M.'s response to a trig. sequ. up to <"t" with carry line>.

① T.M. is first fed a set of examples like

$$= \begin{array}{c} 1001 \\ \square 001 \end{array} ; = \begin{array}{c} 0101 \\ 0101 \end{array} ; = \begin{array}{c} 1110 \\ 1110 \end{array} \text{ etc.}$$

The problems are of R. form  $\begin{array}{c} 1101 \\ 1001 \end{array}$

② T.M. start off by making digit freq. counts and using them for prediction.



③ Predictions are very poor, so T.M. tries digrams next. Some digrams:

$$1\square, \square, \square, \square, \square 1, 0\square, \square, \square, \text{etc.}$$

Each digm. gets a prediction vector assigned to its  $\square$ , thru freq. counts.

Only the digms  $\square, \square, \square, \square$  are of any U in prediction. Since they have a prediction efficiency of 100%, no other digms or trigrams are introduced, until  $\sim$  problems arise.

(SN) The use of  $\square$  or  $\square$  e.g. as monograms or digms., gives T.M. a serious inability, but we will let this ride for a while. 261

Sun July 15, 1956

T.M.

α 59

Monte Carlo searches: from X 974 and α 46 <sup>more</sup> [also previous work]

On α 45 we seem to have worked cases 3) and 4) in correctly case 3) is worked exactly and correctly on th. bottom of X972 and top of X973. "n trials" is correct.

For 4) th. method of th. top of X973 seems right, but it conflicts with <sup>the</sup> corrected method of α 45

for α 45: If the <sup>th</sup>  $i$  element in th. set is correct, then th. expected no. of trials is

$$\begin{aligned}
 & 1 \cdot p_i + 2(1-p_i)p_i + 3(1-p_i)^2 p_i + \dots \\
 & = p_i \sum_{x=1}^{\infty} x(1-p_i)^{x-1} = p_i \left( \frac{1}{1-(1-p_i)} \right)^2 = \frac{1}{p_i}
 \end{aligned}$$

Since th.  $i$ th element is correct  $p_i$  of th. time, th. expected no. of trials for all time is

$$\sum p_i \frac{1}{p_i} = n.$$

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On th. other hand: from X973.02

th. proby of getting th. right choice in <sup>th first</sup> trial is  $\leq p_i^2$ .

th. proby of getting it on th. second <sup>trial</sup> ~~choice~~ is  $(1 \leq p_i^2) \leq p_i$  } No

This is the error. Th. proby. distrib. for th. second choice is not ~~the same~~ th. same as th. first, since knowing we failed th. first time makes it more likely that th. "true" case is more toward th. lower  $p_i$ 's.

α 60

Tues July 10, 1956

α 57.30  
criticisms; cont.

4) shannon suggested that they be given a chance to pick their own problem for this T.M., to guard v.s. ad-hockness - say a sq. root problem, and to do a "hand simulation of it"

List imp. obs. that T.M. ought to have:

- 1. carry line problem soln.
- 2. set of set of sets ... etc.
- 3. to extrapolate  $\Rightarrow \mathbb{R}$ . str  $1 \ \&\&\&\dots 2$ , out into  $\mathbb{R}$ .

→ direction, an arb. distance.

4. Will T.M. think of trying commutativity on various operators? i.e. in  $1+2=3$  to give  $2+1=3$  hyperaprip than  $1+3=2$ .

5. Teaching T.M. to fill in  $< 1 \ \square$  ; 6. omission of  $\square$ .

**SN** On B.G. : Th. "soln." that consists of taking

th. "Best" categorization group that an event belongs to, isn't too bad: In fact it is as general a soln. to  $\mathbb{R}$ . problem as any. Th. gp. that is th. Boolean  $\#$  product of all  $\mathbb{R}$ . gps. an event belongs to, will, in general, be of hy apri.  $U$ , and be th. "Best" gp. available. Th. trouble is, th. evaluation of  $\mathbb{R}$ . prediction params. of a gp. as a function of  $\mathbb{R}$ . ~~prediction~~ params. of  $\mathbb{R}$ . elem gps., n tps and str. from which it was formed, is as difficult as th. B.G. problem!

However, this latter would have to be done anyway, regardless of what sort of B.G. soln. was used.

**Present approach:** Just work on th. deriving of "new gps from old" problem, with a little attention to the ~~shown~~ expected amount of attenuation of aprip with th. no. of operations performed.

A running account of T.M.'s battle with a Tyo. sequ. is on. X939.27 ff. comments up to X957.22 where it is decided that I ought to be able to do up to "+" with carry line, and I should "consolidate gains".

A more detailed description of th. Tng. sequ. up to "+" will follow - on α 60.



in which 01 occurs. If one has an infinitely large discrete universe, or a ~~finite~~ <sup>finite</sup> ~~continuous~~ <sup>finite</sup> continuous one, there are, indeed, an  $\infty$  of ~~the~~ possibl. states of the universe in which 01 occurs.

Also, I think that in the case of learning

$$= \begin{matrix} 1 & 0 & 1 & 1 & 0 & 0 & \dots & 1 & \dots & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & \dots & 1 & \dots & 0 & 0 & 1 & 0 & 0 \end{matrix}$$

there is, essentially, the possibl. of an  $\infty$  of ~~spaces~~ squares between ~~the~~ = and  $\square$ .

Tues, July 10, 1956 : day of talk on T.M. to Dartmouth group: Minsky, McCarthy, McCulloch, Shannon, attended - also partly Bill Schutz.

Criticisms:

- 1) They would like to do a hand simulation of this (Shannon in partic.)
- 2) McCarthy ~~is~~ expressed worry about the length of some of the search processes that will be involved.
- 3) Minsky suggested ad-hock soln. of the str. for and seems quite happy to add all sorts of ad-hock mechanisms, if they seem very useful. I am afraid that
  - a) he is overly impressed by the necessity of built-in mechanisms.
  - b) He isn't aware that ad-hock <sup>mechanisms</sup> ~~is~~ since ~~is~~ in using them, one misses one more chance to pick up the clue of a ~~general~~ <sup>very</sup> general mechanism, that ~~one~~ may be the difference betw. "creative" that and not.

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258.01

The problem about ~~the~~ setting up "classes of classes of classes" etc. in T.M., can, I think, be solved by giving ~~them~~ detailed instructions, in English, then coding them for T.M.

- 1) A class may be defined by a set of instructions telling one if an object is in the class or not - also, it may contain quick-trial rejection <sup>or acceptance</sup> ~~methods~~ that usually work.
- 2) ~~the~~ history of class members
- 3) How class was constructed from other classes of n bps and str.

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T.M.

We also have objects (e.g. permutations) which operate on strs. to produce sets of strs.

If  $S_1, S_2, \dots, S_n$  ~~is~~<sup>is</sup> a set of strs.

then  $G_i \equiv S_i \cdot N_i$  [ $i=1/n$ ] is a set of sets.

I think that there are other useful sets of sets.

3) We probably can use higher order objects like sets of sets of sets.

The concept of "Number": We can use Russell's definition: ~~is~~  $Z \equiv \mathcal{P}$ . set of all pairs of objects. We can have  $\mathcal{P}$ . "betweenness" relation: e.g.  $\langle \mathcal{P}$ . set of all triples  $\rangle$  is "between"  $\mathcal{P}$ .  $\langle$  set of all pairs  $\rangle$  and  $\langle$  the set of all quadruples  $\rangle$ .

We must design T.M. so that this does turn out to be a "useful" concept, and so that it is made ~~is~~ likely that T.M. would "discover" it.

M. suggests that the word "integer" might be difficult to define. This is a set of sets, but it has infinitely many members, which is a difficulty. However, many other of T.M.'s sets might be that of as having ~~is~~ members. E.g. any  $\mathcal{P}$ . diagram 01, may be that of as including all situations 257

Mon July 9, 1956

T.M.

How to get concepts like "number"

E.G. Consider that T.M. ~~finds~~ finds that

$\square$ ,  $1 \square$ ,  $1 B \square$ ,  $1 B B \square$ ,  $1 B B B \square$  are all useful.

would he be able to extrapolate that  $1 B B B B \square$  was probably useful?

To start from R. beginning: ~~we have sets~~

1) we have sets of phenomena. These sets have th. property that they group phenomena in ways that are useful for prediction. i.e. so that th. occurrence of one element of a set can be used as statistical data for th. occurrence of other members of th. set. 2 members of th. same set are "close" to one another in ~~2) we have~~ a "way" that is named by th. set name.

2) 2 sets are "close" if th. ~~usage~~ empirical usefulness of one suggests that th. other is probably useful. "closeness" suggests sets of sets.

I.E. "closeness" suggests "neighborhoods", ~~neighborhoods~~

A closeness relation on a set of pts. is partly defined by a set of point sets, such that all th. members of each point set are close together.

Anyway, in general, the concept of set of sets is useful.

Th. factorization concept also gives sets of sets. Say a ~~group~~ <sup>G</sup> is generated by str.  $S_1$  mult. by ~~intpt~~ <sup>intpt</sup>,  $N_1$ .

$$G_1 = S_1 \cdot N_1$$

If  $S_2$  is close to  $S_1$ , then  $G_2 \equiv S_2 \cdot N_1$  is close to  $G_1$ .

Fri July 6, 1956

T.M.

Methodological Note: It appears that sub-goals (and sec. recurf) are only imp. in T.M. problems involving a search. Eg. in a well defined problem of Th. 1st kind - where one must invert some definite operator. There are many imp. T.M. problems that do not involve searching (This isn't absolutely certain). For this reason, it appears worth while to solve Th. non-search problems first, since they are simpler. A search problem involves all the techniques of a non-search problem, plus the extra problem of devising sub-goals. I think that once the non-search problem is solved, the search problem will be easy to do, using the techniques of abs. construction that were devised on Th. non-search problem.

Th. devising of abstractions sometimes involves searching  
E.g. The inversion of certain xfmns. is very often useful.

For prelim. work, however, we may simply assume that we can carry thru Th. search process successfully - say by means of a very <sup>fast</sup> logical computer

One also feels that if, in th. past,  $\square$  has implied  $\square$ , this fact should be made much of.

A very good reason why I would want  $\square$  to be of type U: In R.W. language, it is the events that do occur, that get words assigned to them, not  $\square$ , which of  $\neq$  zero frequency.

A temporary way to "solve" this problem: (temporary in th. sense of enabling me to continue with the review on T.M.). Th. "ngmst"  $\square$  will have assoc. with it, a U, and a vector, whose components give th. probty. of various ~~explanations of the~~ digits in  $\square$ . This ngmst  $\square$  also has other prediction params. Th. "Rough and dirty" B.G. soln. that will be used at first, is that we take th. ngmst of highest U and ~~prop~~ (meaning? - (since probty is a vector)) or, more exactly, th. "Best" ngmst, and use its probty vector as th. soln. In th. case of several ngmsts all about of = goodness, average their vectors in ~~some~~ way - (or convolute them(?)) - Anyway, if these "Best" vectors differ much, give th. resultant ~~is~~ a low reliability.

It is very probable <sup>previous</sup> that ~~the~~ discn. of ~~what~~ how to make ~~frequency~~ freq. counts in Math T.M. has ~~been~~ looked into this point. X 858A has something to say about this, but not too clearly.

This ~~is~~ approach of suggesting that once take th. "Best" ngmst and use its vector, isn't so bad. If several ngmsts should have their vectors combined to give th. final result, perhaps th. ngmsts themselves could be combined in an optimal way to yield ~~an~~ an even more optimal ngmst.

Th. trouble with th.  $\square$  type representation, is that  $1 \square$ ,  $0 \square$ ,  $0 \square$  could not be members of th. same set. This may not be imp. for very low level problems, but it is certainly imp. for complex problems.

T.M.

In writing a review of work on Math T.M., the foll. prob. has come up:

We are trying to predict the  $\square$  square in

$$= \begin{matrix} 1101 \\ 11\square 1 \end{matrix}$$

We want the rel. probability of

$$= \begin{matrix} 1101 \\ 11\square 1 \end{matrix} \quad \text{and} \quad = \begin{matrix} 1101 \\ 11\square 1 \end{matrix}$$

There are 24 possible adjacent binary diagrams containing  $\square$  — assuming that the 3 poss. completions could be 0, 1, or space.

We would think that the diagrams of greatest U would be  $\begin{matrix} 0 \\ \square \end{matrix}$  and  $\begin{matrix} 0 \\ \square \end{matrix}$ . However, if the diagram  $\begin{matrix} 0 \\ \square \end{matrix}$  is used along with ~~any~~ any other diagram like  $\begin{matrix} 0 \\ \square 1 \end{matrix}$ , of non-zero frequency, prediction is ~~not~~ as good, or almost as good.  $\begin{matrix} 0 \\ \text{null} \end{matrix}$  gives as good prediction as

$\begin{matrix} 0 \\ \square \end{matrix}$ . For n examples, the freq. of occurrence of

~~is~~  $\begin{matrix} 0 \\ \square \end{matrix}$  is zero;  $\begin{matrix} 0 \\ \square \end{matrix}$  is  $\frac{n}{4}$ ;  $\begin{matrix} 0 \\ \text{null} \end{matrix}$  is  $\frac{n}{4}$  (in fact  $\begin{matrix} 0 \\ \text{null} \end{matrix}$  happens if and only if  $\begin{matrix} 0 \\ \square \end{matrix}$ )

For this reason,  $\begin{matrix} 0 \\ \square \end{matrix}$ ,  $\begin{matrix} 1 \\ \square \end{matrix}$ ,  $\begin{matrix} 0 \\ \square \end{matrix}$  and  $\begin{matrix} 1 \\ \square \end{matrix}$  get hy U:

$\begin{matrix} 0 \\ \square \end{matrix}$  etc. get ~~about~~ about the same U as most other diagrams.

It would seem <sup>intuitively</sup> that  $\begin{matrix} 0 \\ \square \end{matrix}$ , etc. ought to get hy U, and that  $\begin{matrix} 1 \\ \square \end{matrix}$  etc. ought to get about zero U. — yet  $\begin{matrix} 1 \\ \square \end{matrix}$  gets as much U as  $\begin{matrix} 1 \\ \square \end{matrix}$ .

Also, the soln. to the B.S. that is to be applied, isn't at all clear. There is a kind of soln. on X 898.24, but it is still in too complex a form to understand what it might mean (even approximately) in ~~most~~ <sup>any</sup> specific cases.

Also, it is felt that if one has statistical info. on 2 mutually incompatible ngn types, ~~like~~ like  $\begin{matrix} 0 \\ \square \end{matrix}$  and  $\begin{matrix} 1 \\ \square \end{matrix}$ , one should make much of this info. At the present time, it is not too clear as to how this should be done. 253

Tues July 3, 1956  
T.M.

T.M. This is ~~because we have~~ because it appears that we can go extremely far, without having to alter the basic abs. methods very much — i.e. str. and ntpsts. Certainly it seems that " + with carry line" is poss., and very probably most hyper operations with little basic modification.

**SU** At the present time, creation of new abstract entities by inversion, is being looked upon as a ~~rather~~ not very special case of the creation of a set by "multiplication" of e.g. str. and ntpsts. E.g. A gp.  $\alpha$  is found to be of hy  $U$ .  $\alpha$  is then found to be factorable into the structure  $S$ , and the ntpst,  $M$ , so  $\alpha = SM$ . If  $M$  is of hy  $U$ , then  $S$  becomes of hy a pri.  $U$  and of hy a pri.

Actually, there is a rather serious search process involved. One can, mathematically, state that for any  $\alpha$  of hy  $U$ , then for any  $M$  of hy  $U$ ,  $\exists SM = \alpha$ , the a pri. of  $S \uparrow$ . However, often, one will not find this  $S, M$  pair, since the computer available, ~~with~~<sup>is</sup> very limited in capacity. The finding of such an  $S, M$  pair will be a real discovery, and will not always occur, even when such a pair exists. The amount of time that one wants to spend looking for such pairs, will be sharply limited, but in general, for any  $M$  of hy  $U$ , many  $SM$ 's will be tried, to see if they yield something close to a gp. of hy  $U$ .

In some cases, the "division" of  $\alpha$  by  $M$  can be carried out to yield  $S$ .

Son July 1, 1956 Hanover, N.H.

T.M.

Some config. eval. methods will assign variances along with values. As  $n$  increases, <sup>effective</sup> variance decreases.

Actually, the variance of an evaluation will depend upon how many moves into the future one's opponent looks.

If one has these variances, one can know just how many moves to pursue each line of play. There is, however, the Q. of whether at any time, it is better to <sup>stop and</sup> spend time evaluating a move or to proceed with further analysis into the future. One may ~~invent~~ <sup>invent</sup> special quickly-applied evaluators, ~~for~~ for preliminary determination of whether to use a hyper powered evaluation scheme, or simply continue tracing into the future.

Is it poss. to make use of some of the work done in deciding on the previous move, to reduce the search process for the present move?

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[SN] Math T.M. starts on ~~T.M. X~~ 729 with discn. of several kinds of probs. to work. X730 has list of some fairly good probs.,  $\Rightarrow$  if they were solved by T.M., we would feel that it was really learning things. Also, it seems not too unreasonable that after learning all that math, T.M. should be able to learn to understand Q's posed in a simple lang. - perhaps, eventually, English.

To understand English, however, it may be nec. to get T.M. to be able to predict analog events in R.W. - then translate them into English, and use the analog events ("concepts") to extrapolate English - or to "understand" it.

A somewhat more detailed "learning sequence" for the initial stages is on X774. [From X774 to X780 is a discn. of whether or not Math T.M. is of much use - i.e. is it likely to demonstrate "creativity" w/o understand English?

As a final remark: A criterion of whether Math T.M. is really being "creative". If, after a long while, T.M. has been producing what would normally be regarded as "creative" results, and has had no new ad-hock abstraction method inserted into his program for a very long time, then we will say that Math T.M. is really being creative.

At the present time, there seems to be reason to believe that we can expect creativity from



Just what would an efficient search procedure be in these games? Would there be much difference <sup>betw.</sup> for chess or checkers?

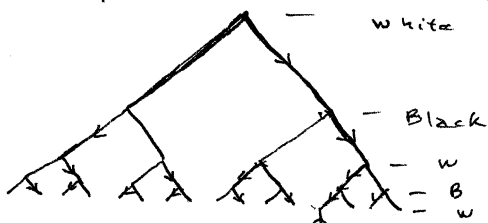
2 ~~search~~ eval. methods can always be compared: Simply take a large no. of ~~pos.~~ configurations that are of reasonable ~~depth~~ <sup>depth</sup> & apply both methods. Then use a 3rd method (or one of 2) that has been "souped up" by, say, a 5 move exhaustive search, to decide which configurations are "really" the best. Compare the orderings of 2 ~~eval~~ methods to be compared, with the "true" ordering.

For ~~an~~ optimum searching, at each point in the search, each move under evaluation will have a certain expected  $G$  and a variance about this  $G$ . Decisions to go ahead with a search line or drop it, can be then made on this basis.

For example, consider game in which each player only has 2 possl. moves each ~~turn~~ turn. E.g.

Blk. and white tokens. 1 dimensional board. Each player places token on North or South end of pile. A player wins if he gets <sup>some of</sup> ~~more~~ his tokens to form occupy all the points of coordinates  $n, z$   $n, z = \text{any integers, } z \text{ may be } < 0, n \text{ must be } > 0$   $n \text{ must be } > 2$ . for  $n=2$ , this may be too easy. This game may be trivial.

To figure out lines of play, one works backward. First one compares all moves in the last row, in pairs. One discards the worst, then compares at next level, etc. The arrows show the choices that were made.



Note: if 2 or more terminal pos. are equally good, we will have some arbitrary choices. Each position has a set

order all configurations. Then one can easily make all choices. A config. has a set of values  $a_1, a_2, \dots, a_n, \dots, a_\infty$ .  $a_i$  is the optimum value obtained after an player exhaustive search  $i$  moves into the future.  $a_\infty$  is the true value of the move - usually  $\infty$  is  $< \infty$ ; or we may have a draw. Usually as  $i \uparrow$ ,  $a_i$  will not be even approx. monotonic - except if  $a_i$  becomes very low or very high - in which case a win or loss, or the game nears its end.

T.M.

and the method that uses the "carry" line.

The above method of "carry line" elim. may not be so good. It seems that in addition problems, one does the carry's in one's head, and it may be expedient to have T.M. learn to do this.

Also, there are probably many processes in which  $a R^I b, b R^{II} c, c R^{III} d > a R^{IV} d$

this suggests it may be

put it another way, one may go from form 'a' to ~~form 'd'~~ O.K. form 'd' thru  $a b c d$ , so that  $a b c d$  is of reasonable proby.

- Then one really only wants  $a d$ .

This can be accomplished by a special operation on the trio of pairs:  $((a, b), (b, c), (c, d)) \equiv (a, d)$ . This is not as general as the set of xfuncs:

$(a, b) \rightarrow (c, d, e) \rightarrow (f, g, h)$  and it is only

$(f, h)$  that one wants. Well - the operation

$(f, g, h) \rightarrow (f, h)$  is certainly simple enuf,

and is by no means a new xfunc or an ad-hock one.

SN

Chess and checker playing machines: ?

A fairly good checker player has been constructed by AP75  
A. Samuel  
It plays out all sequences to 5 moves, then counts men and uses amt. of "centrality" for tie breaking.

On evaluating a position:

Any 2 evaluation methods can be compared. Say method A takes  $t$  seconds to use, and B takes  $kt$  sec. ( $k > 1$ ) Suppose that there are  $m$  alternative moves on the average. Then play out all positions  $\log_m k$  moves in advance and use method A. Compare win probability with B, when B is used immediately.

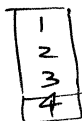
It may, indeed, be true that few eval. methods are as "good" as a very simple method such as

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01 246.32 For example: I was worried about the learning of "addition without carry line." Intuitively, th. soln. is as follows:

$$\begin{array}{r} +1101 \\ 10101 \\ \hline 11110 \\ 10100 \leftarrow \text{soln} \end{array}$$

We can factor this in to th. ntpst  $(+1101, 0101, /10100)$  and th. str



[5N] Tom has suggested, that to simplify explanation, write str. as a vector, with ~~each~~ each vector component giving th. cart. coords of th. displacement of that component relative to th. first component.

e.g.  $\begin{bmatrix} 1 & 2 \end{bmatrix} \equiv (0,0), (1,0) \quad [x, y]$

$\begin{bmatrix} 1 \\ 2 \end{bmatrix} \equiv (0,0), (0,-1)$

$\begin{bmatrix} 2 & 1 \end{bmatrix} \equiv (1,0), (0,0) \equiv (0,0), (-1,0)$

In th. last case, we illustrate th. fact that any constant 2 dim. vector can be added to all components of str. and leave th. str. invariant. Th. "str." is ~~the~~ th. part that remains invariant under such an xform.

$\begin{bmatrix} 1 & 2 \end{bmatrix} \times (a, b) \equiv \begin{bmatrix} a & b \end{bmatrix}$

$\begin{bmatrix} 2 & 1 \end{bmatrix} \times (a, b) \equiv \begin{bmatrix} b & a \end{bmatrix}$

$\begin{bmatrix} 1 \\ 2 \end{bmatrix} \times (a, b) \equiv \begin{bmatrix} a \\ b \end{bmatrix}$

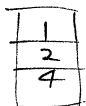
[5N] There is still some trouble with th. i.p. ( ? ) in th. intuitive approach that is being outlined above.

To get

$$\begin{array}{r} +1101 \\ 0101 \\ \hline 10100 \end{array}$$

we mult

$(+1101, 0101, /10100)$  by



Similarly, we can get a little extra trouble.

$0101 + 1101 = 10100$  with

be better to learn than

In fact this

may be better to learn than, since there is some ambig. betw 248

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choices toward hyast  $P_i$ .

The function  $x^r$  is th. "only" <sup>(probably only analytic)</sup> funct. that satisfies  
th. 1 eqn.  $f(x) \cdot f(y) = f(xy)$ . [  $r$  can be anything ]

This means that if the  $P_i$ 's are obtained by Monte Carlo methods and for all  $P_i$ 's,

$$P_i = P_{i1} \cdot P_{i2} \dots$$

suitably ~~is~~ non-linearize ~~the~~  $P_{i1}$  and  $P_{i2}$

$P_{i1}$  and  $P_{i2}$  may, in turn be obtain as random variables. Eventually, at some low level, one may

obtain some random Monte Carlo variables ~~whose~~  
~~whose~~ whose probies one can modify by

~~the~~  $p \rightarrow p^r$ , since the number of such

low level variables is small ~~much smaller~~  
than th. no. of  $P_i$ 's.

General Methodological note: In working, say, on Math T.M., keep operations much closer to intuitive processes than I have been doing. Th. definitions of str's. and utpsts, and their cart. products, ~~should be~~ are things that I feel that I understand intuitively, and so I should be able to do it more exactly.

247.01

.32  
.37

some Induction Postulates:  $\rightarrow$  B Russell "Philosophy" - 1927  
Most recent reference: Russl: "Hum Knowl. - its scope and limits" (P 526) could find it AP75

- 1) post. of quasi-permanence (in time) [ but also in space ]
- 2) post. of separate causal lines. (a thing usually isn't caused by everything)
- 3) " " special-temporal continuity (no action at a dist.)
- 4) " " common causal origin of similar str's.

grouped about a center. [ a center in time or space ] [ this may also involve post. of "possibility of explanation". ]

5) post. of analogy

It would be imp. to see if these "suff" - i.e. can one develop a set of rules of induction that one uses, from them. 247

similar to 1), but in both time and space.