Jin Akiyama

A Friend and His Mathematics

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Abstract. Jin's story from three perspectives.

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1. Defining Moments — notes by MK

In the book [84] entitled "Be Brilliant as Long as You Live", the late Hiroko Oka, an authority on child psychology and mentor of Empress Michiko when she was a student at Seishin Women's University, talks about Jin Akiyama. "I have known Mr. Akiyama since he was five years old. At that time, his mother was worried about his behavior; he did not do as he was told, he did what he wanted instead. He was very different from his older brothers. She brought him to me for an IQ test. He had a charming face, with fair skin and rosy cheeks. He was a very cute boy. He surprised me because he took off his shoes, socks and shirt as soon as he entered my office.[4] During the IQ test, he paid no attention to what he wasn't interested in, but his eyes sparkled when I asked questions about things that interested him. I concluded that there was no problem with his intellect. I advised his mother to send him to a private elementary school, which values students' individual personalities, rather than the usual public school, which values rules."

His mother, following Oka's advice, gave him the opportunity of spending his childhood absorbing what he was interested in. He enjoyed observing bagworms and collecting the colorful threads they weave to knit into pencil caps. He made miniature boats and devised a rubber band mechanism to make them move. He explored the small hill behind his school to look for shards of ancient earthenware. He cared for the family's goat, which he often brought to school.

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[4] Jin hated tight clothes which confined him and he always walked around barefoot.

Fig. 1. Jin barefoot

Oka's assessment was very accurate. She saw into the nature of Jin Akiyama. When he encounters something that really fascinates him, he will give it his full concentration and engage it with his creative energy. Taking off his shirt and shoes was symptomatic of his intolerance for restrictions. This would cause many ups and downs in his later life.

Mathematics was an early fascination, as was art. He studied functional analysis under the guidance of Mitio Nagumo in graduate school at Sophia University. He wrote his first paper entitled "S-Wellposedness of Partial Differential Equations with Constant Coefficients" using Fourier transforms. His result was overtaken, however, by a stronger theorem proved using a new method from distribution theory, and ended up as a mere corollary. He realized that he would have to learn distribution theory, in addition to many other prerequisites, so he would not get to the important research problems in analysis for a very long time.

He was disappointed because he wanted to produce new mathematics as soon as possible. Then he chanced upon Oystein Ore's "The Four-Color Problem" in the library and found that the attractive results there did not require much ground work like classical mathematics. What they required was institution and ingenuity. He concentrated on reading Ore's book, then moved on to 'Graph Theory" by Frank Harary. He got hooked on graph theory. He studied it mostly by himself, but he received a lot of encouragement from Takashi Hamada, his undergraduate mentor at Tokyo University of Science.

After graduation from Sophia, Jin considered doing mathematics in a different country. He found the Japanese research structure too feudal – the student's career options were largely dependent on his professor's recommendation.

He applied to a UNESCO program which sent scientists to developing countries. UNESCO contracted to send him to the Kumasi Institute of Technology in Ghana. The contract was cancelled, however, due to a political upheaval at his destination.

For a couple of years, Jin continued to study graph theory while teaching mathematics and computer science courses at Nippon Ika University. Then he made the bold move of writing directly to Frank Harary, expressing his desire to study graph theory and asking to be accepted as Harary's student. A recommendation letter from the Singaporean graph theorist, Hoon Heng Teh clinched Harary's consent.

The Michigan years were intense. It was a time when Jin lived and breathed mathematics. The body of work in graph theory, that he produced during those years and after, are discussed in the second section of this paper, together with his contribution to the growth of graph theory in Japan.

By the 1990's, Jin became interested in discrete geometry. In 1997, he began a conference series, the Japan Conference on Discrete and Computational Geometry (JCDCG), which continues to this day and which has gained the recognition and support of researchers in that area. Jorge discusses the topics Jin has chosen for his own discrete geometry research in the third section of this paper. Suffice it to say that they are important, graphic and beautiful. He has very good intuition about problems and often gives simple, elegant proofs. I am reminded of a quotation from Felix Klein, "Thus, in a sense, mathematics has been most advanced by those who distinguished themselves by intuition rather than by rigorous proofs."

There is also Jin's important work of popularizing mathematics. The reality in Japan is that there is a decreasing number of high school students who want to study science and engineering in college. Some measures have been introduced to reverse this trend, but with limited success. Jin entered the picture by introducing mathematics to elementary, junior and senior high school students and even to their parents and the general public as something to be experienced with enjoyment and wonder. Mari-Jo has more to say about this in the fourth section of this paper. He introduced manipulative mathematical models in his TV shows, his traveling exhibit, *Mathematical Art*, and his permanent exhibit *Mathematics Wonderland*. The catch phrase is "Let's touch mathematics" and he has created mathematical models they can touch—slides in cycloid and circular arc shapes so that they can experience which slide will bring a ball down faster, a cradle pinball device that shows them how the normal distribution takes shape, and functional vehicles with non-circular wheels of constant width, among many others.

Jin has written many math books for various levels from elementary to advanced. His books entice the readers to the topic, appeal to their curiosity, and foster discovery and analytical thought. The advanced level books are in graph theory, combinatorics, and discrete geometry. A number of them have been translated to Korean and Mandarin.

As if the preceding were not enough for one lifetime, Jin is a regular columnist in several magazines and newspapers, as well as a TV commentator. Some columns are devoted to mathematics, others to education, art, literature, cinema, music, and some others to life. He has won awards for his writing. An essay he wrote in 1993, based on a collection of humorous answers submitted by students, was chosen as one of the best essays of the year by the Japan Association of Novelists. Another essay on the history of his hairstyle, written in 1995, was among that year's best essays selection of the Japan Essayist Club.

I have known Jin for almost 60 % of his life. I am thankful for the good fortune of having met him and the privilege of becoming his trusted friend. On his sixtieth year, I wish him continuing success.[1]

2. Engaging Graph Theory — notes by MK

When Jin first presented a graph theory paper on the cycle multiplicities for certain graphs at a convention of the Japan Mathematics Society in 1974, there were fewer than ten people in the audience. He looked unconventional for a Japanese mathematician at that time – long hair like a hippie and a beard like Ho Chi Minh's. He was warm and friendly and mingled with ease.

[1] The author wishes to thank Reggie and Akiko Marcelo, Haruhide Matsuda for their assistance in the preparation of Section 1.

He had worked for almost two years, 1977–1979, at the University of Michigan under the guidance of Frank Harary and obtained substantial results. During those two years, he did nothing but graph theory; Harary's motto was "Another day, another paper." What Jin accomplished was more like "Another month, another paper." Harary arranged for him to visit universities throughout the United States and encouraged him to participate in conferences all over the world to present his research results.

Fig. 2. Jin in Michigan

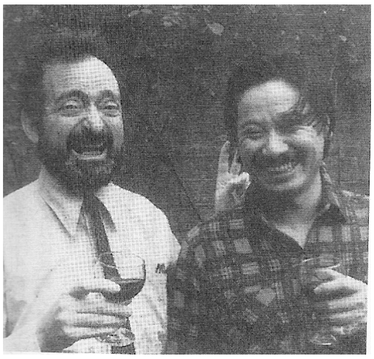


Fig. 3. Jin and Frank

Fig. 4. Jin and Claude in Paris taken by Vašek

Jin says he never worked so hard on mathematics as he did when he was in Michigan. To save time, he would cook enough curry rice for a week and save it in the freezer – this was the only thing he knew how to cook. One of his friends complained that he was beginning to smell like curry and joked that his complexion was turning yellowish.

Jin took advantage of a conference on circuits and systems held in Tokyo in 1979 to hold a one-week graph theory seminar in Nikko. He invited Vašek Chvátal, David Avis, C. L. Liu, András Recski and other participants of the Tokyo conference. Incidentally, this was the first graph theory seminar I ever attended. Jin introduced me to Vašek and I had the temerity to ask him "Do you have some results in graph theory?" This is still an embarrassing memory.

After this seminar, Jin organized many research meetings for young people who were interested in graph theory. In addition to these meetings, he also organized regular seminars held on Saturdays so that anyone in the vicinity of Tokyo could attend. The number of participants in the first year was about ten but it grew to more than twenty in the following year. There were many foreign researchers who lectured at these seminars, including some important names in graph theory like Chvátal, Avis, Harary, Claude Berge, László Lovász, Béla Bollobás, Peter Frankl, Michel Deza, Adrian Bondy, Robin Wilson, Zoltán Füredi, Guan Mei-Gu and Koh Khee-Meng. Most of these foreign graph theorists came to Japan at Jin's invitation. The talks on the most recent developments in graph theory and discussions with the speakers stimulated many young researchers to a degree they had never experienced before in Japan. The drinking sessions after the seminars, which sometimes lasted longer than the seminars, were great for bonding.

2.1. Graph Theory Conferences

Two international graph theory conferences at Hakone contributed greatly to improving the quality of graph theory in Japan and to expanding the range of research areas. This is how the first one came about: In 1983, an international graph theory conference was held in Singapore. It was one of the few conferences which merited the participation of both Frank Harary and Claude Berge. Our Singaporean hosts were very careful to honor both equally.

The only participants from Japan were Jin and I. During this conference, it was suggested that a similar conference be held a few years later in Asia. Other Asian countries had serious financial problems, so "rich" Japan was expected to host it. However, graph theorists in Japan were mostly young people who had no experience running an international conference, much less raising the money for it. After the discussion, Jin said "We will do our best." Clearly this was an activity he was interested in. He would devote his energy to it. Upon our return to Japan, he asked for support from the participants at the regular graph theory seminar. He was firmly determined to hold the conference and the young researchers responded positively to his call.

Jin solved the biggest problems of hosting the conference – getting the fractious group of graph theorists to work together, and finding the venue and the funds. At the time, he was a very popular and influential lecturer at Sundai, a network of schools that prepare students for the college entrance examinations. He was able to secure Sundai's seminar house for free and to get other special arrangements like free telephone calls and volunteers from Sundai's staff. The place was ideal for a week-long conference, as it had facilities for accommodations and meetings. It was harder to get the Organizing Committee to gel. There were strong personalities and personal differences among the members. Many meetings involving rounds of consensus building were held before agreement could be reached on the details of running the conference. Meetings were also held with Hakone authorities on the participation of the community; so the people of Hakone, although they had nothing to do with mathematics, also helped, as did many students from different universities around Tokyo, who were Jin's "fans." Thus, with many of the conference problems solved and the cooperation of the Japanese graph theorists, the First Japan Conference on Graph Theory was held in June 1986 in Hakone. It was his charisma at work. He can inspire people to do great things.

Fig. 5. Paul dishing out open problems at the First Hakone Conference 1986

Around a hundred foreign researchers attended the conference. This was a record number of foreign participants in a mathematics-related conference in Japan. In acknowledgement of the generosity of the Hakone community, Ron Graham gave lectures to junior high school students in Hakone. Paul Erdős also gave lectures to the students in Sundai. The conference proceedings were published in *Discrete Mathematics*, Vol. 72, 1988.

Four years later, in 1990, the Second Japan Conference on Graph Theory was held, also at the seminar house in Hakone. Since this was the same year and about the same time as ICM in Kyoto, many foreign researchers participated. With the experience of the first conference behind us, we were more confident and more relaxed about organizing and running this conference.

The two international conferences in Hakone, aside from expanding the research areas of the graph theory group in Japan, also gained them collaborators worldwide.

2.2. The Journal Graphs and Combinatorics

Let us return to the 1983 Singapore conference. In addition to the discussion on the succeeding conference, there was also a discussion about publishing an international journal on graph theory from Asia. It had been a long-standing dream of Hoon Heng Teh of the National University of Singapore, who was president of the Southeast Asian Mathematics Society at the time. Those present at the meeting agreed that the number of journals devoted to graph theory were not sufficient for the increasing number of graph theory papers being produced. They all strongly felt the need for a new journal. Several problems surfaced: who would the editor be, where would the editorial office be located and who would look for and negotiate with publishing firms. Jin accepted the challenge and responded, “I will do it.” He had the confidence to accept the responsibility because by this time he already had several years experience as member of the editorial board of the *Journal of Graph Theory.*

After returning to Japan, Jin approached several publishing companies. He did not get a favorable response from most of them. Fortunately, after lengthy negotiations, Springer-Verlag expressed interest. Perhaps it was because they did not have a journal in this area and they were anticipating the research results that were to come from an emerging Asia and in particular, a modernizing China. Jin’s Chinese connections were Wang Yuan and Wang Jian Fang of Academia Sinica in Beijing. They agreed to serve on the Editorial Board of the new journal and introduced Jin to the leading Chinese graph theorists and combinatorists. The new Tokyo office of Springer-Verlag pushed for the project and so the journal, *Graphs and Combinatorics*, came to be in 1985, one year earlier than originally planned. Hoon Heng Teh was the editor-in-chief and Jin the managing editor. The editorial office of the journal was in Tokyo.

Fig. 6. Launching of Graphs and Combinatorics

A few lines from one of messages that appeared in the first issue of the journal are the following:

> *The commitment of Springer-Verlag to publish the journal has turned a dream into reality. We sincerely look to our friends all over the world to give us their support and we will try our very best to make this journal worthy of its existence. -H.H. Teh*

Twenty years have passed since the first issue, and the journal is now recognized as an ISI publication.

2.3. Selected Results from Jin's Papers

Jin's research in graph theory covers many areas. We single out three which he concentrated on: factors and factorization, path invariants, a graph and its complements with common properties.

Factors and Factorizations

A spanning subgraph possessing some given property is called a *factor*. There are two kinds of factors, degree factors and component factors. Some examples are the following. Let a, b, k be integers such that $0 \leq a \leq b$ and $1 \leq k$. Then a spanning subgraph F of a graph G is called an $[a, b]$-*factor* if $a \leq \deg_F(x) \leq b$ for all vertices x of G, where $\deg_F(x)$ denotes the degree of x in F, and a $[k, k]$-factor is usually called a *k-regular factor*. An $[a, b]$-graph and a k-regular graph are defined similarly. More generally, for a set $\mathcal{I}$ of integers, a spanning subgraph H of G is called an *$\mathcal{I}$-factor* if $\deg_H(x) \in \mathcal{I}$ for all $x \in V(G)$.

If the edge set $E(G)$ of a graph G is decomposed into disjoint $\mathcal{I}$-factors $E(G) = F_1 \cup F_2 \cup \cdots \cup F_r$, then we say G is *$\mathcal{I}$-factorable* and this decomposition is called a *$\mathcal{I}$-factorization* of G.

Let $\mathcal{S}$ be a set of graphs. Then a spanning subgraph F of a graph is called an *$\mathcal{S}$-factor* if each component of F is in $\mathcal{S}$. A $\{P_n \mid n \geq 2\}$-factor is called a *path-factor*. The number of isolated vertices of a graph G is denoted by $iso(G)$.

It is known that *a graph G has a $\{K_2, C_n \mid n \geq 3\}$-factor if and only if $iso(G-S) \leq |S|$ for all $S \subset V(G)$* ([90],[91]). Jin, Avis and Era [16] proved that *a graph G has a path-factor if and only if $iso(G - S) \leq 2|S|$ for all $S \subset V(G)$.*

Later, the result was generalized as follows: *Let $n \geq 2$ be an integer. Then a graph G has a $\{K_{1,1}, K_{1,2}, \ldots, K_{1,n}\}$-factor if and only if $iso(G - S) \leq n|S|$ for all $S \subset V(G)$* [65]. The same proof technique as in [16] was used.

It is obvious that a path-factor is a special $[1, 2]$-factor. The authors considered a $\{1, 2\}$-factorization and mentioned that every regular graph can be decomposed into $[1, 2]$-factors. From this result, Jin conjectured that *for every integer $k \geq 1$, there exists an integer $\Phi(k)$ such that for any integer $r \geq \Phi(k)$, every r-regular graph is $\{k, k + 1\}$-factorable.* This conjecture was proved by Era [74] in 1985 and a sharp bound for $\Phi(k)$ was obtained by Egawa [72] in the following year.

It is possible that not only regular graphs but also some almost regular graphs are $\{k, k + 1\}$-factorable. In fact, in 1985, Jin and I [34] proved that *for an even integer $k \geq 2$, and $s \geq 0$ and $t \geq 1$ integers, every $[(6k + 2)t + ks, (6k + 4)t + ks]$-graph is*

$\{k, k+1\}$-factorable. In particular, every $[6k+2, 6k+4]$-graph can be decomposed into six $[k, k+1]$-factors.

It is very difficult to characterize graphs having P_3-factors because the question of determining whether a given graph has a P_3-factor is an NP-complete problem. Jin and I [32] conjectured that *every 3-connected cubic graph of order $3n$ has a P_3-factor*. Note that there exist 2-connected cubic graphs of order $3n$ that have no P_3-factor (Figure 7).

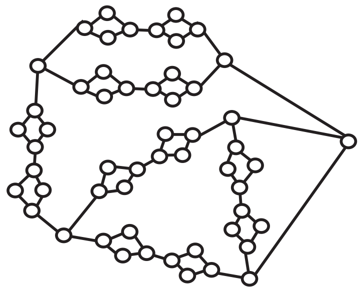


Fig. 7. A 2-connected cubic graph of order 54 having no P_3-factor.

On the other hand, Jin and I proved [32] that *every 3-connected cubic graph of order $4n$ has a P_4-factor containing any two given edges.*

If a graph G has no R-factor for some given graph R, it is natural to ask how many R's can be packed into G, that is, find the largest number of vertex-disjoint subgraphs $H_1, H_2, \ldots, H_k$ of G such that every H_i is isomorphic to R. This problem is called the *R-packing problem.* The graph $Int(R; G)$ is defined as follows: Each vertex of $Int(R; G)$ represents the set of vertices of a subgraph of G that is isomorphic to R, and two vertices of $Int(R; G)$ are adjacent if they represent vertex sets that intersect. If a maximum independent set of $Int(R; G)$ can be found in polynomial time, then the above problem is solved. If $Int(R; G)$ is a *perfect* graph, i.e., the chromatic number of each induced subgraph F is equal to the order of largest clique of F, then its maximum independent set can be obtained in polynomial time ([78], [93]), and so can the solution of the problem. Let H_1 and H_2 be vertex-disjoint graphs and for each $i = 1, 2$, let C_i be a clique in H_i. Assume that $|C_1| = |C_2|$ and let $f : V(C_1) \to V(C_2)$ be a bijection. A graph can be obtained from the union of H_1 and H_2 by identifying each $v \in V(C_1)$ with $f(v) \in V(C_2)$. This operation is called *clique identification.* Jin and Vašek [18] considered the P_3-packing problem, and proved that *for every graph G, the following three conditions are equivalent. (i) $Int(P_3; G)$ is perfect. (ii) G contains none of the six graphs in the Figure 8 as an induced subgraph, and no cycle of order at least seven. (iii) $Int(P_3; G)$ can be obtained from complements of bipartite graphs by repeated clique identifications.*

The spring edition of the *Journal of Graph Theory* (1985) is a special volume on graph factorization. It starts with a survey of graph factorization by Jin and myself [33]. We also wrote a book entitled "Factors and Factorizations of Graphs" [36].

In the 1980's, Jin gave lectures at a number of universities in China at the invitation of Wang Jian Fang of Academia Sinica and Guan Meigu of Shandong Normal University, thus making factors and factorizations of graphs a very popular topic among Chinese graph theorists.

Jin especially remembers his first visit to Chufu, the birthplace of Confucius. At the train station, he was looking for the person who was supposed to pick him up. He was

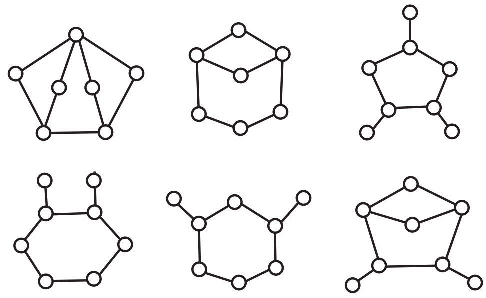


Fig. 8. The six forbidden graphs.

expecting someone with a car. He couldn't find any car but there was a person who was eyeing him. He carefully approached the person. The only way they could communicate was in written kanji. Finally, Jin understood that his mode of transportation would be a horse-drawn carriage. He enjoyed the feeling that he slipped into the Confucius era.

Path Invariants

The *arboricity* $arb(G)$ of a graph G is the minimum number k for which $E(G)$ can be decomposed into k edge disjoint forests. A *linear forest* is a forest each of whose components is a path, and a *star forest* is a forest each of whose components is a star, where path and star must be of order at least two. The *linear arboricity* of a graph G, which was introduced by Jin, Exoo and Harary [23], is the minimum number k for which $E(G)$ can be decomposed into k edge disjoint linear forests. What motivated Jin to study linear arboricity are its applications, which include techniques for search and retrieval in electronic databases. The *star arboricity* of a graph, introduced by Jin and myself [32], can be defined analogously.

The arboricity of G can be calculated by using the formula for arboricity obtained by Nash-Williams [83]. For example, we can show that the arboricity of an r-regular graph is $\lceil (r+1)/2 \rceil$. On the other hand, there are no formulas for linear arboricity and star arboricity.

It is easy to show that every cubic graph can be decomposed into two $[1,2]$-factors. However, in this decomposition, some components of a $[1,2]$-factor might be a cycle. Actually, this can be avoided. Jin, Exoo and Harary proved that *the linear arboricity of every cubic graph is two* [22] and also that *the linear arboricity of every 4-regular graph is three* [23].

A short and elegant proof of this theorem was later obtained by Jin and Vašek [17] while drinking at Rekan, a very small pub in Kabukicho. The whiskey bottle with their names on it is still there more than twenty-five years after.

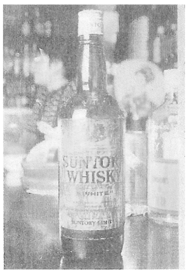


Fig. 9. Whiskey bottle in Rekan

Since the linear arboricity of a graph is at least its arboricity, the linear arboricity of an r-regular graph is at least $\lceil (r+1)/2 \rceil$. From this observation, Jin proposed an interesting conjecture: *The linear arboricity of every r-regular graph is* $\lceil (r+1)/2 \rceil$ [20,22].

The above conjecture was very attractive for many graph theorists. Tomasta [88] proved that the linear arboricity of a 6-regular graph is four. Then Enomoto and Péroche [73] proved that *the linear arboricity of every 5-regular graph is three.*

In the same paper, the authors showed that the linear arboricity of every 6-regular graph and 10-regular graph are four and six, respectively. Since the conjecture was proposed, more than 30 papers on linear arboricity have been published. By using Lovász's local lemma, Alon [64] proved that *for any real number $\epsilon > 0$, the linear arboricity of every graph with sufficiently large maximum degree $\Delta = \Delta(\epsilon)$ is at most $(\frac{1}{2} + \epsilon)\Delta$. Moreover, the linear arboricity conjecture is true for every graph G with an even [odd] maximum degree Δ and with girth at least* 50Δ $[100\Delta]$. Jian-Liang Wu [94] determined that *the linear arboricity of a planar graph with maximum degree Δ is at most* $\lceil (\Delta+1)/2 \rceil$.

The following explains the difficulty of the Linear Arboricity Conjecture. Suppose that a $(2s+1)$-regular graph G, which is an odd regular graph, has linear arboricity $\lceil (2s+1+1)/2 \rceil = s+1$. Then $E(G)$ can be decomposed into $s+1$ linear forests $F_1 \cup F_2 \cup \cdots \cup F_{s+1}$. Then for every vertex v of G,

$$2s+1 = \deg_G(v) = \deg_{F_1}(v) + \deg_{F_2}(v) + \cdots + \deg_{F_{s+1}}(v),$$

which implies that there exists an integer k such that $\deg_{F_i}(v) = 2$ for all $i \in \{1, 2, \ldots, s+1\} - \{k\}$ and $\deg_{F_k}(v) = 1$. That is, every F_i must be a path-factor of G and v an endvertex of exactly one of path-factors, say F_k. Therefore, if the conjecture is true for an odd regular graph G, then G has a path-factorization with the property that for every vertex v, v is an endvertex of exactly one path-factor.

For a vertex subset X of a graph G, the subgraph of G induced by X is denoted by $\langle X \rangle_G$. The *path chromatic number* $\chi(P_\infty; G)$ of a graph G, sometimes called the *vertex linear arboricity,* is the minimum number m such that $V(G)$ can be partitioned into m disjoint subsets $X_1 \cup X_2 \cup \cdots \cup X_m$, where every $\langle X_i \rangle_G$ is a linear forest. The *k-path chromatic number* $\chi(P_k; G)$ is defined to be the minimum number m for which $V(G)$ can be partitioned into m disjoint subsets $X_1 \cup X_2 \cup \cdots \cup X_m$ such that any component of every $\langle X_i \rangle_G$ is a path of order at most k. The path chromatic number and the k-path chromatic number were introduced in the paper [19] by Jin, Era, Gervacio and Watanabe. They proved *(i)The 2-path chromatic number of an r-regular graph is at most* $\lfloor (r+1)/2 \rfloor$. *(ii) The path chromatic number of every outerplanar graph G is at most 2. In particular, every outerplanar graph has a vertex subset of $\lceil n/2 \rceil$ vertices that induces a linear forest.*

(iii) For every integer $k \geq 2$, there exists a planar graph G with $\chi(P_k; G) = 4$. Matsumoto [82] proved a related result: *The path chromatic number of a graph with maximum degree Δ is at most $\lfloor 1 + \Delta/2 \rfloor$.*

The following conjecture was first posed by M.O. Albertson and D. Berman [60] (1979): *Every planar graph of order $n \geq 4$ has a set of $\lceil n/2 \rceil$ vertices that induces a forest.* It is true for outerplanar graphs by the previously mentioned theorem of Jin et al. Later it was independently conjectured by Jin and Watanabe [59] (1987). It is shown in [59] that there exist planar graphs of order n in which a maximal induced forest has order $\lceil n/2 \rceil$, and thus the above conjecture is sharp, if it is true (Figure 11(a)).

One of the models in Jin's *Mathematical Art* exhibit invites the public to verify this conjecture by a circle avoidance game played on an electrical network.

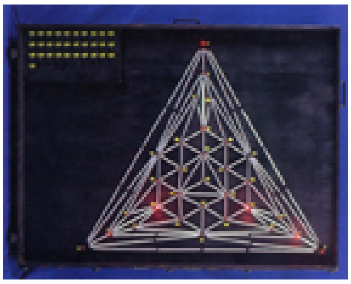


Fig. 10. Circle avoidance game

The following conjecture was first made by Jin and Watanabe [59](1987): *Every bipartite planar graph of order $n \geq 4$ has an induced forest of order at least $\lceil (5n)/8 \rceil$.* They gave a series of graphs showing the sharpness of the bound if the conjecture is true (Figure 11(b)). The same conjecture was later made independently by Albertson and Haas [61](1998).

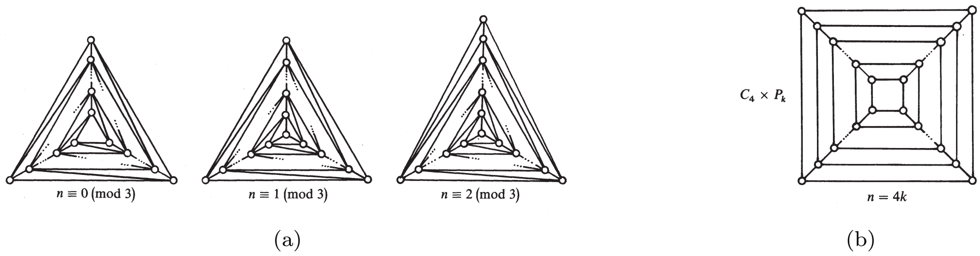


Fig. 11. The graphs which show the sharpness of the conjectures

For a triangle-free planar graph, Salavatipour [87] recently obtained a lower bound for the size of induced forests: *Every triangle-free planar graph on n vertices has an induced forest of size at least* $(17n + 24)/32$.

A Graph and its Complement with Common Properties

Ramsey theory discusses graphs in which either the graph itself, G, or its complement, $\overline{G}$, has a specified property. As a variation on this idea, Jin considered graphs, G, and their complements, $\overline{G}$, and obtained criteria under which both G and $\overline{G}$ have a common specified property. Some of these results follow. Jin and Harary [27] proved that *a graph G of order n satisfies the condition $\kappa(G) = \kappa(\overline{G}) = 1$ if and only if G is a graph with either (i) $\kappa(G) = 1$ and $\Delta(G) = n - 2$ or (ii) $\kappa(G) = 1$, $\Delta(G) \leq n - 3$ and G has a cut-vertex v with endline e and endvertex u such that $G - u$ contains a spanning complete bipartite subgraph.*

The *girth* of a graph is the length of a shortest cycle in it. It is not difficult to see that if both G and $\overline{G}$ have girth at least four, then $G = \overline{G} = C_5$. From this, Jin and Harary [28] proved that *both a graph G and its complement $\overline{G}$ have girth three if and only if G contains one of seven graph of order 5 in Figure 12 as an induced subgraph.*

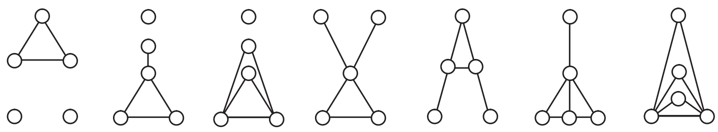


Fig. 12. The seven induced subgraphs.

Jin, Ando, and Harary [15] characterized the graphs G for which G and its complement $\overline{G}$ are interval graphs:

Both G and $\overline{G}$ are interval graphs if and only if G contains none of the seven graphs in Figure 13 as an induced subgraph.

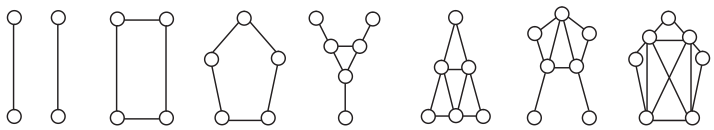


Fig. 13. The seven forbidden subgraphs.

Jin, Exoo and Harary [21] proved that *(i) If G has two endvertices, then $\overline{G}$ has at most two endvertices. (ii) A graph G of order $p \geq 4$ and $\overline{G}$ have two endvertices if and only if G is of the form $F + K_2 \circ K_1$, where F is a graph of order $p - 4$. (iii) No self-complementary graph has exactly one endvertex. (iv) If G is a self-complementary graph with no endvertices, then G is a block.* Using this result, Jin and Harary [29] also determine the number of self-complementary blocks: *For any positive integer $p \geq 5$, the number of self-complementary blocks of order p is $s_p - s_{p-4}$, where s_p is the number of all self-complementary graphs of order p.* Harary referred to this theorem as "the oyster theorem" because it was proved while he and Jin were attending an AMS meeting in Biloxi, Mississippi, where they feasted on a lot of oysters.

A graph G is called *contraction critically k-connected* if it is k-connected but none of the graphs obtained by contracting one of its edges is k-connected. Jin, Ando and Egawa [14] proved that *for $k \geq 4$, if both G and $\overline{G}$ are contraction critically k-connected, then their order is less than $k^{5/3} + 4k^{3/2}$.* In addition, they show that the exponent 5/3 in the above theorem is sharp for $k \geq 2 \times 10^6$.

The distance between two vertices x and y of a graph is denoted by $d(x, y)$. The *eccentricity* $ecc(v)$ of a vertex v is defined to be $\max_{x \in V(G)} d(v, x)$. For a graph G, G_{ecc} is the graph with vertex set $V(G)$ such that two vertices x and y are adjacent if $d(x, y) =$

$ecc(x) = ecc(y)$ in G. Jin, Ando and Avis [13] proved that $G_{ecc} = \overline{G}$ *if and only if* $2 \leq ecc_G(x) \leq 3$ *for all vertices* $x \in V(G)$ *and no two vertices* x *and* y *with* $ecc_G(x) = ecc_G(y) = 3$ *have a common neighbour.*

Jin's first graph theory paper was published in 1974. At that time he was working in isolation, since graph theory, as an area of mathematical research, had not yet established a foothold in Japan. His return to Japan from Michigan marked the growth of graph theory in Japan, so that today, graph theory is taught in every major Japanese university. The Hakone conferences and the journal *Graphs and Combinatorics* established the presence of Japanese graph theorists in the consciousness of the international graph theory community.[1]

3. An Original Slant on Discrete Geometry — notes by JU

Jin Akiyama is 60! This gives us a wonderful excuse to celebrate and reflect on the life and times of one of our most cherished and admirable colleagues.

I met Jin in 1986 during the First Japan Conference on Graph Theory (FJCGT) held in Hakone. At that time Jin was mainly interested in graph theory, but was beginning to work on problems of a more geometric nature (incidentally, it was through Vašek Chvátal that I met him.) My talk at FJCGT was on a result on points and circles I had recently obtained with Victor Neumann-Lara. Since Jin was the main organizer, throughout the conference he was very busy and was not able to attend my presentation. During a conversation Vašek had with Jin, he told Jin about my talk, and afterwards in a very polite way, Jin talked to me and apologized for not attending my lecture. It was then that he asked me to work on the following problem, which as far as I know was one of the first problems on geometric graphs on bicolored point sets:

Let P_{2n} be the set of $2n$ vertices of a convex polygon such that n of them are colored red and n are colored blue. An alternating path $\mathcal{P}$ of P_{2n} is a simple polygonal path whose vertices are the elements of P_{2n} such that every second element of $\mathcal{P}$ is blue and the remaining points are red. It is not hard to see that such a path does not always exist. His question was: *Given a bicolored point set* P_{2n}*, determine if it has an alternating path.*

Before the conference was over we were able to obtain an $O(n^2)$ time algorithm to solve this problem. This gave rise to our first joint paper [58] and was the beginning of a long and fruitful collaboration that has resulted in many papers in discrete geometry and combinatorics, and more important to me, a long and enduring friendship. During numerous visits to Japan and Tokai University, I have had the privilege of working with Jin and many of his collaborators and students, including Kiyoshi Hosono, Mikio Kano, Chie Nara, Gisaku Nakamura, Mari-Jo Ruiz, Toshinori Sakai, Masatsugu Urabe, among others.

Jin first became interested in combinatorial geometry around 1985. His initial results in the area were presented in two papers written with M. Kano and M-J. Ruiz. In the first, they treat the construction of graphs formed from squares and regular hexagons, while in the second they show that any induced subgraph of the regular triangular mesh on the plane, whose every vertex belongs to a triangular face, contains a perfect matching.

[1] The author wishes to thank Haruhide Matsuda for checking and compiling all the references of Section 2.

This was followed by a paper with N. Alon in which they generalized a well-known result that any set of $2n$ points, n are colored red and n are colored blue, in general position on the plane contains a plane perfect matching. This was followed by a series of papers that studied a variety of problems in discrete and computational geometry, such as two papers published with the present author; the first on balanced colorings of lattice points and the second the result mentioned above on alternating paths on bicolored point sets.

In 1993 Jin wrote the book (in Japanese) *Introduction to Discrete Mathematics*, co-authored with Ronald Graham. In this book, he covers among others, the following topics which are fundamental to discrete and computational geometry: convex hulls, alternating paths, balanced colorings of point sets, separability of convex sets, art gallery problems, Euclidean Steiner minimal trees and packing problems of geometric objects.

We believe that one of the reasons why Jin embraced discrete and computational geometry so wholeheartedly was that many problems in this area are not difficult to explain, and their solutions, while not easy to discover, are often very elegant and relatively easy to explain to the non-initiated in mathematics. This makes these problems ideally suited for use in one of important aspects of Jin's work, his *Mathematical Art*, and in radio and television programs.

Jin encouraged some of us to work on problems such as dissections of geometric figures; that is, given a geometric figure (e.g. a rectangle or a box), how can we cut it and reassemble the pieces to form a second geometric object? Others include origami-type problems on folding paper, problems such as how to cut a cake so that each child at a birthday party gets the same amount of both cake and icing, and how to wrap a box using a rectangular sheet of paper with the smallest area possible. At the time of this writing, two papers are scheduled to appear in the *American Mathematical Monthly* which are typical of Jin's creativity: universal measuring boxes with triangular bases, and tile-makers and semi tile-makers. The first of these two papers arises from a very practical and important problem in Japan: how to design a measuring box without graduations that is capable of measuring a variety of integer volumes of *sake*. The second paper, "Tile-Makers and Semi-Tile-Makers," introduces an ingenious way to generate tilings of the plane using the surface of a tetrahedron with four congruent triangular faces.

In 1997, Jin organized the first discrete and computational geometry conference in Japan. It was followed in December of the following year by an international conference, the Japan Conference on Discrete and Computational Geometry (JCDCG). Since then JCDCG has been held annually, mostly in Tokyo, at the Yoyogi campus of Tokai University. By popular request from Asian colleagues, movable variants of this conference series have complemented the stationary model; the Philippines–Japan International Conference on Graph Theory and Discrete Geometry held in Manila in 2001, the Indonesia–Japan International Conference on Discrete Geometry and Combinatorics held in Bandung in 2003, and the China–Japan International Conference on Discrete Geometry, Combinatorics and Graph Theory held in Tianjin/Xian in 2005. JCDCG has become one of the most prominent Asian conference in its field. Regular participants in JCDCG include, among others, Kiyoshi Ando, Takao Asano, Tetsuo Asano, David Avis, Imre Bárány, Sergey Bereg, Vašek Chvátal, Erik Demaine, Greg Frederickson, Naoki Katoh, Ferran Hurtado, John Iacono, Hiroshi Imai, Hiro Ito, Haruhide Matsuda, Mikio Kano, Stefan Langerman, Guizhen Liu, Alberto Márquez, Zhiming Ma, Jiří Matoušek, Hiroshi Maehara, David Rappaport, Joseph O'Rourke, János Pach, Narong Punnim, Kokichi Sugihara, Endre Szemerédi, William Steiger, Xuehou Tan, Takeshi Tokuyama, Godfried Toussaint, Fuji Zhang and Chuanming Zong. Other Japanese researchers working in discrete geometry include Hiroshi Fukuda, Koichi Hirata, Kiyoshi Hosono, Hiro Ito, Atsushi Kaneko, Midori Kobayashi, Takako Kodate, Yoichi Maeda, Nobuaki Mutoh, Gisaku Nakamura, Chie Nara, Toshinori Sakai, Masatsugu Urabe and Mamoru Watanabe.

Fig. 14. Participants at JCDCG '98

Discrete and computational geometry has become a popular research area in Japan. Since this conference is held mostly at the Yoyogi campus of Tokai University located in Shibuya-ku, Tokyo, the names Shibuya and Shibuya Tobu Hotel have become familiar to discrete and computational geometers around the world.

This year's Computational Geometry and Graph Theory conference, KyotoCGGT 2007, is special, for we are celebrating the 60th birthdays of both Jin Akiyama and Vašek Chvátal. We wish them both abundant good health and a happy and productive time in the years to come. Thanks to both of them for the mathematics, kindness and friendship they have shared with us; our lives have been greatly enriched by them.

In the remainder of this section, we review some of the results which Jin established himself or in collaboration with other researchers.

3.1. Point Sets

(a) Tilings and graphs

Jin published his first paper in discrete geometry [37] with M. Kano and M-J. Ruiz. They gave sufficient conditions and one necessary condition for a finite plane figure consisting of squares and hexagons to be tiled with tiles of specific shapes.

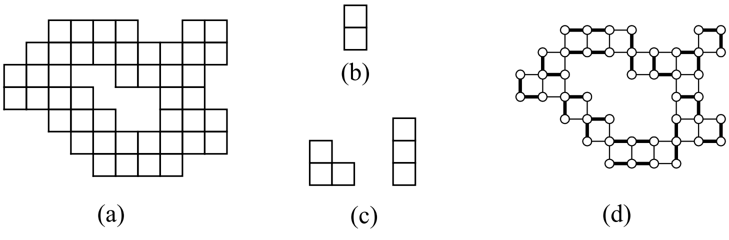


Fig. 15. (a) Tough defective chessboard. (b) Domino. (c) Triominoes. (d) Graph associated with the defective chessboard shown in (a) and its P_3-factor.

A figure obtained from an $m \times n$ chessboard by removing a certain number of unit squares is called a *defective chessboard* (Figure 15(a)). The *order* of a defective chessboard is the number of unit squares in it. A defective chessboard B is said to be *tough* if every domino (Figure 15(b)) in B is contained in a 2×2 square of B. They associated defective chessboards with a graph (Figure 15(d)). A tough chessboard is associated with a graph in which every edge is contained in some C_4. A triomino is the union of three unit squares as shown in Figures 15(c). In [37] they proved the following result: *Every connected tough defective chessboard of order* $3p$ *(resp. even order) can be covered with* p *triominoes (resp. dominoes).* They also discussed similar problems for finite plane figures consisting of regular hexagons [37,35].

(b) Disjoint heterochromatic simplices

A well known result which first appeared as a question in the 1979 Putnam Exam [81] was that of proving the following: *Let* A *be a set of* $2n$ *points in general position in* $\mathbb{R}^2$ *such that* n *of the points are colored red and* n *are colored blue. Then there are* n *pairwise disjoint straight line segments matching the red points to the blue points.*

Jin and N. Alon [12] generalized this result to higher dimensions, proving the following result by using the Ham Sandwich theorem (see Figure 16):

Theorem 1. *Let* A *be a set of* $d \cdot n$ *points in general position in* $\mathbb{R}^d$*, and let* $A = A_1 \cup A_2 \cup \cdots \cup A_d$ *be a partition of* A *into* d *pairwise disjoint sets, each consisting of* n *points. Then there are* n *pairwise disjoint* $(d-1)$*-dimensional simplices, each containing precisely one vertex from each* A_i, $1 \leq i \leq d$.

So, if we regard each A_i as the set of points with color i, then this theorem guarantees the existence of n pairwise disjoint $(d-1)$-dimensional *heterochromatic* simplices, where a heterochromatic simplex is a simplex all of whose vertices have different colors.

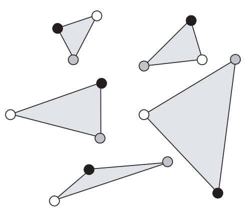


Fig. 16. Five pairwise disjoint heterochromatic simplices in $\mathbb{R}^3$.

They also obtained the following result on *geometric d-hypergraphs* (ordered pairs (V, E) of vertices and edges whose vertices are sets of points in general position in $\mathbb{R}^d$ and whose edges are closed $(d-1)$-dimensional simplices):

Theorem 2. *Every geometric d-hypergraph with* n *vertices and at least* $n^{d-(1/l^{d-1})}$ *edges contains* l *pairwise nonintersecting edges.*

They conjecture that for every integer l, $d \geq 2$, there exists a constant $c = c(l, d)$ such that every geometric d-hypergraph with n vertices and at least $c \cdot n^{d-1}$ edges contains l pairwise nonintersecting edges (see also [69]). It has been shown by J. Pach and J. Törőcsik [85] that this is true for $d = 2$. The best lower bound known to date was obtained by G. Tóth [89]. He proved that any geometric graph with n vertices and more than $2^9(l-1)^2 n$ edges contains l pairwise nonintersecting edges.

(c) Balanced colorings for lattice points

Let P_n be a subset of n elements of the lattice points L of $\mathbb{R}^2$. For every $i \in \mathbb{N}$, let the *row* R_i be the set $\{(x,y) \in P_n : y = i\}$ and the *column* C_i be $\{(x,y) \in P_n : x = i\}$. An m-coloring of P_n is a partitioning of P_n into m subsets $S_1, \ldots, S_m$ called the chromatic classes of the coloring. An m-coloring of P_n is called *almost balanced* if for every row and every column the number of points colored i differs from the number of points colored j by at most one, $i \neq j$. In Figure 17 we show an almost balanced 3-coloring of a point set P_n. The case $m = 2$ was also a problem posed in the 27th International Mathematical Olympiad (1986), which motivated the study of this result, proved by Jin and myself in [57]:

Theorem 3. *Let $P_n \subset L$. Then P_n can always be m-colored with an almost balanced m-coloring, $2 \leq m \leq n$.*

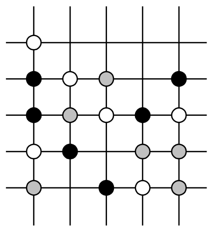


Fig. 17. Almost balanced 3-coloring.

Answering a conjecture posed in [57], Biedl *et al.* [67] proved in 2002 that if P_n is a subset of the set of lattice points in $\mathbb{R}^d$, then P_n can always be m-colored in such a way that on any line parallel to either of the coordinate axes, the difference between the number of points of any two colours is at most $4d - 3$ (this is also a generalization of a similar result by Beck and Fiala [66]).

3.2. Dissections and Related Topics

Most of Jin's papers on polygons and polyhedra result from joint work with his colleague Gisaku Nakamura. At age 79, Nakamura remains very active in research. Jin considers himself fortunate to have this partnership with Nakamura. Incidentally, twenty-five years ago, Jin introduced Nakamura to Vašek and the introduction led to a collaboration between Nakamura and certain Canadian graph theorists.

(a) Dudeney dissections of polygons and polyhedra

A geometric dissection cuts a geometric figure into a finite number of pieces which can be rearranged to form another figure. Many beautiful and important results on dissections have been discovered in the last two millennia [75,76,80]. Among many results on planar dissection, the following result obtained independently by Wallace [92], Bolyai [68] and Gerwien [77] is important: *An arbitrary polygon can be transformed to any other polygon* *of the same area by partitioning it into a finite number of pieces and reassembling the pieces in some suitable way, without turning the pieces over.*

Henry E. Dudeney [71] introduced a partition of an equilateral triangle α into parts that can be reassembled, without turning the pieces over, to form a square β of the same area, moreover the perimeter of the square we obtain comes from the cuts performed along segments in the interior of the triangle (see Figure 18). An examination of Dudeney's method motivated Jin and Nakamura to introduce the notion of Dudeney dissection of a polygon, which they then extended to Dudeney dissections of polyhedra, see [40–43, 45, 31]. In Jin's Dudeney dissections, polygons are converted either into congruent polygons but turned inside out, which he calls *chameleons*, or into other polygons, turned inside out, which he calls *octopuses*. Figure 18 is an example of an octopus, while Figure 19 is an example of a chameleon. In fact, he and Nakamura determined all convex chameleons and octopuses, under the condition that all hinge points are interior to the sides of the polygons. In particular, every triangle or quadrilateral is a chameleon.

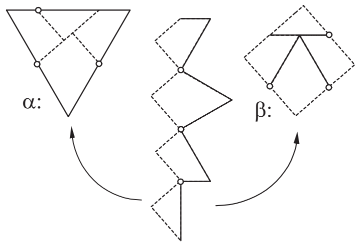


Fig. 18. Dudeney's puzzle.

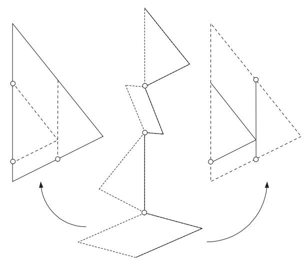


Fig. 19. Chameleon.

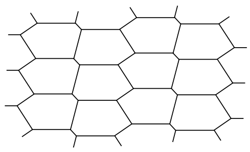


Fig. 20. P_2-tiling by hexagons.

A polygon is called a P_2*-tiler* if its congruent copies and its 180° rotations tile the plane by parallel transformation (see Figure 20). The main theorem used to determine all convex polygons which have Dudeney dissections is the following:

Theorem 4. *Every polygon that has a Dudeney dissection is a* P_2*-tiler.*

According to results from tiling theory [70, 79], the only convex polygons that tile the plane are triangles, quadrilaterals, special kinds of pentagons and three different types of hexagons. Thus, in order to determine all polygons which have Dudeney dissections, it is sufficient to consider only four cases: triangles, quadrilaterals, pentagons and hexagons.

To actually find a Dudeney dissection of α to β, Theorem 4 suggests that a plane P_2-tiling using α be appropriately superimposed on a P_2-tiling of β (see Figure 21).

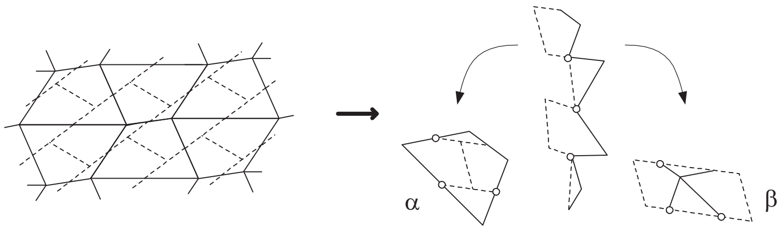


Fig. 21. Dudeney transform of α to β obtained by superimposition of P_2-tilings by α and β.

(b) Sequentially n-divisible dissections of polygons

In [38,48,49], a different problem on dissections was studied. For a given integer $n \geq 2$, dissect a square into a finite number of polygons in such a way that for every k, $2 \leq k \leq n$, the polygons can be reassembled to form k squares of different sizes. Such a dissection is called a *sequentially n-divisible dissection* of a square.

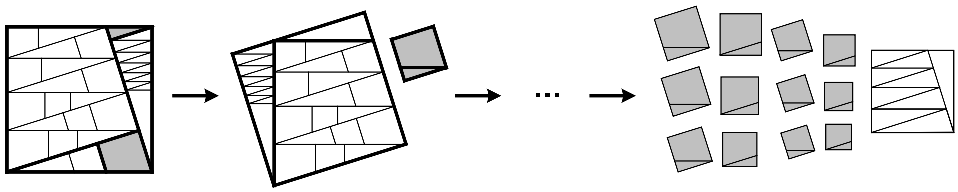


Fig. 22. Sequentially 13-divisible dissection of a square consisting of 33 pieces.

Jin and Nakamura [38] found a sequentially n-divisible dissection of a square with $f(n)$ pieces such that $\frac{f(n)}{n} \to 2$ as $n \to \infty$. Their dissection is shown in Figure 22. It is shown in [48,49] that the Akiyama-Nakamura dissection is asymptotically optimal when restricted to dissections called *purely recursive dissections.*

Jin *et al.* [55] also considered sequentially n-divisible dissections of other polygons and related problems. For example, it is shown in [55] that a convex k-gon P, $k \leq 5$, can be dissected into $k(n-1)+1$ pieces in such a way that the pieces can be reassembled to form m different sized polygons similar to P, $m = 2, 3, \ldots, n$ (Figure 23).

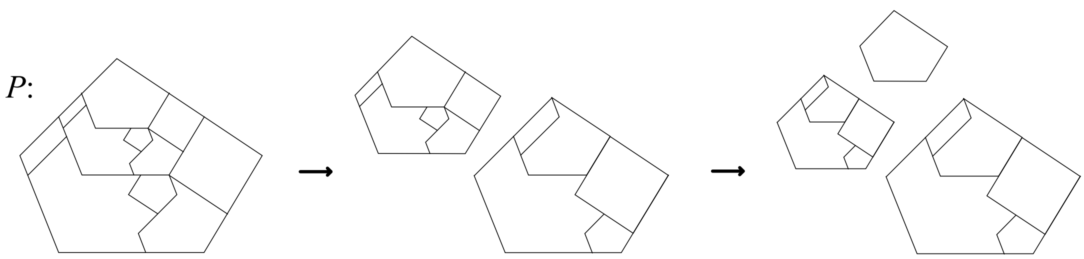


Fig. 23. Sequentially 3-divisible dissection of a pentagon consisting of 11 pieces.

3.3. Foldings of a Regular n-Gon to Convex Polyhedra

A *development* of a convex polyhedron is a *plane* figure obtained by cutting its surface. Cuts are not necessarily confined to the edges of the polyhedron; they are allowed to pass through its faces. A *folding* of a regular n-gon into a convex polyhedron is accomplished by gluing portions of the perimeter of the n-gon together to form the polyhedron. The regular n-gon can be regarded as a development of the polyhedron (Figure 24). Pioneering work by Alexandrov [62] and some very nice results by Alexander, Dyson and O'Rourke [63] (in which they determined all possible foldings of a square to convex polyhedra) motivated Jin to enter this area of research.

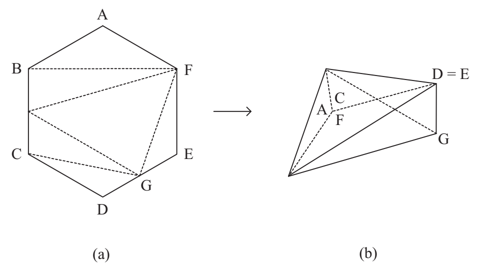


Fig. 24. Folding of a regular hexagon into a tetrahedron.

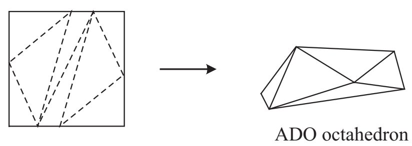


Fig. 25. ADO octahedron folded from a square.

Jin and Nakamura extend the results in [63] by determining all possible convex polyhedra foldable from regular n-gons, $n = 3$ [47], $n = 5$ [44], $n \geq 6$ [46]. Furthermore, they conjecture that the maximum volume convex polyhedron obtained by folding a regular polygon of a unit area is the octahedron shown in Figure 25.

This is the same octahedron having the maximum volume among all convex polyhedra foldable from a square with unit area, which was found by Alexander, Dyson and O'Rourke. This octahedron is called "ADO octahedron" after the authors (Figure 25).

3.4. Universal Measuring Boxes

A measuring cup usually has gradations marked on its sides. A traditional Japanese device used to measure *sake* from 1 to 6 liters is a lidless rectangular box without gradations with a capacity of 6 liters. By tilting the box to align the surface of the liquid with its edges and vertices, 1, 3, and 6 liters can be measured as shown in Figure 26. If a shop clerk needs to give a customer 4 liters of liquid, he would first fill the box by immersing it in the large shop container just once. He would then pour 3 liters into the customer's container. Next he would pour liquid back into the shop container until 1 liter was left. This he would then pour into the customer's container. It is easy to see that a similar procedure could be carried out to obtain any k-liter amount, where k is an integer, $1 \leq k \leq 6$.

Motivated by this, Jin *et al.* [25, 24, 26] proceeded to study the problem of finding boxes whose vertices can be used as markers in a process similar to that described above (with the 6-liter box) to measure from 1 to k liters, where k is an integer determined by the dimensions of the box. Such a box is called a *universal measuring box*, referred to simply as *UMB*.

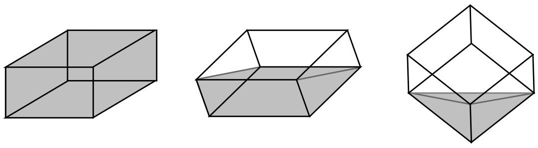


Fig. 26. 6 liters, 3 liters and 1 liter.

They mainly studied three types of UMB's: those with triangular or rectangular bases and plane trapezoidal sides orthogonal to their bases, and those with triangular bases and plane quadrilateral sides not necessarily orthogonal to their bases (see Figures 27, 28, and 29 respectively).

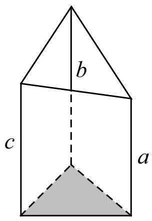


Fig. 27. UMB with triangular base.

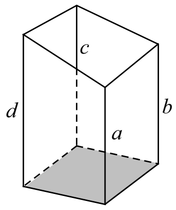


Fig. 28. UMB with rectangular base.

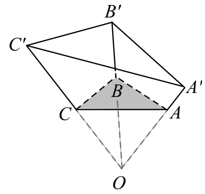


Fig. 29. UMB with triangular base and sides not orthogonal to the base.

The results obtained were surprising. For measuring boxes with a triangular base of area 3, with sides orthogonal to the base, if $\{a, b, c\} = \{12, 13, 16\}$ or $\{a, b, c\} = \{4, 18, 19\}$, the capacity of the box is 41, the boxes are universal, and 41 is the maximum volume possible for such boxes. For boxes with triangular bases non-orthogonal to the sides, a measuring box ABC-$C'A'B'$ (Figure 29) such that the triangular cone $OABC$ has volume 1 and such that $OA'/OA = 2, OB'/OB = 4$ and $OC'/OC = 16$ has capacity 127, and is also a universal measuring box. The solution is also optimal.

For boxes with a rectangular base of area 6 and sides orthogonal to the base, they showed that the box with $(a, b, c, d) = (130, 132, 156, 169)$ has capacity 858 and is universal. It is not known whether this solution is optimal. It would also be interesting to investigate measuring boxes whose bases are various convex polygons.

3.5. Developments of Polyhedra

To conclude, we present two particularly nice results on developments of polyhedra.

(a) Tile-makers and semi-tile-makers

A plane figure f is said to *tile* the plane if the plane can be covered with copies of f with disjoint interiors. A convex polyhedron is a *tile-maker* if every development of it tiles the plane (see Figure 30). In [11] Jin determines all convex polyhedra that are tile-makers. He includes several dihedral polyhedra (polyhedra with volume 0 in which the bottom and top surfaces are considered as distinct from one another. For example, think of a rectangular piece of paper folded in half; this generates a *flat* rectangle with two different faces). The dihedral polyhedra in Jin's result are polyhedra whose faces are equilateral triangles, isosceles right triangles, half equilateral triangles or rectangles (Figure 31), and tetrahedral polyhedra with four congruent triangular faces.

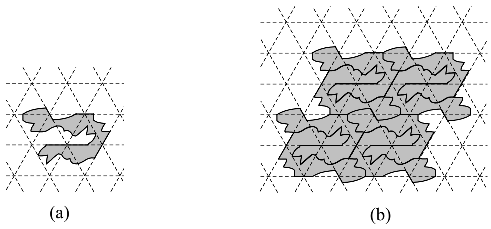


Fig. 30. (a) Development of a regular tetrahedron. (b) Tiling of the plane using copies of the development in (a).

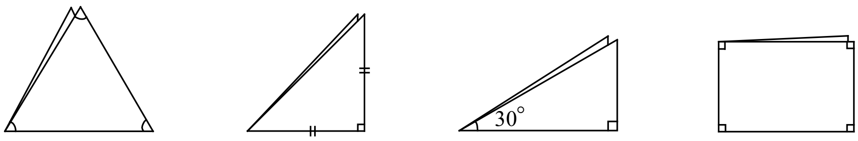


Fig. 31. All convex dihedra which are tile-makers[1]

A development of a polyhedron is called an *edge-development* if the cuts on the surface of the polyhedron are restricted to be only along its edges. A convex polyhedron such that all its edge-developments tile the plane is called a *semi-tile maker*. In [11] Jin conjectures that the set of all semi-tile-maker polyhedra consists of all polyhedra that are tile-makers plus cubes and regular octahedral polyhedra.

(b) Double packable solids

Of all Jin's work, one of my favorite subjects is that of *double packable* solids. This topic has real-world applications in the design of shipping and packing boxes. The "double" part of the term refers to the fact that these boxes fulfill *two* important properties; the box must tile the space, and there must exist a development of the box that tiles the plane.

The advantages of such a box are evident; the plane tiling property minimizes waste of the sheets of material used to make the boxes, while the space tiling property minimizes wasted space once the assembled boxes are filled with product and stacked for shipping. In a memorable invited lecture at the 1997 Canadian Conference on Computational Geometry, Jin introduced the concept of double packable solids, proving that the cube and the tetrahedron shown in Figure 32 are double packable. In a paper with Nakamura [39], it is proved that there are an infinite number of double packable solids with $4, 5, 6, 7, 8$ faces, one with 9 faces, and one with 12 faces. It is not known whether any double packable solid with 10, 11 and 13 or more faces exists. [1]

[1] The faces of these figures are shown slightly apart in the diagram to indicate that the figures are dihedral, but in reality they consist of two congruent layers glued along each of the edges.

[1] The author would like to thank Toshinori Sakai for his invaluable help during the writing of this manuscript.

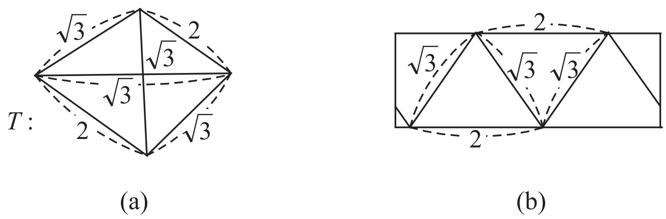


Fig. 32. (a) Tetrahedron T which fills the space. (b) Development of T which tiles the plane.

4. Eye Witness Report: Jin Akiyama in Action — notes by MJR

Wherever Jin Akiyama travels in Japan, he has fans who give him the TV star treatment – they ask for his autograph, they pose with him for photographs, they signal his presence to their companions by a look or a whisper. I have observed these scenes many times. I have also witnessed Jin's ascendance to celebrity status, since I met him at the 1983 Singapore conference, long before he became famous.

From a few guest spots in popular TV shows, he captivated the audiences with his charisma and some surprising down-to-earth mathematics. It did not take long for NHK, Japan's largest TV network, to note his drawing power and offer him a regular TV series on mathematics. This was in 1991 – the series continues to this day [1, 9, 10]. Videos of these programs, also produced by NHK, are top sellers. Radio programs followed in 1994. Audiences follow the mathematics taught in these programs through accompanying workbooks which are sold to the public.

Fig. 33. Broadcasting Radio DJ Math program

Fig. 34. At the Math Samurai TV program

It takes a special talent to hold on audience in a medium that can be turned off at will, especially if what one offers is mathematics. What factors contribute to Jin's success? The charisma is palpable. He is witty – he makes the audience laugh. He has a good grasp of audience psychology – he knows how to make an impact. And most important – his choice of mathematics topics is inspired.

The sources of his materials are varied: his own published results, published journal articles of other authors, mathematics and history of mathematics textbooks, popular mathematics and popular science books. His originality is seen in the way the materials are presented and in the models that are conceived [53, 56, 54, 5].

In one of our conversations, he explains his mission: "There are some rare flowers that are hidden deep in the jungle, only a few have a chance to find them and only those few will get to appreciate their beauty; but, many common flowers are beautiful as well, and they are within everyone's reach, so many are touched by their beauty. The mathematics I choose to do can be compared to the common flowers – it is within everyone's reach. I want to bring the power and beauty of mathematics to many people." In keeping with this mission, the mathematics he presents to the public is within their day-to-day experience, they can see its applications in their lives, it is reachable – not the kind that only mathematicians can understand and appreciate [52, 51]. Indeed, this is the mathematics one will find in his books, will see and hear in his lectures and TV programs, and will experience in his traveling exhibit, *Mathematical Art*, and his permanent exhibit, *Mathematics Wonderland*.

Mathematical Art/Mathematics Wonderland

Just as people cannot appreciate a symphony by reading the score, or a feast by reading the recipes, people cannot appreciate mathematics by reading theorems and memorizing formulas. The symphony must be heard, the feast must be savored. Jin holds that, similarly, mathematics must engage the senses and be experienced. This is the guiding principle of *Mathematical Art* [2–5, 86].

Mathematical Art evolved from the models created for the TV programs. To suit the medium, these are colorful manipulative models that illustrate mathematical principles. A lot of imagination and creative energy went into their conceptualization and production.

Since this exhibit was first launched in Asahikawa in 1998, it has traveled to many cities in Japan under the sponsorship of the National Museum of Science and to neighboring cities in the Asia-Pacific region: Manila and Seoul. It was shown at Makuhari in conjunction with ICME 9 in 2000. Models from *Mathematical Art* form part of the UNESCO traveling exhibit *Experience Mathematics* first shown in Stockholm, then in Copenhagen during ICME 10 in 2004 and subsequently in Paris, Beijing, Athens, Madrid and several cities in Africa [6]. Models from *Mathematical Art* are on permanent exhibit at the Shizuoka Science Museum, while the entire collection of models (more than three hundred) is permanently housed in *Mathematics Wonderland*, in Hokkaido, Northern Japan.

Curves of Constant Width

Among the earliest models developed for Jin's programs are those based on curves of constant width. They provide good examples of his kind of mathematics. The initial inspiration was an article entitled "Why are Manhole Covers Round?" (in Japanese) by his colleague Gisaku Nakamura. To introduce the topic Jin usually asks whether the audience has observed the shape of manhole covers. He then brings out models of manholes and covers of various shapes – round, square, triangular, trapezoidal and one in a strange shape which turns out to be a reuleaux triangle (Figure 35).

He moves the covers around and demonstrates that the square, the triangular and the trapezoidal ones could fall into their associated manholes, whereas the round one and the reuleaux triangle cannot fall in. At this point, Jin urges the audience to think of the reason why this happens and elicits their comments. Audience participation is a strong element of his lectures.

He points out that a circle and a reuleaux triangle have a common property that the other shapes do not possess – they are curves of constant width. Once the audience

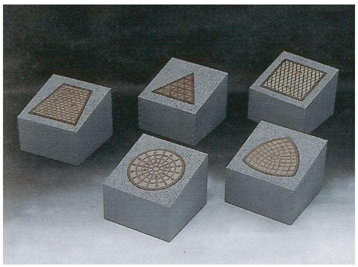


Fig. 35. Manhole covers of various shapes

understands the concept, he follows up with other practical uses of curves of constant width using specially crafted manipulative models. One of these is a drill that makes square holes. It is a modified version of a 200-Kg. Industrial drill invented in the U.S.A. by the Hyatt Co. in 1921. This model is battery operated. The blade of the drill is a reuleaux triangle. It's movement is confined to a space bounded by a square (Figure 36). The audience is invited to operate the drill and to create a square hole on a slab of stiff foam. If you have not seen this before, you will surely be fascinated. The audience usually breaks out in an enthusiastic applause.

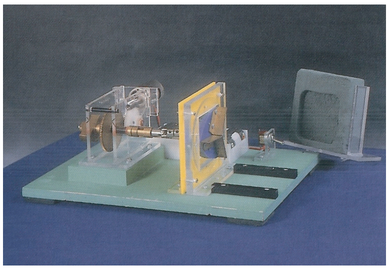


Fig. 36. Square drill

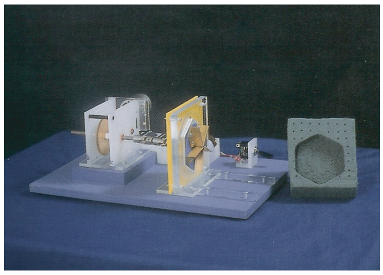


Fig. 37. Hexagonal drill

He shows that the idea can be extended by bringing out another drill which makes hexagonal holes. The blade of this drill is a reuleaux pentagon.

There is more. He shows a model of a rotary engine. The movement of a rotor of constant width regulates the process of compression and decompression which creates the power that causes a vehicle to move.

There are rollers (his own invention) whose wheels have a cross – section of constant width. Jin demonstrates how smoothly this can move along a flat surface, just like vehicles with round wheels.

When these particular models are on exhibit, they are easily among the crowd's favorites and are often photographed.

Mathematics and Music

Sometimes Jin will begin a program by playing his accordion (Figure 38). Since this is totally unexpected in a mathematics lecture, and since Jin is a talented musician, the audience is delighted and applauds.

This is an introduction to a talk on the relationship between mathematics and music. After the short musical performance, he will play some chords and ask the audience to

Fig. 38. Jin playing the accordion

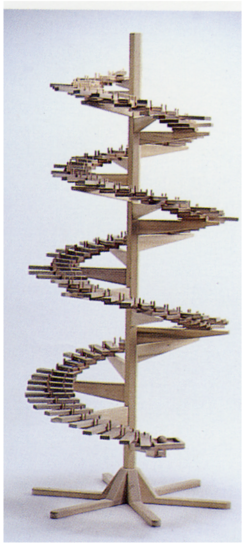


Fig. 39. Spiral xylophone

indicate which chords are discordant and which are harmonious. The audience identifies three harmonious chords. Jin illustrates the notes of the scale on a circular device and shows that the distances between the notes in a harmonious chord form some permutation of the Pythagorean triple 3 – 4 – 5 [8]! The audience is awed.

Sometimes he brings a rather large spiral xylophone (Figure 39) along which balls roll down to produce music. He uses this to explain the role of proportion in harmony.

He might also play a music CD then damage it with a cutter and play it again to show that the music has not been distorted. Again, this is a surprising demonstration. He uses this as an illustration of error-correcting codes.

Since the theory of error-correcting codes may be somewhat daunting for the audience he simply draws an analogy. He invites a participant from the audience to choose an animal from among twelve (Chinese zodiac symbols) on a slide. The participant is instructed to inform the audience of his choice but to keep it from Jin. He then asks the participant to indicate whether the chosen animal is in each of seven successive slides that he will show. He also tells the participant that he can lie at most *once* as he answers. At the end of the question and answer sequence, he identifies the chosen animal; and, if the participant lied, he also identifies the slide being shown when the participant lied. The astonished audience applauds. He explains that this is the way error-correcting codes work, a small distortion in the information can be identified and corrected.

Upon seeing this demonstration, a graduate student, whose research area is quantum codes, remarked "I've never seen it done this way before – a simple demonstration that gets to the gist of the matter."

Mathematics and Art

Always on the lookout for new and intriguing mathematics, Jin's own research results in discrete geometry have become a major part of his programs and his models. The mathematics is deep and significant but he has found ways of presenting it to the audience in non-intimidating terms. It is always the beauty of mathematics that is emphasized.

Among his most recent results is the theorem that states: *Every development of a regular tetrahedron tiles the plane.* The theorem was described by an *American Mathematics Monthly* referee as "a gem." He will demonstrate this theorem.

He begins by explaining the concept of *development* and he has an amusing way of doing it. He has several congruent paper tetrahedra whose surfaces are painted to resemble gindara (codfish) (Figure 40). He asks the audience to pretend that he is a sushi chef – an expert at cutting fish. Participants from the audience can choose to have their fish served in the shape of various polygons – triangle, quadrilateral, pentagon, hexagon. After each choice, he will cut the surface of one of the tetrahedra to come up with the requested figure. He explains that each figure is a development of the original tetrahedron and that infinitely many developments are possible [50, 30].

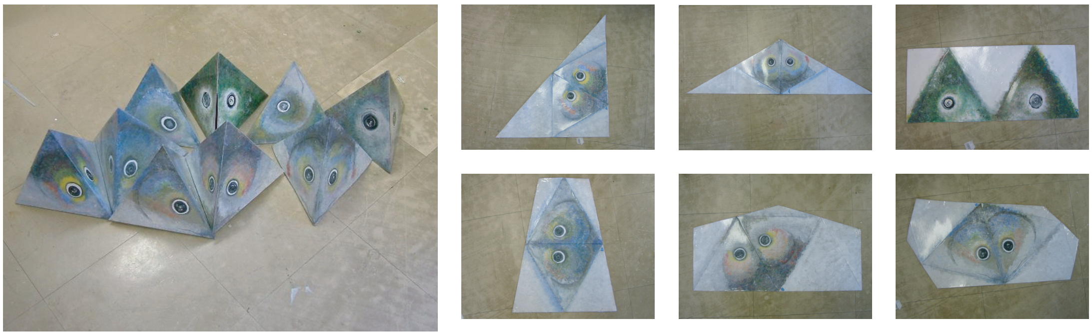


Fig. 40. Gindara and its developments

He is ready to demonstrate the theorem. He takes a regular tetrahedron made of colored paper (actually several layers of paper of various colors). He cuts the surface in random fashion but in such a way that the surface can be lain flat on a plane. To do this, he explains that the cut must pass through each vertex of the tetrahedron. When the cutting is done, he has several congruent pieces of colored paper which he can lay on a flat surface in interlocking fashion so that the pieces begin to cover the surface without gaps or overlaps. He has created a tiling of the plane!

This particular lecture is entitled "You Can Be an Artist Like Escher." [7]. Each member of the audience is provided with a tetrahedron and scissors and invited to create Escher-like patterns that tile the plane. The audience plunges into this activity much like kindergarten students in an art class.

Jin himself has created some beautiful collages from these developments and has shown them in his exhibitions (Figure 41).

Reversible Solids

My own personal favorites from among Jin's models come from research results he obtained with Gisaku Nakamura in a series of published papers on Dudeney dissections of polyhedra. In these papers, they identified many convex polyhedra that can be dissected, hinged and turned inside out to obtain other convex polyhedra [40, 41]. In some cases, the resulting polyhedron is congruent to the original [42, 43, 45].

A lecture on this topic commences with Jin showing two truncated octahedra and a rectangular plastic box. He says his problem is to pack the octahedra in the box (Figure

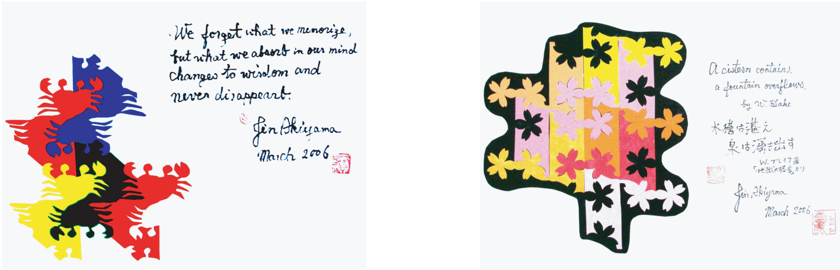
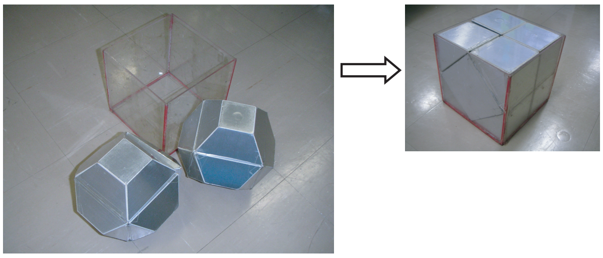


Fig. 41. Collage from several copies of a development of a regular tetrahedron

42(a)). When you see the octahedra and the box, this seems to be an impossible task; but the audience is in for a surprise. He reverses the octahedra into two rectangular solids that fit perfectly in the box (Figure 42(b)). He gives a short spiel on creative problem solving. He says that to solve a problem it is important to ask the right questions and he quotes Georg Cantor: "The art of asking the right questions in mathematics is more important than the art of solving them."



(a) (b)

Fig. 42. Rectangular solids in the rectangular box

He then shows a rhombic dodecahedron painted to look like a fox. With one pull of a string, the dodecahedron reverses into a rectangular solid painted to look like a snake (Figure 43). The snake swallowed the fox! The audience loves this.

Fig. 43. The snake swallows the fox

A similar demonstration starts with a truncated octahedron painted to look like a pig which with one twist of a rod reverses into a rectangular solid looking like a slab of ham (Figure 44). This makes the audience laugh.

These whimsical models are actually quite sophisticated. They are the work of Yasuyuki Yamaguchi, an industrial designer, and Minoru Kanzaki, a sculptor, colleagues of Jin from the Tokai University, School of Art and Design in Hokkaido.

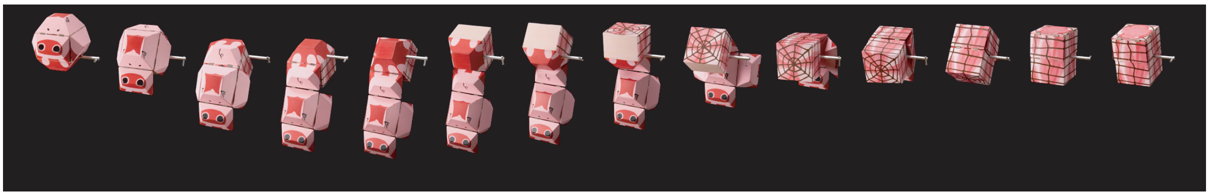


Fig. 44. From pig to ham

Endnote

The public's response to Jin is overwhelming. More than 49,000 viewers flocked to the three-week *Mathematical Art* exhibit at the National Science Museum in Ueno, more than 66,000 came to the month-long exhibit in Hiroshima, approximately 17,000 visited the two-day ICME exhibit in Makuhari. His TV programs are highly rated. He receives hundreds of fan mail. His exhibits are discussed enthusiastically in Japanese websites.

Jin is not one who rests on his laurels. He is at work twelve to fourteen hours a day including Sundays. Only a few can keep up with his pace. Every week he crisscrosses Japan giving lectures. Magazine and TV interviewers are always knocking at his door. Although he may be tired in private, once the lecture or the taping begins, his energy level rises several notches and never flags. He is a seasoned professional, a real trouper.

At the opening party of *Mathematical Art* in Asahikawa, I was asked to give some remarks as one of the invited guests from abroad. I remember challenging Jin to keep surprising us with his inventiveness, and he has not disappointed since.

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