

UNIVERSIDADE DE LISBOA  
Faculdade de Ciências  
Departamento de Informática



**FORMAL VERIFICATION OF PARALLEL C+MPI  
PROGRAMS**

**Nuno Alexandre Dias Martins**

**DISSERTAÇÃO**

**MESTRADO EM ENGENHARIA INFORMÁTICA**  
Especialização em Engenharia de Software

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Dissertação orientada pelo Prof. Doutor Eduardo Resende Brandão Marques  
e co-orientado pelo Prof. Doutor Vasco Manuel Thudichum de Serpa Vasconcelos

2013



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## Resumo

O Message Passing Interface (MPI) [6] é o padrão de referência para a programação de aplicações paralelas de alto desempenho em plataformas de execução que podem ir até centenas de milhares de *cores*. O MPI pode ser utilizado em programas C ou Fortran, sendo baseado no paradigma de troca de mensagens. De acordo com o paradigma *Single Program, Multiple Data* (SPMD), um único programa define o comportamento de vários processos, utilizando chamadas a primitivas MPI, como por exemplo para comunicações ponto-a-ponto ou para comunicações colectivas. O uso de MPI levanta questões de fiabilidade, uma vez que é muito fácil escrever um programa contendo um processo que bloqueie indefinidamente enquanto espera por uma mensagem, ou que o tipo e a dimensão dos dados enviados e esperados por dois processos não coincidam. Assim, não é possível garantir à partida (em tempo de compilação) uma série de propriedades fundamentais sobre a execução de um programa.

Lidar com este desafio não é de todo trivial. A verificação de programas MPI utiliza tipicamente técnicas avançadas como a verificação de modelos ou execução simbólica [9, 40]. Estas abordagens deparam-se frequentemente com o problema de escalabilidade, dado o espaço de estados do processo de verificação crescer exponencialmente com o número de processos considerados. A verificação em tempo útil pode estar limitada a menos de uma dezena de processos na verificação de aplicações *real-world* [42]. A verificação é ainda adicionalmente complicada por outros aspectos, como a existência de diversos tipos de primitivas MPI com diferentes semânticas de comunicação [40], ou a dificuldade em destrinçar o fluxo colectivo e individual de processos num único corpo comum de código [2].

A abordagem considerada para a verificação de programas MPI é baseada em *tipos de sessão multi-participante* [19]. A ideia base passa por especificar o protocolo de comunicação a ser respeitado por um conjunto de participantes que comunicam entre si trocando mensagens. A partir de um protocolo expresso desta forma, é possível extrair por sua vez o protocolo local de cada um dos participantes, segundo uma noção de projecção de comportamentos. Se para cada participante (processo) no programa MPI se verificar a aderência ao protocolo local respectivo, são garantidas propriedades como a ausência de condições de impasse e a segurança de tipos. A verificação desta aderência é feita sobre programas C que usam MPI, usando a ferramenta VCC da Microsoft Research [5].

Para codificar protocolos, foi utilizada uma linguagem formal de descrição de protocolos, apropriada à expressão dos padrões mais comuns de programas MPI. A partir de um protocolo expresso nessa linguagem, é gerado um *header* C que exprime o tipo num formato compatível com a ferramenta VCC [5]. Para além da codificação protocolo, a verificação é ainda guiada por um conjunto de contratos pré-definidos para primitivas MPI e por anotações no corpo do programa C. As anotações no programa são geradas automaticamente ou, em número tipicamente mais reduzido, introduzidas pelo programador.

Os protocolos que regem as comunicações globais num programa MPI são especificados numa linguagem de protocolos, desenhada especificamente para o efeito no contexto do projecto MULTICORE em complemento ao trabalho desta tese, e em associação um *plugin* Eclipse que verifica a boa formação dos protocolos e que gera a codificação do protocolo na linguagem VCC. As ações básicas dos protocolos descrevem no caso geral primitivas MPI, por exemplo para comunicação ponto-a-ponto ou comunicação colectiva. Os valores associados a ações podem ser do género inteiro ou vírgula flutuante, bem como vectores. Além disso, qualquer um destes géneros pode ser *refinado* com imposição de restrições aos valores dos dados. As ações básicas são compostas através de operadores de sequência, iteração, e ainda de fluxo de controlo coletivo em correspondência a escolhas ou ciclos executados de forma síncrona por todos os participantes.

A partir da especificação de um protocolo, a sua tradução no formato VCC define uma função de projecção. A função de projecção toma como argumento o índice do processo MPI, conhecido como *rank*, e devolve a codificação de um protocolo local a ser verificado para execução do participante, em linha com o enunciado pela noção de projecção [19]. Esta codificação reflecte de resto todas as características gerais da especificação do protocolo, em termos de ações básicas, o uso de tipos refinados, e operadores de composição. O processo de verificação tem por fim certificar a aderência do programa C+MPI face ao protocolo, para cada participante. Entre a inicialização e o término das comunicações MPI, a verificação deve operar a redução progressiva do protocolo até ao protocolo vazio. As reduções são definidas mediante pontos de chamadas a primitivas MPI e características do fluxo de controlo de programa. Para manter o estado, a verificação manipula uma variável “fantasma” desde o ponto de entrada da programa (a função `main()`) que representa o protocolo. Para além da aderência ao protocolo, são ainda verificados aspectos complementares, como por exemplo se os dados usam regiões válidas de memória.

Esta verificação usa um corpo base de definições, a que chamamos a “MPI anotada”. A MPI anotada compreende a lógica de protocolos e um corpo de contratos para um conjunto de primitivas MPI. A lógica de protocolos permite definir a estrutura de protocolos e definir as regras de redução, enquanto que os contratos das primitivas definem casos base para redução via ações de comunicação. Este corpo base pode ser importado para o contexto de verificação de um programa em particular, mediante a inclusão de um *header*

C, a versão anotada do *header* convencional da MPI (`mpi.h`) [6]. Usando esta lógica base, o programa C pode ser anotado para verificação. As anotações relacionam-se com uma diversidade de aspectos que impactam da verificação do programa, tais como o fluxo de controlo colectivo, contratos de funções, invariantes de ciclos, asserções respeitantes ao uso de memória, ou a declaração de assunções até aí implícitas no comportamento do programa.

O processo de anotação é um desafio para um programador, já que requer o domínio de uma lógica complexa de verificação. Para automatizar o processo, foi desenvolvido um anotador que gera uma parte significativa das anotações necessárias, transformando código C usando a biblioteca clang/LLVM [4]. O seu funcionamento baseia-se por anotações de alto nível para identificação de fluxo de controlo relevante e marcas de anotação simples introduzidas pelo programador, por forma a gerar em correspondência a um conjunto mais vasto e complexo de anotações. Após este processo automático, há anotações complementares que têm de ser introduzidas manualmente para a verificação bem sucedida de um programa. Estas últimas relacionam-se com aspectos diversos que ou são de inferência complexa, por exemplo o uso de memória, ou ainda não tratados na aproximação atual com um processo automatizado.

Esta aproximação à verificação de programas C+MPI foi testada com um conjunto de exemplos tirados de livros de texto. Além de demonstrar a aplicabilidade da aproximação geral considerada, é apresentada uma análise do esforço de anotação e do tempo de verificação. O esforço de anotação mede a comparação entre o número de anotações automáticas face ao número de anotações manuais, verificando-se no caso geral que o número de anotações manuais é inferior ao número das automáticas. O tempo de verificação diz respeito ao tempo de execução da ferramenta VCC para o código anotado final de um programa. A análise de escalabilidade do mesmo face a um número crescente de processos permitiu identificar casos distintos: casos em que o tempo de execução é insensível ao número de processos e outros em que este tempo cresce exponencialmente face ao número de processos.

Em conclusão, é definida uma metodologia para a verificação formal de programas MPI e demonstrada a sua aplicabilidade, combinando os paradigmas da teoria de tipos de sessão multi-participante e da verificação dedutiva de programas. Para lidar com uma maior gama de programas MPI, em particular programas *real-world*, ao longo do texto foram discutidos vários desafios que se colocam para uma evolução da metodologia, de tipo conceptual ou relacionados com a automação e escalabilidade. Para lidar com esses desafios, propõe-se na parte final um conjunto de linhas gerais para trabalho futuro.

**Palavras-chave:** MPI, programação paralela, verificação, tipos de sessão



## Abstract

Message Passing Interface (MPI) is the de facto standard for message-based parallel applications. Two decades after the first version of its specification, MPI applications routinely execute on supercomputers and computer clusters.

MPI programs incarnate the Single Program Multiple Data (SPMD) paradigm. A single body of code, written in C or Fortran, defines the behavior of all participant processes in a program. The number of processes is defined at runtime, and any partial distinction of behavior between participants is based on the unique rank (numerical identifier) of each process. The communication between processes is defined through point-to-point or collective communication primitives. As a result, programs may easily exhibit intricate behaviors mixing collective and participant-specific flow, making it difficult to verify a priori desirable properties like absence of deadlocks or adherence to a desired communication protocol.

In line with the concern for verifiable program behavior in MPI, the theory of multi-party session types provides a framework for well-structured communication protocols by an arbitrary number of participant processes. By construction, a multi-party global protocol declares a desired interaction behavior that guarantees properties such as type safety and absence of deadlocks, and implicitly defines the individual local protocols per participant. Provided that the actual program specification of each participant conforms to the corresponding local protocol, the safety properties and the intended choreography of the global protocol are preserved by the program.

This thesis proposes the application of multi-party session type theory to C+MPI programs, coupled with the use of deductive software verification. A framework is proposed for the expression of multi-party session protocols in the MPI context and their verification using a deductive software tool. The framework also addresses concerns for automation through semi-automatic annotation of verification logic in program code. An evaluation of the applicability of the approach is conducted over a sample set of programs.

**Keywords:** MPI, parallel programming, formal verification, session types



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# Chapter 1

## Introduction

### 1.1 Motivation

Message Passing Interface (MPI) [6] is a standard for the programming of high performance parallel applications that can be deployed on execution platforms with hundreds of thousands of cores. Based on the message-passing paradigm, the MPI standard has C and Fortran bindings. These programs are encoded according to the *Single Program, Multiple Data* (SPMD) paradigm, i.e., a single program defines the behavior of the various processes at runtime. Any partial distinction of behavior between participants is based on the unique rank (a numerical identifier) of each process.

It is quite easy to write a MPI program that leads to execution errors such as blocked processes waiting for a message, data races between the process and the MPI environment, type/size between message senders and receivers, or adherence to a desired communication protocol. No guarantees exist in advance, at compilation time, for key fundamental properties regarding the execution of a program, a well-acknowledged problem by active research [10].

Several methodologies are employed in the formal verification of MPI programs, such as model checking and symbolic execution [9, 40, 39]. These approaches typically face a scalability problem, given that the state space for verification grows exponentially with the number of processes. Verifying real-world applications may restrict that state space to the interactions to no more than a few processes, as in [42]. The verification is further complicated by various additional aspects such as the existence of diverse kinds of MPI primitives with different communication semantics [40], or the difficulty in distinguishing the collective and individual behavior of processes in a single program [2].

### 1.2 Proposal

This thesis presents an approach for the formal verification of MPI programs based on multi-party session types [19]. The theory considers the specification of a global interac-

tion protocol among multiple participants, from which we can derive an endpoint protocol for each individual participant. A well-formed protocol is by construction type-safe and contains no deadlocks [19]. The same properties are preserved for a program that complies with that protocol, e.g., as in Session-C [31]. The idea considered in this thesis is to apply the same reasoning to MPI programs.

To verify the compliance of a MPI program against a given protocol specification, deductive software verification is employed over MPI programs written in C [5, 35, 20]. The approach requires the annotation of the target program with a verification logic for properties of interest. Thus, in the case at stake, annotations must allow for the expression of the target protocol and the necessary logic for verifying compliance of the program with the protocol. For this purpose the VCC verifier is employed [5].

The overall methodology comprises the distinct steps of protocol specification and protocol verification. The focus of this thesis is on protocol verification, although the overall traits of protocol specification are also described. A protocol specification is encoded in a formal language that is based on multi-party session types but also captures some of the main traits of MPI programs, such as the diversity of (point-to-point or collective) communication primitives and collective control flow. A well-formed protocol specification is converted to the form of the VCC logic automatically, and the latter is used in the verification process. Through complementary annotation of the target program, the compliance of the target program against the specified protocol can be verified.

The implementation of the verification framework comprises some base verification logic and a tool for semi-automated annotation of programs. The base verification logic defines how protocols can be expressed and matched, plus function contracts for a subset of MPI primitives which are called the “annotated MPI”. In order to automate the annotation process as much as possible, a significant part of program annotations is automatically generated by an annotator tool.

## 1.3 Thesis structure

The remainder of this thesis is structured as follows:

- **Chapter 2. Background** — Some background context is provided on MPI, multi-party session type theory and tools, the VCC software verifier, and related work in the formal verification of MPI programs.
- **Chapter 3. Methodology**— In this core chapter, the approach for formal verification of MPI programs is described. The various aspects of protocol specification, protocol translation to verification logic, program verification, and semi-automated program annotation are covered. Two MPI programs are used as running examples in the discussion.



- **Chapter 4. Design and Implementation** — The design and implementation of the verification framework is explained, regarding the base VCC logic for protocol verification and the tool for semi-automated generation of program annotations.
- **Chapter 5. Evaluation** — An evaluation is made considering a sample set of MPI programs. Results are presented and discussed regarding the effort for annotating programs and the execution time of the verification process.
- **Chapter 6. Conclusion** — The final chapter makes a summary of the contributions of this thesis and identifies directions for future work.
- **Appendix A. Evaluation examples** — The evaluation examples of Chapter 5 are provided in full, regarding the protocol specification in VCC format, plus the annotated source code. In complement and due to space restraints, the source code of the “annotated MPI” and the annotator tool are provided as an annex to this thesis in the form of a ZIP archive.



# Chapter 2

## Background

This chapter presents the background context of this thesis, comprising a general description of MPI (Section 2.1), the multi-party session type methodology (Section 2.2), the use of VCC (Section 2.3), and a brief survey of related work on formal verification of MPI programs (Section 2.4).

### 2.1 MPI

MPI is a library specification targeting the development of communication-intensive parallel applications [6], which became the de facto standard for message-based parallel computing. The MPI standard is large, at this point comprising such diverse aspects such as parallel I/O, one-sided communication, process management, beyond an enormous variety of features related to message-passing itself. We concentrate on illustrating the MPI features that will be at stake in this thesis.

MPI programs can be written in C or Fortran, employing the SPMD paradigm. A single program body defines the behavior of the various processes at runtime, which exchange point-to-point messages or engage in collective communication. Any possible distinction of behavior for a process or group processes is done through the process rank, an unique integer identifier of each process at runtime.

An example MPI program in Figure 2.1 illustrates these core traits. It is a toy program to calculate an approximation of  $\pi$  through a numerical integration, adapted from [13]. The process with rank 0 reads the number of intervals for the required numerical integration (line 12) and sends that value to all other processes (l. 13-16) using point-to-point messages (`MPI_Send`, l. 15). In correspondence, all other processes wait to receive the data (using `MPI_Recv`, l. 19). The computation then takes place locally at each process (l. 22–28) and is followed a collective reduction operation (`MPI_Reduce`, l. 31). The reduction operation calculates the sum of all local values computed per process, and yields that result to process 0 only.

Observe that the program in Figure 2.1 can easily be changed to induce erroneous

```
1 int main(int argc, char **argv) {
2     int n, rank, procs, i;
3     double PI25DT = 3.141592653589793238462643;
4     double mypi, pi, h, sum, x;
5     MPI_Status status;
6
7     MPI_Init(&argc, &argv);
8     MPI_Comm_size(MPI_COMM_WORLD, &procs);
9     MPI_Comm_rank(MPI_COMM_WORLD, &rank);
10    if (rank == 0) {
11        printf("Enter the number of intervals: ");
12        scanf("%d",&n);
13        for (i = 1; i < procs; i++) {
14            // Send message to every other process
15            MPI_Send(&n, 1, MPI_INT, i, 0, MPI_COMM_WORLD);
16        }
17    } else{
18        // Receive a message from process 0
19        MPI_Recv(&n, 1, MPI_INT, 0 , 0, MPI_COMM_WORLD, &status);
20    }
21    // Computation
22    h = 1.0 / (double) n;
23    sum = 0.0;
24    for (i = rank + 1; i <= n; i += procs) {
25        x = h * ((double)i - 0.5);
26        sum += (4.0 / (1.0 + x*x));
27    }
28    mypi = h * sum;
29    // Reduction using sum of mypi from all processes
30    // Value becomes available at process 0.
31    MPI_Reduce(&mypi, &pi, 1, MPI_DOUBLE, MPI_SUM, 0, MPI_COMM_WORLD);
32    if (rank == 0) {
33        printf("pi is approximately %.16f, Error is %.16f\n",
34            pi, fabs(pi - PI25DT));
35    }
36    MPI_Finalize();
37    return 0;
38 }
```

Figure 2.1: Example MPI program

behavior. For instance, if we remove the call to `MPI_Recv` (l. 19), process 0 may hang on the first call to `MPI_Send` (l. 15), since the programmer should assume no buffering for `MPI_Send` by the implementation [6]. Process 0 would also hang if instead lines 31 and 32 were swapped, i.e., if the reduction operation (`MPI_Reduce`) was initiated by process 0 alone. All processes need to eventually engage in the reduction, otherwise deadlock will occur.

## 2.2 Multi-party session types

Session types are a representation of a system that ensures the communication is respecting a particular protocol associated with a channel, first introduced in [18]. The structuring concept is that of a *session*, an high-level specification for the disciplined interaction between participants in a concurrent setting. Multi-party session types [19] define a theory for an arbitrary, parametric number of participants, extending previous work that mostly focussed on binary (two-party) session types.

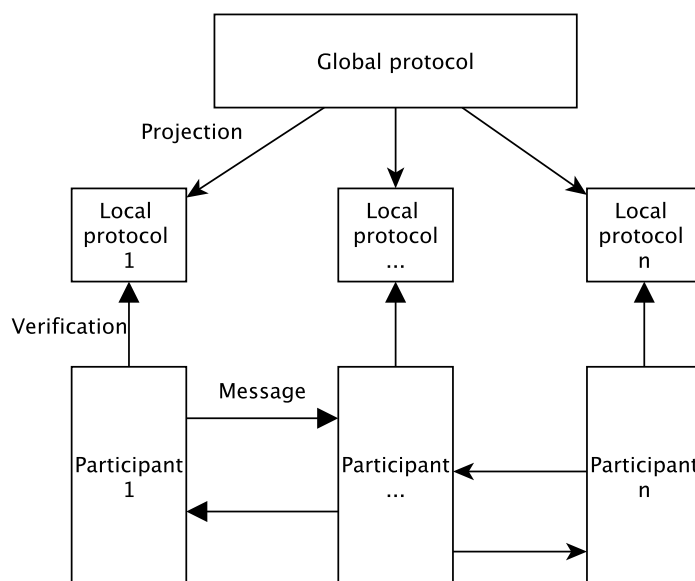


Figure 2.2: Multiparty session type [19]

The intuition for multi-party session types is depicted in Figure 2.2. On top, a global protocol specifies the intended interaction of participant programs that communicate by message-passing, shown at bottom. The global protocol implicitly defines a local protocol per each participant, shown at middle, through a notion of protocol projection. A local protocol respects to the role of a participant in the global choreography. A well-formed protocol verifies by construction type safety, communication safety and deadlock freedom [19]. The same properties are preserved for participant programs, provided the programs are certified as compliant with the corresponding local protocol. Software frameworks like Scribble [17], Session Java [32], or Session C [31], are example instantiations of the theory.

Considering the context of MPI programs, there are a few key aspects that challenge the adoption of the multi-party session type approach. The conformance of participant programs against local protocols cannot be checked modularly per participant, given the SPMD paradigm of MPI. In contrast, participants in Session-C [31] or Session-Java [32] are specified separately. MPI programs also make use of collective operations and exhibit

a collective flow which are not governed by definition by a specific participant, e.g., as defined by the directed choice and iteration operators in [17, 31, 32]. Finally, MPI programs employ synchronized communication, whereas multi-party session type theory and its instantiations assume asynchronous, buffered semantics for communication, which tend to impose less restrictions on the order of messages/operations per each participant.

## 2.3 VCC

VCC [5] is a deductive verifier for C programs. To perform the verification of a program using VCC, the C code must be annotated with several items, such as logical assertions, function contracts, data and loop invariants and ghost code, e.g., as in the Java Modelling Language (JML) [23] and associated verification tools. The format of annotations and associated verification logic is exposed in detail in the next two chapters.

The tool has been developed at Microsoft Research and applied to real-world projects, e.g., Hyper-V [28] comprising 60 thousand lines of operating system-level C and assembly code. VCC takes an annotated program and tries to certify the correctness of the program with respect to the annotations. For that purpose, VCC translates the code and annotations into the intermediate language of the Boogie tool [27], in turn then fed to the Z3 [30] a satisfiability modulo theories (SMT) solver.

There are other similar tools which were considered for use in this work, like Framac [35] or Verifast [20]. An analysis concluded that VCC had the more adequate and comprehensive set of features, such as the possibility of abstract ghost datatypes, user-specified theories, or built-in pointer validity and type-checking [5].

## 2.4 Verification of MPI programs

The current state-of-the-art in MPI program verification has been covered recently in [9]. Overall, the target properties of verification can be diverse, ranging from simple verifications of MPI primitive call arguments [11], traditional interaction properties like deadlock detection [12, 41, 44], up to functional equivalence to serial programs [39]. So are the adopted methodologies, ranging from “traditional” static and dynamic program analysis or a combination of both [2, 43, 44, 12, 15], to model checking and symbolic execution [41, 39, 42]. Some reference of key work in this research area is provided next.

**MPI-Check** MPI-CHECK [11, 12] is a tool to check Fortran 90 MPI [6] programs using compile-time and runtime checks of MPI programs. The focus of the tool is on debugging and requires explicit instrumentation by the programmer. Compile-time checks comprise the validity of calls to MPI primitives and some limited properties of their respective arguments. Runtime checks comprise the validity of runtime arguments, memory safety,

and deadlock detection. For deadlock detection, MPI-CHECK maintains a dependency graph of MPI calls at runtime.

**DAMPI, P<sup>N</sup>MPI and MUST** DAMPI [44] is a dynamic analyzer for MPI [6] programs for detection of deadlocks and resource-leaks in real applications. It operates in automated manner without requiring program changes. The automation comes from using the P<sup>N</sup>MPI [38] interposition layer, that intercepts MPI calls and calls DAMPI monitoring routines. MUST [15], preceded by [22] and Umpire [21], uses a similar approach to DAMPI. Again, P<sup>N</sup>MPI [38] is employed and the focus of runtime checks also concerns resource usage and interaction properties like deadlock detection or message loss.

**ISP and TASS** ISP [43] is a deadlock detection tool which uses a scheduler to explore all possible thread interleavings of an execution, combining model-checking and symbolic execution related techniques. TASS [39, 41] also combines symbolic execution and model checking. Beyond deadlocks and traditional safety properties, TASS is able to verify user-specified assertions for the interaction behavior of the program, so-called collective assertions. Finally, TASS is able to verify functional equivalence between a MPI program and a sequential program.

**Parallel data-flow analysis** Parallel data-flow analysis is a static analysis technique applied in [2]. The work focuses on send-receive matching in MPI source code, which helps identify message leaks and communication mismatch, by constructing a parallel control-flow graph by simple symbolic analysis on the control-flow graph of the MPI program. In [37] the authors discuss extending the technique by combining static and dynamic analysis to improve precision of the data-flow analysis.





# Chapter 3

## Methodology

This chapter describes the methodology for verifying C+MPI programs against communication protocols specified using multi-party session types [19]. The chapter begins with an overview of the approach (Section 3.1) and a discussion of two running examples (Section 3.2). The specification of protocols using multi-party session types (Section 3.3) and the verification of C+MPI programs against these protocols (Section 3.4) is then explained. The chapter ends with the description of the process of annotating programs for verification (Section 3.5).

### 3.1 Overview

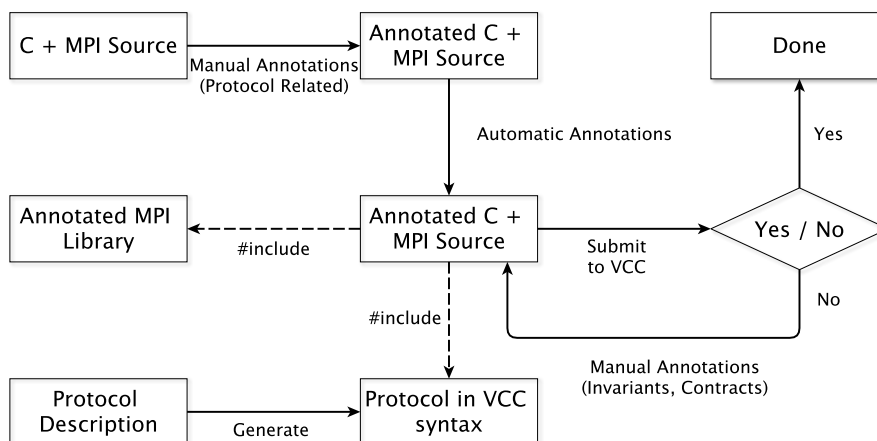


Figure 3.1: Verification approach

The overall approach for the verification of C+MPI programs is illustrated in Figure 3.1. Given a C+MPI program and a protocol description, the goal is to verify compliance of the program against the protocol.

A protocol is specified in a formal language that is based on multi-party session types, and recognized by a plugin module embedded in the Eclipse [1] platform. From a well-formed protocol description, the same module is able to generate a C header file encoding

the protocol in a format understood by the VCC verifier [5]. The C program must include this header, along with another one, a mock MPI header (`mpi.h`) that imports the base logic for verification and MPI function contracts using that base logic.

For verification, a C+MPI program must then be annotated with complementary verification logic. This takes place in semi-automated manner with the help of an automatic annotator tool and comprising different levels of annotation. High-level annotation marks are introduced by the programmer to guide the annotator in generating a larger set of low-level annotations. These high-level marks relate to protocol-related features that the annotator is unable to infer on its own. The output of the annotator is not usually sufficient for VCC to assert that the program complies with the protocol, and manually introduced annotations are required.

## 3.2 Running Examples

Two C+MPI programs are considered as running examples: a program that implements a one-dimensional finite differences algorithm, and an N-body simulation algorithm.

### 3.2.1 Finite differences

The finite differences program concerns an iterative algorithm described in [7]. Considering an initial vector  $X^0$ , successive approximations of a possible problem solution are calculated,  $X^1, X^2, \dots, X^n$ , until a condition of numerical convergence is met or a maximum number of iterations has been reached.

Figure 3.2 lists the main code of the C program, also adapted from [7]. The process number, `procs`, and the rank of each process, are initially obtained using the MPI primitives `MPI_Comm_size` and `MPI_Comm_rank` (lines 5 and 6). In sequence, the participant with rank 0 begins by reading the initial vector  $X^0$  (lines 9–10) and distributing the vector through all participants (line 11), using `MPI_Scatter`. Each participant is responsible for calculating part of the solution, with the same vector length per participant.

Following a ring topology according to the rank of the participant, the program executes a loop (lines 16–37), exchanging point-to-point messages (`MPI_Send`, `MPI_Recv`) between each participant and its left and right neighbours. The purpose of these exchanges is to distribute the border values due to each participant, which are necessary to local computations by the neighbour processes. To avoid deadlocks, a different order of calls to `MPI_Send` and `MPI_Recv` is required for participant 0 (lines 19–22), participant `procs - 1` (lines 23–27) and for other participants (lines 28–32). This is necessary since the behavior of `MPI_Send` and `MPI_Recv` must be assumed as *synchronous* [6], meaning that no buffering exists and that a call to `MPI_Send` by one process will not return before a corresponding `MPI_Recv` call has been issued by another process. For example, if two

```

1  int main(int argc, char** argv) {
2      int procs;           // Number of processes
3      int rank;           // Process rank
4      MPI_Init(&argc, &argv);
5      MPI_Comm_size(MPI_COMM_WORLD, &procs);
6      MPI_Comm_rank(MPI_COMM_WORLD, &rank);
7      ...
8      int psize = atoi(argv[1]);           // Global problem size
9      if (rank == 0)
10         read_vector(work, lsize * procs);
11     MPI_Scatter(work, lsize, MPI_FLOAT, &local[1], lsize, MPI_FLOAT, 0, MPI_COMM_WORLD);
12     int left = (procs + rank - 1) % procs; // Left neighbour
13     int right = (rank + 1) % procs;       // Right neighbour
14     int iter = 0;
15     // Loop until minimum differences converged or max iterations attained
16     while (!converged(globalerr) && iter < MAX_ITER) {
17         ...
18         if (rank == 0) {
19             MPI_Send(&local[1], 1, MPI_FLOAT, left, 0, MPI_COMM_WORLD);
20             MPI_Send(&local[lsize], 1, MPI_FLOAT, right, 0, MPI_COMM_WORLD);
21             MPI_Recv(&local[lsize+1], 1, MPI_FLOAT, right, 0, MPI_COMM_WORLD, &status);
22             MPI_Recv(&local[0], 1, MPI_FLOAT, left, 0, MPI_COMM_WORLD, &status);
23         } else if (rank == procs - 1) {
24             MPI_Recv(&local[lsize+1], 1, MPI_FLOAT, right, 0, MPI_COMM_WORLD, &status);
25             MPI_Recv(&local[0], 1, MPI_FLOAT, left, 0, MPI_COMM_WORLD, &status);
26             MPI_Send(&local[1], 1, MPI_FLOAT, left, 0, MPI_COMM_WORLD);
27             MPI_Send(&local[lsize], 1, MPI_FLOAT, right, 0, MPI_COMM_WORLD);
28         } else {
29             MPI_Recv(&local[0], 1, MPI_FLOAT, left, 0, MPI_COMM_WORLD, &status);
30             MPI_Send(&local[1], 1, MPI_FLOAT, left, 0, MPI_COMM_WORLD);
31             MPI_Send(&local[lsize], 1, MPI_FLOAT, right, 0, MPI_COMM_WORLD);
32             MPI_Recv(&local[lsize+1], 1, MPI_FLOAT, right, 0, MPI_COMM_WORLD, &status)
33         }
34         ...
35         MPI_Allreduce(&localerr, &globalerr, 1, MPI_FLOAT, MPI_MAX, MPI_COMM_WORLD);
36         ...
37     }
38     ...
39     if (converged(globalerr)) { // Gather solution at rank 0
40         MPI_Gather(&local[1], lsize, MPI_FLOAT, work, lsize, MPI_FLOAT, 0, MPI_COMM_WORLD);
41         ...
42     }
43     ...
44     MPI_Finalize();
45     return 0;
46 }

```

Figure 3.2: Finite differences program

processes are assumed and lines 20 and 21 are exchanged, a deadlock will result, since participants 0 and 1 would both be waiting for the reception of a message from each other.

After exchanging messages, and still in the cycle, the global error is calculated with a reduction operation and is then disseminated to all participants using `MPI_Allreduce` (line 35). The loop terminates when there is a convergence condition or after a predefined number of iterations. When the loop terminates, and in case of convergence, the participant 0 aggregates the final solution, receiving from each participant a part of the array by using `MPI_Gather` (lines 39 and 40). Otherwise, if the loop terminates without reaching convergence, no further exchange of messages takes place.

### 3.2.2 N-Body simulation

An N-body simulation computes the trajectories of bodies/particles that interact through gravitational forces at discrete time intervals. The N-body simulation program considered here is adapted from [13].

As in the finite differences example, each process initiates retrieving the number of processes and the process' rank, `procs` and `rank` (lines 18–19). Similarly, the example uses a ring topology: each process locally computes the ranks of its left and right neighbours (lines 22–23) for subsequent point-to-point message exchanges.

The initial message exchange is an all-gather operation, so that all processes get the number of particles handled by all other processes (line 29). Then all processes locally calculate the total number of particles (`totpart`, line 31). The outcome is as if each process in the group had executed a `MPI_Send` primitive to all processes, including itself, and executed a `MPI_Recv` primitive from all processes, again including itself.

The program's main loop (lines 36–60) will execute the n-body simulation for a fixed number of iterations, as defined by the `cnt` variable. An inner loop is defined (lines 40–57) for a transmission of data and computation of forces. The transmission of data will occur from the process `rank` for its right neighbour, receiving the data from its left neighbour. This communication behavior will be repeated `procs - 2` times (line 42). In order to avoid deadlocks, a different behavior is needed concerning the order of use of the `MPI_Send` and `MPI_Recv` primitives for the participant with `rank` even (lines 44–47) and for other participants (lines 49–52).

These communications are necessary to share the particles information and locally compute the forces. After the `for` loop (line 40–57) and the computation step, the changes in position are calculated through the `ComputeNewPos` procedure (line 59). This function computes the new positions for the local particles and internally adjusts the global time-step with a global reduction (`MPI_AllReduce`) operation.

```

1  int main( int argc, char *argv[])
2  {
3      float    particles[MAX_PARTICLES * 4]; /* Particles on ALL nodes */
4      float    pv[MAX_PARTICLES * 6];      /* Particle velocity */
5      float    sendbuf[MAX_PARTICLES * 4], /* Pipeline buffers */
6              recvbuf[MAX_PARTICLES * 4];
7      int      counts[MAX_P],              /* Number on each processor */
8              displs[MAX_P];              /* Offsets into particles */
9      int      rank, procs, npart, i, j,
10             offset;                      /* location of local particles */
11      int      totpart,                    /* total number of particles */
12             cnt;                          /* number of times in loop */
13      float    sim_t;                      /* Simulation time */
14      int      pipe, left, right;
15      MPI_Status statuses[2];
16
17      MPI_Init( &argc, &argv );
18      MPI_Comm_rank( MPI_COMM_WORLD, &rank );
19      MPI_Comm_size( MPI_COMM_WORLD, &procs );
20
21      /* Get the best ring in the topology */
22      left = (procs + rank - 1) % procs;
23      right = (rank + 1) % procs;
24      ...
25      npart = atoi(argv[1]);
26      ...
27      npart = npart / procs;
28      /* Get the sizes */
29      MPI_Allgather( &npart, 1, MPI_INT, counts, 1, MPI_INT, MPI_COMM_WORLD );
30      ... /* calculate displacements */
31      totpart = displs[procs-1] + counts[procs-1];
32      /* Generate the initial values */
33      ...
34      cnt = 10;
35      sim_t = 0.0;
36      while (cnt > 0)
37      {
38          /* Load the initial sendbuffer */
39          ...
40          for (pipe=0; pipe<procs; pipe++)
41          {
42              if (pipe != procs-1) {
43                  if (rank == 0) {
44                      MPI_Send( sendbuf, npart * 4, MPI_FLOAT, right, 0,
45                               MPI_COMM_WORLD);
46                      MPI_Recv( recvbuf, npart * 4, MPI_FLOAT, left, 0,
47                               MPI_COMM_WORLD, &statuses[0] );
48                  } else {
49                      MPI_Recv( recvbuf, npart * 4, MPI_FLOAT, left, 0,
50                               MPI_COMM_WORLD, &statuses[0] );
51                      MPI_Send( sendbuf, npart * 4, MPI_FLOAT, right, 0,
52                               MPI_COMM_WORLD);
53                  }
54              }
55              /* Compute forces (2D only) */
56              ...
57          }
58          /* Once we have the forces, we compute the changes in position */
59          sim_t += ComputeNewPos( particles, pv, npart, max_f, MPI_COMM_WORLD);
60      }
61      ...
62      MPI_Finalize();
63      return 0;
64  }
65
66  double ComputeNewPos( Particle particles, ParticleV pv, int npart,
67                       double max_f, MPI_Comm commring)
68  {
69      int i;
70      float a0, a1, a2;
71      static float dt_old = 0.001, dt = 0.001;
72      float dt_est, new_dt, dt_new;
73      ...
74      /* Re-Calculate the time-step control */
75      ...
76      /* Reduce to the minimum time-step control in the participants */
77      MPI_Allreduce( &dt_est, &dt_new, 1, MPI_FLOAT, MPI_MIN, commring);
78      ...
79      return dt_old;
80  }

```

Figure 3.3: N-body simulation program

### 3.3 Protocol specification

A specific language was designed to describe protocols for global communications in MPI programs. This base been done in the context of the MULTICORE research project, in a complementary line of work to that of this thesis.

#### 3.3.1 Syntax

The base syntax of the language for protocol specification is defined by the grammar shown in Figure 3.4.

$T ::= \text{skip}$	terminated protocol
$\text{message } i \ x: D$	point-to-point comm.
$\text{broadcast } i \ x: D \mid \text{scatter } i \ x: D \mid \dots$	collective comm.
$T; T$	sequence
$\text{foreach } x: i..i \ \text{do } T$	repetition
$\text{loop } T$	collective loop
$\text{choice } T \ \text{or } T$	collective choice
$\text{val } x: D$	variable
$D ::= \text{int} \mid \text{float} \mid D[i] \mid \{x: D \mid p\} \mid \dots$	index types
$i ::= x \mid n \mid i + i \mid \max(i, i) \mid \text{length}(i) \mid i[i] \mid \dots$	index terms
$p ::= \text{true} \mid i \leq i \mid p \ \text{and } p \mid a(i, \dots i) \mid \dots$	index propositions

Figure 3.4: Protocol communication grammar

The simplest syntactic term is `skip`, representing an empty (or terminated) protocol. A set of communication operators are then defined in correspondence to MPI point-to-point messaging, the **message** operator, or collective communication, like the **broadcast** and **scatter** operators and a few others.

Protocols can be composed sequentially using the base sequence operator `;` or the **foreach** repetition operator. Collective control flow, collective loops and choices, are defined by **loop** and **choice** operators, respectively. These collective flow operators represent consensual agreement between *all* participants. Finally, the **val** operator represents an abstract value computed at runtime.

Values associated to message exchange can be of primitive type, like **int** or **float**, or vectors. Furthermore, types can be refined through the imposition of constraints over the domain of values, e.g., `{ x:int | x % 5 == 0 }` stands for the refined type with values ranging over the domain of integer numbers that are multiples of 5.

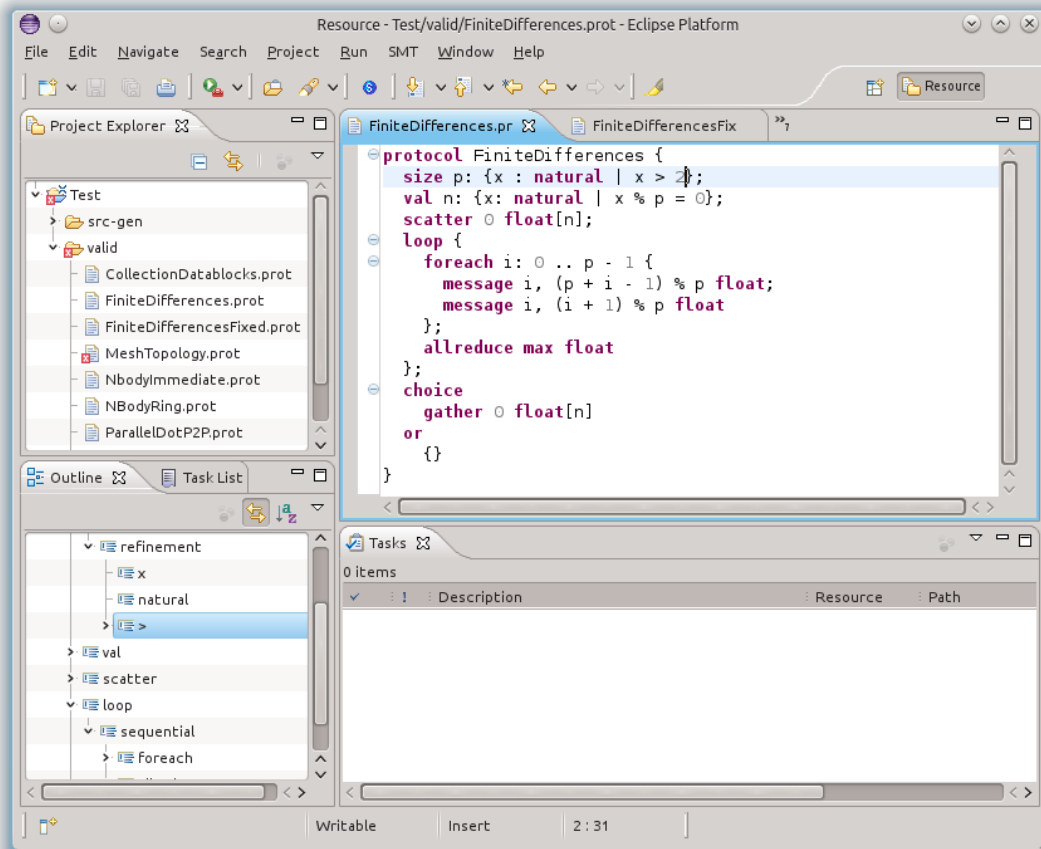


Figure 3.5: Eclipse plugin for protocol specification

### 3.3.2 Eclipse plugin module

The Eclipse plugin module that recognizes the protocol language has been developed by César Santos. It is implemented using the Xtext framework [46] and the SMTLib [3] library. A screenshot of the tool is shown in Figure 3.5.

### 3.3.3 Examples

**Finite Differences protocol** The protocol for the finite differences program is given in Figure 3.6. Line 2 introduces the number of processes over variable  $p$  and line 3 introduces the problem size variable  $n$ . Both are runtime parameters, that are matched, respectively, by a call to `MPI_Comm_size` and by an abstract value in the C+MPI program flow ( $n$ ). The remaining structure corresponds to the pattern of communication in the C+MPI program. An initial data scatter operation (line 4) operation is specified, followed by a collective loop (lines 5–11) where all processes exchange messages with their left and right neighbors (lines 6–9) before engaging collectively in a global reduction (line 10). Lastly, participant 0 gathers the data from all participants in a collective choice, corre-

```

1 protocol FiniteDifferences {
2   size p: positive;           // process number
3   val n: {x: natural | x % p == 0}; // problem size
4   scatter 0 float[n];
5   loop {
6     foreach i: 0 .. p - 1 {
7       message i, (i + 1) % p float;
8       message i, (p + i - 1) % p float
9     };
10    allreduce max float
11  };
12  choice
13    gather 0 float[n]
14  or
15    {}
16 }

```

Figure 3.6: Protocol for the Finite Differences program

sponding to the case of numerical convergence of the finite differences algorithm (recall that the final solution is discarded otherwise).

**N-Body pipeline protocol** The protocol for the N-body simulation program is presented in Figure 3.7. We omit an explanation, given that the syntactic traits are similar to those in the finite differences example, even if the example is slightly more intricate.

### 3.4 Protocol verification

A protocol specification is translated to VCC verification logic and the target program for verification must be annotated. The following aspects are at stake, described in this section:

1. A protocol specification is translated to a C header file that encodes the protocol. This header file must be included in the main C file of the program to verify;
2. The frogman's main C file must also include the standard MPI header `mpi.h`. In reality, the MPI header contains a mock definition of annotated MPI primitives with verification contracts;
3. The programs's C code in turn must be additionally modified with the insertion of annotations that allow the verification of the program against the protocol specification;



```

1 protocol nbodypipe {
2   size p : positive; // number process
3   val n : positive; // problem size
4   allgather int ;
5   loop {
6     loop {
7       choice {
8         foreach i: 0..p-1 {
9           message i, (i+1)%p float[n/p];
10        };
11      }
12     or
13     {}
14   };
15   allreduce min float;
16 }
17 }

```

Figure 3.7: Protocol for the N-Body simulation program

4. The generation of a significant part of program annotations is automated, but some still need to be inserted by the user. The manual annotations are either simple “hints” that guide the automatic generation of a larger set of annotations, or required by VCC to complement program verification.

### 3.4.1 VCC syntax

We begin by introducing some key aspects of the syntax used in the VCC verification logic. In summary, these are as follows:

- All annotations are enclosed in blocks of the form `\_ (annotation block)`. Annotated code may be processed normally by a C compiler, filtering out any annotations, using the following C preprocessor macro:

```
#define _(a) /* blank macro expansion */
```
- Logical clauses of the form **requires** condition and **ensures** condition are used to specify pre and post-conditions in a function contract, respectively, as usual in design-by-contract frameworks (e.g., JML [23]). Likewise, invariant condition expresses an invariant over data structures or loops, the **ghost** keyword is used in association to “ghost” definitions, and the **pure** keyword indicates that a (C or ghost) function has no semantically observable side effects.
- A definition of the form `(\lambda type x; f[x])` encodes an anonymous type domain function.

```

1  _(ghost _(pure) \SessionType ftype (\binteger rank)
2  _(ensures \result ==
3  seq(
4  action(size(), intRef(\blambda \binteger y; y>0, 1)),
5  abs(body(\blambda \binteger p;
6  seq(
7  action(val(), intRef(\blambda \binteger x; x>0 && x%p==0, 1)),
8  abs(body(\blambda \binteger n;
9  seq(
10 action(scatter(0), floatRef(\blambda float v; \btrue, n)),
11 seq(
12 loop(
13 seq(
14 foreach(0, p-1,
15 body(\blambda \binteger i;
16 seq(
17 message(i, (p+i-1)%p,
18 floatRef(\blambda float v; \btrue, 1))[rank],
19 message(i, (i+1)%p,
20 floatRef(\blambda float v; \btrue, 1))[rank])
21 action(allreduce(MPI_MAX), floatRef(\blambda float v; \
22 true, 1))))),
23 choice(
24 action(gather(0), floatRef(\blambda float v; \btrue, n)),
25 skip()
26 )))))))
27 )

```

Figure 3.8: VCC projection function for the finite differences example

### 3.4.2 The projection function

A protocol specification is translated to a C header file, containing a VCC ghost function with the following signature:

```
\SessionType ftype(\binteger rank)
```

We call it the protocol projection function. Given a process rank as argument, it defines a local (endpoint) view of the protocol for the given rank of VCC type `\SessionType`. In line with the notion of projection in the theory of multi-party session types [19], the projection is the local protocol for verification per each participant in the global protocol.

The projection functions for the two running examples are provided in Figure 3.8 and Figure 3.9. In spite of a different syntactic context and other details we explain in Chapter 4, we can in any case observe that the projection functions resemble the original

```

1  -(ghost _(pure) \SessionType ftype (\integer rank)
2  -(ensures \result ==
3    seq(action(size(), intRef(\lambda \integer procs; procs > 0,
4      1)),
5    seq(abs(body(\lambda \integer procs;
6      seq(action(val(), intRef(\lambda \integer size; size > 0,
7        1)),
8      seq(abs(body(\lambda \integer size;
9        seq(action(allgather(), anyInt(procs))),
10       seq(loop(
11         seq(loop(
12           seq(choice(
13             seq(foreach(0, procs-1,
14               body(\lambda \integer i;
15                 seq(message(i, (i+1)%procs,
16                   anyFloat(size * 4))[rank],
17                   skip()))),
18             skip()),
19             skip()),
20             skip()))),
21         seq(action(allreduce(MPI_MIN), anyFloat(1)),
22           skip()))),
23       skip()))),
24     skip())));

```

Figure 3.9: VCC projection function for the N-body example

protocol specifications (Figures 3.6 and 3.7). A crucial difference exists though: the projected protocol will differ per participant according to point-to-point message exchanges. We have that the `\SessionType` term returned by `ftype` takes the `rank` parameter in consideration. A term of the form `message(from, to, dt)[rank]`, defining a point-to-point message exchange, evaluates (projects) to:

- `action(send(to), dt)` if `rank` equals `from`, encoding a message sent (from `rank`) to participant `to`;
- `action(recv(from), dt)` if `rank` equals `to`, encoding a message received (by `rank`) from process `from`;
- and `skip()` otherwise, i.e., no message exchange.

### 3.4.3 The verification process

The verification of a program against a projected protocol analyses the program's control flow between the initialization and termination points of the MPI library, respectively calls to `MPI_Init` and `MPI_Finalize`. The protocol is obtained through the projection function (`ftype`, described earlier), upon initialization. It must then be progressively reduced such that in the end the protocol is congruent to `skip()`. For example, an empty collective loop `—loop{}`—or a *foreach* without any possible unfolding `—foreach(0, -1, ...)`— are both congruent to `skip()`. To verification state manipulates a **ghost** variable of `\SessionType`, declared in the program's `main()` function. The contracts of `MPI_Init` and `MPI_Finalize` summarize the overall logic of verification:

```
int MPI_Init(... _(ghost mpi_glue_t gd) _(out \SessionType
    typeOut))
    _(ensures typeOut == ftype(gd->rank))
    ...
int MPI_Finalize(... _(ghost \SessionType typeIn))
    _(ensures congruence(typeIn, skip()))
    ...
```

In the contract of `MPI_Init`, the `gd` term encodes the process rank within the verification logic, as shown, which is then used in the invocation `ftype`. In the contract of `MPI_Finalize`, the `congruence` predicate expresses the congruence between two instances of `\SessionType`.

### 3.4.4 MPI function contracts

Between initialization and termination, the verification must deal with the progressive reduction of the protocol, including calls in the program to MPI communication primitives. The contracts of the latter define the base cases for reduction. Typically, the reduction defined by a primitive works by extracting (requiring) a communication action prefix from the input protocol, and yielding (ensuring) as output a protocol continuation. Other complementary aspects are also at stake, such as verifying that buffer arguments supplied to MPI primitives are valid memory regions.

We illustrate these aspects with the following fragment of the contract for `MPI_Send`:

```
int _MPI_Send(void *buf, int count, MPI_Datatype datatype, ...
    _(ghost \SessionType type_in)
    _(out \SessionType type_out))
    _(requires actionType(first(type_in)) == send(dest))
    _(requires actionLength(first(type_in)) == count)
    _(requires refTypeCheck(refType(first(type_in)), buf, count))
    _(requires datatype == MPI_INT
        ==> \thread_local_array ((int *) buf, count))
```

```

_(requires datatype == MPI_FLOAT
  ==>\thread_local_array((float *)buf, count))
...
_(ensures type_out == next(type_in))
...

```

The contract above stipulates (in the order shown) that:

- The first action of the input protocol is `send(dest)`, where `dest`, as shown in the function signature, is the function parameter that identifies the destination rank;
- The dimension of the array in the action is equal to the `count` parameter;
- The data to be transmitted, each element of the `buf` argument, must verify the restrictions of type refinement and is a valid memory region;
- Finally, as a post-condition, the protocol to check afterwards is the continuation of the starting type.

Note that the contract defines `_MPI_Send`, not `MPI_Send`. The latter is in reality defined as a C processor macro that expands to a call to `_MPI_Send` with the necessary complementary ghost arguments:

```

#define MPI_Send(a1, a2, a3, a4, a5, a6) \
  _MPI_Send(a1, a2, a3, a4, a5, a6 (__mpi_glue_data__) (ghost
    _type) (out _type))

```

In the current verification framework, we define contracts for the point-to-point MPI operations `MPI_Send` and `MPI_Recv` plus some of the most common collective MPI primitives, e.g., `MPI_Bcast`, `MPI_Allreduce` or `MPI_Scatter`; The set of annotated operations is detailed in Chapter 4. Some common and important features found in MPI programs [6] are not yet supported, including non-blocking operations (e.g., `MPI_Isend` and `MPI_Irecv`) or communicator creation (only the global top-level communicator `MPI_COMM_WORLD` is supported).

### 3.4.5 Collective loops and choices

Given the SPMD nature of MPI programs, a program typically specifies some collective flow control, in the form of loops and conditional statements (choices), apart from rank-dependent behaviour. These must be accounted for in terms of the verification logic, matching terms of the form **loop**  $T$  and **choice**  $T$ . The related annotations can for the most part be automatically generated, through a process described later in this chapter. We focus now on its meaning and the associated verification process. Consider a fragment of the annotated finite differences program as illustration:

```

_(ghost \SessionType body = loopBody(_type);)
_(ghost \SessionType cont = next(_type);)
while (!converged(globalerr) && iter < MAX_ITER)
  ...
{
  _(ghost _type = body;)
  ...
  _(assert congruence(_type, skip()))
}
_(ghost _type = cont);
...

```

We recall that the loop is a collective one. Each process executes the loop exactly the same number of the times until a maximum number of iterations or numerical convergence is attained. The annotations above illustrates the verification logic. Just before the loop, two ghost variables are defined for the loop body (`body`) and its continuation (`cont`); for this to work, the protocol must have a prefix of the form `loop(T)` (and, if so, `T` becomes referenced by `body`). The loop body protocol is then matched by assertions within the loop, i.e., each loop iteration must reduce it to a term that is congruent with `skip()`. After the loop, the verification proceeds with the loop continuation (`cont`). The case of collective choices is handled similarly.

### 3.4.6 For-each protocols

A **foreach** protocol encodes repetition and it may be matched by an actual loop. In that case, the required annotations are similar to those for collective loops or choices. However, this is not always the case, since the projection of a **foreach** loop does not necessarily define an iteration for all participants. To illustrate these traits, consider a protocol where participant 0 sends a message to every other participant:

```
foreach i = 1 .. procs-1 { message 0 i float }
```

The required annotations are as follows:

```

if (rank == 0) {
  _(ghost \SessionType body = foreachBody(_type);)
  _(ghost \SessionType cont = next(_type);)
  for (to=1; to < procs; to++) {
    ...
  }
  _(ghost _type = body;)
  ...
  MPI_Send (... to ...); // send message
  ...
  _(assert congruence(_type, skip()))
} else {

```

```

    MPI_Recv (... 0 ...); // receive message
  }
}

```

As shown, the required annotations and verification for the program flow associated to rank 0 are similar to the case of collective loops. However, for all other participants the entire `foreach` loop is matched only by (the contract of) the `MPI_Recv` operation, since the projection of the protocol defines only the receipt of a message for those participants, i.e., the protocol is asserted as congruent to a term of the form `recv(0, ...)`. Apart from congruence, some “loop unfolding” logic supports this type of verification, described in Chapter 4.

## 3.5 Program annotation

After describing how protocols are overall specified and verified, we now explain the process of annotation over programs. There are three different levels of annotation, which we describe in this section:

1. The programmer may introduce simple protocol-related annotations to identify special control flow in the program, for instance collective loops;
2. Automatically generated annotations are inserted, guided by the protocol-related annotations and some program analysis;
3. Finally, complementary manual annotations are required for a number of reasons, (e.g., memory usage, unstated functional assumptions, or function contracts, which we discuss).

### 3.5.1 Protocol-related annotations

To identify some protocol-related features and guide verification in that sense, the programmer must introduce high-level, simple annotation marks in the program.

The first annotation mark of this kind has the form `_collective_(expr)` where `expr` is the C expression for the condition of a collective loop or choice. That is, the programmer changes

```
if (expr) { ... } else { ... }
```

to

```
if (_collective_(exptr)) { ... } else { ... }
```

when a collective choice is a stake, and

```
while (expr) { ... }
```

to

```
while (_collective_(expr)) { ... }
```

for a collective loop, which can also be a **do-while** or a **for** loop. These marks would otherwise be complex to infer automatically. Non-trivial program analysis is required to assert a given program fragment or expression is in fact collective, and our verification framework does not allow us to define built-in logic to express the simultaneous behavior of all processes (e.g., collective assertions as in [39]). A similar argument can be made for the other two types of mark described below.

The second type of annotation mark is `_foreach_(var, expr)` and applies to the identification of loops that match a **foreach** loop. We have that `var` identifies the iteration variable, and that `expr` is the loop condition expression. For instance, the programmer may change:

```
for (i=0; i < n; i++) { ... }
```

to

```
for (i=0; _foreach_(i, i < n); i++) { ... }
```

The final type of annotation mark has the form `_apply_(expr)`. This relates to the protocol action primitive **val**, used to model “injection” in the protocol of a specific value at runtime. For instance, variable `n` in the finite differences protocol (line 3, Figure 3.6) corresponds to the variable `psize` of the C program (line 8, Figure 3.2).

### 3.5.2 Automated generation of annotations

An annotator tool has been implemented to generate annotations automatically, greatly reducing the annotation effort. The annotator feeds only on protocol-related annotation marks in a C program, and generates another C program that essentially expands the simple annotation marks to the more complex logic discussed in Section 3.4. The process of annotation is illustrated in Table 3.1, concerning `_collective_` and `_foreach_` marks. The `_apply_` mark is maintained and dealt with by the definition of a function contract. The implementation of the annotator tool is described in Chapter 4.

### 3.5.3 Complementary manual annotations

The verification of a program typically requires complementary annotations that must be introduced manually by a programmer. This happens for a number of distinct reasons, which we now illustrate with examples and a discussion.

A major reason for manual annotations concerns memory usage. VCC requires fine-grained detail on memory usage, particularly on loops and function contracts, as a major aim of the tool is verifying memory safety. For instance, in the finite differences program we need to complement the annotations in the main loop as follows:



Original code	Transformed code
<pre> /* Collective choices */ if( _collective_(expr)) {   ... } else {   ... } </pre>	<pre> _(ghost \SessionType _cTrue = choiceTrue(_type)); _(ghost \SessionType _cFalse = choiceFalse(_type)); _(ghost \SessionType _cCont = next(_type)); if (expr) {   _(ghost _type = cTrue;)   ...   _(\assert congruence(_type, skip())) } else {   _(ghost _type = cFalse;)   ...   _(\assert congruence(_type, skip())) } _(ghost _type = cCont;) </pre>
<pre> /* Collective loops */ while( _collective_(expr)) {   ... } /* similarly for do-while and for loops */ </pre>	<pre> _(ghost \SessionType _lBody = loopBody (_type)); _(ghost \SessionType _lCont = next(_type)); while (expr) _ampi_loop {   _(ghost _type = _lBody;)   ...   _(\assert congruence(_type, skip())) } _(ghost _type = lCont;) </pre>
<pre> /* foreach */ int var = ...; ... while( _foreach_(var, expr)) {   ... } /* similarly for do-while and for loops */ </pre>	<pre> int var = ...; ... _(ghost STMap0      fBody = foreachBody(_type)); _(ghost \SessionType fCont = foreachCont(_type)); while(expr) {   _(ghost _type = fBody[var];)   ...   _(\assert congruence(_type, skip())) } _(ghost _type = fCont;) </pre>
<pre> /* Functions with MPI calls */ int functionName (int arg1) {   ...   MPI_call (...);   ...   return arg1; } </pre>	<pre> #define functionName (a) _functionName(a _ampi_arg) ... int _functionName (int arg1 _ampi_arg_decl) _ampi_func {   ...   MPI_call (...);   ...   _ampi_on_return   return arg1; } </pre>

Table 3.1: Automatic generation of annotations

```

while (!converged(globalerr) && iter < MAX_ITER)
  _(writes &globalerr)
  _(writes \array_range(local, (unsigned) lpsize + 2))
{
  ...
}

```

The two writes clauses above identify memory that is updated within the loop, the `globalerr` variable and the array pointed to by variable `local`.

Manual annotations may also reflect implicit assumptions made by the programmer, which must be made explicit for verification of the program. For example, in the N-body program we have to introduce such an annotation, after the call to `MPI_Comm_size`, as

follows below:

```
...
MPI_Comm_size( MPI_COMM_WORLD, &procs );
_(assume procs <= MAX_P)
...
```

The program defines `MAX_P` as the maximum number of bodies, and `procs` corresponds to the number of processes. There is an implicit programmer assumption that `MAX_P` exceeds `procs` at runtime, as the program will fail to execute properly otherwise. Likewise, the program's verification will fail too, if the assumption is not made explicit.

A final reason for manual annotations concerns function contracts, which are required for modular (compositional) verification. The function contracts comprise manual annotations due to aspects discussed above, but also to the protocol verification itself. Consider a fragment of the contract for the `ComputeNewPos` function in the N-body example, as follows:

```
float ComputeNewPos( Particle particles, ParticleV pv, int npart,
                    float max_f, MPI_Comm commring
                    _ampi_arg_decl)
...
_(writes \array_range (particles, (unsigned) (npart * 4)))
_(writes \array_range (pv, (unsigned) (npart * 6)))
_(requires npart < MAX_PARTICLES)
_(requires first(_type) == action(allreduce(MPI_MIN), anyFloat
  (1)))
_(ensures _type_out == next (_type))
{
...
MPI_Allreduce( ... );
...
}
```

As shown also, `ComputeNewPos` contains a single MPI call to `MPI_Allreduce`. The contract of `MPI_Allreduce` in fact guarantees that a protocol reduction occurs through an action prefix `action(allreduce(MPI_MIN), anyFloat(1))`, but this has to be necessarily stated in the contract of `ComputeNewPos`. To deal with this burden, automated function inlining within the annotator logic may be particularly helpful in the future (given that MPI programs are typically not recursive in what regards communication logic).

### 3.5.4 Examples

An excerpt of the final result of the annotation process for our two running examples is provided in Figure 3.10 (finite differences program) and Figure 3.11 (N-body simulation program).

```

1  int main(int argc, char** argv _ampi_arg_decl) {
2  int procs;           // Number of processes
3  int rank;           // Process rank
4  MPI_Init(&argc, &argv);
5  MPI_Comm_size(MPI_COMM_WORLD, &procs);
6  MPI_Comm_rank(MPI_COMM_WORLD, &rank);
7  ...
8  int psize = atoi(argv[1]);           // Global problem size
9  _apply_(psize);
10 if (rank == 0)
11     read_vector(work, lsize * procs);
12 MPI_Scatter(work, lsize, MPI_FLOAT, &local[1], lsize, MPI_FLOAT, 0, MPI_COMM_WORLD);
13 int left = (procs + rank - 1) % procs; // Left neighbour
14 int right = (rank + 1) % procs;       // Right neighbour
15 int iter = 0;
16 // Loop until minimum differences converged or max iterations attained
17 _(\ghost \SessionType lBody = loopBody(_type);)
18 _(\ghost \SessionType lCont = next(_type);)
19 while (!converged(globalerr) && iter < MAX_ITER)
20     _(\writes &globalerr)
21     _(\writes \array_range(local, (unsigned) lsize + 2))
22 {
23     _(\ghost _type = lBody;)
24     if (rank == 0) {
25         MPI_Send(&local[1], 1, MPI_FLOAT, left, 0, MPI_COMM_WORLD);
26         MPI_Send(&local[lsize], 1, MPI_FLOAT, right, 0, MPI_COMM_WORLD);
27         MPI_Recv(&local[lsize+1], 1, MPI_FLOAT, right, 0, MPI_COMM_WORLD, &status);
28         MPI_Recv(&local[0], 1, MPI_FLOAT, left, 0, MPI_COMM_WORLD, &status);
29     } else if (rank == procs - 1) {
30         MPI_Recv(&local[lsize+1], 1, MPI_FLOAT, right, 0, MPI_COMM_WORLD, &status);
31         MPI_Recv(&local[0], 1, MPI_FLOAT, left, 0, MPI_COMM_WORLD, &status);
32         MPI_Send(&local[1], 1, MPI_FLOAT, left, 0, MPI_COMM_WORLD);
33         MPI_Send(&local[lsize], 1, MPI_FLOAT, right, 0, MPI_COMM_WORLD);
34     } else {
35         MPI_Recv(&local[0], 1, MPI_FLOAT, left, 0, MPI_COMM_WORLD, &status);
36         MPI_Send(&local[1], 1, MPI_FLOAT, left, 0, MPI_COMM_WORLD);
37         MPI_Send(&local[lsize], 1, MPI_FLOAT, right, 0, MPI_COMM_WORLD);
38         MPI_Recv(&local[lsize+1], 1, MPI_FLOAT, right, 0, MPI_COMM_WORLD, &status);
39     }
40     ...
41     MPI_Allreduce(&localerr, &globalerr, 1, MPI_FLOAT, MPI_MAX, MPI_COMM_WORLD);
42     ...
43     _(\assert congruence(_type, skip()))
44 }
45 _(\ghost _type = lCont;)
46 ...
47 _(\ghost \SessionType cTrue = choiceTrue(_type);)
48 _(\ghost \SessionType cFalse = choiceFalse(_type);)
49 _(\ghost \SessionType cCont = next(_type);)
50 if (converged(globalerr)) { // Gather solution at rank 0
51     _(\ghost _type = cTrue)
52     MPI_Gather(&local[1], lsize, MPI_FLOAT, work, lsize, MPI_FLOAT, 0, MPI_COMM_WORLD)
53     ...
54     _(\assert _type == skip())
55 } else {
56     _(\ghost _type = cFalse;)
57     ...
58     _(\assert congruence(_type, skip()))
59 }
60 _(\ghost _type = cCont)
61 MPI_Finalize();
62 return 0;
63 }

```

Figure 3.10: Annotated finite differences program

```

1  int main( int argc, char *argv[])
2  {
3      ...
4      MPI_Init( &argc, &argv );
5      MPI_Comm_rank( MPI_COMM_WORLD, &rank );
6      MPI_Comm_size( MPI_COMM_WORLD, &procs );
7      _(assume procs <= MAX_P)
8      ...
9      npart = npart / procs;
10     _apply_(npart);
11     /* Get the sizes */
12     MPI_Allgather( &npart, 1, MPI_INT, counts, 1, MPI_INT, MPI_COMM_WORLD );
13     ...
14     cnt = 10;
15     sim_t = 0.0;
16     _(ghost \SessionType lb = loopBody(_type));
17     _(ghost \SessionType lc = next(_type));
18     while (cnt > 0)
19         _(writes \array_range (particles, (unsigned) (npart * 4)))
20         _(writes \array_range (pv, (unsigned) (npart * 6)))
21     {
22         _(ghost _type = lb;)
23         ...
24         _(ghost \SessionType lb1 = loopBody(_type));
25         _(ghost \SessionType lc1 = next(_type));
26         for (pipe=0; pipe<size; pipe++)
27             _(writes \array_range (particles, (unsigned) (npart * 4)))
28             _(writes \array_range (pv, (unsigned) (npart * 6)))
29         {
30             _(ghost _type = lb1;)
31             _(ghost \SessionType ct = choiceTrue(_type));
32             _(assert congruence(choiceFalse(_type), skip()))
33             _(ghost \SessionType cc = next(_type));
34             if (pipe != procs-1) {
35                 _(ghost _type = ct;)
36                 if (rank == 0) {
37                     MPI_Send( sendbuf, npart * 4, MPI_FLOAT, right, 0, MPI_COMM_WORLD);
38                     MPI_Recv( recvbuf, npart * 4, MPI_FLOAT, left, 0, MPI_COMM_WORLD, &statuses[0] );
39                 } else {
40                     MPI_Recv( recvbuf, npart * 4, MPI_FLOAT, left, 0, MPI_COMM_WORLD, &statuses[0] );
41                     MPI_Send( sendbuf, npart * 4, MPI_FLOAT, right, 0, MPI_COMM_WORLD);
42                 }
43                 _(assert congruence(_type, skip()))
44             }
45             _(ghost _type = cc;)
46             /* Compute forces (2D only) */
47             ...
48             _(assert congruence(_type, skip()))
49         }
50         _(ghost _type = lc1;)
51         /* Once we have the forces, we compute the changes in position */
52         sim_t += ComputeNewPos( particles, pv, npart, max_f, MPI_COMM_WORLD _ampi_arg);
53         _(assert congruence(_type, skip()))
54     }
55     ...
56     _(ghost _type = lc;)
57     MPI_Finalize();
58     return 0;
59 }
60
61 double ComputeNewPos( Particle particles, ParticleV pv, int npart,
62     float max_f, MPI_Comm commring, _ampi_arg_decl)
63     _ampi_func
64     _(writes \array_range (particles, (unsigned) (npart * 4)))
65     _(writes \array_range (pv, (unsigned) (npart * 6)))
66     _(requires npart < MAX_PARTICLES)
67     _(requires first(_type) == action(allreduce(MPI_MIN),anyFloat(1)))
68     _(ensures _type_out == next (_type))
69     _(requires commring == MPI_COMM_WORLD)
70 {
71     ...
72     MPI_Allreduce( &dt_est, &dt_new, 1, MPI_DOUBLE, MPI_MIN, commring);
73     ...
74     _ampi_on_return
75     return dt_old;
76 }

```

Figure 3.11: Annotated N-body simulation program

# Chapter 4

## Design and Implementation

This chapter presents the core design and implementation aspects of the verification framework, comprising the base logic of the “annotated MPI” library (Section 4.1), and the annotator tool for automatic generation of verification annotations in the body of a program (Section 4.2). The source code for both is provided as an annex to this thesis.

### 4.1 Annotated MPI library

#### 4.1.1 Mock MPI header

Primitive	Description
<code>MPI_Init</code>	Initialize the MPI execution environment
<code>MPI_Finalize</code>	Finalize the MPI execution environment
<code>MPI_Comm_size</code>	Get number of processes
<code>MPI_Comm_rank</code>	Get process rank
<code>MPI_Send</code>	Blocking send
<code>MPI_Recv</code>	Blocking receive
<code>MPI_Bcast</code>	Broadcasts from one process to all other processes
<code>MPI_Scatter</code>	Scatters data from one process to all processes
<code>MPI_Gather</code>	Gathers data from all processes to one process
<code>MPI_Allgather</code>	All-to-all gather
<code>MPI_Reduce</code>	Reduction with result available to a single process
<code>MPI_Allreduce</code>	Reduction with result available to all processes

Table 4.1: Annotated MPI — supported communication primitives

The “annotated MPI” is a core body of definitions that can be imported in the context by an annotated C program, through the inclusion of a mock MPI header, `mpi.h`. The program may be compiled normally by using the concrete header of a MPI implementation in the place of the mock one. However, for verification purposes, the program should only contain calls to the subset of MPI communication primitives listed in Table 4.1. In

terms of size, the annotated MPI library comprises 27 files with approximately 1500 lines of verification logic.

### 4.1.2 Parameterization

In order to verify MPI programs, two core program parameters must be parameterised: the process rank and the number of processes. This kind of “glue” must be “known” by the verifier at all times, and is defined through the `mpi_glue_t` data structure shown in Figure 4.1. For every annotated MPI function, a ghost parameter of type `_mpi_glue_t` is introduced, “propagating” this parameterization.

```
typedef struct
{
  int procs; /* Number of processes */
  int rank; /* Process rank */
  _(invariant procs > 1)
  _(invariant rank >= 0)
  _(invariant rank < procs)
  _(invariant procs < 32768)
} mpi_glue_t;
```

Figure 4.1: Overall parameterization

On Figure 4.1, the constraints state that the number of processes lies between 2 to 32678, and that the value of rank is greater or equal to 0 and lower than the number of processes. Some additional but necessarily compatible constraints may be placed in the body of a program in the form of assumptions, e.g., stating a lower upper bound for the number of processes or actually making that number constant. Note also that MPI does allow for more than 32768 processes, but this bound was judged to be quite sufficient for the current stage of work and the verification experiments we conducted (Chapter 5). It should be stressed that the verification operated over MPI programs *really* is parametric according to these constraints. That is, an MPI program is checked against a protocol specification *for every* possible number of processes and *every possible* process rank (every process expressed by the program, up to the number of processes), as opposed to a particular choice of values for them.

### 4.1.3 Refinement types

```
_(ghost typedef \bool \IRefinement[\integer]);
_(ghost typedef \bool \FRefinement[float]);
```

Figure 4.2: Support for refinement types

In association to protocols, restrictions on the value domains of transmitted data can be imposed through *refinement types*. The syntax shown in Figure 4.2 defines refinement types as logical predicates, i.e., as maps from of integers (`\IRefinement`) or floats (`\FRefinement`) to boolean values. Typically, these can be expressed as anonymous functions of the form `\lambda type; predicate in VCC`. For example, `\lambda integer x; \true` is an instance of `\IRefinement` that imposes no value restrictions; so is `\lambda integer x; x > 0 && x < 100`, but which restrains the value of an integer to the (0,100) interval. The use of refined types is illustrated on the example projection functions of Chapter 3 (§ 3.4.2, Figures 3.8 and 3.9).

#### 4.1.4 Session type representation

Protocol projections are expressed as `\SessionType` terms, cf. § 3.4.2. The definition is provided on Figure 4.3. Using VCC terminology [5], `\SessionType` is an “inductively defined type”. The definition can be interpreted as a grammar where `\SessionType` is the root symbol, resembling the protocol specification grammar recognized by the Eclipse/Xtext framework (§ 3.3, Figure 3.4):

- A `\SessionData` term represents message data, with an associated refined type and length;
- An `\Action` term represents a communication action and associated arguments, if any, including `send` and `recv` to represent projections and the support collective communication primitives (e.g., `bcast`, `reduce`);
- A `\SessionType` term can either be: the terminated protocol (`skip`); a communication action (`action`); a protocol sequence (`seq`); a collective choice or loop (`choice` and `loop`); a for-each protocol (`foreach`); or, finally, a function abstraction (`val`).

The meaning and use of `STMap`, `STMap0` and `struct _vcc_math_type_STMap` is quite intricate in Figure 4.3, due to some VCC technicalities. To clarify, beginning `STMap0`, an instance of `STMap0` is a map from integers to `\SessionType` terms, e.g., as in `\lambda integer i; action(bcast(i), ...)`. An `STMap` instance merely wraps with the later with a `body` constructor, for the verification logic did not seem to work otherwise. For instance, a for-each protocol instance has the following general form: `foreach(lo, hi, body(\lambda integer i; T))`. Finally, `struct _vcc_math_type_STMap` is just the VCC way of making a forward declaration to `STMap`.

```

_(datatype \SessionData
{
  case intRef    (\IRefinement, \integer);
  case floatRef (\FRefinement, \integer);
})
_(datatype \Action
{
  case size      ();
  case val       ();
  case send      (\integer);
  case recv      (\integer);
  case bcast     (\integer);
  case gather    (\integer);
  case scatter   (\integer);
  case reduce    (\integer, MPI_Op);
  case allreduce (MPI_Op);
  case allgather ();
})
_(datatype \SessionType
{
  case skip      ();
  case action    (\Action, \SessionData);
  case seq       (\SessionType, \SessionType);
  case choice    (\SessionType, \SessionType);
  case loop      (\SessionType);
  case foreach   (\integer, \integer, struct _vcc_math_type_STMap);
  case abs       (struct _vcc_math_type_STMap);
})
_(ghost typedef \SessionType STMap0[\integer];)
_(datatype STMap {
  case body(STMap0);
})
)

```

Figure 4.3: Session type representation

```

_(pure STMap0 message(\integer from, \integer to, \SessionData sd);)
_(axiom \forall \integer from, to; \forall \SessionData sd;
  from != to ==> message(from,to,sd)[from] == action(send(to),sd))
_(axiom \forall \integer from, to; \forall \SessionData sd;
  from != to ==> message(from,to,sd)[to] == action(recv(from),sd))
_(axiom \forall \integer from, to, rank; \forall \SessionData sd;
  rank != from && rank != to ==> message(from,to,sd)[rank] == skip())

```

Figure 4.4: Message projections



### 4.1.5 Protocol projection

Protocol projections are defined by inferring `recv` and `send` actions from message terms in VCC projection functions (cf. § 3.4.2). The logic is defined as shown on Figure 4.4. The message function takes integers representing the source (`from`) and destination (`destination`) of the message, plus a session data argument (`sd`); it yields back a map from integers (process ranks) to `\SessionType` terms (an instance of `STMap0`). In line with the intended nature of projections, the projection of a message for a given rank either yields a `send(from)` action, a `recv(to)` action, or `skip()`.

### 4.1.6 Structural congruence

```

_(pure \bool congruence(\SessionType, \SessionType);)
_(axiom \forall \SessionType t; congruence(t, t) // (1)
_(axiom \forall \SessionType t1,t2,t3;
  congruence(t1,t2) && congruence(t2,t3) ==> congruence(t1,t3)) // (2)
_(axiom \forall \SessionType t; congruence(seq(skip(),t), t)) // (3)
_(axiom \forall \SessionType t1,t2,t3;
  congruence(seq(seq(t1,t2),t3), seq(t1,seq(t2,t3)))) // (4)
_(axiom \forall \SessionType t;
  \forall \integer lo,hi;
  \forall STMap b;
  lo > hi ==> congruence(seq(foreach(lo,hi,b),t),t)) // (5)
_(axiom \forall \SessionType t;
  \forall \integer lo,hi;
  \forall STMap0 f;
  lo <= hi && congruence(f[lo],skip()) ==>
  congruence(seq(foreach(lo,hi,body(f)),t),
    seq(foreach(lo+1,hi,body(f)),t))) // (6)

```

Figure 4.5: Structural congruence predicate

A structural congruence predicate is defined to identify semantically equivalent protocols. This helps the verification logic with term “rewriting” for protocol reductions. The definition is shown on Figure 4.5. It comprises axioms (numbered in comments): for (1) reflexive and (2) transitive congruence of terms; (3) recognition of the terminated protocol; (4) rewriting of `seq` terms; and, finally, (5, 6) termination of for-each iterations.

### 4.1.7 Protocol reductions

The reduction of a `\SessionType` instance occurs through the logic of MPI function contracts (cf. § 3.4.4), plus automatically generated annotations (§ 3.5.2, § 4.2, Table 3.1) for collective choices and loops (§ 3.4.5) and for-each protocols (§ 3.4.6). The associated definitions are shown on Figure 4.6. They comprise the extraction of action prefixes

(prefix), protocol continuations (next), and bodies for collective blocks and for-each iterations (loopBody, choiceTrue, choiceFalse, foreachBody).

```

_(axiom \forall \Action p;
  \forall \SessionData mt;
  \forall \SessionType t;
  first(seq(action(p, mt), t)) == action(p, mt))
_(axiom \forall \Action p;
  \forall \SessionData mt;
  \forall \SessionType t;
  next(seq(action(p, mt), t)) == t)
_(axiom \forall \SessionType t;
  \forall STMap0 f;
  \forall \integer lo, hi;
  lo <= hi ==>
    first(seq(foreach(lo, hi, body(f)), t)) == first(seq(f[lo],
      seq(foreach(lo+1, hi, body(f)), t))))
_(axiom \forall \SessionType t;
  \forall STMap0 f;
  \forall \integer lo, hi;
  (lo <= hi && !congruence(f[lo], skip())) ==>
    next(seq(foreach(lo, hi, body(f)), t))
    ==
    next(seq(f[lo], seq(foreach(lo+1, hi, body(f)), t))))
_(axiom \forall \SessionType t1, t2;
  next(seq(loop(t1), t2)) == t2)
_(axiom \forall \SessionType t1, t2, t3;
  next(seq(choice(t1, t2), t3)) == t3)
_(axiom \forall \SessionType t;
  \forall \SessionData sd;
  next(seq(action(val(), sd), t)) == t)
_(axiom \forall \SessionType t1, t2;
  loopBody(seq(loop(t1), t2)) == t1)
_(axiom \forall \SessionType t1, t2, t3;
  choiceTrue(seq(choice(t1, t2), t3)) == t1)
_(axiom \forall \SessionType t1, t2, t3;
  choiceFalse(seq(choice(t1, t2), t3)) == t2)
_(axiom \forall \SessionType t;
  \forall STMap0 f;
  \forall \integer a, b;
  foreachBody(seq(foreach(a, b, body(f)), t)) == f)

```

Figure 4.6: Protocol reduction logic

## 4.2 Program annotator

This section describe the implementation of the annotator tool. The annotator source code makes use of the Clang/LLVM framework as base infrastructure. Some Clang/LLVM background is provided first, which is then followed by an overall description of the operation and implementation of the annotator.

```
1 int main(int argc, const char **argv) {
2     CXIndex Index = clang_createIndex(0,0);
3     CXTranslationUnit tu = clang_parseTranslationUnit(Index, 0,
4         argv, argc, 0, 0, CXTranslationUnit_None);
5     CXCursor cursor = clang_getTranslationUnitCursor (tu);
6     clang_visitChildren(cursor, cursorCallback, NULL);
7     clang_disposeTranslationUnit(tu);
8     clang_disposeIndex(Index);
9     ...
10    return 0;
11 }
12 ...
13 CXChildVisitResult cursorCallback(CXCursor cursor, CXCursor parent,
14     CXClientData client_data){
15     CXCursorKind kind = clang_getCursorKind(cursor);
16     ...
17     if (kind == CXCursor_WhileStmt) {
18         // Handle 'while' loops
19         WhileStmt* stmt = (WhileStmt*) cursor.data[1];
20         ...
21     }
22     ...
23     return CXChildVisit_Recurse;
24 }
```

Figure 4.7: Example use of a Clang/LLVM cursor

### 4.2.1 The Clang/LLVM framework

Clang [4] is an open-source compiler for the C family of programming languages, built on top of the LLVM infrastructure [24] for optimization and code generation. This combination provides provide allows portability and high-performance code generation for many target platforms. Furthermore, Clang/LLVM provides an infrastructure to write tools for static analysis and transformation of programs, in the form of source code or LLVM intermediate bytecode.

The Clang C/C++ development API provides an abstract syntax tree (AST) abstraction for C programs, and associated AST traversal mechanisms, known as visitors and cursors.

Visitors allow a syntactic-driven AST traversal in the traditional sense, but backward-compatible support is not guaranteed by the head development team for future. Such guarantee exists for cursors, the choice for this work, even if it performs AST traversal using a more low-level procedural callback mechanism.

Figure 4.7 briefly demonstrates AST traversal using cursors. At line 2, the `Index` variable represents a set of translation units compiled and linked together. Line 3 is the main entry point for the Clang C API, providing the ability to parse a source file into a translation unit that can then be queried by other functions in the API. Line 5 returns the top cursor of the source file. Finally, in line 6, the function that will start the iteration on cursor is invoked and the callback function `cursorCallback` is invoked in the process. The callback function may process AST information, as shown for a `while` statement in lines 16–20, and then instruct the AST traversal to either recurse (as in line 22), stop, or resume at the same AST level.

## 4.2.2 Implementation

Figure 4.8 illustrates the operation of the annotator. The tool takes as input the C+MPI source code of a program, and yields back transformed source code with automatically generated annotations.

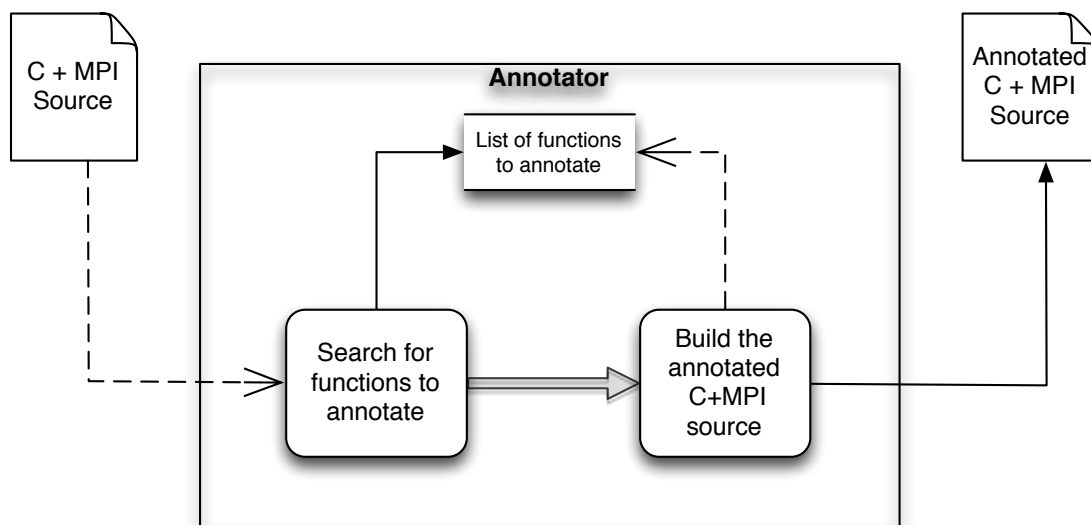


Figure 4.8: Annotator tool — overall operation

Before generating any annotated source code, the tool needs to determine the functions that need to be annotated. The base case is that of a function that uses MPI primitives or that contain protocol-related annotations introduced by the programmer (e.g., `collective_`, see Table 3.1). The annotator has to consider however if there are client (callee) functions of the later case. This cycle goes on, building a list of functions to annotate and associated

information, until it is asserted that no more functions need to be accounted.

After completing the list of functions to annotate, the tool proceeds with the generation of annotations in the output file. This stage illustrated in Figure 4.9 with an excerpt of the tool's source code. The fragment at stake relates to the generation of annotations of for-each protocol annotations (the `_foreach_` case of Table 3.1) in the Clang cursor callback function that produces the output file.

```

1  CXChildVisitResult anCyclesVisitor(CXCursor cursor, CXCursor parent,
   CXClientData client_data) {
2  CXCursorKind kind = clang_getCursorKind(cursor);
3  ...
4  if (kind >= CXCursor_WhileStmt && kind <= CXCursor_ForStmt) { // loop
5  ...
6  clang_visitChildren(cursor, aFindForeach, ad);
7  ...
8  if (ad->condExpr == 0) { // for & while loops
9      int lVarIdx = ad->gvCtr++;
10     if (ad->foreachVarname != 0){ // Foreach-case
11         // Annotations for extracting loop body and continuation
12         ad->out << "_(\ghost \SessionType _lBody" << lVarIdx
13             << " = " << "foreachBody" <<"(_type);)\n";
14         ad->out << "_(\ghost \SessionType _lCont" << lVarIdx
15             << " = " << "foreachCont" <<"(_type);)\n";
16     }else{
17         ...
18     }
19     handleCycle(kind, cursor, (Stmt*) cursor.data[1], lVarIdx, ad);
20     ad->out << "_(\ghost _type = _lCont" << lVarIdx << ");\n";
21     ...
22     return CXChildVisit_Continue;
23 } else { // do-while loop
24     ...
25 }
26 }
27 ...
28 }

```

Figure 4.9: Annotator tool — generation of annotations



# Chapter 5

## Evaluation

This chapter describes the verification framework’s evaluation over a sample set of MPI programs. The chapter begins with a summary of the chosen MPI programs for evaluation (Section 5.1). Results are then presented and discussed with respect to two metrics: the annotation effort required by verification (Section 5.2), and the execution time of VCC (Section 5.3). The VCC protocol projections and annotated program code for the sample programs are provided in Appendix A.

### 5.1 Sample MPI programs

The sample program set comprises four programs, adapted from textbooks [7, 13, 34]. Two of them were presented in Chapter 3 (§ 3.2): the finite differences [7] and N-body simulation programs [13]. The other two examples are from [34]: a vector dot product computation, and a linear system solver using the Jacobi method.

The examples had to be preliminarily adjusted in some aspects. The most important one was converting the use of non-blocking communication primitives (`MPI_Isend`, `MPI_Irecv`, `MPI_Wait` and `MPI_Waitall`) onto code that uses only blocking primitives (`MPI_Send` and `MPI_Recv`). Other aspects had to be adjusted due to limitations of VCC. For instance, VCC can not handle C functions with variable number of arguments like `printf`. VCC also can not handle floating point logic in association to flow control, e.g., the tool will crash if it finds a block of the type `if (a < b) { . . . }` where `a` and `b` are of type `float` or `double`. The latter type of logic had to be replaced with function calls and mock declarations (e.g., the `converged` function in the finite differences example).

### 5.2 Annotation effort

#### 5.2.1 Annotation process

All the examples were annotated according to the methodology described in Chapter 3. That is, part of the annotations were generated automatically and the complementary ones

were introduced manually.

The annotation took in account an arbitrary number of processes, except for the vector dot product case, due to a general shortcoming in the actual framework. Essentially, using the current framework, it is not possible to annotate functions' contracts to match a for-each protocol with a parametric number of iterations, but instead only a constant number of iterations. The finite differences and N-body example, also use for-each protocols, but only at the level of the `main` function. The issue can be dealt with automated function inlining (see § 3.5.3 for a discussion), or further work in the verification logic. The following code fragment illustrates the technical difficulty, using a fragment of the contract of the `Read_vector` function in the example at stake:

```

void Read_vector (...) {
...
  _(requires
    _ampi_rank == 0 ==>
      first(_type) == action(send(1),anyFloat(n_bar)) &&
      first(next(_type)) == action(send(2), anyFloat(n_bar)) &&
      first(next(next(_type))) == action(send(3), anyFloat(n_bar))
  )
  _(ensures
    _ampi_rank == 0 ==>
      next(next(next(_type))) == _type_out
  )
  _(requires
    _ampi_rank > 0 ==>
      first(_type) == action(recv(0),anyFloat(n_bar))
  )
  _(ensures
    _ampi_rank > 0 ==>
      next(_type) == _type_out
  )
...

```

The pre and post-condition expressing protocol reduction for rank 0 are only valid for 4 processes, since the corresponding protocol fragment to match is a simple for-each protocol, encoding messages sent from participant 0 to all other processes:

```

...
foreach(1,p-1,
  body(\lambda \integer q;
    message(0, q, anyFloat(n/p))[rank])
)

```



Program	LOC	F	A	M	A / M	M / F
Finite differences	256	8	38	17	2.2	2.1
Jacobi iteration	429	9	55	56	1.0	7.0
N-body simulation	362	14	52	26	2.0	3.7
Vector dot product	357	6	38	30	1.3	5.0

LOC: lines of code; F: annotated functions; A: automated annotations; M: manual annotations

Table 5.1: Annotation effort

## 5.2.2 Results

Table 5.1 summarizes the annotation effort. For each example, it indicates the number of code lines in each program, the number of annotated C functions, the number of automatically and manually generated annotations and their ratio, plus the ratio of manual annotations per function. In all examples except the Jacobi iteration, the number of automatically generated annotations is greater than the number of manual annotations. The results also hint at the correlation the ratio between automatic and manual annotations (A/M) and the effort of annotating function contracts manually (M/F). In the Jacobi iteration and vector dot product examples, communication code is split modularly into several functions, leading to more manual annotations. Note that the F column in the table does not refer only to contracts of functions that contain MPI calls. In fact, the finite differences example has all communication code in the `main` function, and the N-body simulation example in just two functions (`main` and `ComputeNewPos`). In these last two examples there is a much higher A/M ratio.

## 5.3 Verification time

### 5.3.1 Test setup and execution

The execution time of VCC for each of examples was measured on a Windows 7 machine with two Intel x86 2.8 GHz cores and 4 GB of RAM. For each example, separate timings imposing were took, the restriction, that is, a VCC assumption, of a fixed number of processes, powers of 2 from 4 to 64. The exception was the vector dot product that was annotated assuming a constant number of 4 processes, for the reasons discussed in the previous section.

Program	4	8	16	32	64
Finite differences	14.8	28.0	70.1	260.0	timeout
Jacobi iteration	11.0	16.0	12.4	11.5	12.0
N-body simulation	6.1	8.3	42.8	timeout	timeout
Vector dot product	2.3	N/A	N/A	N/A	N/A

Table 5.2: VCC execution time (seconds)

### 5.3.2 Results

The average times in seconds of 5 VCC executions per example are shown on Table 5.2. The timeout entries indicate that the verification did not end after 5 minutes of waiting. The first observation is a stable performance in the Jacobi iteration example, which can be explained by the exclusive use of collective communication operations, no for-each loops, and a reduced distinction between the operations of several participants. On the contrary, the finite differences and N-body simulation make use of both point-to-point and collective communications plus for-each loops, and the verification time grows exponentially with the number of processes.

# Chapter 6

## Conclusion

### 6.1 Summary of contributions

This thesis presented a verification methodology for C+MPI programs, combining the theory of multi-party session types with a deductive software verification approach. The essential idea was that of expressing communication protocols as multi-party session types, and verifying the adherence of C+MPI programs to such protocols using deductive software verification. The work is also described in peer-reviewed publications over the period of this thesis [25, 26]. In detail, the core contributions were as follows:

1. The methodology has been characterized from protocol specification to verification. This comprised the translation of protocol specifications to the deductive verification logic, the overall verification process and logic, and the annotation of a program for verification.
2. Deductive verification logic has been defined in the VCC framework comprising the definition of protocols as multi-party session types and corresponding reduction through verification. A tool has also been developed to automatically generate an important fraction of C program annotations required for verification.
3. An evaluation has been conducted using sample MPI programs. Even if the set of programs considered is still small in number and constrained in terms of features, it was sufficient to illustrate the applicability of the approach and some core challenges in the verification of MPI programs.

### 6.2 Future work

For future work, the following challenges are considered to be of core relevance:

1. Real-world programs must be considered, as opposed to simple textbook examples, to identify extra requirements and measure the overall applicability and scalability of this approach.

2. The support of a larger MPI subset is required, for instance comprising primitives for non-blocking communications or communicator creation, which are quite common in MPI programs, even in simple textbook examples.
3. More automated program annotation techniques must be developed, to alleviate the burden of manual annotation by the programmer. For instance, even relatively simple techniques such as function inlining may in principle greatly reduce the number of manual annotations in a program.
4. The inference and distinction between rank-dependent and collective program behavior may in turn also alleviate the need for protocol-related annotations, and possibly lead to a more robust verification approach. A number of concepts and techniques may be useful in that regard, e.g., parallel control-flow graphs [2], collective assertions [39], or program slicing [45].

# Appendix A

## Evaluation examples

### A.1 Finite differences

#### A.1.1 Protocol projection

```
1  _(ghost _(pure) \SessionType ftype (\binteger rank)
2  _(ensures \result ==
3  seq(
4  action(size(), intRef(\blambda \binteger y; y>0, 1)),
5  abs(body(\blambda \binteger p;
6  seq(
7  action(val(), intRef(\blambda \binteger x; x>0 && x%p==0, 1)),
8  abs(body(\blambda \binteger n;
9  seq(
10 action(scatter(0), floatRef(\blambda \bfloat v; \btrue, n)),
11 seq(
12 loop(
13 seq(
14 foreach(0, p-1,
15 body(\blambda \binteger i;
16 seq(
17 message(i, (p+i-1)%p,
18 floatRef(\blambda \bfloat v; \btrue, 1))[rank],
19 message(i, (i+1)%p,
20 floatRef(\blambda \bfloat v; \btrue, 1))[rank]))),
21 action(allreduce(MPI_MAX), floatRef(\blambda \bfloat v; \btrue, 1)))
22 ),
23 choice(
24 action(gather(0), floatRef(\blambda \bfloat v; \btrue, n)),
25 skip()
26 )))))))
27 )
```

#### A.1.2 Program code

```
1 #include <stdio.h>
2 #include <stdlib.h>
3 #include <math.h>
4 #include <mpi.h>
5
6 #define MAX_SIZE 1024
7 #define MAX_ITER 100
8 #define MAX_ERR 0.0001
9
10 #include "fdiff.h"
11
12 void read_array(float *data, int size)
13     _(requires \thread_local_array(data, (unsigned) size))
14     _(writes \array_range (data, (unsigned) size))
15 ;
16
17 void write_array(float *data, int size);
18
19 void compute(float *data, float dataNIter[], int size)
20     _(requires size >= 0)
21     _(requires \thread_local_array(data, (unsigned) size))
22     _(writes \array_range(dataNIter, (unsigned) size))
23 ;
24
25 float maxerror(float *data, float dataNIter[], int size)
26     _(requires size >= 0)
27     _(requires \thread_local_array(data, (unsigned) size))
28     _(requires \thread_local_array(dataNIter, (unsigned) size))
29 ;
30
31 int converged(float err);
32
33 void swap(float x[], float y[], int size)
34     _(writes \array_range(x, (unsigned) size))
35     _(writes \array_range(y, (unsigned) size))
36 ;
37 void read_array(float *data, int size) {
38     int i;
39
40 #ifndef _VCC_LIMITATIONS_
41     for (i = 0; i < size; i++)
42         scanf("%f", &data[i]);
43 #else
44     for (i = 0; i < size; i++)
45         data[i] = 31;
46 #endif
47 }
48
49 void write_array(float *data, int size) {
```

```
50  int i;
51  #ifndef _VCC_LIMITATIONS_
52  for (i = 0; i < size; i++)
53  printf("%f\n", data[i]);
54  #endif
55  }
56
57  void compute(float *data, float dataNIter[], int size) {
58  int i;
59  for (i = 1; i < size - 1; i++)
60  dataNIter[i] = (data [i - 1] + 2 * data [i] + data [i + 1]) / 4;
61  }
62
63  float maxerror(float *data, float dataNIter[], int size) {
64  float error = 0;
65  int i;
66  for (i = 1; i < size - 1; i++)
67  #ifndef _VCC_LIMITATIONS_
68  error += fabs(dataNIter[i] - data [i]);
69  #else
70  error = dataNIter[i] - data [i];
71  #endif
72  return error;
73  }
74
75  int converged(float err) {
76  #ifndef _VCC_LIMITATIONS_
77  return err < MAX_ERR;
78  #else
79  return 1;
80  #endif
81  }
82
83  void swap(float x[], float y[], int size) {
84  int i;
85  for (i = 0; i < size; i++) {
86  float swap_data = x[i];
87  x[i] = y[i];
88  y[i] = swap_data;
89  }
90  }
91
92  int read_size(int procs)
93  _(ensures \result >0 && \result % procs == 0 && \result < MAX_SIZE-2)
94  ;
95
96  int main(int argc, char** argv _ampi_arg_decl) {
97  int np; // Number of processes
98  int me; // Process rank
```

```

99
100 MPI_Init(&argc, &argv);
101 MPI_Comm_rank(MPI_COMM_WORLD, &me);
102 MPI_Comm_size(MPI_COMM_WORLD, &np);
103
104 _(assume _ampi_procs == 16)
105
106 if (argc < 2) // too few arguments
107 return 1;
108
109 int psize = read_size(np); // Global problem psize
110 _apply_(psize);
111
112 if (psize <= 0 || psize >= MAX_SIZE - 2 || psize % np != 0)
113 return 1; // returns in case of an invalid
114 // psize
115 float work[MAX_SIZE]; // Initial input data (used only
116 // by rank 0)
117 if (me == 0)
118 read_array(work, psize);
119
120 // Scatter input data
121 // Two extra slots in local buffer are required for boundary exchange
122 int lsize = psize / np; // Local problem psize
123 float local[MAX_SIZE]; // Local data buffer per
124 // process.
125 MPI_Scatter(work, lsize, MPI_FLOAT, &local[1], lsize, MPI_FLOAT, 0,
126 // MPI_COMM_WORLD);
127
128 float globalerr = 999.0f;
129
130 // Loop, until finite differences converge to a minimum error.
131 int iter = 0;
132 int left = (np + me - 1) % np; // Left neighbor rank
133 int right = (me + 1) % np; // Right neighbor rank
134
135 _(ghost \SessionType lb = loopBody(_type);)
136 _(ghost \SessionType lc = next(_type);)
137
138 while (!converged(globalerr) && iter < MAX_ITER)
139 {
140 _writes &globalerr
141 _writes \array_range(local, (unsigned) lsize + 2)
142 {
143 _ghost _type = lb;)
144 MPI_Status status; // MPI status data
145
146 if (me == 0) {
147 MPI_Send(&local[1], 1, MPI_FLOAT, left, 0, MPI_COMM_WORLD);

```



```

144     MPI_Send(&local[lsize], 1, MPI_FLOAT, right, 0, MPI_COMM_WORLD);
145     MPI_Recv(&local[lsize+1], 1, MPI_FLOAT, right, 0, MPI_COMM_WORLD,
             &status);
146     MPI_Recv(&local[0], 1, MPI_FLOAT, left, 0, MPI_COMM_WORLD, &
             status);
147     } else if (me == np-1) {
148     MPI_Recv(&local[lsize+1], 1, MPI_FLOAT, right, 0, MPI_COMM_WORLD,
             &status);
149     MPI_Recv(&local[0], 1, MPI_FLOAT, left, 0, MPI_COMM_WORLD, &
             status);
150     MPI_Send(&local[1], 1, MPI_FLOAT, left, 0, MPI_COMM_WORLD);
151     MPI_Send(&local[lsize], 1, MPI_FLOAT, right, 0, MPI_COMM_WORLD);
152     } else {
153     MPI_Recv(&local[0], 1, MPI_FLOAT, left, 0, MPI_COMM_WORLD, &
             status);
154     MPI_Send(&local[1], 1, MPI_FLOAT, left, 0, MPI_COMM_WORLD);
155     MPI_Send(&local[lsize], 1, MPI_FLOAT, right, 0, MPI_COMM_WORLD);
156     MPI_Recv(&local[lsize+1], 1, MPI_FLOAT, right, 0, MPI_COMM_WORLD,
             &status);
157     }
158     float nextLocal[MAX_SIZE]; // Local data for the next
             iteration
159     compute(local, nextLocal, lsize);
160     float localerr = maxerror(local, nextLocal, lsize);
161     MPI_Allreduce(&localerr, &globalerr, 1, MPI_FLOAT, MPI_MAX,
             MPI_COMM_WORLD);
162     swap (local, nextLocal, lsize + 2);
163     iter++;
164     _ (assert congruence(_type, skip()))
165     }
166     _ (ghost _type = lc;)
167
168
169     _ (ghost \SessionType ct = choiceTrue(_type);)
170     _ (ghost \SessionType cf = choiceFalse(_type);)
171     _ (ghost \SessionType cc = next(_type);)
172     if (converged(globalerr)) {
173         _ (ghost _type = ct)
174         // Gather data at rank 0 for converged solution
175         MPI_Gather(&local[1], lsize, MPI_FLOAT, work, lsize, MPI_FLOAT, 0,
             MPI_COMM_WORLD);
176
177         if (me == 0)
178             write_array(work, psize);
179         _ (assert congruence(_type, skip()))
180     } else {
181         _ (ghost _type = cf;)
182     #ifndef _VCC_LIMITATIONS_
183         printf ("failed to converge after %d iterations!", MAX_ITER);

```

```

184 #endif
185   _(\assert congruence(_type, skip()))
186   }
187   _(\ghost _type = cc;)
188
189   MPI_Finalize();
190
191   return 0;
192 }

```

## A.2 N-body simulation

### A.2.1 Protocol projection

```

1  _(\pure \SessionData anyFloat(\integer len) _(\ensures \result == floatRef
2     (\lambda float n; \true, len)));
3  _(\pure \SessionData anyInt(\integer len) _(\ensures \result == intRef(\
4     lambda \integer n; \true, len)));
5  _(\pure \SessionData natural(\integer len) _(\ensures \result == intRef
6     (\lambda \integer n; n >= 0, len)));
7
8  _(\ghost _(\pure) \SessionType ftype (\integer rank)
9     _(\ensures \result ==
10        seq(action(size(), intRef(\lambda \integer procs; procs > 0, 1)),
11            seq(abs(body(\lambda \integer procs;
12                seq(action(val(), intRef(\lambda \integer size; size > 0, 1)),
13                    seq(abs(body(\lambda \integer size;
14                        seq(action(allgather(), anyInt(procs)),
15                            seq(loop(
16                                seq(loop(
17                                    seq(choice(
18                                        seq(foreach(0, procs-1,
19                                            body(\lambda \integer i;
20                                                seq(message(i, (i+1)%procs, anyFloat(
21                                                    size * 4))[rank],
22                                                        skip()))),
23                                                    skip()),
24                                                    skip()),
25                                                    skip()))),
26                                                seq(action(allreduce(MPI_MIN), anyFloat(1)),
27                                                    skip()))),
28                                                    skip()))),
29                                                    skip()))),
30                                                    skip()))),
31                                                    skip()))),
32                                                    skip()))),
33                                                    skip()))),
34                                                    skip()))),
35                                                    skip()))),
36                                                    skip()))),
37                                                    skip()))),
38                                                    skip()))),
39                                                    skip()))),
40                                                    skip()))),
41                                                    skip()))),
42                                                    skip()))),
43                                                    skip()))),
44                                                    skip()))),
45                                                    skip()))),
46                                                    skip()))),
47                                                    skip()))),
48                                                    skip()))),
49                                                    skip()))),
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68                                                    skip()))),
69                                                    skip()))),
70                                                    skip()))),
71                                                    skip()))),
72                                                    skip()))),
73                                                    skip()))),
74                                                    skip()))),
75                                                    skip()))),
76                                                    skip()))),
77                                                    skip()))),
78                                                    skip()))),
79                                                    skip()))),
80                                                    skip()))),
81                                                    skip()))),
82                                                    skip()))),
83                                                    skip()))),
84                                                    skip()))),
85                                                    skip()))),
86                                                    skip()))),
87                                                    skip()))),
88                                                    skip()))),
89                                                    skip()))),
90                                                    skip()))),
91                                                    skip()))),
92                                                    skip()))),
93                                                    skip()))),
94                                                    skip()))),
95                                                    skip()))),
96                                                    skip()))),
97                                                    skip()))),
98                                                    skip()))),
99                                                    skip()))),
100           skip()))),
101       );
102 )

```

### A.2.2 Program code

```
1 #include <mpi.h>
2 #include <stdlib.h>
3 #include <stdio.h>
4 #include <math.h>
5 #include <string.h>
6
7 extern float drand48(void);
8 extern float fsqrt(float v);
9
10 #define MAX_SIZE 100
11 #define MAX_PARTICLES 4000
12 #define MAX_P      8
13
14 #define valParticle(v,p,e) ((v)[((p) * 4) + (e)])
15 #define valParticleV(v,p,e) ((v)[((p) * 6) + (e)])
16
17 #include "nbodypipe.h"
18
19 int lessThan (float a, float b);
20
21 int greaterThan (float a, float b);
22
23 int equalsToZero (float a);
24
25 /* Pipeline version of the algorithm... */
26 /* we really need the velocities as well... */
27 /*typedef struct {
28     float x, y, z;
29     float mass;
30     } Particle;
31 */
32
33 // x, y, z, mass
34 typedef float *Particle;
35
36 /* We use leapfrog for the time integration ... */
37 /* typedef struct {
38     float xold, yold, zold;
39     float fx, fy, fz;
40     } ParticleV;
41 */
42
43 // xold, yold, zold, fx, fy, fz
44 typedef float *ParticleV;
45
46 void InitParticles(Particle particles, ParticleV pv, int npart);
47 float ComputeForces( Particle myparticles, Particle others,
48     ParticleV pv, int npart );
49 float ComputeNewPos( Particle particles, ParticleV pv, int npart,
```

```

50         float max_f, MPI_Comm commring _ampi_arg_decl);
51
52
53 int
54 main( int argc, char *argv[] _ampi_arg_decl) {
55     float particles[MAX_PARTICLES * 4]; /* Particles on ALL nodes
56     */
57     float pv[MAX_PARTICLES * 6]; /* Particle velocity */
58     float sendbuf[MAX_PARTICLES * 4], /* Pipeline buffers */
59     recvbuf[MAX_PARTICLES * 4];
60     int counts[MAX_P], /* Number on each processor
61     */
62     displs[MAX_P]; /* Offsets into particles */
63     int rank, procs, npart, i, j,
64     offset; /* location of local particles */
65     int totpart, /* total number of particles
66     */
67     cnt; /* number of times in loop */
68     float sim_t; /* Simulation time */
69     int pipe, left, right;
70     MPI_Status statuses[2];
71
72     _(assume _ampi_procs <= MAX_P)
73     MPI_Init( &argc, &argv );
74     MPI_Comm_rank( MPI_COMM_WORLD, &rank );
75     MPI_Comm_size( MPI_COMM_WORLD, &procs );
76
77     /* Get the best ring in the topology */
78     left = (procs + rank - 1) % procs;
79     right = (rank + 1) % procs;
80
81     /* Everyone COULD have a different size ... */
82     if (argc < 2) {
83     #ifndef _VCC_LIMITATIONS_
84     fprintf( stderr, "Usage: %s n\n", argv[0] );
85     MPI_Abort( MPI_COMM_WORLD, 1 );
86     #else
87     return 1;
88     #endif
89     }
90     npart = atoi(argv[1]);
91
92     if (npart <= 0 || npart > MAX_PARTICLES || npart % procs != 0) {
93     #ifndef _VCC_LIMITATIONS_
94     fprintf( stderr, "%d is too many; max is %d\n",
95     npart*procs, MAX_PARTICLES );
96     MPI_Abort( MPI_COMM_WORLD, 1 );
97     #else
98     return 1;
99     }

```

```

96 #endif
97     }
98
99     npart = npart / procs;
100     _apply_(npart);
101     /* Get the sizes and displacements */
102     MPI_Allgather( &npart, 1, MPI_INT, counts, 1, MPI_INT,
103                 MPI_COMM_WORLD );
104     //    _(assume \forall int i; i >= 0 && i < procs ==> counts [i] <
105         MAX_PARTICLES)
106     displs[0] = 0;
107     for (i=1; i<procs; i++)
108         _(writes \array_range(displs, (unsigned) procs))
109         {
110             displs[i] = _(unchecked) (displs[i-1] + counts[i-1]);
111         }
112     totpart = _(unchecked) (displs[procs-1] + counts[procs-1]);
113
114     /* Generate the initial values */
115     InitParticles( particles, pv, npart);
116     offset = displs[rank];
117     cnt     = 10;
118
119     sim_t = 0.0f;
120     _(ghost \SessionType lb = loopBody(_type);)
121     _(ghost \SessionType lc = next(_type);)
122     while (cnt > 0)
123         _(writes \array_range (particles, (unsigned) (npart * 4)))
124         _(writes \array_range (pv, (unsigned) (npart * 6)))
125         _ampi_loop
126         {
127             _(ghost _type = lb;)
128             cnt--;
129             float max_f, max_f_seg;
130
131             /* Load the initial sendbuffer */
132             memcpy( sendbuf, particles, (unsigned long) (npart * sizeof(Particle))
133                 );
134             max_f = 0.0f;
135             _(ghost \SessionType lb1 = loopBody(_type);)
136             _(ghost \SessionType lc1 = next(_type);)
137             for (pipe=0; pipe < procs; pipe++)
138                 _(writes \array_range (particles, (unsigned) (npart * 4)))
139                 _(writes \array_range (pv, (unsigned) (npart * 6)))
140                 _ampi_loop
141                 {
142                     _(ghost _type = lb1;)
143                     _(ghost \SessionType ct = choiceTrue(_type))
144                     _(assert congruence(choiceFalse(_type), skip()))

```

```

142     _(ghost \SessionType cc = next(_type))
143 if (pipe != procs-1) {
144     _(ghost _type = ct)
145     if (rank == 0) {
146         MPI_Send( sendbuf, npart * 4, MPI_FLOAT, right, 0,
147         MPI_COMM_WORLD);
148         MPI_Recv( recvbuf, npart * 4, MPI_FLOAT, left, 0,
149         MPI_COMM_WORLD, &statuses[0] );
150     } else {
151         MPI_Recv( recvbuf, npart * 4, MPI_FLOAT, left, 0,
152         MPI_COMM_WORLD, &statuses[0] );
153         MPI_Send( sendbuf, npart * 4, MPI_FLOAT, right, 0,
154         MPI_COMM_WORLD);
155     }
156     _(assert congruence(_type, skip()))
157 }
158
159     _(ghost _type = cc;)
160     /* Compute forces (2D only) */
161     max_f_seg = ComputeForces( particles, sendbuf, pv, npart );
162     if (greaterThan(max_f_seg, max_f)) max_f = max_f_seg;
163     /* Push pipe */
164     //     if (pipe != size-1)
165     //     MPI_Waitall( 2, request, statuses );
166     memcpy( sendbuf, recvbuf, _(unchecked) ((unsigned long) counts[pipe]
167     * sizeof(Particle)) );
168     _(assert congruence(_type, skip()))
169 }
170 _(ghost _type = lc1;)
171 /* Once we have the forces, we compute the changes in position */
172 sim_t += ComputeNewPos( particles, pv, npart, max_f, MPI_COMM_WORLD
173     _ampi_arg );
174
175 /* We could do graphics here (move particles on the display) */
176 _(assert congruence(_type, skip()))
177 }
178 _(ghost _type = lc;)
179 MPI_Finalize();
180 return 0;
181 }
182
183 void InitParticles( Particle particles, ParticleV pv, int npart )
184     _(writes \array_range (particles, (unsigned) (npart * 4)))
185     _(writes \array_range (pv, (unsigned) (npart * 6)))
186     _(requires npart < MAX_PARTICLES)
187 {
188     int i;
189     for (i=0; i<npart; i++) {

```

```

189     valParticle(particles, i, 0) = drand48();
190     valParticle(particles, i, 1) = drand48();
191     valParticle(particles, i, 2) = drand48();
192     valParticle(particles, i, 3) = 1.0f;
193     valParticleV(pv, i, 0) = valParticle(particles, i, 0);
194     valParticleV(pv, i, 1) = valParticle(particles, i, 1);
195     valParticleV(pv, i, 2) = valParticle(particles, i, 2);
196     valParticleV(pv, i, 3) = 0;
197     valParticleV(pv, i, 4) = 0;
198     valParticleV(pv, i, 5) = 0;
199     }
200 }
201
202 float ComputeForces( Particle myparticles, Particle others,
203     ParticleV pv, int npart )
204     _(requires \thread_local_array (myparticles, (unsigned) (npart * 4)))
205     _(requires \thread_local_array (others, (unsigned) (npart * 4)))
206     _(writes \array_range (pv, (unsigned) (npart * 6)))
207     _(requires npart < MAX_PARTICLES)
208 {
209     float max_f, rmin;
210     int i, j;
211
212     max_f = 0.0f;
213     for (i=0; i<npart; i++) {
214         float xi, yi, mi, rx, ry, mj, r, fx, fy;
215         rmin = 100.0f;
216         xi = valParticle(myparticles, i, 0);
217         yi = valParticle(myparticles, i, 1);
218         fx = 0.0f;
219         fy = 0.0f;
220         for (j=0; j<npart; j++) {
221             rx = xi - valParticle(others, j, 0);
222             ry = yi - valParticle(others, j, 1);
223             mj = valParticle(others, j, 2);
224             r = rx * rx + ry * ry;
225             /* ignore overlap and same particle */
226             if (equalsToZero(r)) continue;
227             if (lessThan(r, rmin)) rmin = r;
228             /* compute forces */
229             r = r * fsqrt(r);
230             fx -= mj * rx / r;
231             fy -= mj * ry / r;
232         }
233         valParticleV(pv, i, 3) += fx;
234         valParticleV(pv, i, 4) += fy;
235         /* Compute a rough estimate of (1/m)|df / dx| */
236         fx = fsqrt(fx*fx + fy*fy)/rmin;
237         if (greaterThan(fx, max_f)) max_f = fx;

```

```

238     }
239     return max_f;
240 }
241
242
243 float ComputeNewPos( Particle particles, ParticleV pv, int npart,
244     float max_f, MPI_Comm commring, _ampi_arg_decl)
245     _ampi_func
246     _(writes \array_range (particles, (unsigned) (npart * 4)))
247     _(writes \array_range (pv, (unsigned) (npart * 6)))
248     _(requires npart < MAX_PARTICLES)
249     _(requires first(_type) == action(allreduce(MPI_MIN),anyFloat(1)))
250     _(ensures _type_out == next (_type))
251     _(requires commring == MPI_COMM_WORLD)
252
253 {
254     int i;
255     float a0, a1, a2;
256     static float dt_old = 0.001f, dt = 0.001f;
257     float dt_est, new_dt, dt_new;
258
259     /* integration is a0 * x^+ + a1 * x + a2 * x^- = f / m */
260     a0 = 2.0f / (dt * (dt + dt_old));
261     a2 = 2.0f / (dt_old * (dt + dt_old));
262     float minus1 = 4.0f - 5.0f;
263     a1 = minus1 * (a0 + a2);      /* also -2/(dt*dt_old) */
264
265     for (i=0; i<npart; i++) {
266         float xi, yi;
267         /* Very, very simple leapfrog time integration. We use a variable
268            step version to simplify time-step control. */
269         xi = valParticle(particles, i, 0);
270         yi = valParticle(particles, i, 1);
271         valParticle(particles, i, 0) = (valParticleV(pv, i, 3) - a1 * xi -
272             a2 *
273             valParticleV(pv, i, 0)) / a0;
274         valParticle(particles, i, 1) = (valParticleV(pv, i, 4) - a1 * yi -
275             a2 *
276             valParticleV(pv, i, 5)) / a0;
277         valParticleV(pv, i, 0) = xi;
278         valParticleV(pv, i, 1) = yi;
279         valParticleV(pv, i, 3) = 0;
280         valParticleV(pv, i, 4) = 0;
281     }
282
283     /* Recompute a time step. Stability criteria is roughly
284        2/fsqrt(1/m |df/dx|) >= dt. We leave a little room */
285     dt_est = 1.0f/fsqrt(max_f);
286     /* Set a minimum: */

```



```

285   if (lessThan(dt_est, 1.0e-6f)) dt_est = 1.0e-6f;
286   MPI_Allreduce( &dt_est, &dt_new, 1, MPI_FLOAT, MPI_MIN, commring);
287   /* Modify time step */
288   if (lessThan(dt_new, dt)) {
289     dt_old = dt;
290     dt     = dt_new;
291   }
292   else if (greaterThan(dt_new, 4.0f * dt)) {
293     dt_old = dt;
294     dt     *= 2.0f;
295   }
296
297   _ampi_on_return
298   return dt_old;
299 }

```

## A.3 Parallel Jacobi iteration

### A.3.1 Protocol projection

```

1  /* Global type:
2
3  size procs: {x:nat | N % x == 0}.
4  val(MAX_DIM) ->
5  val(N) ->
6  scatter 0 float[MAX_DIM * N].
7  scatter 0 float[N].
8  allgather float[N].
9  loop
10   allgather float[N].
11   end.
12  choice
13   gather float[N].end,
14   end.
15  end
16  */
17  -(pure \SessionData anyFloat(\integer len)
18   -(ensures \result == floatRef(\lambda float n; \true, len));)
19
20  -(ghost -(pure) \SessionType ftype (\integer rank)
21   -(ensures \result ==
22    seq(action(val(), intRef(\lambda \integer maxDim; maxDim > 0,1)),
23     seq(abs(body(\lambda \integer maxDim;
24     seq(action(size(), intRef(\lambda \integer procs; procs > 0 &&
25     maxDim % procs == 0, 1))),
26     seq(abs(body(\lambda \integer procs;
27     seq(action(val(),intRef(\lambda \integer size; size > 0 &&
28     size % procs == 0 && size <= maxDim, 1))),
29     seq(abs(body(\lambda \integer size;

```

```

28         seq(action(scatter(0),anyFloat(maxDim * size)),
29         seq(action(scatter(0),anyFloat(size)),
30         seq(action(allgather()),anyFloat(size)),
31         seq(loop(seq(action(allgather()),anyFloat(size)), skip()
           )),
32         seq(choice(seq(action(gather(0),anyFloat(size)),skip()
           , skip()),
33         skip())))))))
34         skip()))),
35         skip()))),
36         skip()))
37     );
38 )

```

### A.3.2 Program code

```

1  #include <stdio.h>
2  #include <stdlib.h>
3  #include <mpi.h>
4  #include <math.h>
5
6
7  #define MAX_DIM 1024
8  #define MAX_ITER 1000
9
10 #include "parallel_jacobi.h"
11 #ifndef _VCC_LIMITATIONS_
12 // The original definitions from Peter Pacheco
13 typedef float MATRIX_T[MAX_DIM][MAX_DIM];
14 #define val(m,r,c) ((m)[(r)][(c)])
15 #define MATRIX_DECL(var) float var[MAX_DIM][MAX_DIM]
16 #else
17 typedef float* MATRIX_T;
18 #define val(m,r,c) ((m)[((r) * MAX_DIM) + (c)])
19 #define MATRIX_DECL(var) float var[MAX_DIM * MAX_DIM]
20 #endif
21
22 void Jacobi_iteration(
23     MATRIX_T A_local /* in */,
24     float* x_local /* out */,
25     float* b_local /* in */,
26     float* x_old /* in */,
27     int n /* in */,
28     int p /* in */,
29     int my_rank /* in */
30     _ampi_arg_decl_no_type
31 )
32 _ (requires my_rank == _ampi_rank)
33 _ (requires p == _ampi_procs)
34 _ (requires n > 0 && n <= MAX_DIM)

```

```

35  _(requires n % p == 0)
36  _(requires \thread_local_array(A_local, MAX_DIM * MAX_DIM))
37  _(requires \thread_local_array(x_old, MAX_DIM))
38  _(requires \thread_local_array(b_local, MAX_DIM))
39  _(writes \array_range(x_local, MAX_DIM))
40  ;
41
42  int converged(float *x, float *y, int n)
43  _(requires n > 1)
44  _(requires \thread_local_array(x, (unsigned) n))
45  _(requires \thread_local_array(y, (unsigned) n))
46  ;
47
48
49  int Parallel_jacobi(
50      MATRIX_T  A_local    /* in */,
51      float*    x_local    /* out */,
52      float*    b_local    /* in */,
53      int       n          /* in */,
54      int       max_iter   /* in */,
55      int       p          /* in */,
56      int       my_rank    /* in */
57      _ampi_arg_decl
58  )
59  _ampi_func
60  _(requires my_rank == _ampi_rank)
61  _(requires p == _ampi_procs)
62  _(requires n > 0 && n <= MAX_DIM)
63  _(requires n % p == 0)
64  _(requires max_iter > 0 && max_iter <= MAX_ITER)
65  _(requires \thread_local_array(A_local, MAX_DIM * MAX_DIM))
66  _(requires \thread_local_array(b_local, MAX_DIM))
67  _(writes \array_range(x_local, MAX_DIM))
68  _(requires
69      first(_type) == action(allgather(),anyFloat(n)) &&
70      loopBody(next(_type)) == seq(action(allgather(),anyFloat(n)),skip())
71  )
72  _(ensures _type_out == next(next(_type)))
73  ;
74
75  void Read_matrix(
76      char*      prompt    /* in */,
77      MATRIX_T  A_local    /* out */,
78      int       n          /* in */,
79      int       my_rank    /* in */,
80      int       p          /* in */
81      _ampi_arg_decl)
82  _ampi_func
83  _(requires my_rank == _ampi_rank)

```

```

84  _(requires p == _ampi_procs)
85  _(requires n > 0 && n <= MAX_DIM)
86  _(requires n % p == 0)
87  _(writes \array_range(A_local, MAX_DIM * MAX_DIM))
88  _(requires first(_type) == action(scatter(0), anyFloat(MAX_DIM*n)))
89  _(ensures _type_out == next(_type))
90  ;
91
92
93  void Read_vector(
94      char*  prompt    /* in */,
95      float  *x_local  /* out */,
96      int    n          /* in */,
97      int    my_rank   /* in */,
98      int    p          /* in */
99      _ampi_arg_decl)
100  _ampi_func
101  _(requires p == _ampi_procs)
102  _(requires my_rank == _ampi_rank)
103  _(requires n % p == 0)
104  _(requires n > 0 && n <= MAX_DIM)
105  _(writes \array_range(x_local, (unsigned) n))
106  _(requires first(_type) == action(scatter(0), anyFloat(n)))
107  _(ensures _type_out == next(_type))
108  ;
109
110  void Print_matrix(
111      char*  title     /* in */,
112      MATRIX_T A_local /* in */,
113      int    n          /* in */,
114      int    my_rank   /* in */,
115      int    p          /* in */
116      _ampi_arg_decl)
117  _ampi_func
118  _(requires p == _ampi_procs)
119  _(requires my_rank == _ampi_rank)
120  _(requires n > 0 & n <= MAX_DIM)
121  _(requires n % p == 0)
122  _(requires \thread_local_array(A_local, MAX_DIM*MAX_DIM))
123  _(requires first(_type) == action(gather(0), anyFloat(MAX_DIM*(n))))
124  _(ensures _type_out == next(_type))
125  ;
126
127  void Print_vector(
128      char*  title     /* in */,
129      float  x_local[] /* in */,
130      int    n          /* in */,
131      int    my_rank   /* in */,
132      int    p          /* in */

```

```

133     _ampi_arg_decl)
134 _ampi_func
135 _(requires p == _ampi_procs)
136 _(requires my_rank == _ampi_rank)
137 _(requires n > 0 & n <= MAX_DIM)
138 _(requires n % p == 0)
139 _(requires \thread_local_array(x_local, (unsigned) n))
140 _(requires first(_type) == action(gather(0), anyFloat(n)))
141 _(ensures _type_out == next(_type))
142 ;
143
144 #ifndef _VCC_LIMITATIONS_
145 // The original definitions from Peter Pacheco
146 typedef float MATRIX_T[MAX_DIM][MAX_DIM];
147 #define val(m,r,c) ((m)[(r)][(c)])
148 #define MATRIX_DECL(var) float var[MAX_DIM][MAX_DIM]
149 #else
150 typedef float* MATRIX_T;
151 #define val(m,r,c) ((m)[((r) * MAX_DIM) + (c)])
152 #define MATRIX_DECL(var) float var[MAX_DIM * MAX_DIM]
153 #endif
154 int main(int argc, char** argv _ampi_arg_decl) {
155     int      p;
156     int      my_rank;
157     MATRIX_DECL(A_local);
158     float    x_local[MAX_DIM];
159     float    b_local[MAX_DIM];
160     int      n;
161     int      max_iter;
162     int      converged;
163
164     MPI_Init(&argc, &argv);
165     _apply_(MAX_DIM);
166     MPI_Comm_size(MPI_COMM_WORLD, &p);
167     MPI_Comm_rank(MPI_COMM_WORLD, &my_rank);
168
169     if (argc < 3) {
170         return 1; // invalid number of arguments
171     }
172
173     n = atoi(argv[0]);
174     max_iter = atoi(argv[2]);
175
176     if (n <= 0 || n > MAX_DIM || n % p != 0 || max_iter <= 0 || max_iter
177         >= MAX_ITER) {
178         return 1; // invalid arguments
179     }
180     _apply_(n);
181     Read_matrix("Enter the matrix", A_local, n, my_rank, p _ampi_arg);

```

```

181     Read_vector("Enter the right-hand side", b_local, n, my_rank, p
182               _ampi_arg);
183
184     converged = Parallel_jacobi(A_local, x_local, b_local, n,
185                               max_iter, p, my_rank _ampi_arg);
186     _(ghost \SessionType ta = choiceTrue(_type);)
187     _(ghost \SessionType tb = choiceFalse(_type);)
188     _(ghost \SessionType tc = next(_type);)
189     if (converged)
190     {
191         _(ghost _type = ta;)
192         Print_vector("The solution is", x_local, n, my_rank, p _ampi_arg);
193     }
194     else
195     {
196         _(ghost _type = tb;)
197         if (my_rank == 0) {
198             #ifndef _VCC_LIMITATIONS_
199                 printf("Failed to converge in %d iterations\n", max_iter);
200             #endif
201         }
202         _(assert congruence(_type, skip()))
203         _(ghost _type = tc;)
204         MPI_Finalize();
205         _ampi_on_return
206     } /* main */
207
208     #define Swap(x,y) {float* temp; temp = x; x = y; y = temp;}
209
210     int Parallel_jacobi(
211         MATRIX_T A_local /* in */,
212         float* x_local /* out */,
213         float* b_local /* in */,
214         int n /* in */,
215         int max_iter /* in */,
216         int p /* in */,
217         int my_rank /* in */
218         _ampi_arg_decl
219     )
220     {
221         int i_local, i_global, j;
222         int n_bar;
223         int iter_num;
224         float x_temp1[MAX_DIM];
225         float x_temp2[MAX_DIM];
226         float* x_old;
227         float* x_new;
228

```

```

229     n_bar = n/p;
230
231     /* Initialize x */
232     MPI_Allgather(b_local, n_bar, MPI_FLOAT, x_temp1,
233                 n_bar, MPI_FLOAT, MPI_COMM_WORLD);
234     x_new = x_temp1;
235     x_old = x_temp2;
236
237     iter_num = 0;
238     _(ghost \SessionType lb = loopBody(_type);)
239     _(ghost \SessionType lc = next(_type);)
240     do
241         _(invariant iter_num <= max_iter) // verification: to avoid
242           overflow warning
243         _(invariant \thread_local_array(x_new, MAX_DIM))
244         _(invariant \thread_local_array(x_old, MAX_DIM))
245     {
246         _(ghost _type = lb;)
247         iter_num++;
248
249         /* Interchange x_old and x_new */
250         Swap(x_old, x_new);
251
252         Jacobi_iteration(A_local, x_local, b_local, x_old, n, p, my_rank
253                        , _ampi_arg_name);
254
255         MPI_Allgather(x_local, n_bar, MPI_FLOAT, x_new,
256                     n_bar, MPI_FLOAT, MPI_COMM_WORLD);
257         _(assert congruence(_type, skip()))
258     } while ((iter_num < max_iter) && converged(x_new,x_old,n));
259     _(ghost _type = lc;)
260     _ampi_on_return
261     return 1;
262 } /* Jacobi */
263
264 void Read_matrix(
265     char*    prompt /* in */,
266     MATRIX_T A_local /* out */,
267     int      n      /* in */,
268     int      my_rank /* in */,
269     int      p      /* in */,
270     _ampi_arg_decl)
271 {
272     int      i, j;
273     MATRIX_DECL(temp);
274     int      n_bar;
275
276     n_bar = n/p;

```

```

276
277     /* Fill dummy entries in temp with zeroes */
278 #ifndef _VCC_LIMITATIONS_
279     for (i = 0; i < n; i++)
280         _(writes \array_range(temp, MAX_DIM * MAX_DIM)) {
281             for (j = n; j < MAX_DIM; j++)
282                 _(writes \array_range(&val(temp,i,n), MAX_DIM-(unsigned) n)) {
283                     val(temp,i,j) = 0.0f;
284                 }
285             }
286
287     if (my_rank == 0) {
288         printf("%s\n", prompt);
289         for (i = 0; i < n; i++)
290             for (j = 0; j < n; j++)
291                 scanf("%f",&temp[i][j]);
292     }
293 #endif
294     MPI_Scatter(temp, n_bar*MAX_DIM, MPI_FLOAT, A_local,
295               n_bar*MAX_DIM, MPI_FLOAT, 0, MPI_COMM_WORLD);
296
297     _mpi_on_return
298 } /* Read_matrix */
299
300 void Read_vector(
301     char* prompt    /* in */,
302     float *x_local /* out */,
303     int n           /* in */,
304     int my_rank    /* in */,
305     int p          /* in */,
306     _mpi_arg_decl)
307 {
308
309     int i;
310     float temp[MAX_DIM];
311     int n_bar;
312
313     n_bar = n/p;
314
315     if (my_rank == 0) {
316 #ifndef _VCC_LIMITATIONS_
317         printf("%s\n", prompt);
318         for (i = 0; i < n; i++)
319             scanf("%f", &temp[i]);
320 #endif
321     }
322
323     MPI_Scatter(temp, n/p, MPI_FLOAT, x_local, n/p, MPI_FLOAT,
324               0, MPI_COMM_WORLD);

```



```
325
326     _mpi_on_return
327 } /* Read_vector */
328
329 void Print_matrix(
330     char*    title    /* in */,
331     MATRIX_T A_local  /* in */,
332     int      n        /* in */,
333     int      my_rank  /* in */,
334     int      p        /* in */,
335     _mpi_arg_decl)
336 {
337
338     int      i, j;
339     MATRIX_DECL(temp);
340     int      n_bar;
341
342     n_bar = n/p;
343
344     _(assume \arrays_disjoint(A_local, MAX_DIM*MAX_DIM, temp, MAX_DIM*
345     MAX_DIM))
346     MPI_Gather(A_local, n_bar*MAX_DIM, MPI_FLOAT, temp,
347     n_bar*MAX_DIM, MPI_FLOAT, 0, MPI_COMM_WORLD);
348
349     if (my_rank == 0) {
350     #ifndef _VCC_LIMITATIONS_
351     printf("%s\n", title);
352     for (i = 0; i < n; i++) {
353     for (j = 0; j < n; j++)
354     printf("%4.1f ", temp[i][j]);
355     printf("\n");
356     }
357     #endif
358     }
359     _mpi_on_return
360 } /* Print_matrix */
361
362 void Print_vector(
363     char*    title    /* in */,
364     float    x_local[] /* in */,
365     int      n        /* in */,
366     int      my_rank  /* in */,
367     int      p        /* in */,
368     _mpi_arg_decl)
369 {
370
371     int      i;
372     float    temp[MAX_DIM];
373     int      n_bar;
```

```

373
374     n_bar = n/p;
375
376     MPI_Gather(x_local, n_bar, MPI_FLOAT, temp, n_bar, MPI_FLOAT,
377               0, MPI_COMM_WORLD);
378
379     if (my_rank == 0) {
380 #ifndef _VCC_LIMITATIONS_
381         printf("%s\n", title);
382         for (i = 0; i < n; i++)
383             printf("%4.1f ", temp[i]);
384         printf("\n");
385 #endif
386     }
387     _ampi_on_return
388 } /* Print_vector */

```

## A.4 Parallel dot product

### A.4.1 Protocol projection

```

1  _(ghost _(pure) \SessionData anyFloat(\integer len)
2      _(ensures \result == floatRef(\lambda float v; \true, len));
3  )
4  _(ghost _(pure) \SessionType ftype (\integer rank)
5  _(ensures \result ==
6      seq(action(size(), intRef(\lambda integer p; p == 4, 1)),
7      seq(abs(body(\lambda integer p;
8          seq(action(val(),intRef(\lambda integer n; n > 0 && n % p == 0, 1))
9
10         seq(abs(body(\lambda integer n;
11             seq(action(bcast(0), intRef(\lambda integer v; v == n, 1)),
12             seq(foreach(1,p-1, body(\lambda integer q;
13                 seq(message(0,q,anyFloat(n/p))[rank],skip()))),
14                 seq(message(0,q,anyFloat(n/p))[rank],skip()))),
15             seq(action(allreduce(MPI_SUM), anyFloat(1)),
16             seq(foreach(1,p-1, body(\lambda integer q;
17                 seq(message(q,0,anyFloat(1))[rank],skip()))),
18                 skip()))))))) ,
19         skip()))),
20     skip()))
21 );
22 )

```

### A.4.2 Program code

```

1  /* parallel_dot1.c -- Computes a parallel dot product. Uses
   MPI_Allreduce.

```

```
2  *
3  * Input:
4  *     n: order of vectors
5  *     x, y: the vectors
6  *
7  * Output:
8  *     the dot product of x and y as computed by each process.
9  *
10 * Note: Arrays containing vectors are statically allocated. Assumes
    that
11 *     n, the global order of the vectors, is evenly divisible by p, the
12 *     number of processes.
13 *
14 * See Chap 5, pp. 76 & ff in PPMPI.
15 */
16 #include <stdio.h>
17 #include <mpi.h>
18
19 #define MAX_LOCAL_ORDER 100
20 #define FAKE_USER_INPUT 60
21
22 #include "parallel_dot.h"
23
24 void Read_vector(char* prompt, float local_v[], int n_bar, int p,
25                int my_rank _mpi_arg_decl)
26 ;
27
28 float Parallel_dot(float local_x[], float local_y[], int n_bar
29                  _mpi_arg_decl)
29 _mpi_func
30 _{(requires n_bar > 0)
31 _{(requires \thread_local_array(local_x, (unsigned) n_bar))
32 _{(requires \thread_local_array(local_y, (unsigned) n_bar))
33
34 _{(requires
35     first(_type) == action(allreduce(MPI_SUM),anyFloat(1))
36     )
37 _{(ensures
38     next(_type) == _type_out
39     )
40 ;
41
42 void Print_results(float dot, int my_rank, int p _mpi_arg_decl)
43 _mpi_func
44 _{(requires p == _mpi_procs)
45 _{(requires my_rank == _mpi_rank)
46 _{(requires _mpi_procs == 4)
47
48 _{(requires
```

```

49  _mpi_rank == 0 ==>
50  first(_type) == action(recv(1), anyFloat(1)) &&
51  first(next(_type)) == action(recv(2), anyFloat(1)) &&
52  first(next(next(_type))) == action(recv(3), anyFloat(1))
53  )
54  _(ensures
55  _mpi_rank == 0 ==> next(next(next(_type))) == _type_out
56  )
57
58  _(requires
59  _mpi_rank > 0 ==> first(_type) == action(send(0),anyFloat(1))
60  )
61  _(ensures
62  _mpi_rank > 0 ==> next(_type) == _type_out
63  )
64  ;
65
66  float Serial_dot(float x[], float y[], int n)
67  _(requires n > 0)
68  _(requires \thread_local_array(x, (unsigned) n))
69  _(requires \thread_local_array(y, (unsigned) n))
70  ;
71
72  void Scan_vector(float v[], int len)
73  _(writes \array_range (v, (unsigned) len))
74  ;
75
76  int
77  main(int argc, char* argv[] _mpi_arg_decl)
78  {
79  float local_x[MAX_LOCAL_ORDER];
80  float local_y[MAX_LOCAL_ORDER];
81  int n;
82  int n_bar; /* = n/p */
83  float dot;
84  int p;
85  int my_rank;
86
87  _(assume _mpi_procs == 4)
88  MPI_Init(&argc, &argv);
89  MPI_Comm_size(MPI_COMM_WORLD, &p);
90  MPI_Comm_rank(MPI_COMM_WORLD, &my_rank);
91
92  if (my_rank == 0) {
93  #ifndef _VCC_LIMITATIONS_
94  printf("Enter the order of the vectors\n");
95  scanf("%d", &n);
96  #else
97  n = FAKE_USER_INPUT;

```

```

98 #endif
99     }
100
101     _(assume n > 0 && n < MAX_LOCAL_ORDER)
102     _(assume n % p == 0)
103     _apply_(n);
104     MPI_Bcast(&n, 1, MPI_INT, 0, MPI_COMM_WORLD);
105
106     n_bar = n/p;
107
108     _(assert
109     my_rank == 0 ==> first(_type) == action(send(1),anyFloat(n_bar))
110     )
111     Read_vector("the first vector", local_x, n_bar, p, my_rank _mpi_arg)
112     ;
113     Read_vector("the second vector", local_y, n_bar, p, my_rank _mpi_arg
114     );
115     dot = Parallel_dot(local_x, local_y, n_bar _mpi_arg);
116     Print_results(dot, my_rank, p _mpi_arg);
117
118     MPI_Finalize();
119     _mpi_on_return
120     return 0;
121 } /* main */
122
123 void Scan_vector(float v[], int len) {
124 #ifndef _VCC_LIMITATIONS_
125     printf("Enter %s\n", prompt);
126     int i;
127     for (i = 0; i < len; i++)
128         scanf("%f", &v[i]);
129 else
130     int i;
131     for (i = 0; i < len; i++)
132         v[i] = 27;
133 #endif
134 }
135
136 void Read_vector(
137     char* prompt /* in */,
138     float local_v[] /* out */,
139     int n_bar /* in */,
140     int p /* in */,
141     int my_rank /* in */
142     _mpi_arg_decl)
143     _mpi_func
144     _(requires n_bar > 0 && n_bar < MAX_LOCAL_ORDER)

```

```

145  _(requires \thread_local_array(local_v, (unsigned) n_bar))
146  _(requires p == _mpi_procs)
147  _(requires my_rank == _mpi_rank)
148  _(writes \array_range (local_v, (unsigned) n_bar))
149  _(requires _mpi_procs == 4)
150
151  _(requires
152    _mpi_rank == 0 ==>
153    first(_type) == action(send(1),anyFloat(n_bar)) &&
154    first(next(_type)) == action(send(2), anyFloat(n_bar)) &&
155    first(next(next(_type))) == action(send(3), anyFloat(n_bar))
156  )
157  _(ensures
158    _mpi_rank == 0 ==> next(next(next(_type))) == _type_out
159  )
160
161  _(requires
162    _mpi_rank > 0 ==> first(_type) == action(recv(0),anyFloat(n_bar))
163  )
164  _(ensures
165    _mpi_rank > 0 ==> next(_type) == _type_out
166  )
167
168
169  {
170    int i, q;
171    float temp[MAX_LOCAL_ORDER];
172    MPI_Status status;
173
174    if (my_rank == 0) {
175      Scan_vector (local_v, n_bar);
176
177      // send to 1
178      Scan_vector (temp, n_bar);
179      MPI_Send(temp, n_bar, MPI_FLOAT, 1, 0, MPI_COMM_WORLD);
180
181      // send to 2
182      Scan_vector (temp, n_bar);
183      MPI_Send(temp, n_bar, MPI_FLOAT, 2, 0, MPI_COMM_WORLD);
184
185      // send to 3
186      Scan_vector (temp, n_bar);
187      MPI_Send(temp, n_bar, MPI_FLOAT, 3, 0, MPI_COMM_WORLD);
188    } else {
189      MPI_Recv(local_v, n_bar, MPI_FLOAT, 0, 0, MPI_COMM_WORLD,
190              &status);
191    }
192    _mpi_on_return
193  } /* Read_vector */

```

```
194
195
196 /*****
197 float Serial_dot(
198     float x[] /* in */,
199     float y[] /* in */,
200     int n /* in */) {
201     int i;
202     float sum = 0.0f;
203
204     for (i = 0; i < n; i++)
205         sum = sum + x[i]*y[i];
206     return sum;
207 } /* Serial_dot */
208
209
210 /*****
211 float Parallel_dot(
212     float local_x[] /* in */,
213     float local_y[] /* in */,
214     int n_bar /* in */
215     _mpi_arg_decl) {
216
217     float local_dot;
218     float dot = 0.0f;
219
220     local_dot = Serial_dot(local_x, local_y, n_bar);
221     MPI_Allreduce(&local_dot, &dot, 1, MPI_FLOAT,
222                 MPI_SUM, MPI_COMM_WORLD);
223     _mpi_on_return
224     return dot;
225 } /* Parallel_dot */
226
227
228 /*****
229 void Print_results(
230     float dot /* in */,
231     int my_rank /* in */,
232     int p /* in */
233     _mpi_arg_decl) {
234     int q;
235     float temp;
236     MPI_Status status;
237
238     if (my_rank == 0) {
239 #ifndef _VCC_LIMITATIONS_
240         printf("dot = \n");
241         printf("Process 0 > %f\n", dot);
242         MPI_Recv(&temp, 1, MPI_FLOAT, 1, 0, MPI_COMM_WORLD,
```

```
243         &status);
244     printf("Process %d > %f\n", 1, temp);
245     MPI_Recv(&temp, 1, MPI_FLOAT, 2, 0, MPI_COMM_WORLD,
246             &status);
247     printf("Process %d > %f\n", 2, temp);
248     MPI_Recv(&temp, 1, MPI_FLOAT, 3, 0, MPI_COMM_WORLD,
249             &status);
250     printf("Process %d > %f\n", 3, temp);
251 #else
252     MPI_Recv(&temp, 1, MPI_FLOAT, 1, 0, MPI_COMM_WORLD,
253             &status);
254     MPI_Recv(&temp, 1, MPI_FLOAT, 2, 0, MPI_COMM_WORLD,
255             &status);
256     MPI_Recv(&temp, 1, MPI_FLOAT, 3, 0, MPI_COMM_WORLD,
257             &status);
258 #endif
259     } else {
260         MPI_Send(&dot, 1, MPI_FLOAT, 0, 0, MPI_COMM_WORLD);
261     }
262     _mpi_on_return
263 } /* Print_results */
```







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