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ON RULED THREE-DIMENSIONAL VARIETIES OF ORDER FIVE

Every algebraic three-dimensional variety of order not greater than three is known to be ruled. The determination of those of order four which are ruled was carried out by TOGLIATTI ⁽¹⁾. In this paper, we consider the classification of the non-composite, non-conical, three-dimensional varieties, V_3^5 , of order five, which are generated by right lines but not by planes.

If such a variety belongs to a space of more than four dimensions, its curve sections are of genus two, at most, and the variety is, accordingly, ruled. If V_3^5 is a ruled hypersurface in four dimensions it is known ⁽²⁾ that it must contain a multiple curve or a multiple surface and our classification is based on the nature of this multiple locus. We denote by p the genus of the generic plane sections of V_3^5 and we find that the variety is ruled under the following conditions:

$p=6$. There is (α) a triple curve C^5 of order five and genus one of which V_3^5 is the locus of the bisecant lines or (β) a fourfold right line or (γ) a triple conic, or a triple right line, along which two sheets of V_3^5 touch each other.

$p=5$. There is a double plane π and also (α) a fourfold right line in π or (β) a triple line in π along which two sheets of V_3^5 touch each other or (γ) a triple line skew to π or (δ) a triple conic with one point in π .

$p=4$. If the double surface is two planes with just one point in common, V_3^5 is ruled.

If the double surface is two planes π_1 and π_2 with a line l in common, V_3^5 is ruled if (α) l is a fourfold line or (β) there is, in π_1 , a triple line along which two sheets of V_3^5 touch or (γ) there is a triple line skew to π_1 but meeting π_2 or (δ) there is a triple conic which meets π_1 and π_2 , each in one point.

⁽¹⁾ TOGLIATTI: *Sulle varietà a tre dimensioni e di quart'ordine che son luoghi di almeno ∞^2 rette*. Rendiconti dei Lincei, Ser. 5, Vol. 30, 1921, Sem. 1, pp. 252-5, Sem. 2, pp. 22-5.

⁽²⁾ C. SEGRE: *Preliminari di una teoria delle varietà luoghi di spazi*. Rendiconti di Palermo, Vol. 30, 1910, pp. 87-121.

If the double surface is a quadric, V_3^5 is ruled (α) if there is on the quadric a triple conic or a triple right line along which two sheets of V_3^5 touch each other or (β) if there is a triple conic, or a triple right line, which does not lie on the quadric.

$p \leq 3$. The variety is always ruled except when the multiple surface is three planes with a line l in common. In this last case, V_3^5 is ruled if l is a fourfold line.