Shuihudi's Bamboo Strips of Qin Dynasty and Mathematics in Pre-Qin Period

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The bamboo strips of the laws of the Qin Dynasty unearthed in 1975 from Tomb 11 (which was buried in about 217 BCE) of Shuihudi 睡虎地 Site are very valuable for the researches on the history of science and technology of China; But their importance has not been paid enough attention. Although Mr. Guo Shirong 郭世荣 and Mr. Feng Lisheng 冯立升 have discussed the relationships between mathematics and social economy with the references of bamboo strips from Shuihudi and some Han remains, their discussions were brief and general, and were mainly limited to the Qin and Han Dynasties. In this paper, I would like to present my discoveries of their great significance on rebuilding the history of mathematics in the pre-Qin period.

It has been believed in the academia that the *Nine Chapters on the Mathematical Procedures* 九章算术, which formed the main pattern of Chinese traditional mathematics, was the most complete collection of the mathematical methods and achievements of the pre-Qin period and the Qin and Han Dynasties; However, the portion of the pre-Qin mathematics in it has hardly been defined. Mr. Qian Baocong 钱宝琮 believed that most of the first five chapters were handed down from the pre-Qin period in the consideration that there had been social demands for this kind of mathematical knowledge; Mr.Guo Shuchun 郭书春, moreover, thought that most of its contents were handed down from the pre-Qin period; in addition to the social demands, he got this conclusion by analyzing the consistency of the book's organization to Liu Hui's 刘徽 records of the compiling of *Nine Chapters* 九章 which was done in the third century CE, and considering Liu Hui's devotion to the pursuit of truth. Anyway, their reasoning processes were too simple and general to fill the gap of their arguments and their conclusions. I plan to prove that the main mathematical methods in the *Nine Chapters* were handed down from pre-Qin period by discussing and synthesizing the mathematical information in Qin laws on bamboo strips unearthed from Shuihudi Site, the *Nine Chapters* and its preface and commentary written by Liu Hui, *Suanshu Shu* 算数书 (Writings on Reckoning) unearthed from a Western Han tomb buried in c.186 BCE and literature handed down from the ancient time.

Observing the Pre-Qin Mathematics through the Strict Demands to Accounting and Statistics

Statistics and accounting are very important for governments to effectively manage the nation; the "*Ji* 计" (calculating), "*Kuai* 会" and "*Kuaiji* 会计" (accounting or accountant) and "*Jishu* 计数" (counting) numerously mentioned in the pre-Qin historic literature were all actually including the statistics and accounting jobs, but had wider scope than present-day statistics and accounting (relevant management jobs were also included, e.g. assessment and distribution). Statistics and accounting required the staff members to be proficient to the relevant calculating methods. The strict requirements to the statistics and accounting works recorded in the Qin laws reflected the high mathematical level needed by the officials in charge of these jobs.

In *Xiaolü* 效律 (statutes concerning checking) of the Qin laws in Shuihudi bamboo strips, the punishment to the accounting mistakes were: If the error was less than 220 cashes, the Overseer of the office would be blamed; if the error was between 220 and 2200 cashes, he would be fined a shield; if the error was more than 2200 cashes, he would be fined a set of armor. If one household or

one piece of livestock (cattle or horse) was miscounted, he would be fined a shield; two or more were miscounted, he would be fined a set of armor.

Moreover, if the difference between the numbers on account and the actual numbers was more than those limited by the law, or the account was canceled while it should have been kept, the difference and the lost account should be evaluated; if the loss was less than 22 cashes, the officials concerned would be forgiven; if the loss was between 22 and 660 cashes, the Overseer of the office would be fined a shield; if the loss was more than 660 cashes, he would be fined a set of armor, and he would be furthermore charged with the value of what he had canceled. Miscounting one household or one piece of livestock (or more) was serious mistake; if he traced this mistake himself, the punishment would be reduced by one grade. From these articles, we can see that the punishment of the Qin laws to the economic mistakes was very severe. These strict articles prevented the officials from corrupting and urged them to grasp management abilities, including high-level knowledge of mathematics and abilities of calculating.

The Qin laws defined the responsibilities of the superior officials of the staff who made mistakes on accounting. It was also recorded in *Xiaolü* that if the county defender's accountants and the functionaries of the county defender's office made punishable mistakes, the county magistrate and his assistant should also take responsibility as in the case of other offices; the clerk of the Controller of Horses compiled the accounts of a stud-farm; if the accounts had mistakes, the Controller of Horses should take the responsibility just as in the case he should take the responsibility of the accounting mistakes of the office. It seems that the law of ascertaining responsibilities of officials whose subordinates made mistakes was also very strict. That *Xiaolü* did not avoid Qin Shihuang's name hints that it might be completed before the first year of Qin Shihuang's reign (246 BCE) and issued in the Warring States period. Large amounts of counting and calculating should be used in economic managements especially in statistics and accounting; the strict articles in the Qin laws suggested that there must have been many officials and clerks who were skillful on relevant methods of counting and calculating. Therefore, we should not underestimate the mathematical achievements in the Warring States Period.

The Origins of the Algorithm of Proportion and Proportional Allocation in the *Nine Chapters on the Mathematical Procedures*

Taking the proportion method as the key algorithm, the *Sumi* 粟米 (foodstuffs and treated foodstuffs) of *Nine Chapters on the Mathematical Procedures* were about the mathematical methods of conversion of foodstuffs and treated foodstuffs and other related methods. This chapter was started with a conversion table; by comparative studies on this table, the "*Mi* 米" section of *Shuowen Jiezi* 说文解字 (Explaining Radicals and Analyzing Compound Characters, ca. CE 100), the *Canglü* 仓律 (statutes on granaries) of Shuihudi Qin bamboo strips and the *Suanshu Shu* 算数书 on bamboo strips of the Western Han Dynasty from Zhangjiashan 张家山 Cemetery, Hubei 湖北 Province, we can confirm some right viewpoints as well as correct wrong opinions in our academic field. The *Nine Chapters* has a noticeable mistake which occurred all over this book, which was that the rates of *Zuomi* 糳米 (a kind of hulled millet) and the corresponding *Zuofan* 糳饭 (cooked *Zuo*). In this book, all "*Zuo* 糳" should have been "*Hui* 毅 (highly hulled millet)" : This implied that this mistake must have existed when the *Nine Chapters* was completed in the later period of the Western Han Dynasty, therefore this chapter could not be directly written by officials and clerks in charge of grains or compiled with textual materials left by them, but was completed by some scholars who had not actually practiced this job. The rate of "*Bai* 粺 (a kind of hulled millet)" recorded in the *Nine Chapters* was different from that in *Shuowen Jiezi*, and *Suanshu Shu* claimed that the changing of this rate by former scholars based on *Shuowen Jiezi* was wrong and showed that the *Sumi* Chapter of the *Nine Chapters* had much earlier origins. In his preface of the commentary on the *Nine Chapters*, Liu Hui marked that Zhang Cang 张仓 (252 or earlier–152 BCE) and Geng Shouchang 耿寿昌 (Middle of 1st century BCE) collected the remains of the pre-Qin *Nine Chapters on the Mathematical Procedures* survived from the "Burning books and burying Confucian scholars alive" conducted by Qin Shihuang. Working on the basis of the remains of the book, they made some deletions and supplements, and adjusted its subjects. They rewrote the book with many expressions of that time and finally they formed the Han version of the *Nine Chapters on the Mathematical Procedures*. The fact that *Canglü* 仓律 written before 246 BCE recorded the ratios of conversions of grains

processed in different grades of fineness reflected the actual demands existing at that time; Referring to the algorithm and questions recorded in *Suanshu Shu*, we can infer that the mathematical methods in *Sumi* Chapter of the *Nine Chapters* must have emerged in pre-Qin period and its contents might also have been handed down from that time.

The *Gongren Cheng* 工人程 (Norms of Productivity for the Laborers) regulated the proportions of the efficiencies of laborers in different ages, genders and strengths, and *Canglü* defined the standards of food allotment to the laborers in different statuses, such as *Lichen* 隶臣 (bond-servants), *Liqie* 隶妾 (female bond-servants), *Xiao Liqie* 小隶妾 (young female bond-servants) and infant laborers when they were on official duties, and the standards were being changed according to situations and times, the proportions of which had values with fractional numbers, showing that the complicated fraction operation was thoroughly mastered by the officials and clerks in charge of these issues. In these official activities, many situations might occur; for example, to know how long and how many people a given amount of grains could supply, and when a given amount of grains could not supply a given number of people for a given duration of time, how much more grains should be transported in, and so on. Under these situations, calculating methods including the methods of distribution by proportion (called *Cuifen Shu* 衰分术 in ancient time) must have been applied, which were suitable for fractional operations, and the relevant questions were more complicated than many questions in *Cuifen* 衰分 (Distribution by Proportion) Chapter of the *Nine Chapters*. Obviously, the methods of distribution by proportion emerged in the pre-Qin period. Referring to Liu Hui's records, we can judge that the *Cuifen* Chapter of the *Nine Chapters* must have had pre-Qin origins.

The Issues of *Shanggong* and *Junshu* Observed from the Angle of Qin Bamboo Strips and Pre-Qin Literature

The *Shanggong* 商功 (Construction Consultations) Chapter of the *Nine Chapters* gave algorithms of volume calculating in many shapes and the conversions of different kinds of work under different environments. The typical problem of *Junshu* 均输 (transportation based on fair burden of levies) was to calculate the amounts of foodstuff of laborer distributed among the units (for example, several counties) on principle that each person, household, or *suan* 算 (a unit of tax, usually taking an adult laborer as a standard) should bear equal burden which is considered to be influenced by several factors such as distance, price of good, price of labor, etc. Very complicated methods for these issues were recorded in this book. In the *Gougu* 勾股 (Right-angled Triangles) Chapter, methods of surveying and measuring height, depth, width and distances were recorded. Considering and consulting to other references, we can get the conclusion that these methods emerged in the pre-Qin period.

As noted by *Guo Yu* 国语 (Discourse on the States), Confucius narrated that the former Kings levied taxes in different ranks according to the fertility of the farmlands, the physical strengths of the laborers (of which age was an important factor), the distances of the taxpayers to their destinations and so on. This is clearly thought of "transportation based on fair burden of levies," in which at least three parameters have been involved. It was also pointed out in the *Chapter of Wang Zhi* 王制 (Regulations of King) of *Xun Zi* 荀子 that when the taxes were levied, the status of markets, resources of mountains and lakes, fertility of farmlands, distances of the taxpayers to their destinations and the prices of grains and goods should be considered. Here five factors were given and at least the last three must be referred to when the quota of certain taxpayer was defined. It was noted in *Zuo Zhuan* 左传 (Commentary on Spring and Autumn Annals attributed to Zuoqiu Ming 左丘明) that when the Yi 沂 City was built in the 11th year of Duke Xuan's reign (598 BCE) and Chengzhou 成周 City was built in the 32nd year of Duke Zhao's reign (510 BCE), plans were made and many issues and problems were considered and analyzed in advance, such as the topography of the sites, the shapes of the city walls, the nature of the soils, the distances of two relevant places, the work amounts of all involved jobs, the time needed and the relevant manpower, goods, grains and the budgets, the matching of planks and poles used in building tamped-earth walls, the matching of amounts of earth being transported and tamped and the corresponding manpower assigned, the total amounts of earth and other building materials needed in the whole construction (no excess, no deficit) and the examination of the officials and clerks in charge of the details of the constructions. In these issues, the calculation of volumes of three-dimensional objects, the conversion of work amounts in different types and the coordination and matching of the factors related to the engineering were involved. For example,

digging, transporting and tamping earth belonged to the same workflow; from the view of mathematical calculating, they were a set of parameters related to and influencing each other, while digging earth was related to the firmness of earth, transporting earth was related to the distances from the earth quarries to the building sites. To make all the links in the workflow be matched well, the assigning of manpower had to be planned under the principle that the amount of earth processed in each link in a given time must be equal to that in other two links. Methods based on this principle had similar mathematical model with the *Junshu Shu* 均输术 (method of transportation based on fair burden of levies) in the *Nine Chapters*. The difference was just that one was to even up the basic burdens on taxpayers while the other to equalize the earth processed in every link of a workflow. In fact, the factors considered in planning large-scale constructions at that time were many more than those mentioned in the *Nine Chapters*; not only the methods in *Junshu* type but also those in *Shanggong* were applied, or even the geometric measuring method in *Gougu*. In the Warring States period, some large-scale irrigation works were conducted in areas with very complex terrains where indirect surveying and measuring techniques had to be used, even possibly including Gougu Theorem 勾股定理 (Pythagorean Theorem).

However, the narrations in these ancient literatures could only prove or hint that the mathematical methods similar to those in the *Nine Chapters* were required or applied in the pre-Qin period; the articles of the law on the unearthed Qin bamboo strips confirmed that the sophisticated mathematical methods like those in the *Nine Chapters* have been used at latest in the Warring-States period.

The punishments to the mistake-makers were also defined in the *Yaolü* 徭律 (statutes on statute labor); For example, the city wall built by the drafted statute laborers should be guaranteed for one year. If the wall was eroded or collapsed within this year, the *Sikong* 司空 (Controller of Works) in charge of the whole project and the *Junzi* 君子 (Officer in charge of a specific task) in charge of building this section of wall would be adjudicated and the laborers would be ordered to build this section of wall again and the time used in the rebuilding would not be included in the normal term of corvée. The work amounts of routine engineering in the counties and construction projects in the counties which were reported to and approved by the higher officials should be estimated in advance by the officials of the counties. If the difference of the actual time of construction and the estimated term proved to be an excess or a deficit of two days or more, the responsible officers will be adjudicated as "lack of perspicacity." As for the engineering and project in administrative divisions higher than county, the punishments were correspondingly charged. The estimation of the work amounts must be done by the Controller of Works in charge of the whole project together with *Jiang* 匠 (Craftsman) but not by the Craftsman only. If mistakes were made in estimating, the estimators would be adjudicated according the statutes, and then the number of laborers needed for the project would be re-calculated under actual situation. These severe rules to the project planning and quality and process examining set strict demands to the grasping of the mathematical methods related to the types in *Shanggong*, *Junshu*, *Gougu* and so on, and therefore, propelled their development, if these methods were not mature enough when these laws and rules began to go into effect. The strict punishments of the Qin laws to the responsible officials of these incorrect estimations, which might be second to none in Chinese history, would be seen as the expression of the maturity of these mathematical methods. Then, it can be proven that the typical methods recorded in the Chapters of *Shanggong* and *Junshu*, and some methods in the Chapter of *Gougu* had been fully developed in the pre-Qin period; referred to Liu Hui's commentaries and the algorithms recorded in *Suanshu Shu*, the main contents of the *Nine Chapters on the Mathematical Procedures* are certified to emerge before the unification of the Qin Dynasty.

The *Nine Chapters* and Pre-Qin Mathematics Observed from Qin Laws

The above discussions showed that in the Warring States period, the enacting and implementing of laws must have been based on the mathematical knowledge highly developed at that time. On the other hand, some states (especially the Qin State) had strict laws that urged the progresses of mathematics by demanding accurate and precise data. From the view of social background, the requirement to high-level mathematical knowledge in the Han Dynasty was not as strong as that in the Warring States period. Therefore, albeit the extant *Nine Chapters* was completed in the later stage of the Western Han Dynasty, the main methods and operations in it could only emerge in the Warring States period or earlier, and some of them might emerge in the Qin Dynasty but

not in the Han; the influences of the Legalists to the mathematics also occurred in the pre-Qin period or the Qin Dynasty but not in the Han Dynasty.

As early as in the Western Zhou Dynasty (11th Century–771 BCE), mathematics became one of the core courses studied by the children of the noble people; At least some counting and calculating methods based on decimal place-value notation system and four arithmetic operations of integers had been invented and applied by that time. In Spring and Autumn period, fractions were widely used in calculation, four fundamental operations based on the Multiplication Table and application of counting rods were popularized and mathematical methods similar to those in the Chapters of *Cuifen*, *Shanggong*, *Junshu* and *Gougu* of the *Nine Chapters* were invented and used, which laid firm foundation for the future development of Chinese mathematics. In the Warring States period, Schools of Mohists and Logicians conducted researches on rules of logic reasoning and on some concepts and propositions of or related to mathematics. These reflected that the theoretical and logical researches had been good enough to create the mathematical methods in the *Nine Chapters*. The new-discovered *Suanshu Shu*, which was completed before 186 BCE, did not have direct textual relationship with the *Nine Chapters*; it is a collection of selected contents from many mathematical works, which must have been written or edited much earlier and have more contents not quoted in *Suanshu Shu*. However, just the mathematical knowledge collected in it which was popular in the early stage of the Western Han or earlier periods has made it conceivable to date the creation of the mathematical methods in the *Nine Chapters* back to the pre-Qin period.

In short, in the light of the primitive accumulation, the needed conditions for mathematical reasoning, the social economic backgrounds and social demands for the development of mathematical knowledge, together with Liu Hui's commentaries and the nature and contents of *Suanshu Shu* unearthed from Zhangjiashan Han cemetery, we can take the conclusion that the main mathematical methods in the *Nine Chapters* emerged in the pre-Qin period. In the demonstration of this opinion, the Qin laws in Shuihudi bamboo strips provided us a reliable exact temporal mark to confirm that many typical methods especially those complex ones were created in Pre-Qin Period.

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