Chapter 11: Limits of Propagation (Costas Array)

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ECLiPSe ELearning Overview

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Outline

1. Problem
2. Program
3. Search
4. Improvements

What we want to introduce

- Improving propagation does not always pay
- For some problems, simple backtracking is best
- CP may not always be the best method
- CP should always be fastest way to model problem
- Consider time to target
  - Time required to *run* program
  - Time required to *write* program
- Problem: Costas Array (Antenna design, sonar systems)

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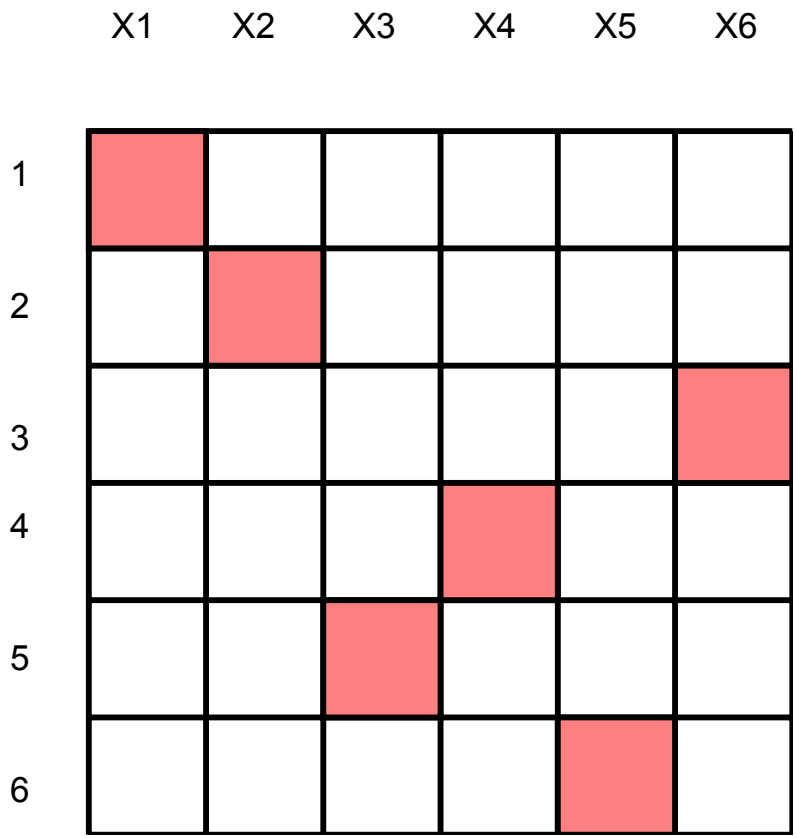
Problem Definition

Costas Array (Wikipedia)

A Costas array (named after John P. Costas) can be regarded geometrically as a set of N points lying on the squares of a NxN checkerboard, such that each row or column contains only one point, and that all of the N(N - 1)/2 vectors between each pair of dots are distinct.

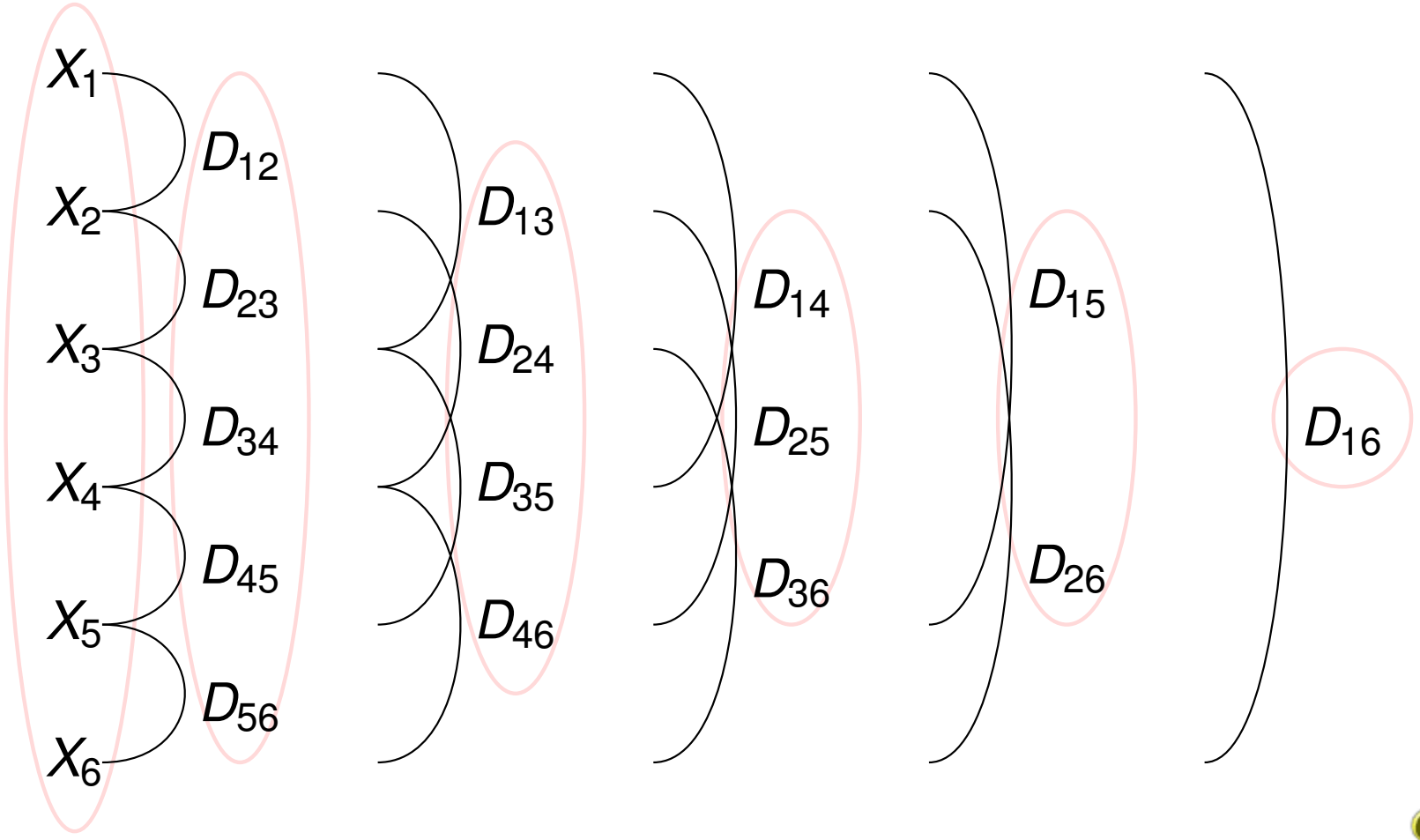
Example (Size 6)



Model

- A variable for each column, ranging from 1 to N
- A list of N variables for the columns
- A difference variable between each ordered pair of variables
- `alldifferent` constraint between variables
- `alldifferent` constraints for all differences

Model


X_1
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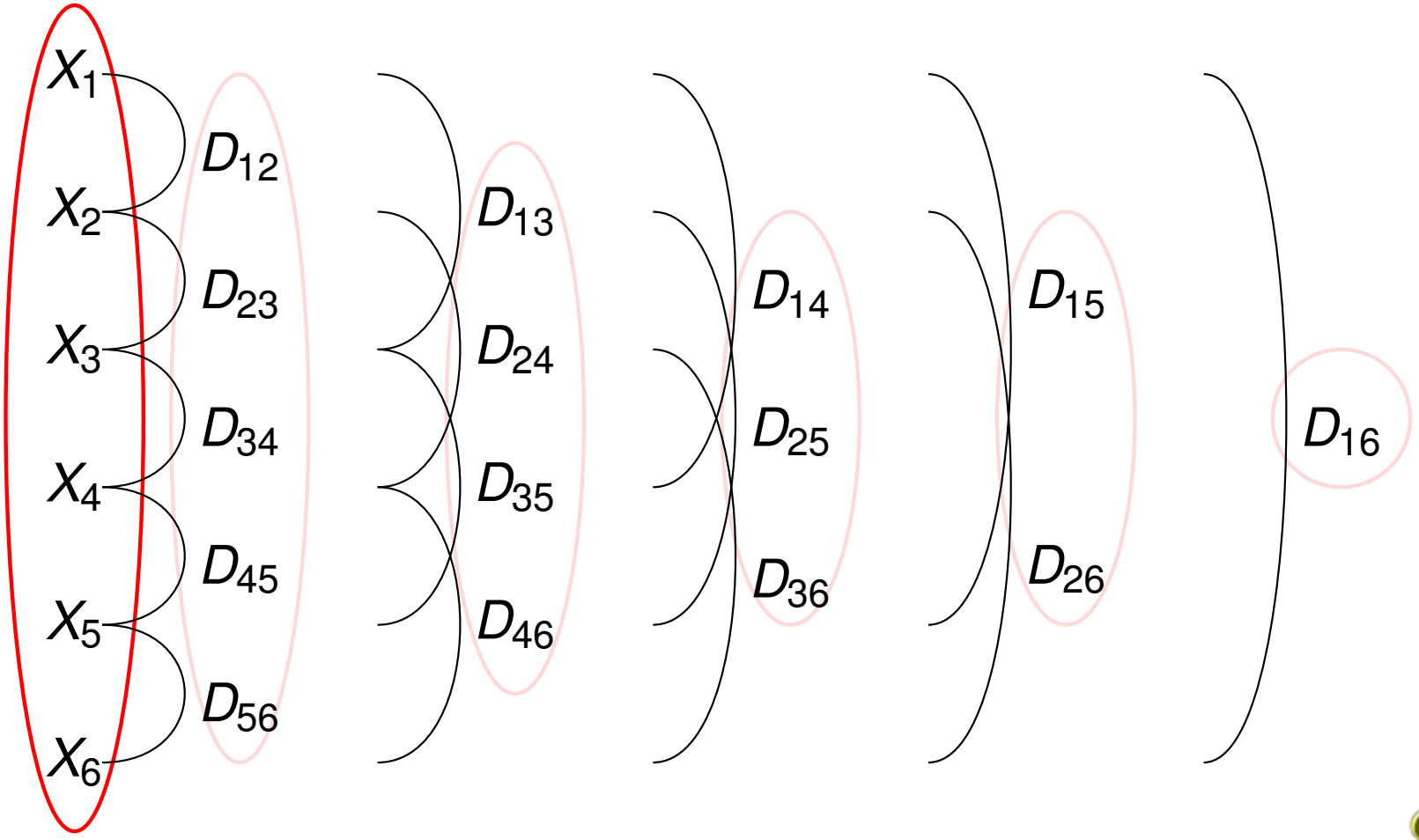
Model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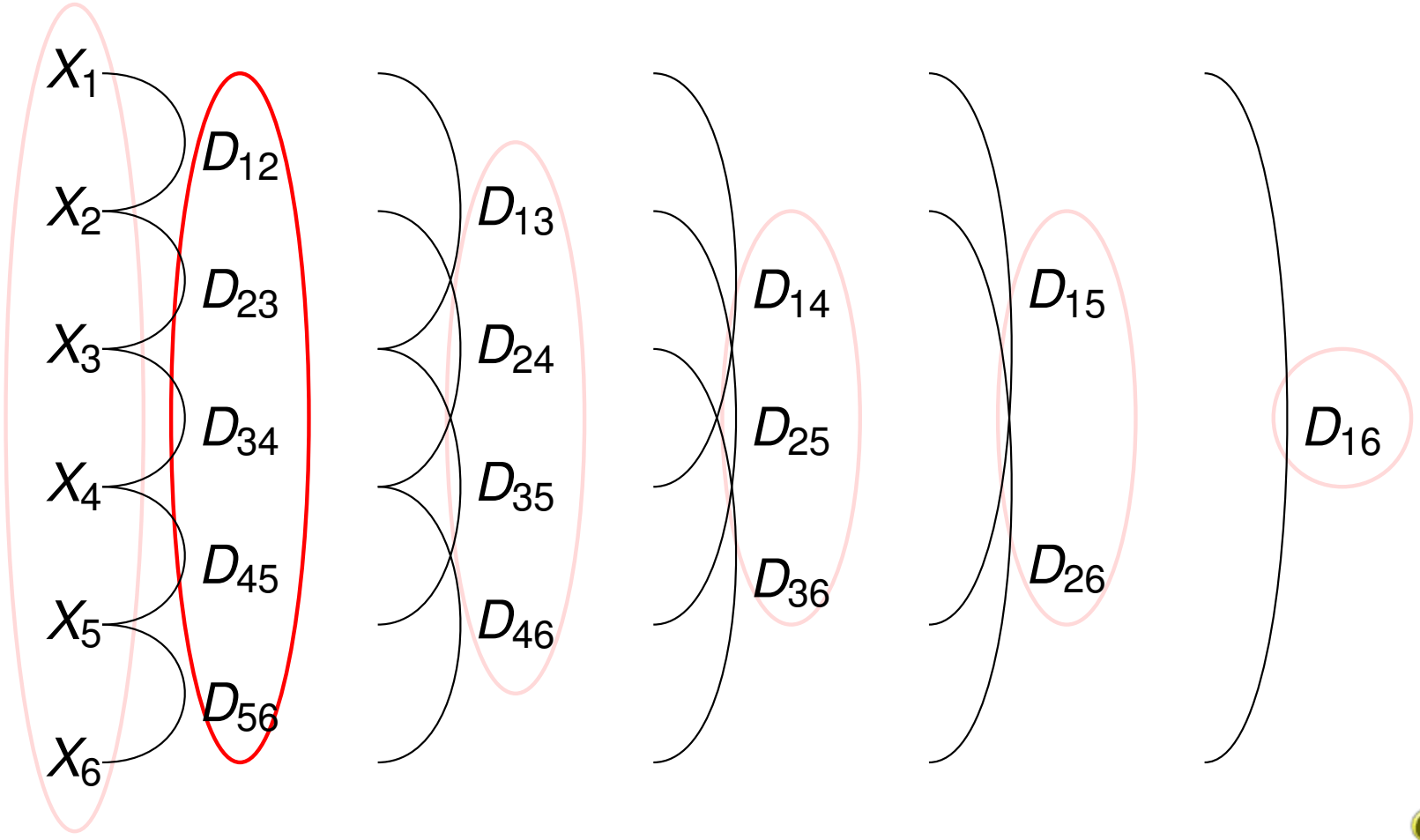
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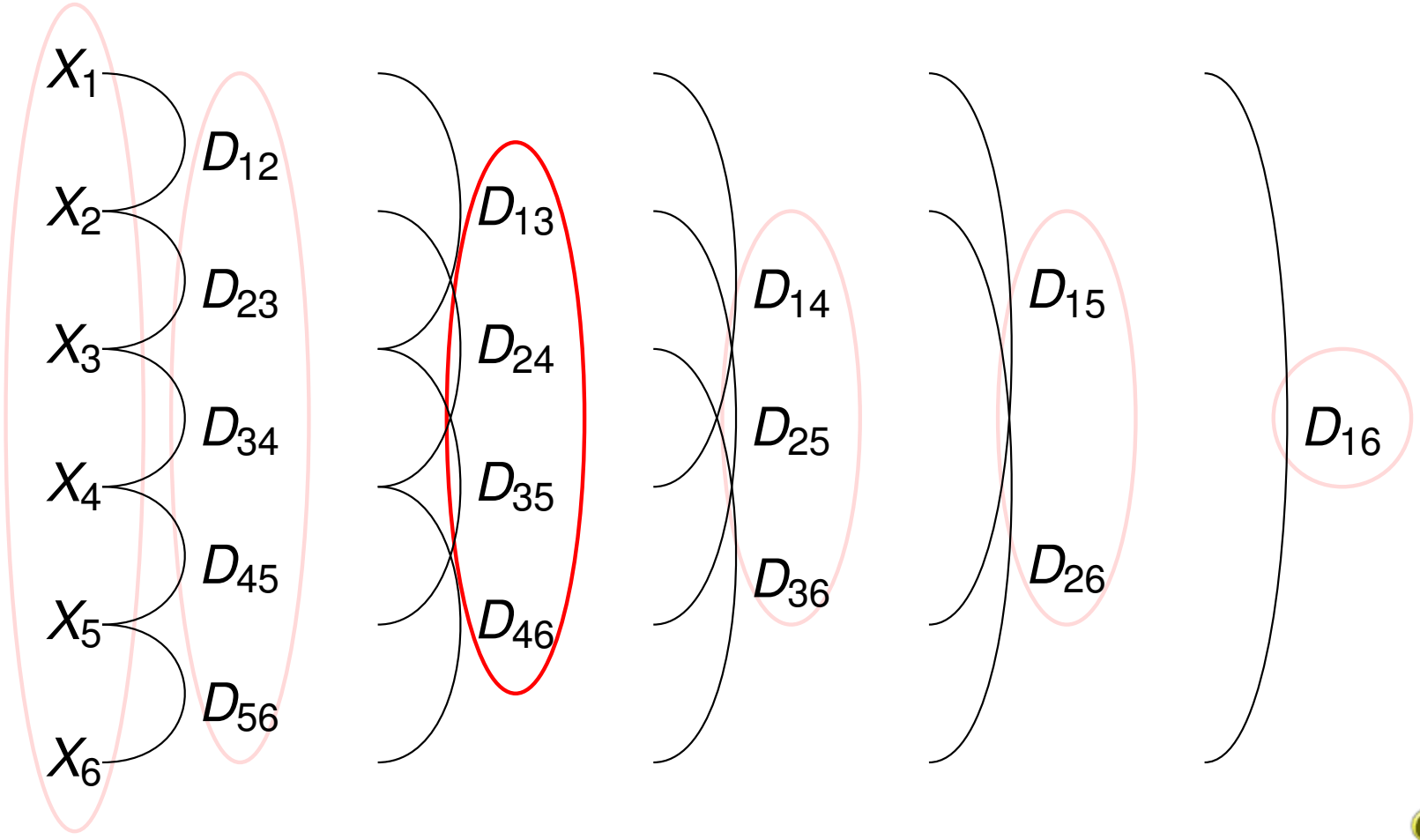
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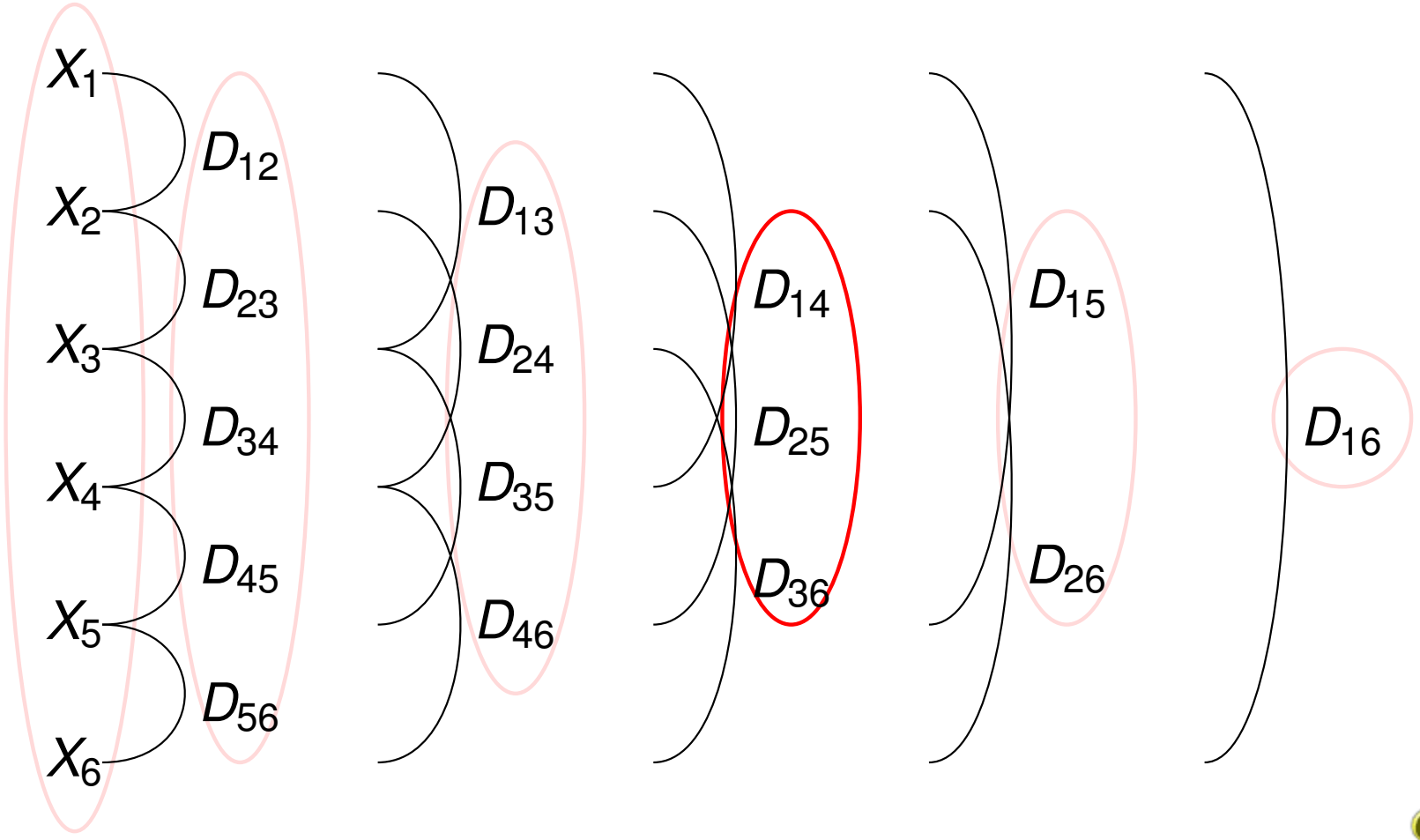
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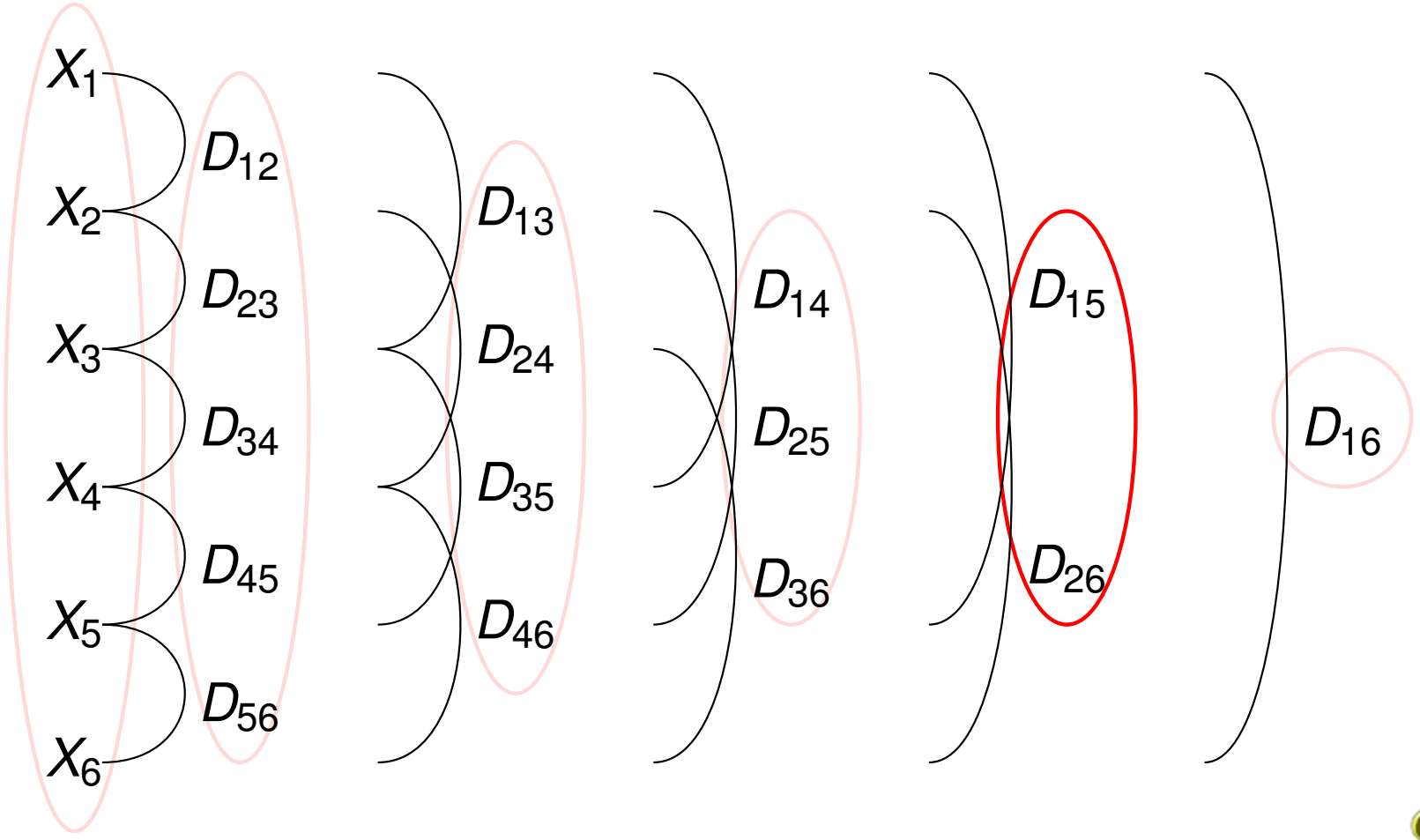
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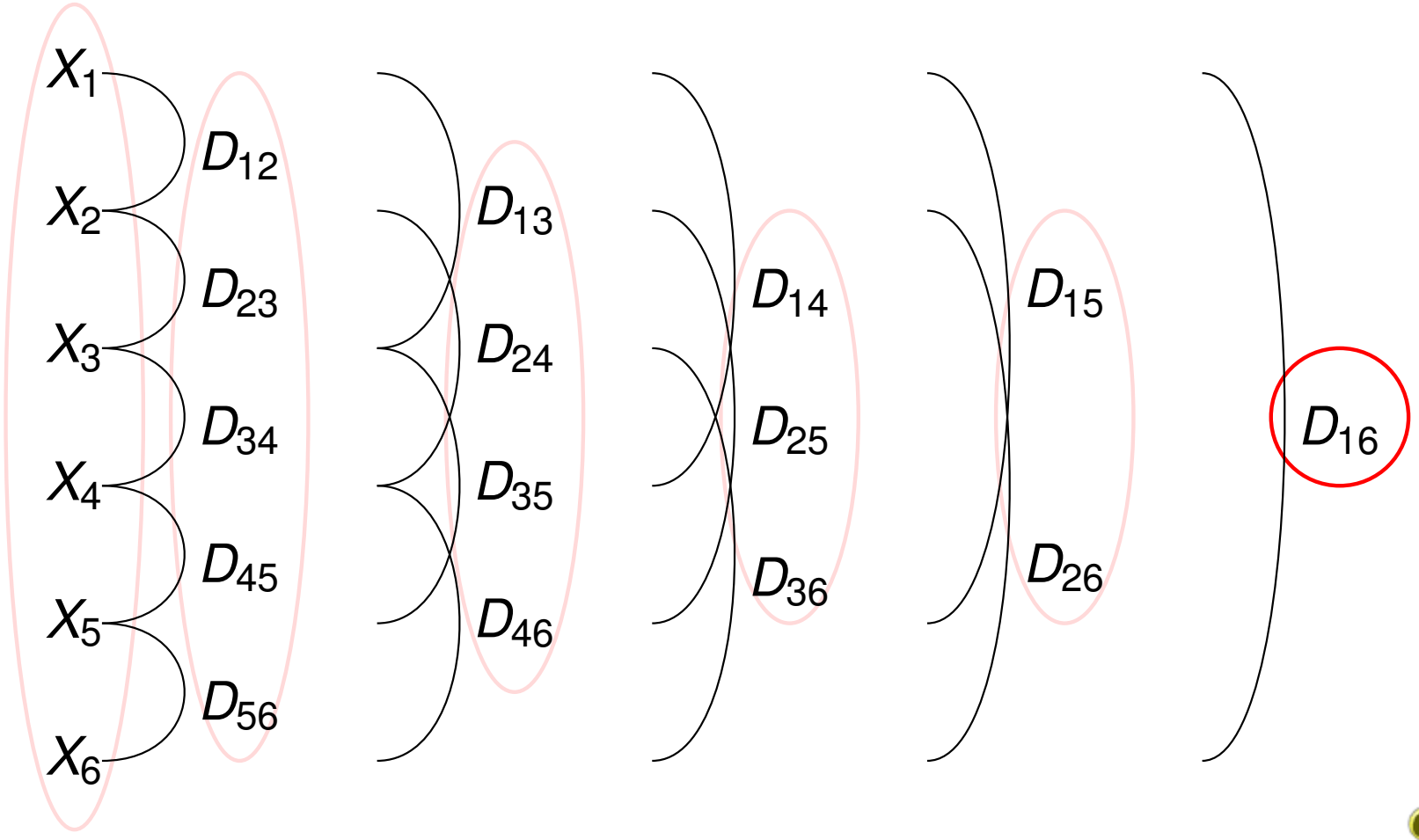
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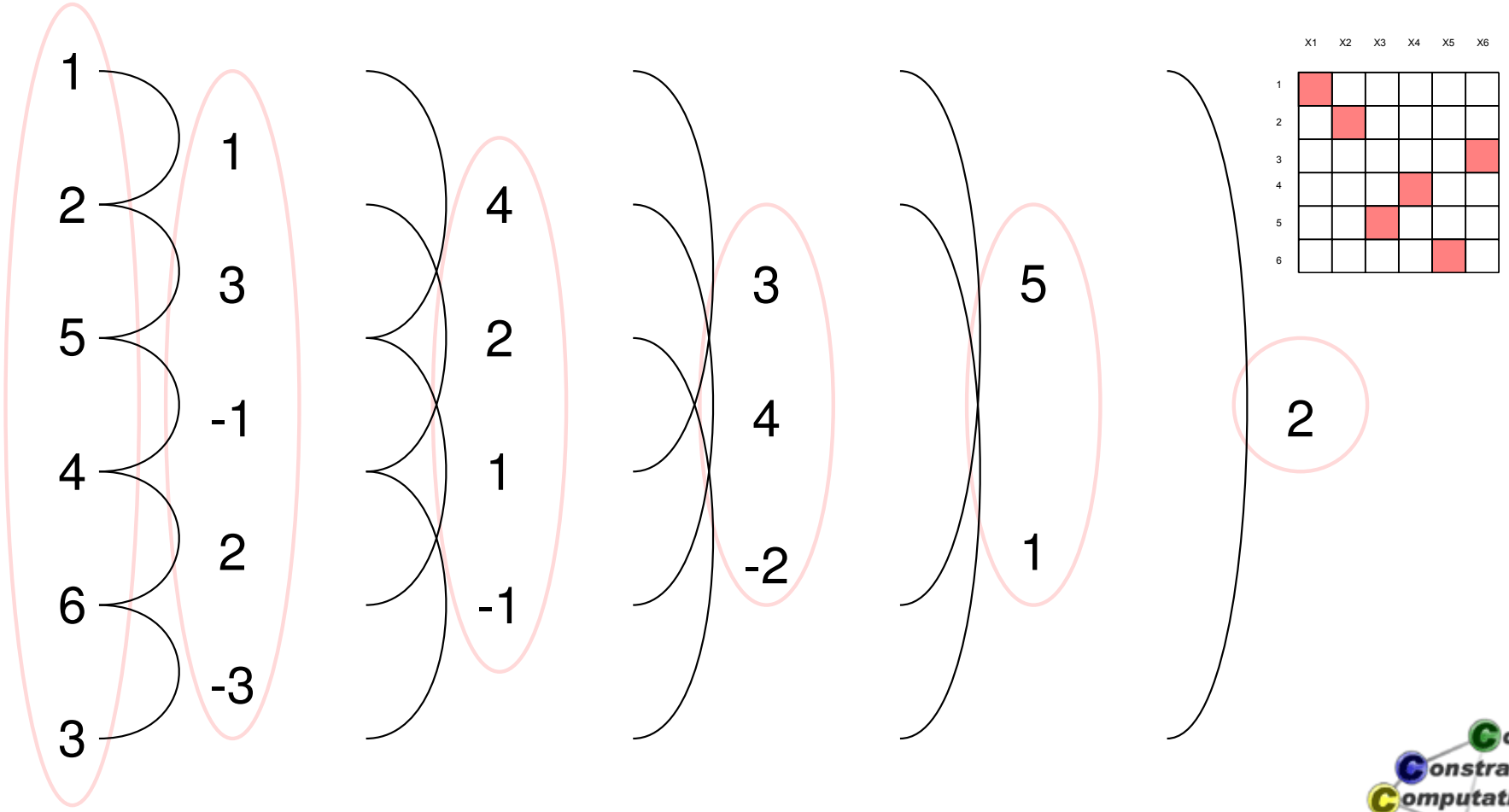
Example



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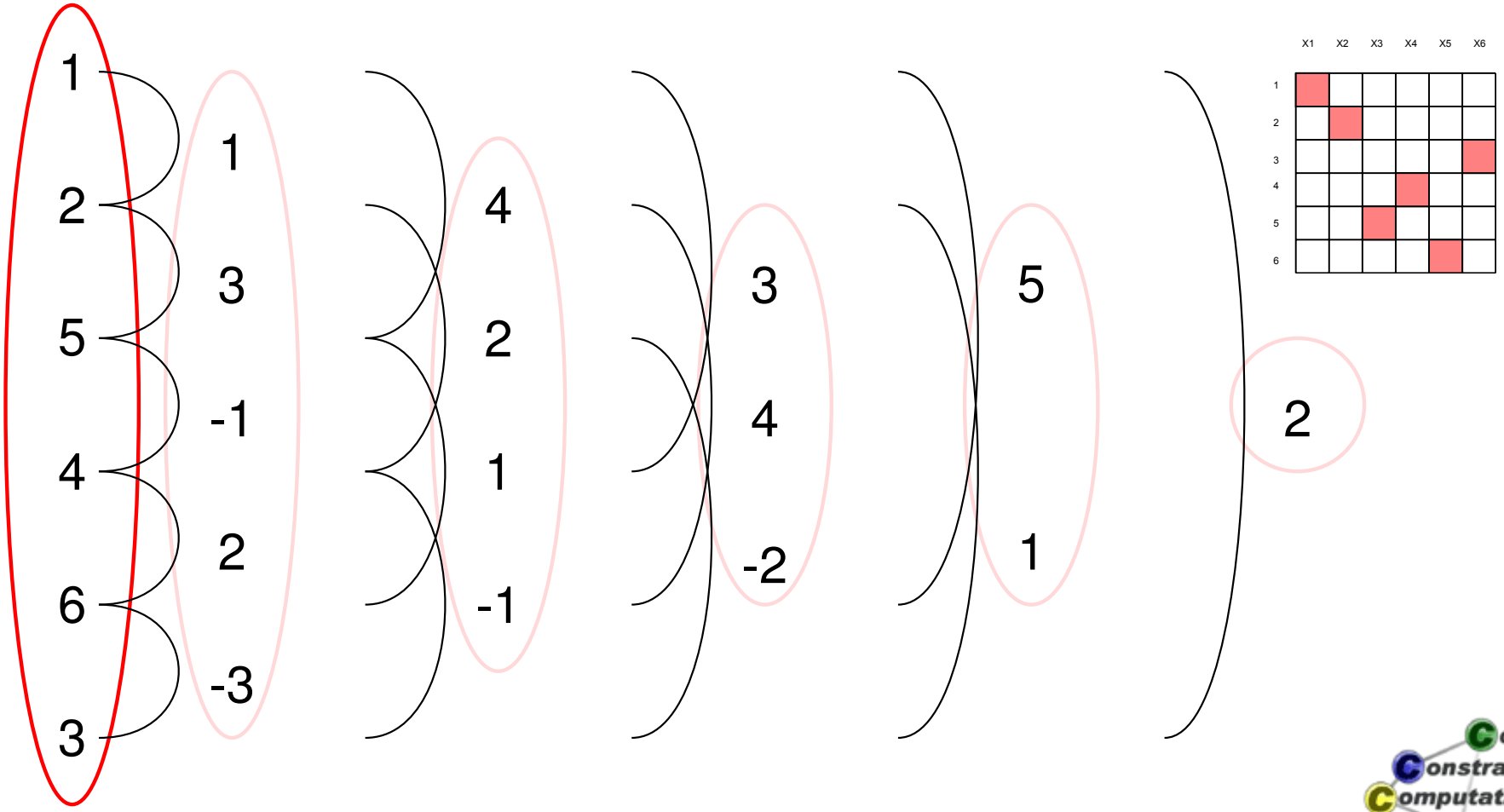

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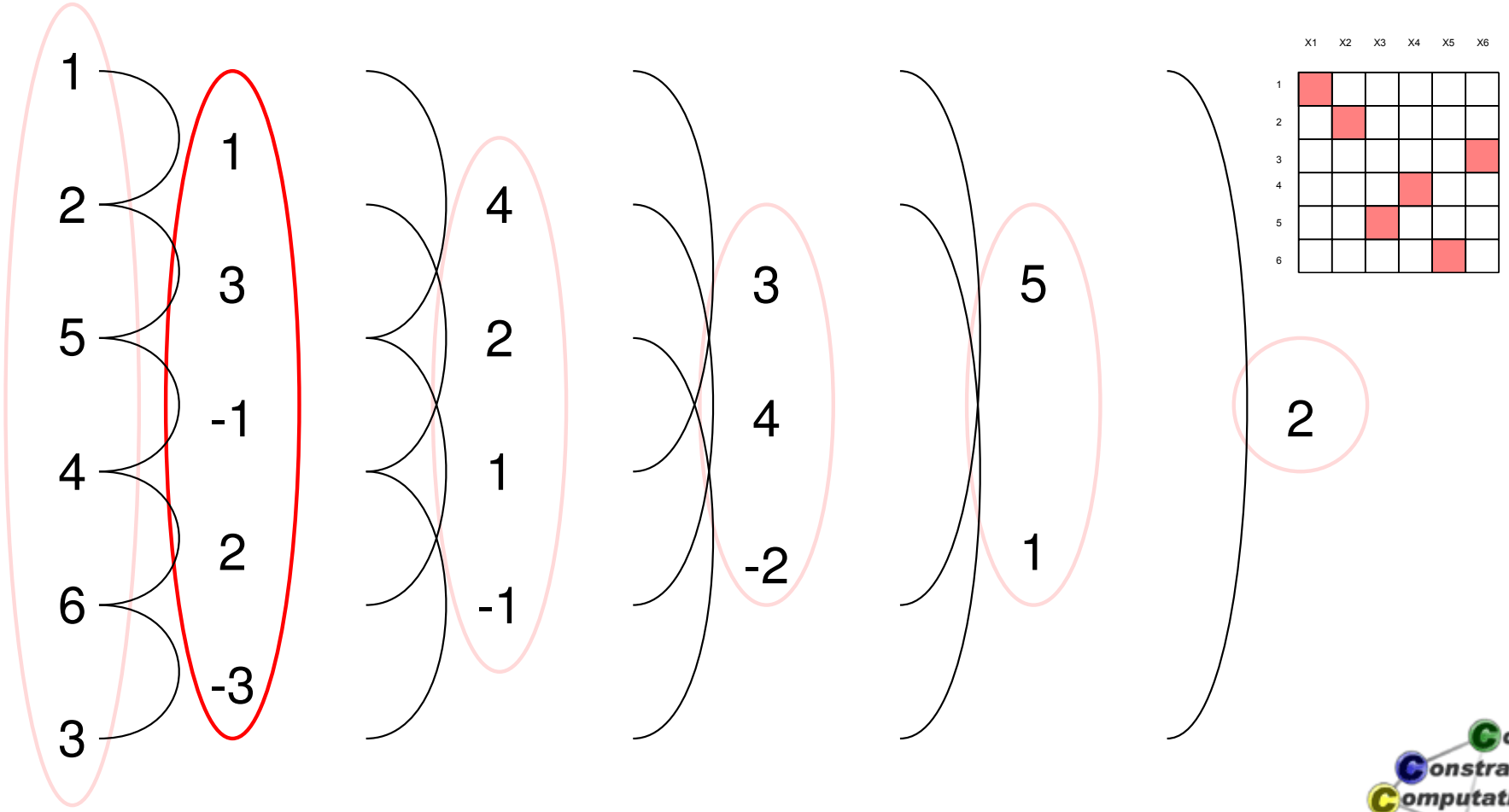

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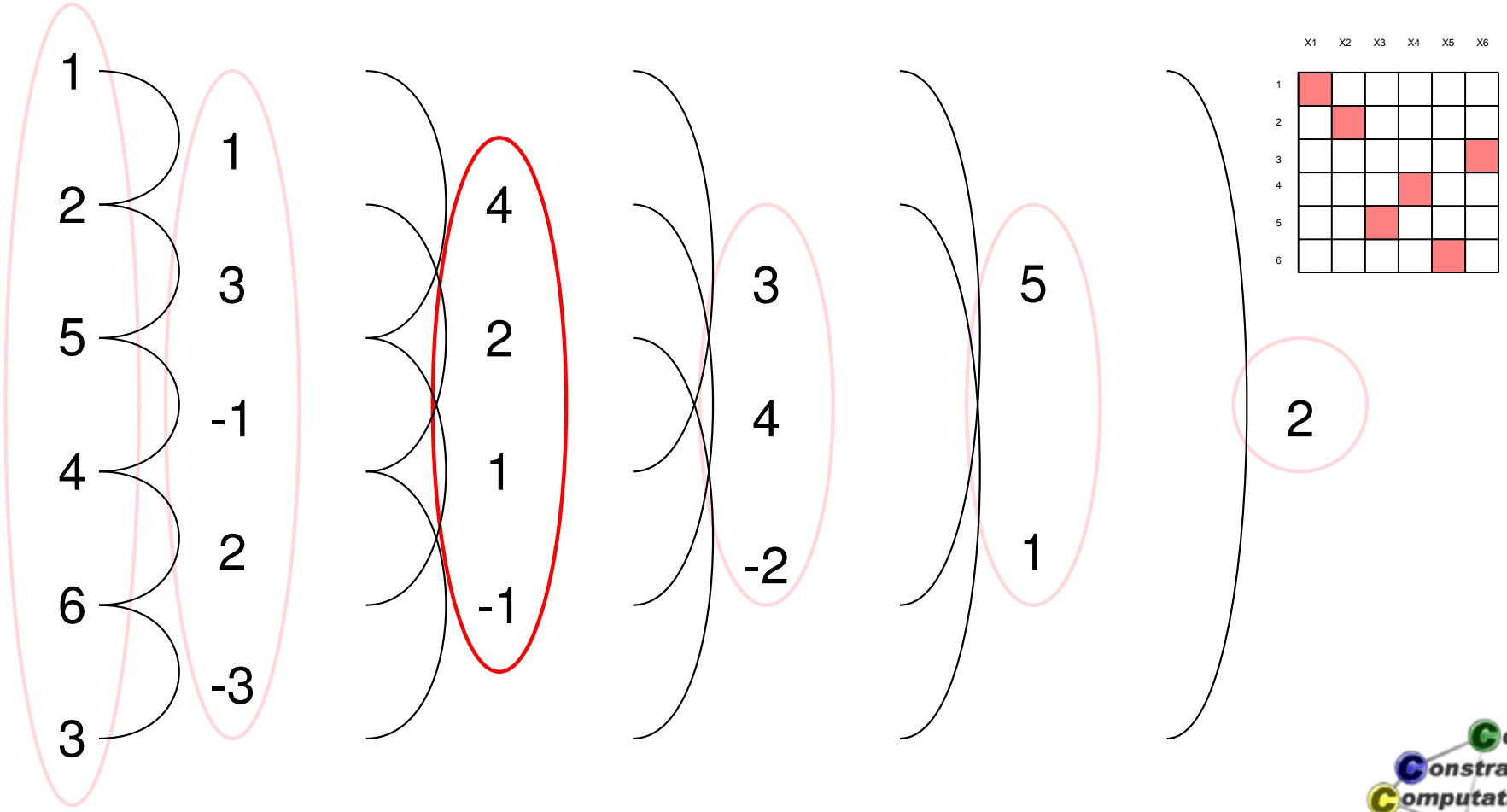

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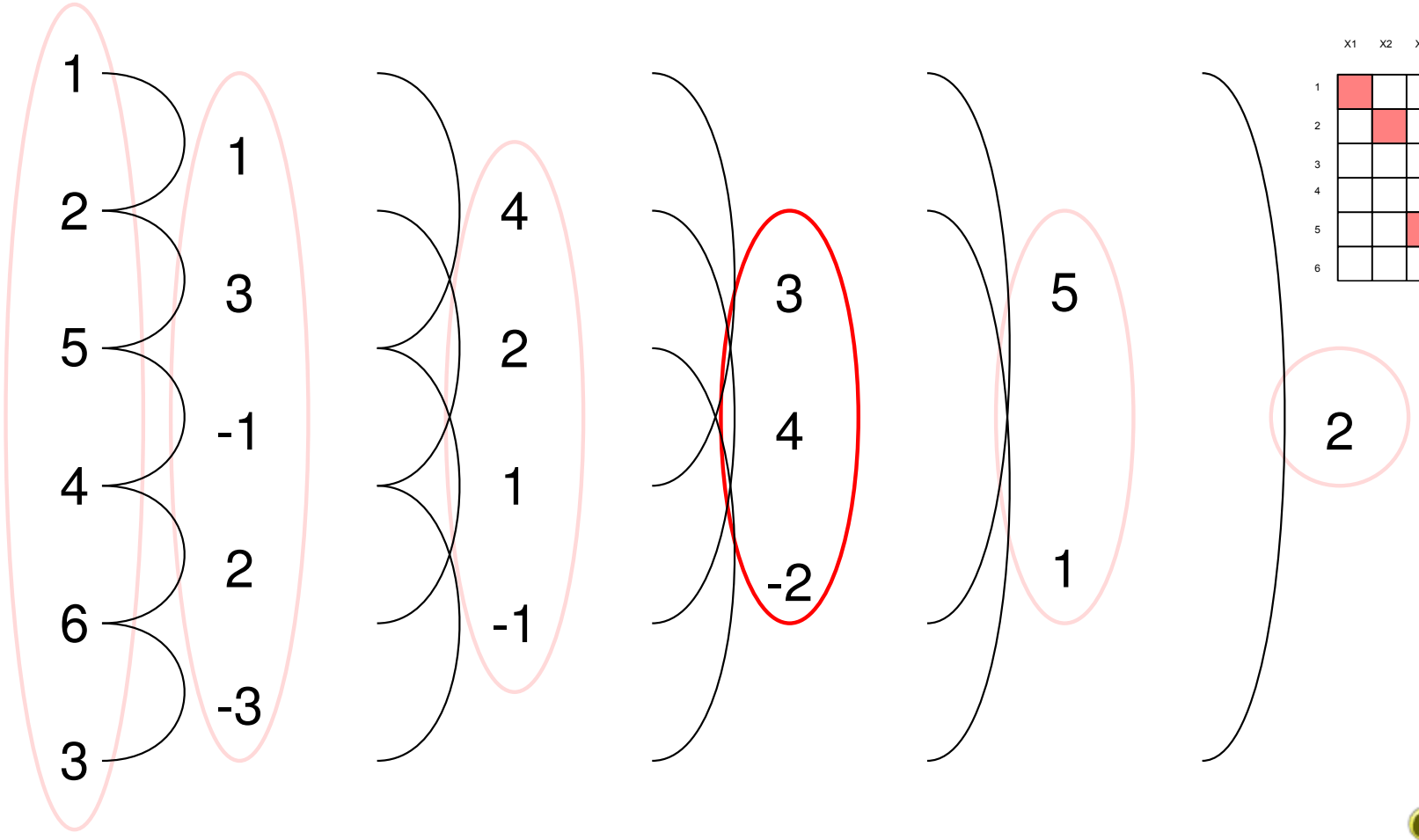

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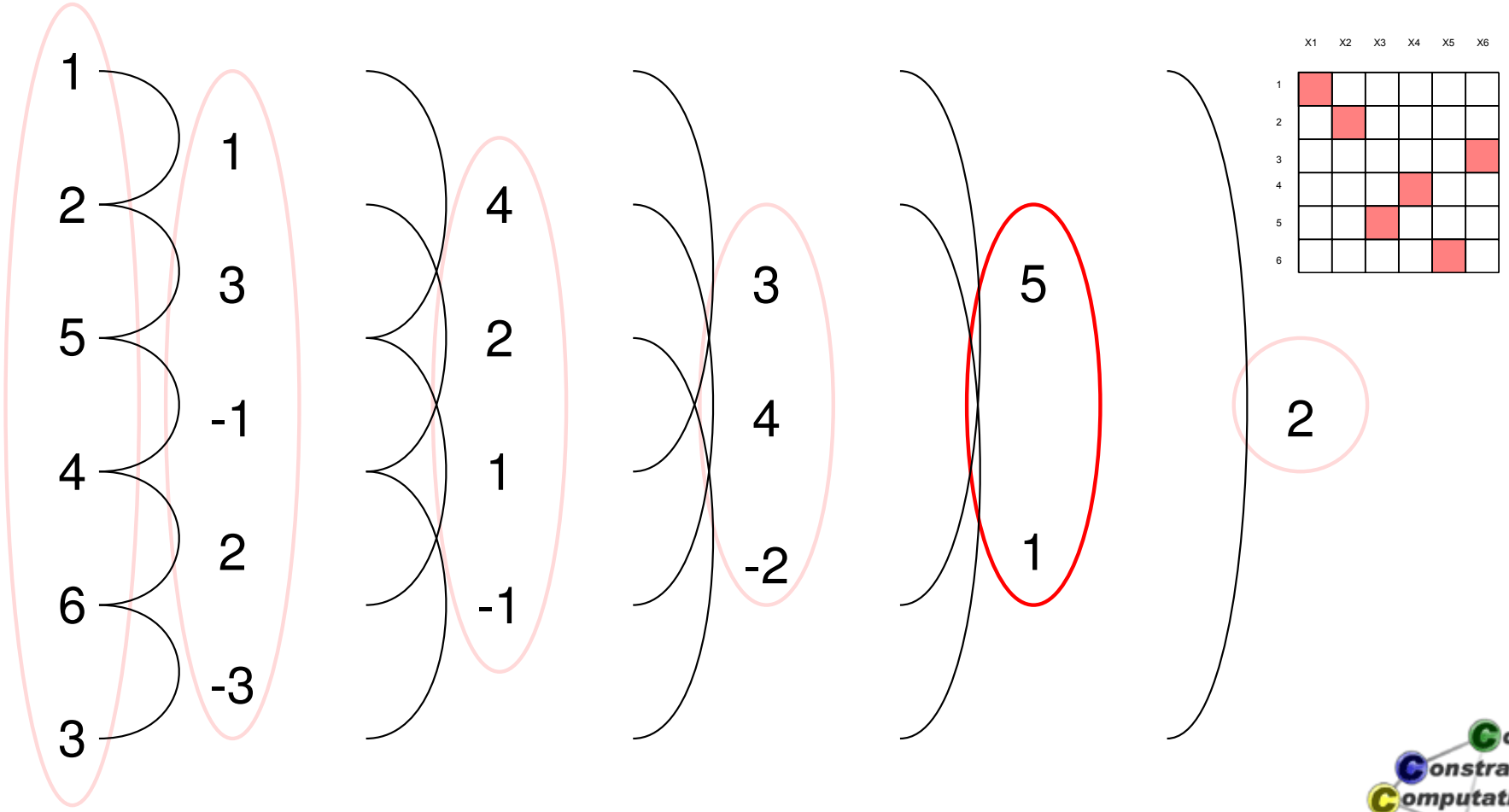

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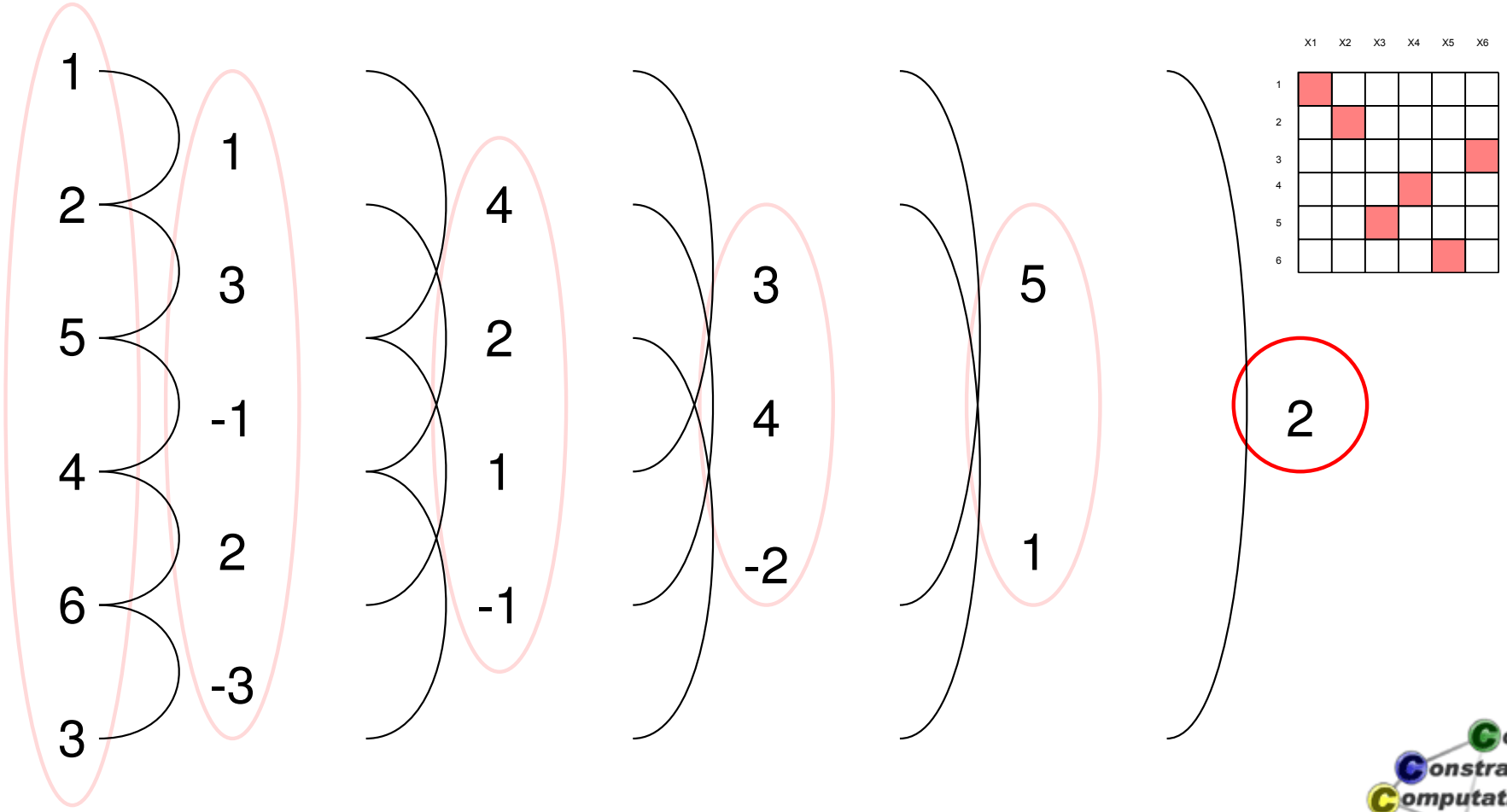

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Declarations

```
:-module(costas).

:-export(top/0).

:-lib(ic).
```

Main Program

```
top:-
        (for(N,3,20) do
            costas(N,_)
        ).

costas(N,L):-
        length(L,N),
        L :: 1..N,
        alldifferent(L),
        L = [_|L1],
        diffs(L,L1),
        search(L,0,first_fail,indomain,
               complete,[]).
```

Differences

```
diffs(_,[]).
diffs(L,[H|T]):-
        diff_pairs(L,[H|T],Diffs),
        alldifferent(Diffs),
        diffs(L,T).

diff_pairs(_,[],[]).
diff_pairs([X|X1],[Y|Y1],[D|D1]):-
        X #= Y+D,
        diff_pairs(X1,Y1,D1).
```

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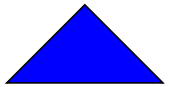
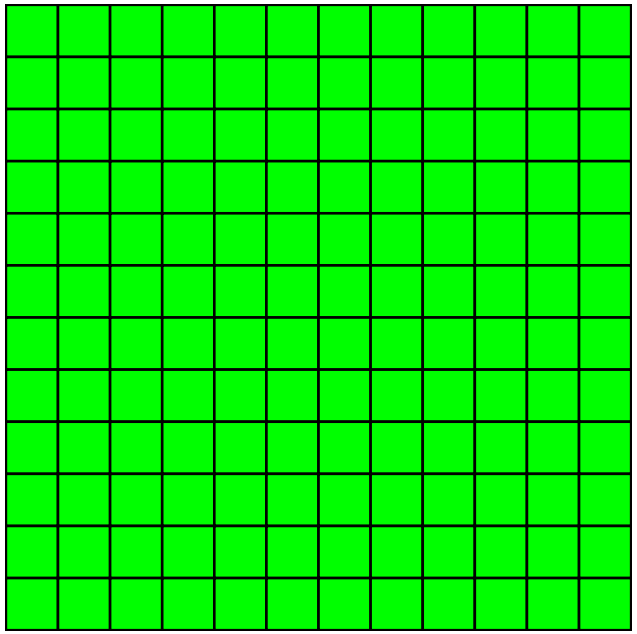
Basic Model





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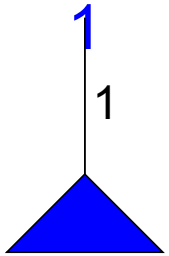
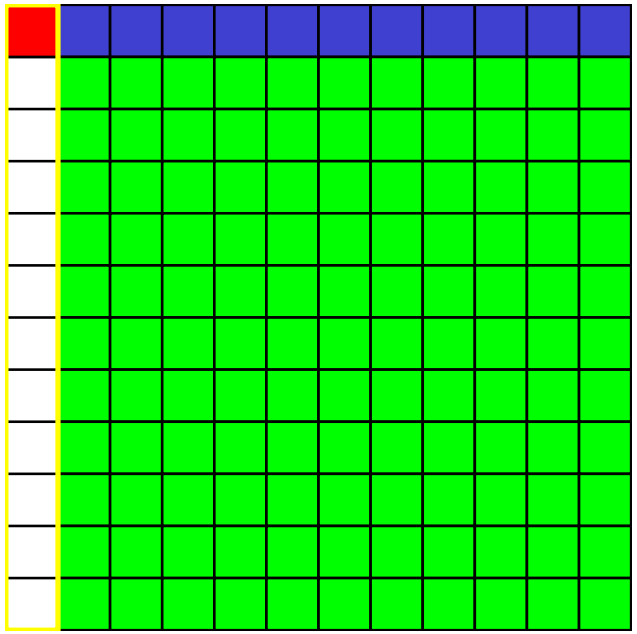
Basic Model





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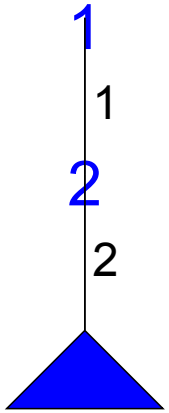

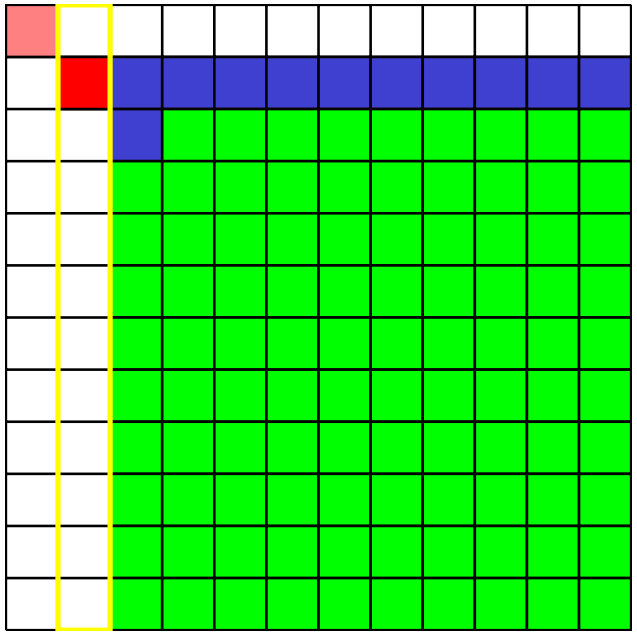

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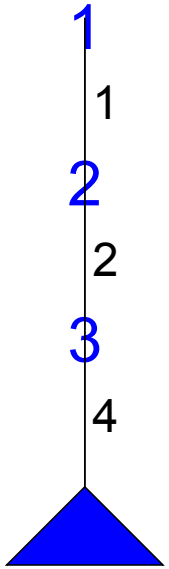

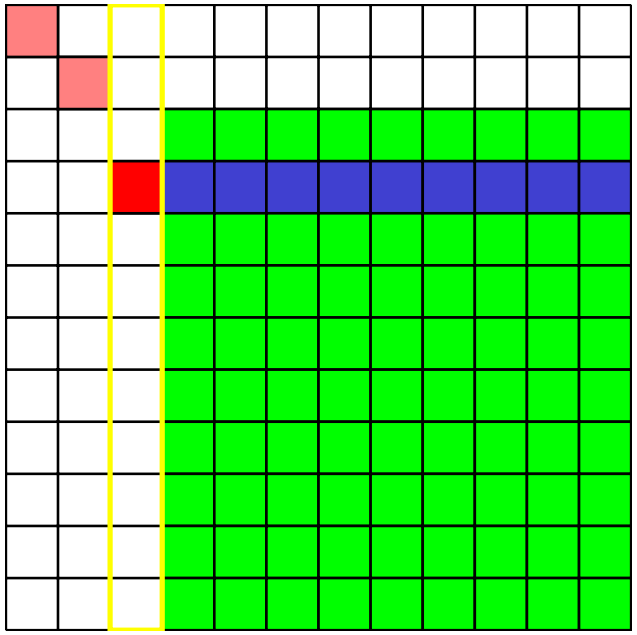

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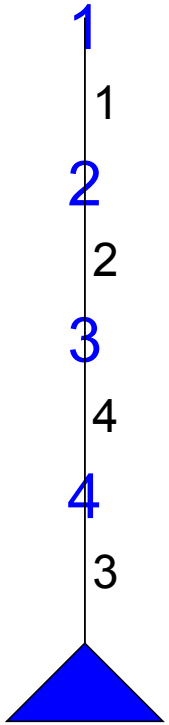

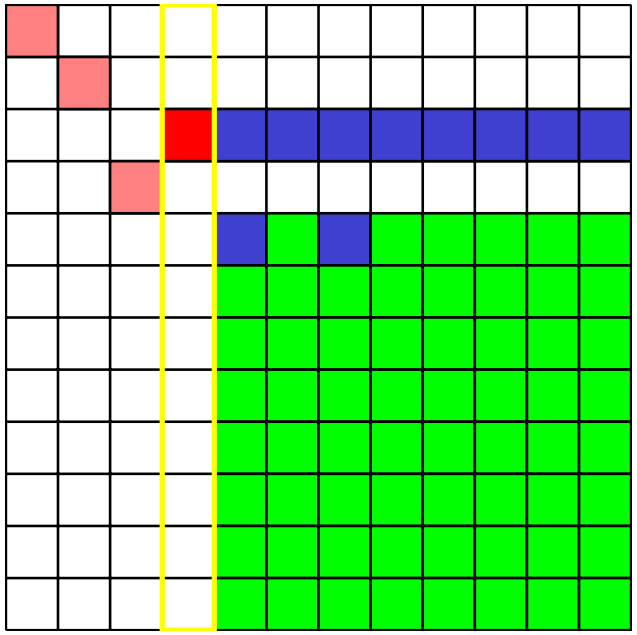

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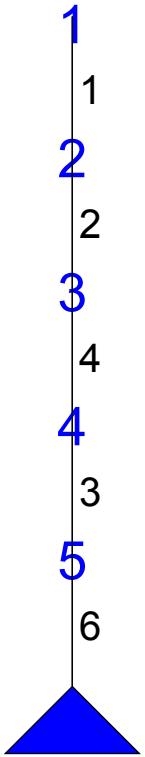

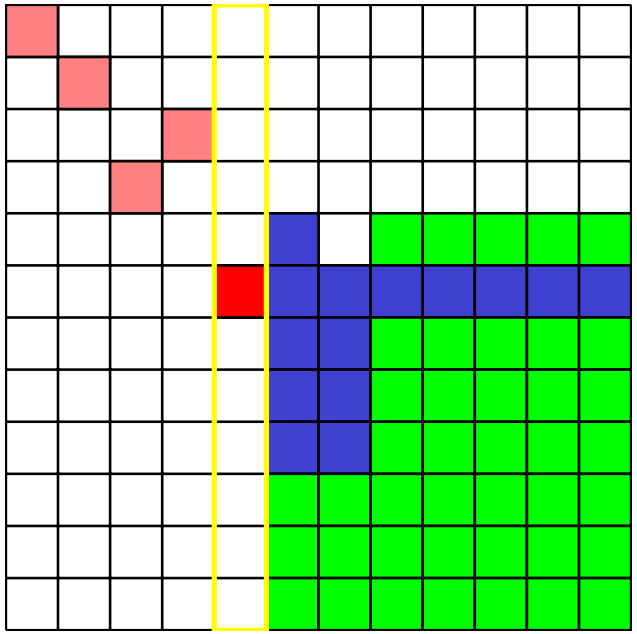

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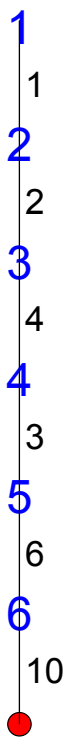

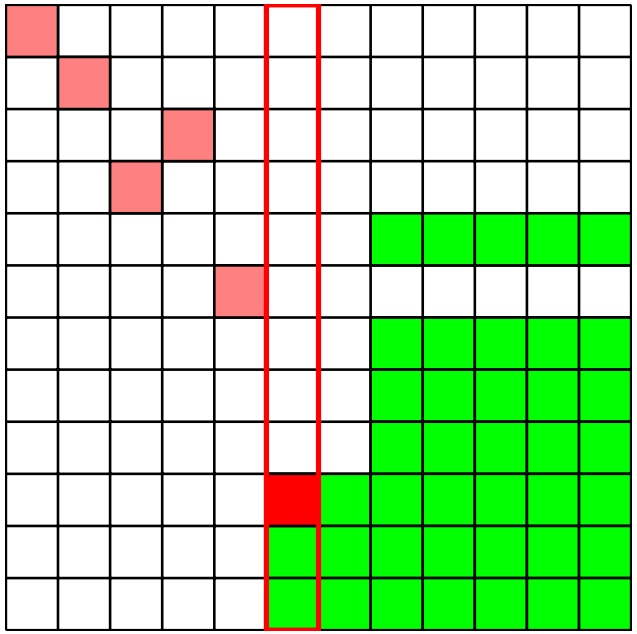

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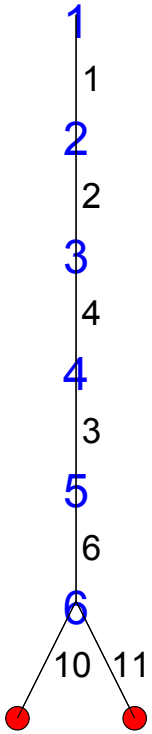

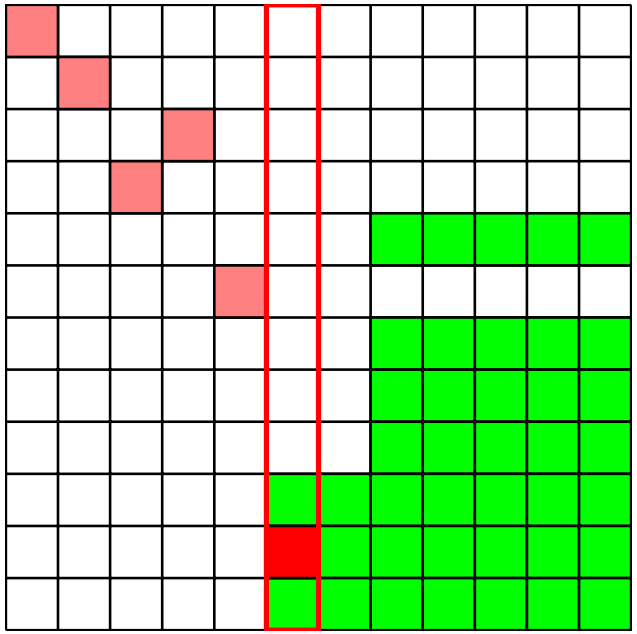

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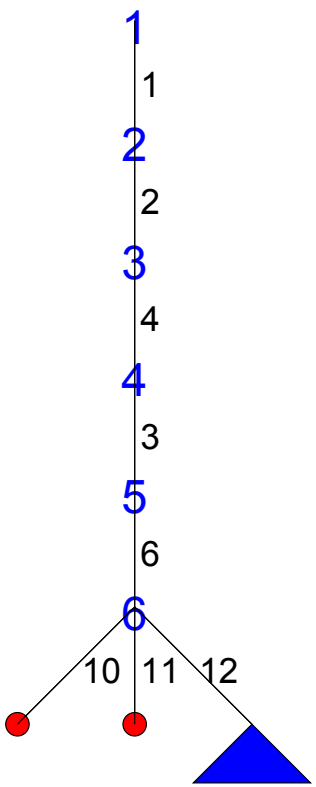

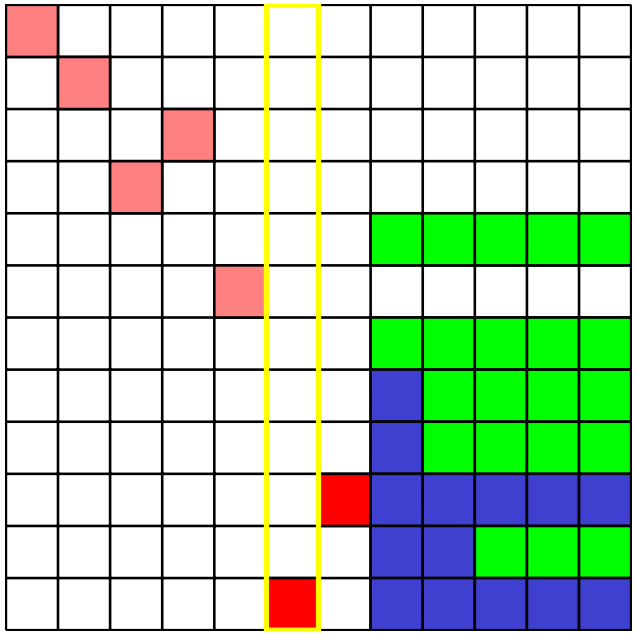

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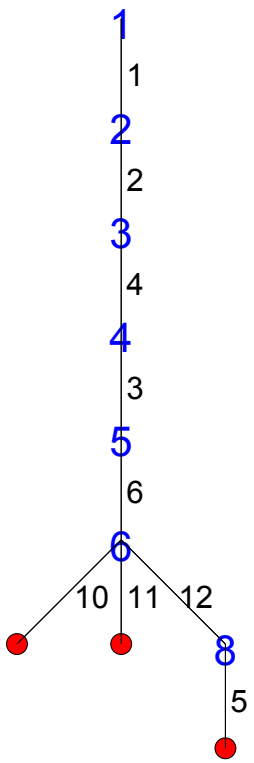

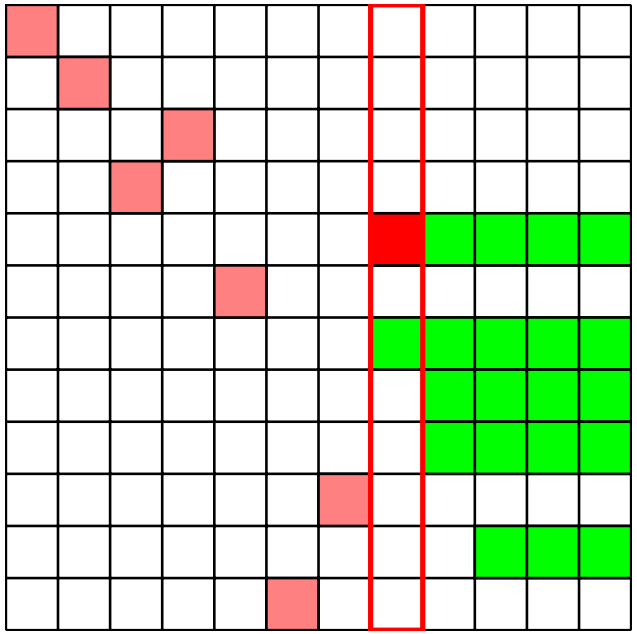

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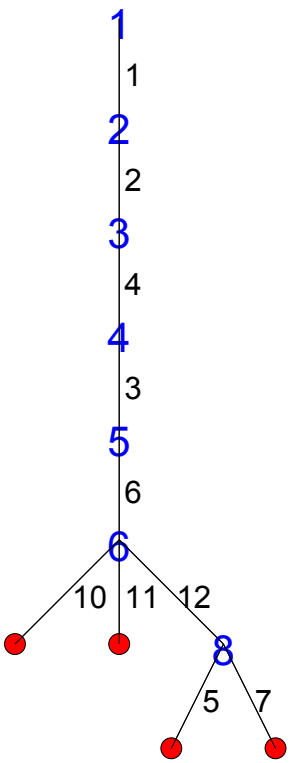

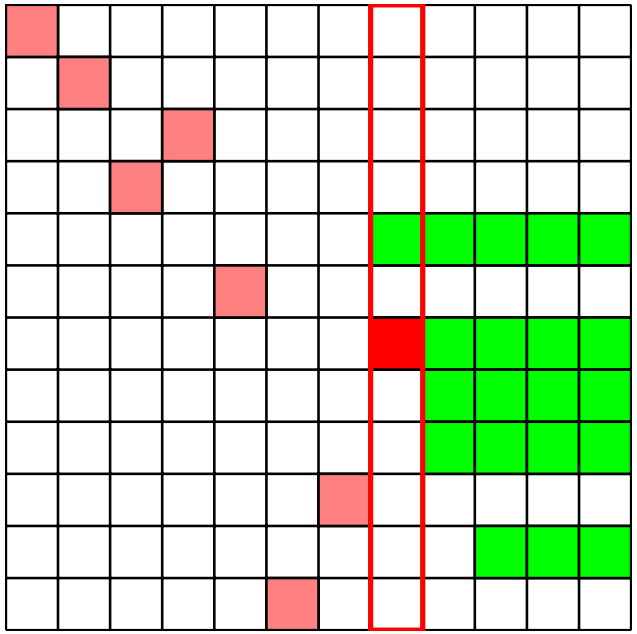

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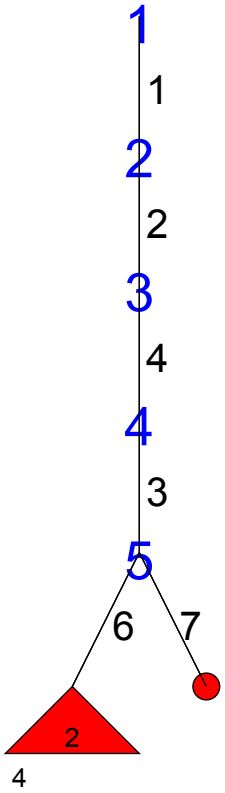

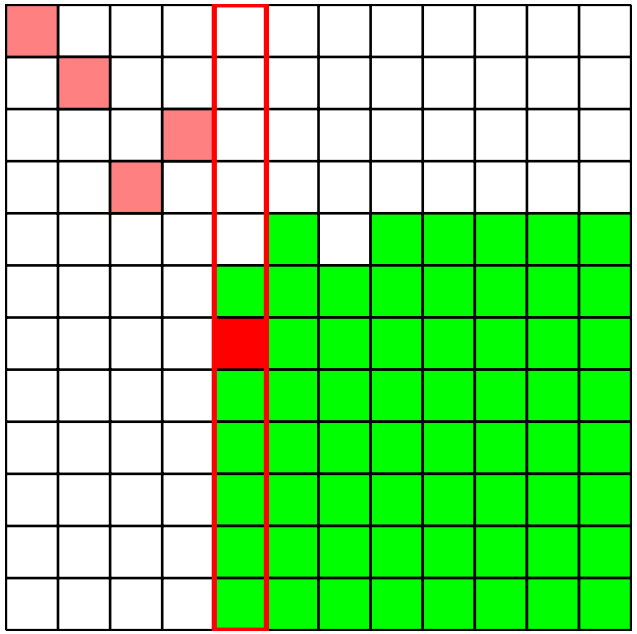

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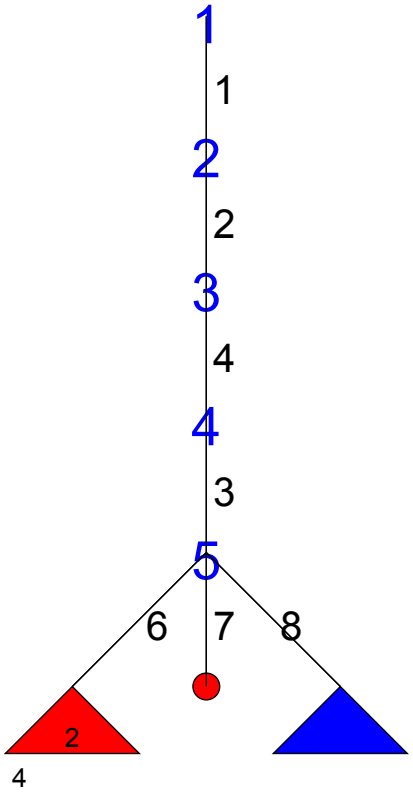

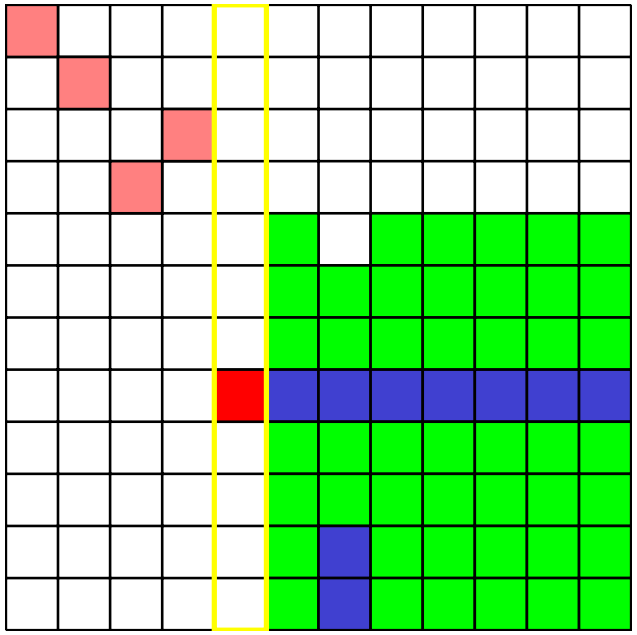

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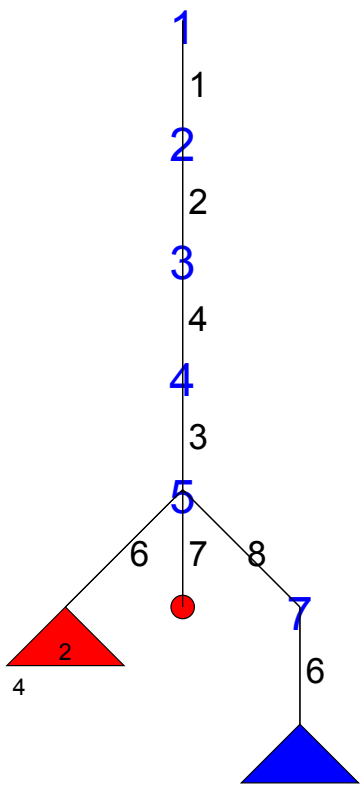

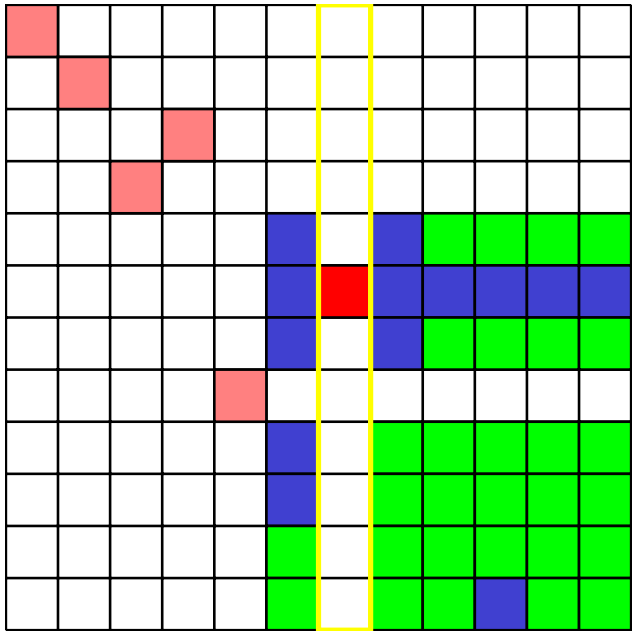

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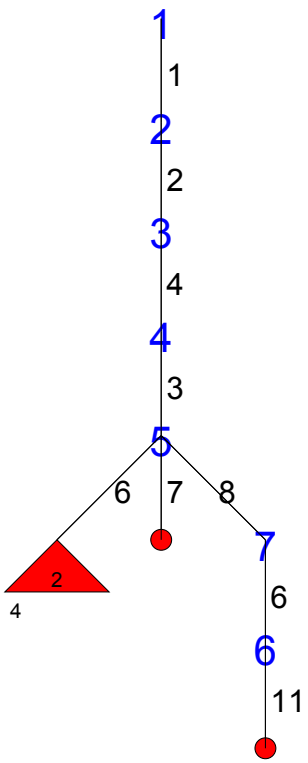

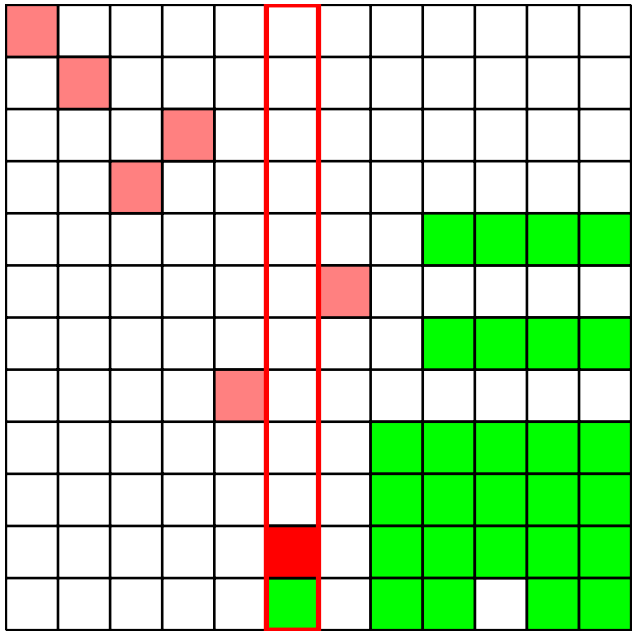

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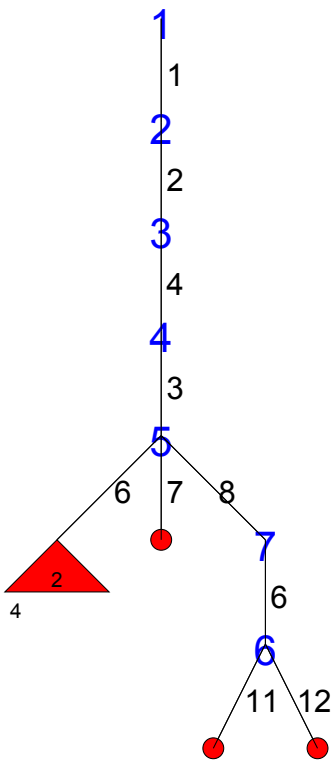
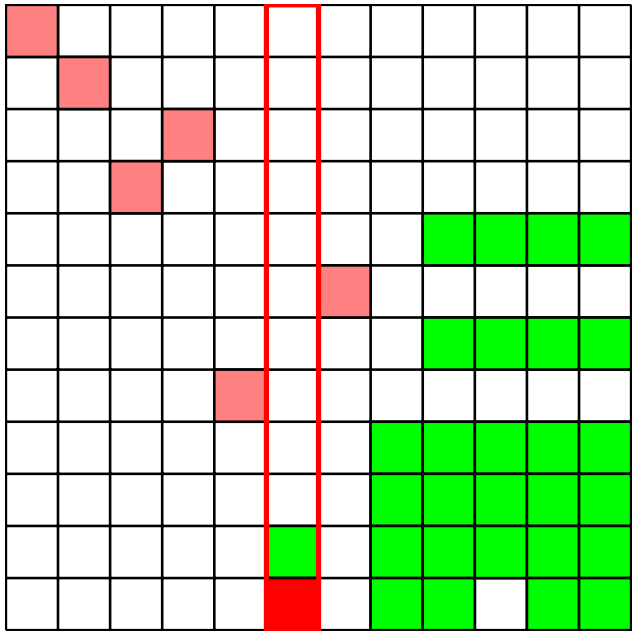
Basic Model





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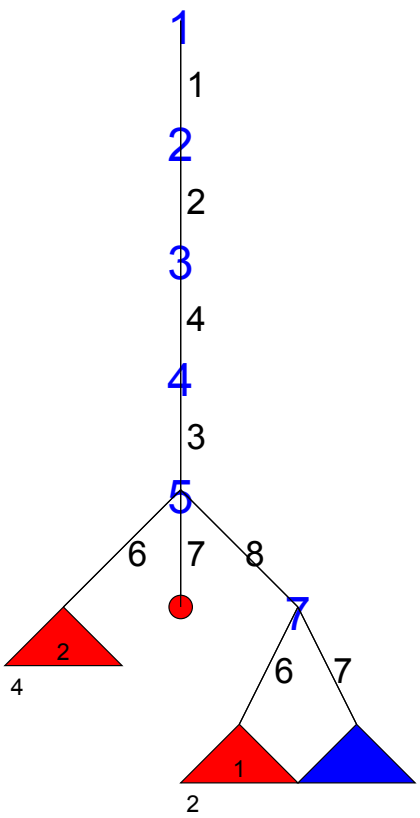

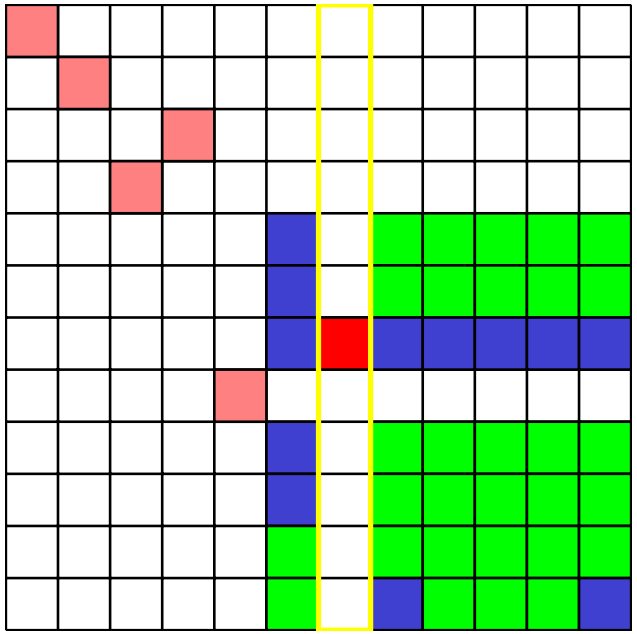

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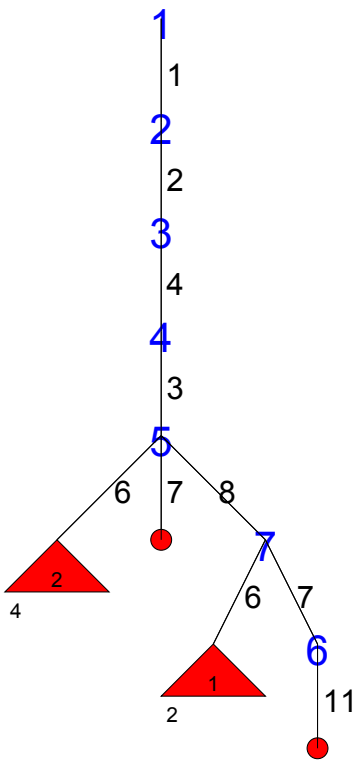

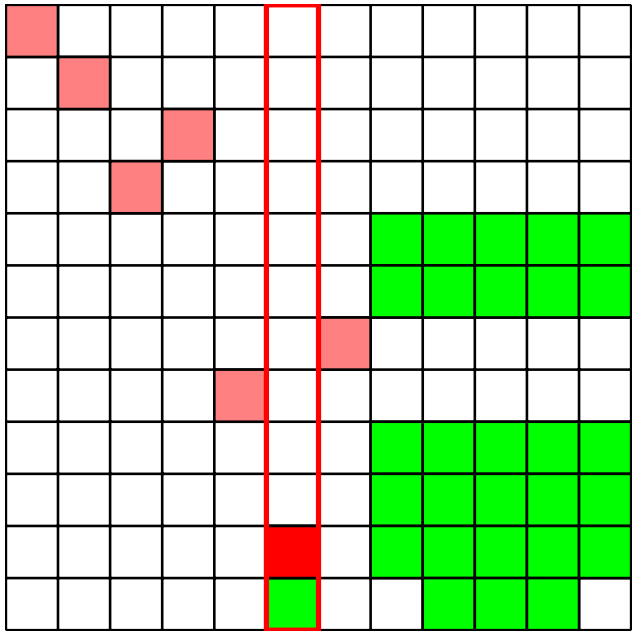

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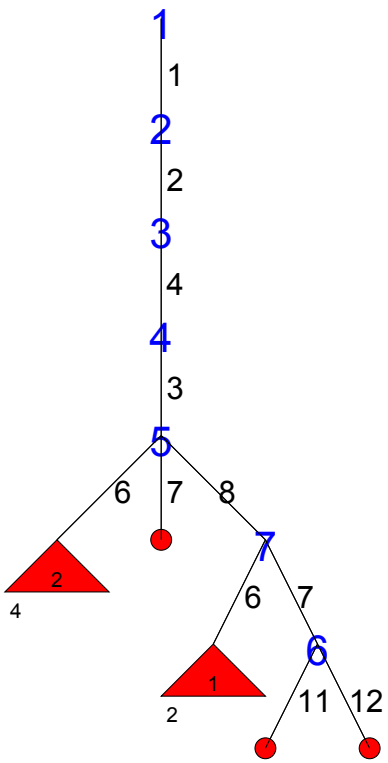

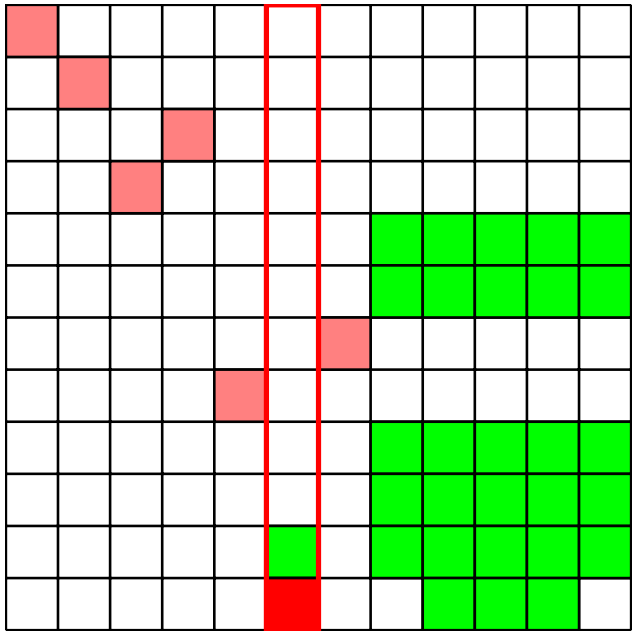

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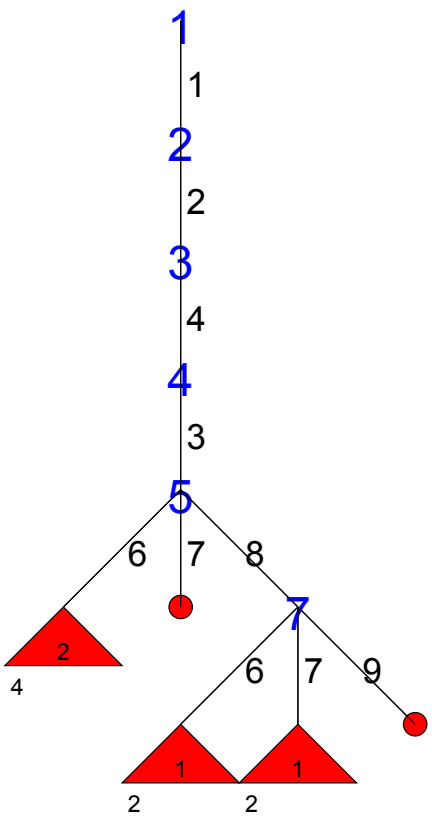

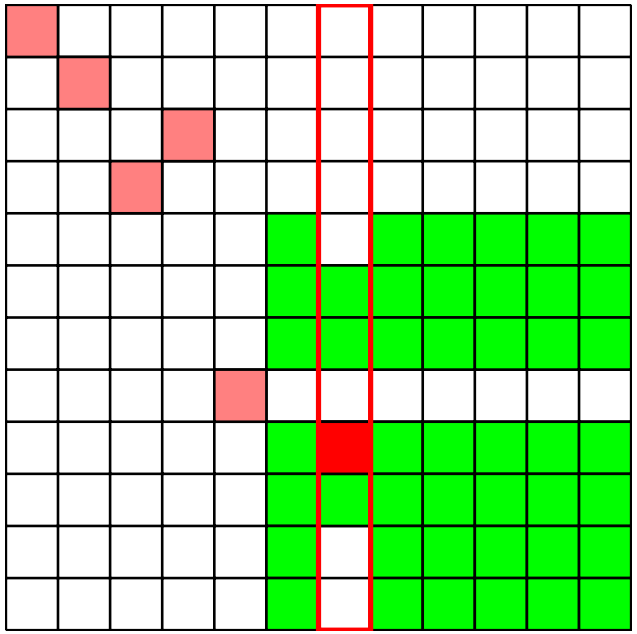

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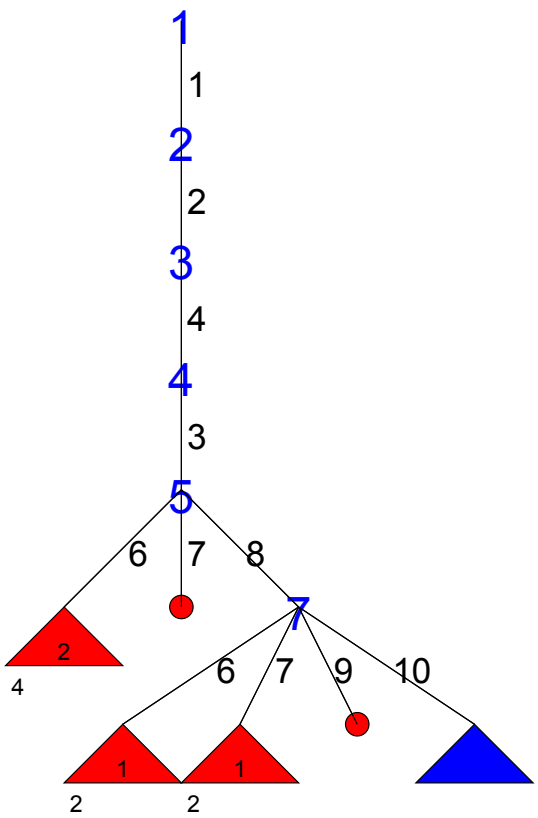

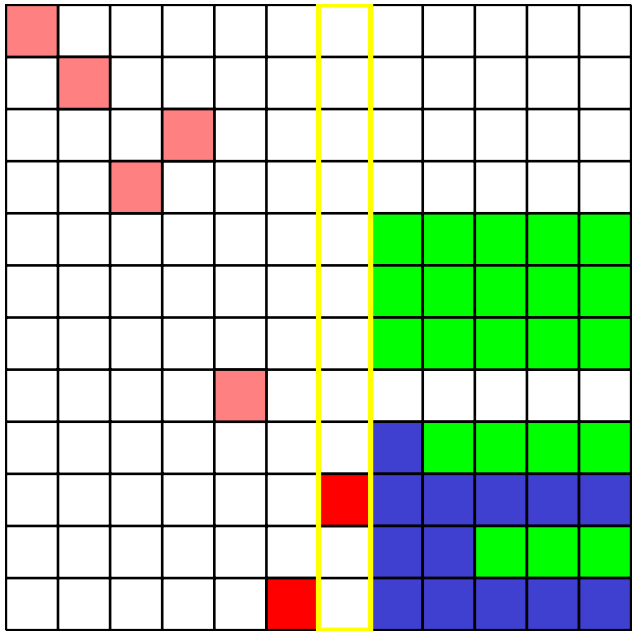

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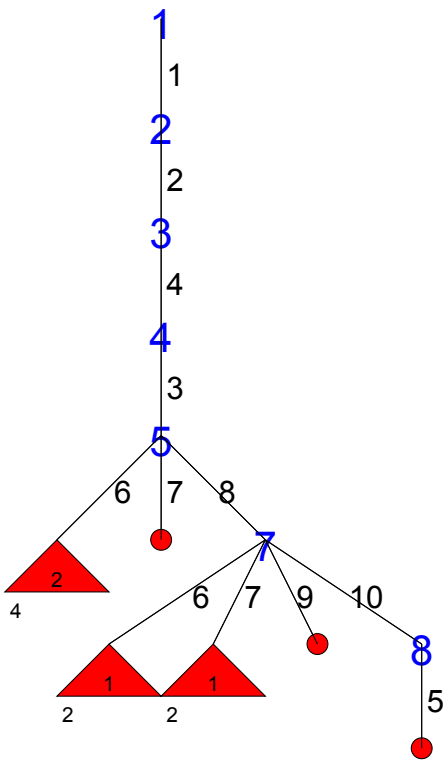

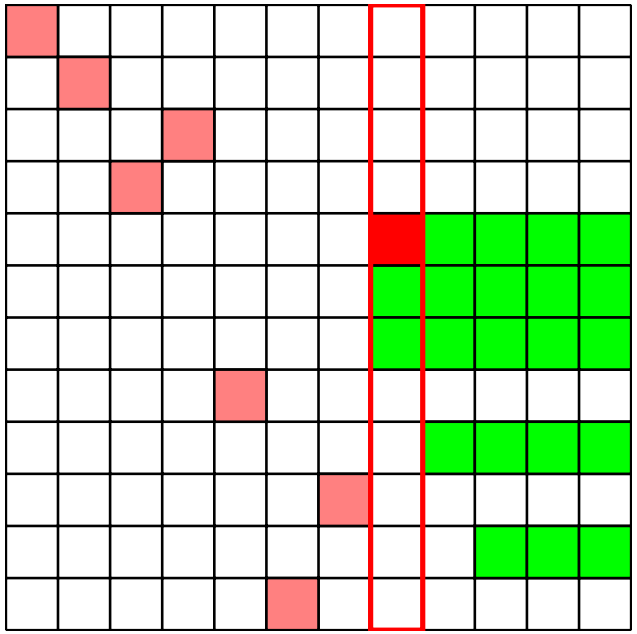

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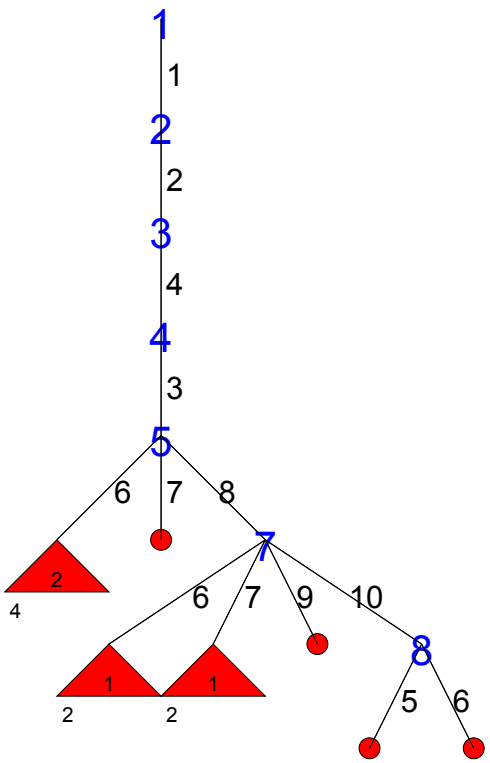

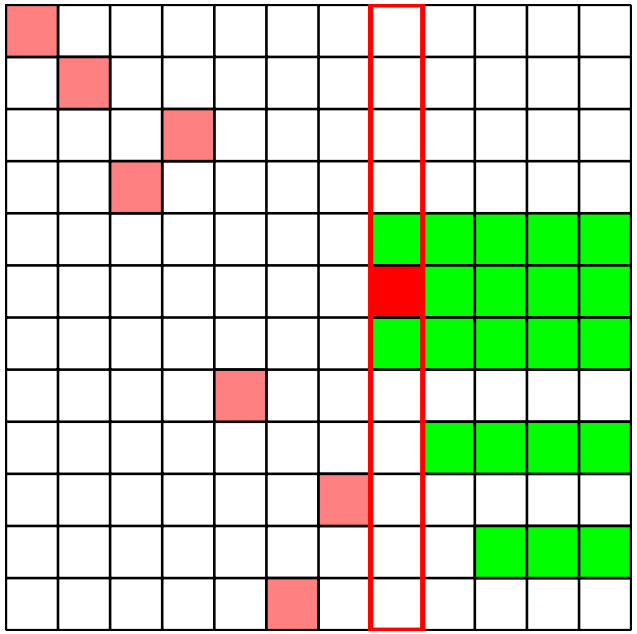

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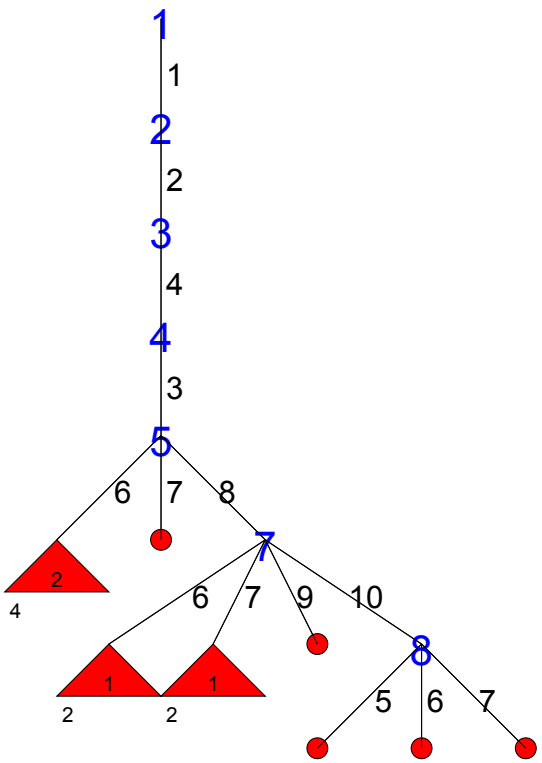

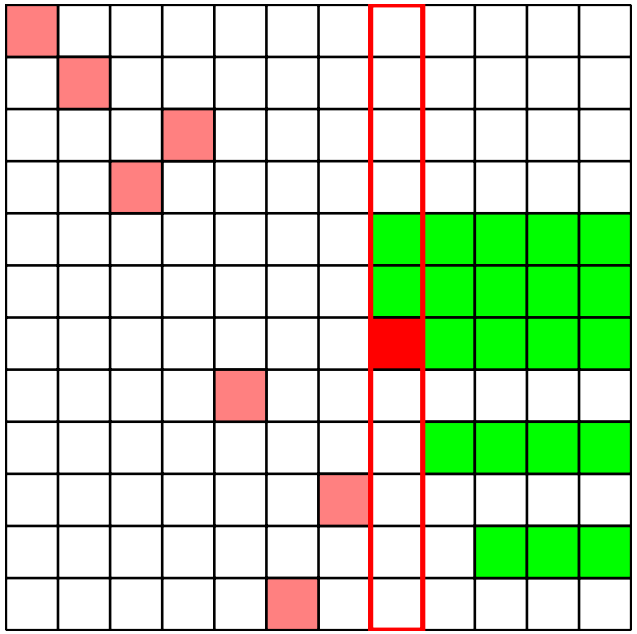

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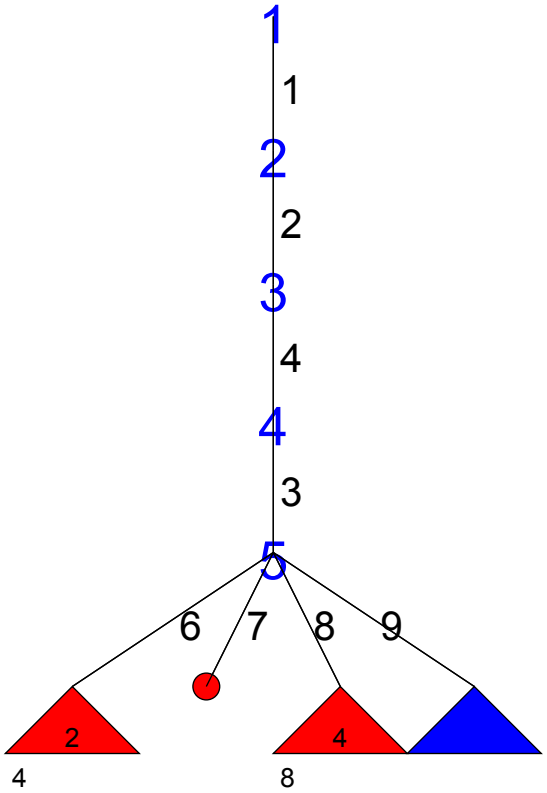

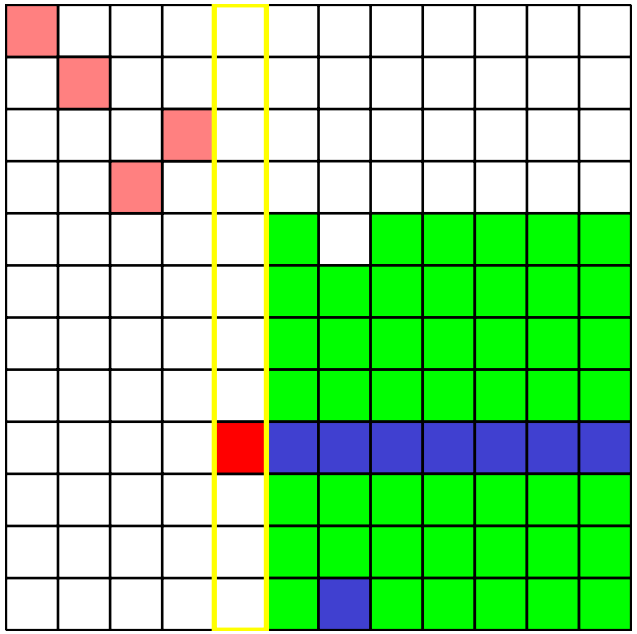

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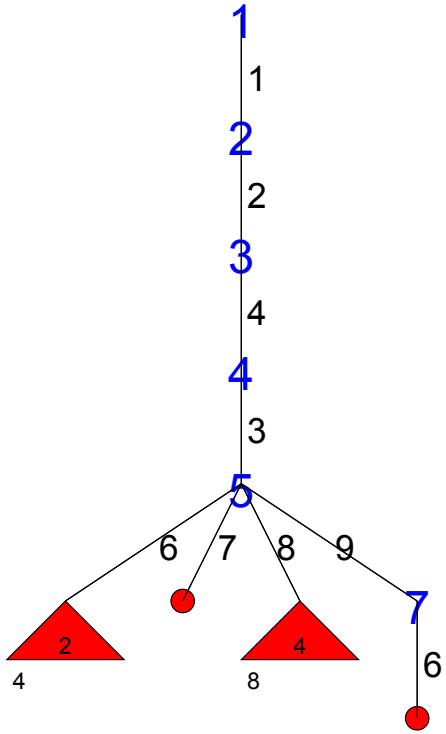

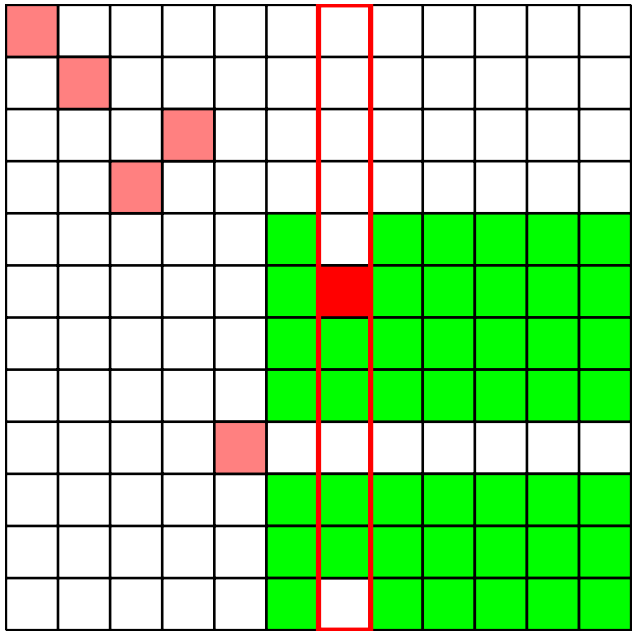

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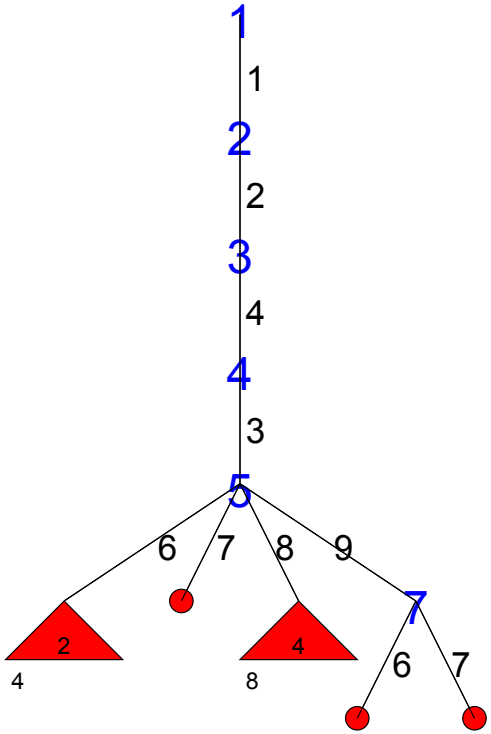

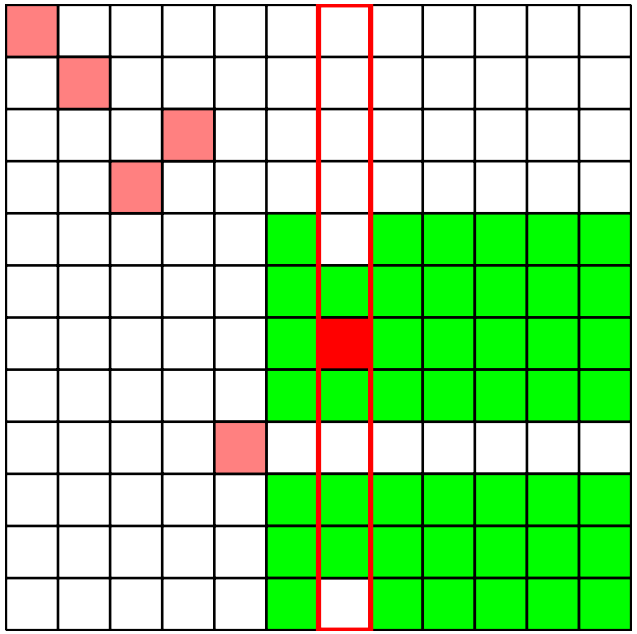

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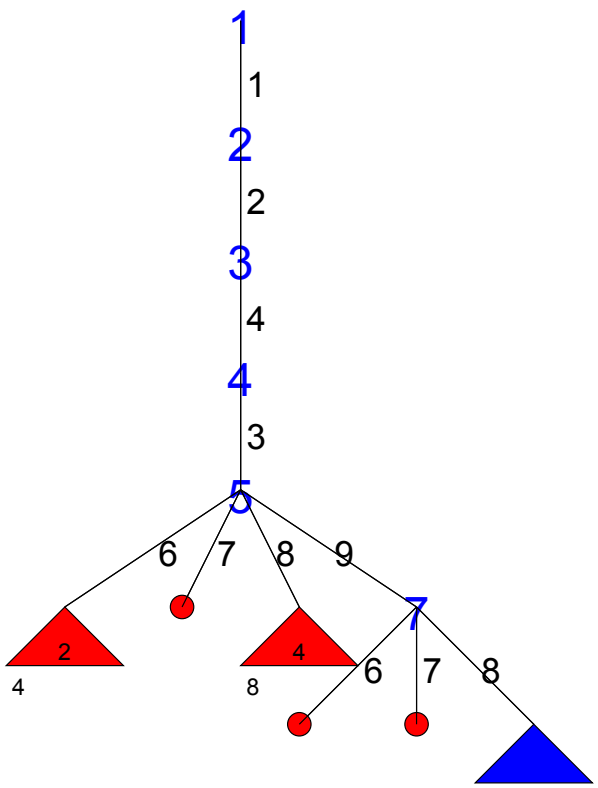

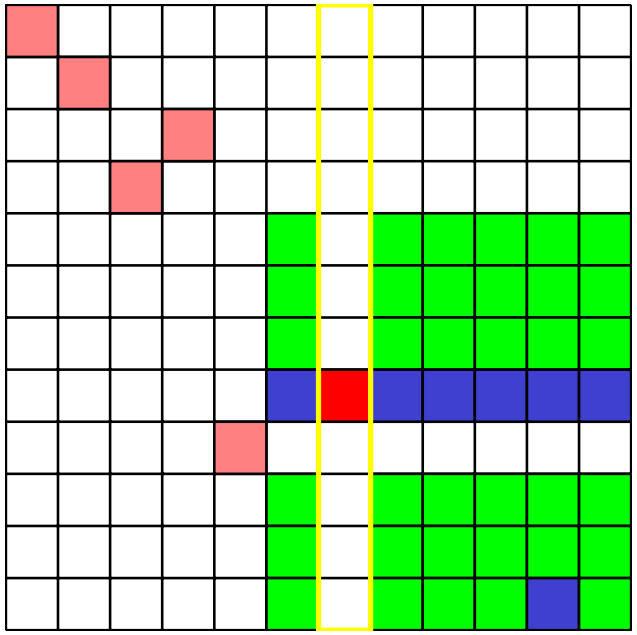

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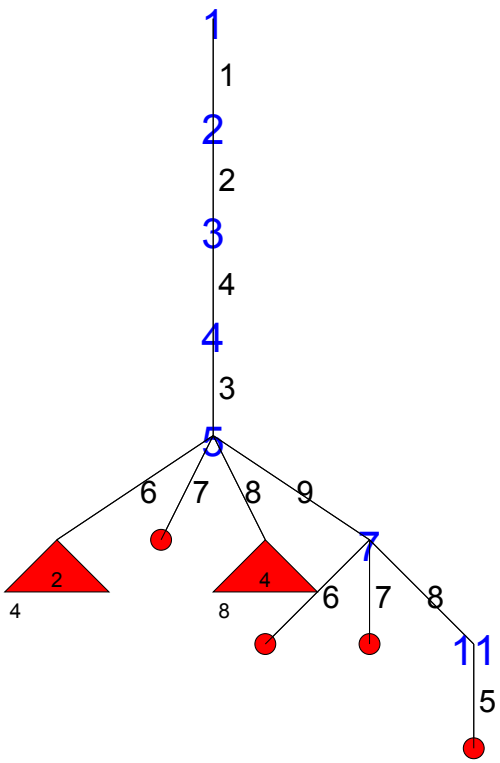

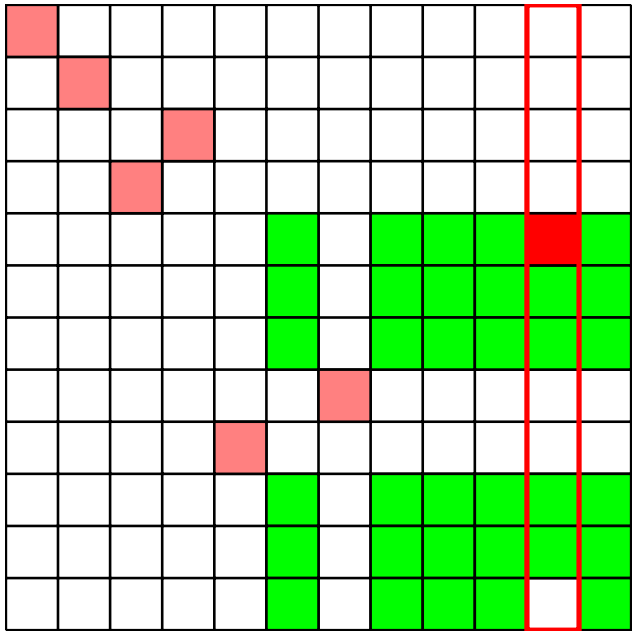

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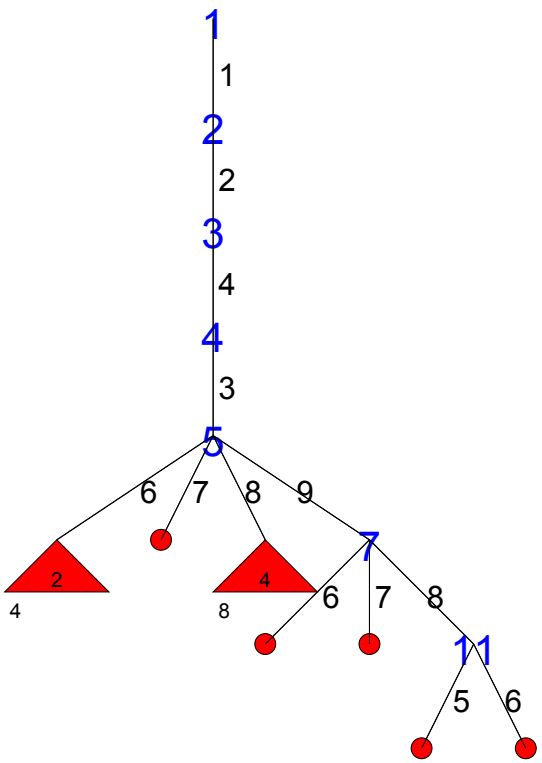

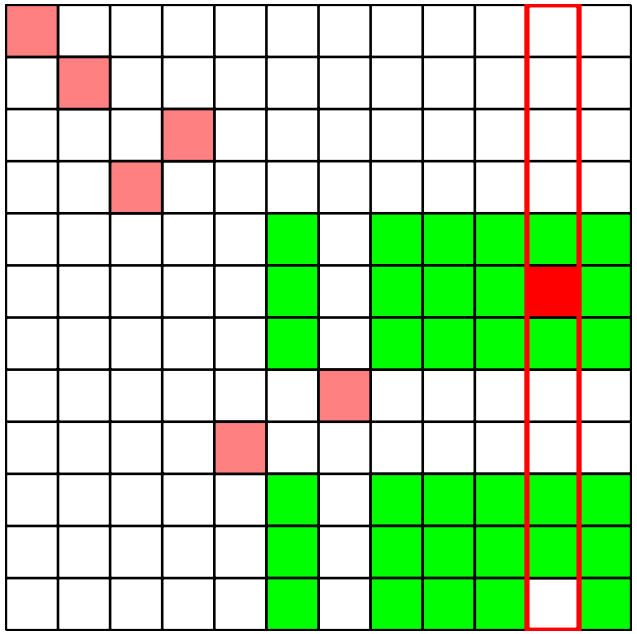

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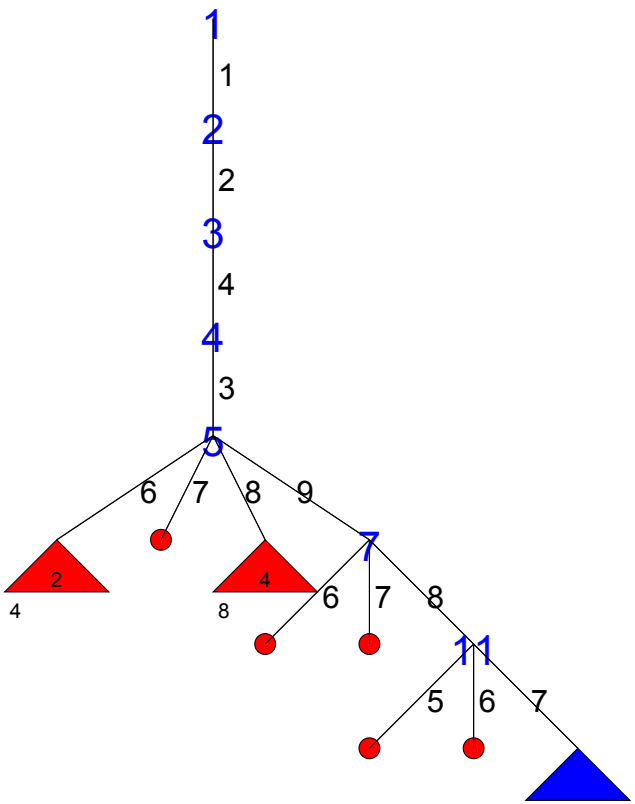

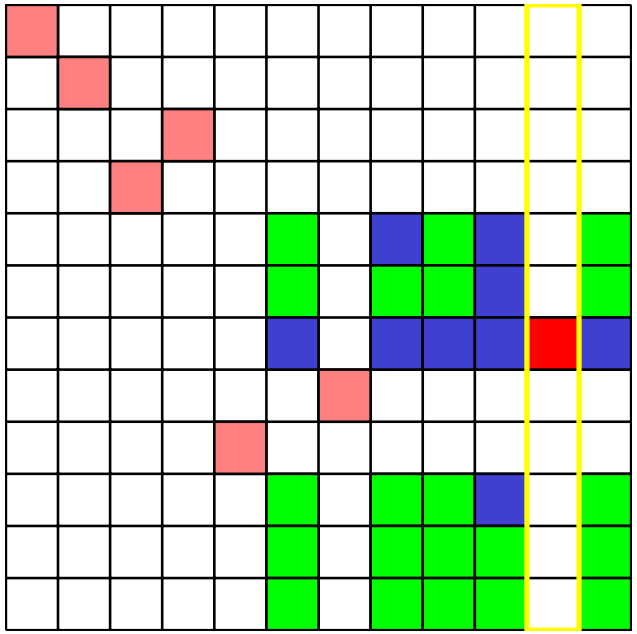

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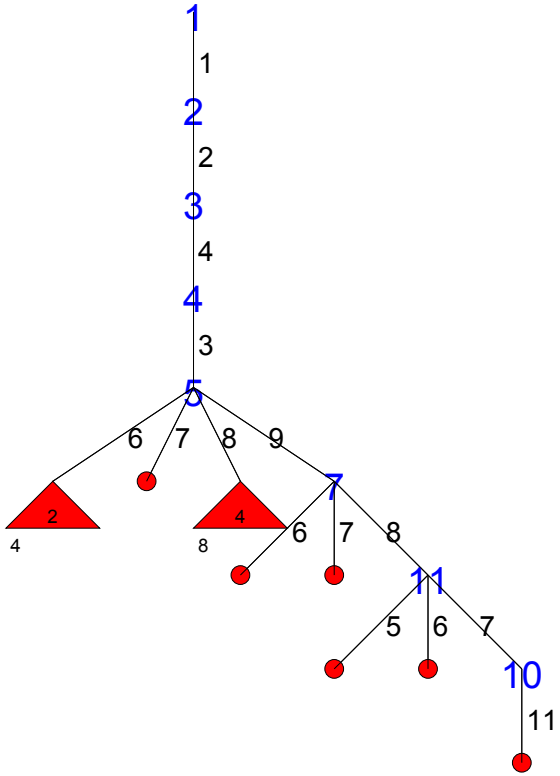

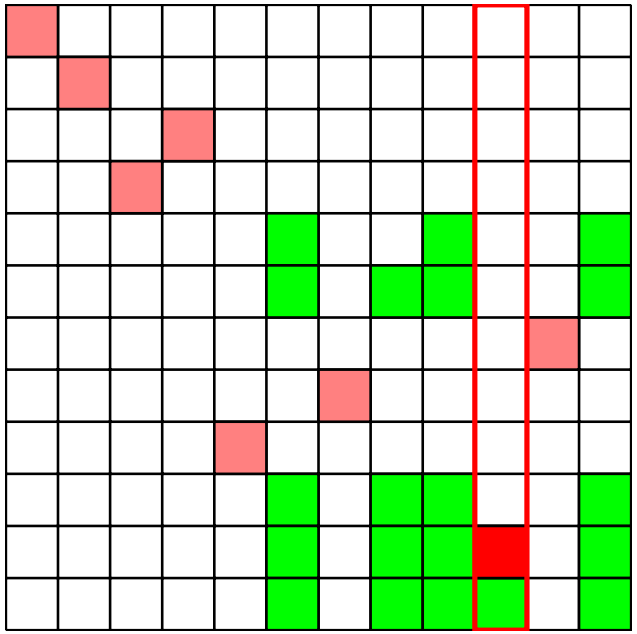

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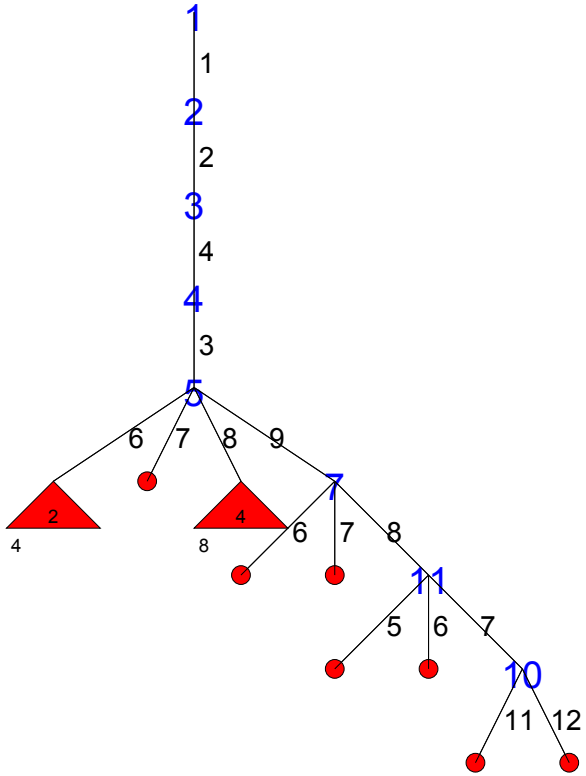

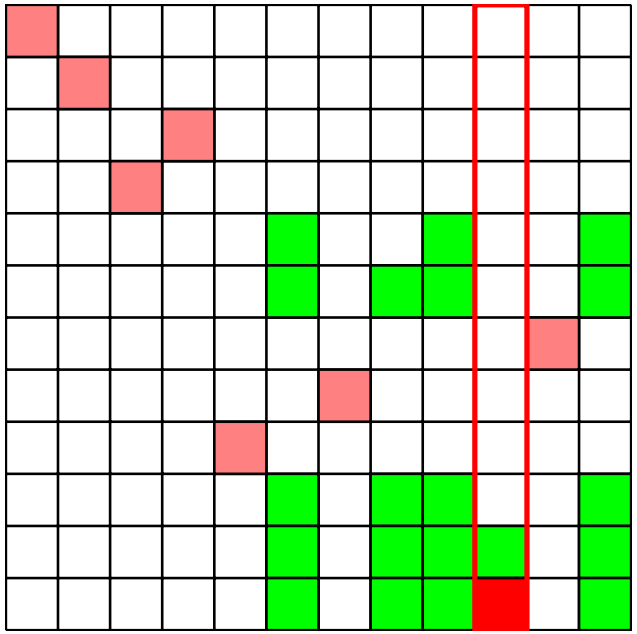

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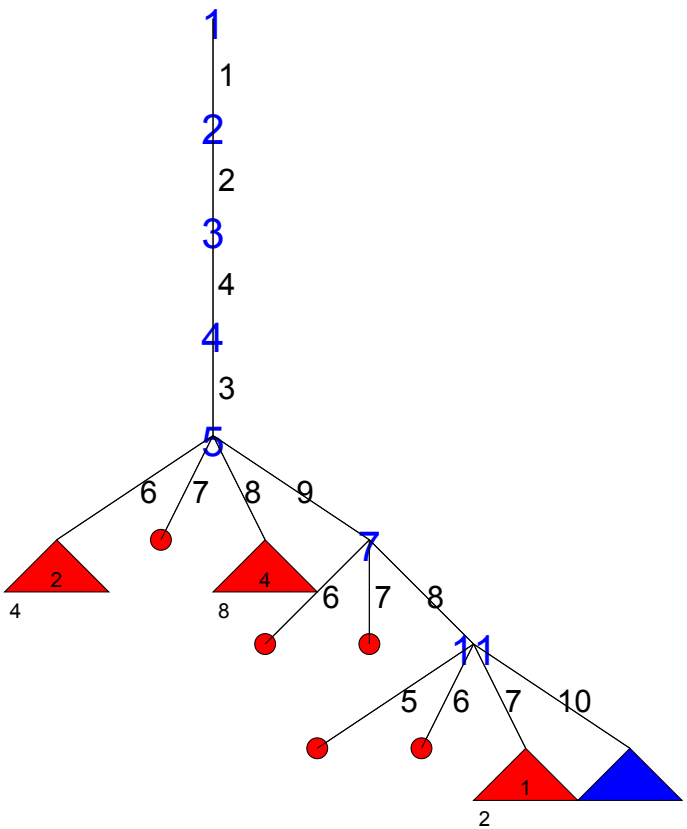

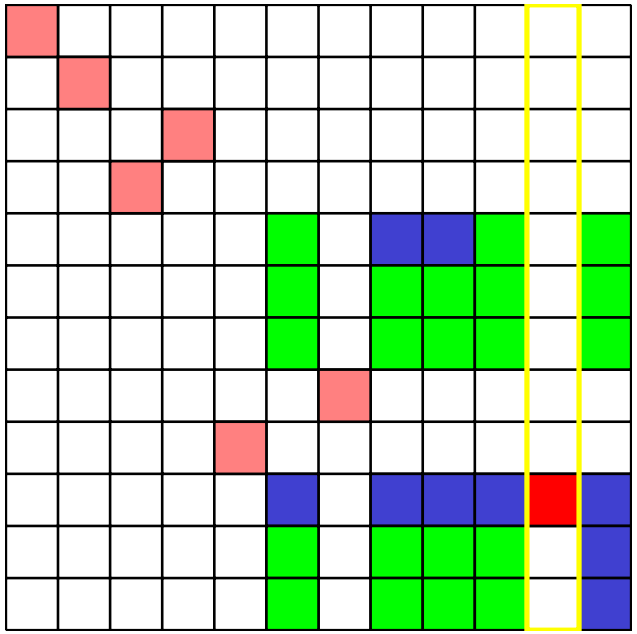

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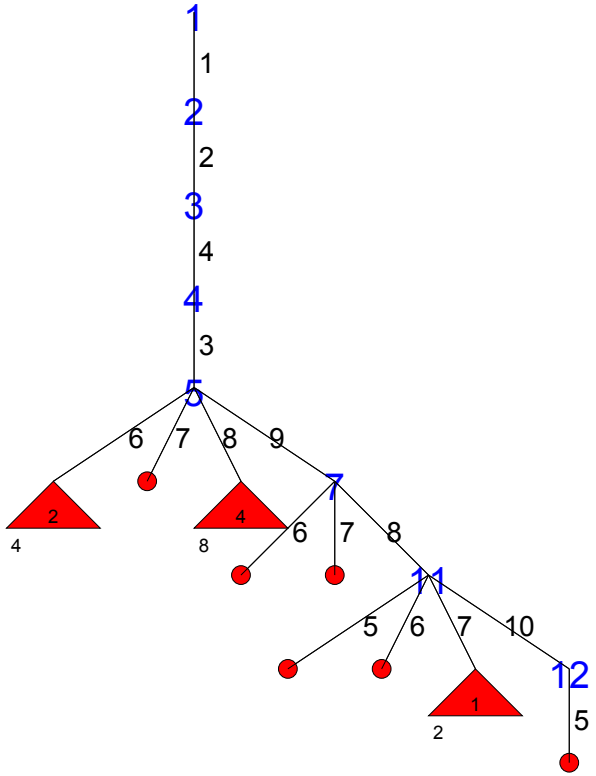

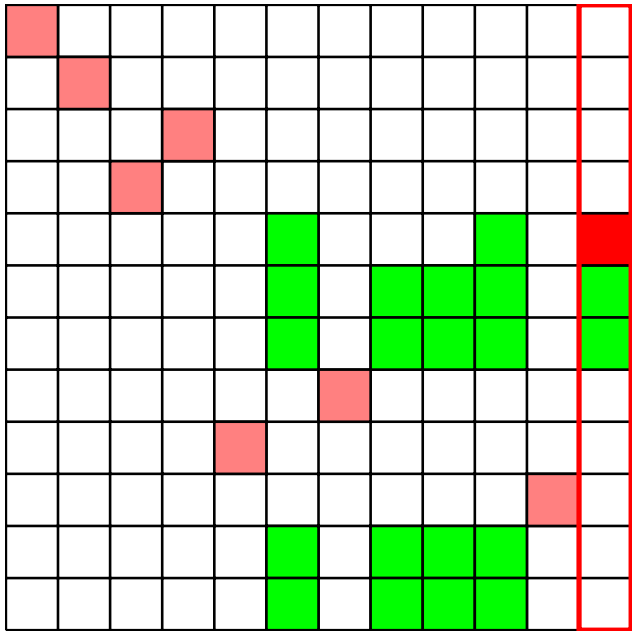

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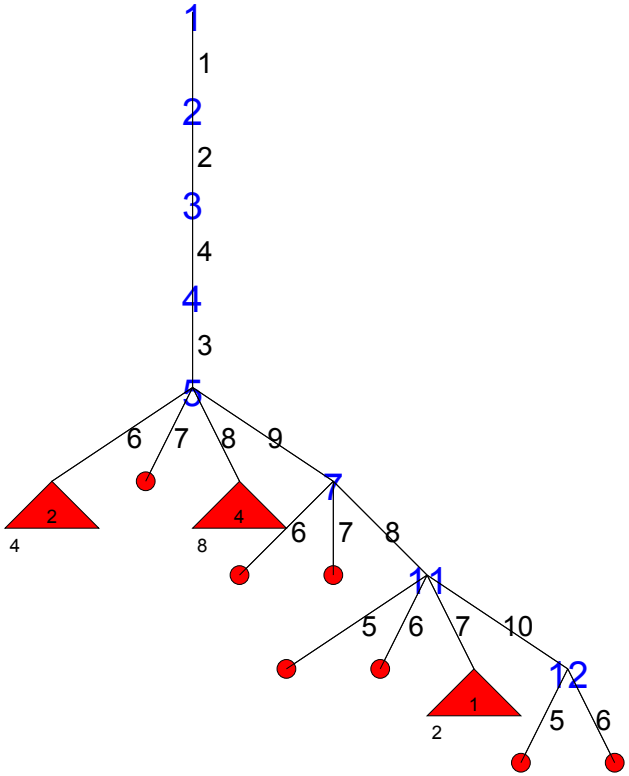

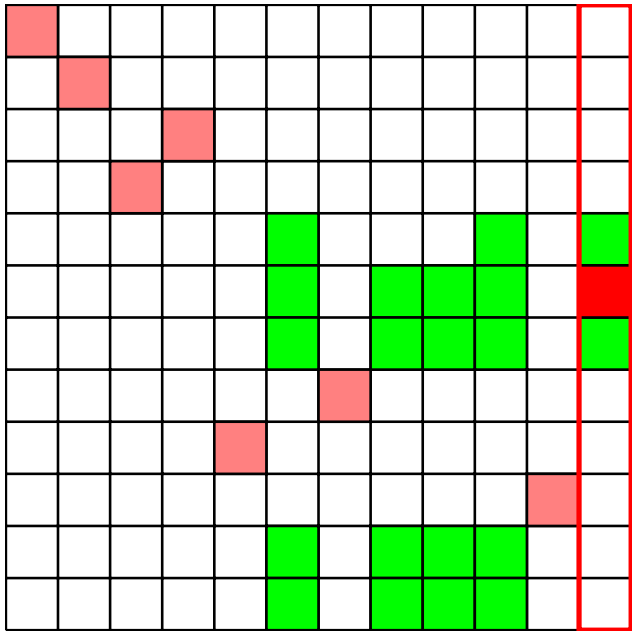

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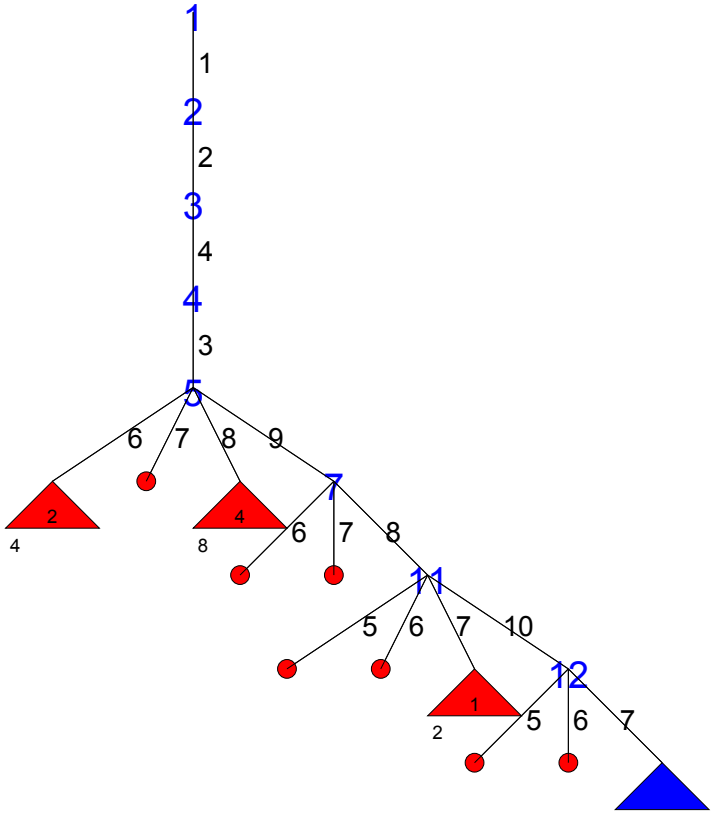

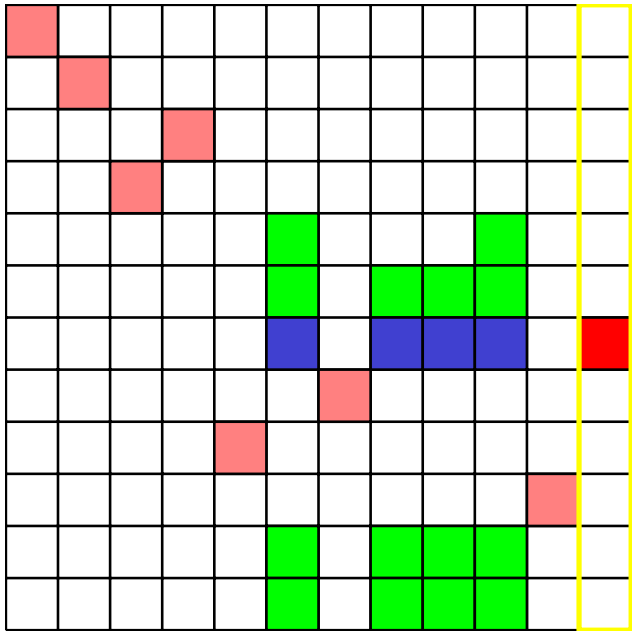

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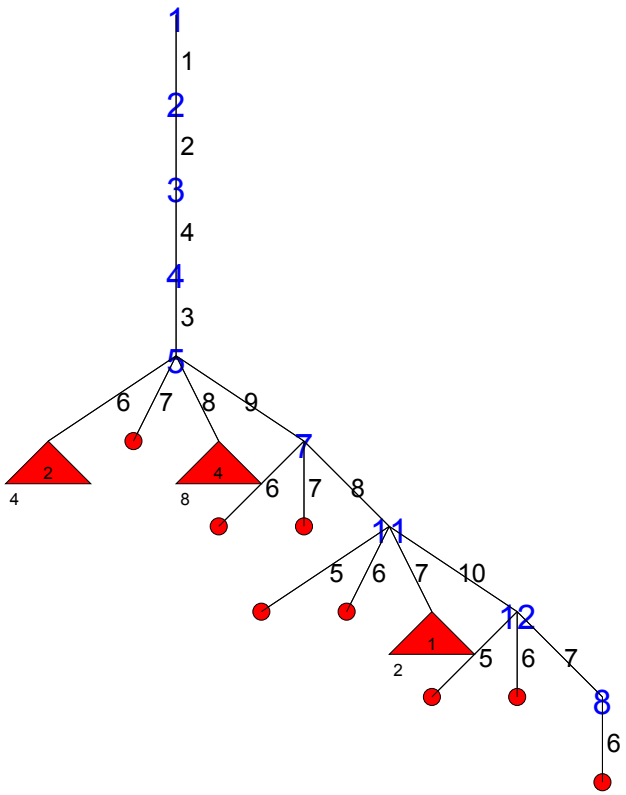

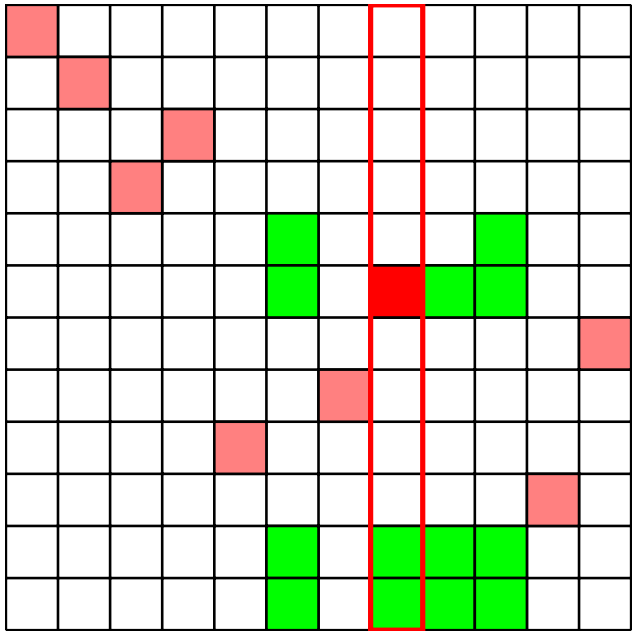

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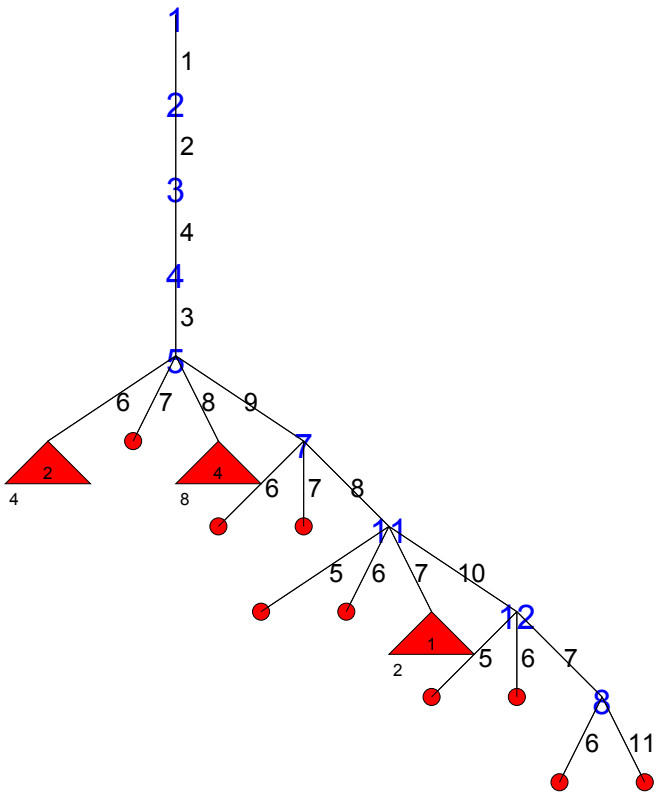

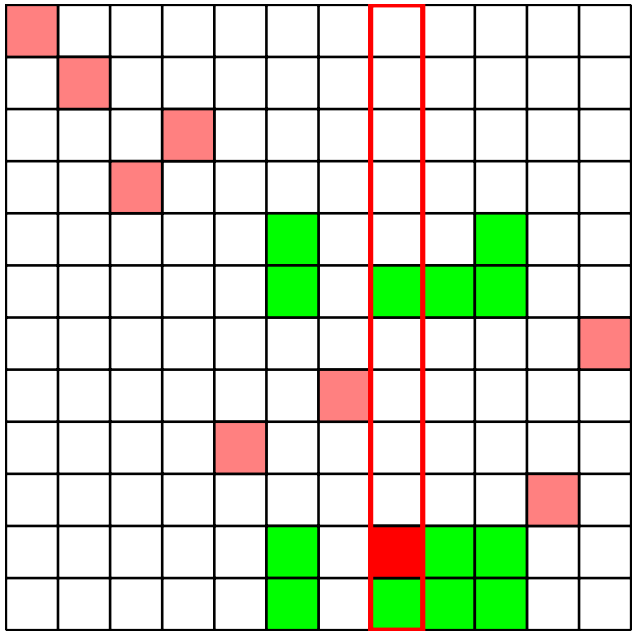

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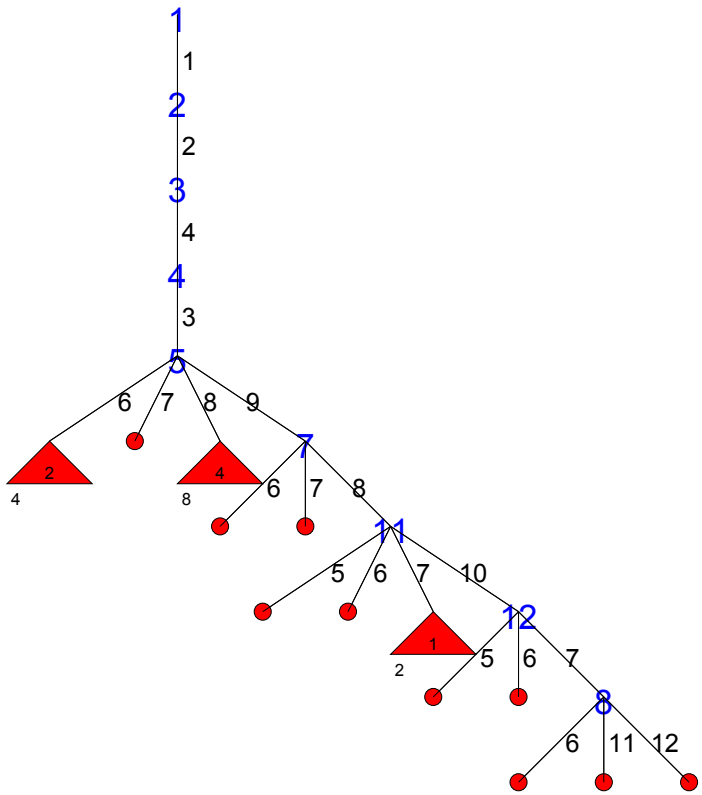

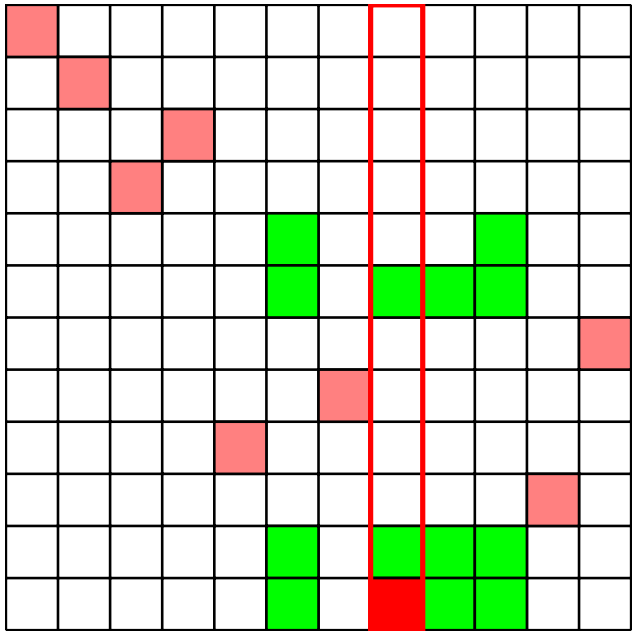

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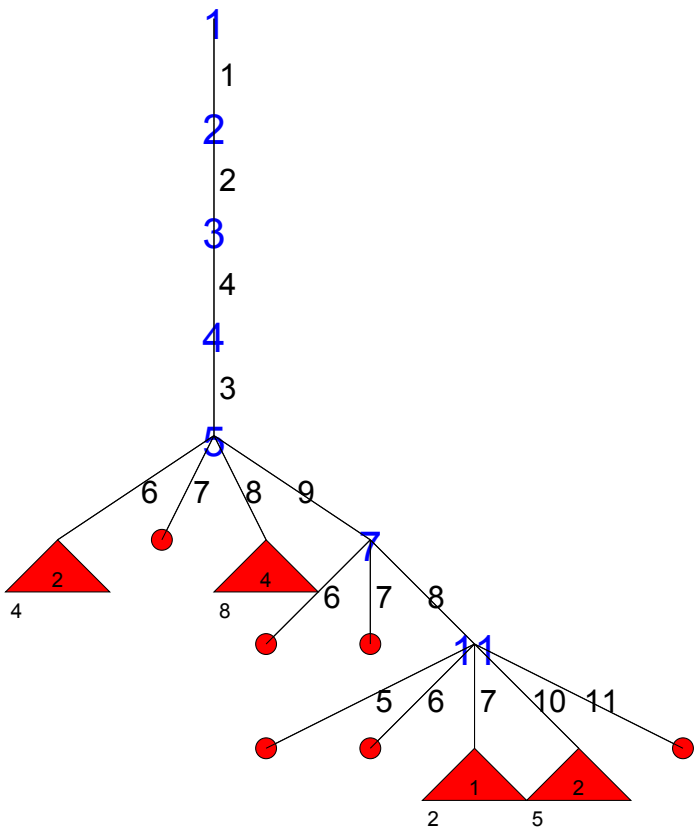

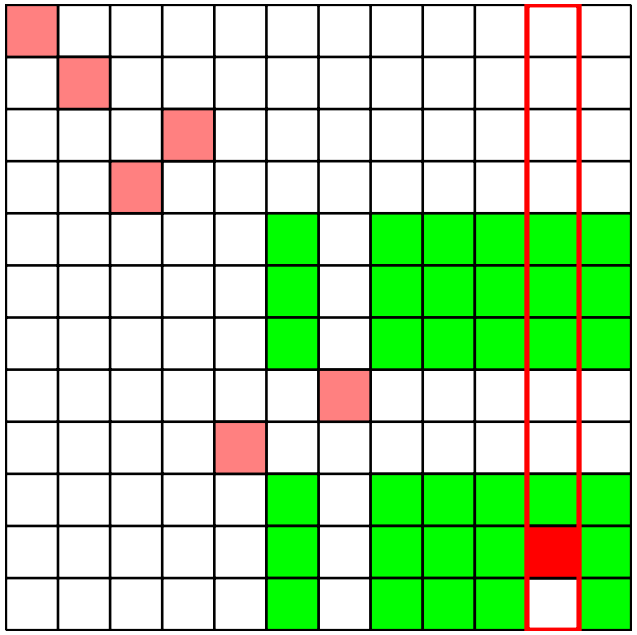

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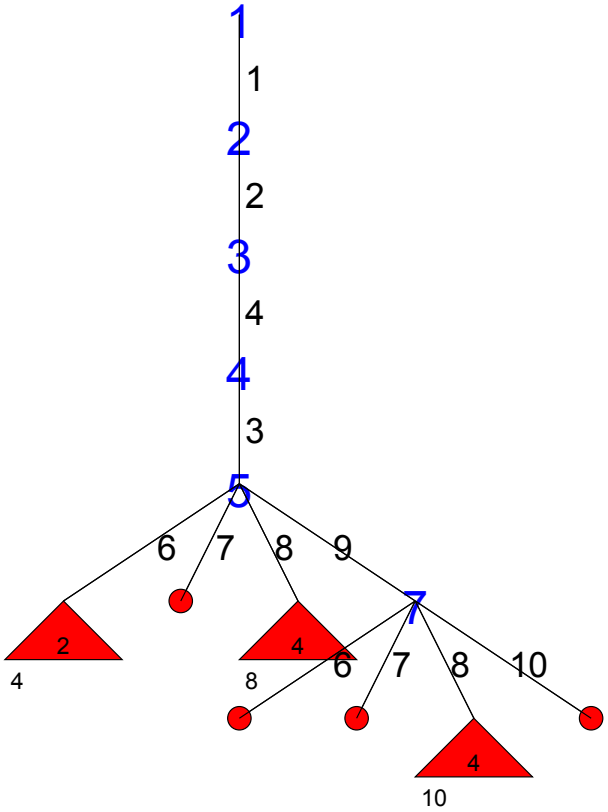

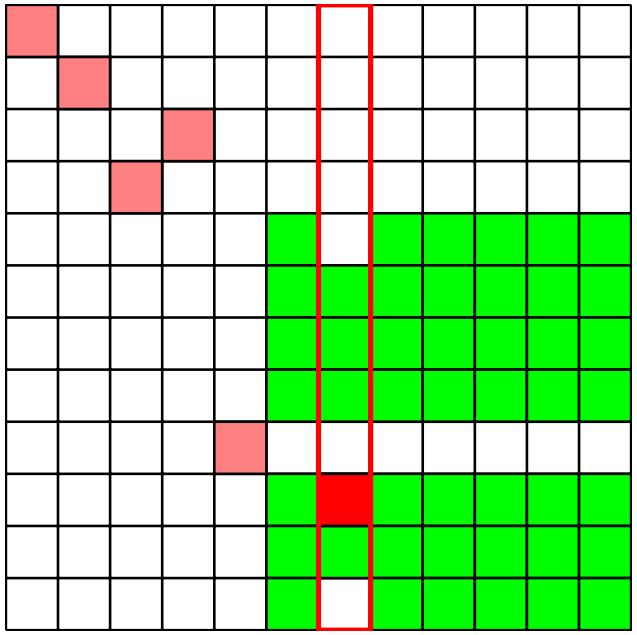

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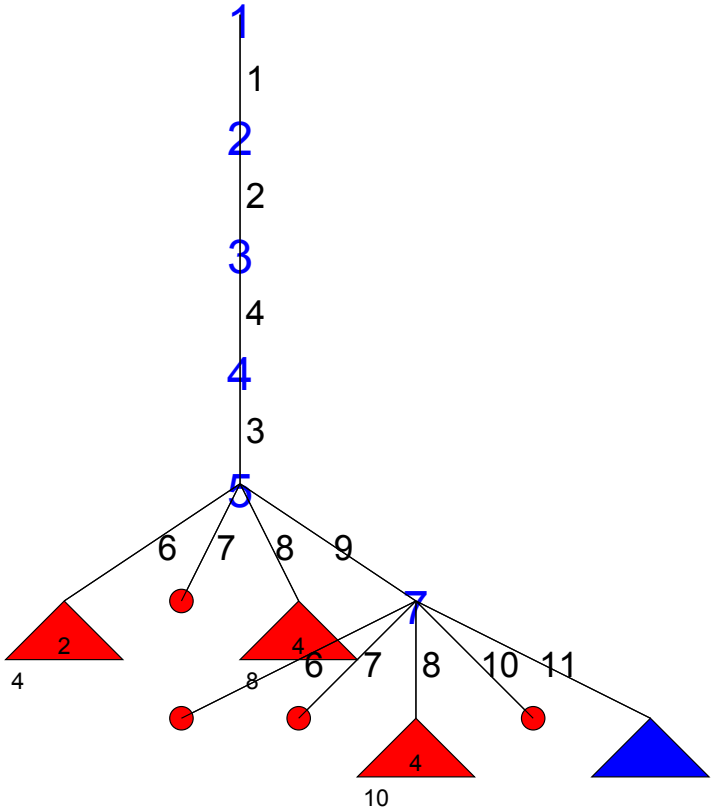

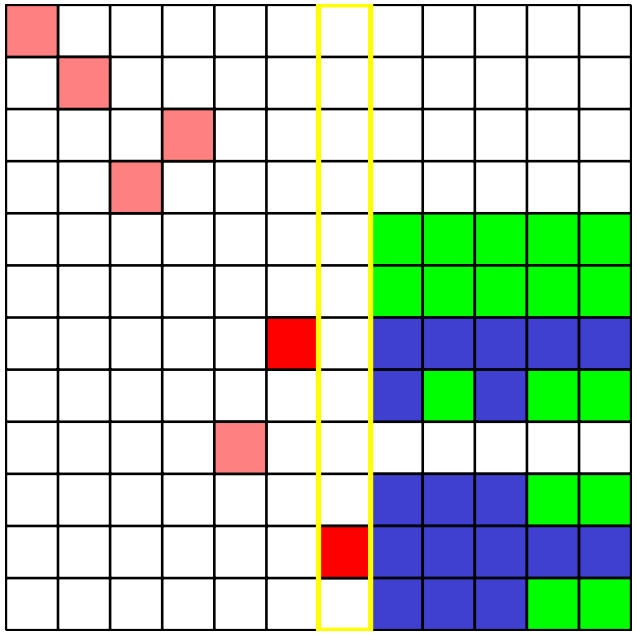

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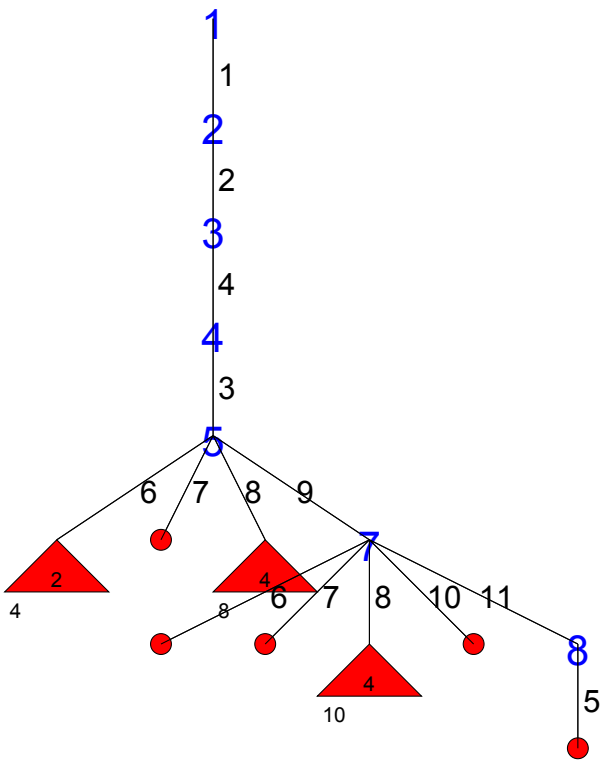

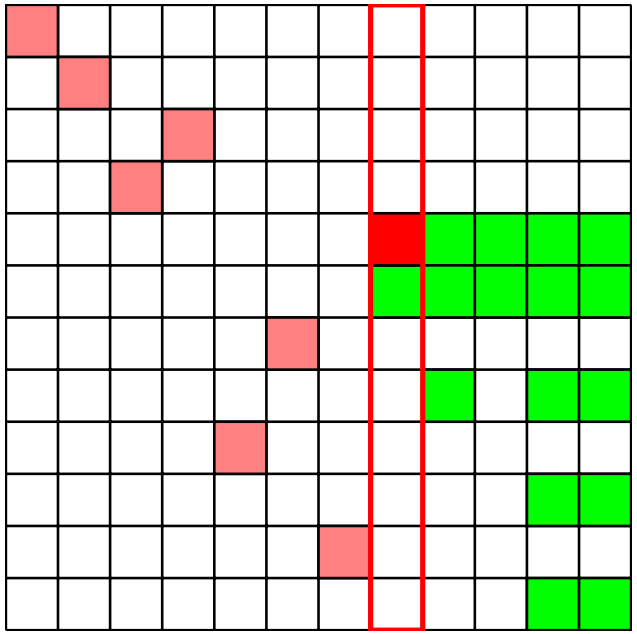

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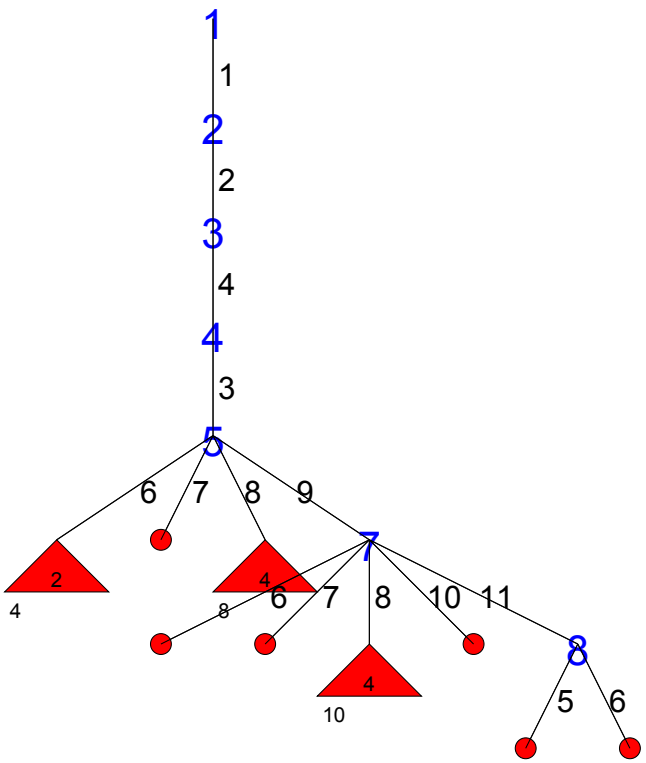

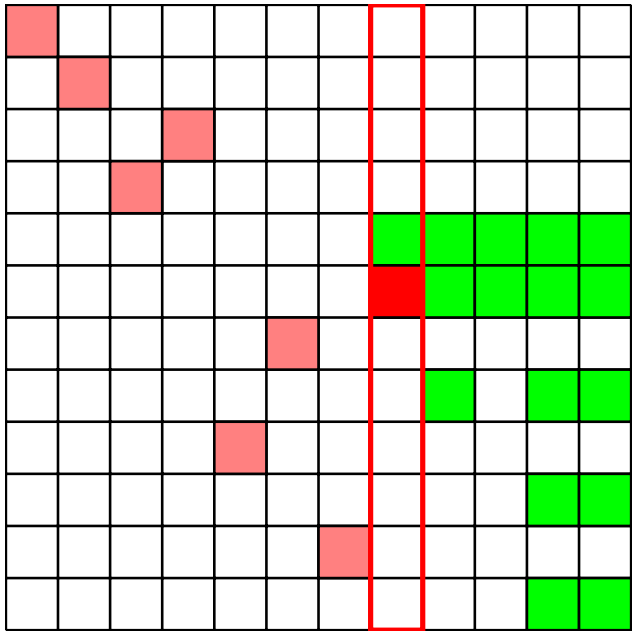

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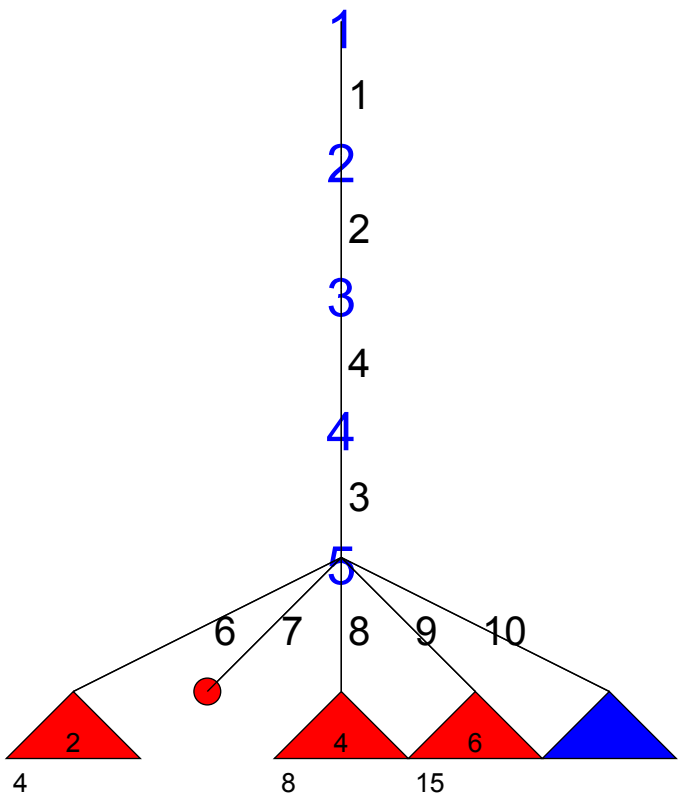

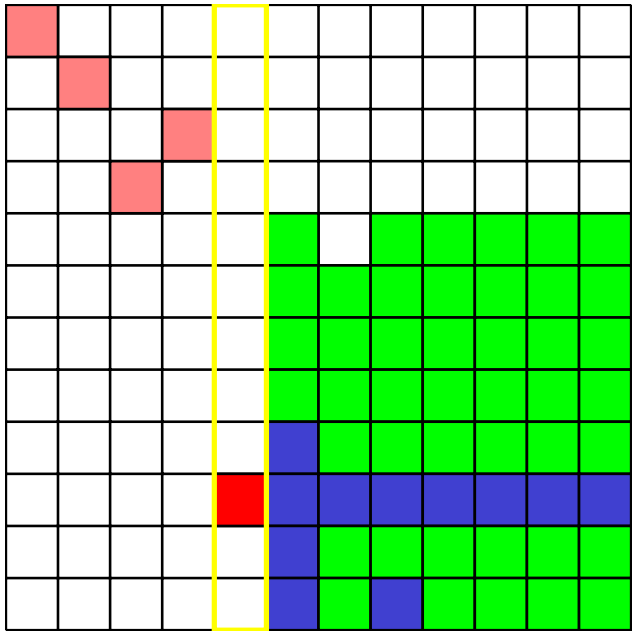

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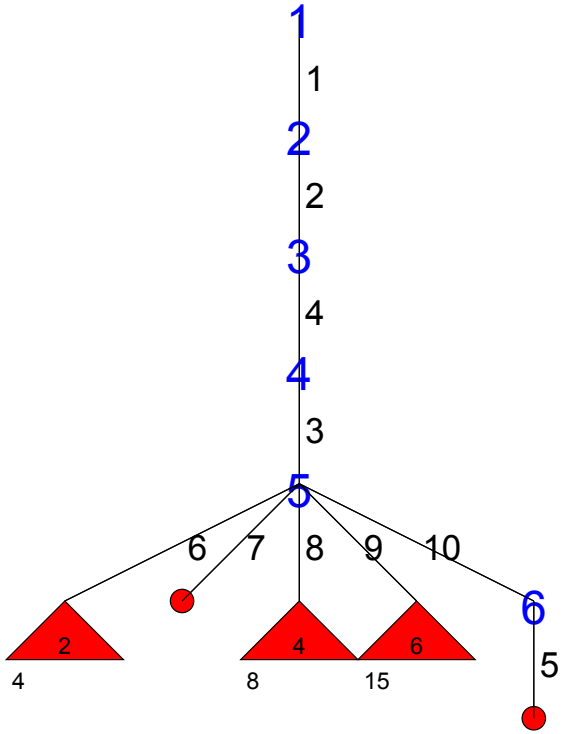

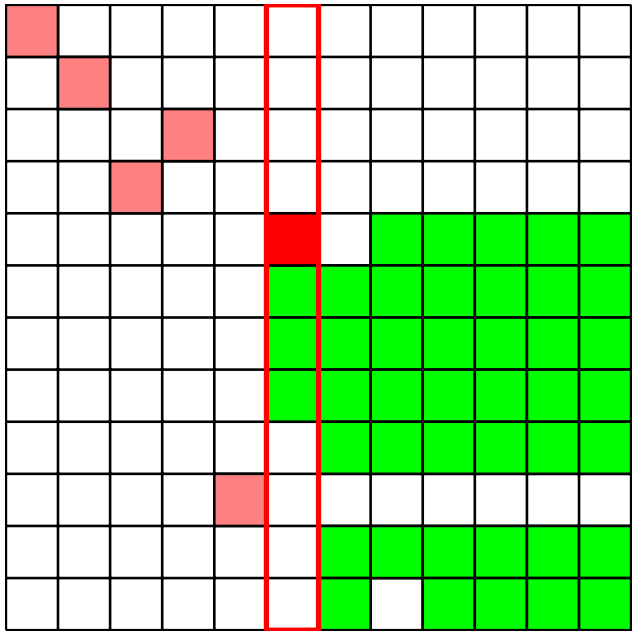

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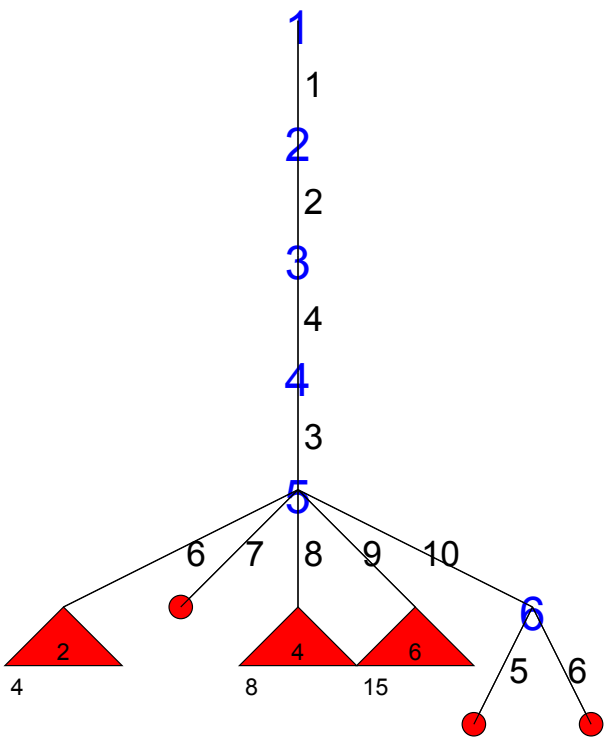

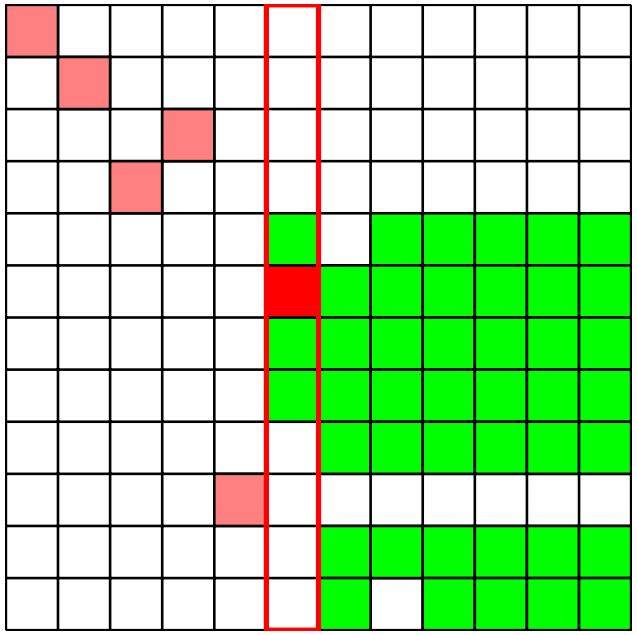

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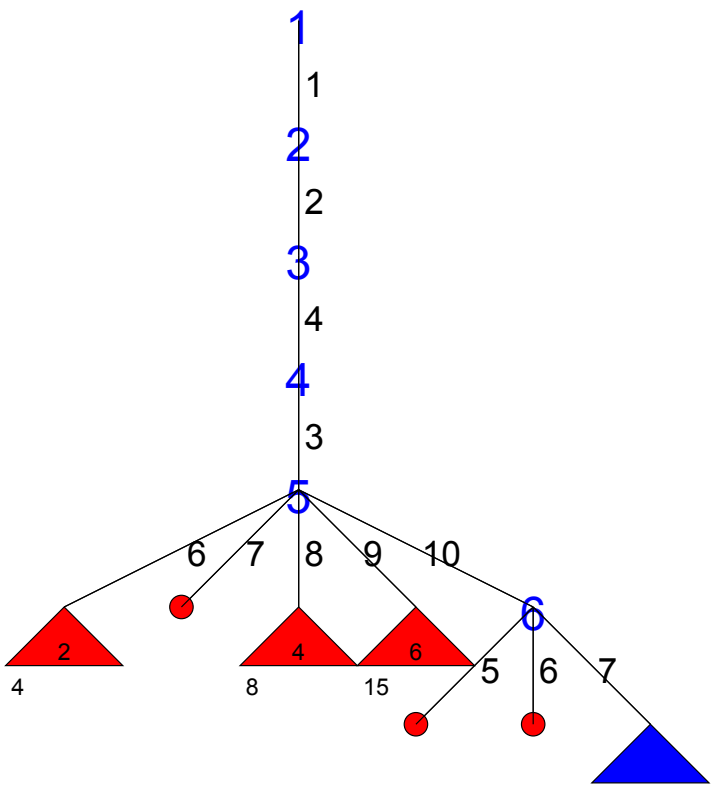

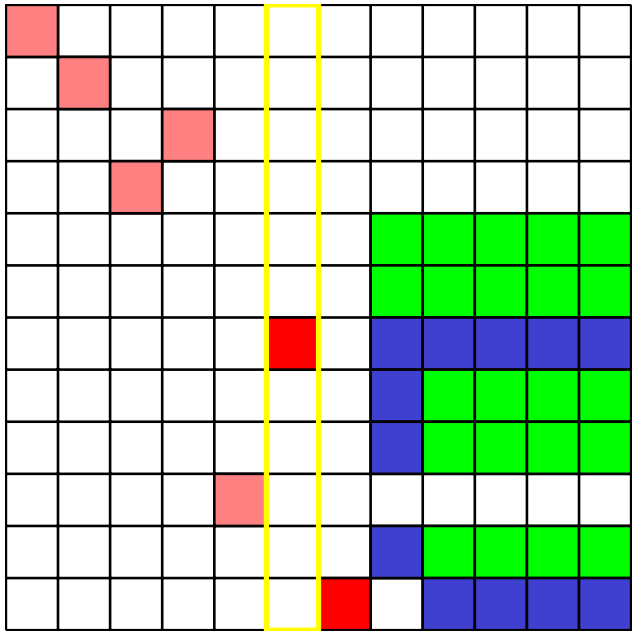

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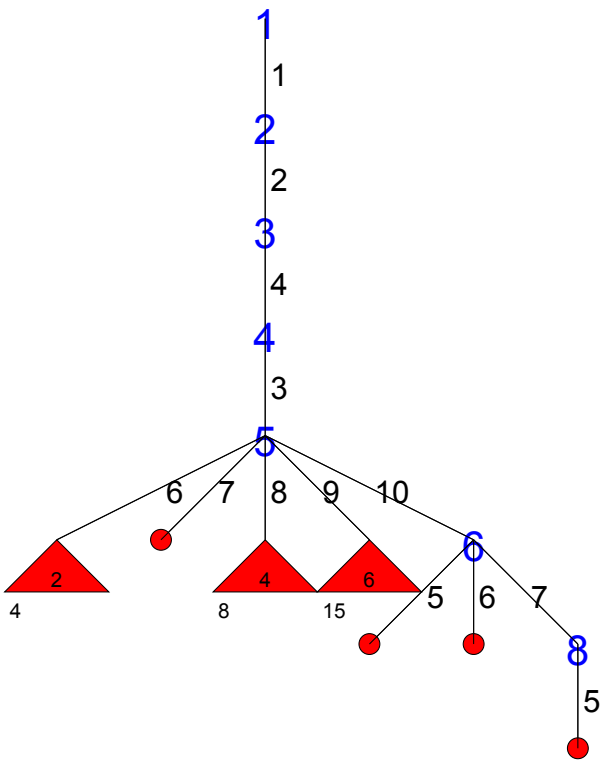

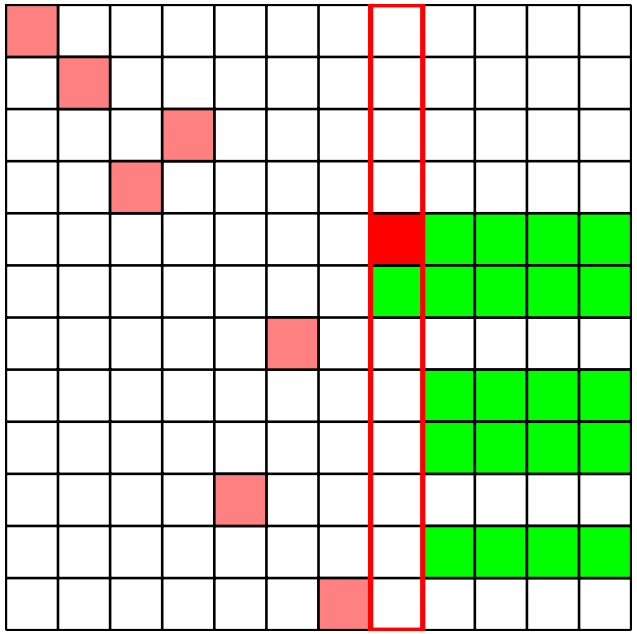

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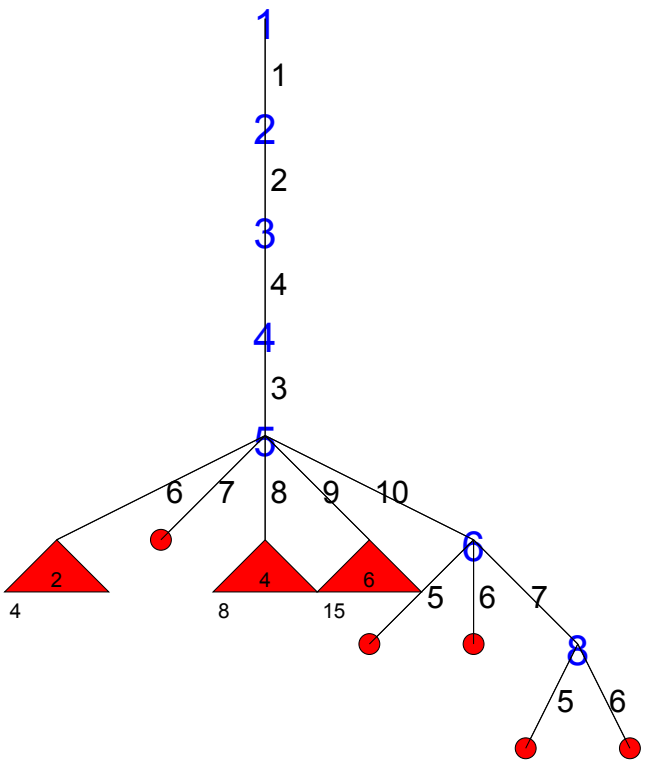

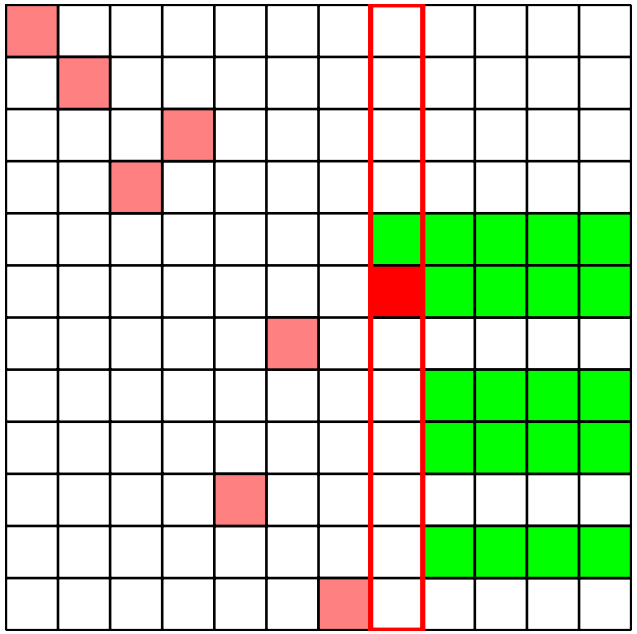

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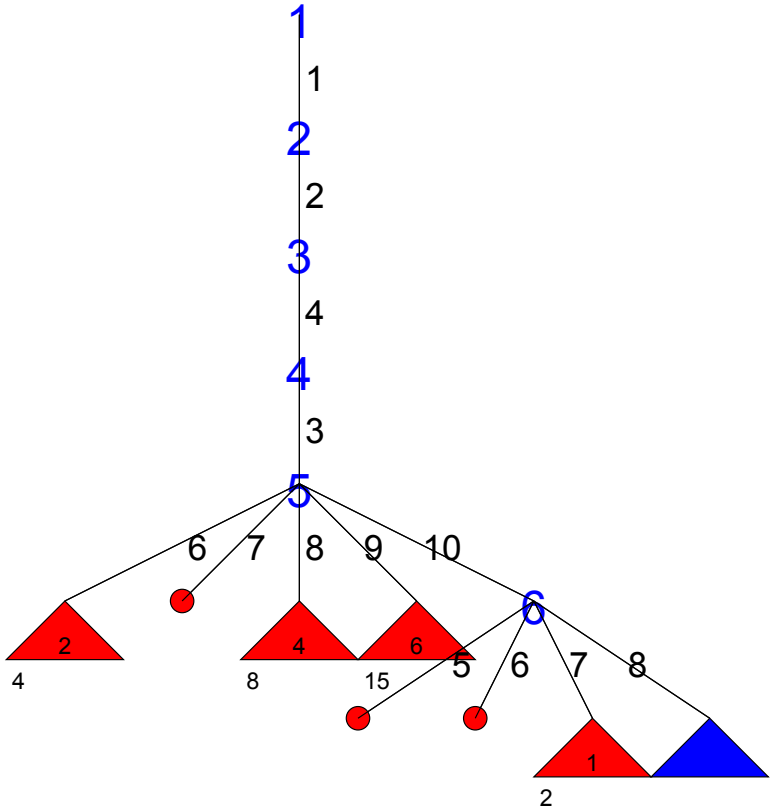

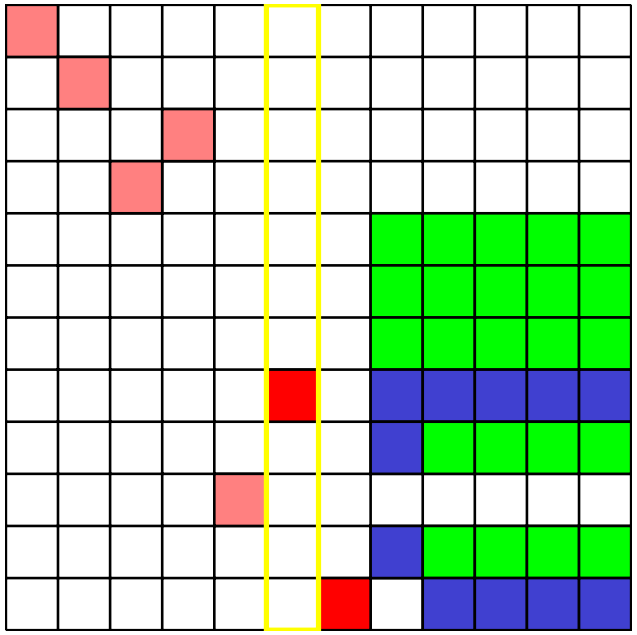

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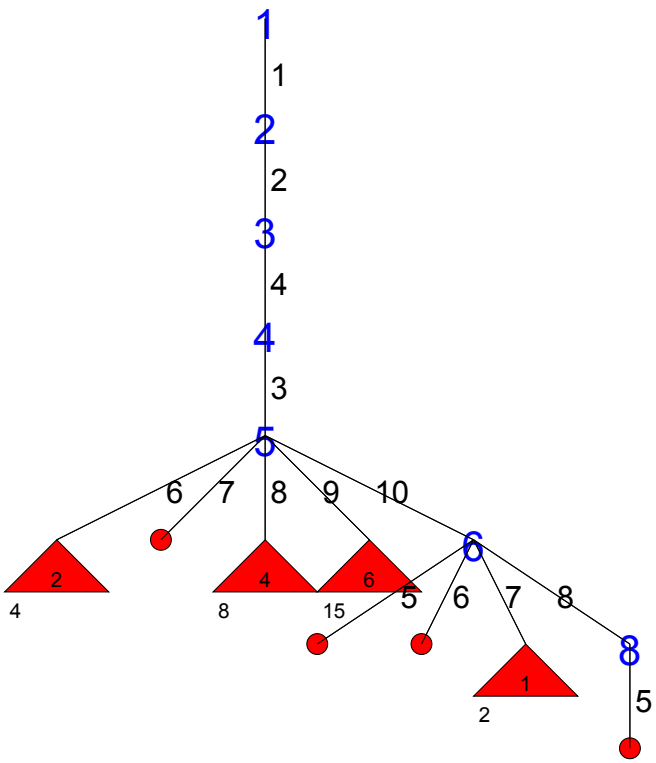

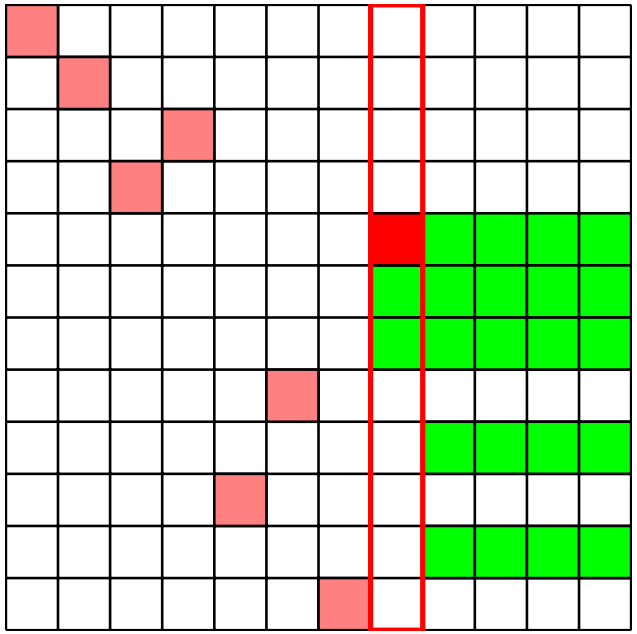

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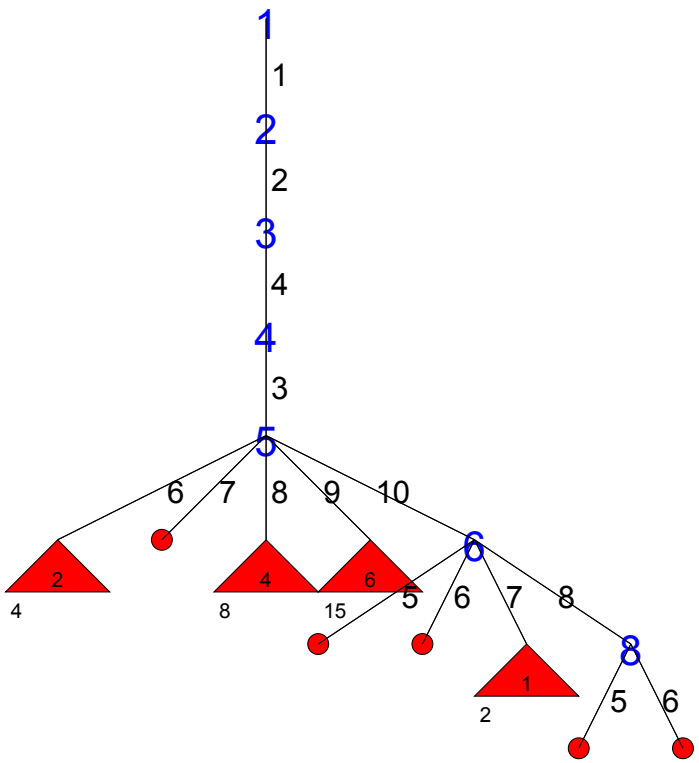

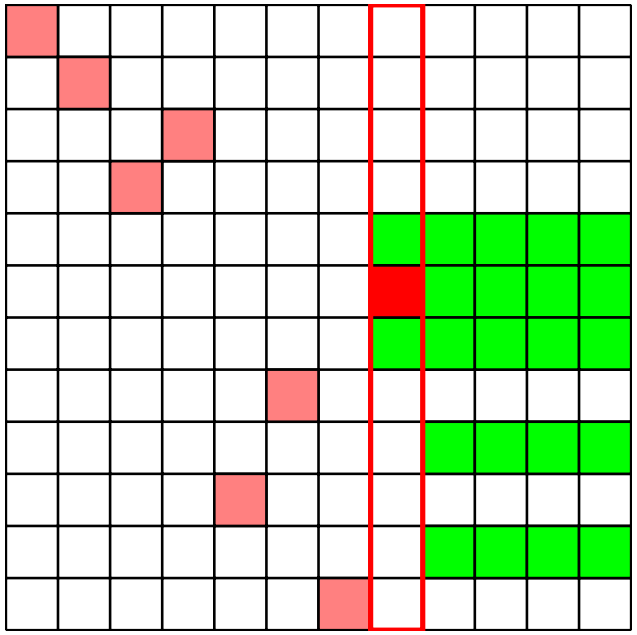

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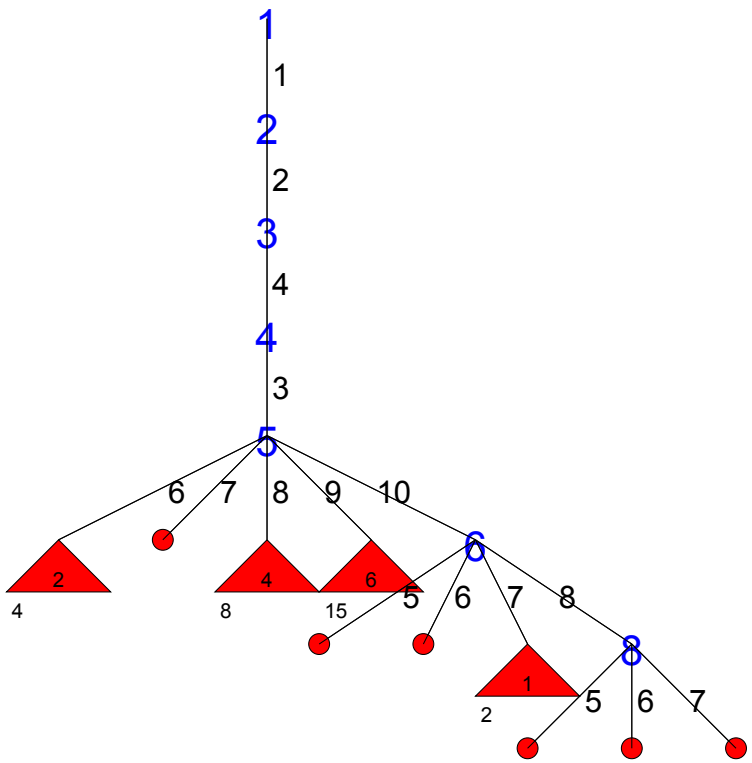

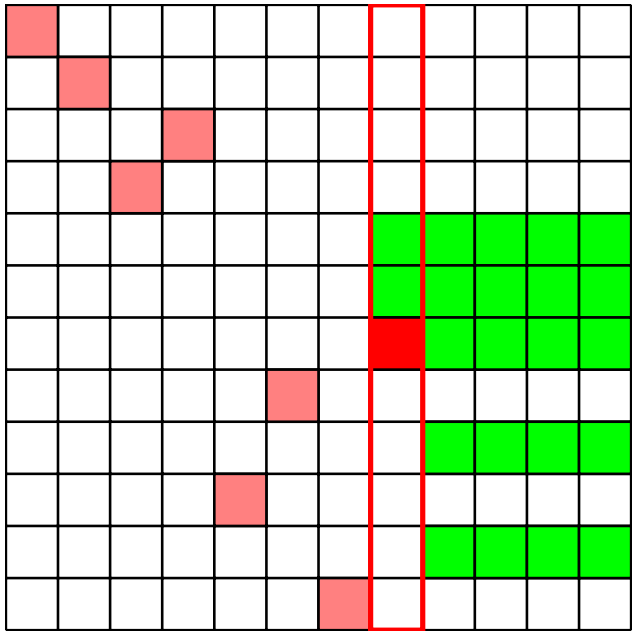

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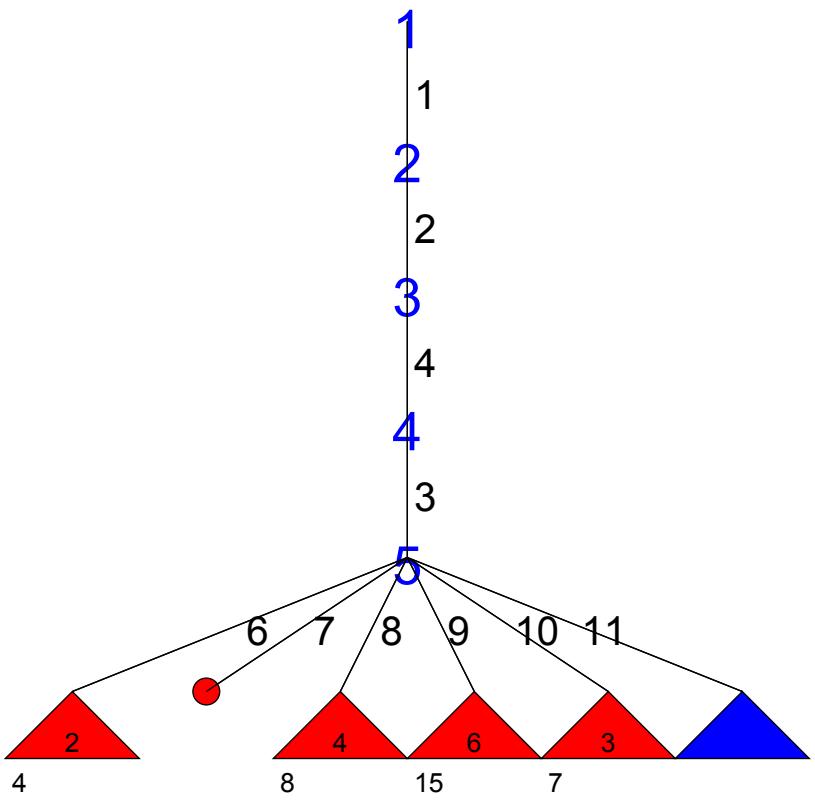

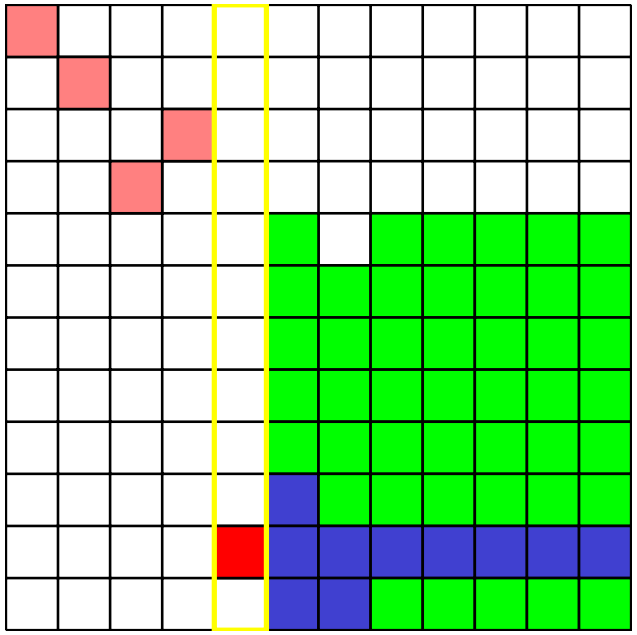

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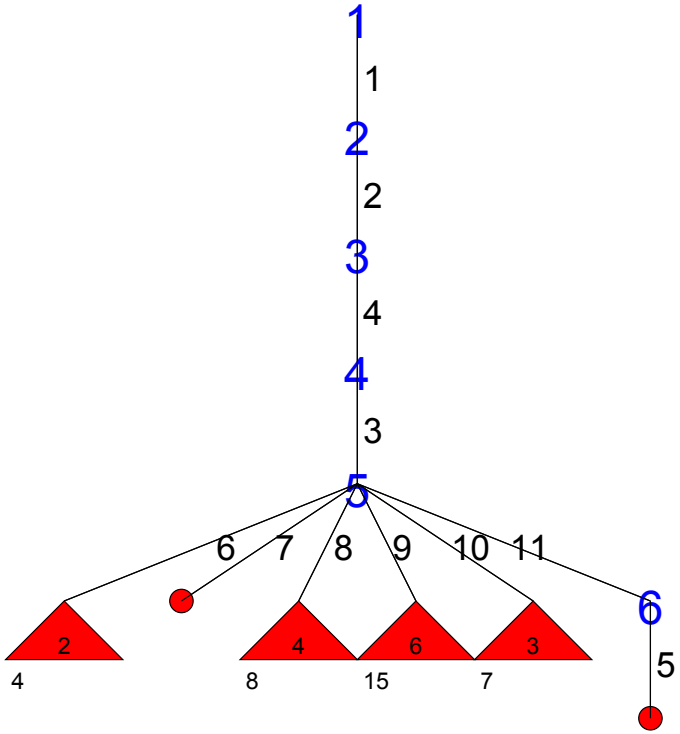

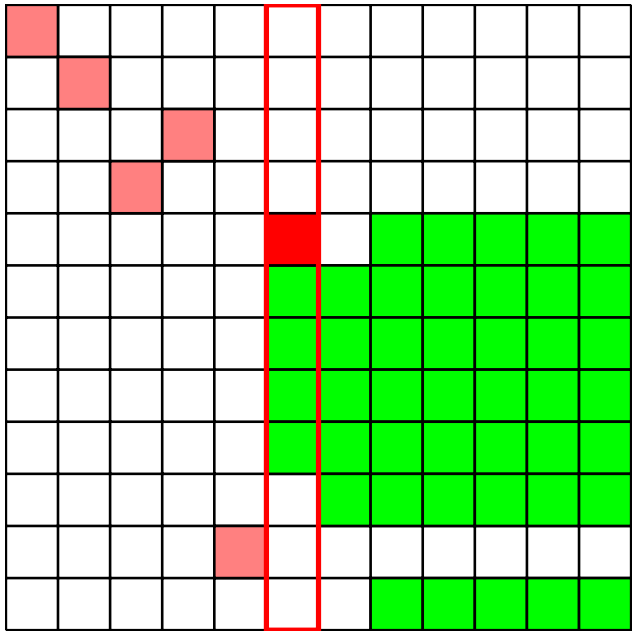

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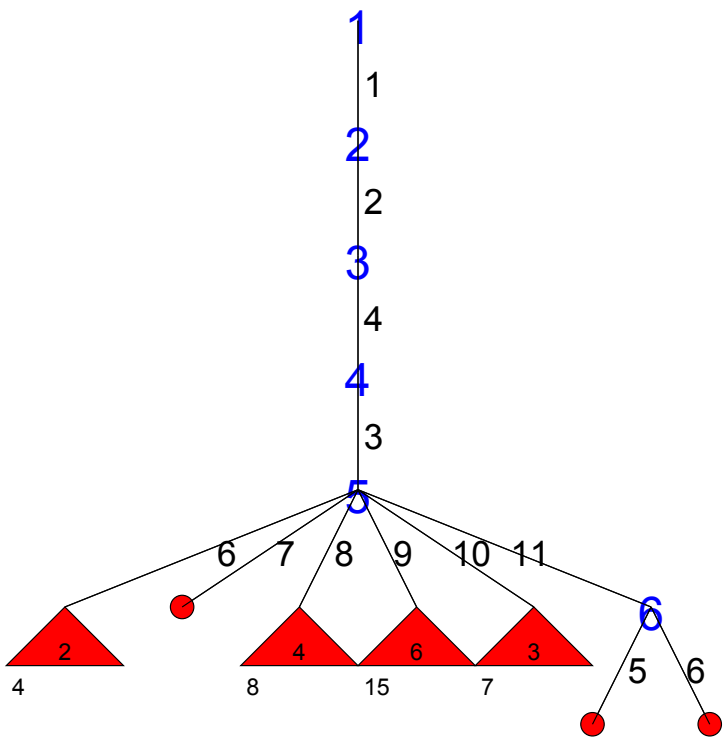

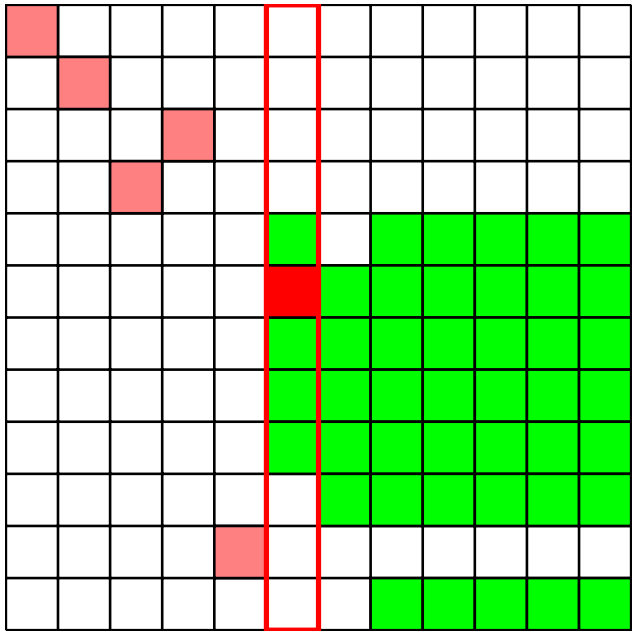

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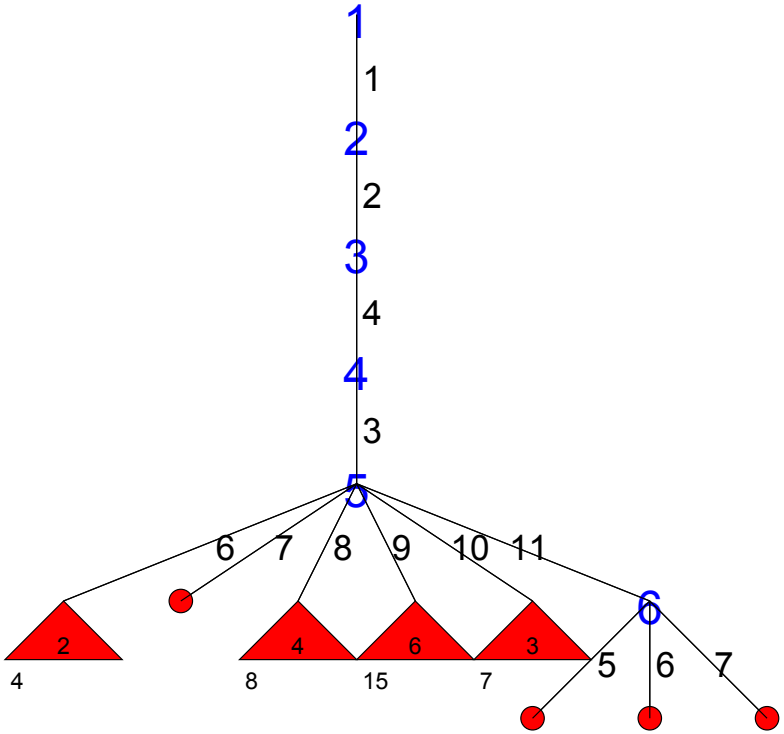

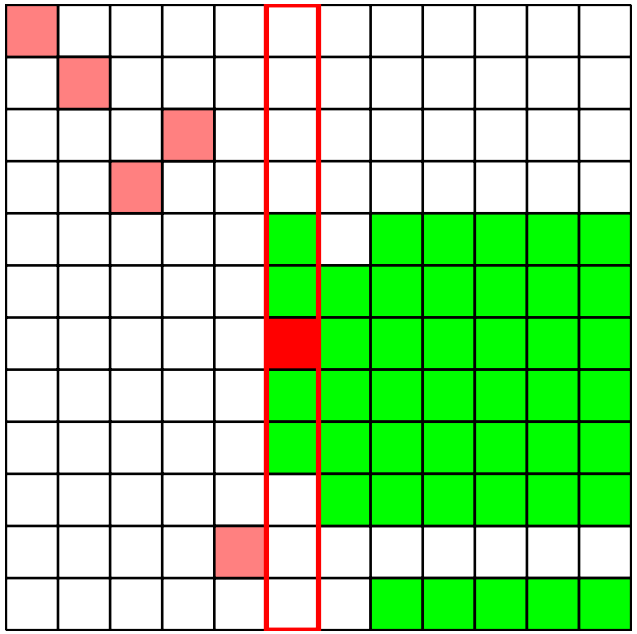

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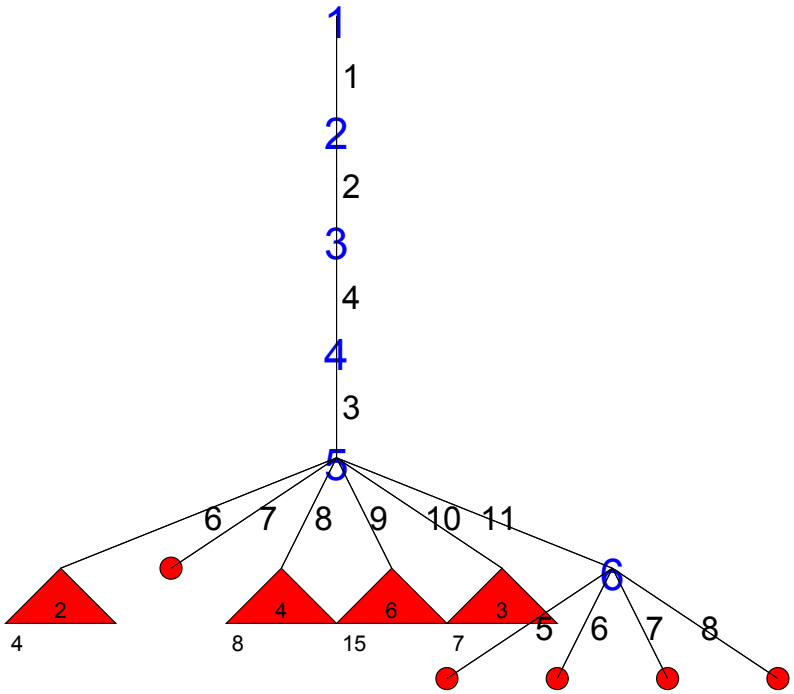

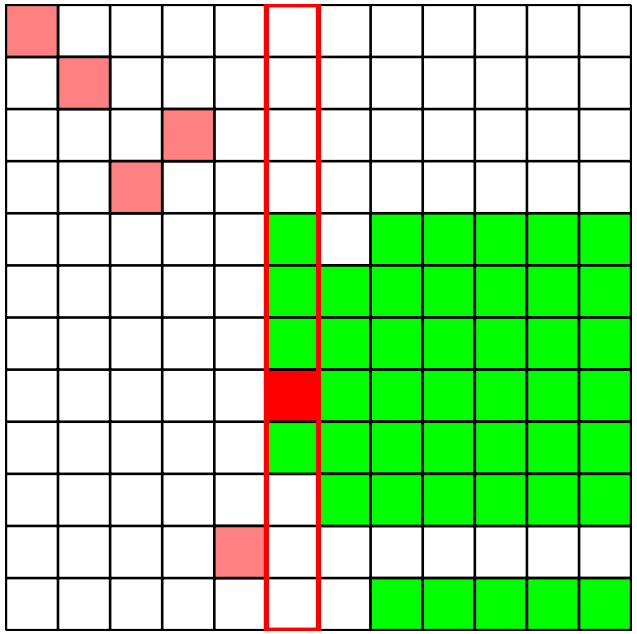

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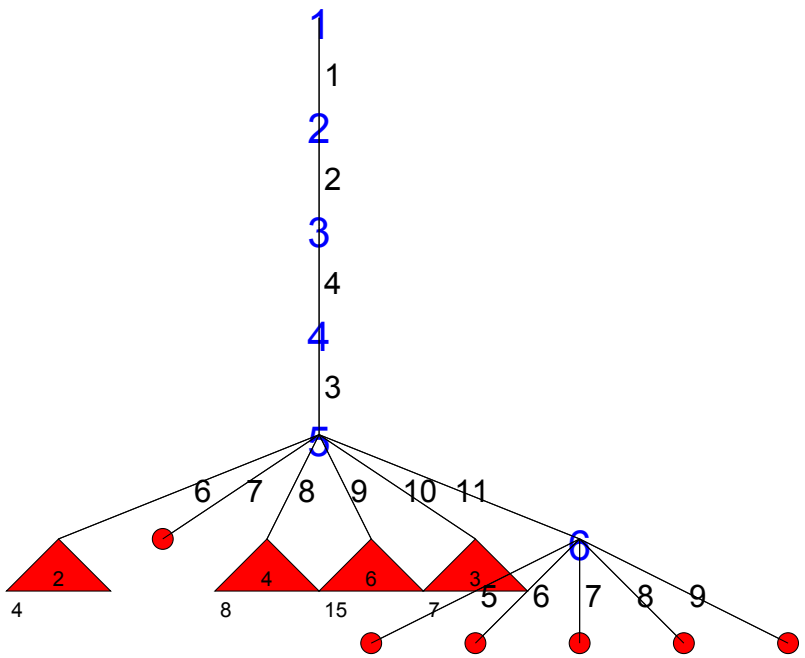

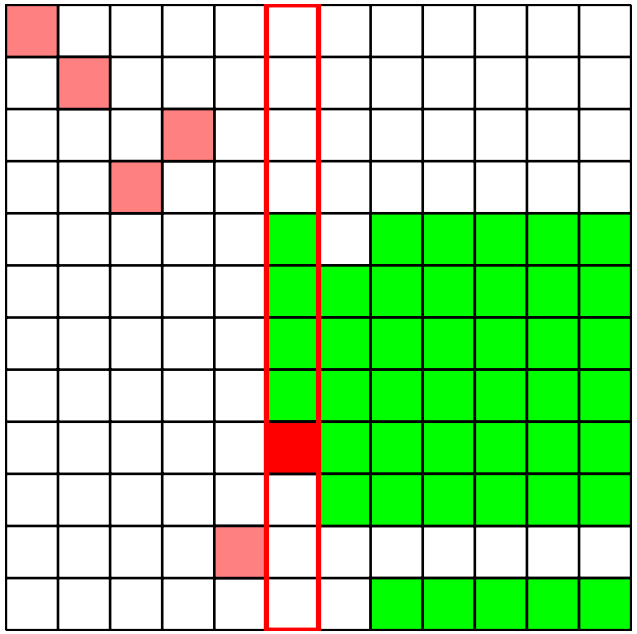

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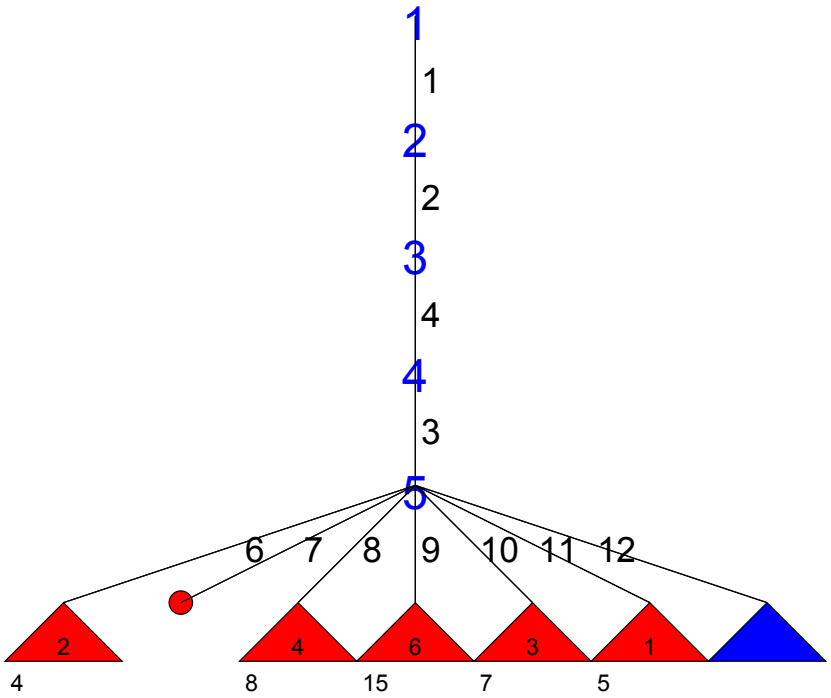

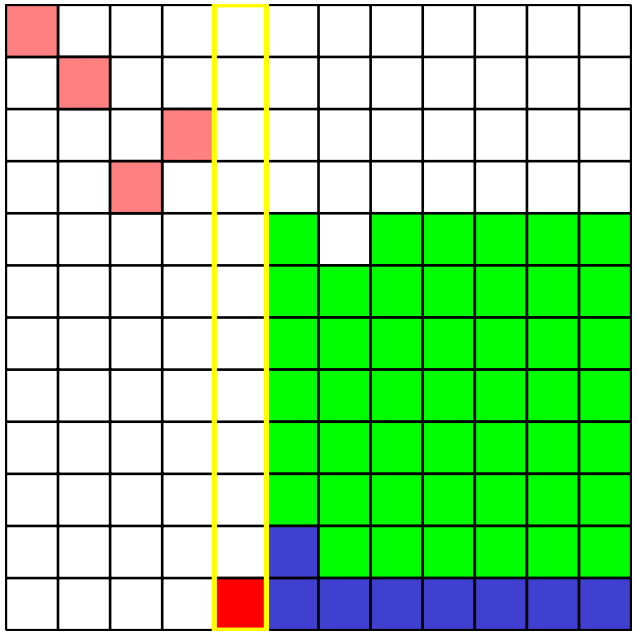

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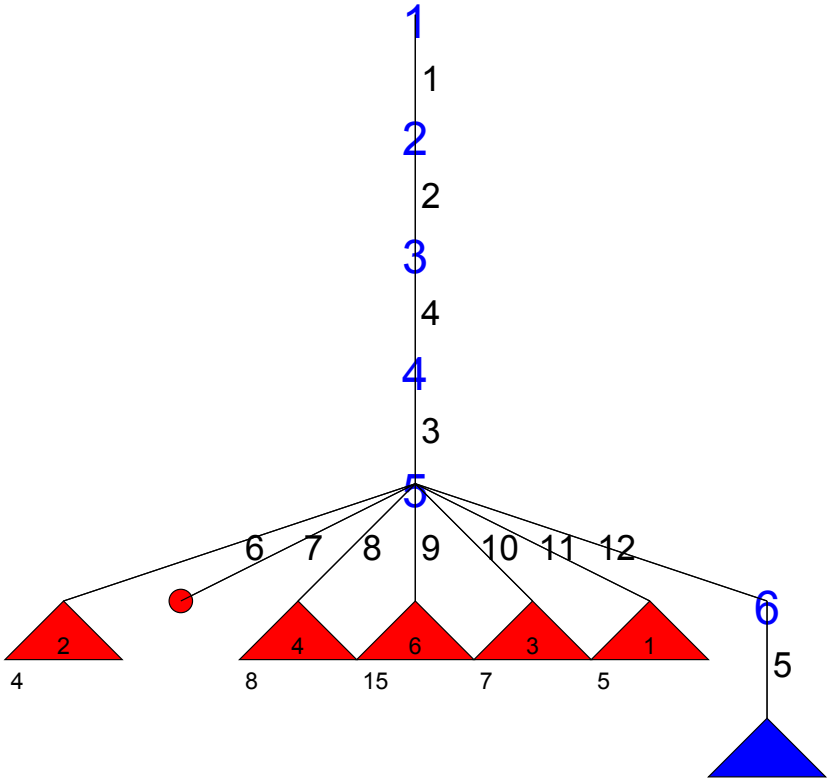

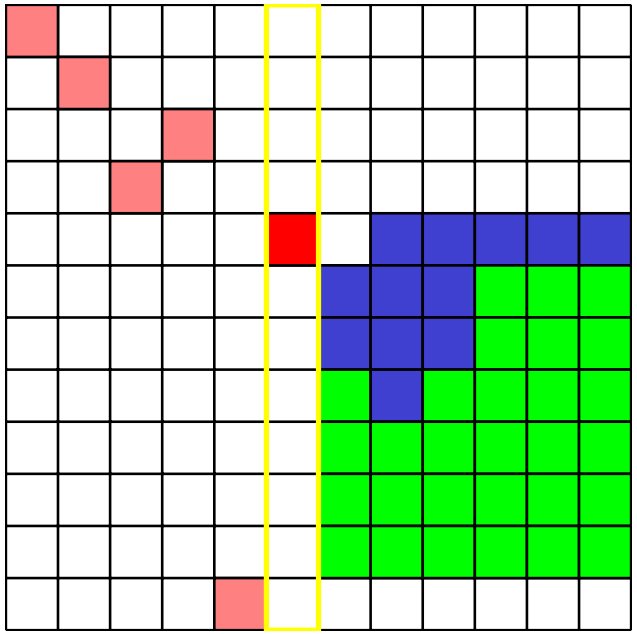

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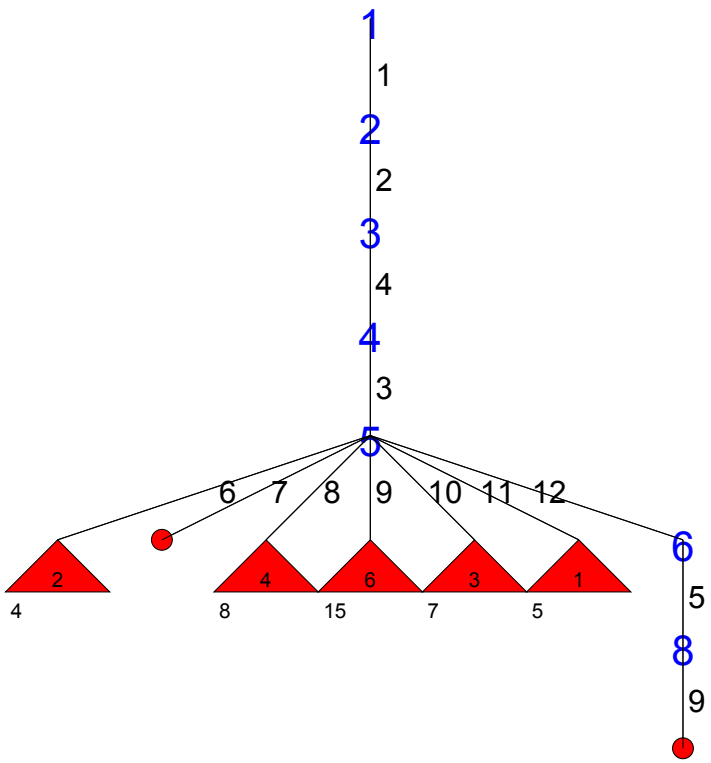

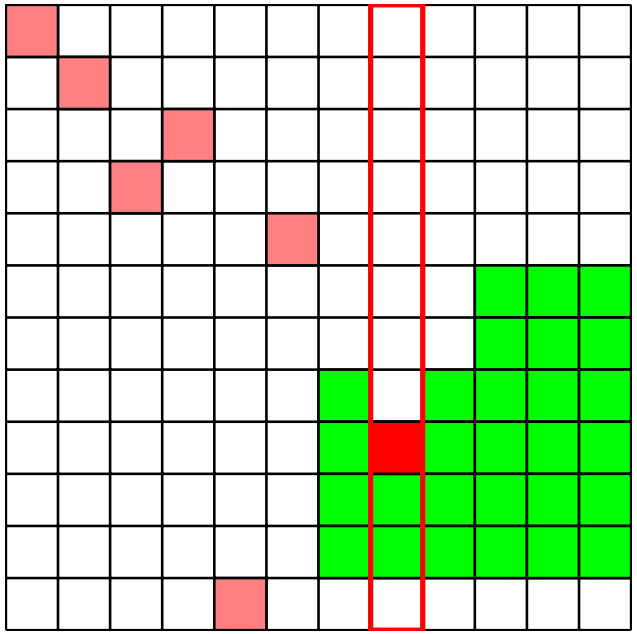

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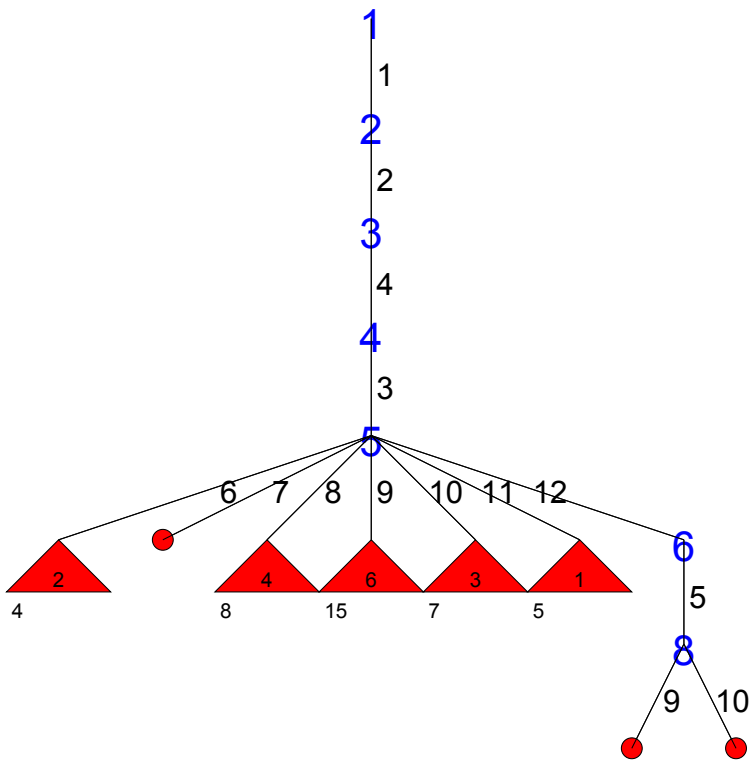

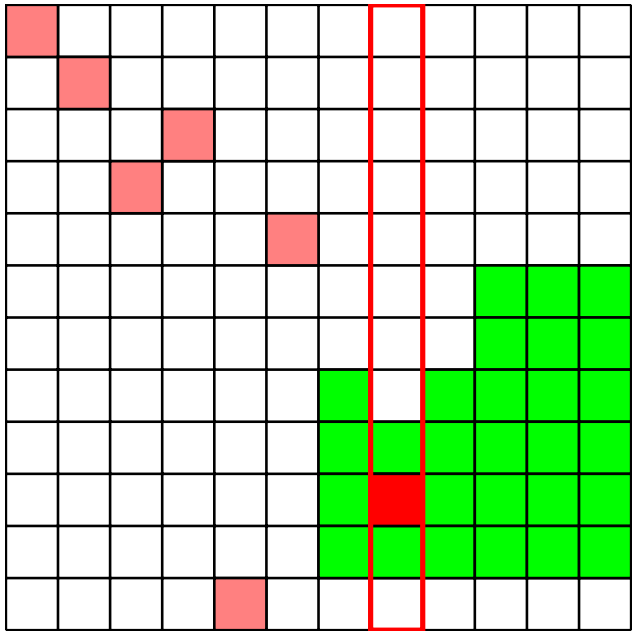

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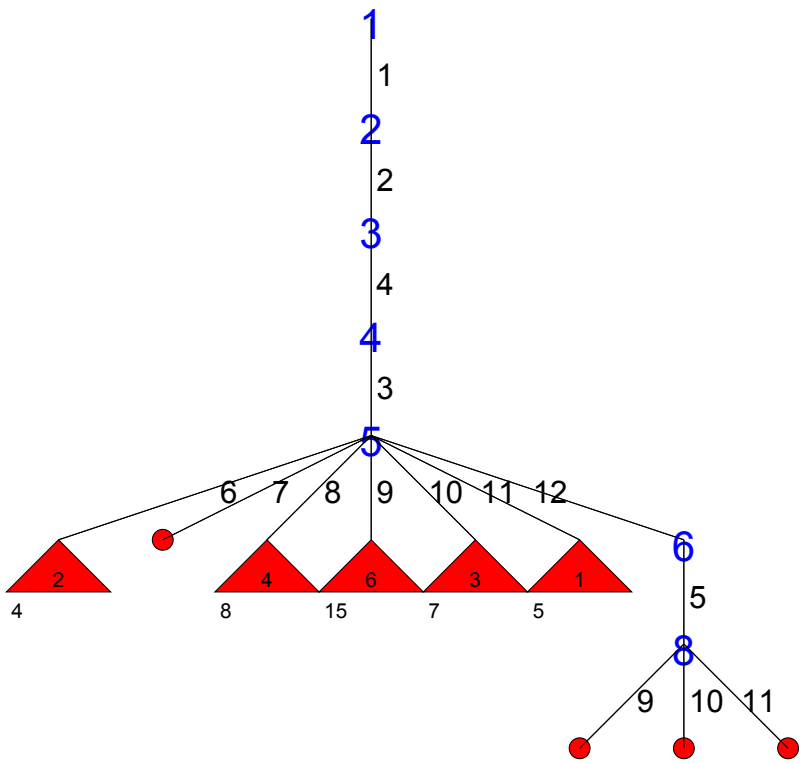

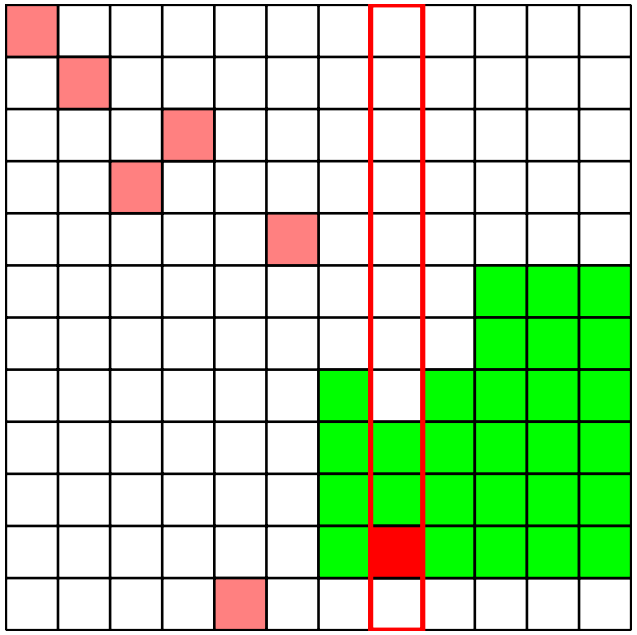

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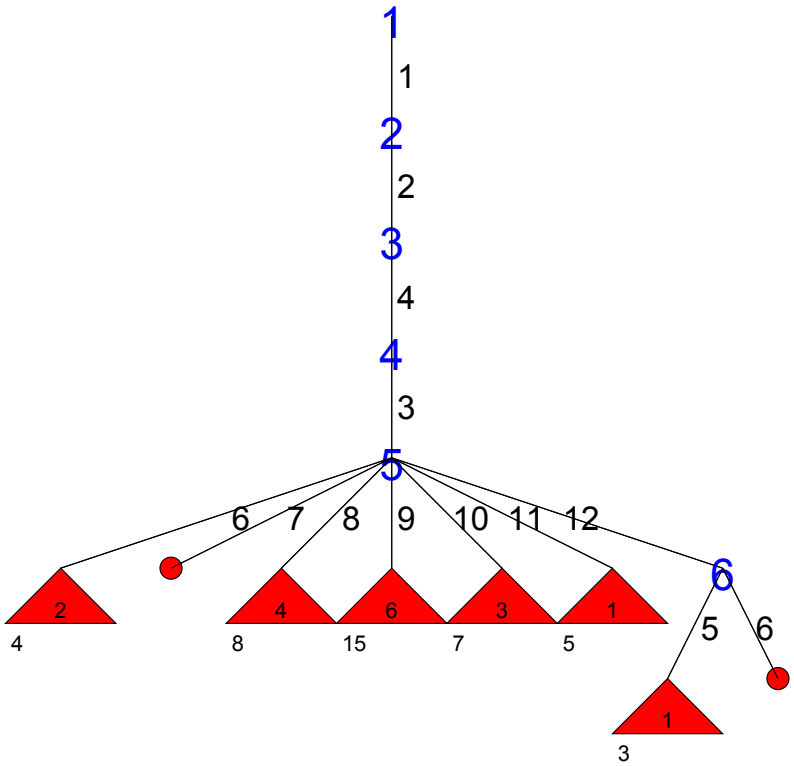

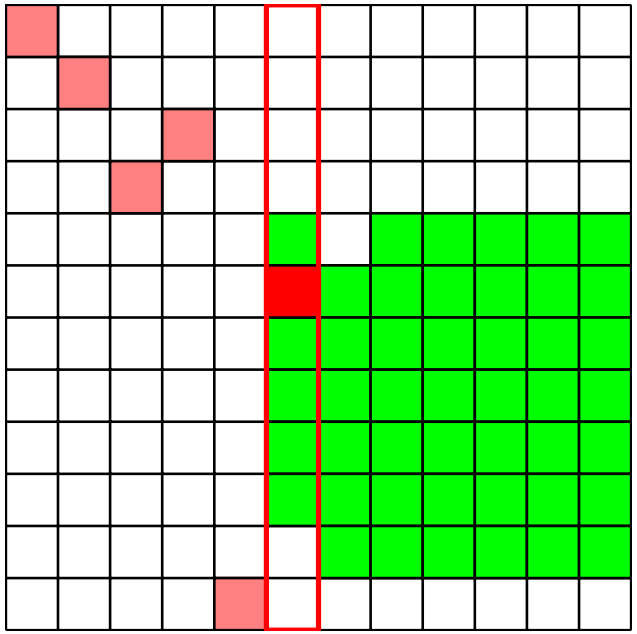

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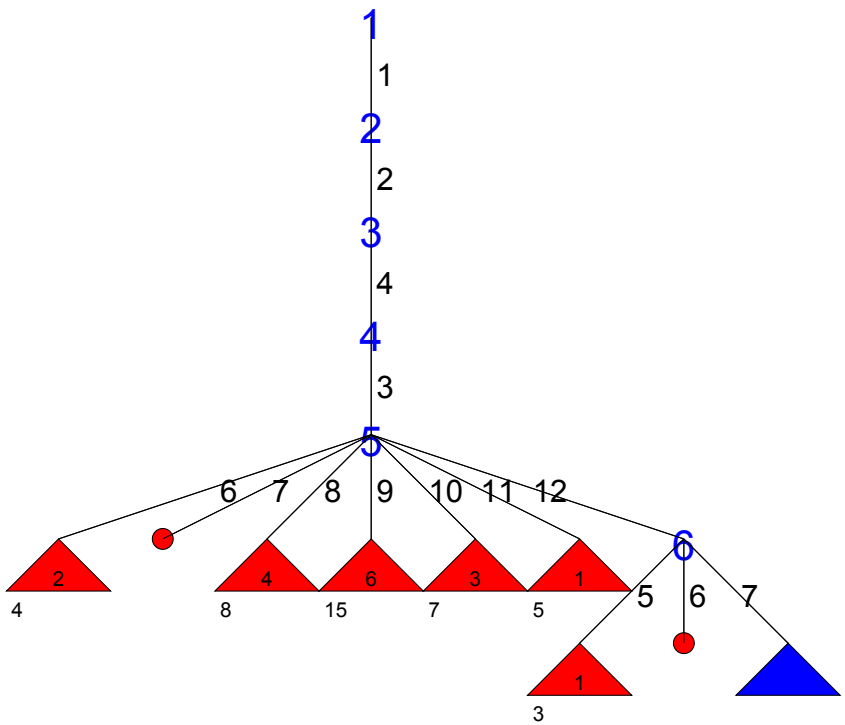

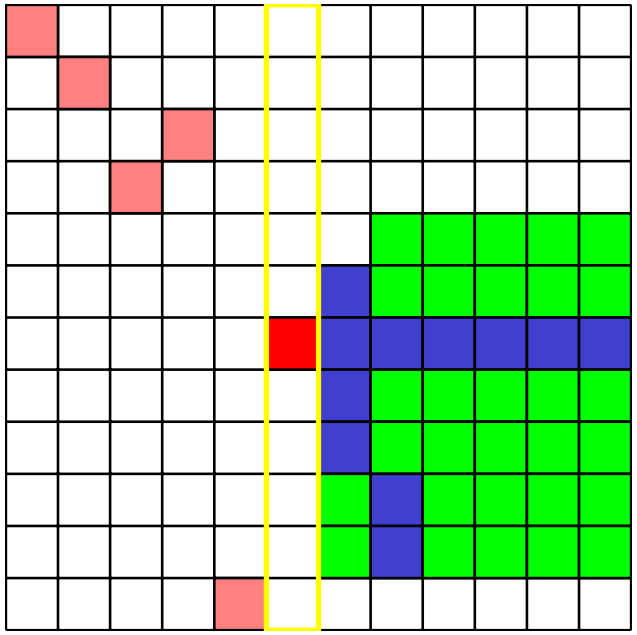

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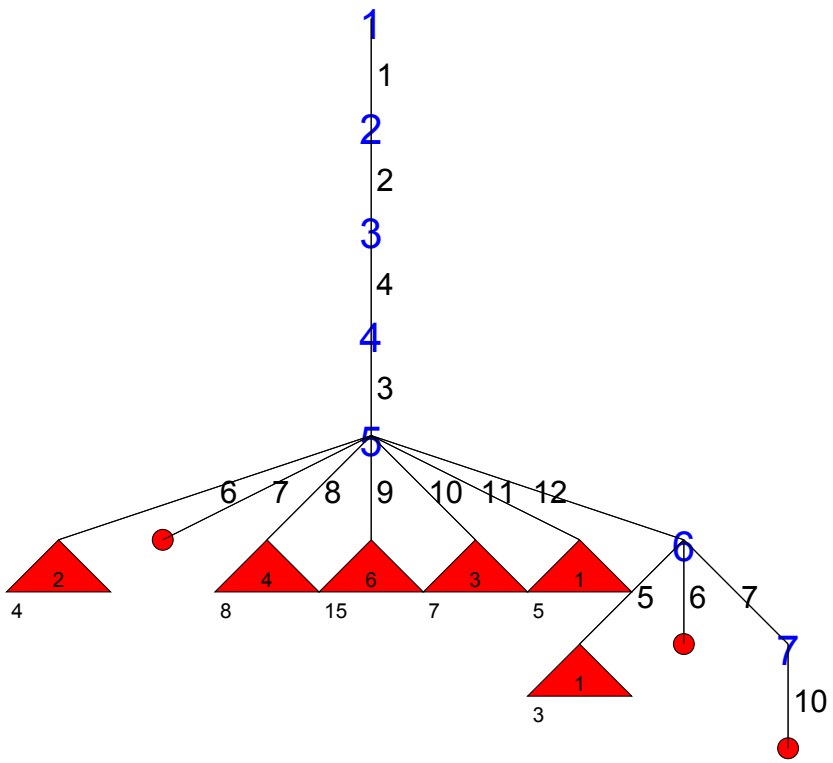

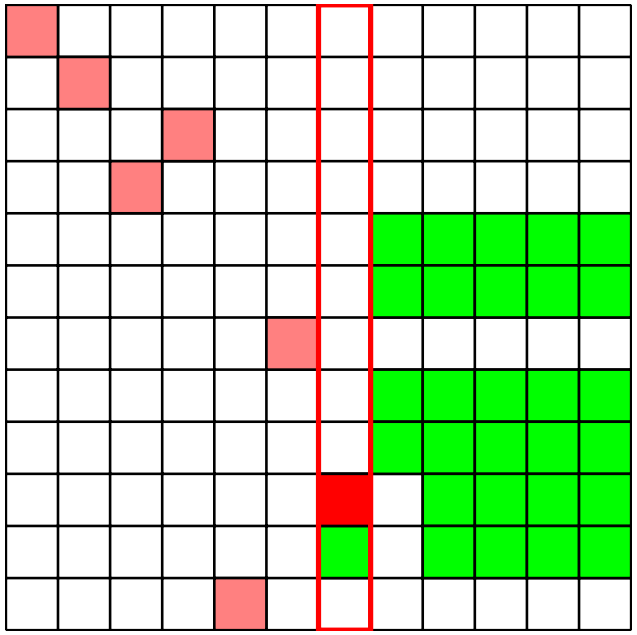

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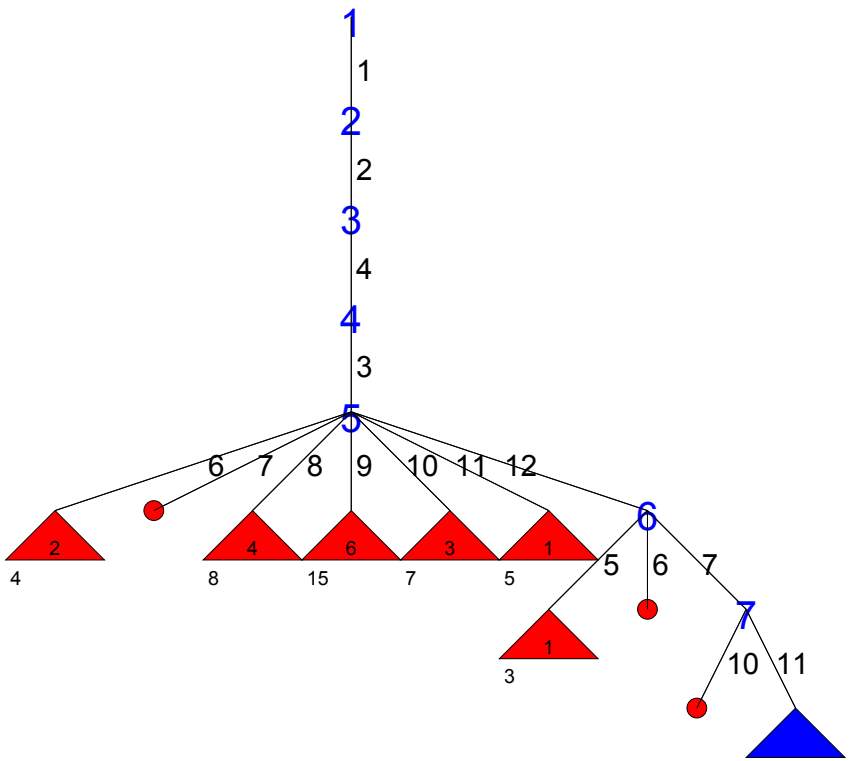

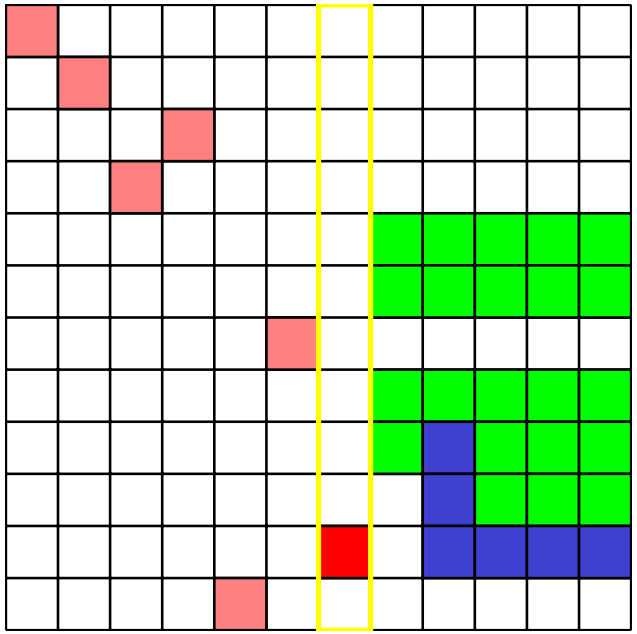

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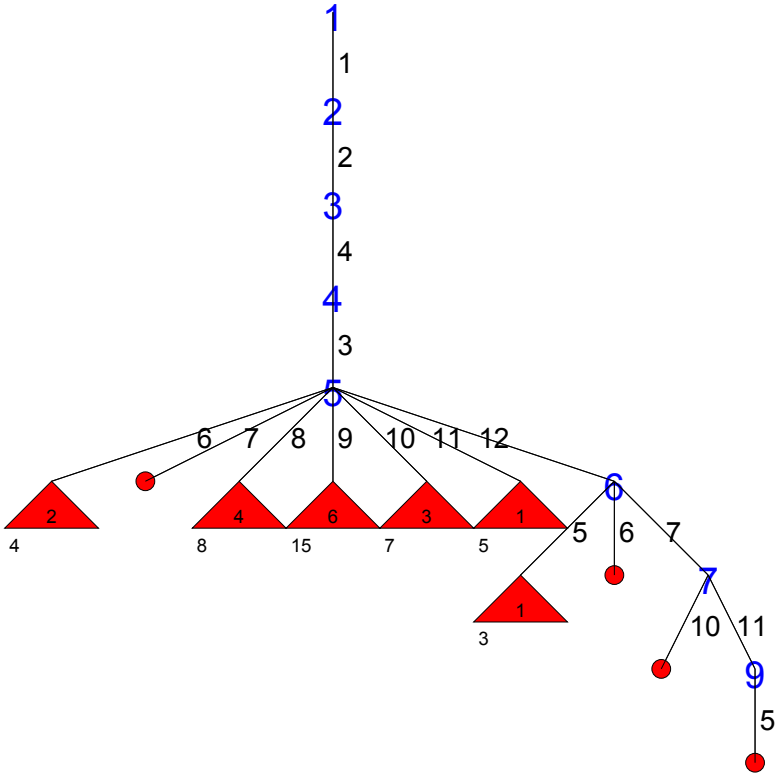

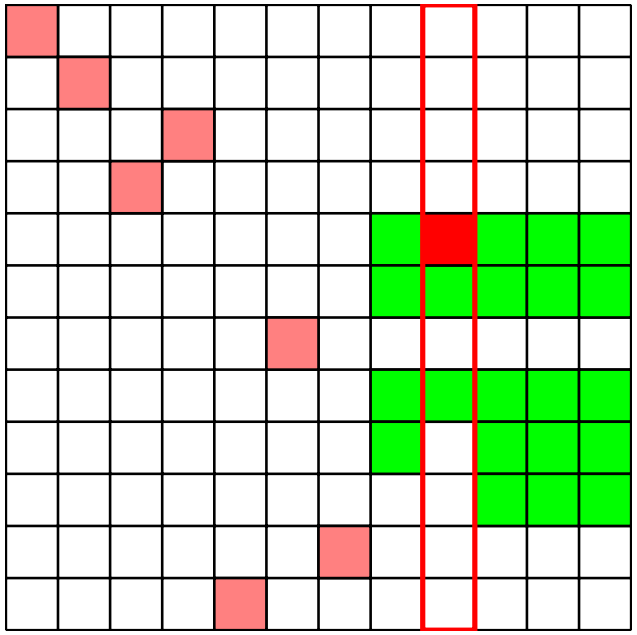

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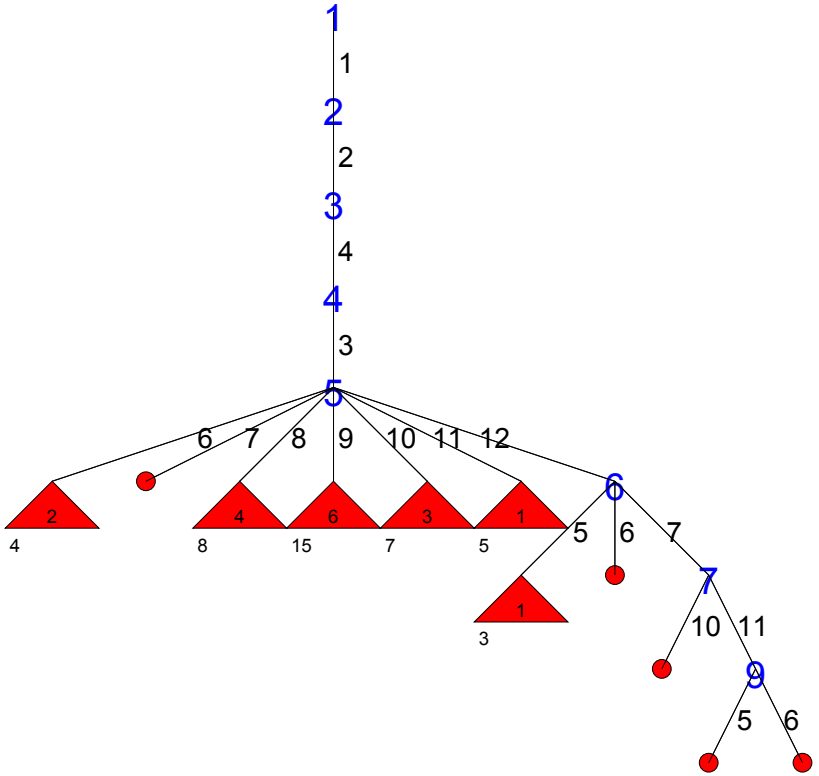

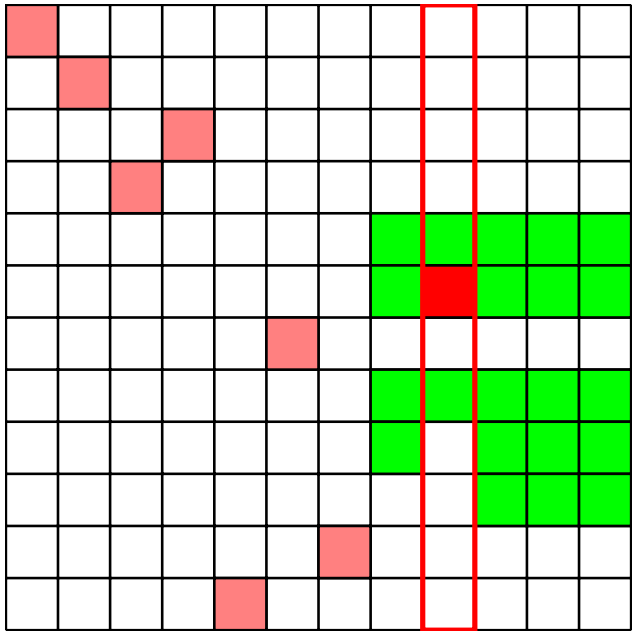

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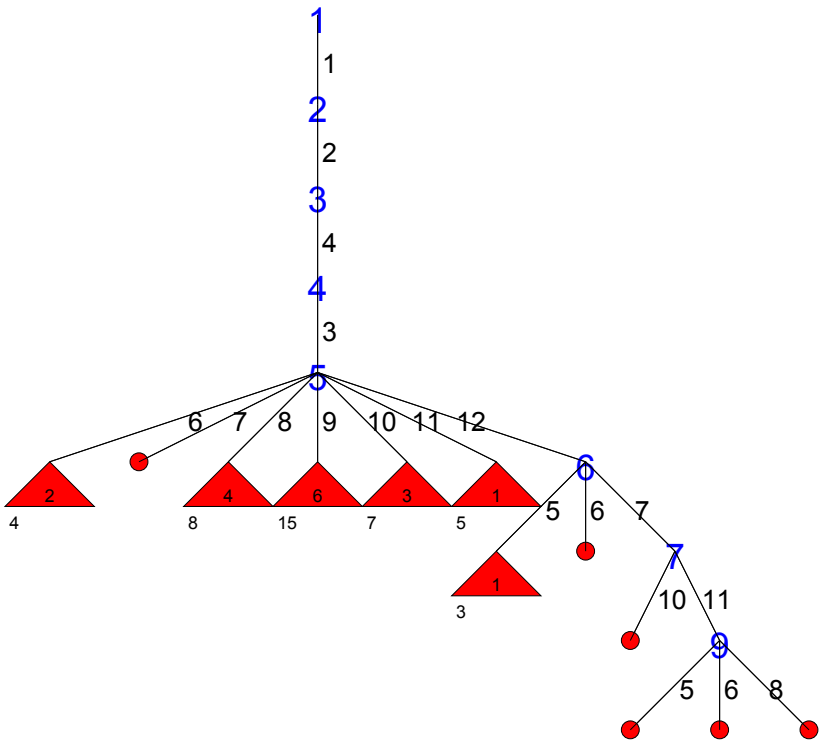

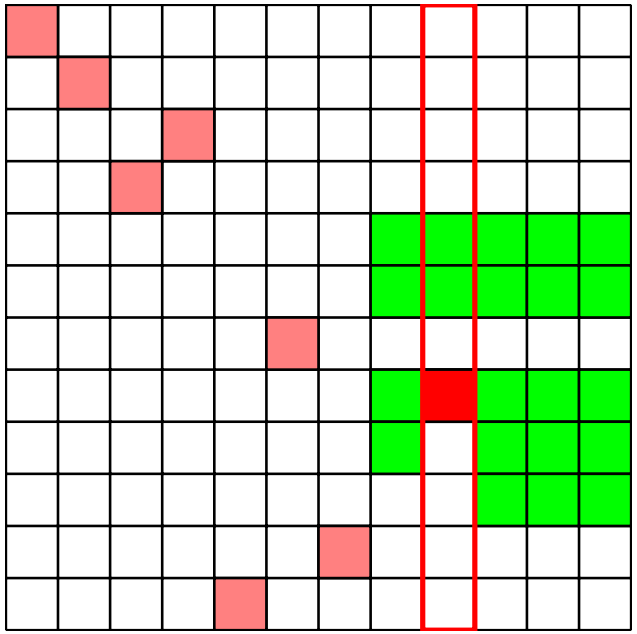

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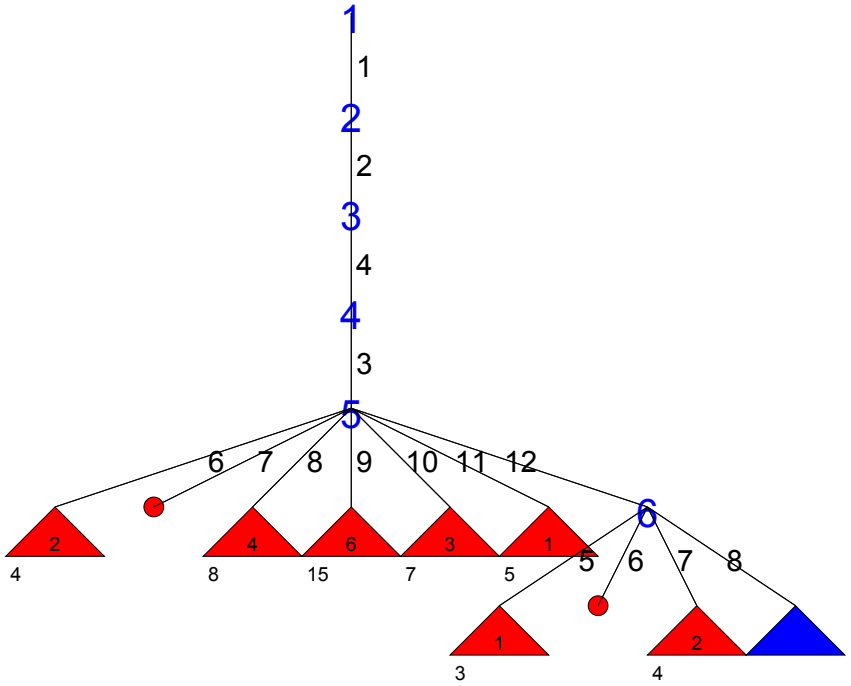

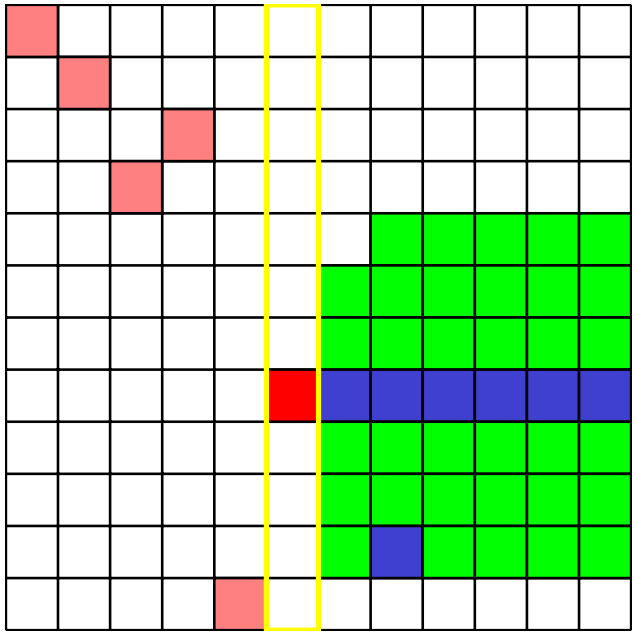

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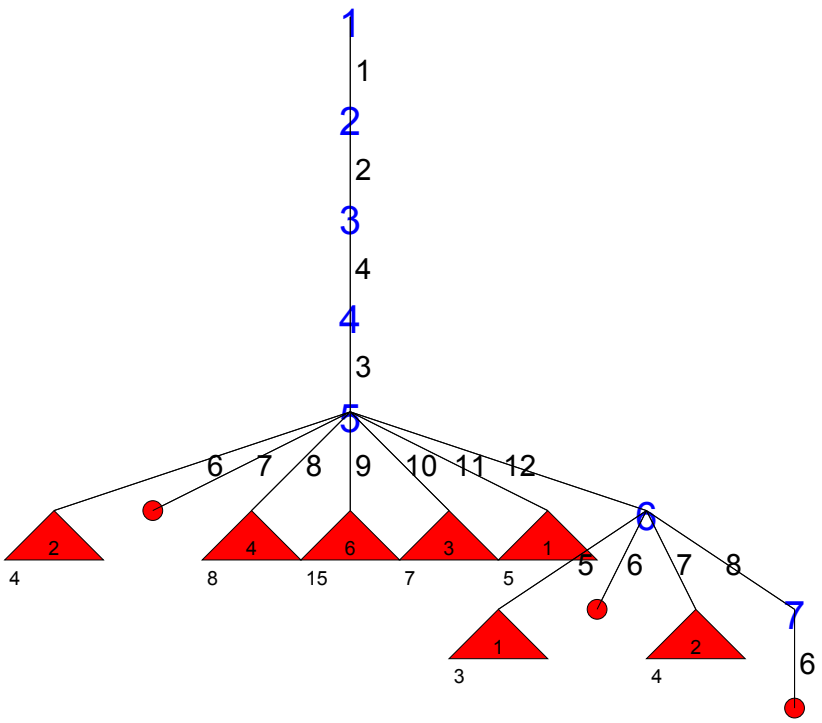

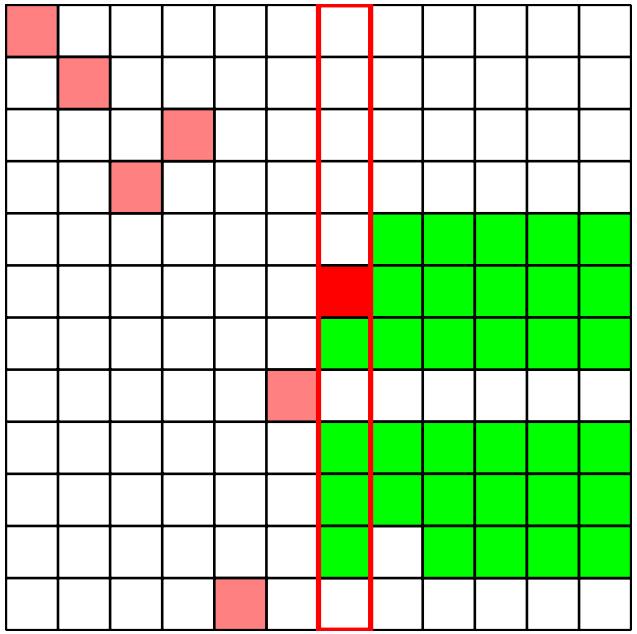

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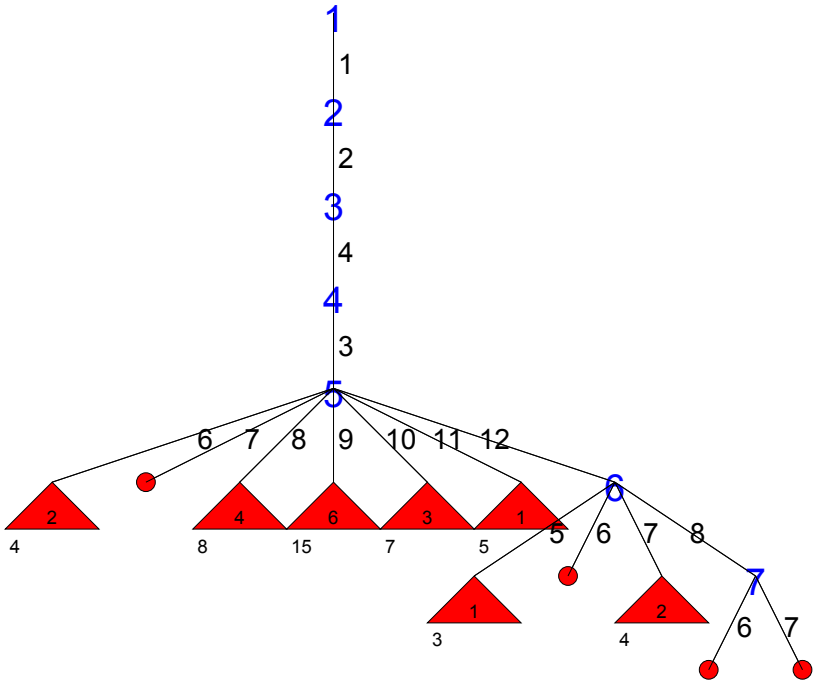

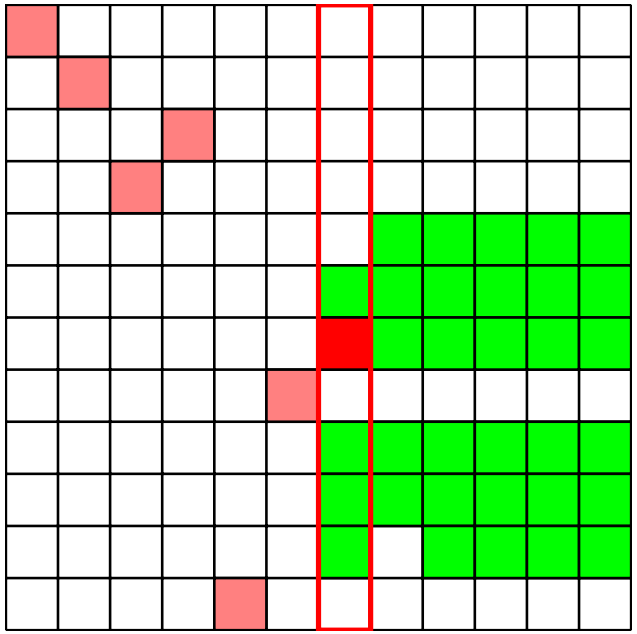

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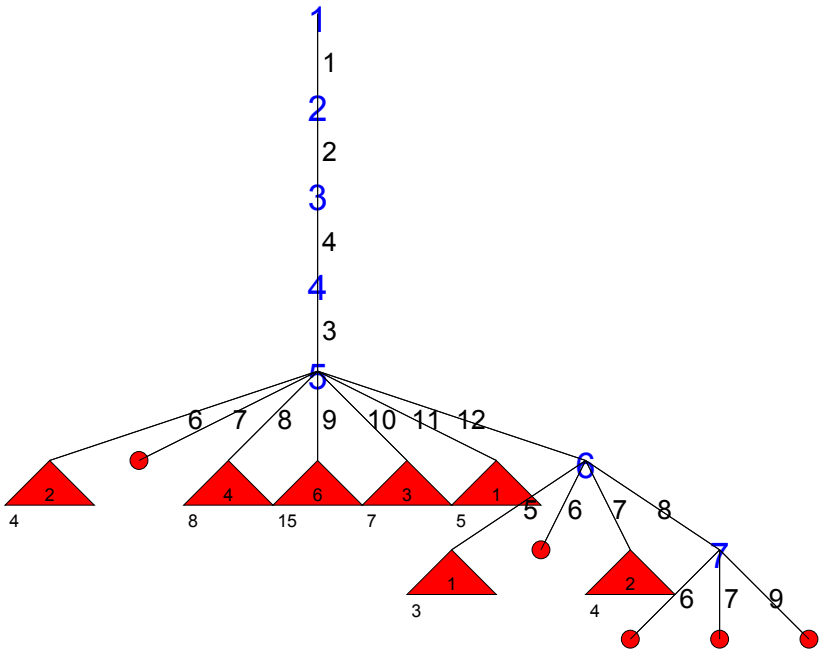

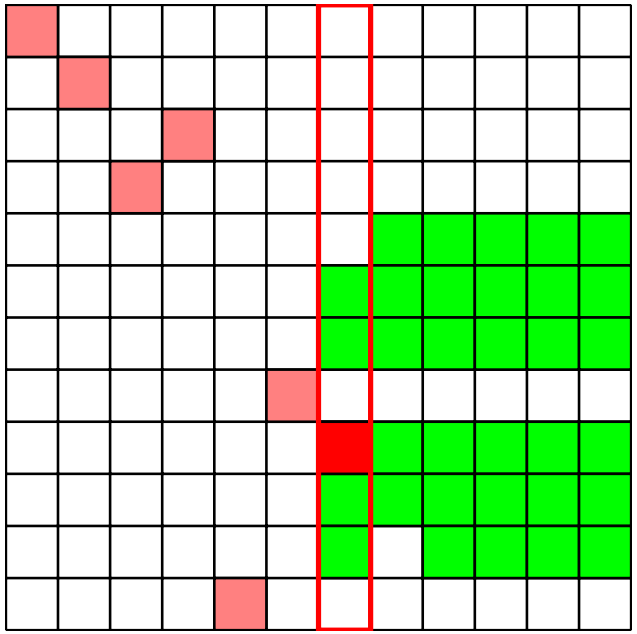

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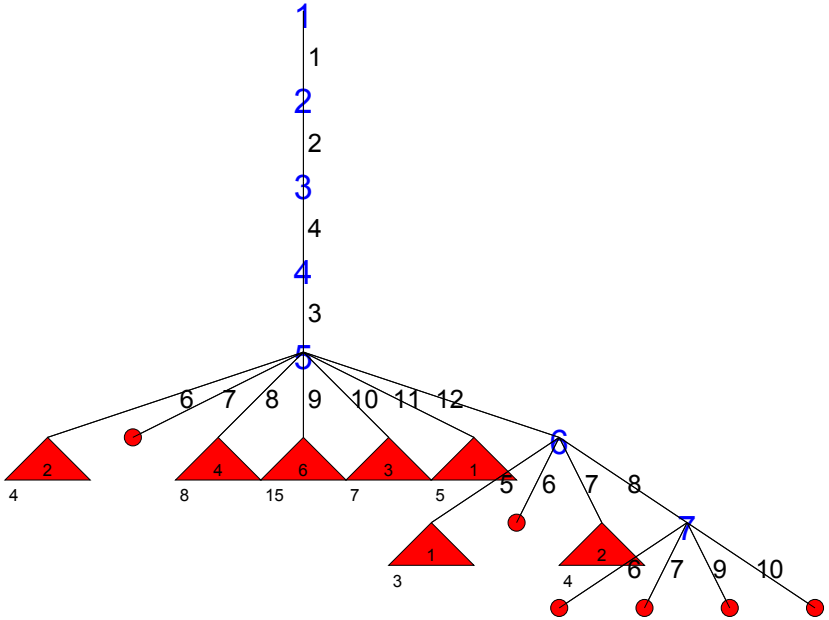

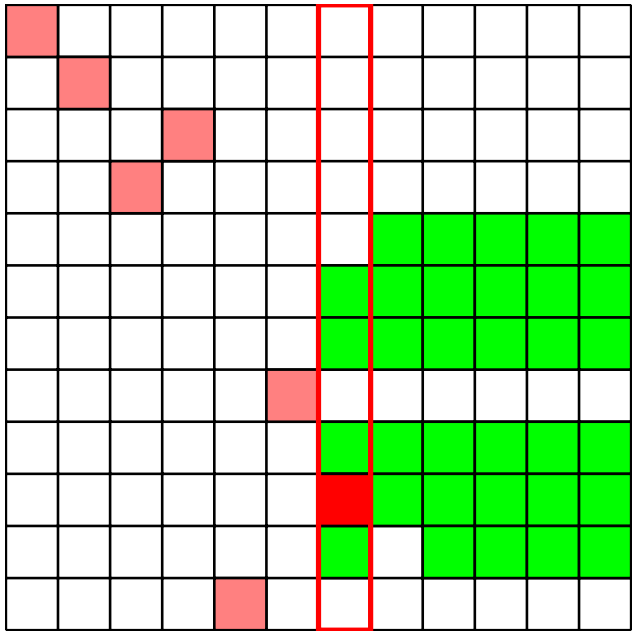

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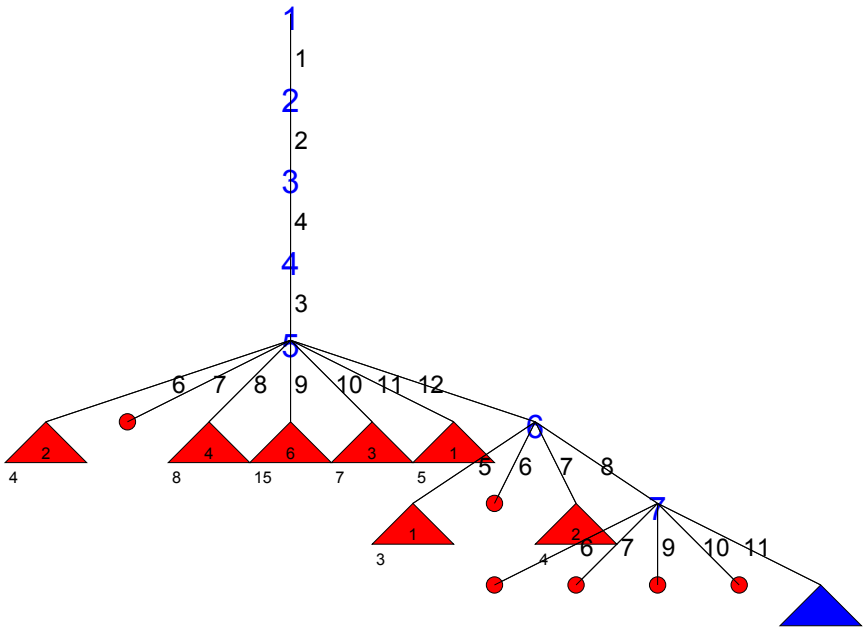

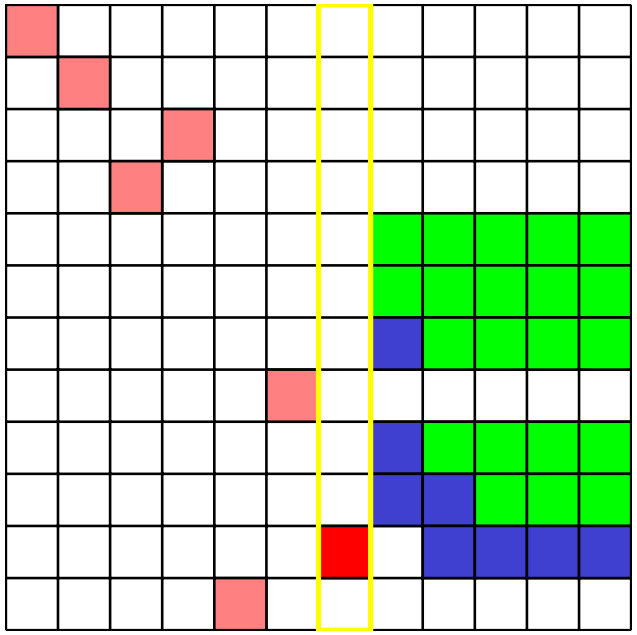

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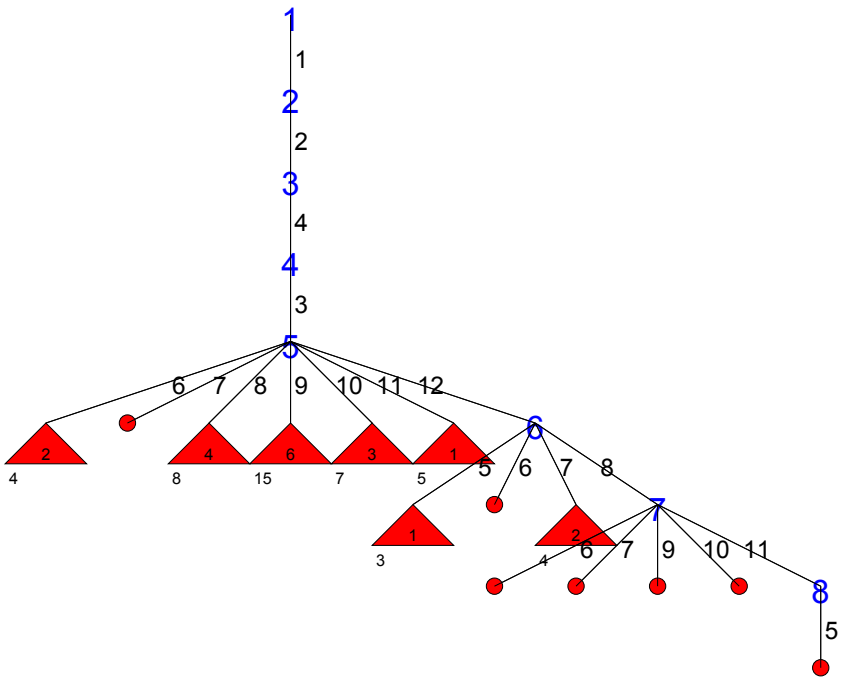

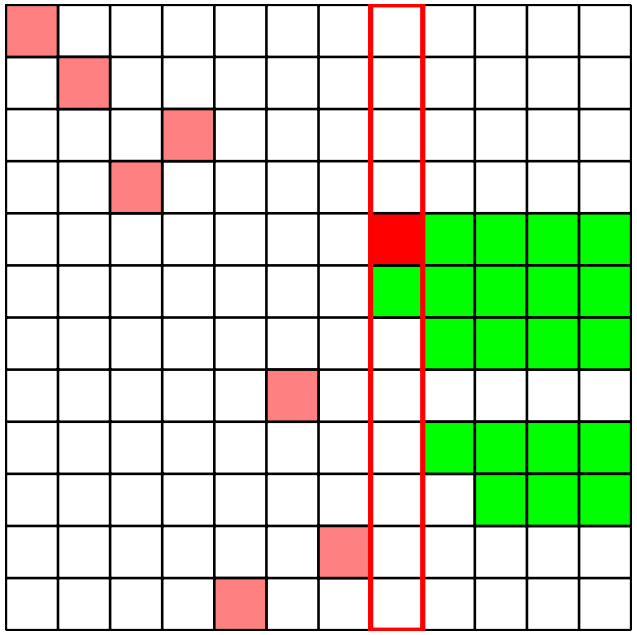

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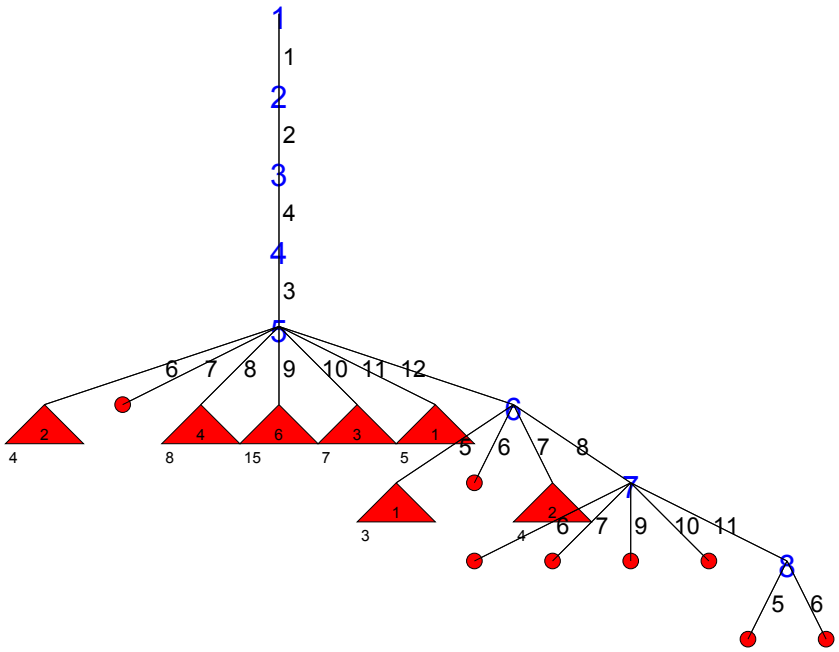
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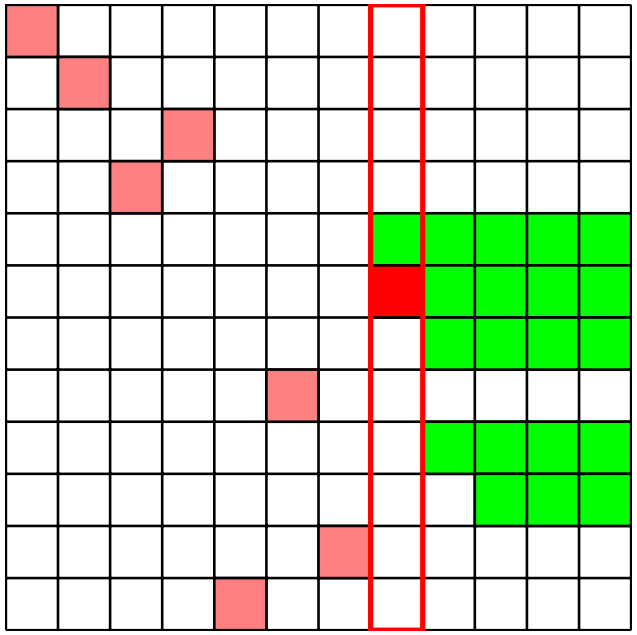

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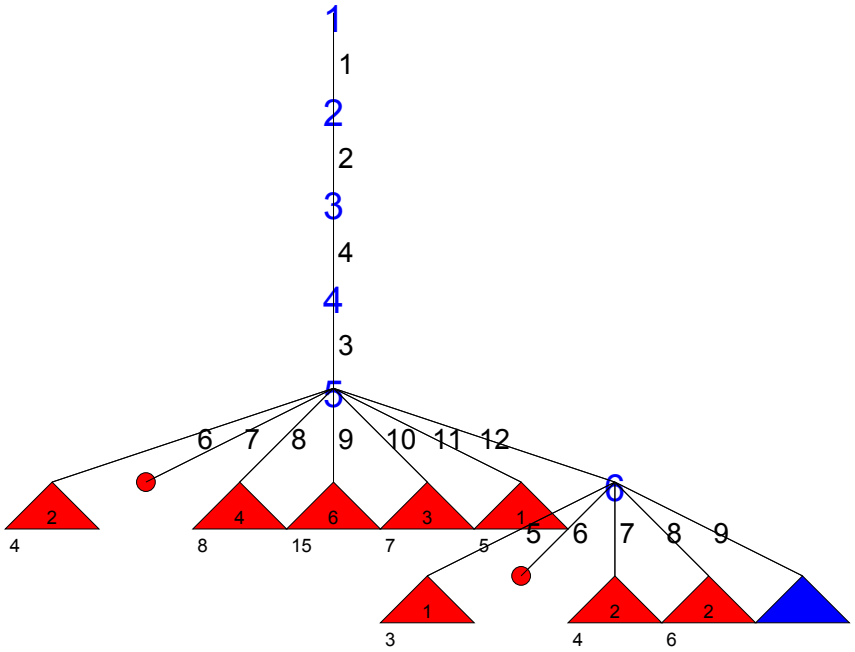

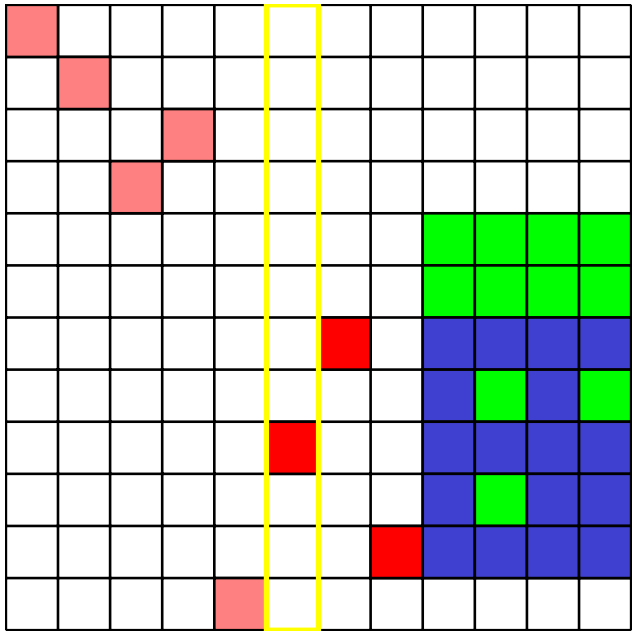

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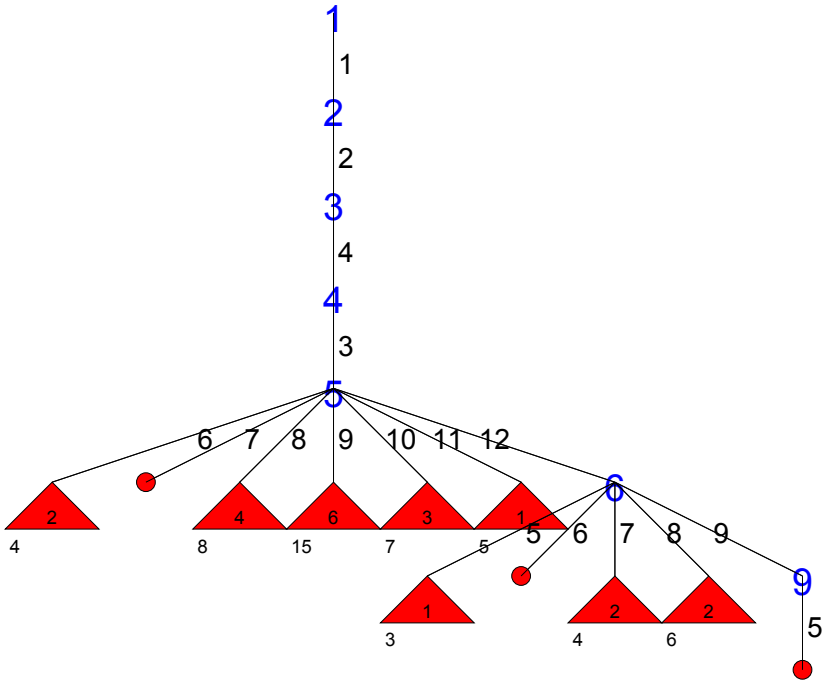

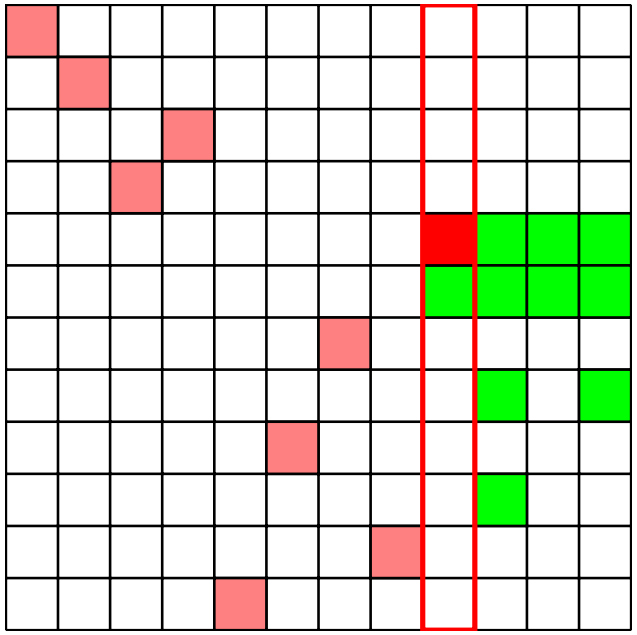

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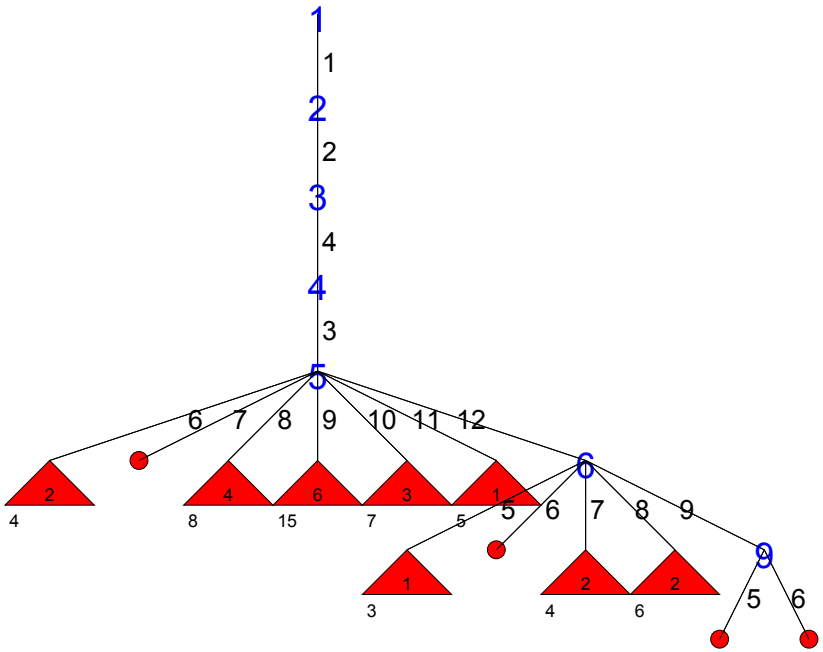

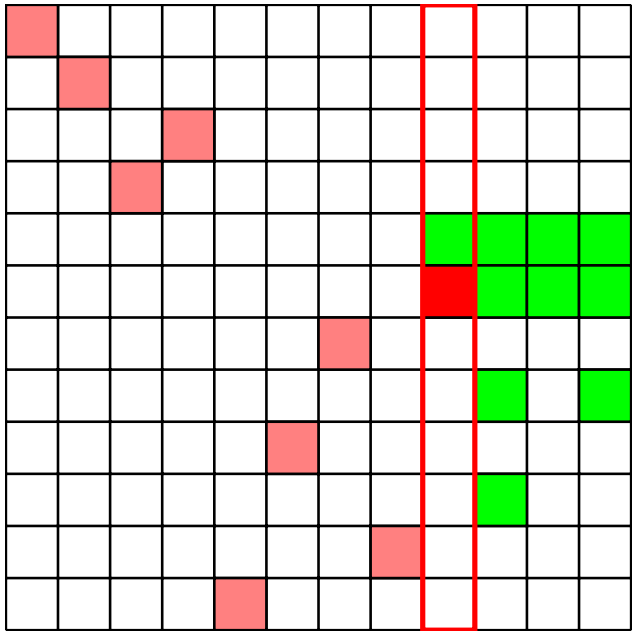

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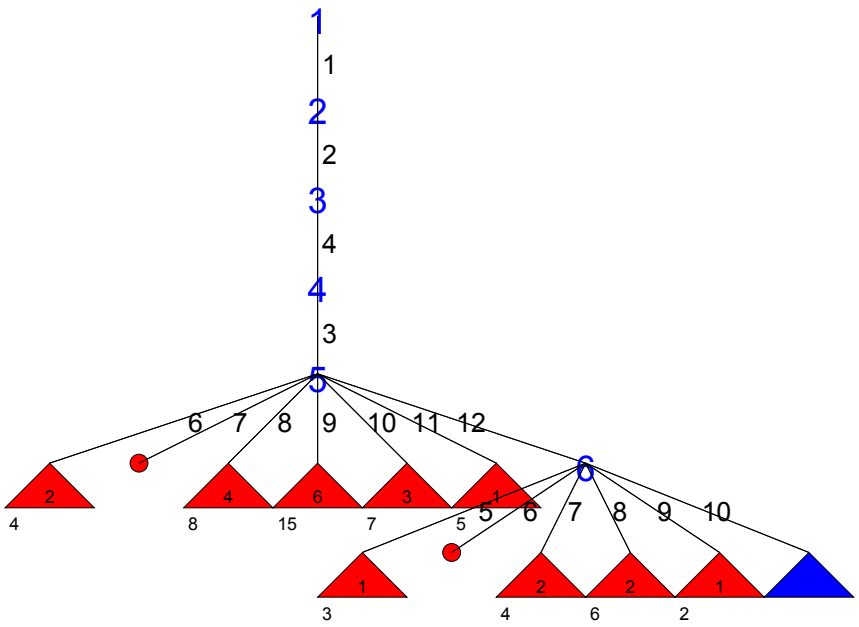

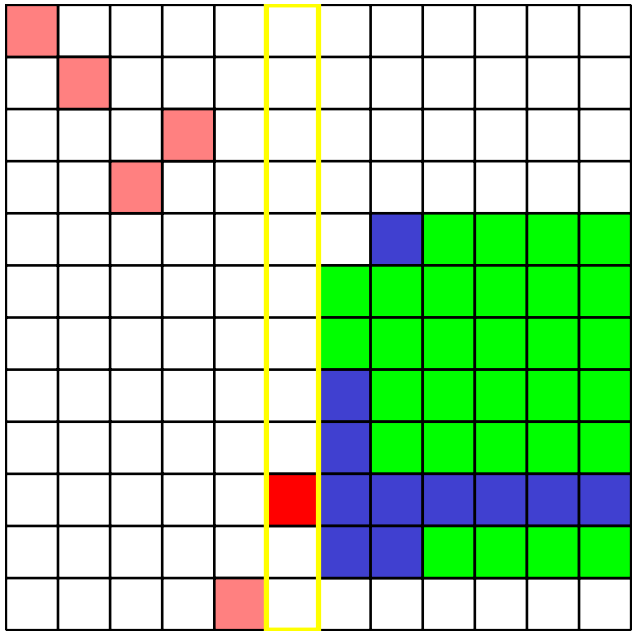

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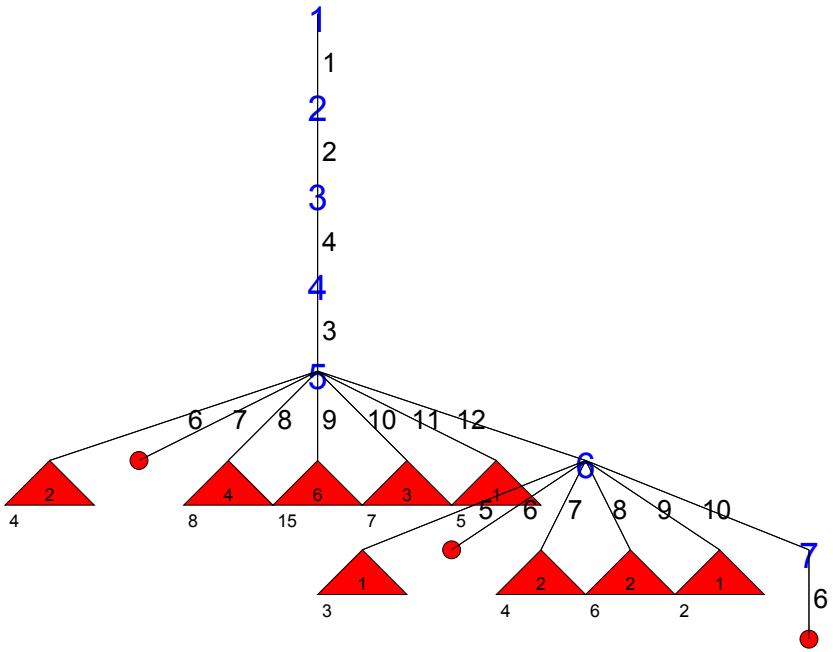

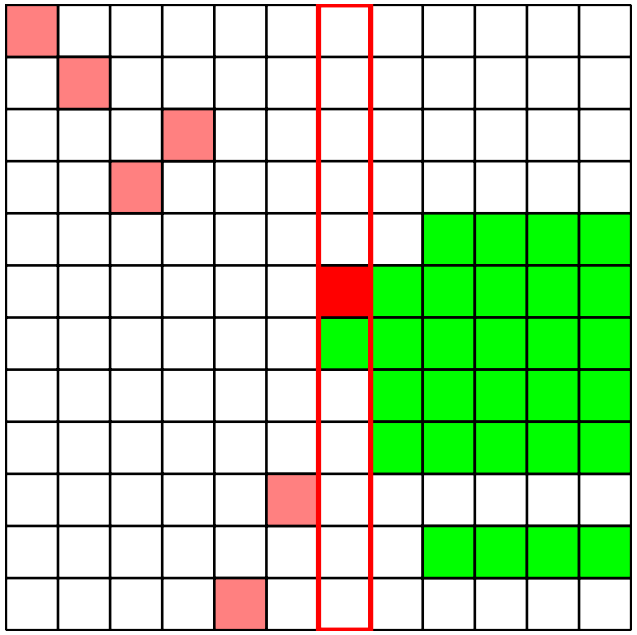

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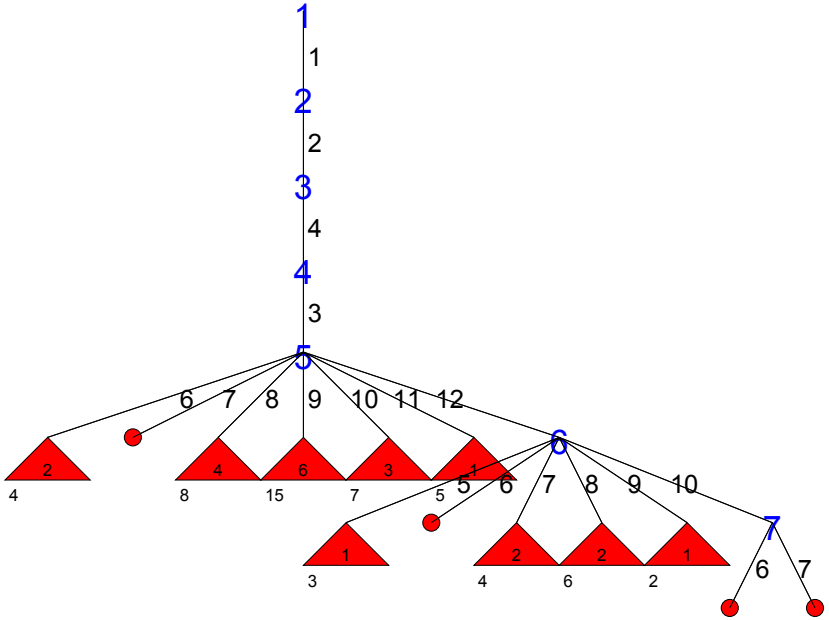

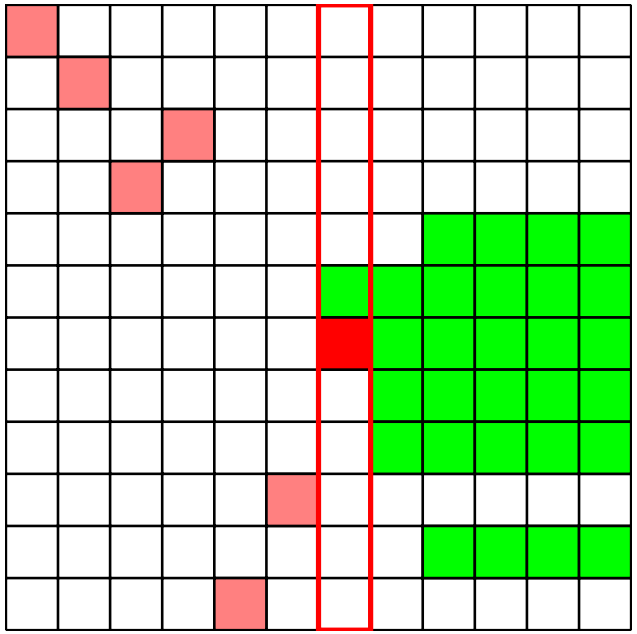

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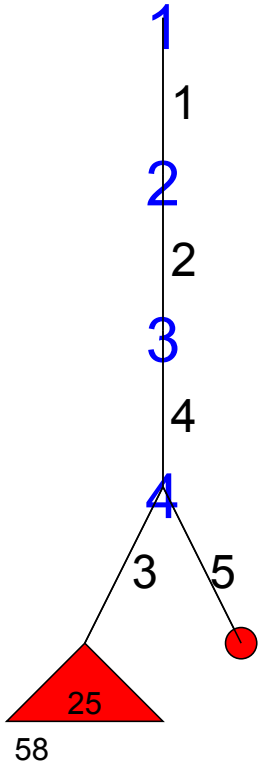

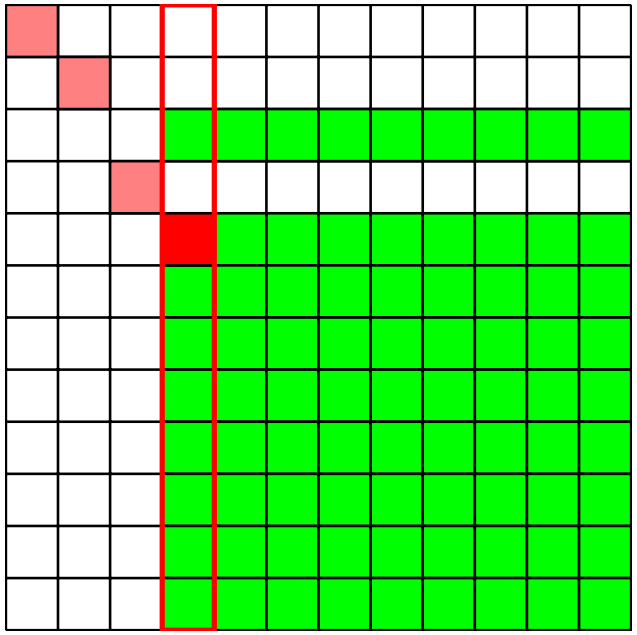

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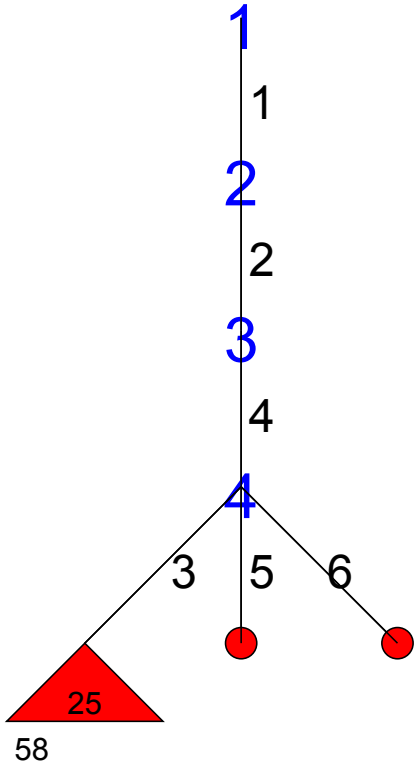

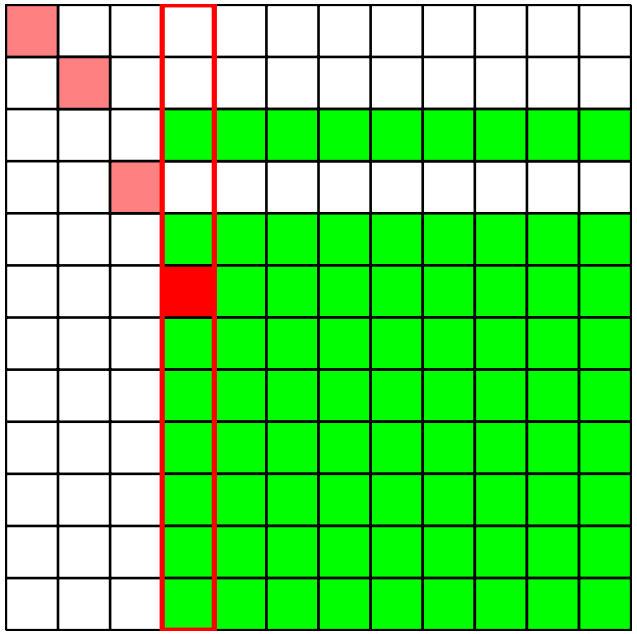

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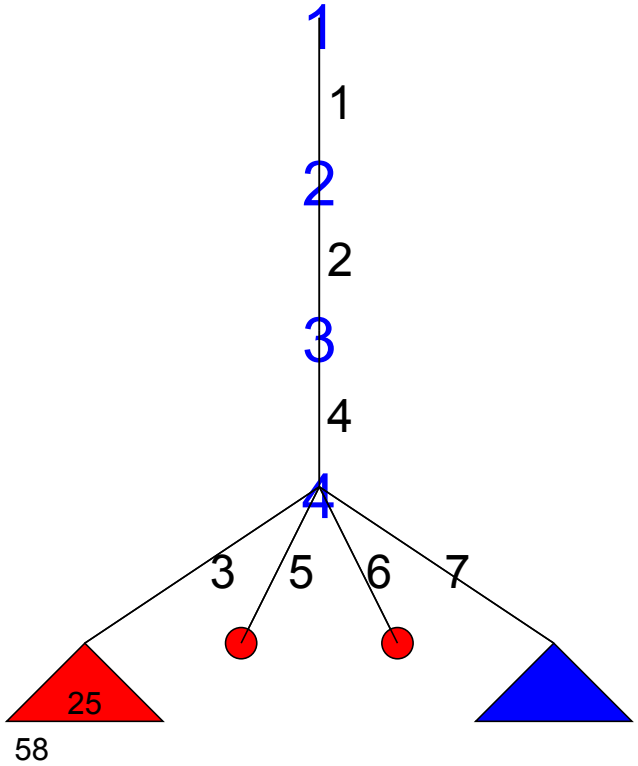

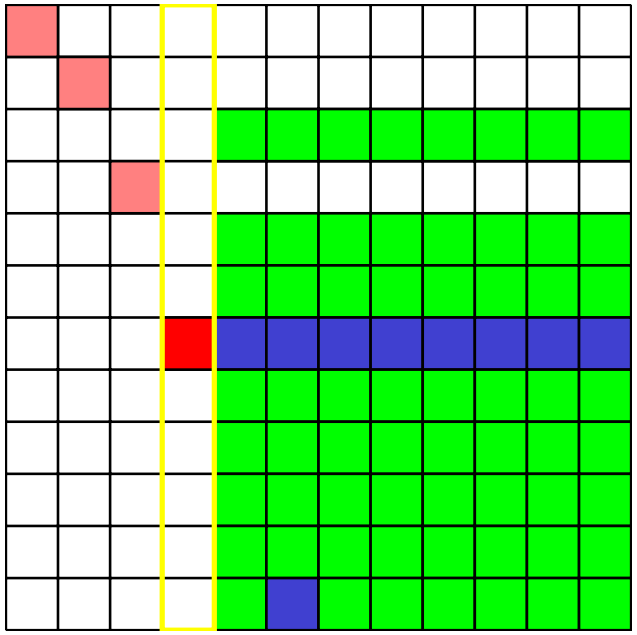

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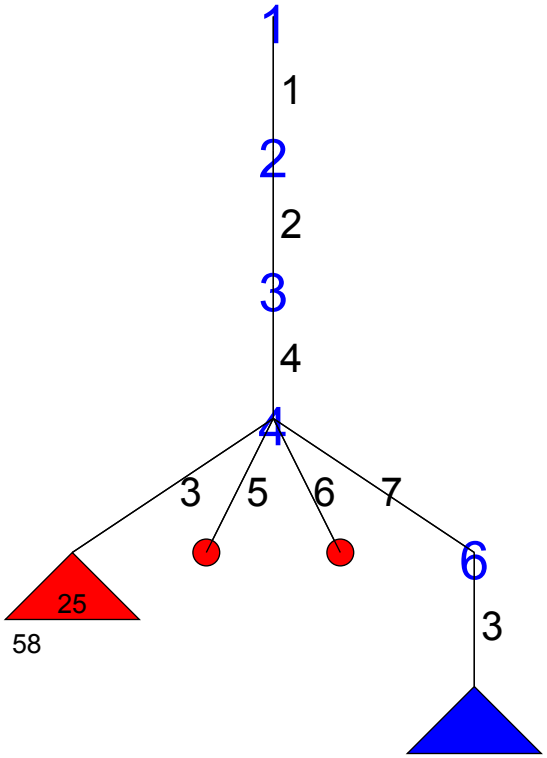

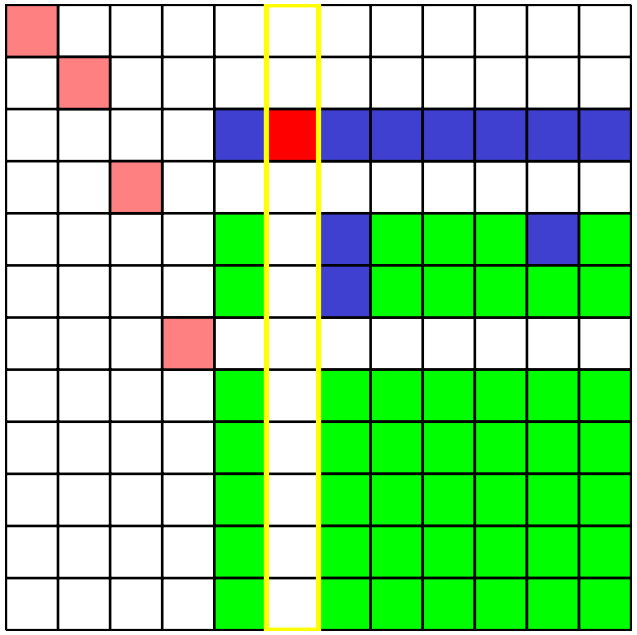

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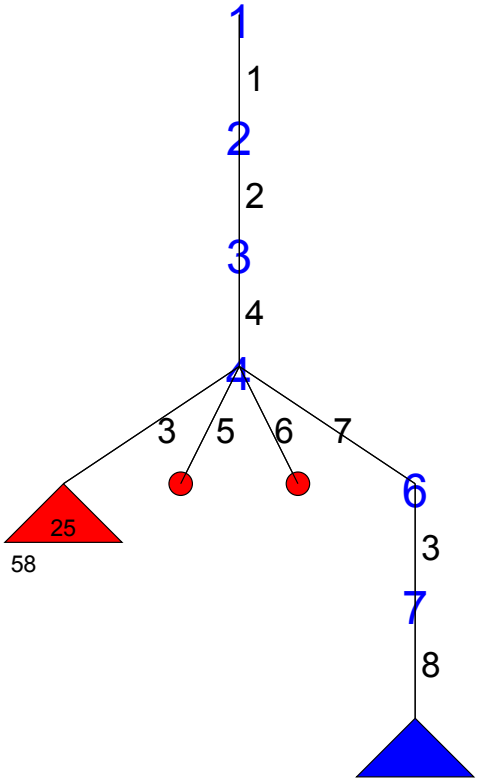

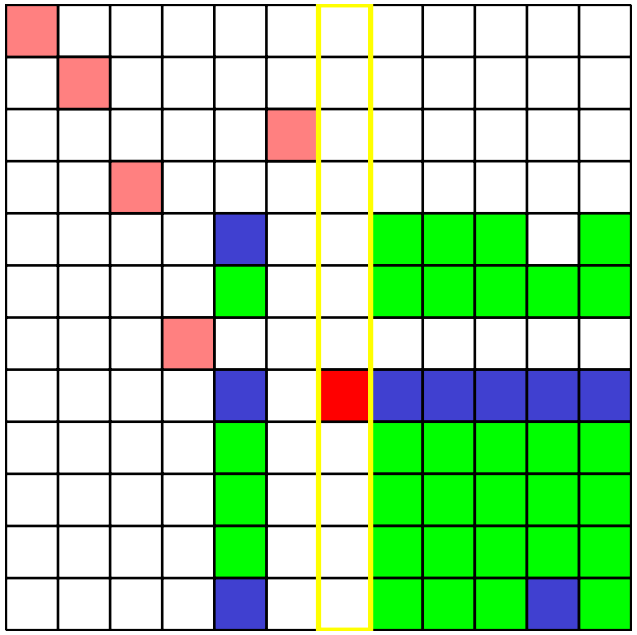

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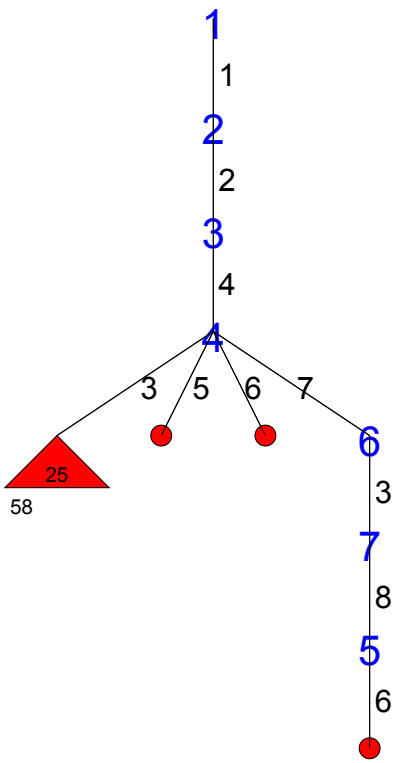

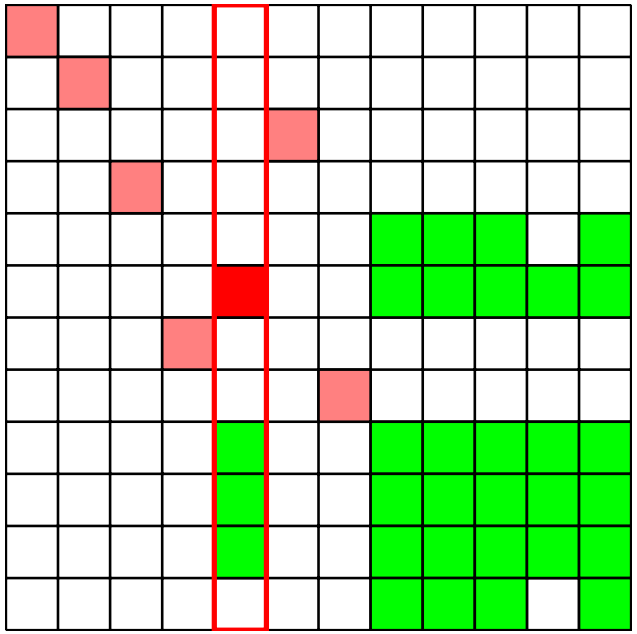

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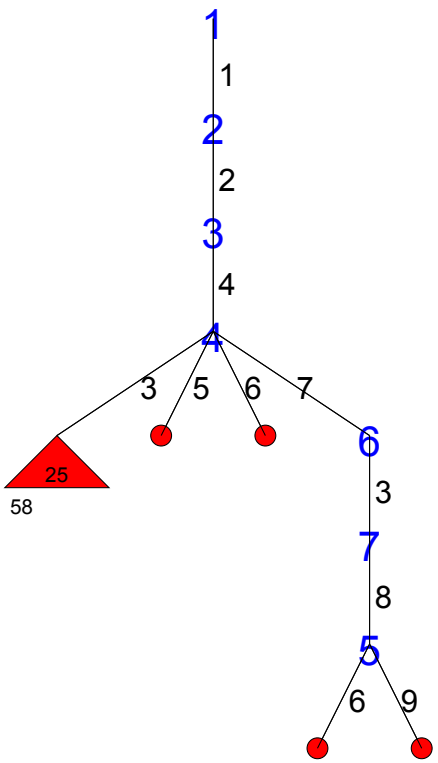

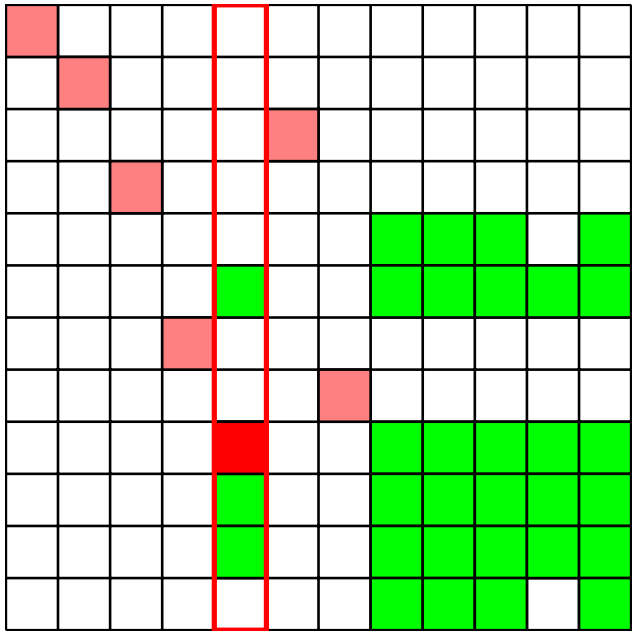

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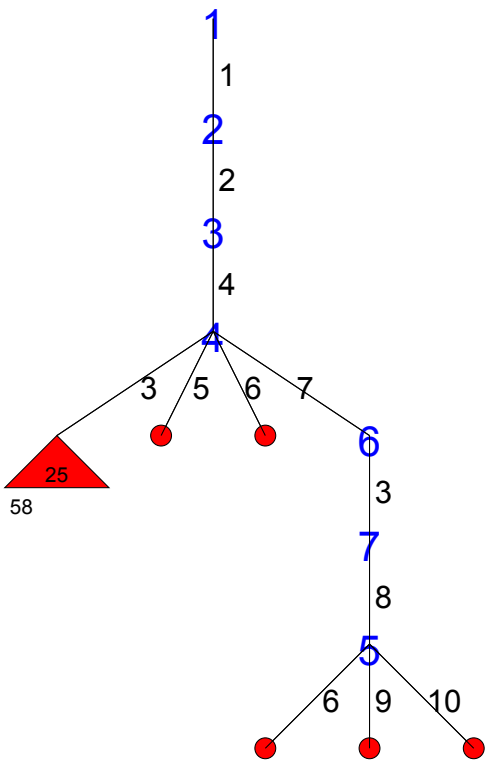

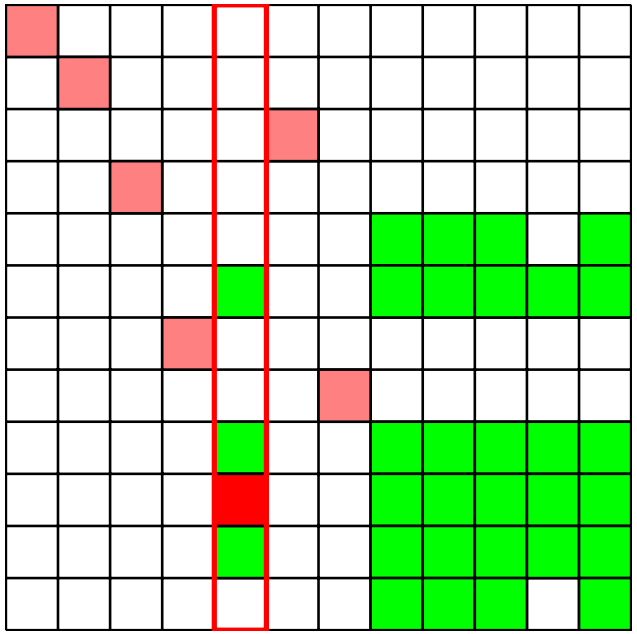

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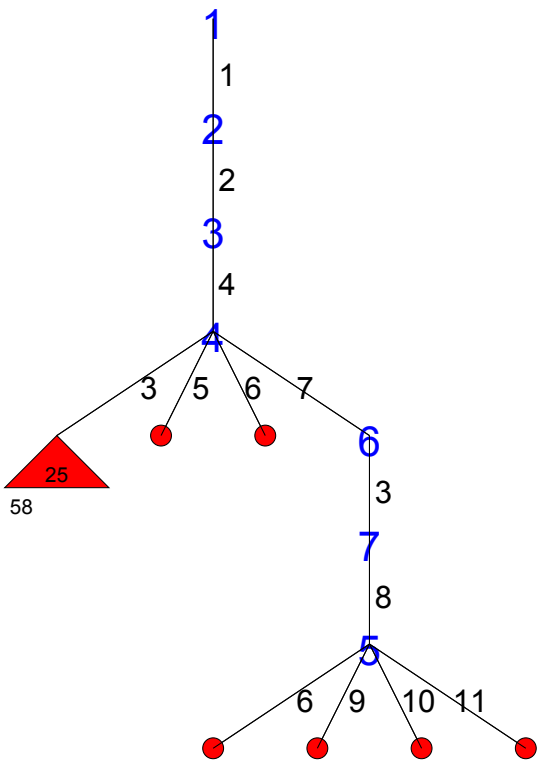

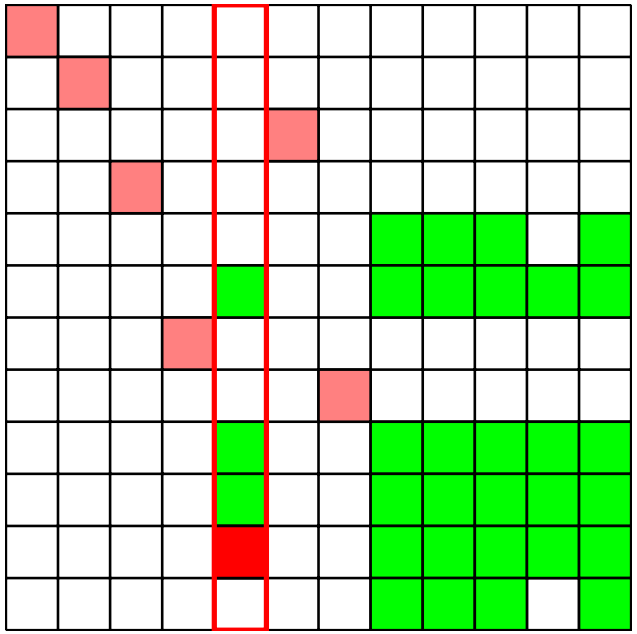

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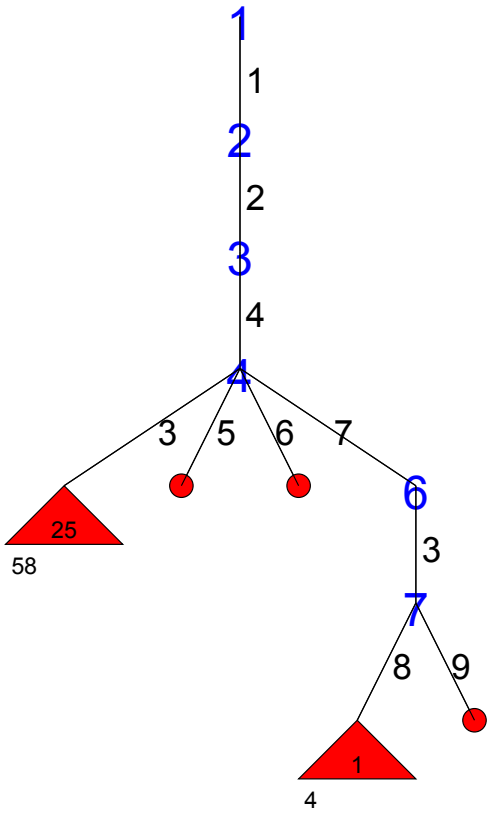

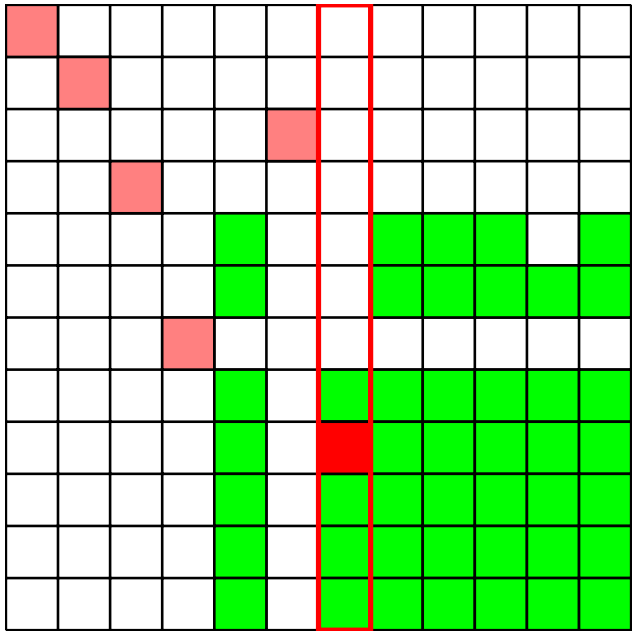

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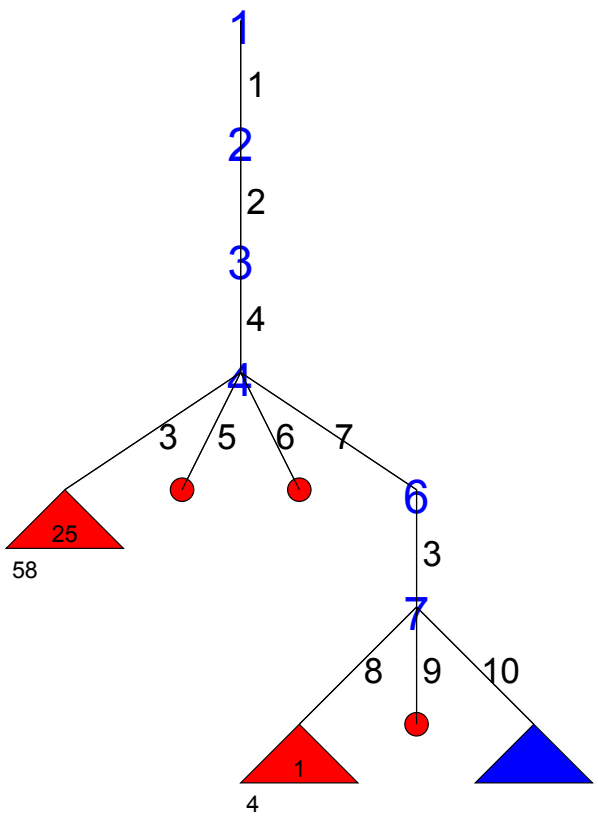

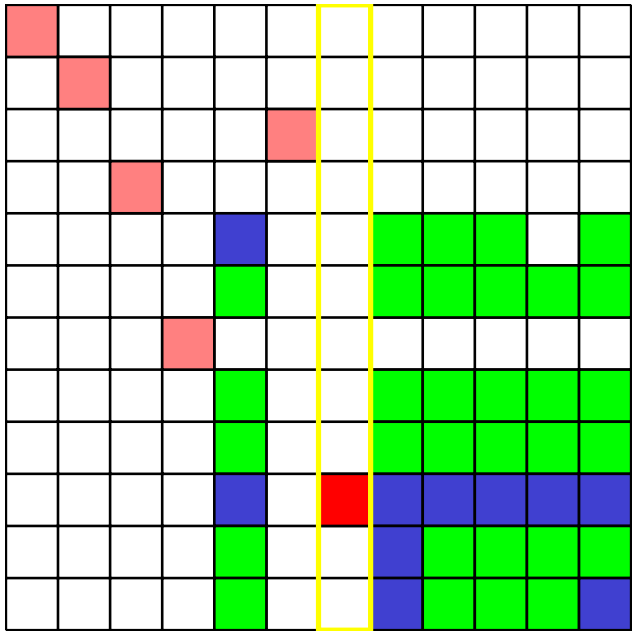

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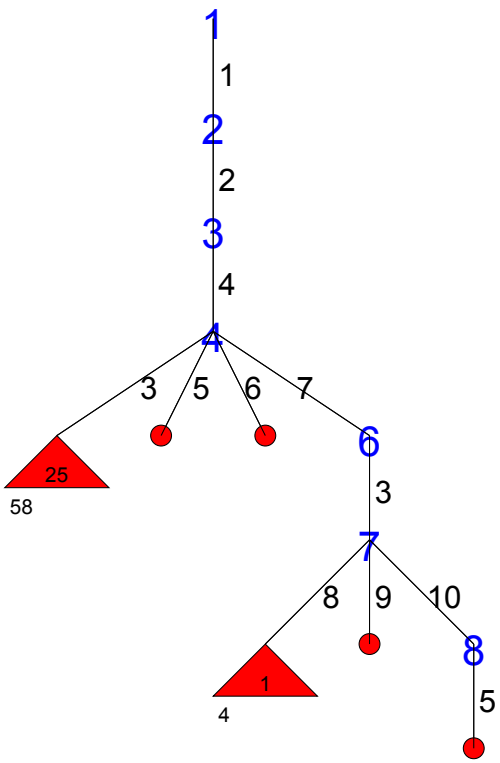

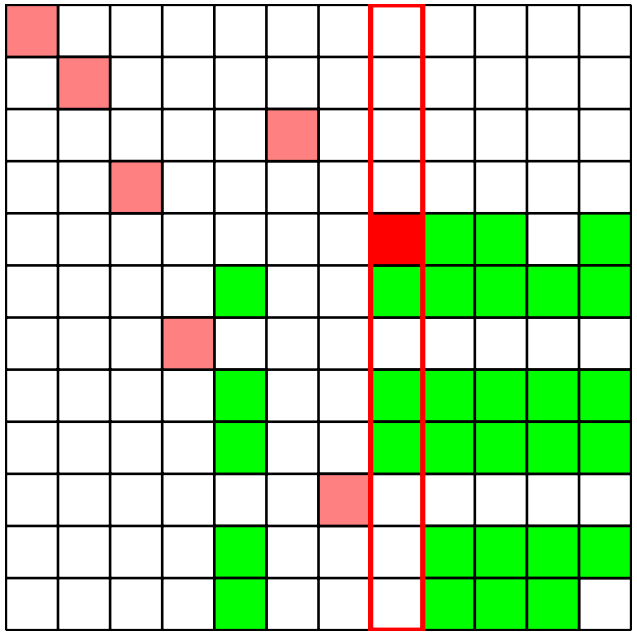

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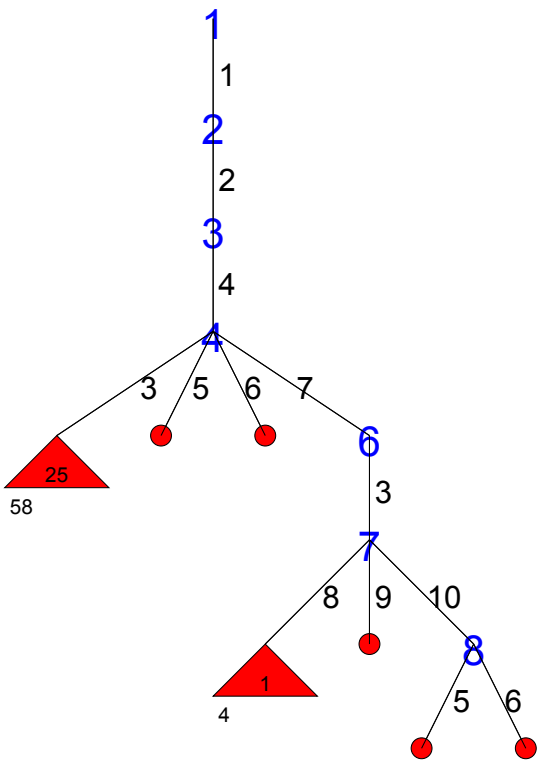

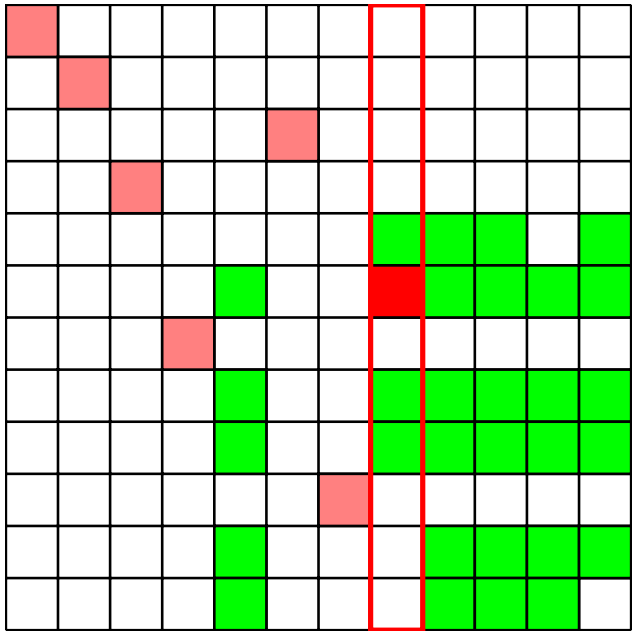

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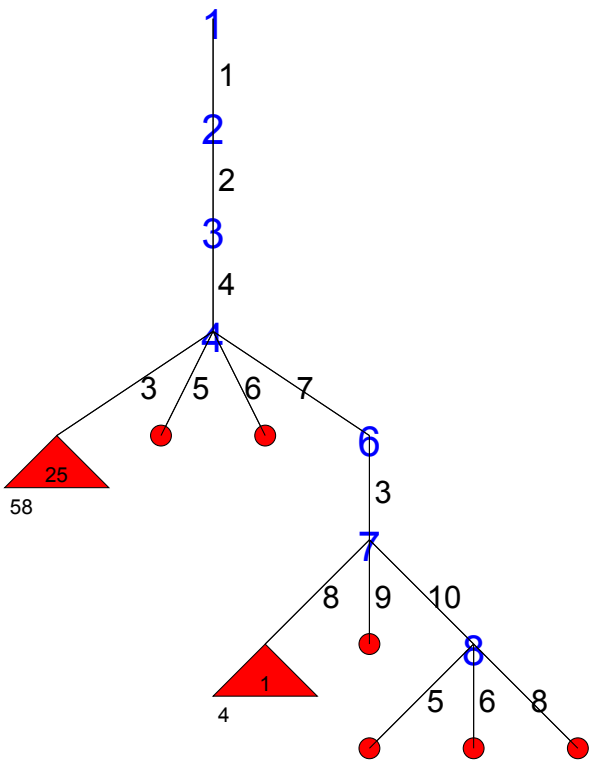

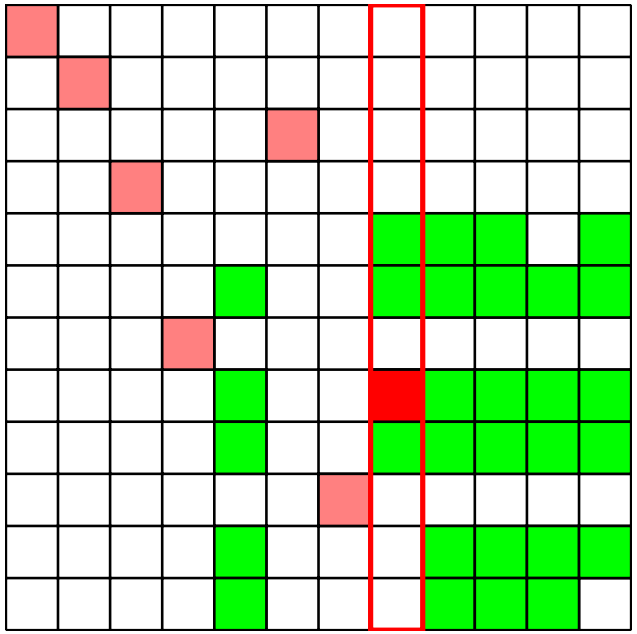

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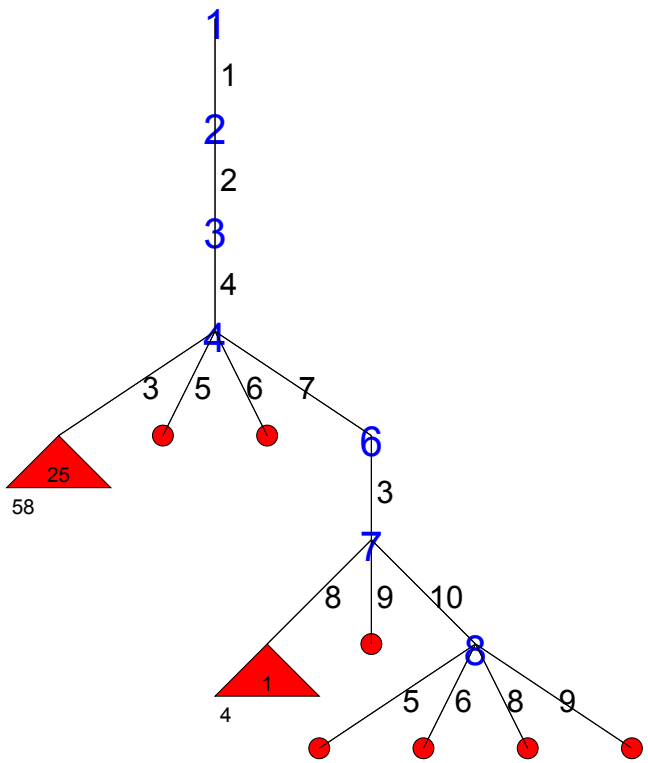

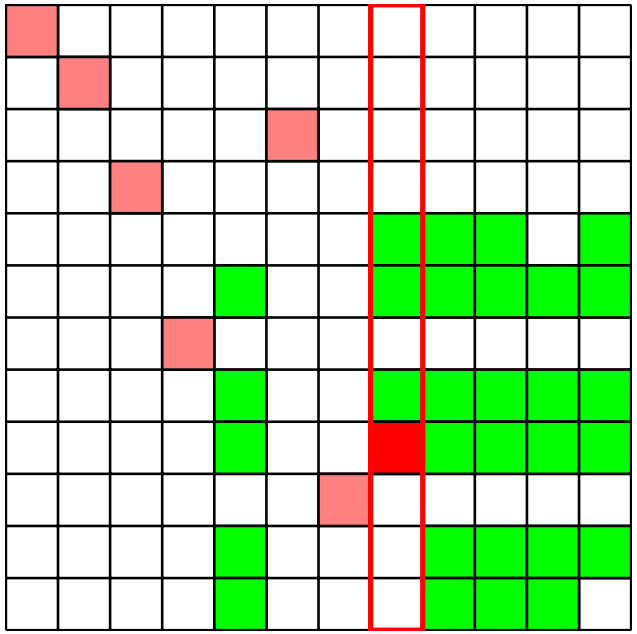

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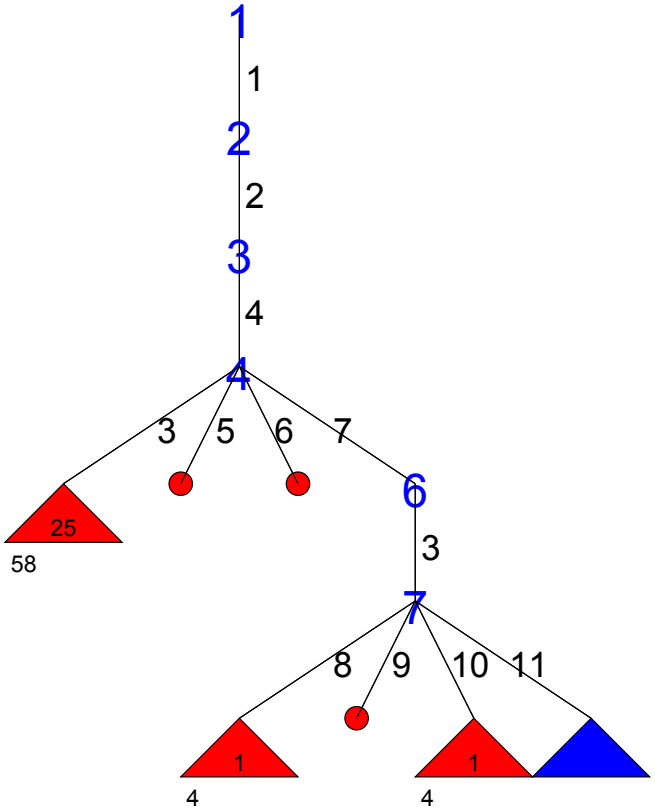

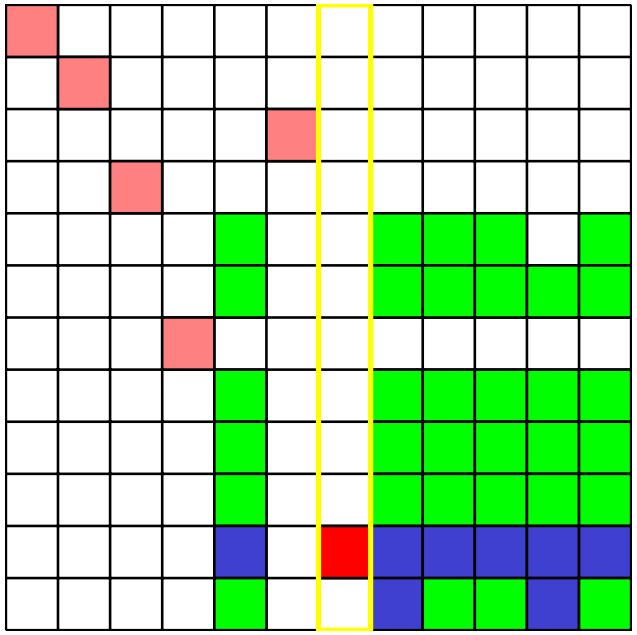

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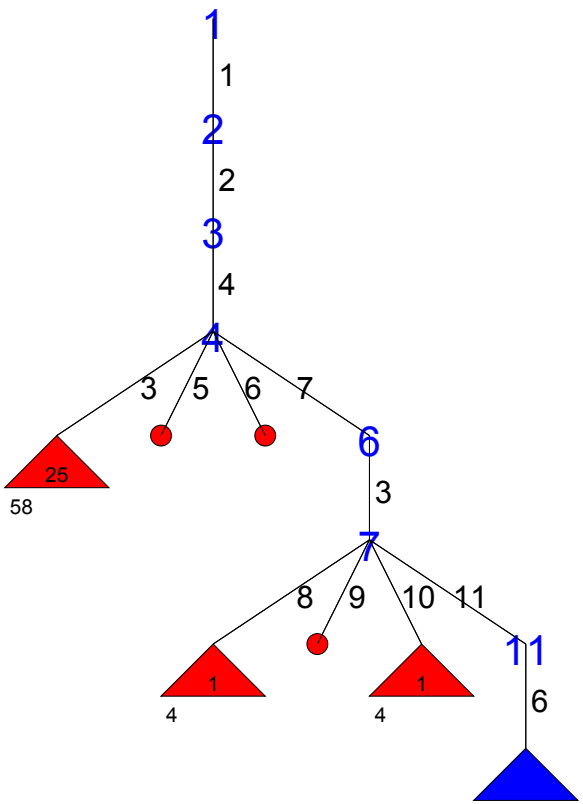

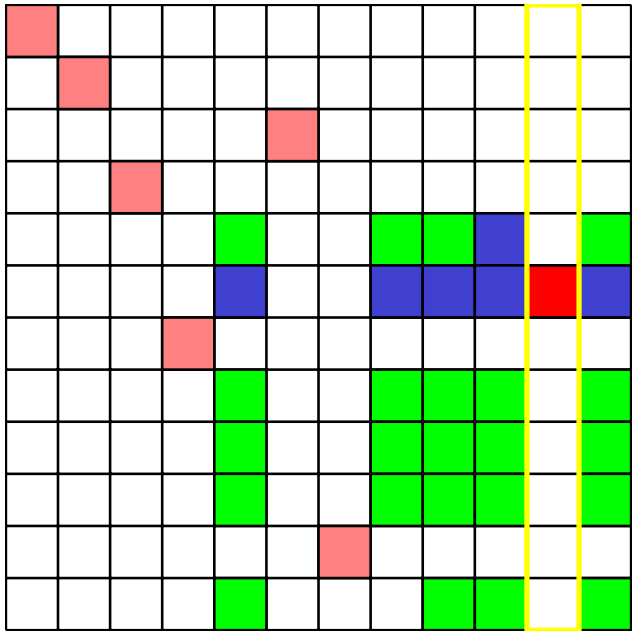

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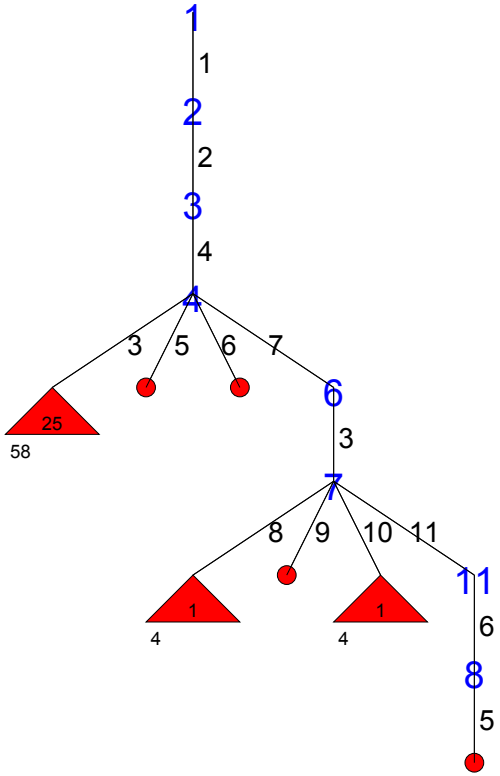

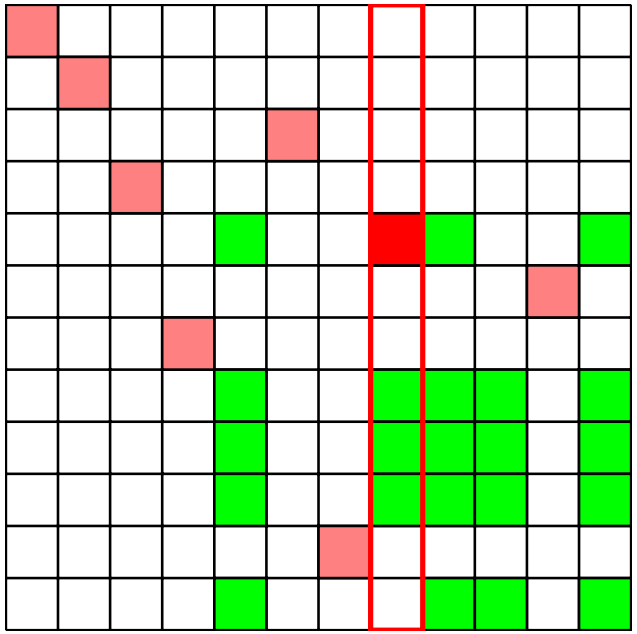

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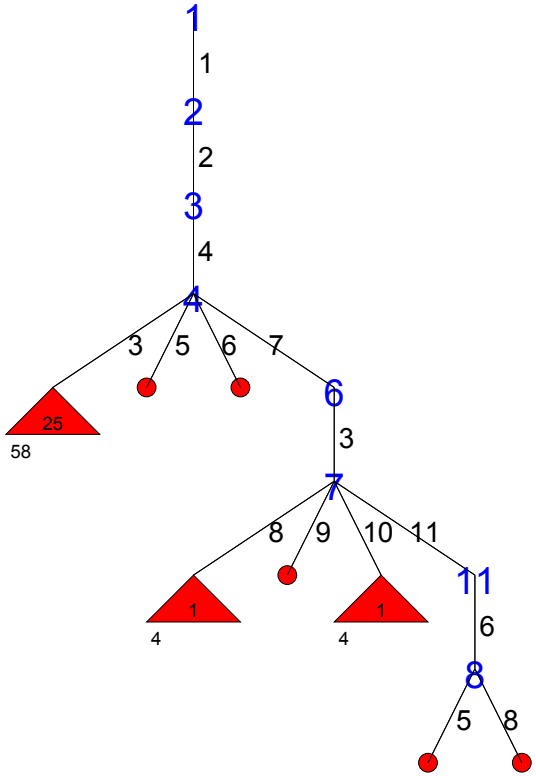

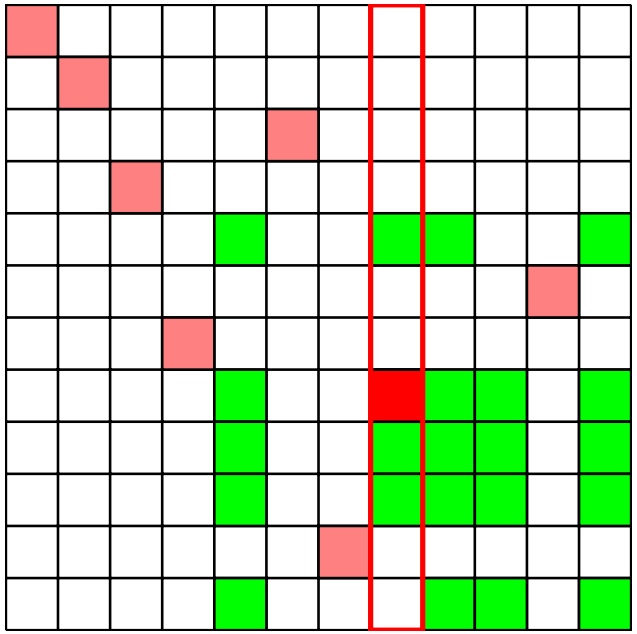

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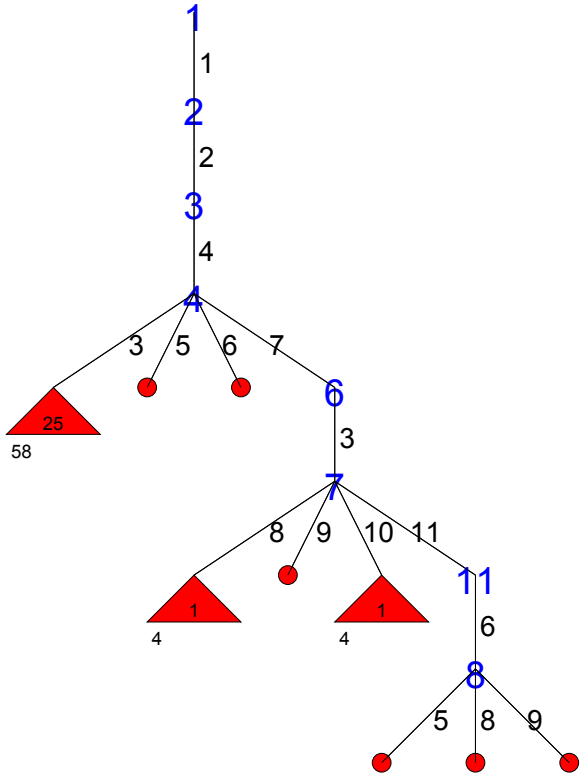

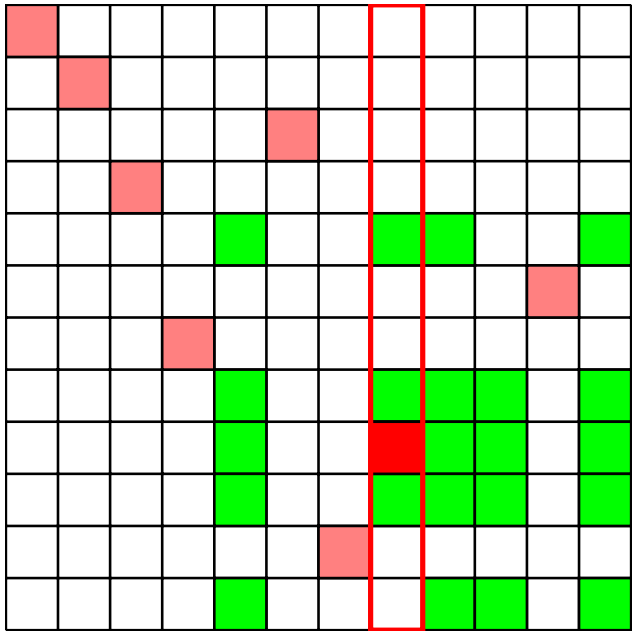

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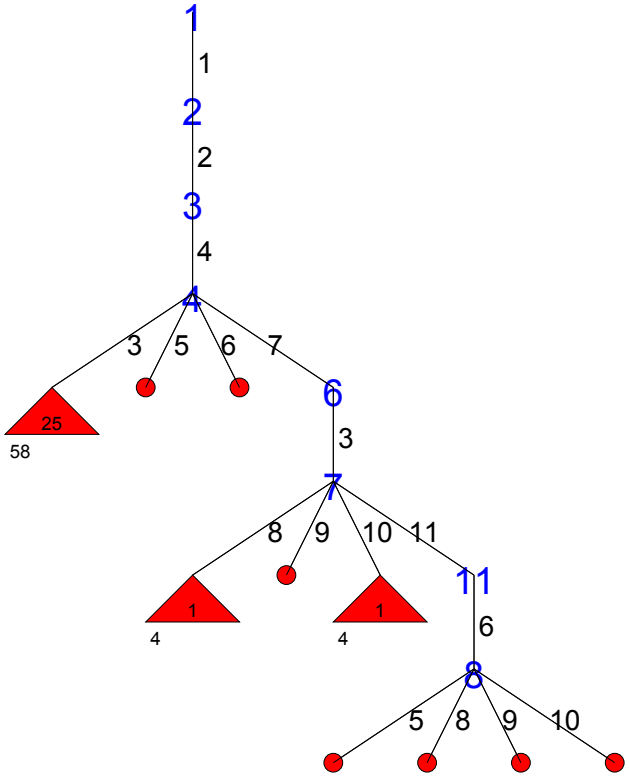

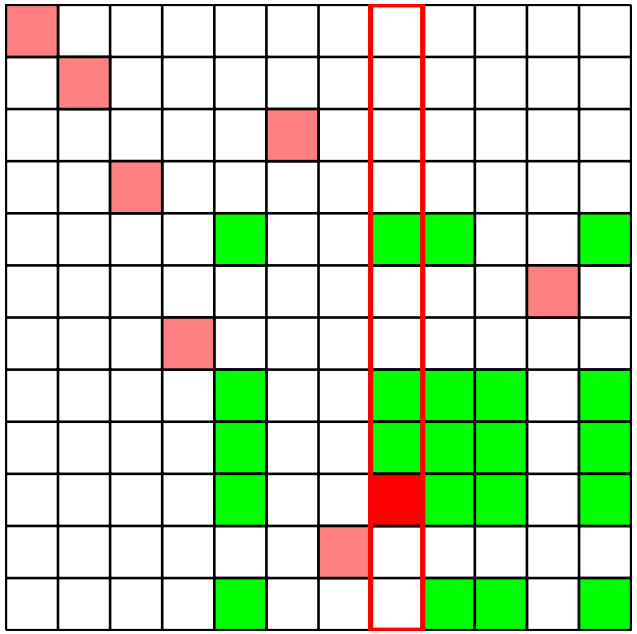

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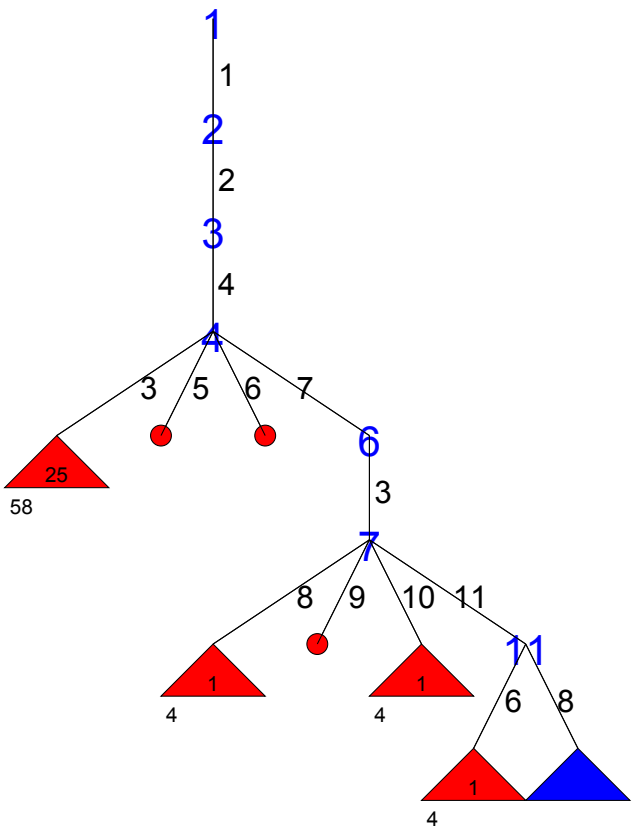

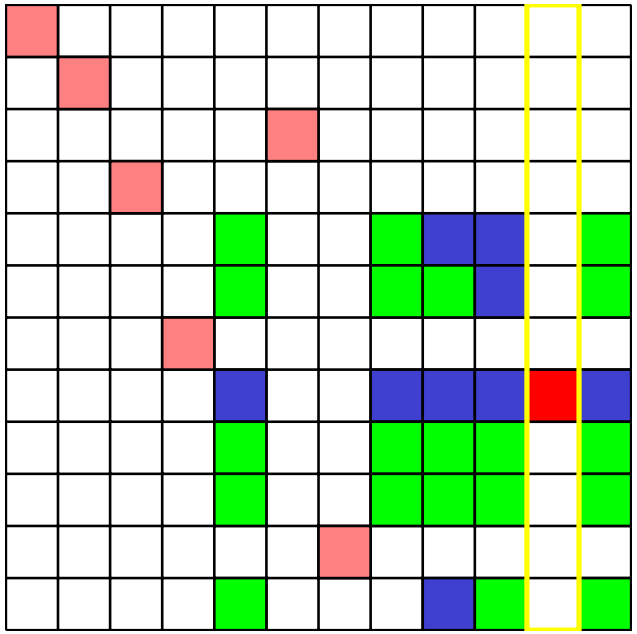

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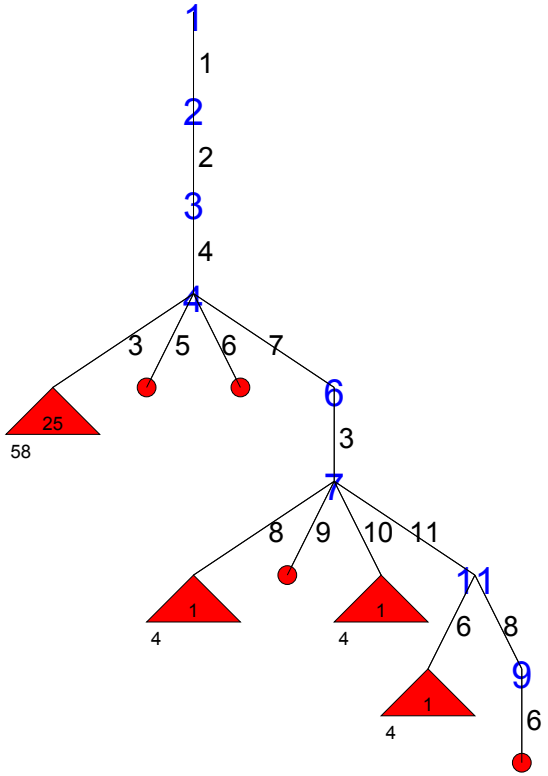

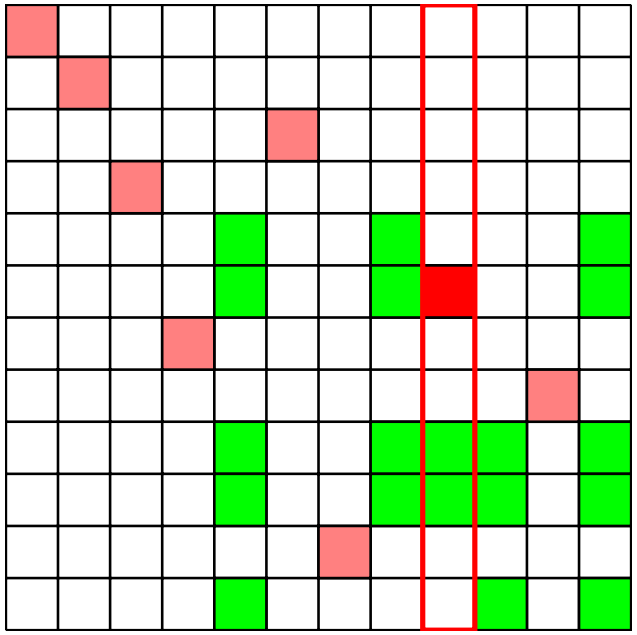

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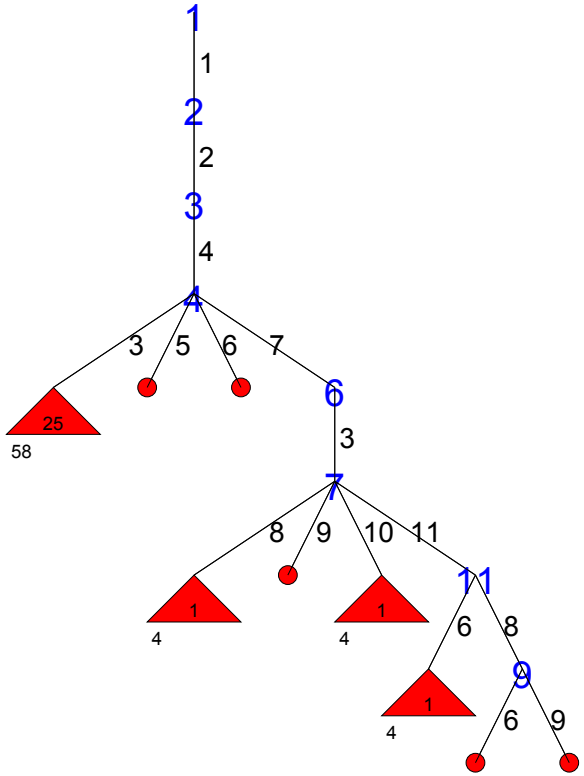

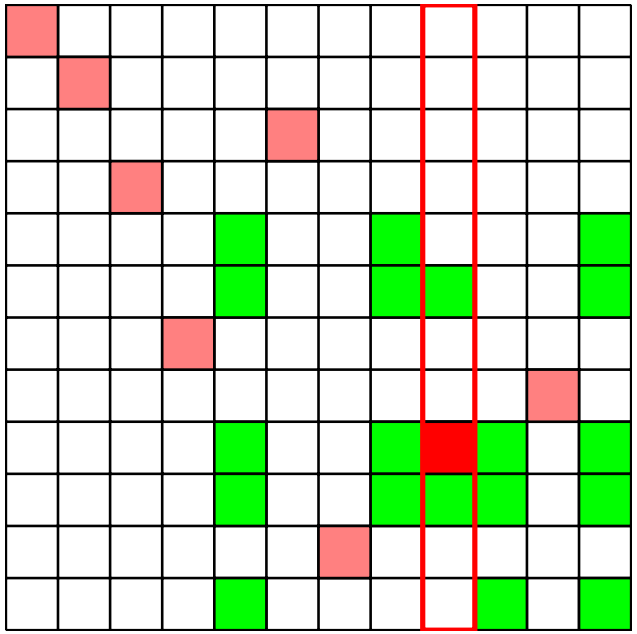

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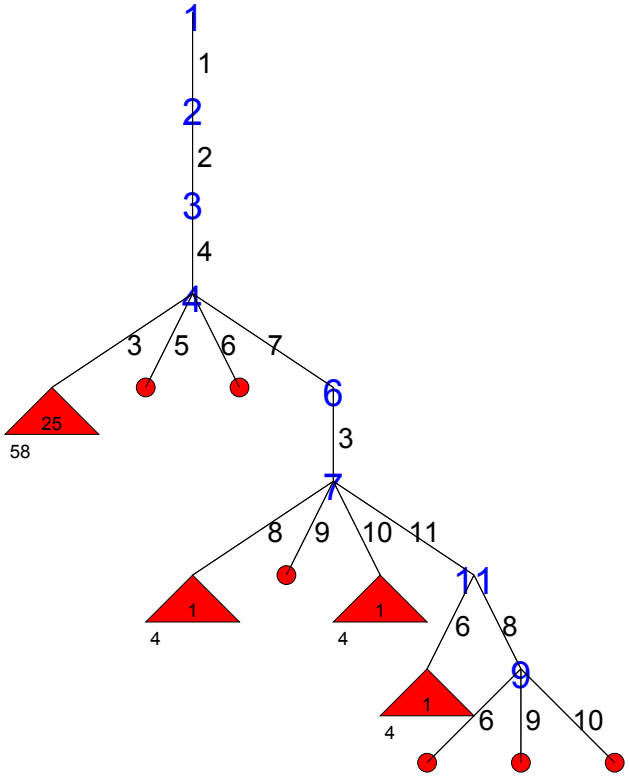

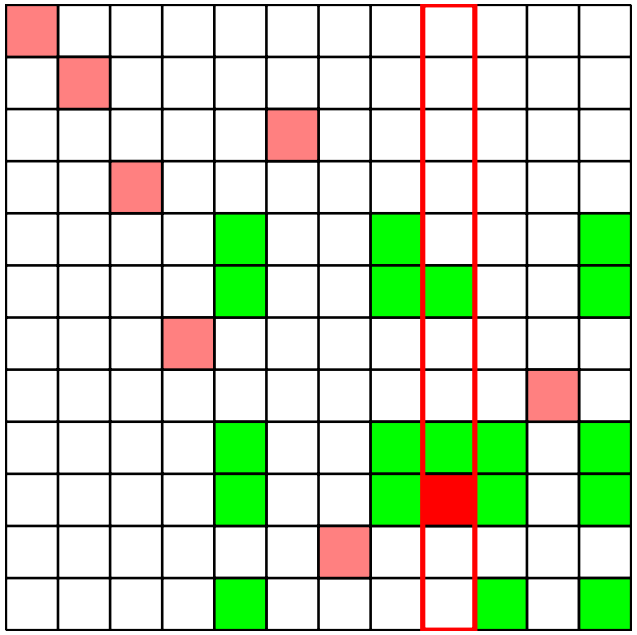

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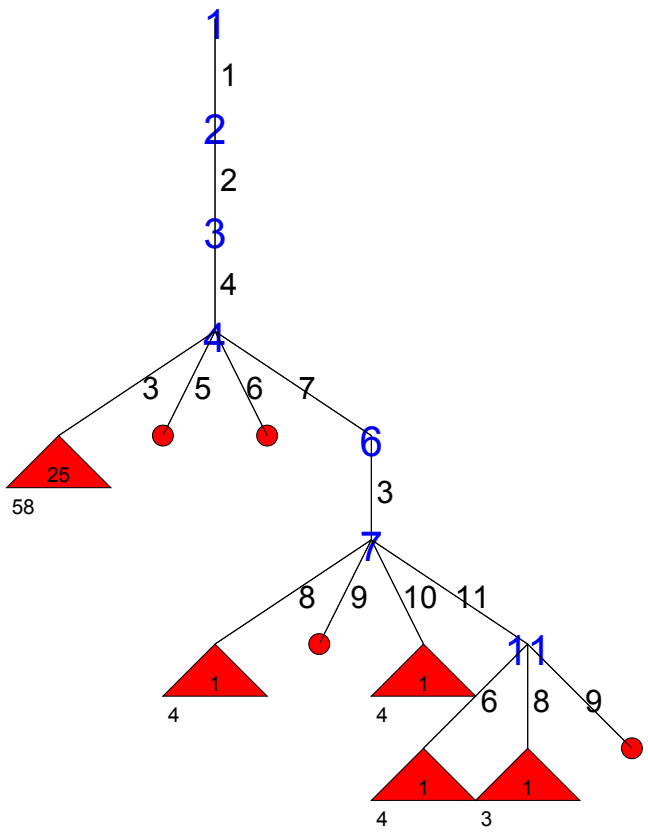

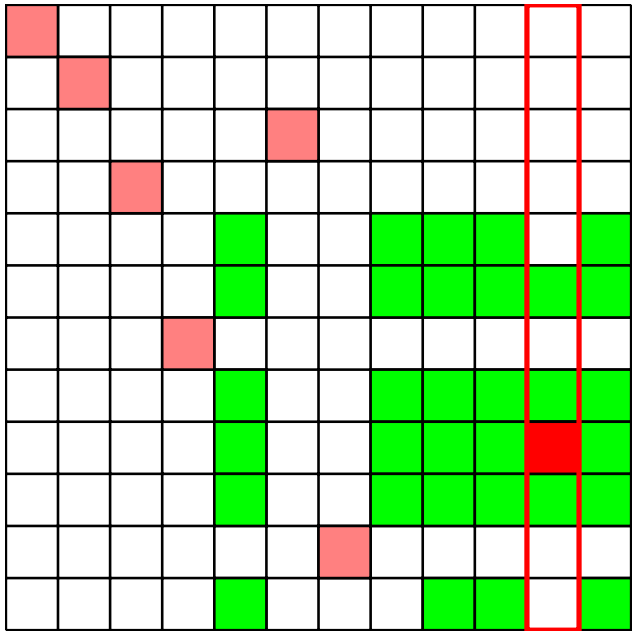

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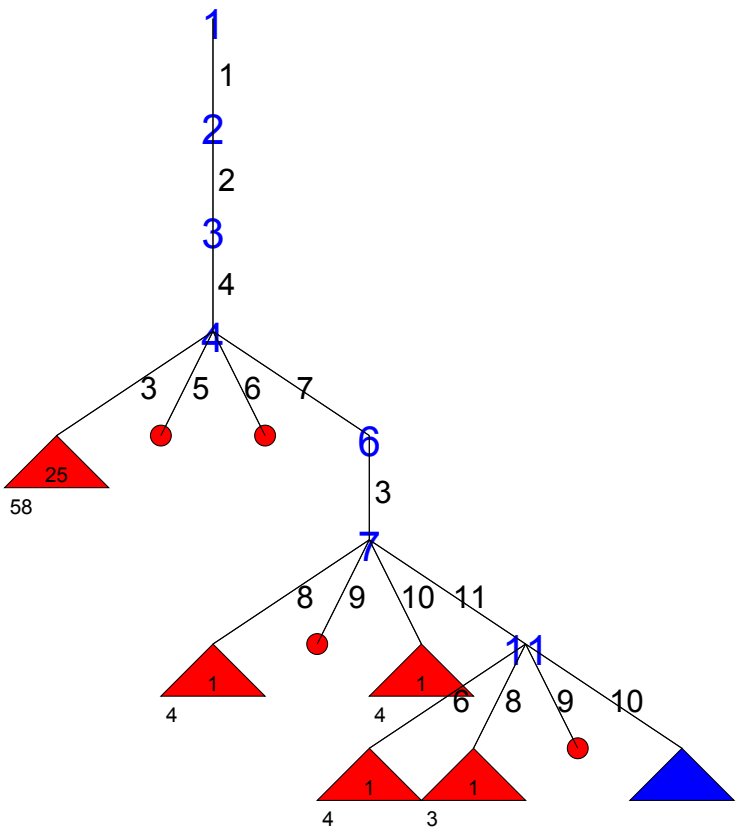

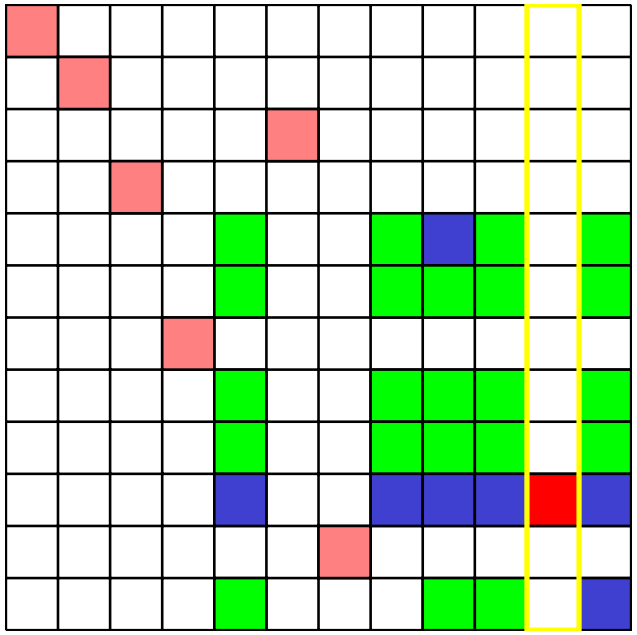

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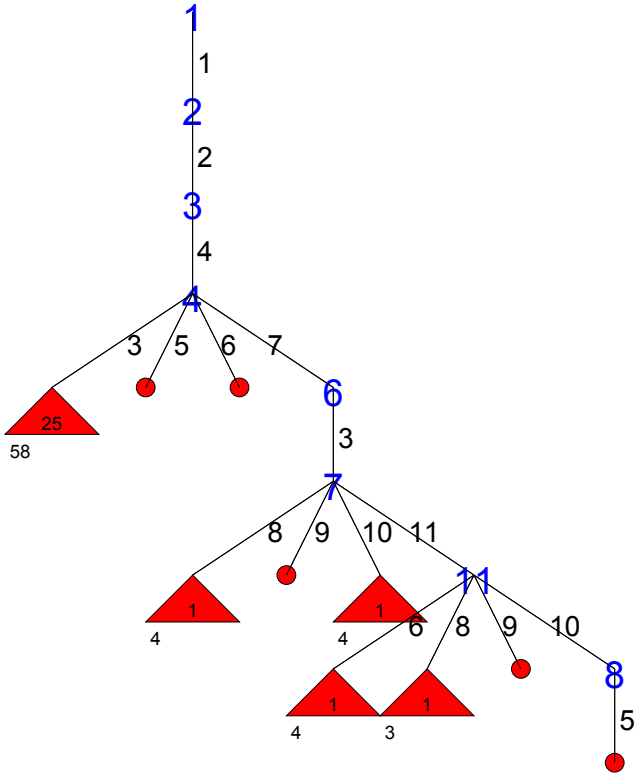

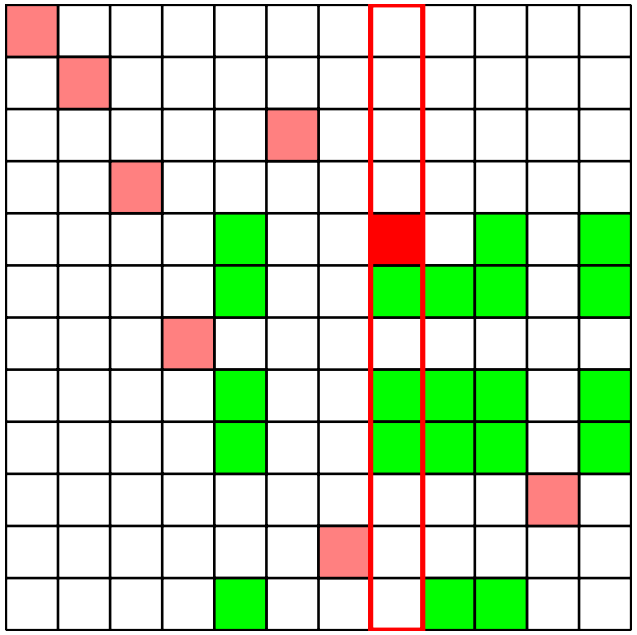

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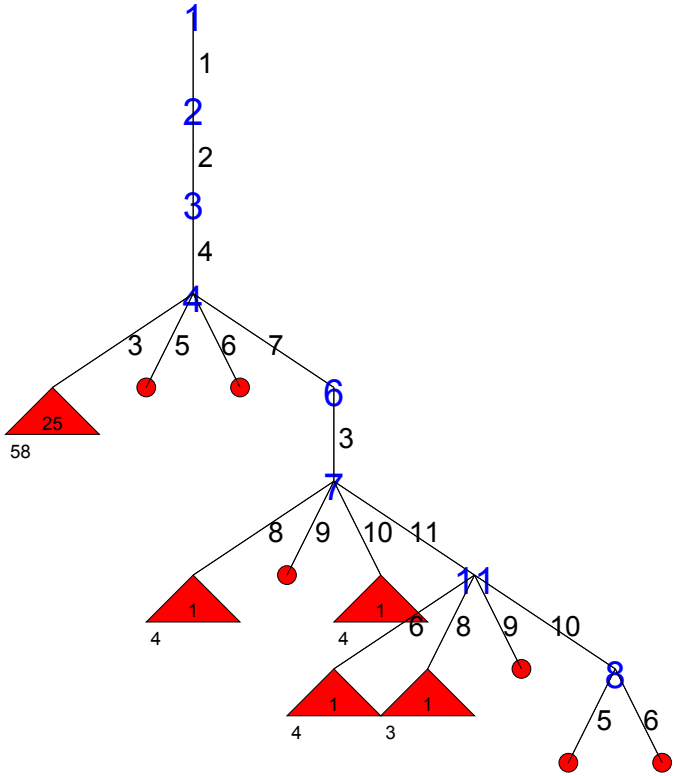

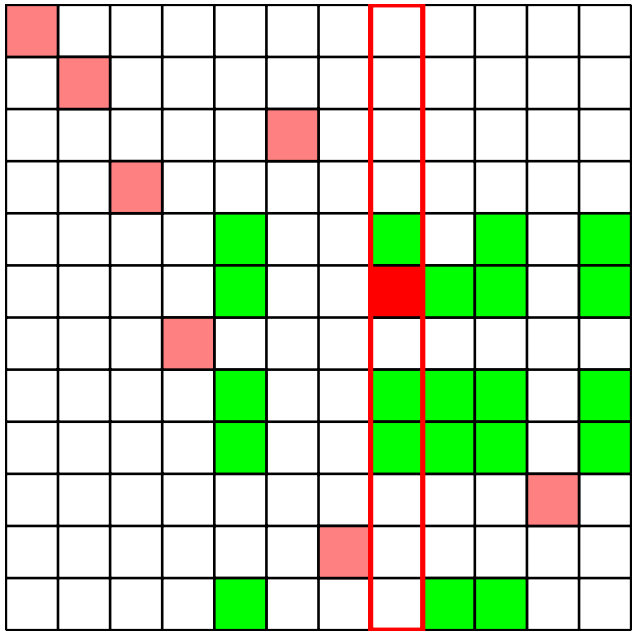

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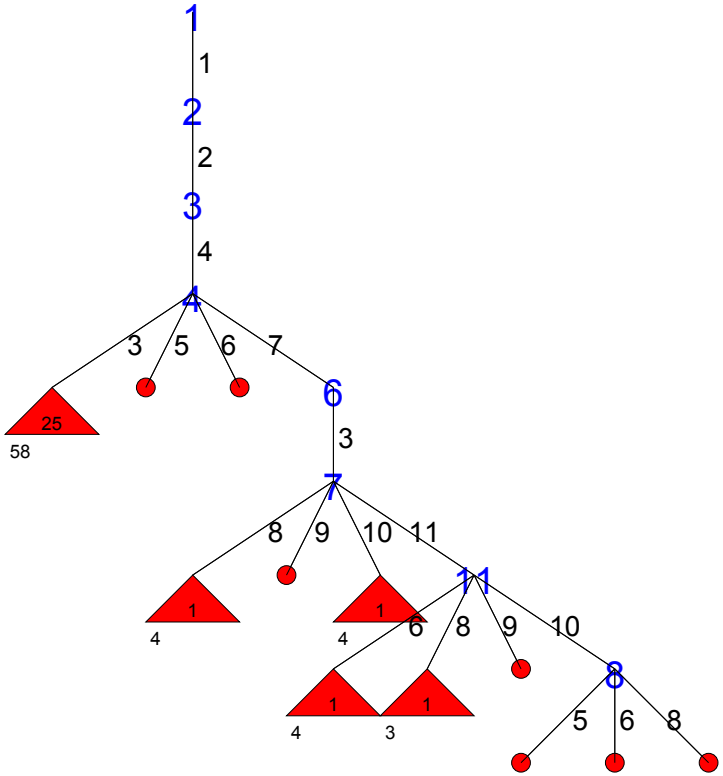

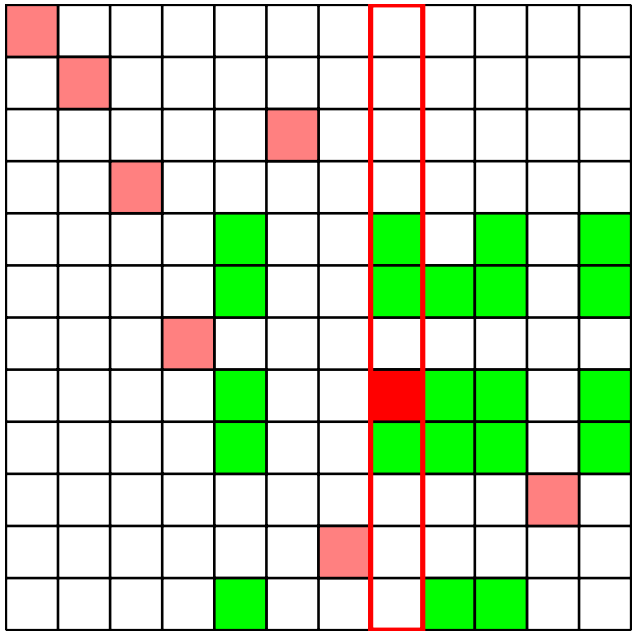

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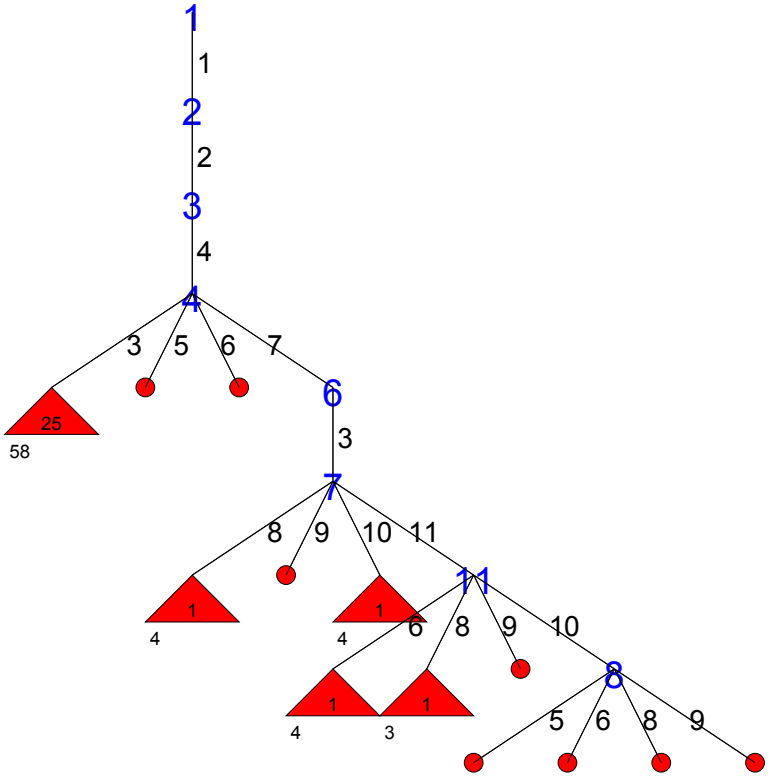

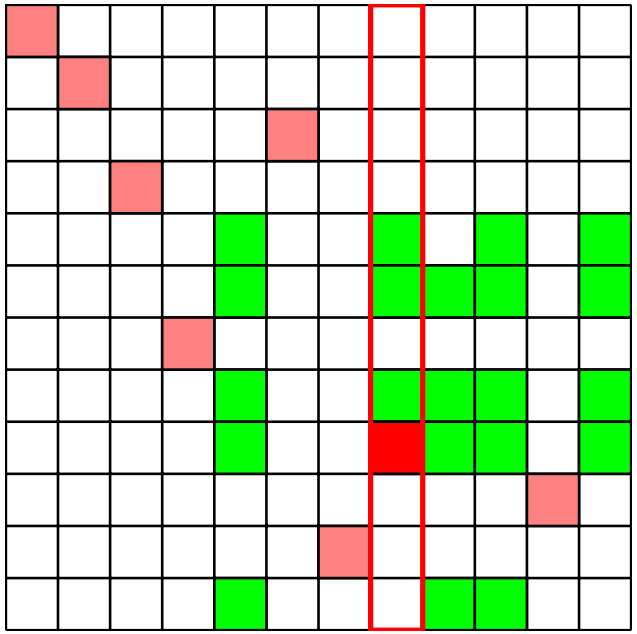

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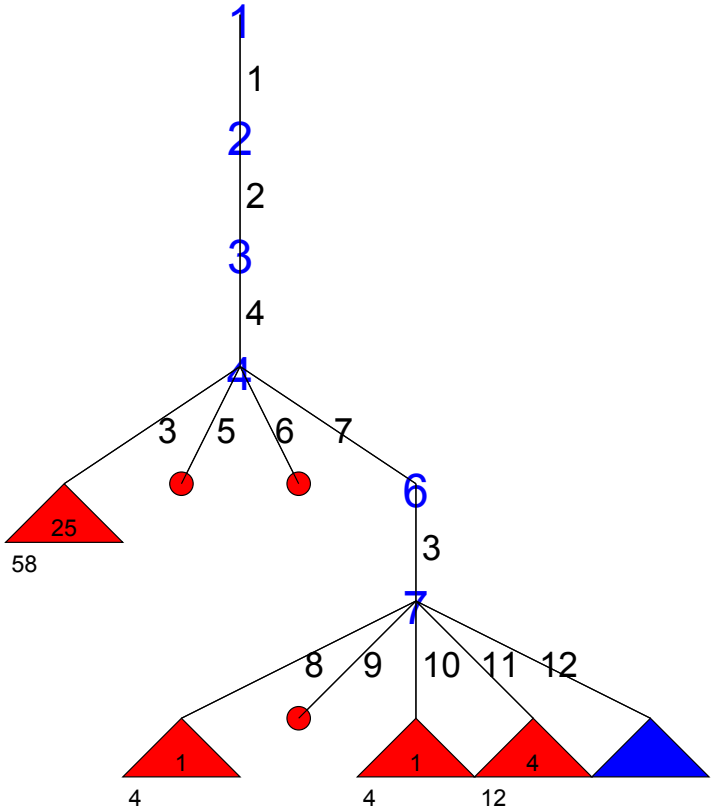

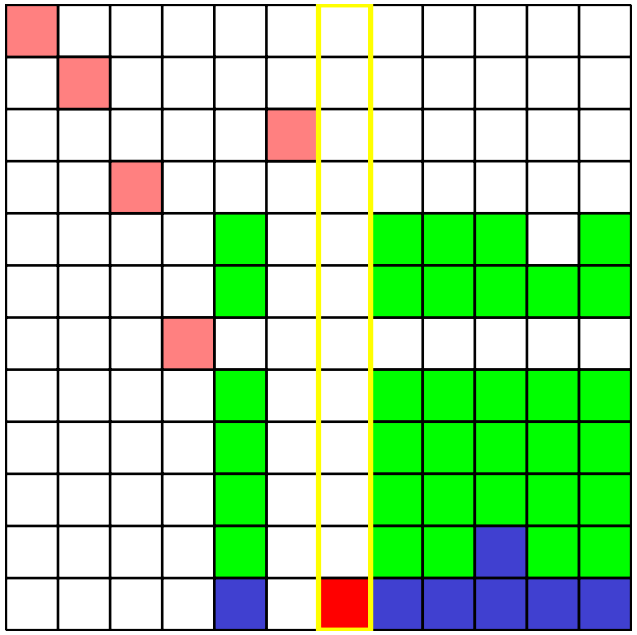

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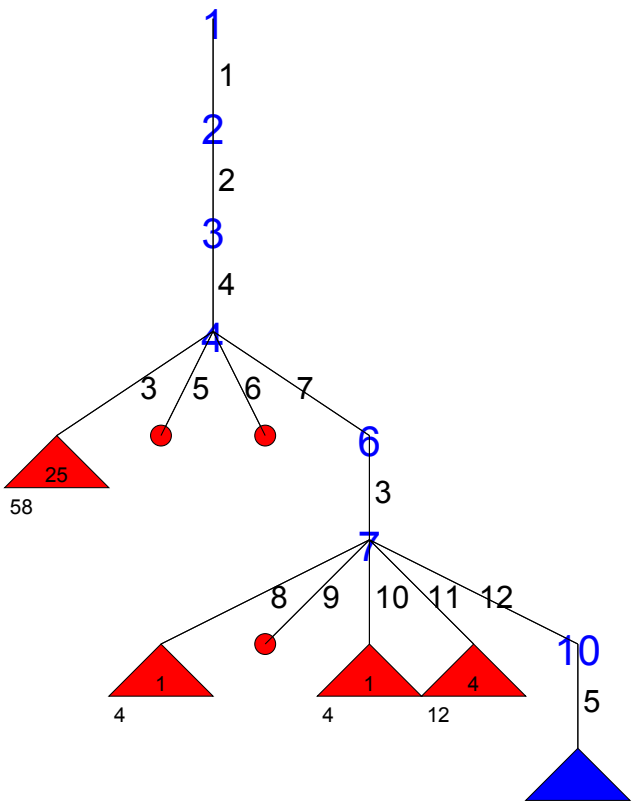

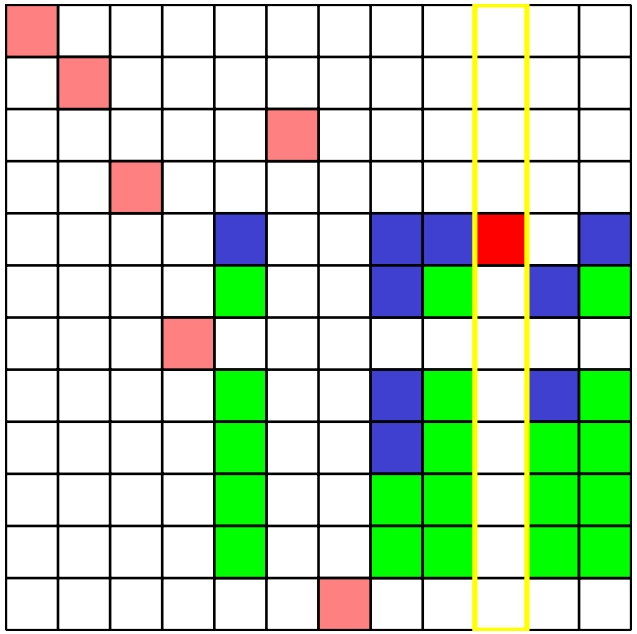

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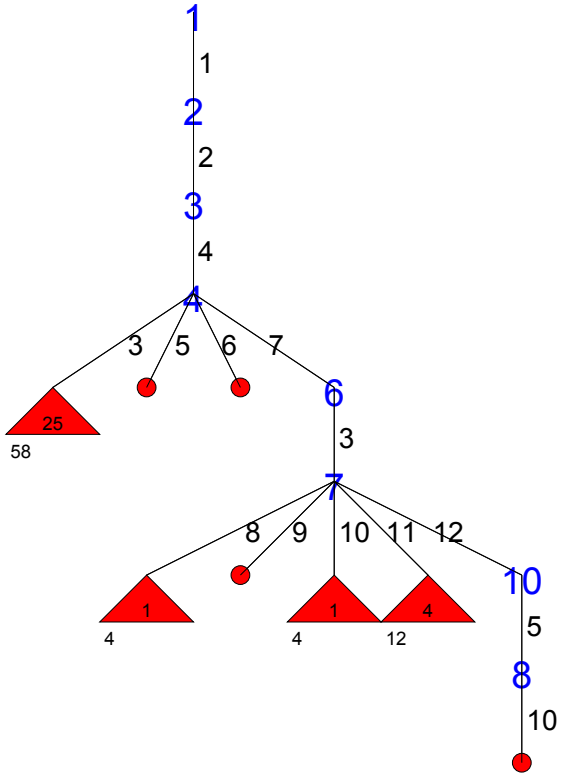

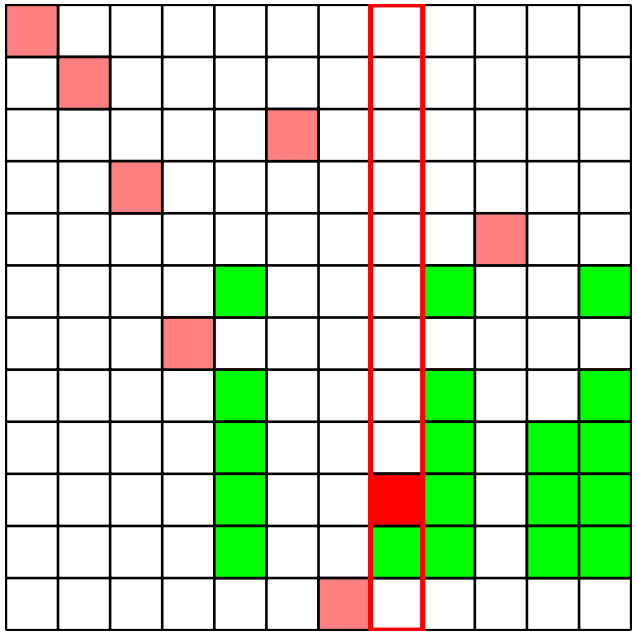

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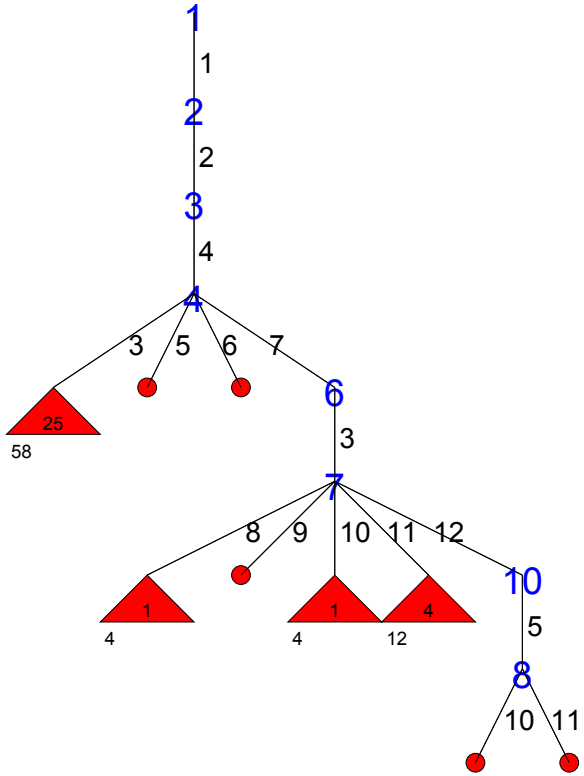

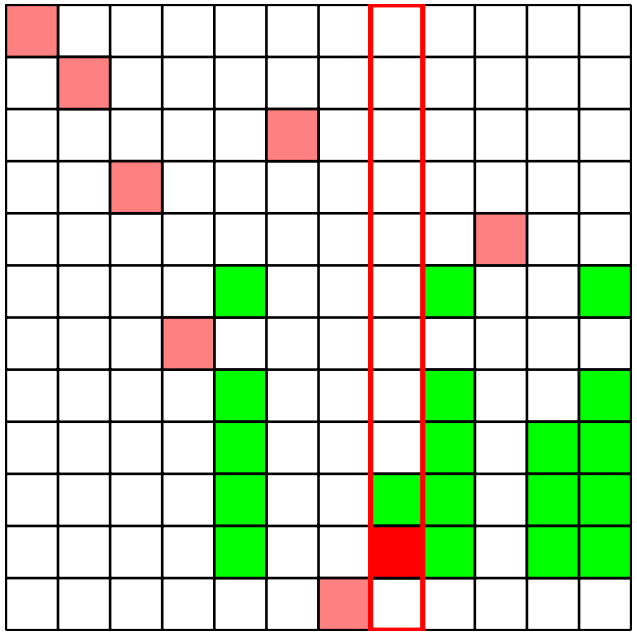

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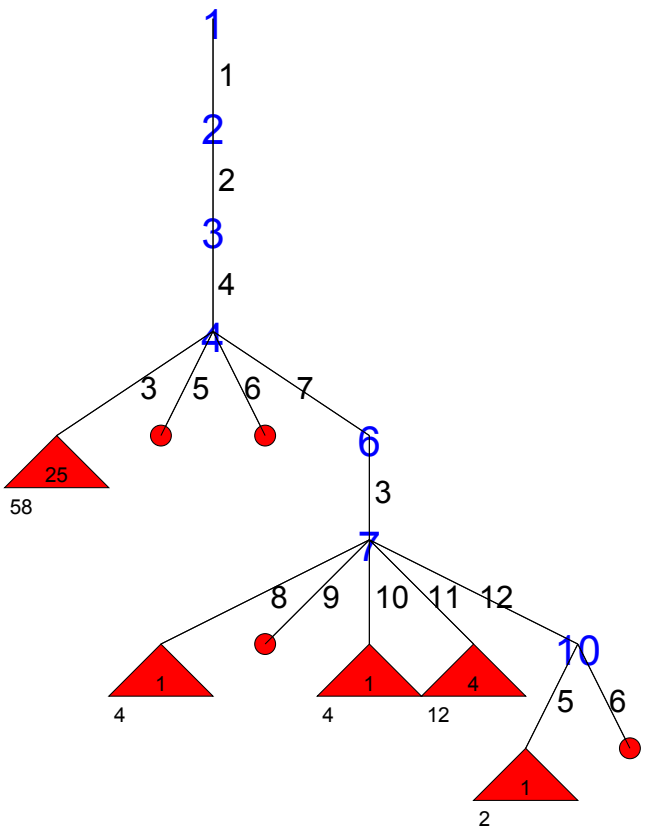

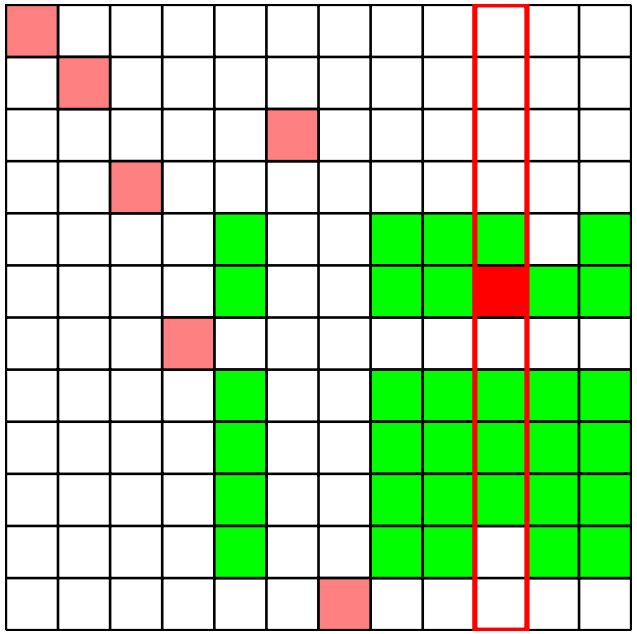

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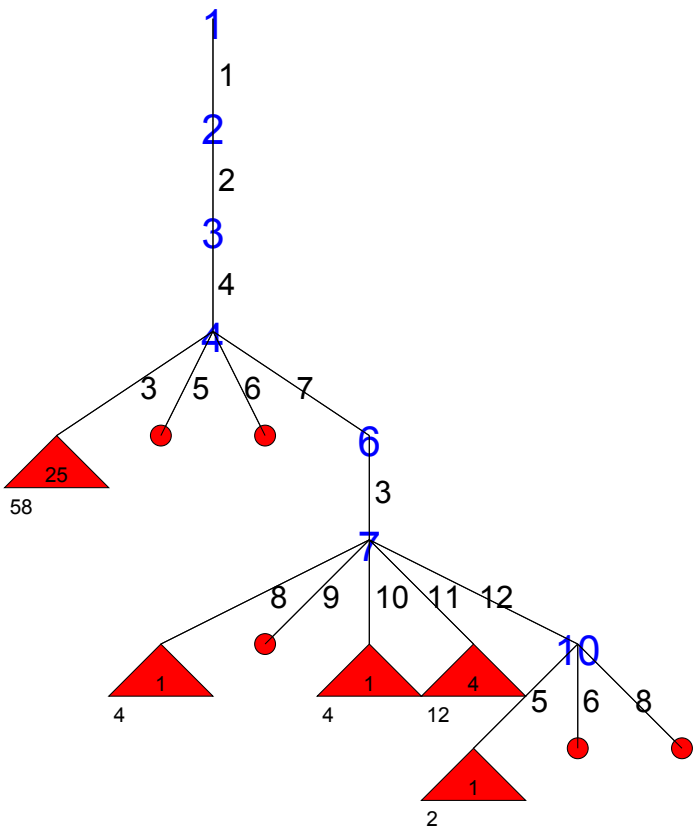

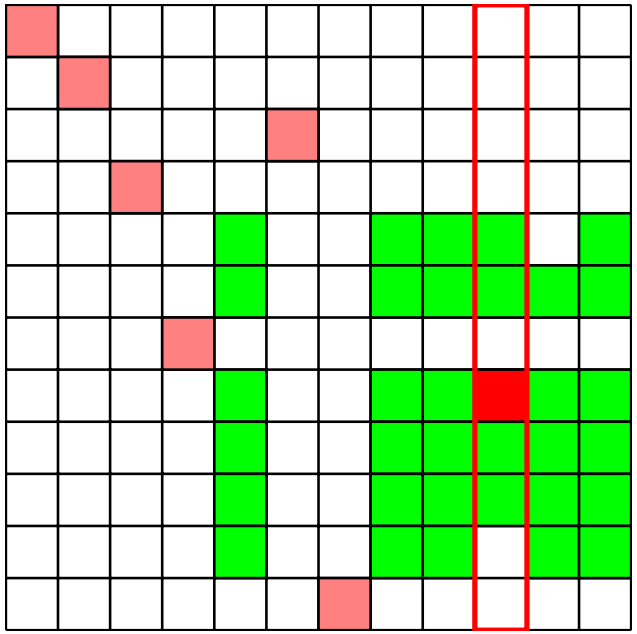

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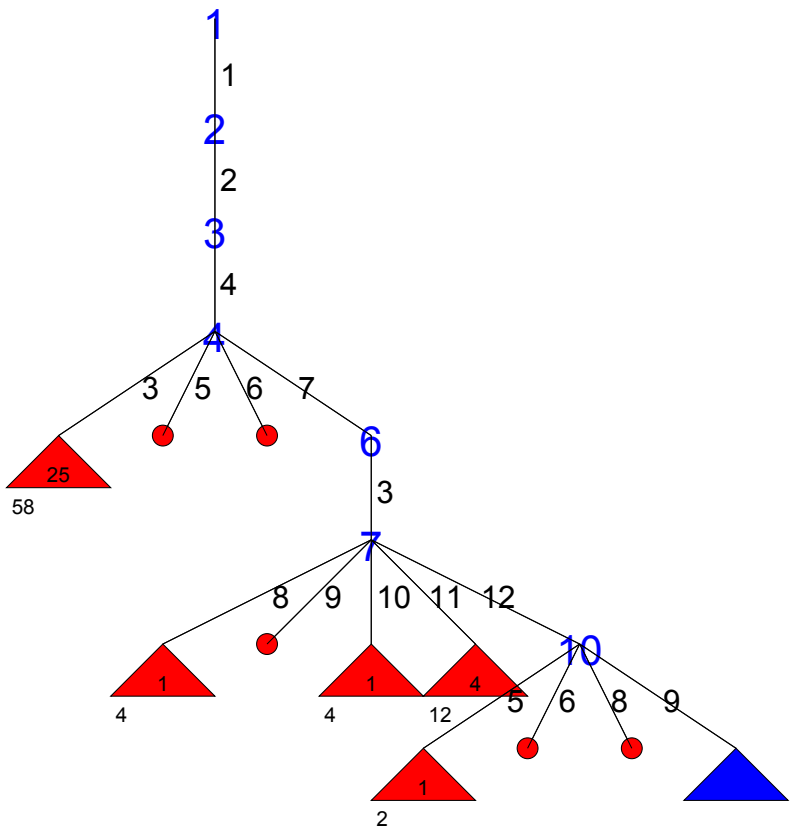

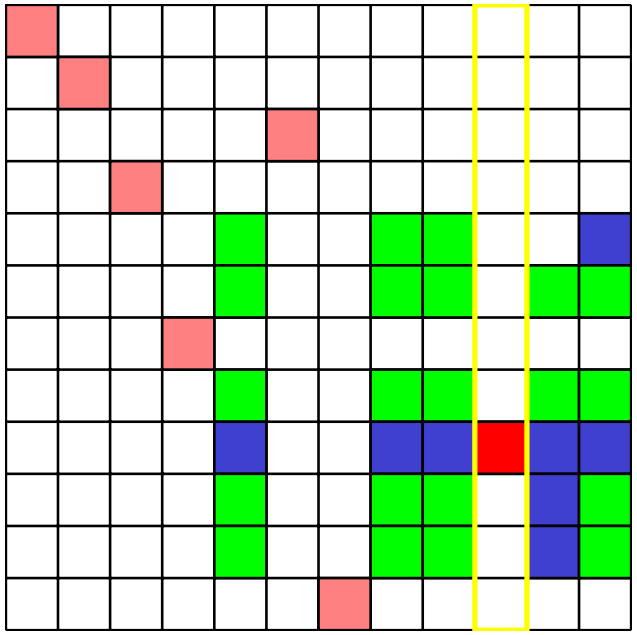

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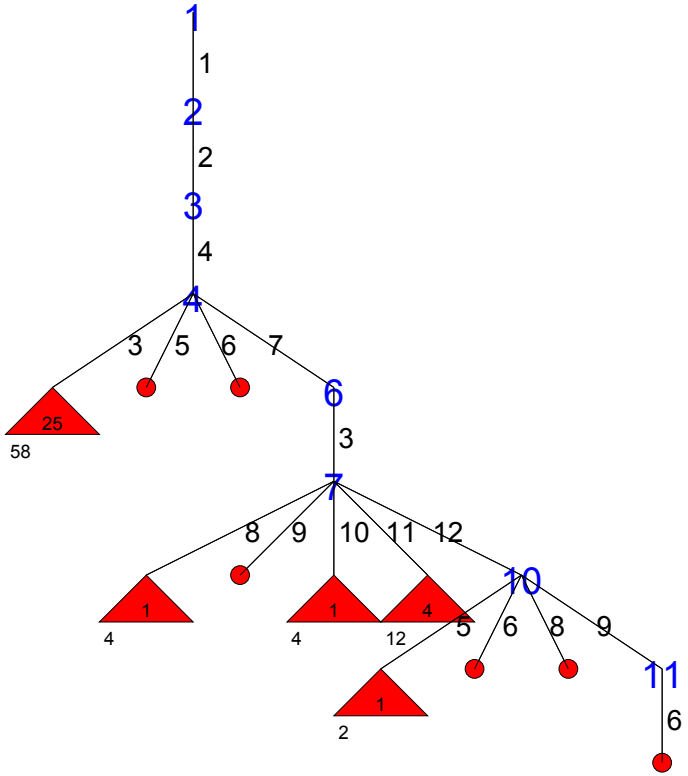

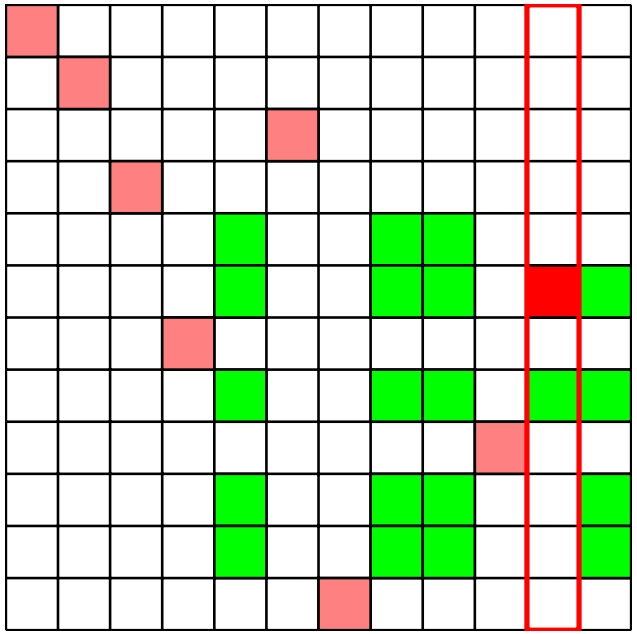

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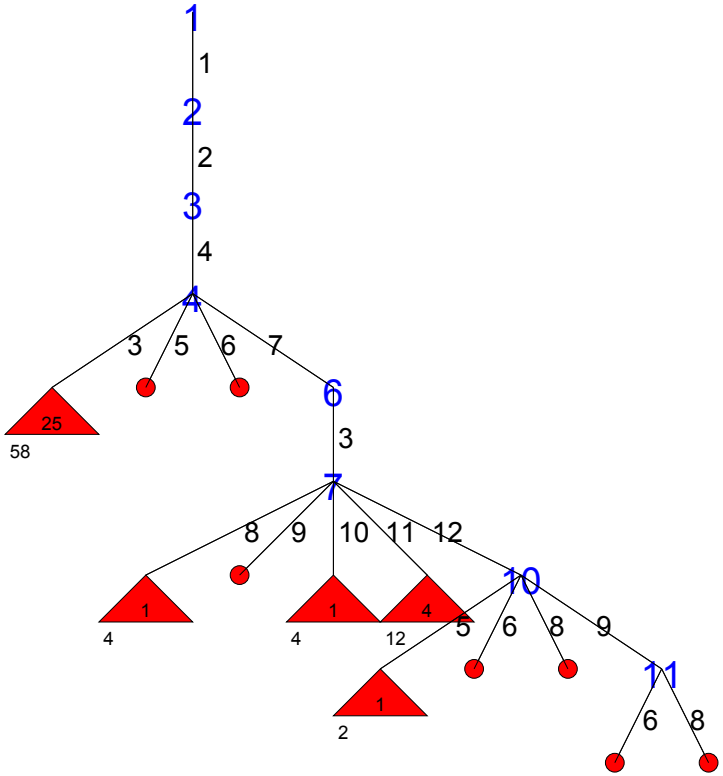

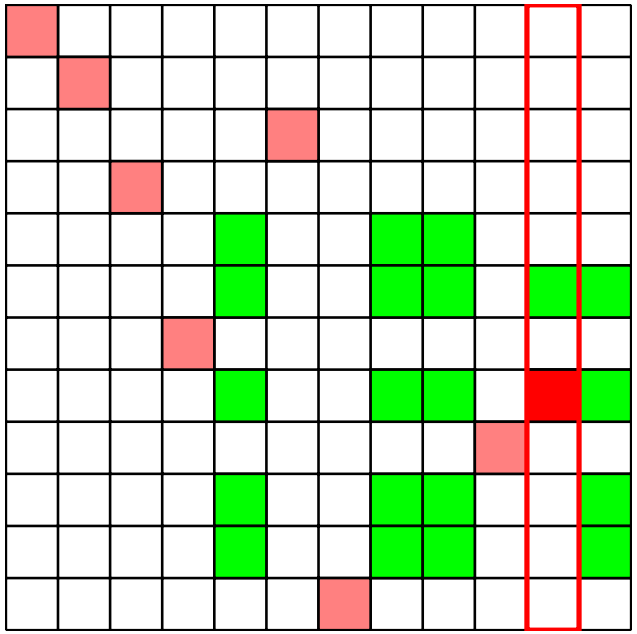

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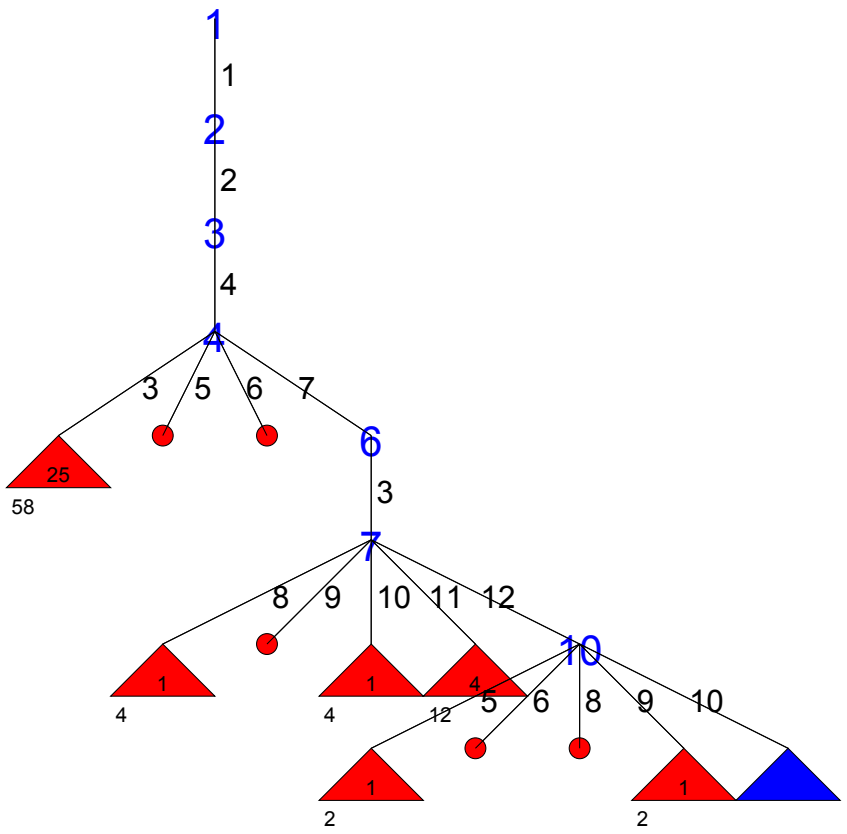

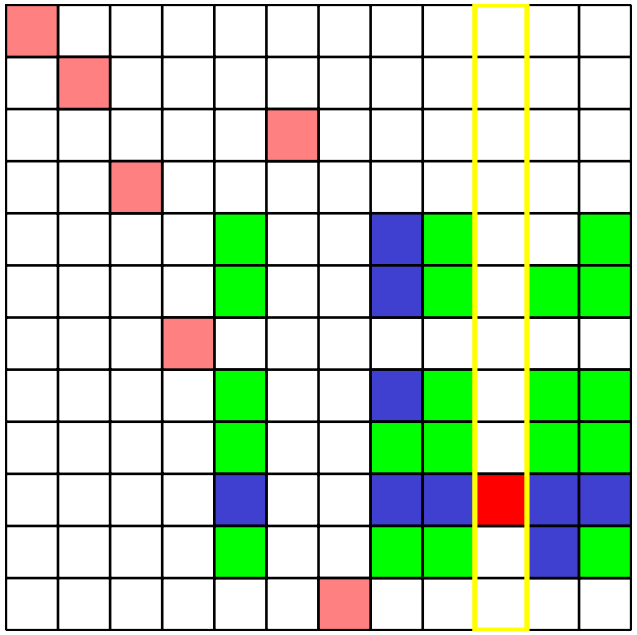

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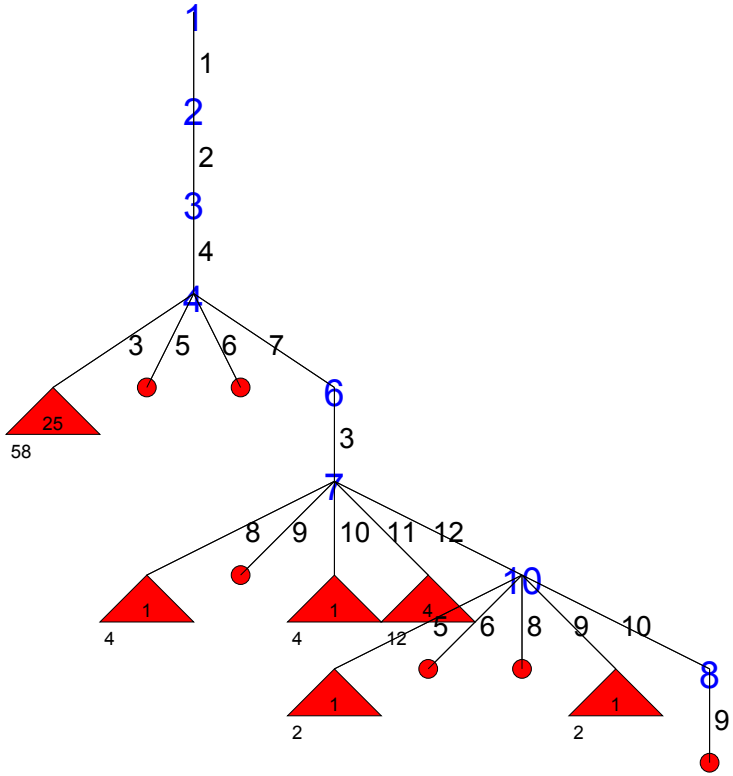

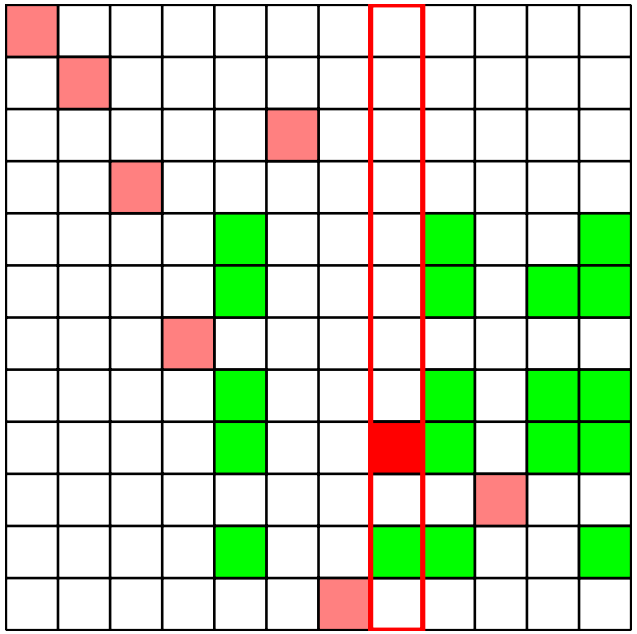

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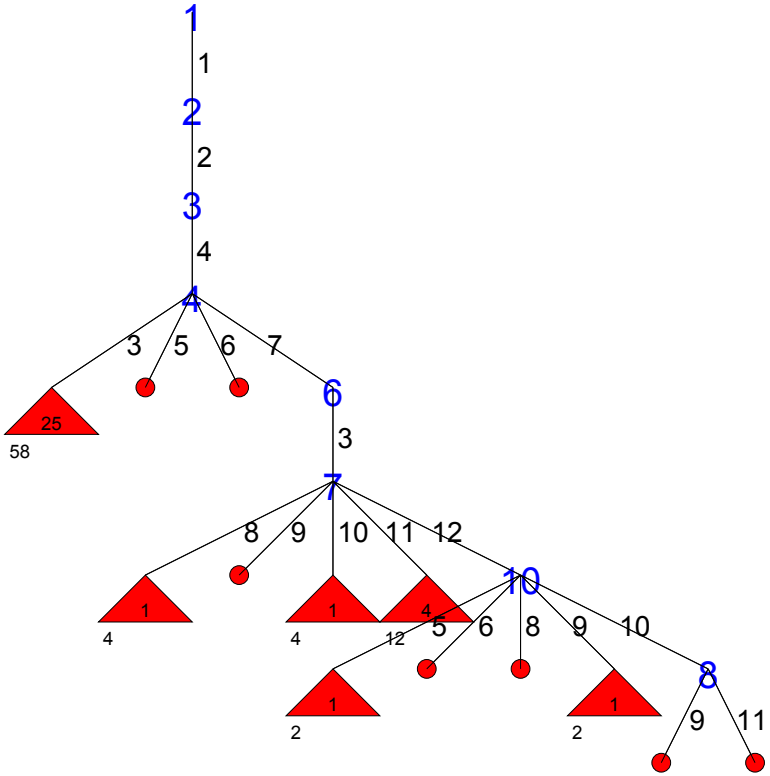

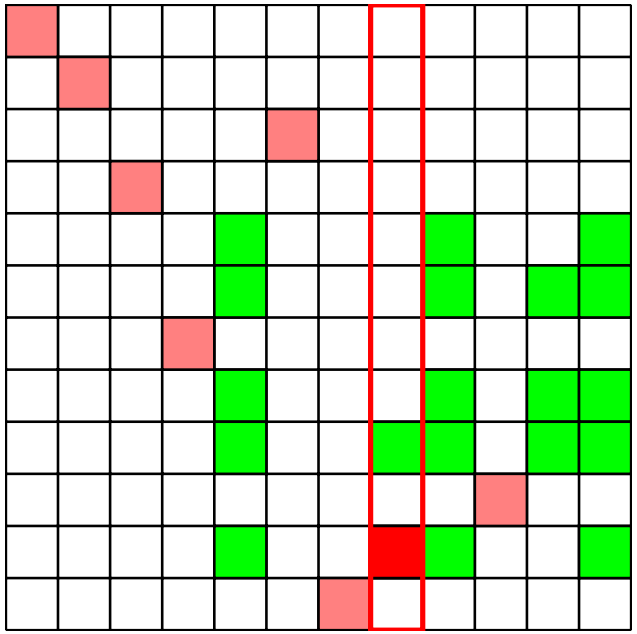

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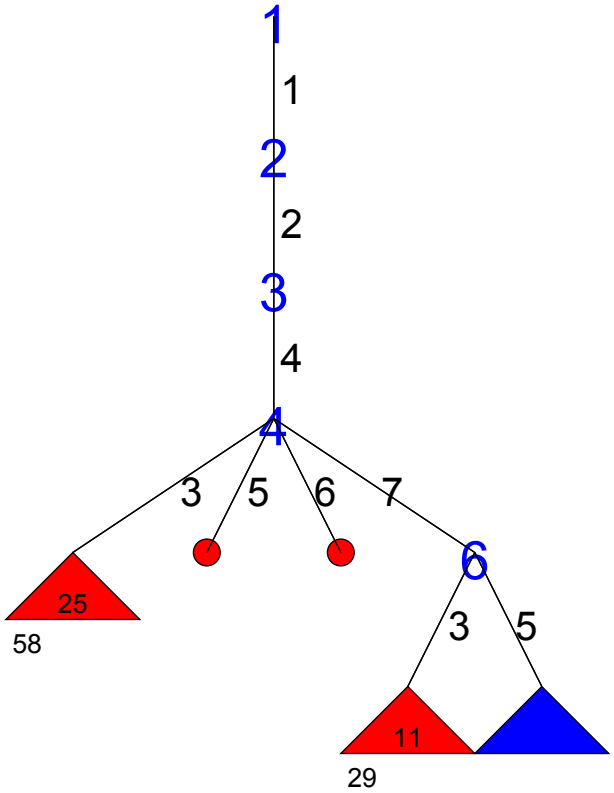

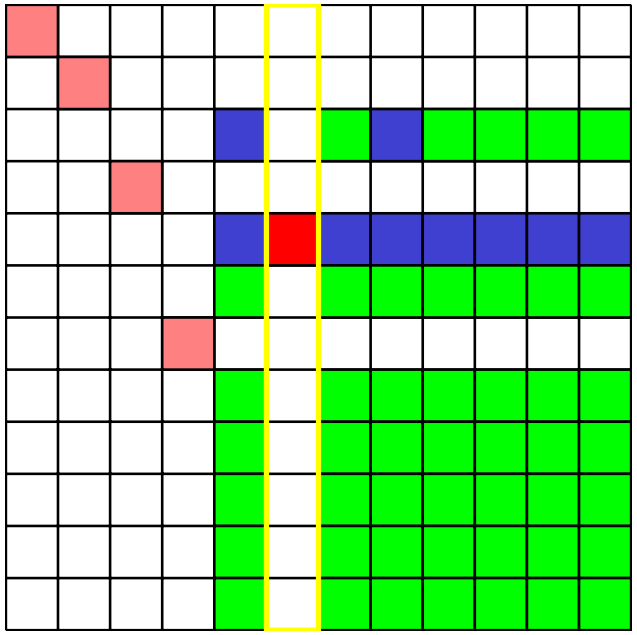

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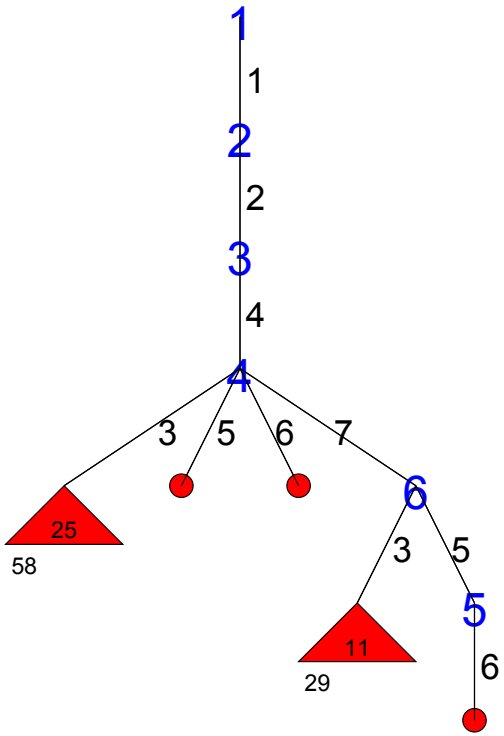

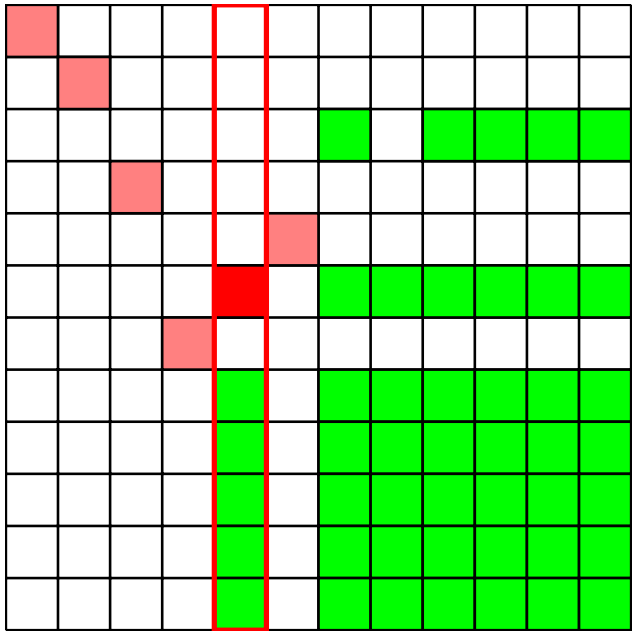

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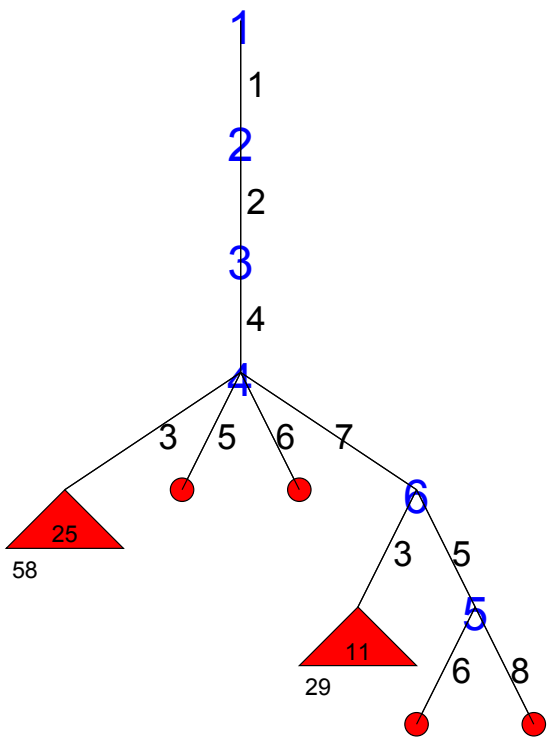

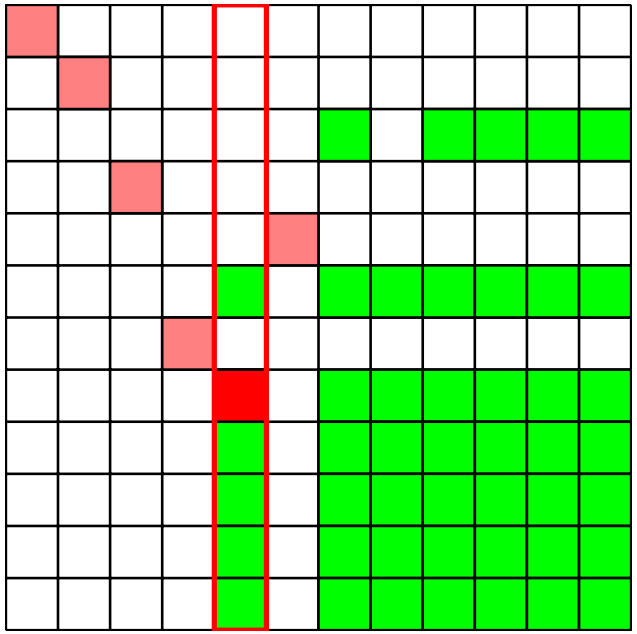

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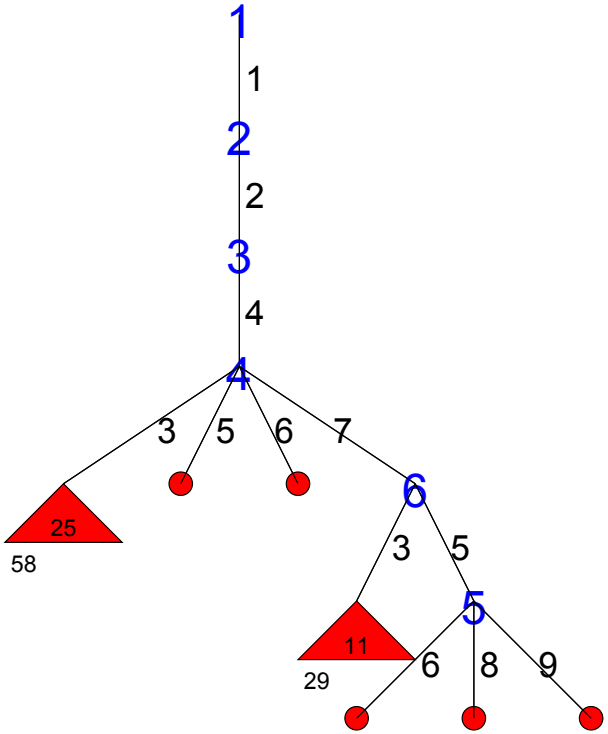

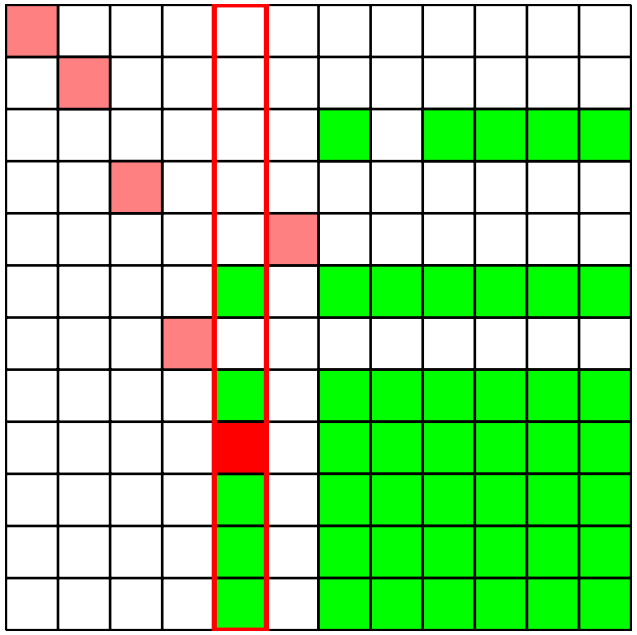

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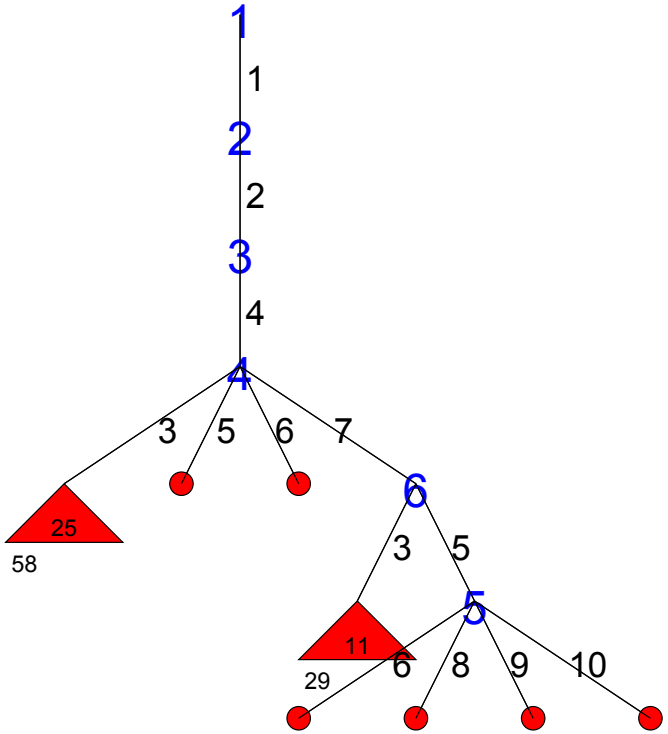

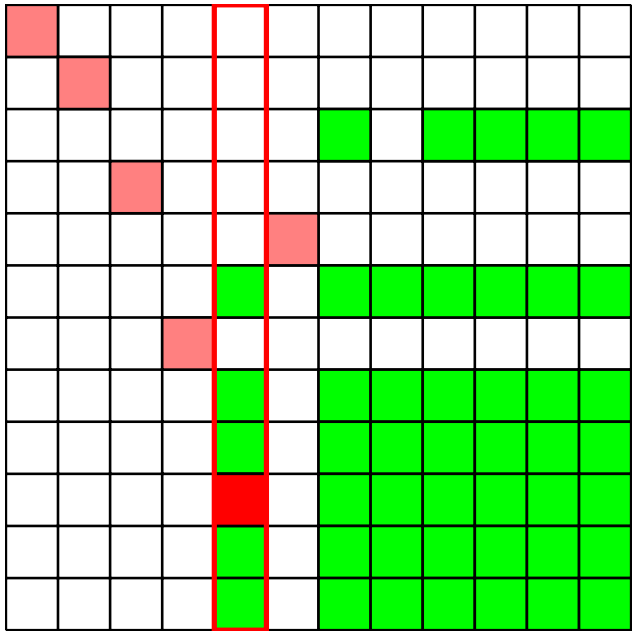

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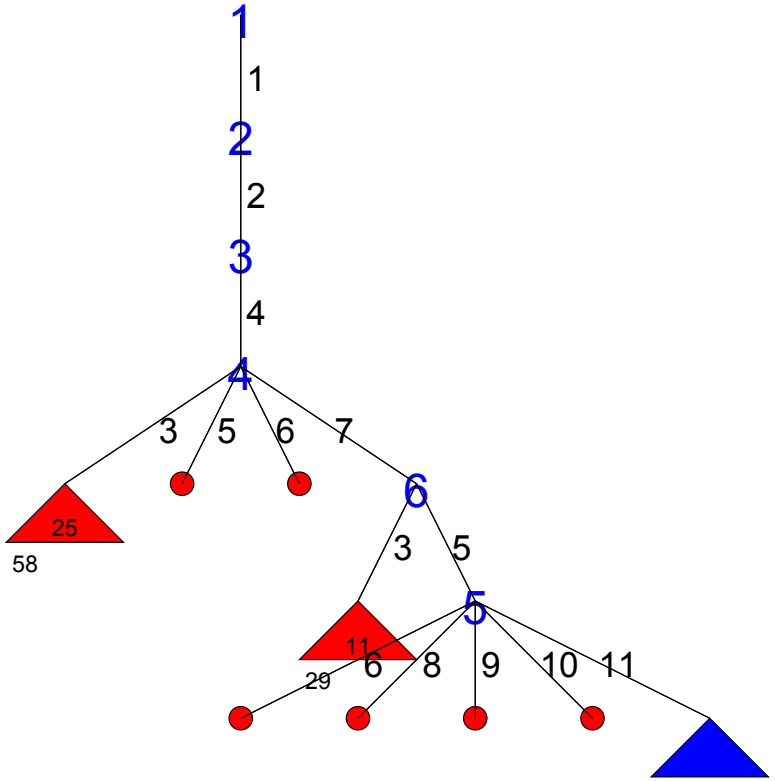

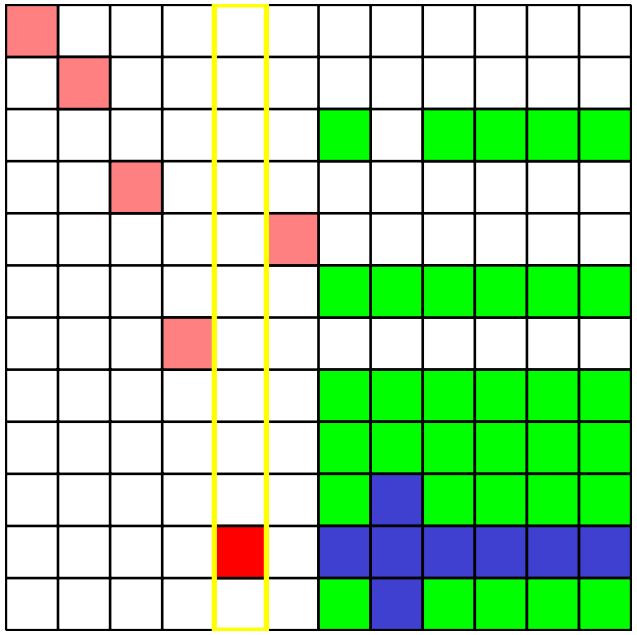

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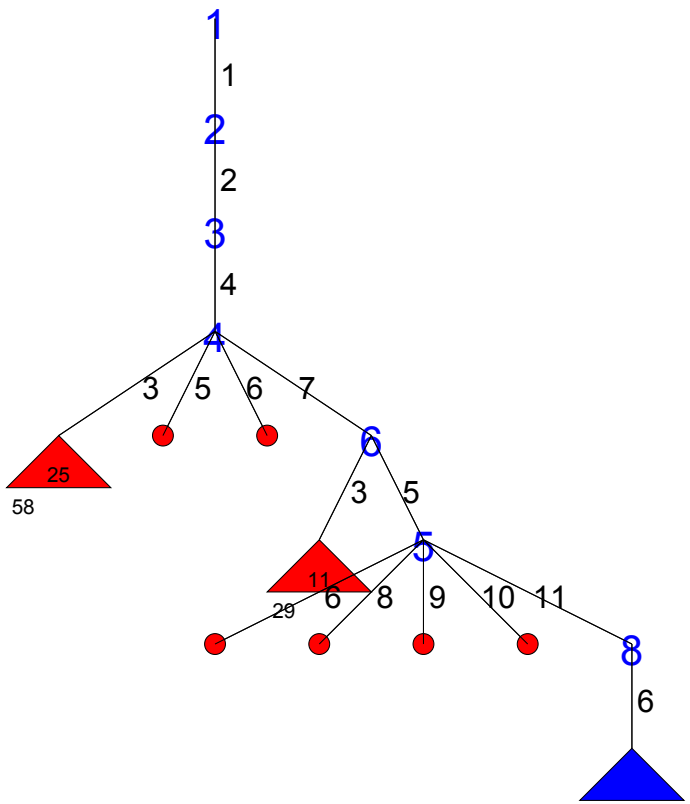

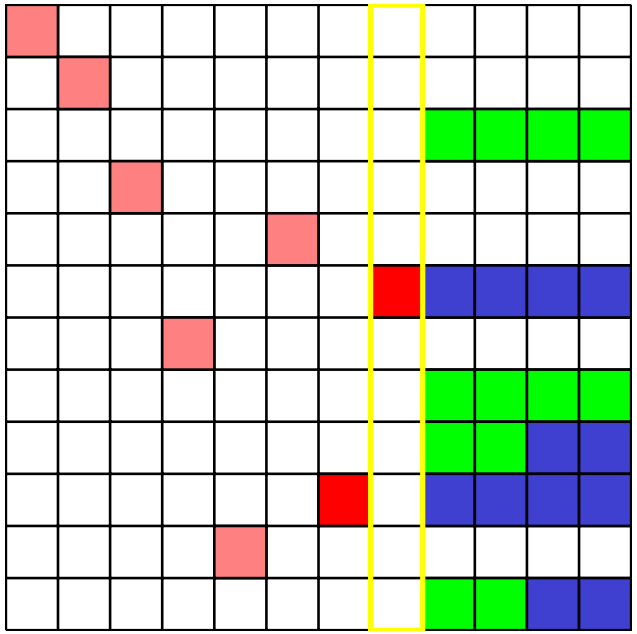

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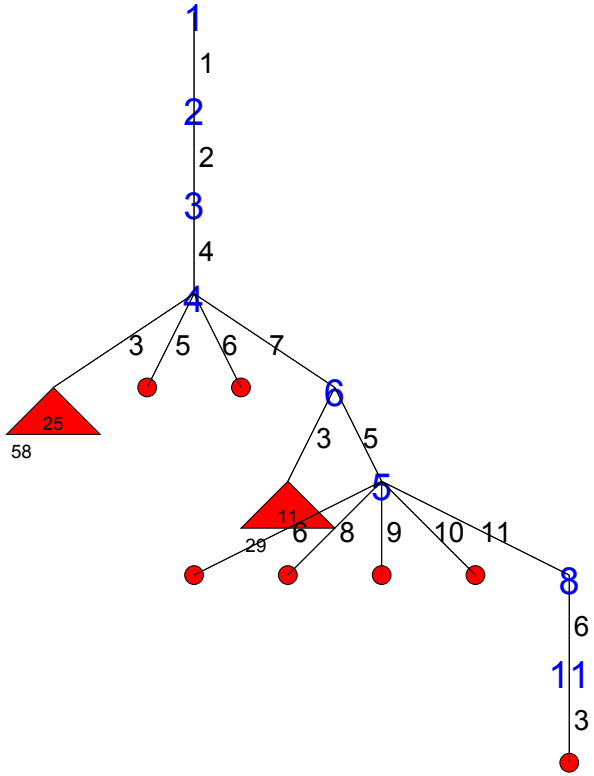

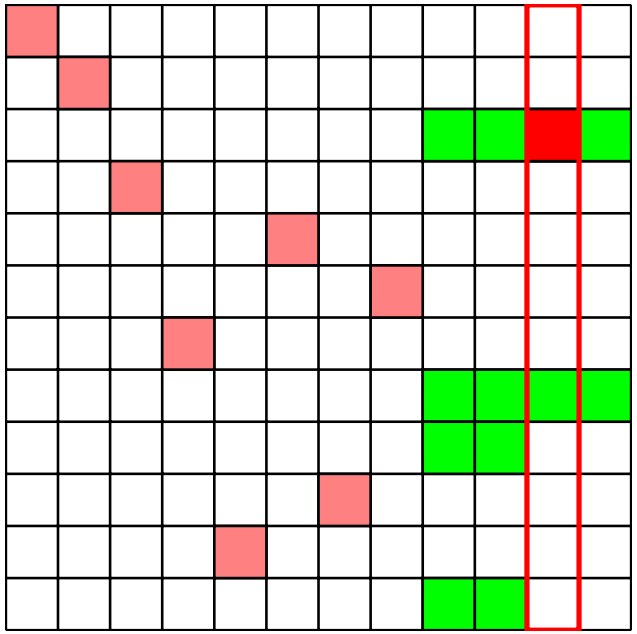

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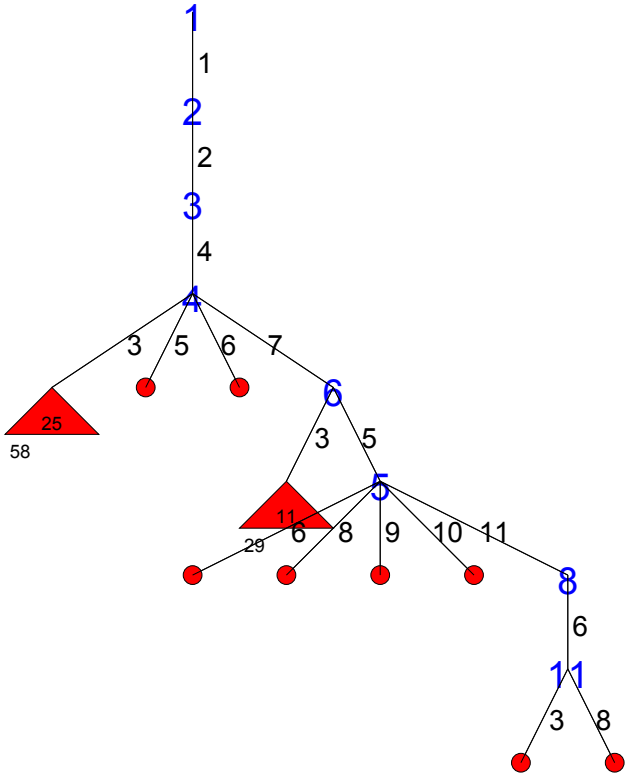

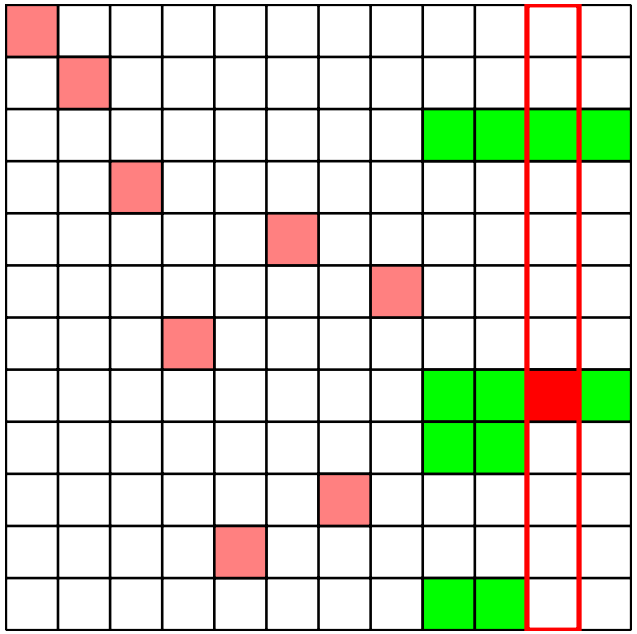

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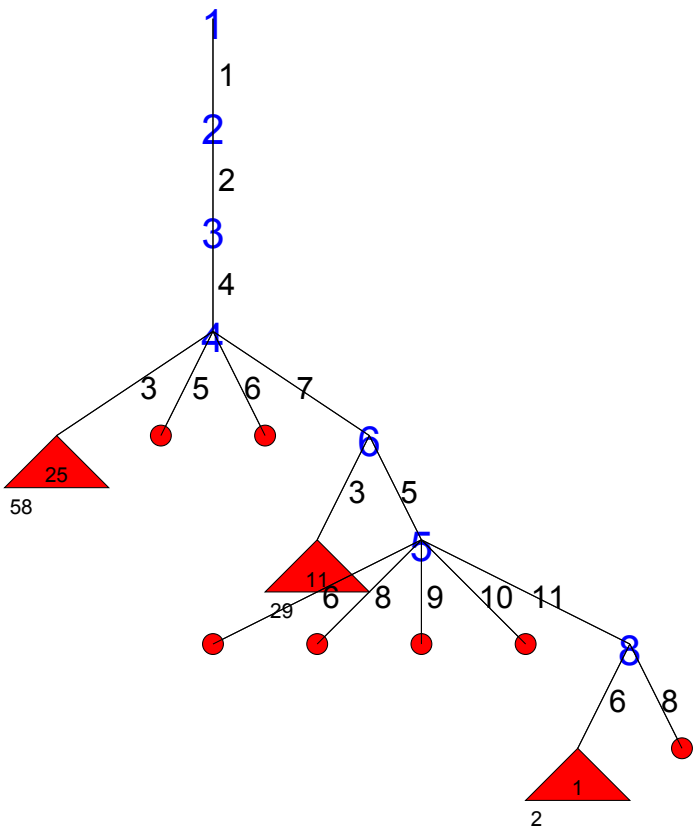

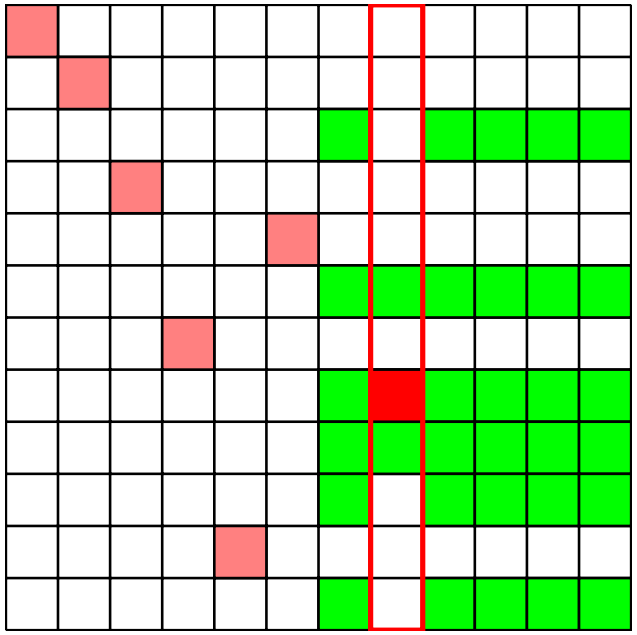

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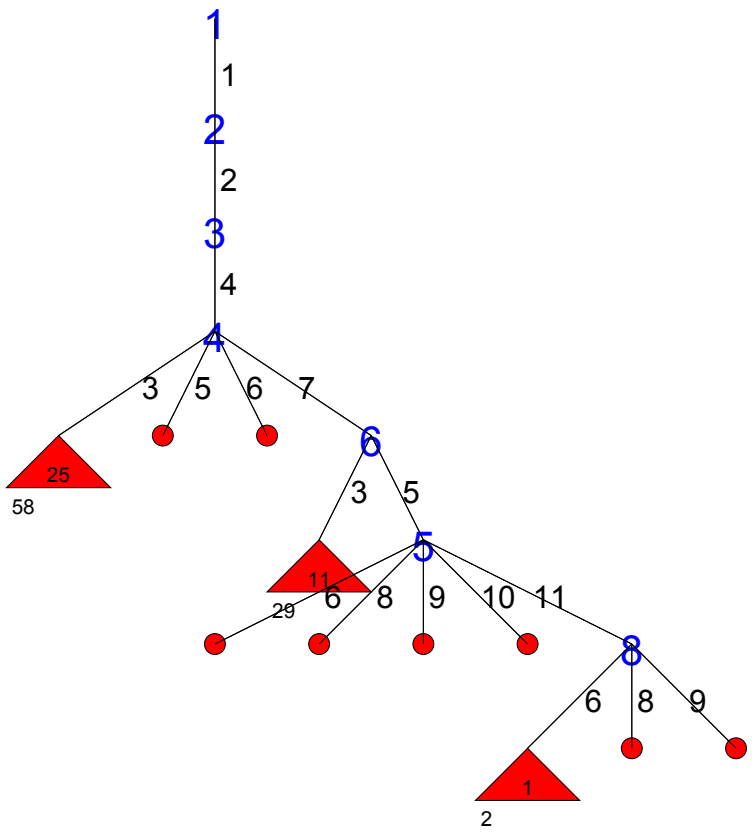

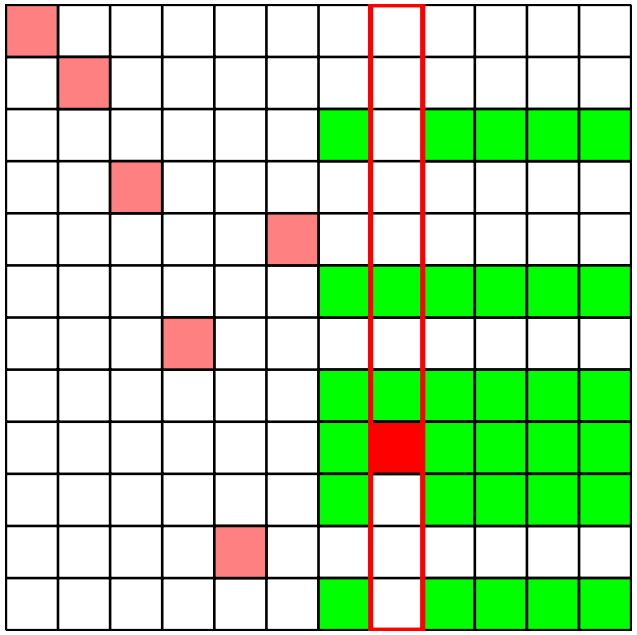

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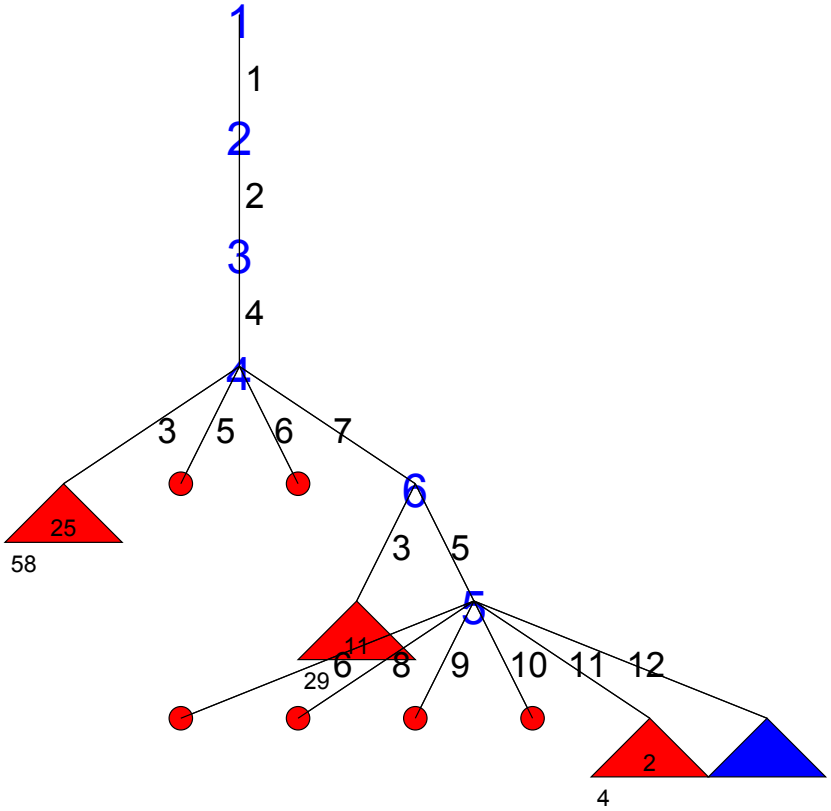

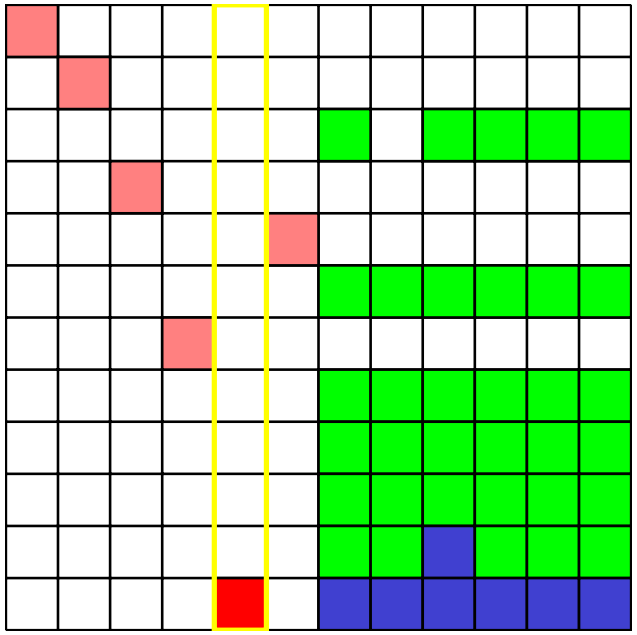

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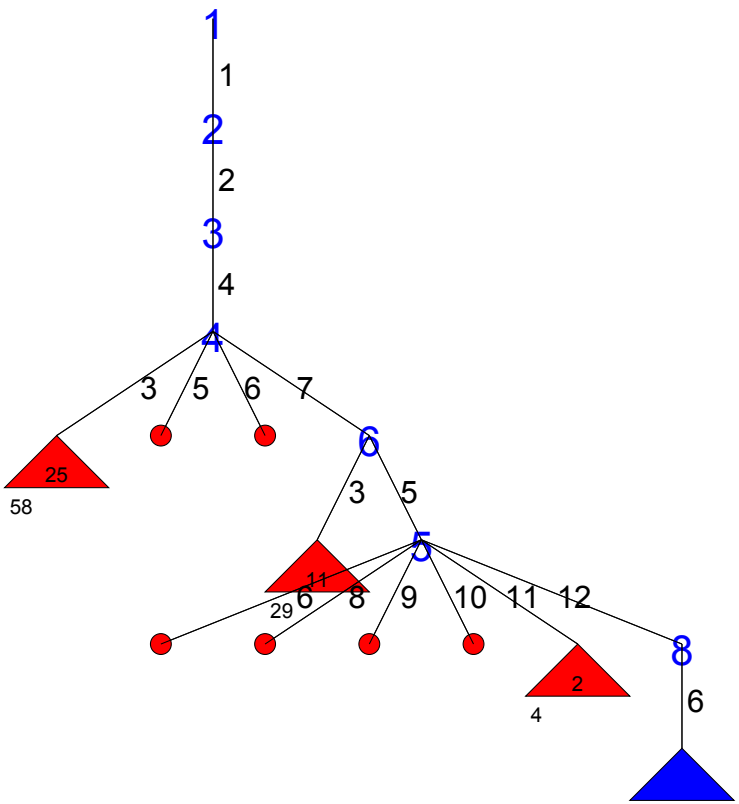

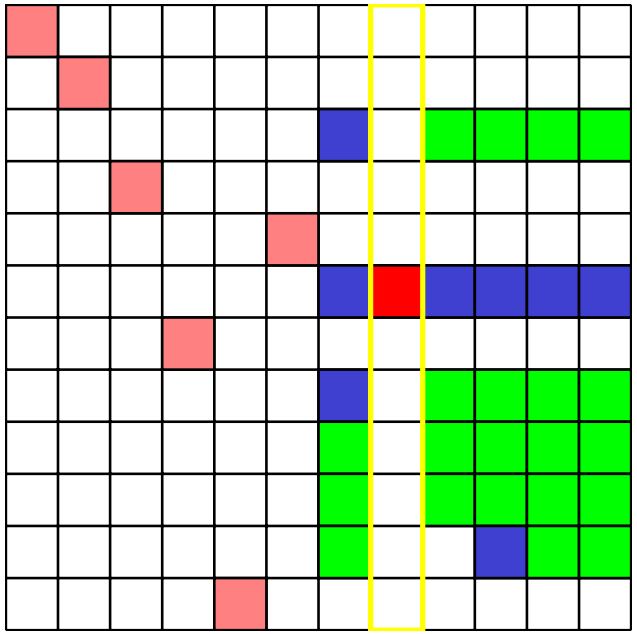

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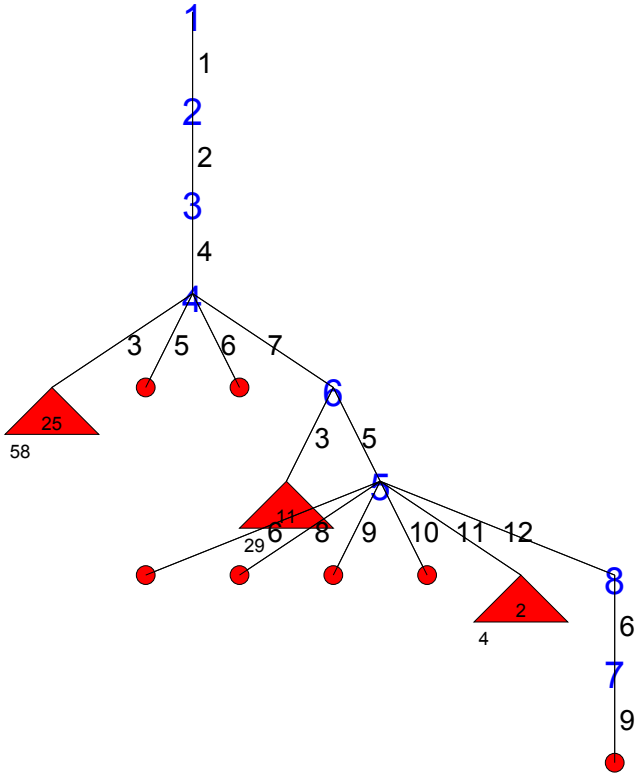

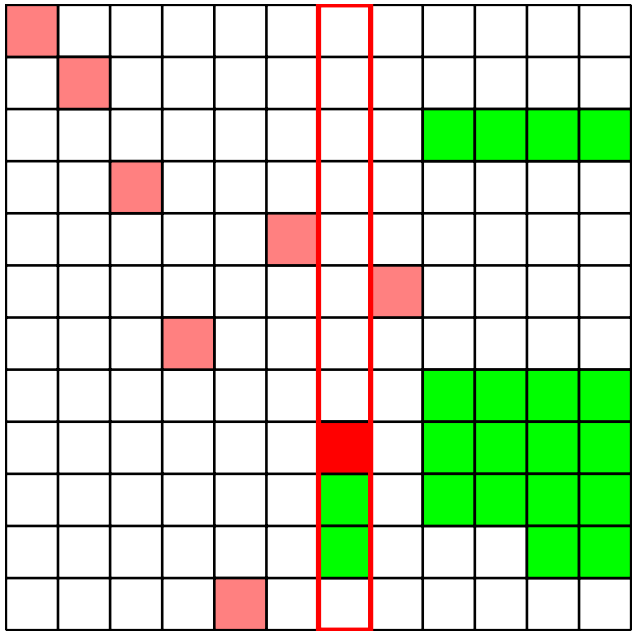

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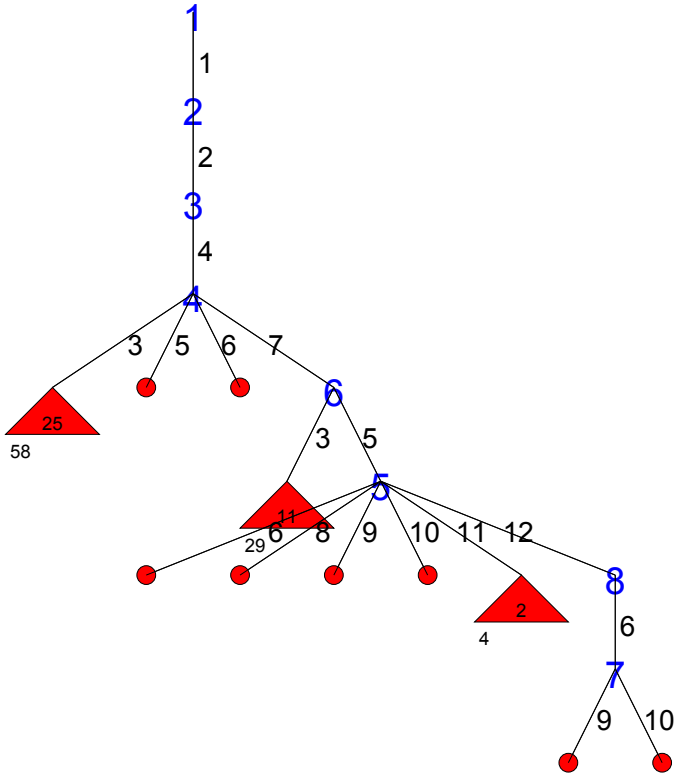

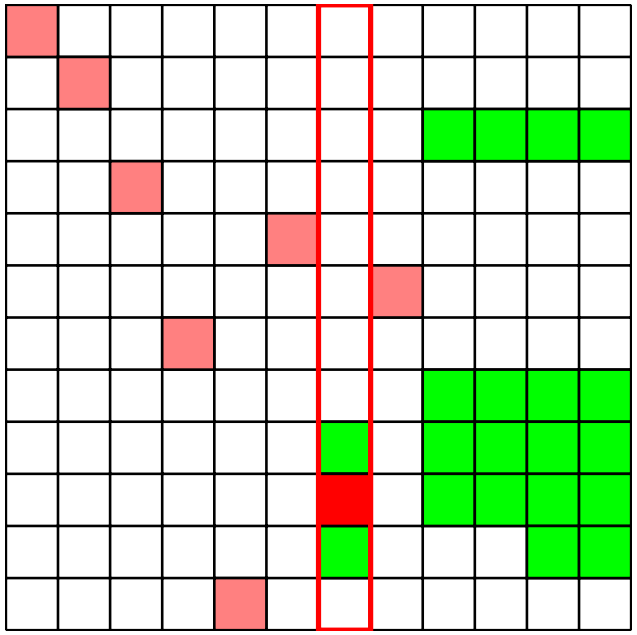

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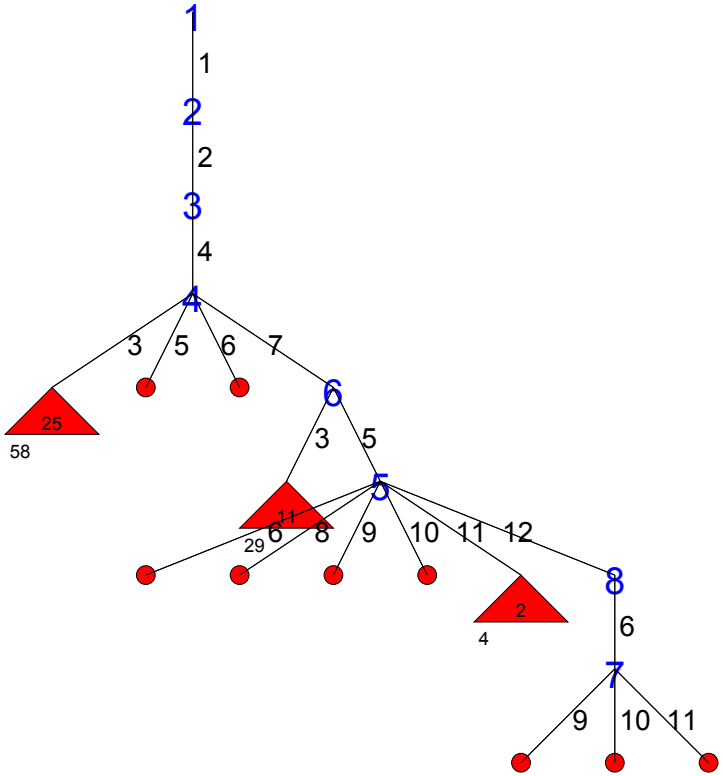

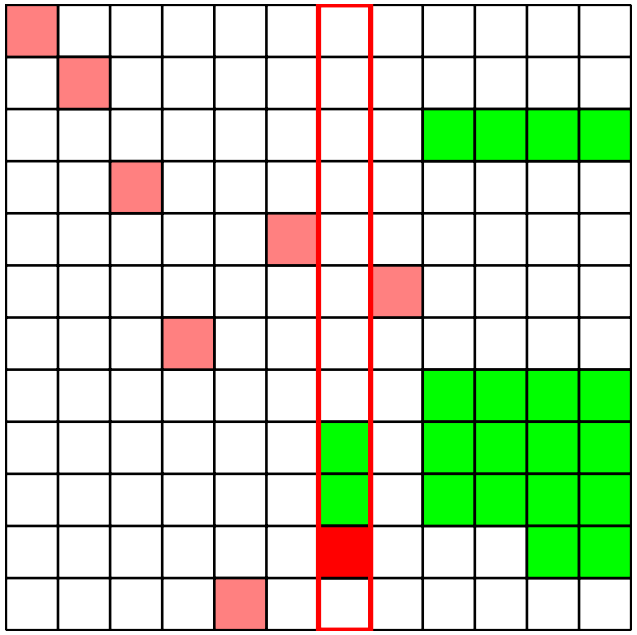

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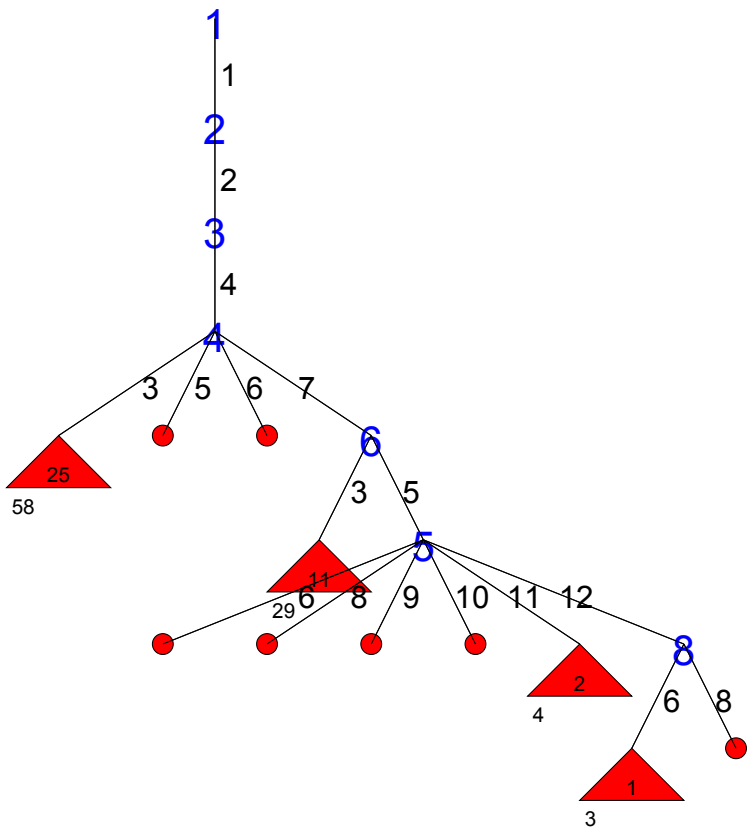

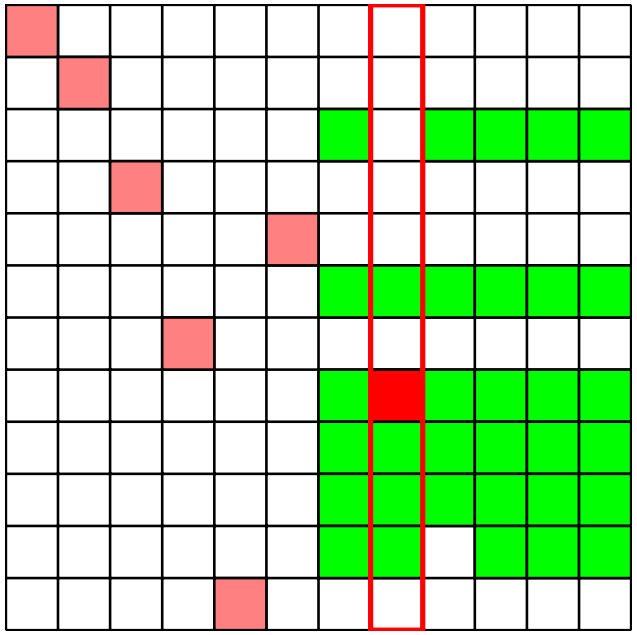

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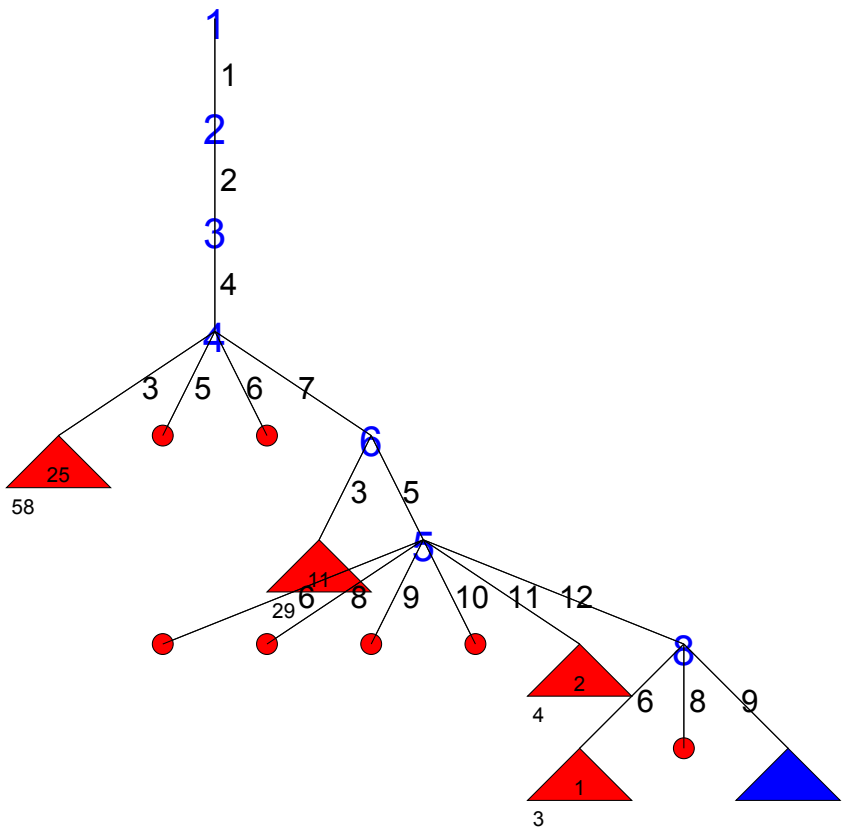

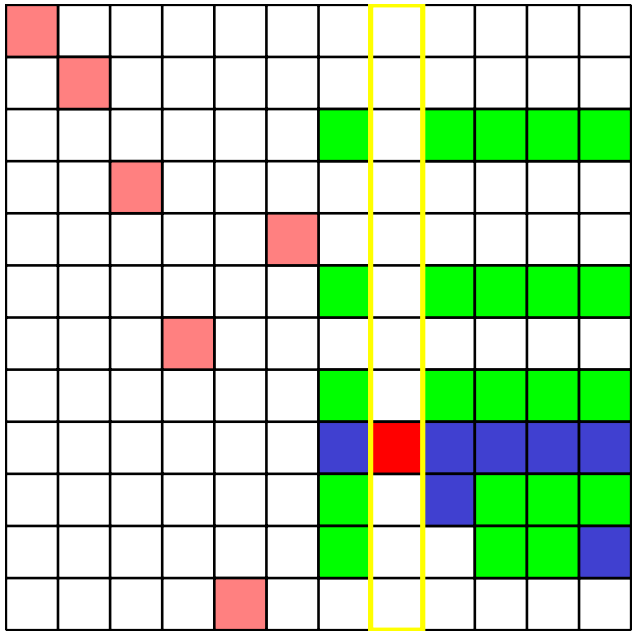

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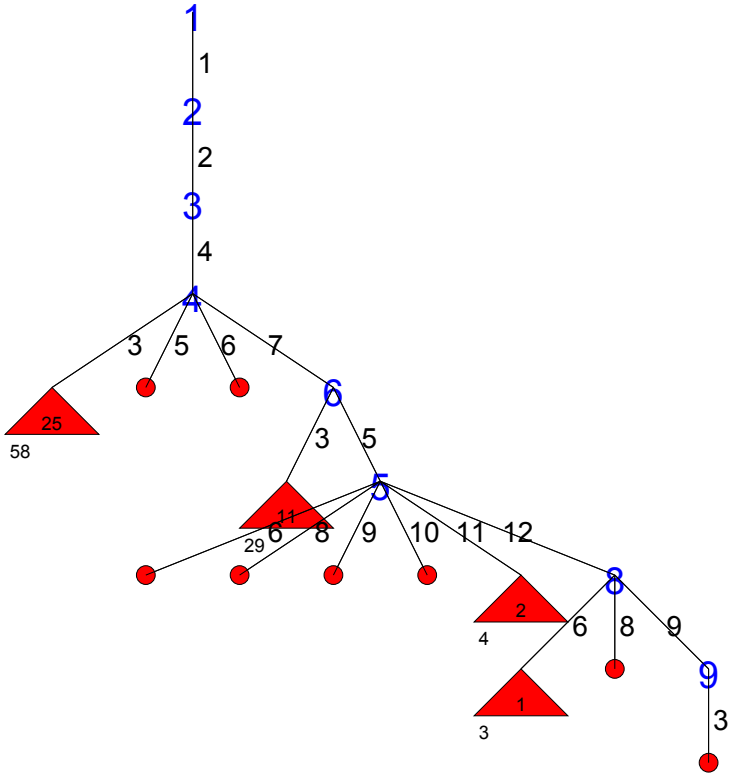

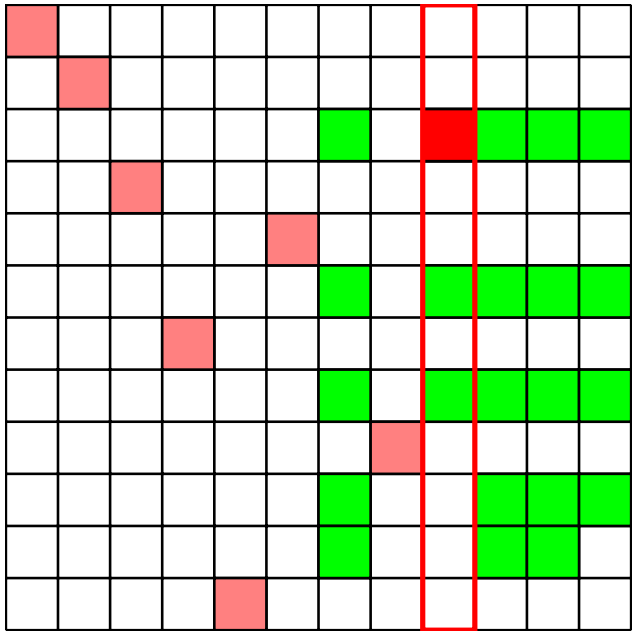

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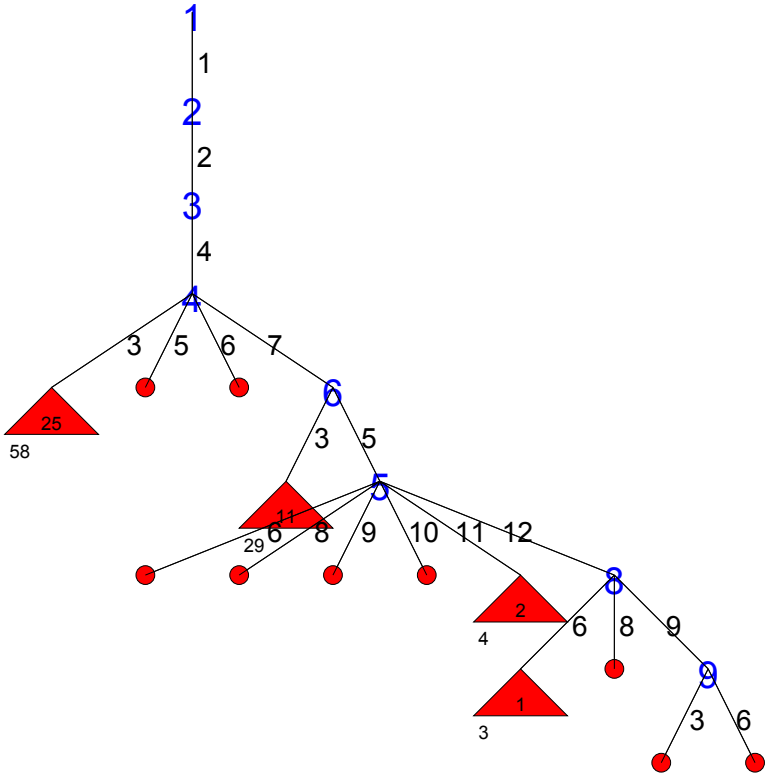

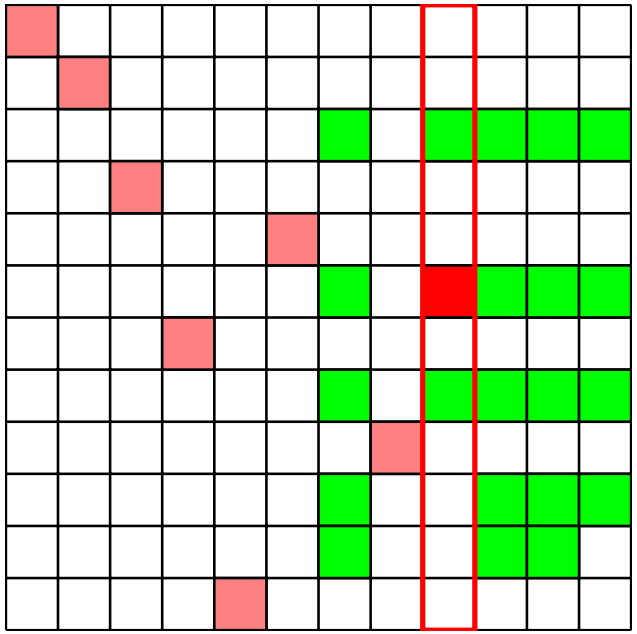

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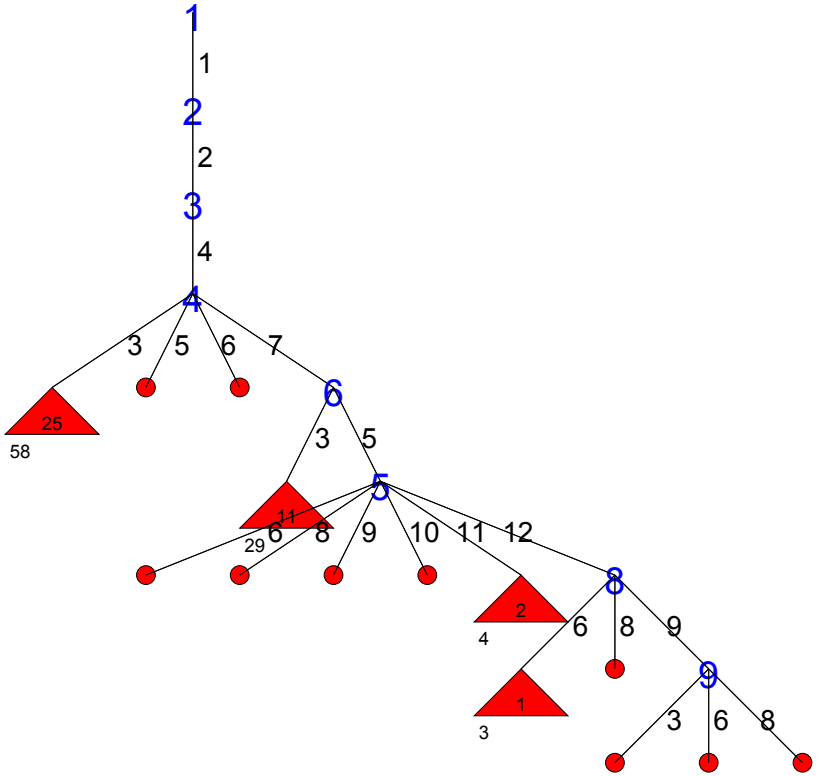

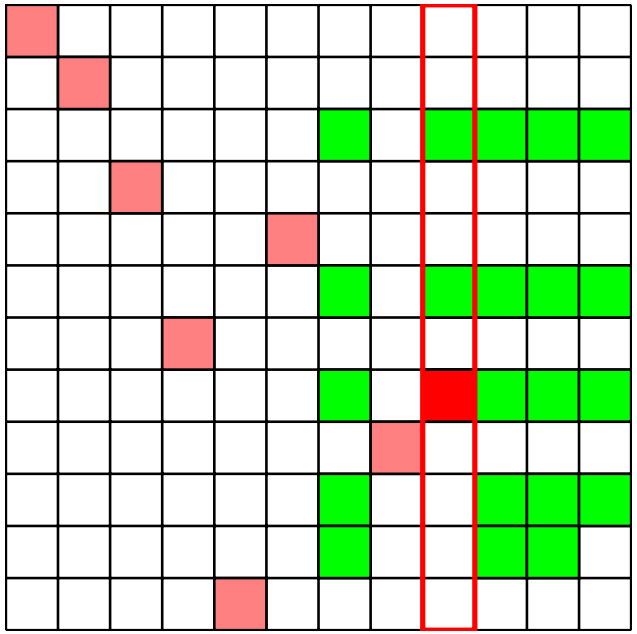

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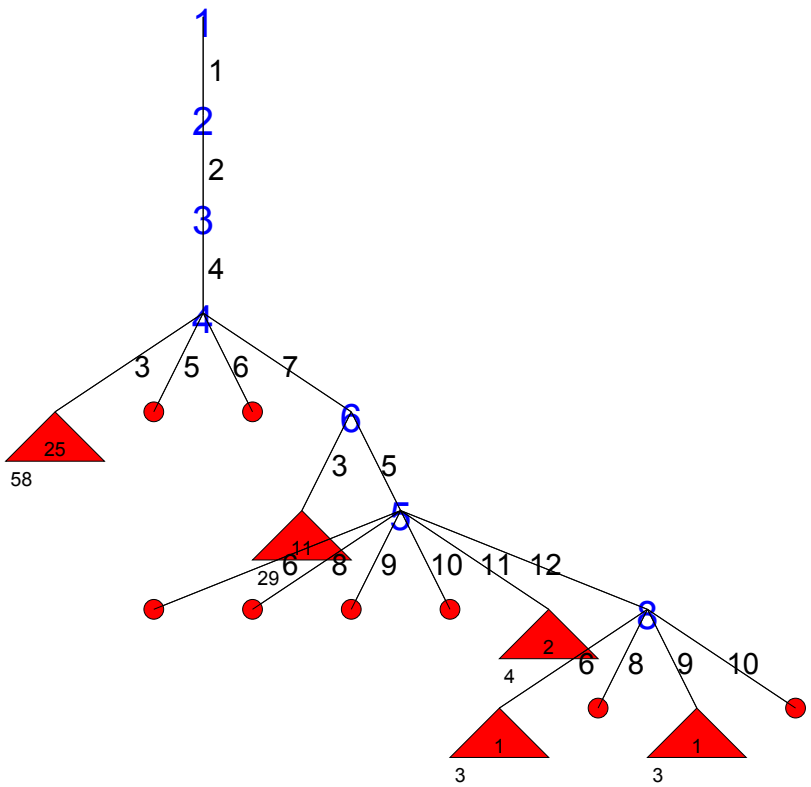

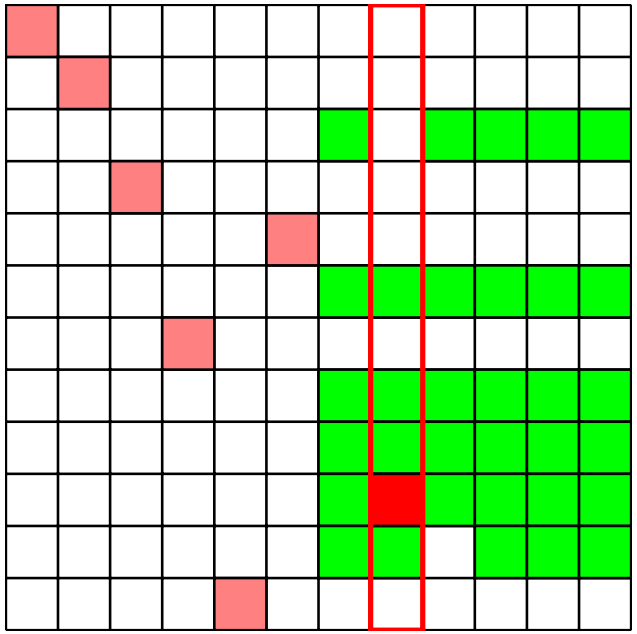

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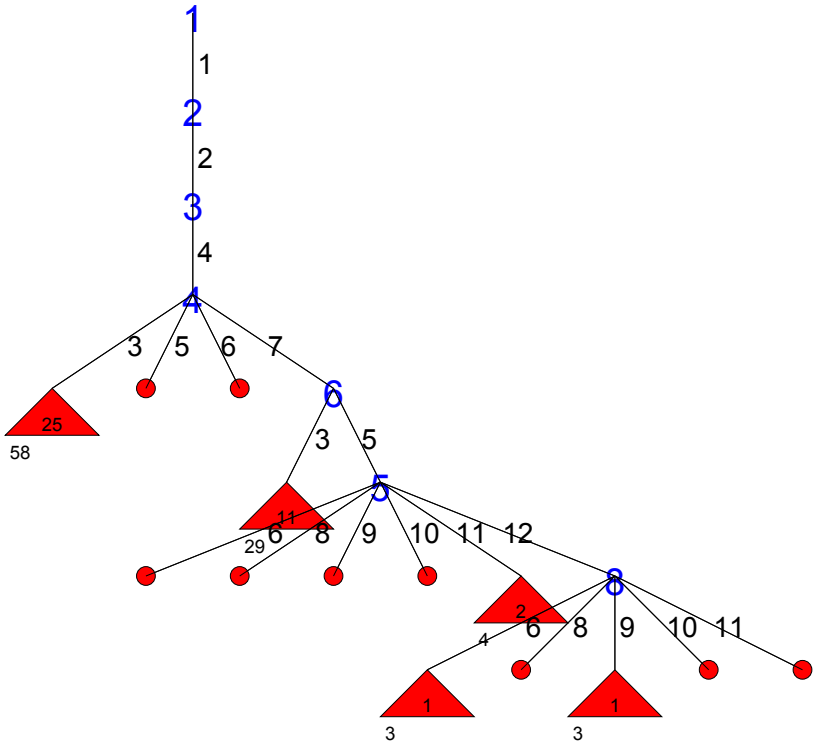

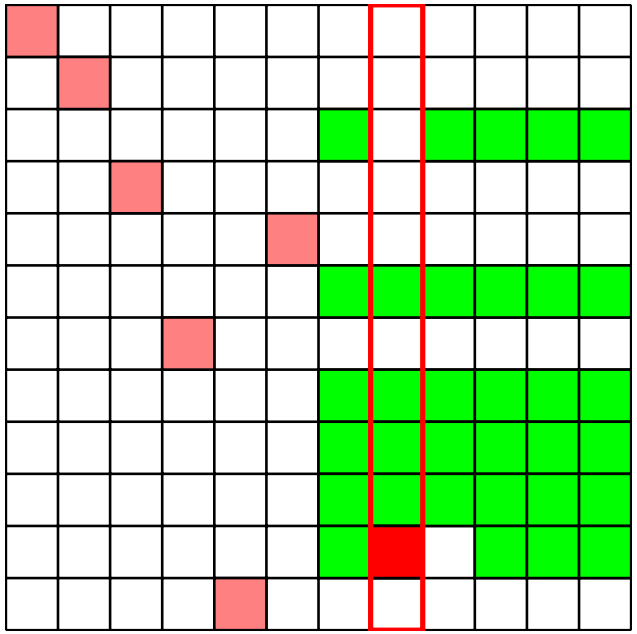

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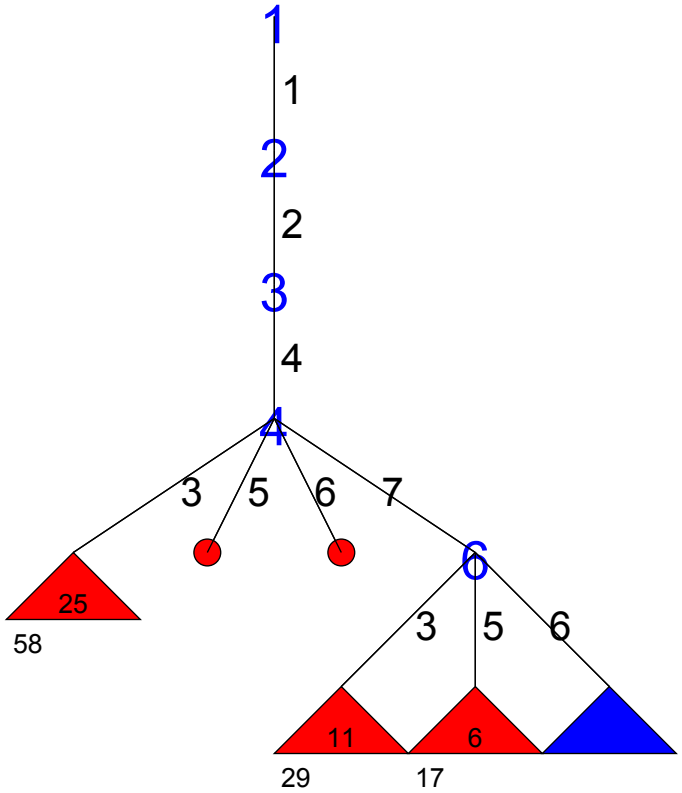

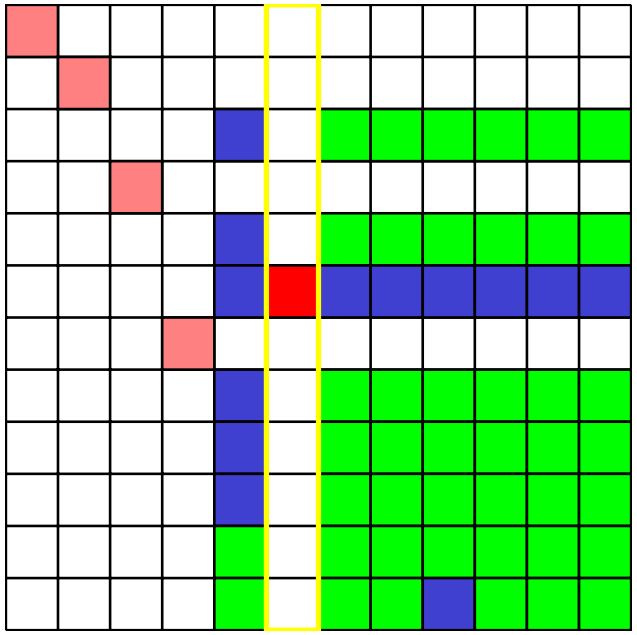

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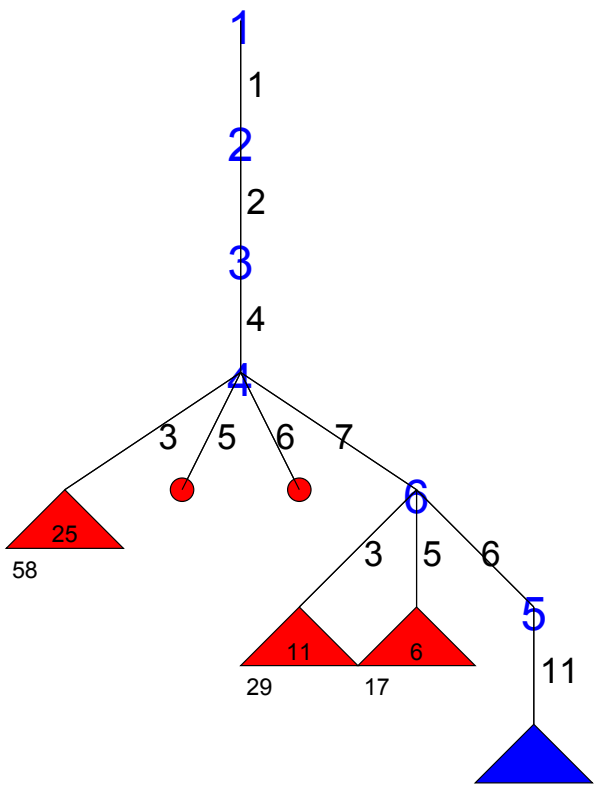

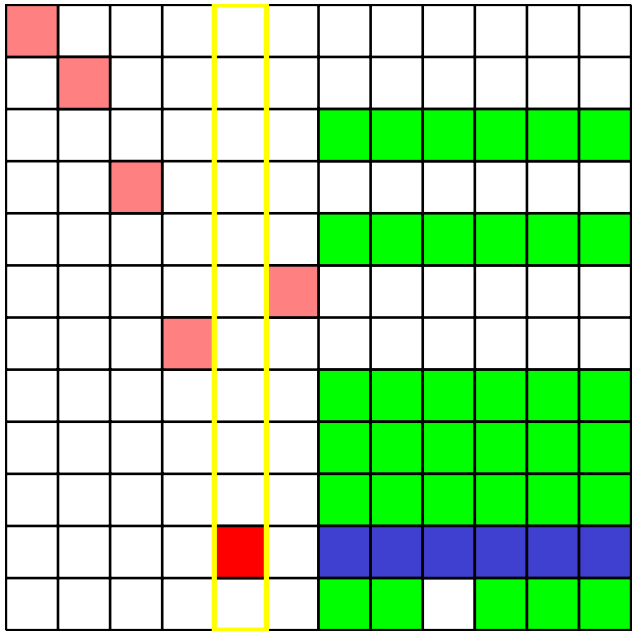

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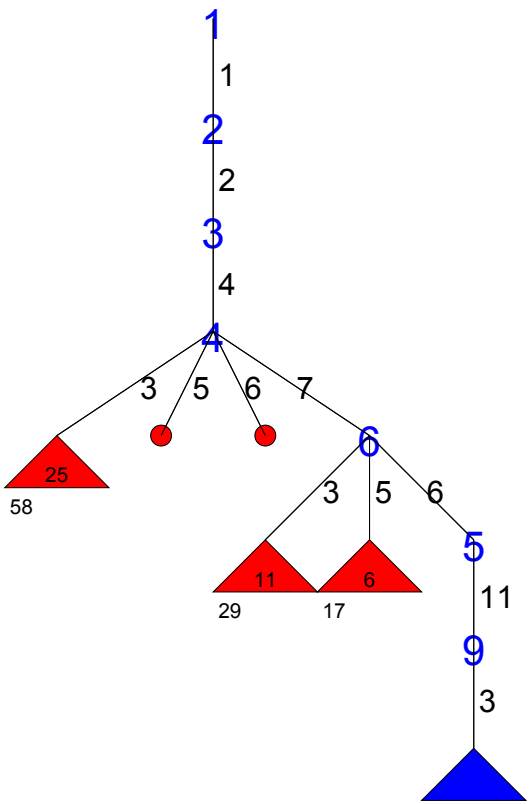

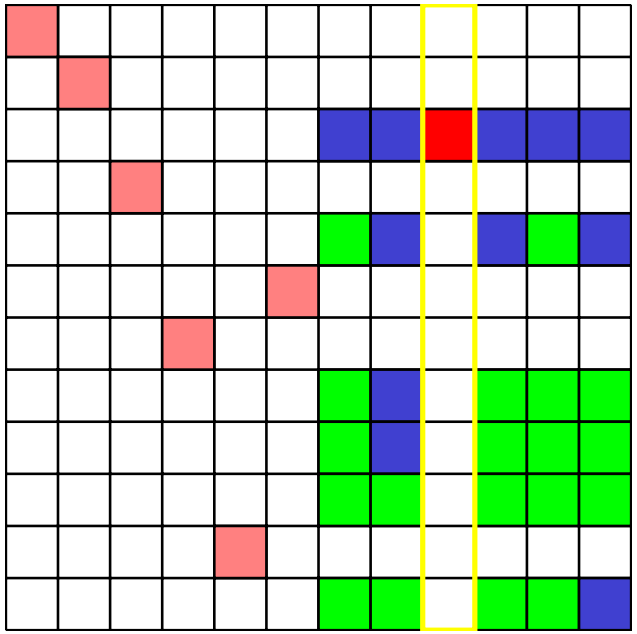

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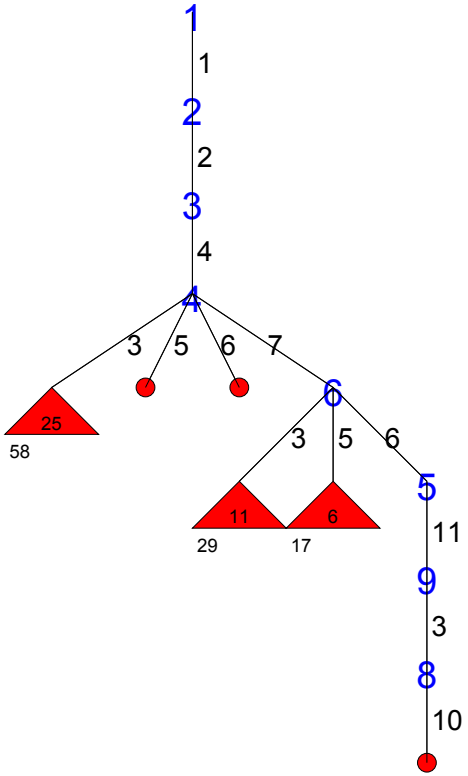

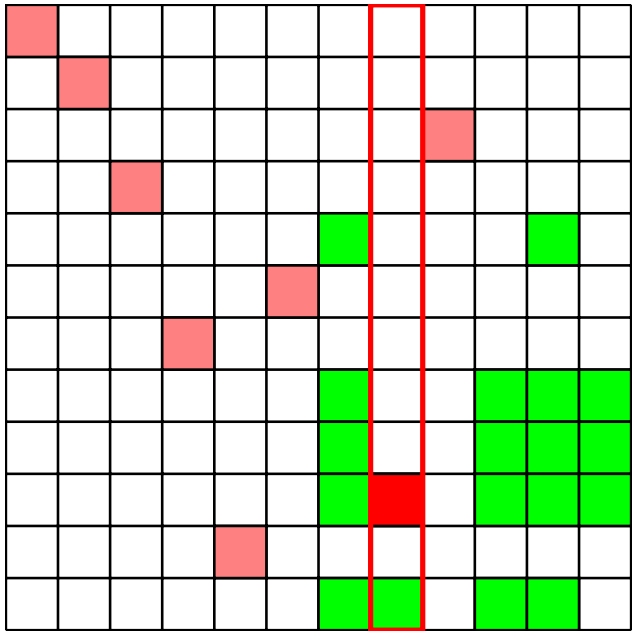

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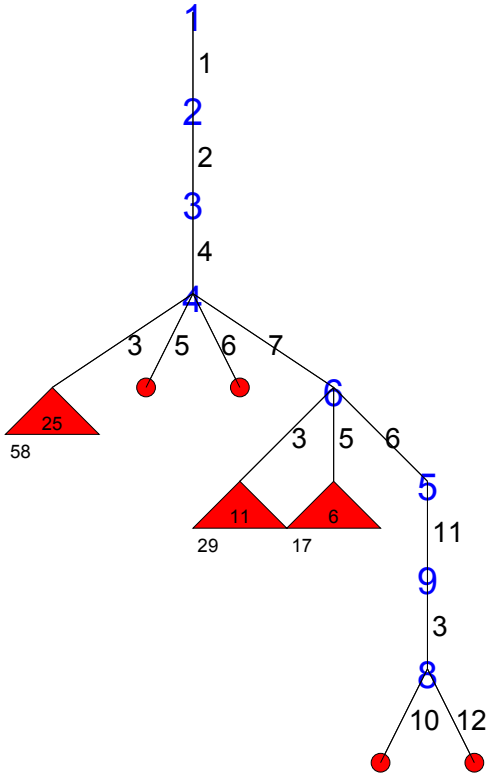

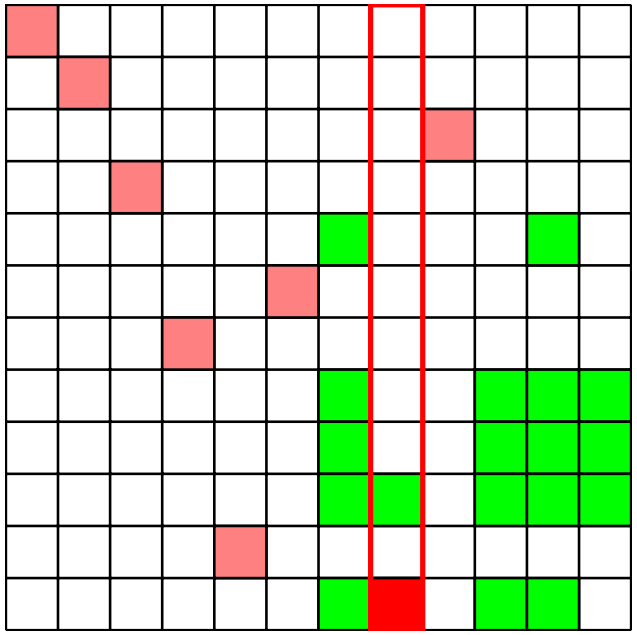

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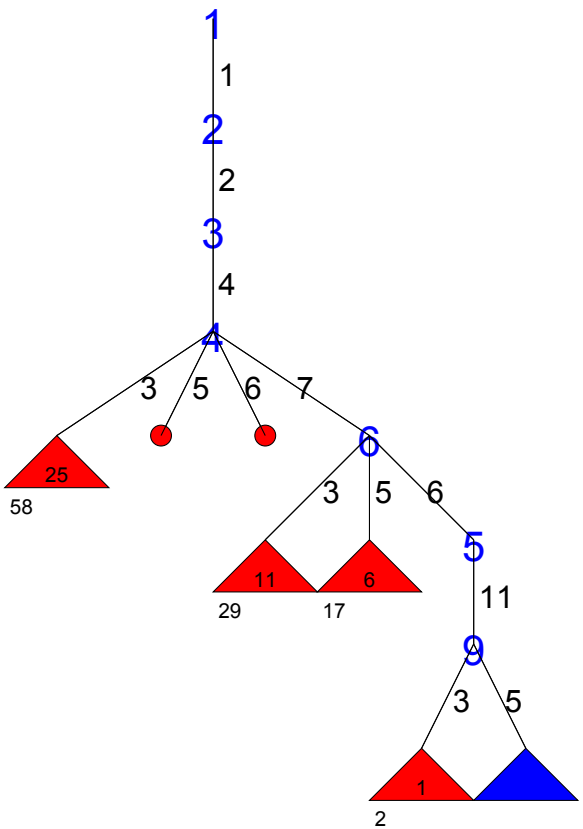

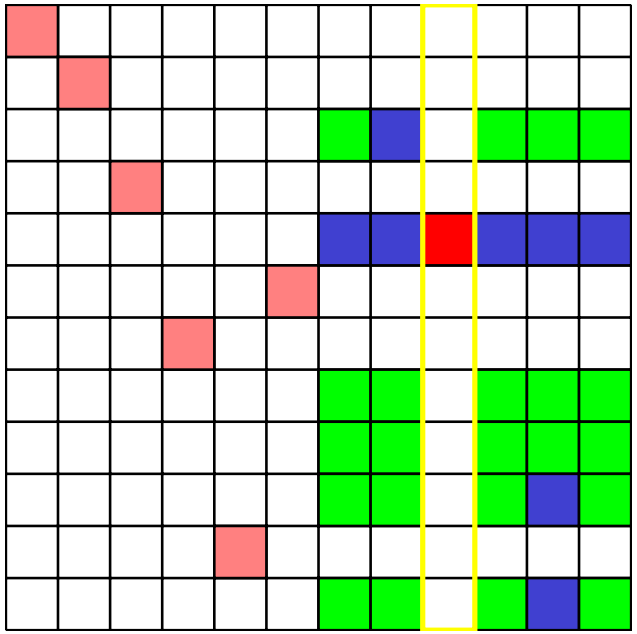

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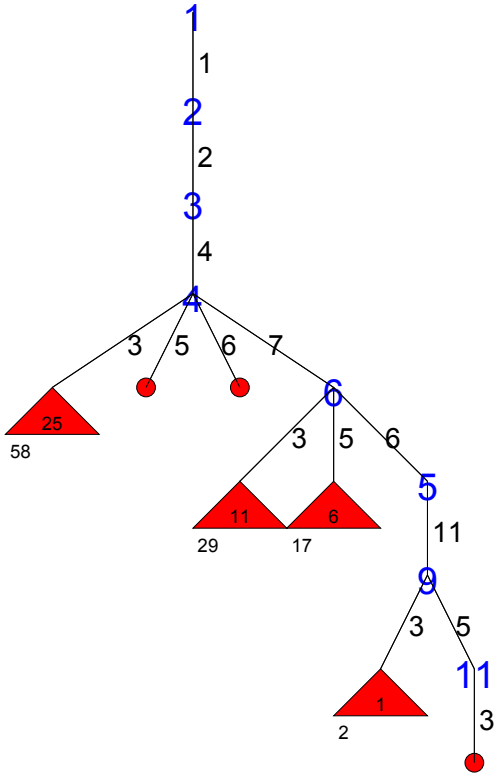

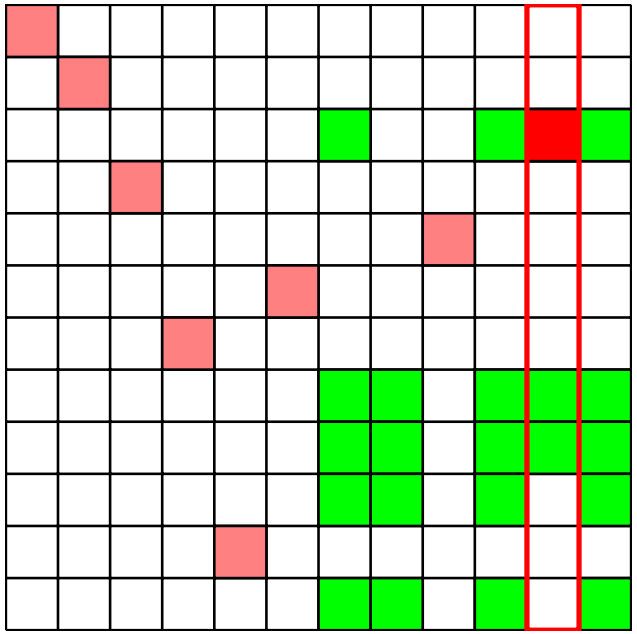

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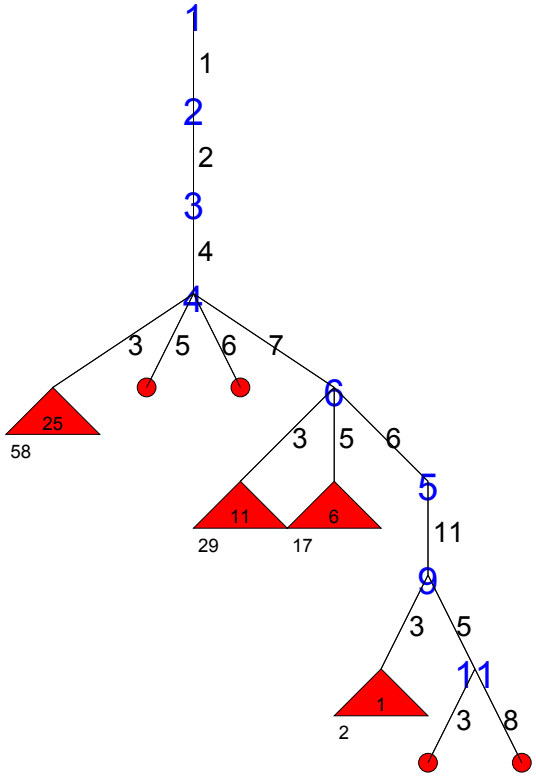

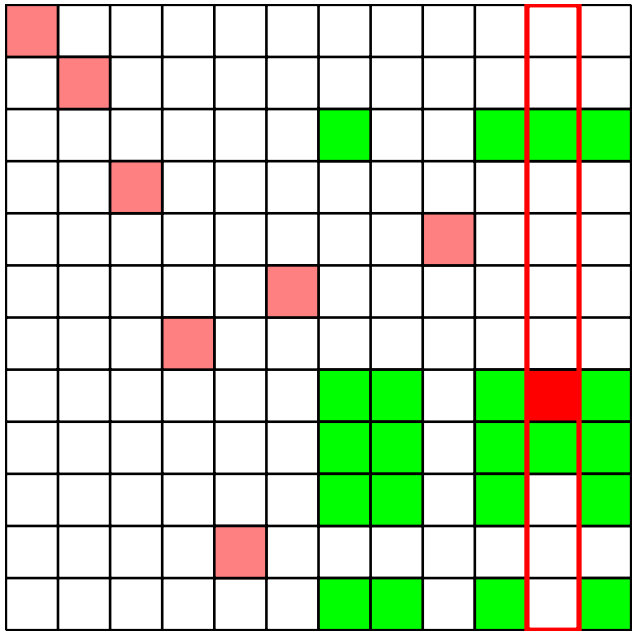

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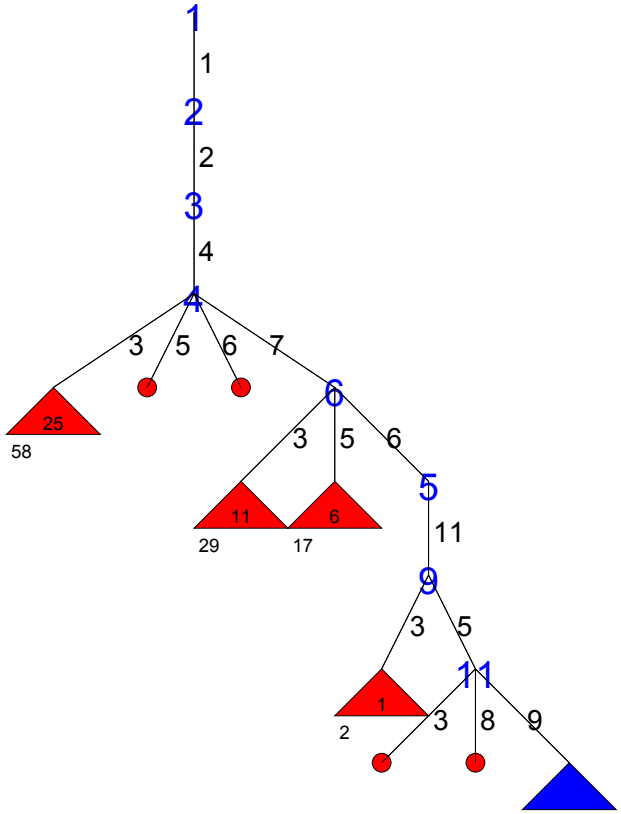

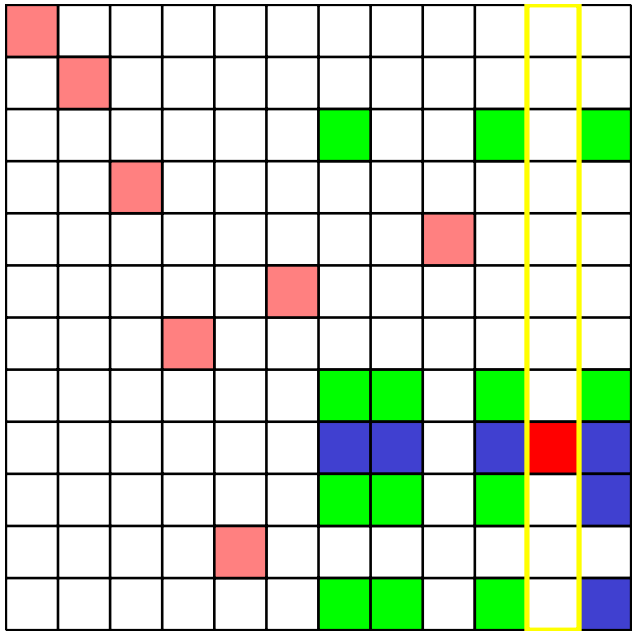

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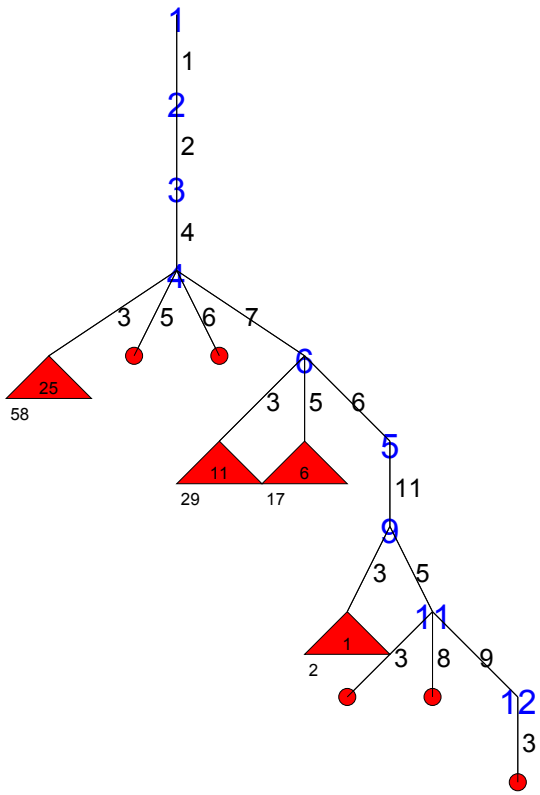

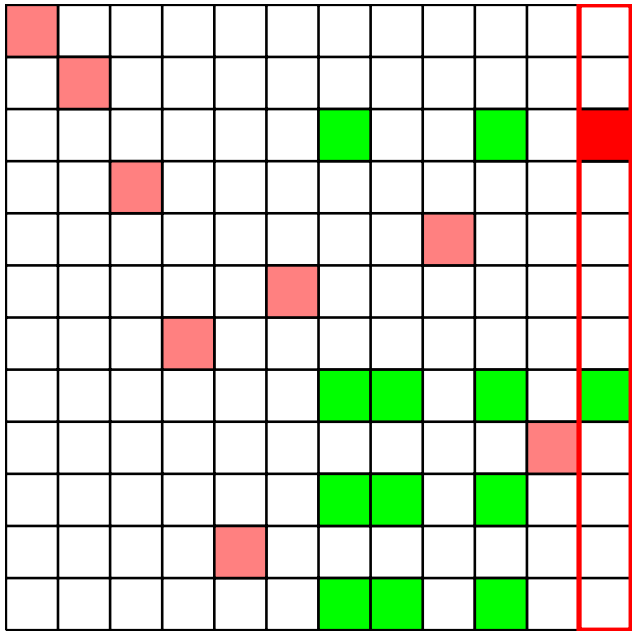

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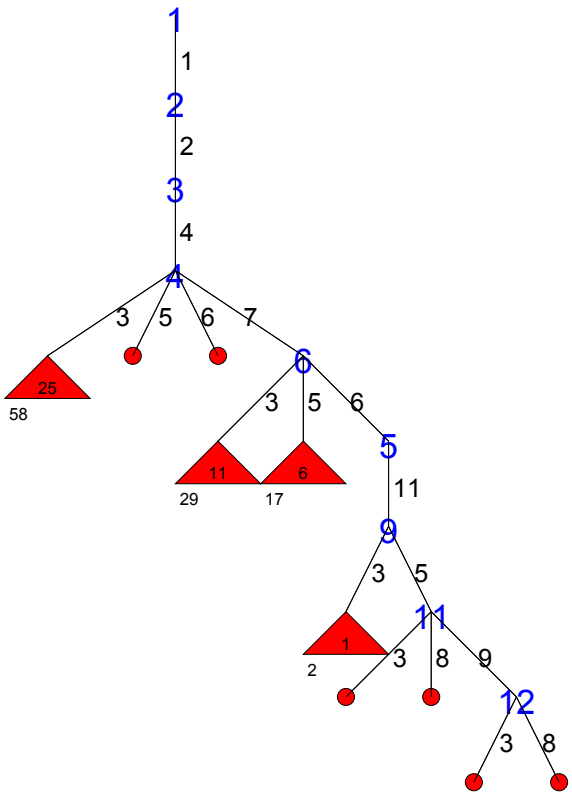

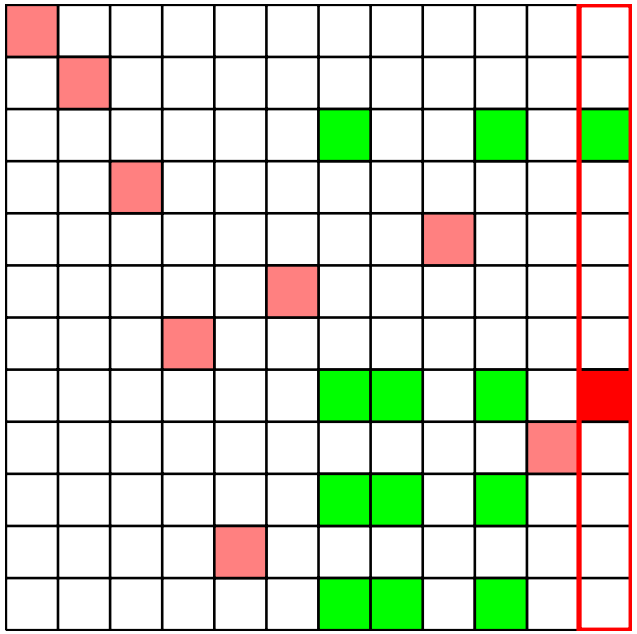

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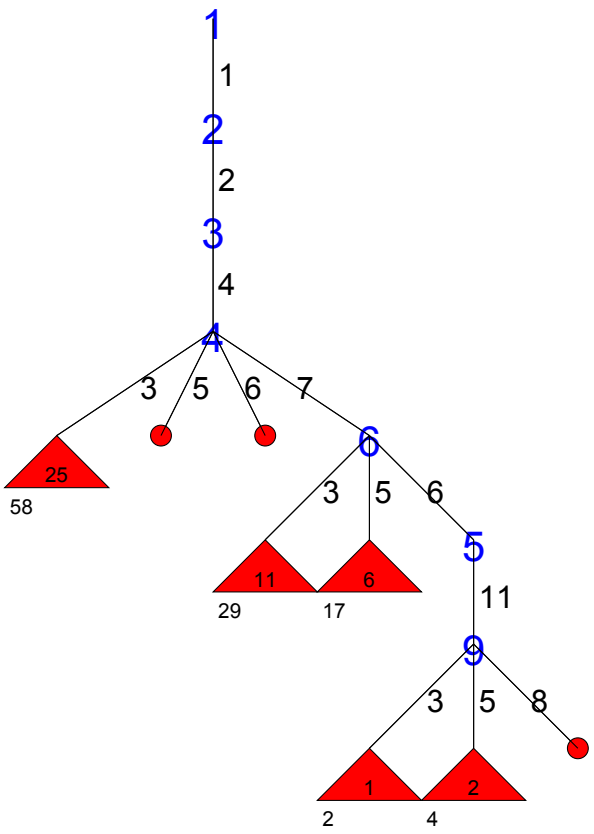

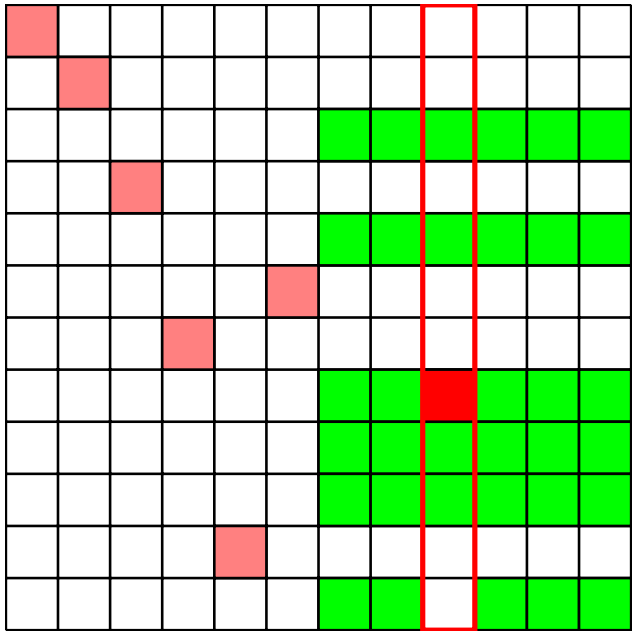

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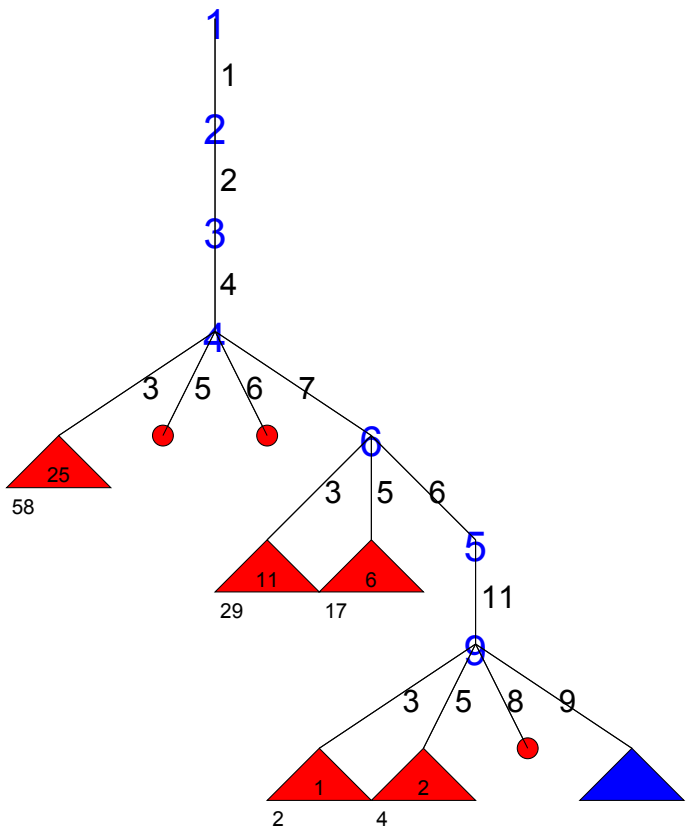

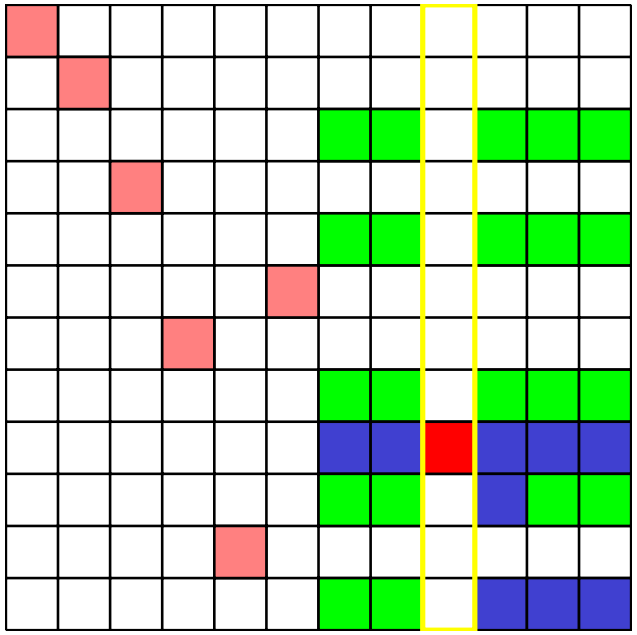

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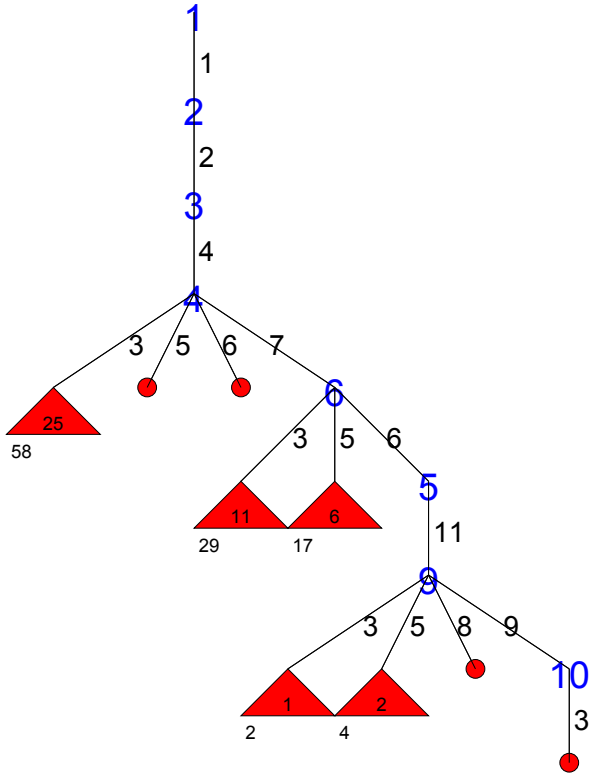

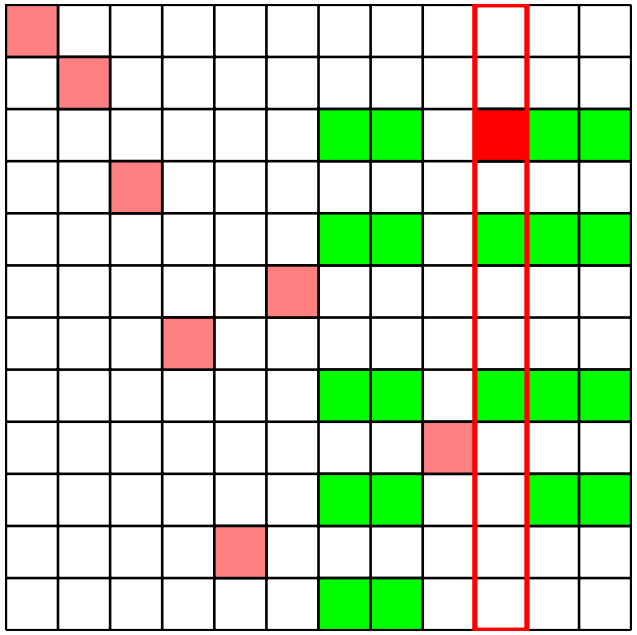

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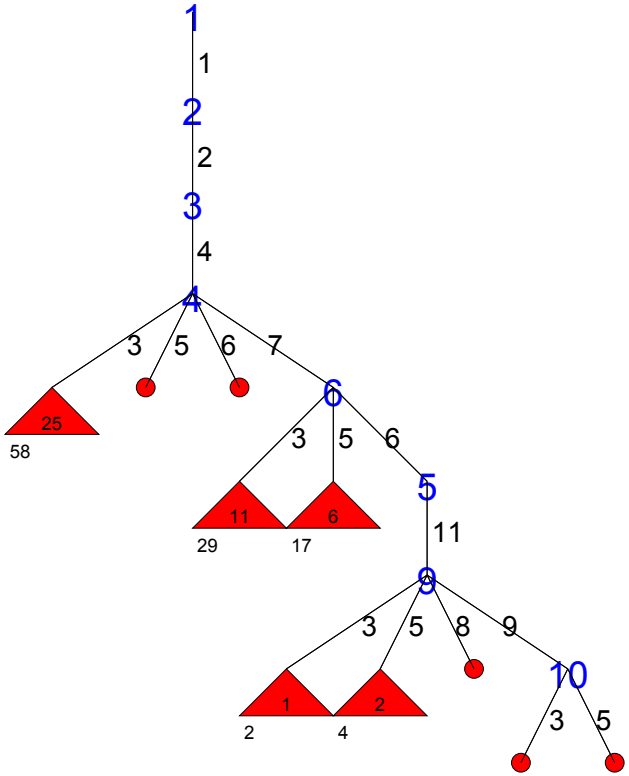

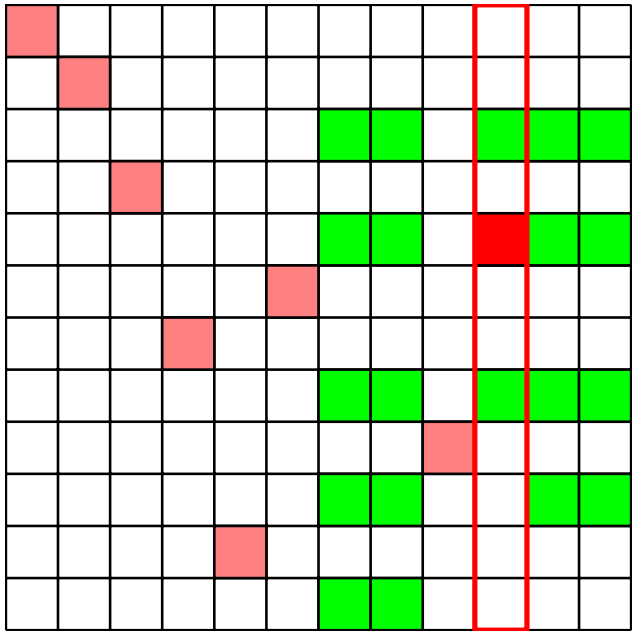

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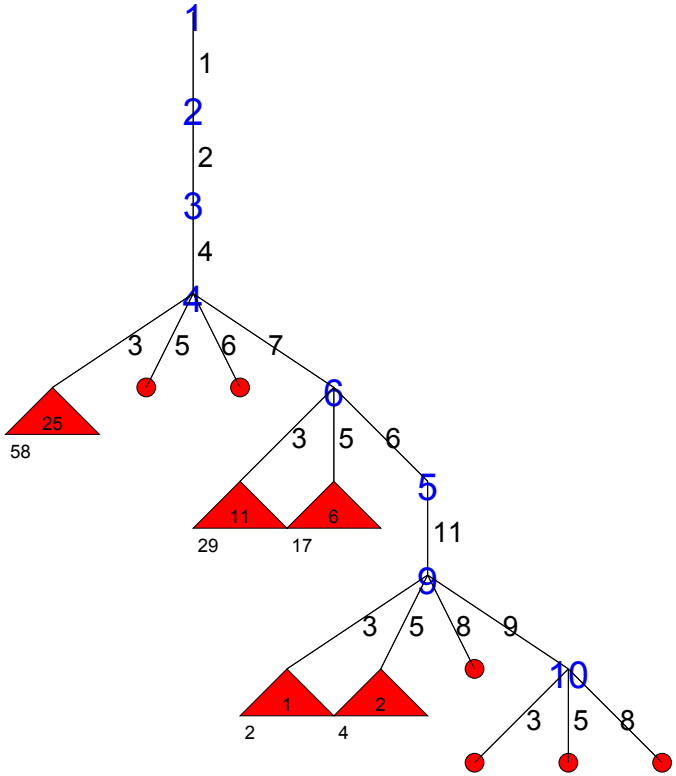

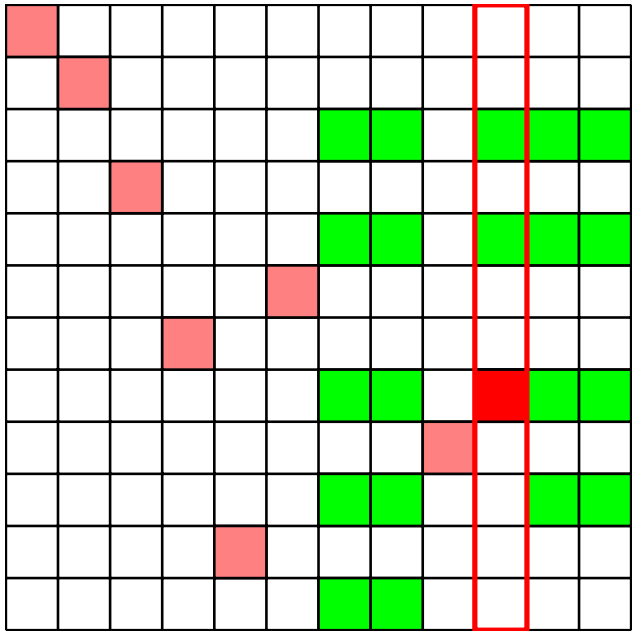

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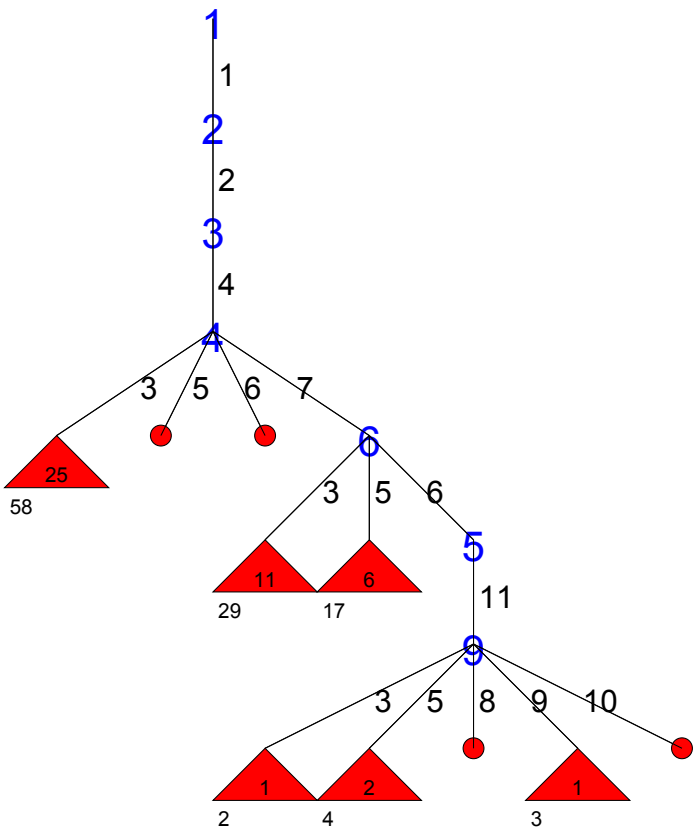

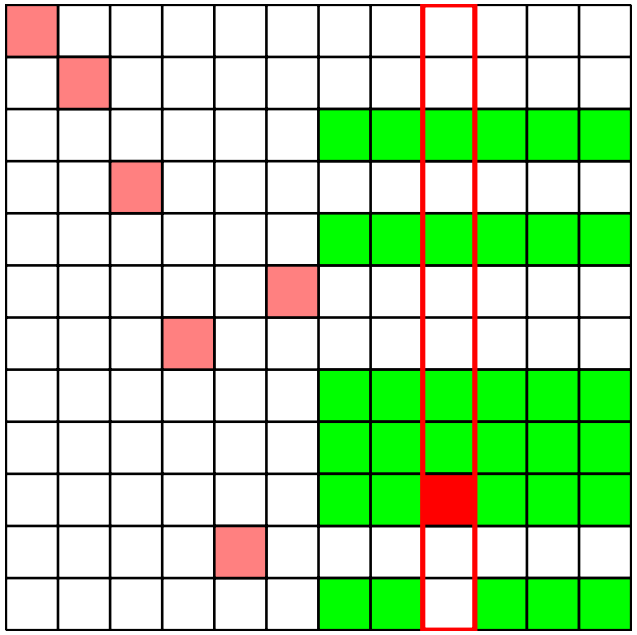

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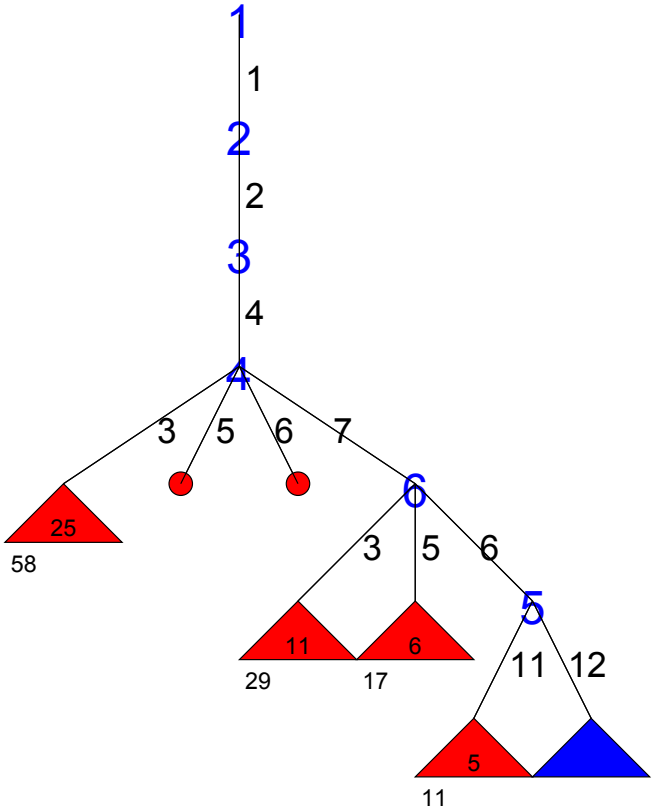

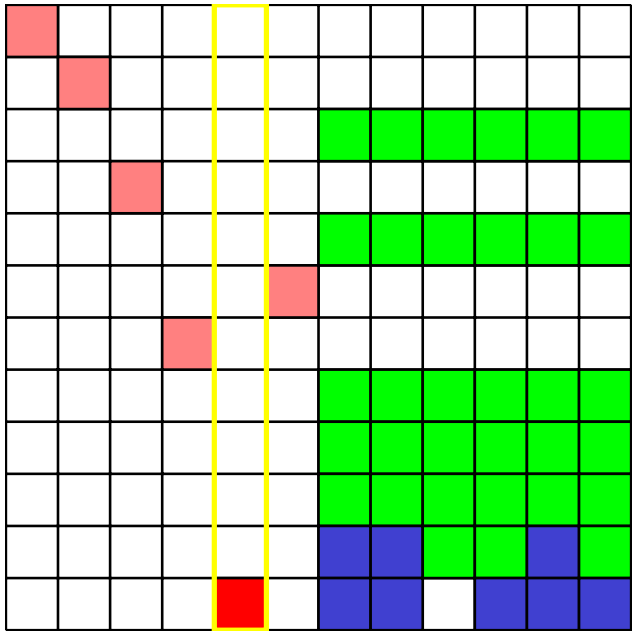

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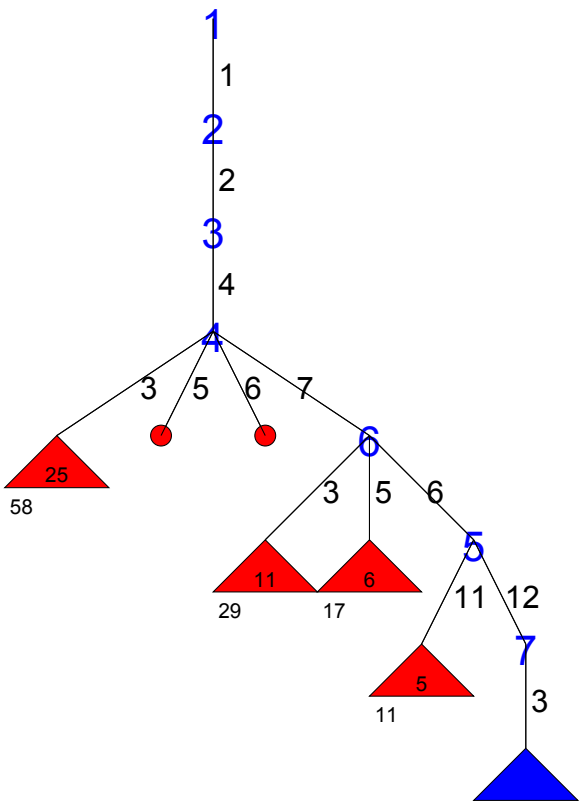

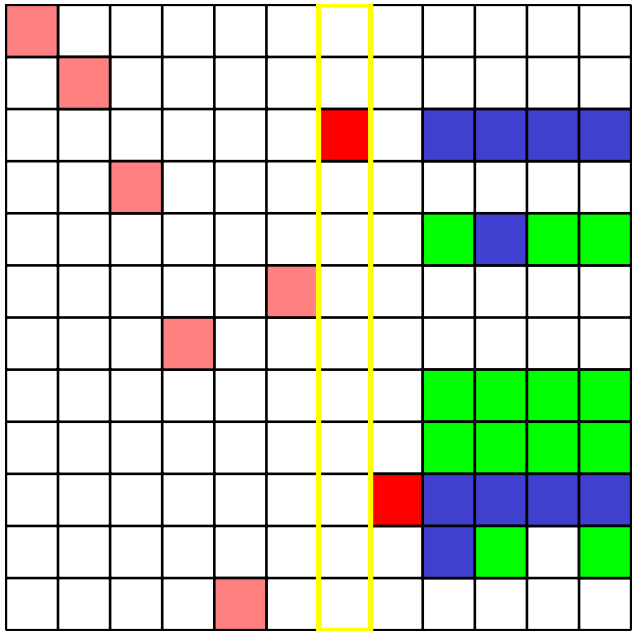

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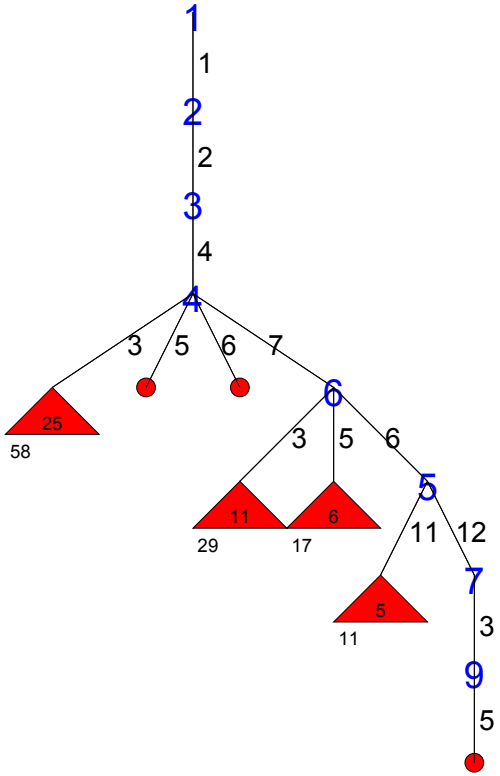

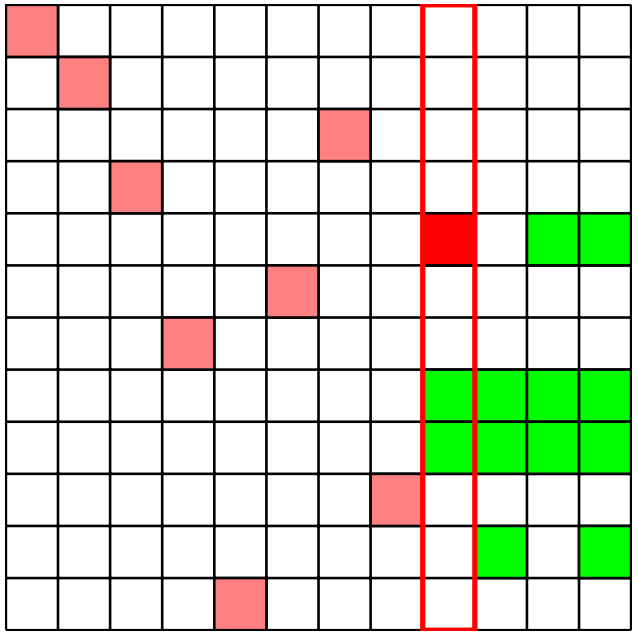

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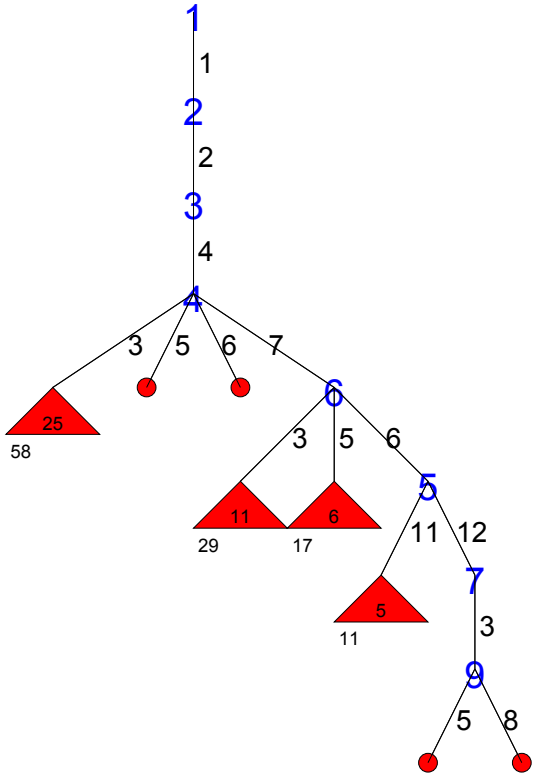

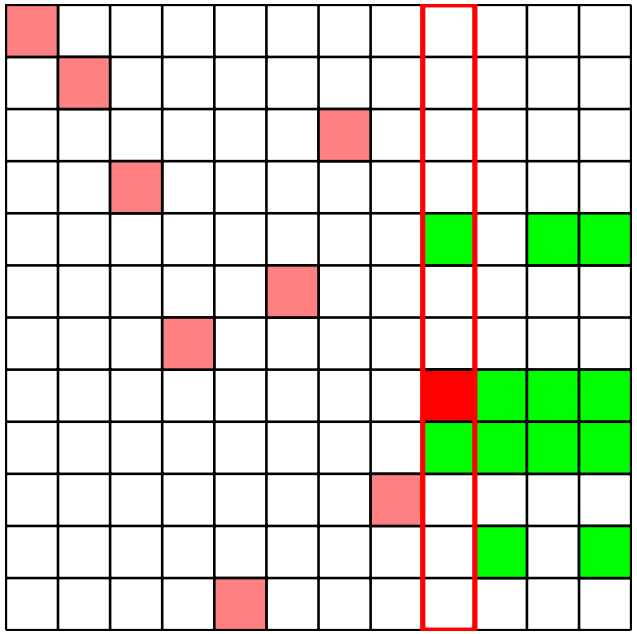

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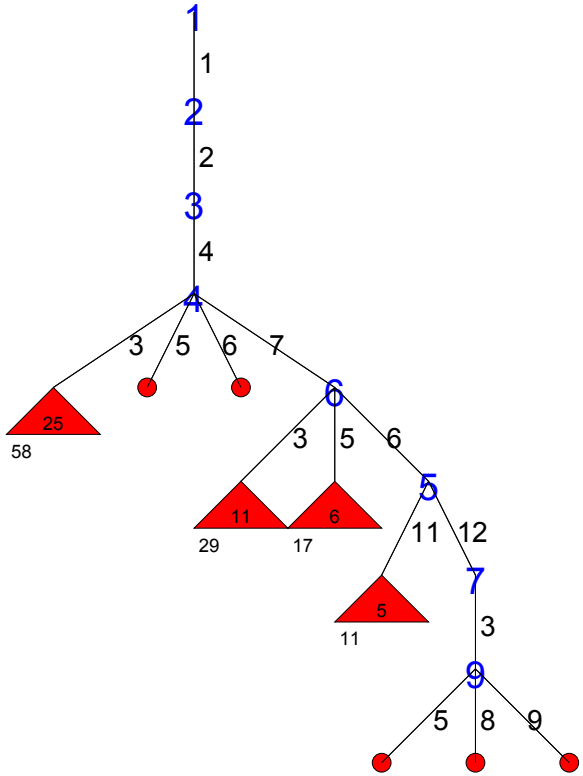

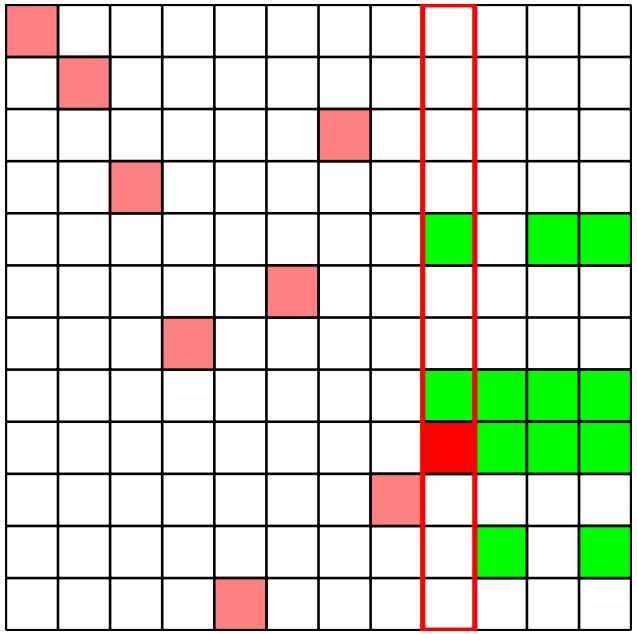

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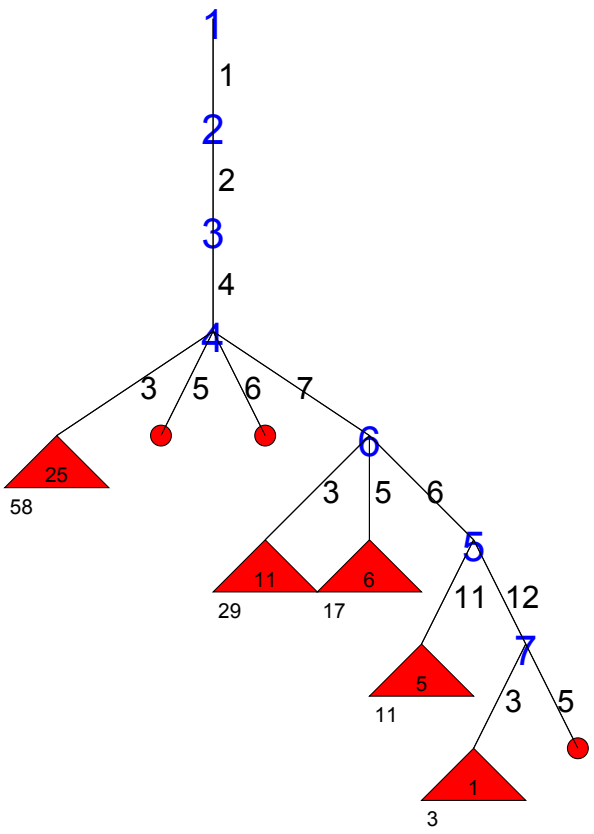

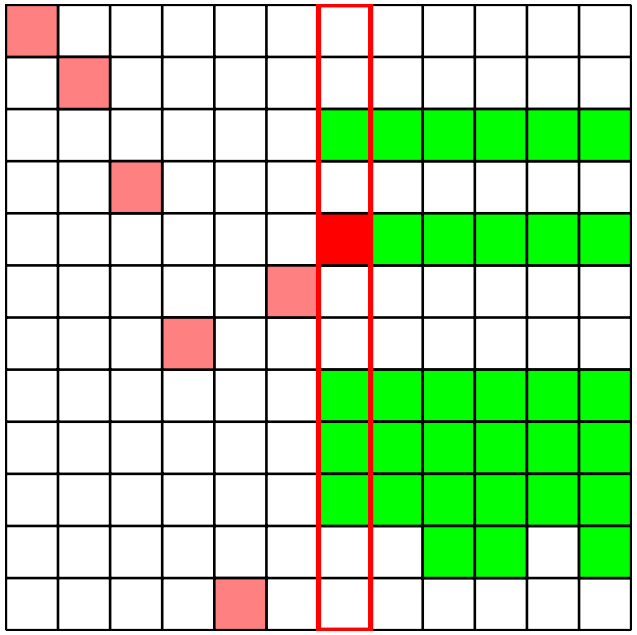

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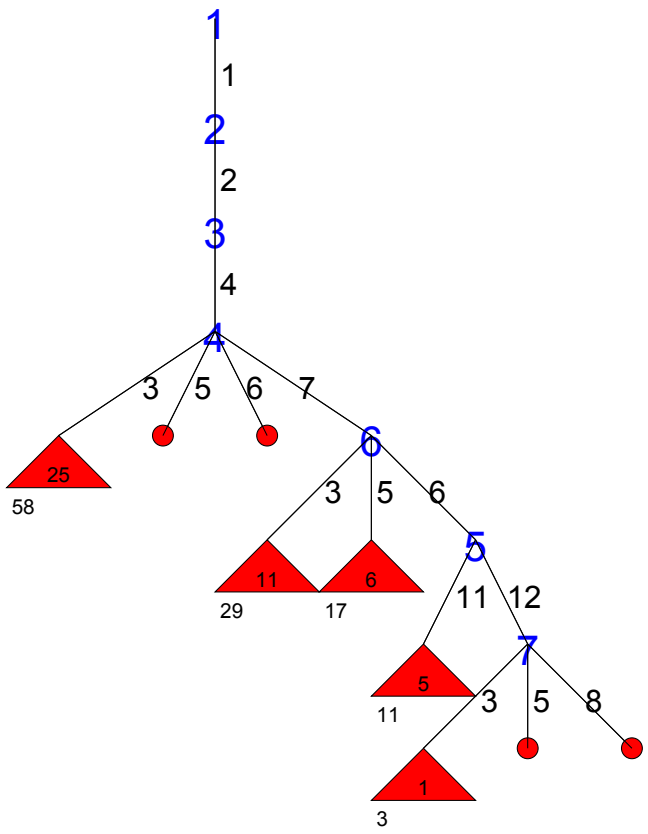

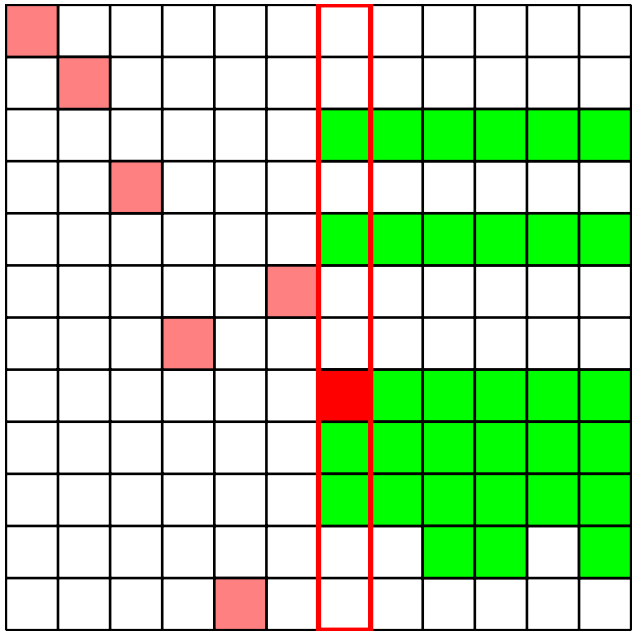

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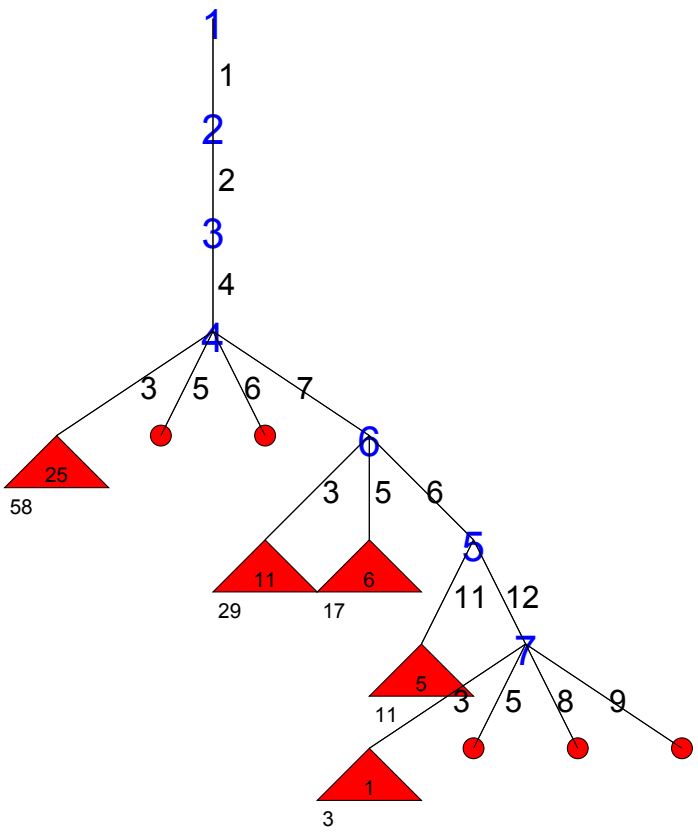

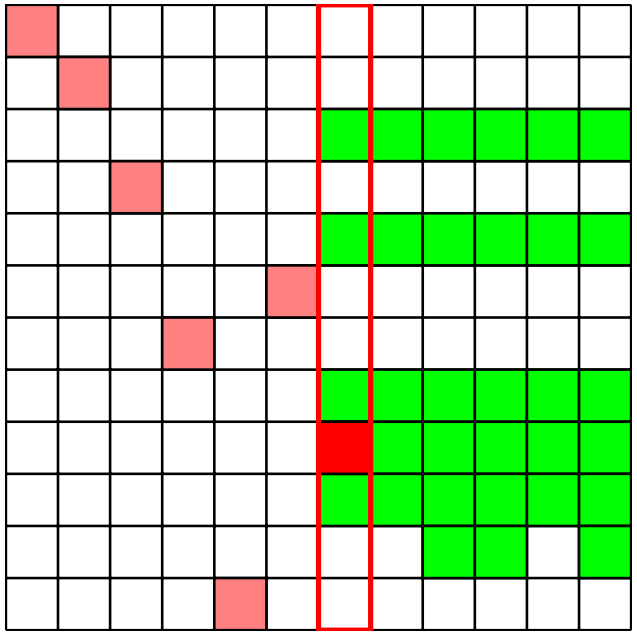

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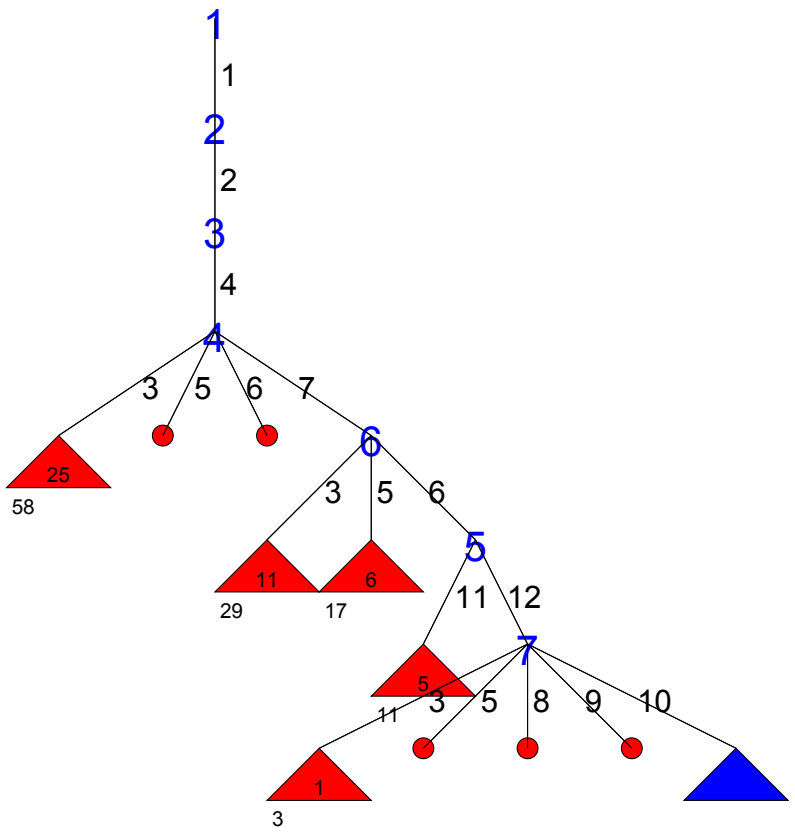

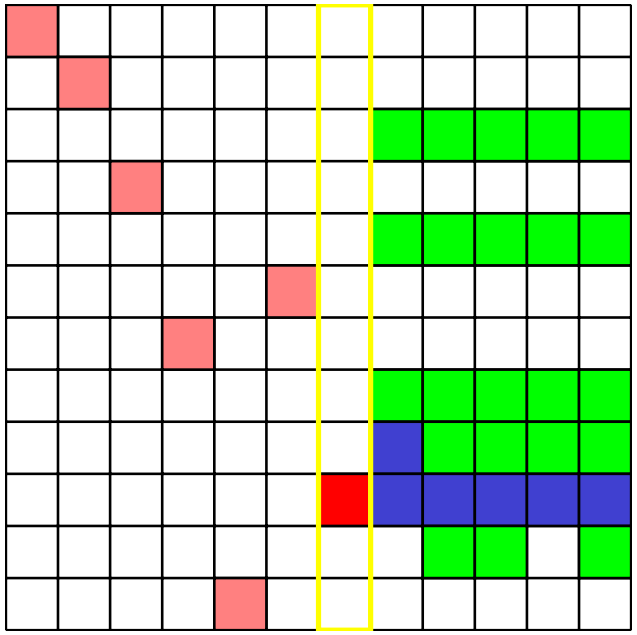

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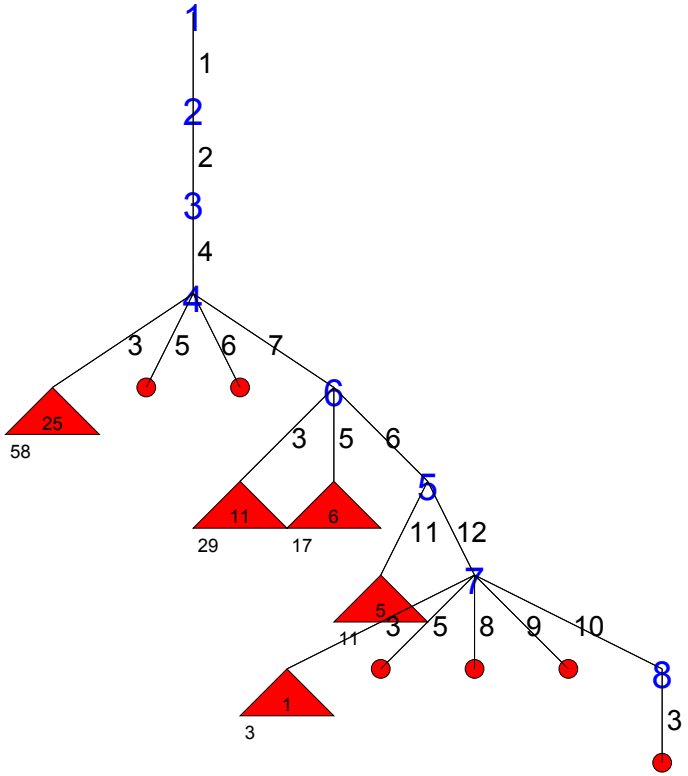

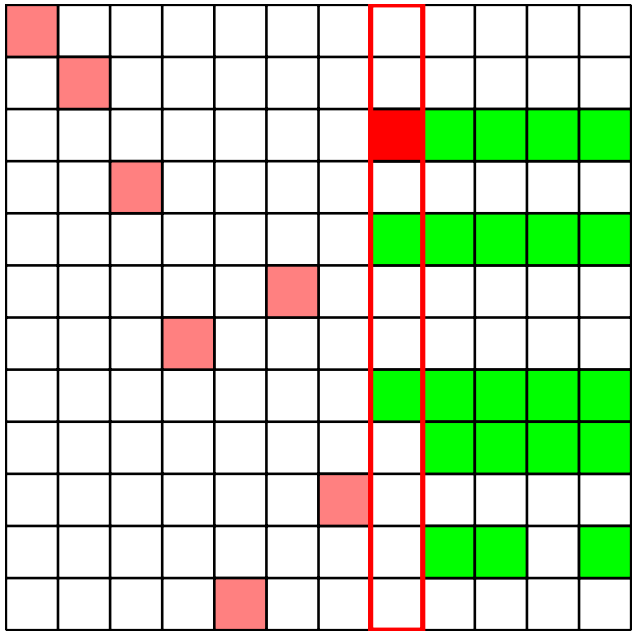

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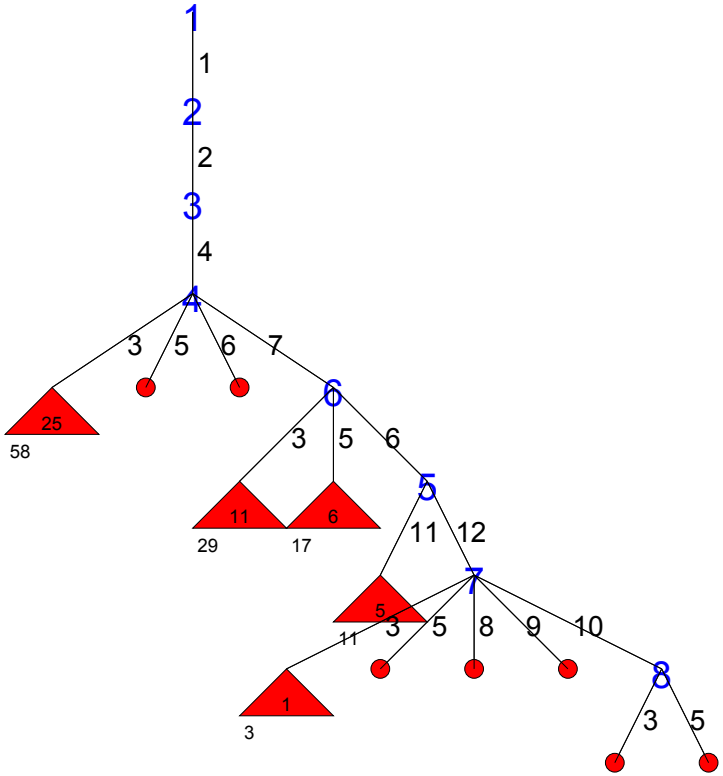

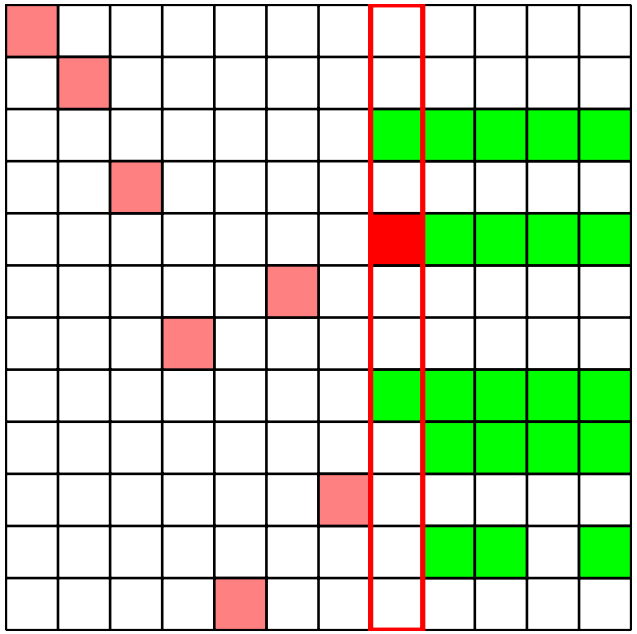

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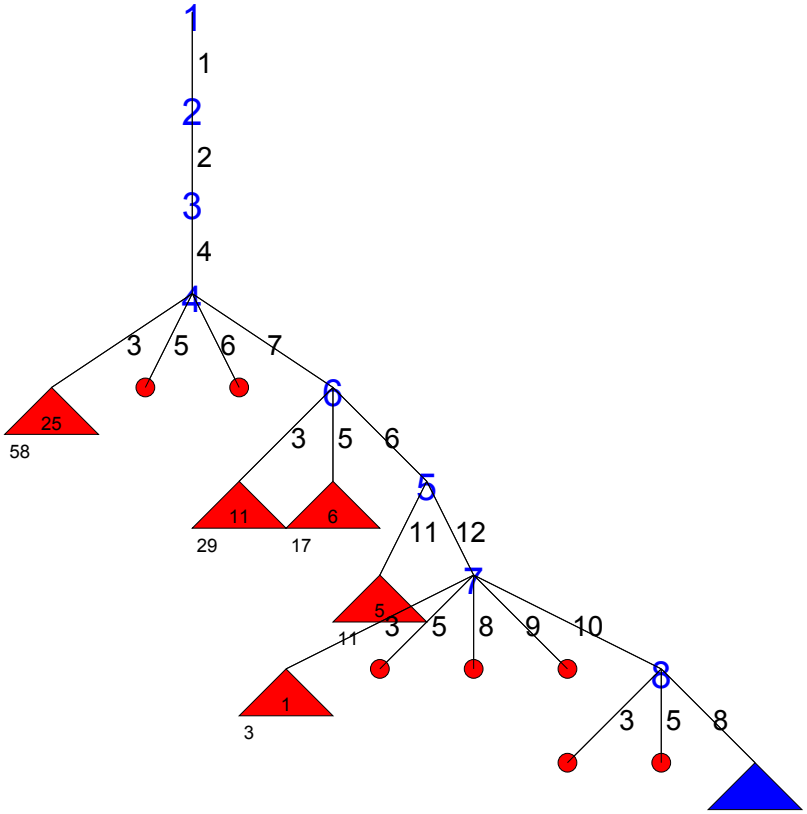

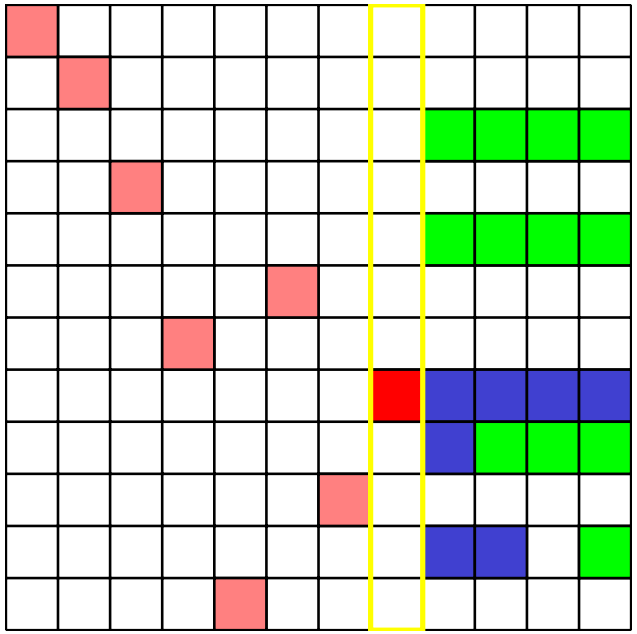

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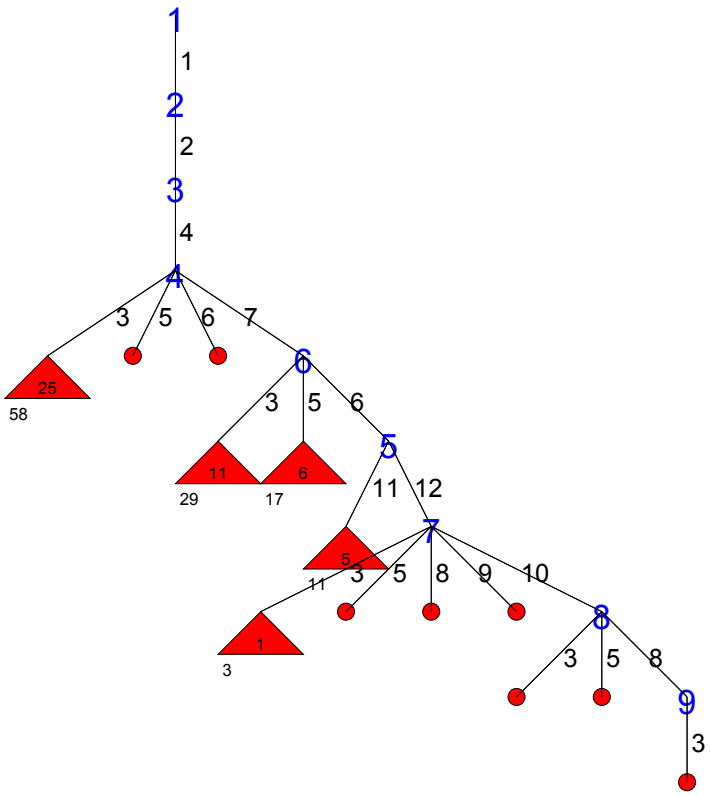

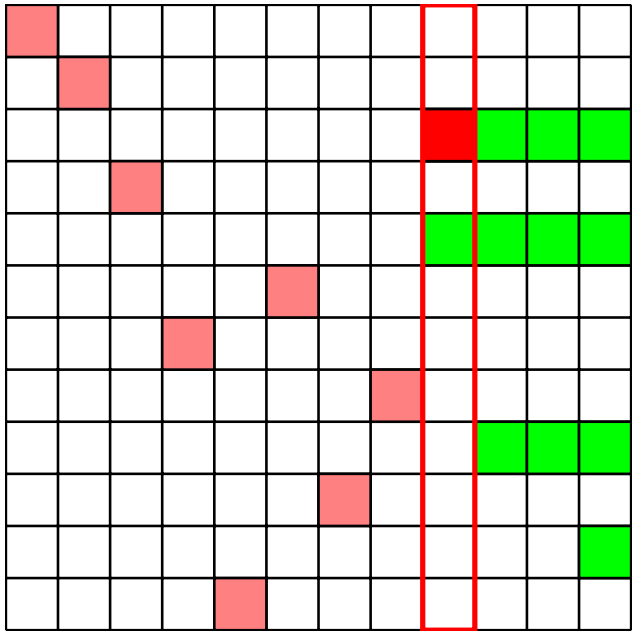

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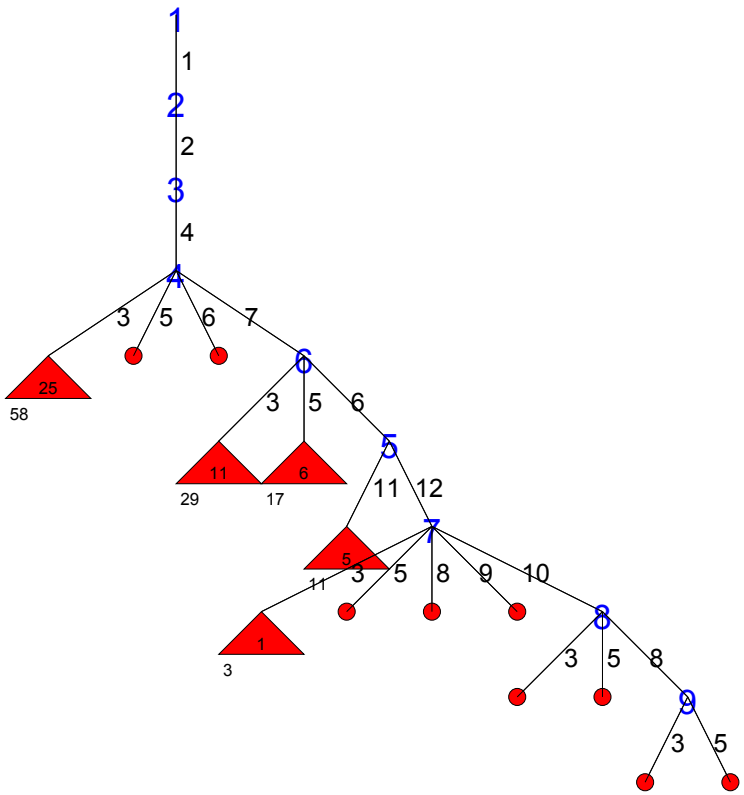

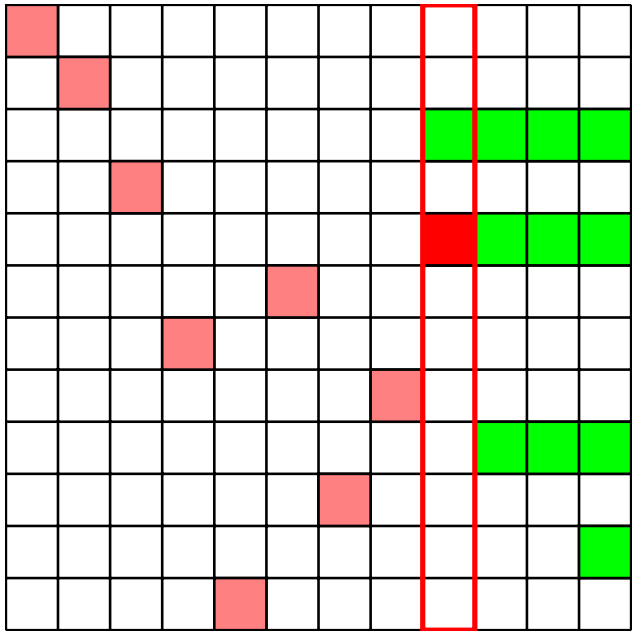

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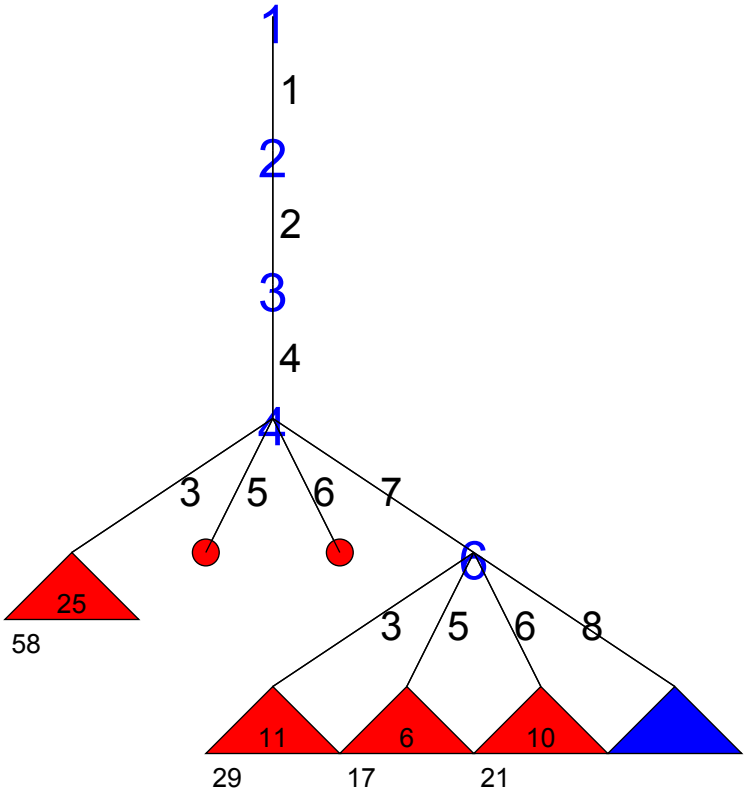

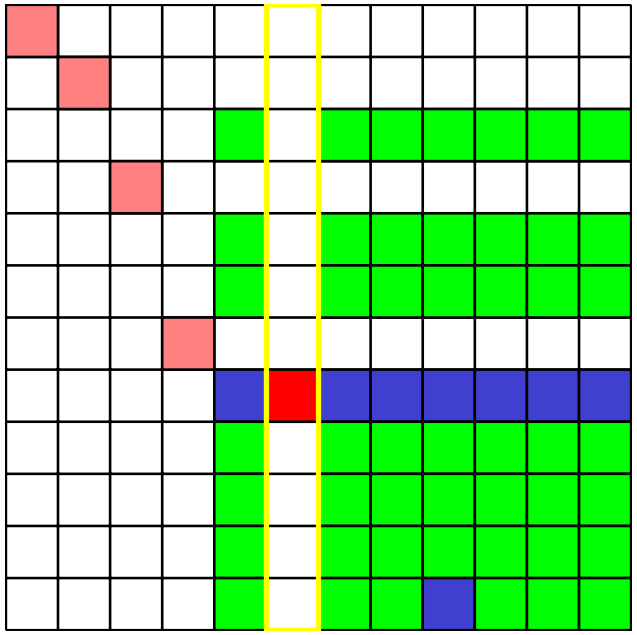

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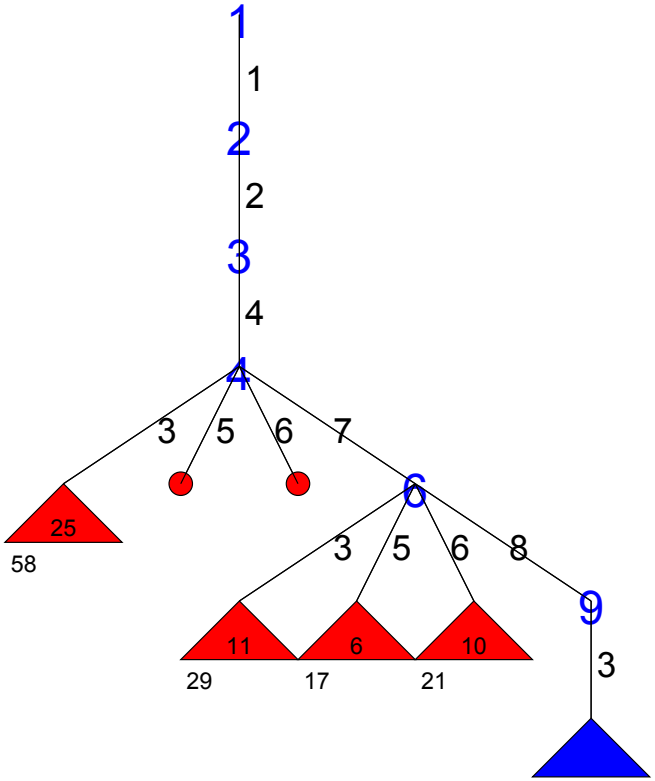

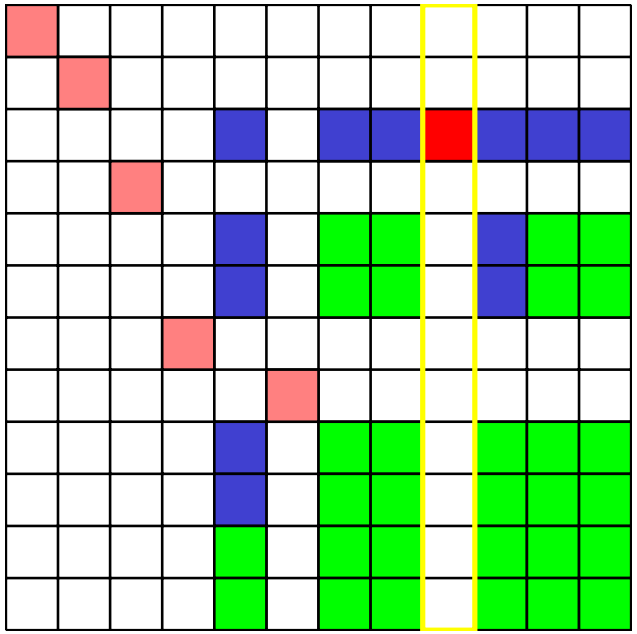

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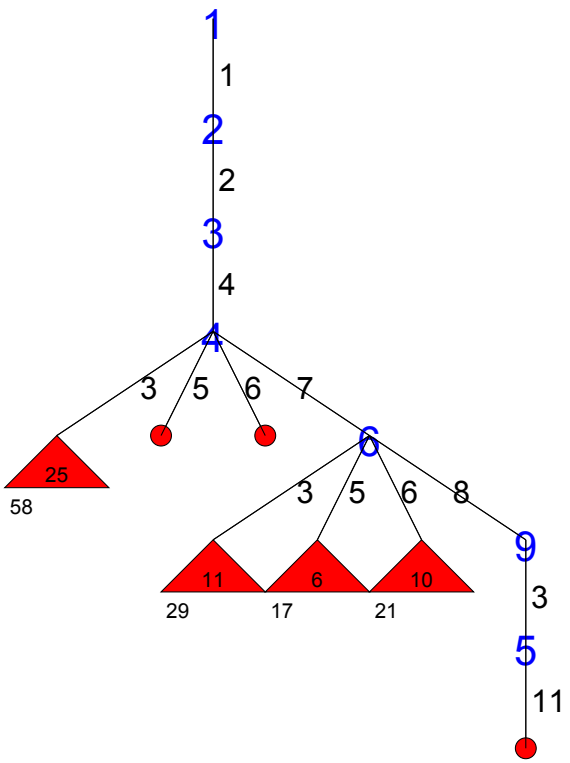

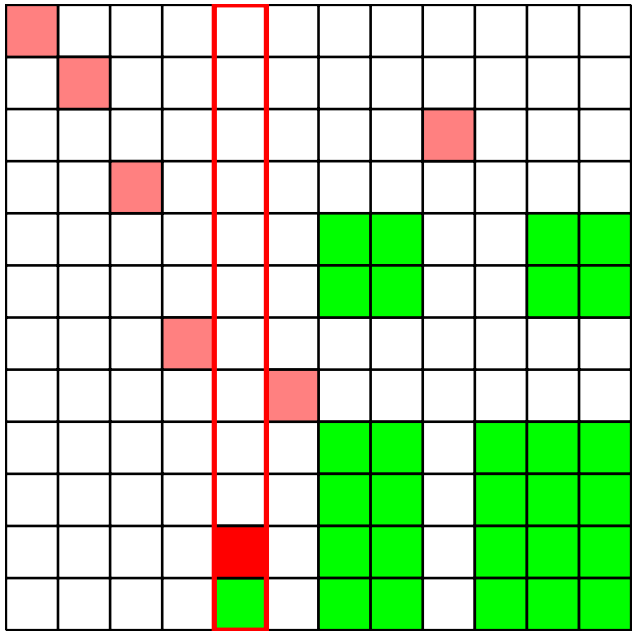

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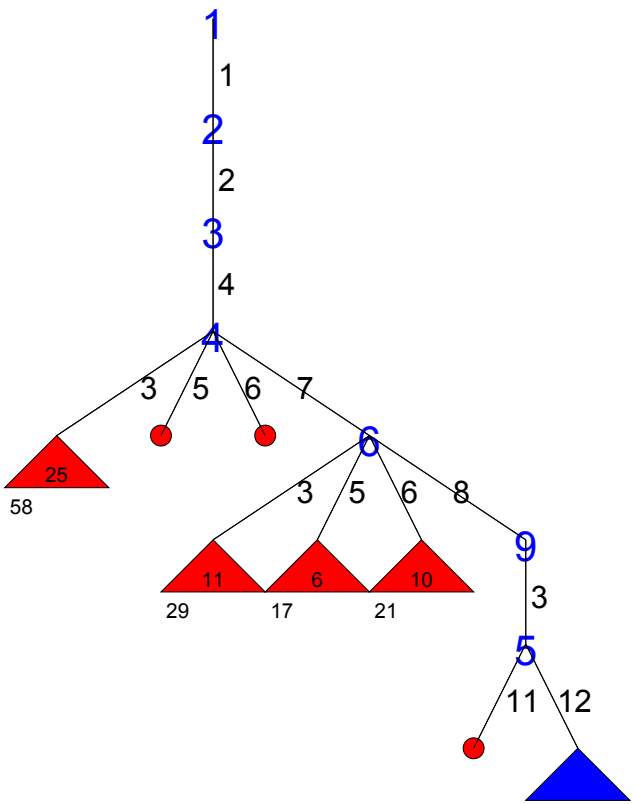

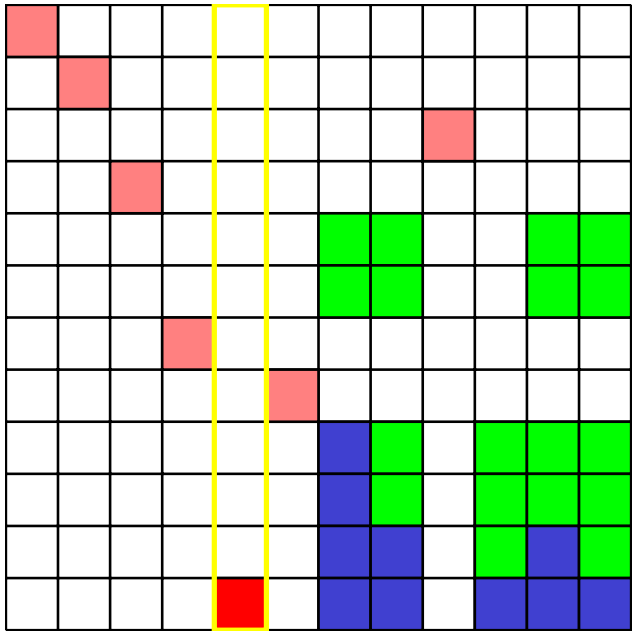

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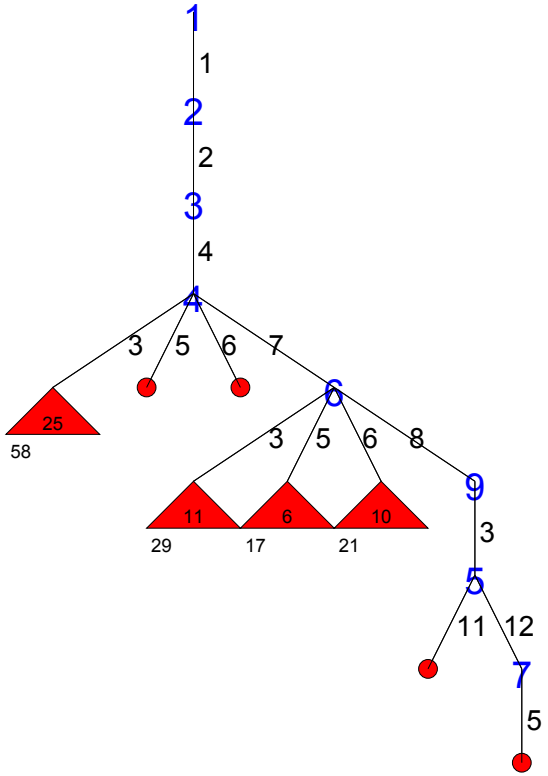

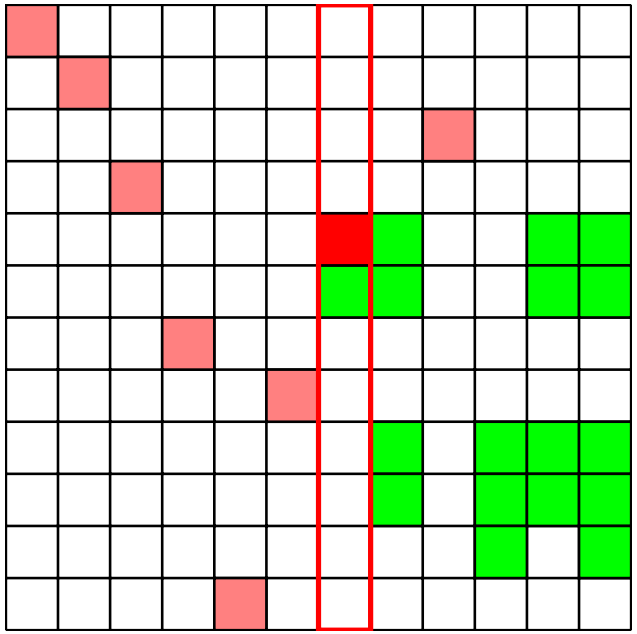

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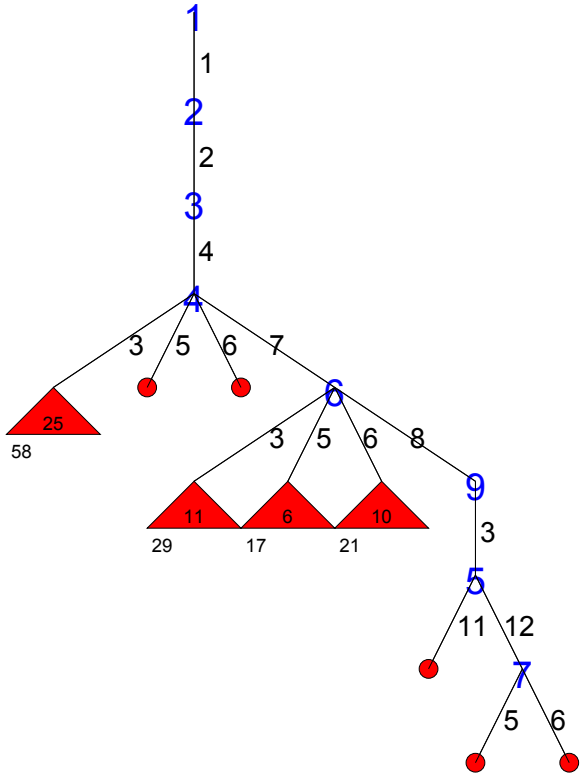

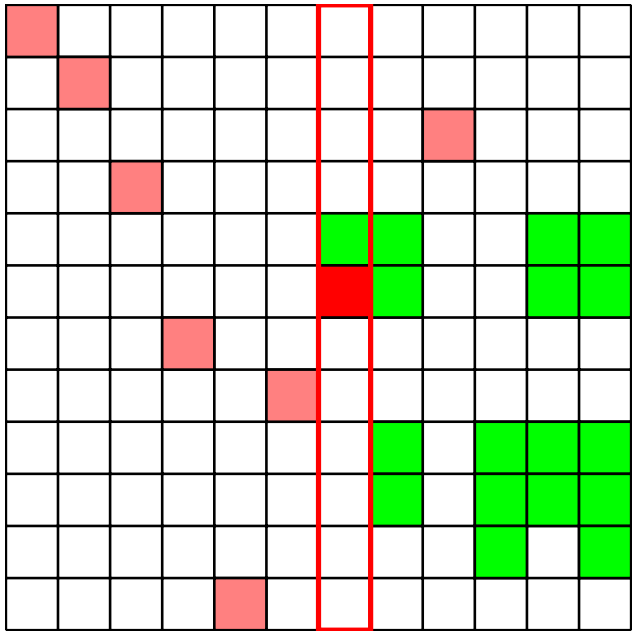

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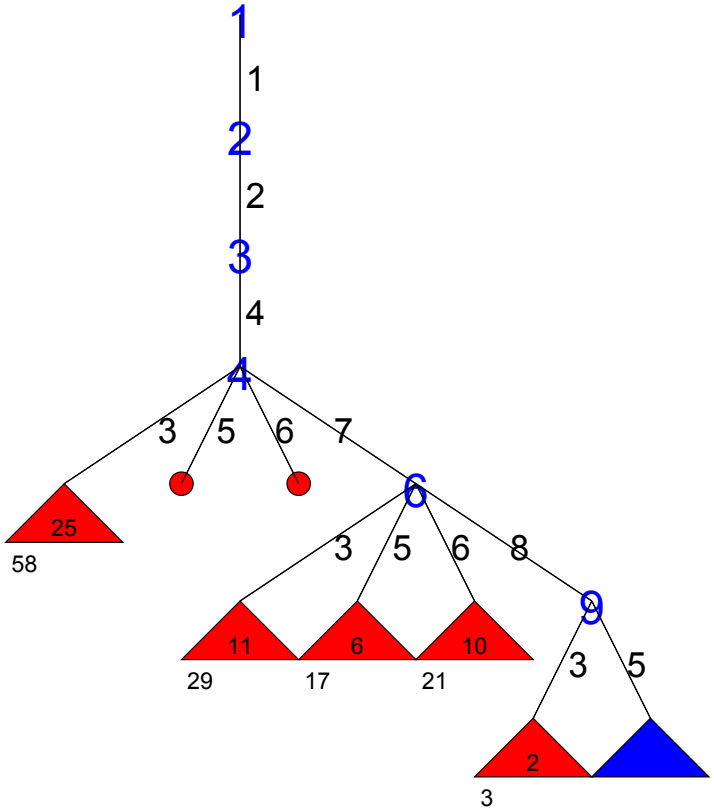

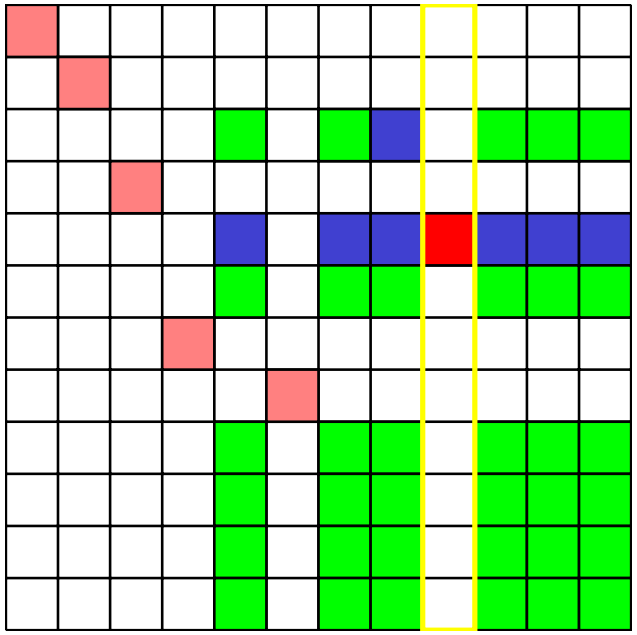

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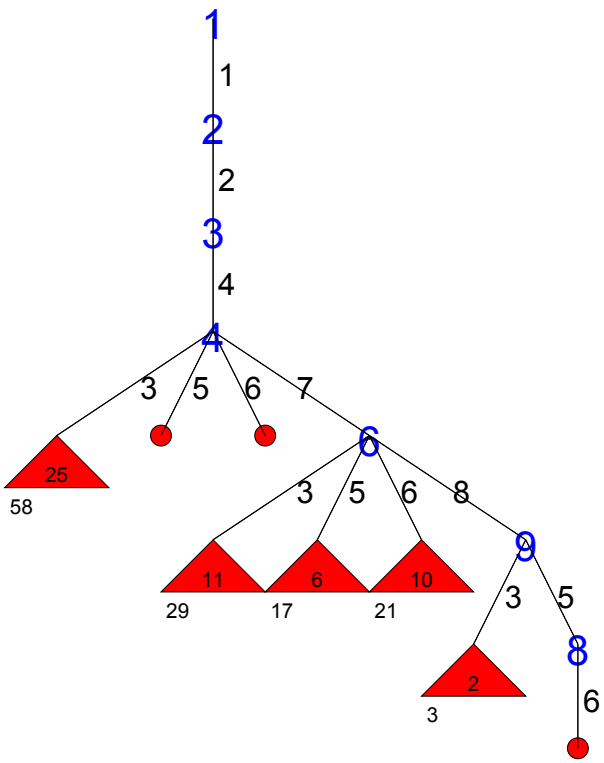

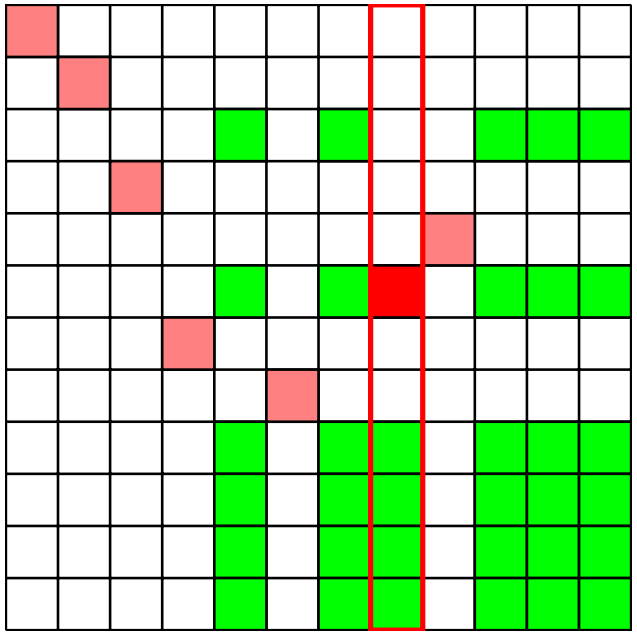

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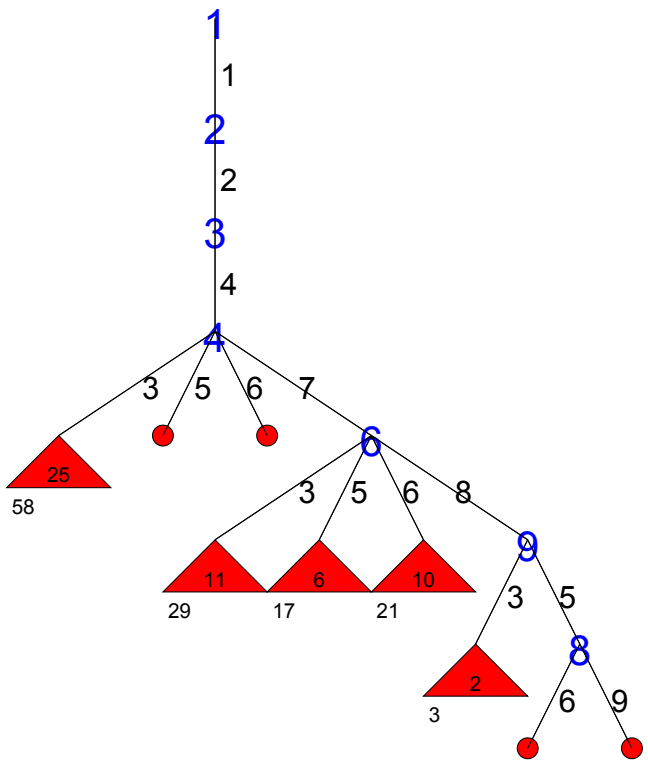

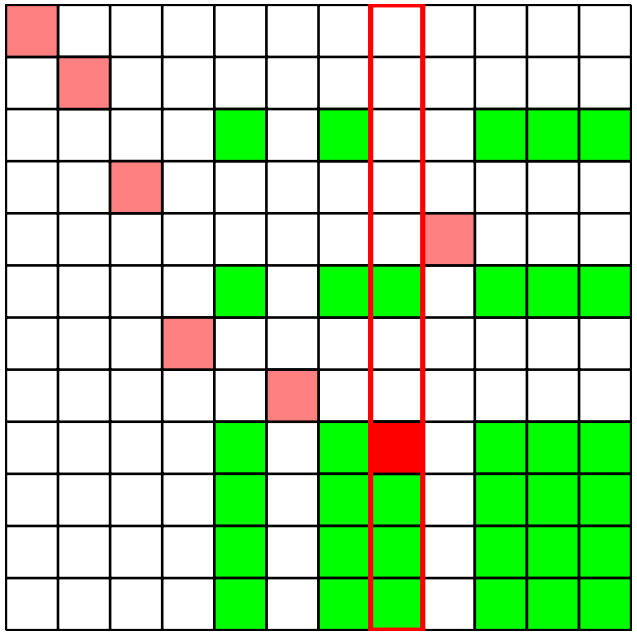

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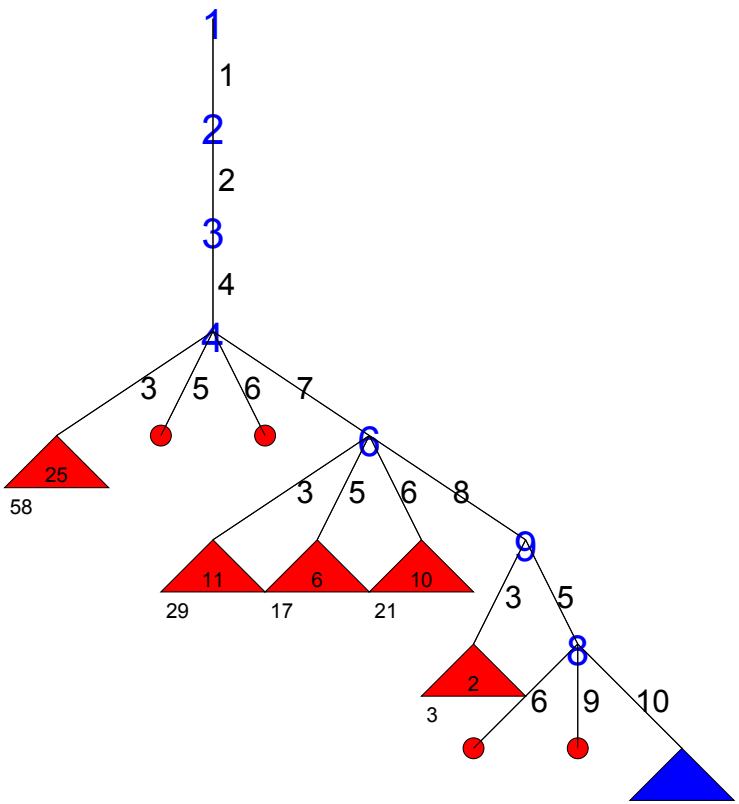

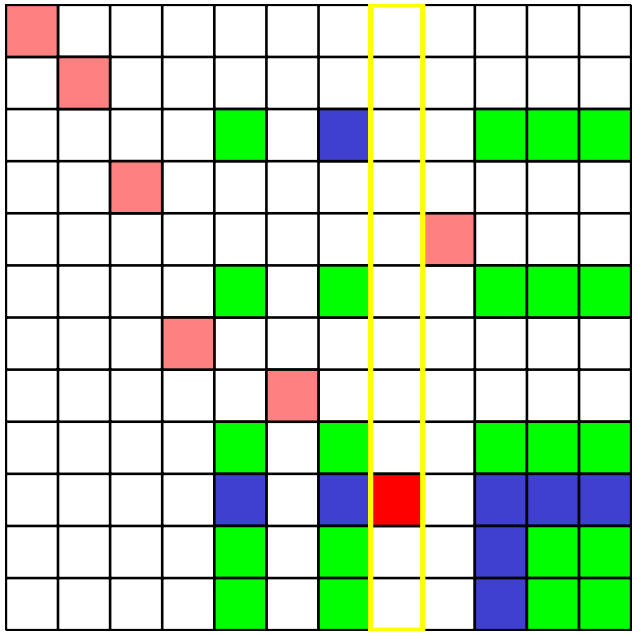

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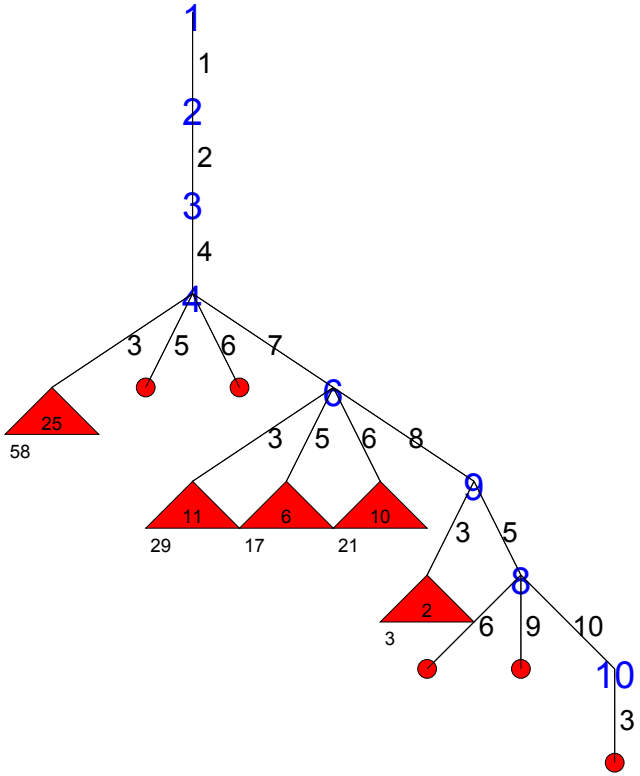

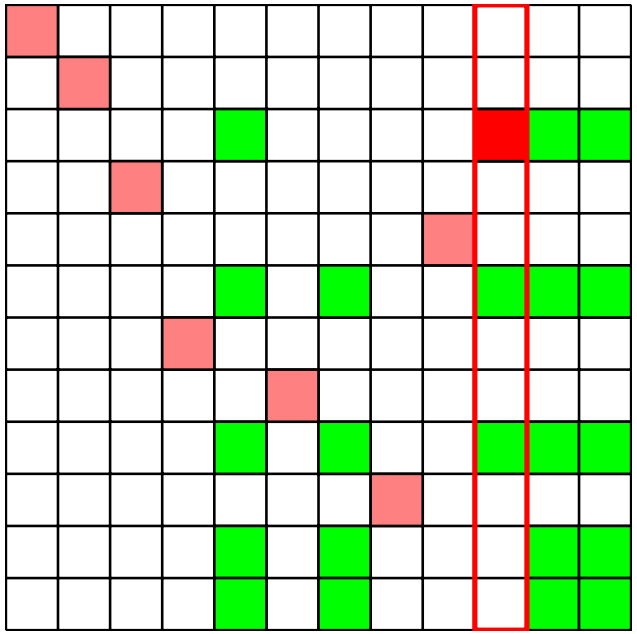

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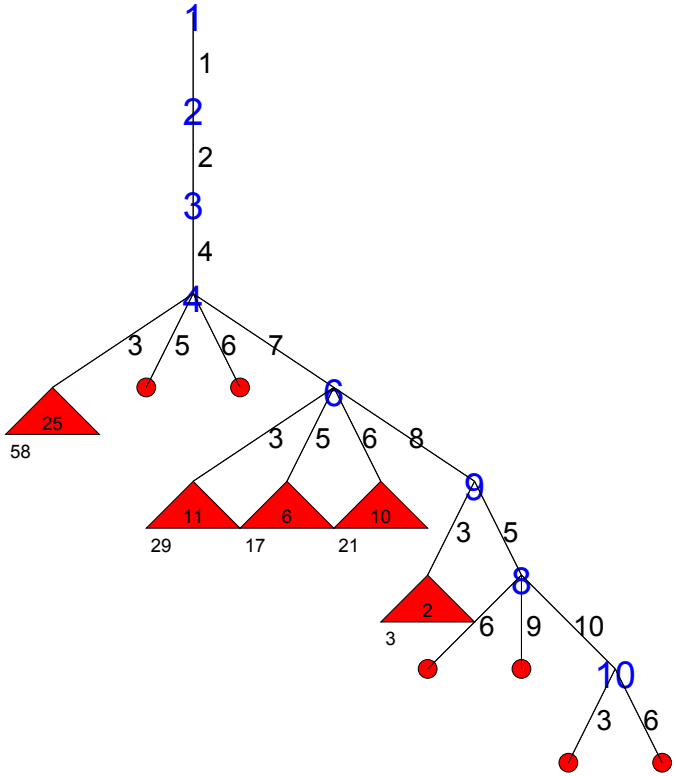

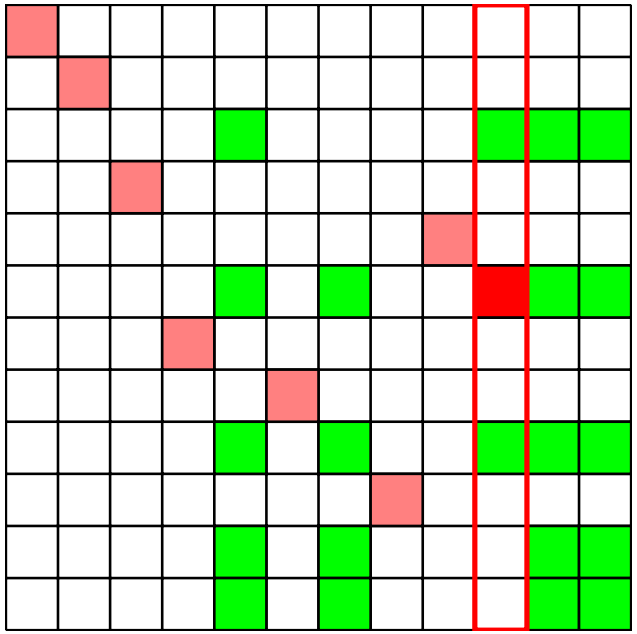

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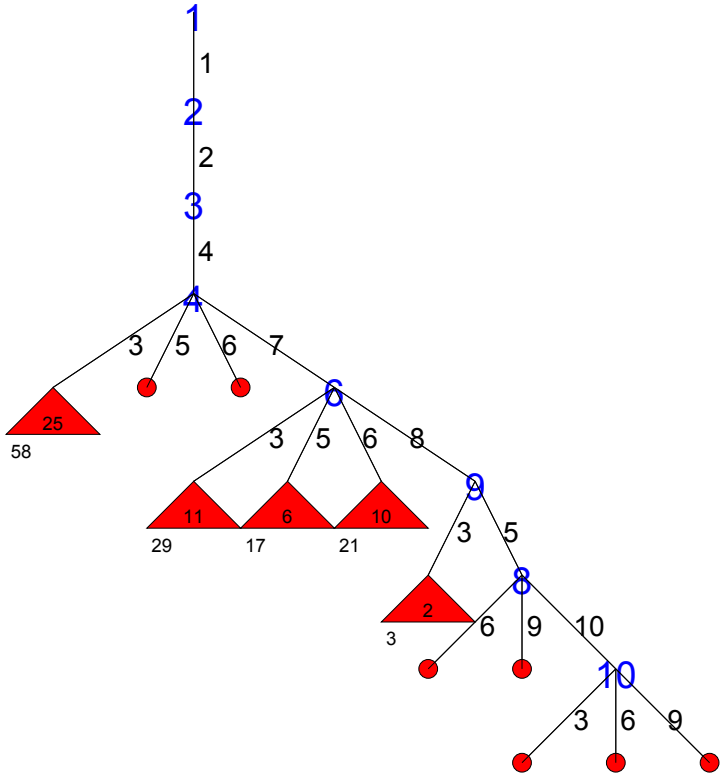

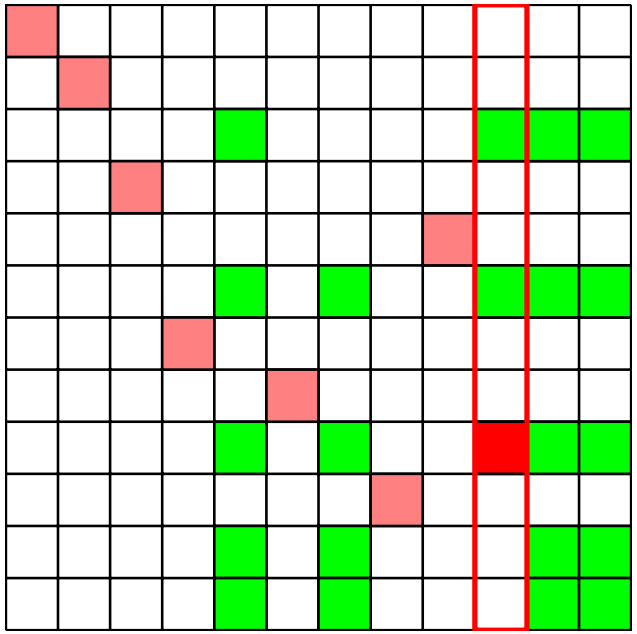

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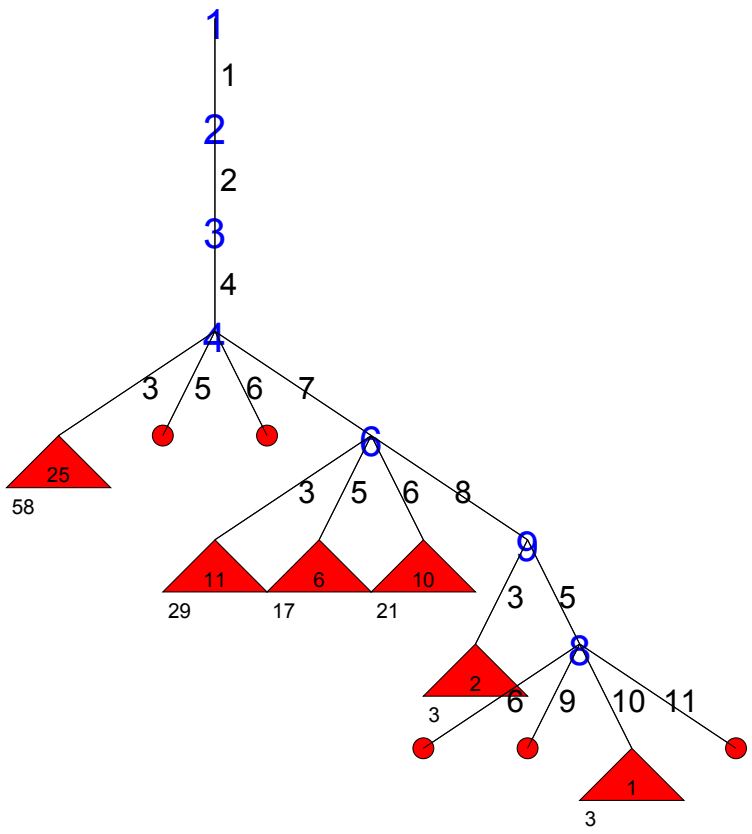

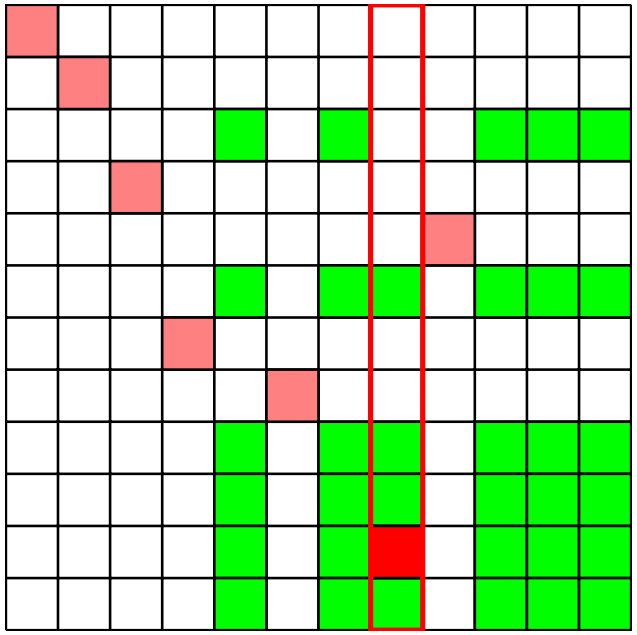

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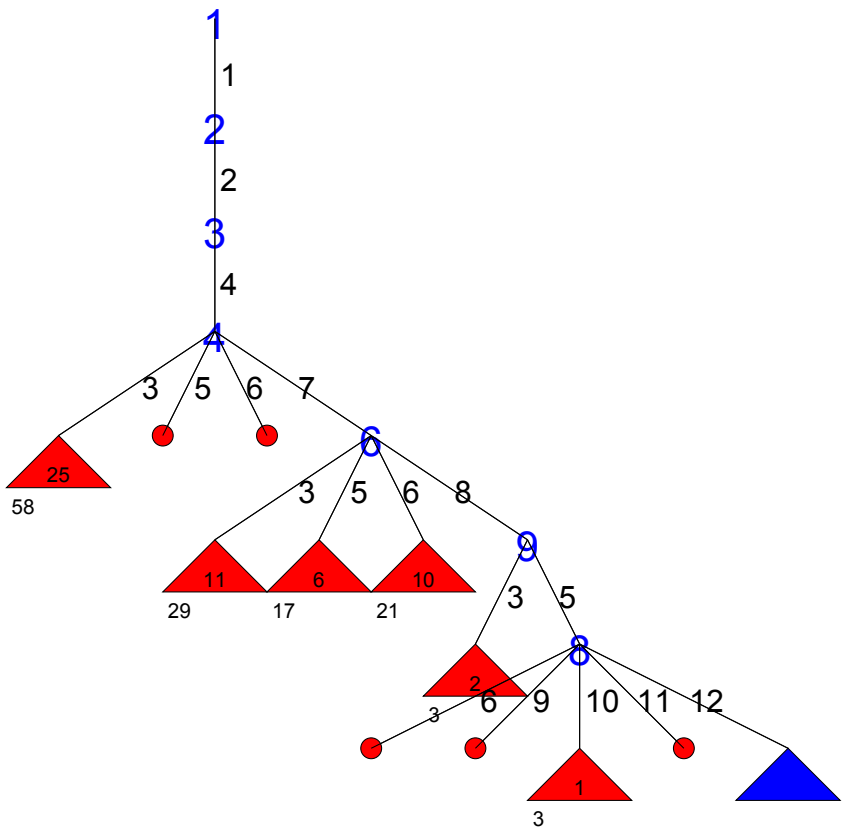

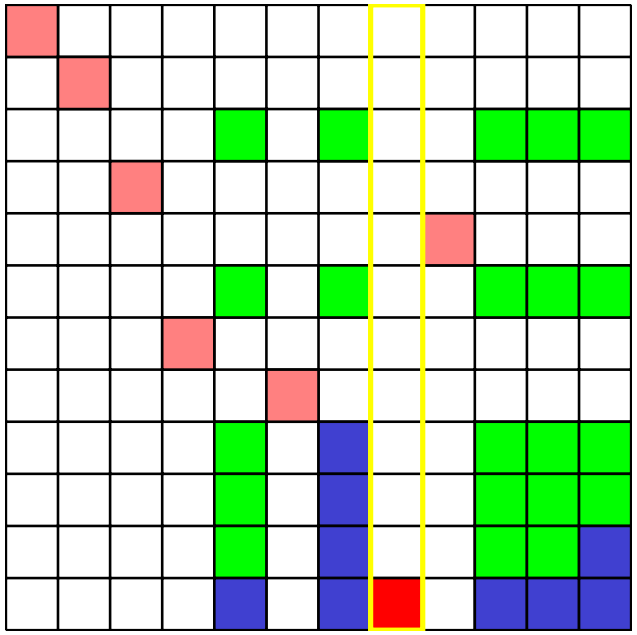

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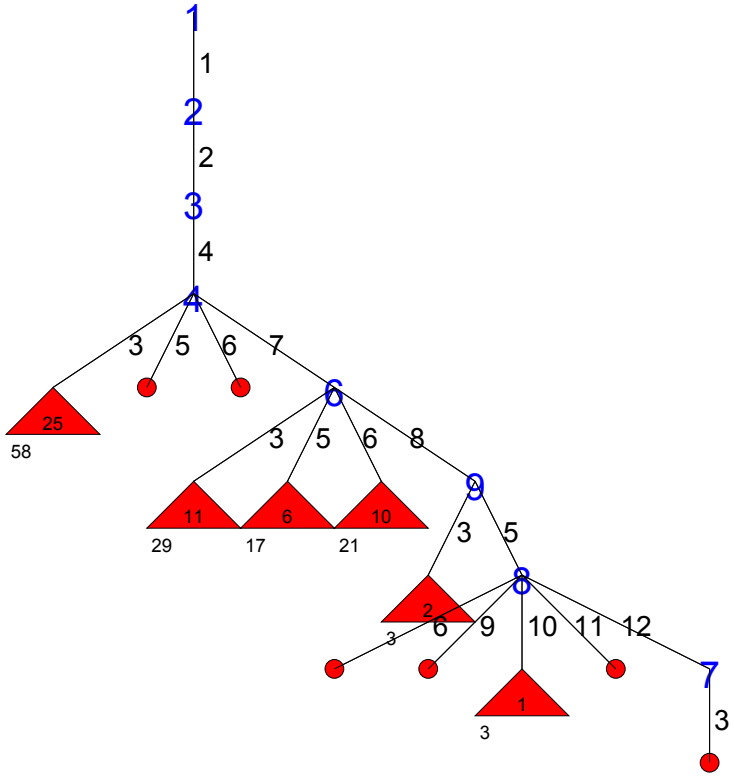

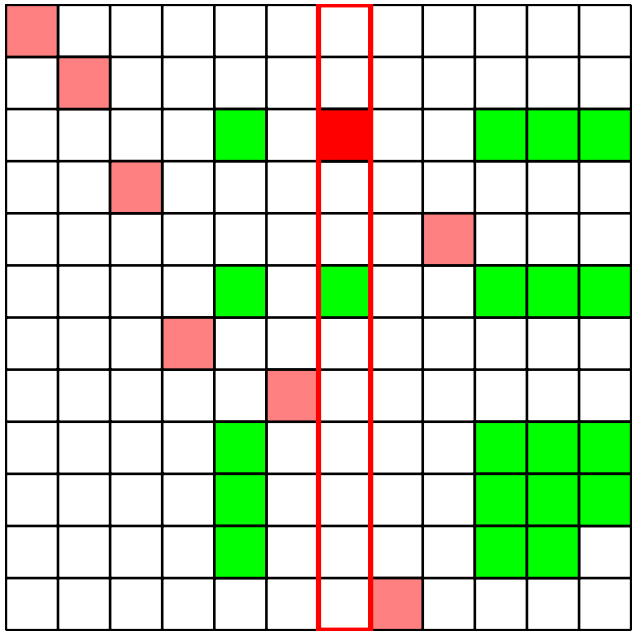

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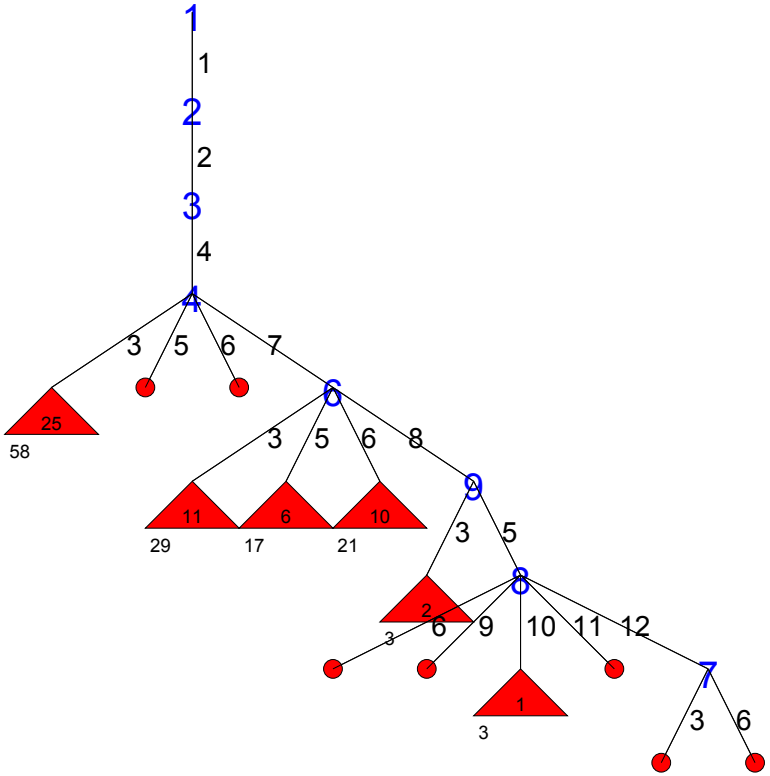
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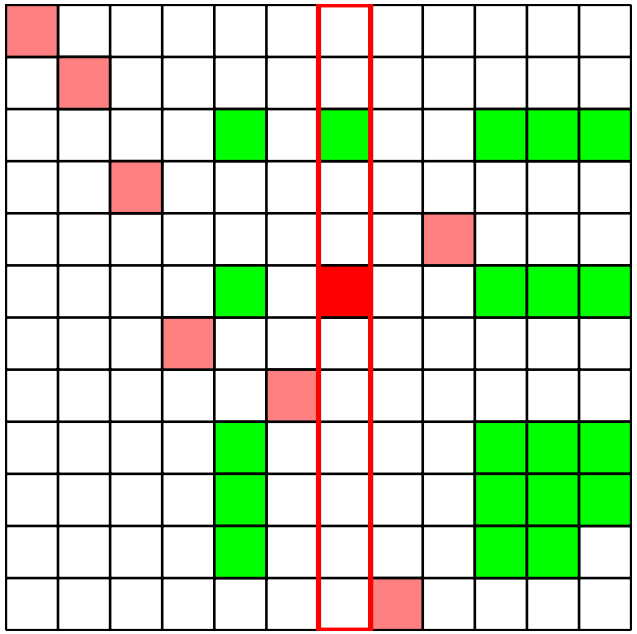

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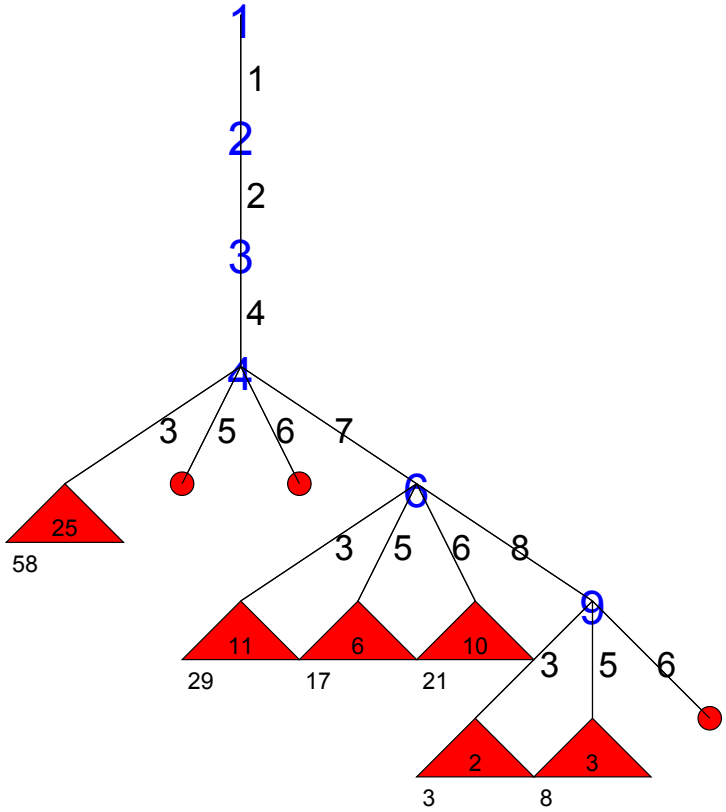

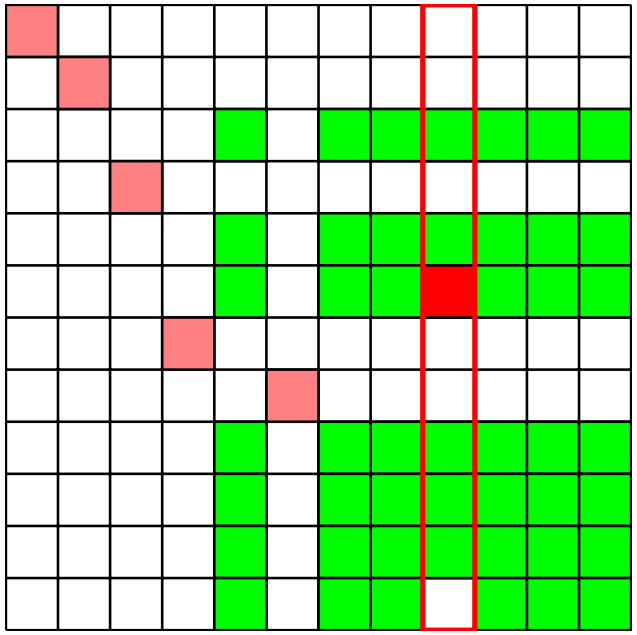

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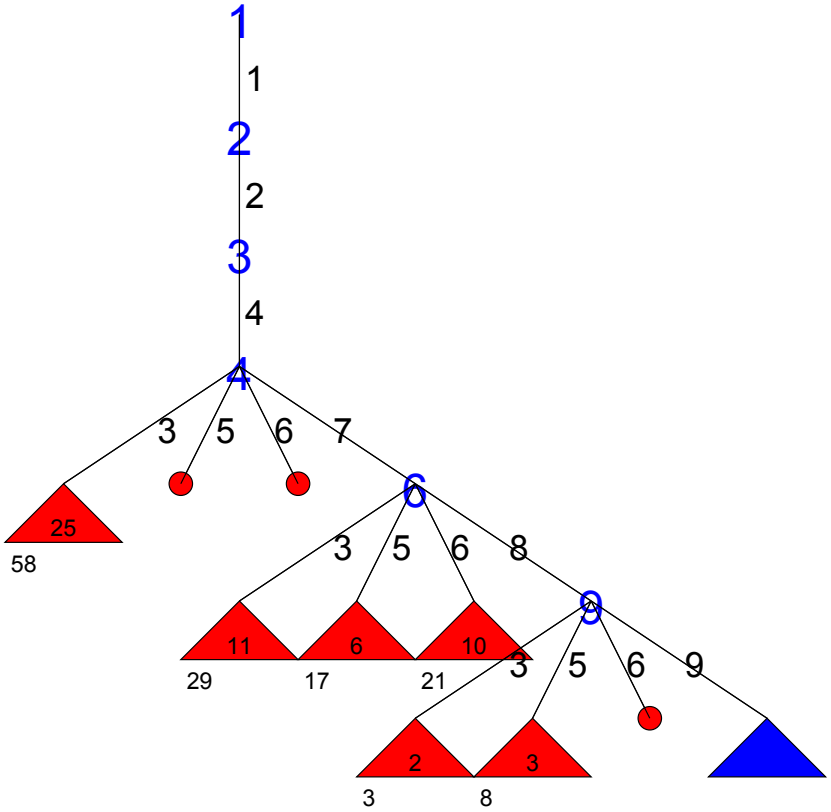

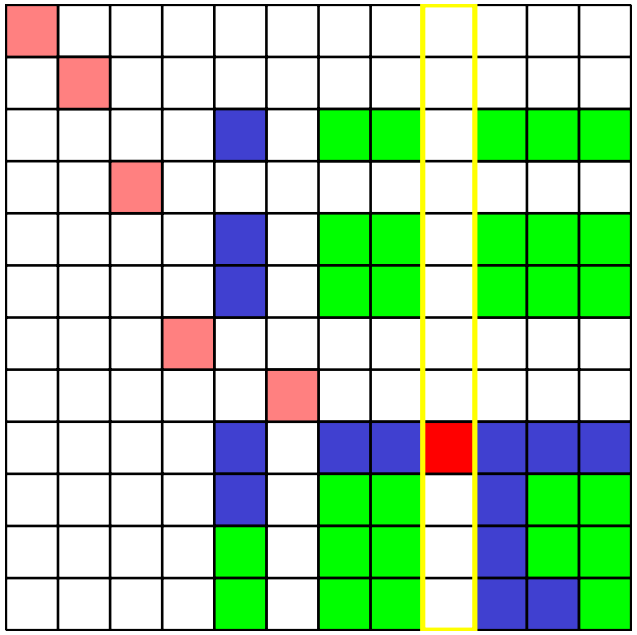

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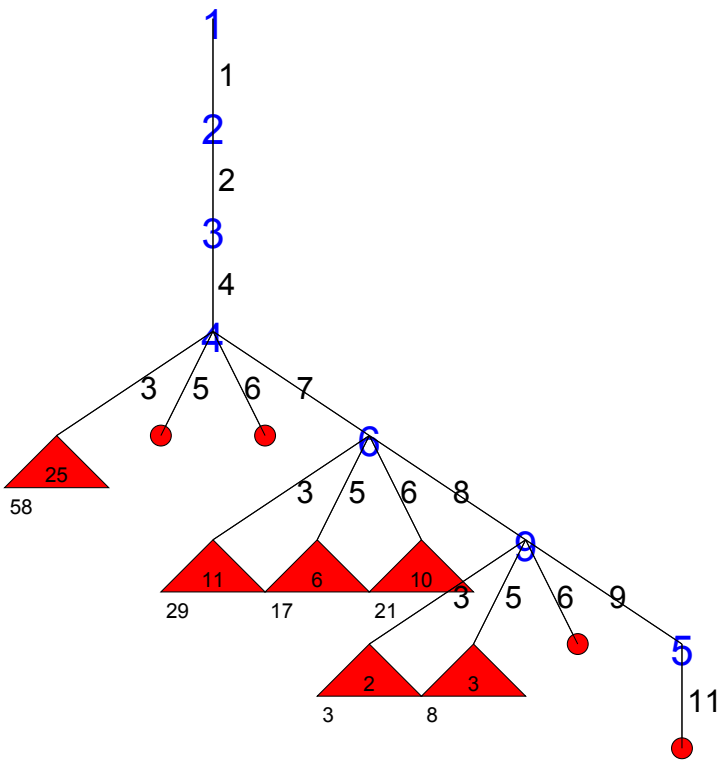

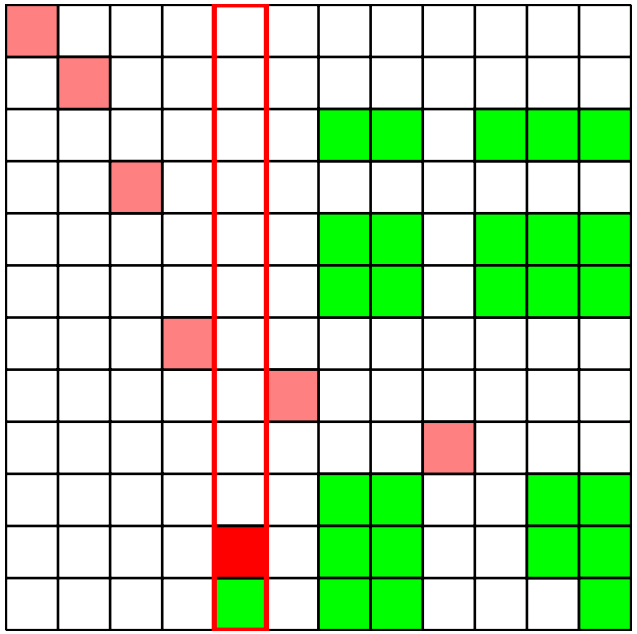

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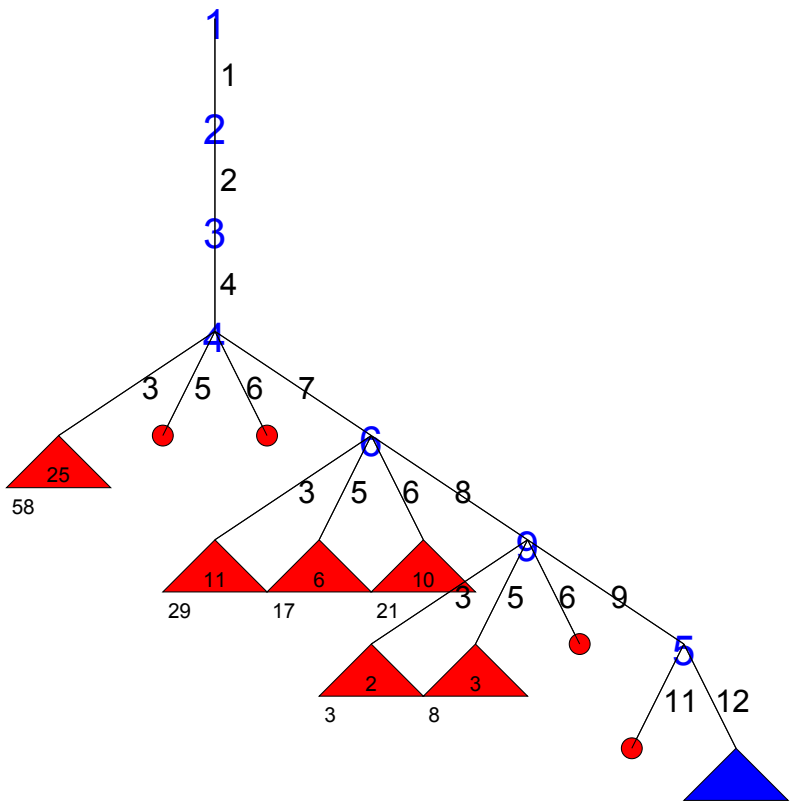

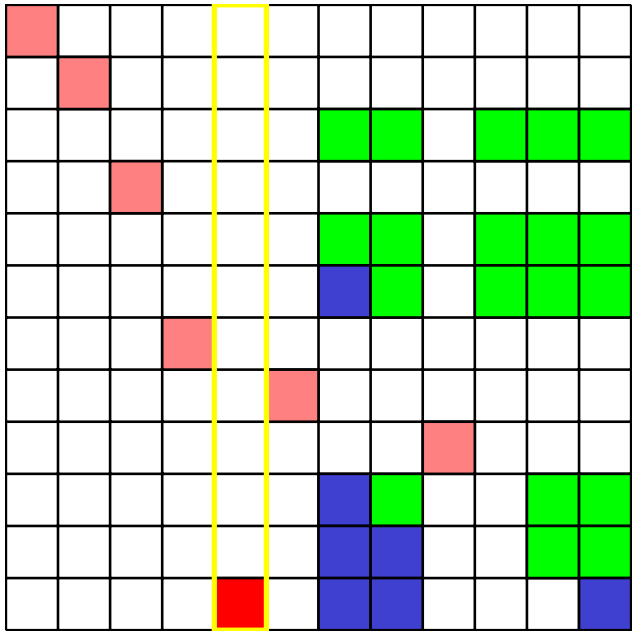

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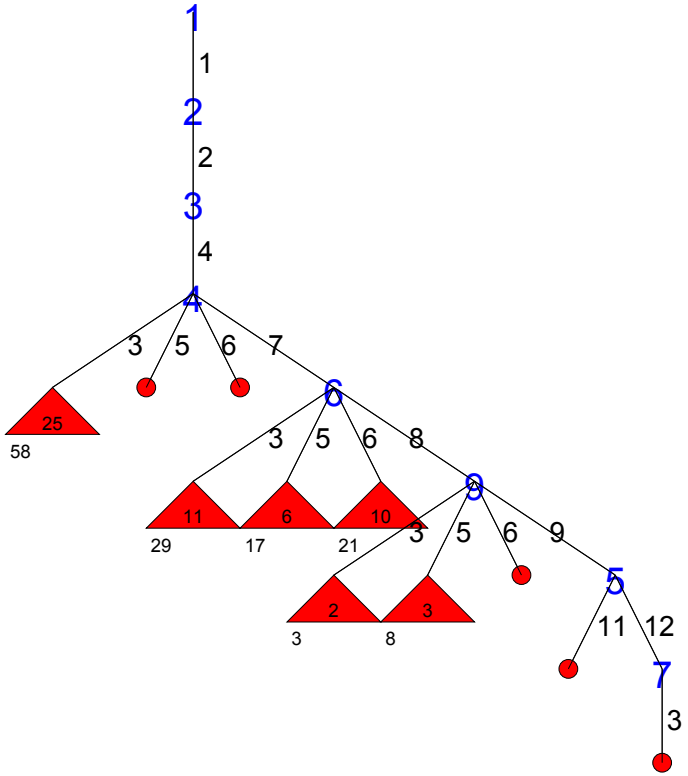

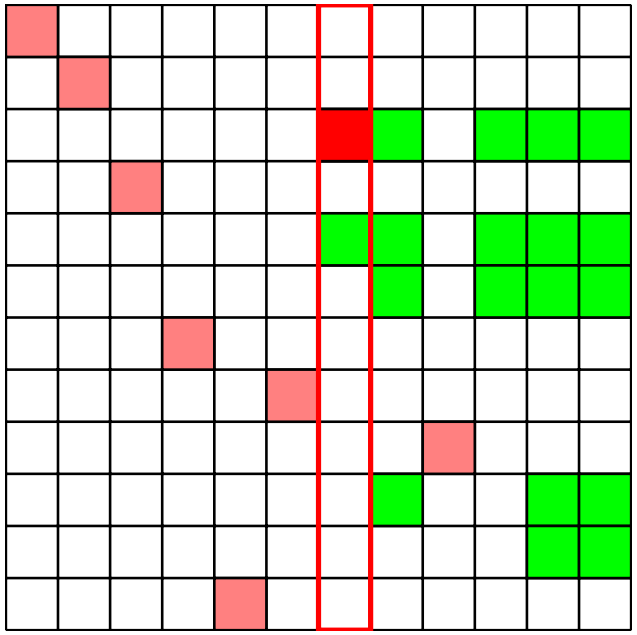

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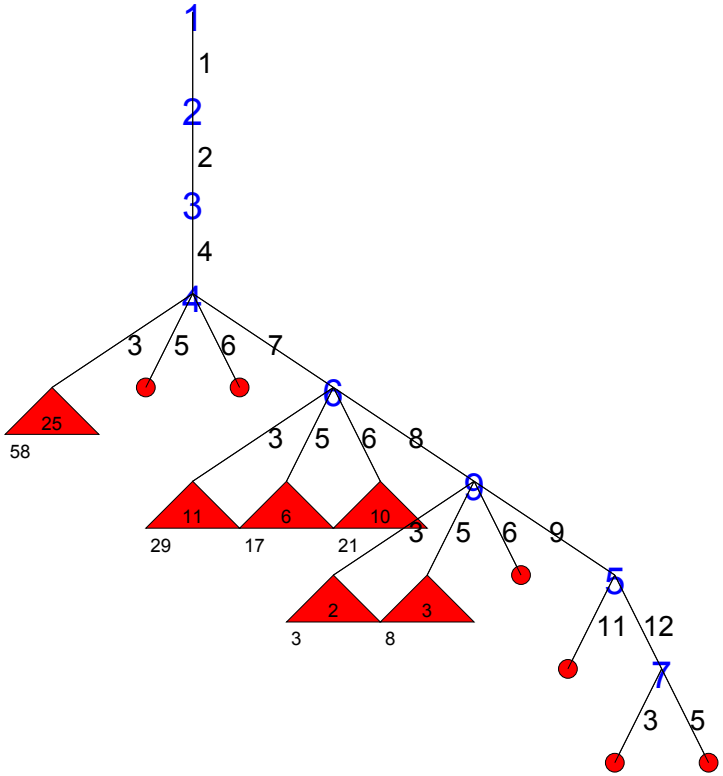

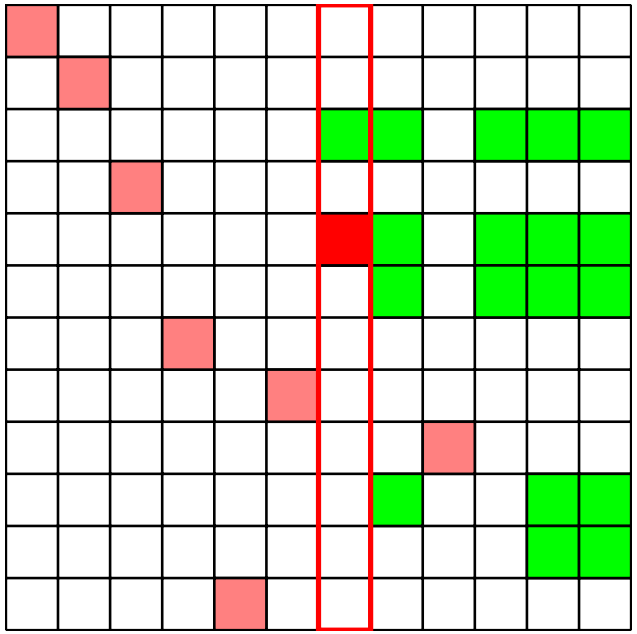

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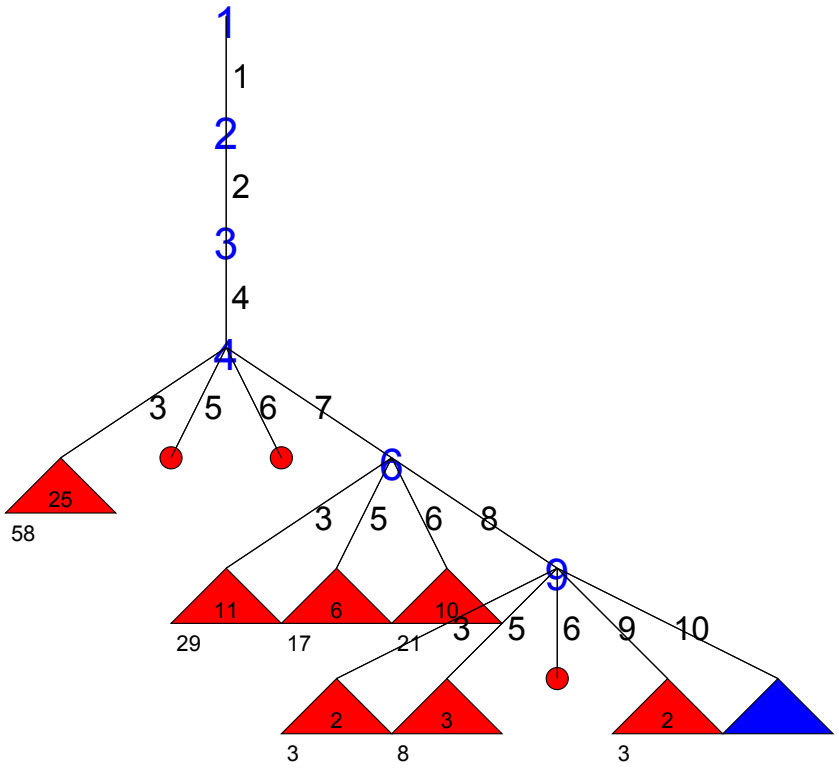

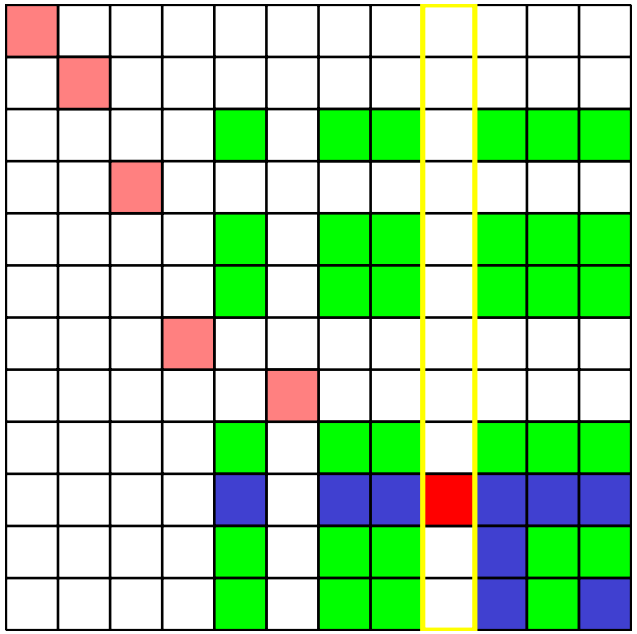

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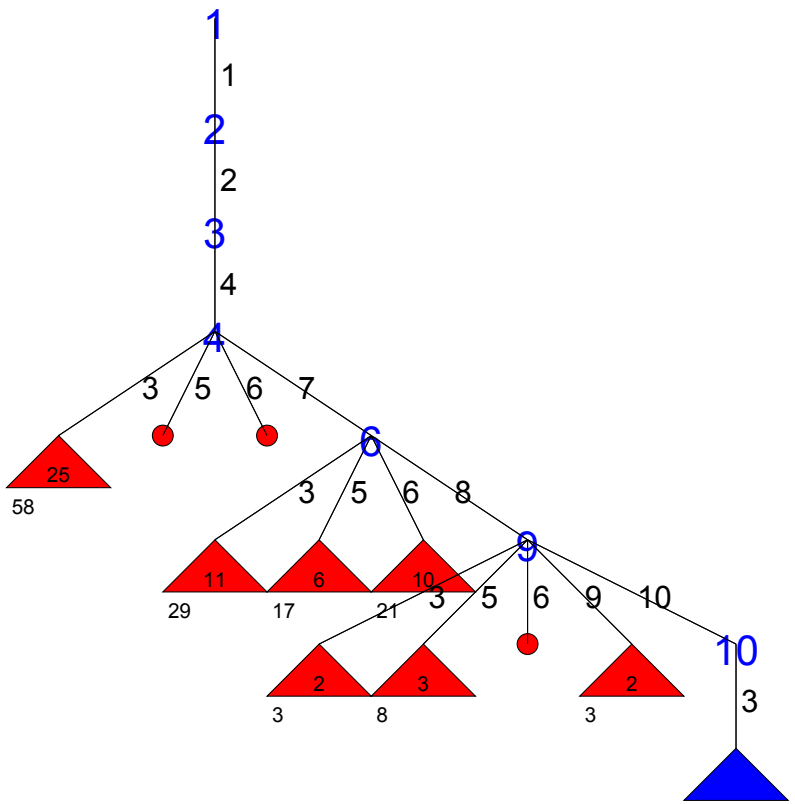

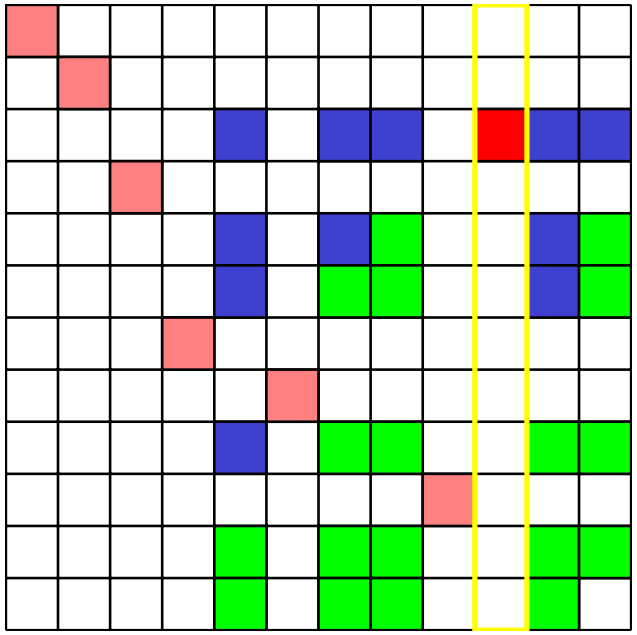

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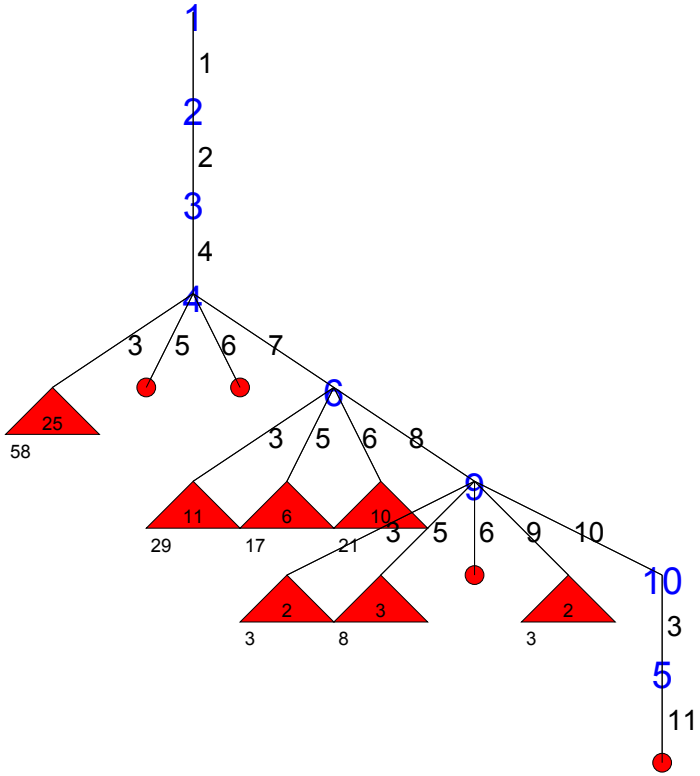
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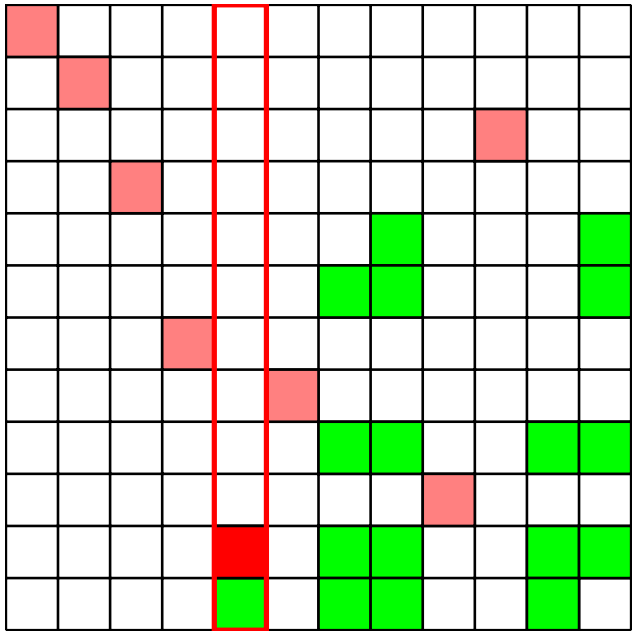

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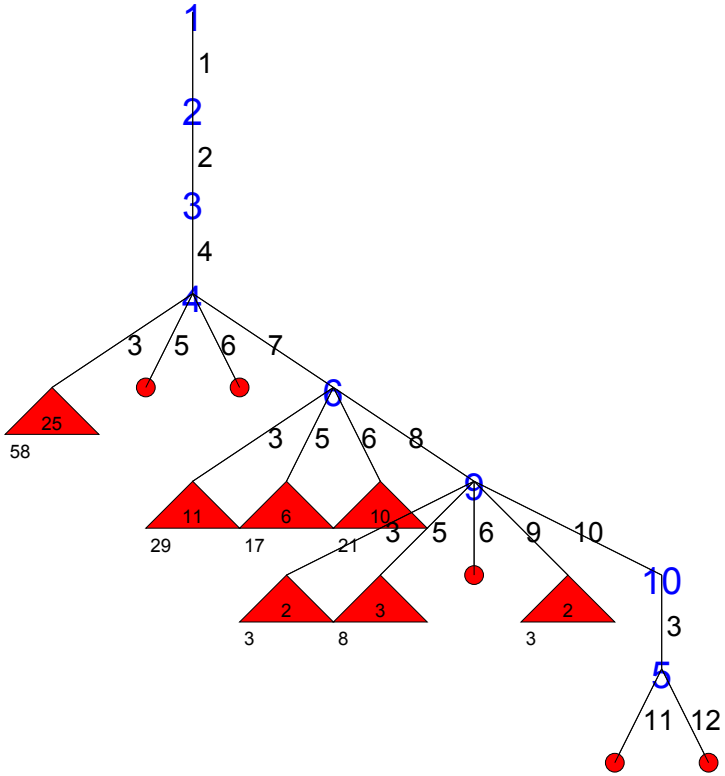

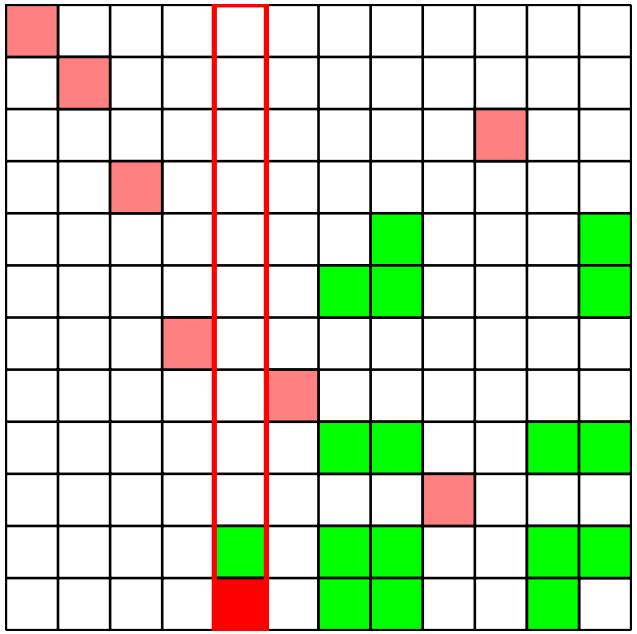

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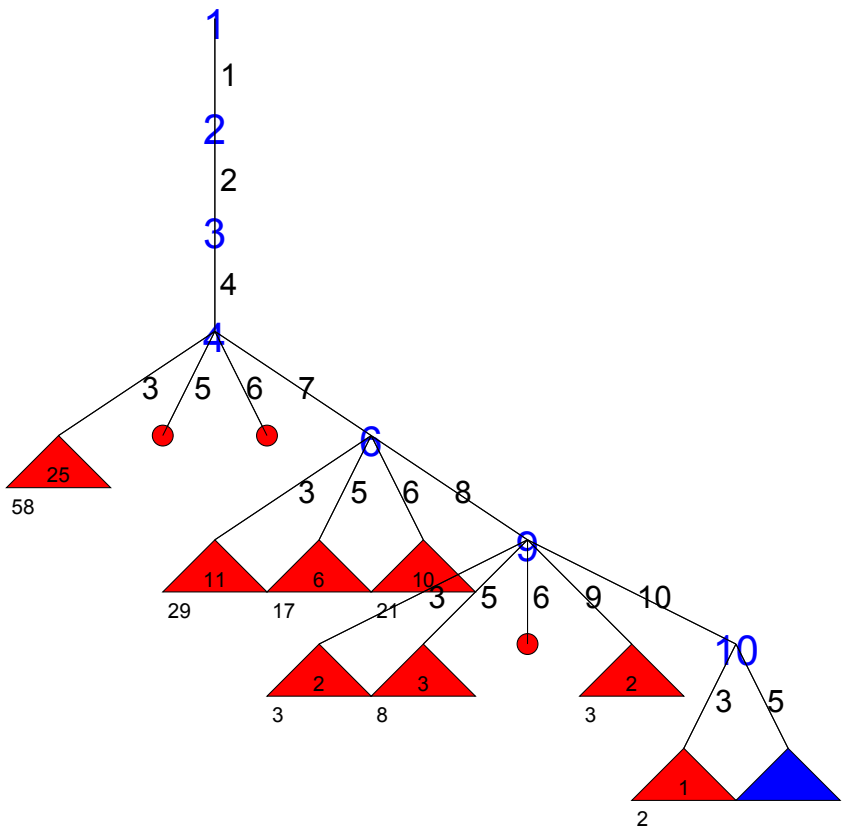

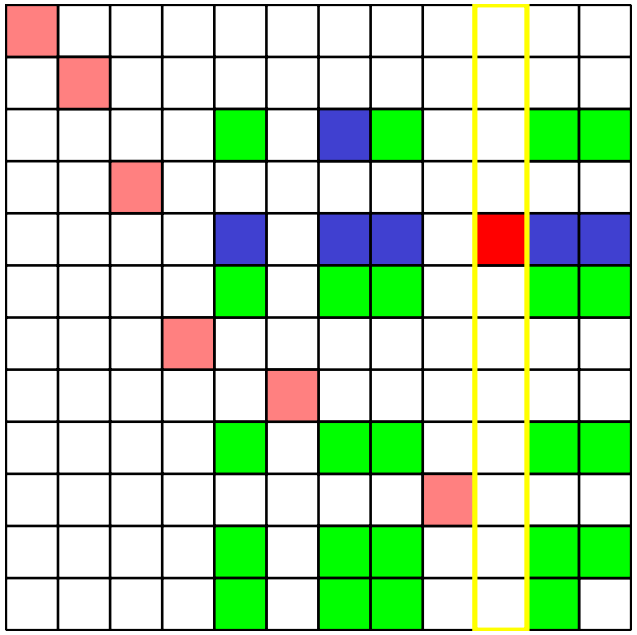

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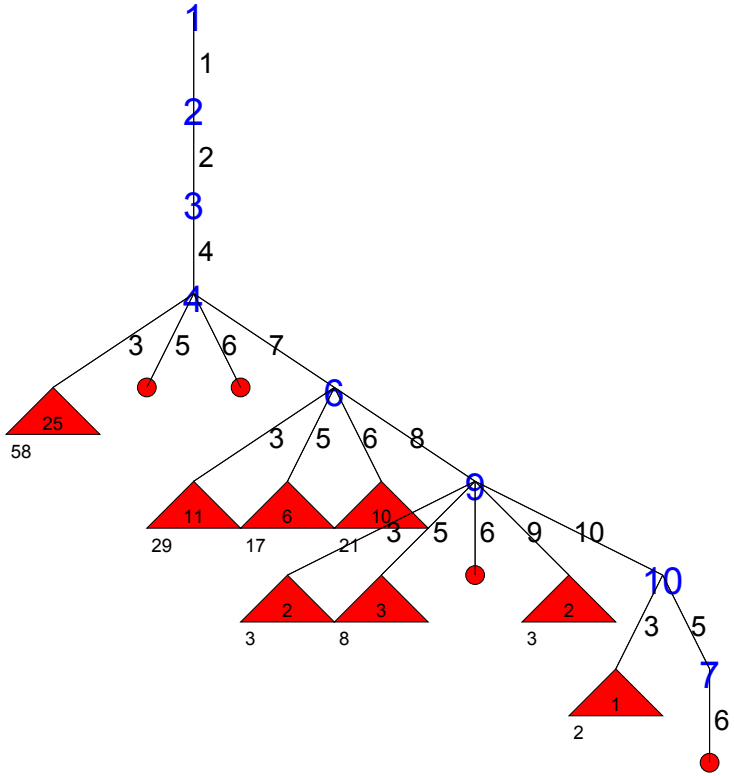
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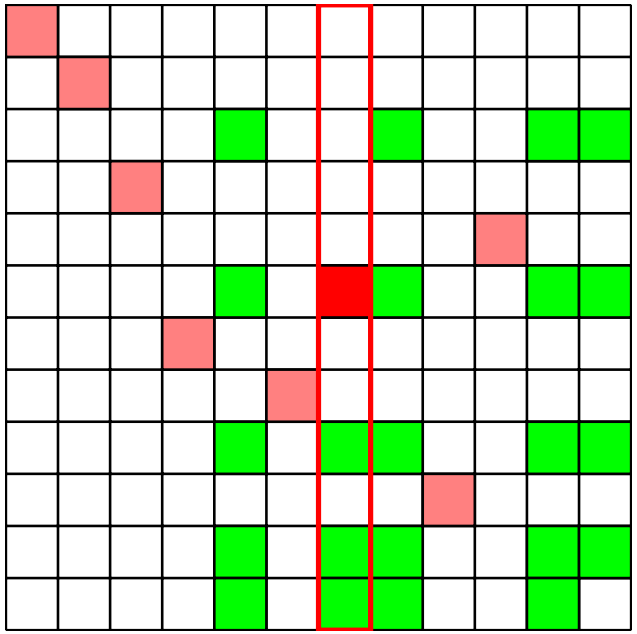

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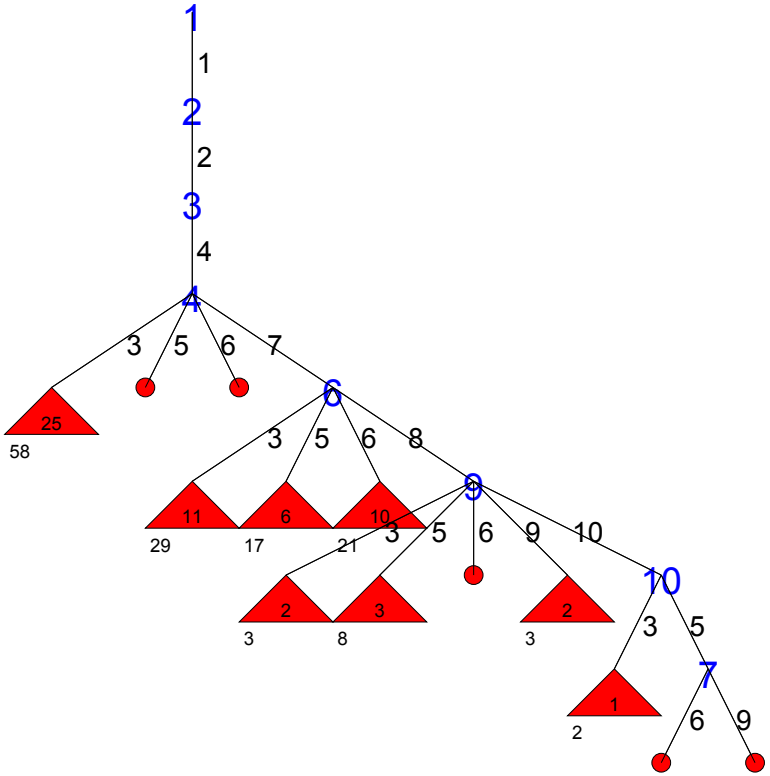
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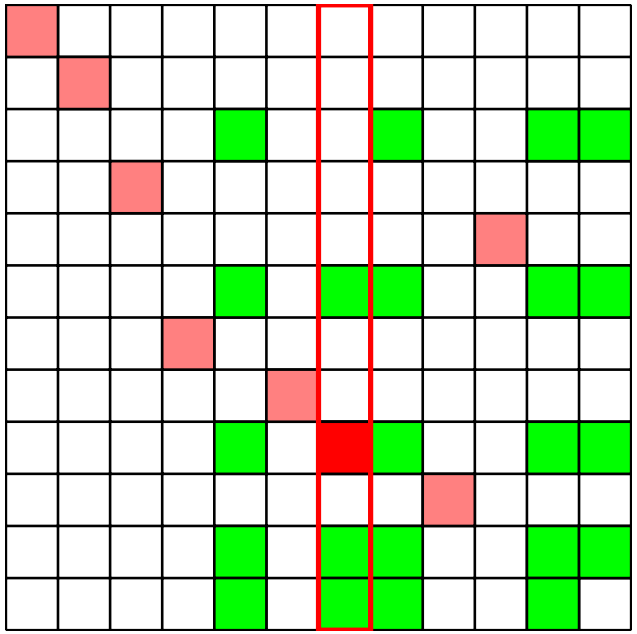

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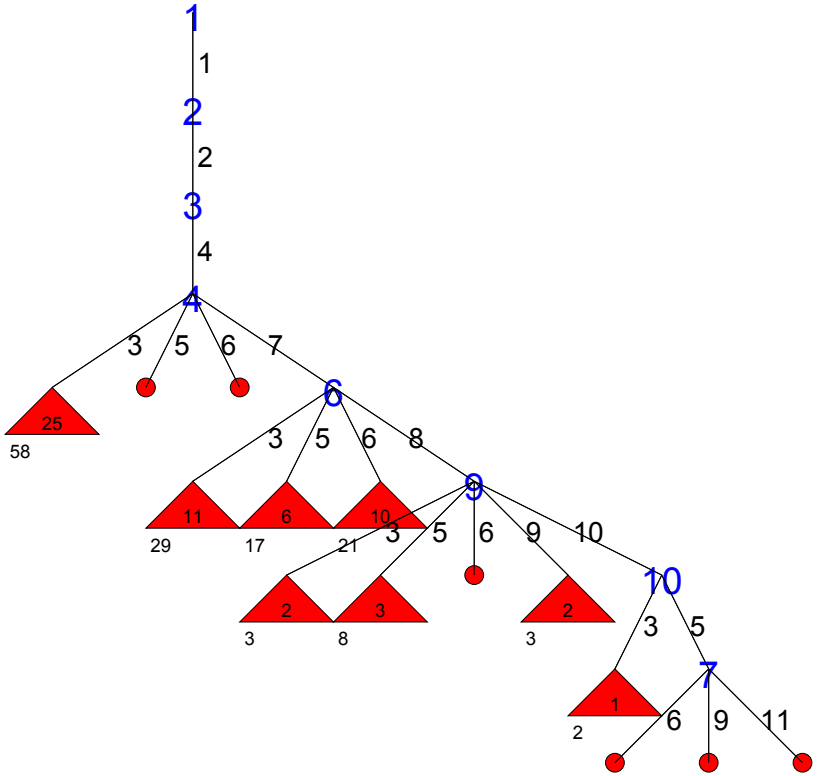
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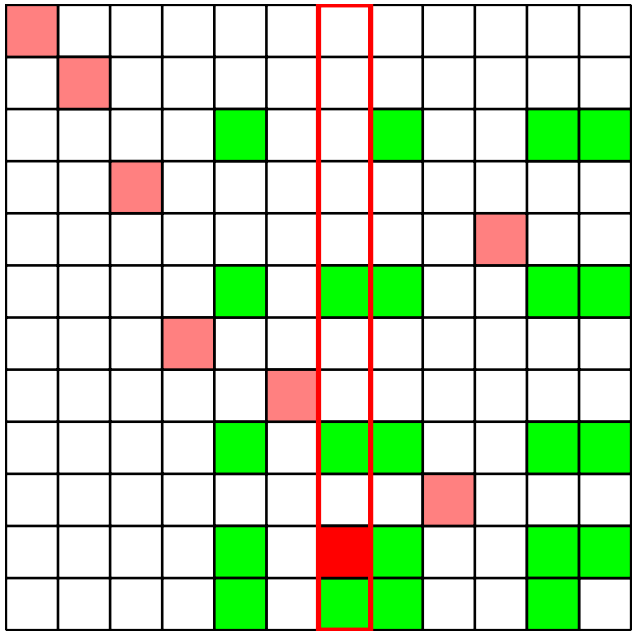

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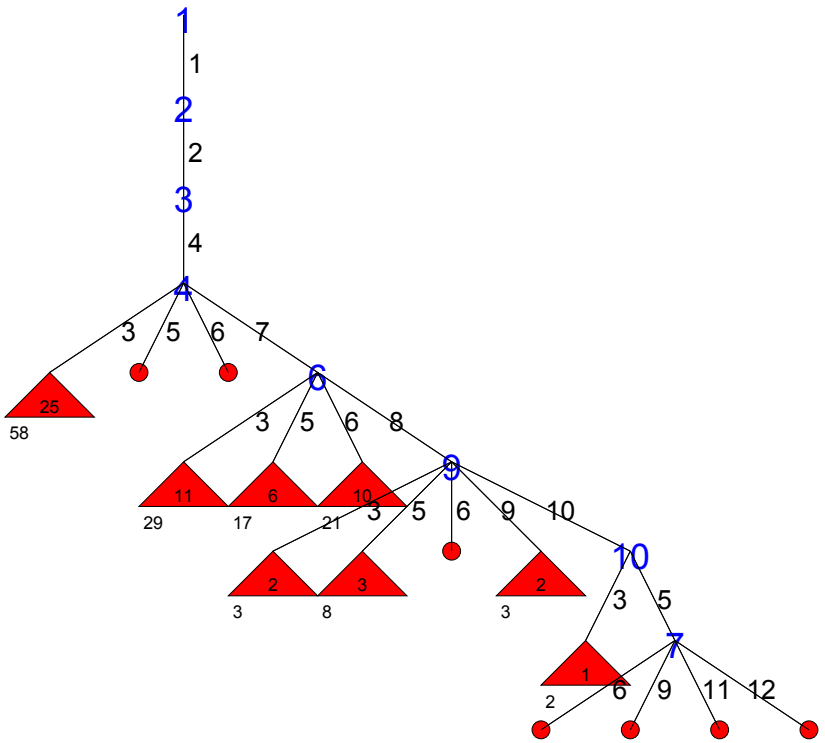
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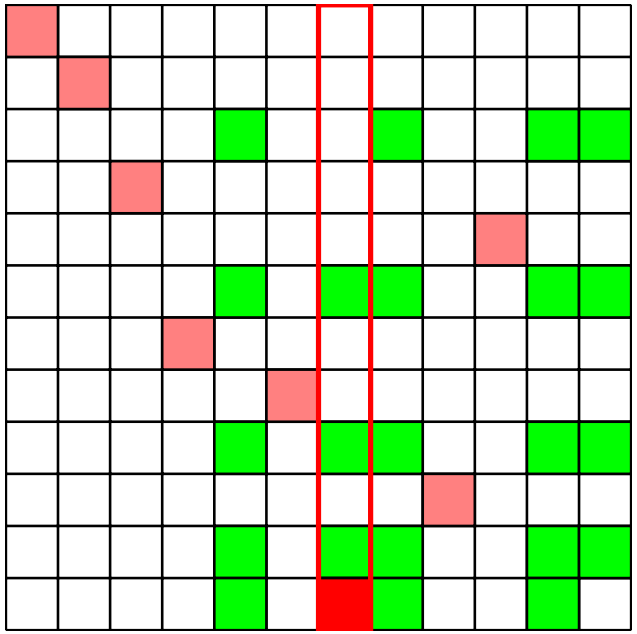

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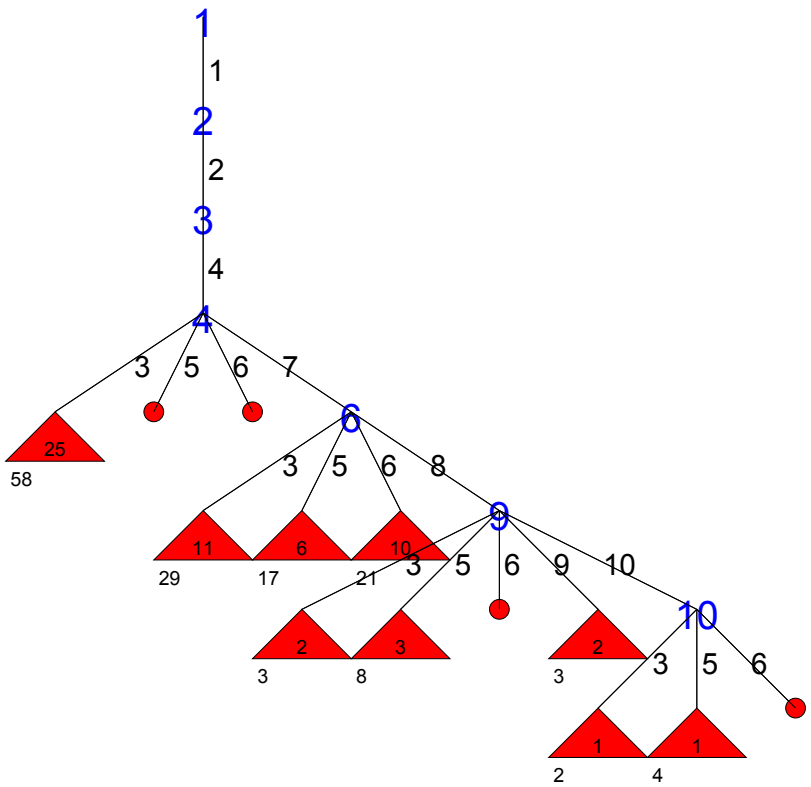

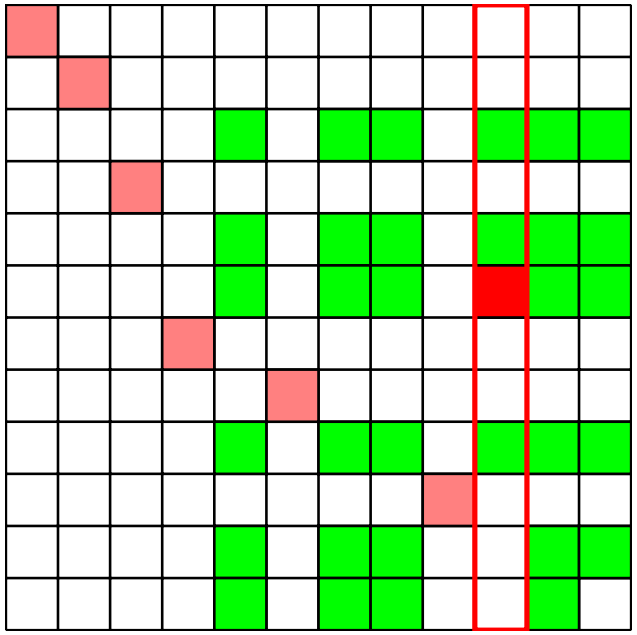

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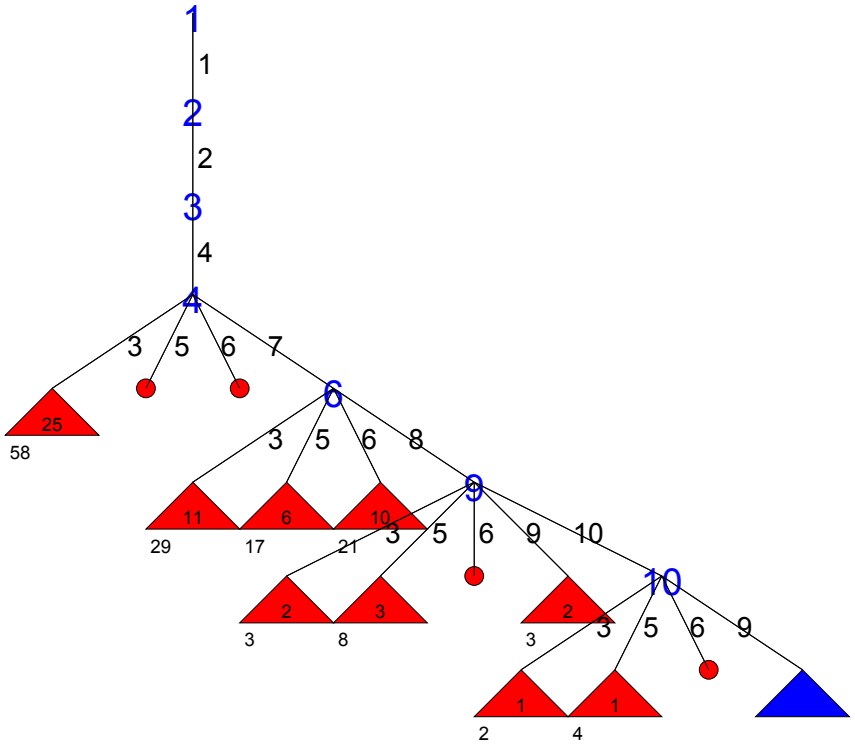
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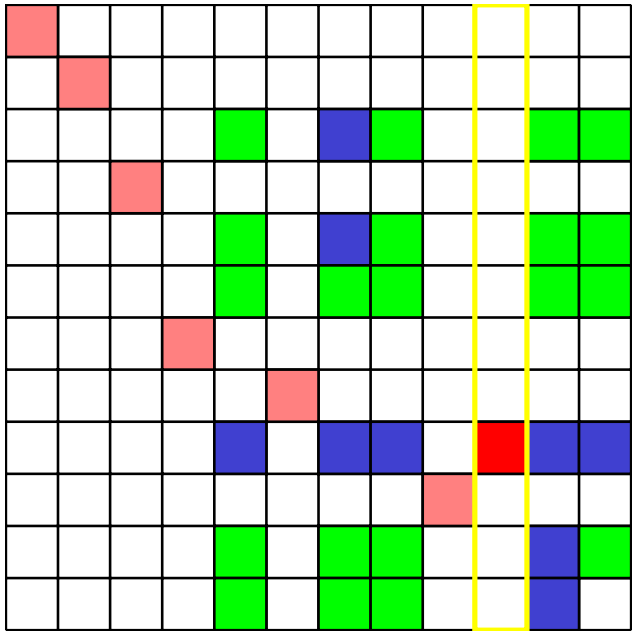

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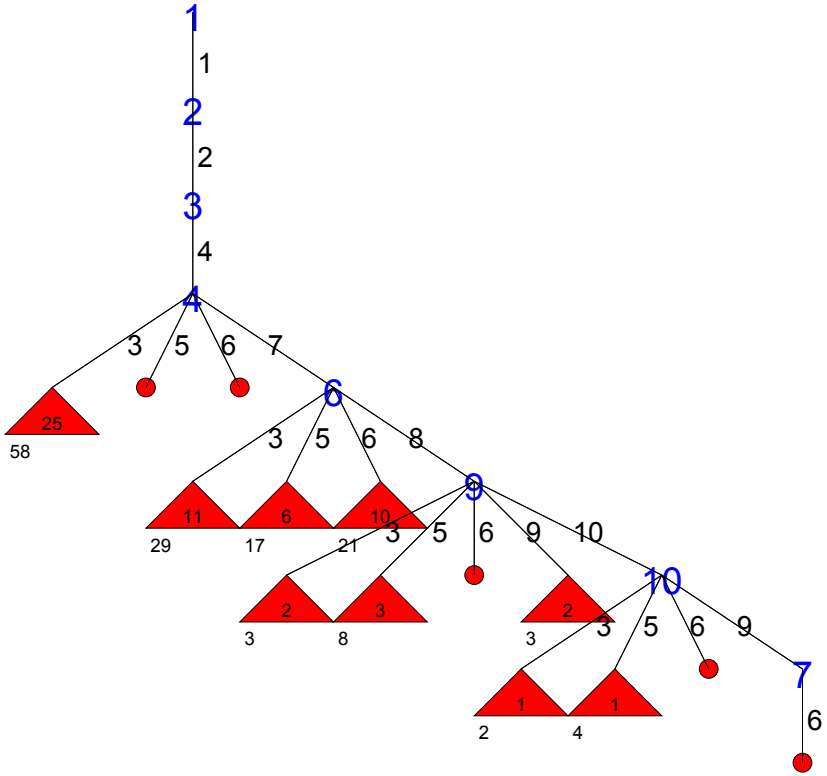
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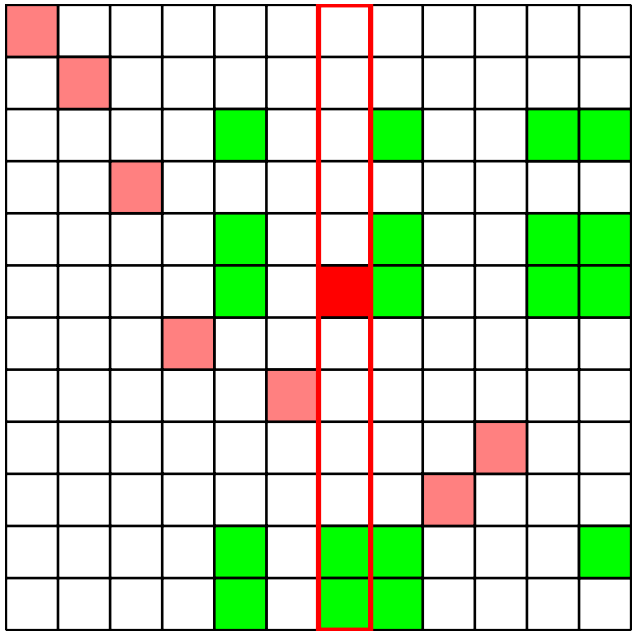

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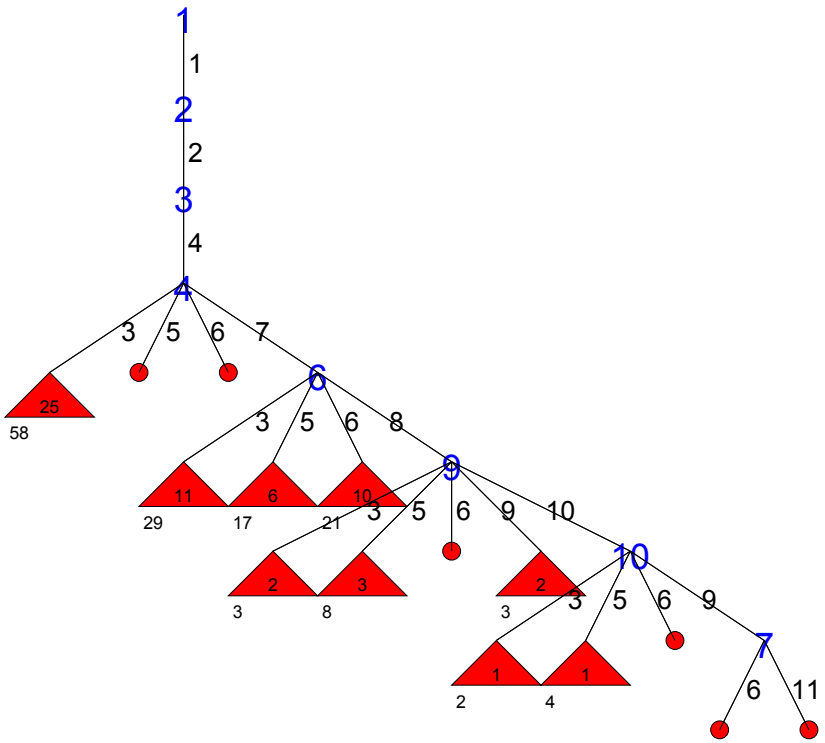
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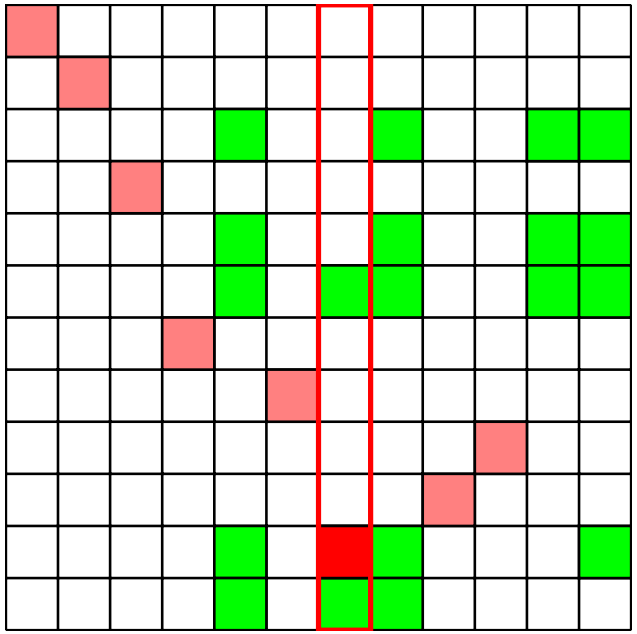

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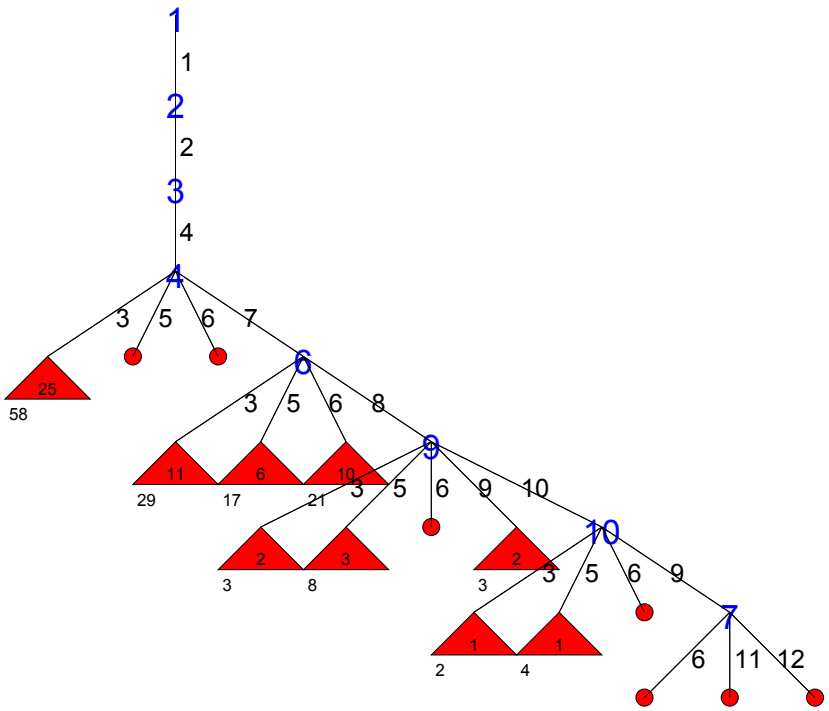

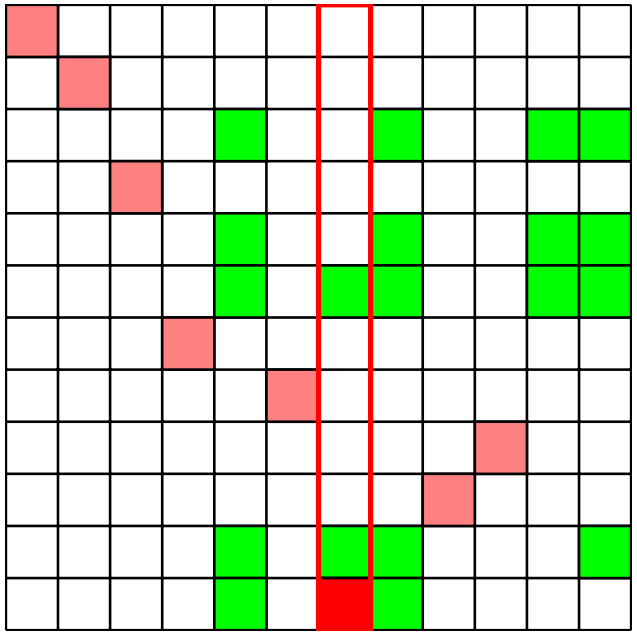

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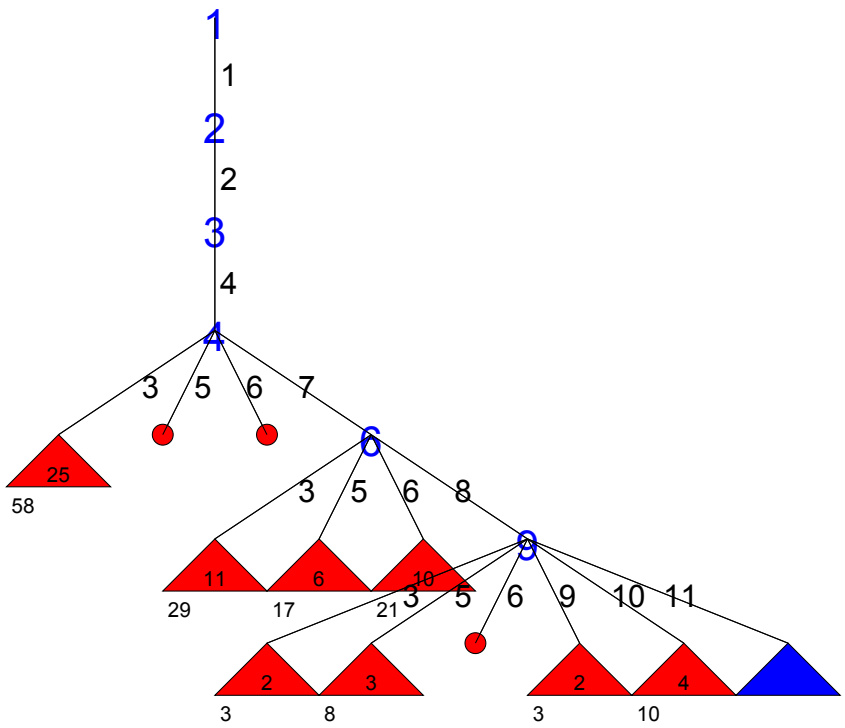

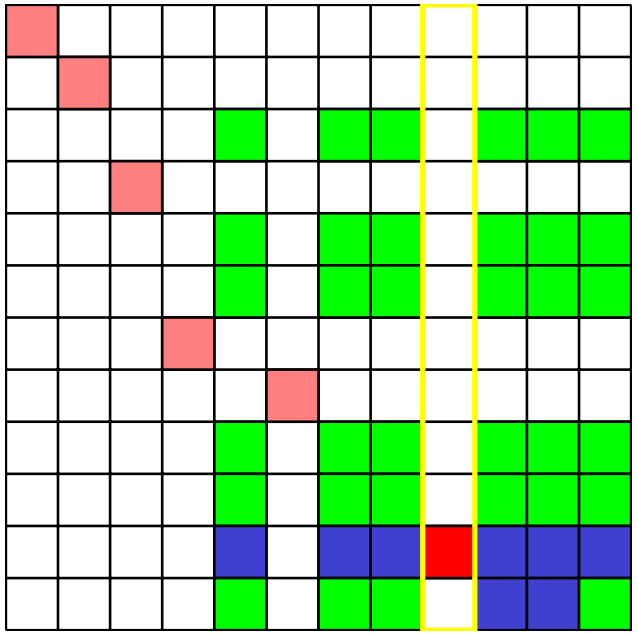

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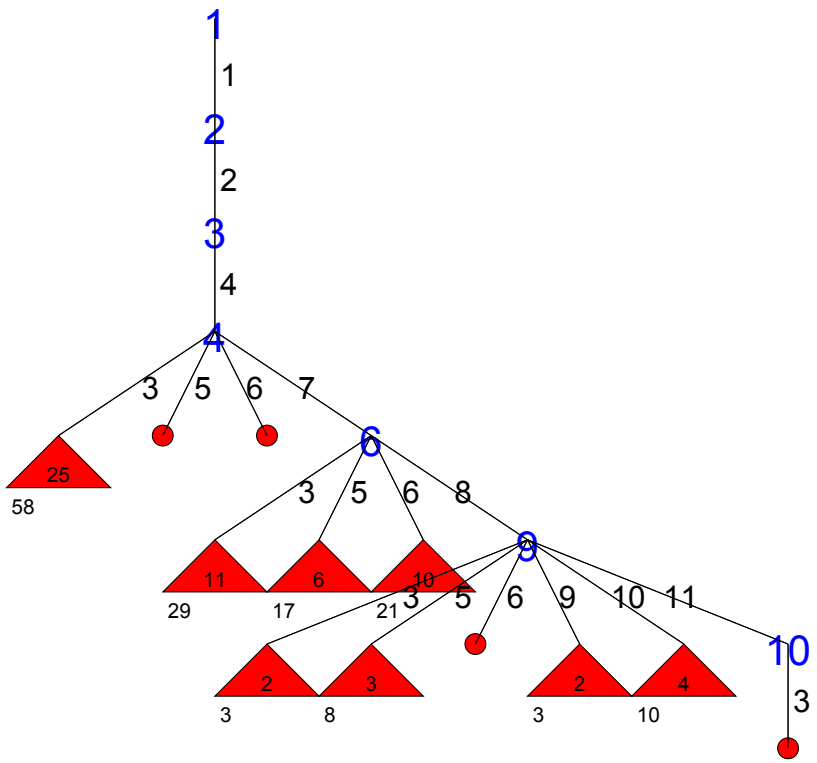

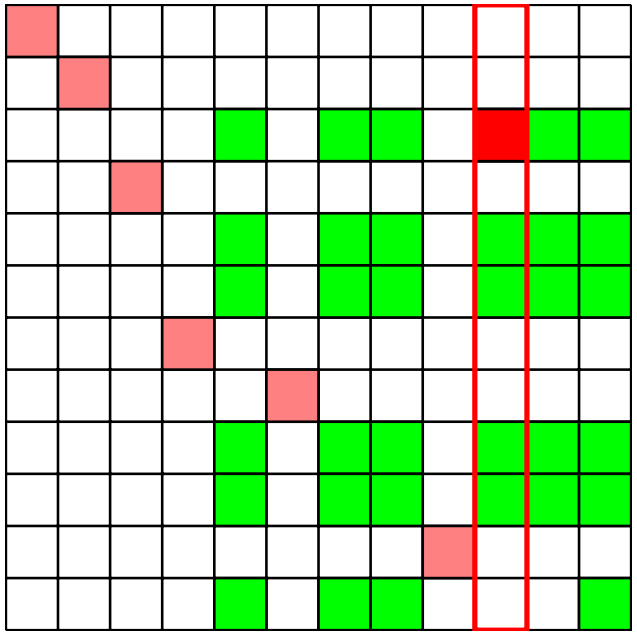

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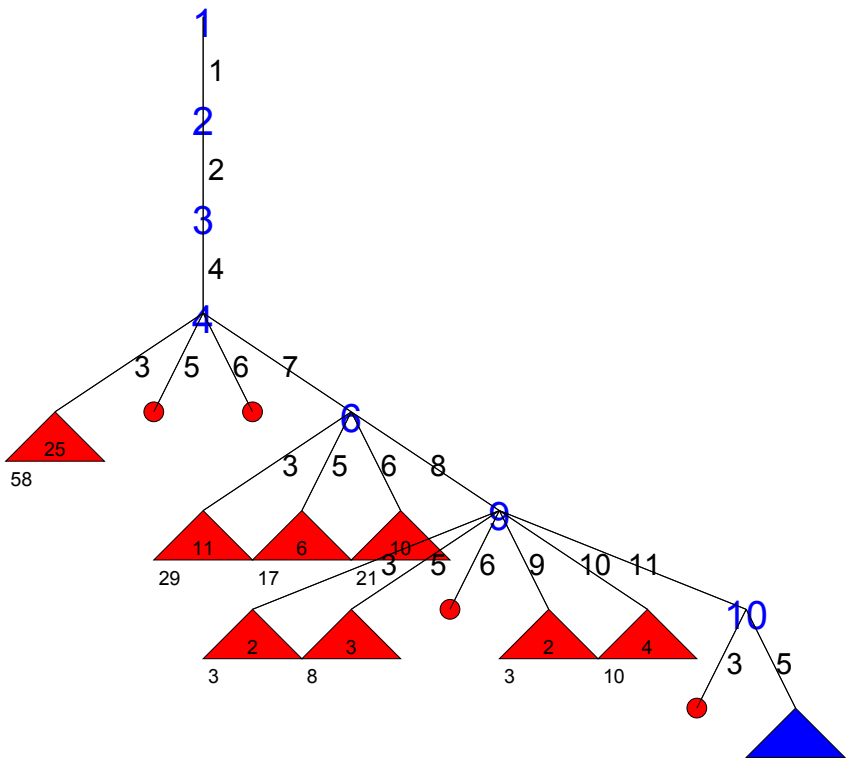
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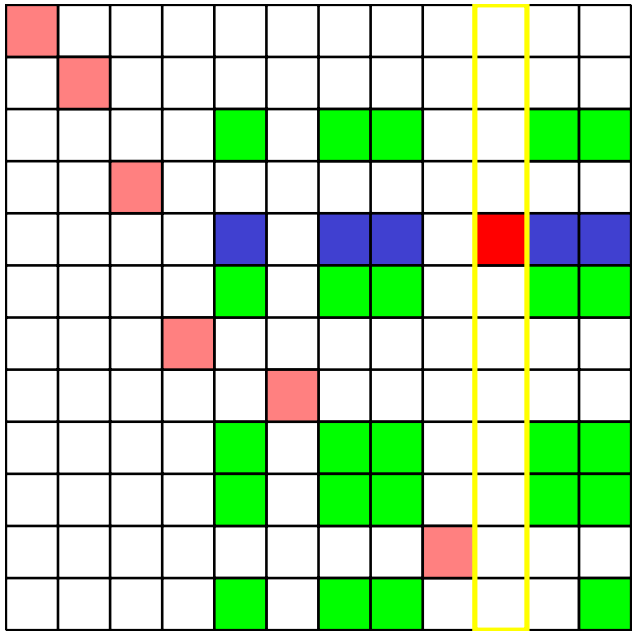

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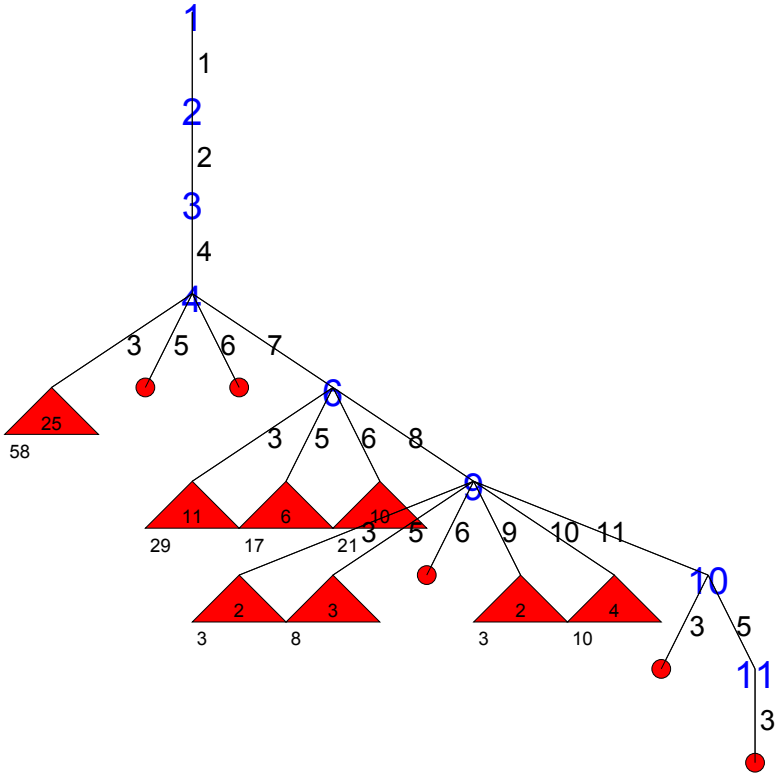
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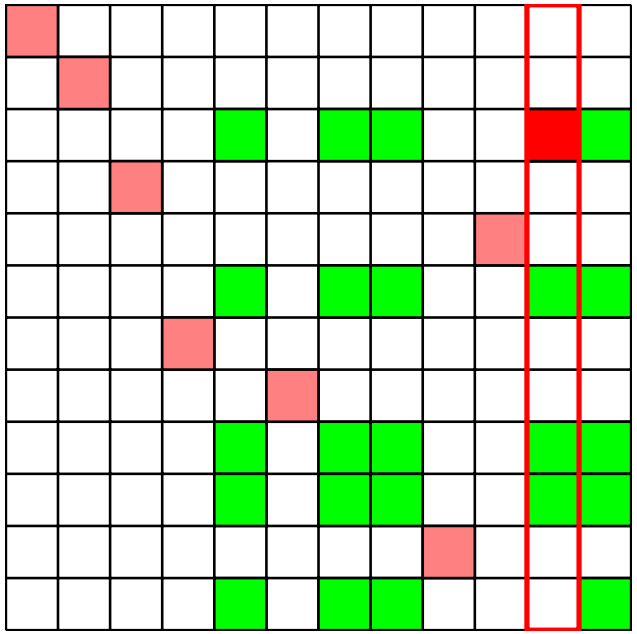

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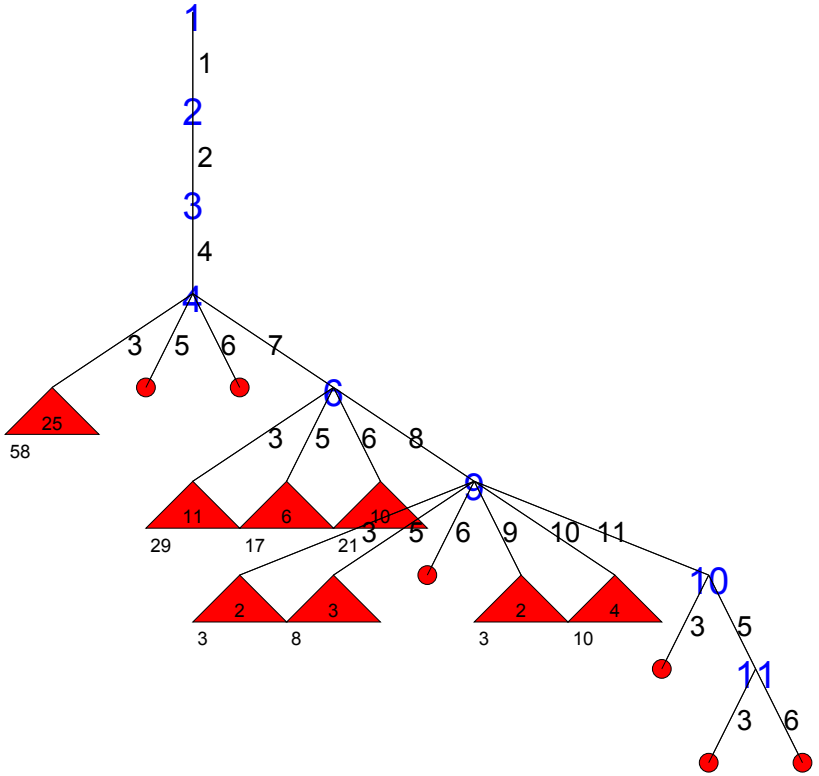
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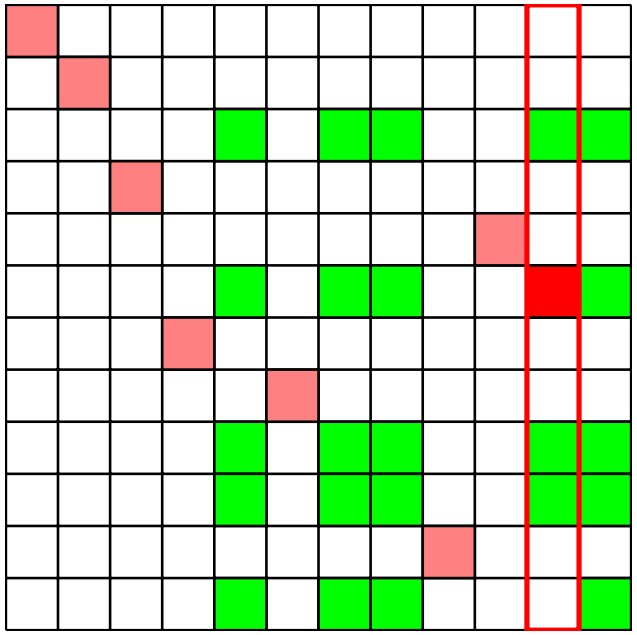

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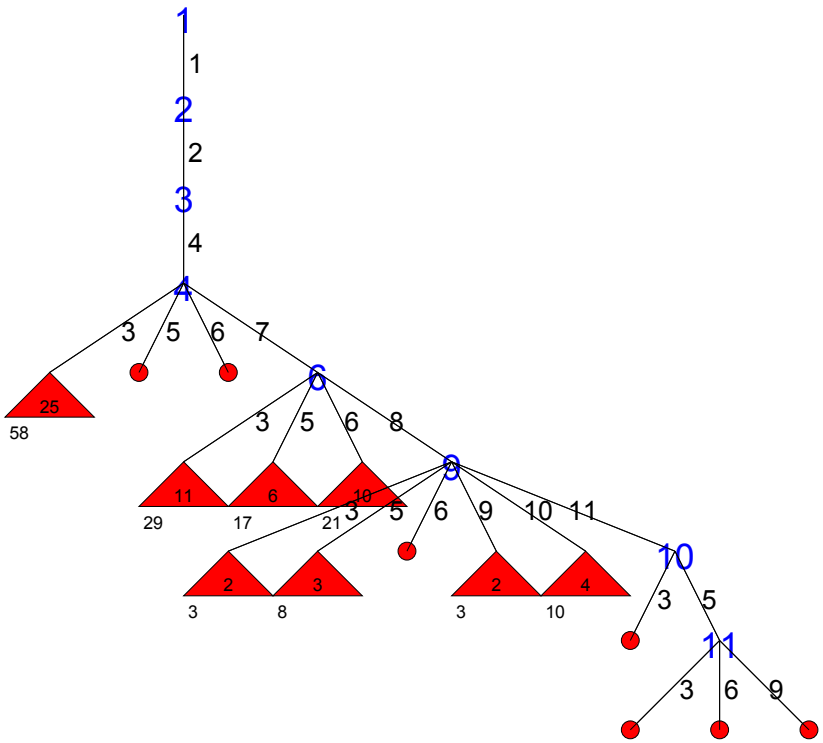
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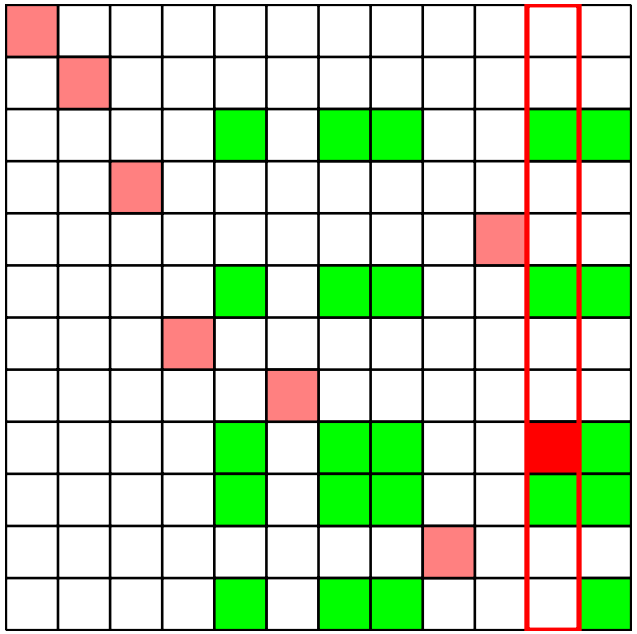

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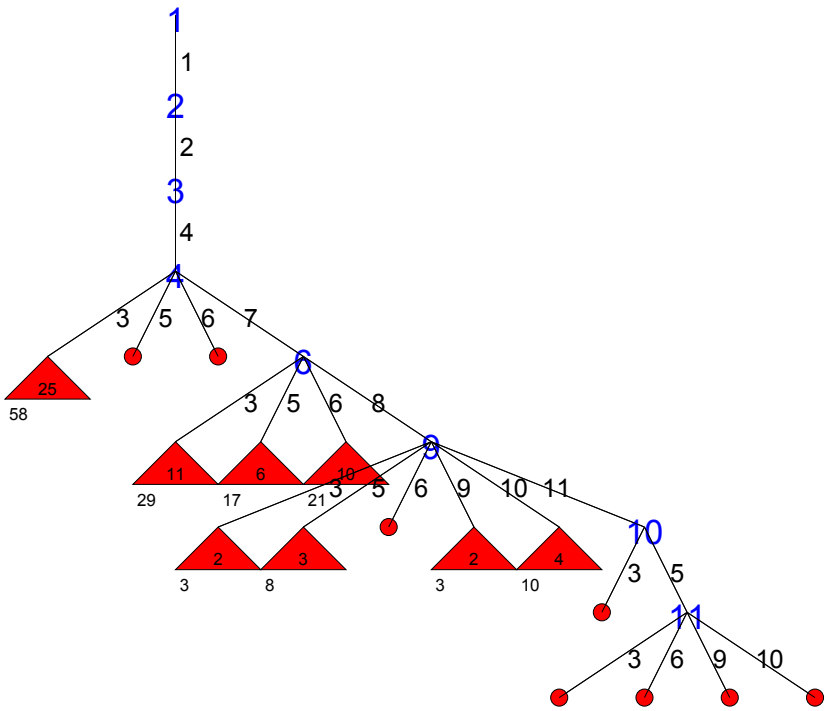
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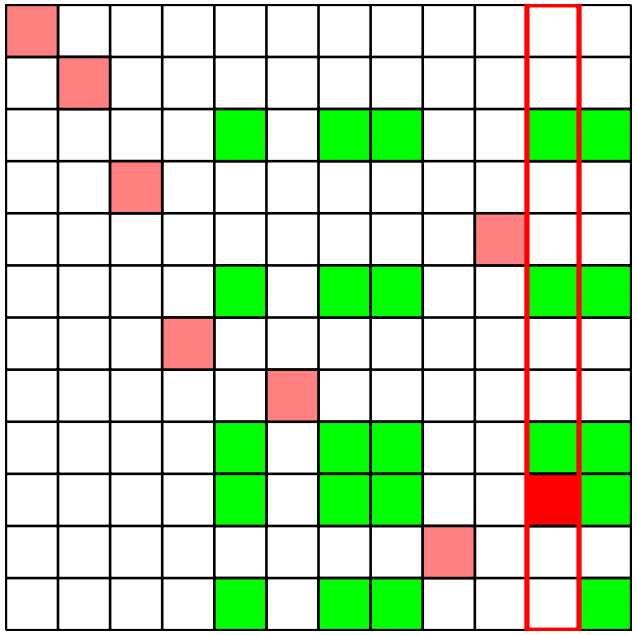

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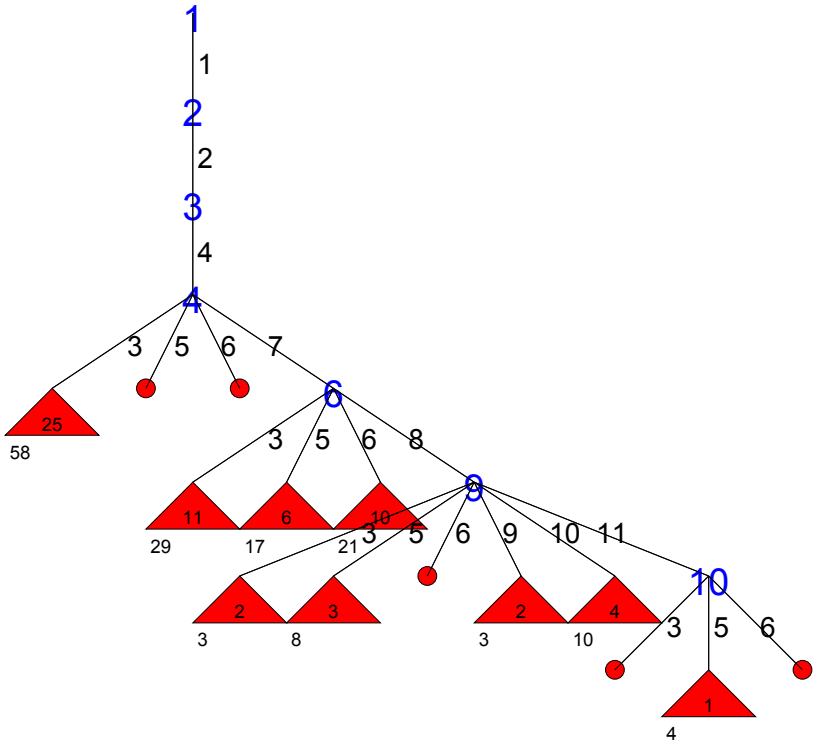
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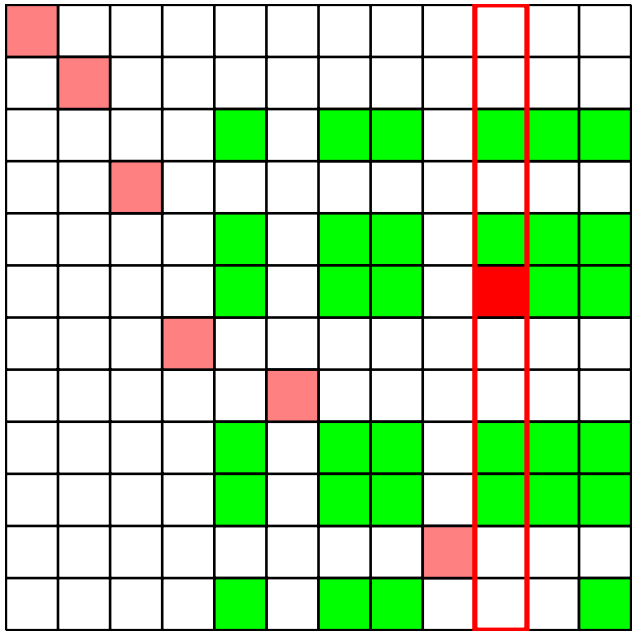

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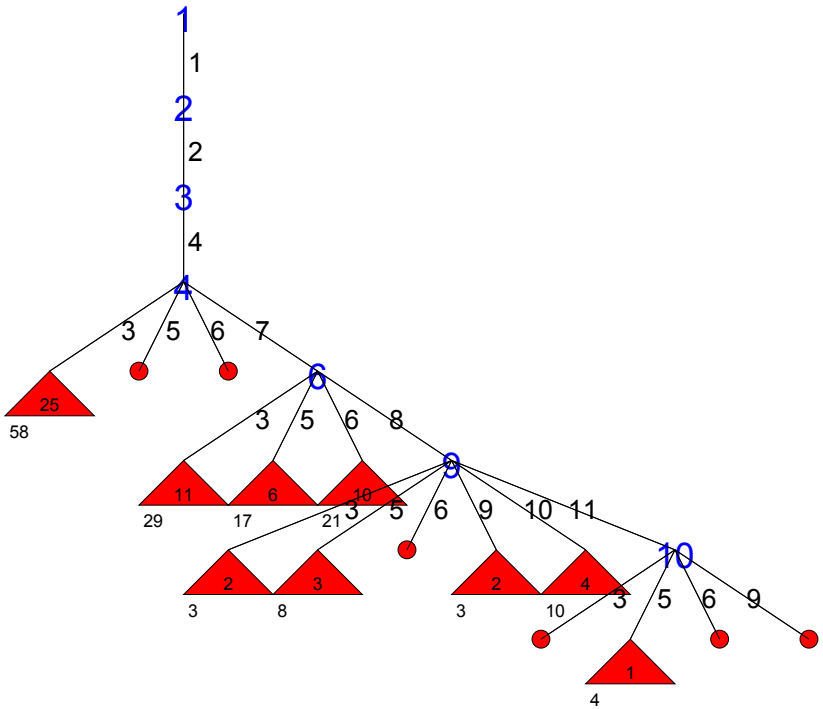
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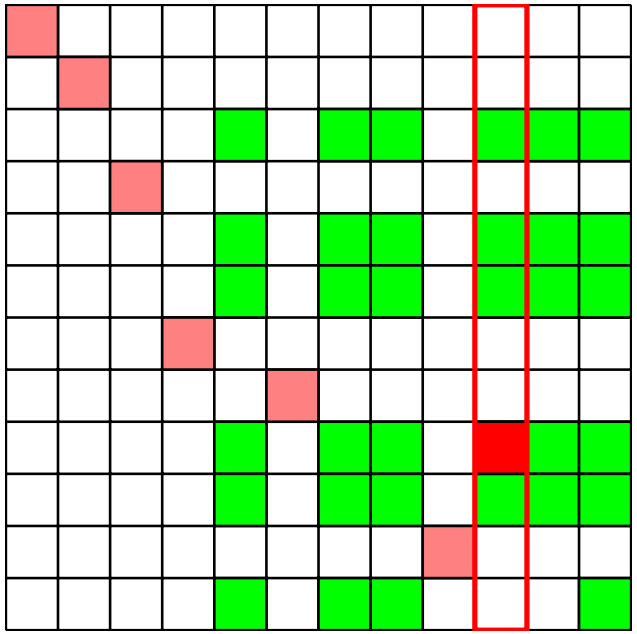

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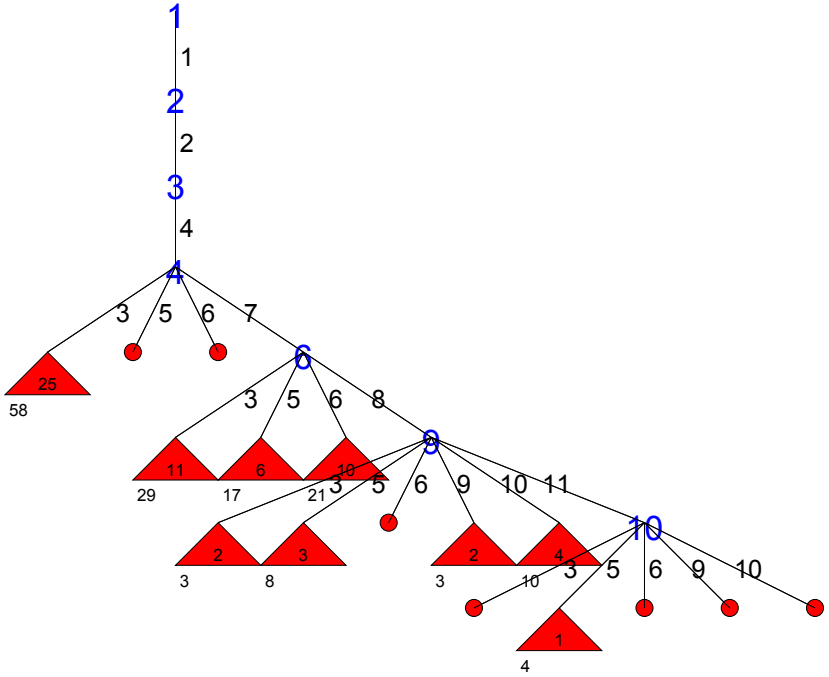
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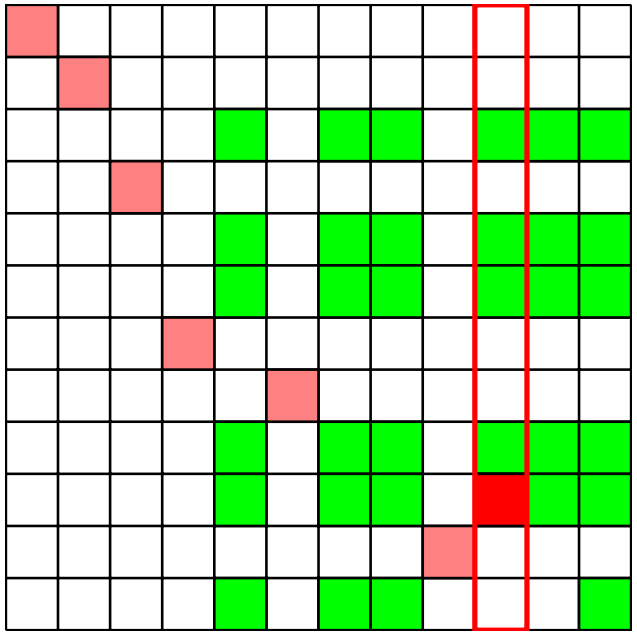

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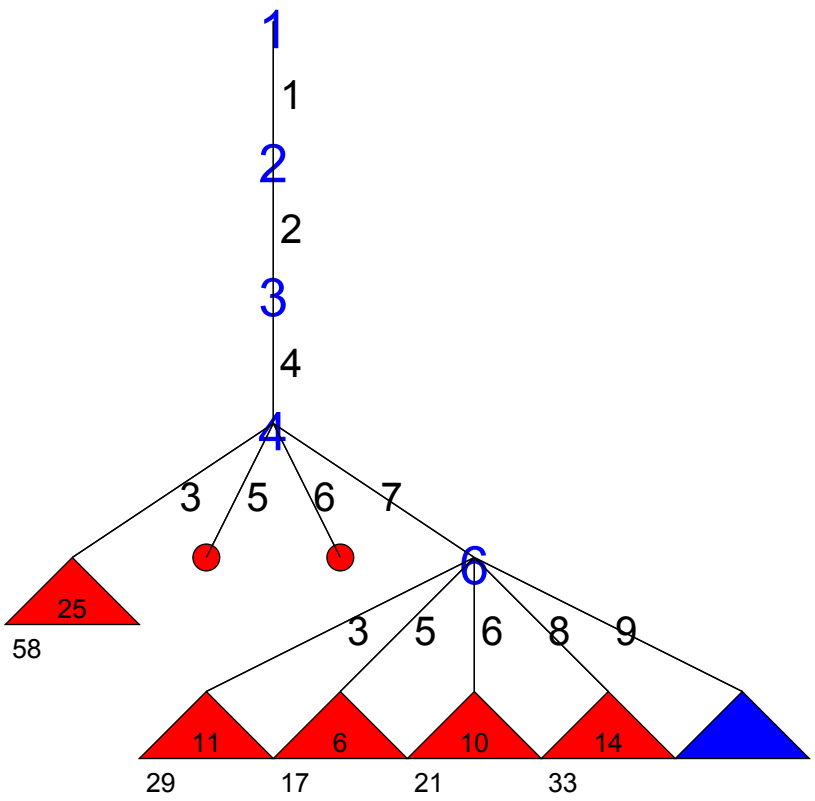

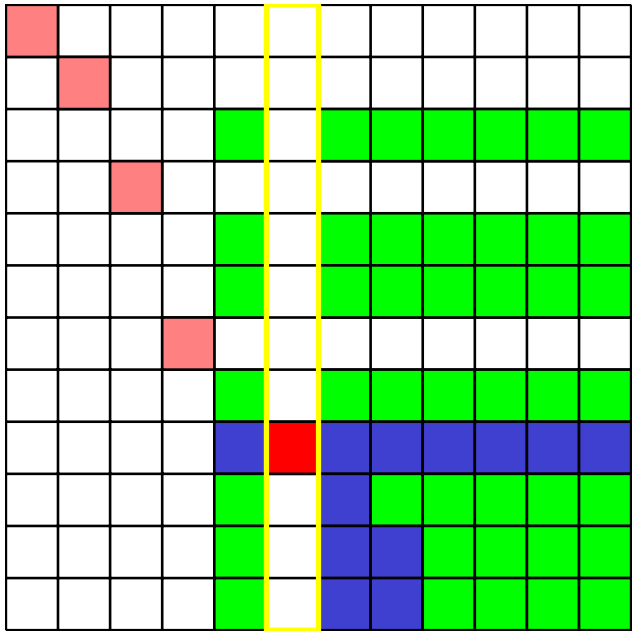

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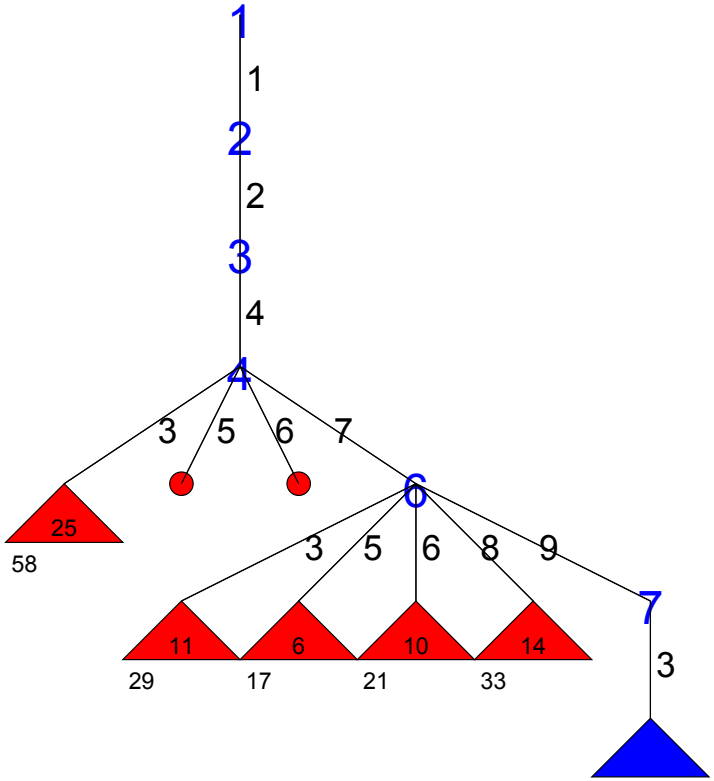

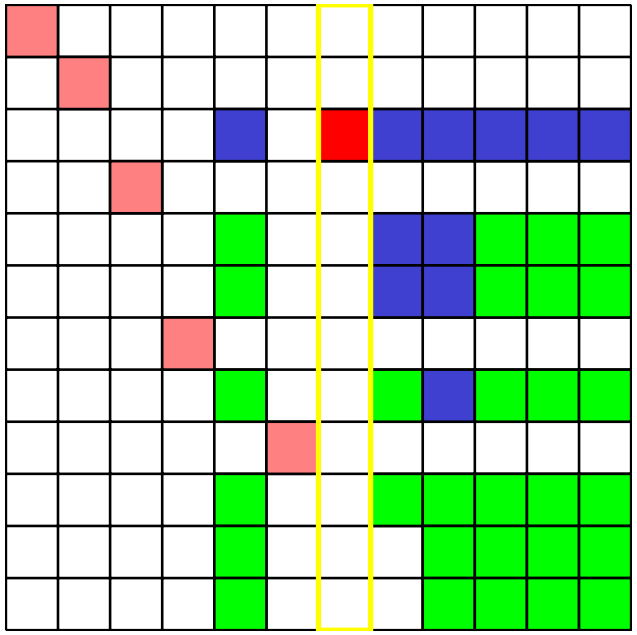

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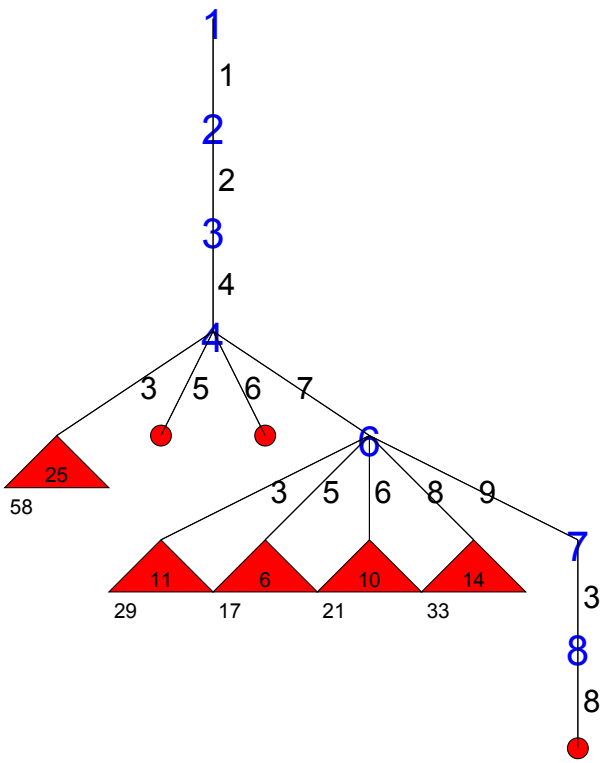

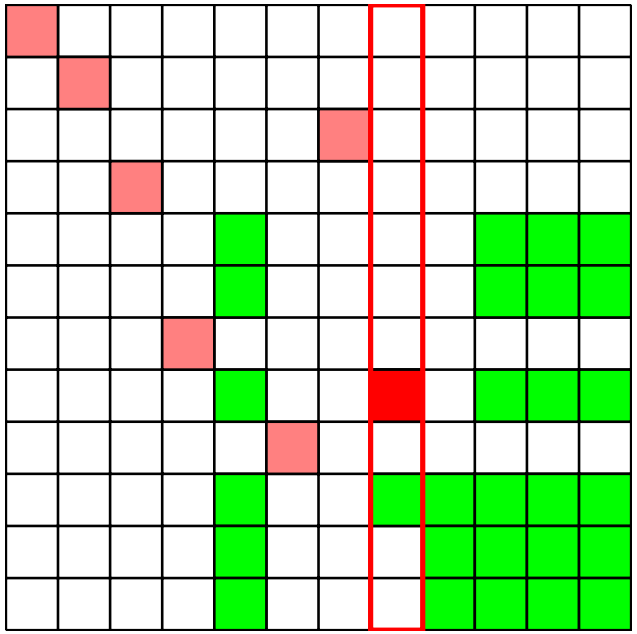

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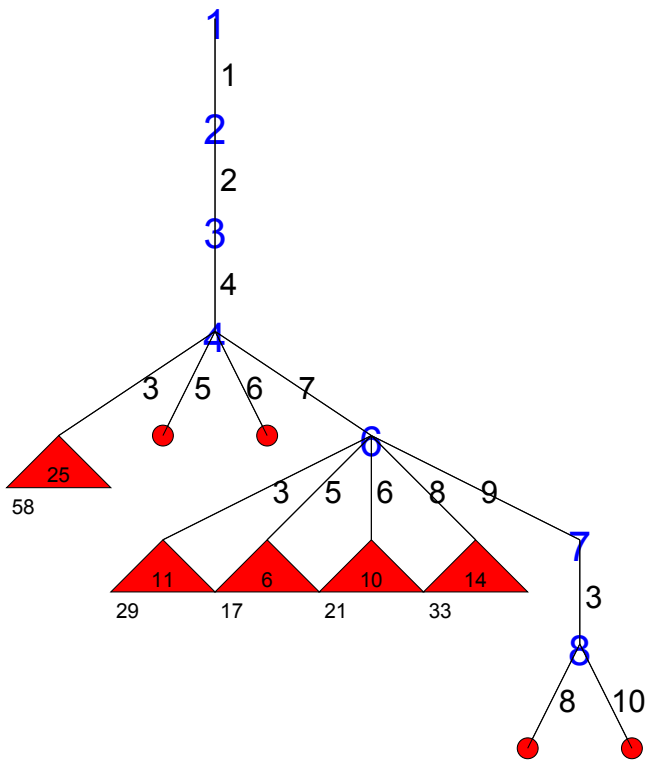

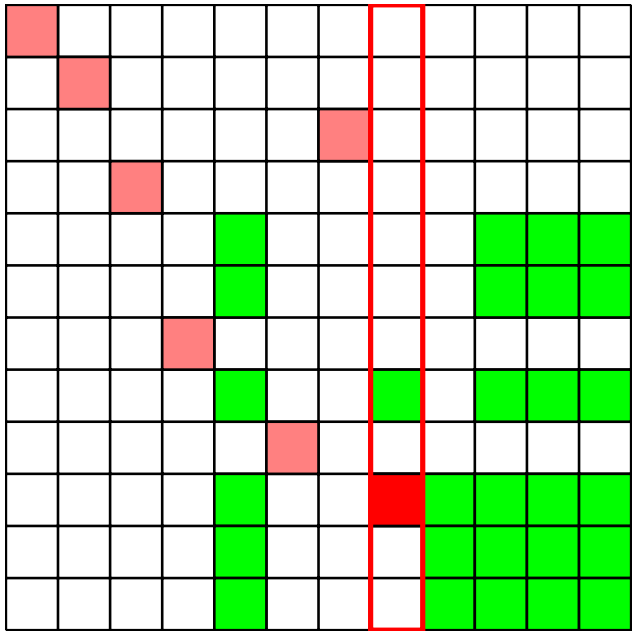

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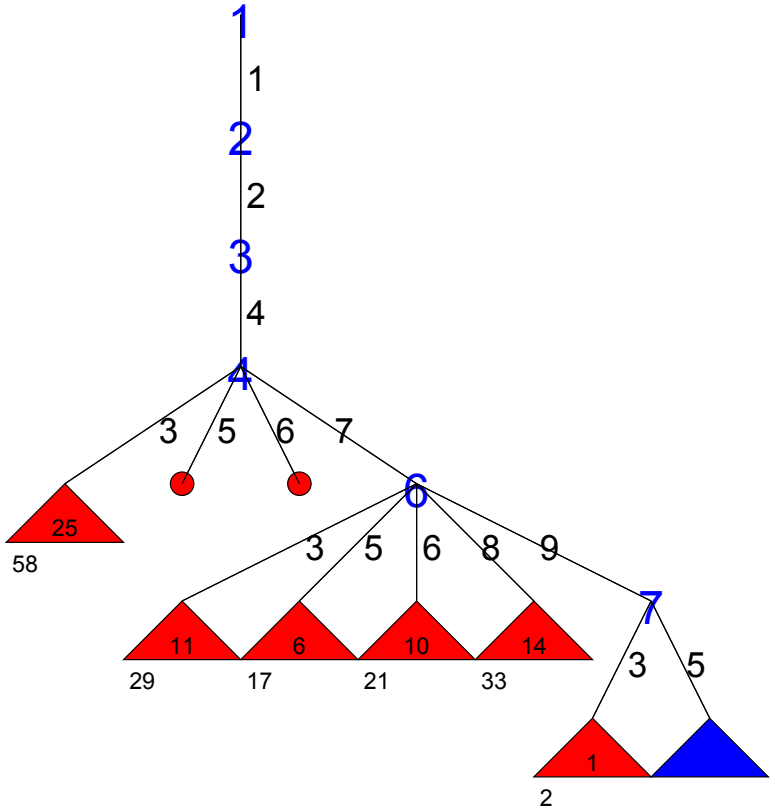

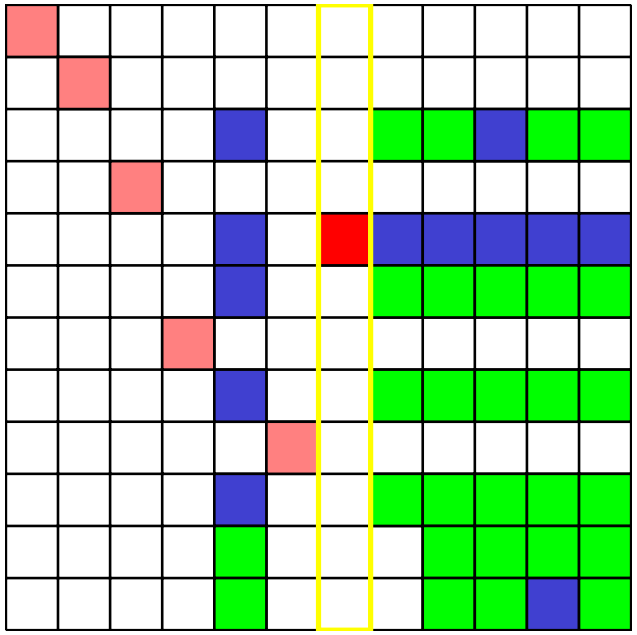

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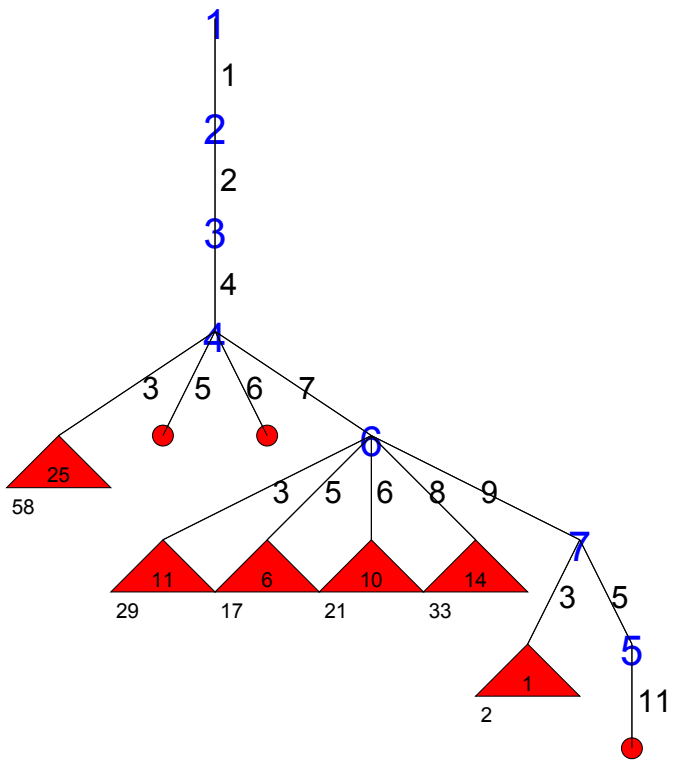

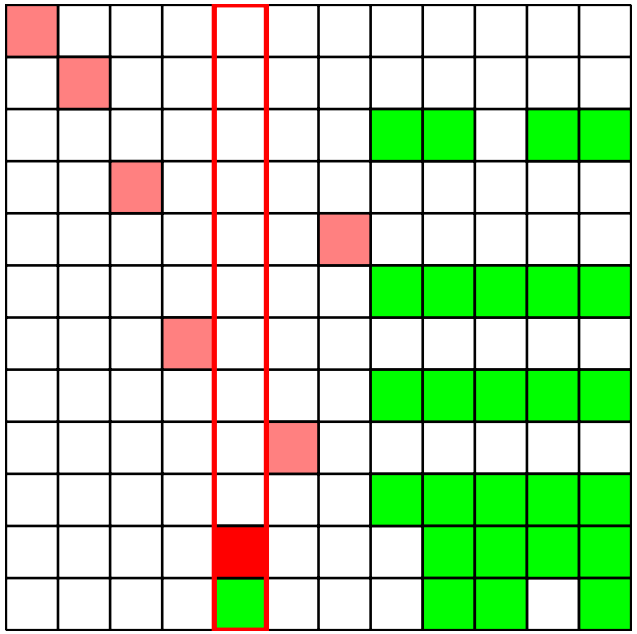

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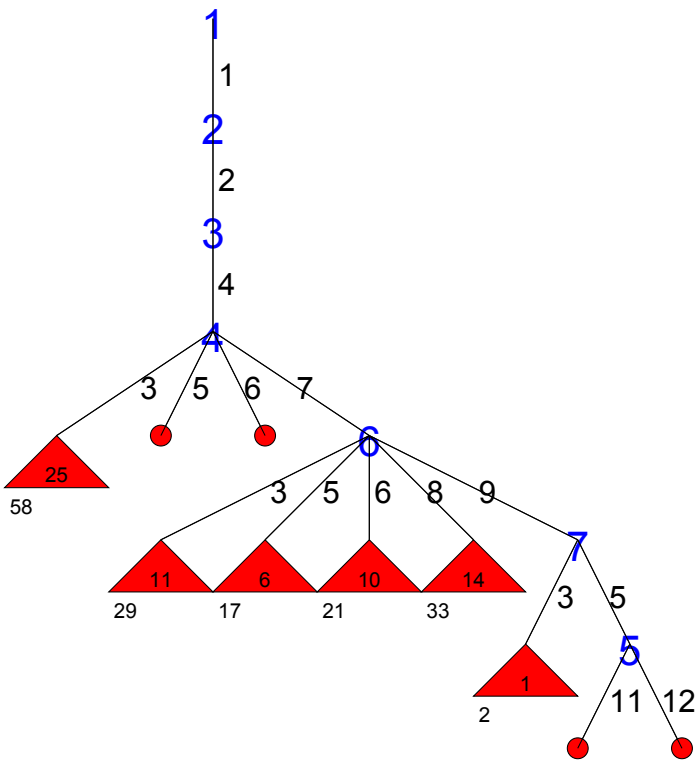

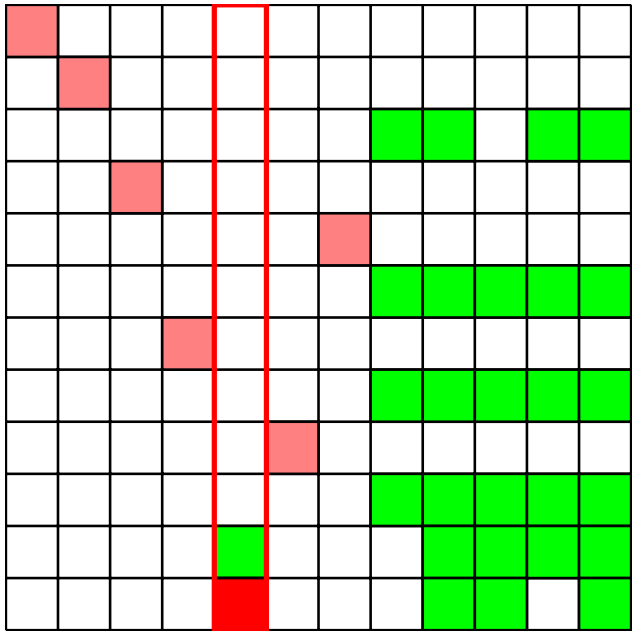

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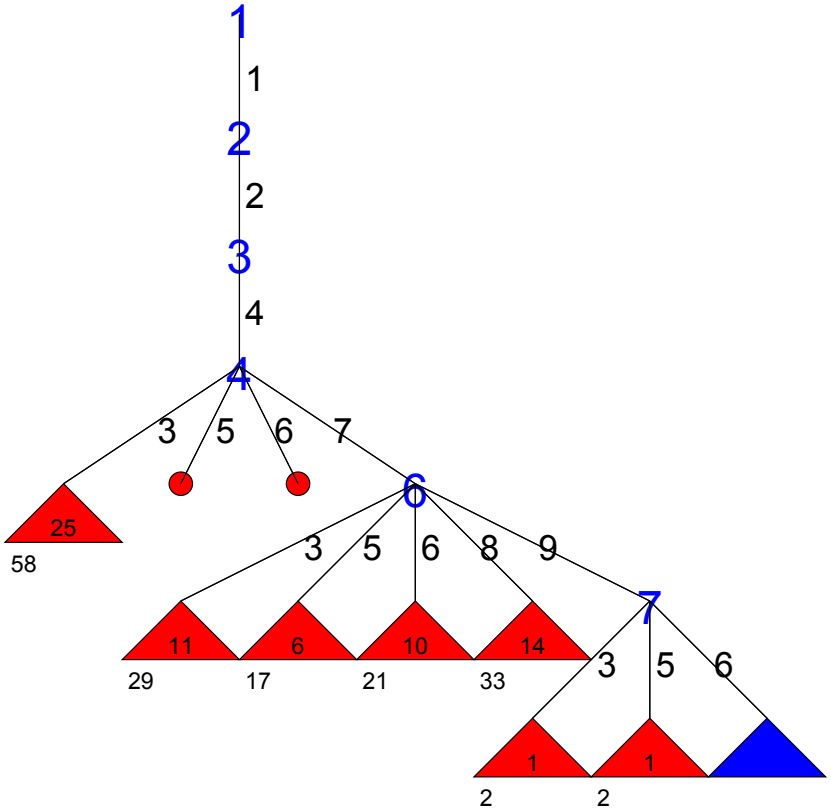

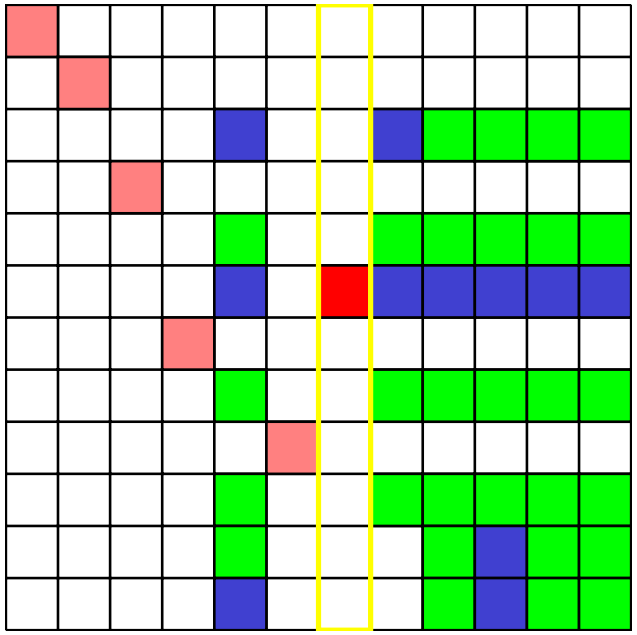

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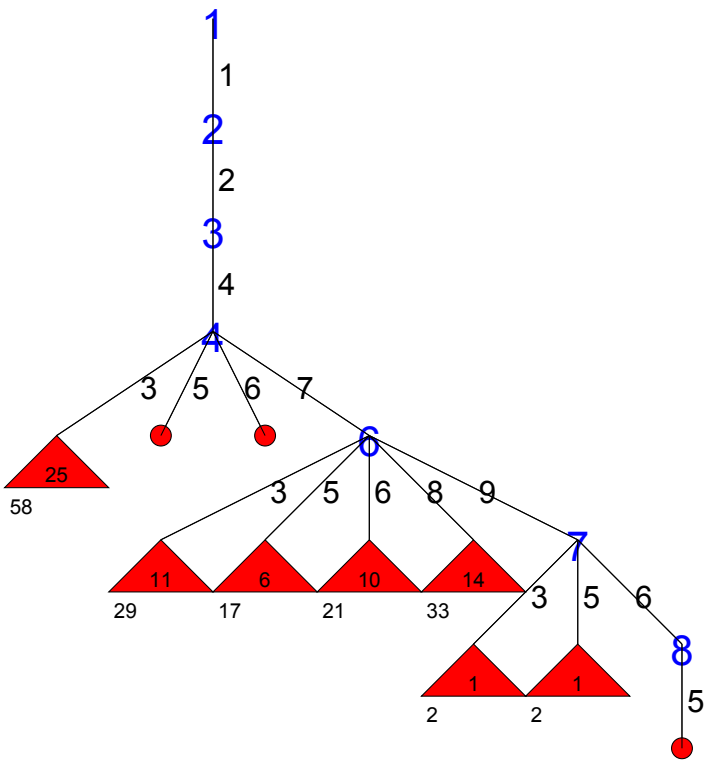

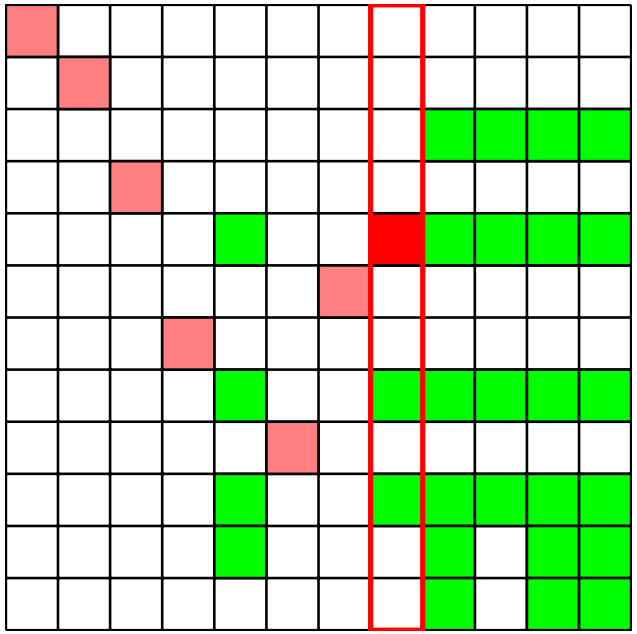

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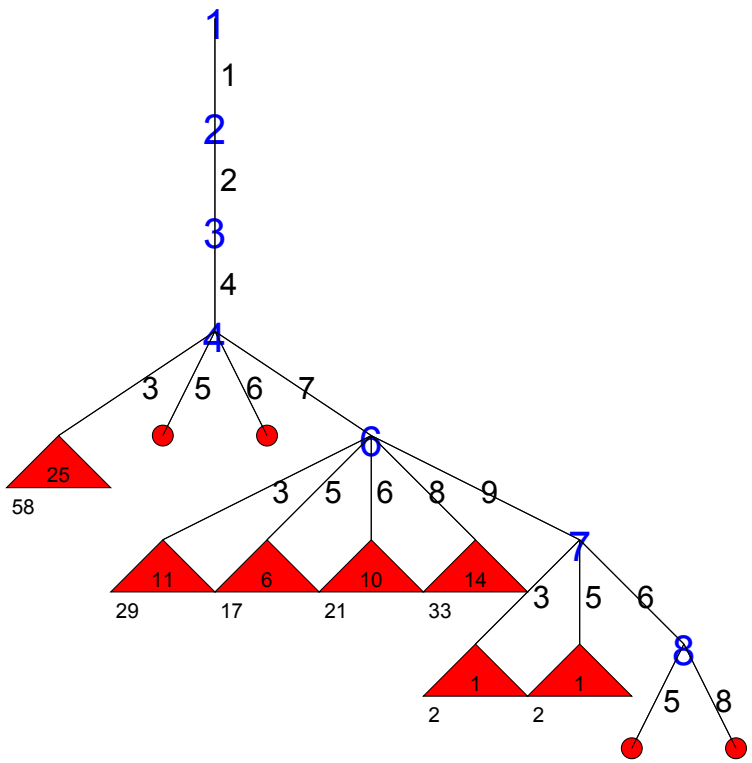

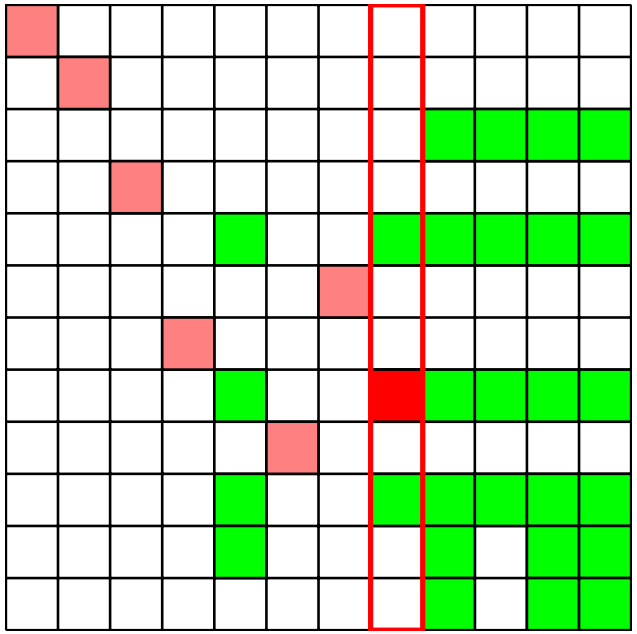

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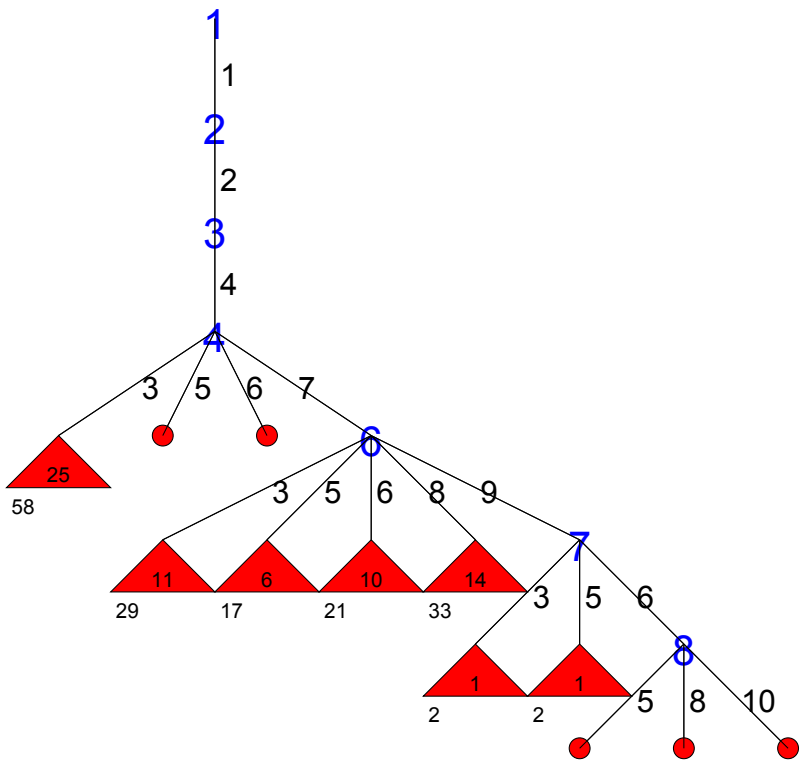

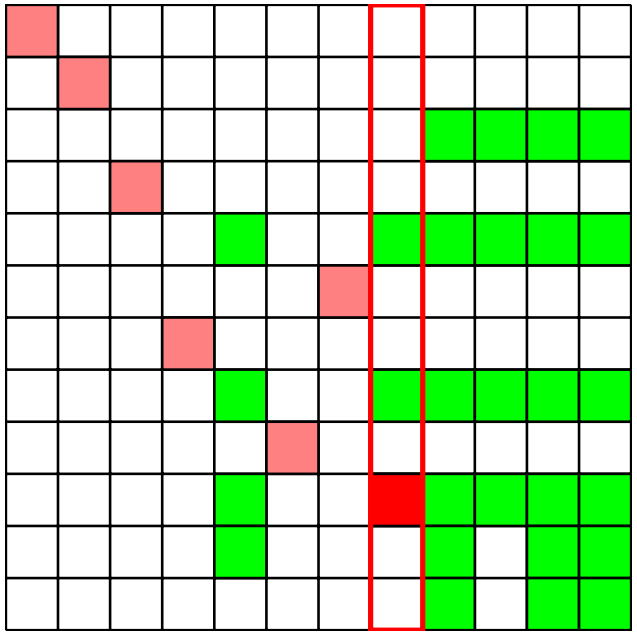

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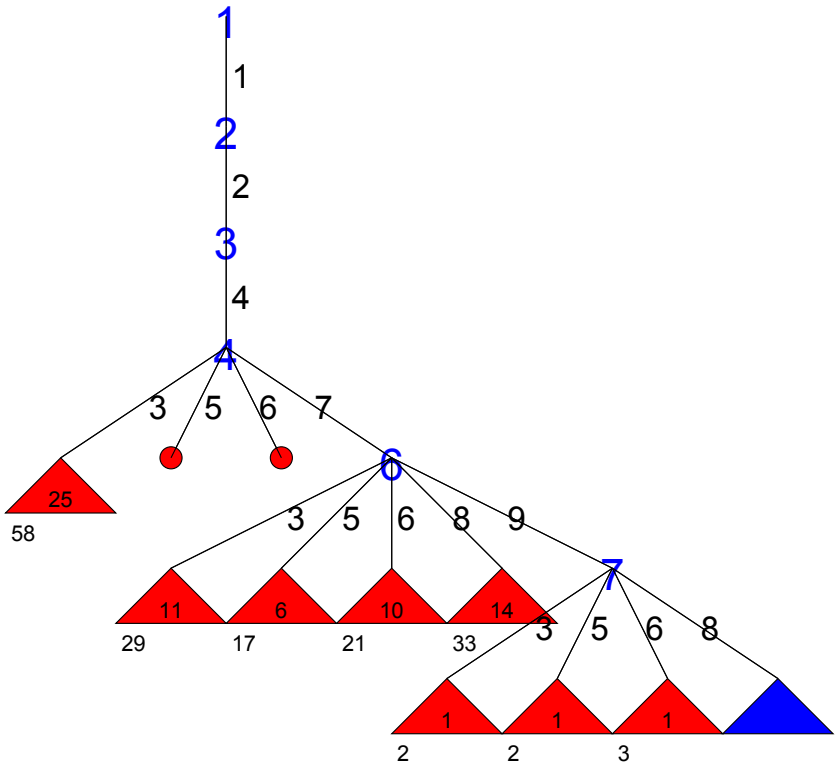

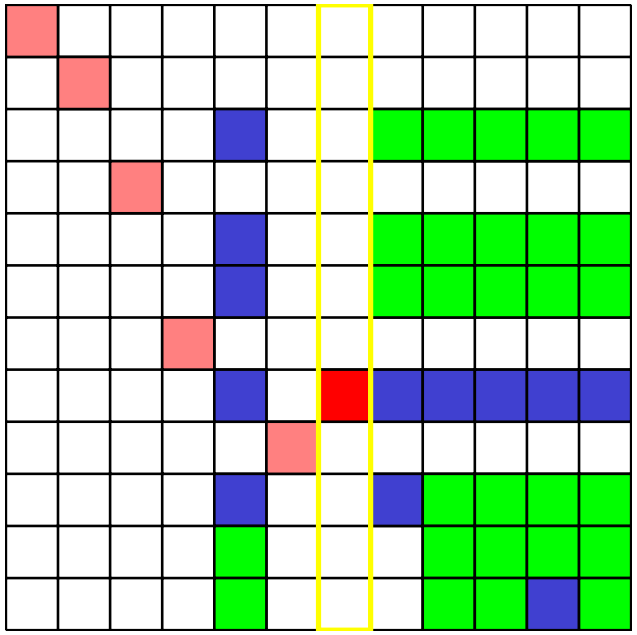

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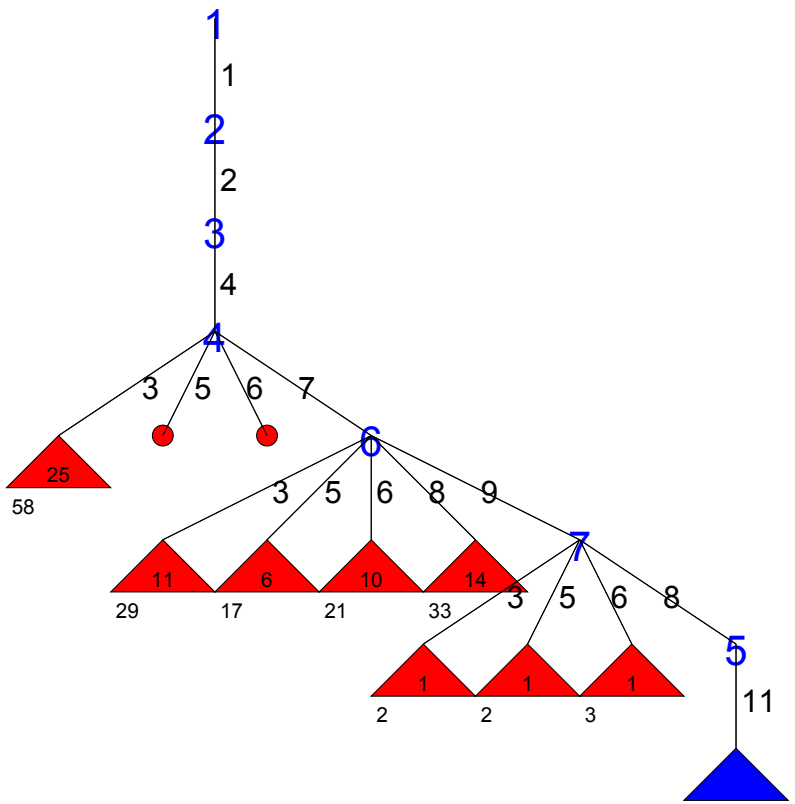
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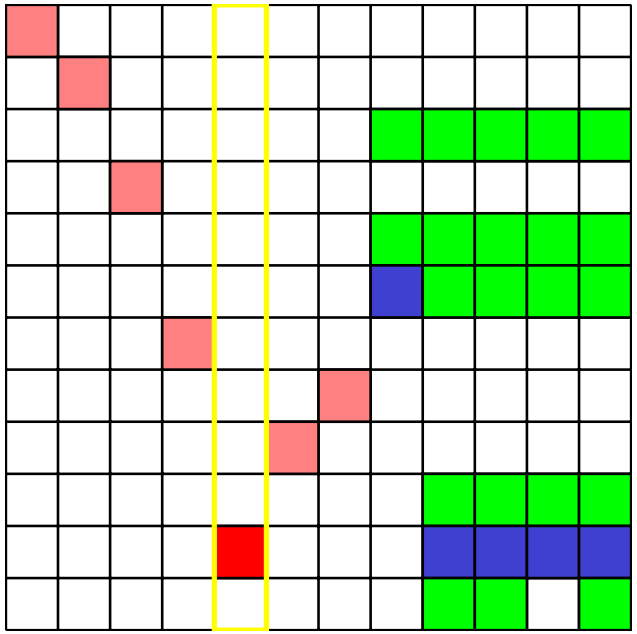

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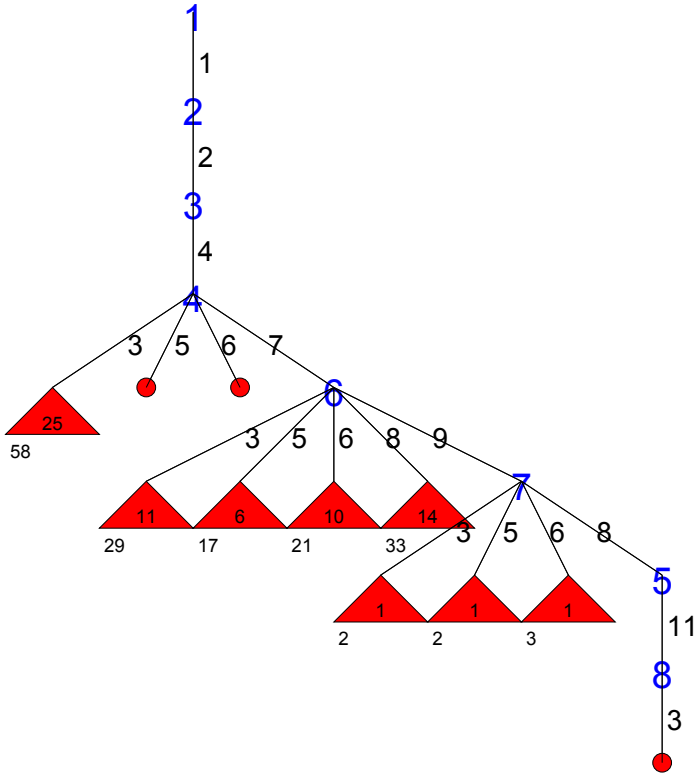

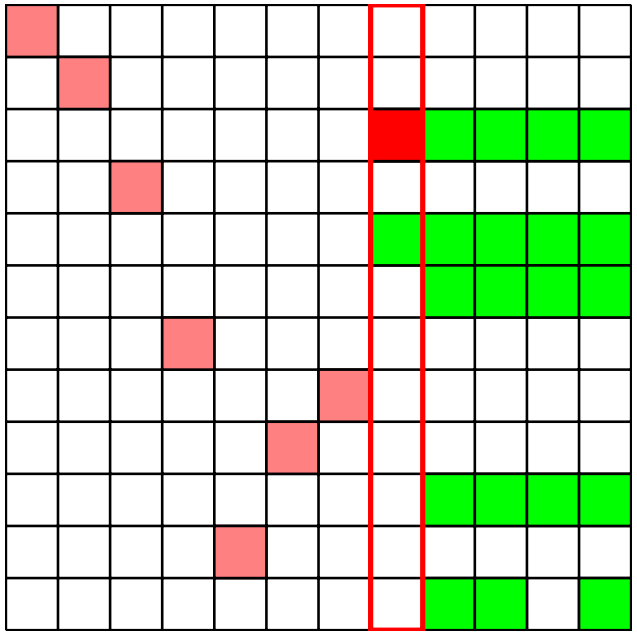

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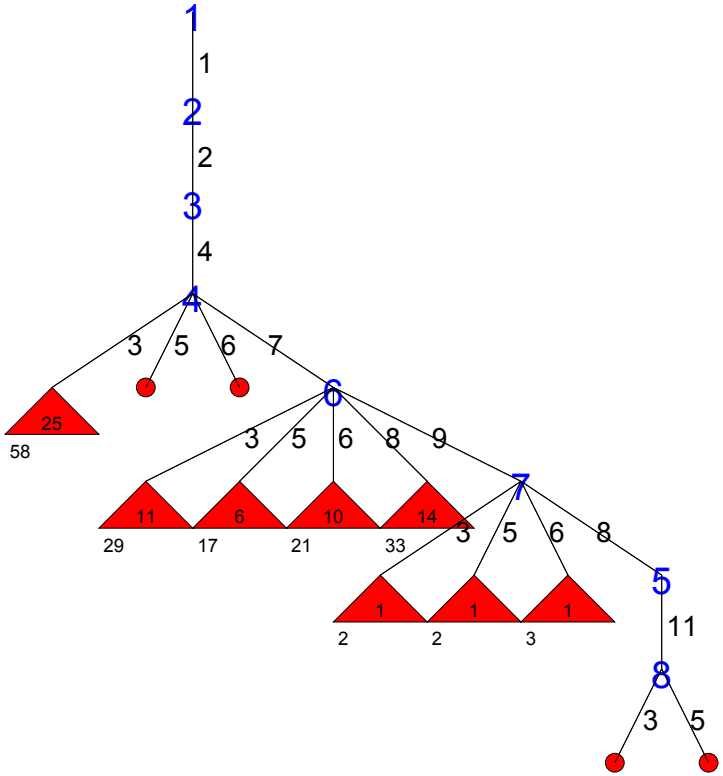
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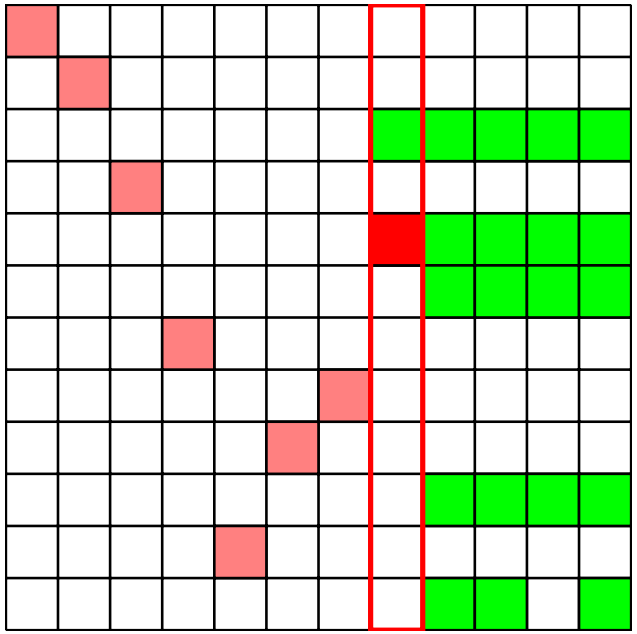

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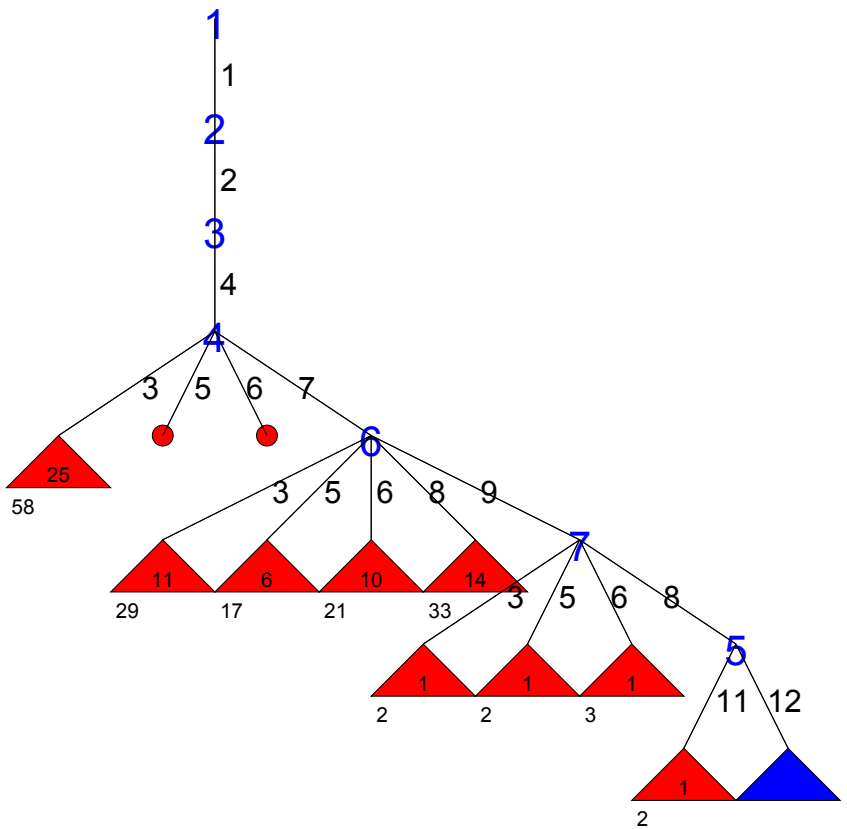

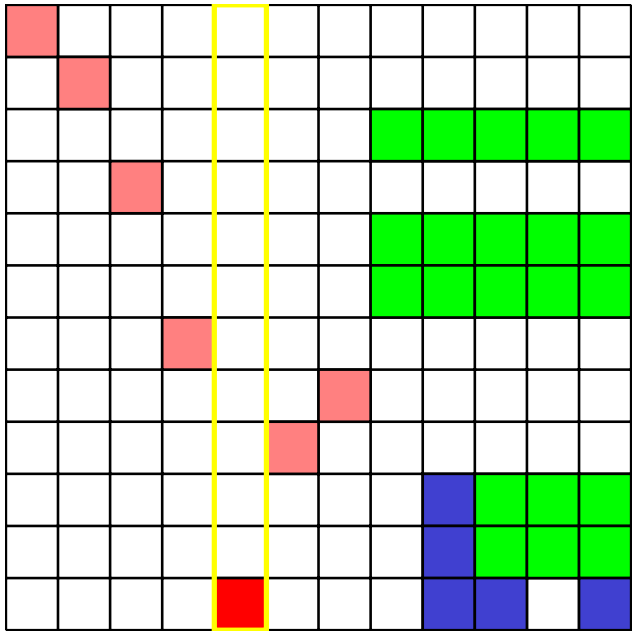

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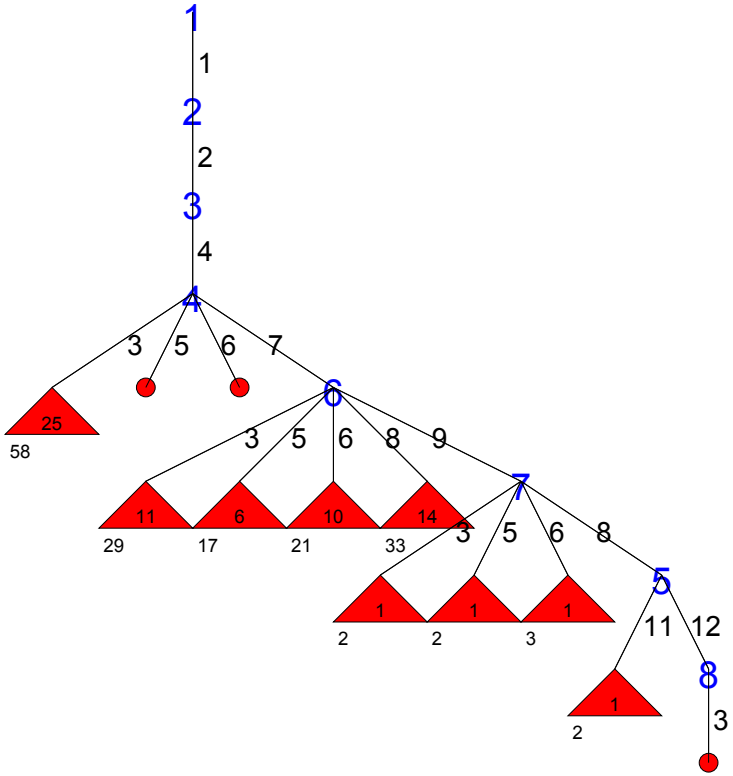
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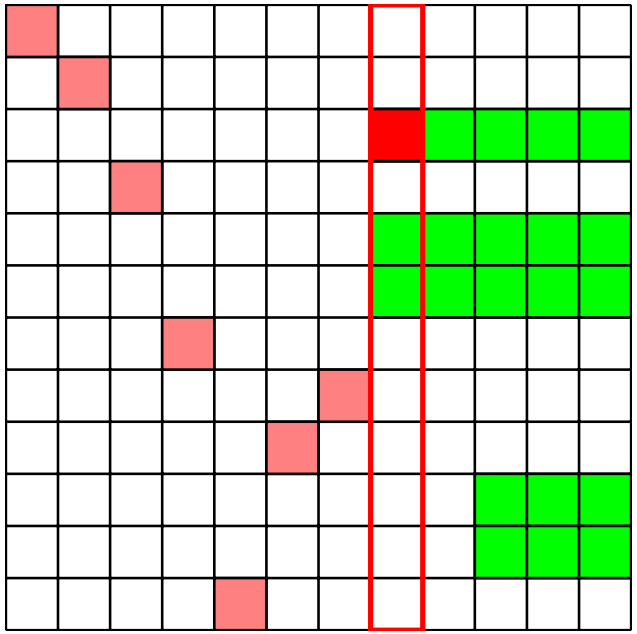

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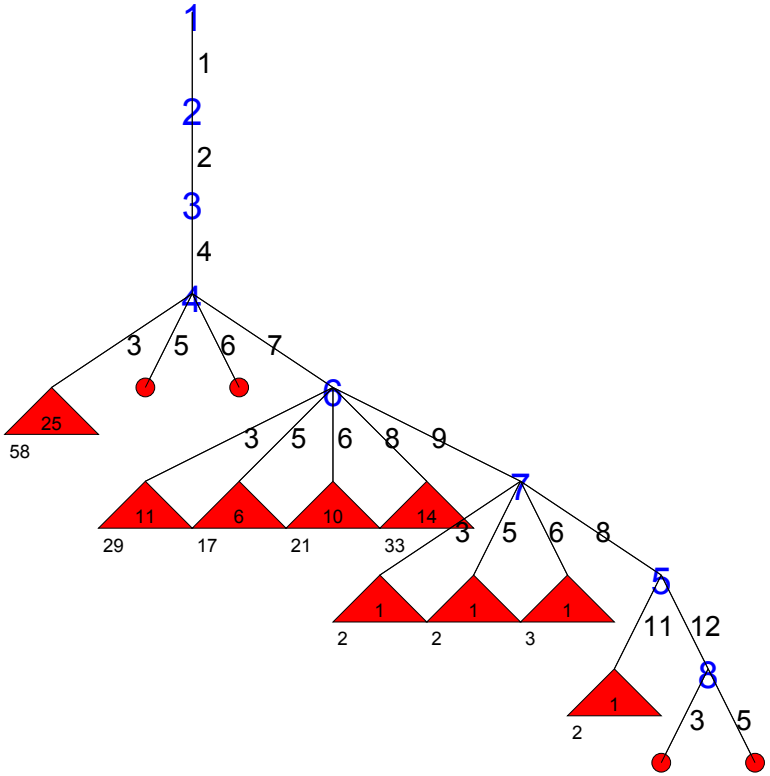
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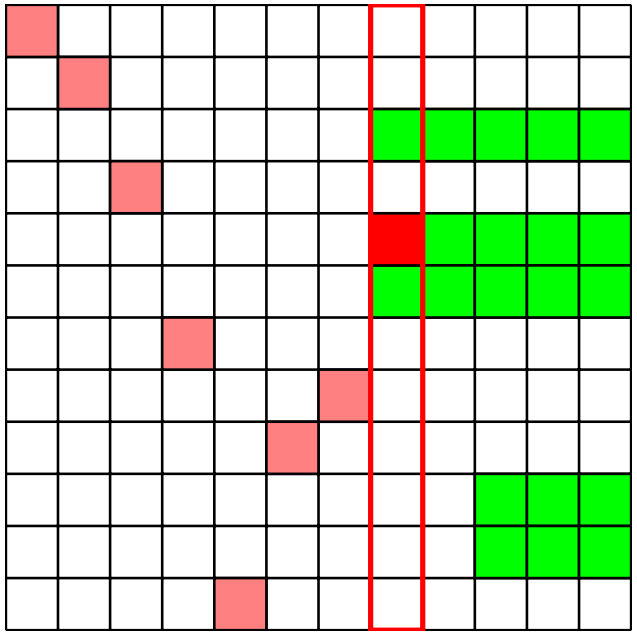

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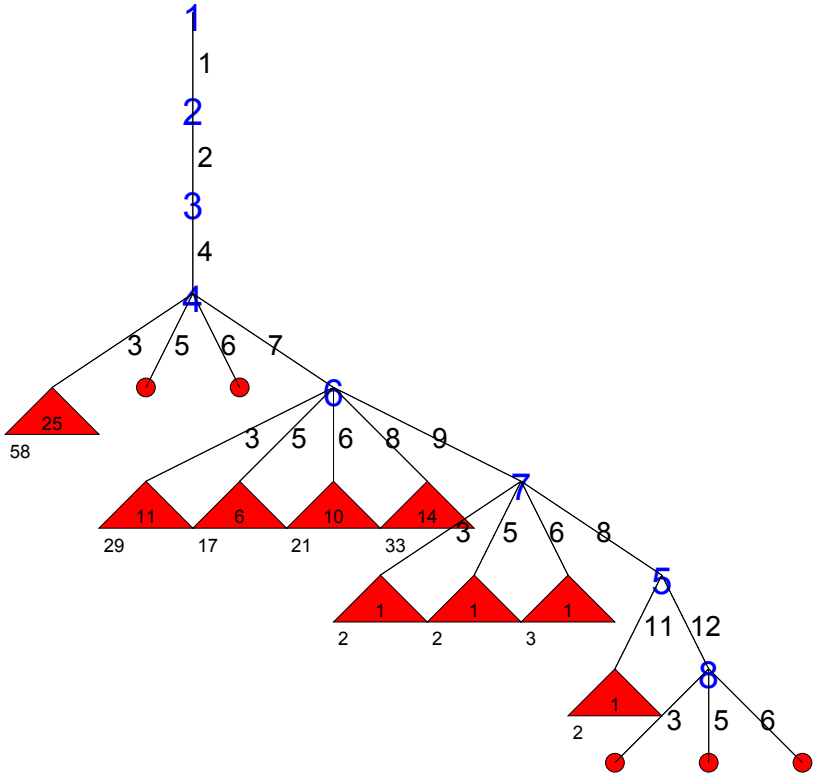
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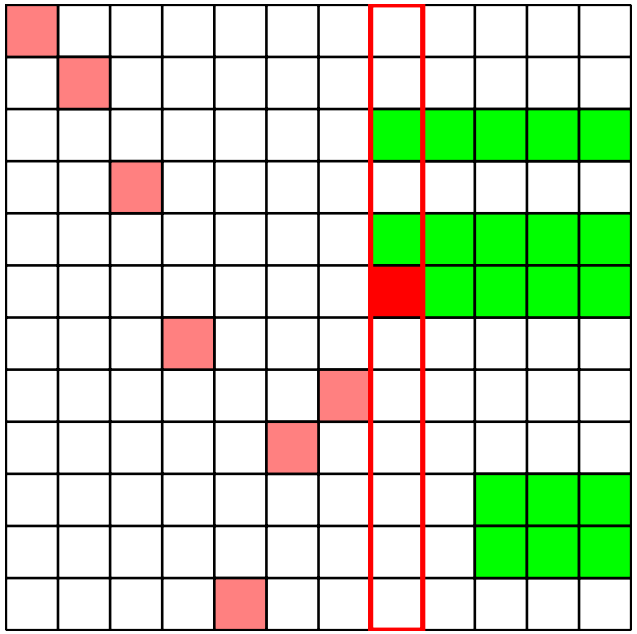

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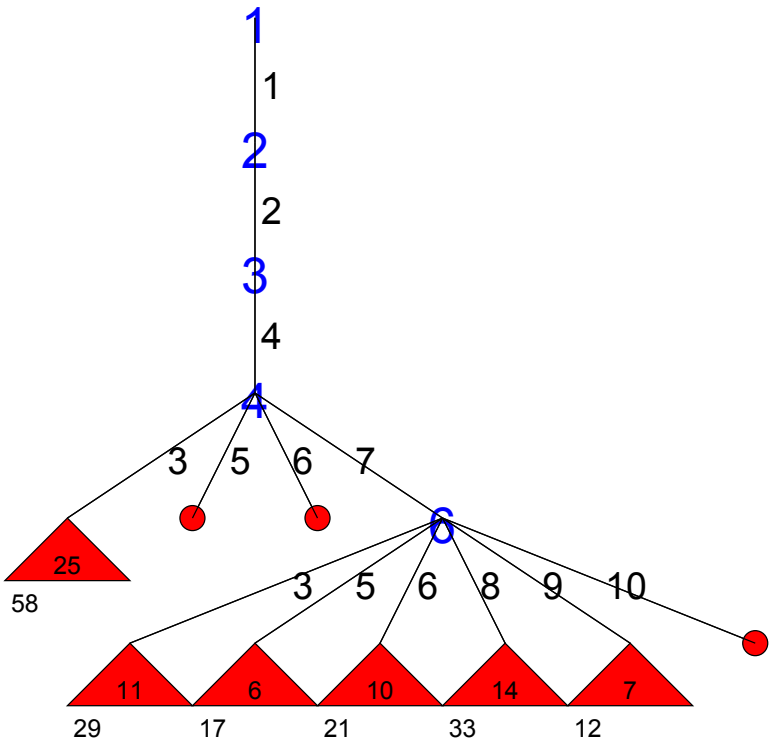

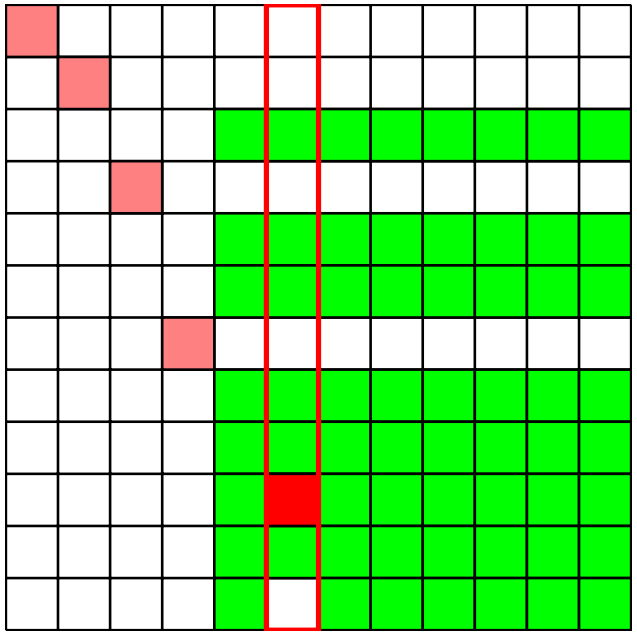

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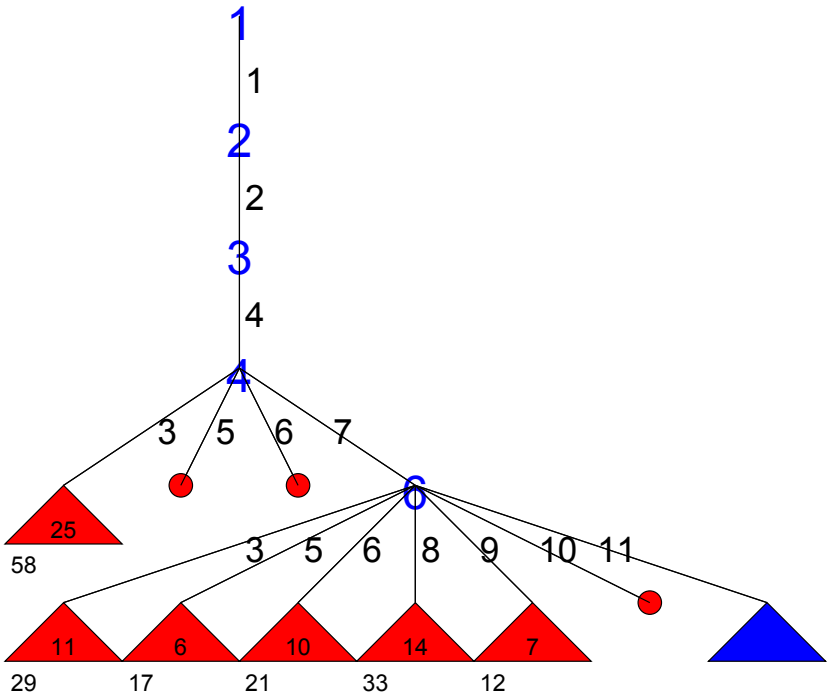

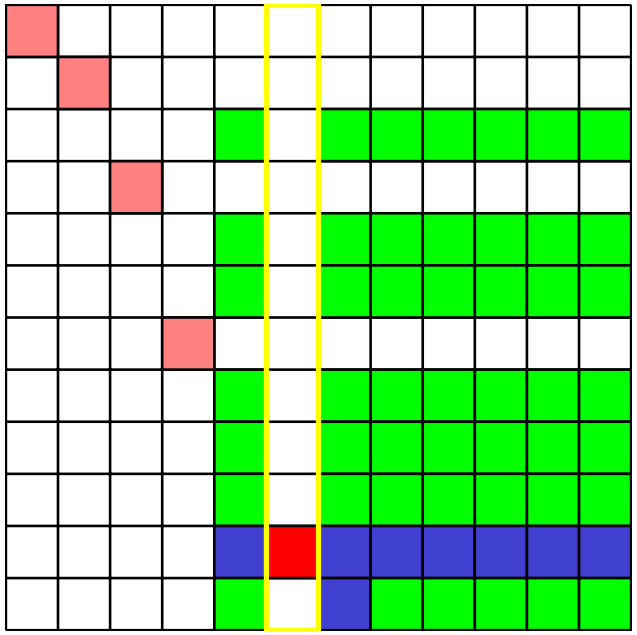

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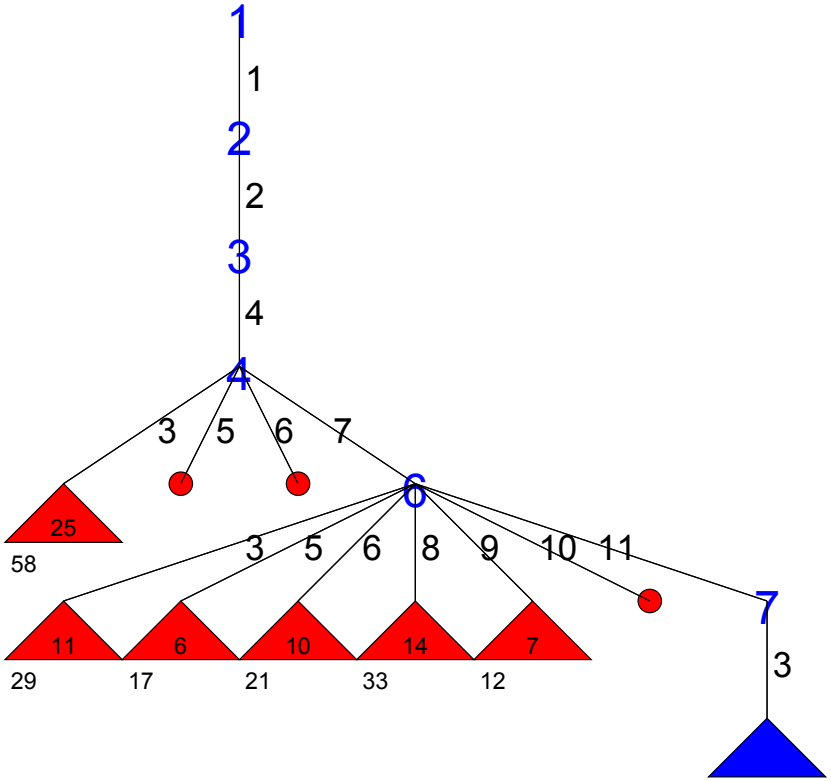

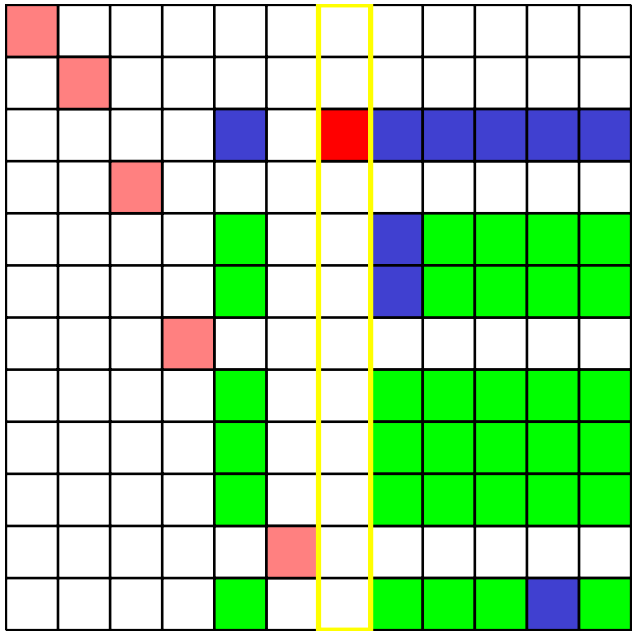

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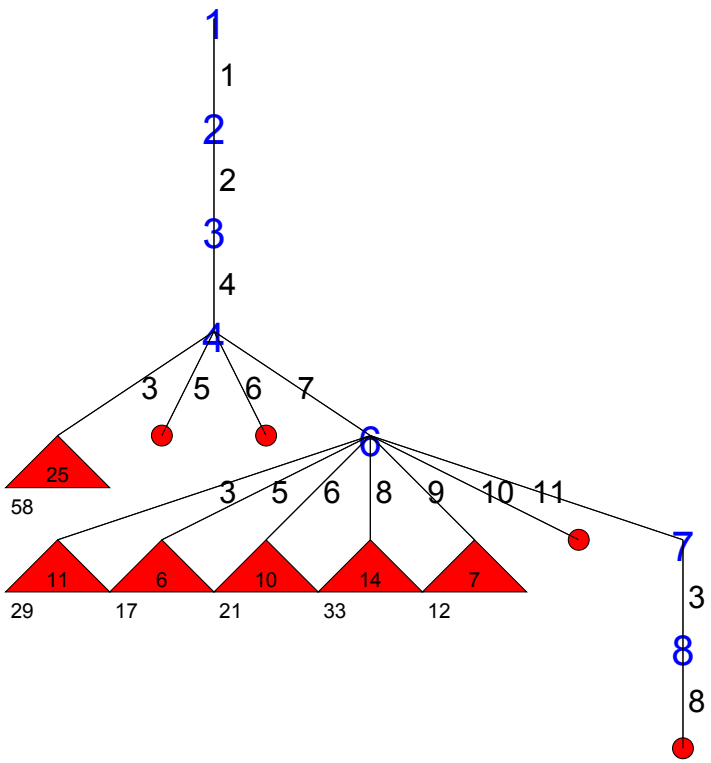

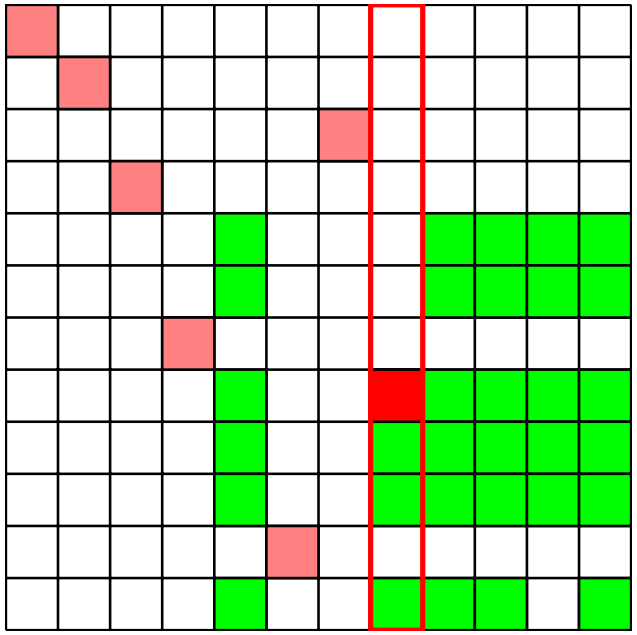

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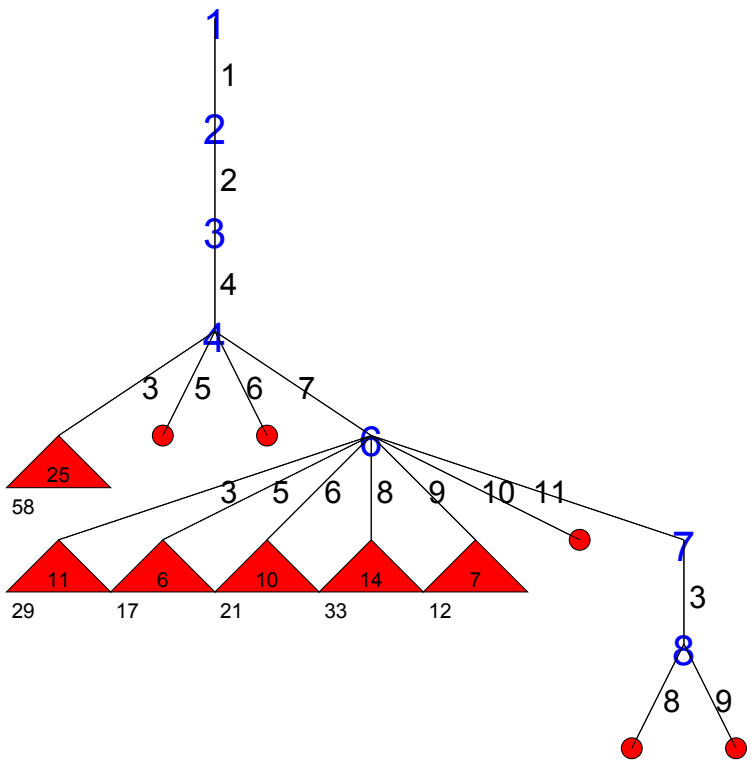

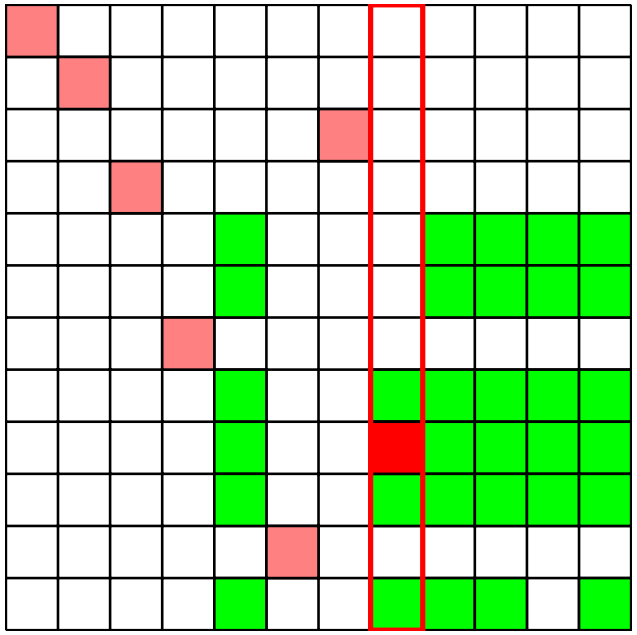

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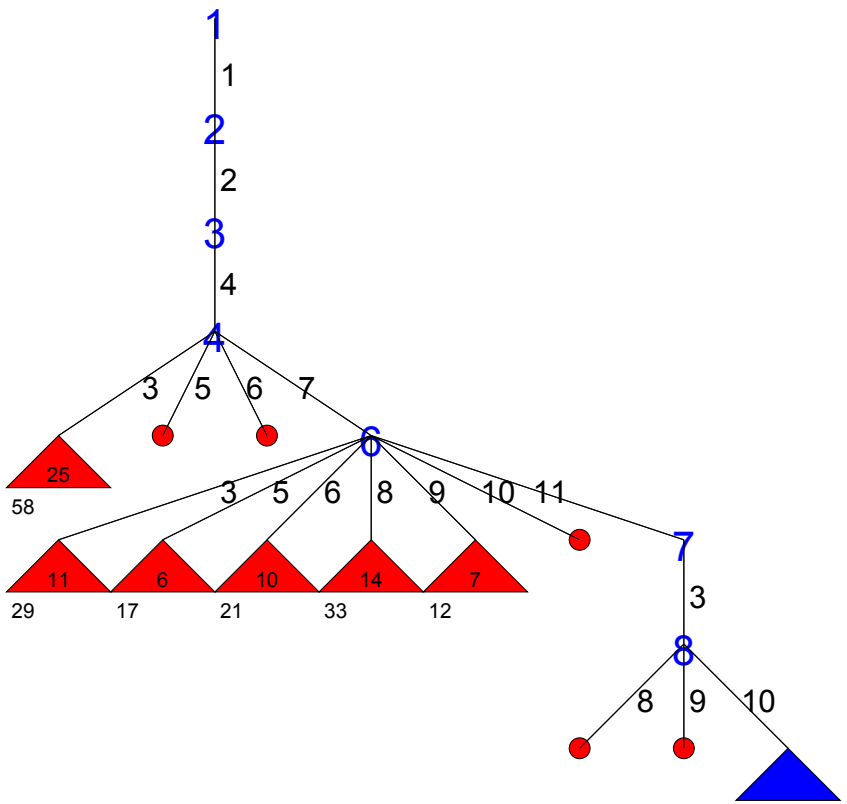
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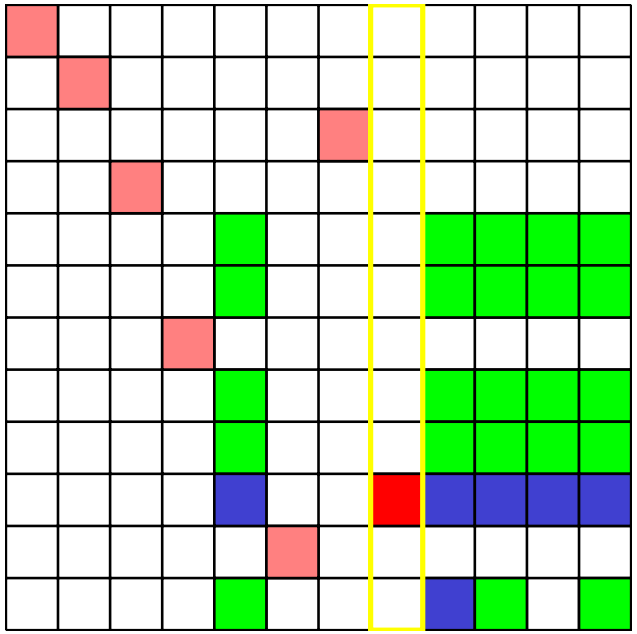

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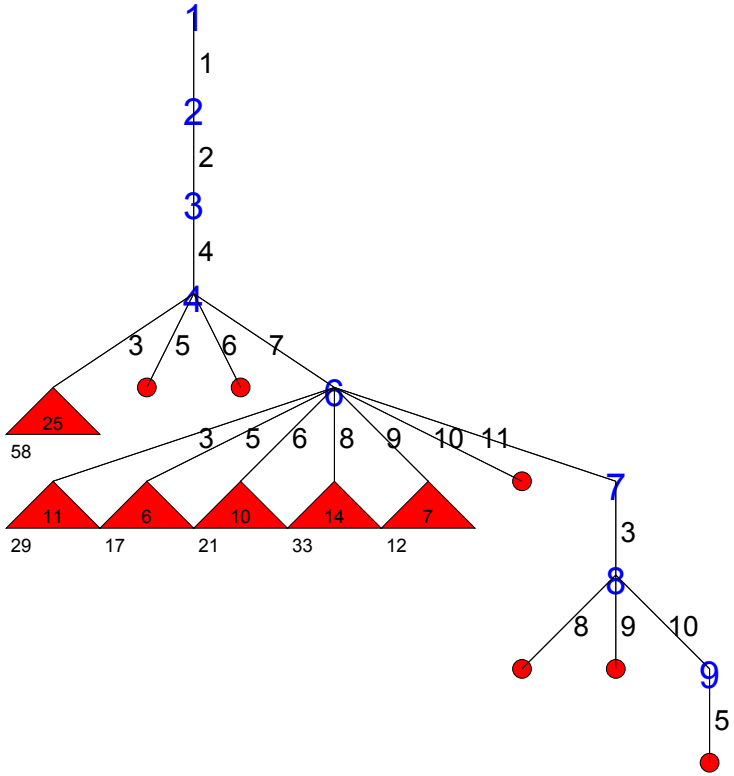

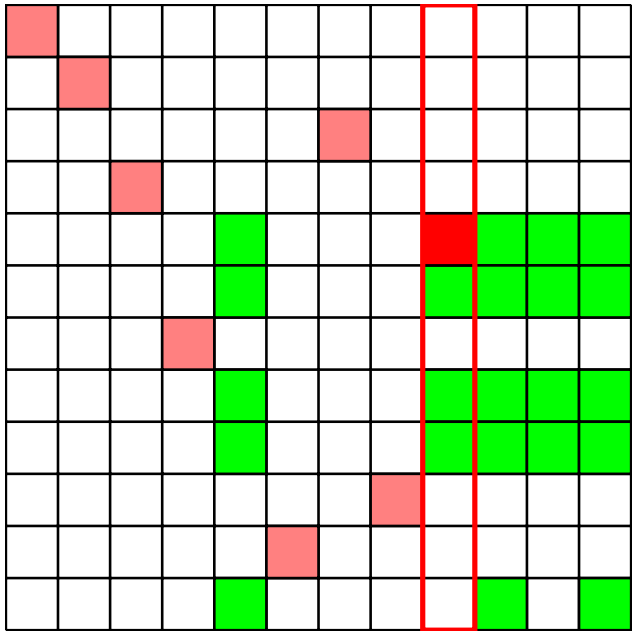

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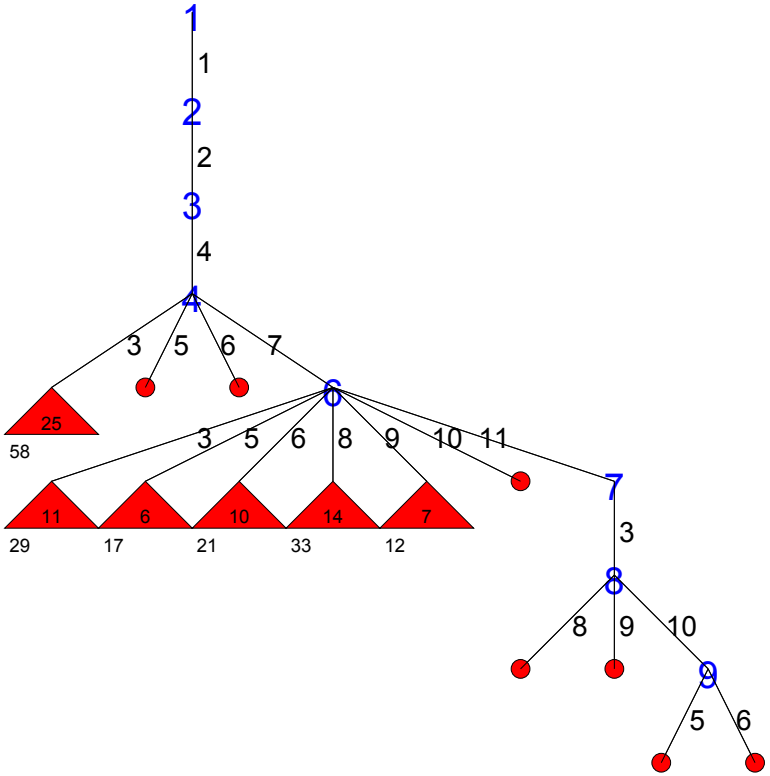
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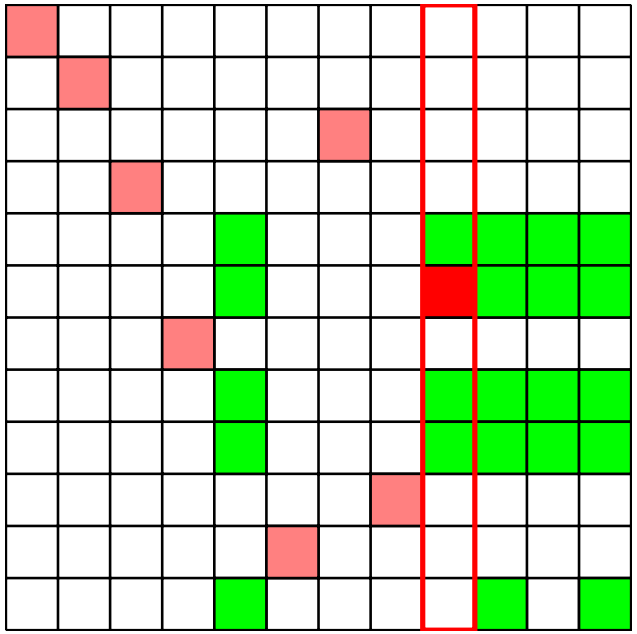

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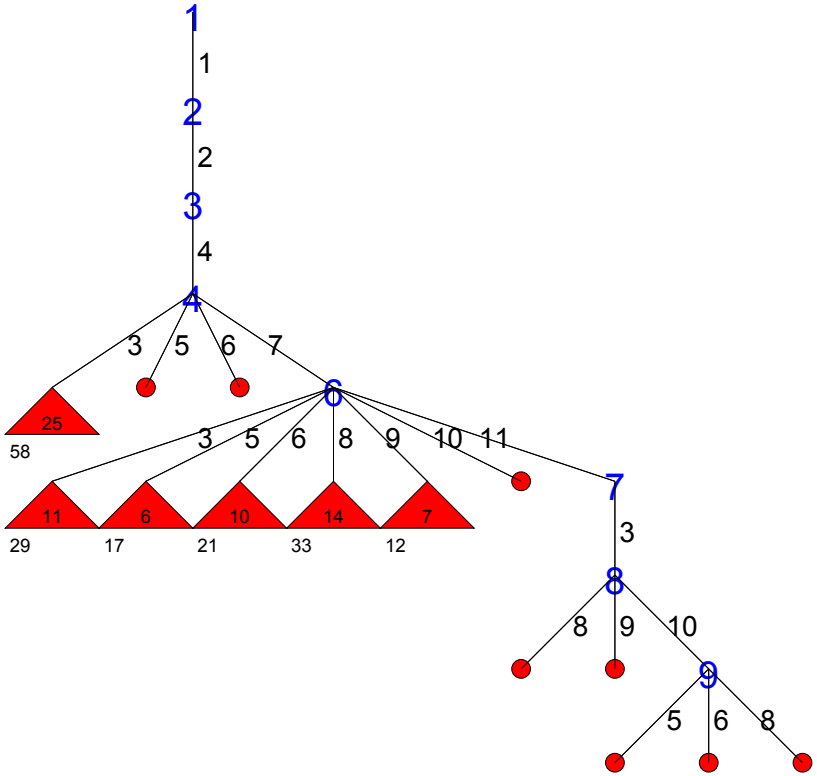
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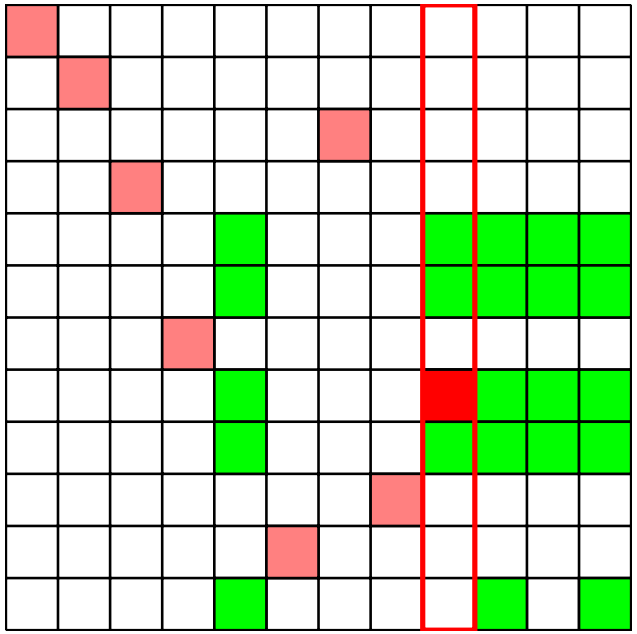

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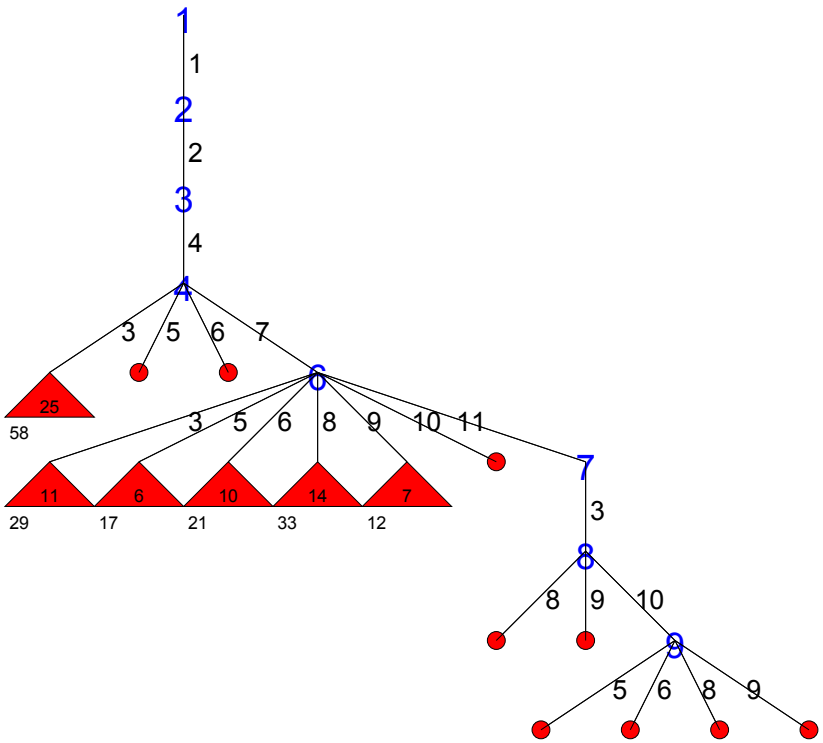
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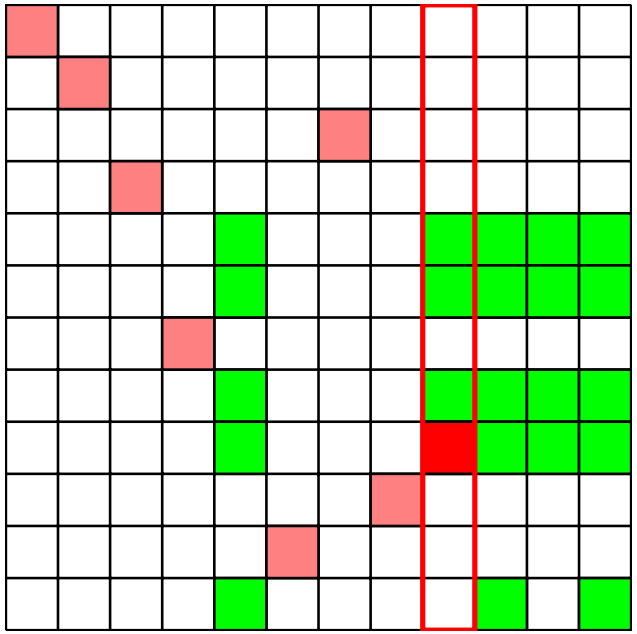

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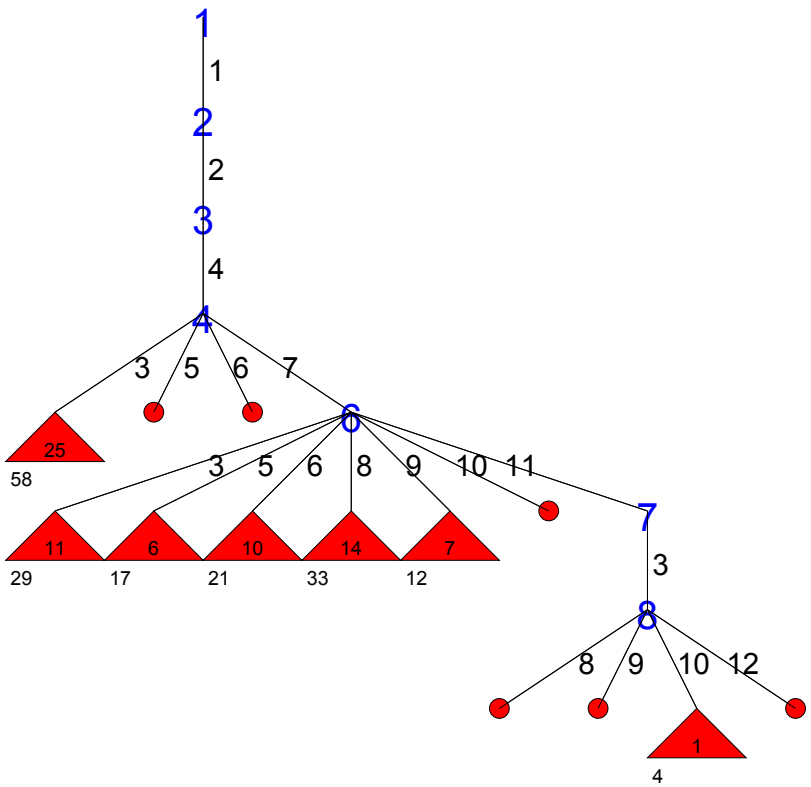

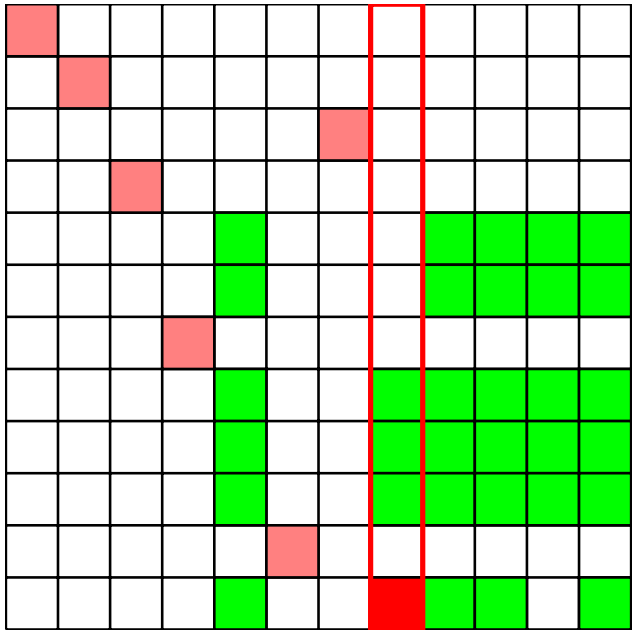

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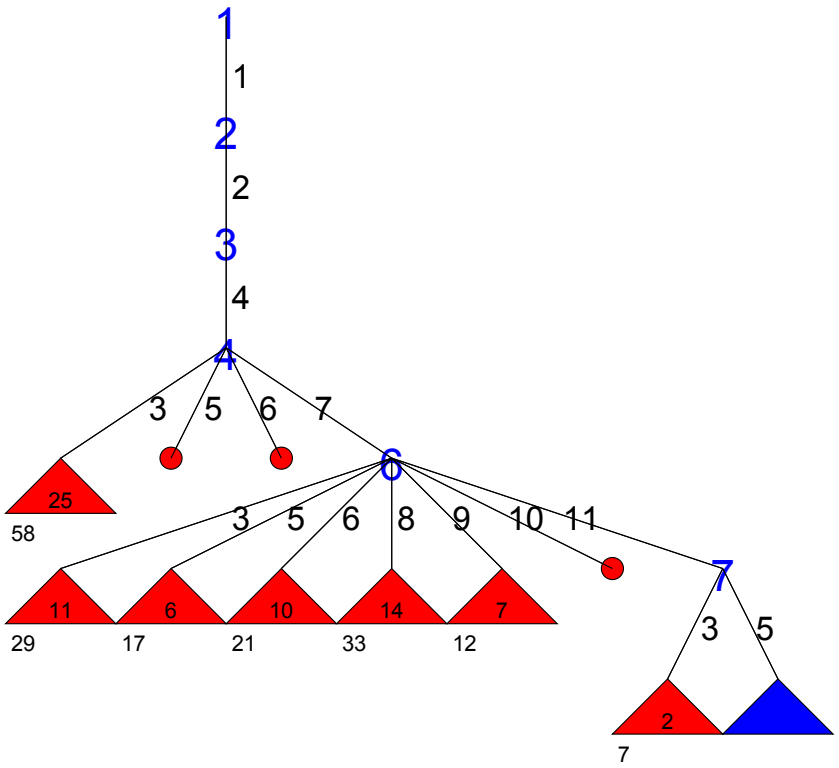

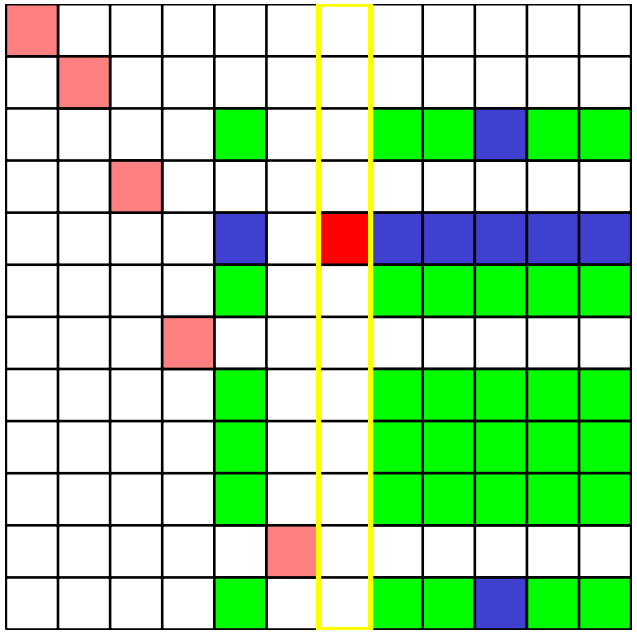

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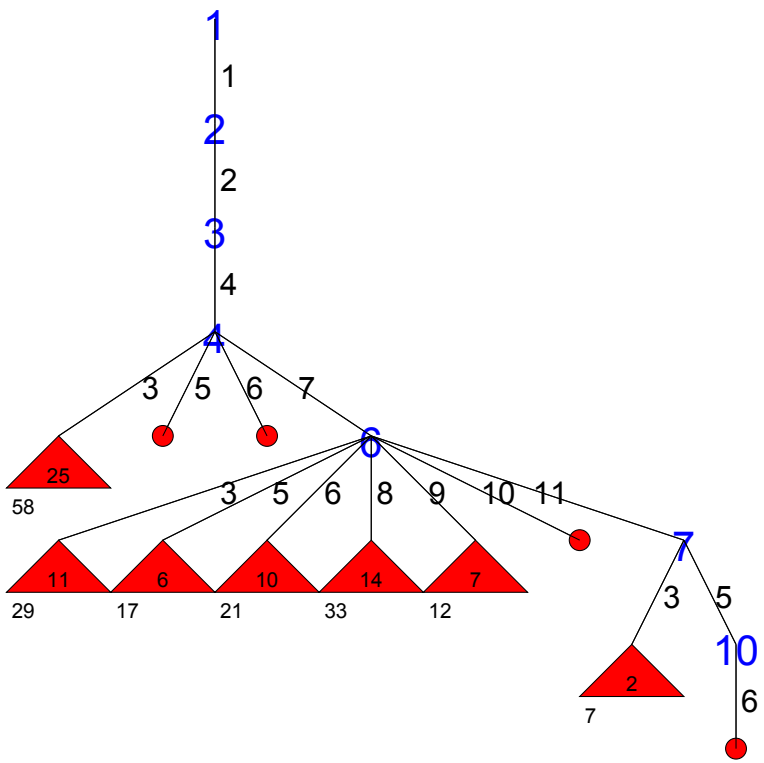

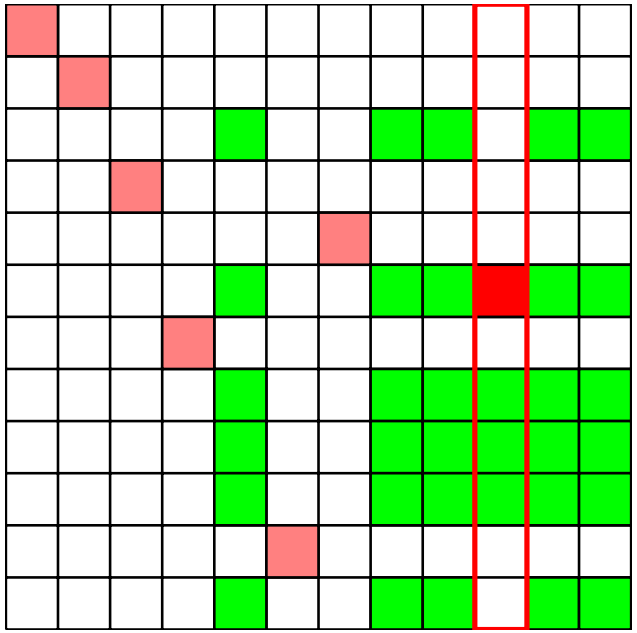

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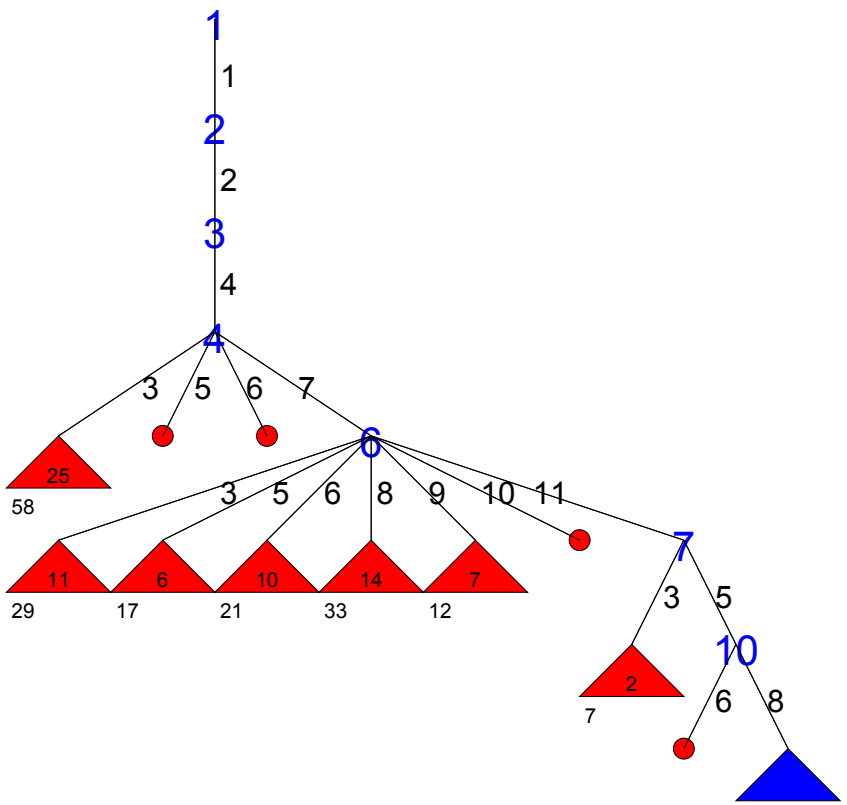

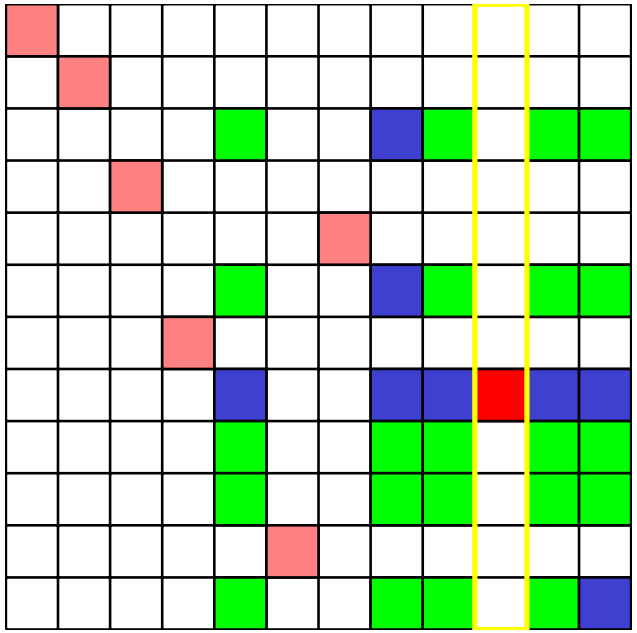

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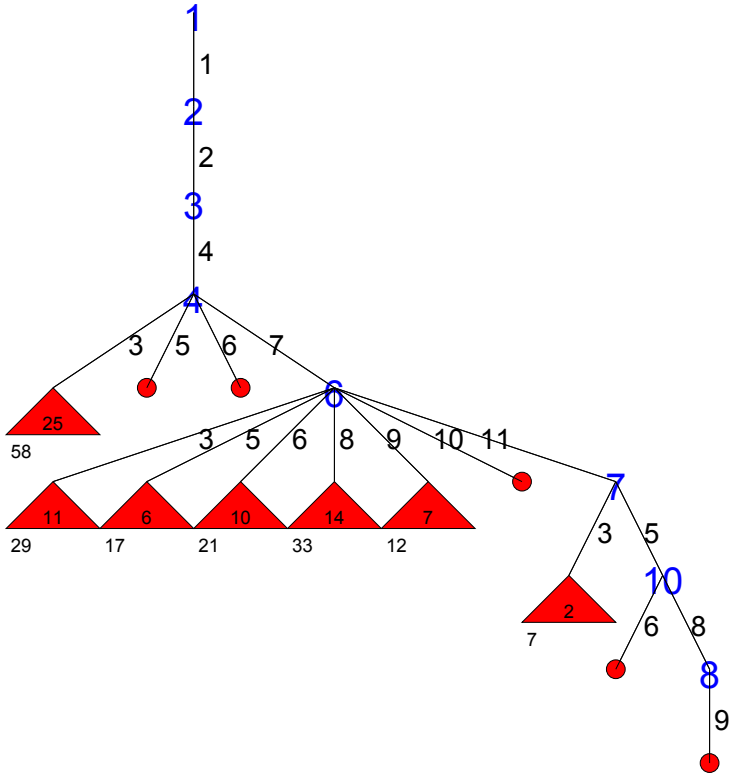

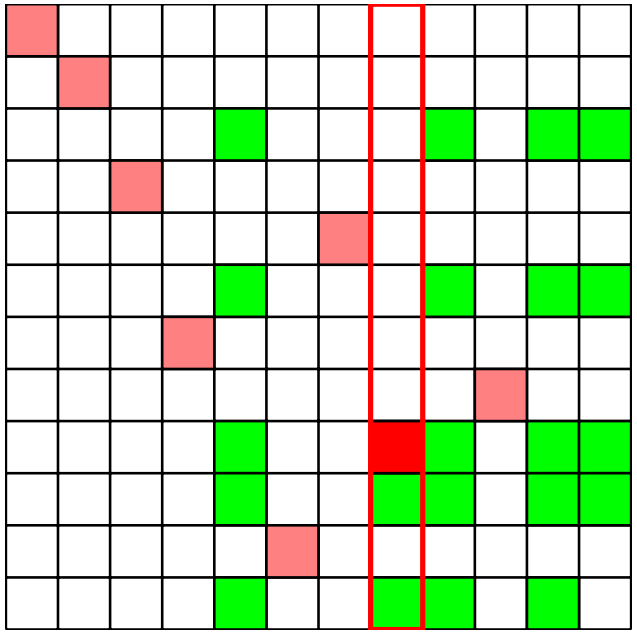

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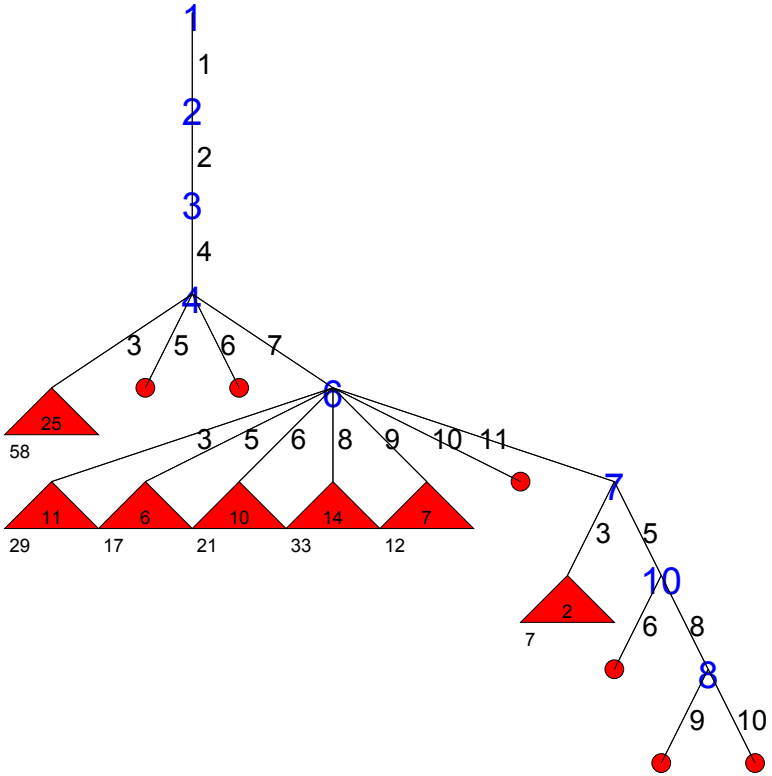
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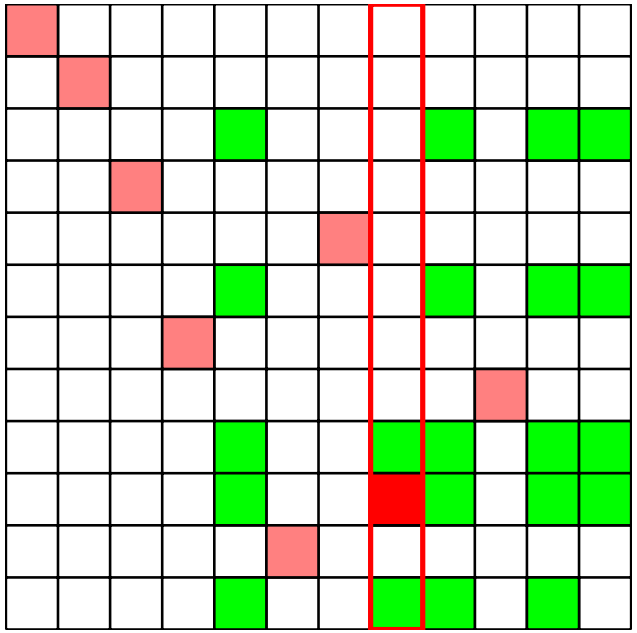

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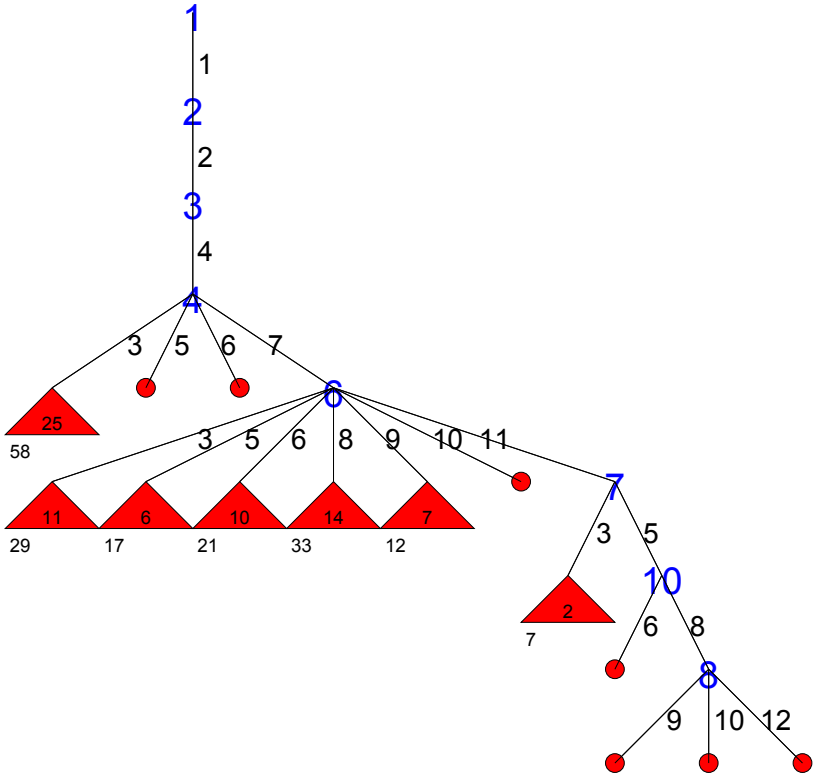

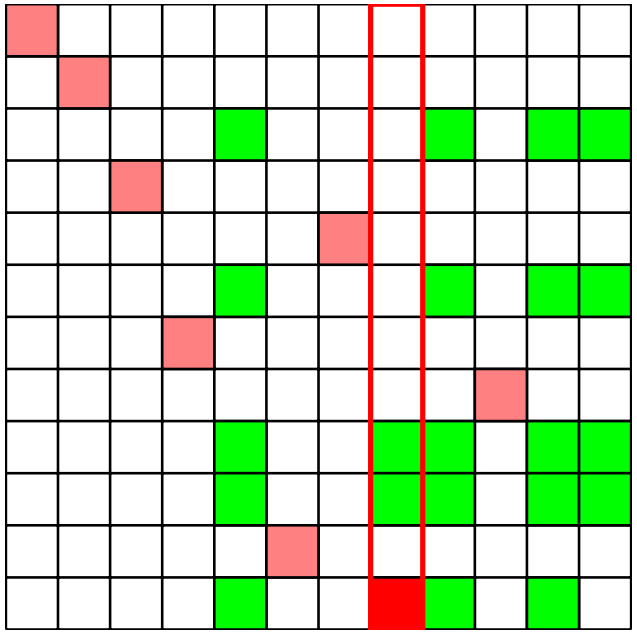

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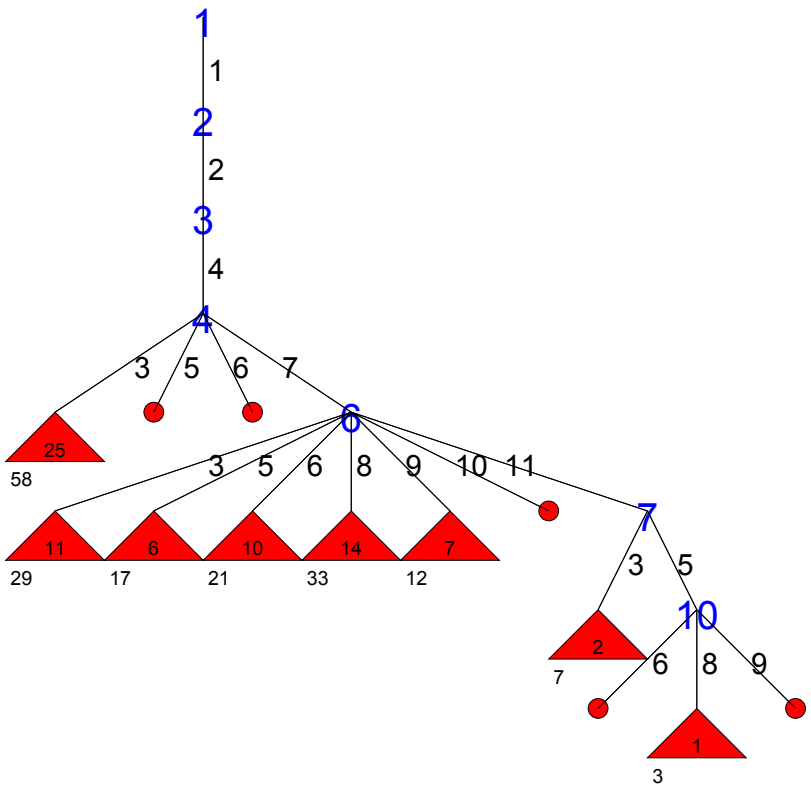

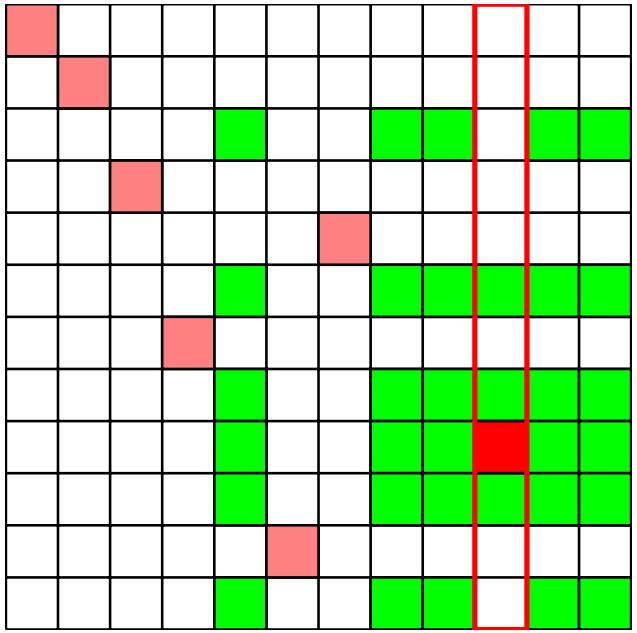

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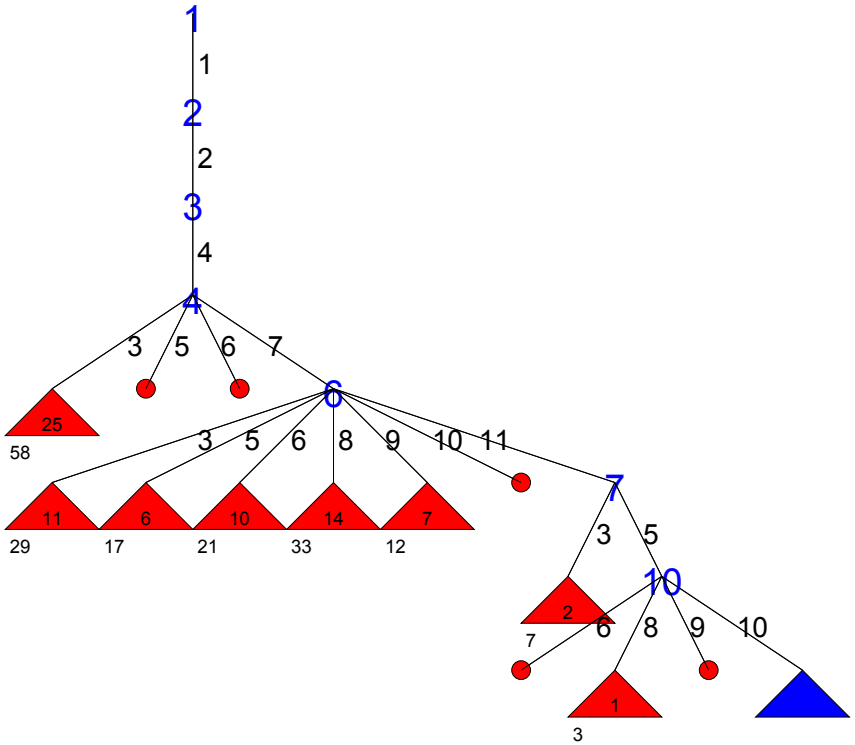

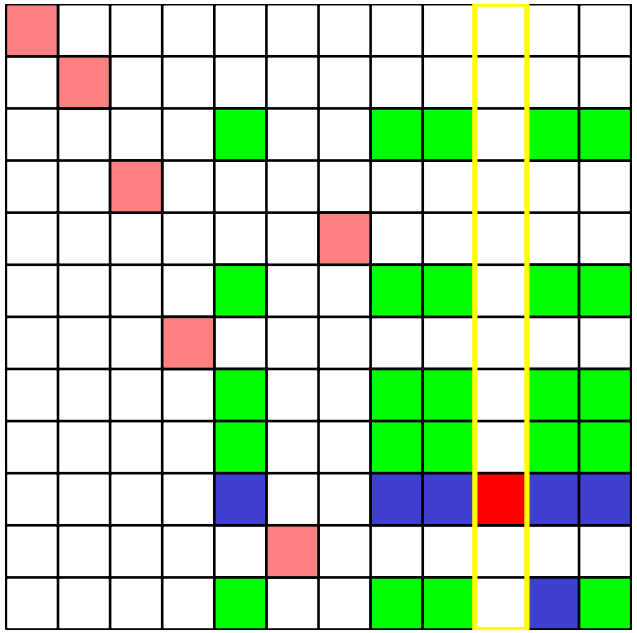

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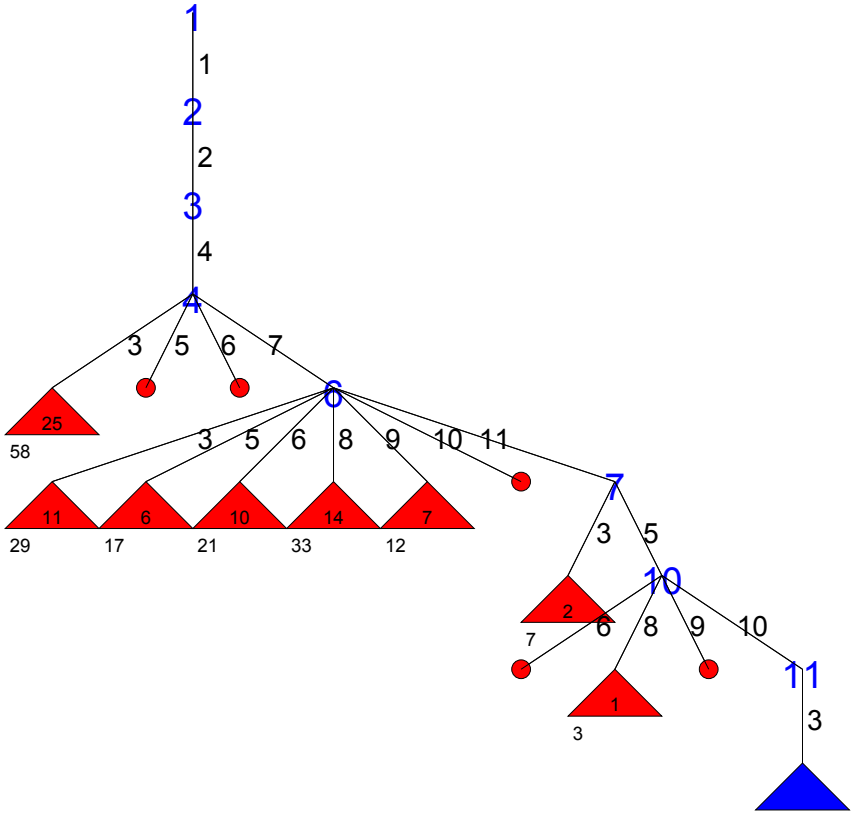

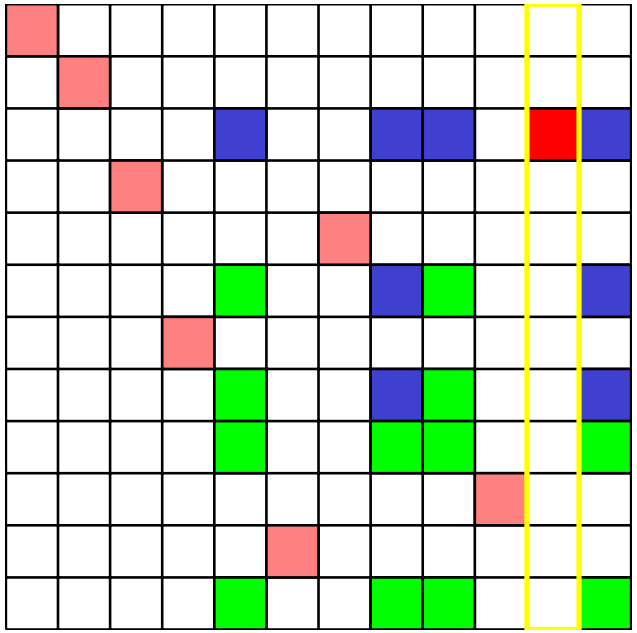

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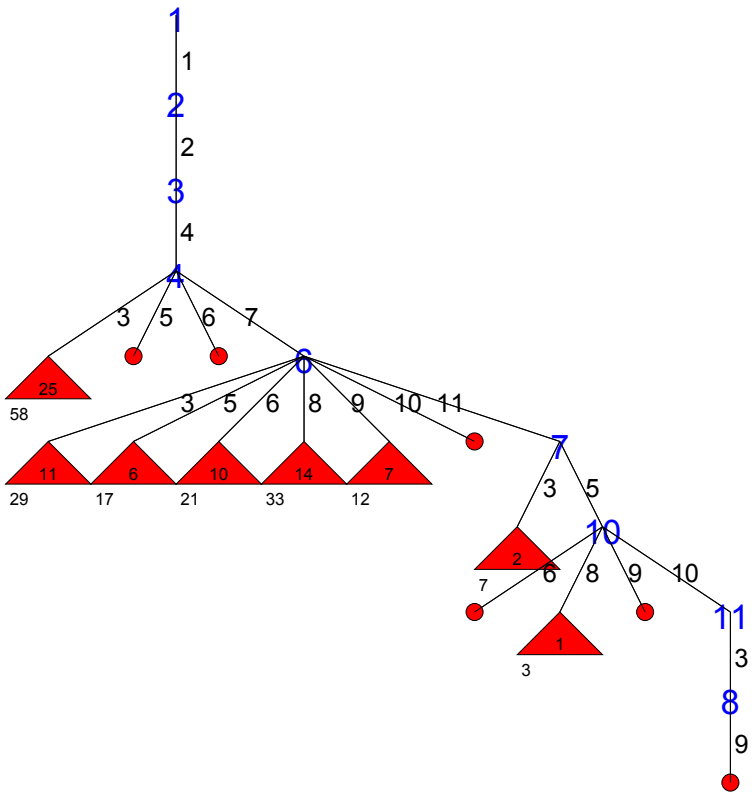

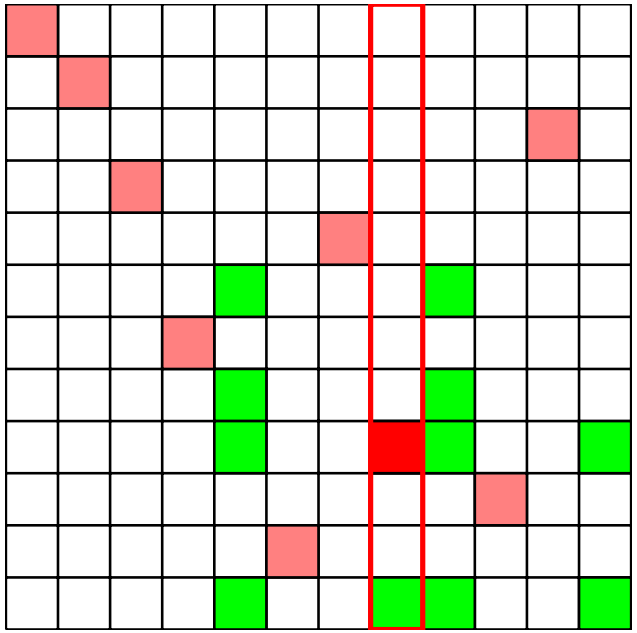

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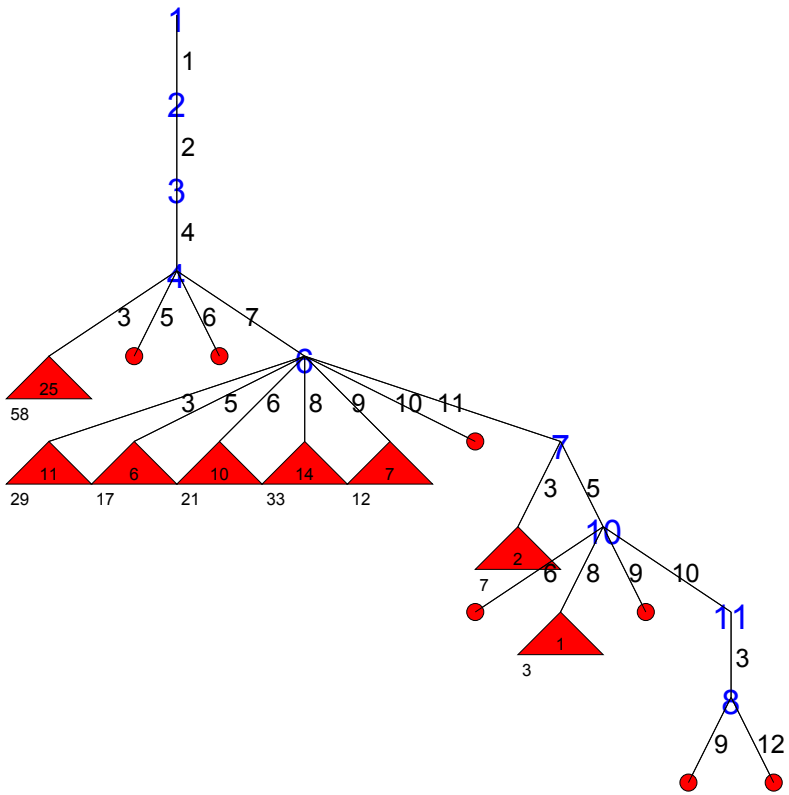

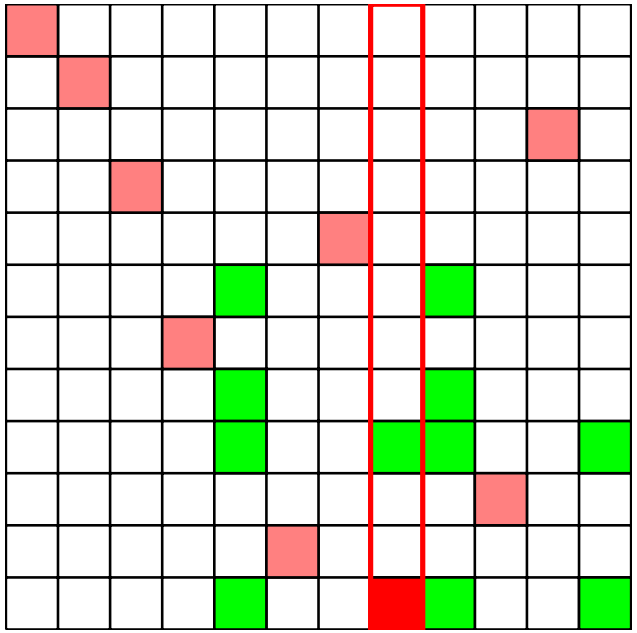

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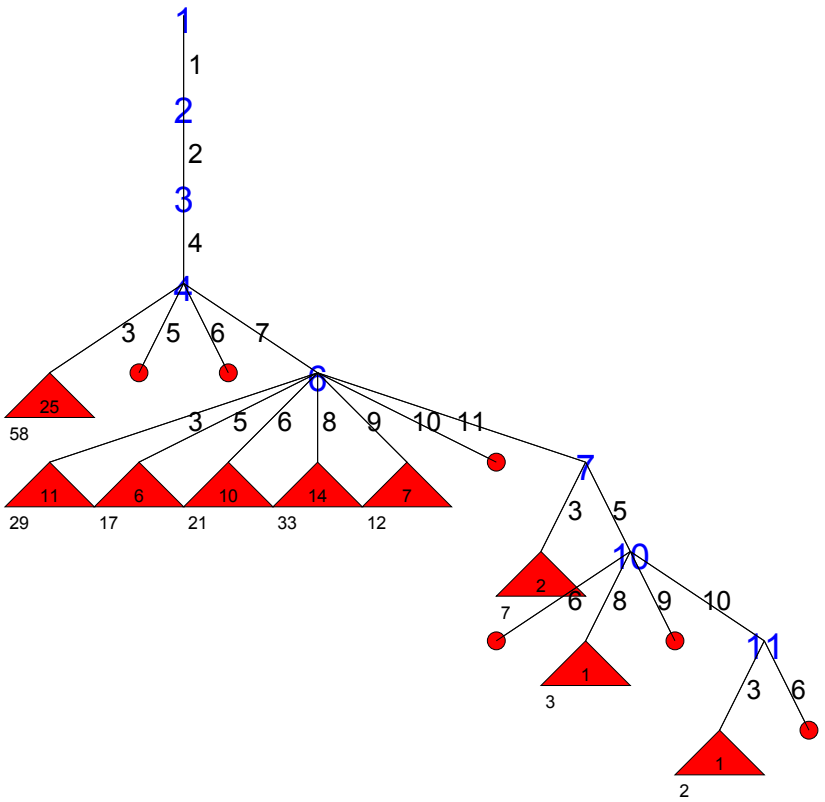

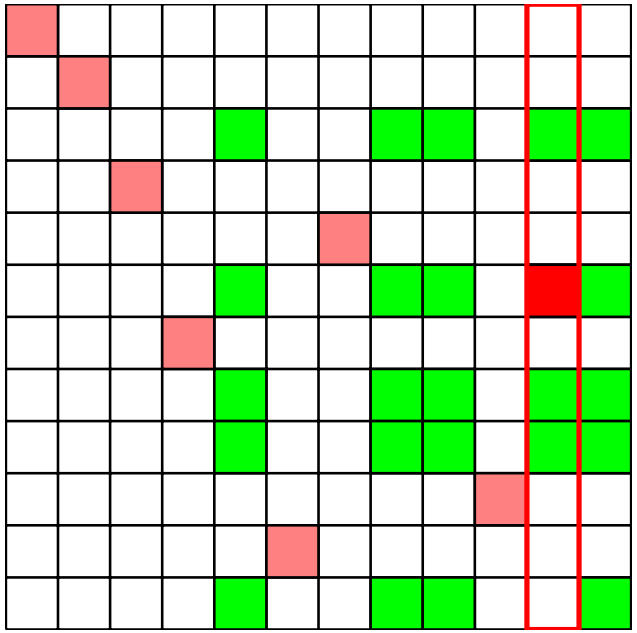

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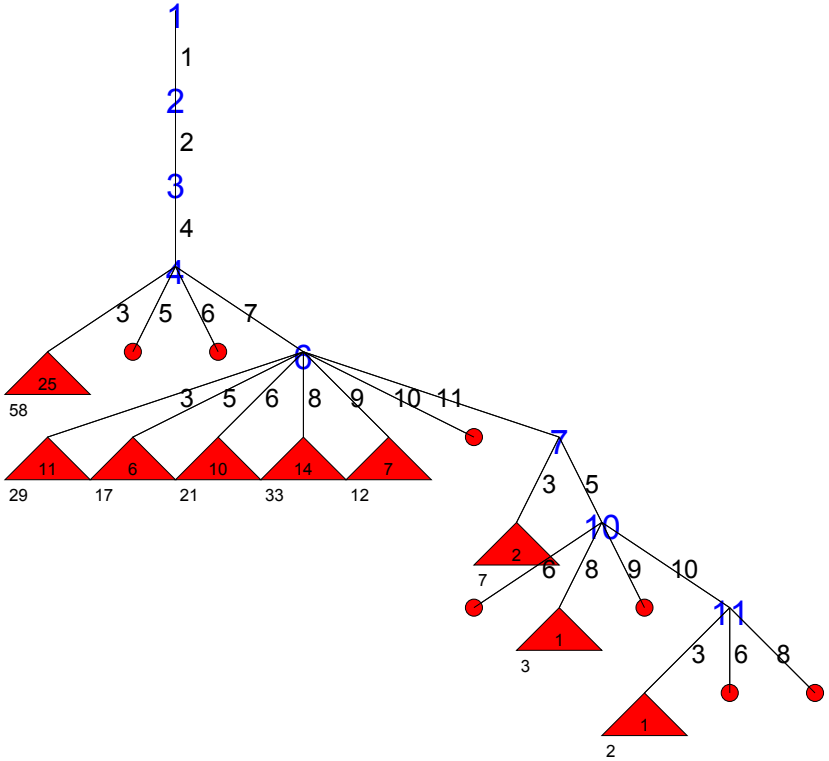

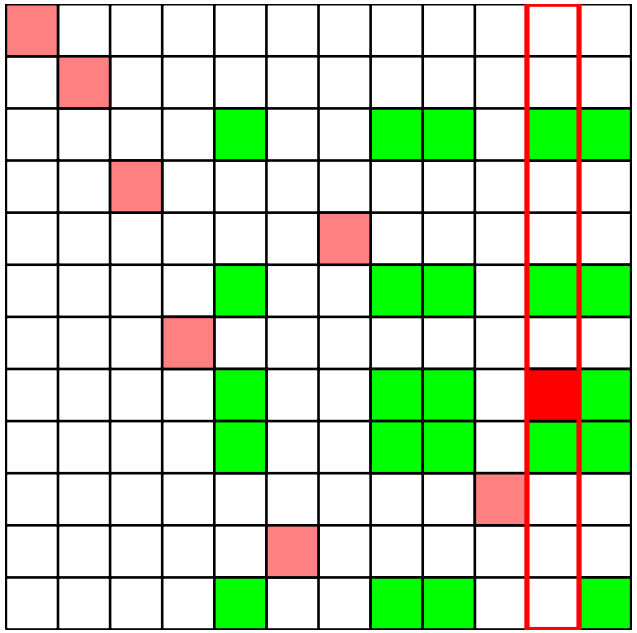

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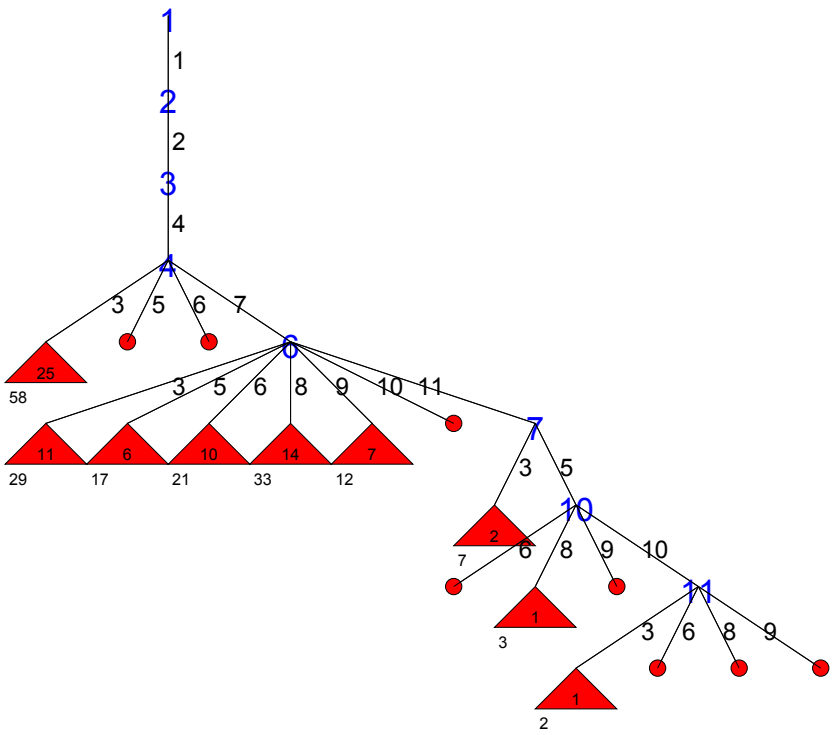

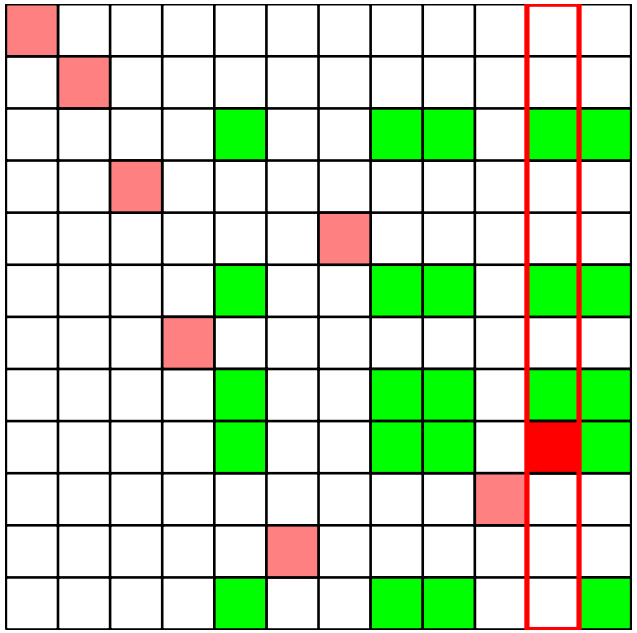

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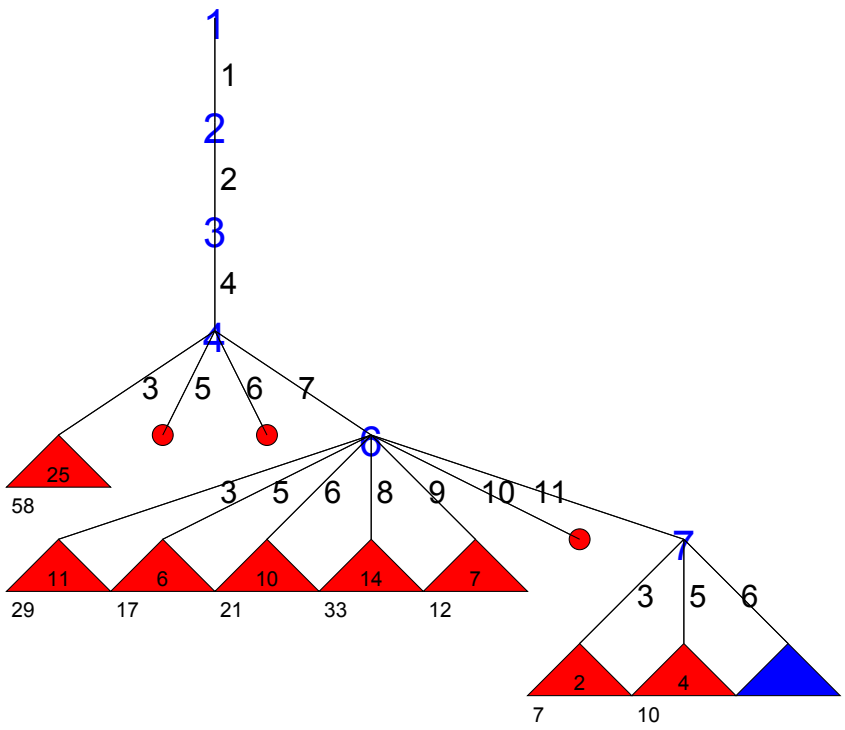

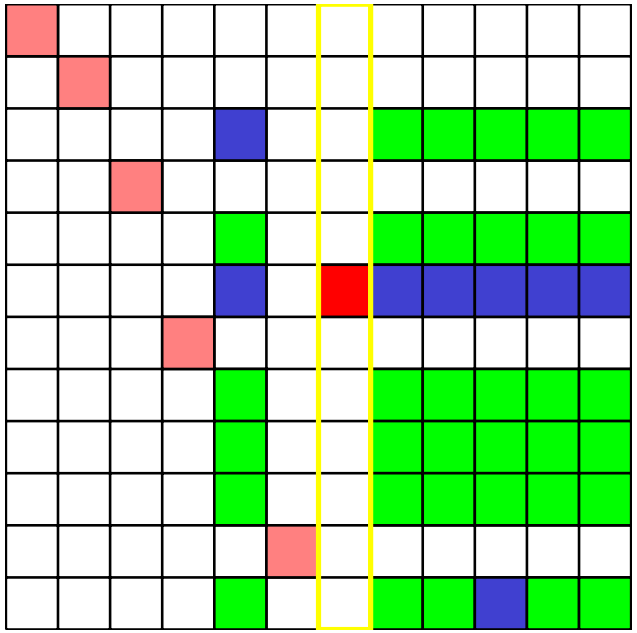

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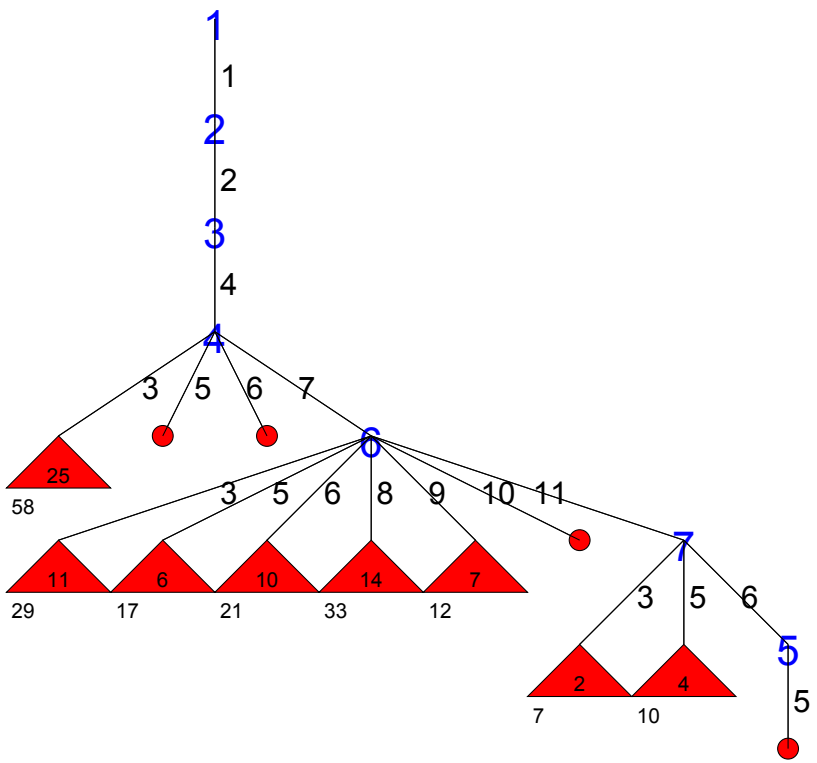
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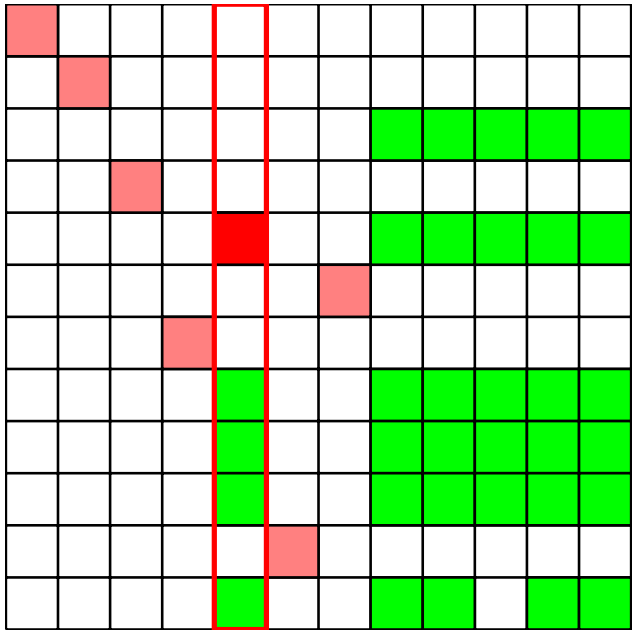

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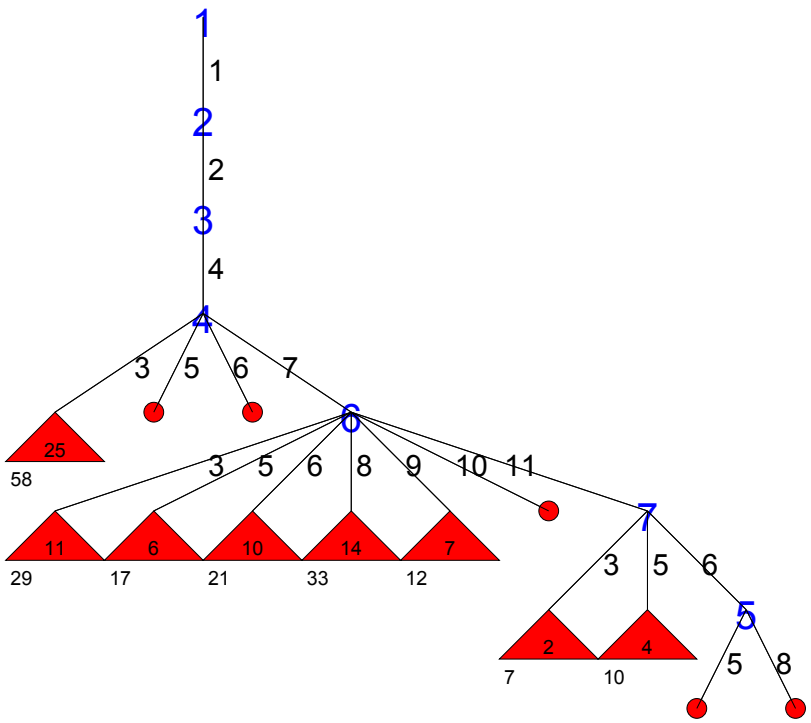

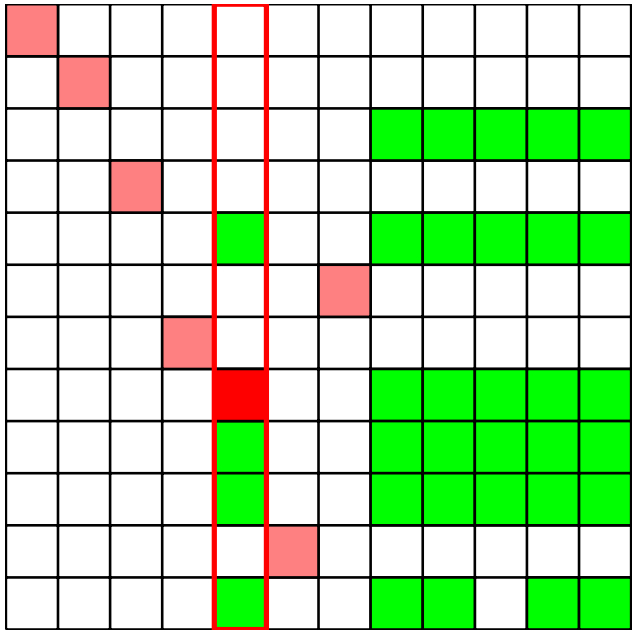

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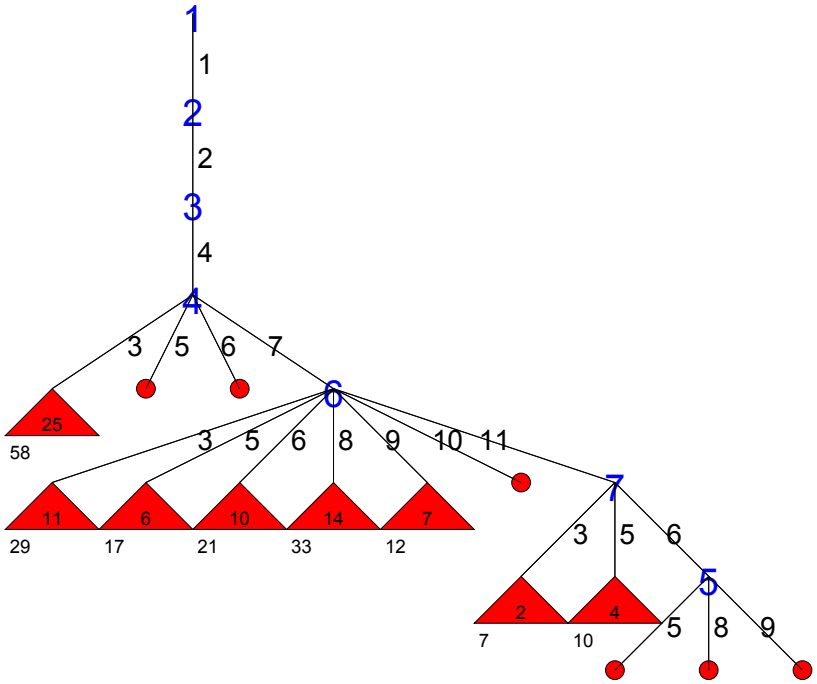

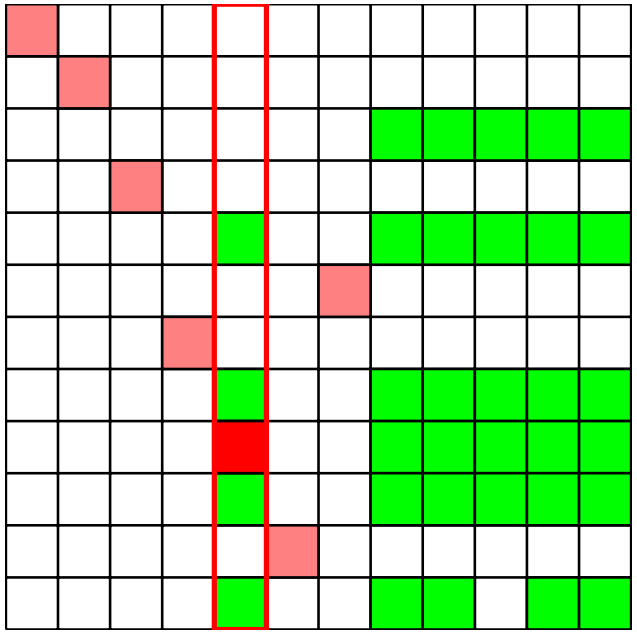

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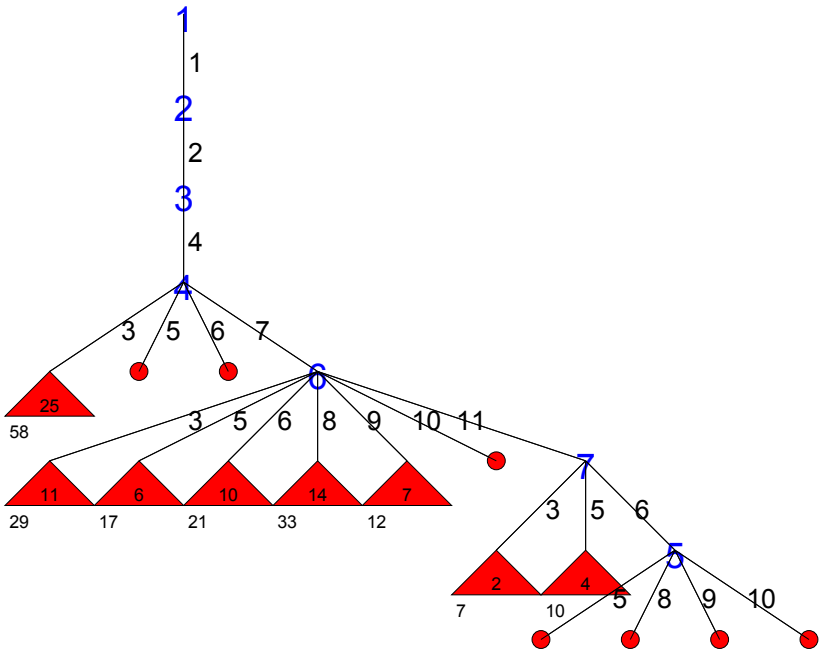
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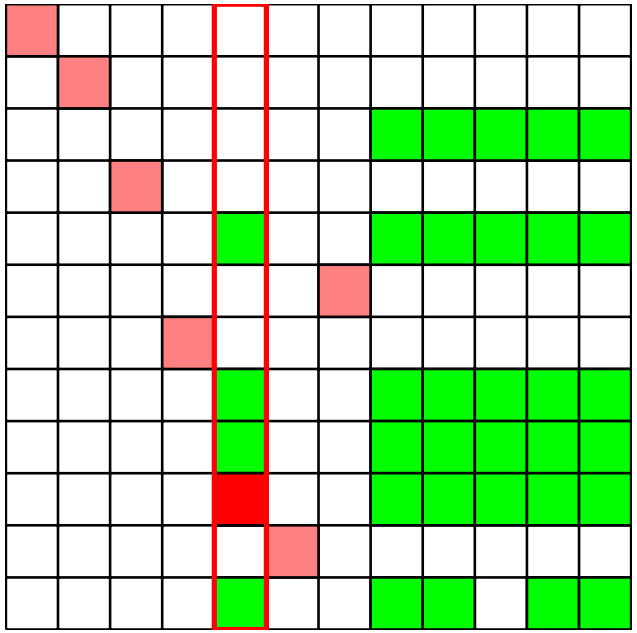

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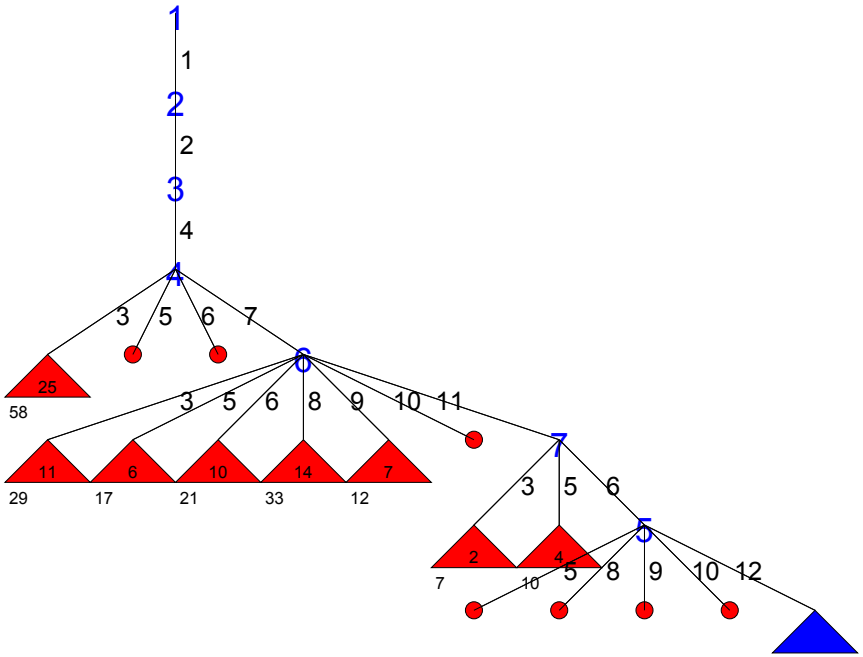

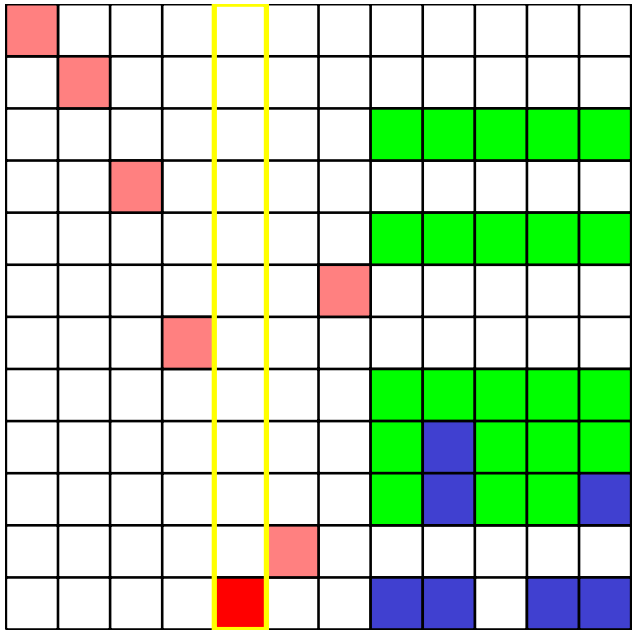

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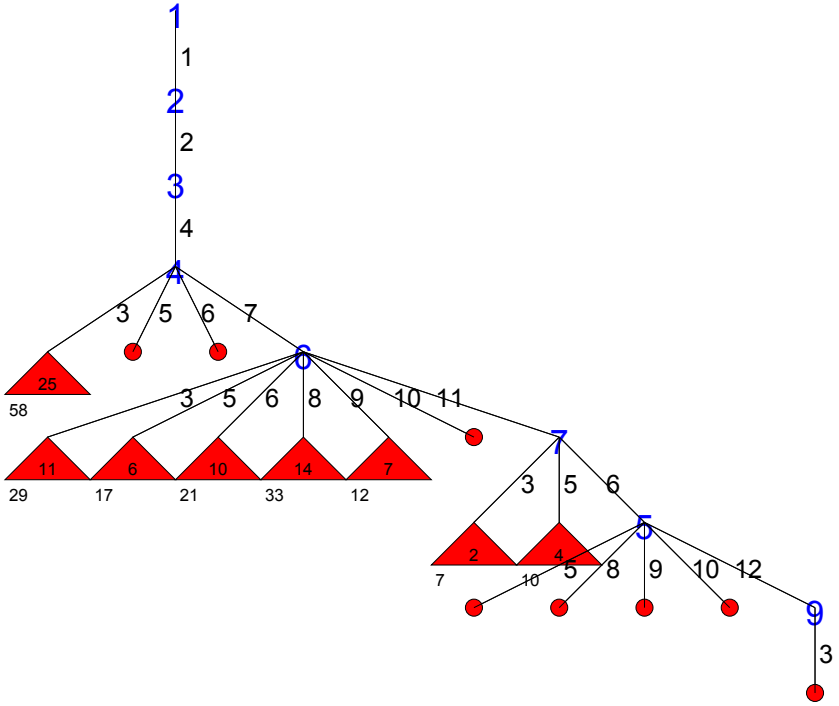

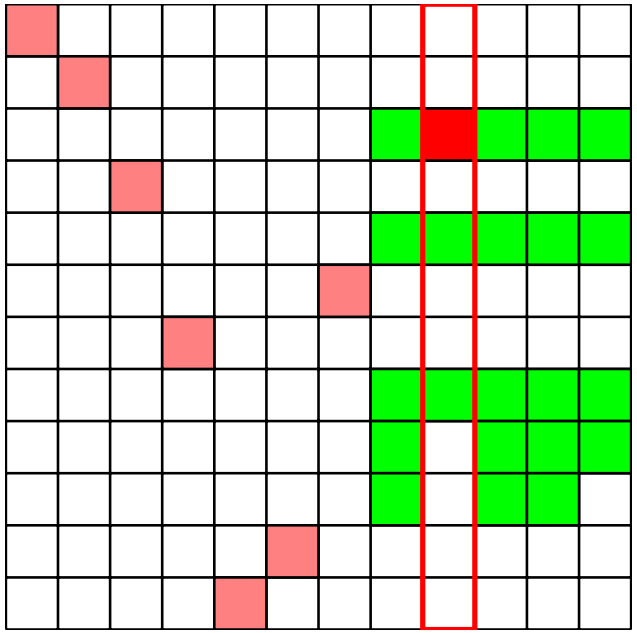

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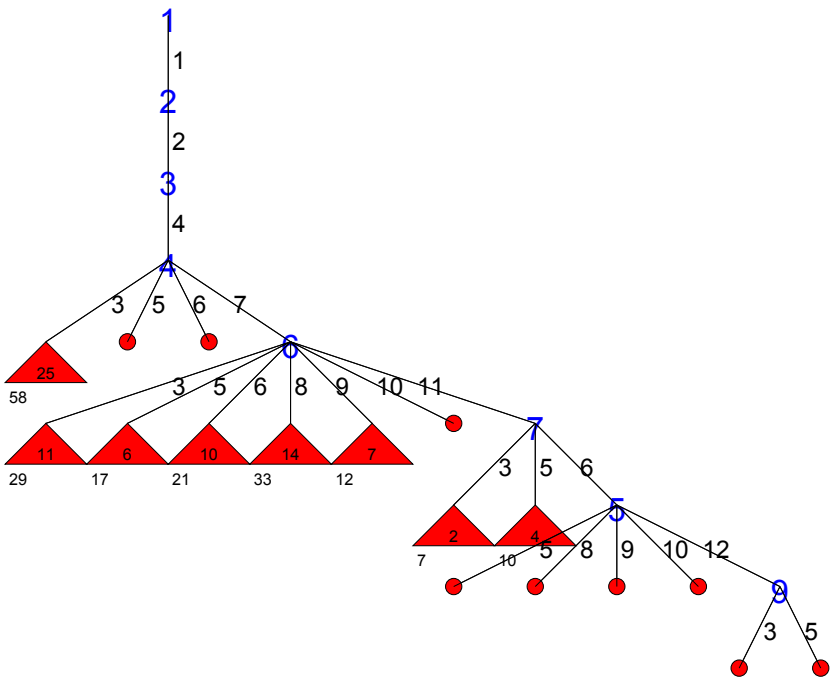

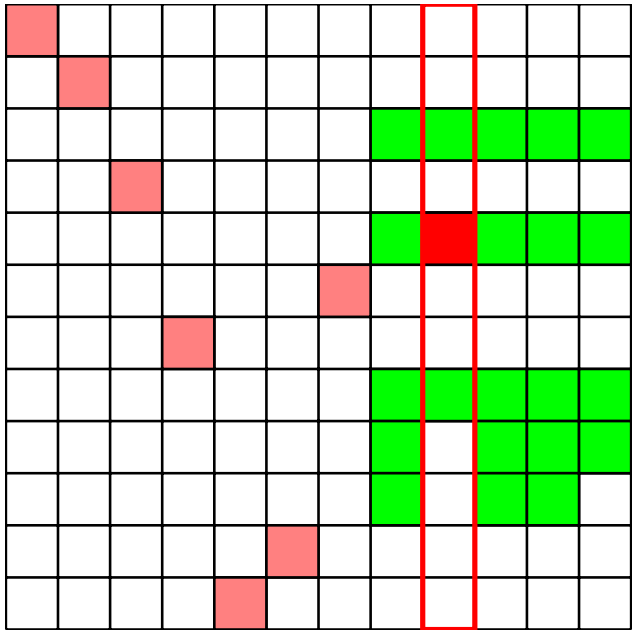

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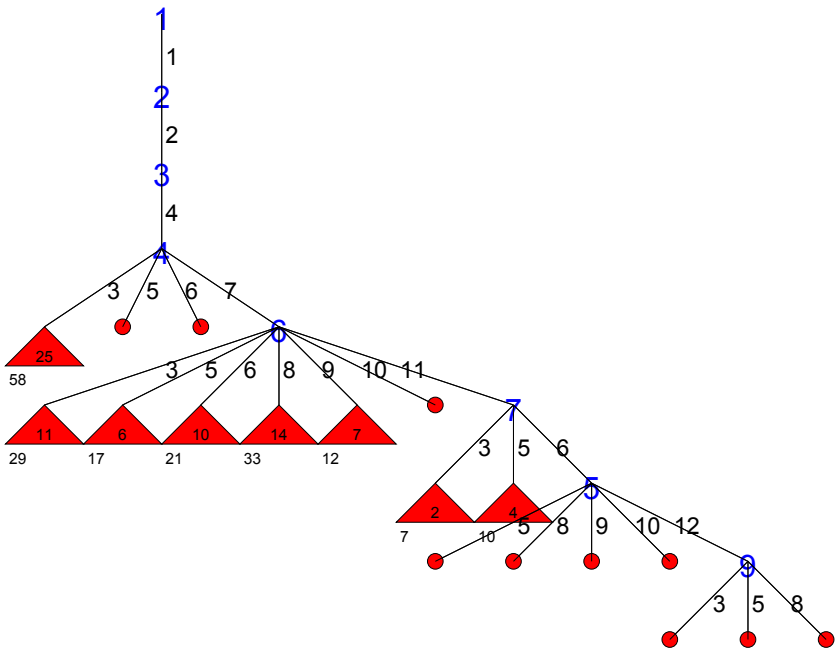

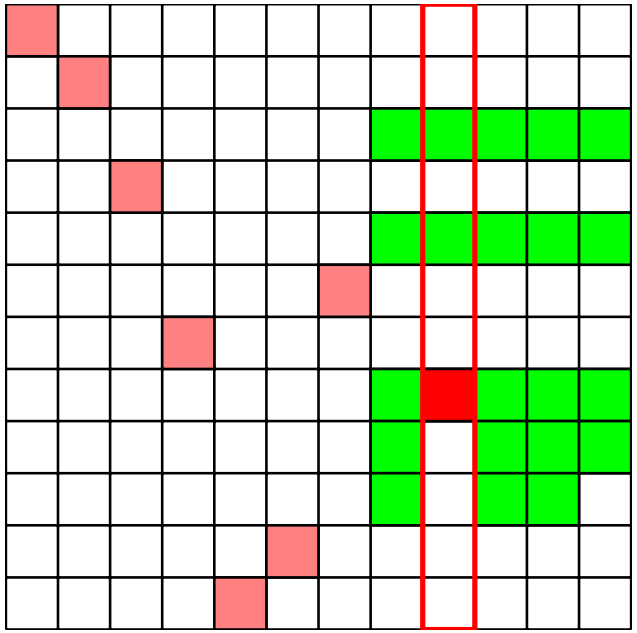

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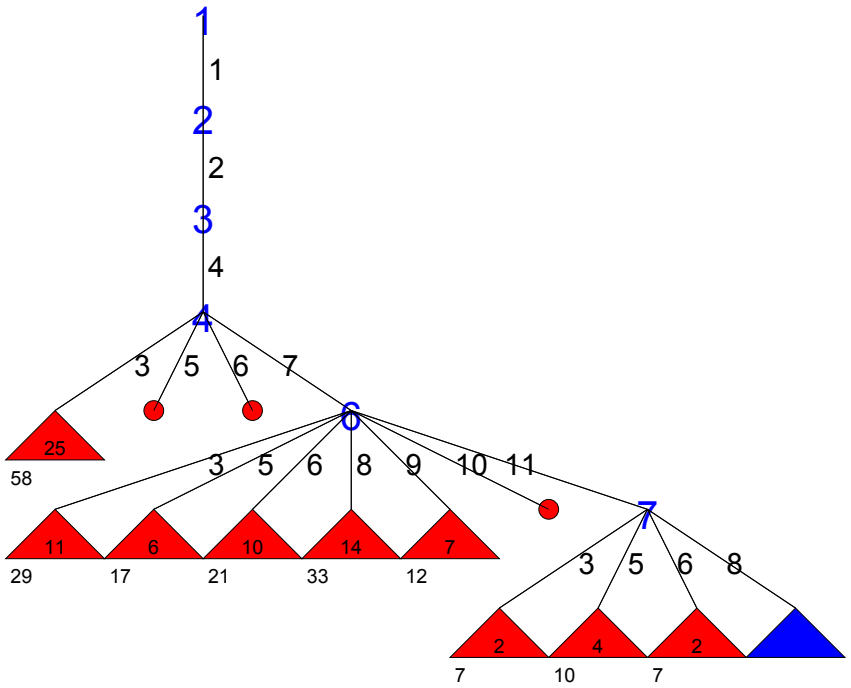

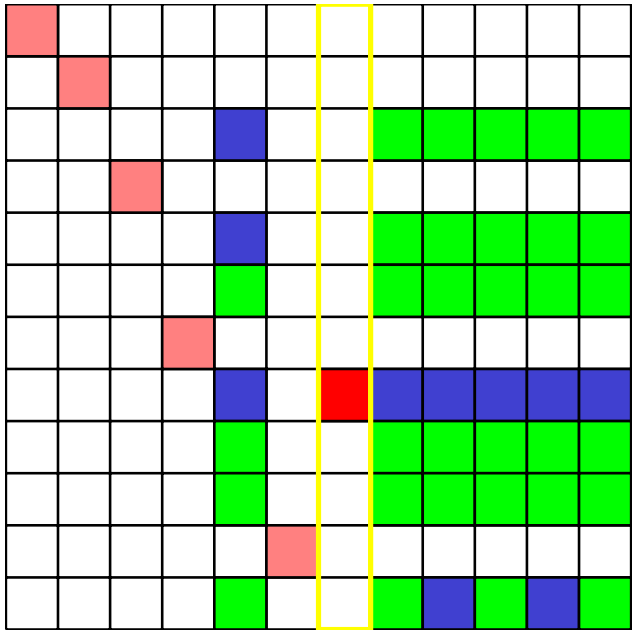

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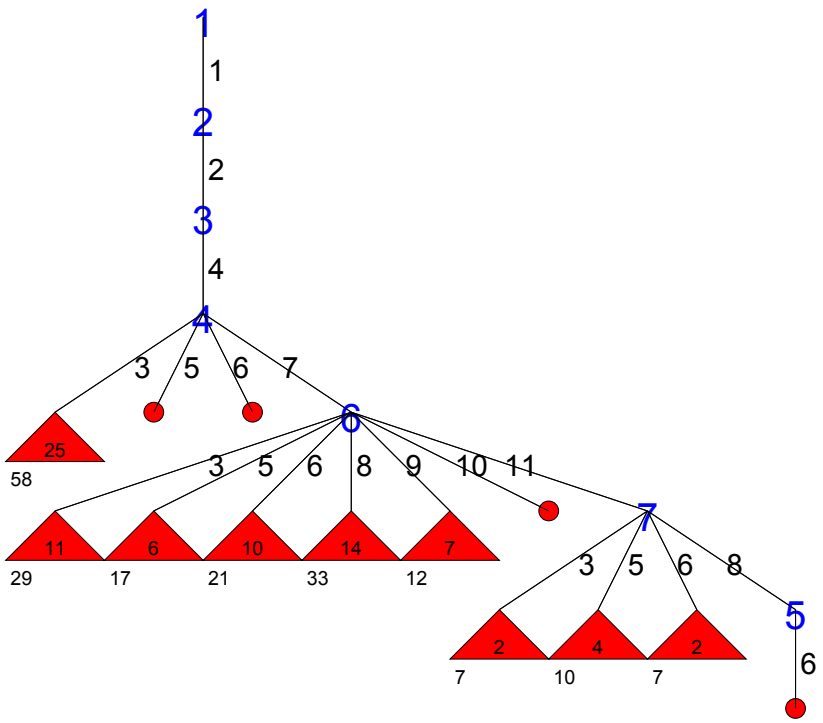

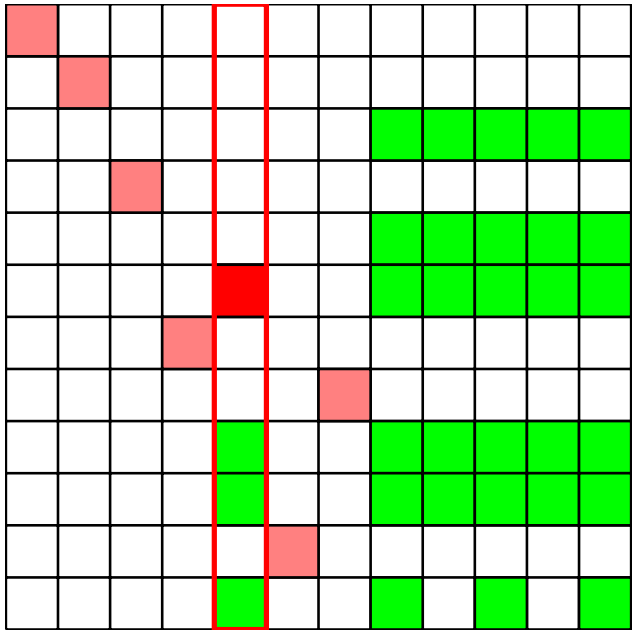

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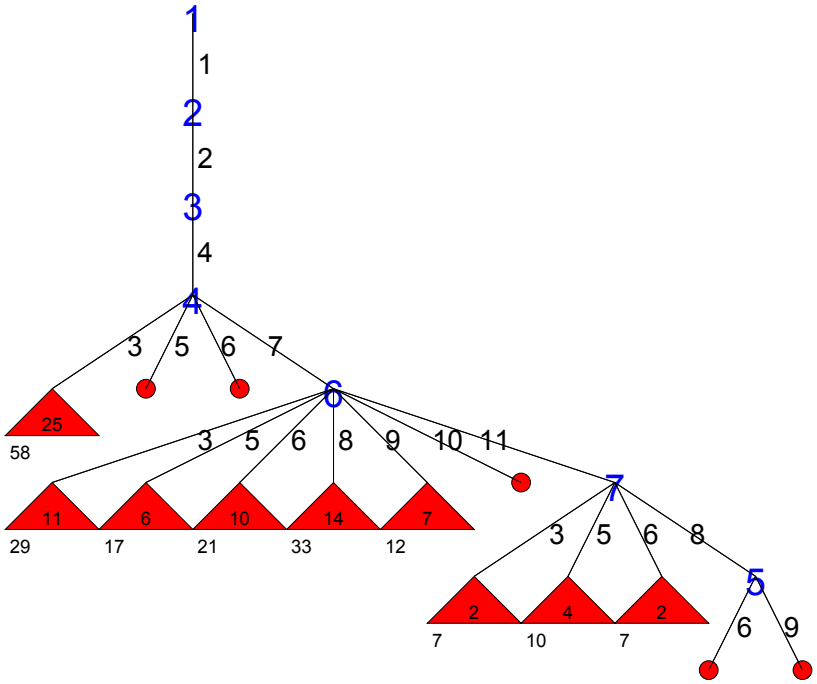

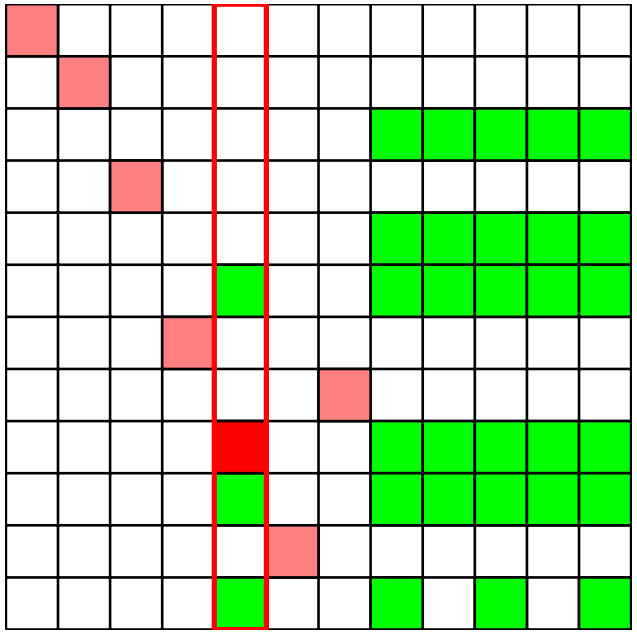

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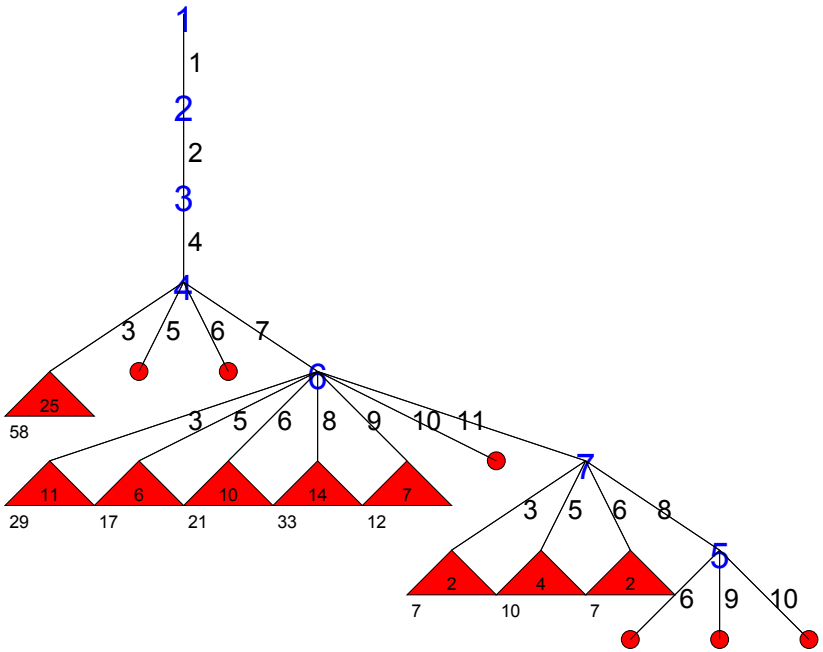

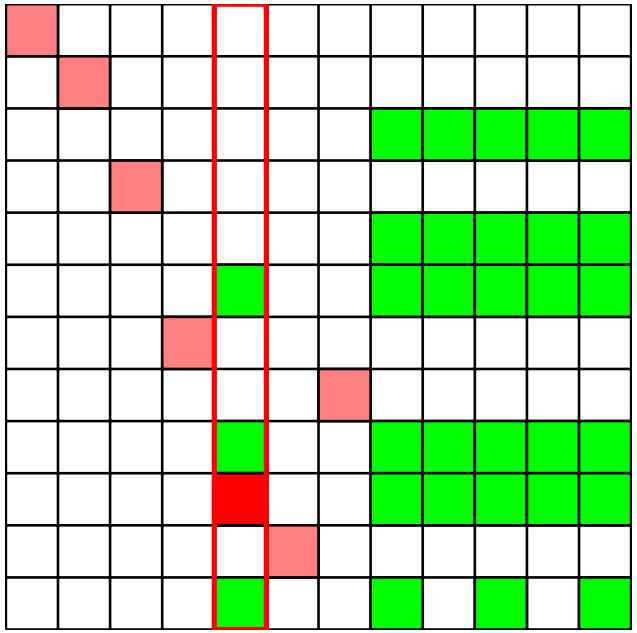

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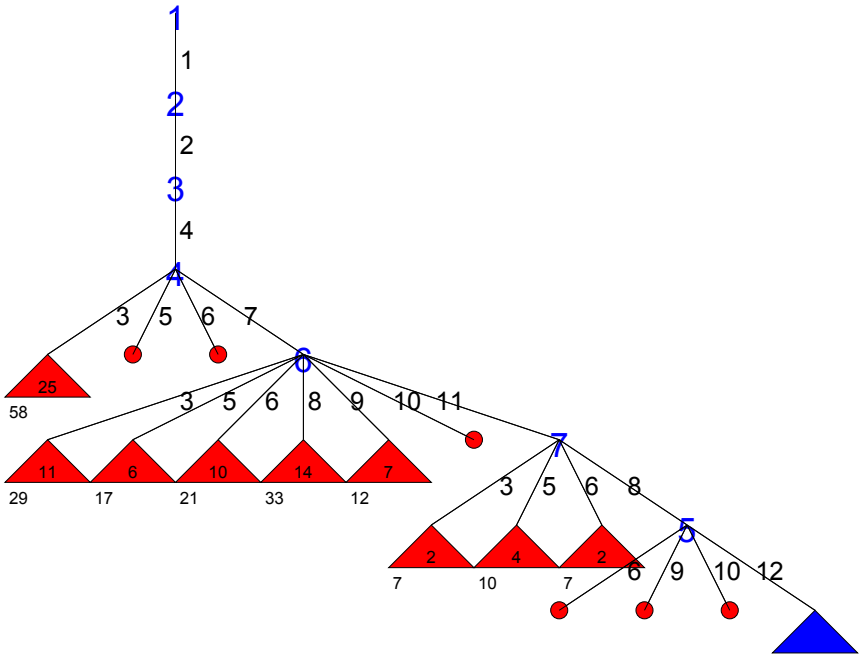

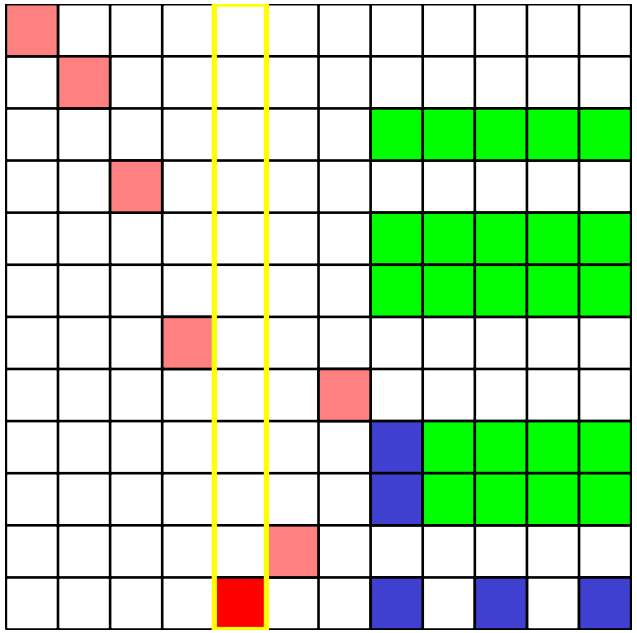

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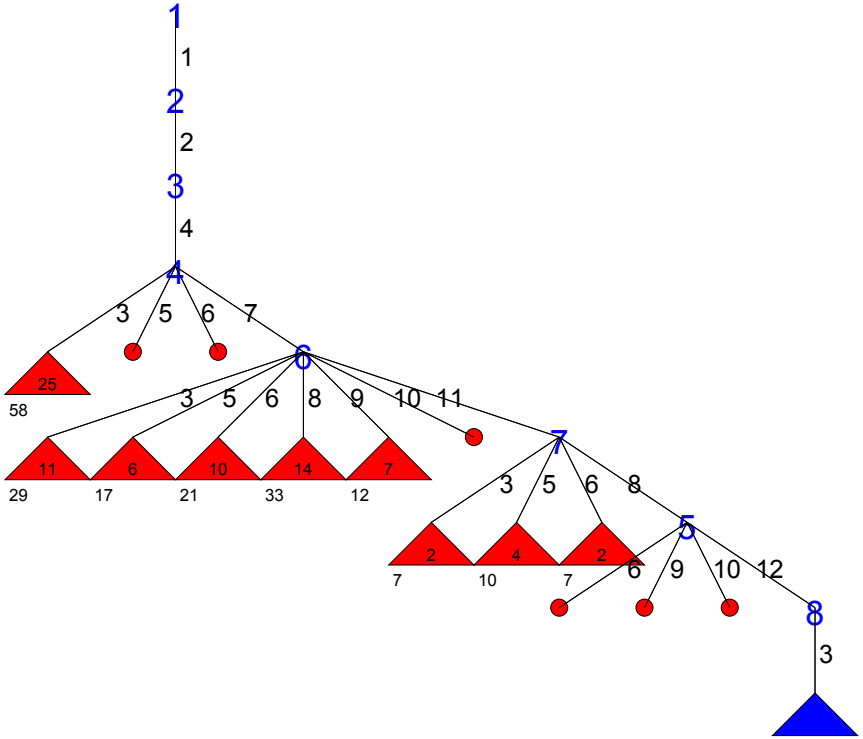

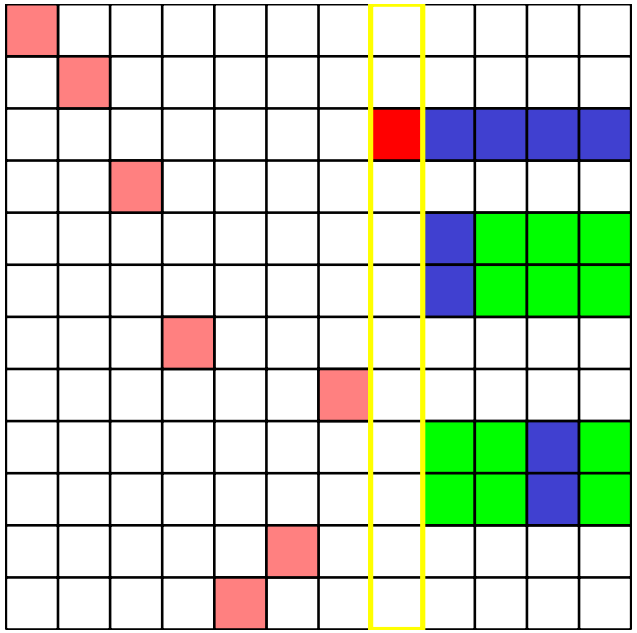

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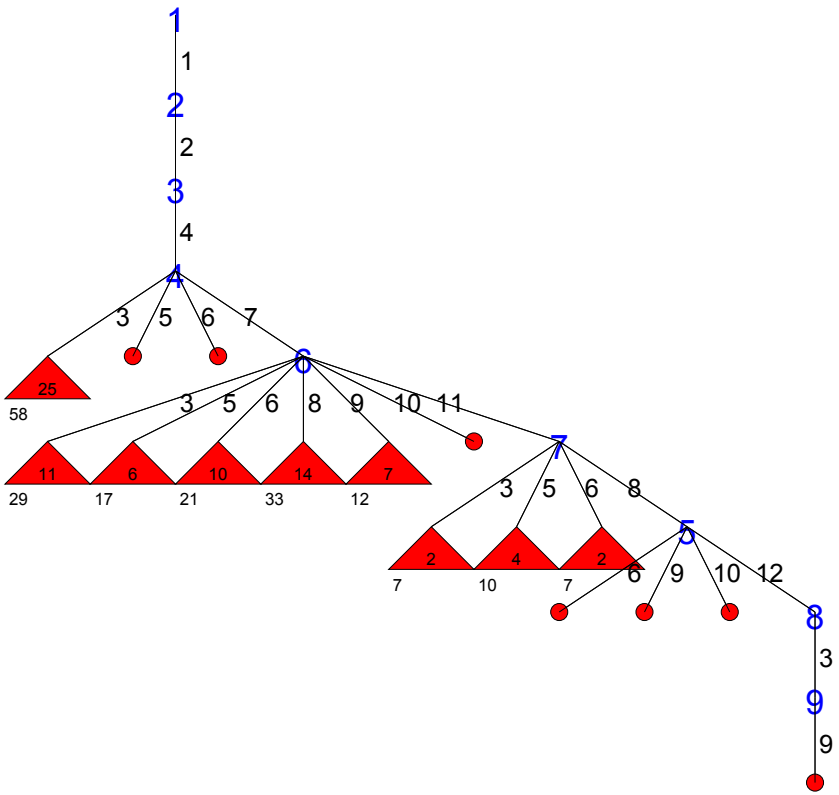

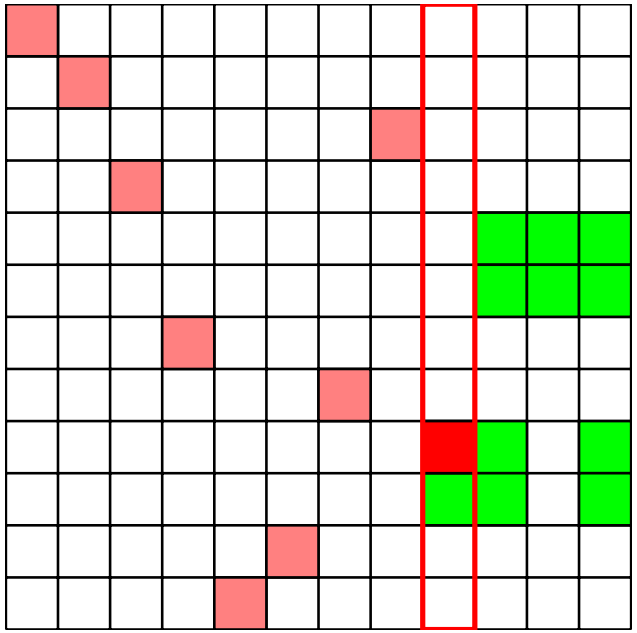

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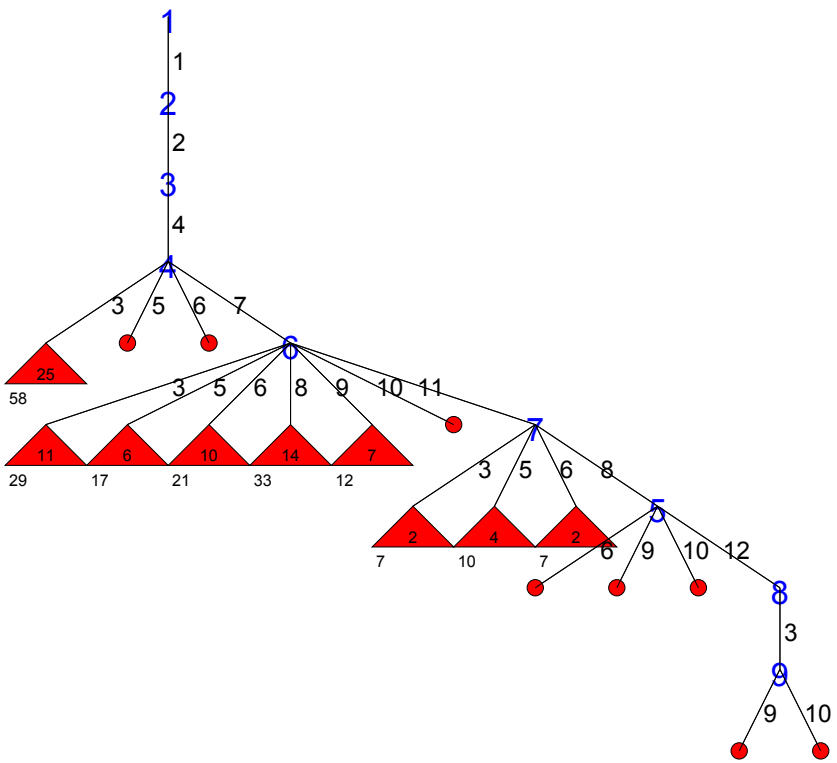
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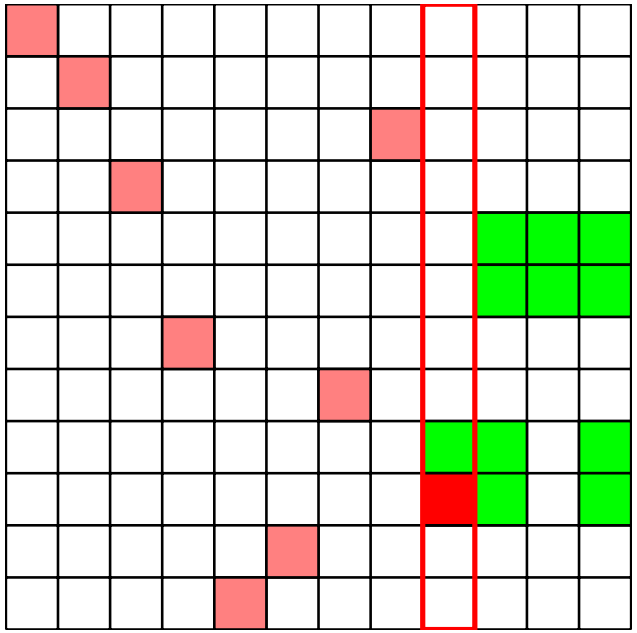

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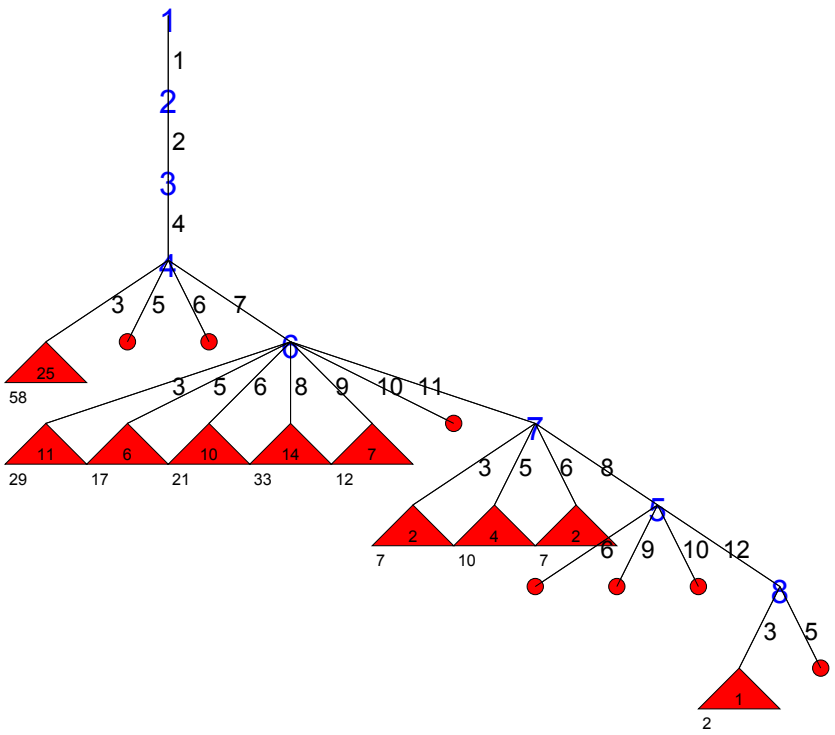

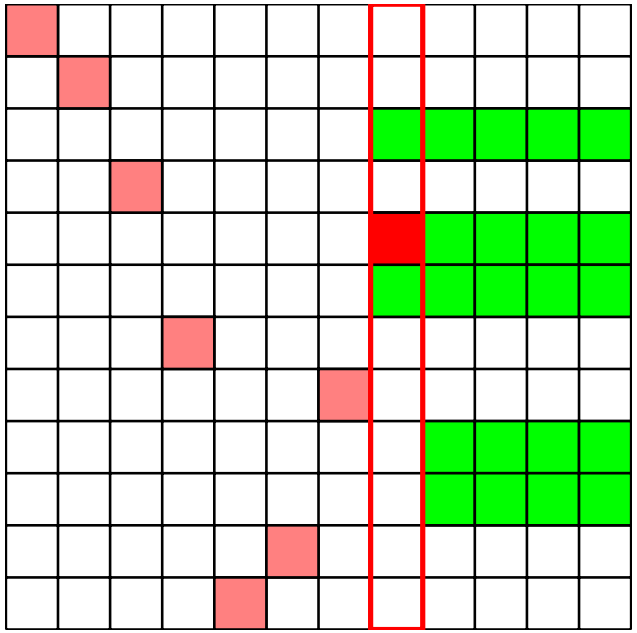

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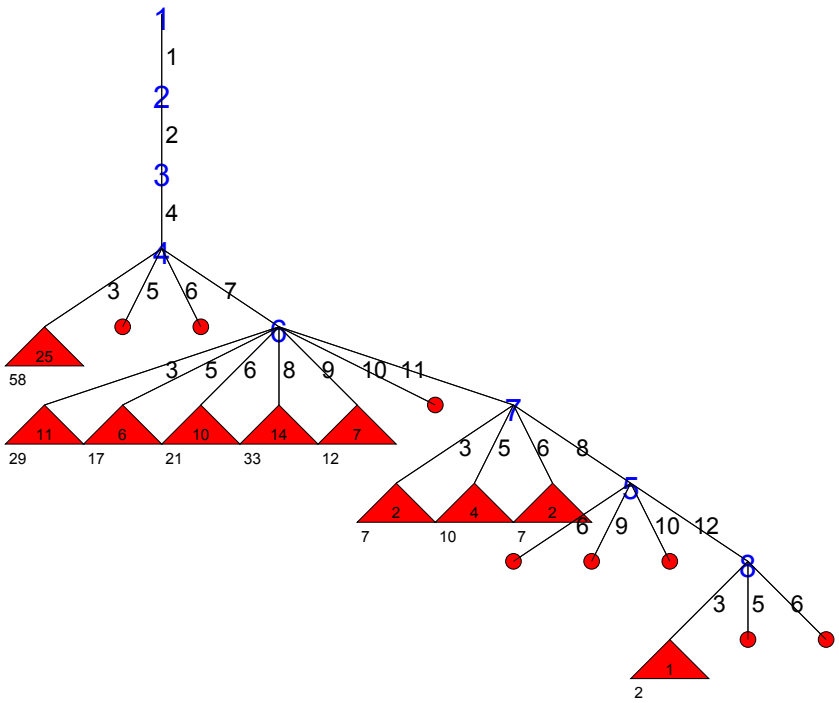

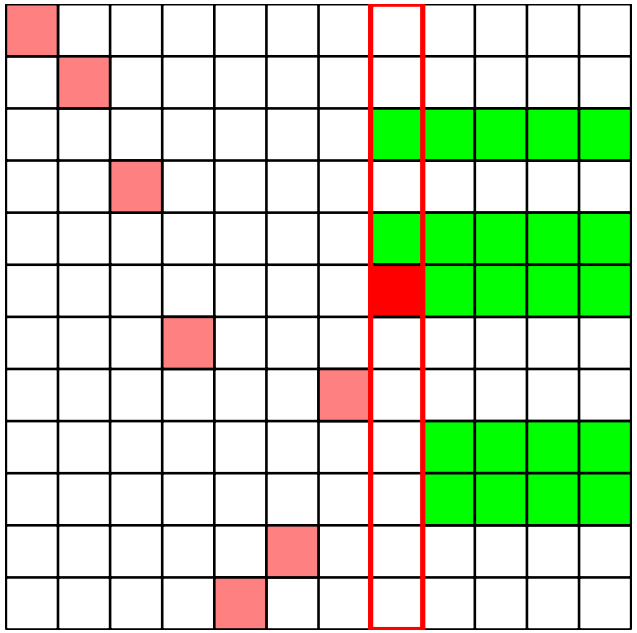

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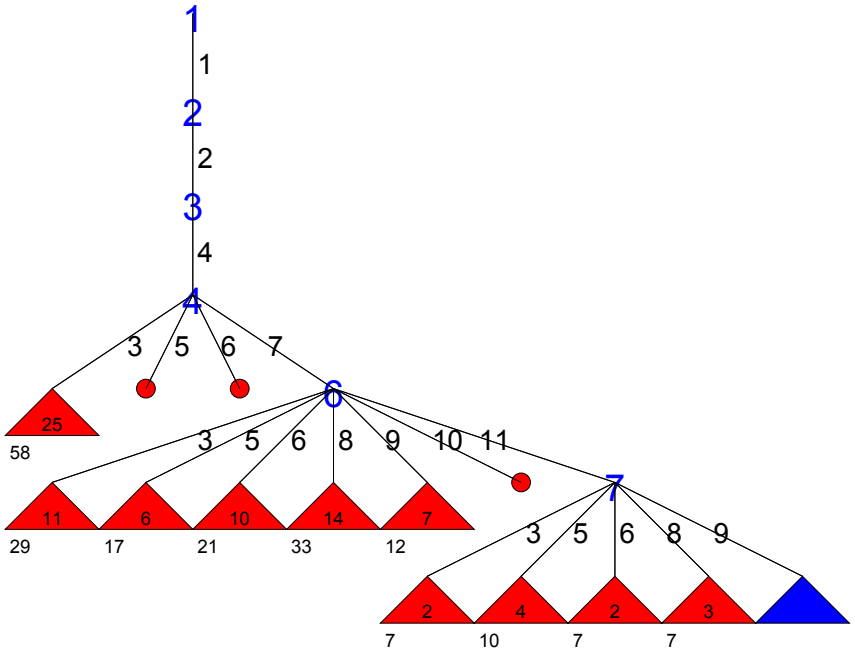

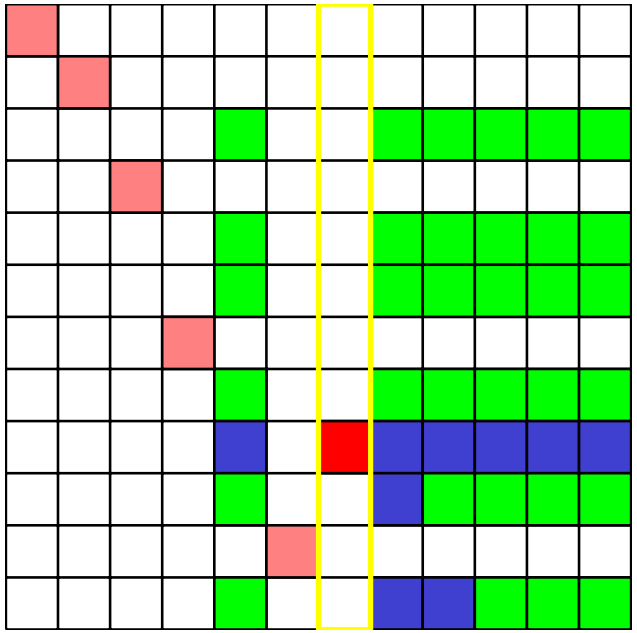

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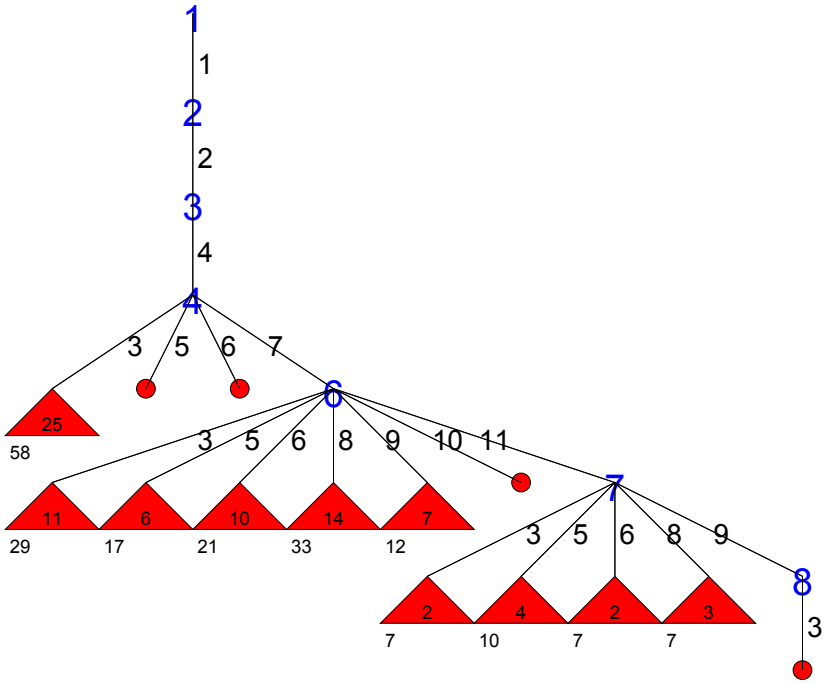

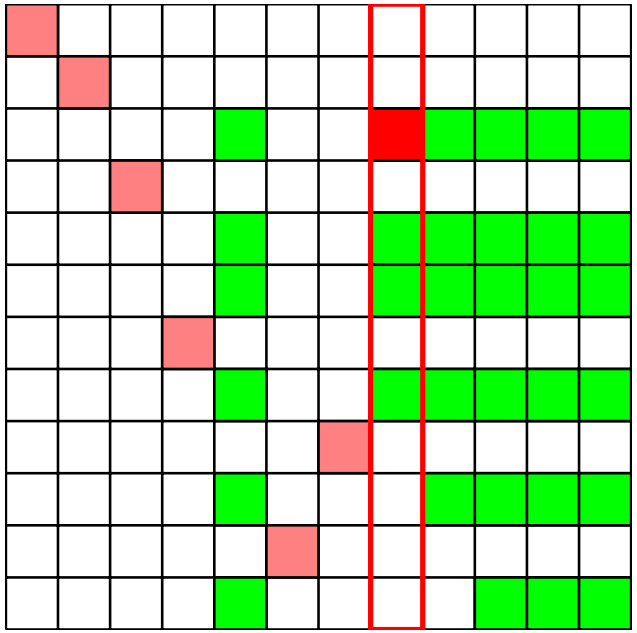

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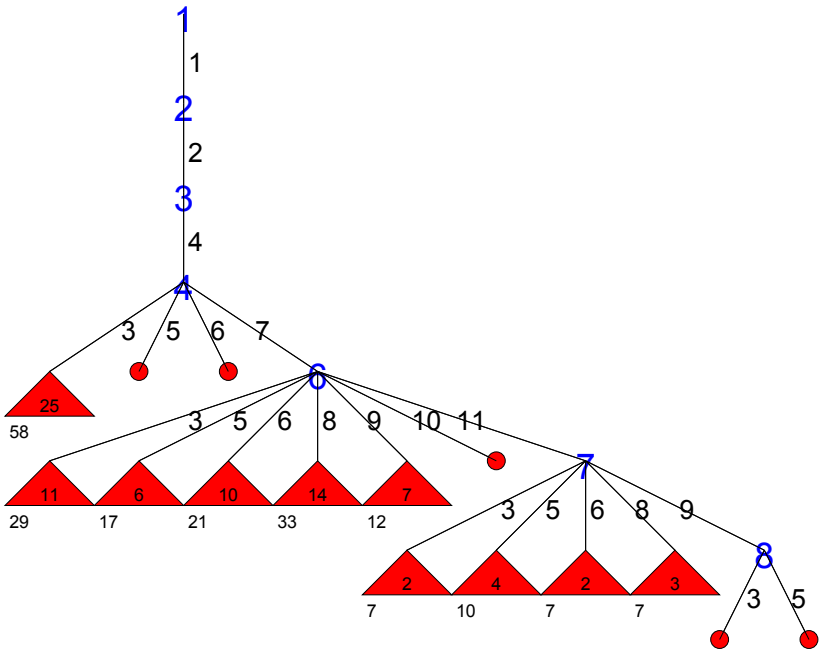

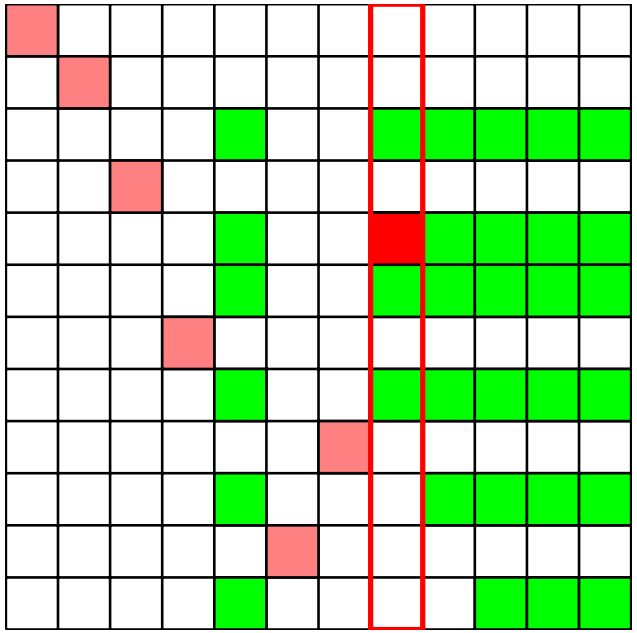

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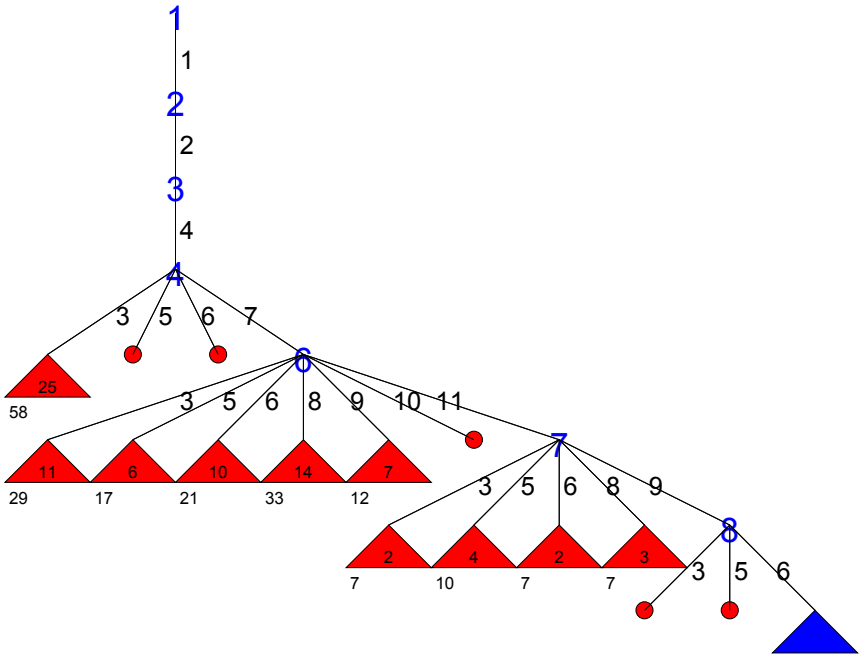
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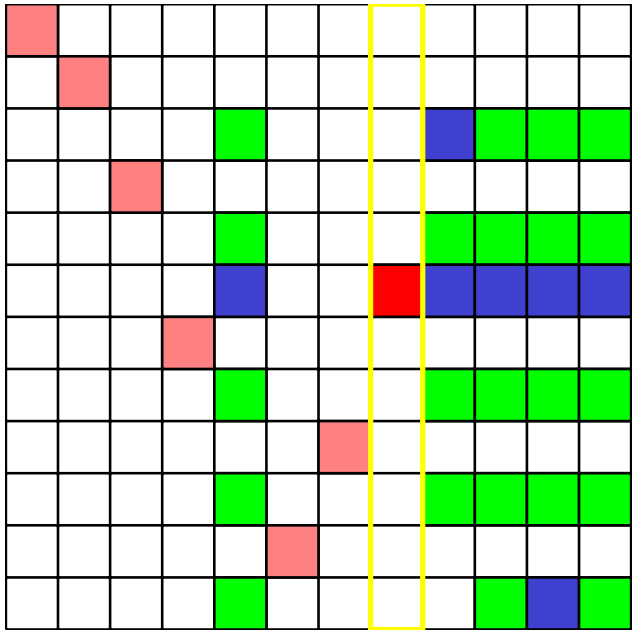

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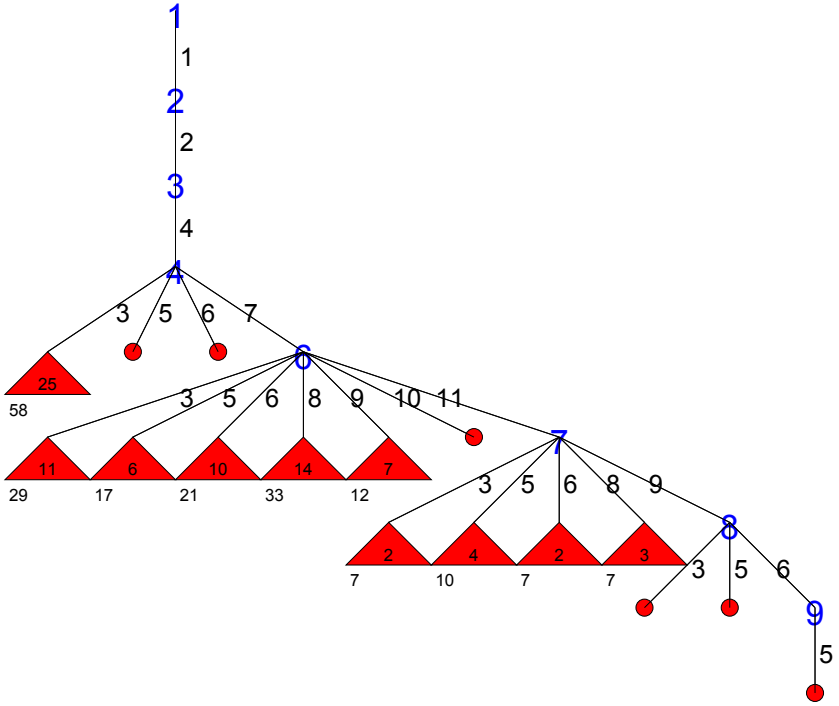

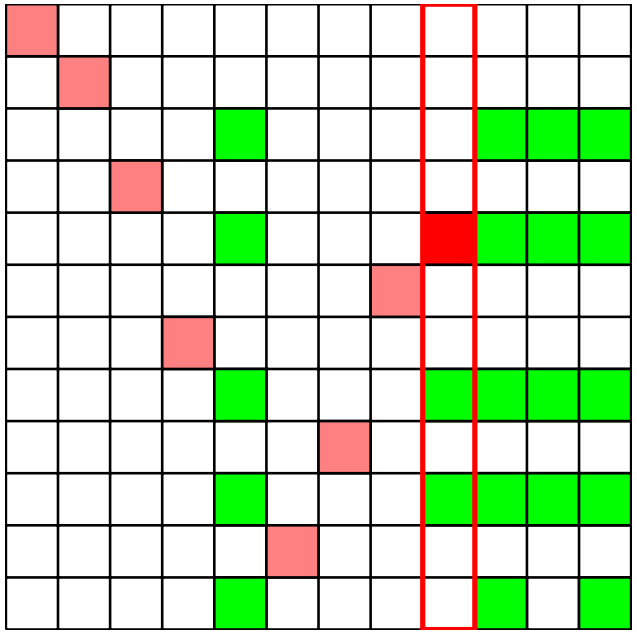

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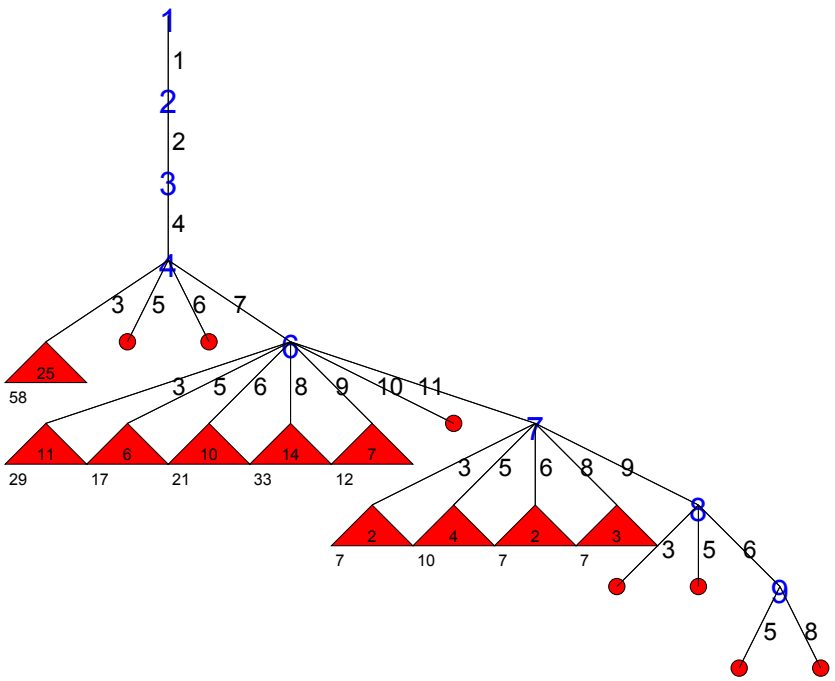

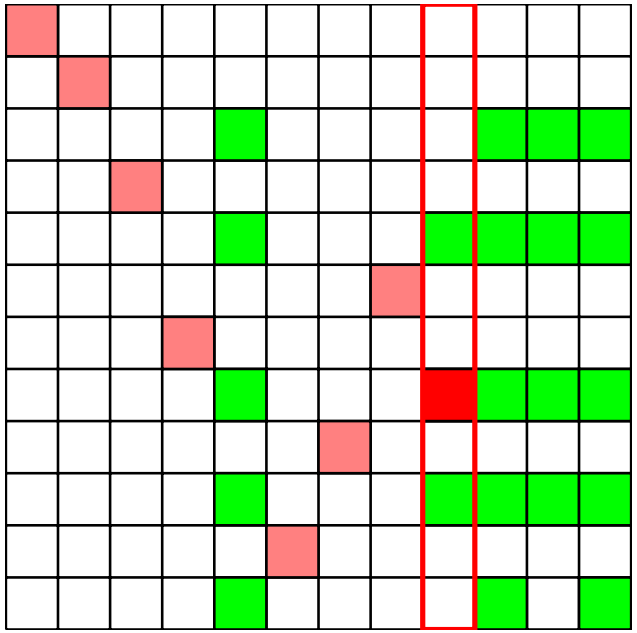

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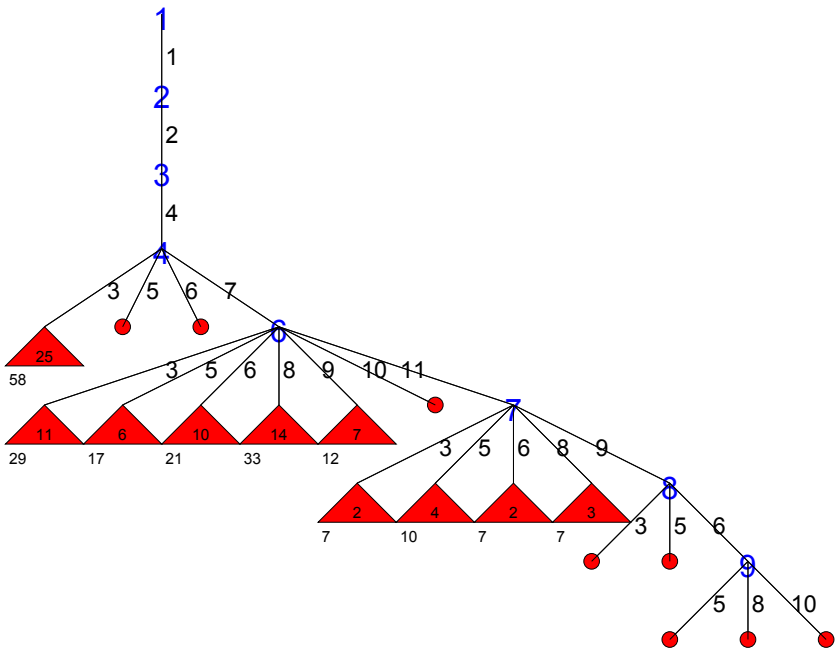

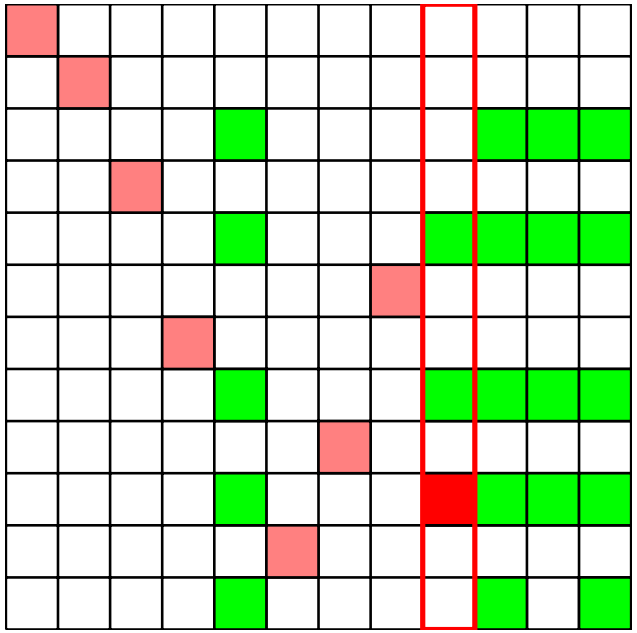

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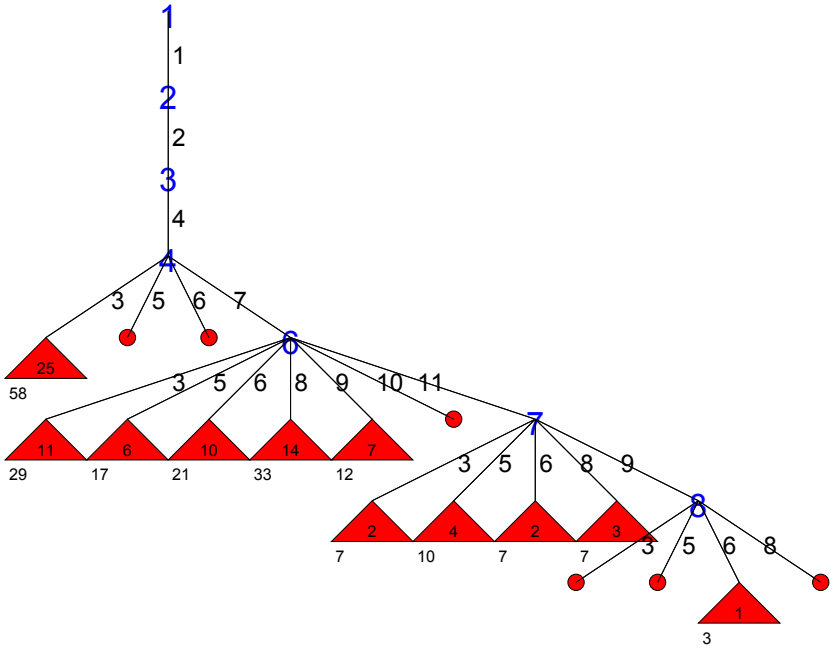

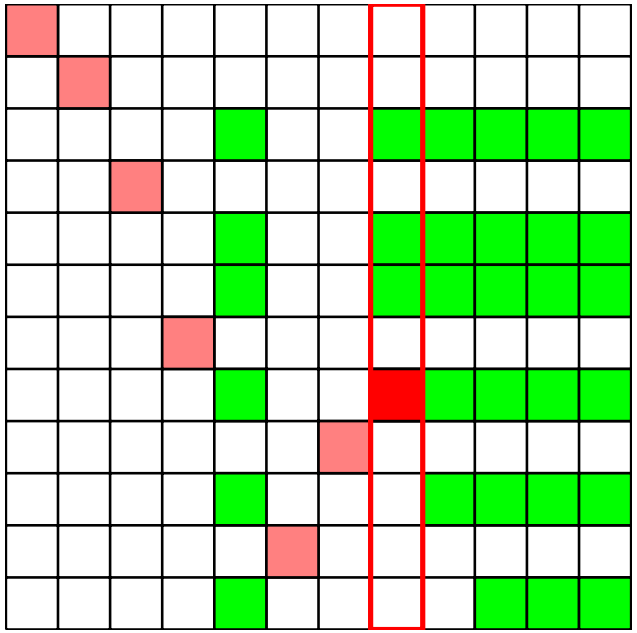

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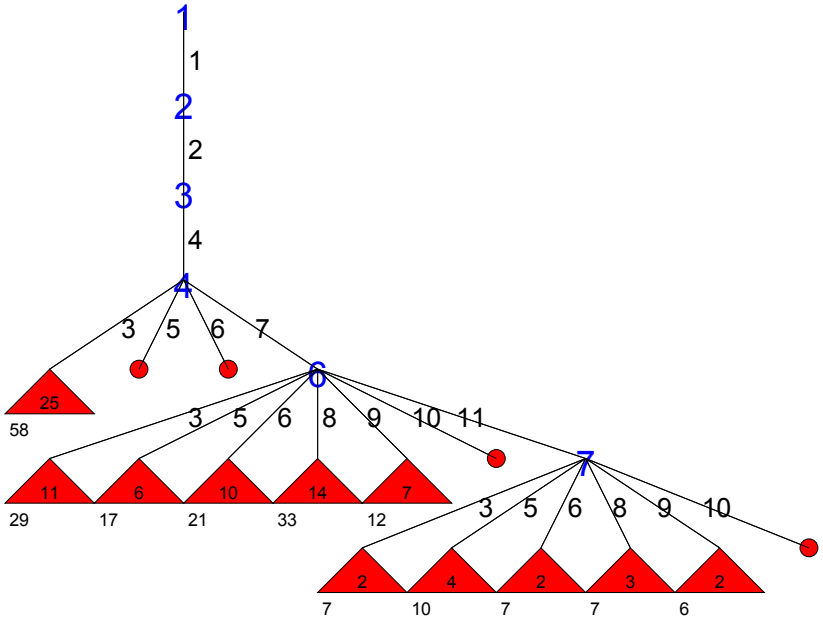

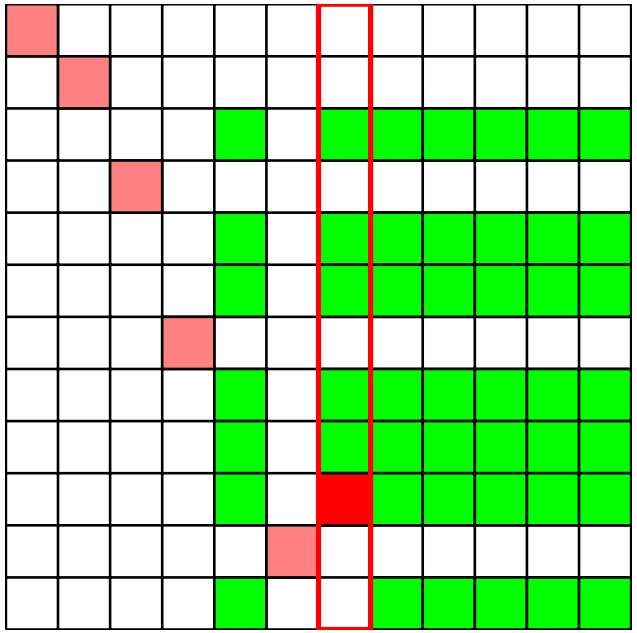

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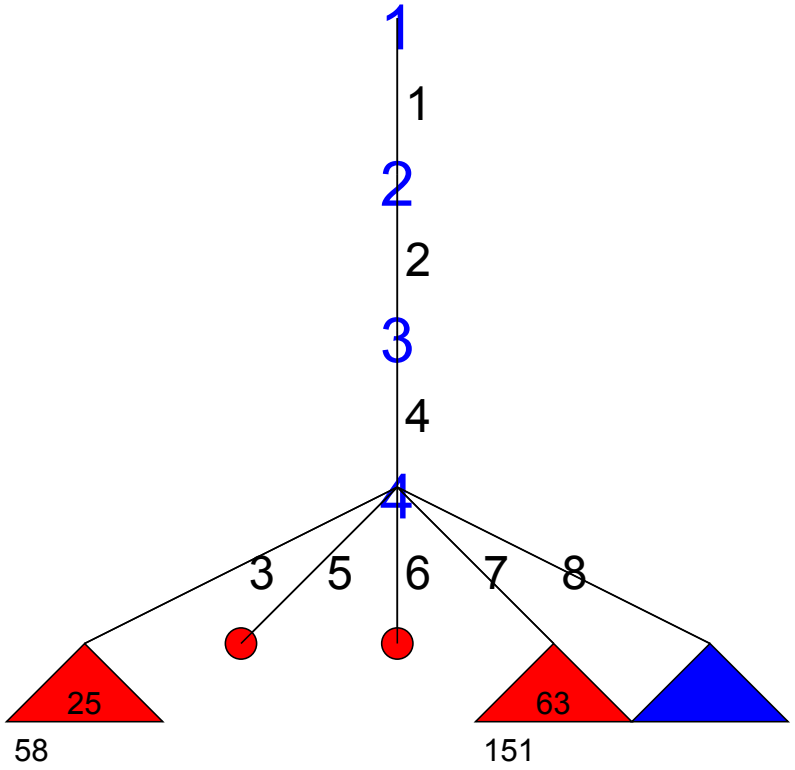

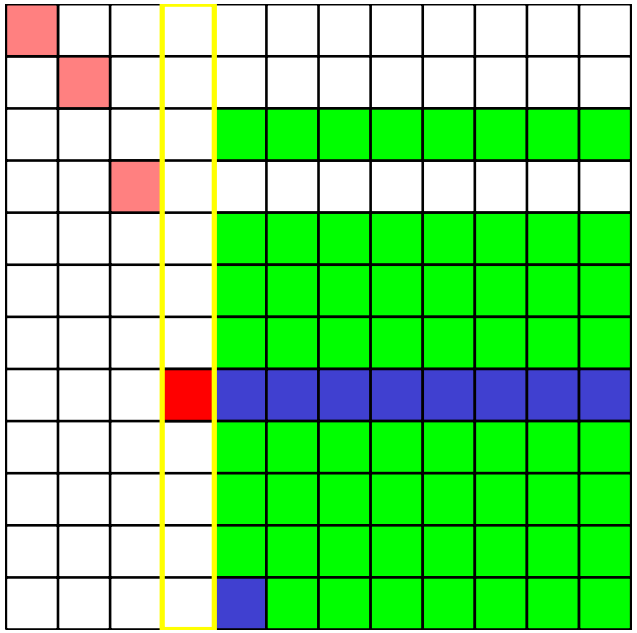

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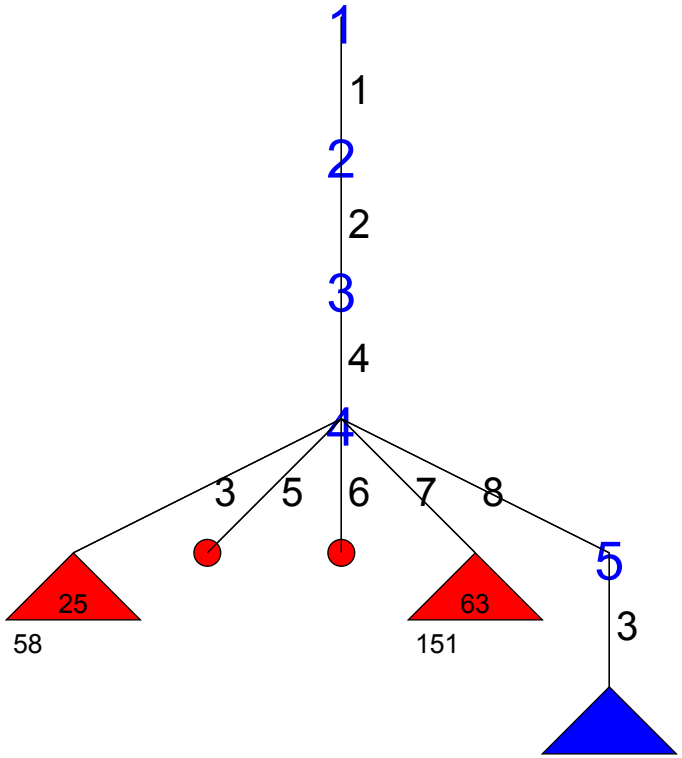

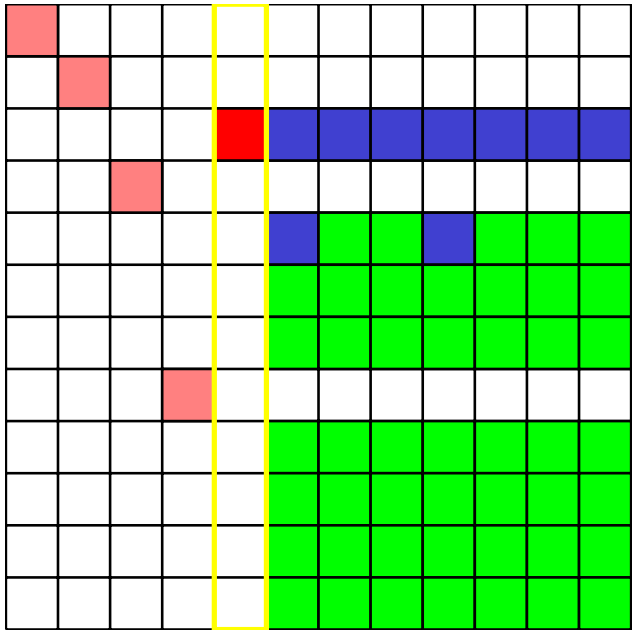

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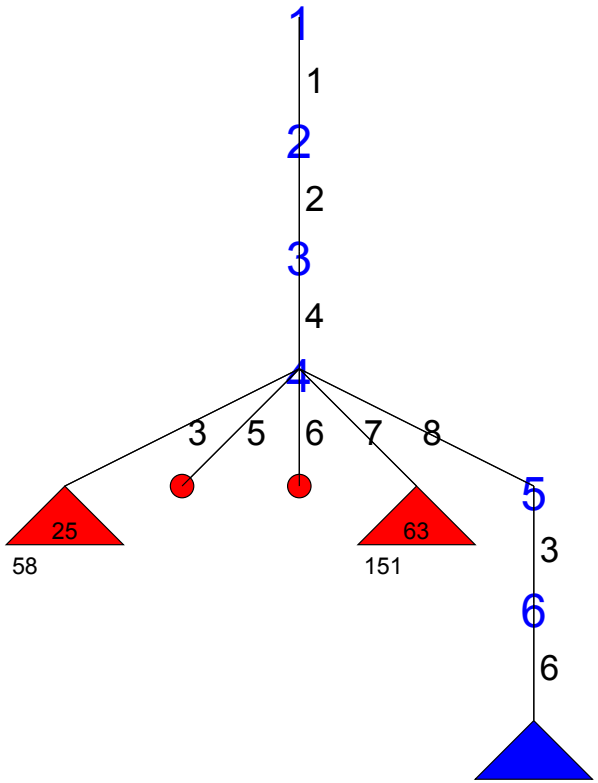

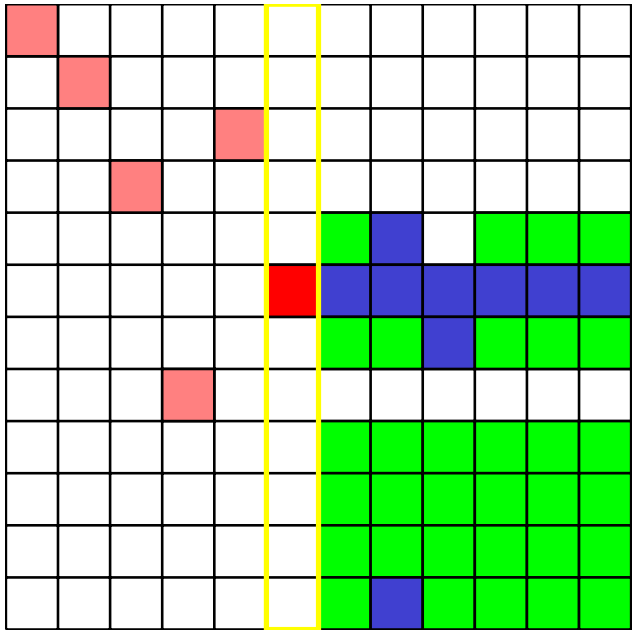

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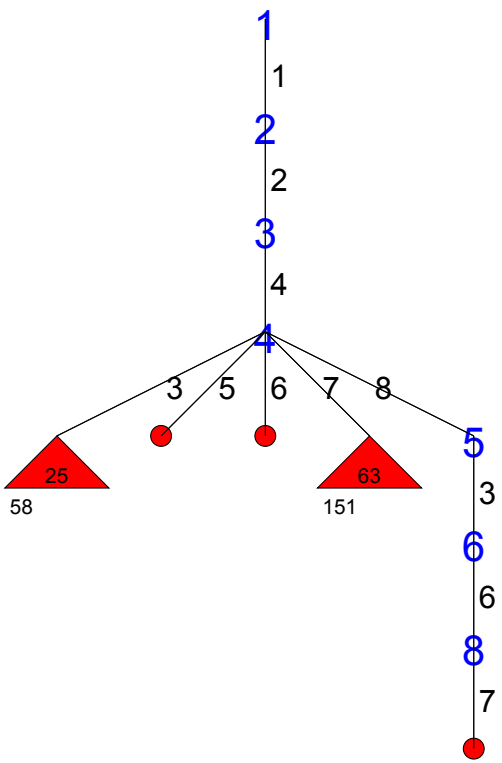

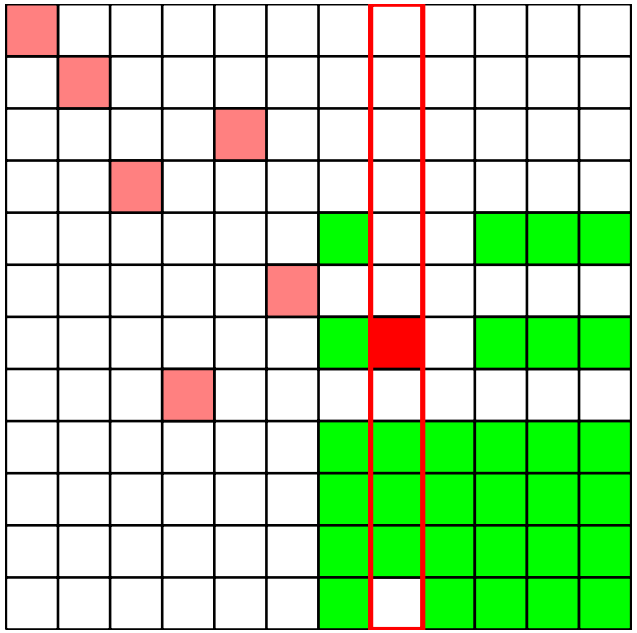

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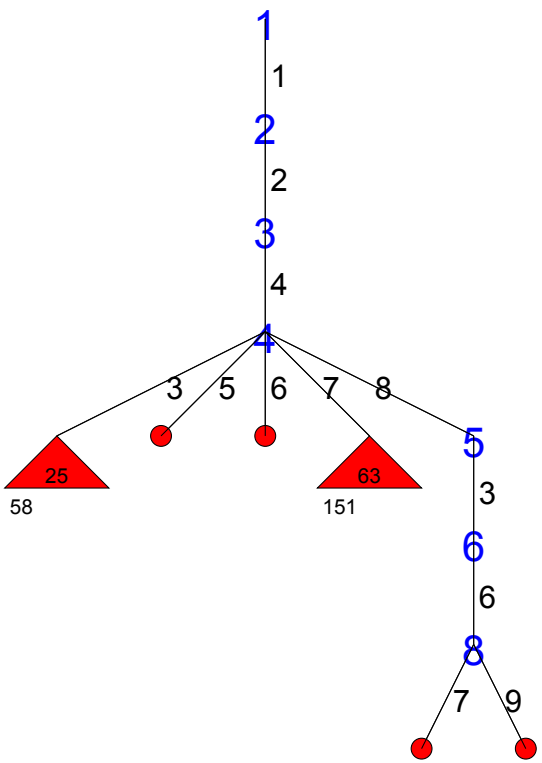

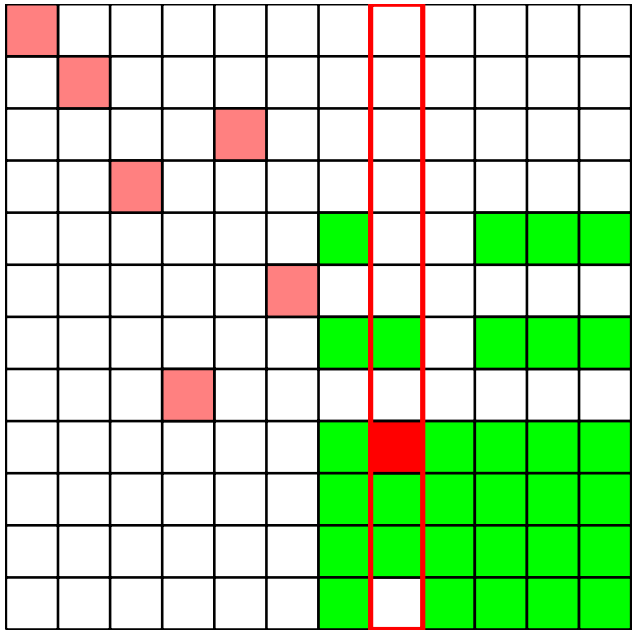

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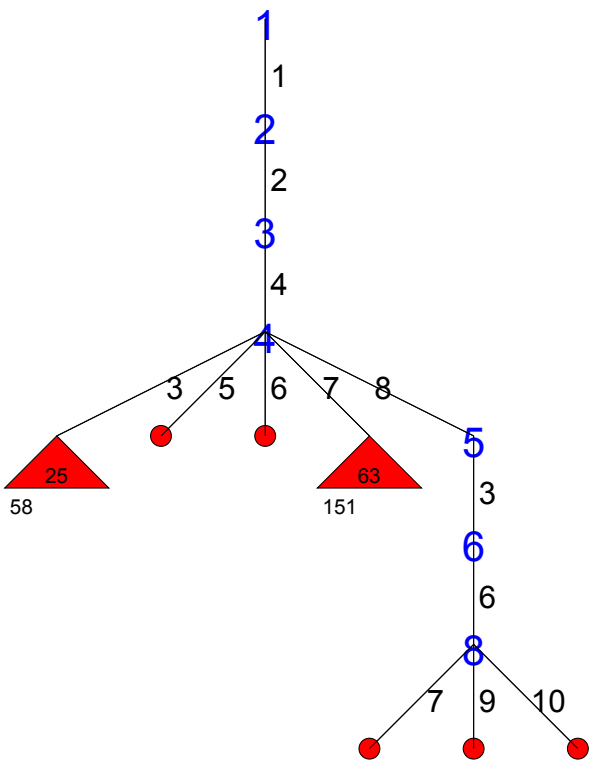

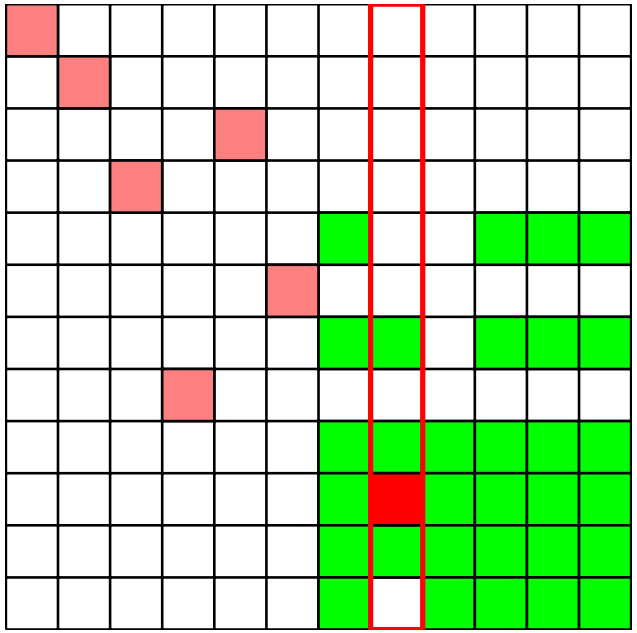

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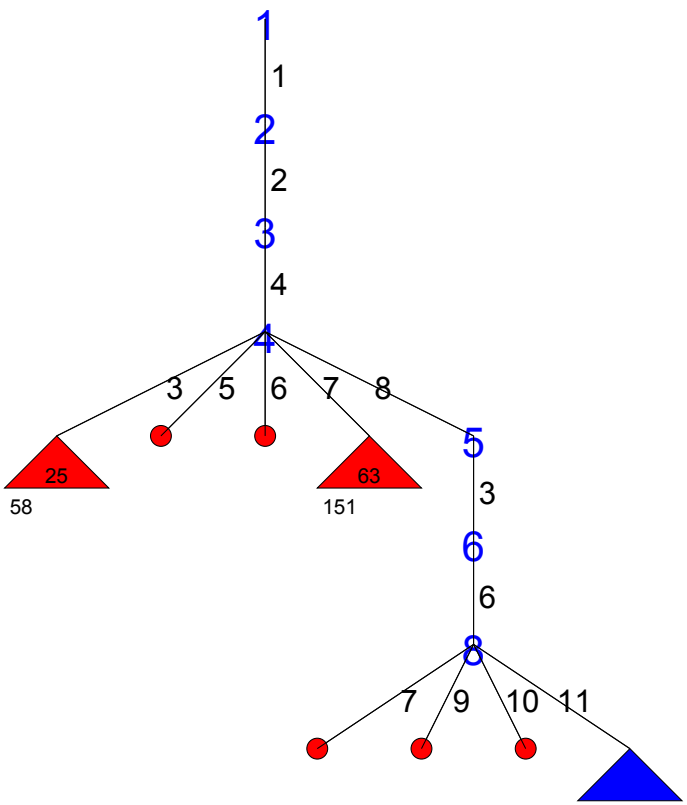

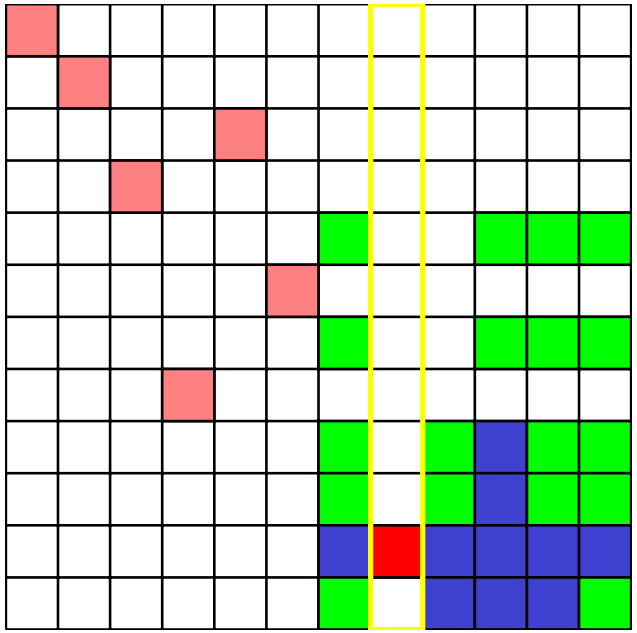

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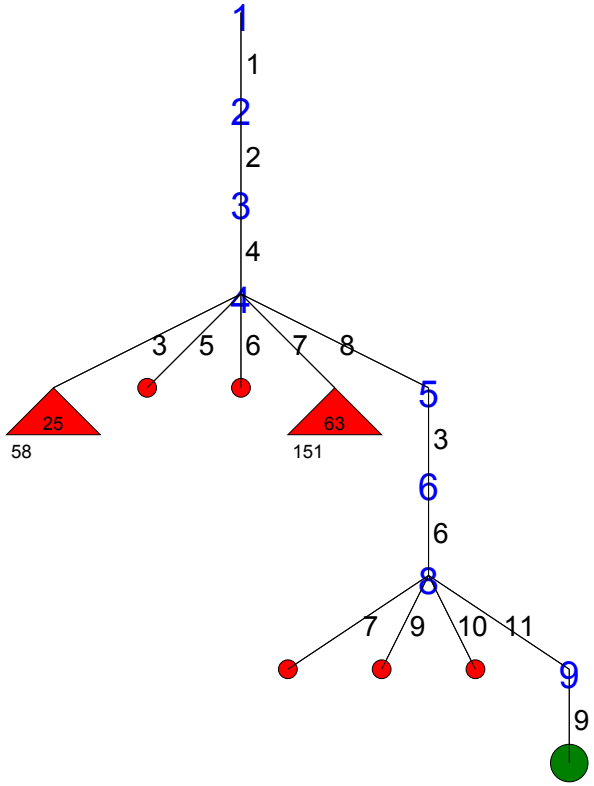
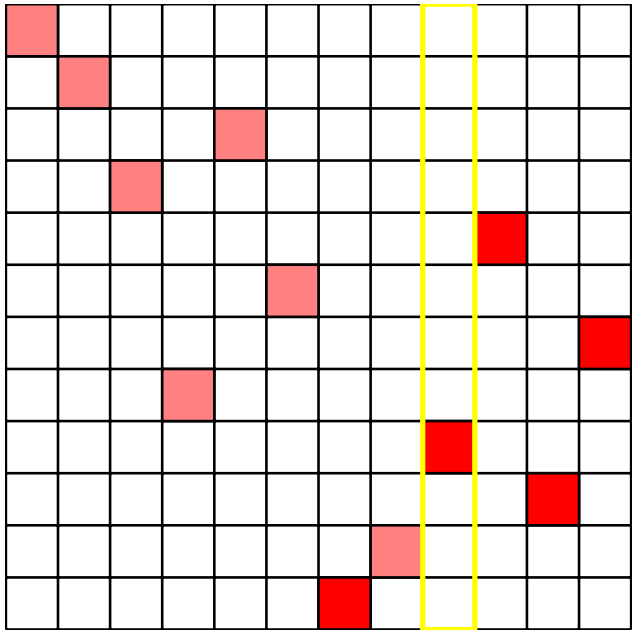
Basic Model



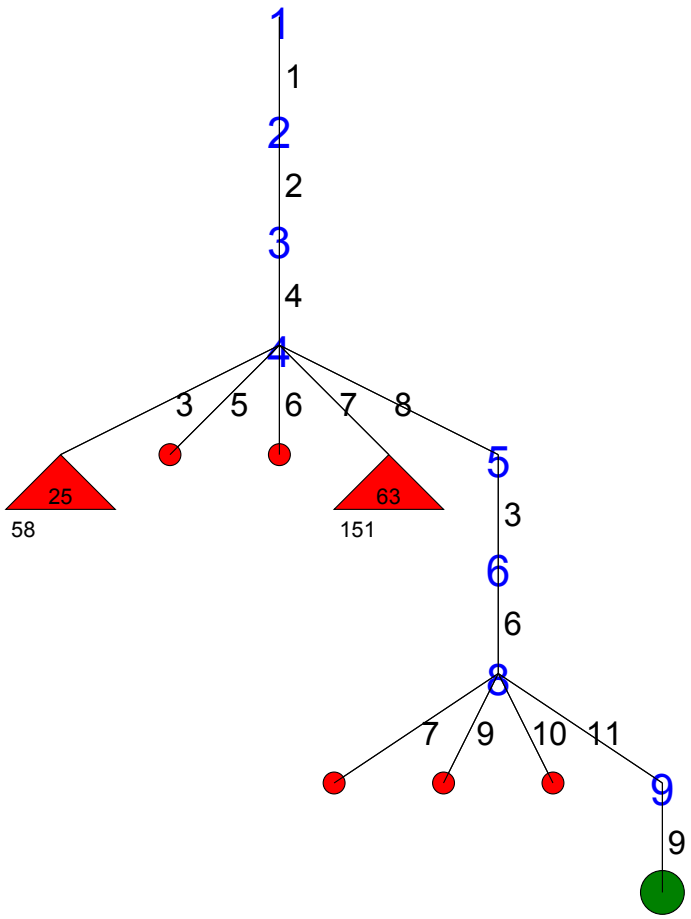


Back to Start

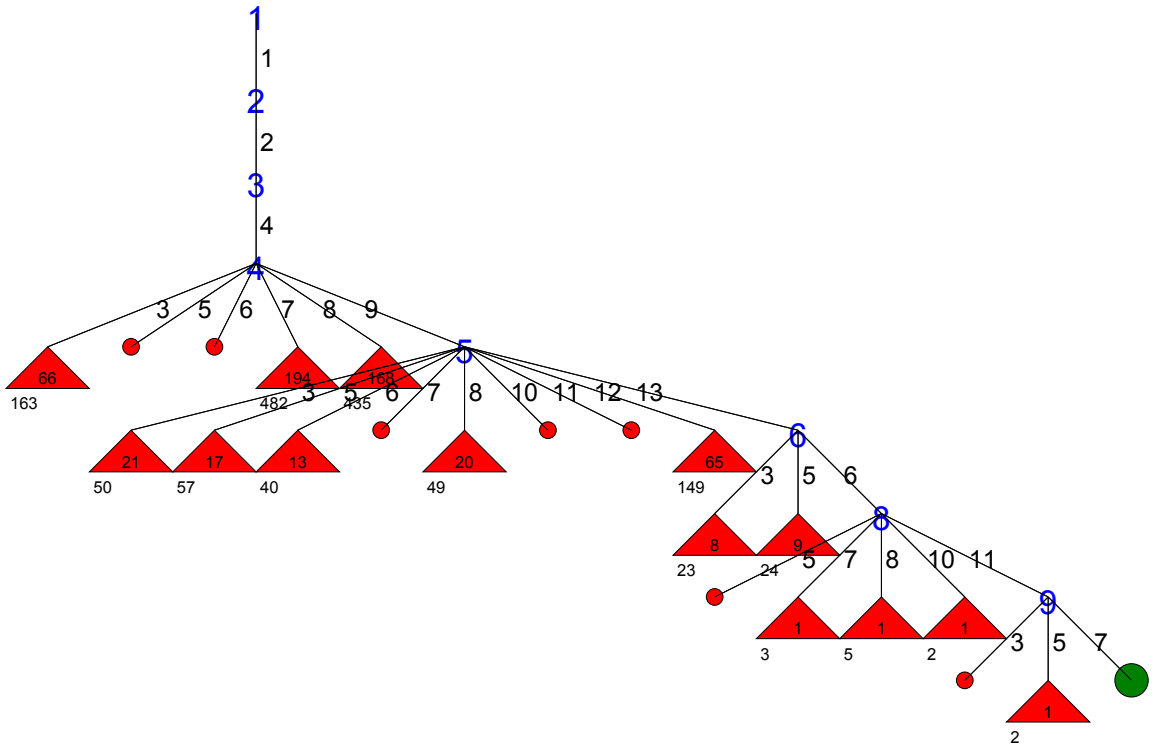
Other Problem Sizes

	Basic Model	
Size	Backtrack	Time
10	4	0.00
11	118	0.08
12	50	0.05
13	335	0.36
14	5008	6.23
15	47332	68.92
16	157773	271.22
17	1641685	3278.19
18	115745	283.97

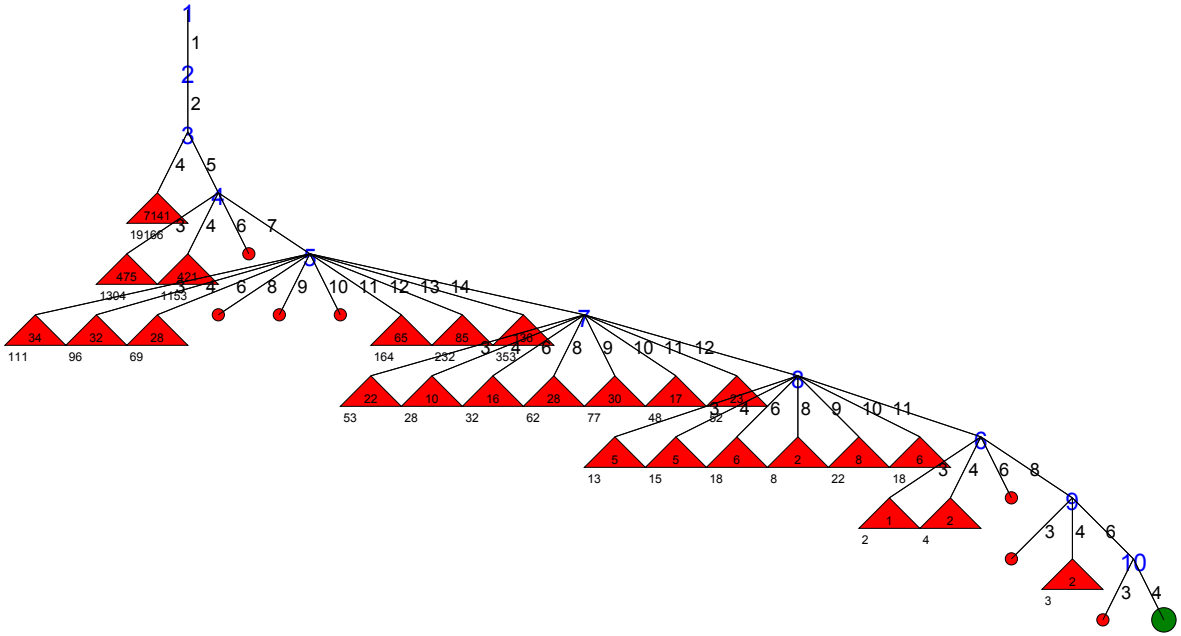
Search tree (Size 12)



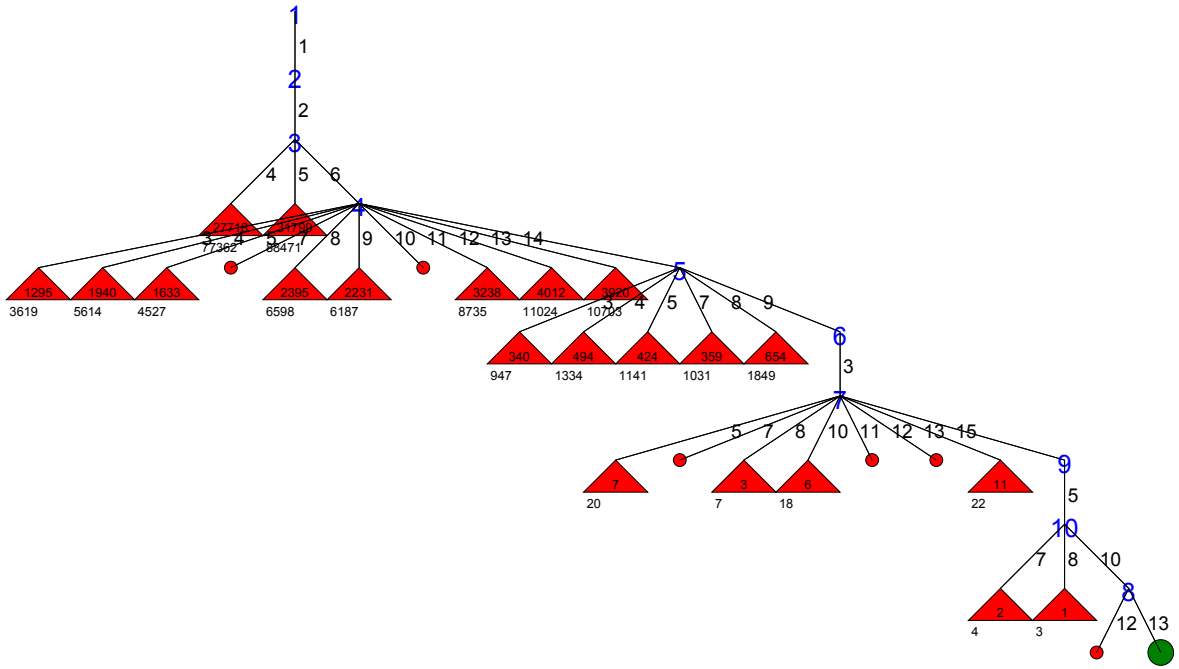
Search tree (Size 13)



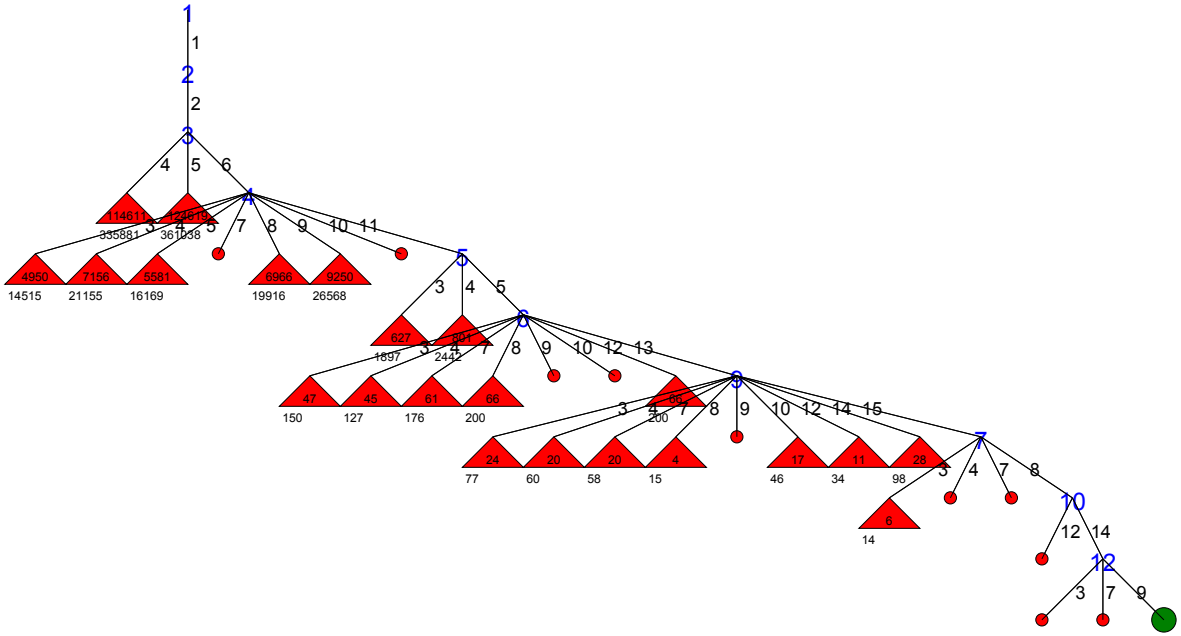
Search tree (Size 14)



Search tree (Size 15)



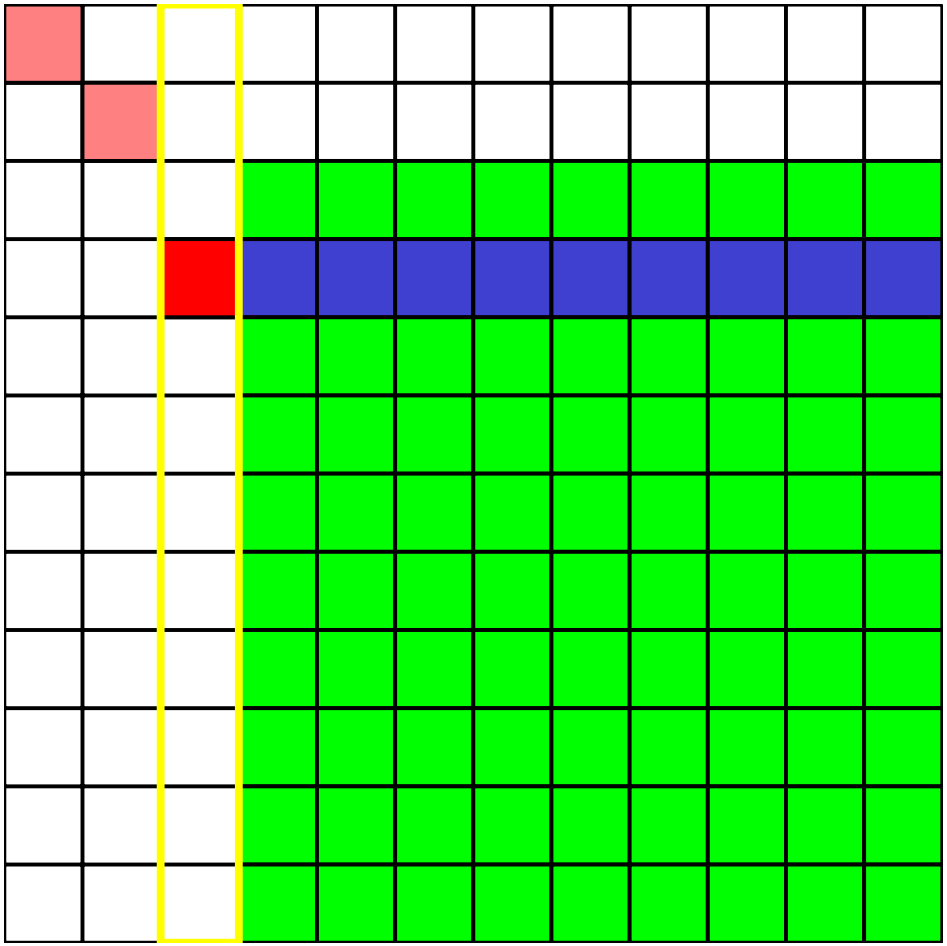
Search tree (Size 16)



Observation

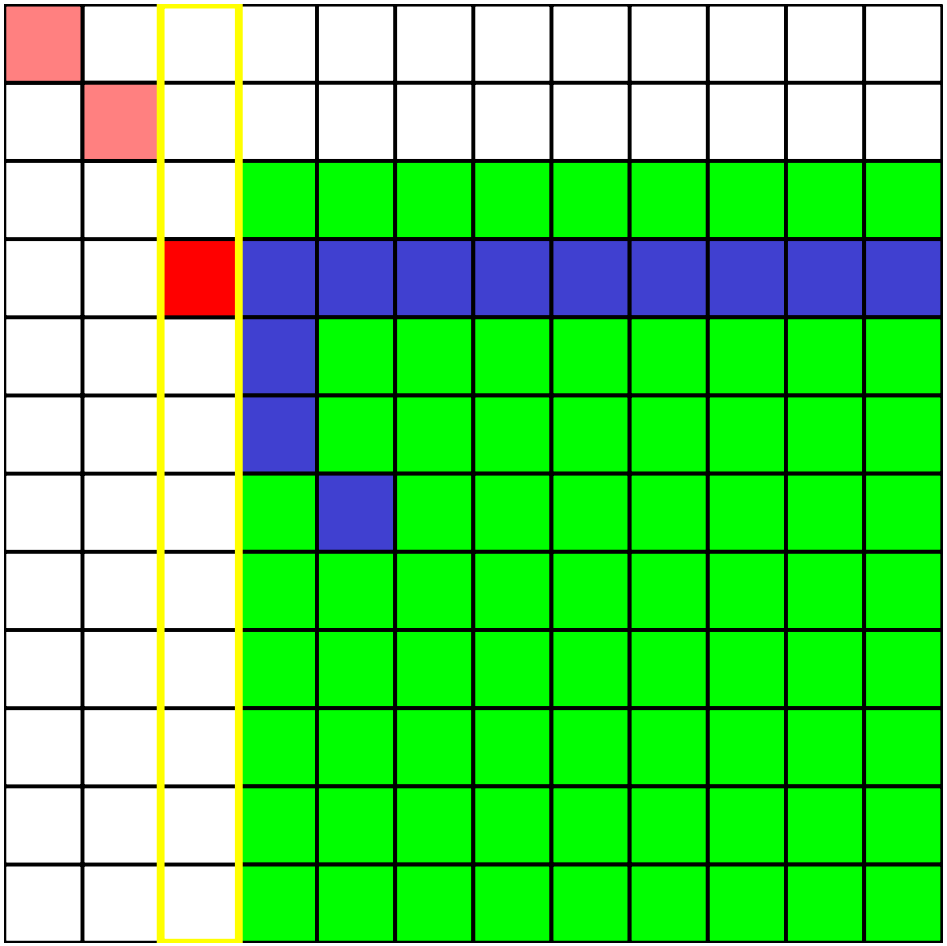
- Problem becomes harder with increasing size
- Failures occur from level 3 down
- Deep backtracking required to undo wrong choices
- Value selection not working, have to explore all choices
- Increase not uniform

Missing Propagation



The model is doing this

Missing Propagation



It could be doing that!

Outline

1 Problem

2 Program

3 Search

4 Improvements
- Adding Constraints
- Change of Search Strategy

Changed Differences

```
diffs(_,[]).
diffs(L,[H|T]):-
        diff_pairs(L,[H|T],Diffs,Triples),
        impose_triples(Triples,[]),
        alldifferent(Diffs),
        diffs(L,T).

diff_pairs(_,[],[],[]).
diff_pairs([X|X1],[Y|Y1],[D|D1],[t(X,Y,D)|T]):-
        X #= Y+D,
        diff_pairs(X1,Y1,D1,T).
```

Changed Differences

```
impose_triples([],_).
impose_triples([t(X,Y,D)|R],Others):-
        suspend(impose_triple(D,R),4,D->inst),
        suspend(impose_triple(D,Others),4,D->inst),
        impose_triples(R,[t(X,Y,D)|Others]).
```

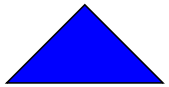
Changed Differences

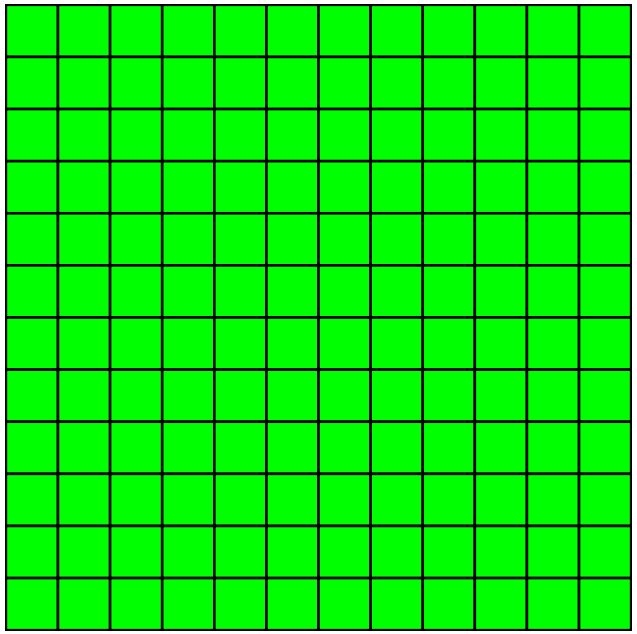
```
impose_triple(_D,[]).
impose_triple(D,[t(X,Y,_)|R]):-
        (var(X) ->
            suspend(impose_one_triple12(D,X,Y),
                    4,X->inst)
        ;
            impose_one_triple12(D,X,Y)
        ),
%       ...
        impose_triple(D,R).

impose_one_triple12(D,X,Y):-
        V is X-D,
        Y #\= V.
```

Further Model Improvements

- DC consistent `alldifferent` between variables
- (DC consistent `alldifferent` between differences)
- DC difference constraint

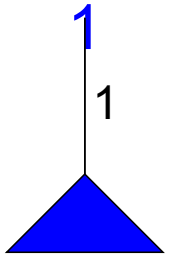
Improved Model

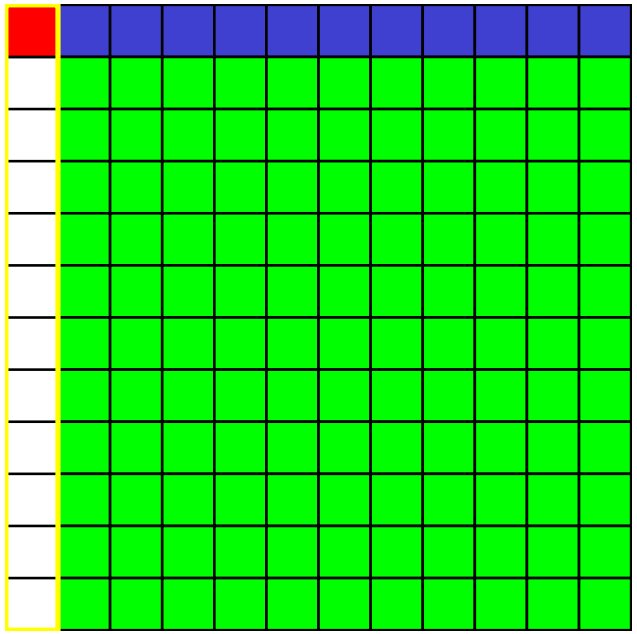




Skip Animation

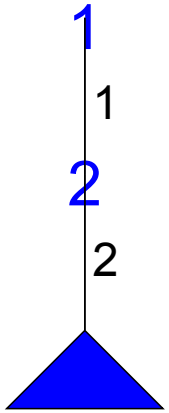
Improved Model

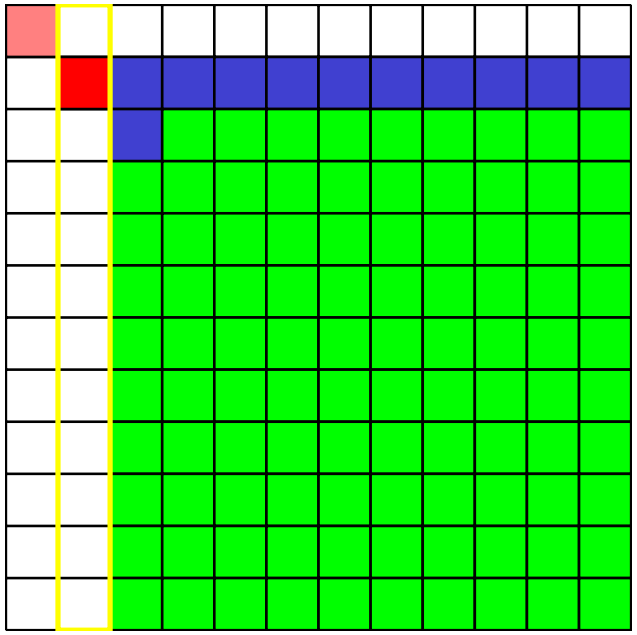




Back to Start
Skip Animation

Improved Model





Back to Start
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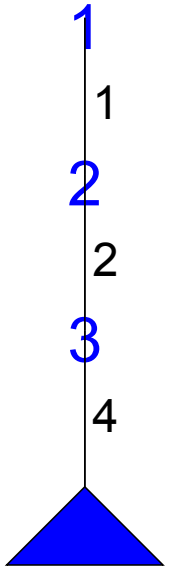
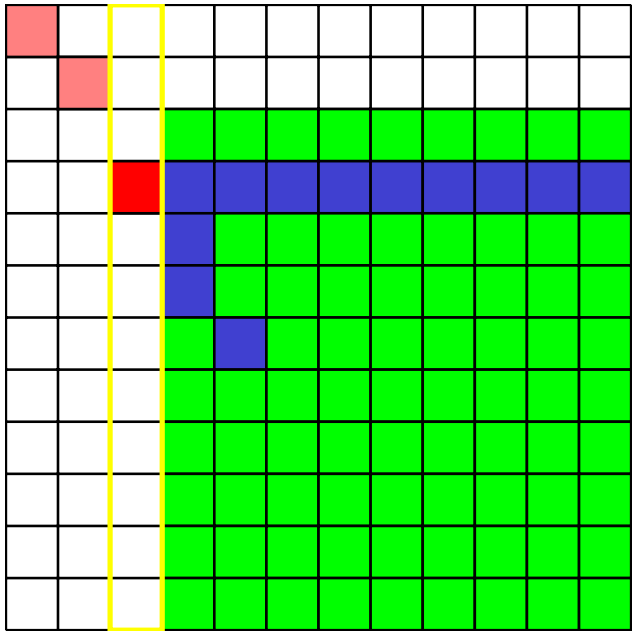
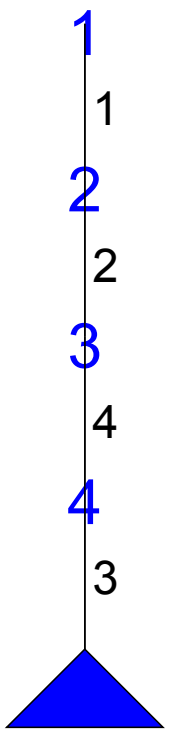
Improved Model

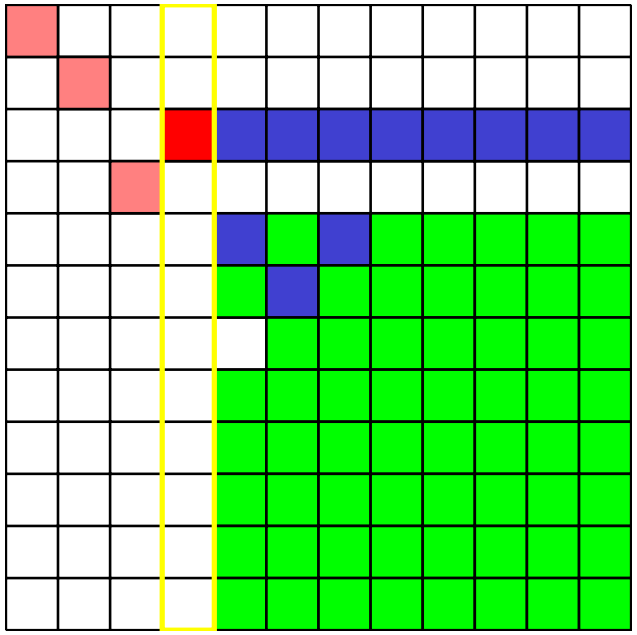




Back to Start
Skip Animation

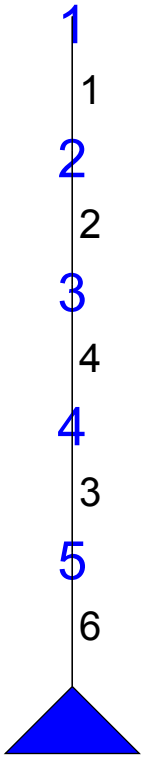
Improved Model

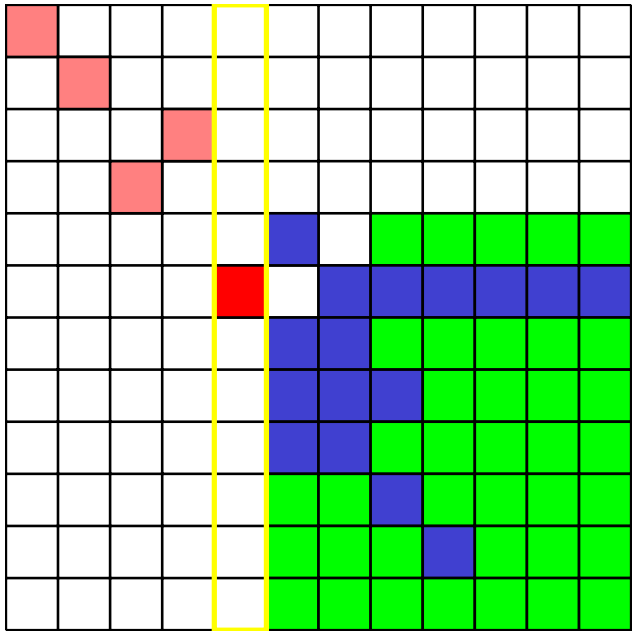




Back to Start
Skip Animation

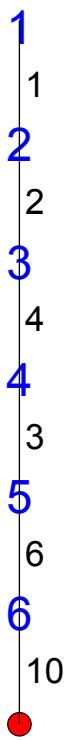
Improved Model

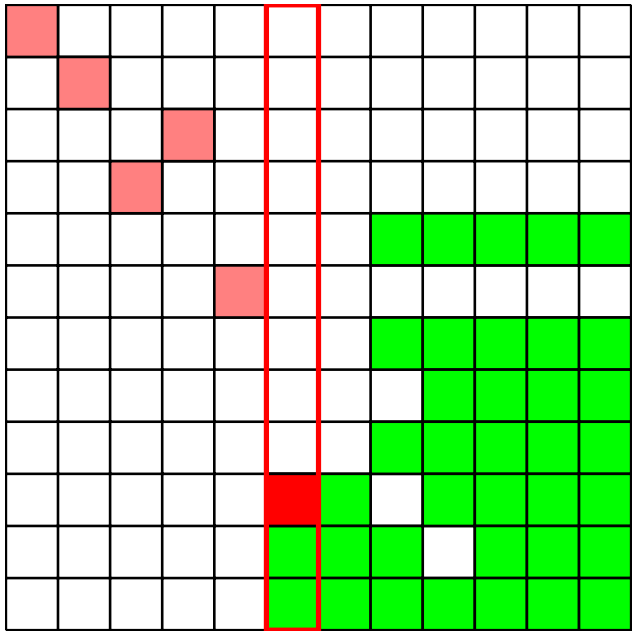




Back to Start
Skip Animation

Improved Model





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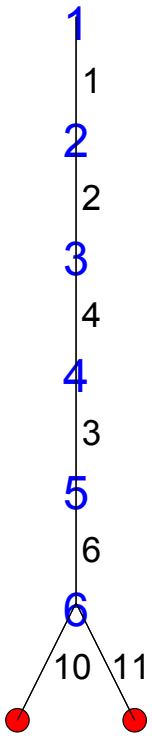
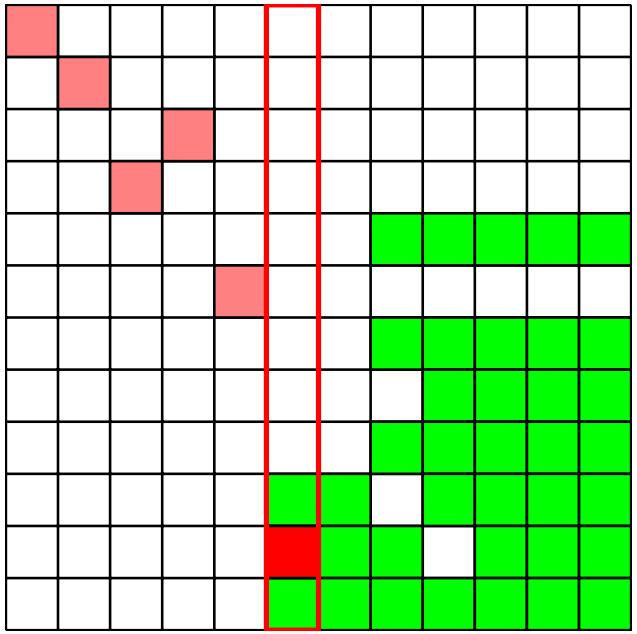
Skip Animation

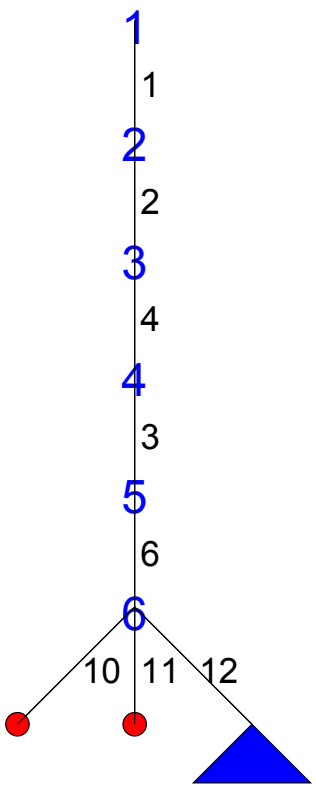
Improved Model

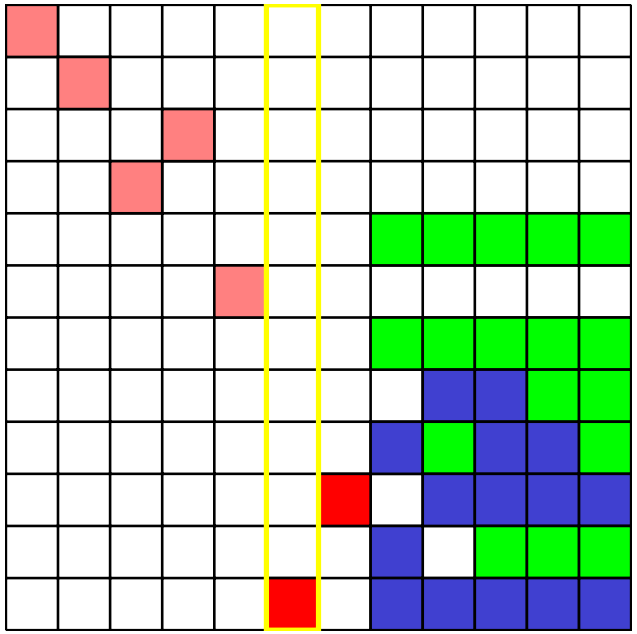




Back to Start
Skip Animation

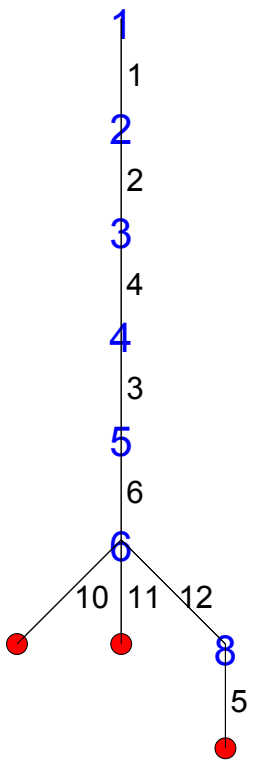
Improved Model

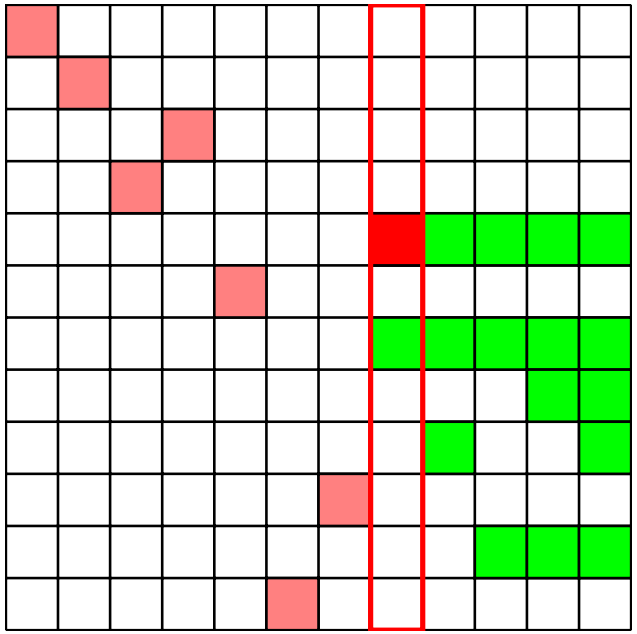




Back to Start
Skip Animation

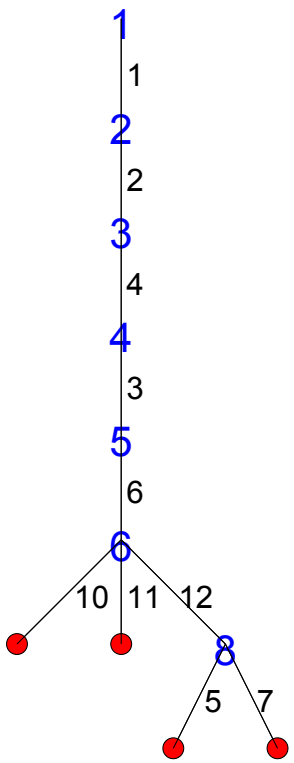
Improved Model

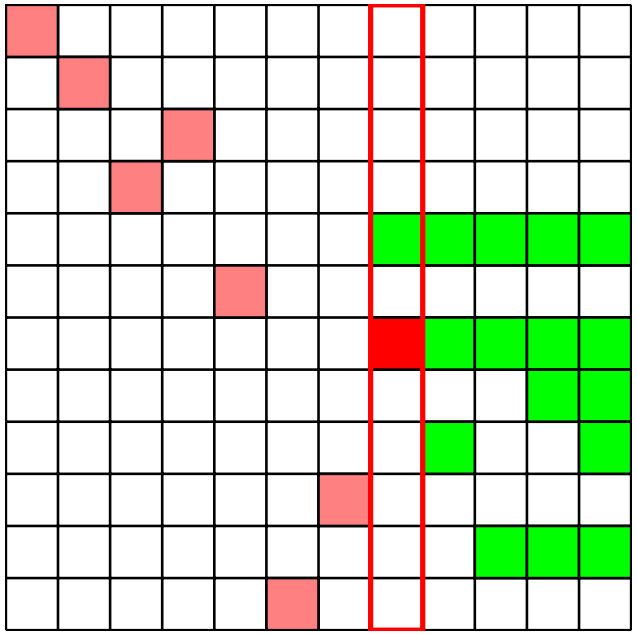




Back to Start
Skip Animation

Improved Model





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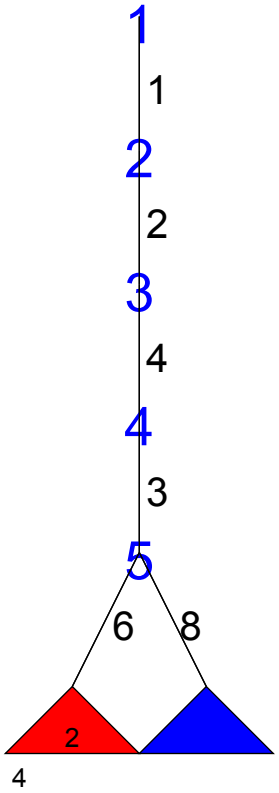
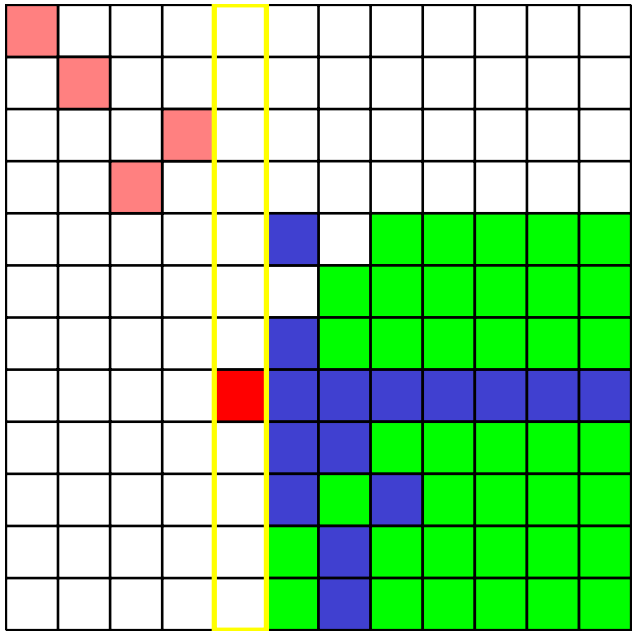
Skip Animation

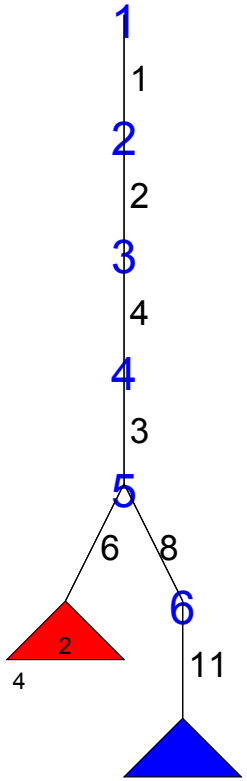
Improved Model

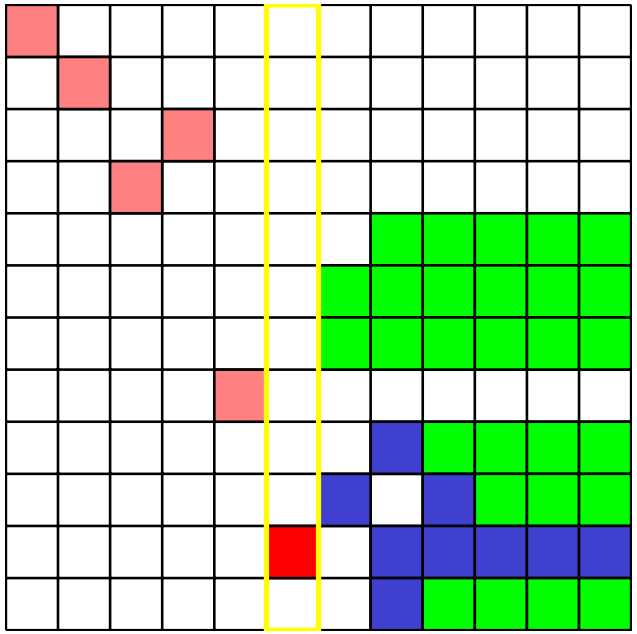




Back to Start
Skip Animation

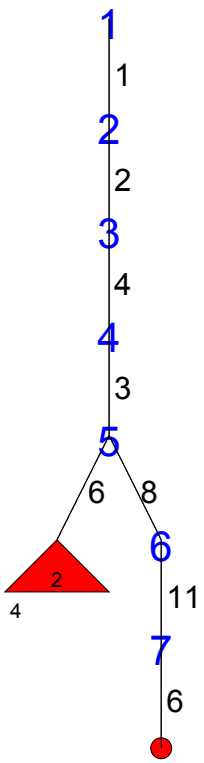
Improved Model

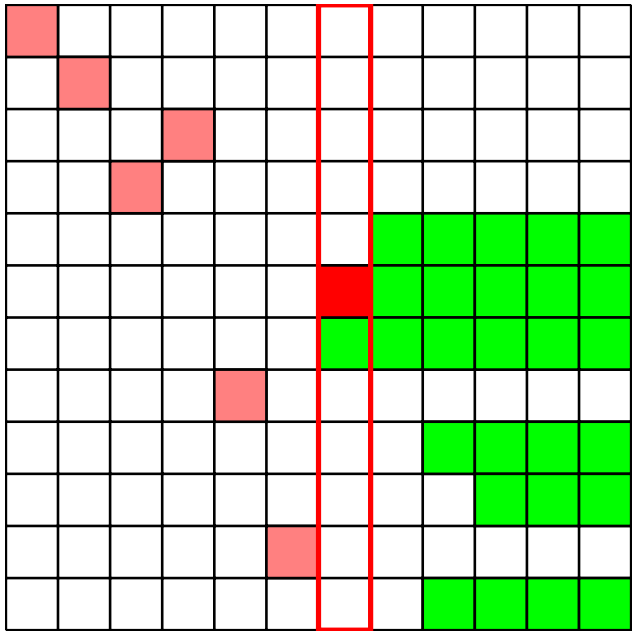




Back to Start
Skip Animation

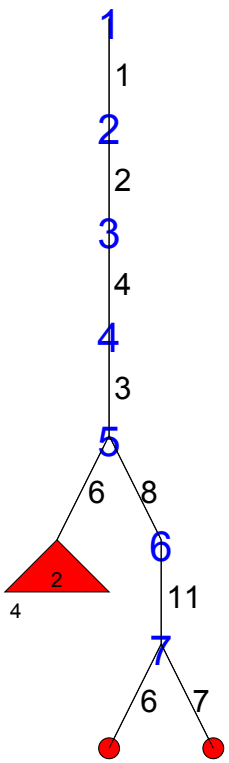
Improved Model

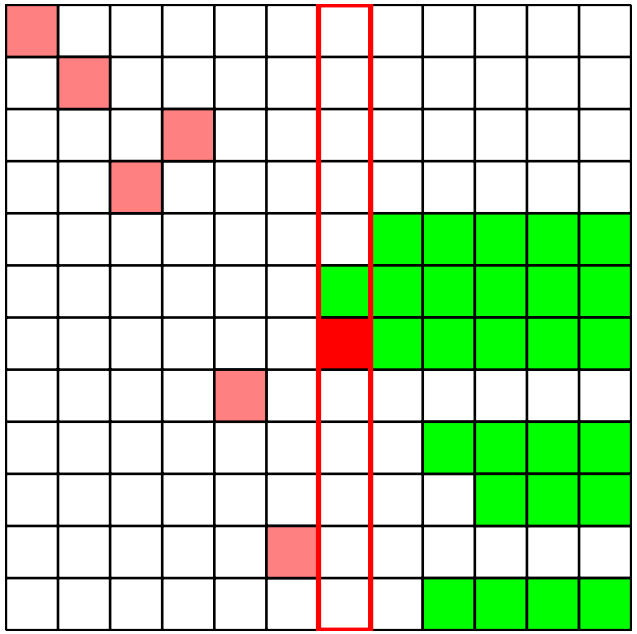




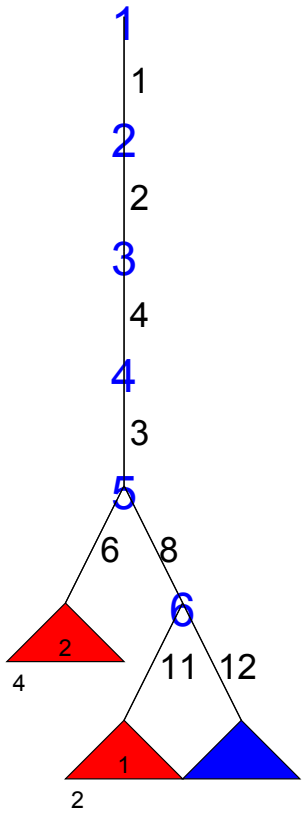
Back to Start
Skip Animation

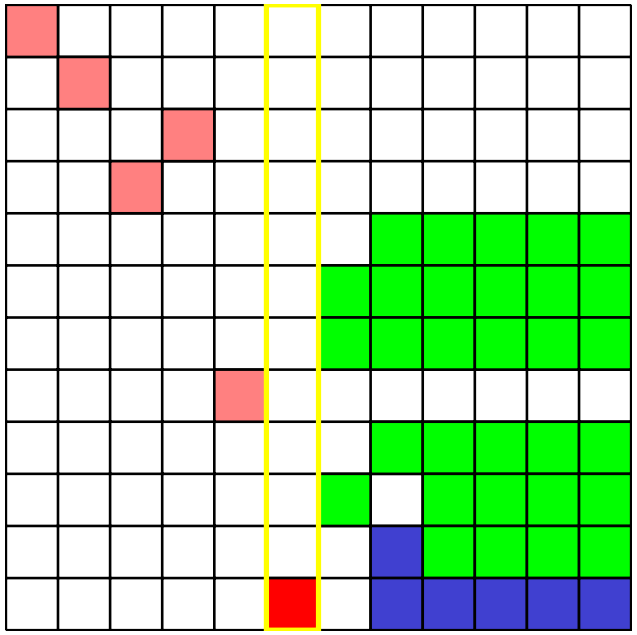
Improved Model





Improved Model





Back to Start
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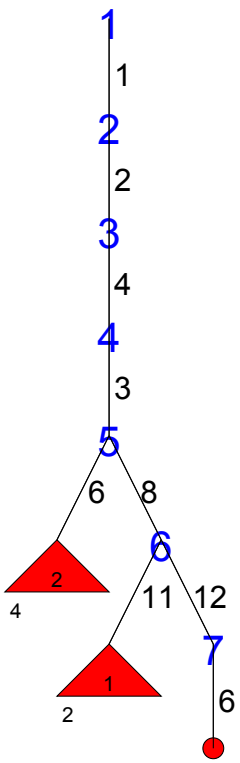
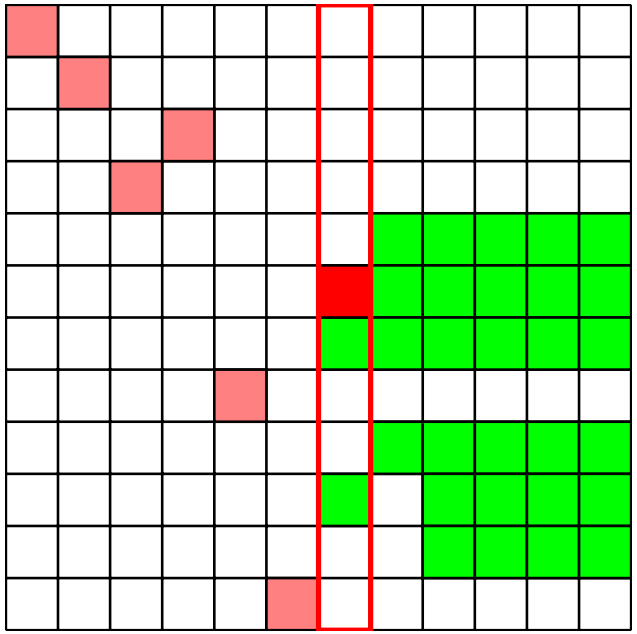
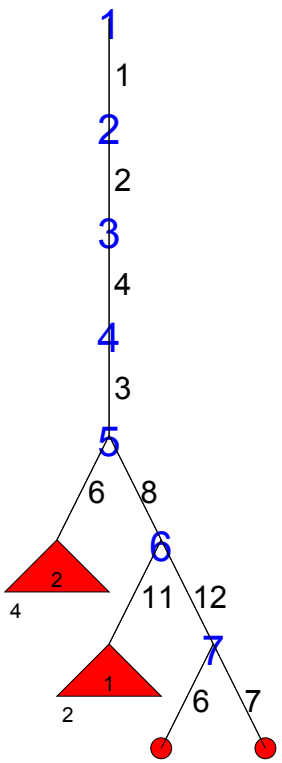
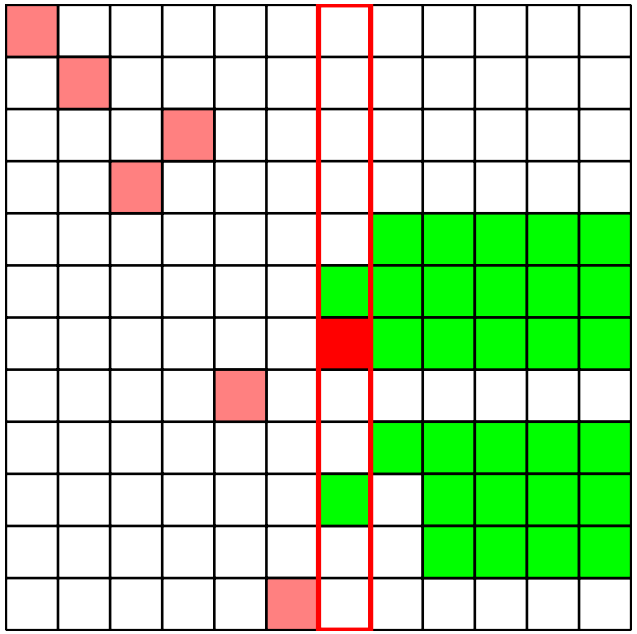
Improved Model



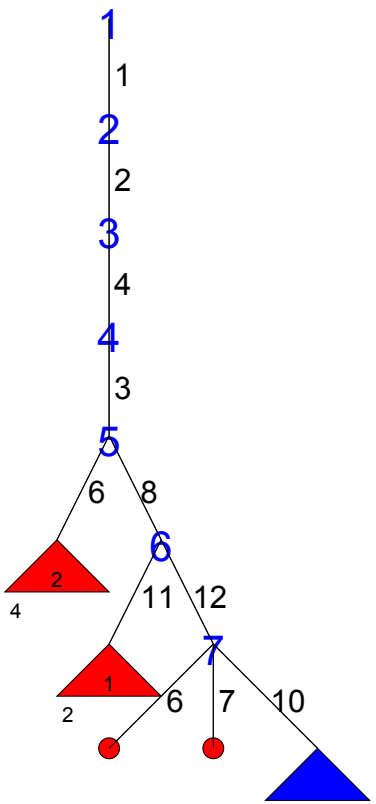


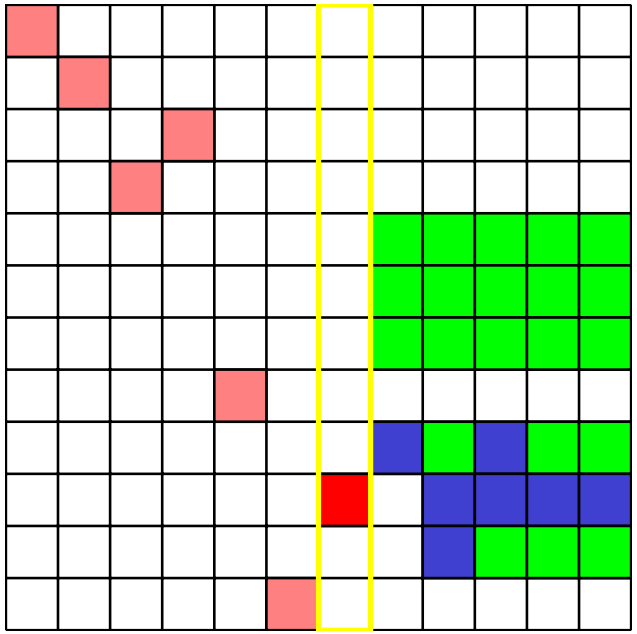
Improved Model



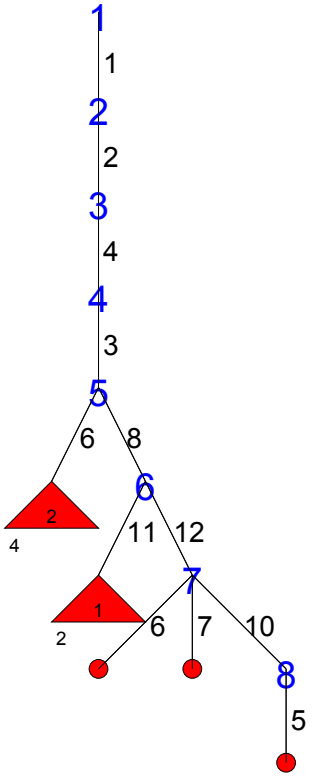


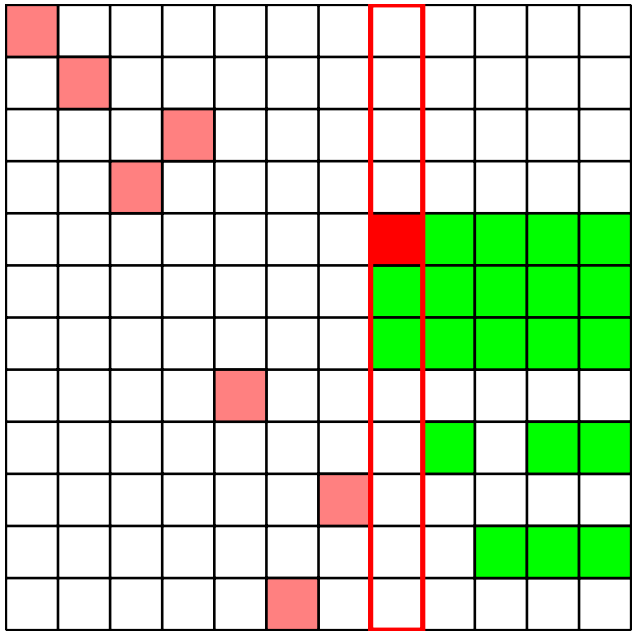
Improved Model



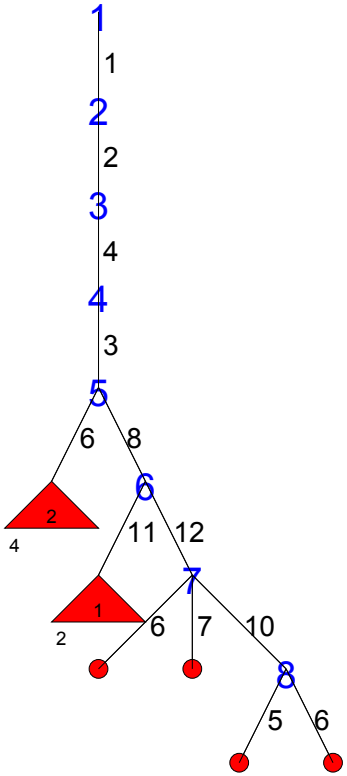


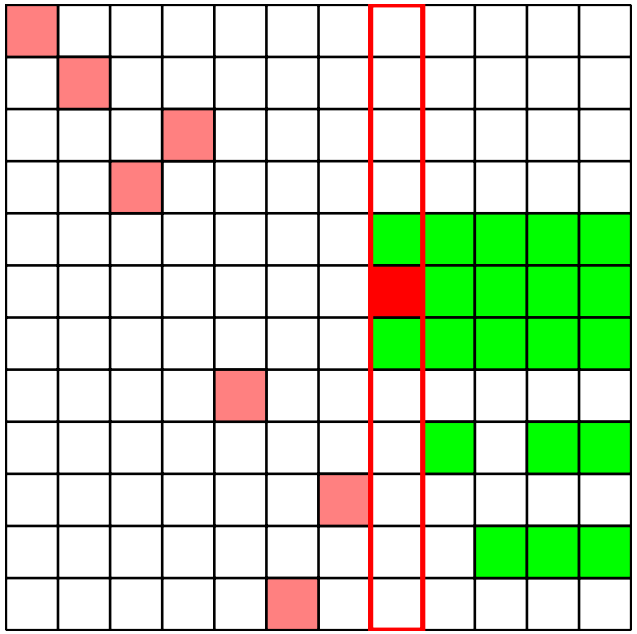
Improved Model



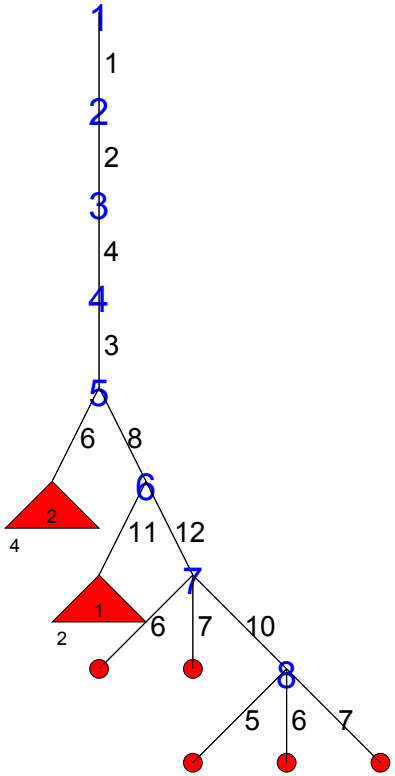


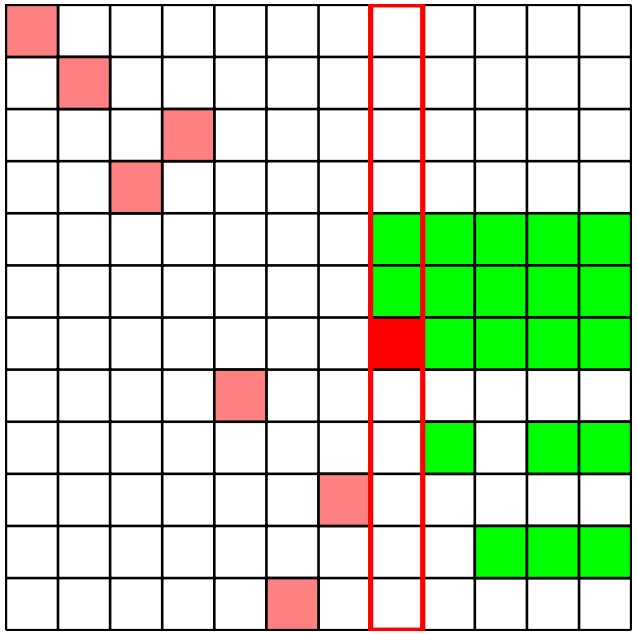
Improved Model





Improved Model





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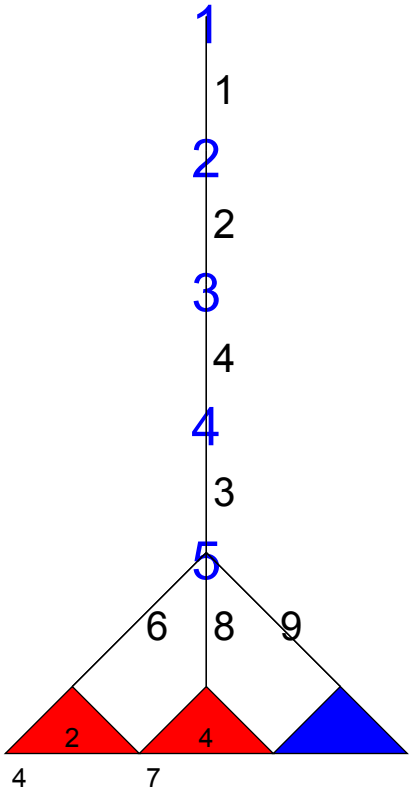

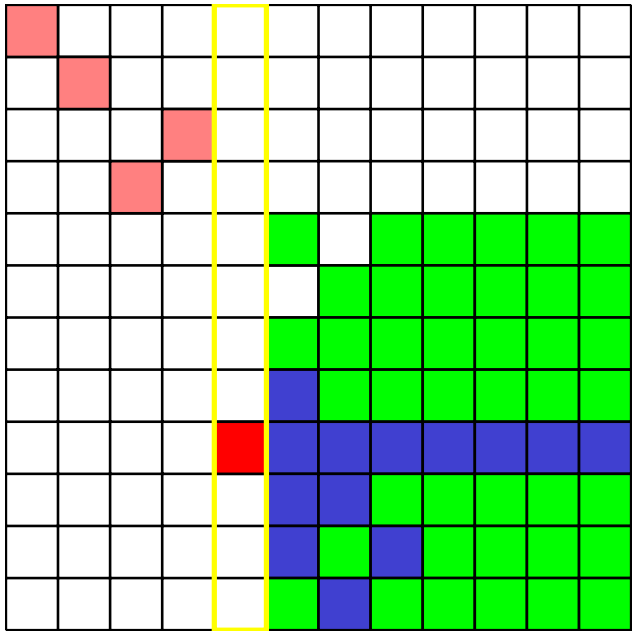

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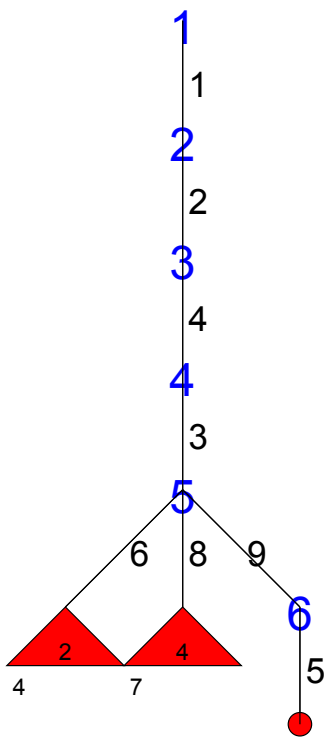

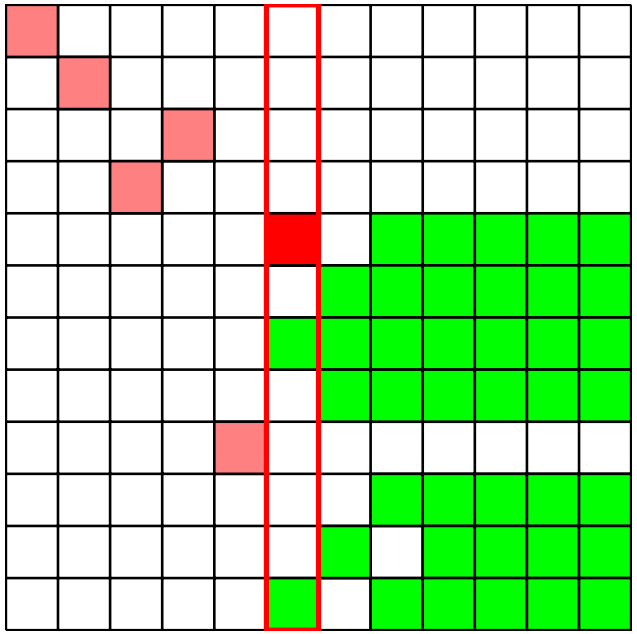

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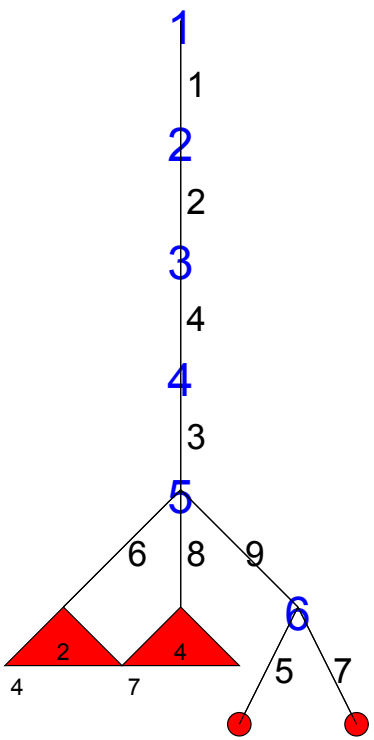

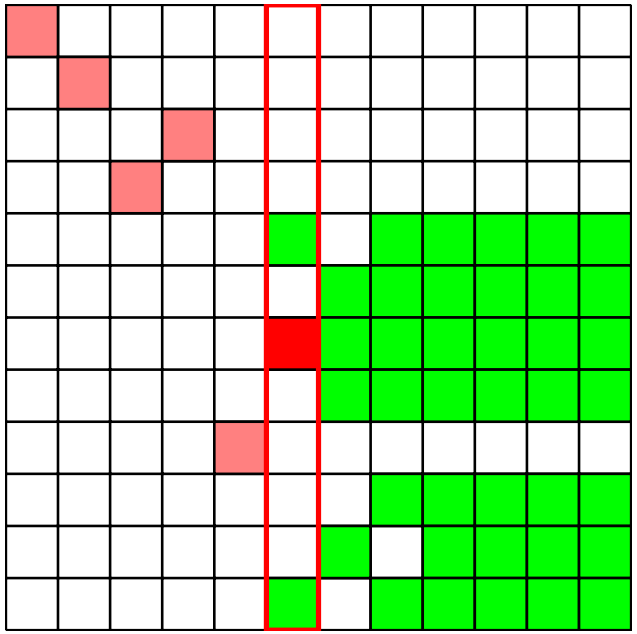

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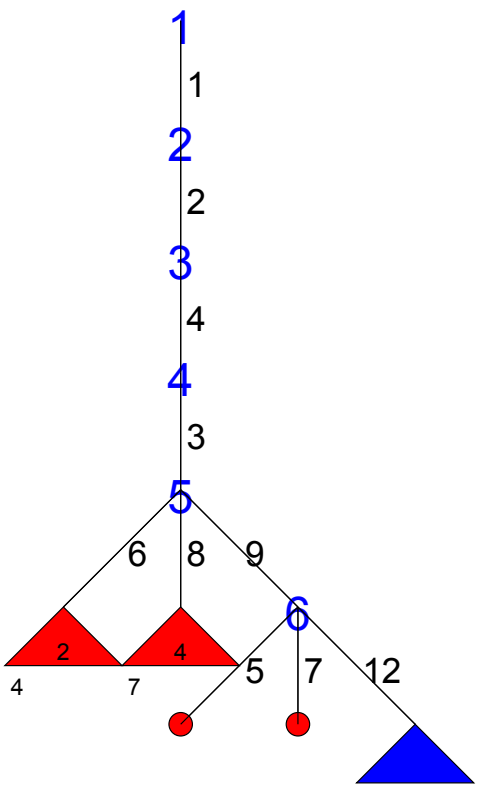

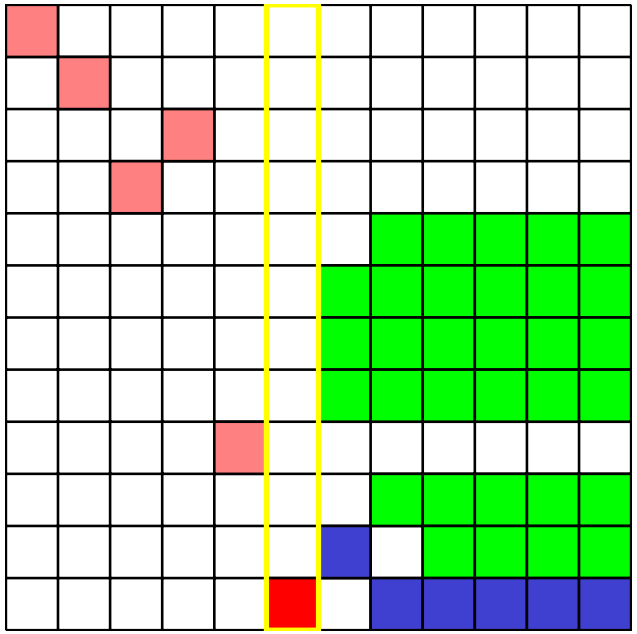

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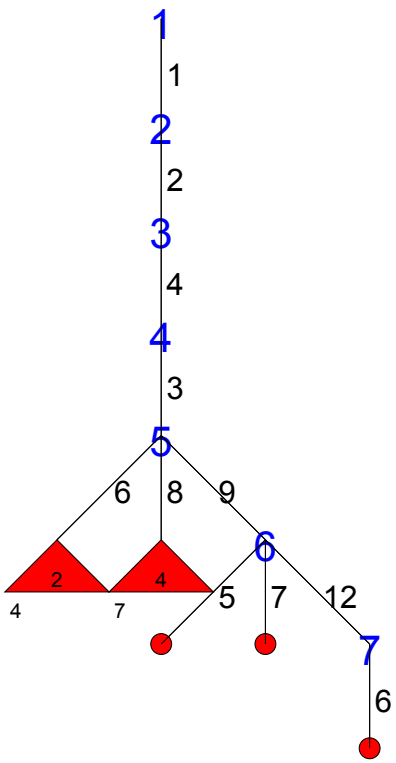

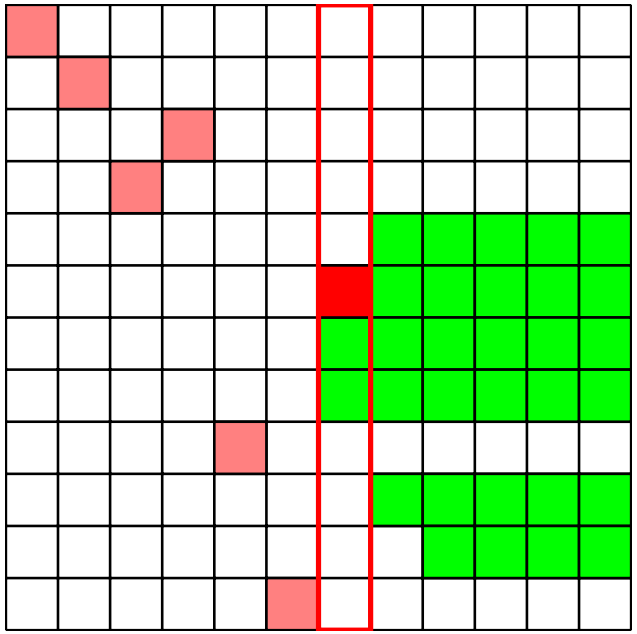

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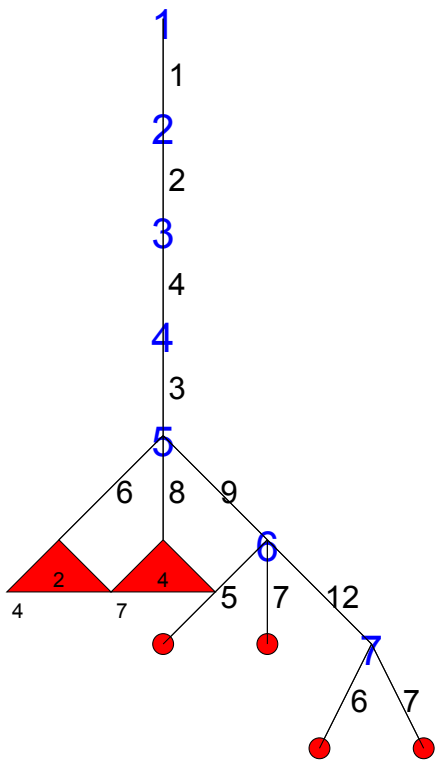

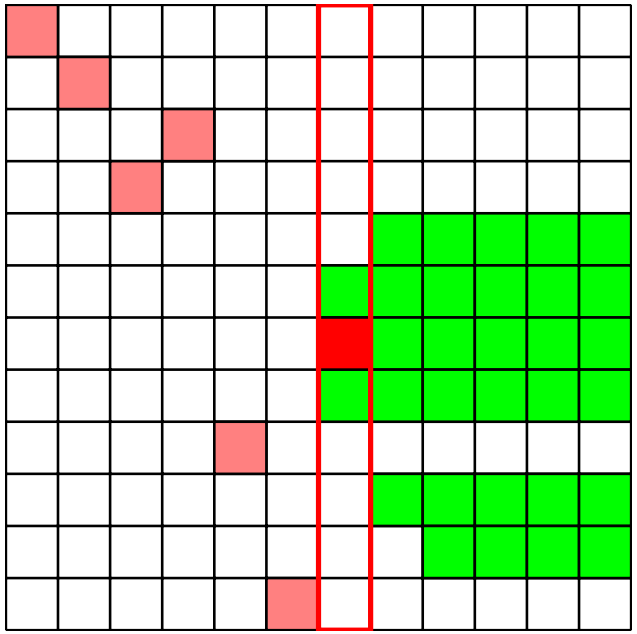

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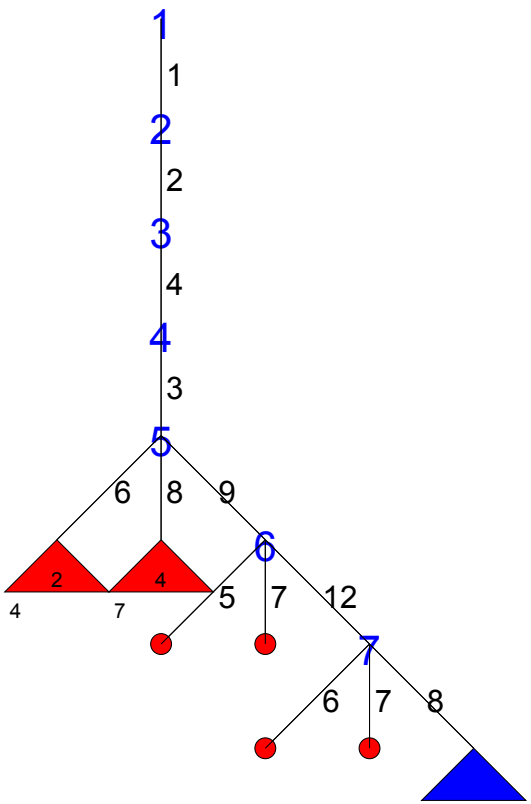

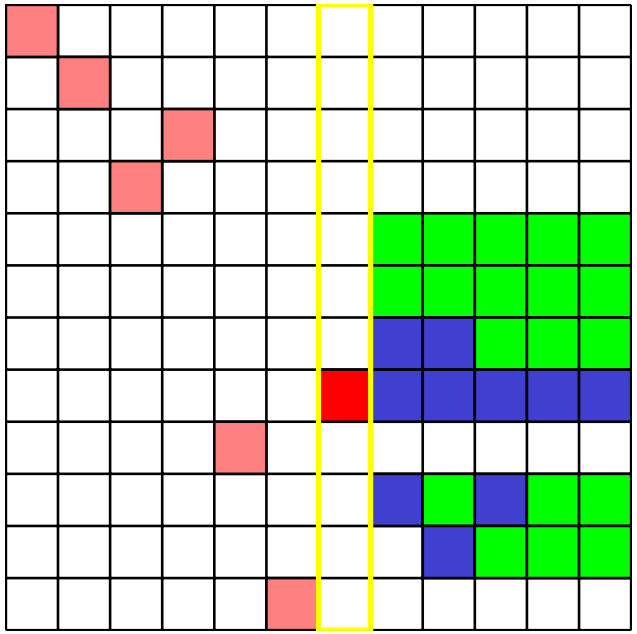

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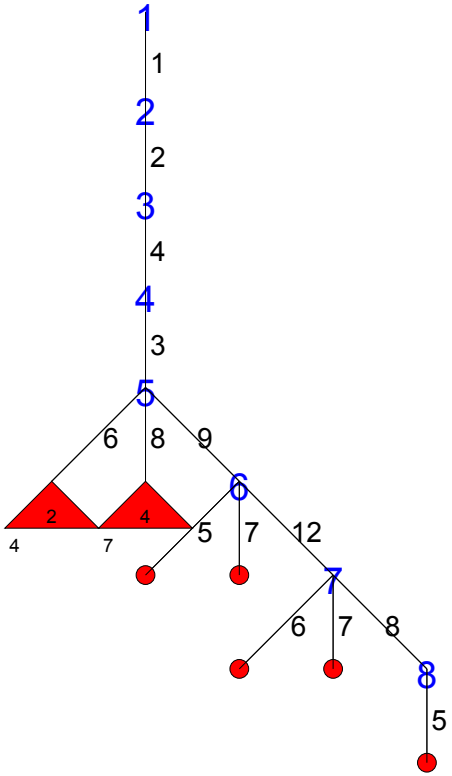

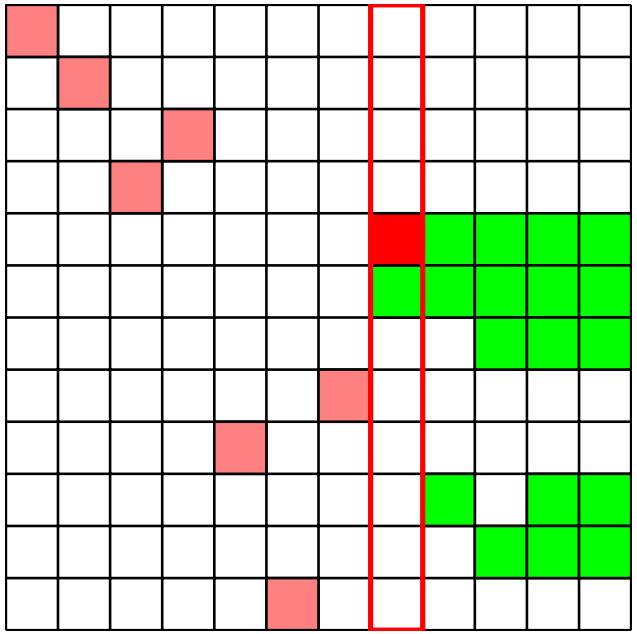

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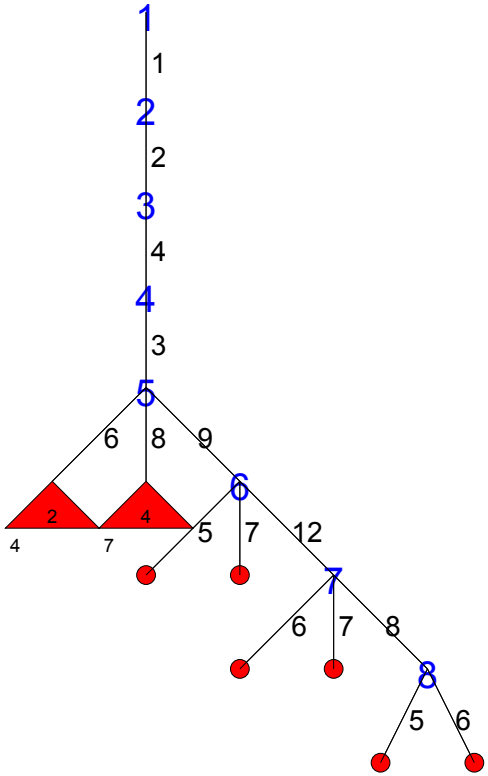

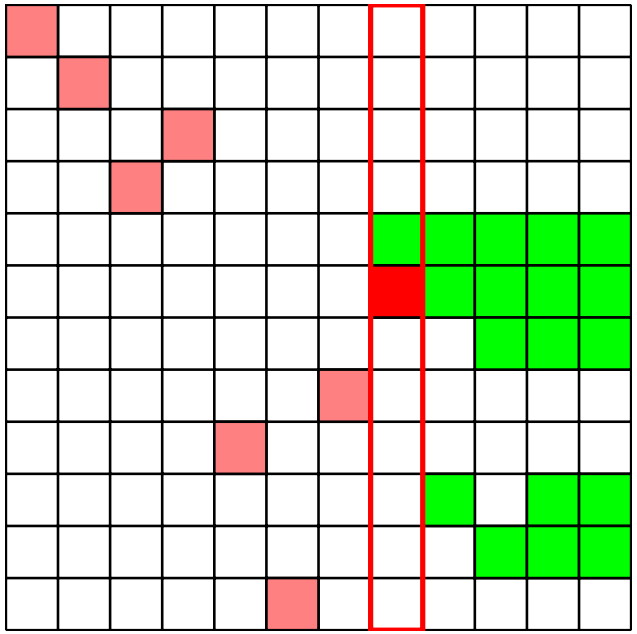

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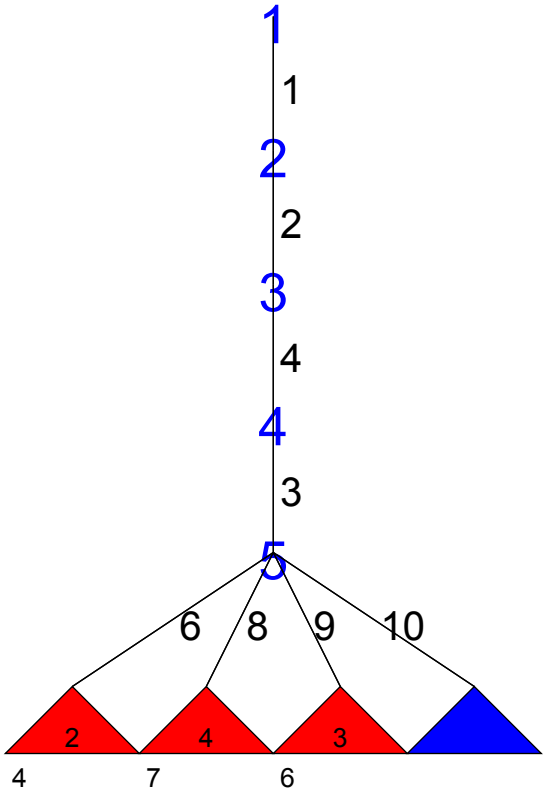

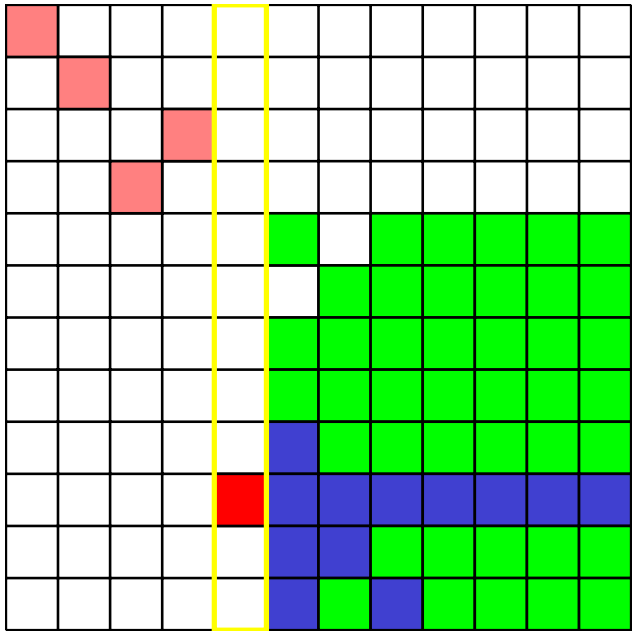

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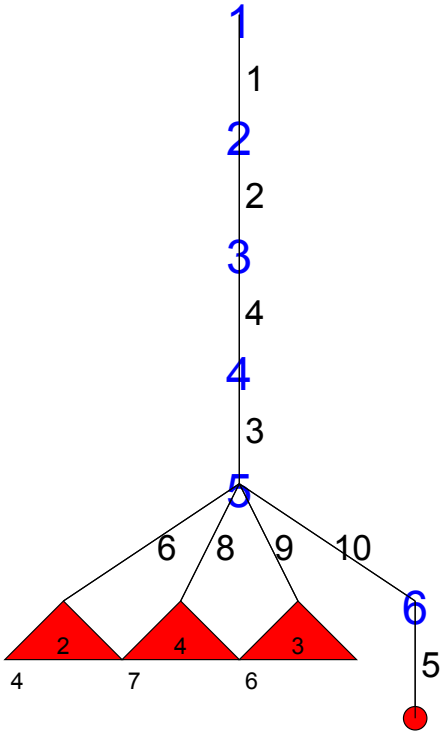

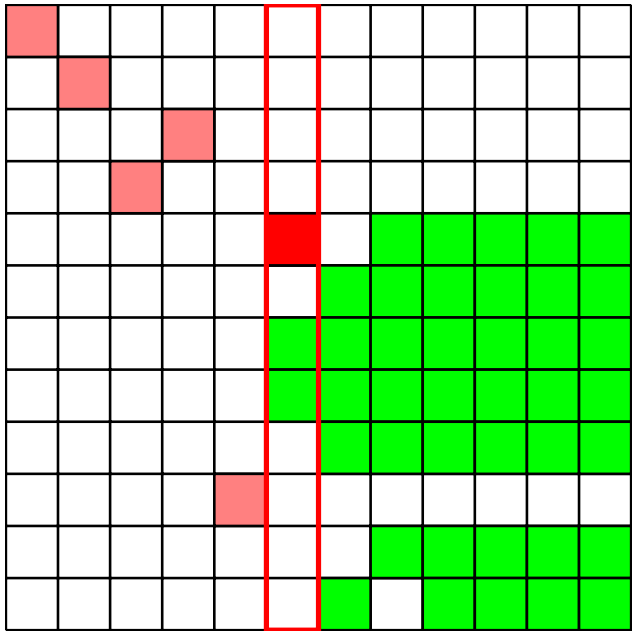

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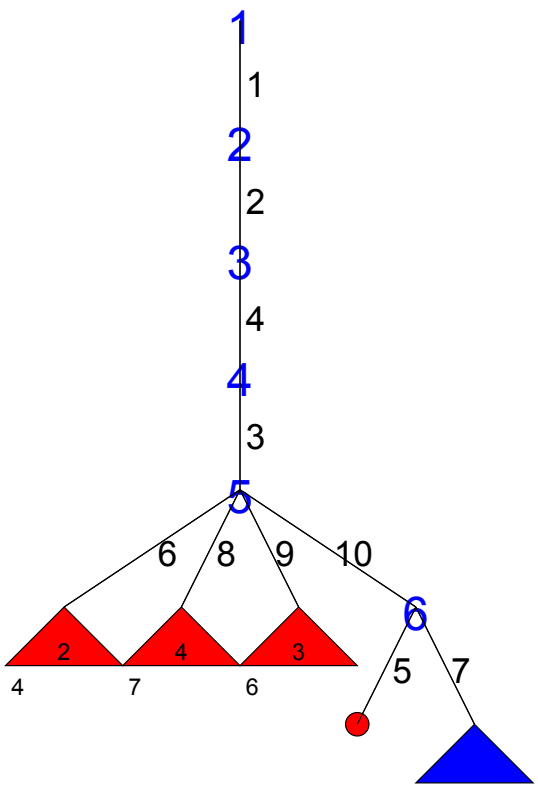

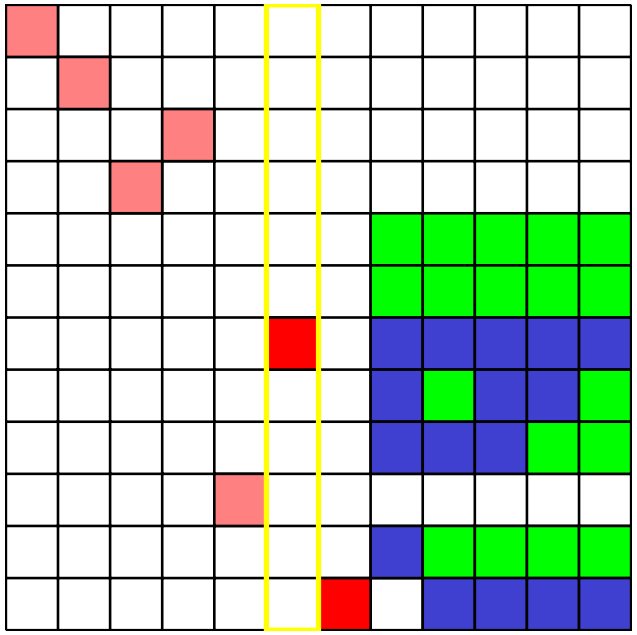

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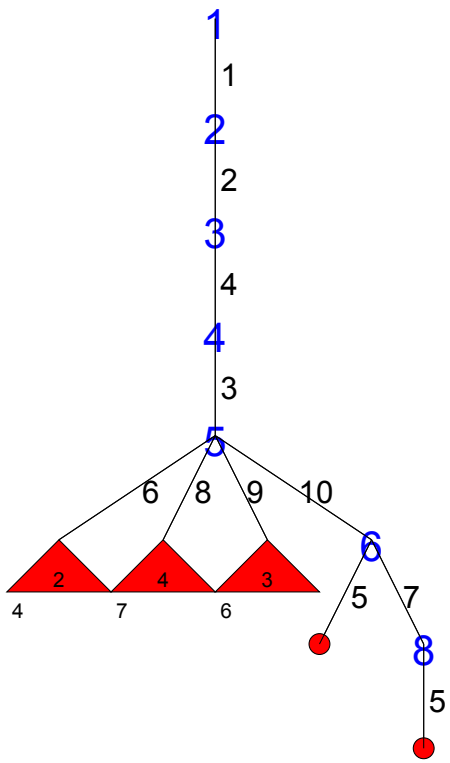

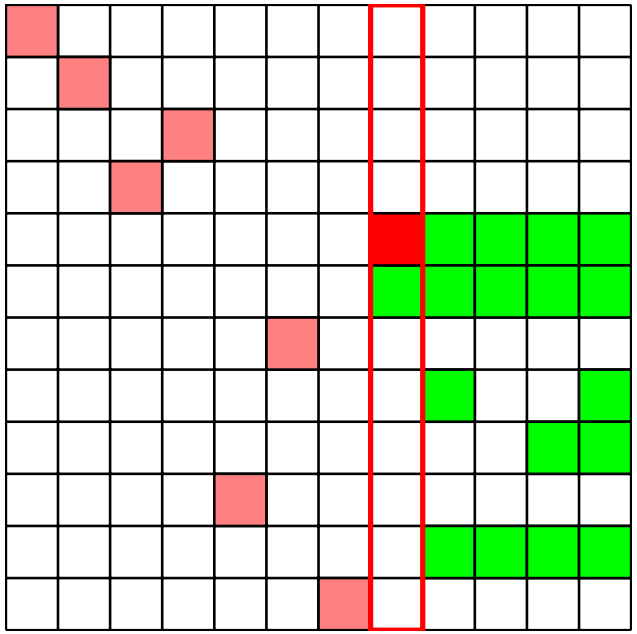

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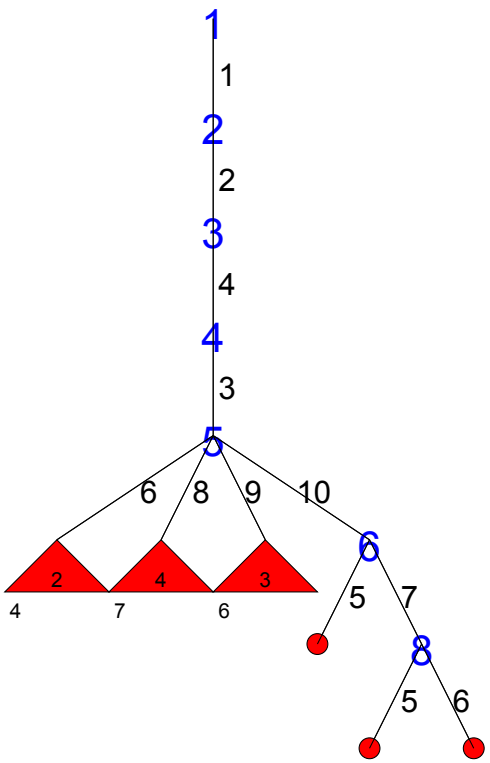

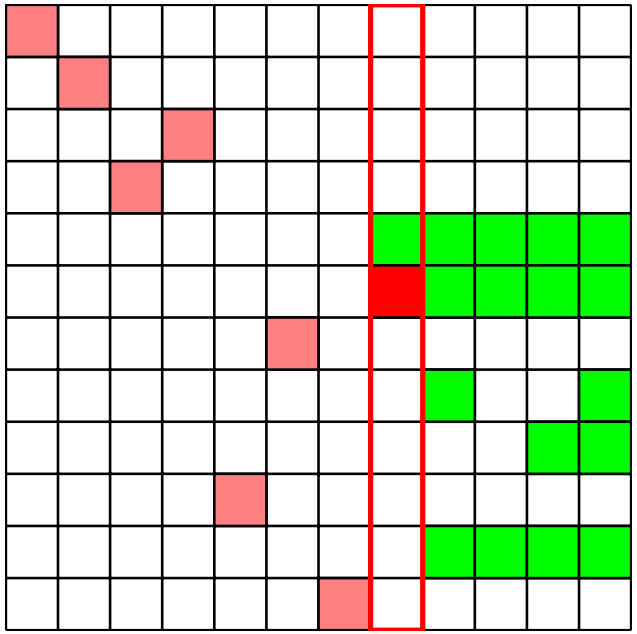

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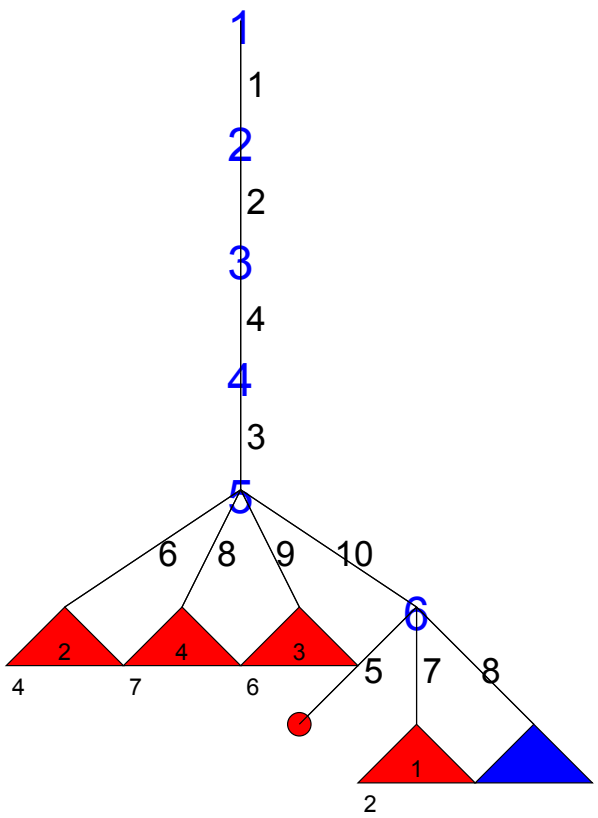

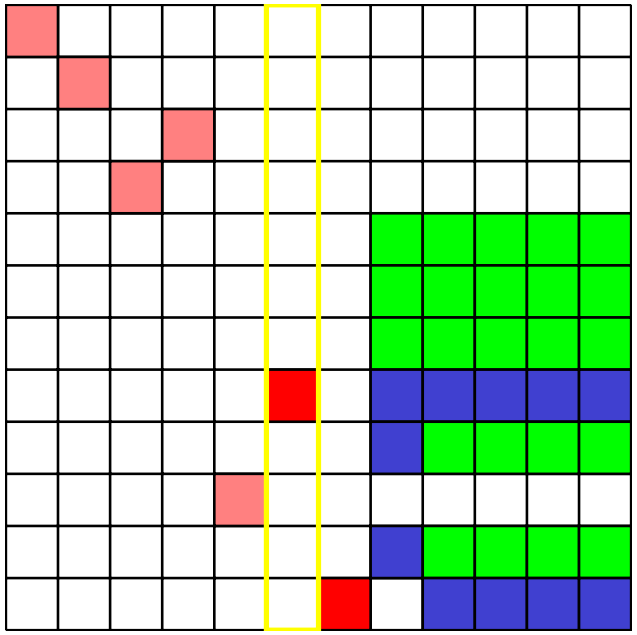

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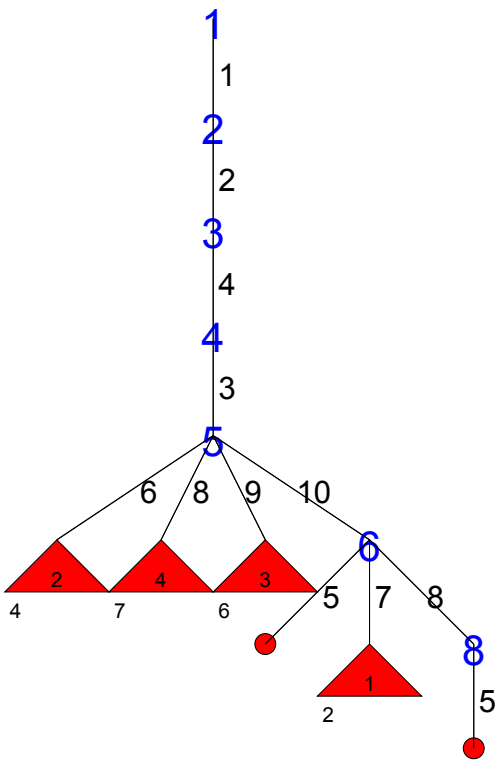

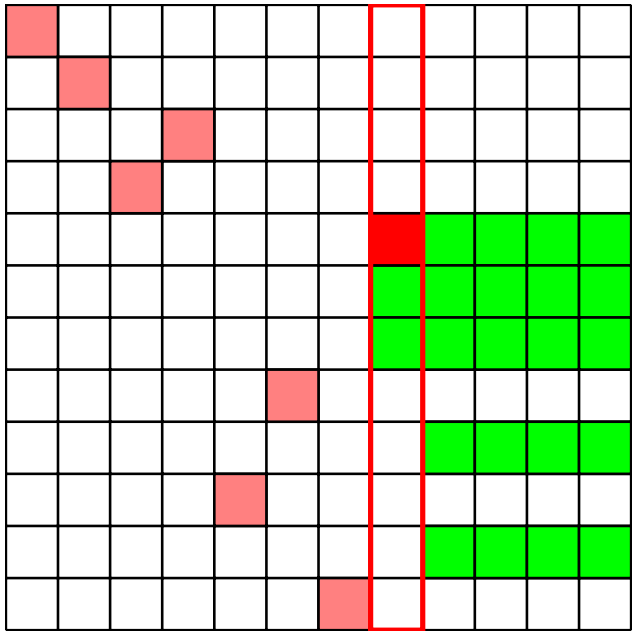

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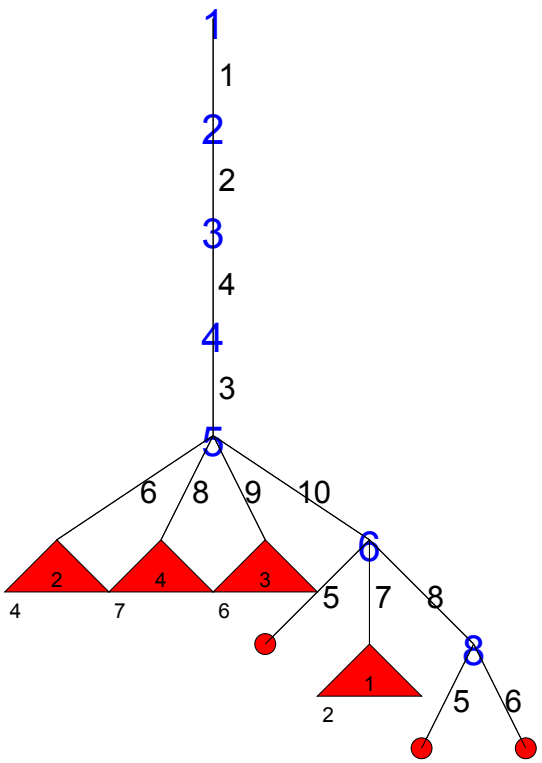

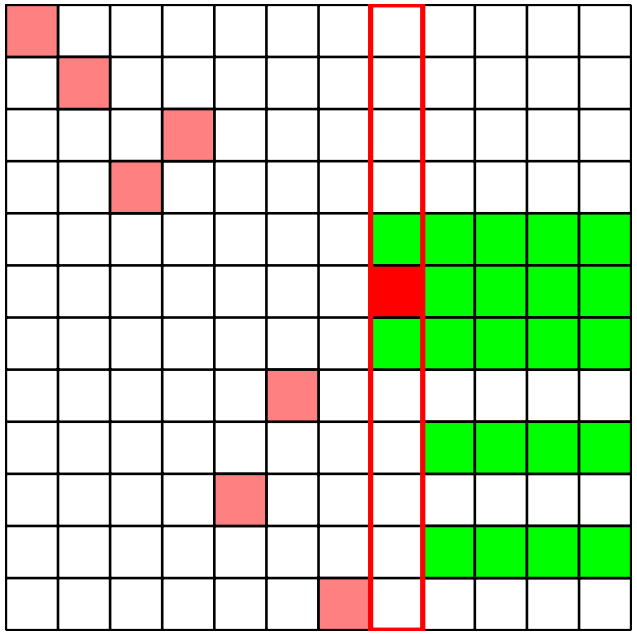

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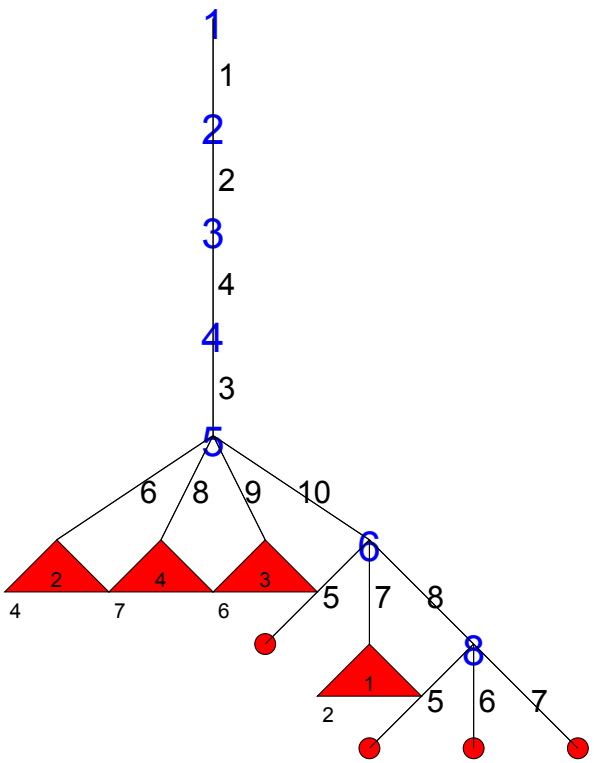

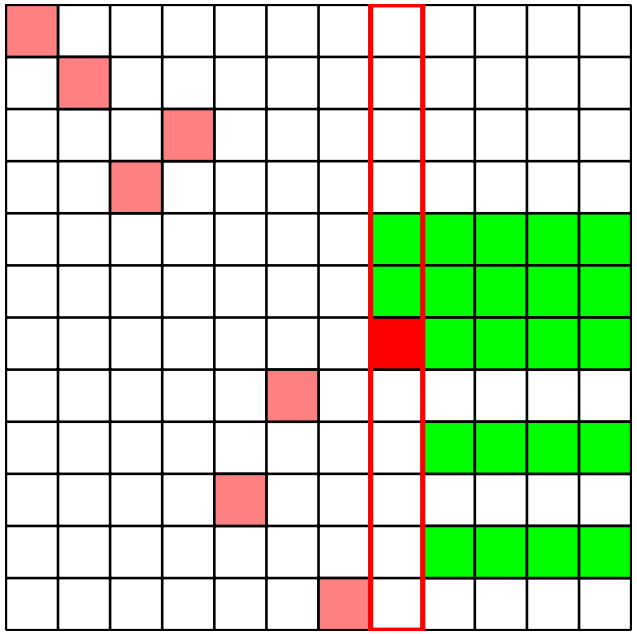

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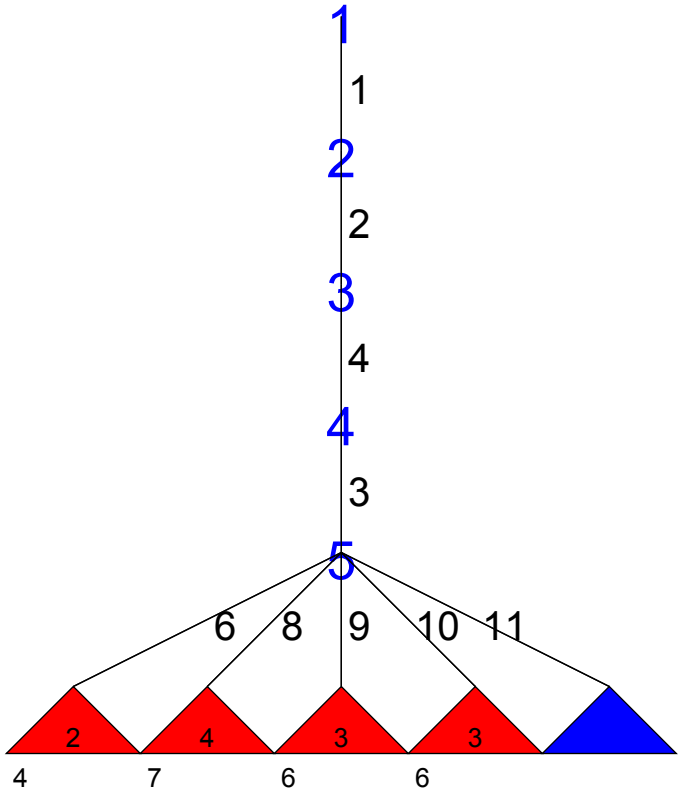

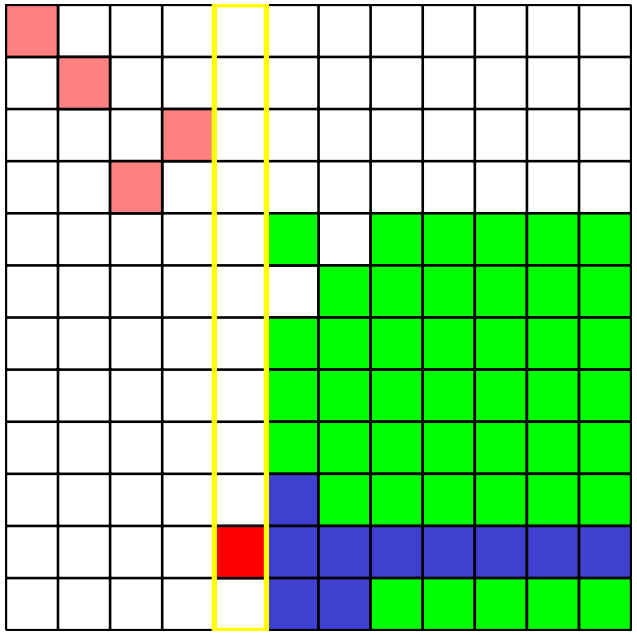

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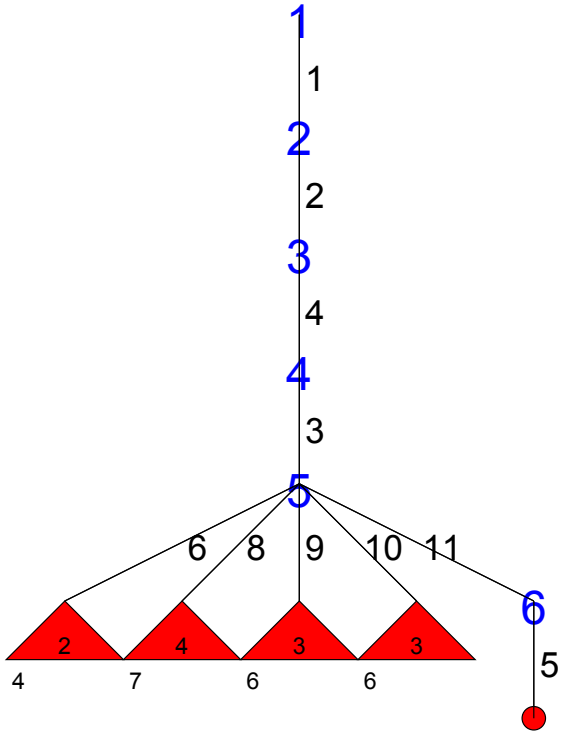

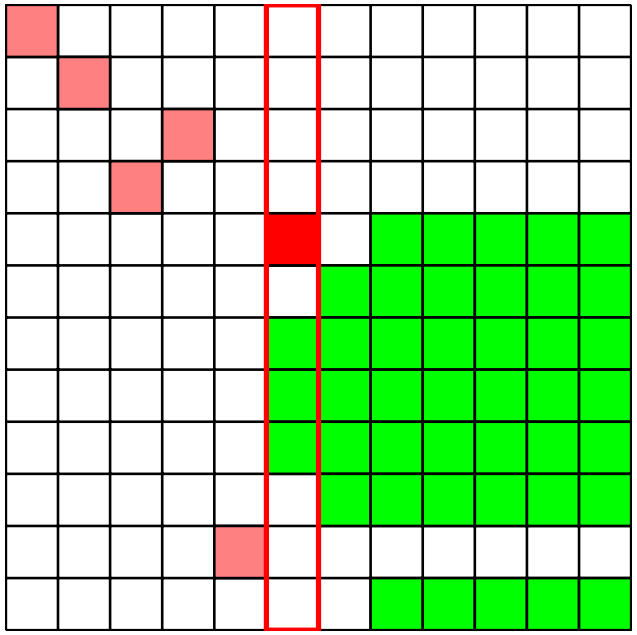

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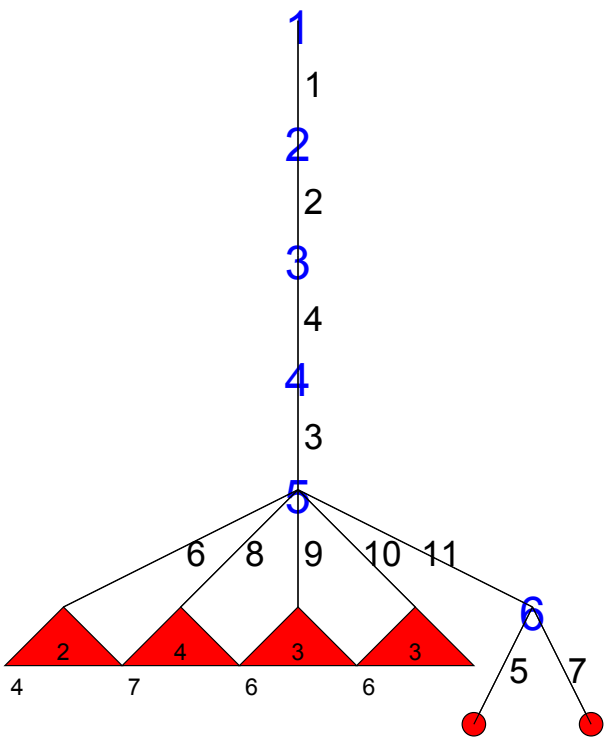

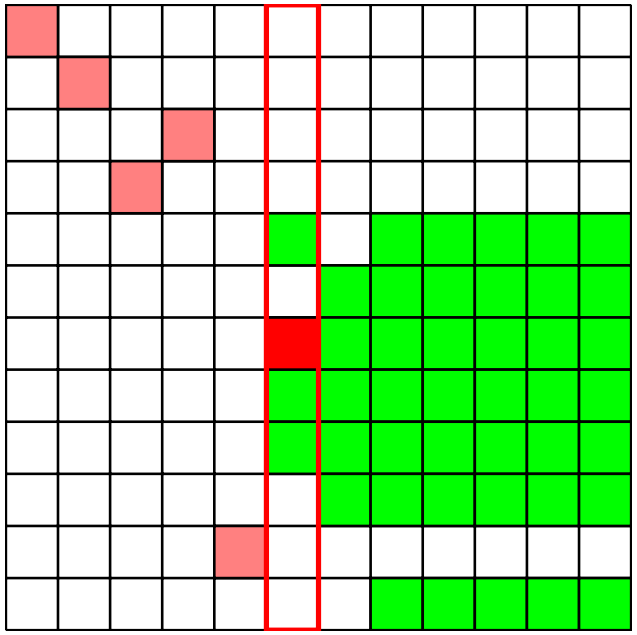

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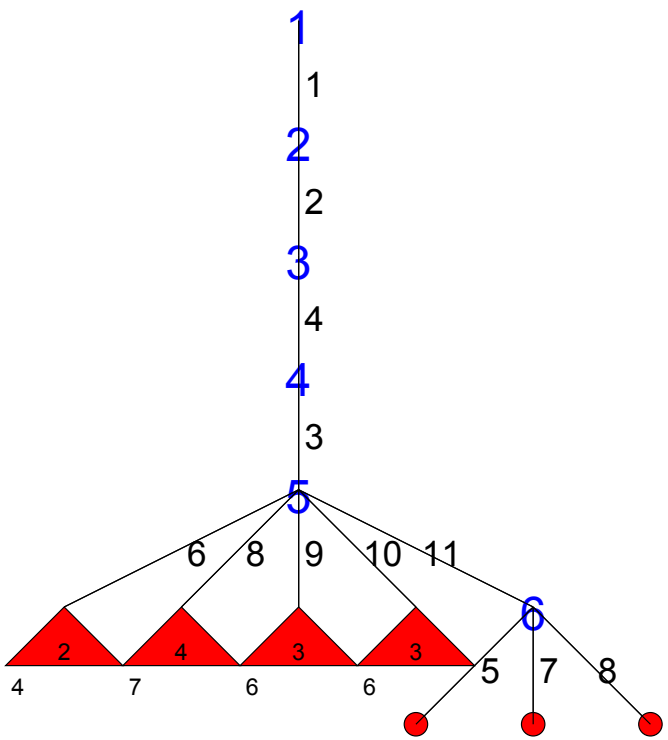

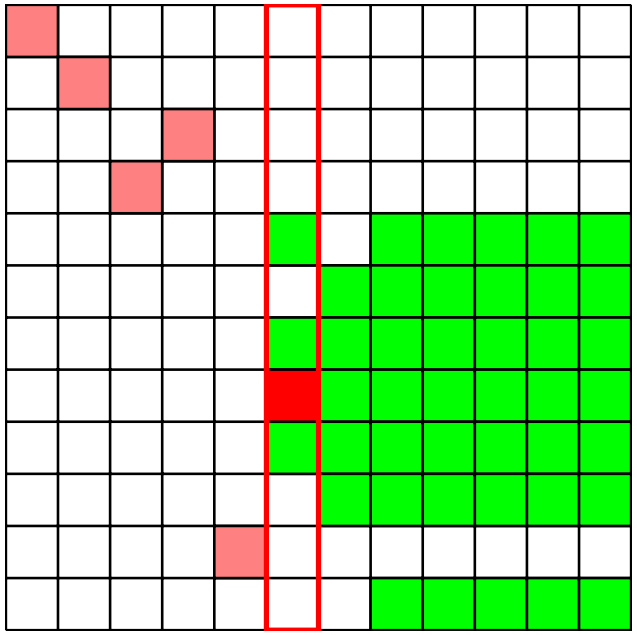

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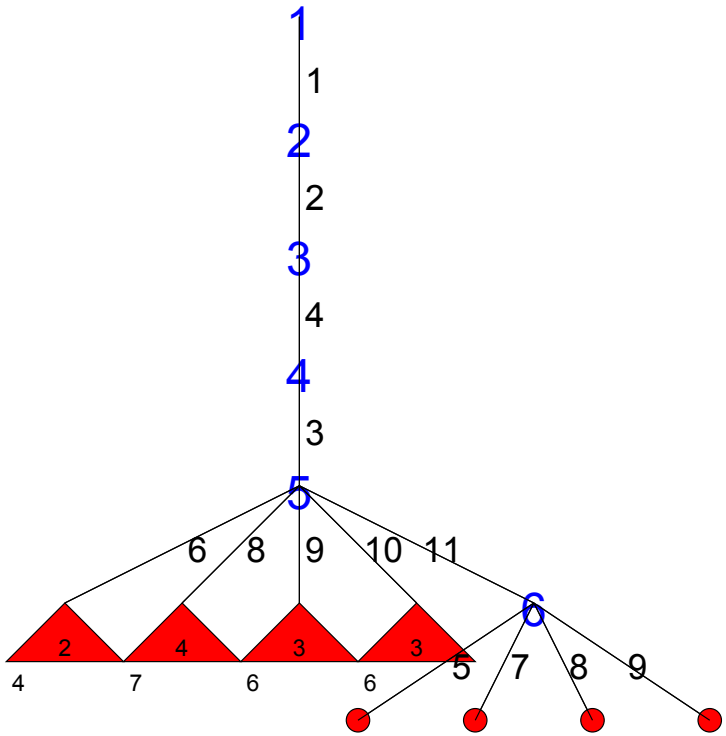

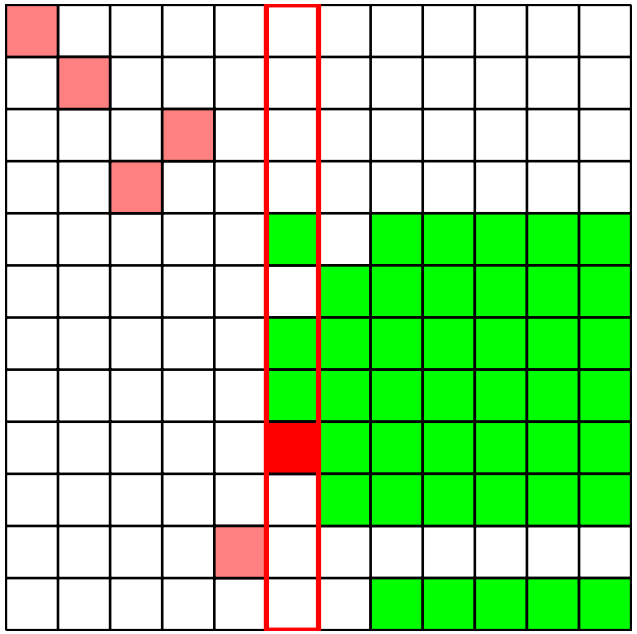

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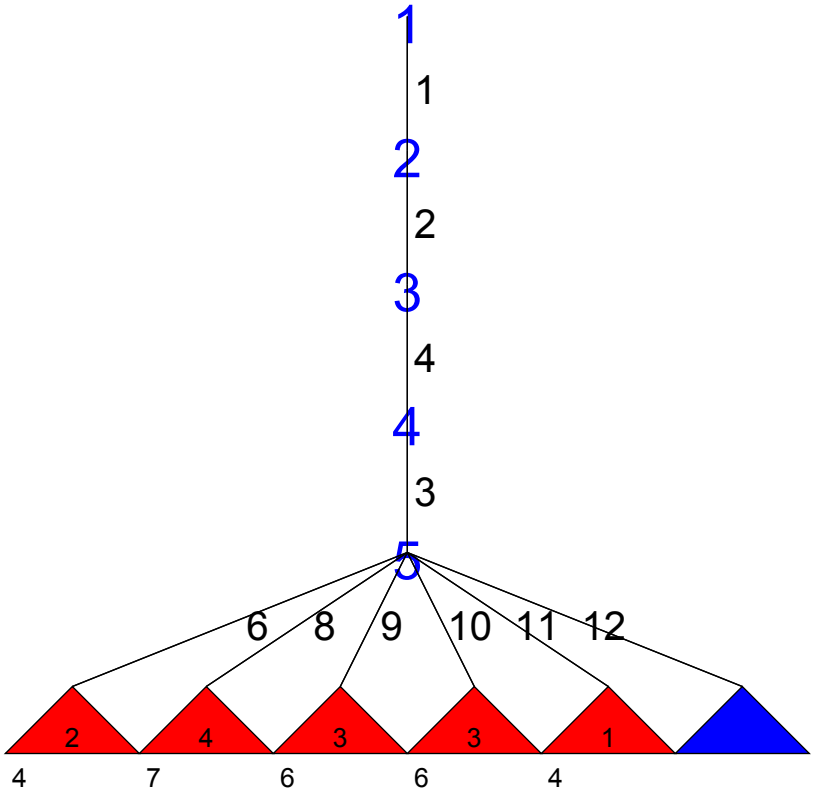

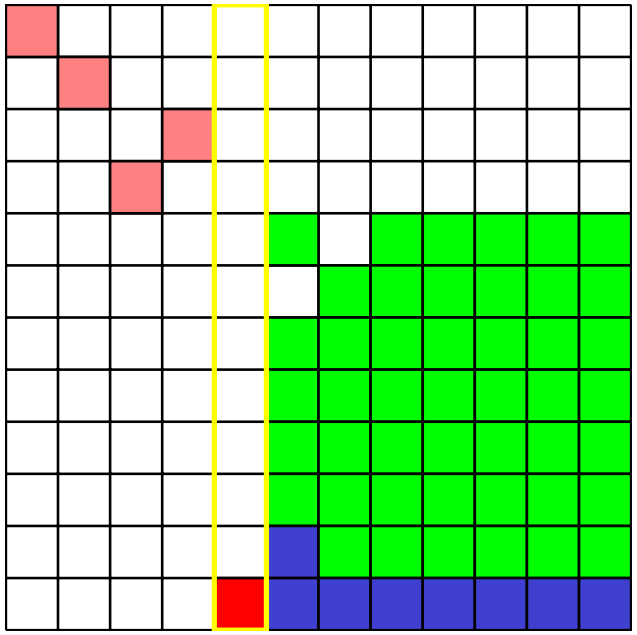

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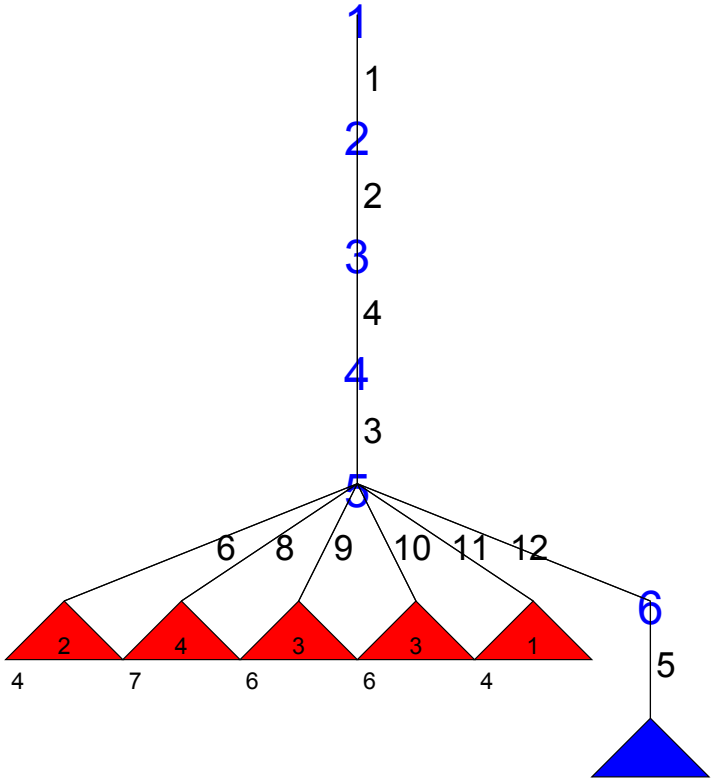

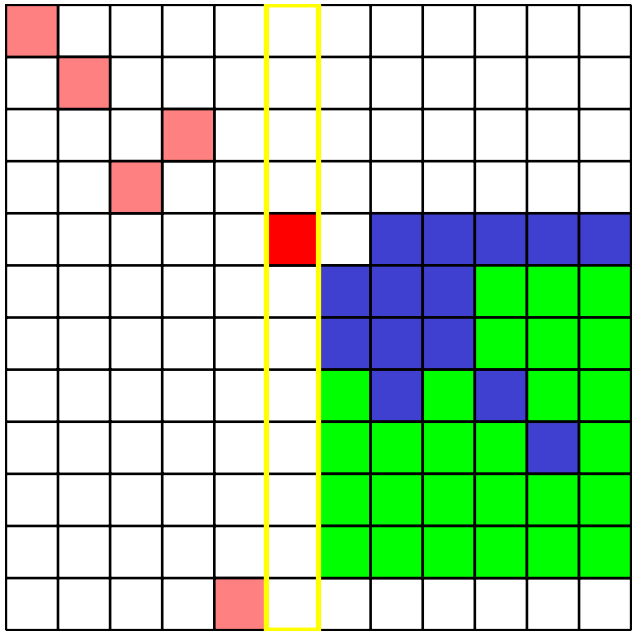

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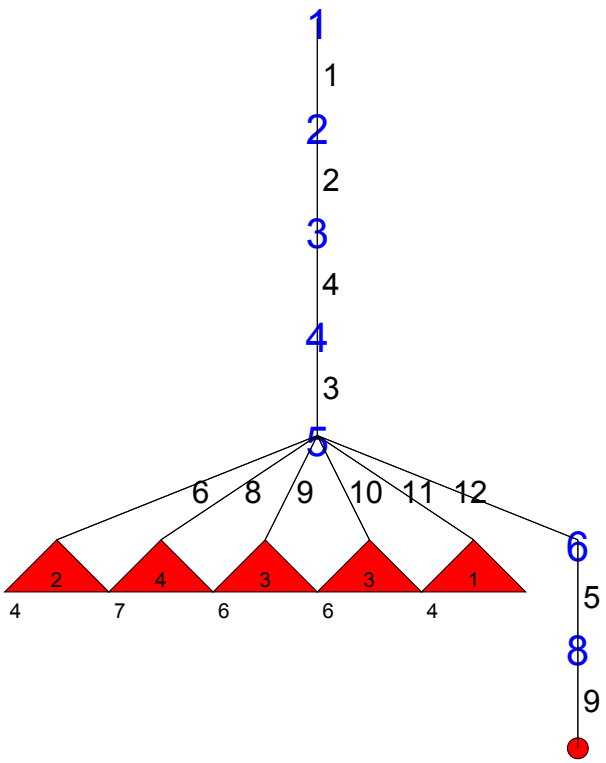

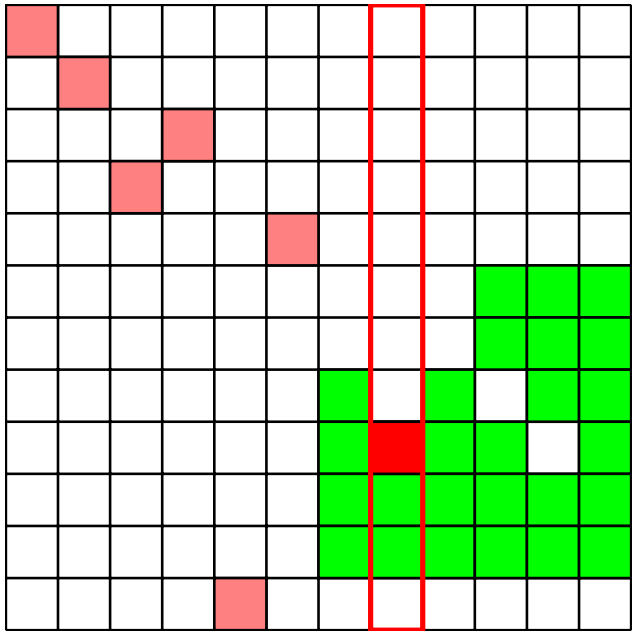

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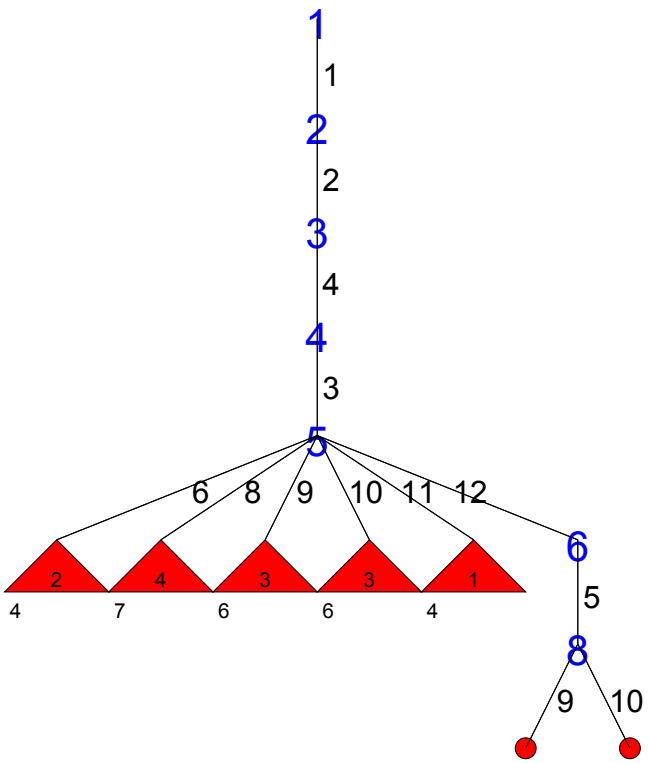

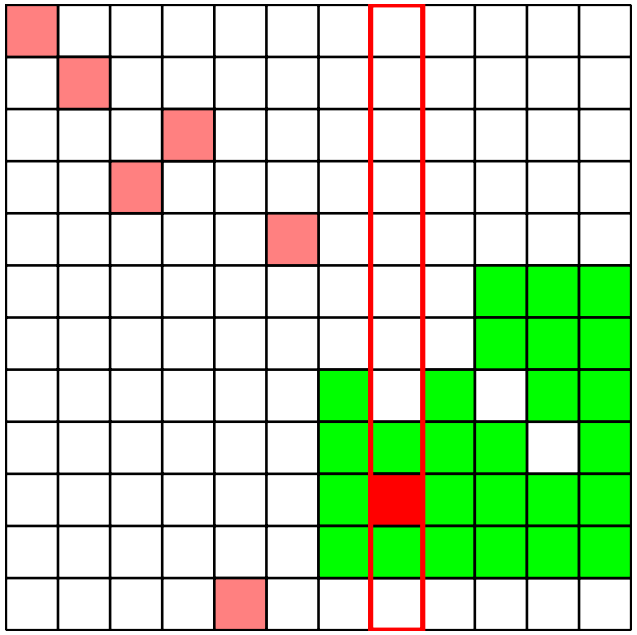

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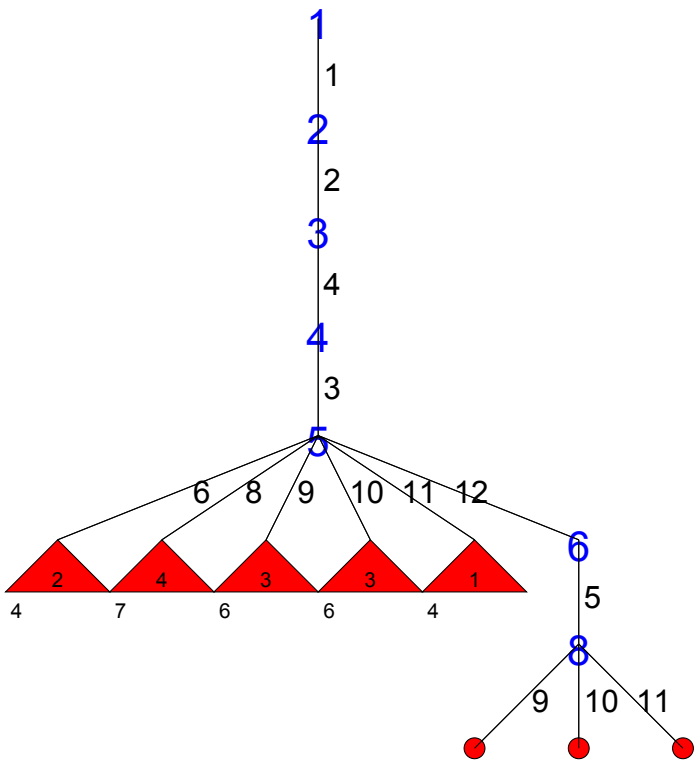

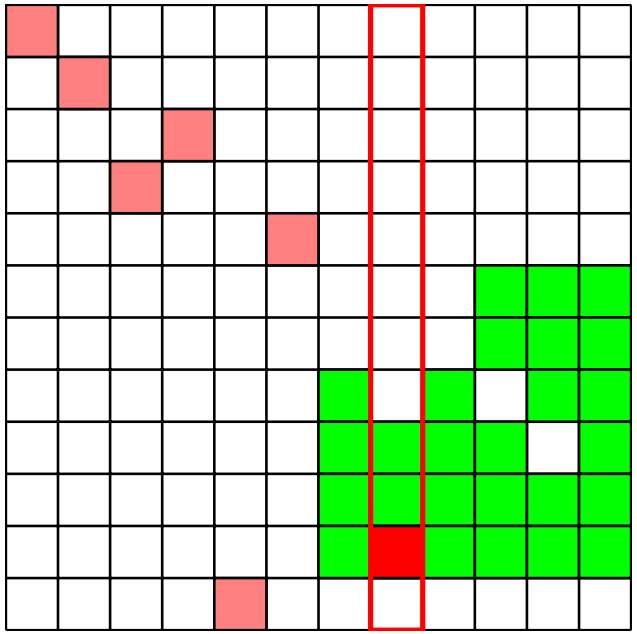

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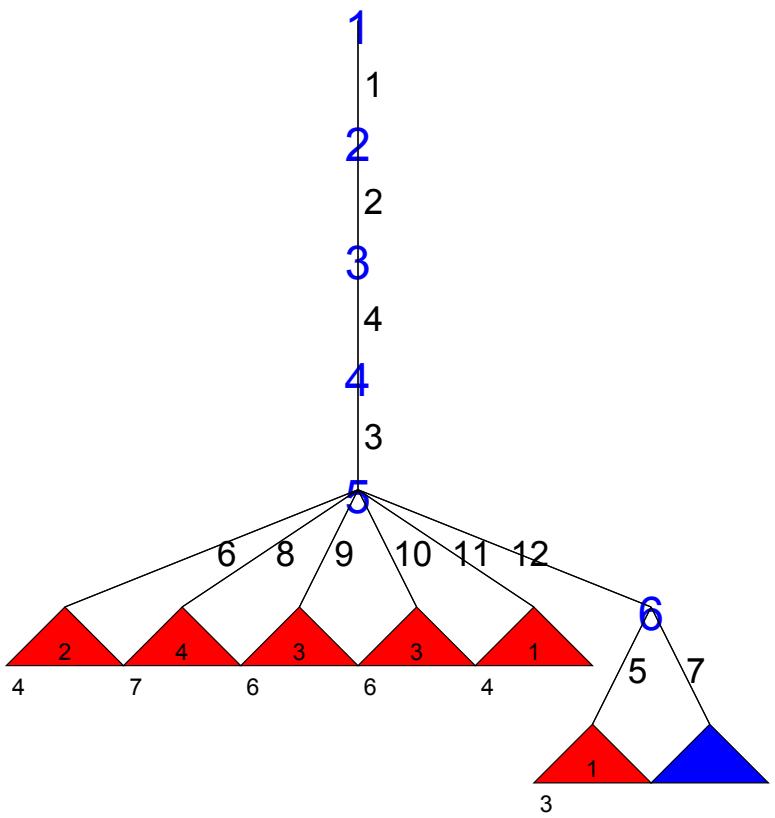

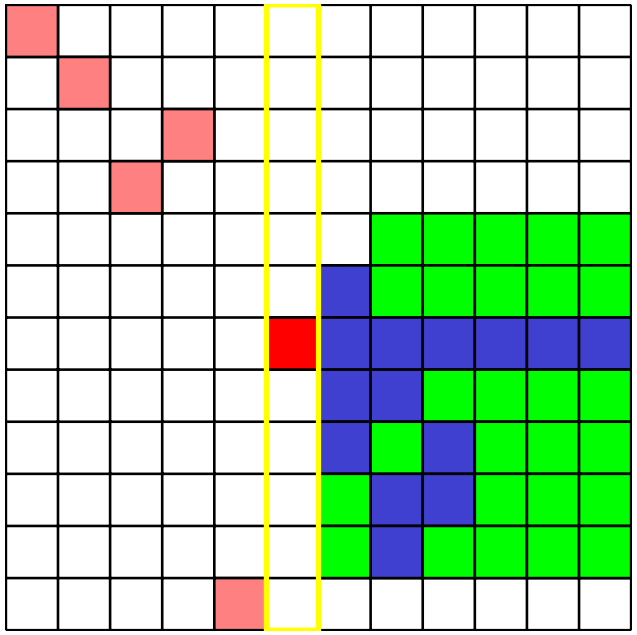

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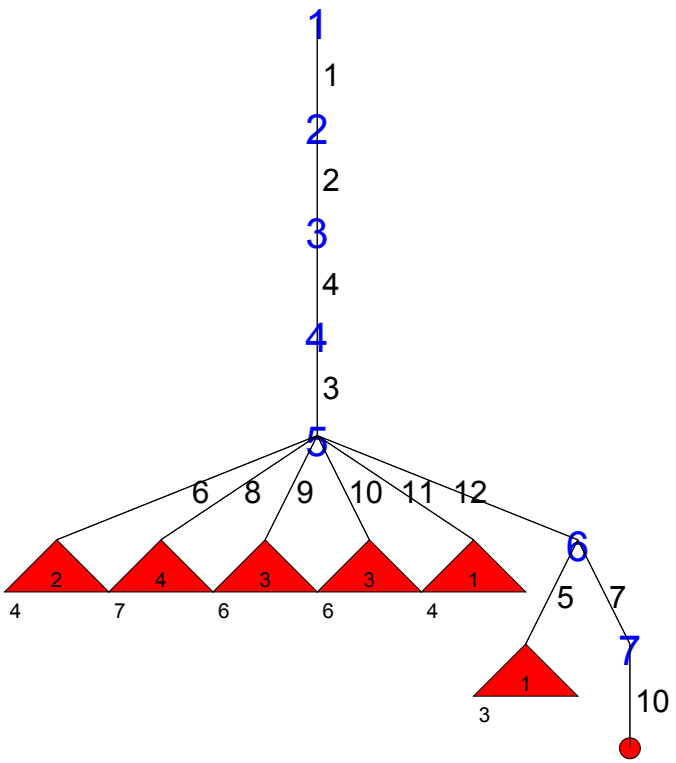

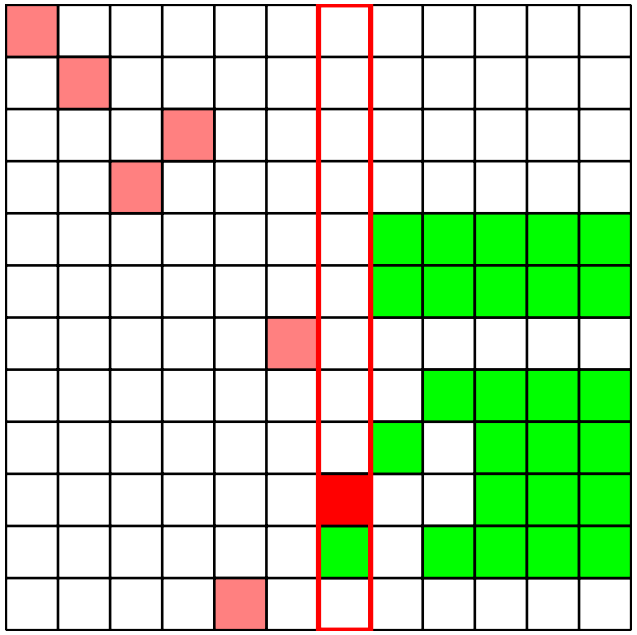

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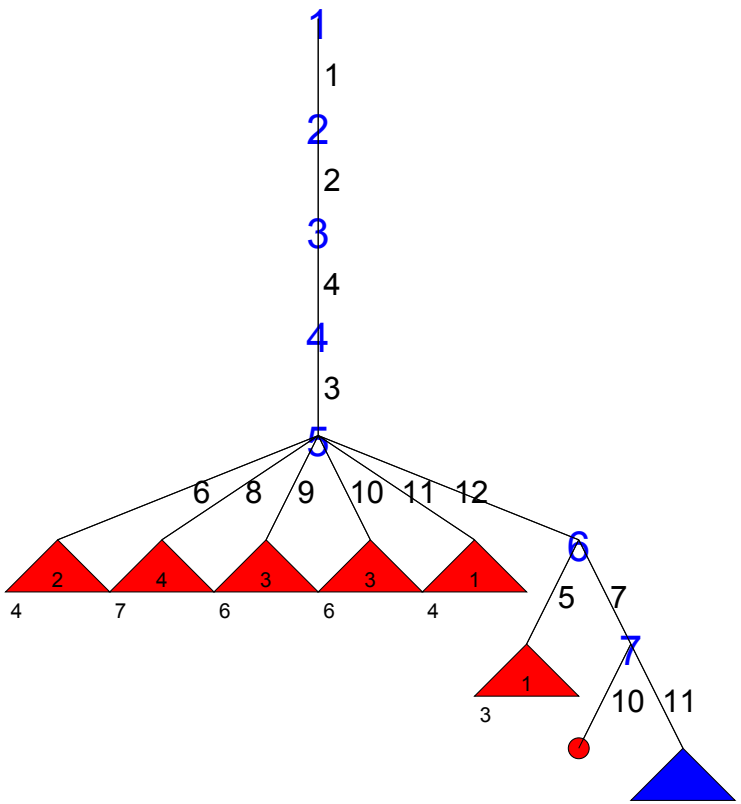

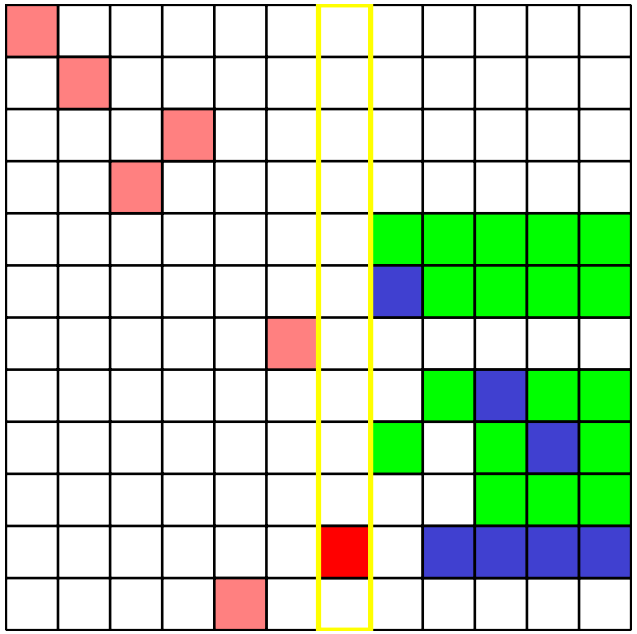

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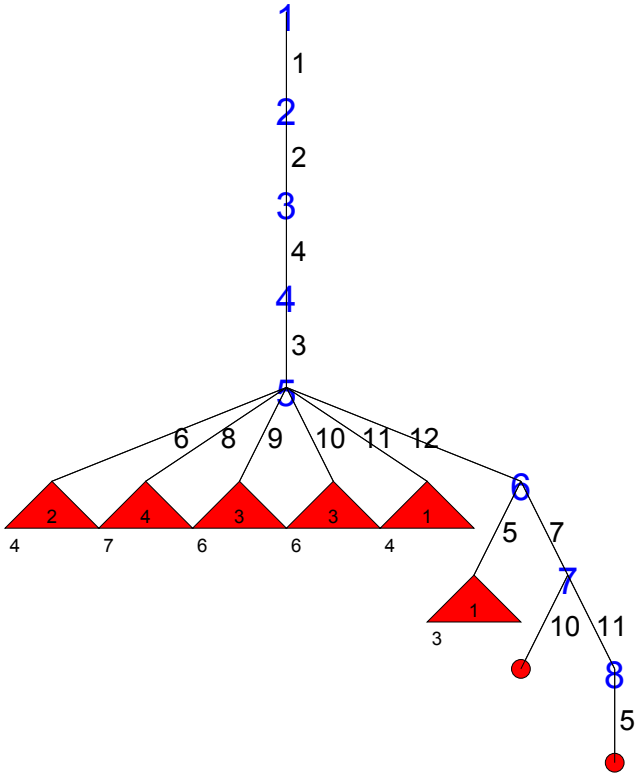

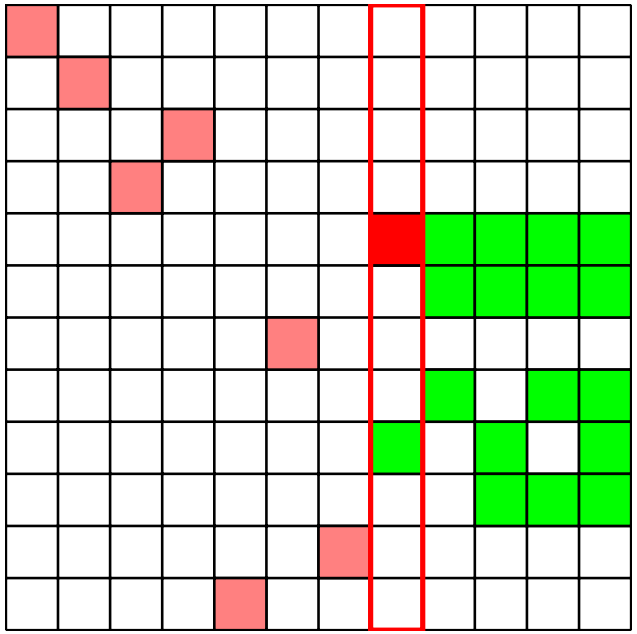

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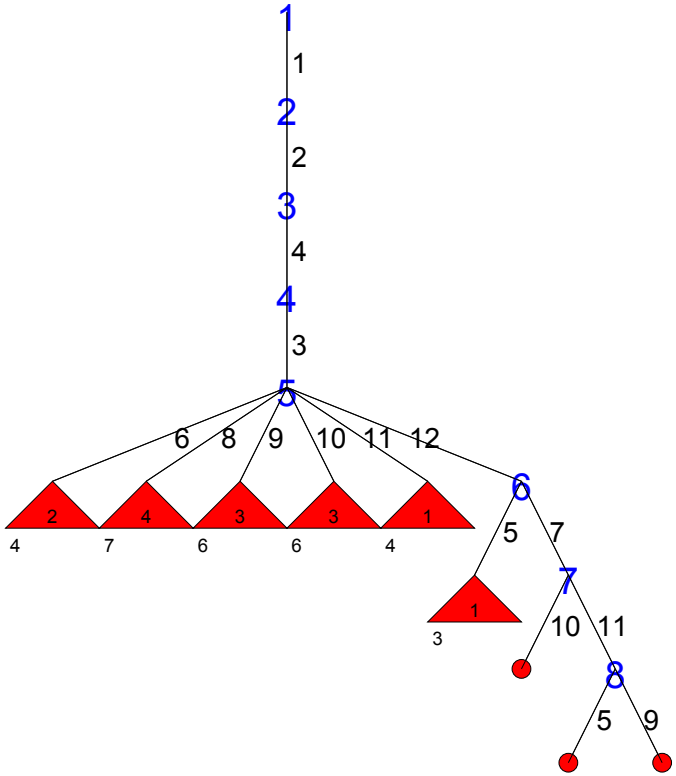

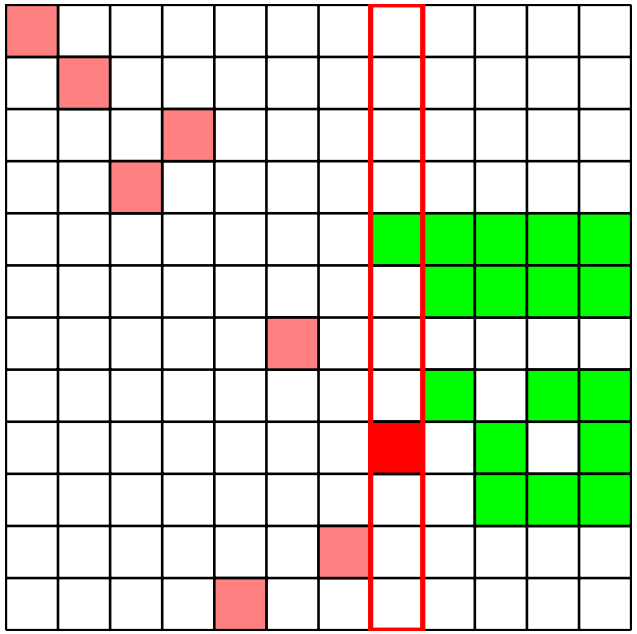

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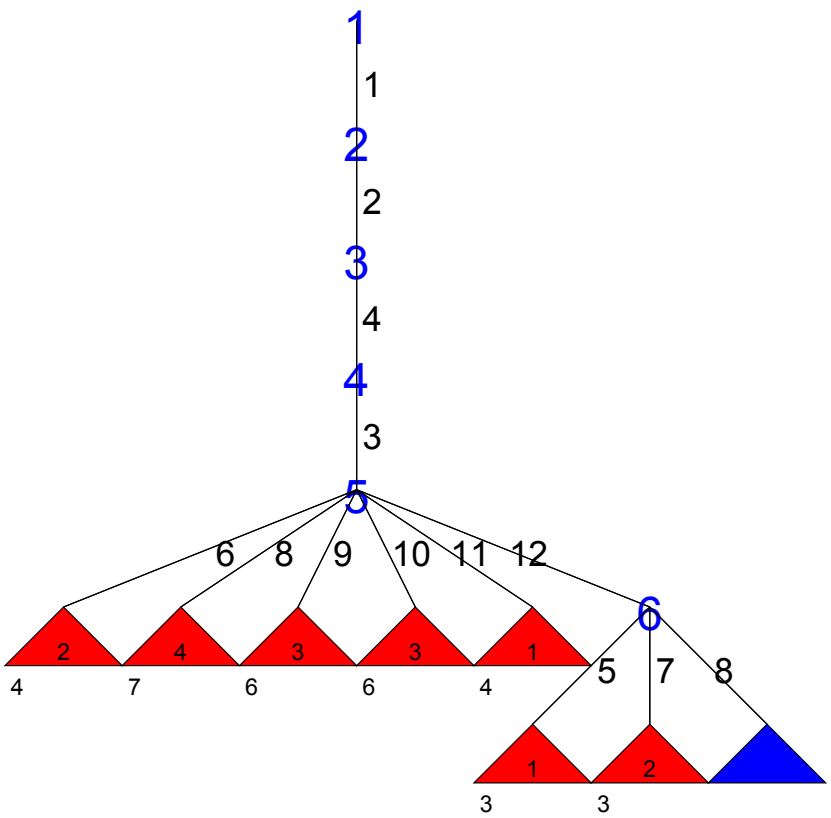

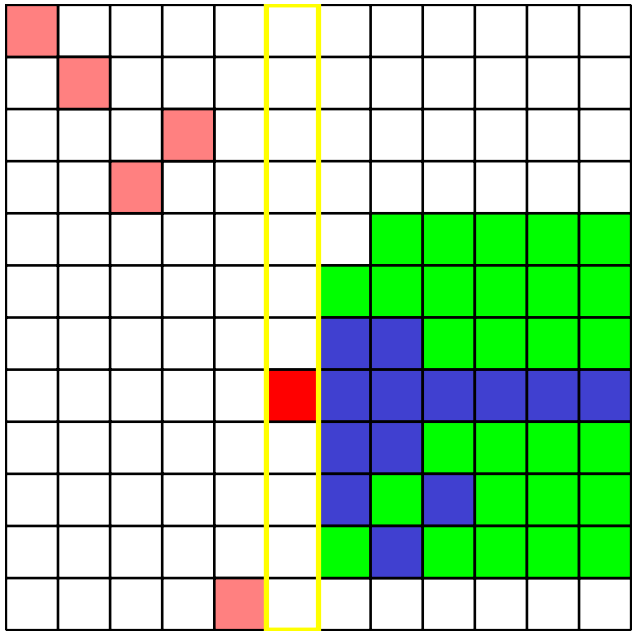

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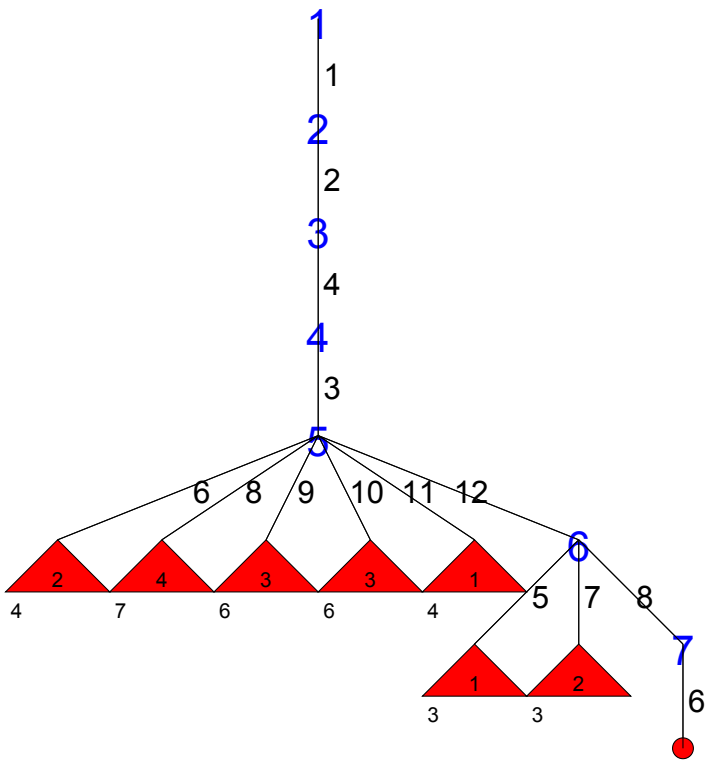

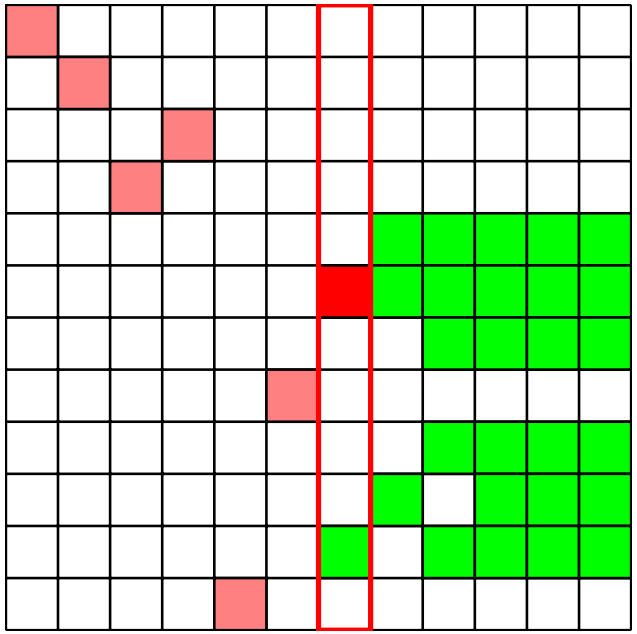

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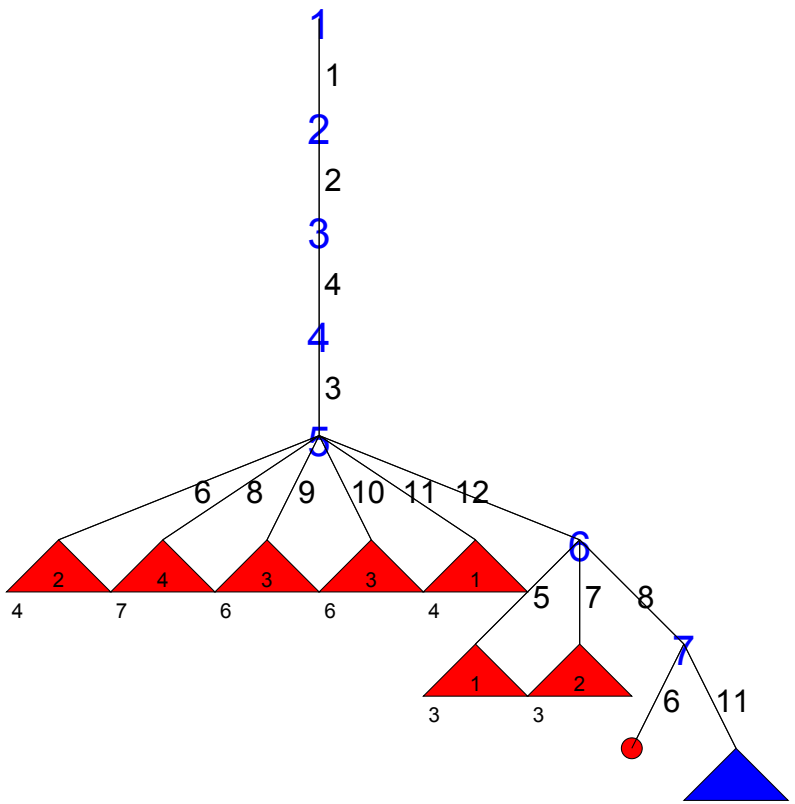

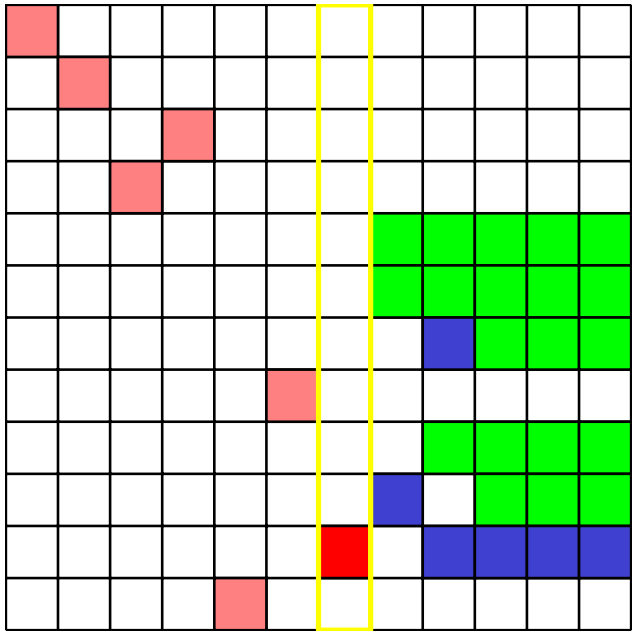

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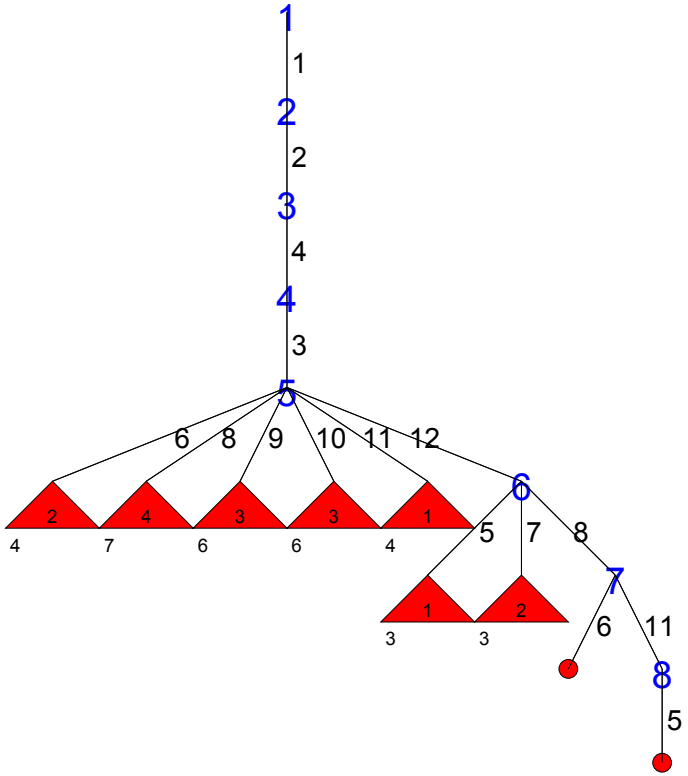

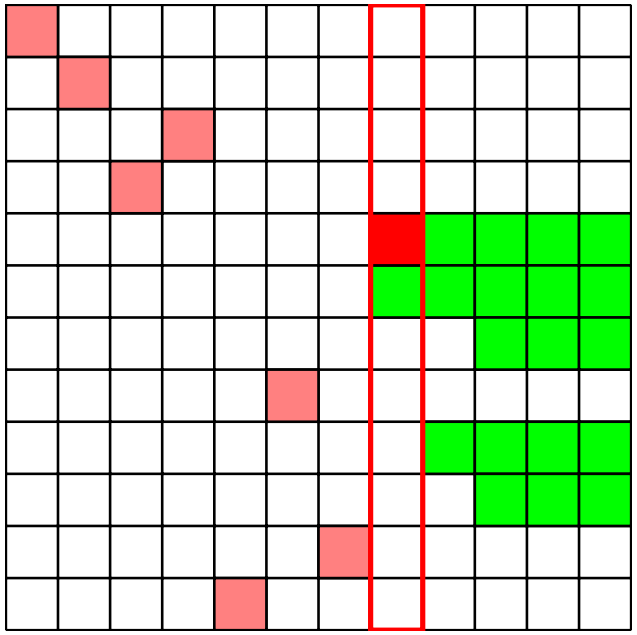

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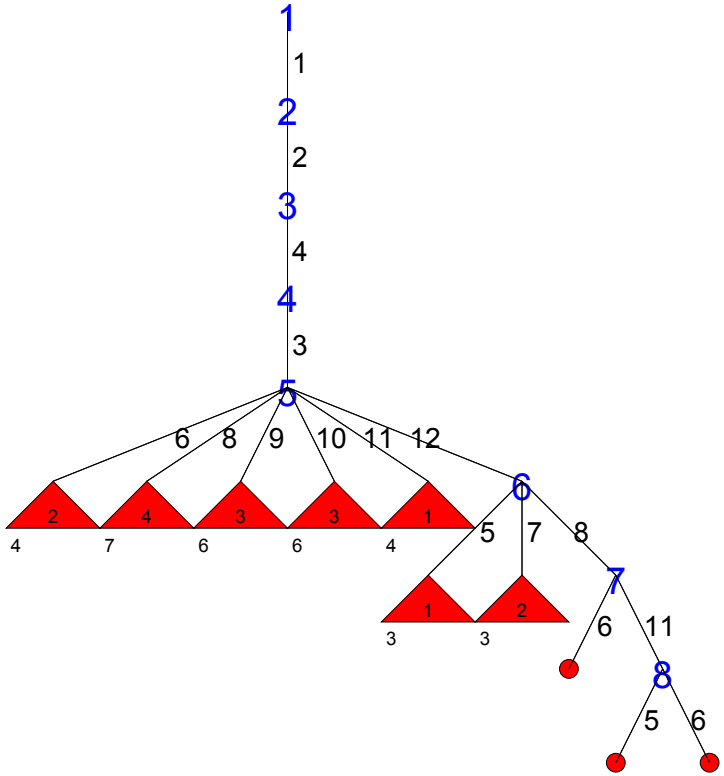

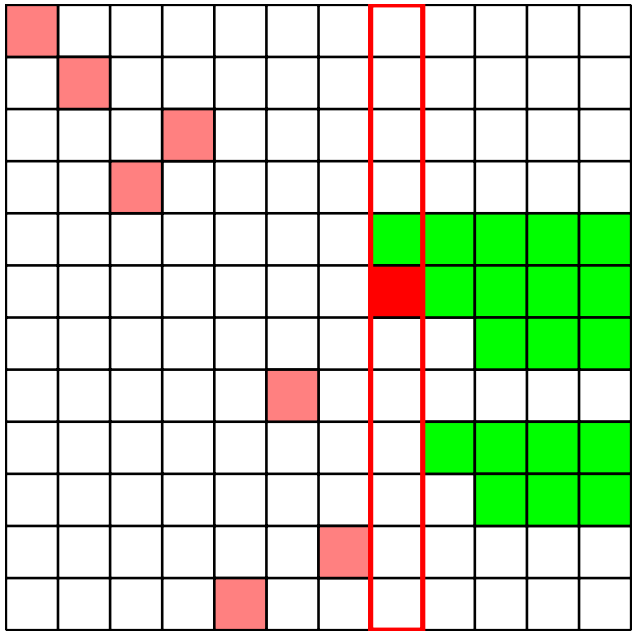

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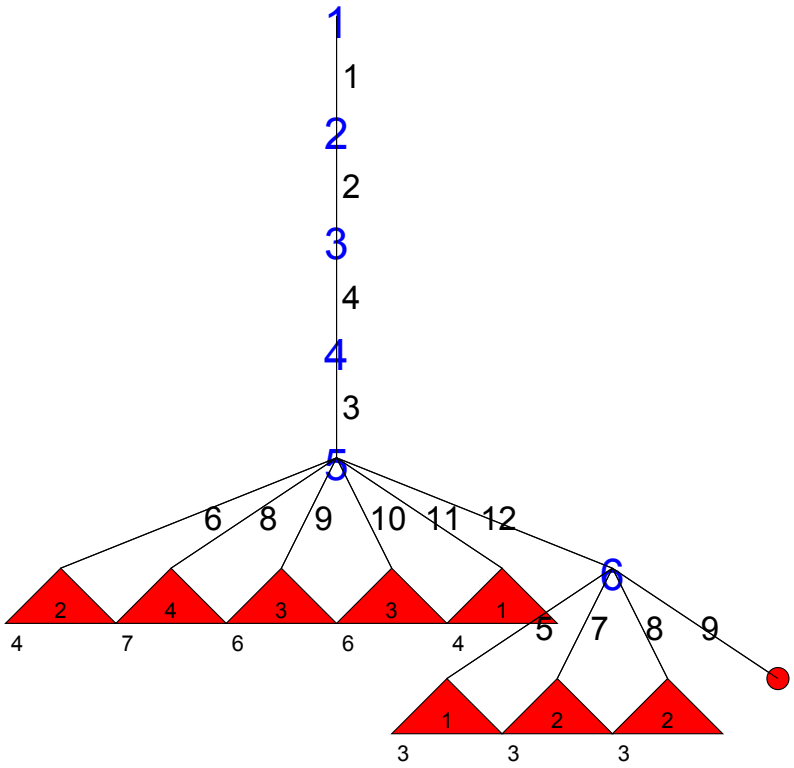

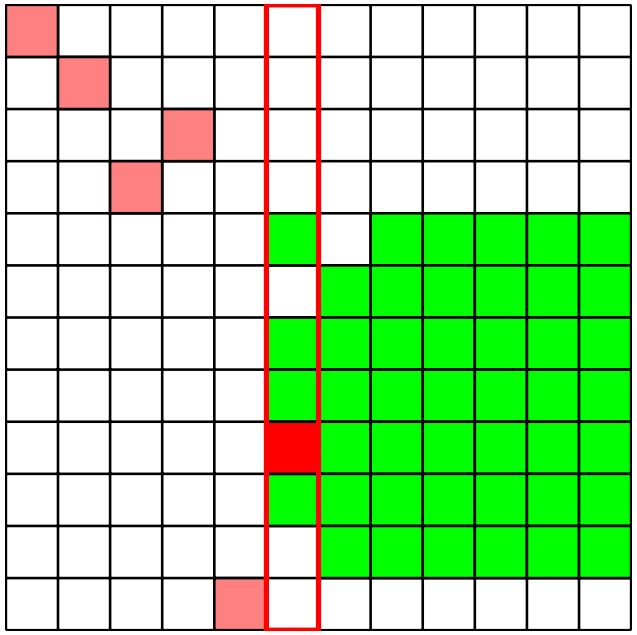

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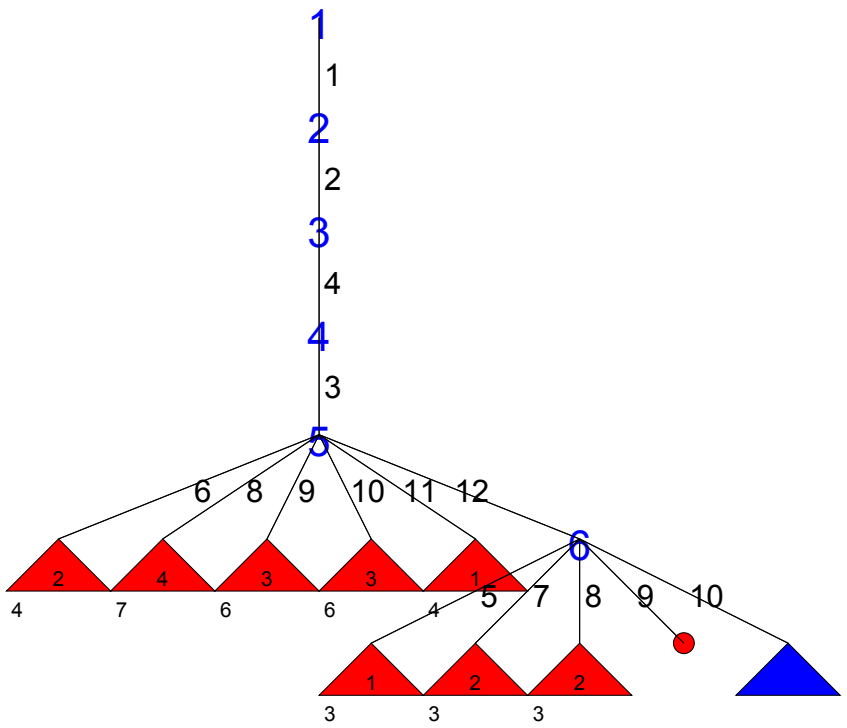

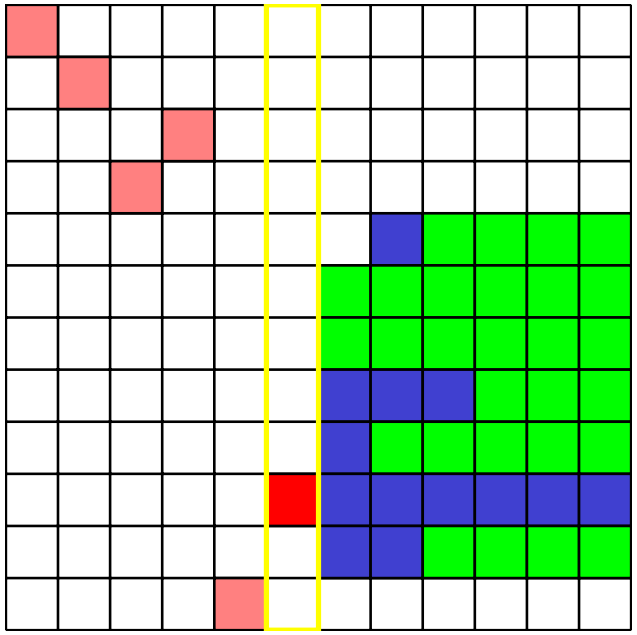

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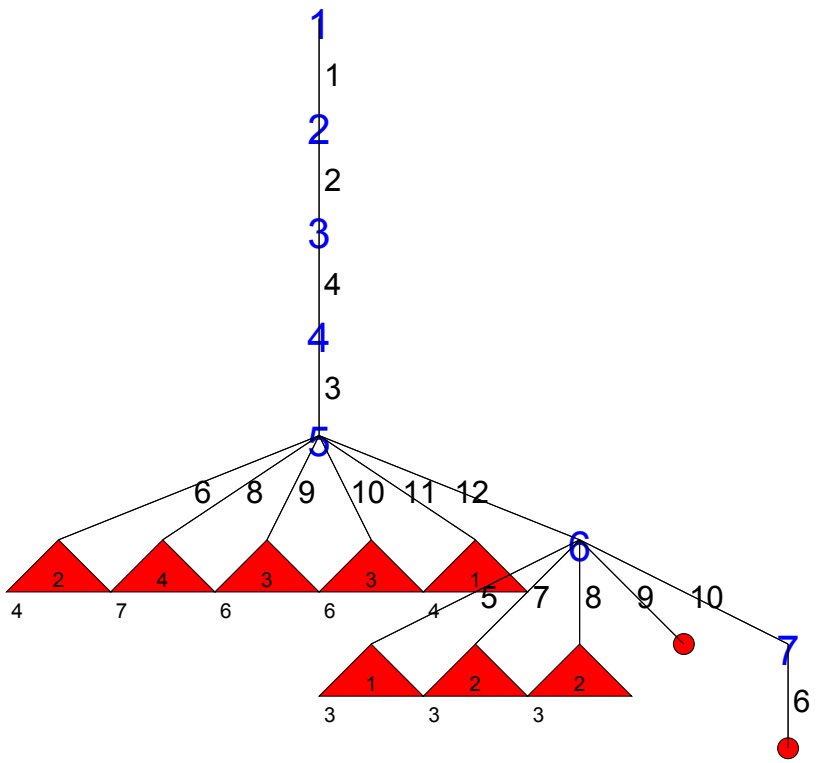

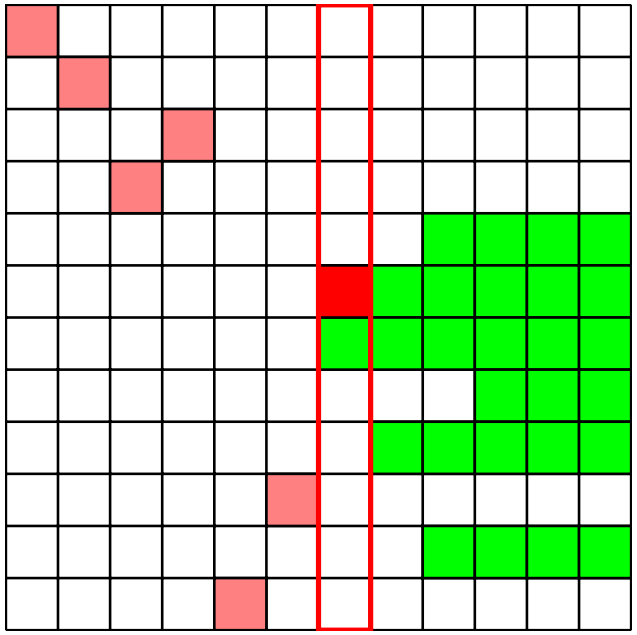

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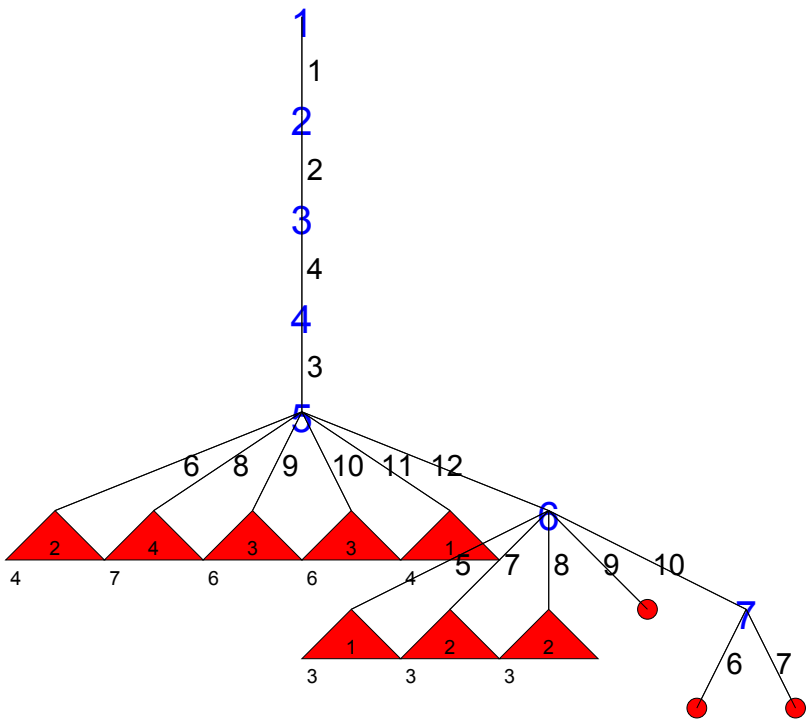

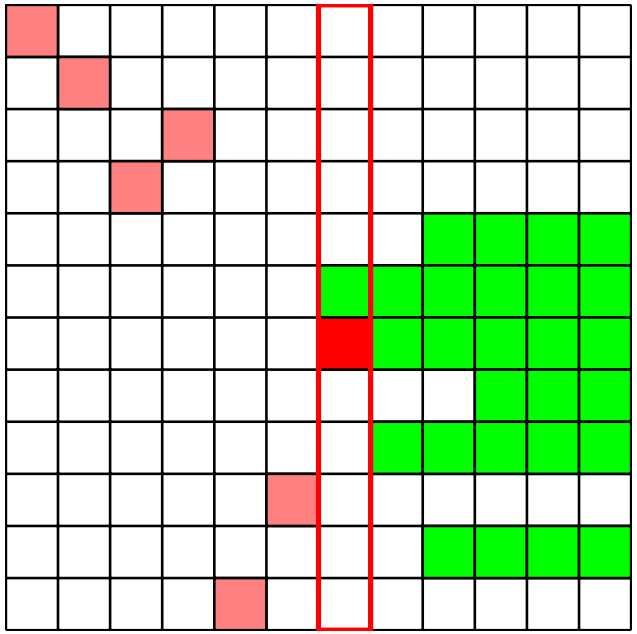

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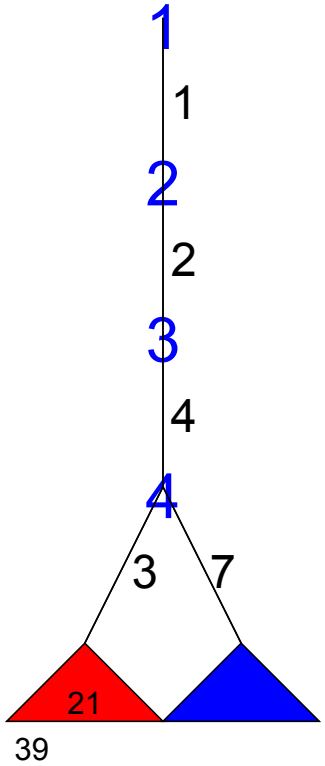

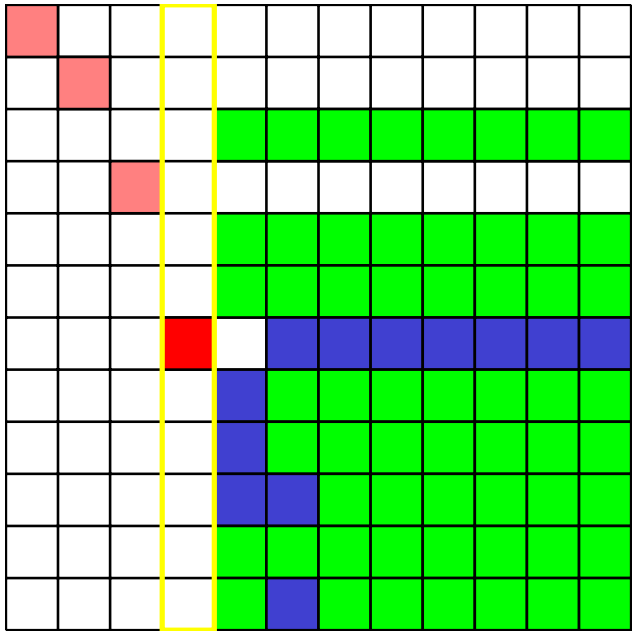

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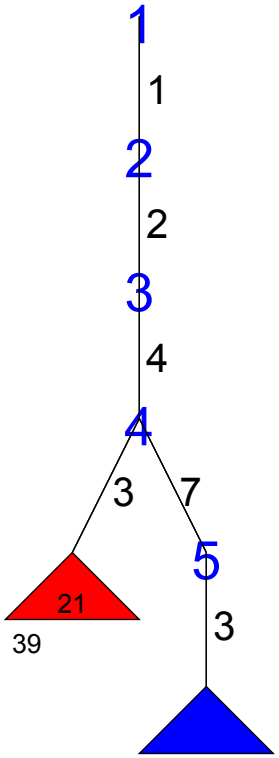

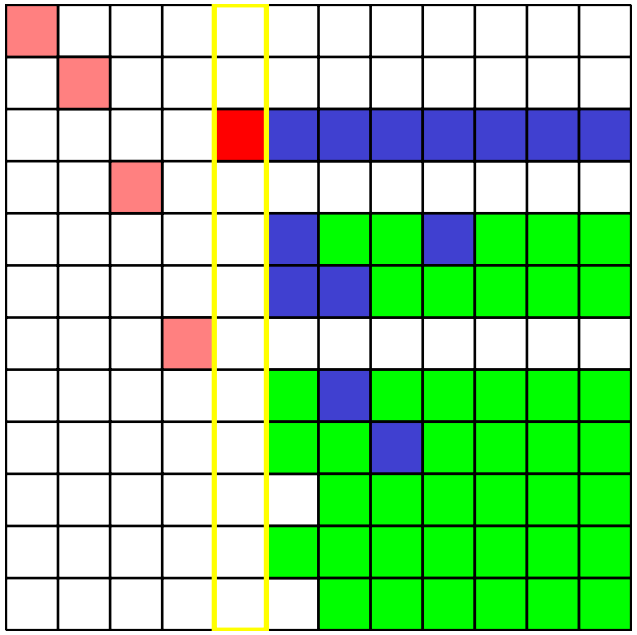

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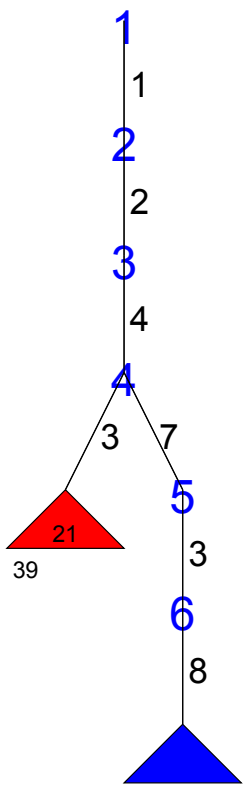

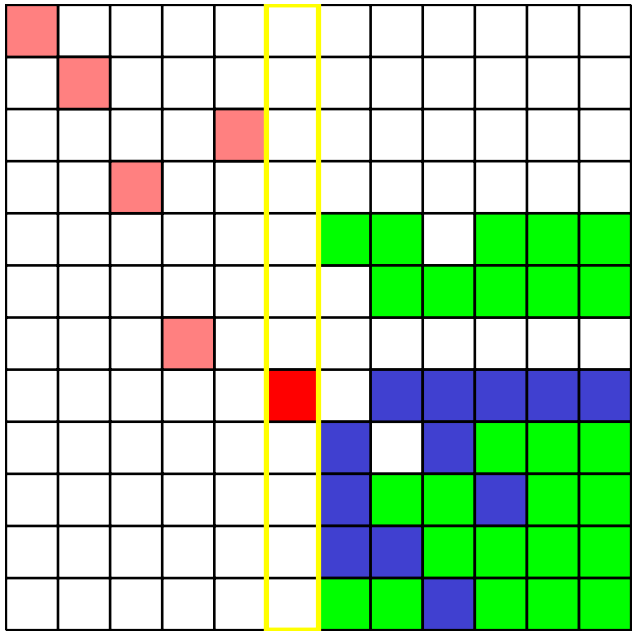

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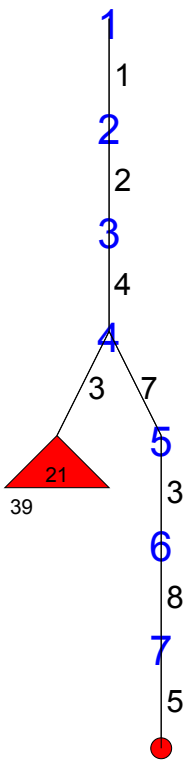

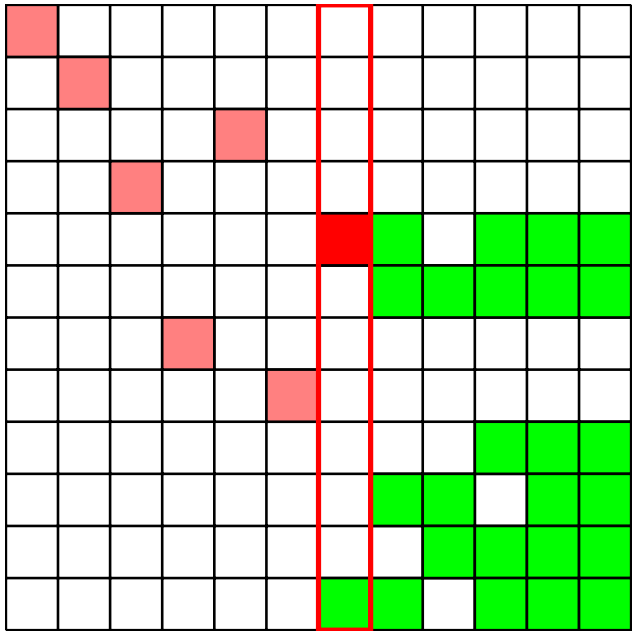

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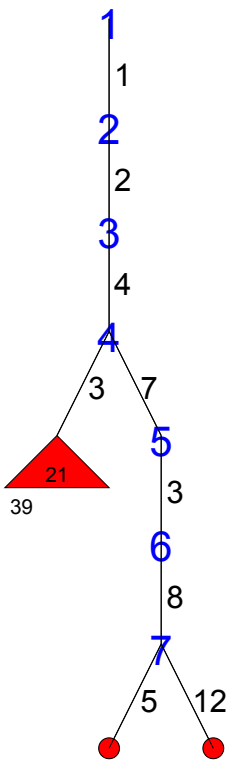

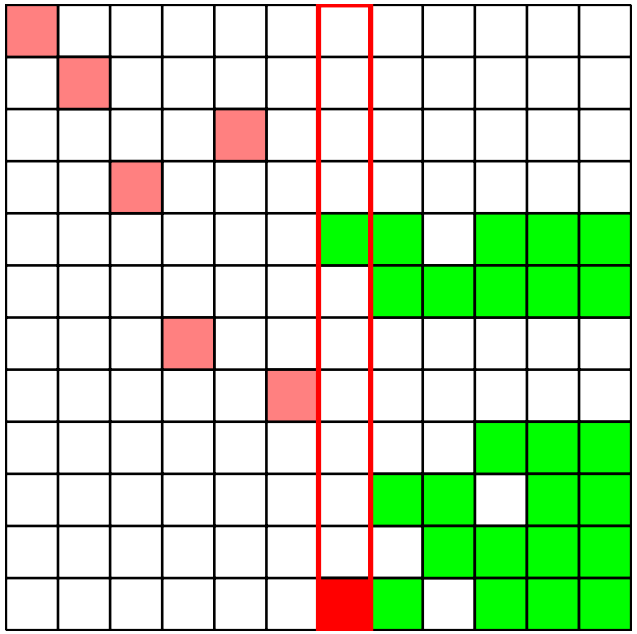

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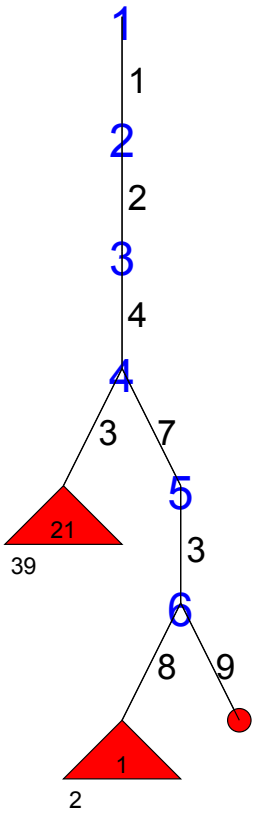

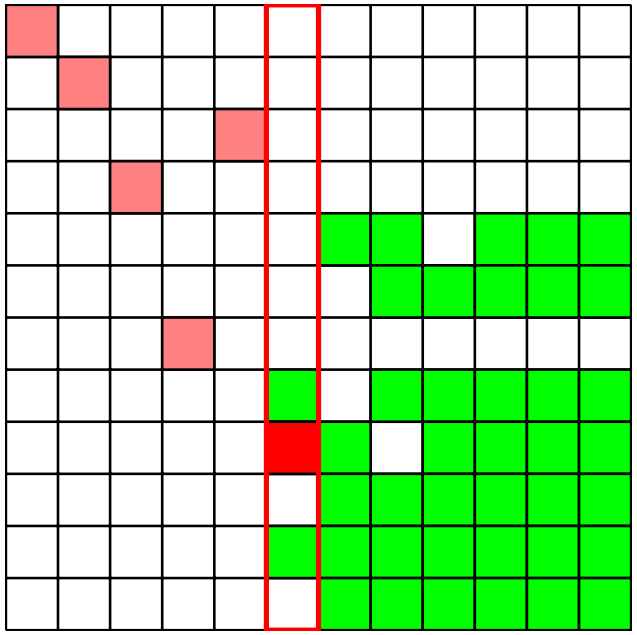

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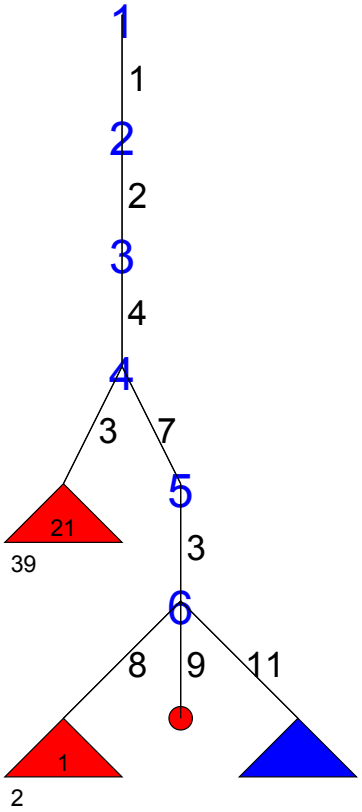

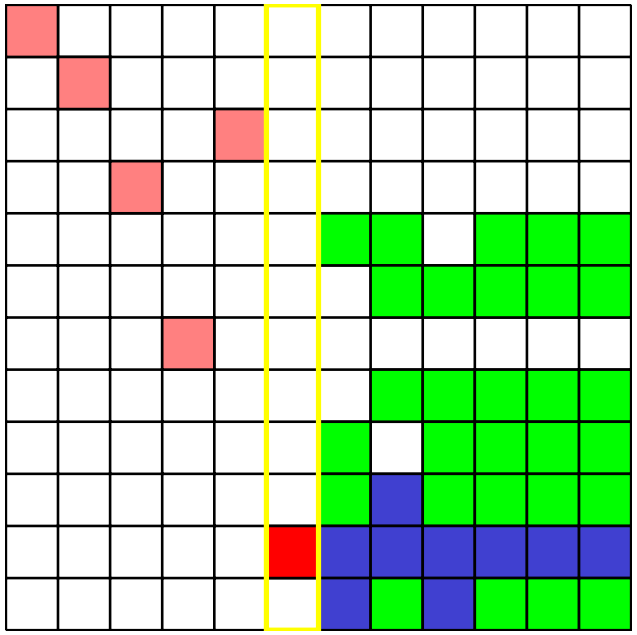

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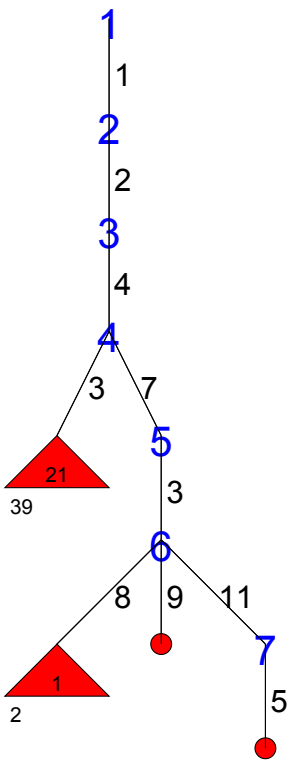

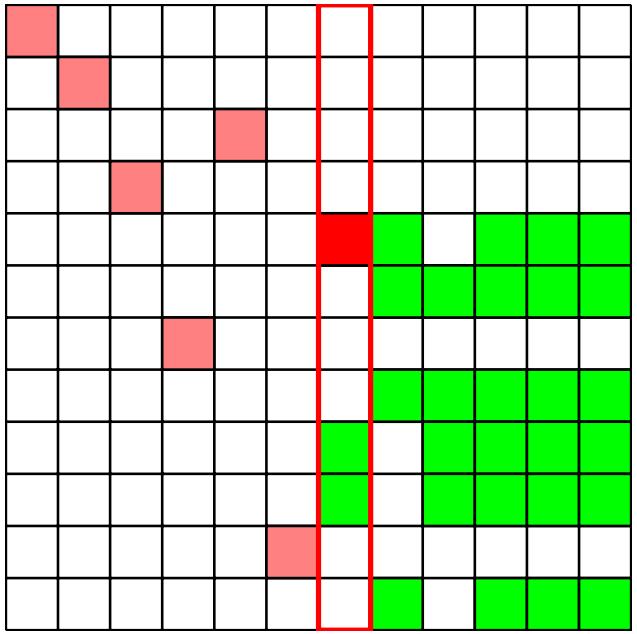

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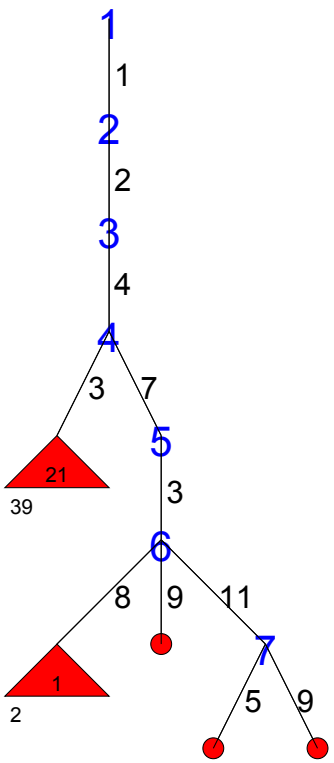

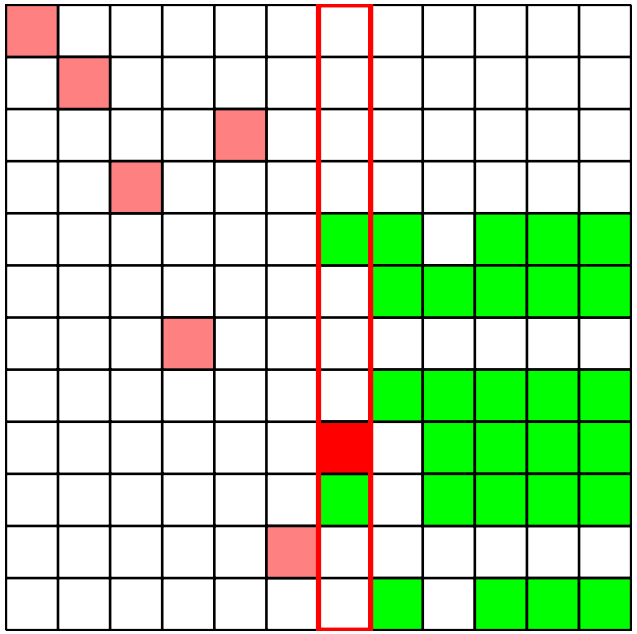

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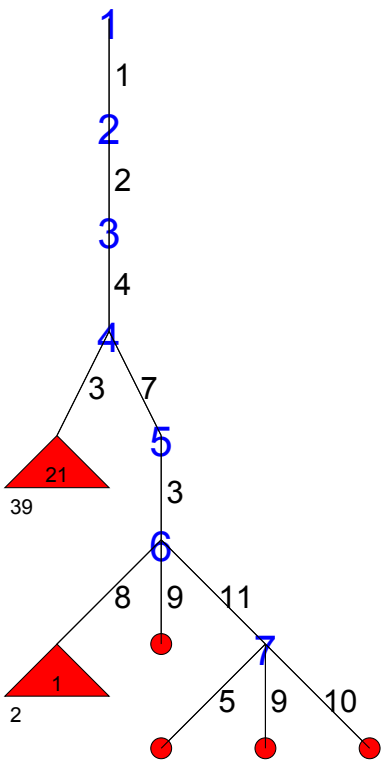

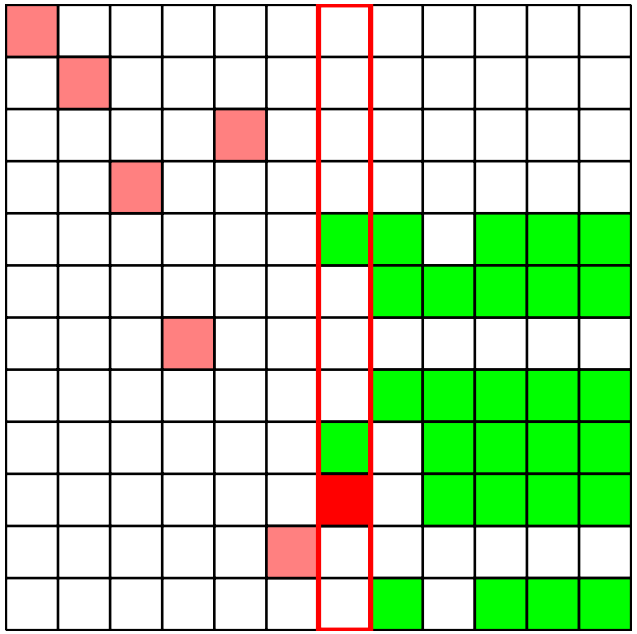

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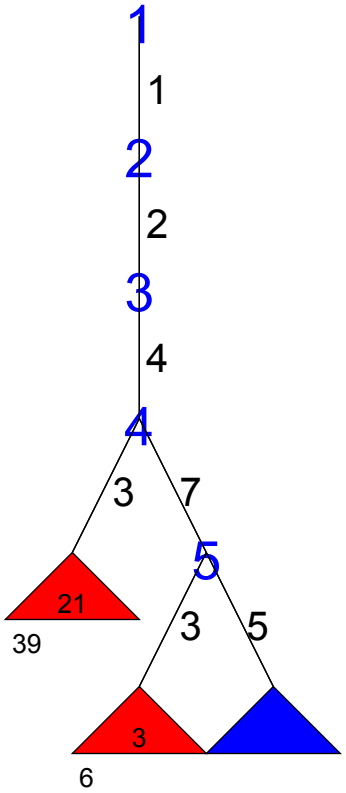

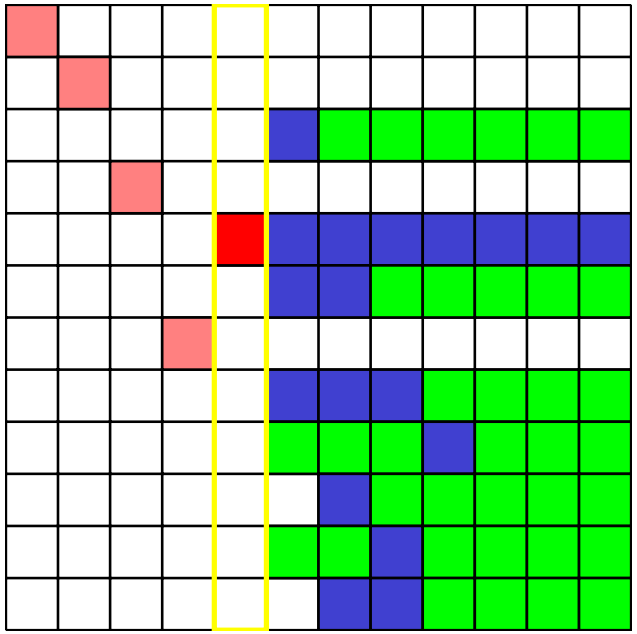

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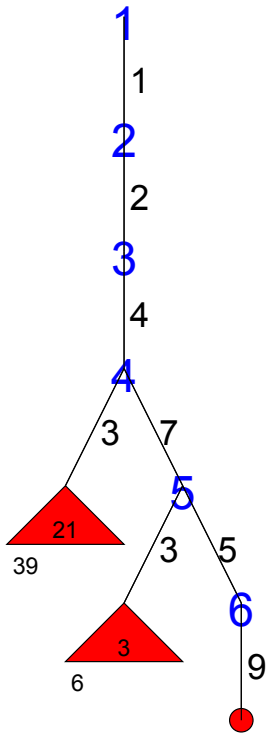

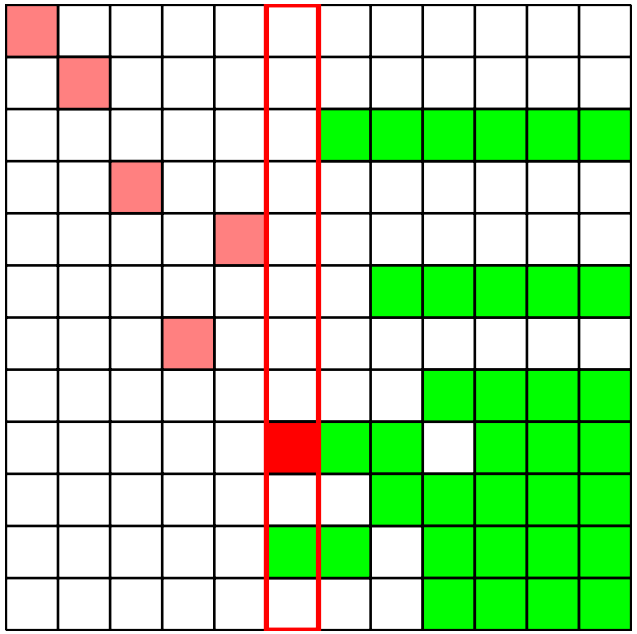

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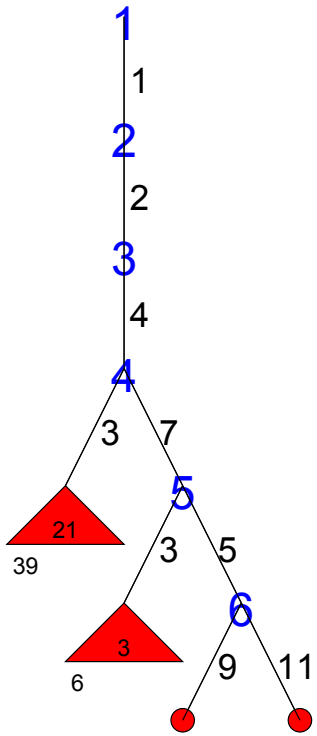

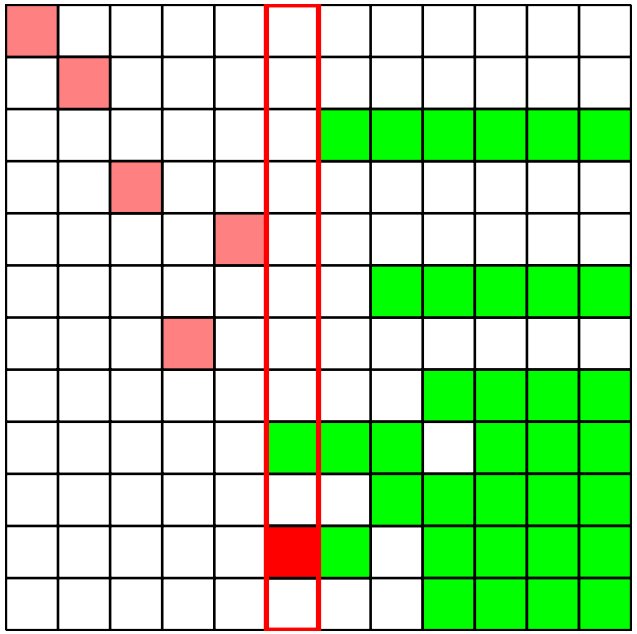

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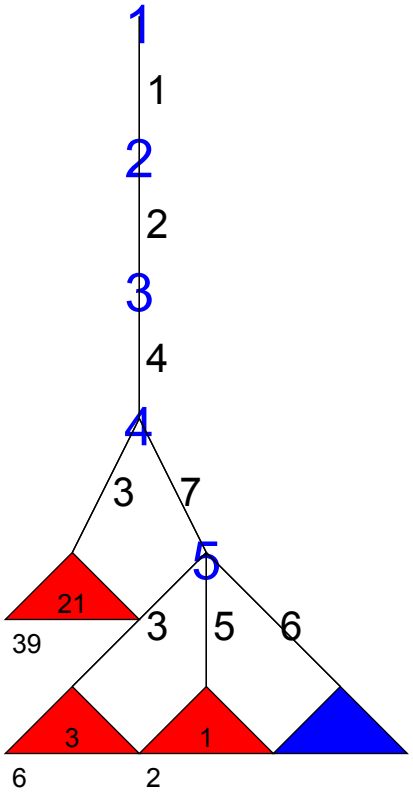

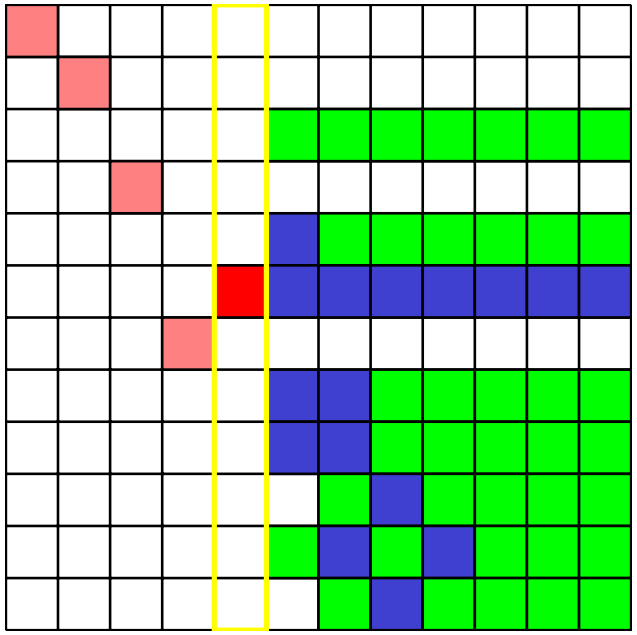

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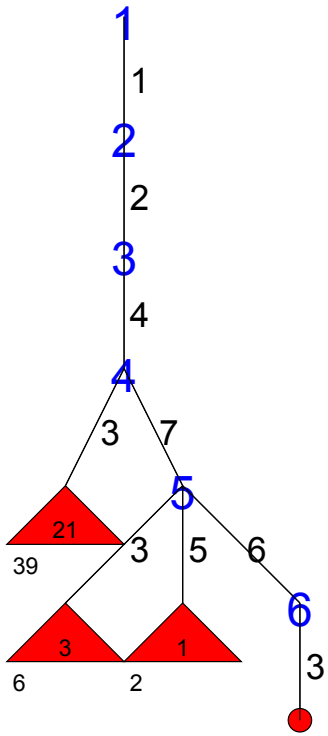

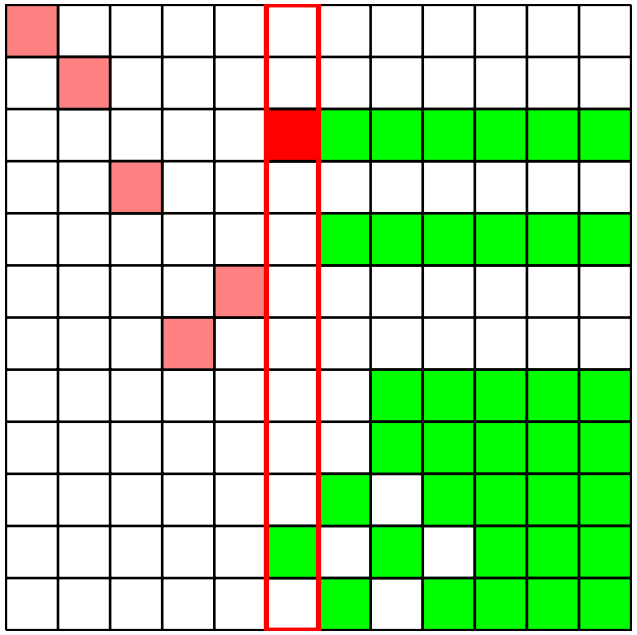

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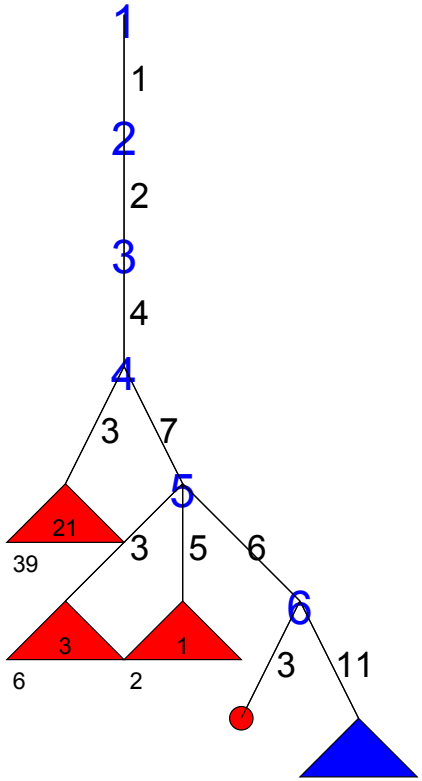

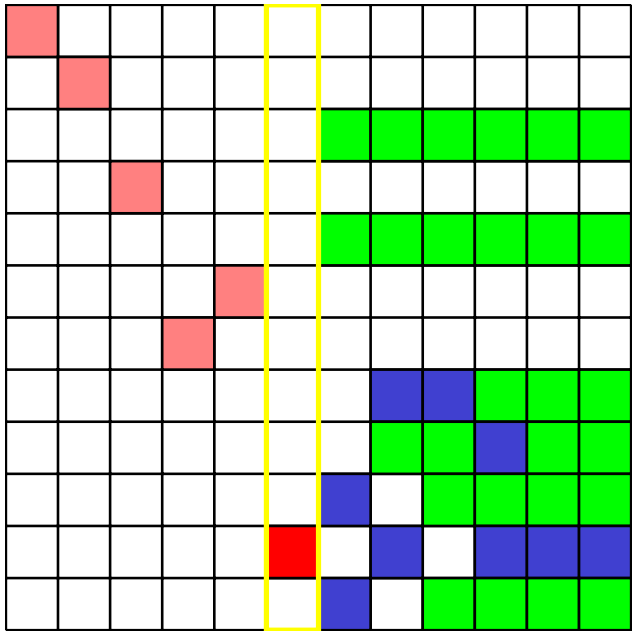

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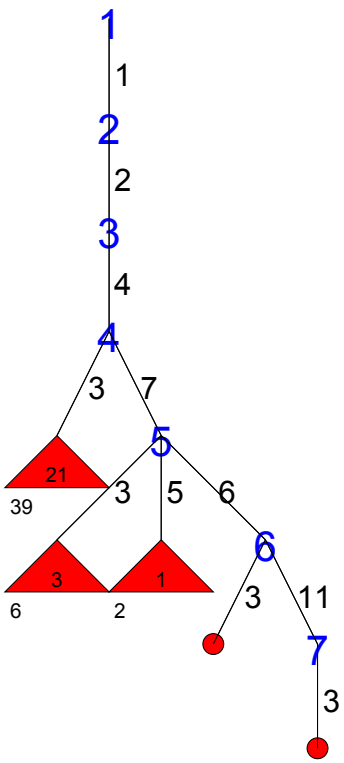

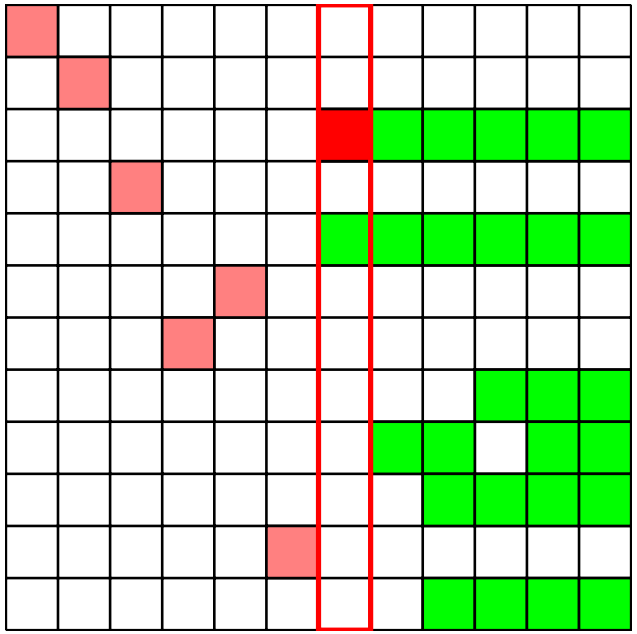

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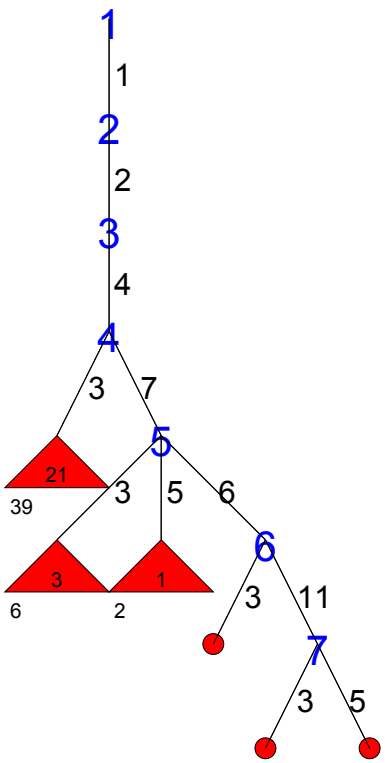

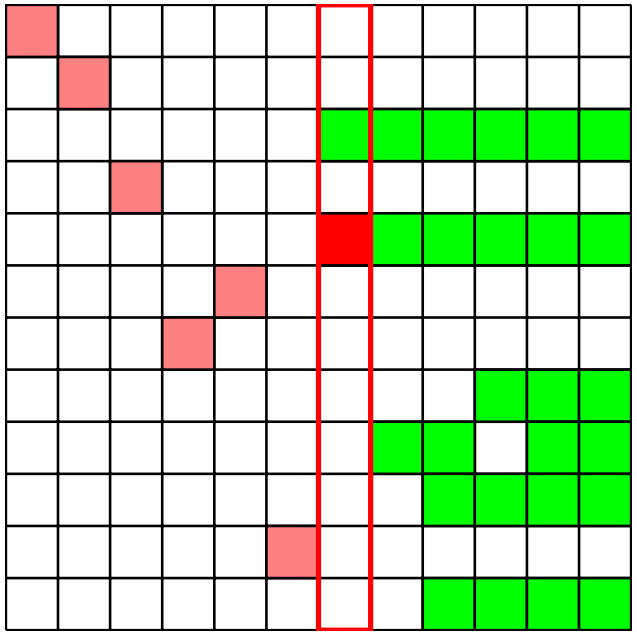

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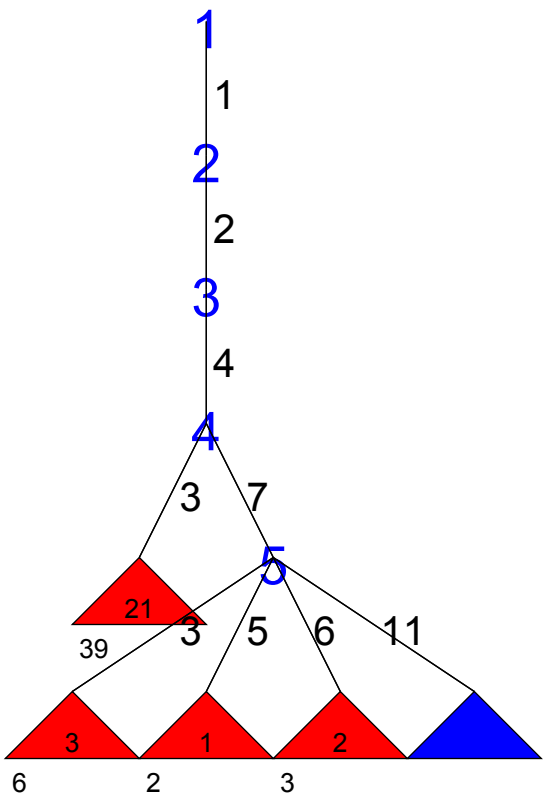

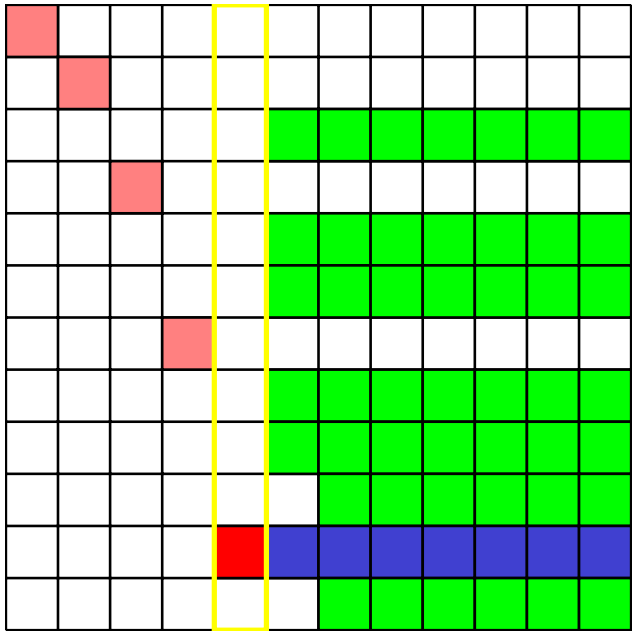

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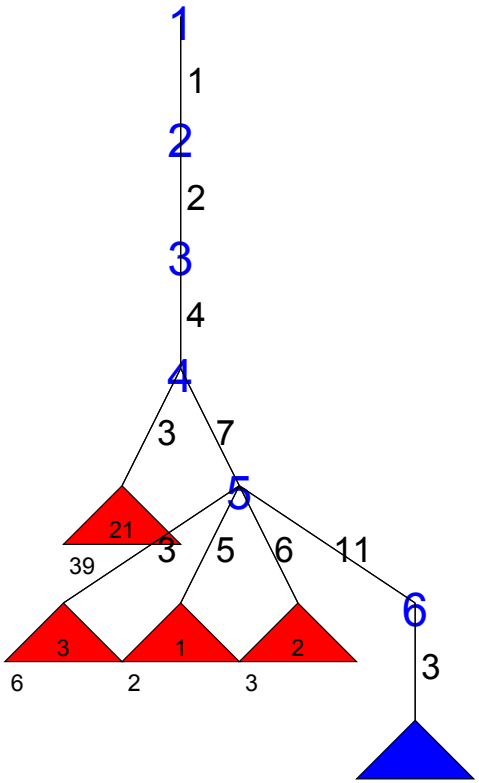

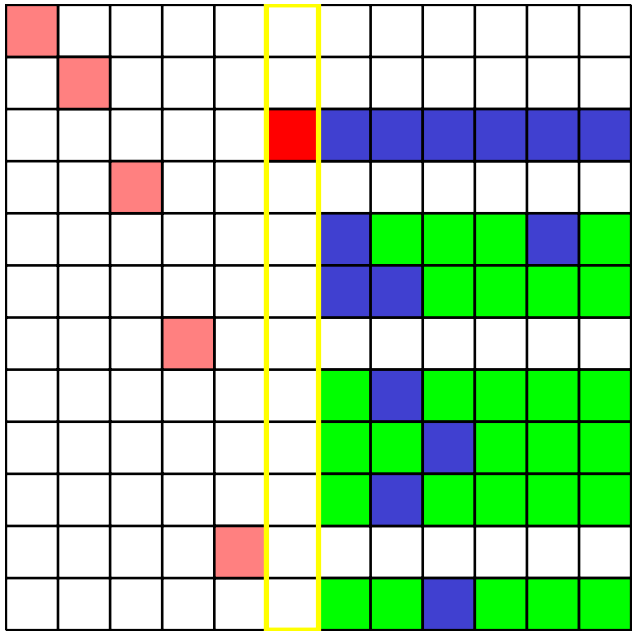

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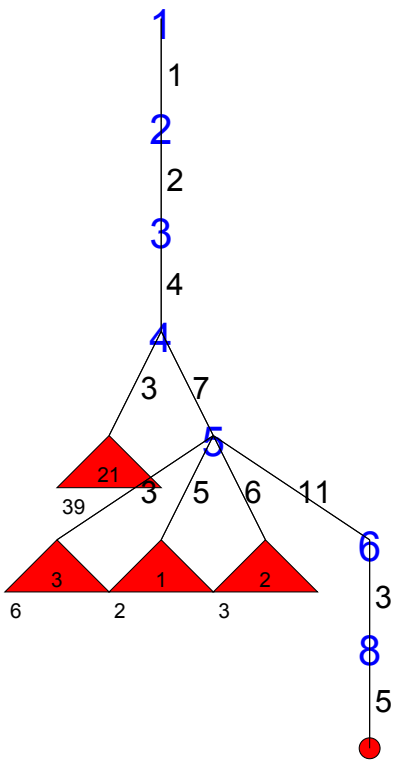

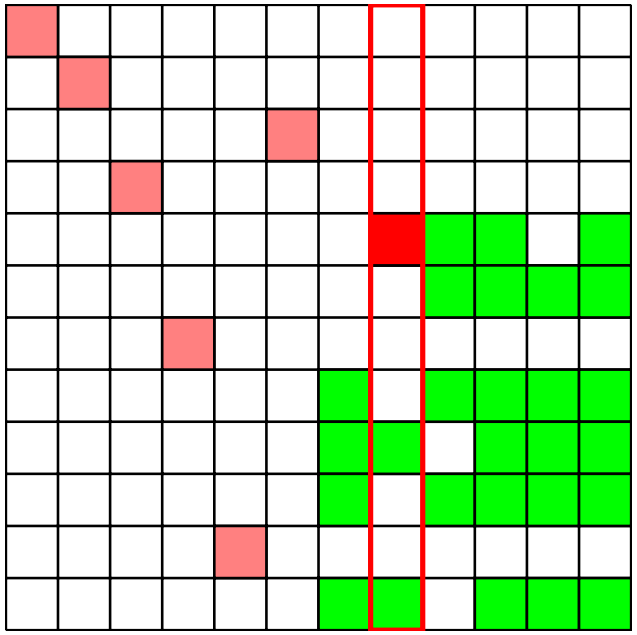

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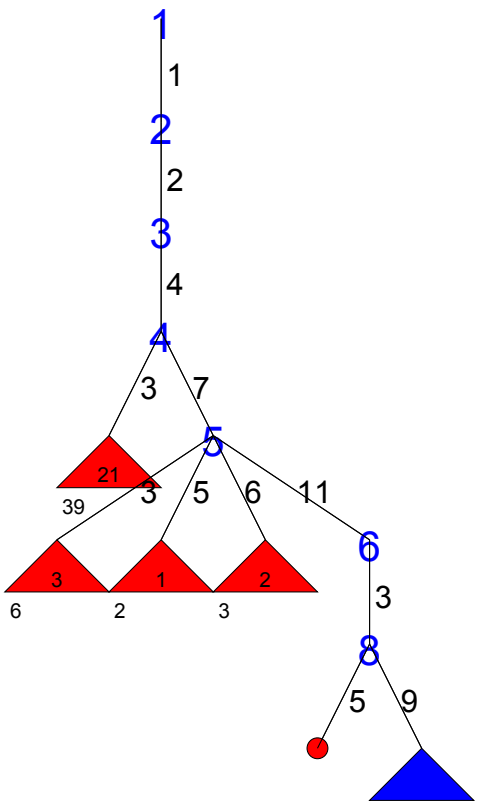

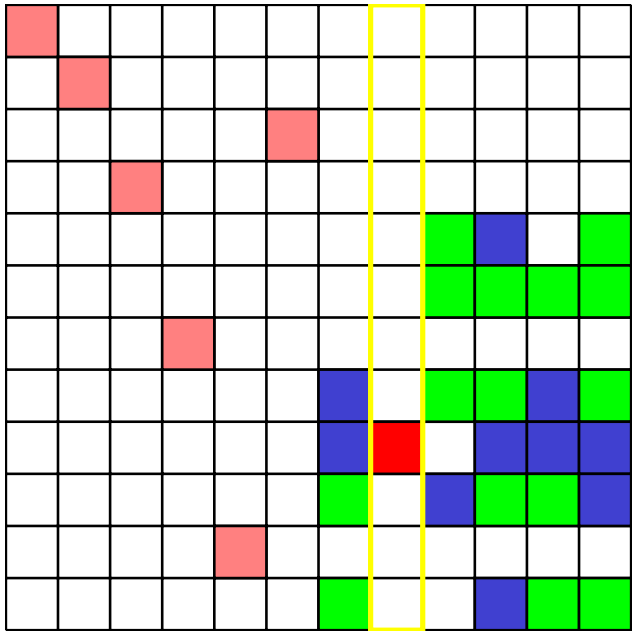

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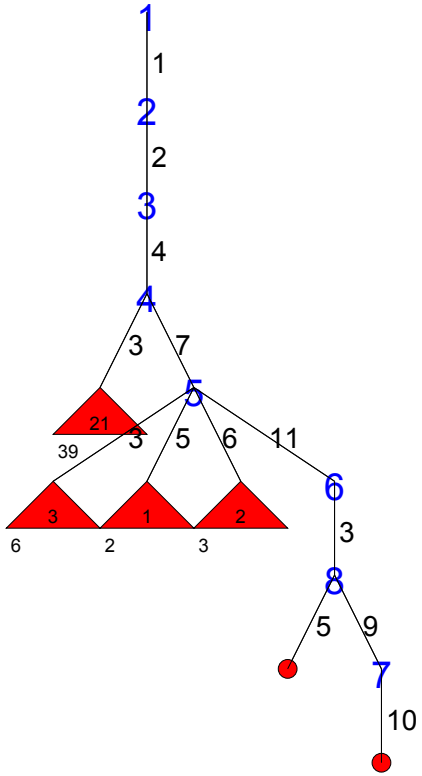

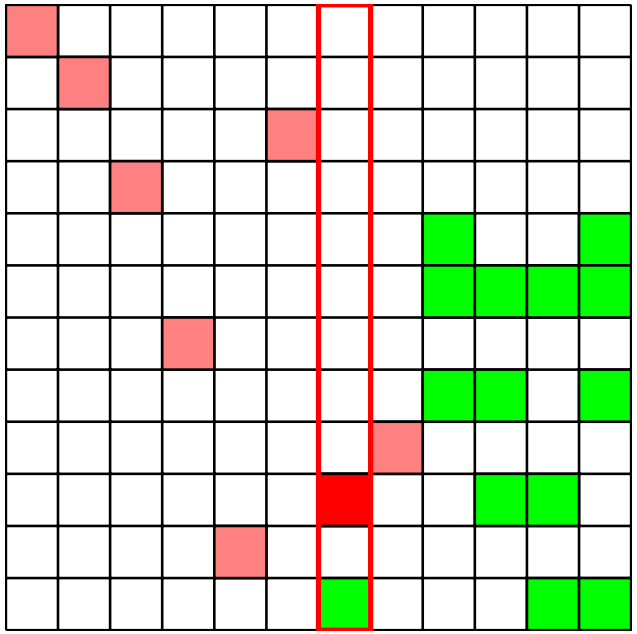

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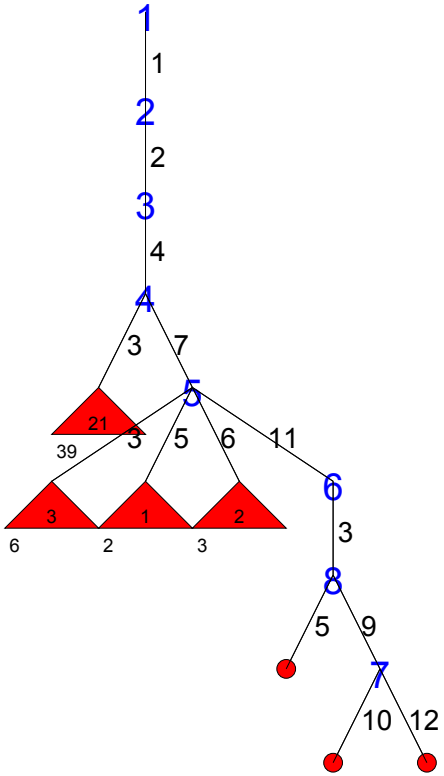

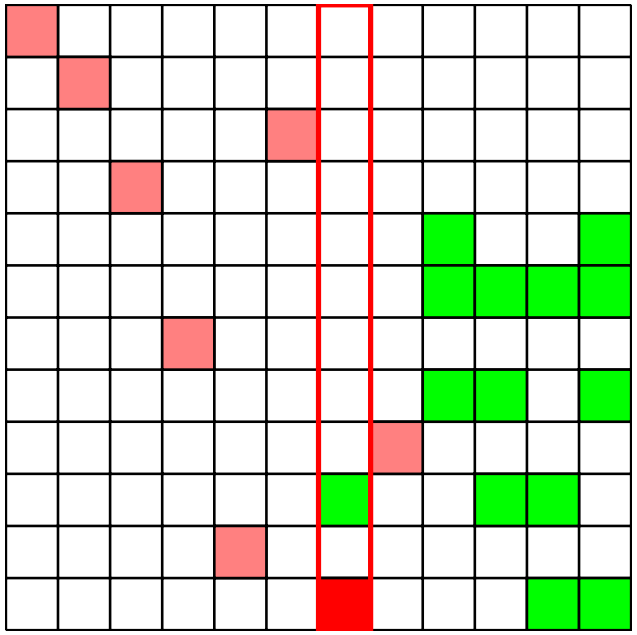

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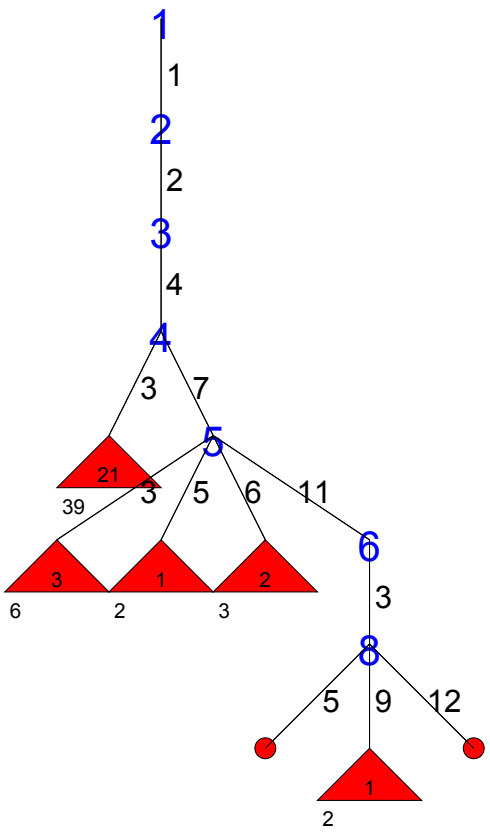

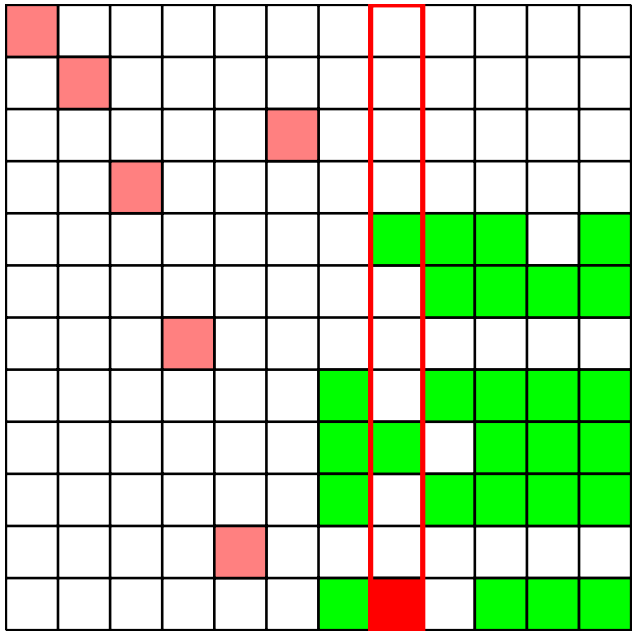

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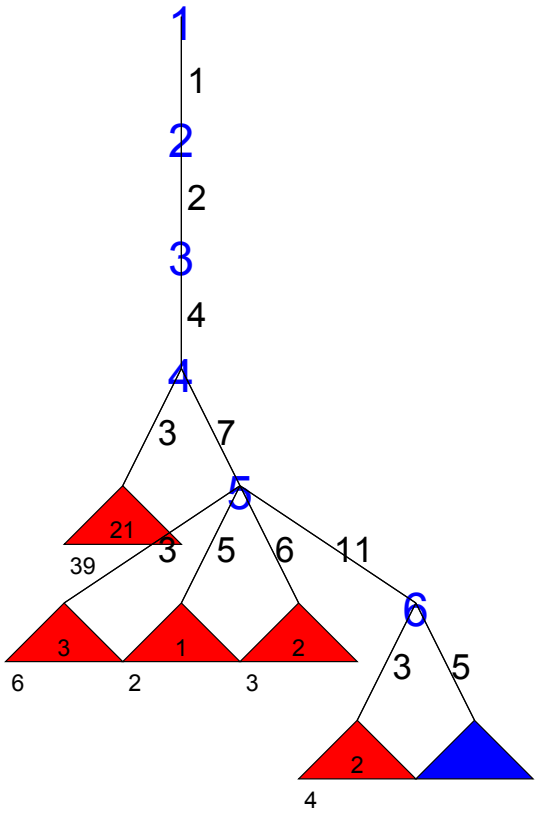

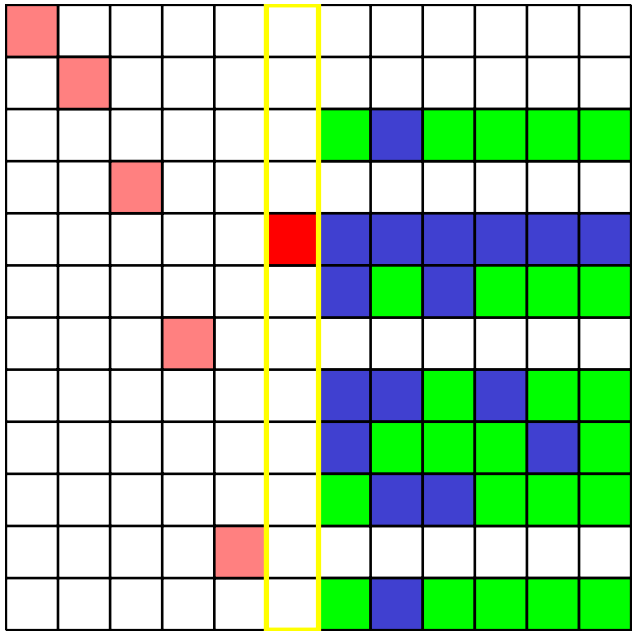

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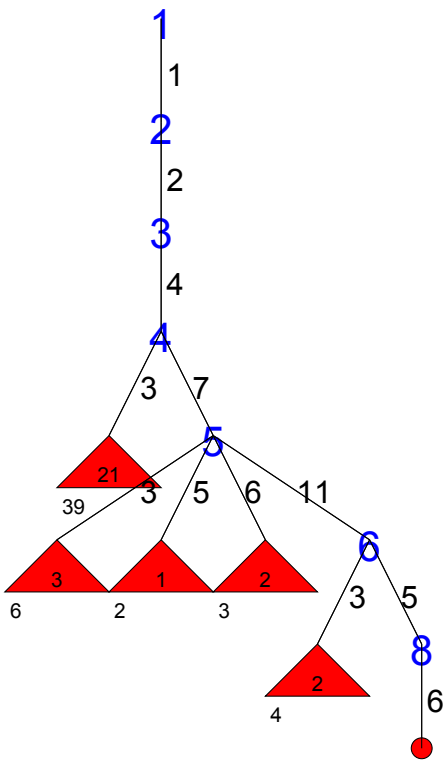

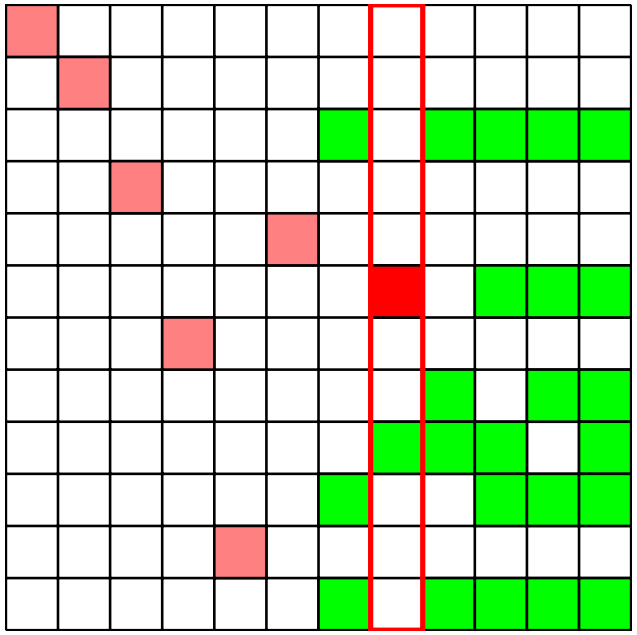

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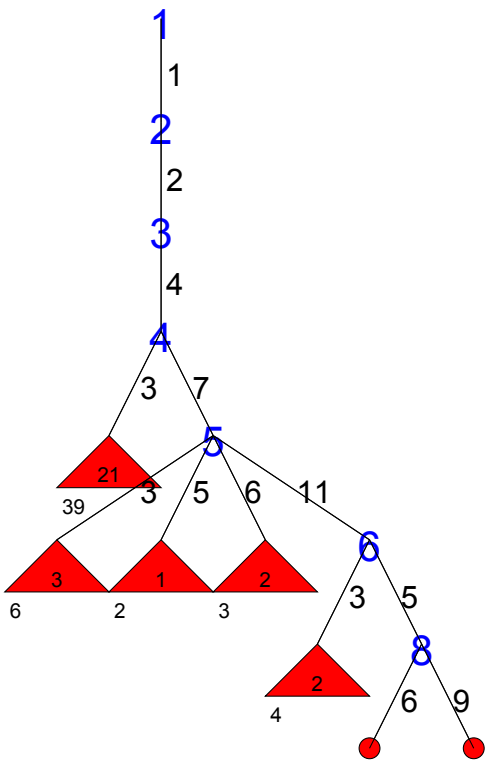

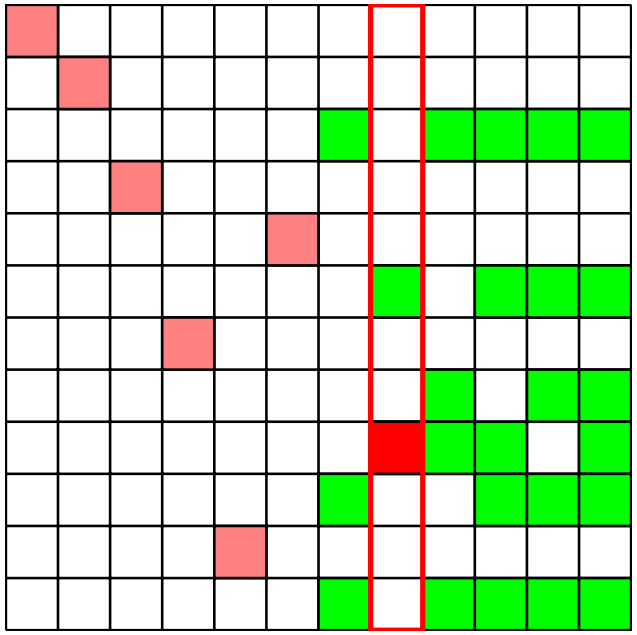

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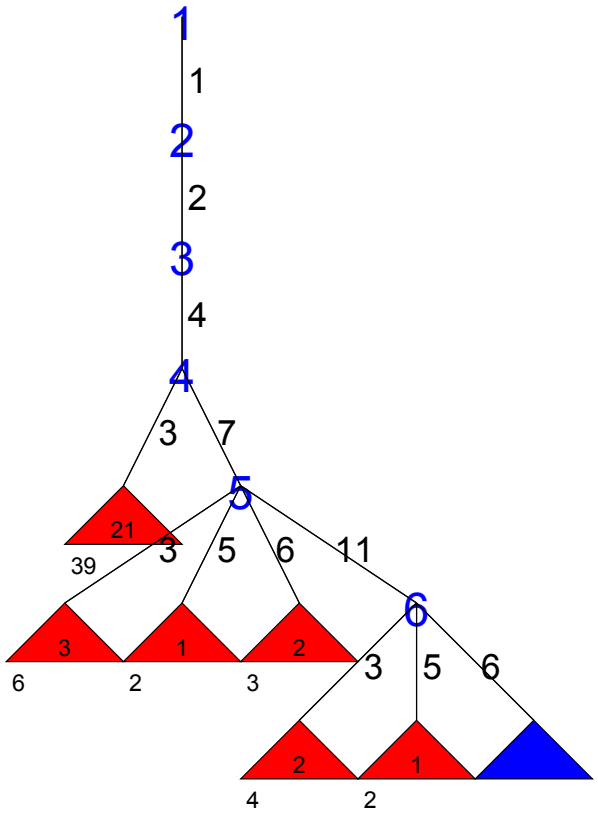

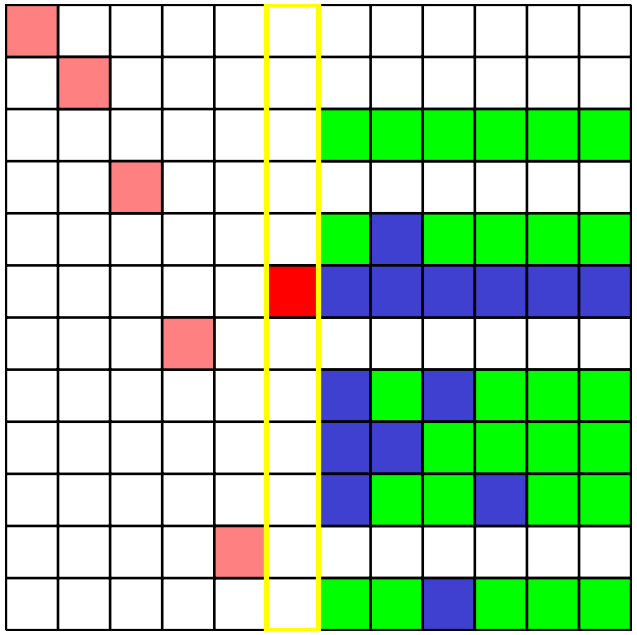

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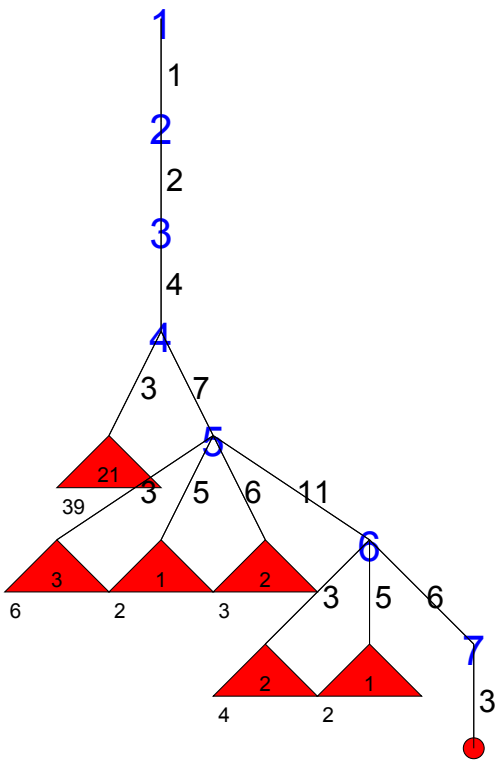

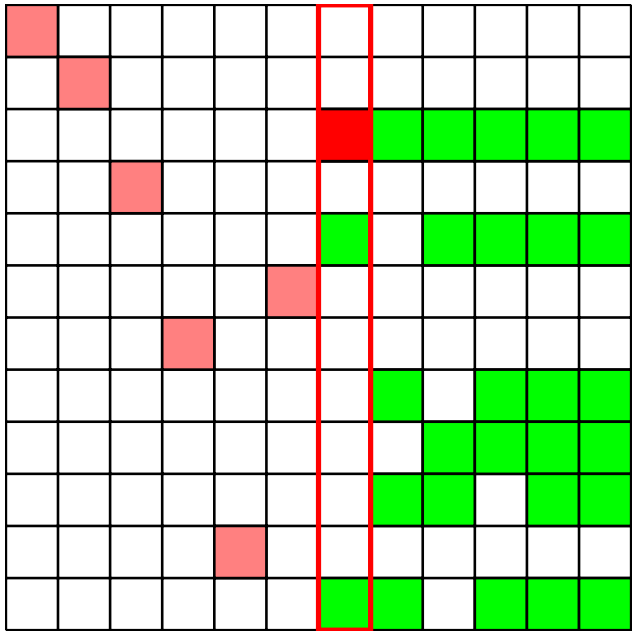

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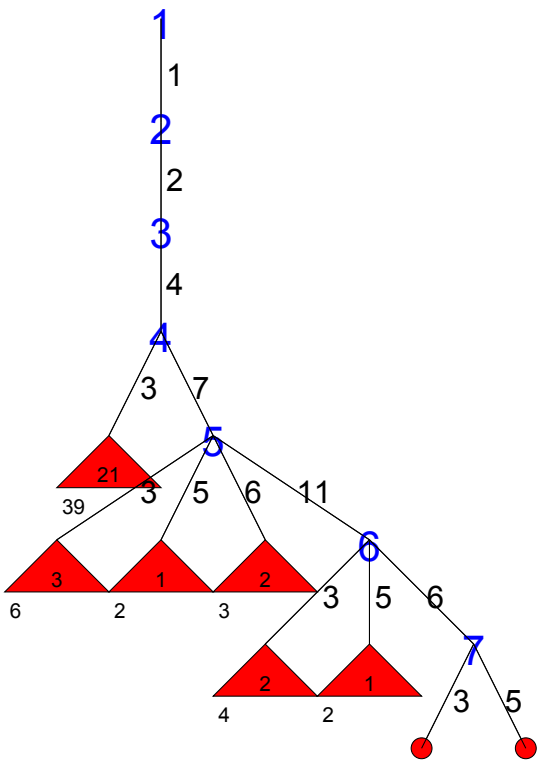

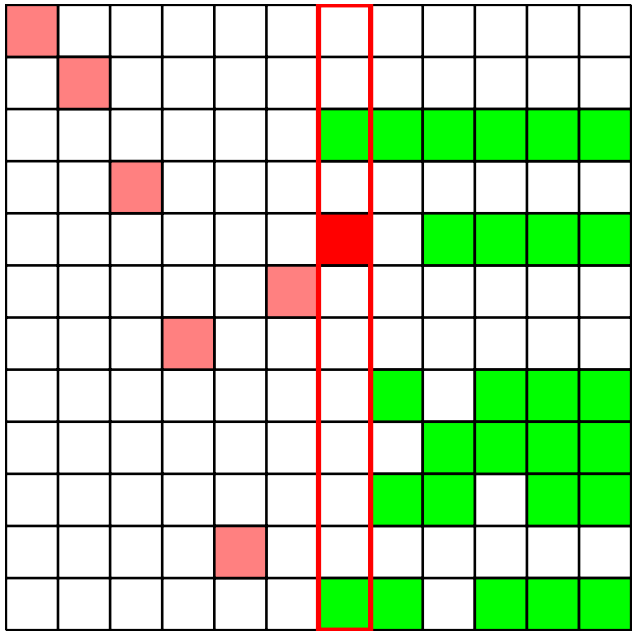

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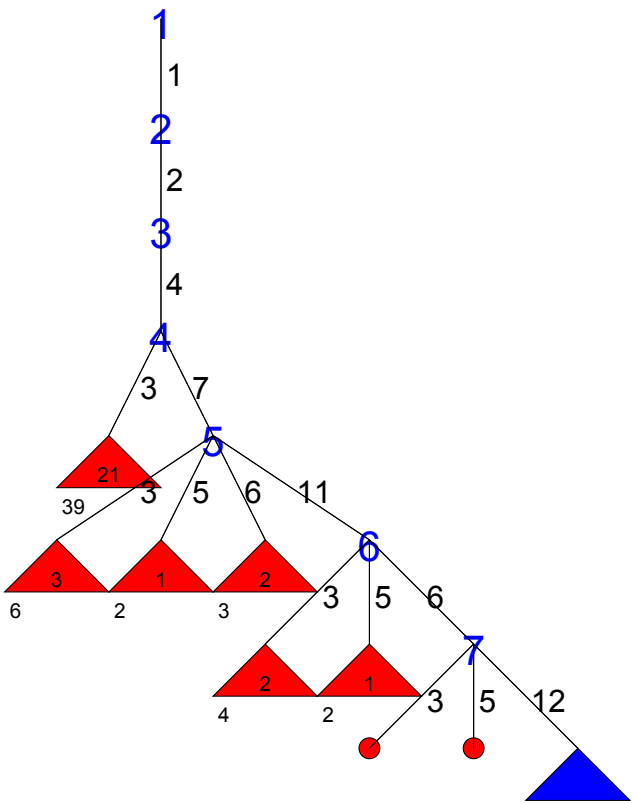

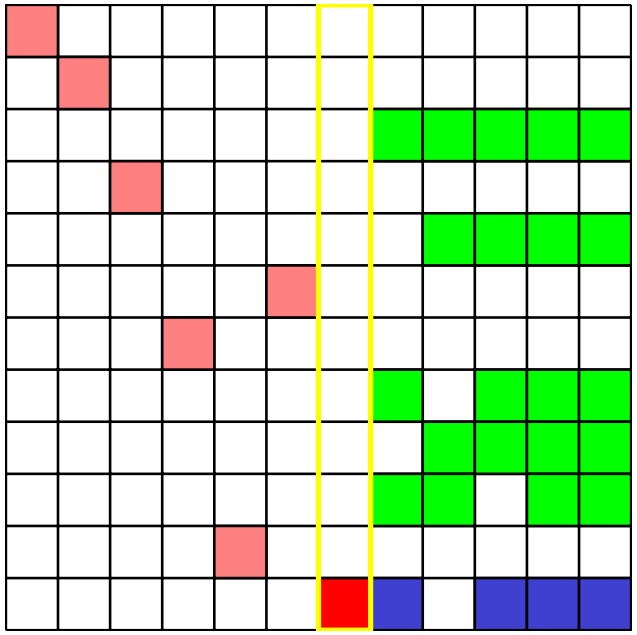

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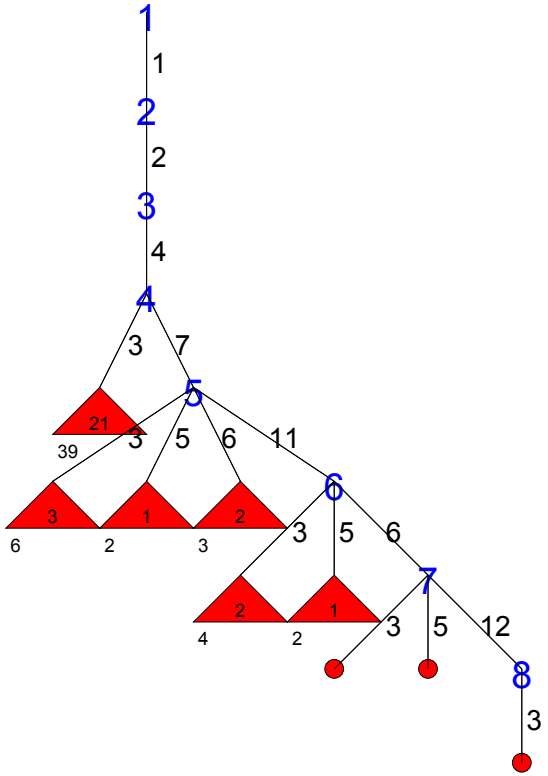

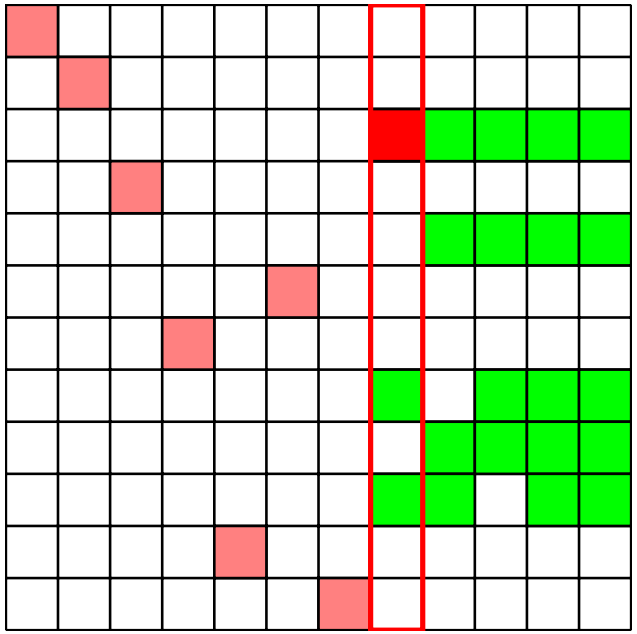

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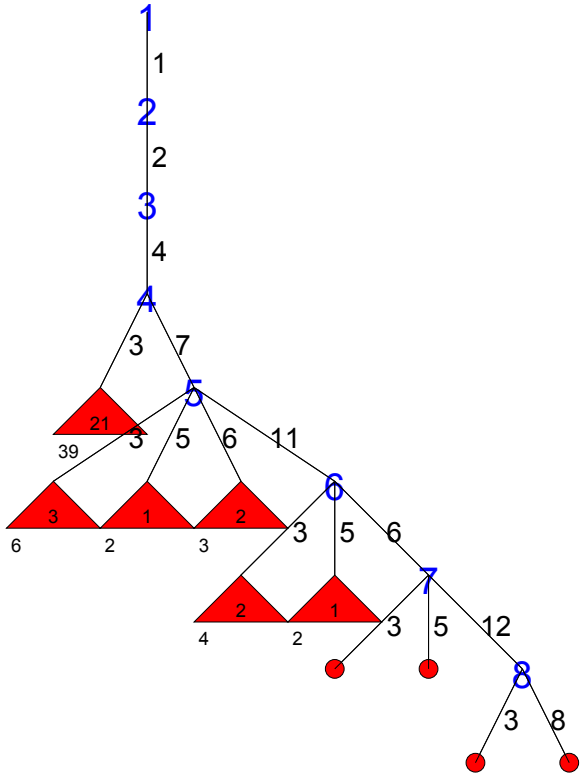

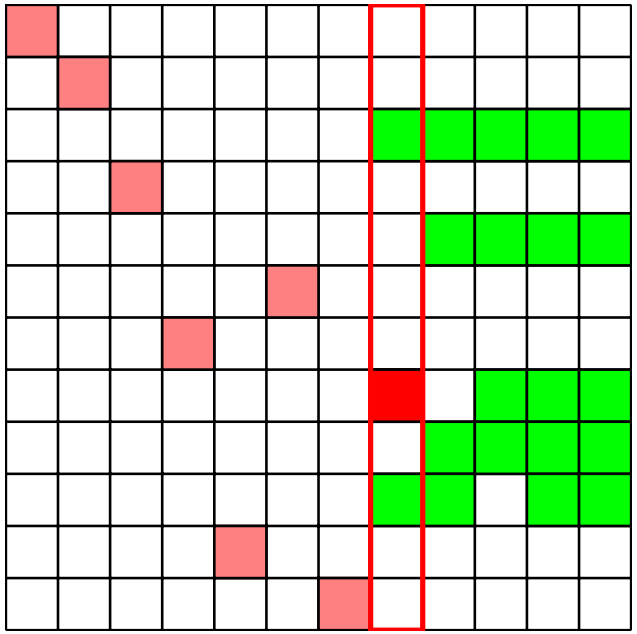

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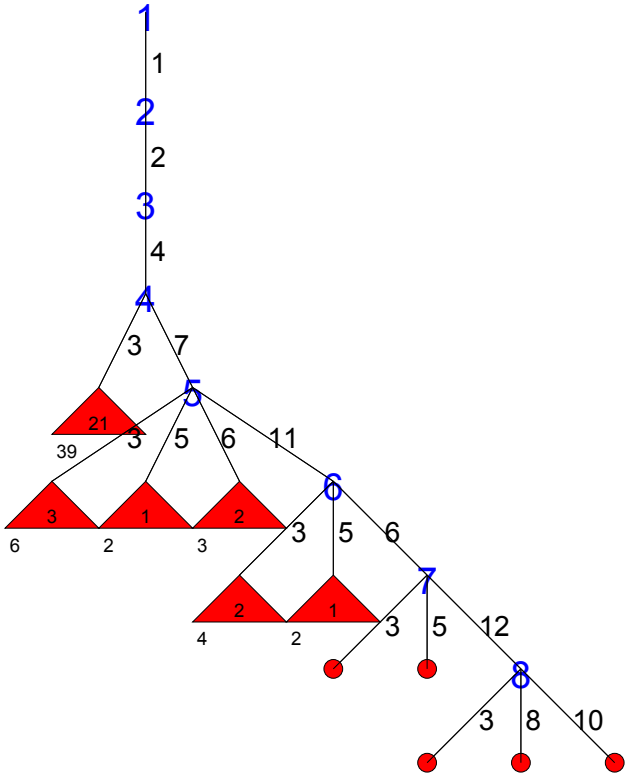

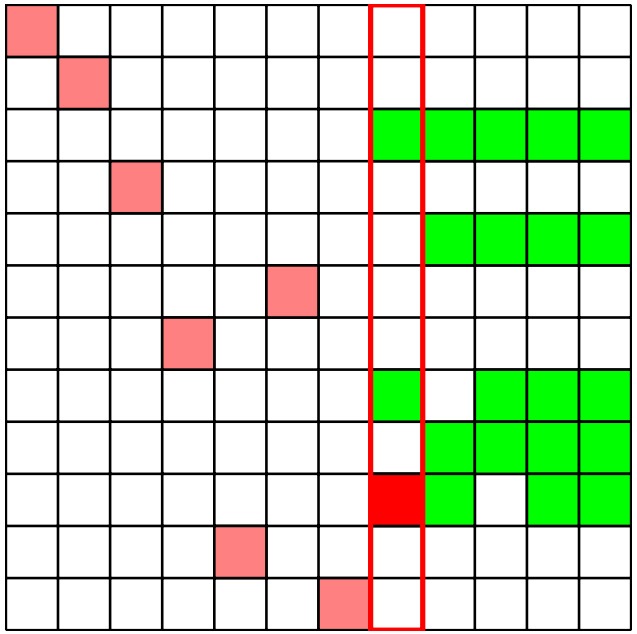

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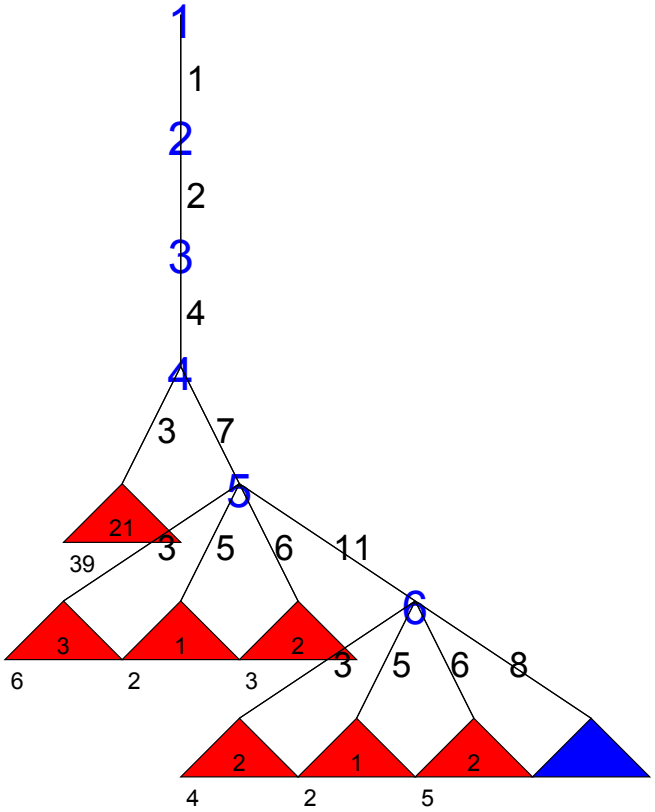

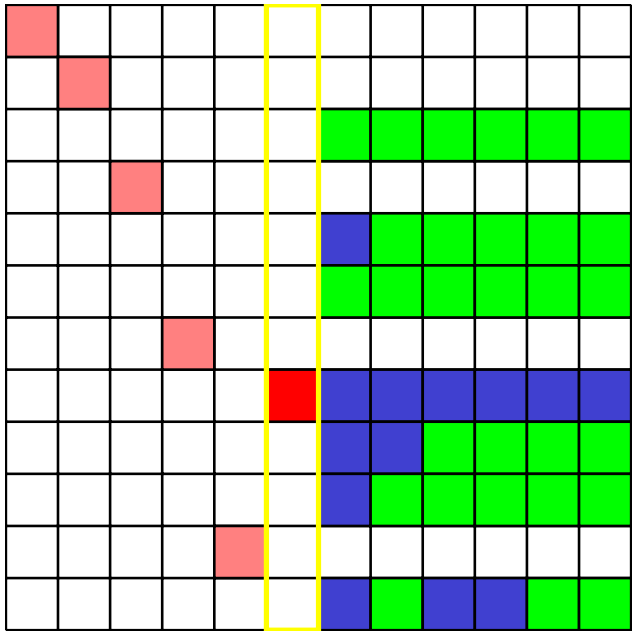

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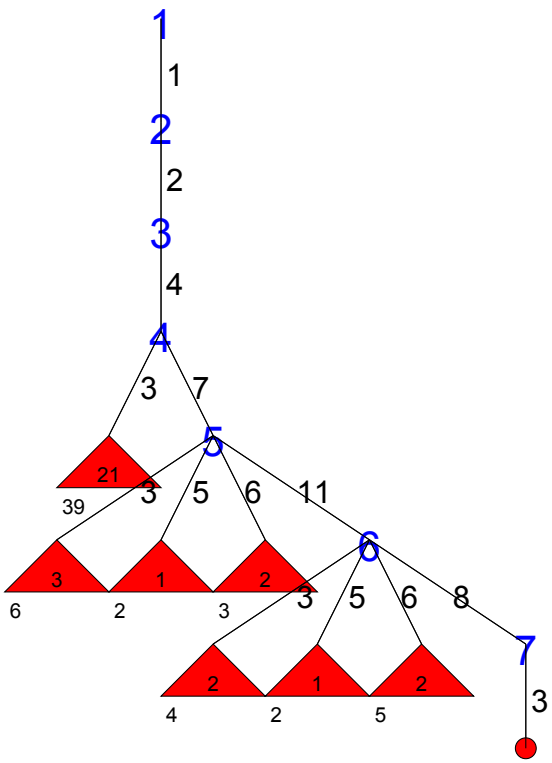

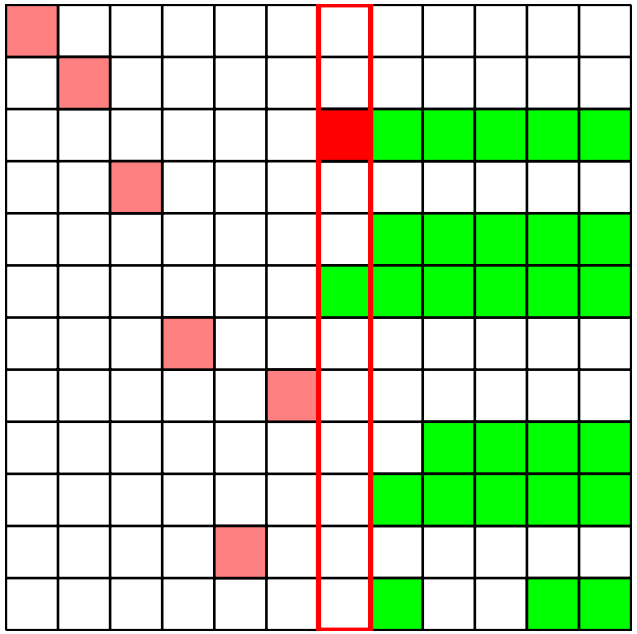

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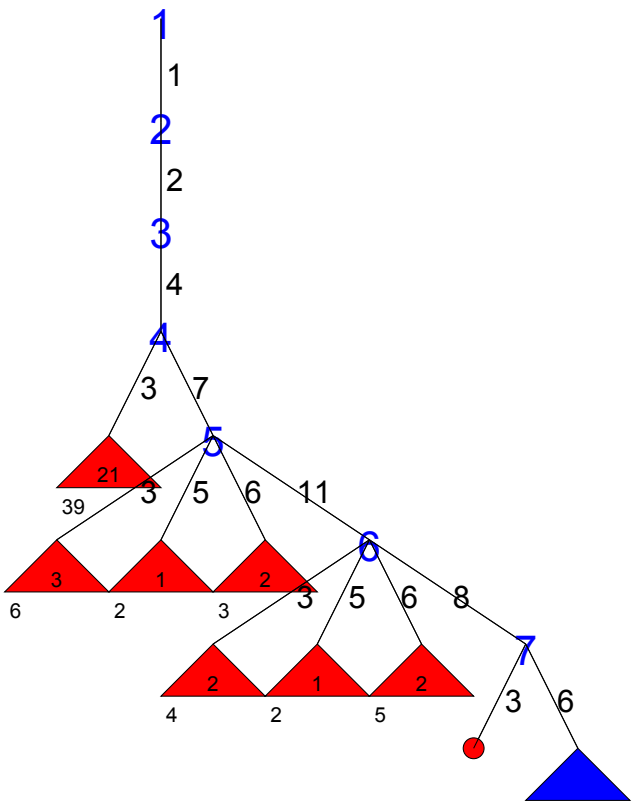

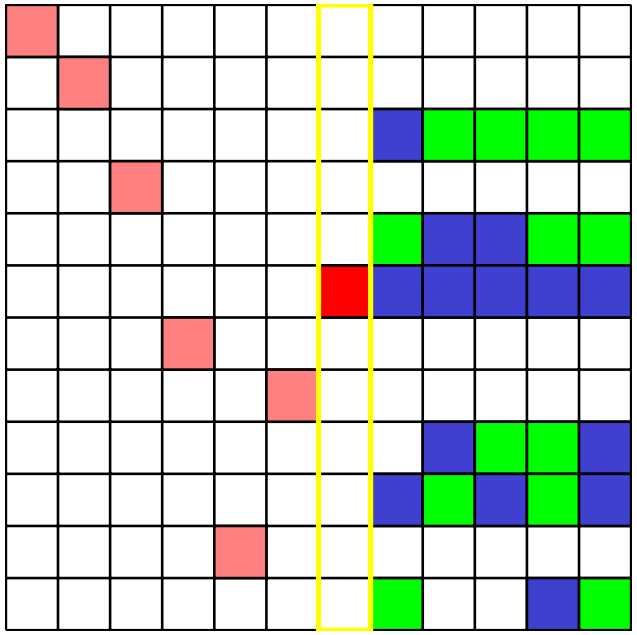

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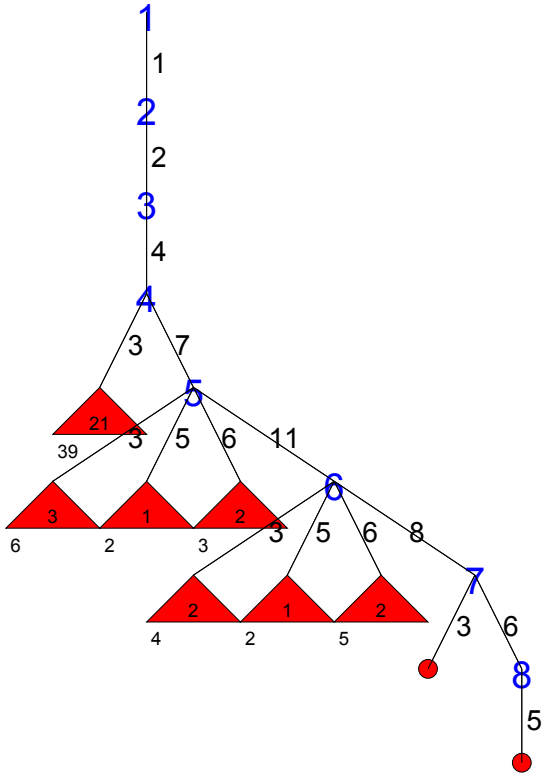

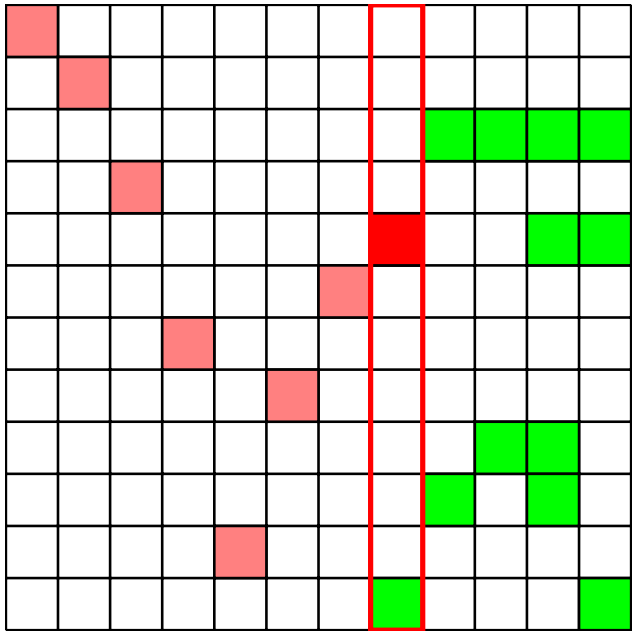

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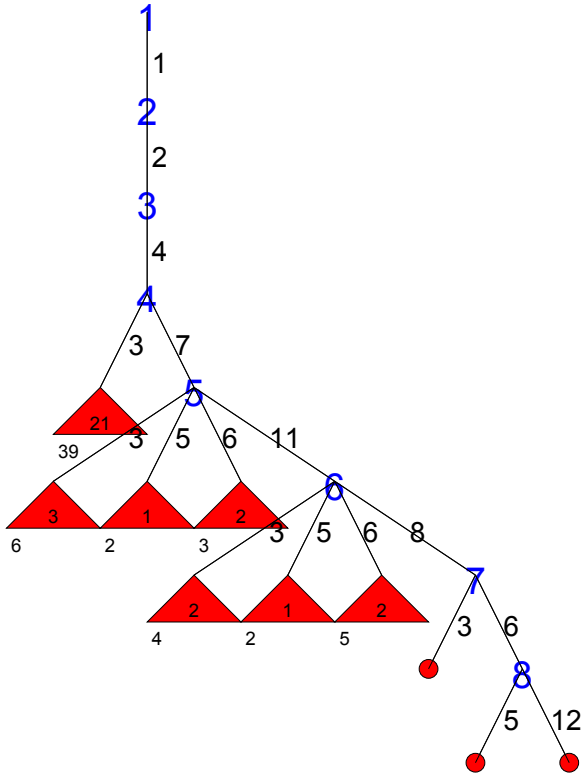

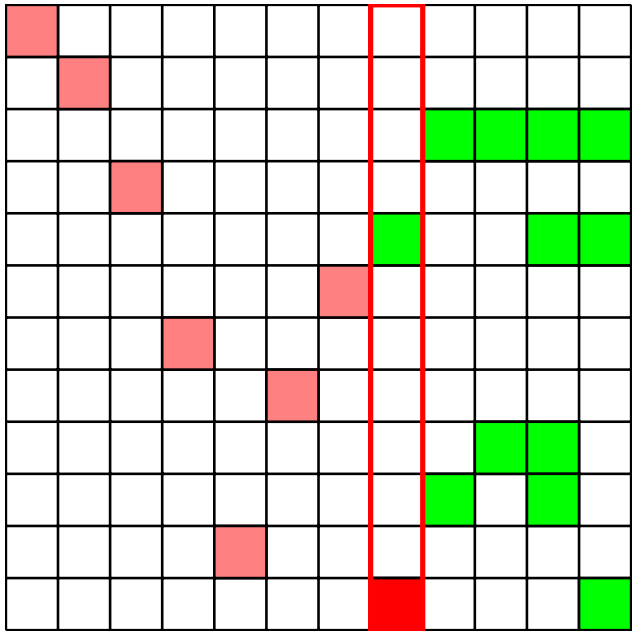

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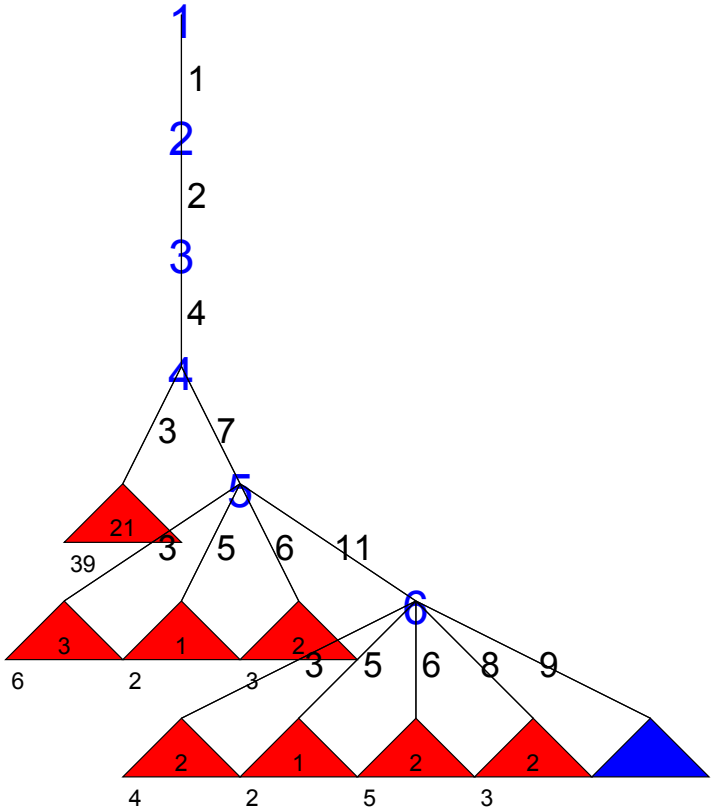

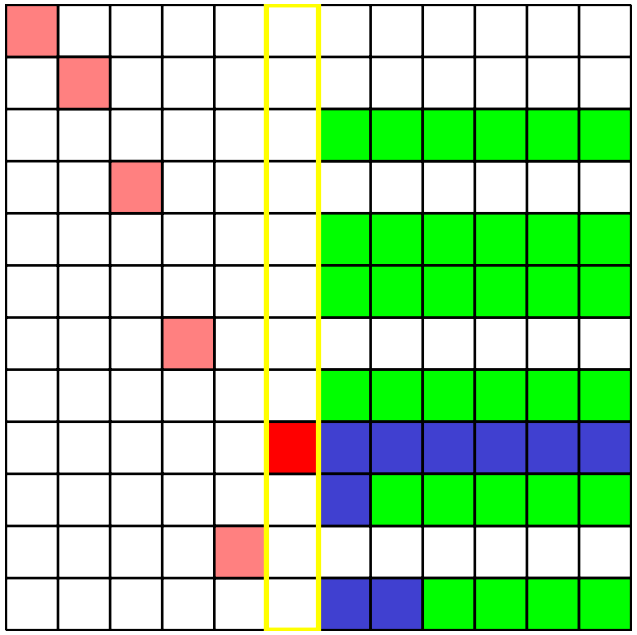

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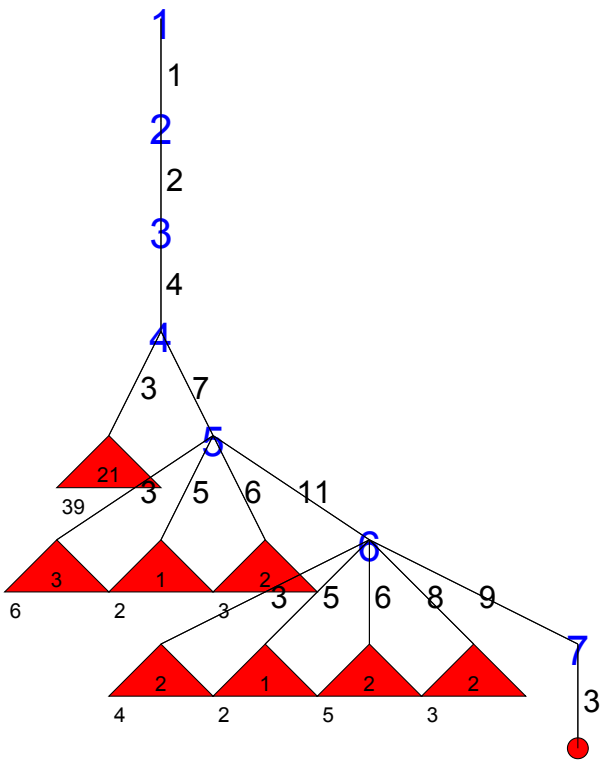

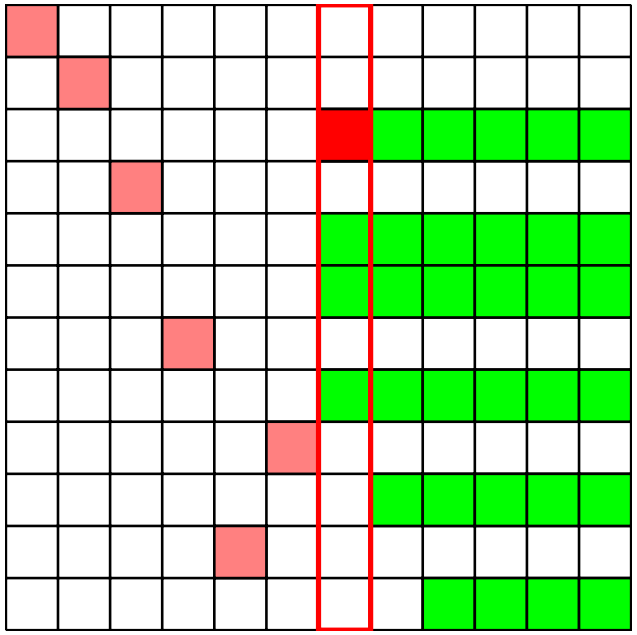

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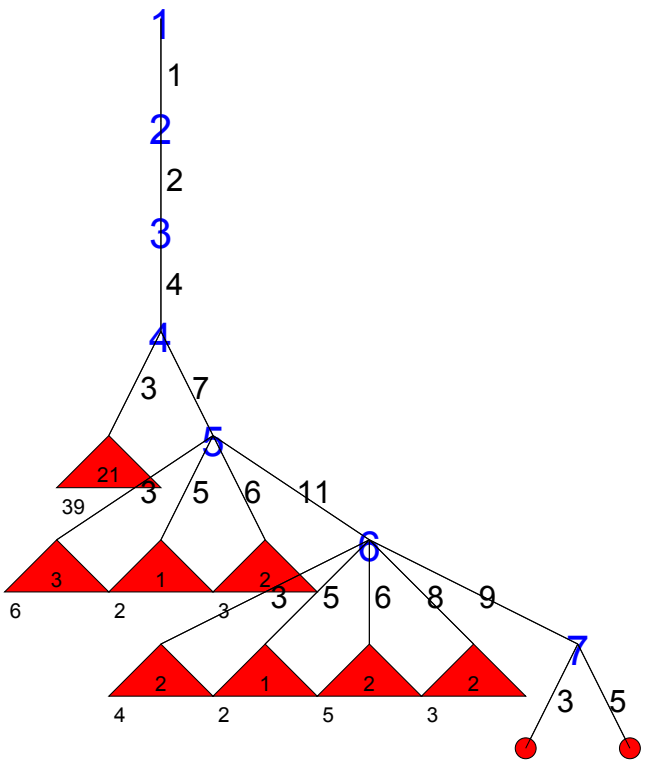

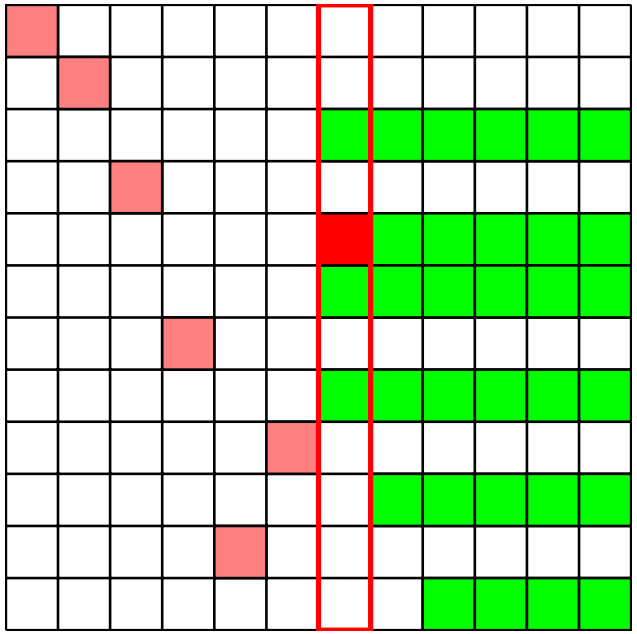

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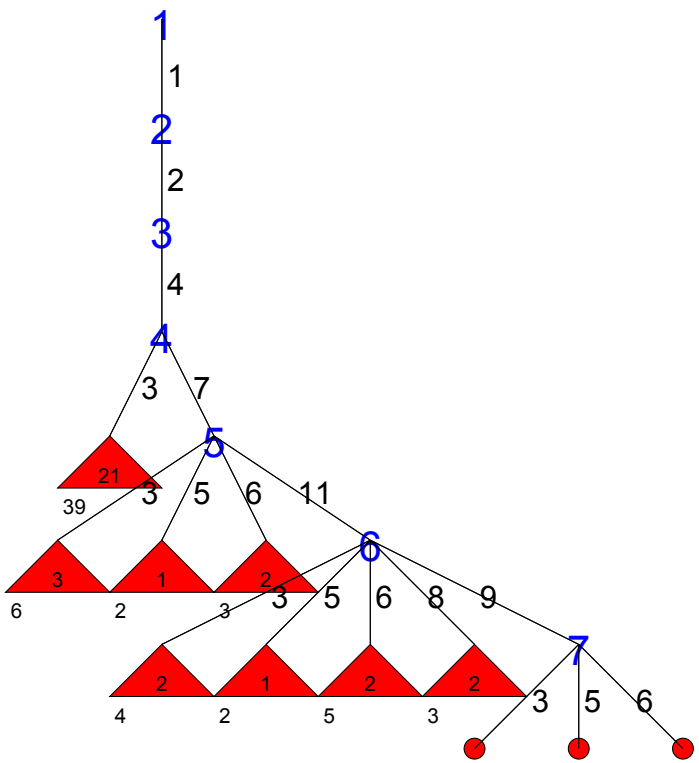

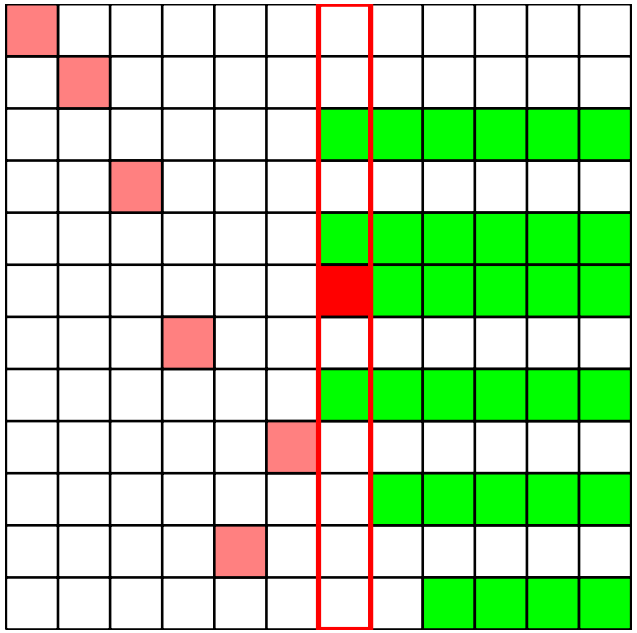

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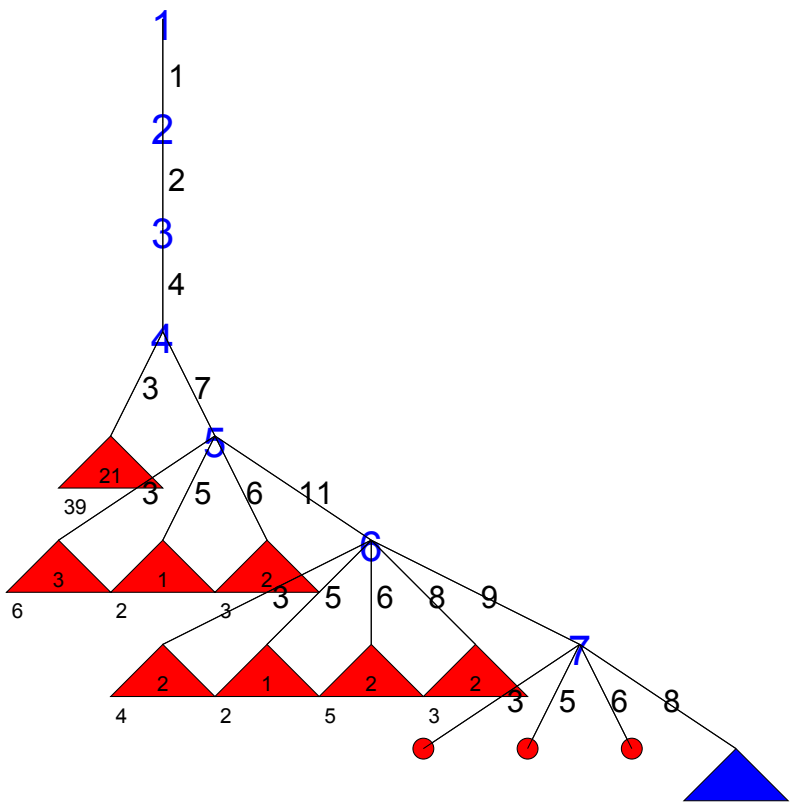

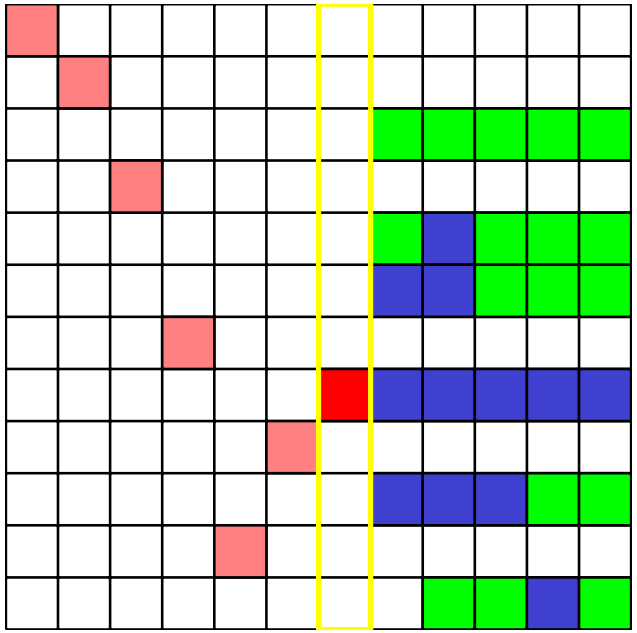

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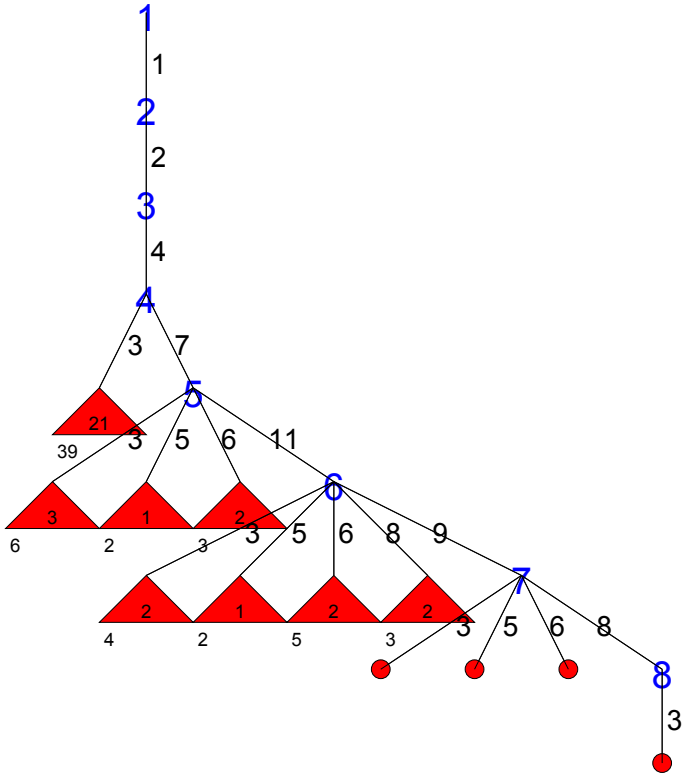

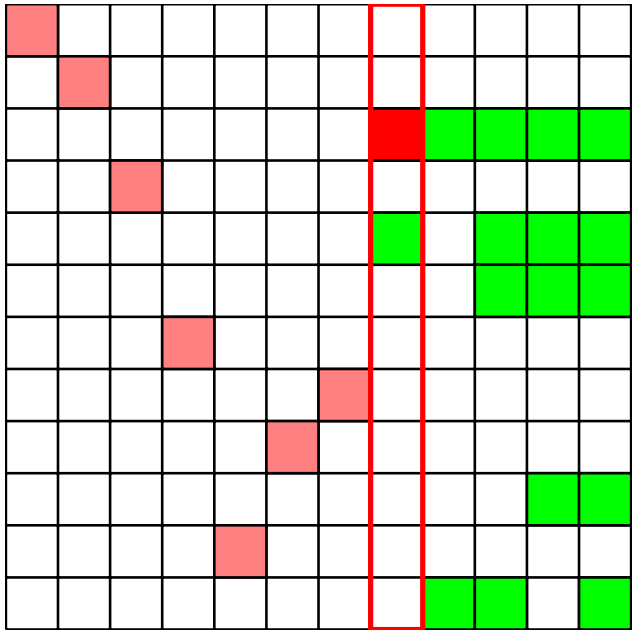

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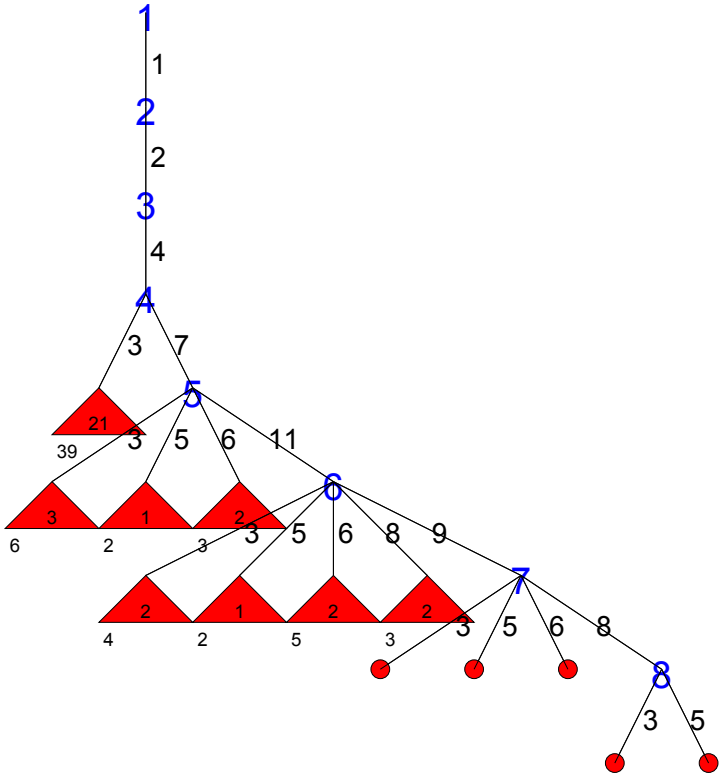

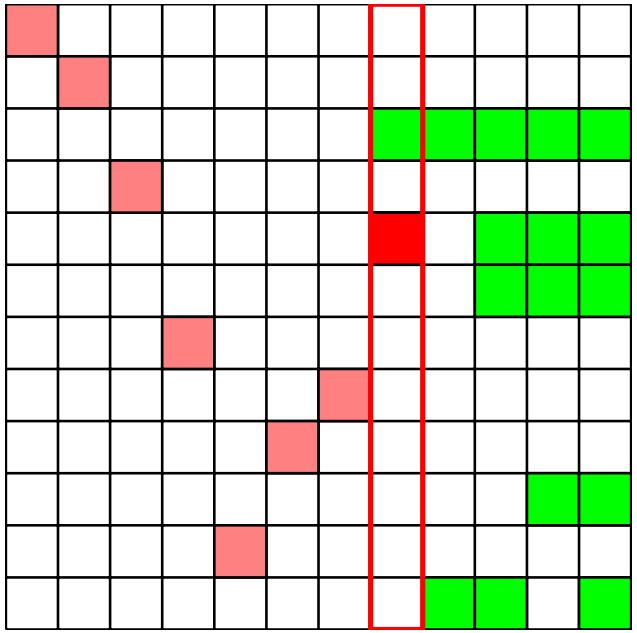

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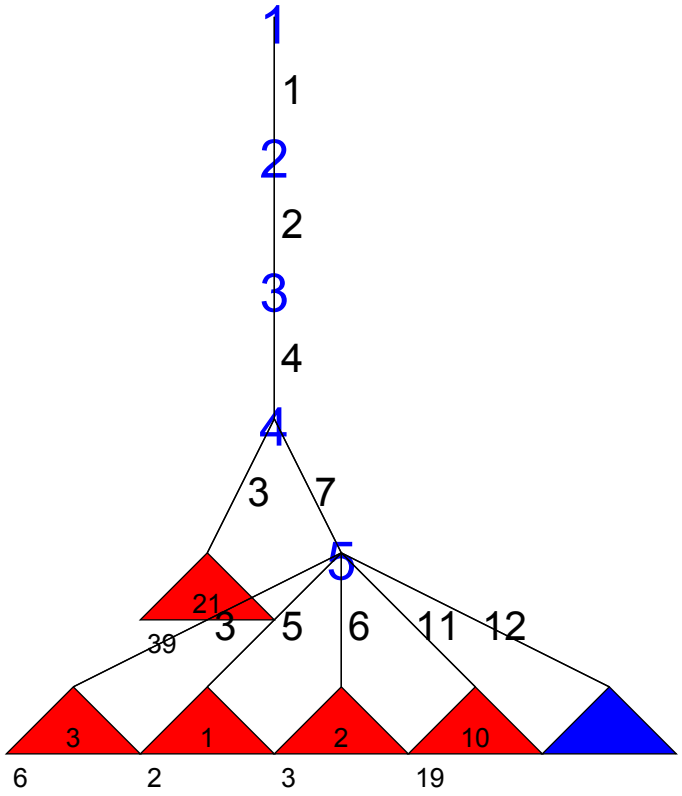

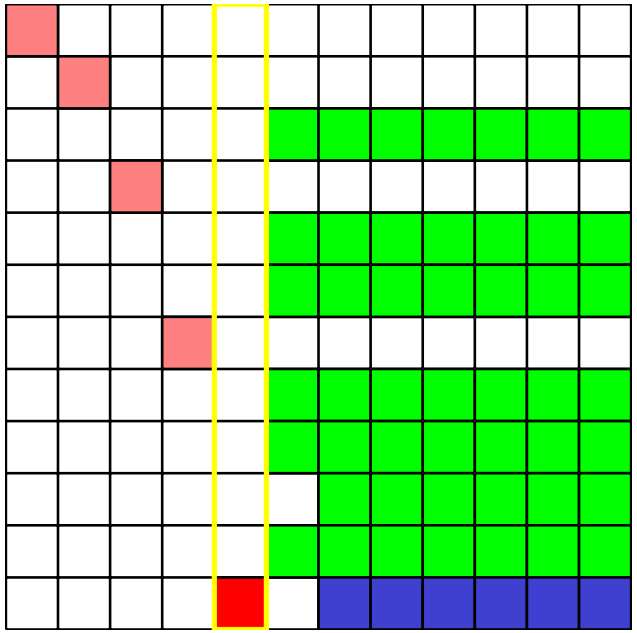

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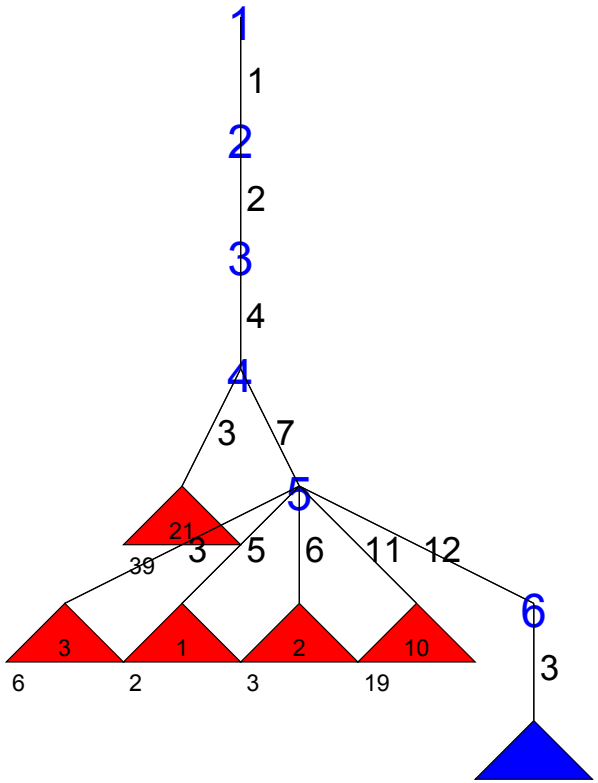

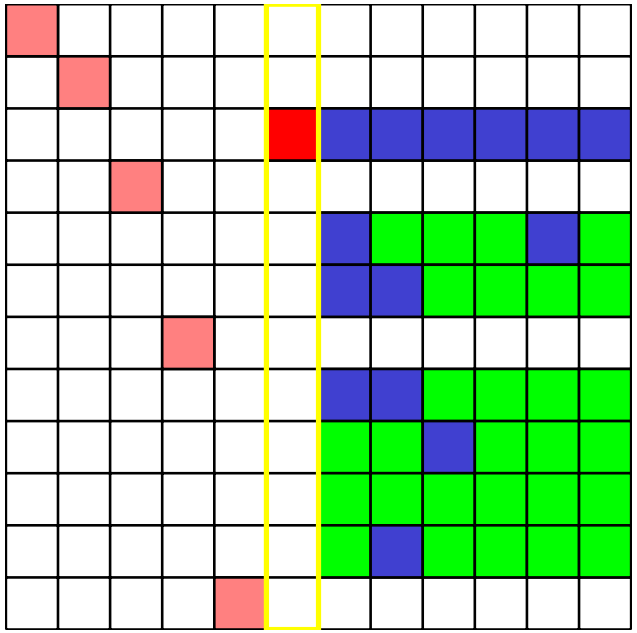

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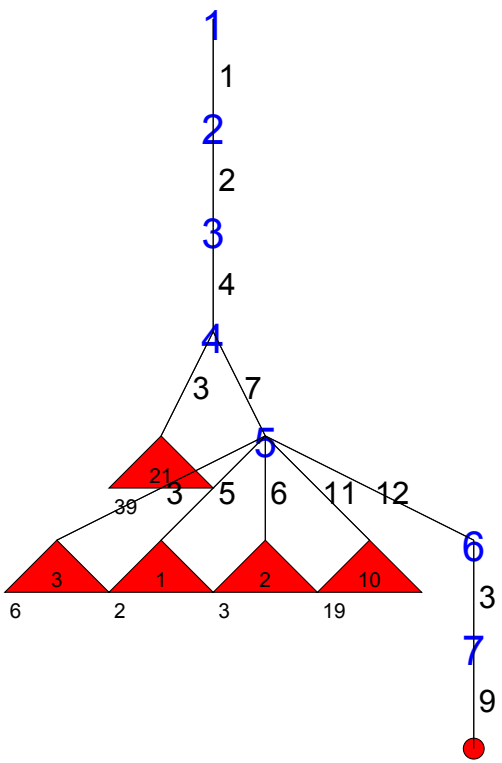

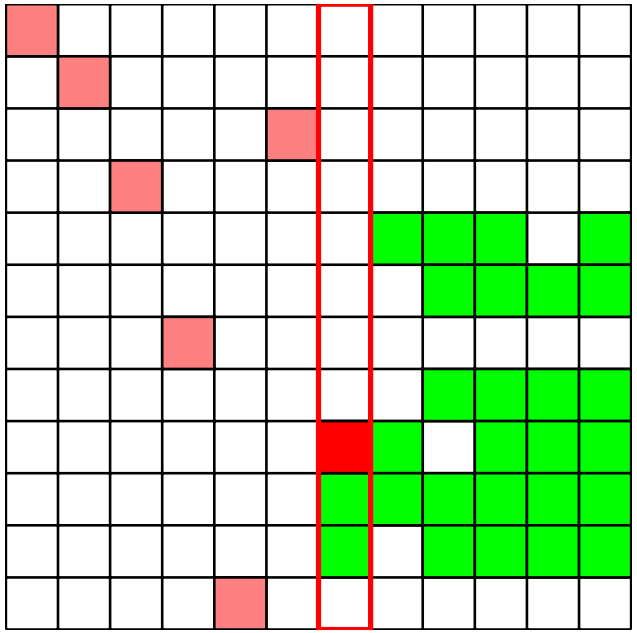

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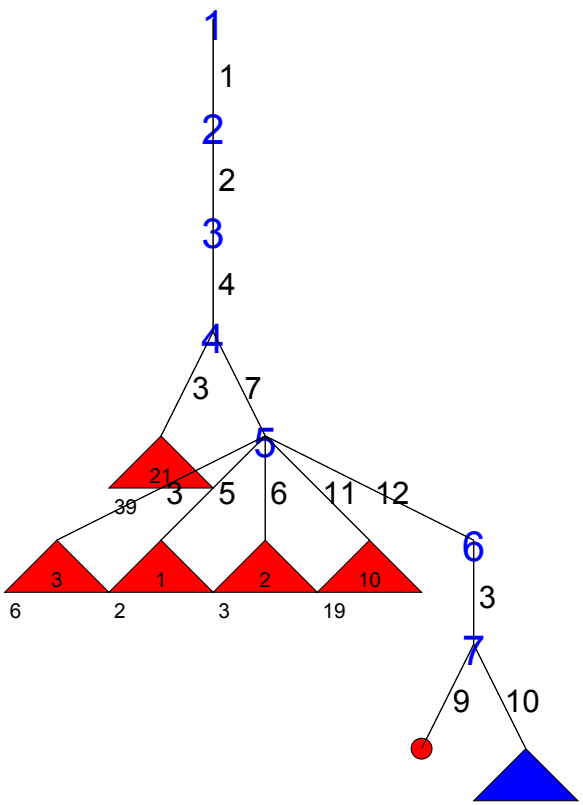

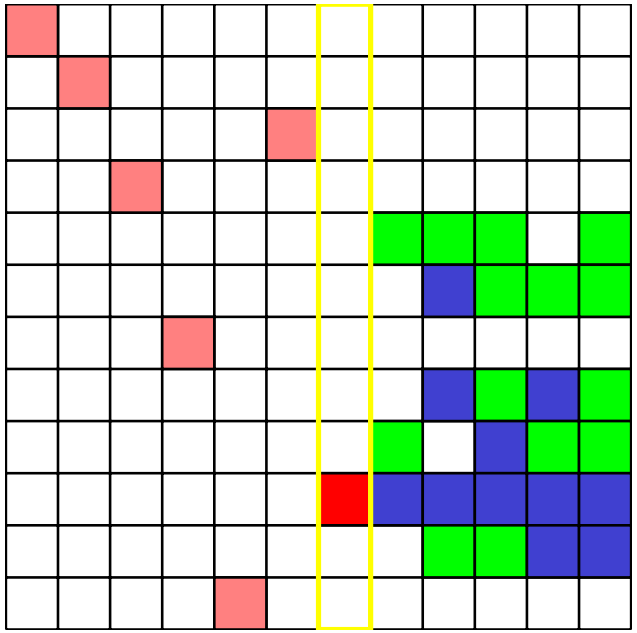

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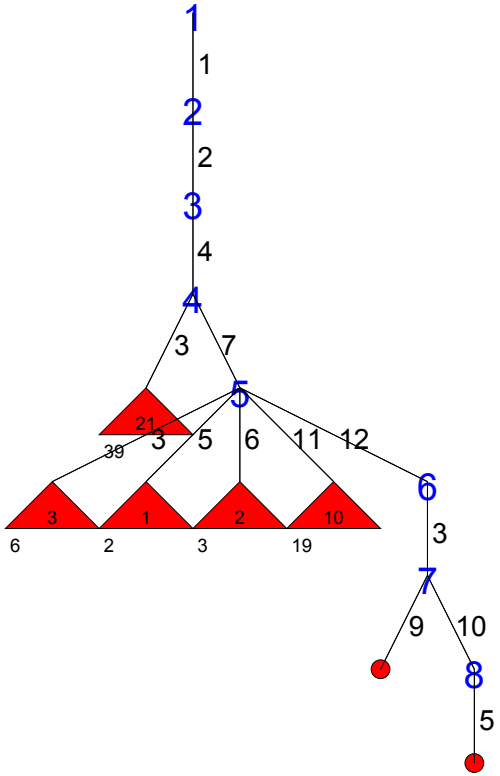

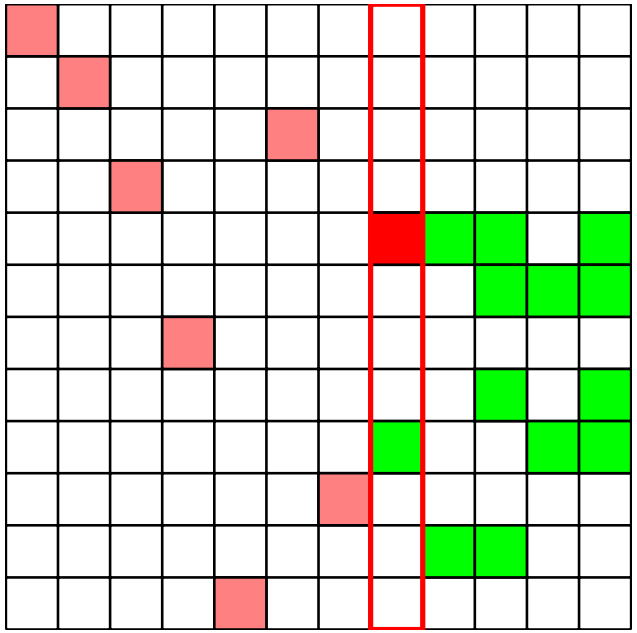

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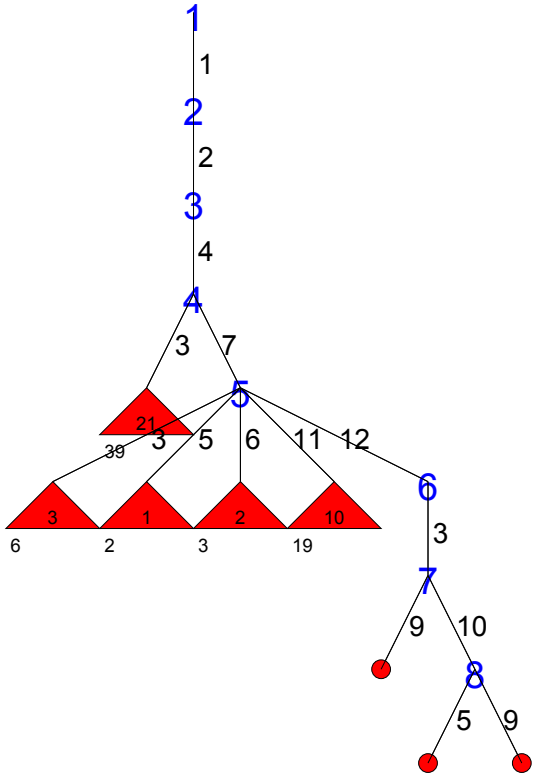

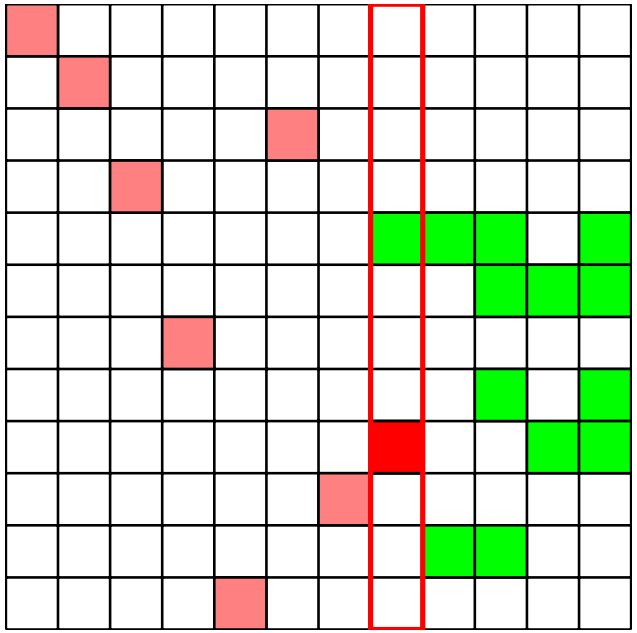

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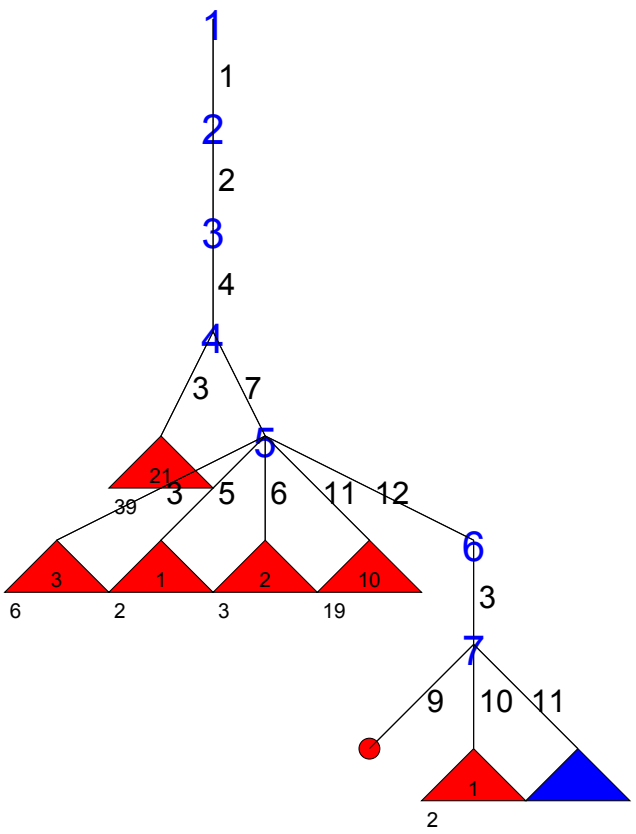

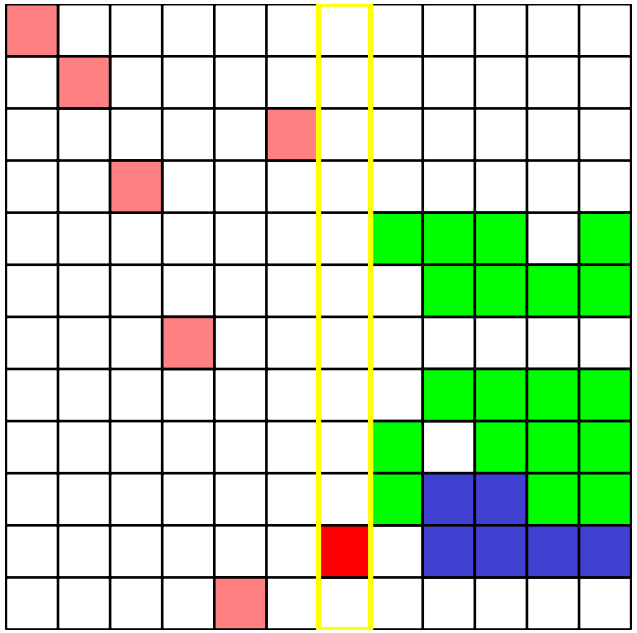

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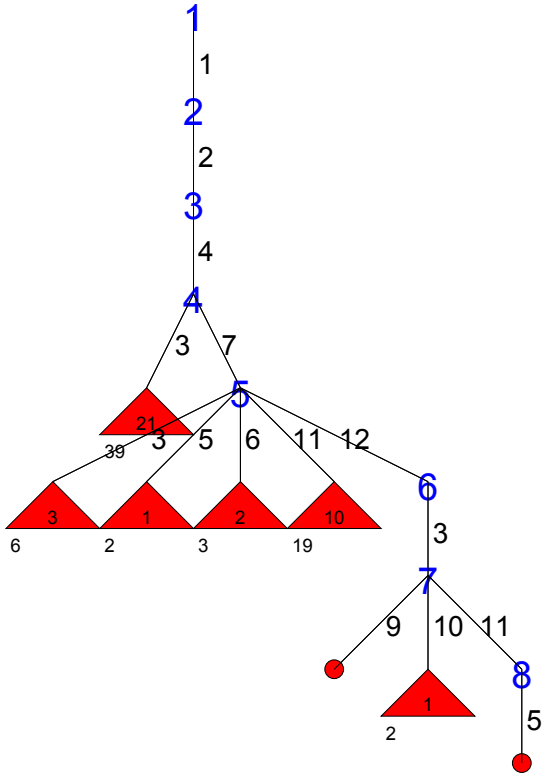

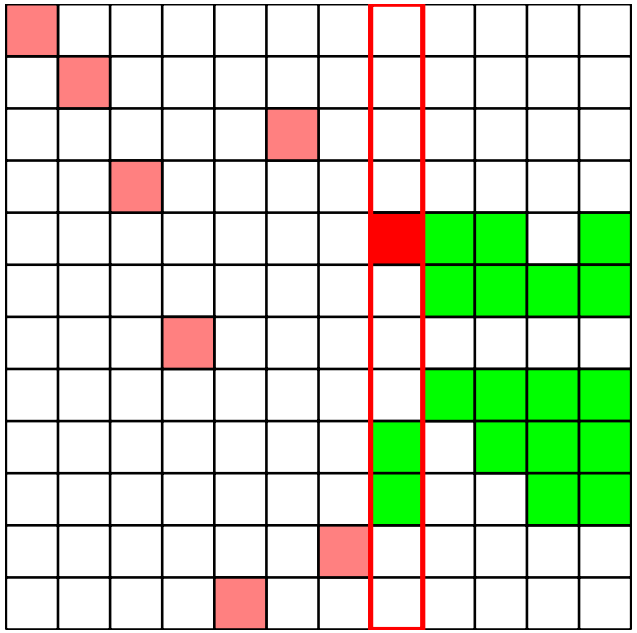

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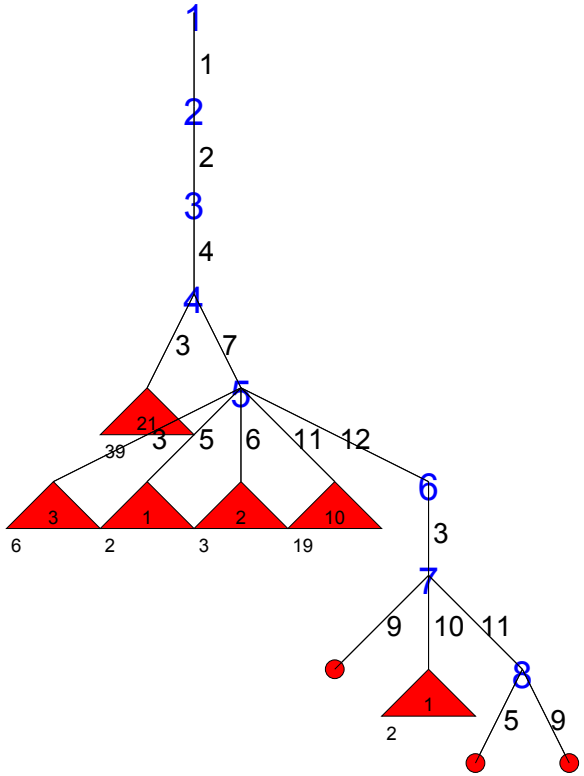

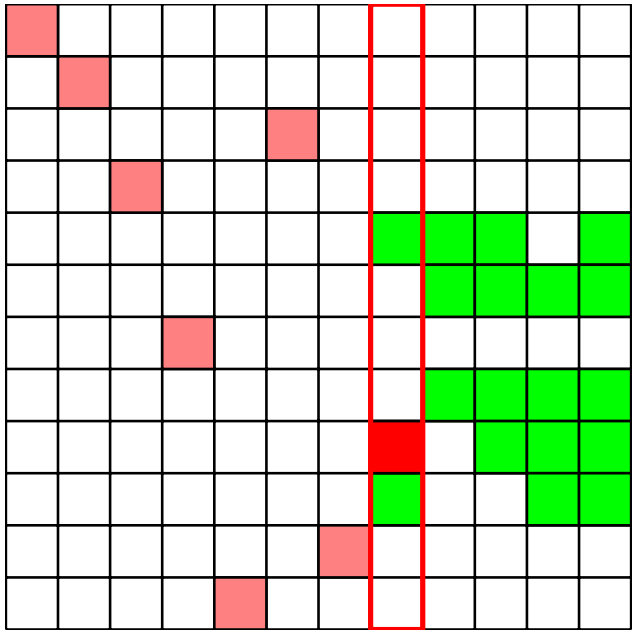

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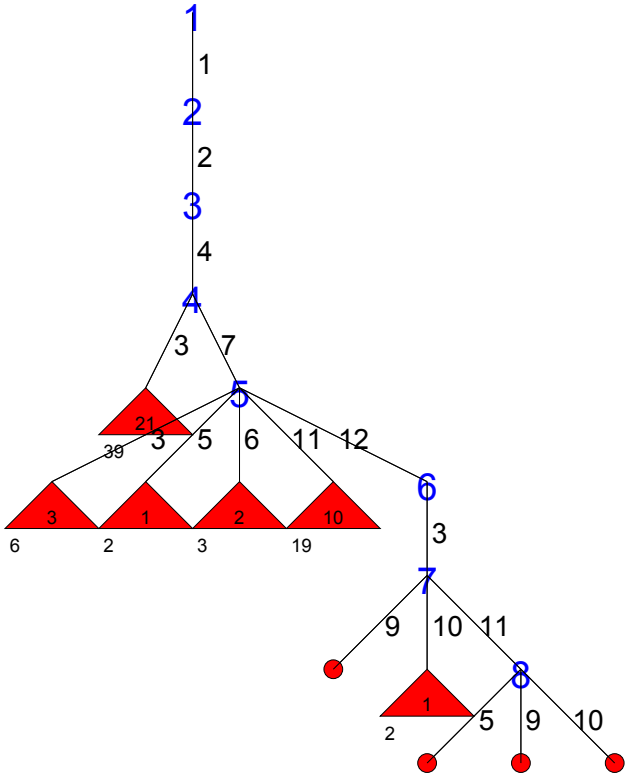

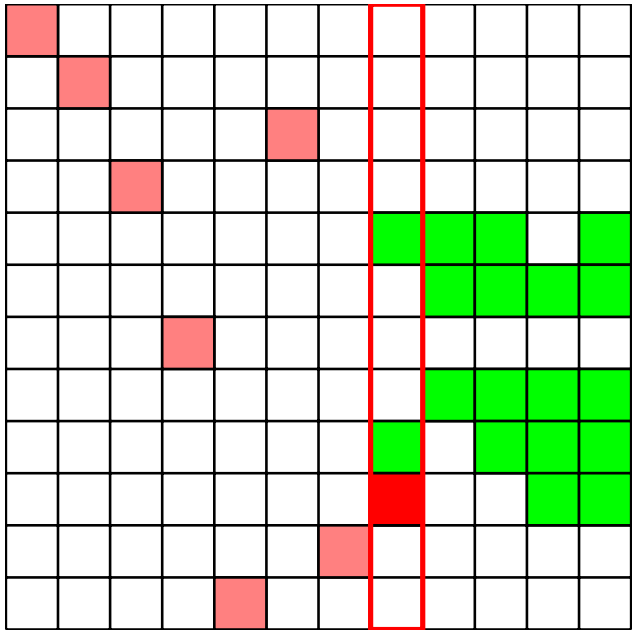

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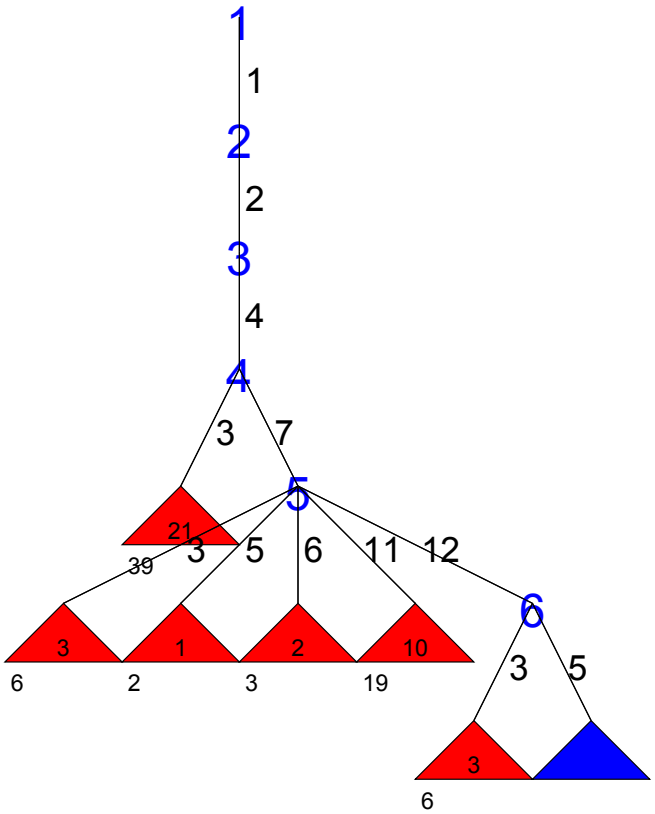

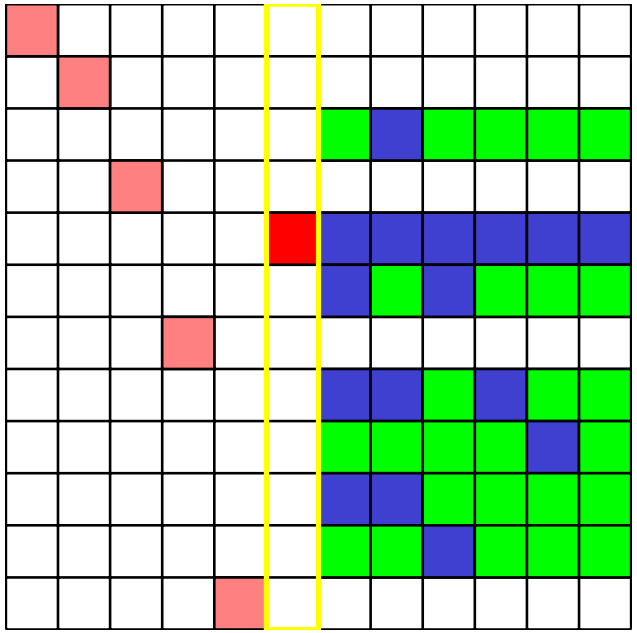

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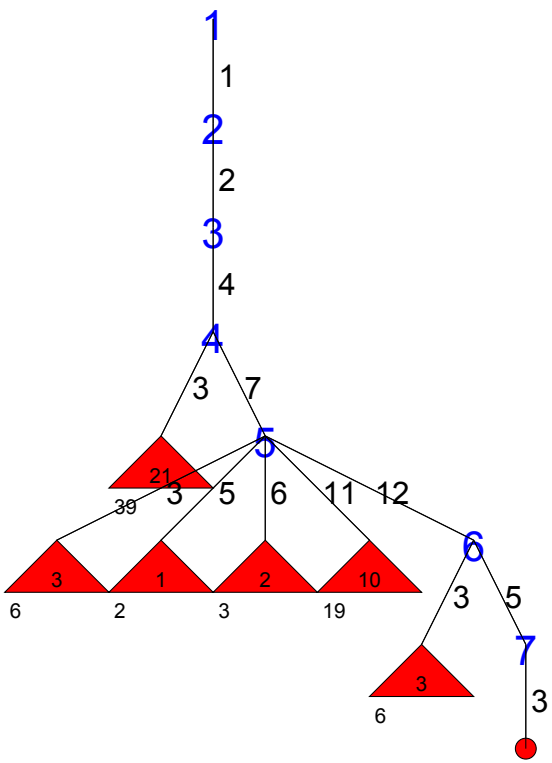

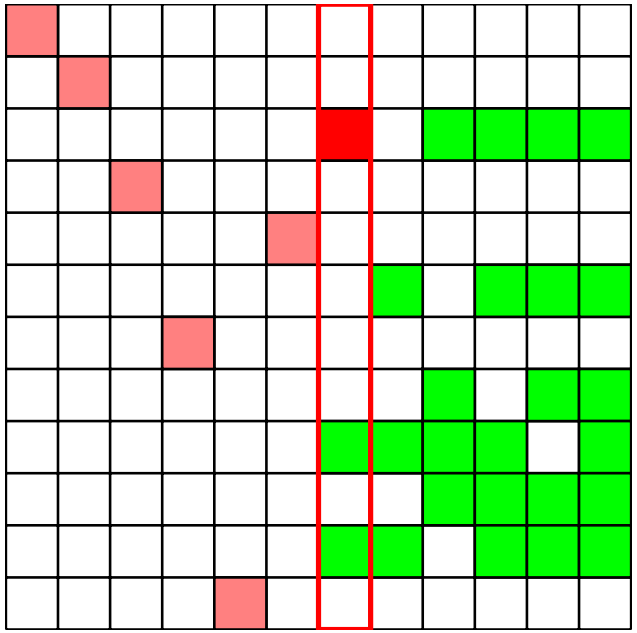

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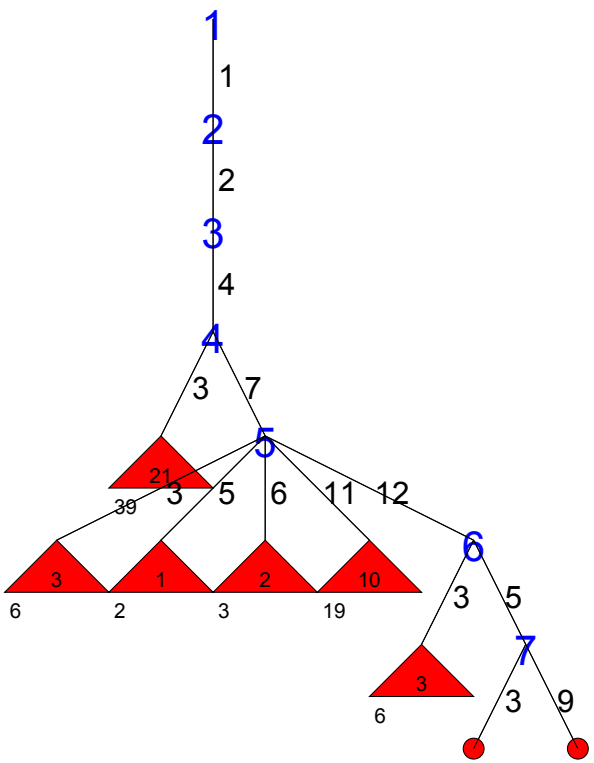

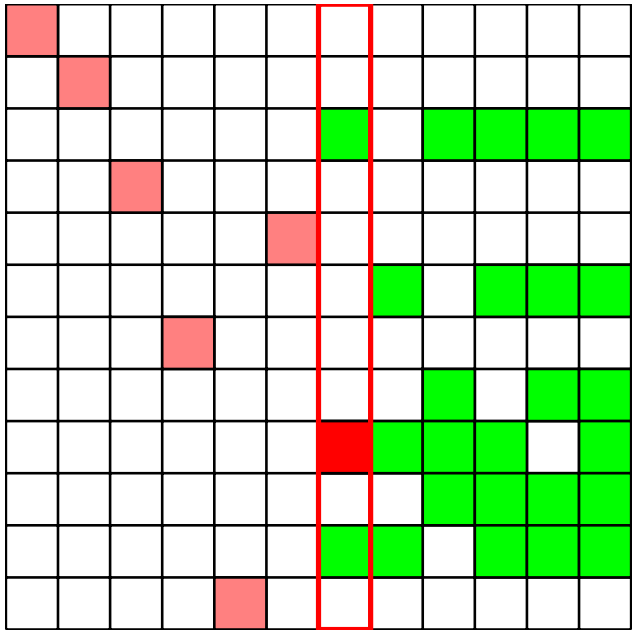

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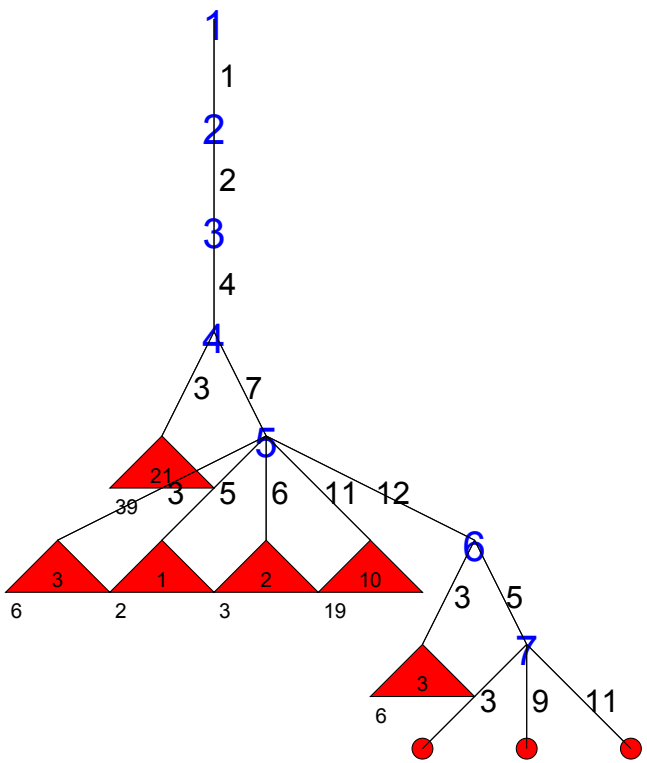

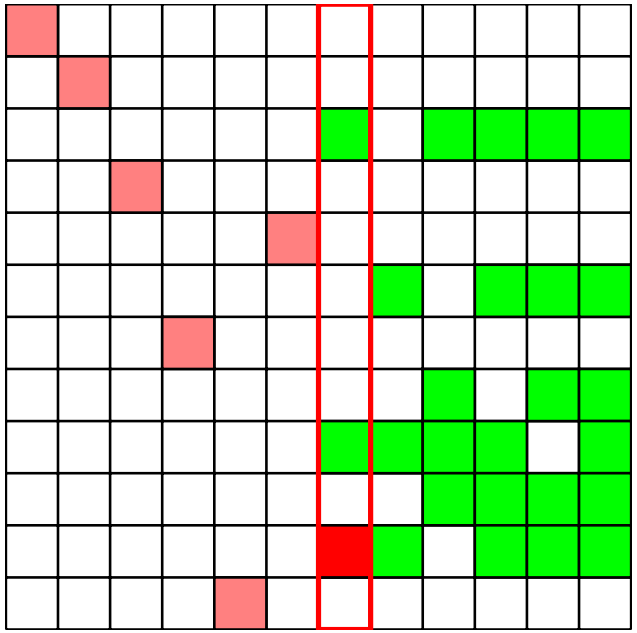

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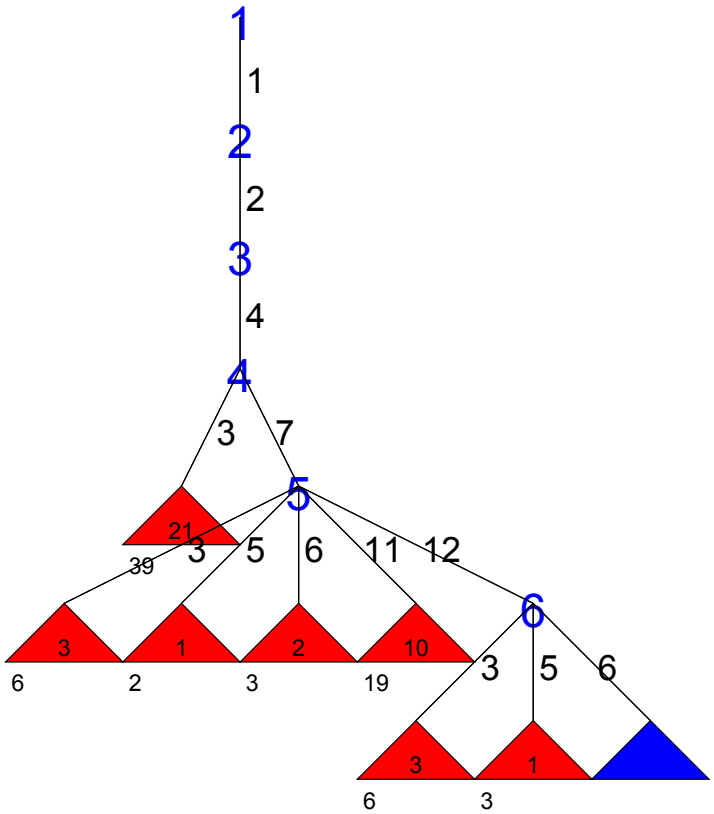

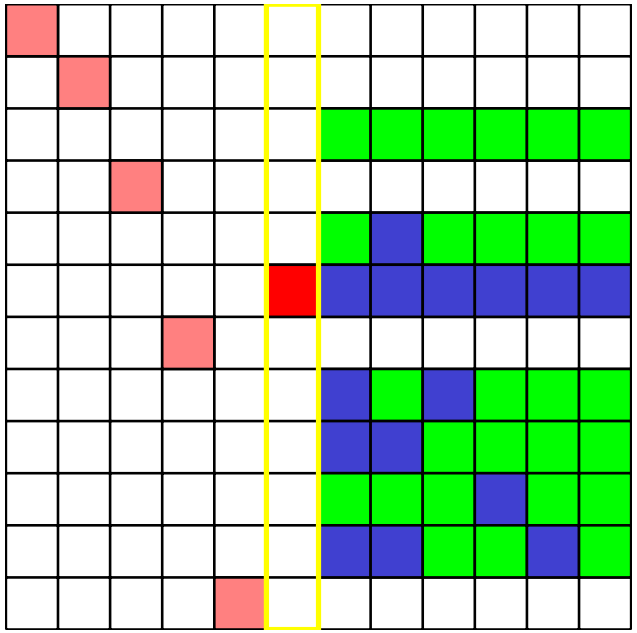

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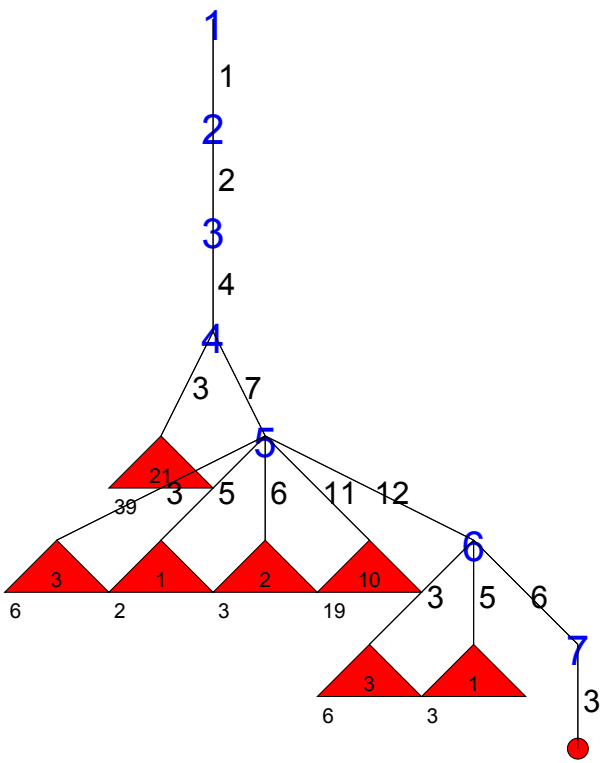

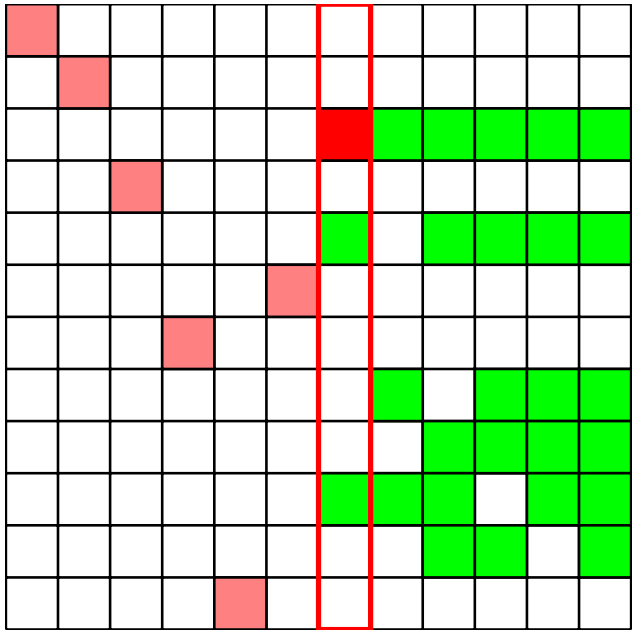

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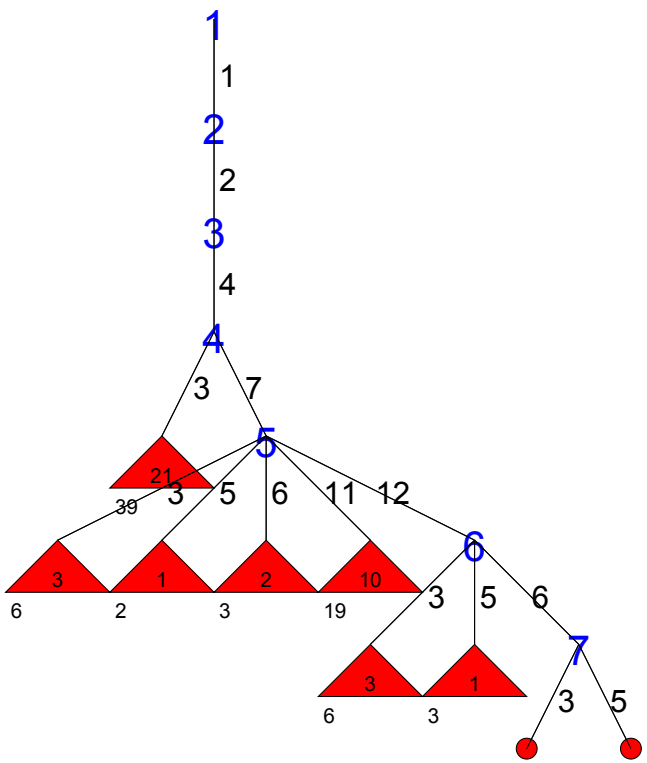

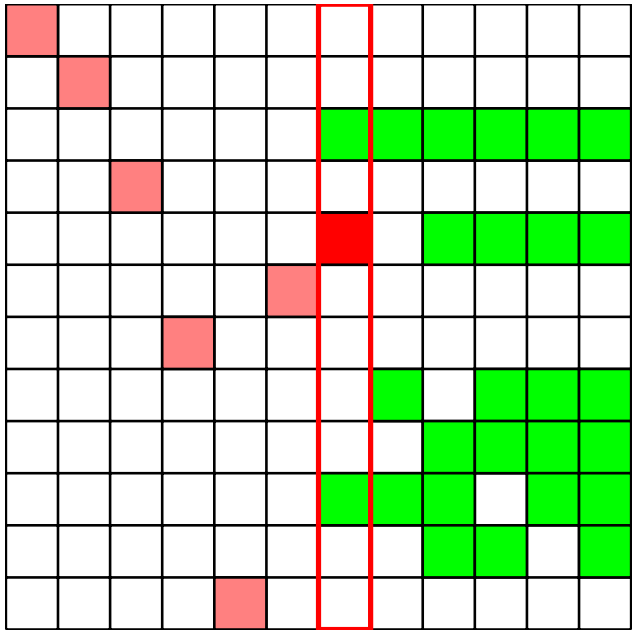

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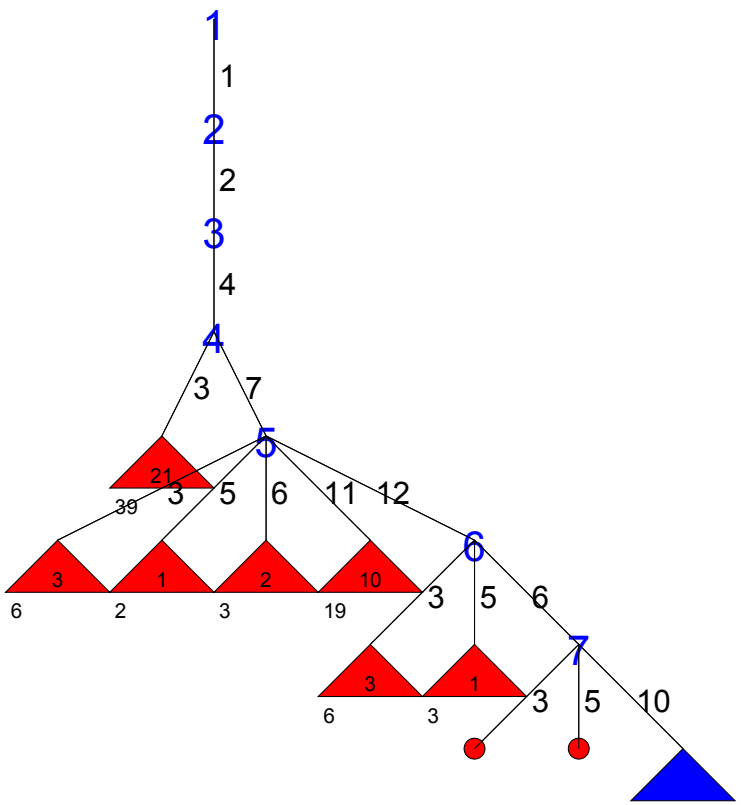

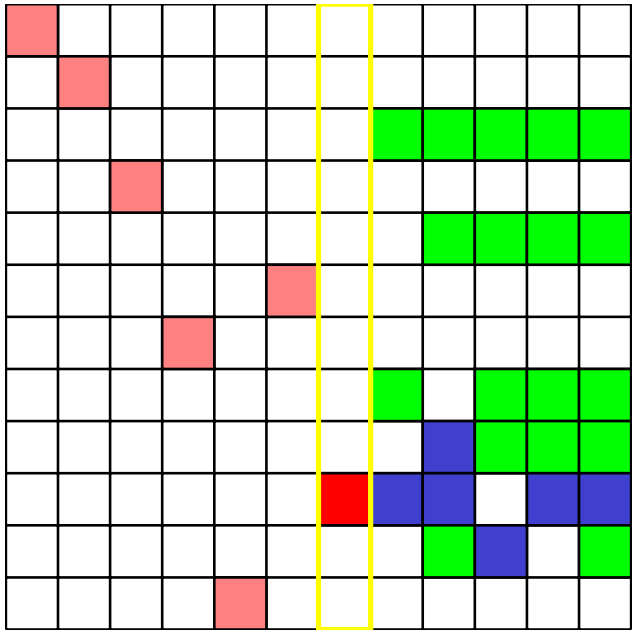

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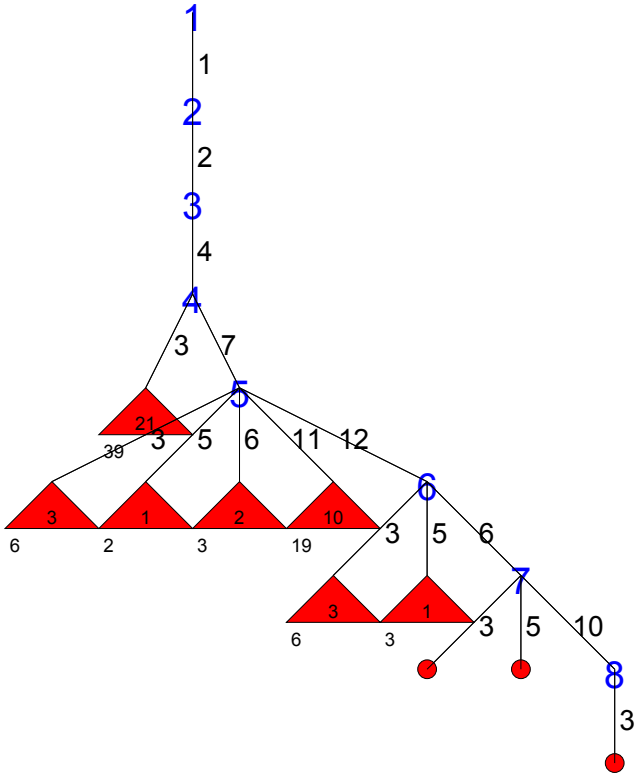

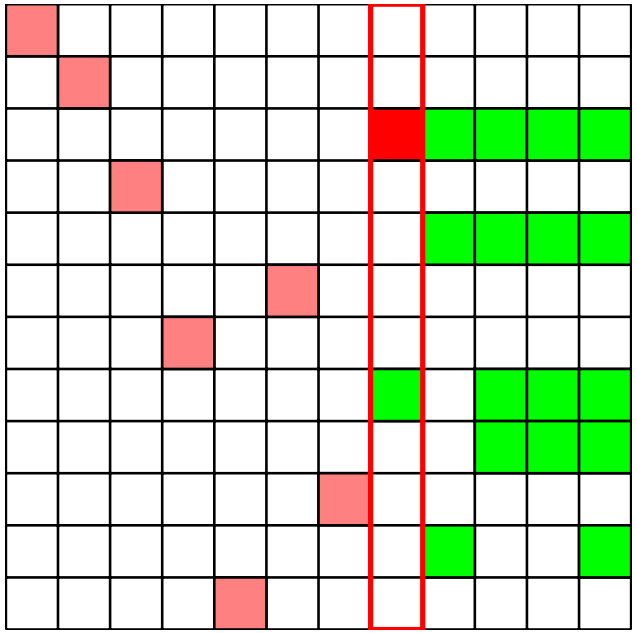

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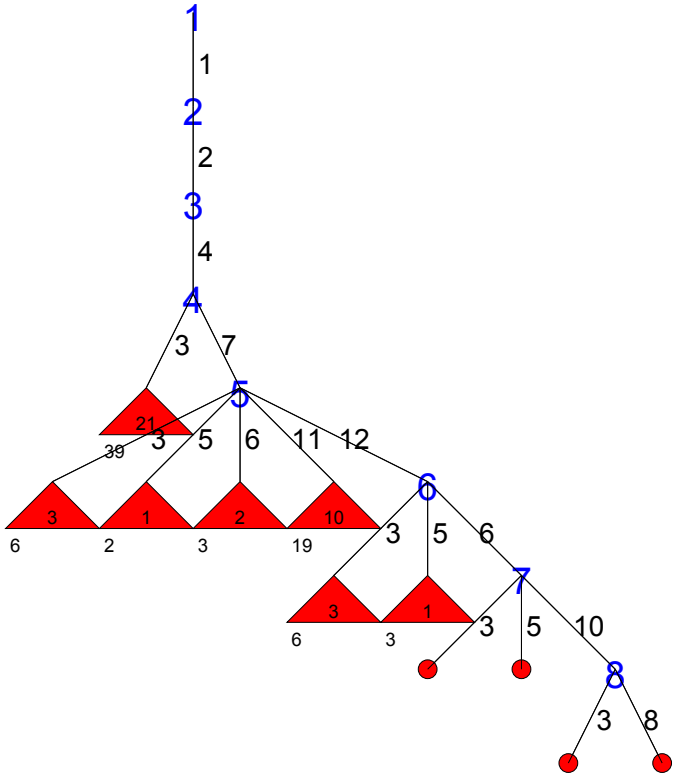

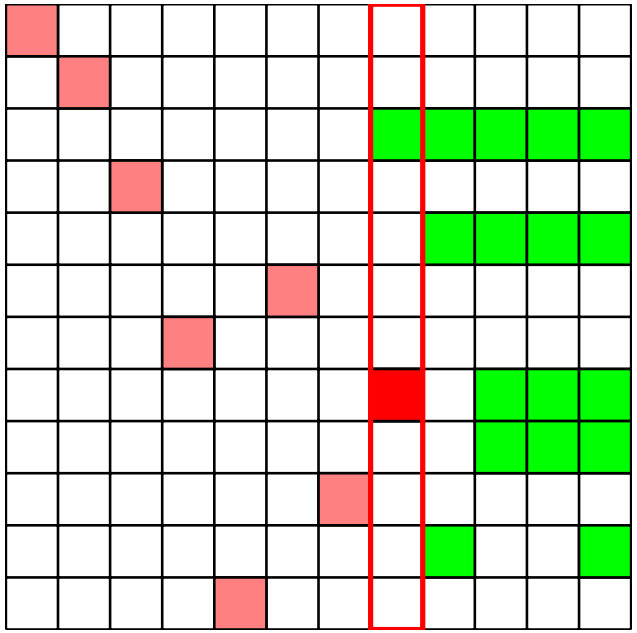

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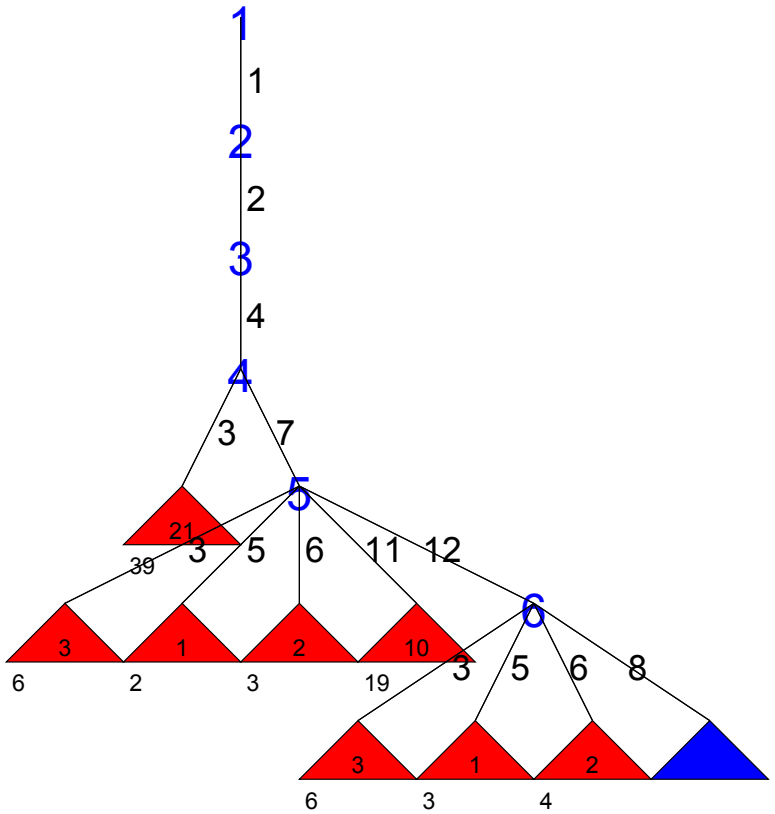
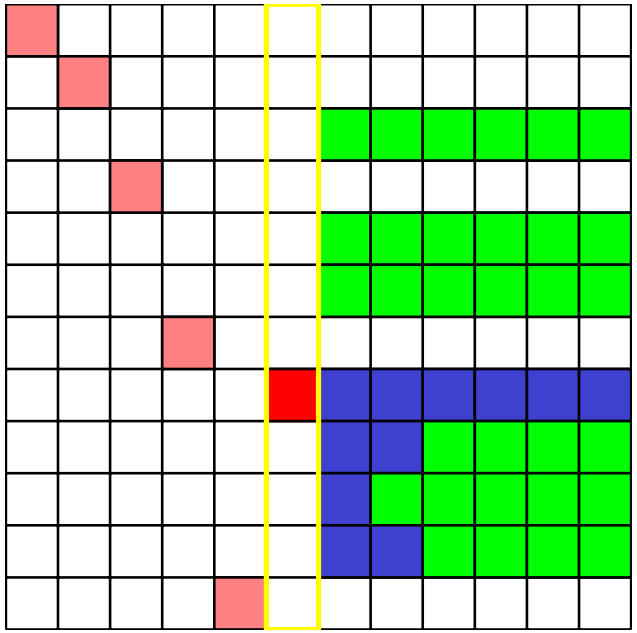
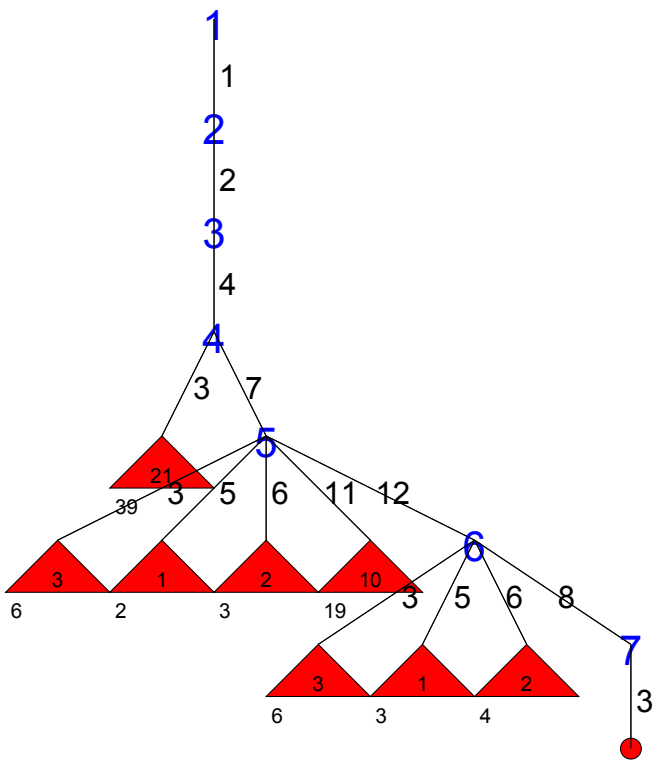
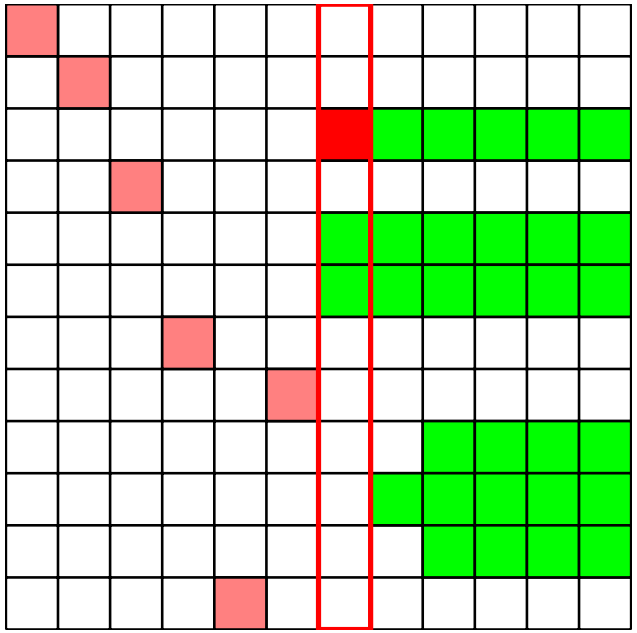




Back to Start
Skip Animation

Improved Model





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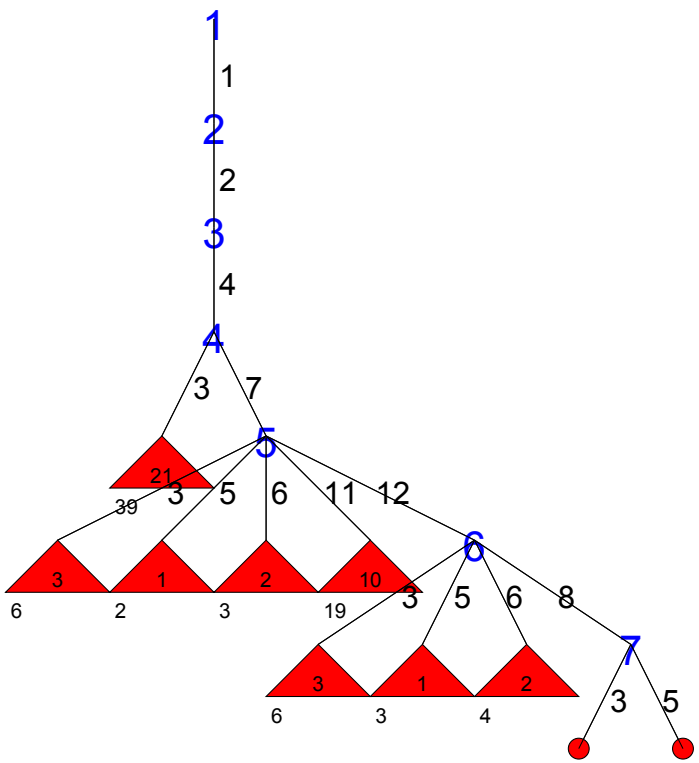

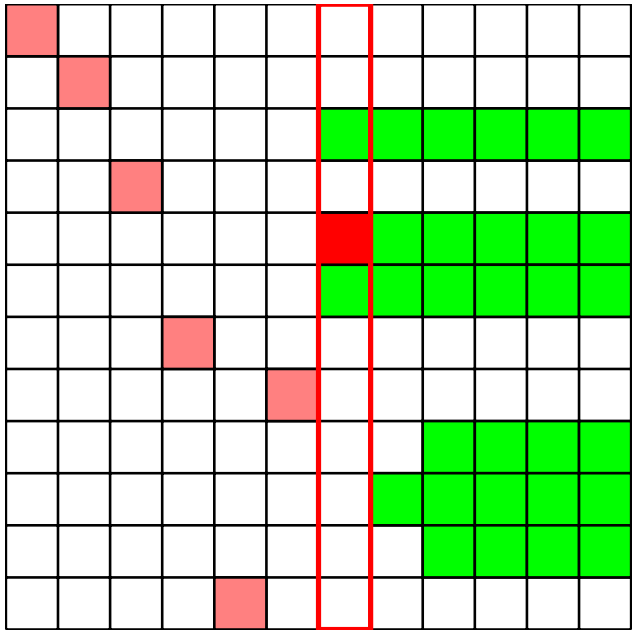

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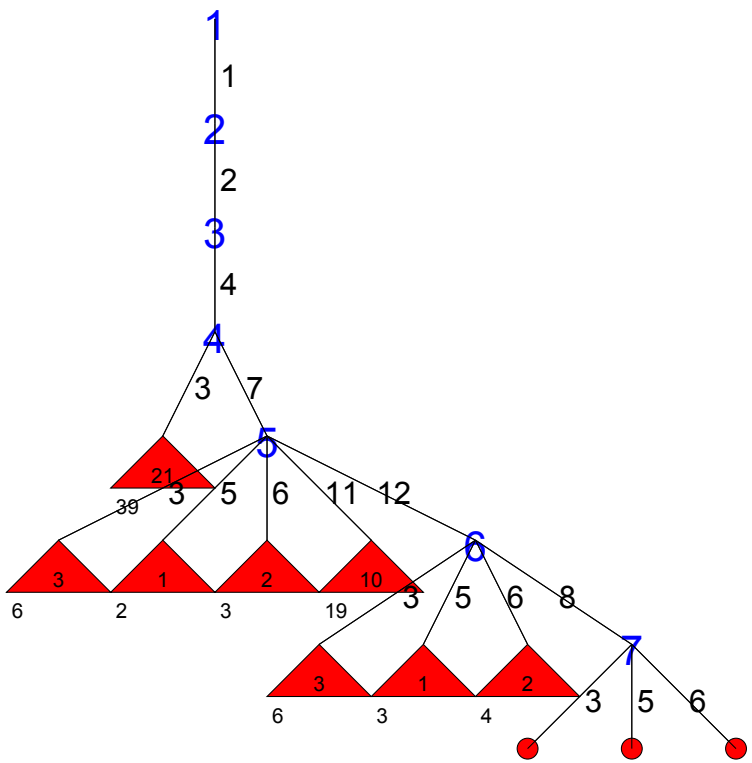

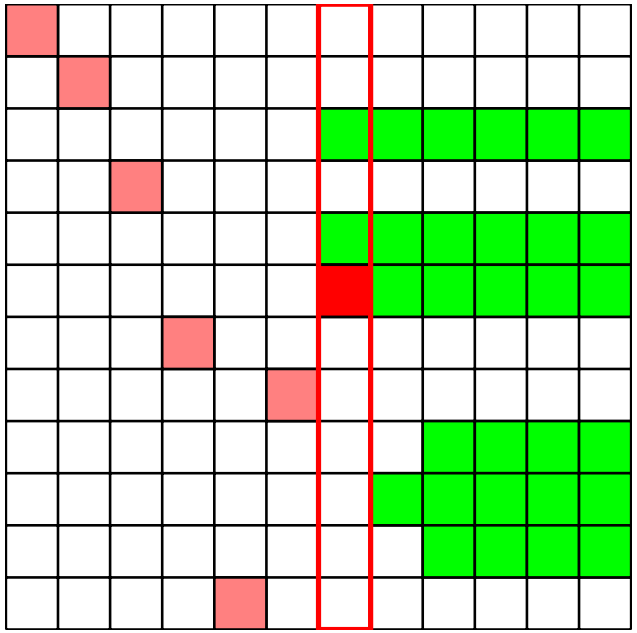

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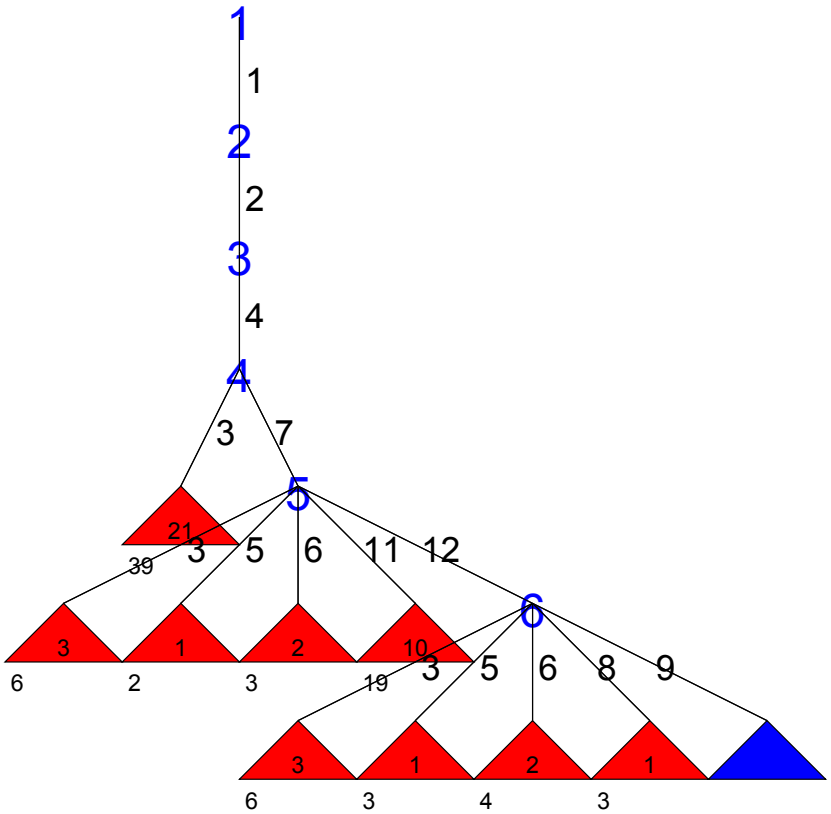

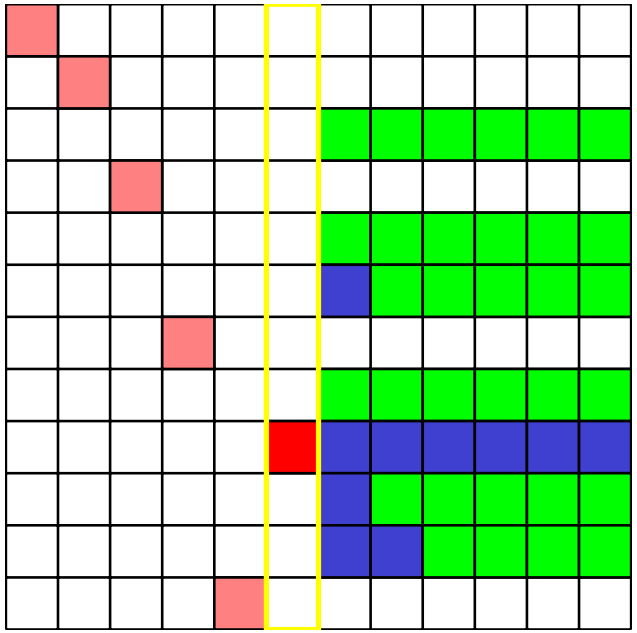

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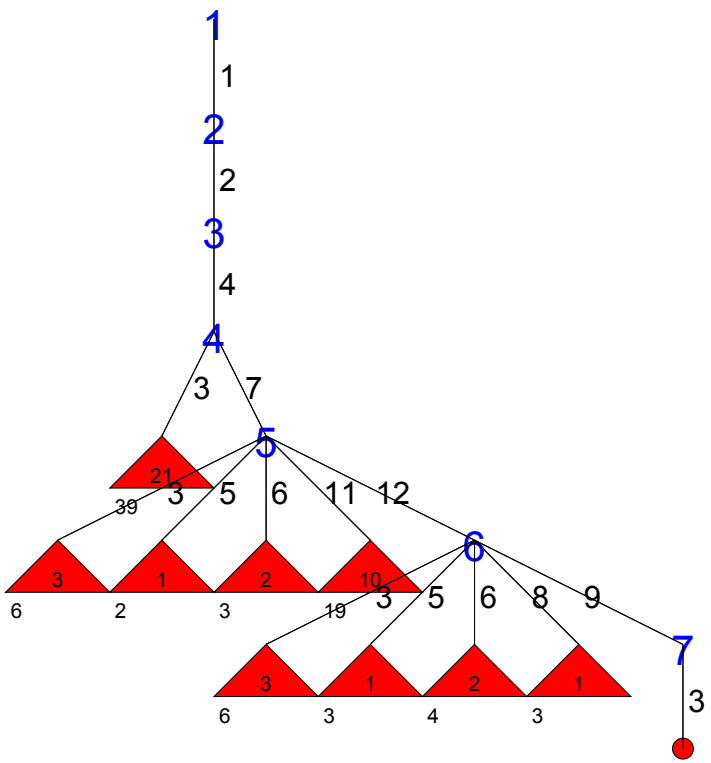

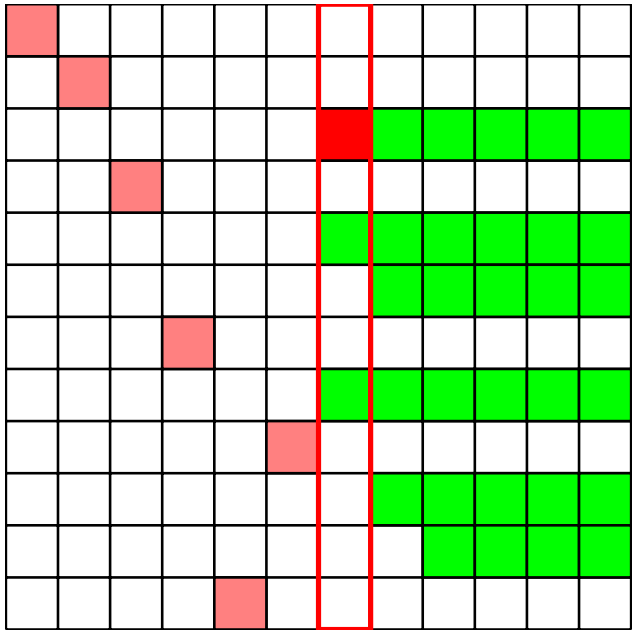

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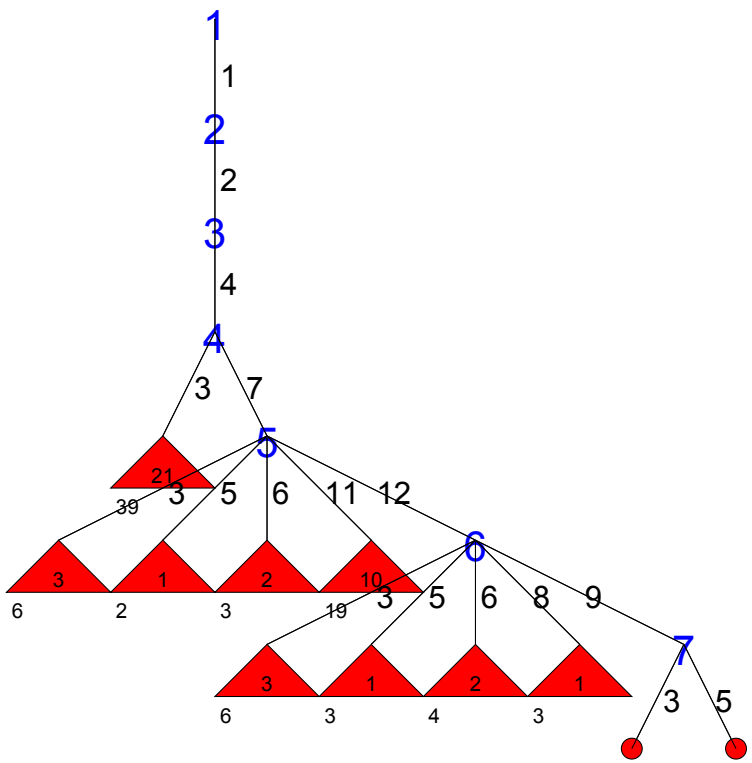

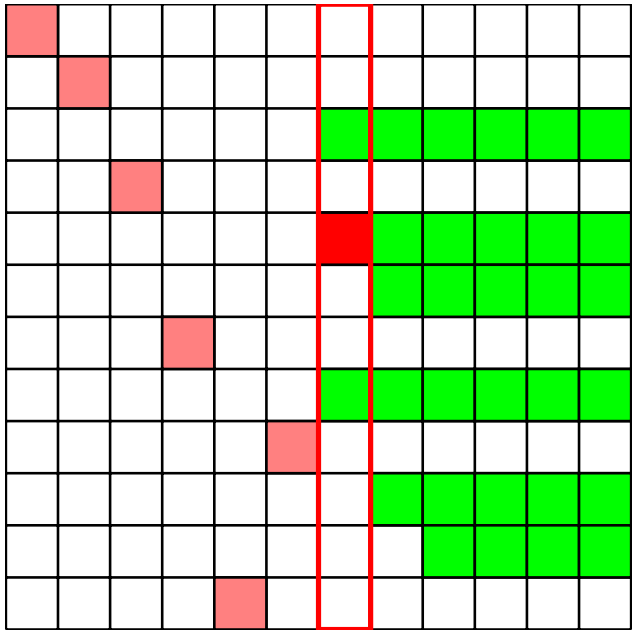

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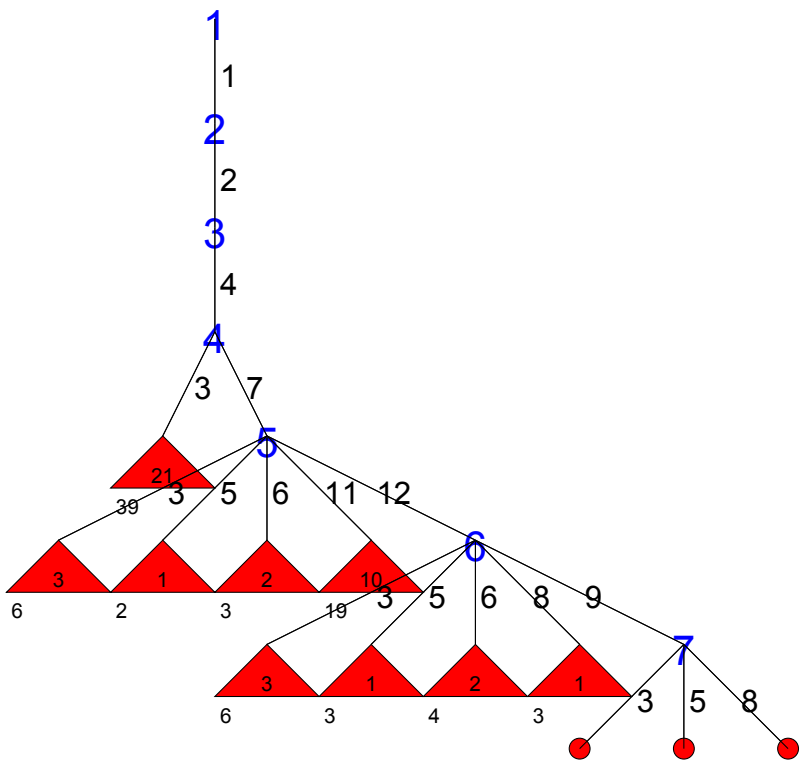

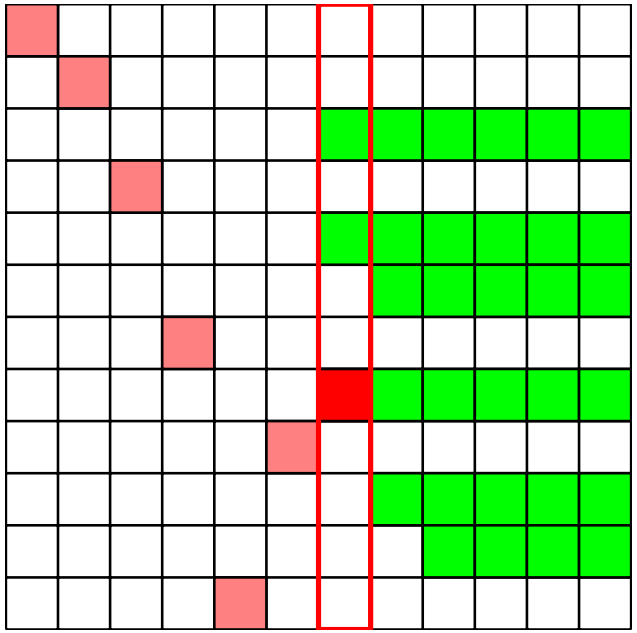

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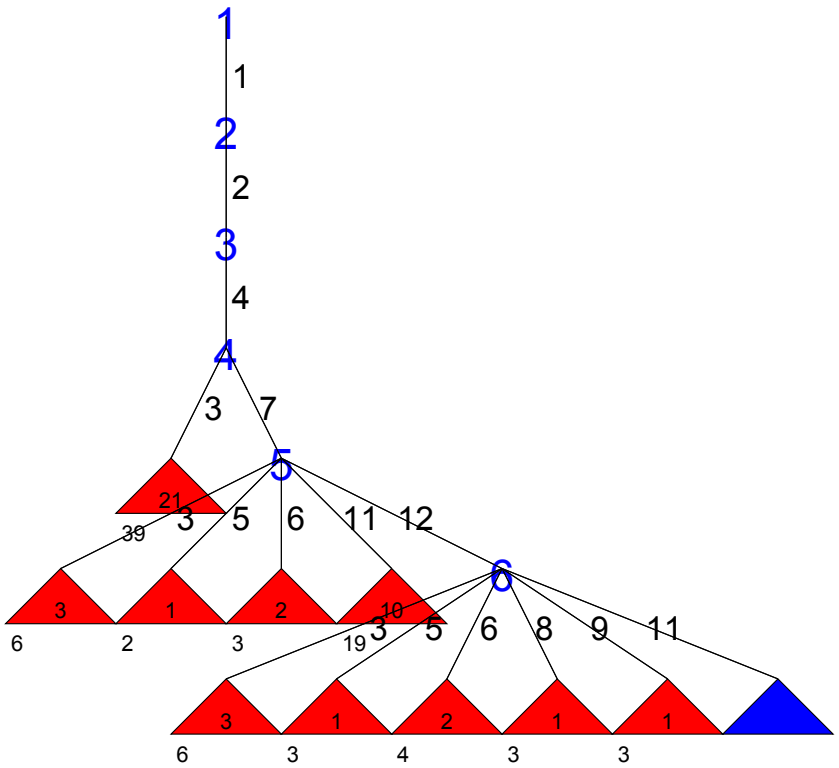

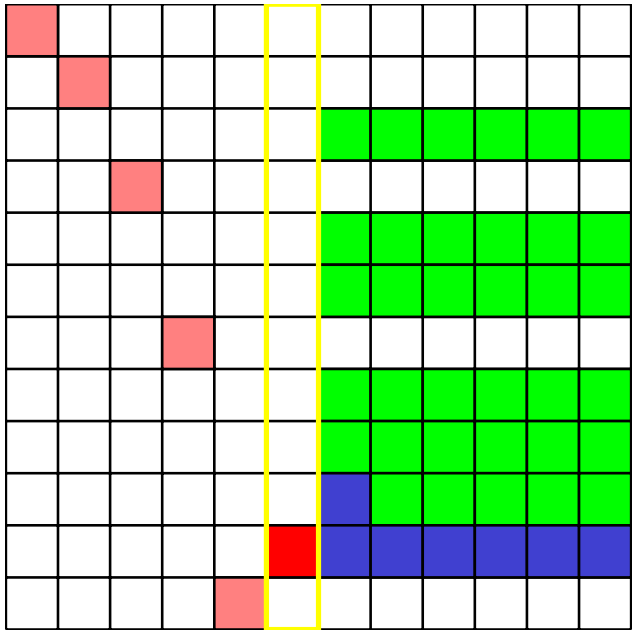

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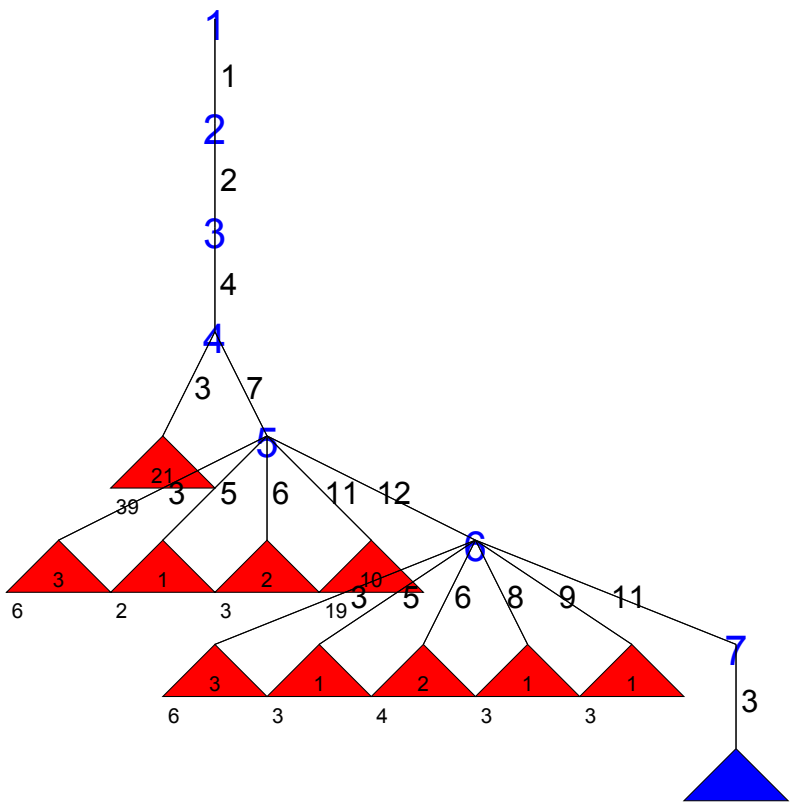

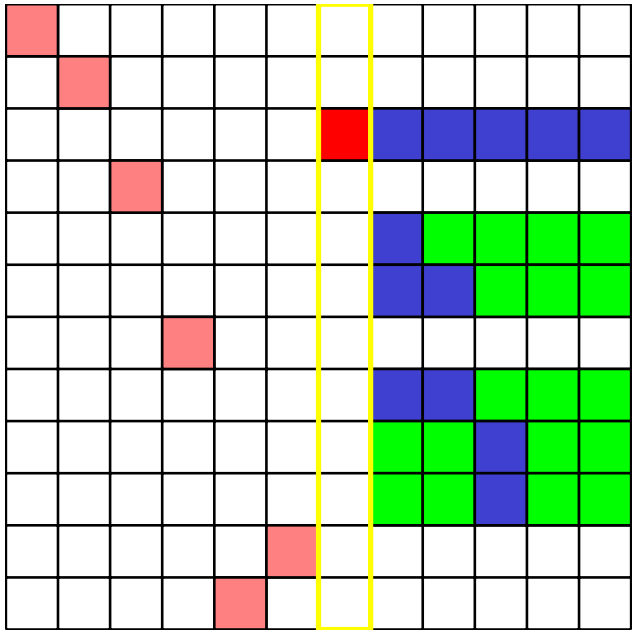

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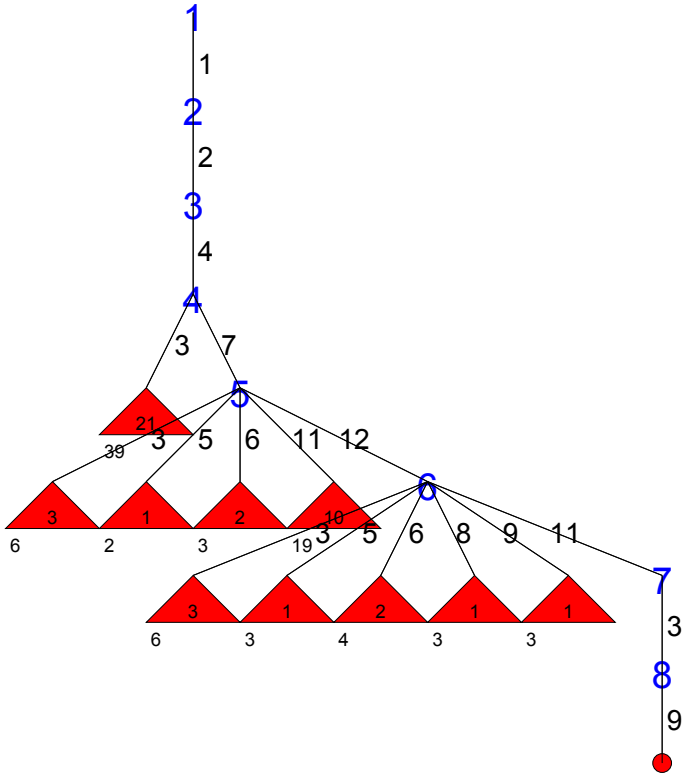

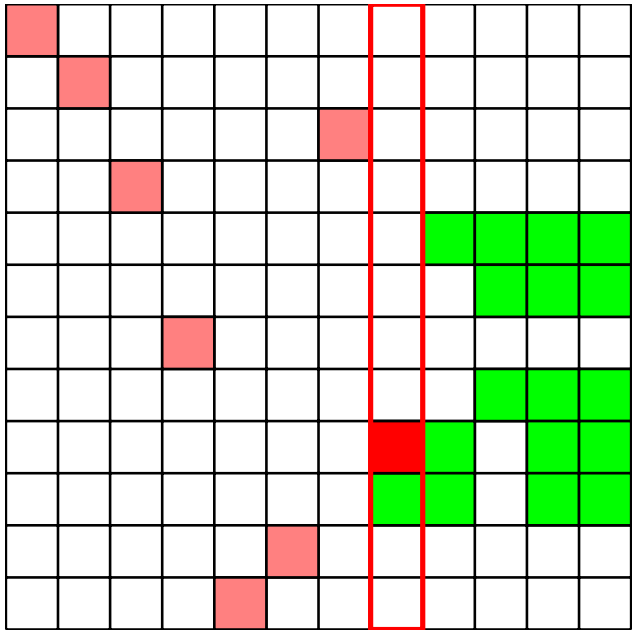

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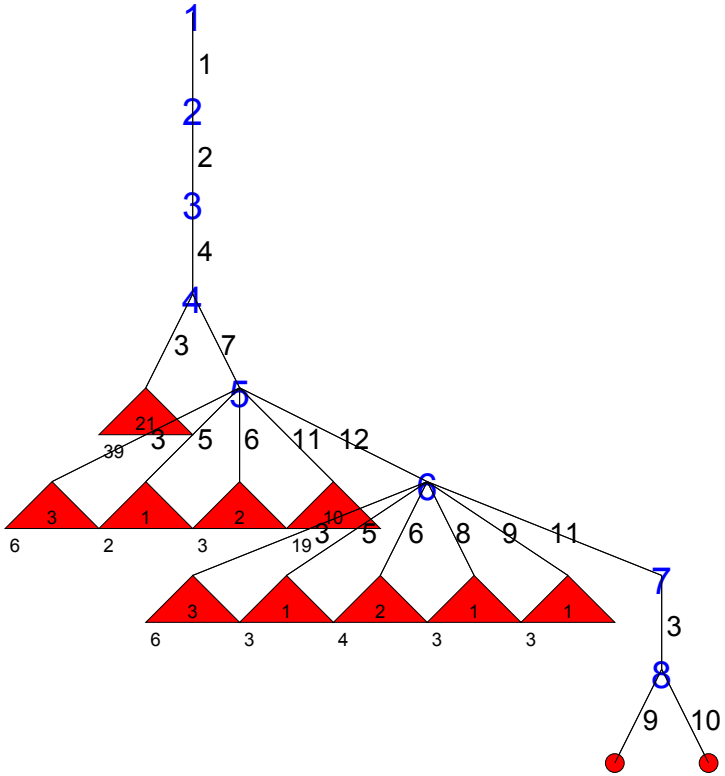

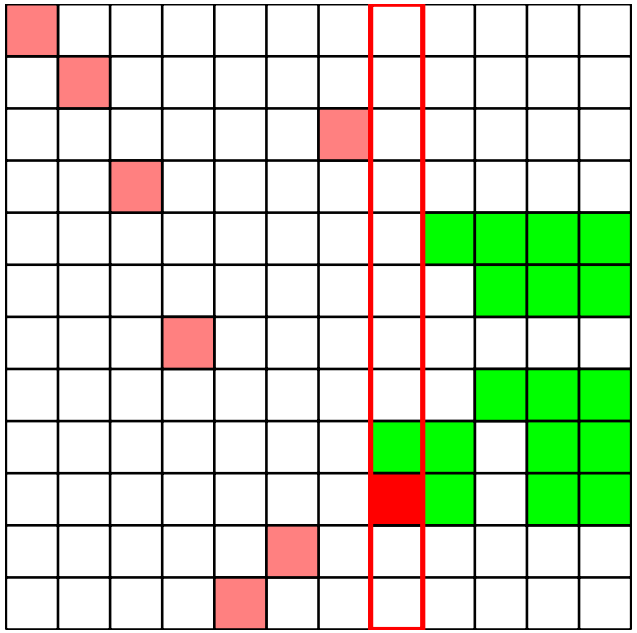

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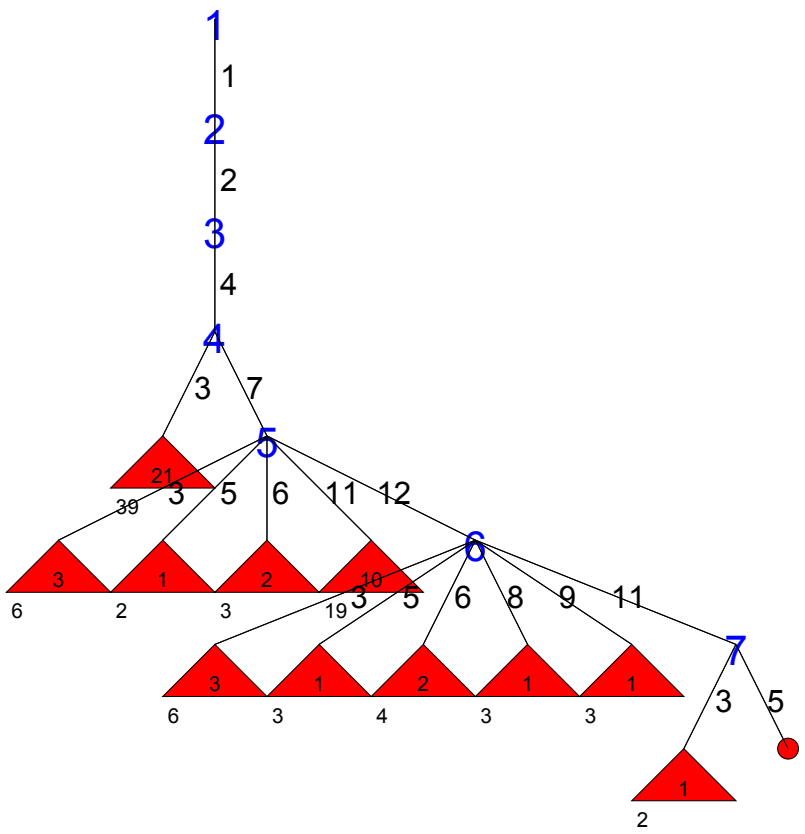

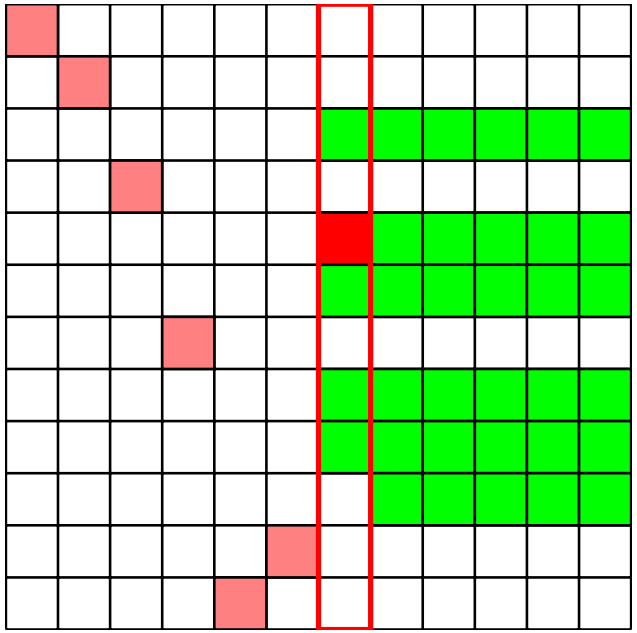

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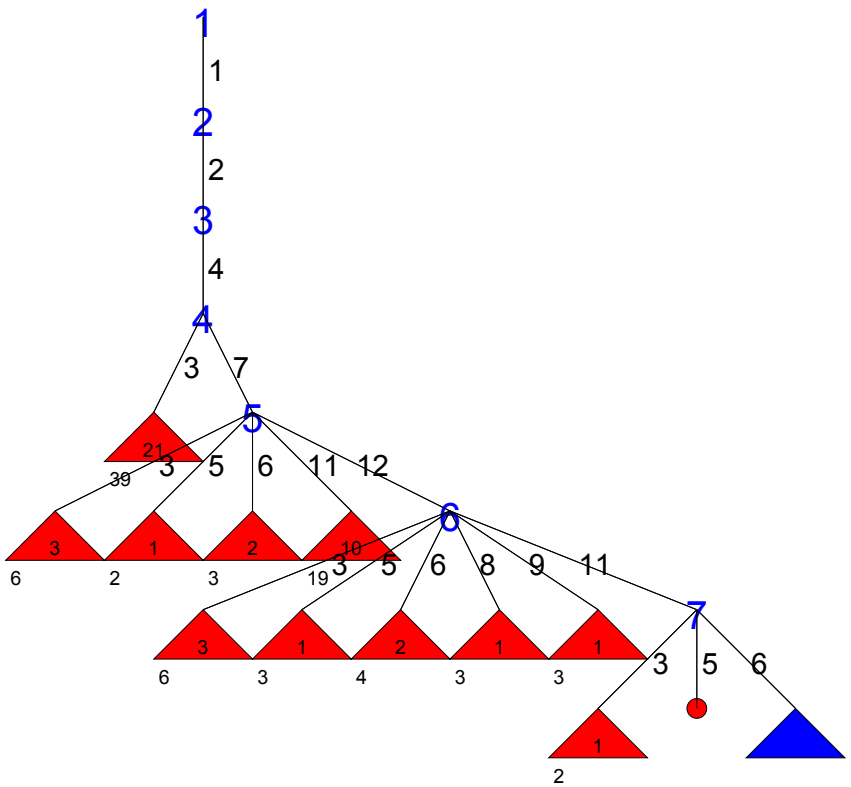

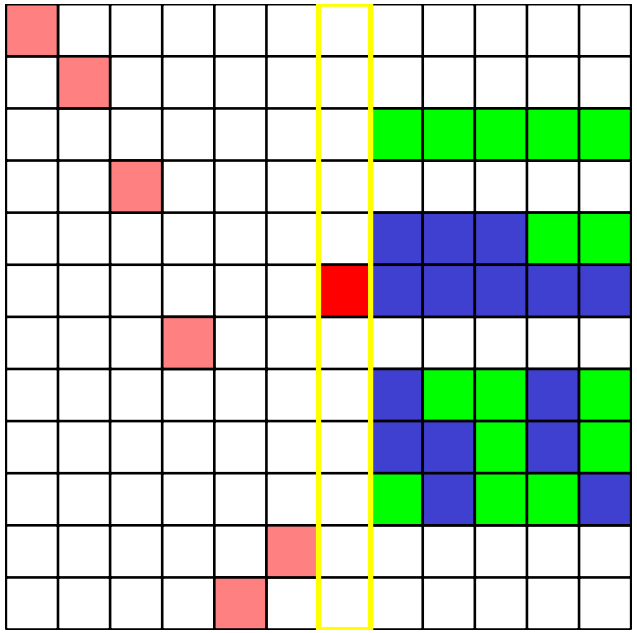

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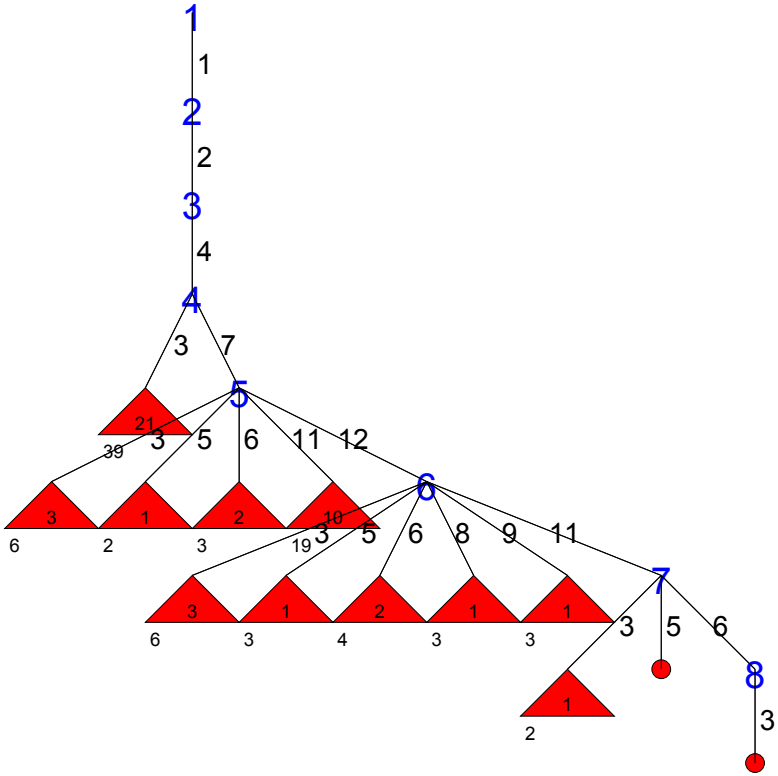

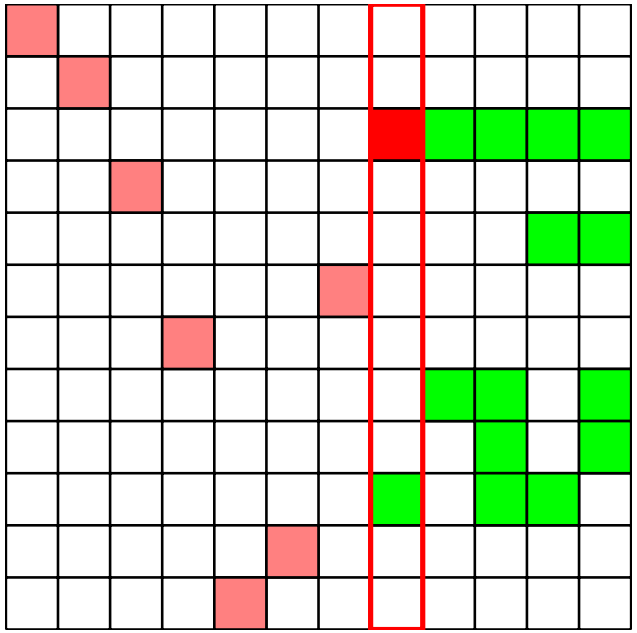

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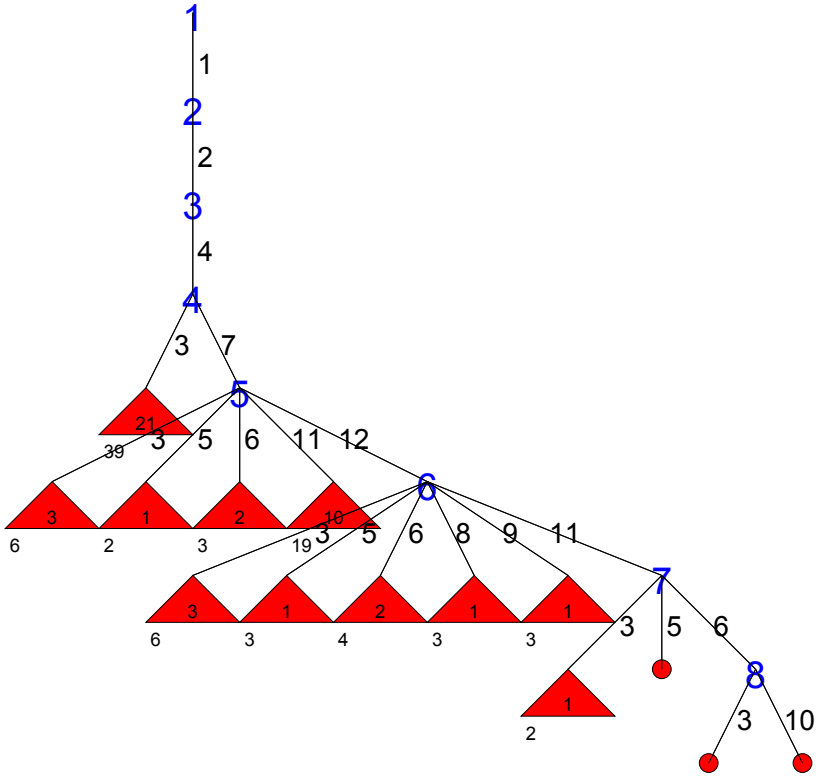

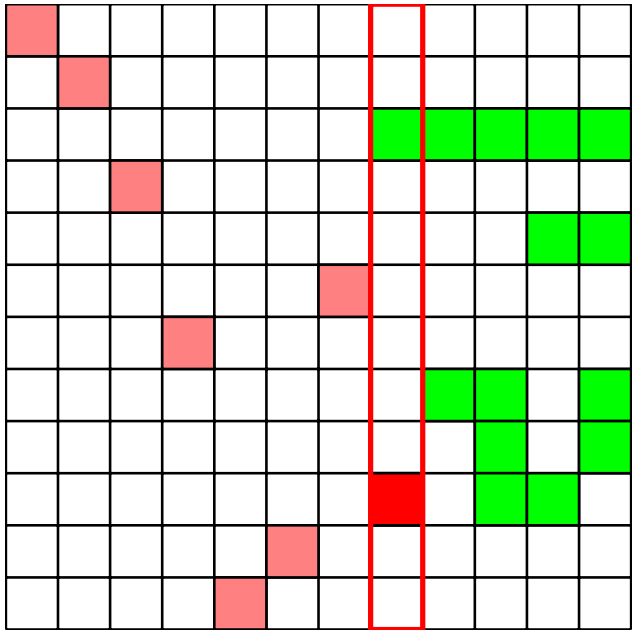

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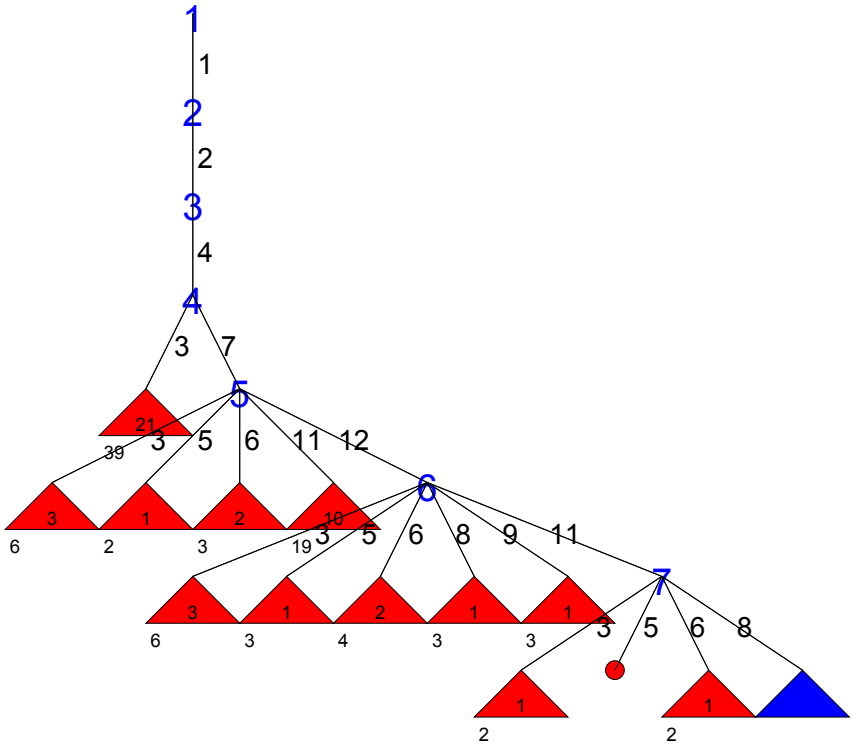

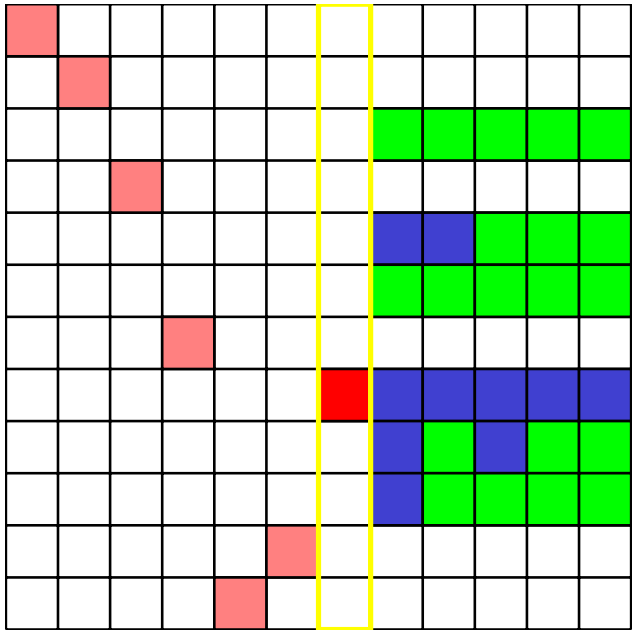

Back to Start
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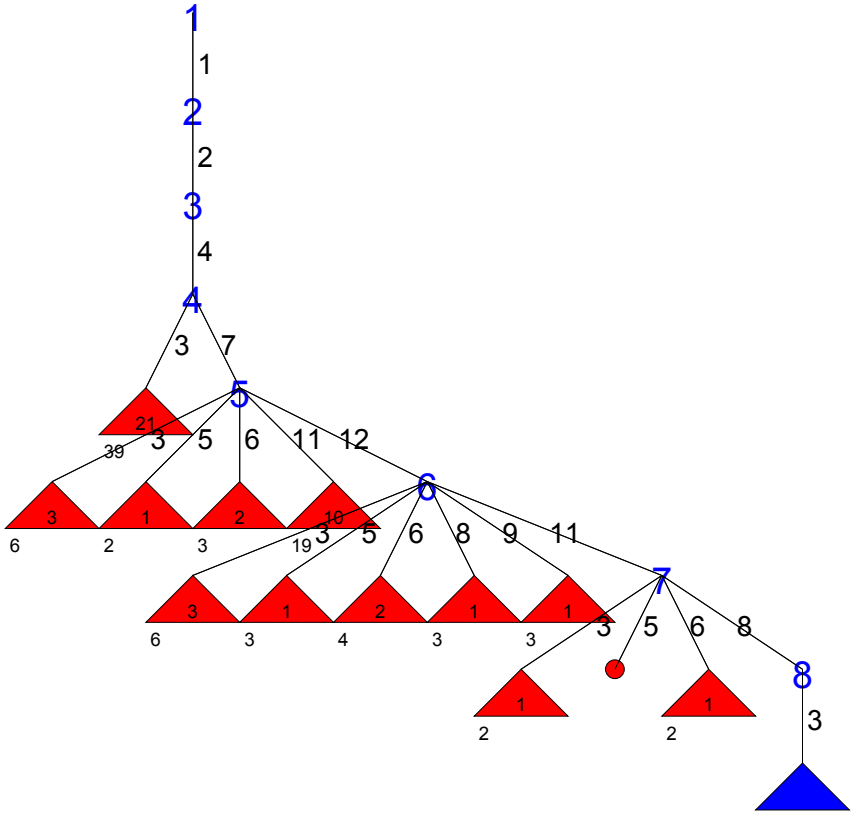
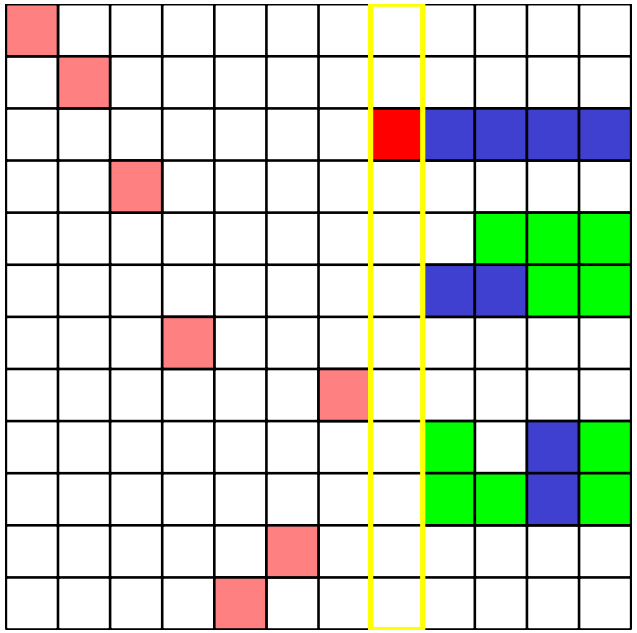
Improved Model





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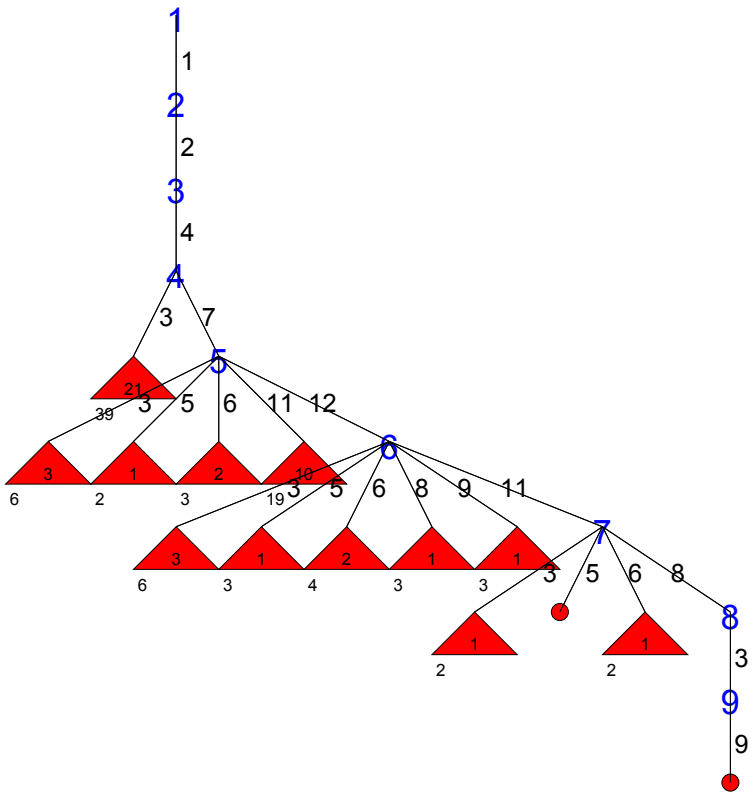

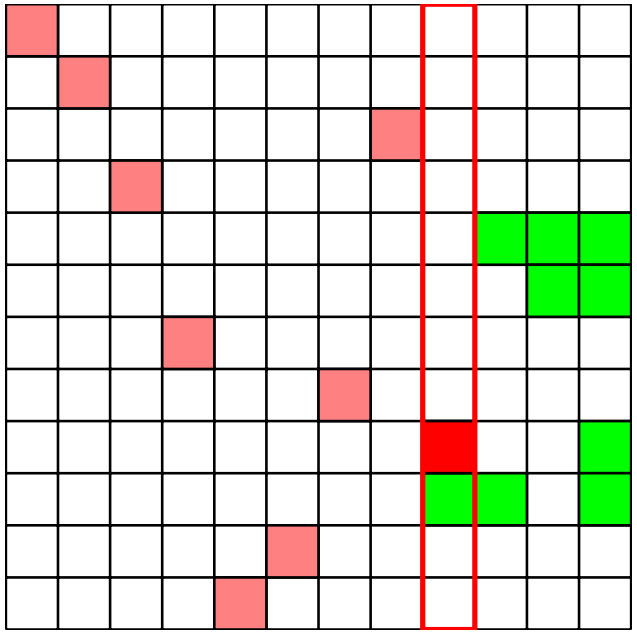

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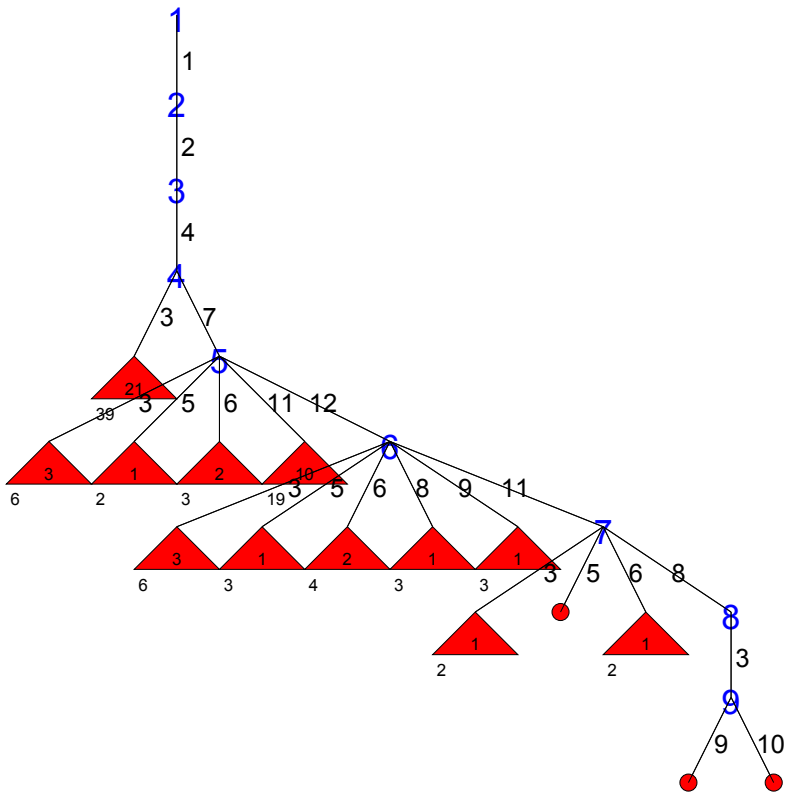

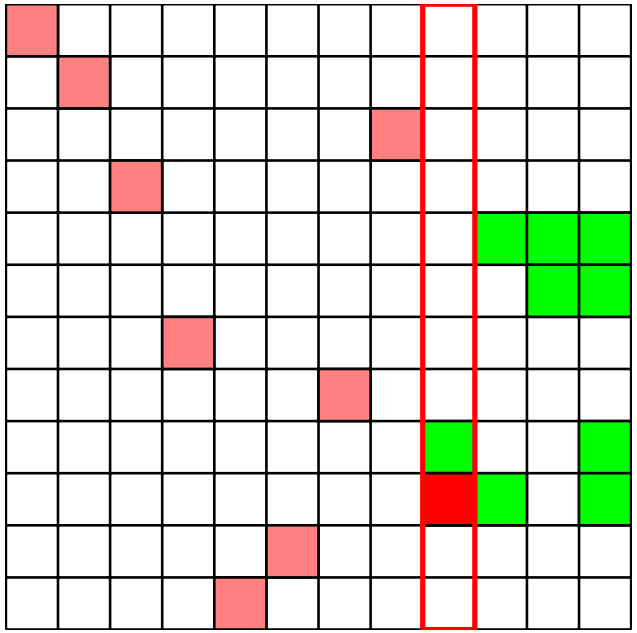

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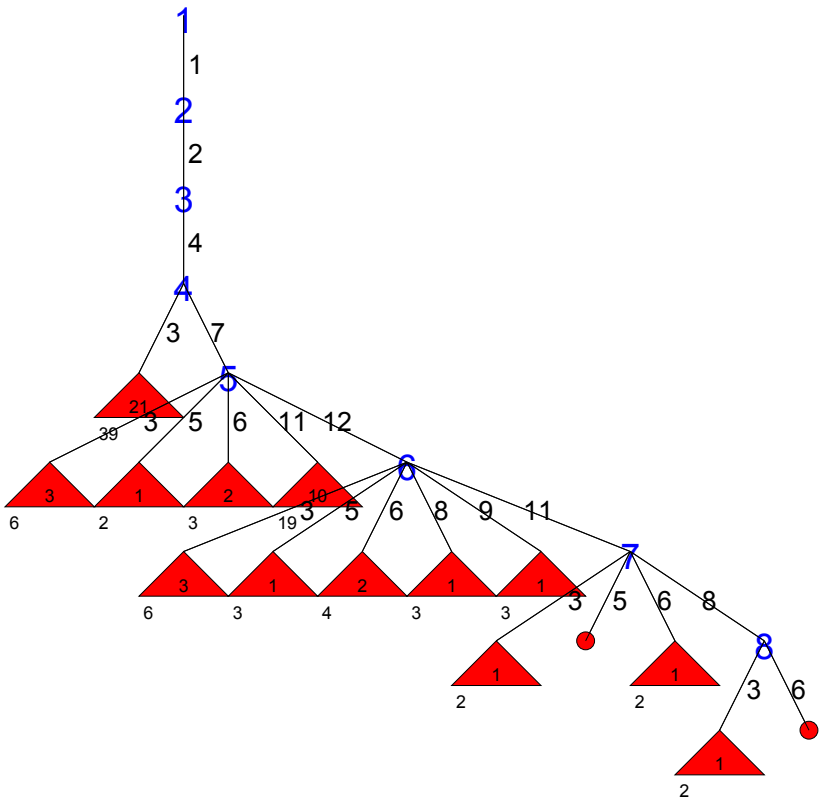

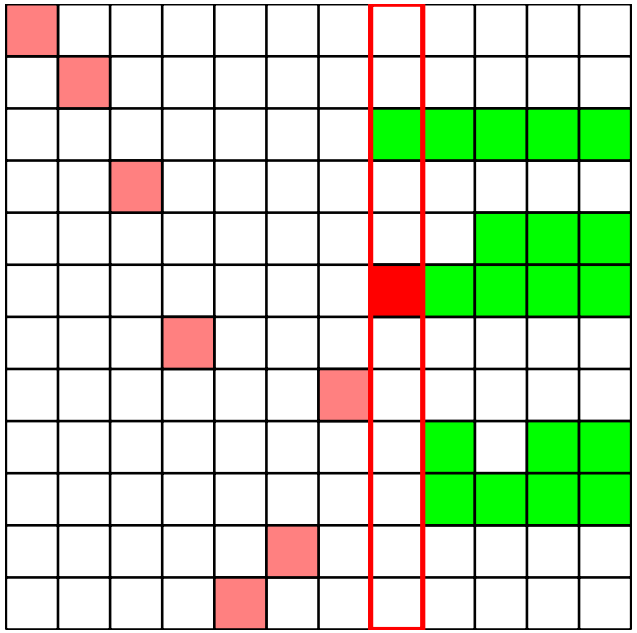

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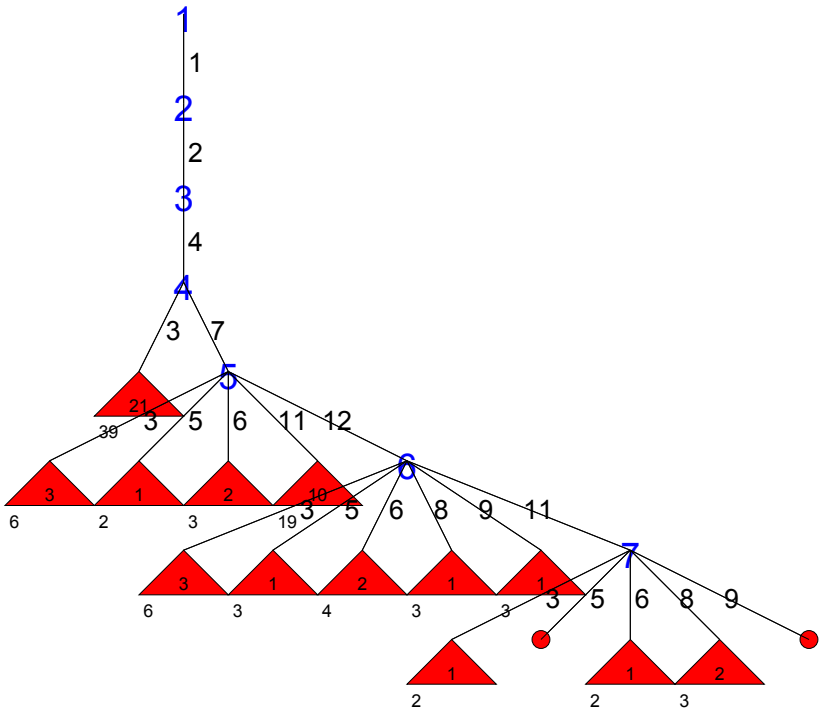

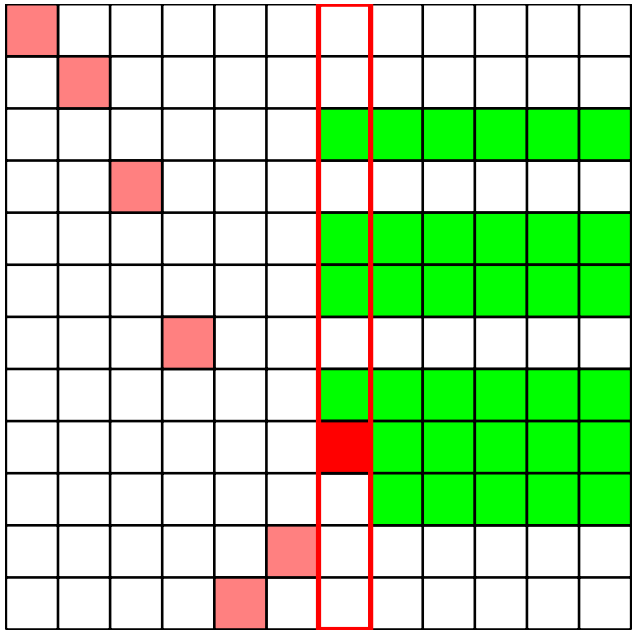

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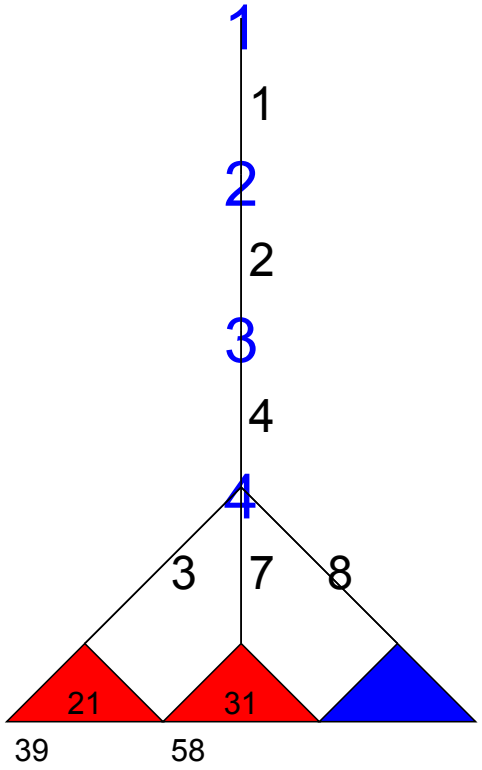

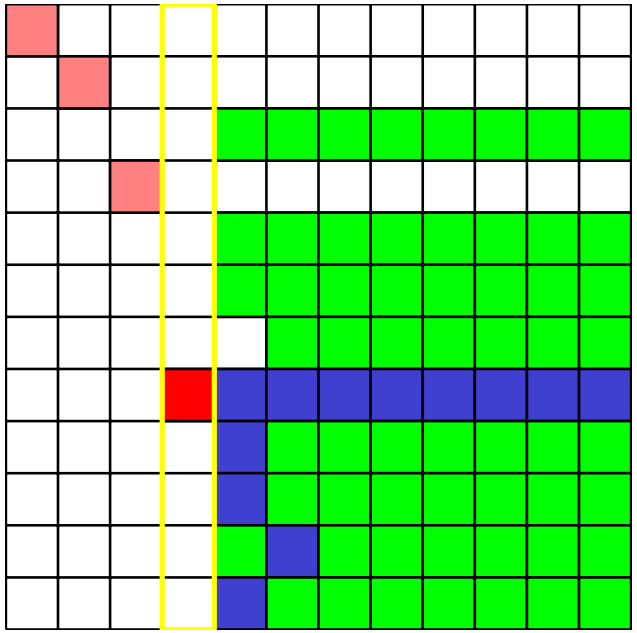

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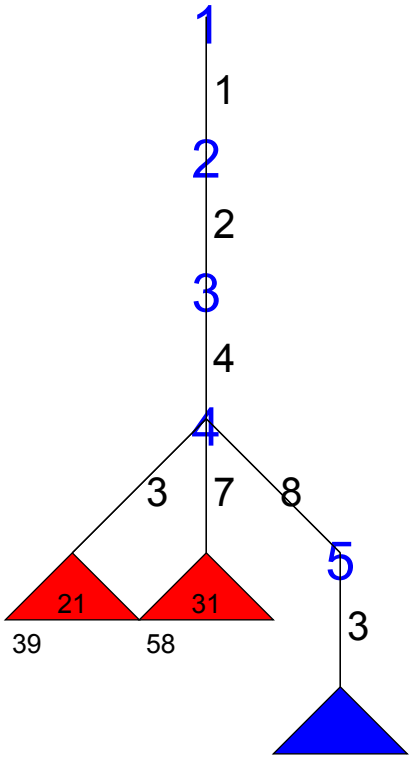

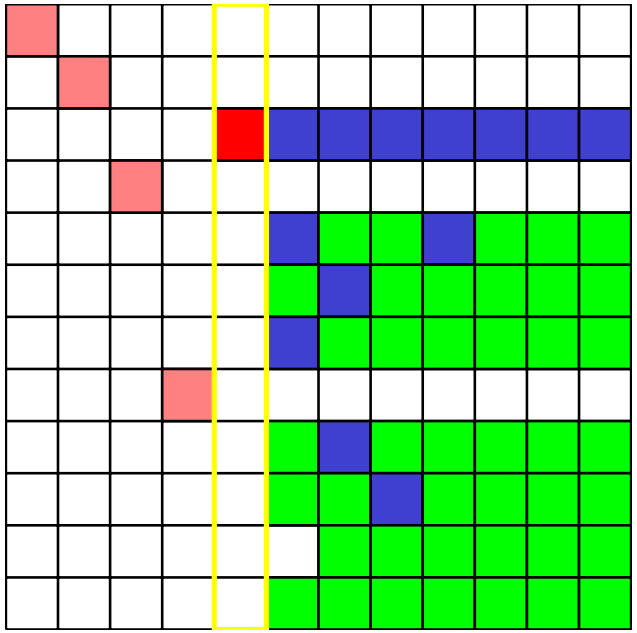

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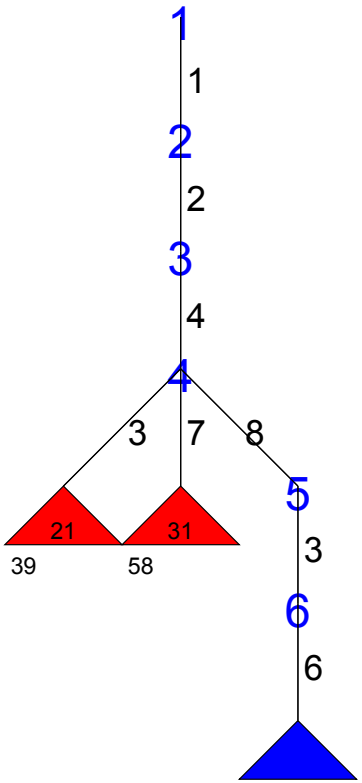

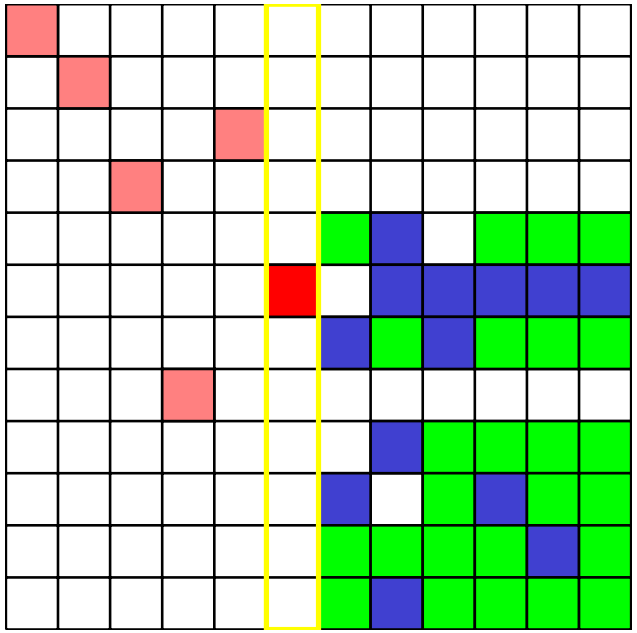

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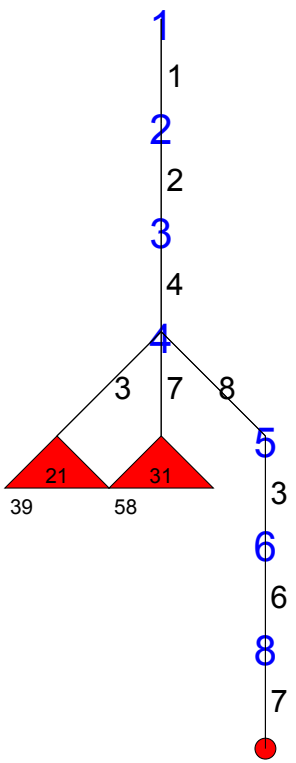

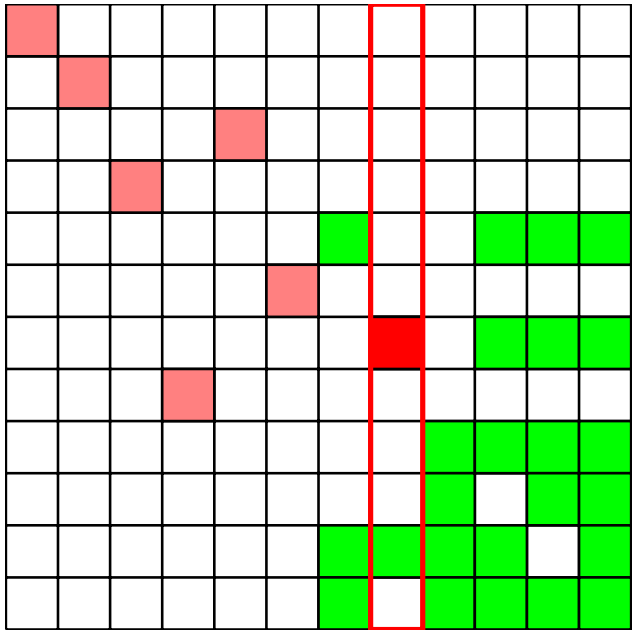

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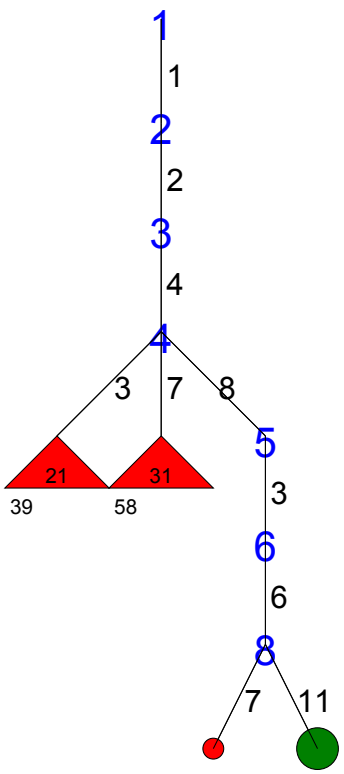

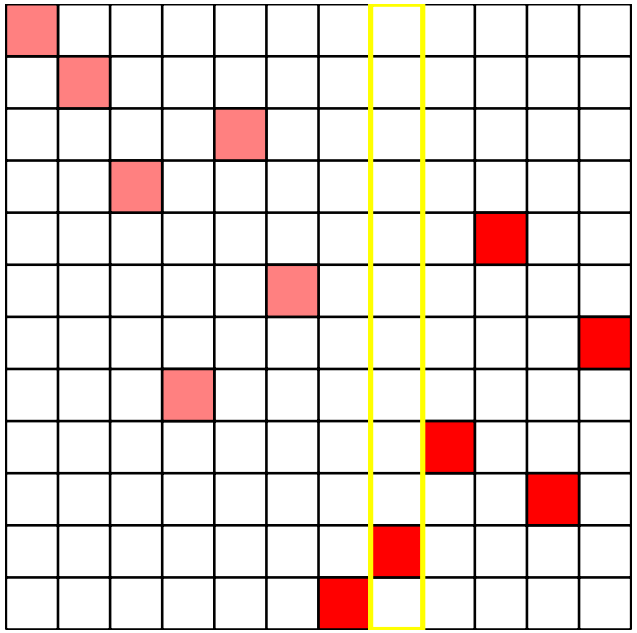

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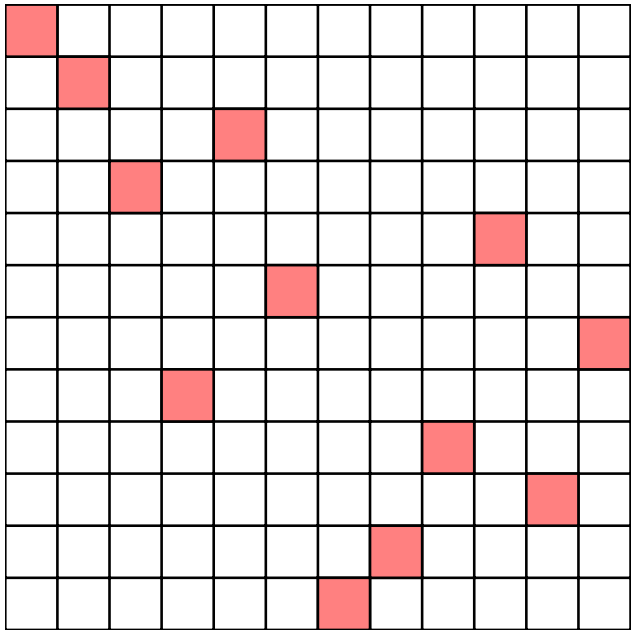
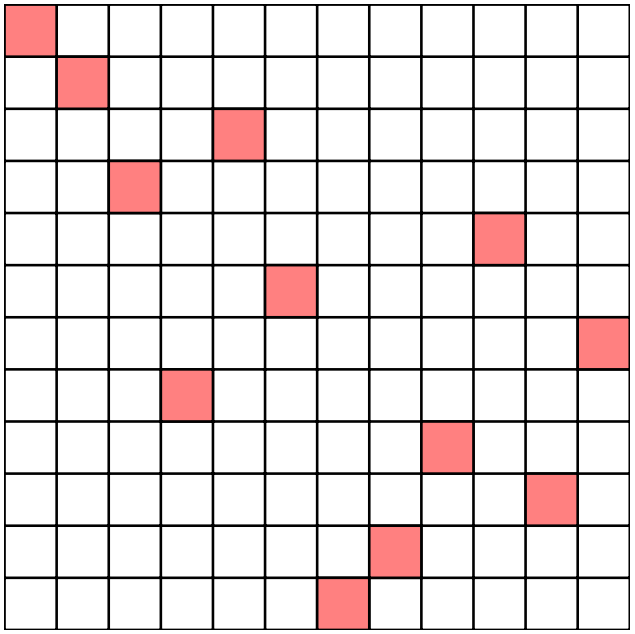
Comparison (Solutions)

Initial Model

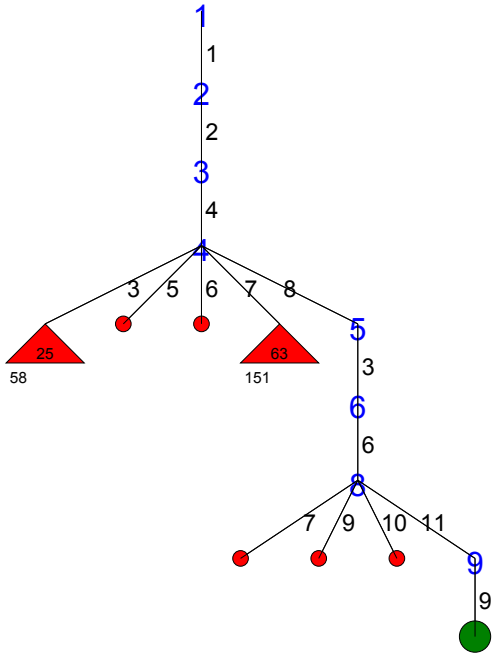


Improved Model

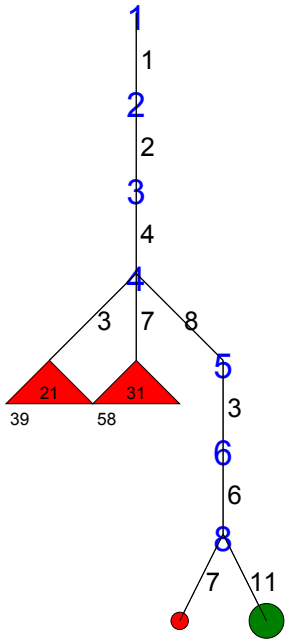


Comparison (Search Trees)

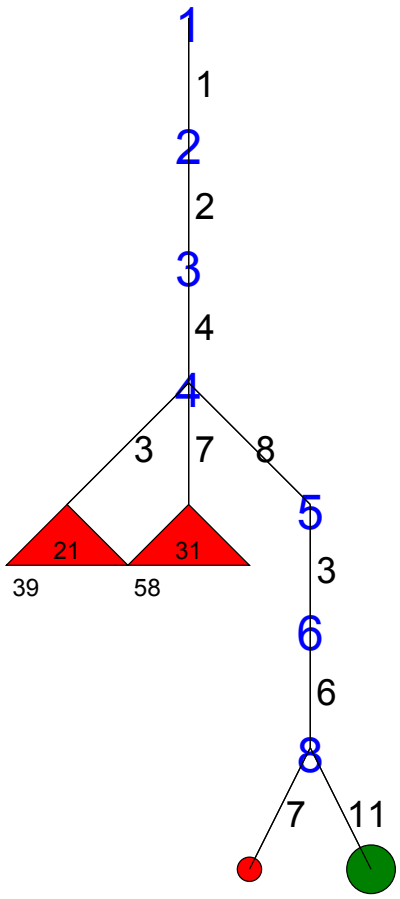
Initial Model



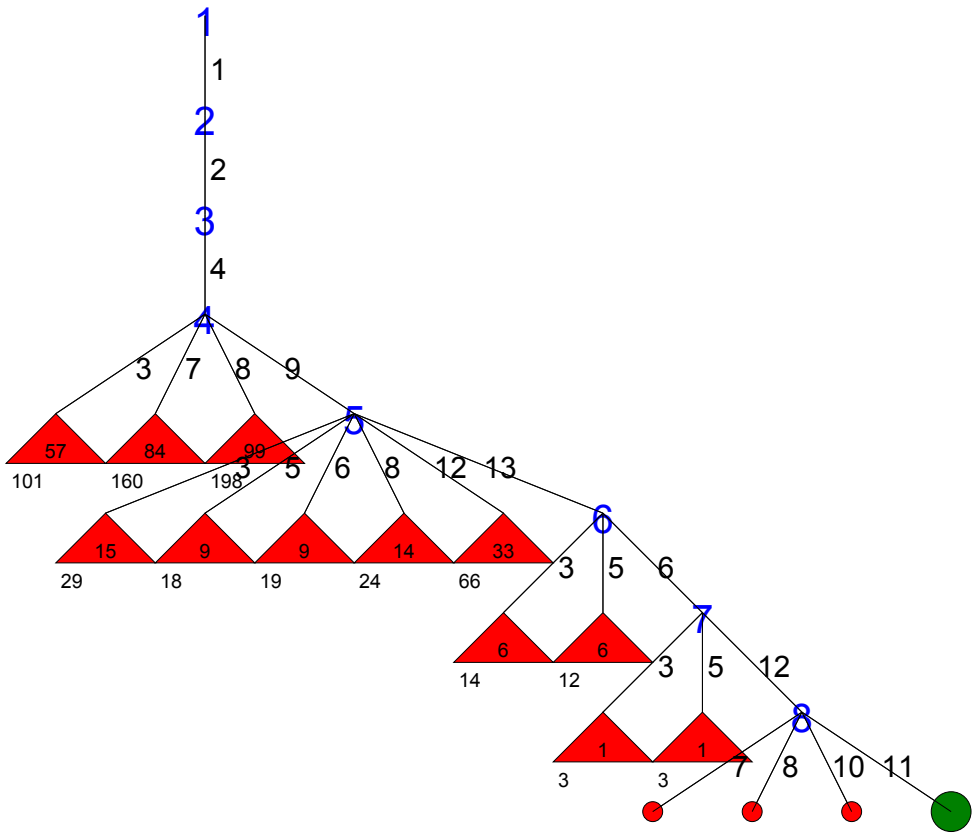
Improved Model



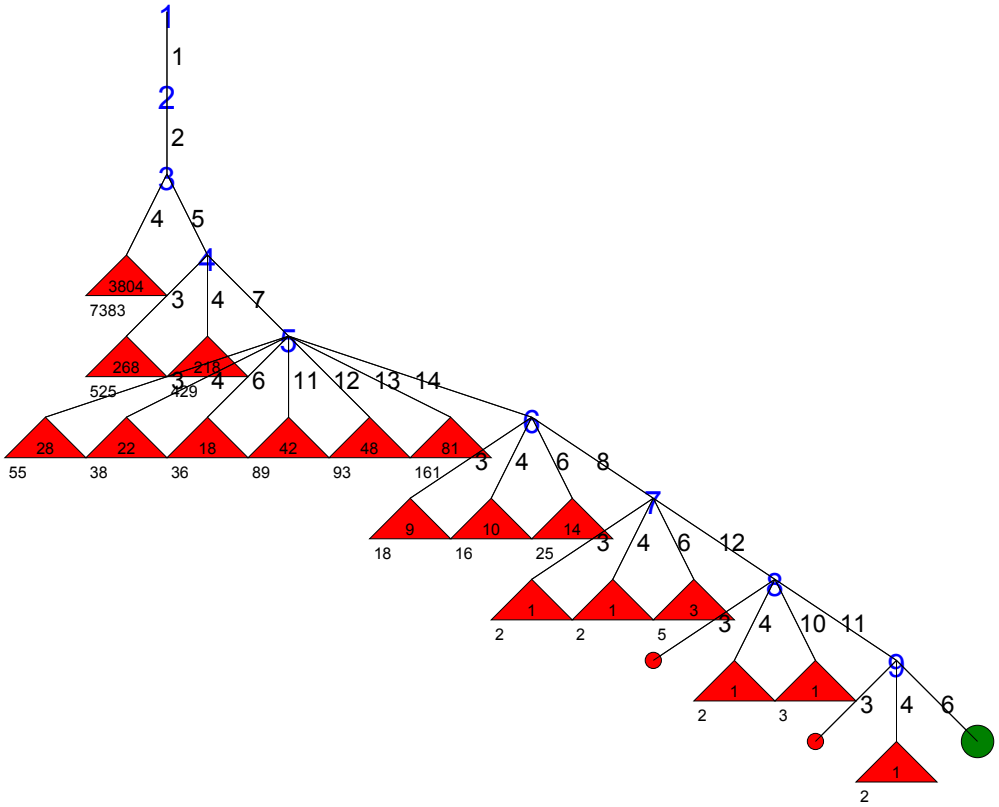
Search tree (Size 12)



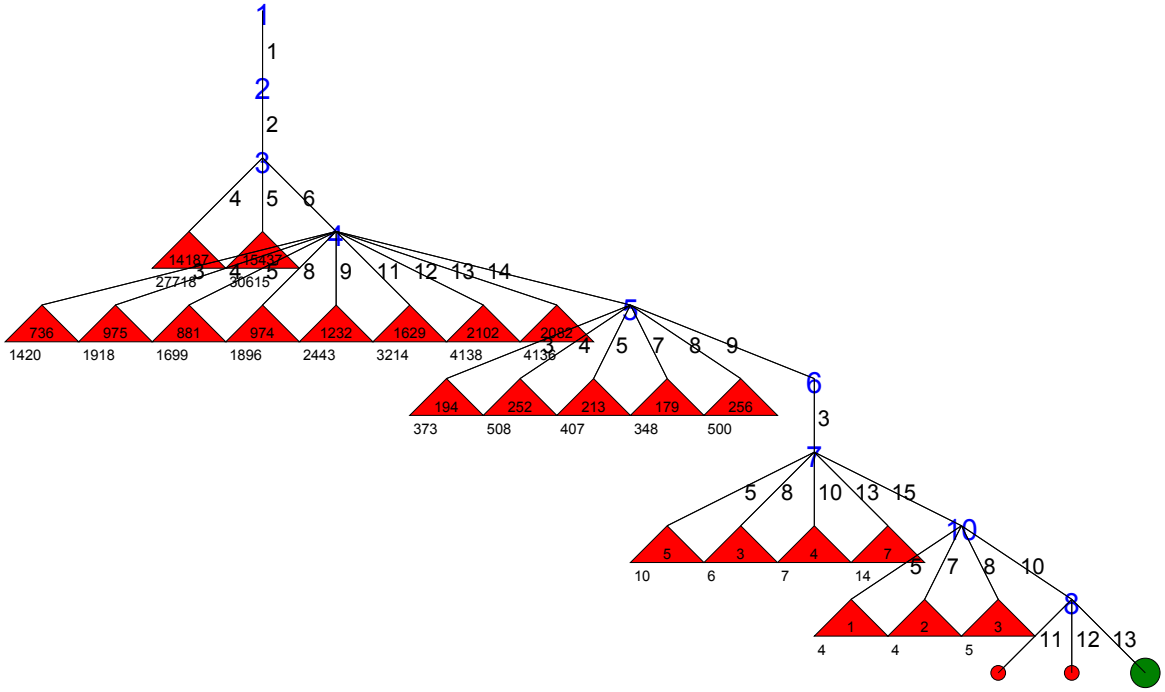
Search tree (Size 13)



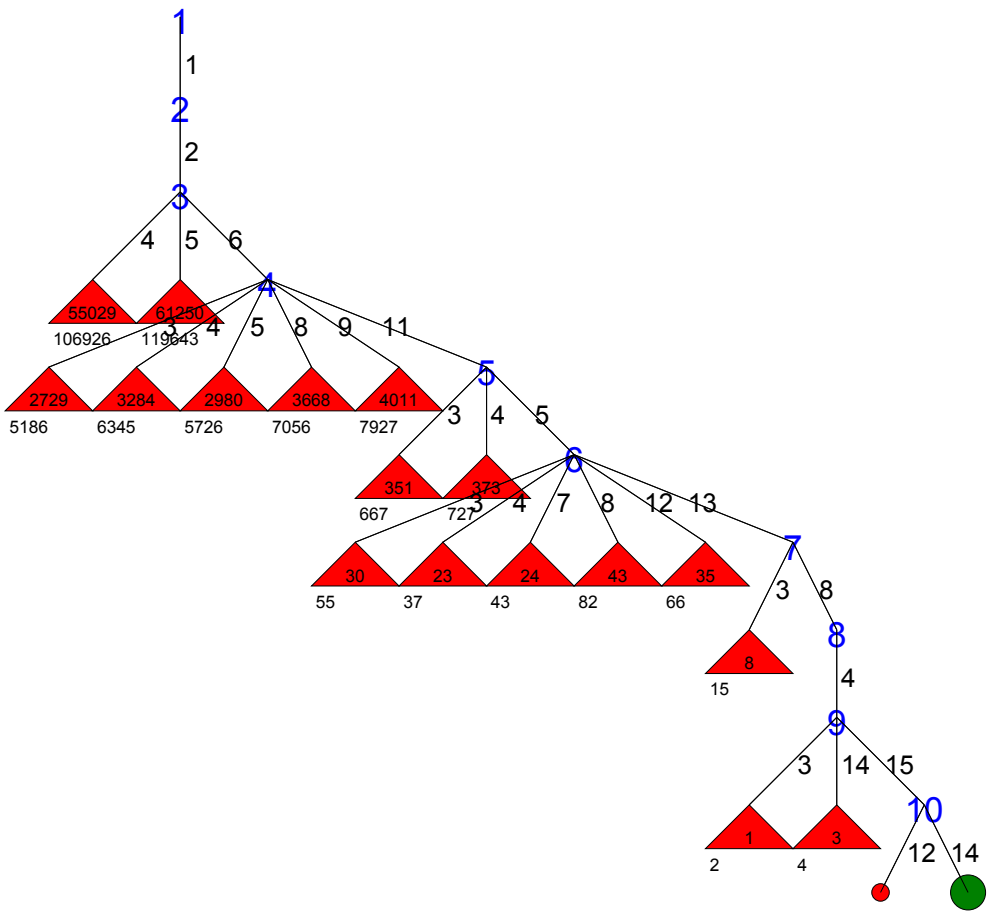
Search tree (Size 14)



Search tree (Size 15)

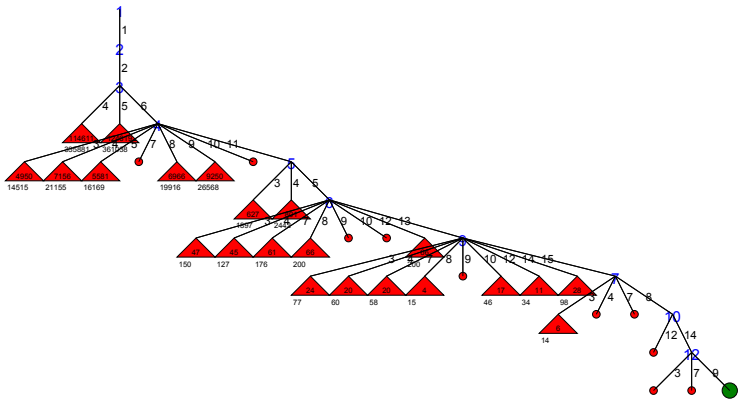


Search tree (Size 16)

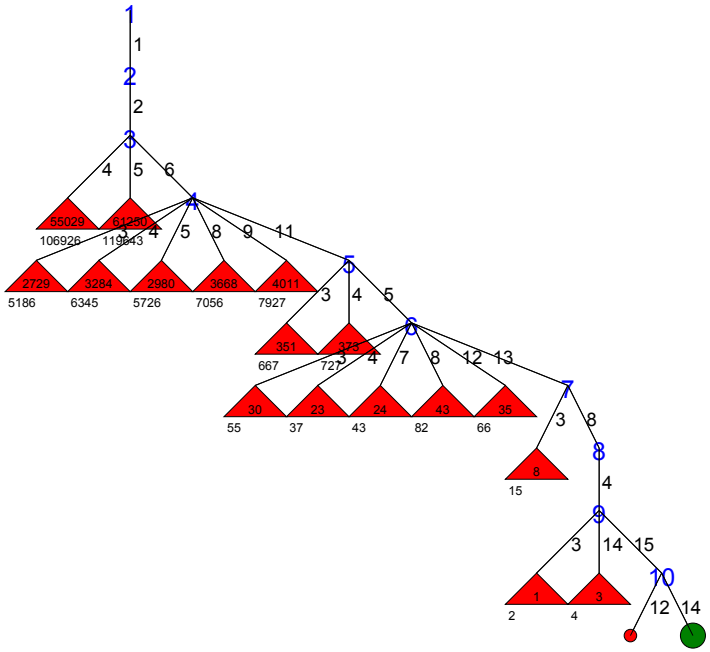


Comparison (Search Tree, size 16)

Initial Model



Improved Model



Other Problem Sizes

	Basic Model		Improved Model	
Size	Backtrack	Time	Backtrack	Time
10	4	0.00	4	0.16
11	118	0.08	77	1.44
12	50	0.05	31	0.94
13	335	0.36	216	6.22
14	5008	6.23	2875	95.94
15	47332	68.92	25820	1046.75
16	157773	271.22	84161	4099.52
17	1641685	3278.19	825590	49371.02
18	115745	283.97	55102	4530.83

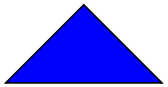
Observation

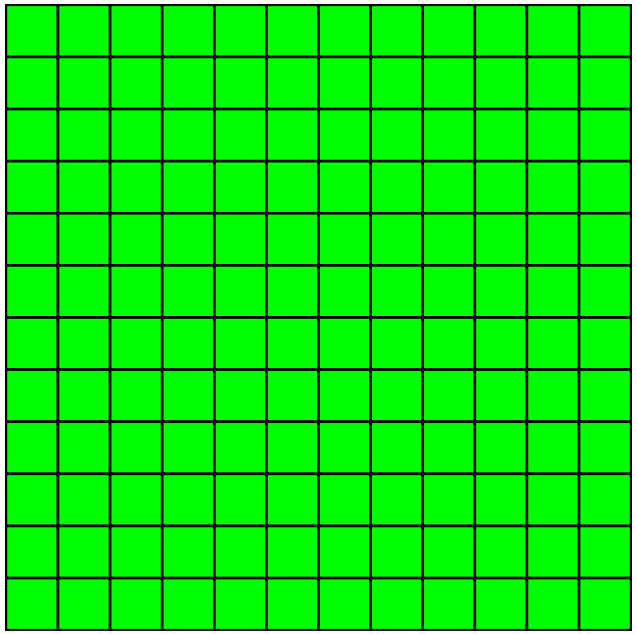
- Changes reduce backtracks by 50%
- But, run times explode
- Being clever does not always pay
- Or, perhaps, we did not make the right improvements?

Change of Search Strategy

- Idea: Make more difficult choices first
- Reorder variables to start from middle
- Assign values starting in middle

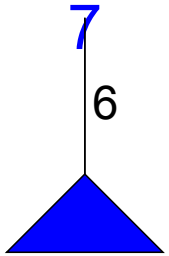
Labeling From Middle

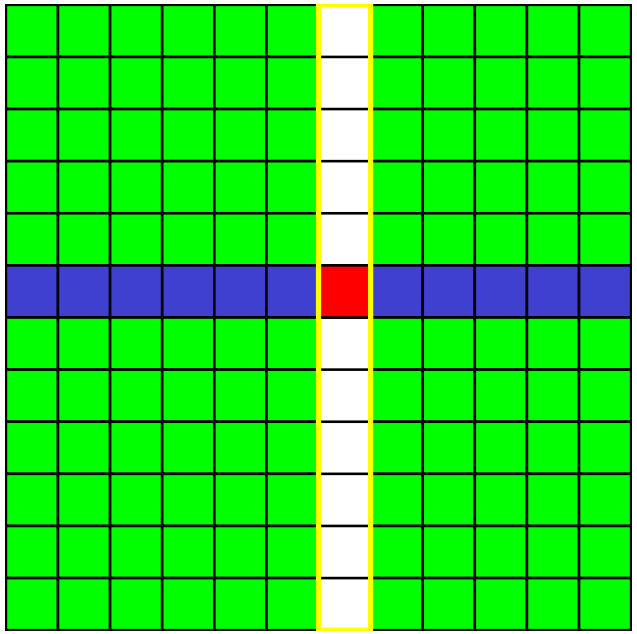




Skip Animation

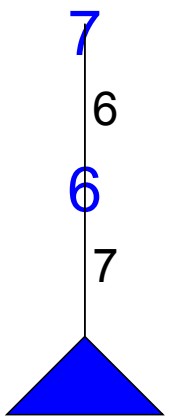
Labeling From Middle

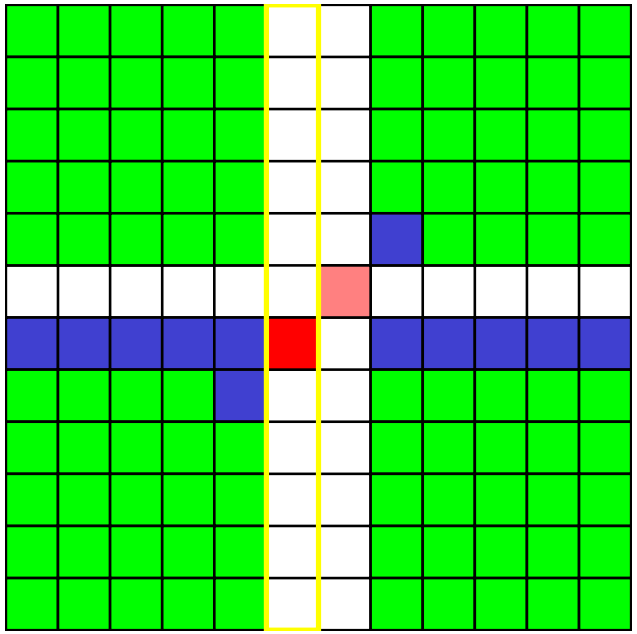




Back to Start
Skip Animation

Labeling From Middle





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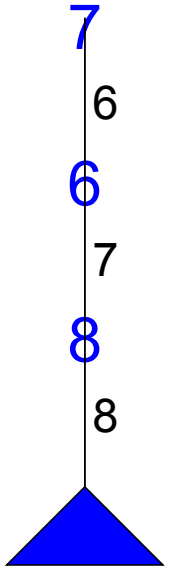
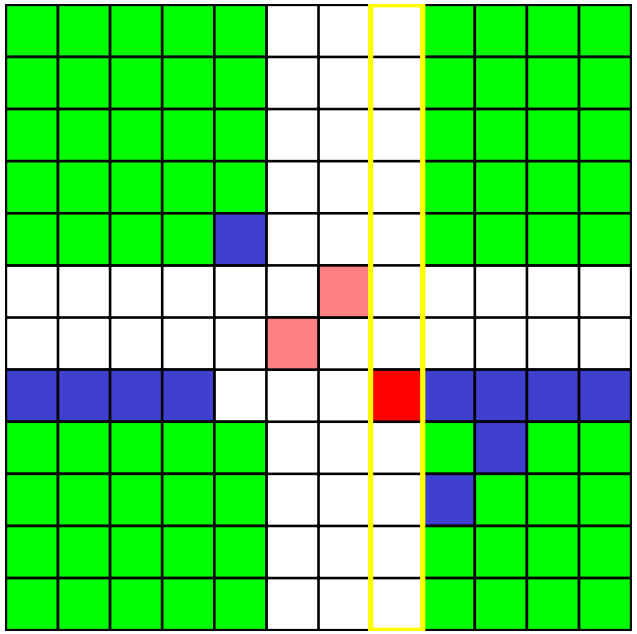
Skip Animation

Labeling From Middle





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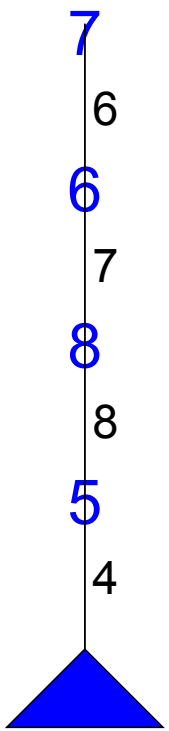
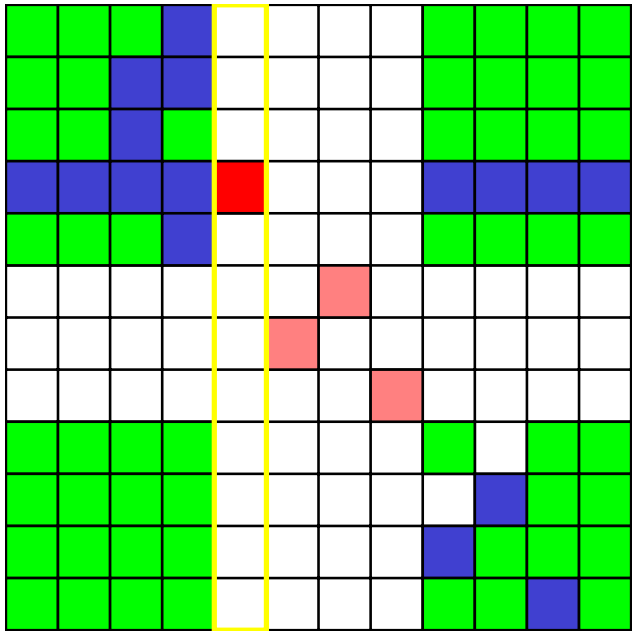
Skip Animation

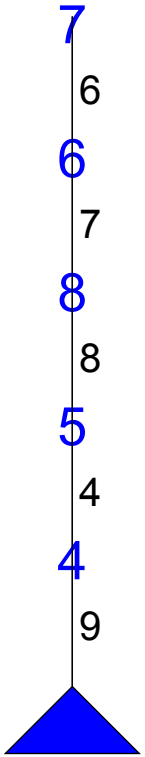
Labeling From Middle

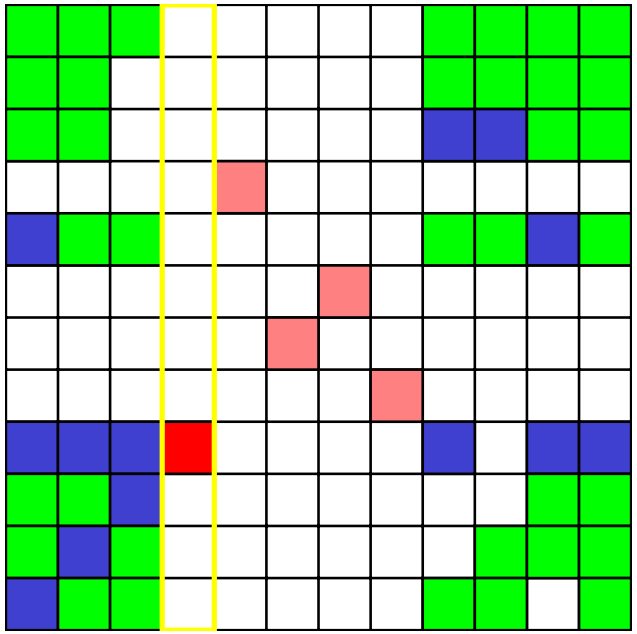




Back to Start
Skip Animation

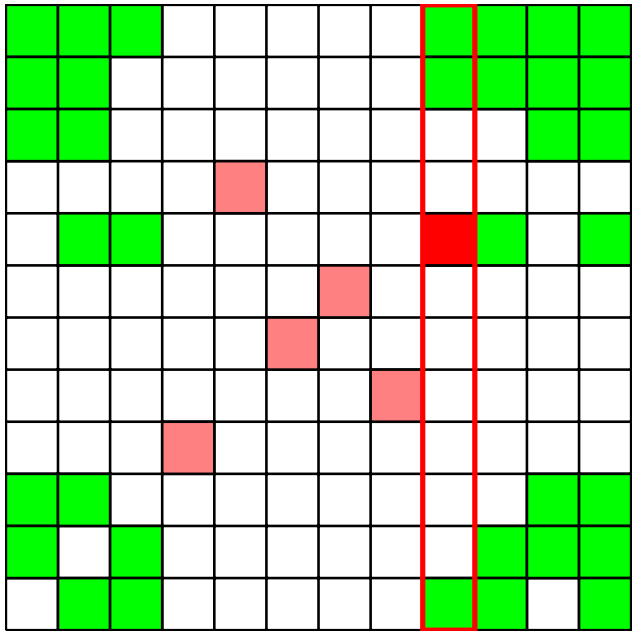
Labeling From Middle





Back to Start
Skip Animation

Labeling From Middle



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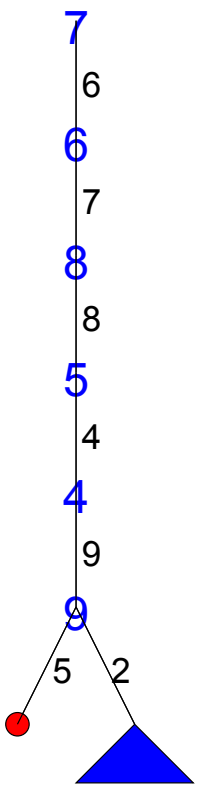
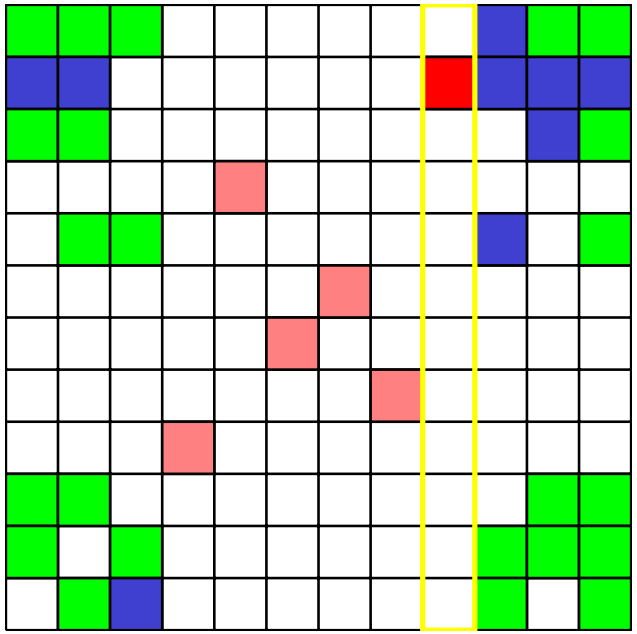
Skip Animation

Labeling From Middle





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Skip Animation

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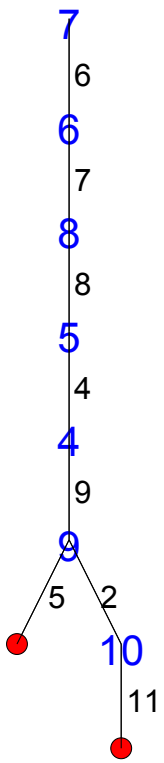

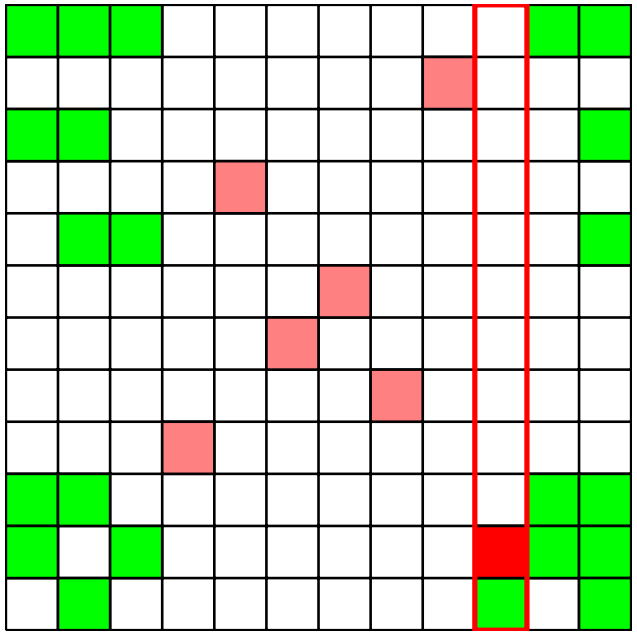

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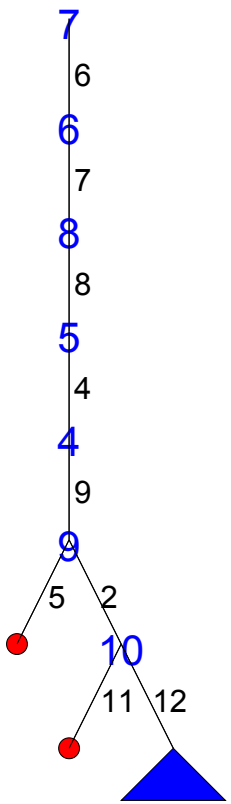

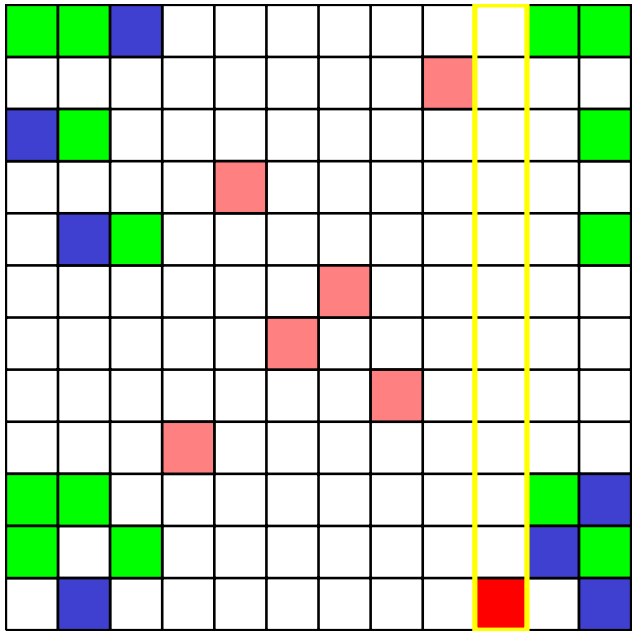

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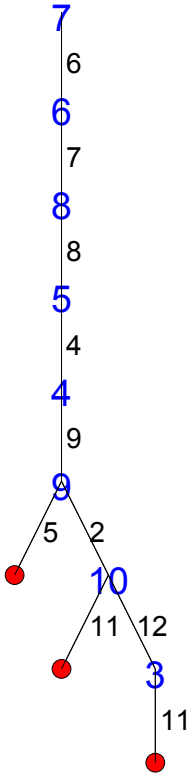

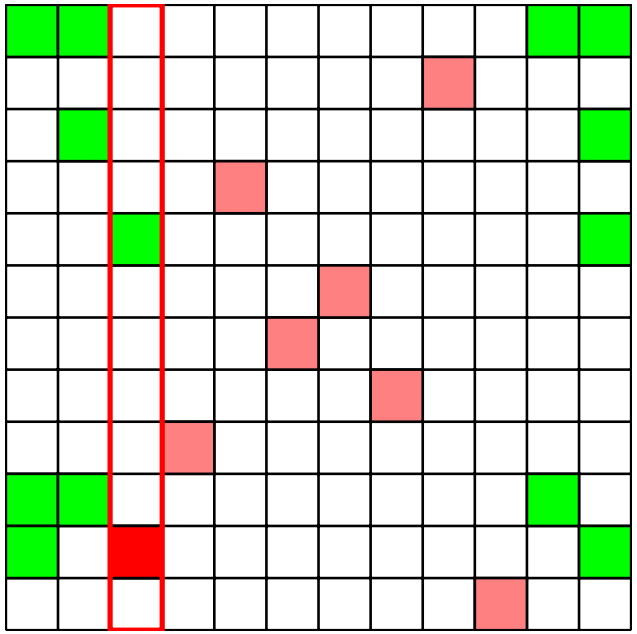

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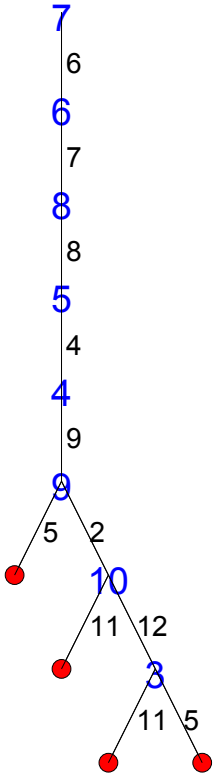

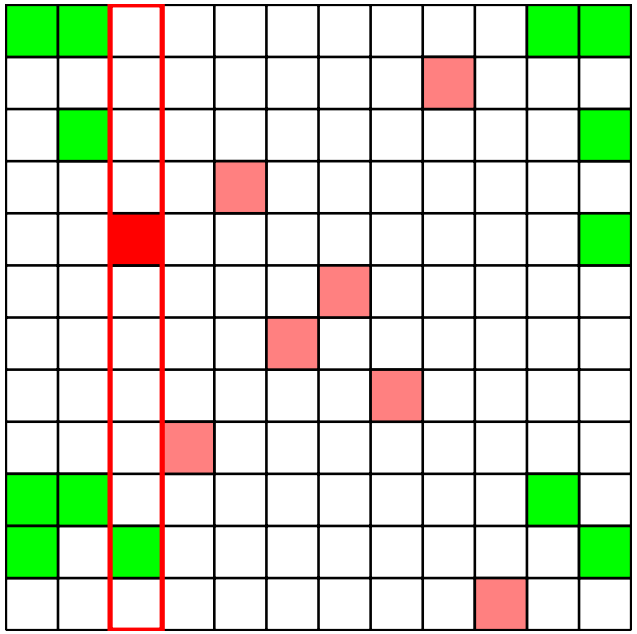

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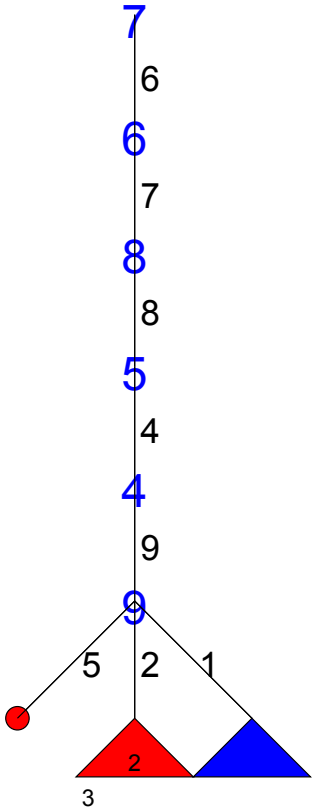

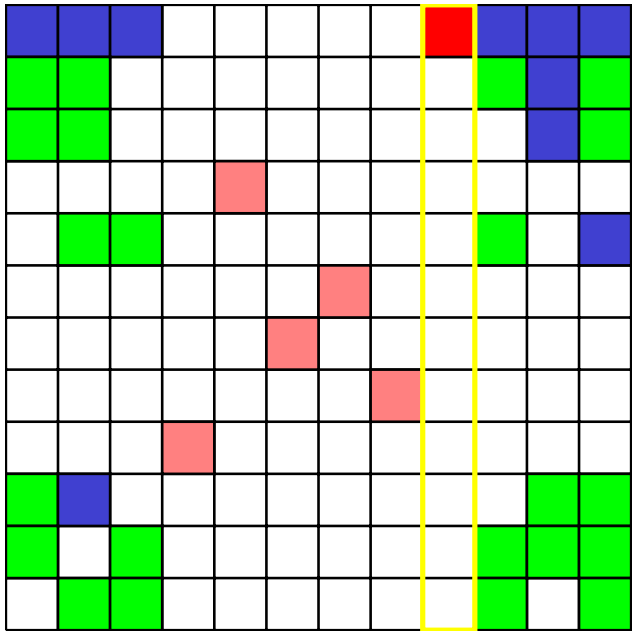

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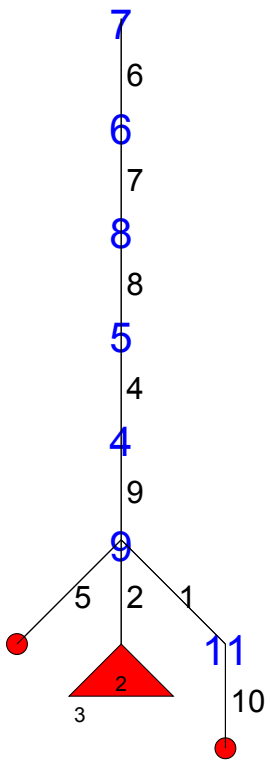

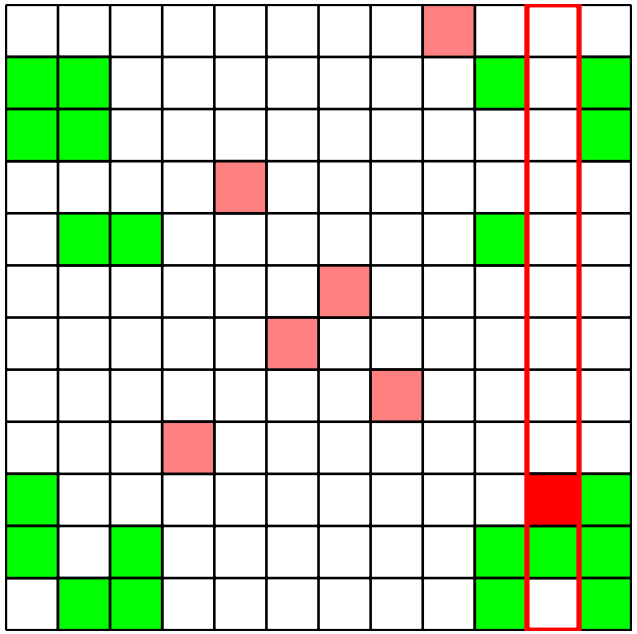

Back to Start
Skip Animation

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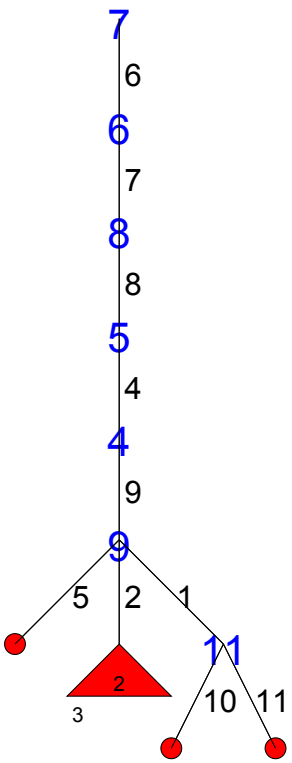
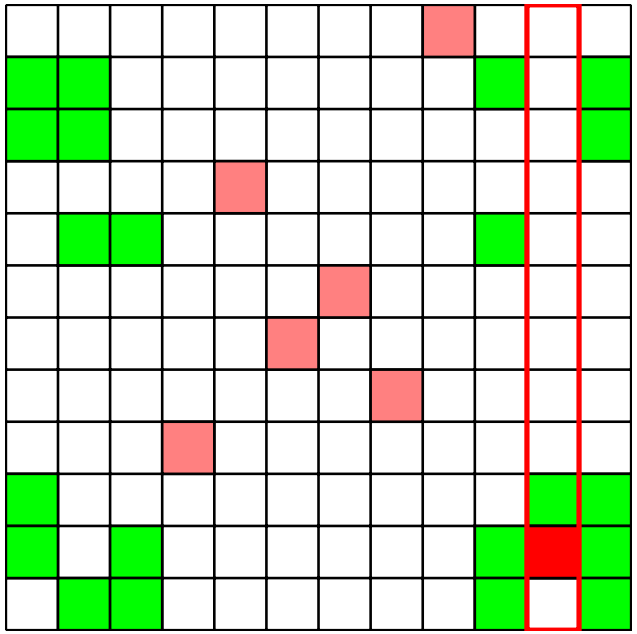
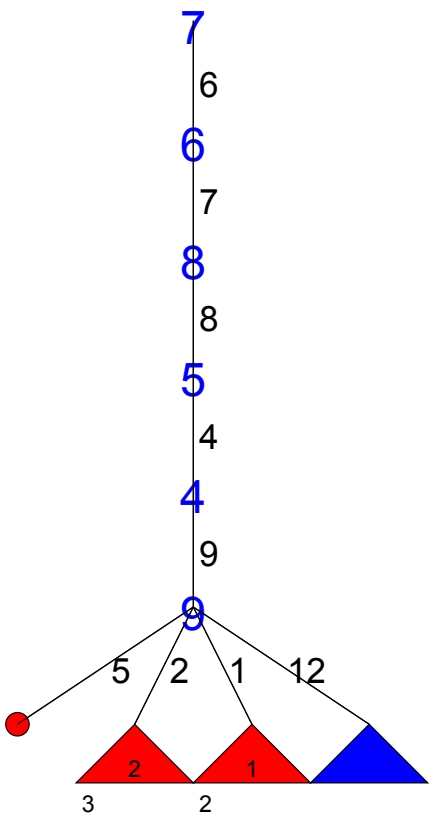
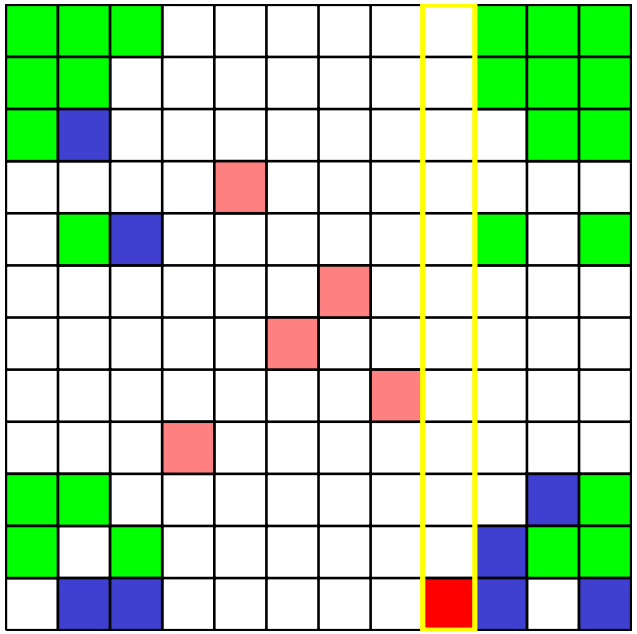




Back to Start
Skip Animation

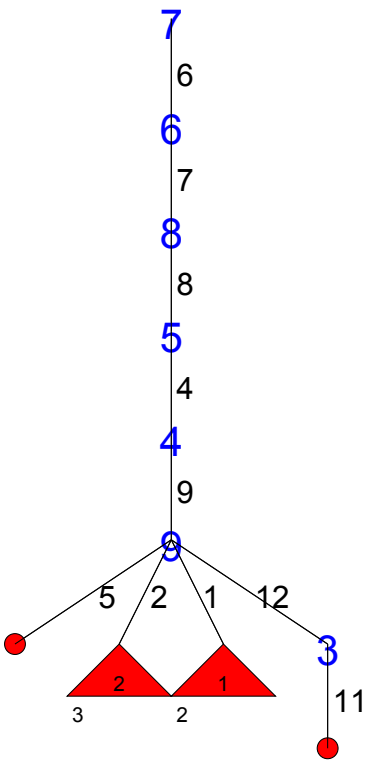
Labeling From Middle

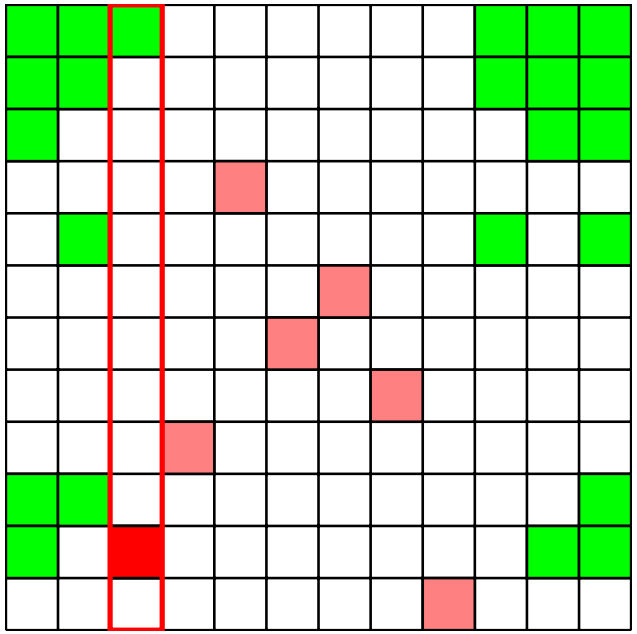




Back to Start
Skip Animation

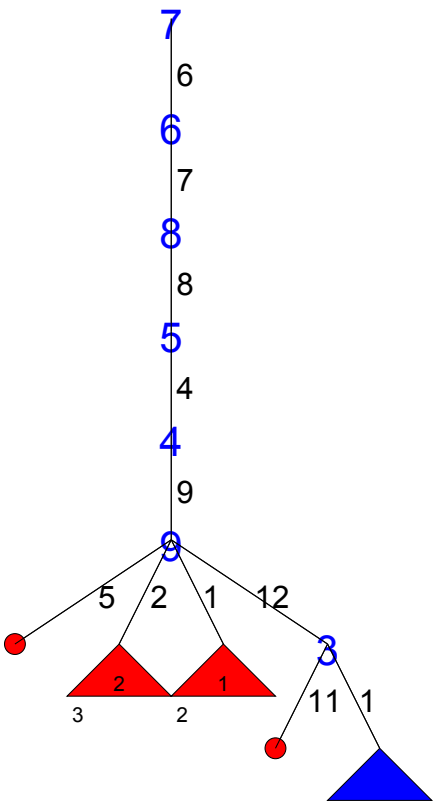
Labeling From Middle

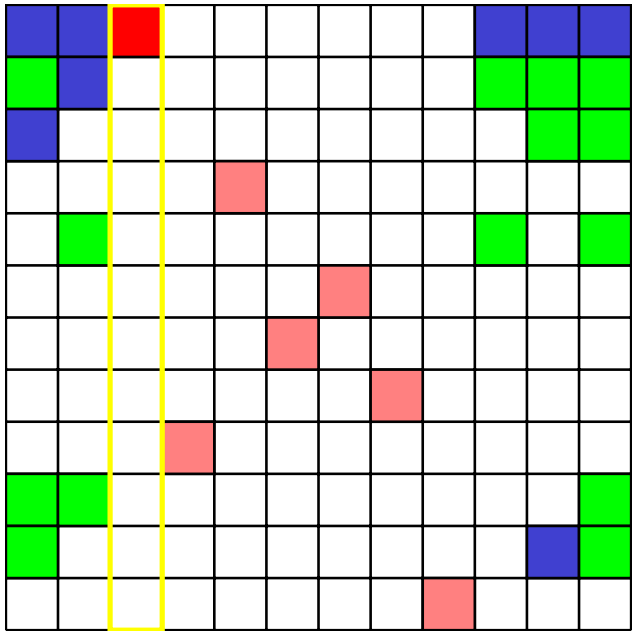




Back to Start
Skip Animation

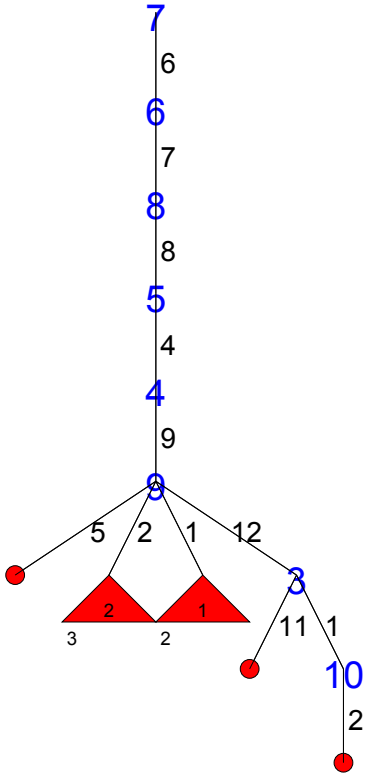
Labeling From Middle

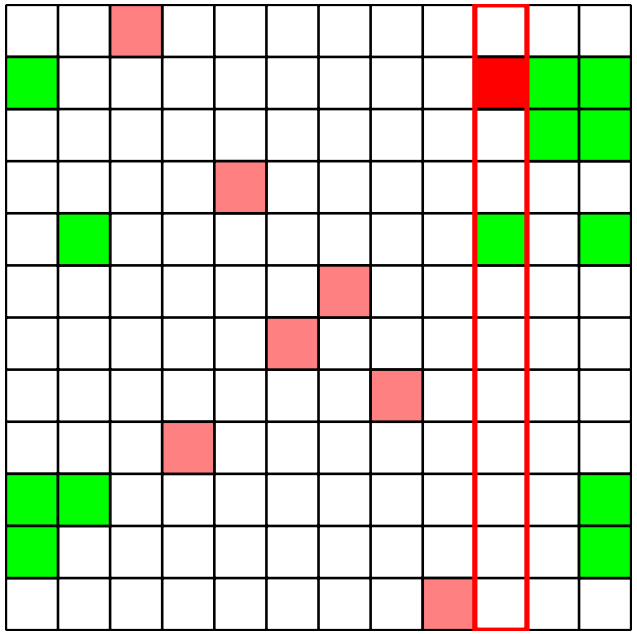




Back to Start
Skip Animation

Labeling From Middle





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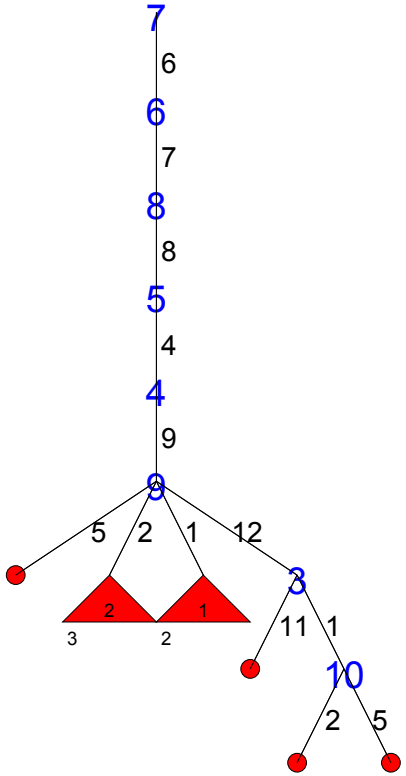

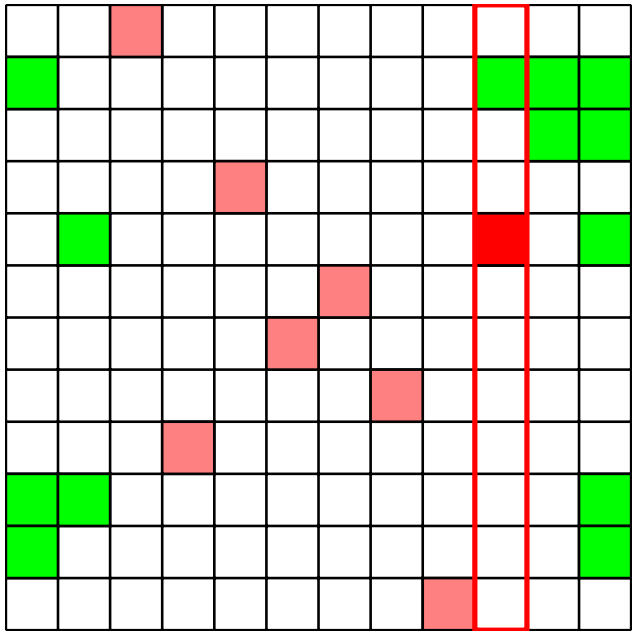

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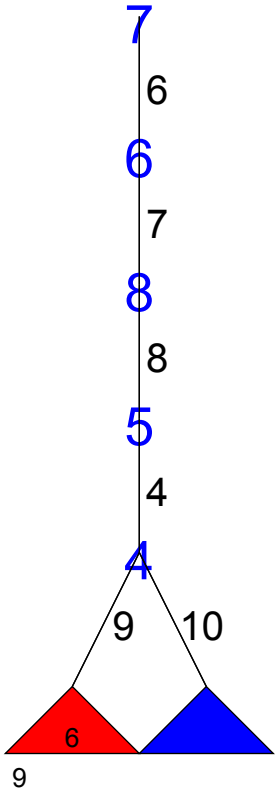

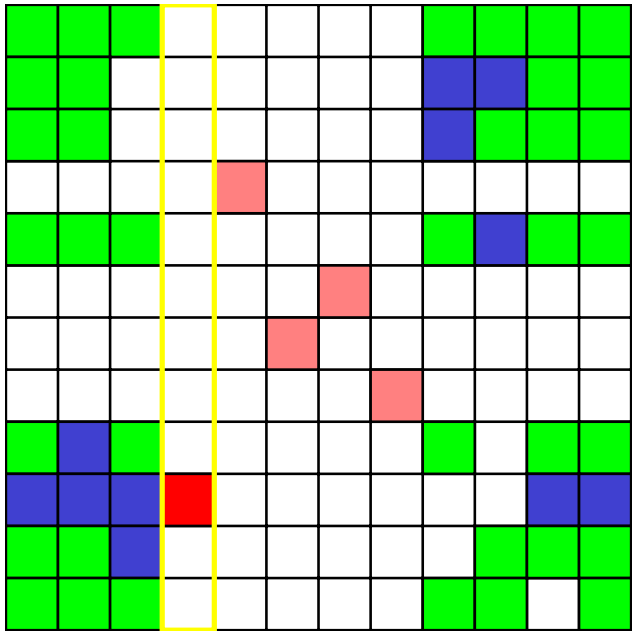

Back to Start
Skip Animation

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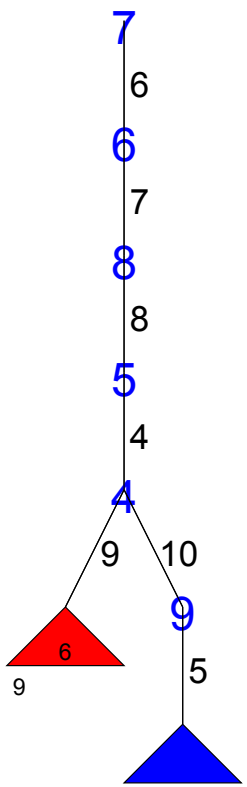

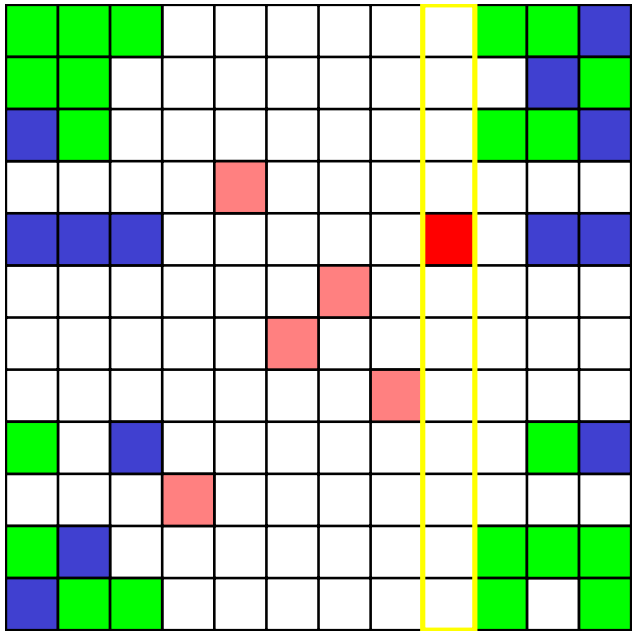

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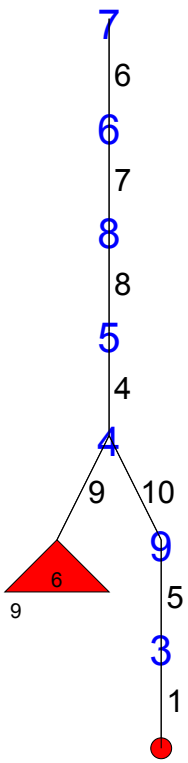

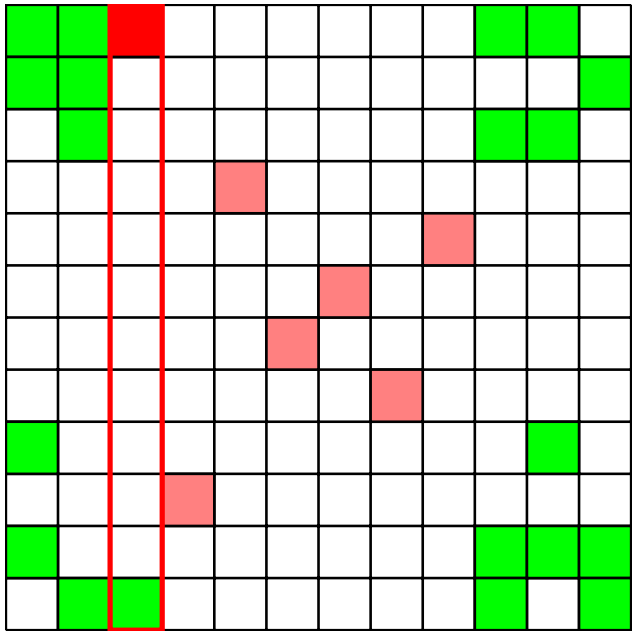

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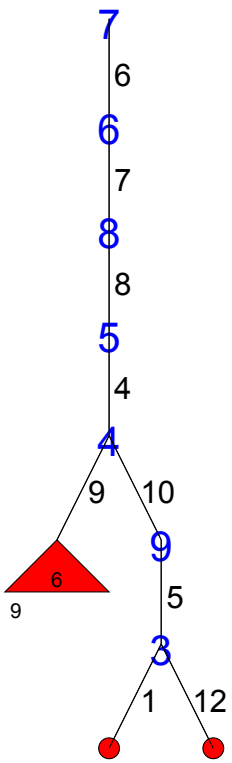

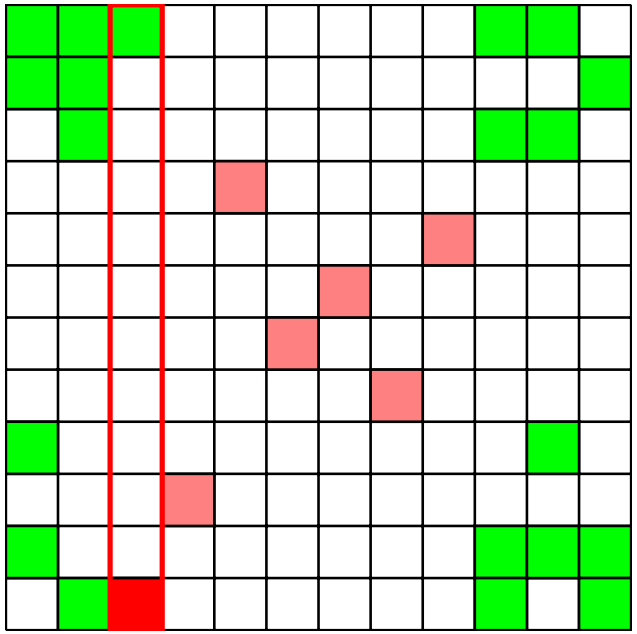

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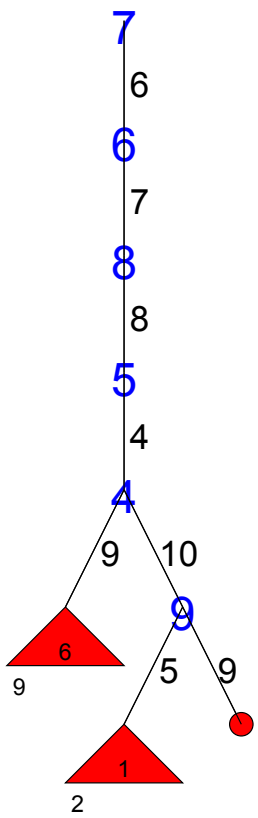

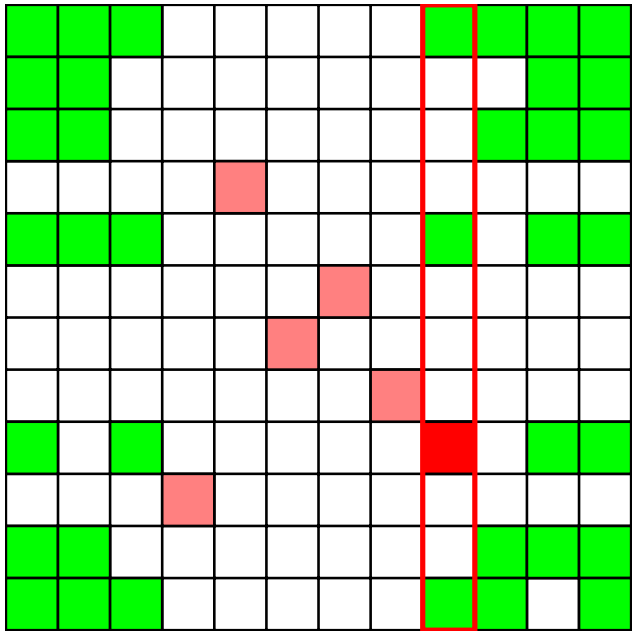

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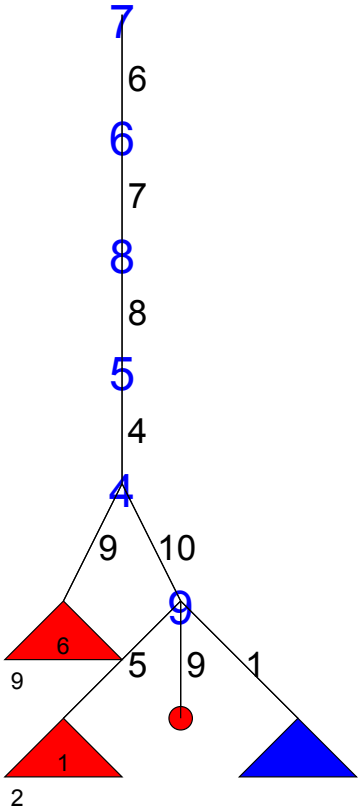

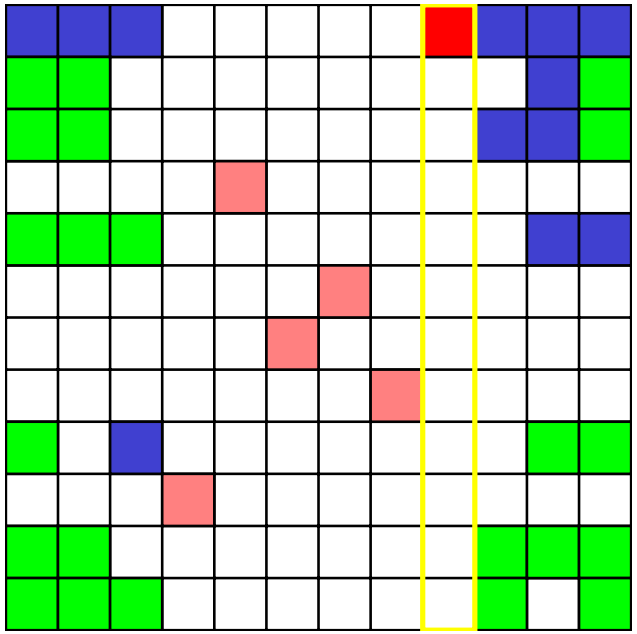

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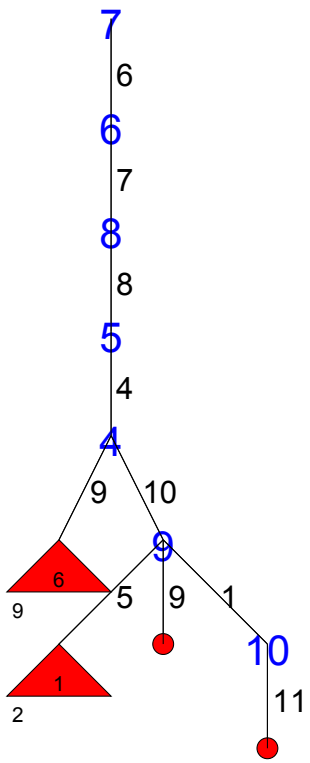

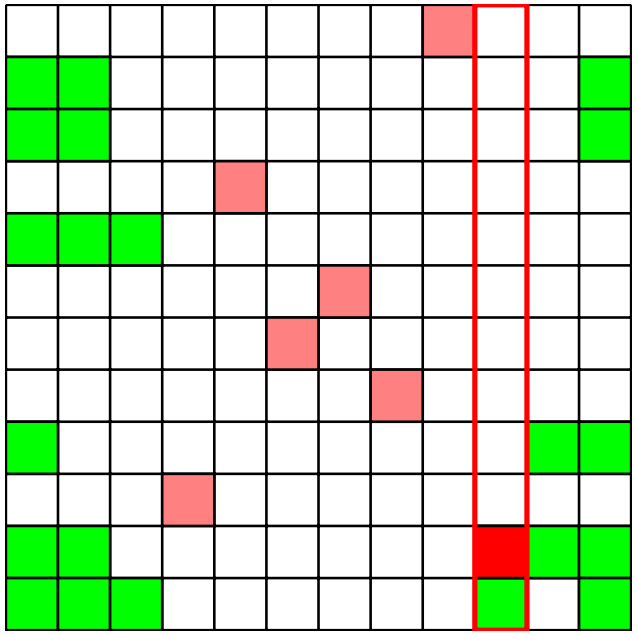

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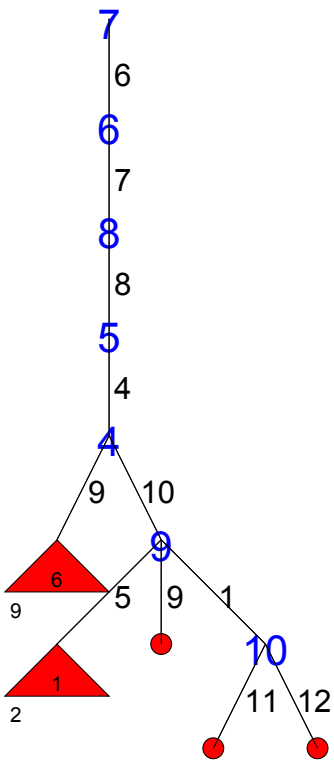

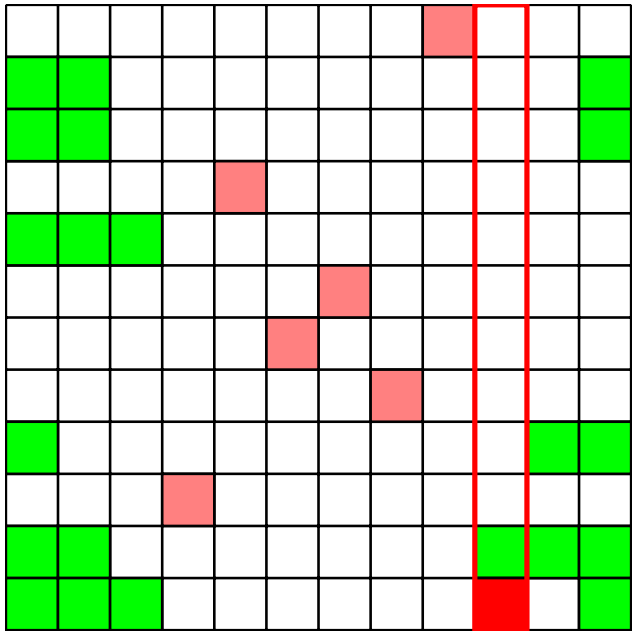

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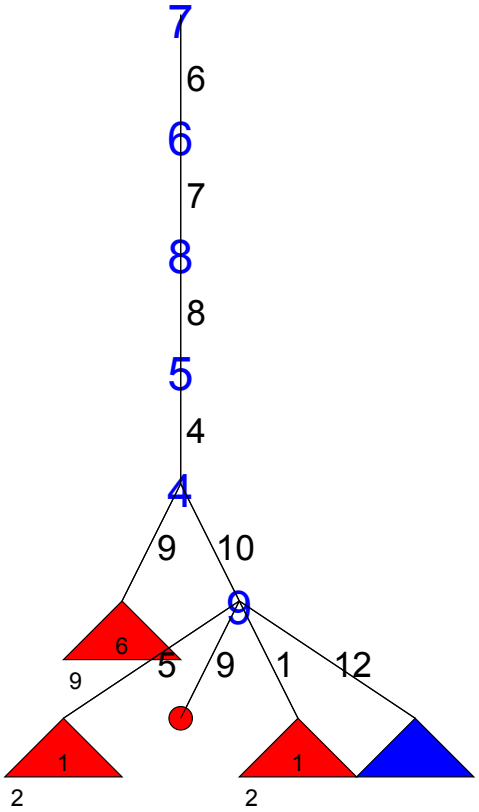

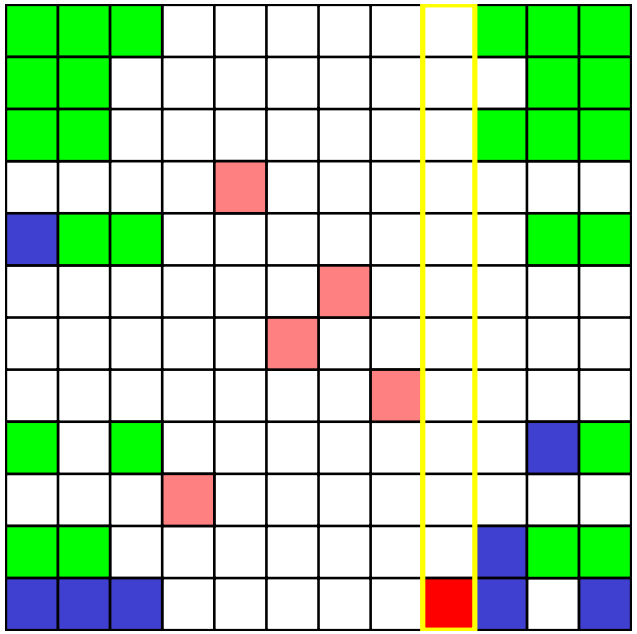

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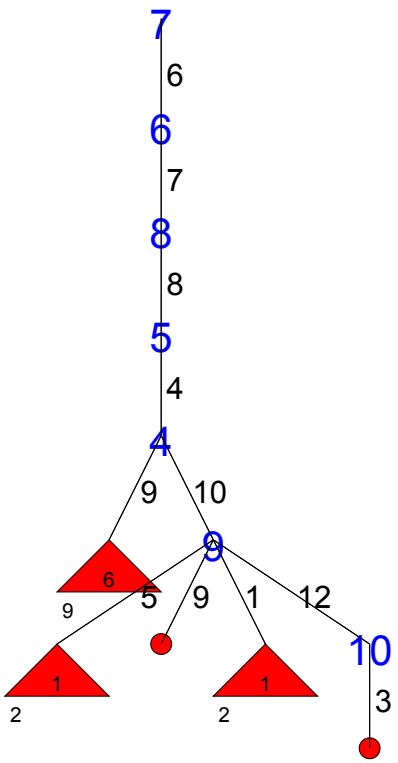

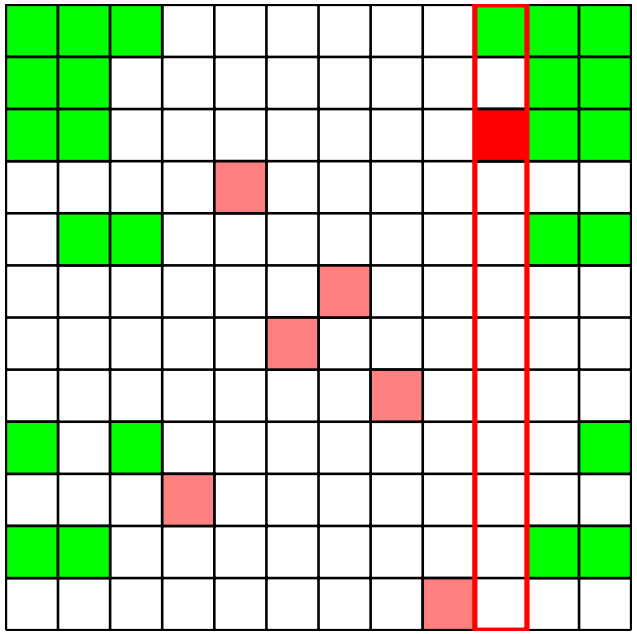

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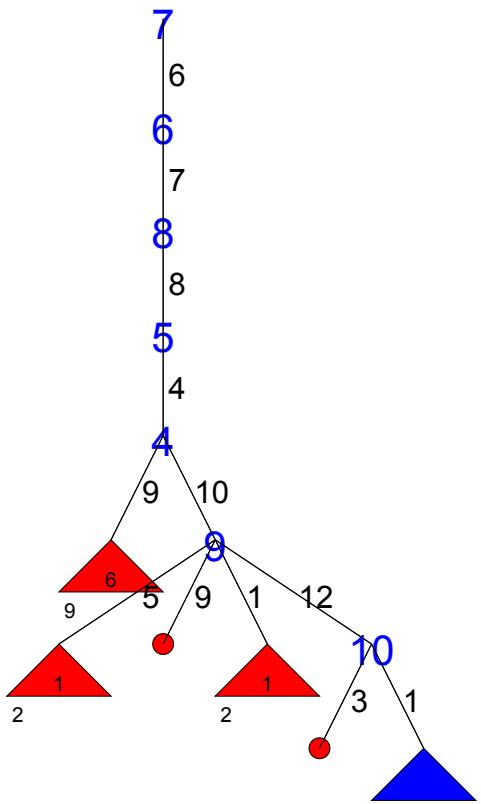

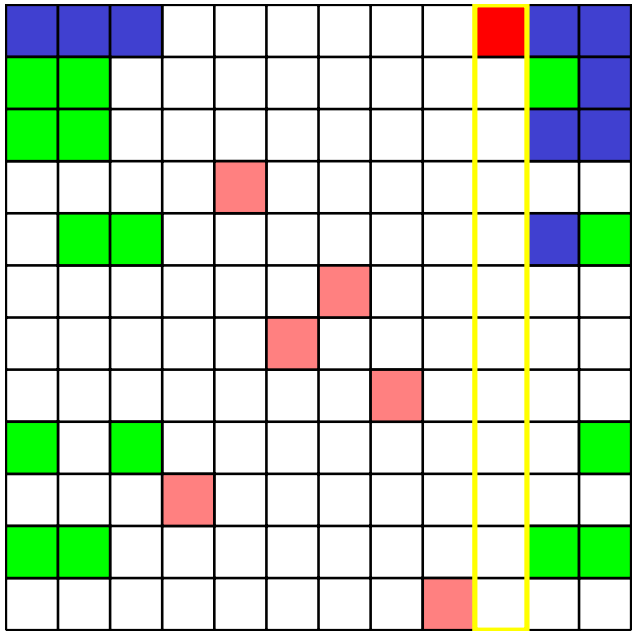

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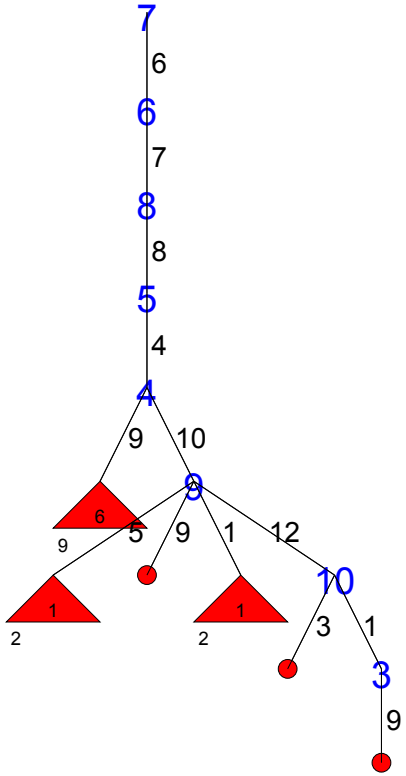

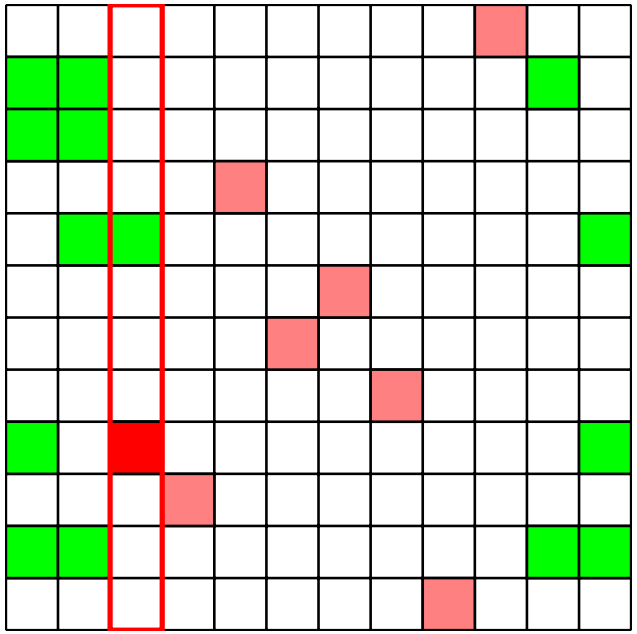

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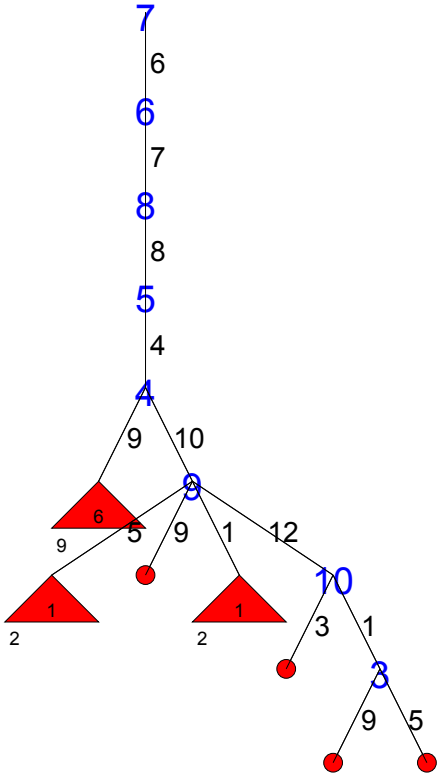

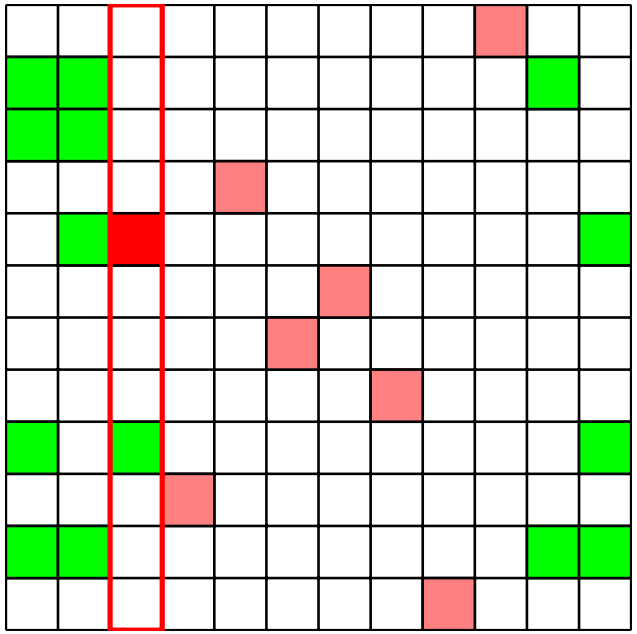

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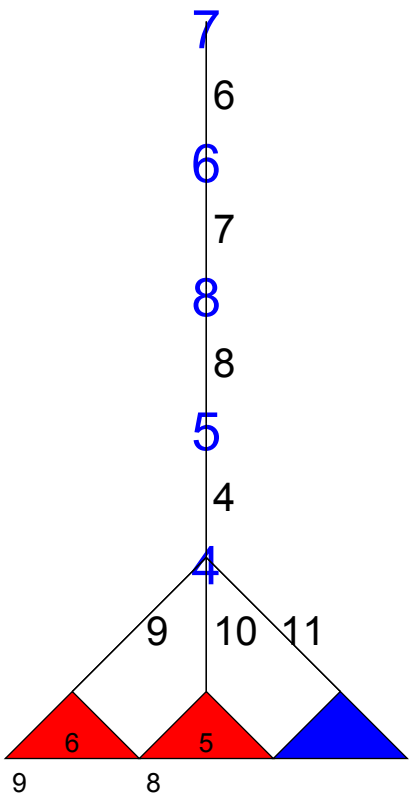

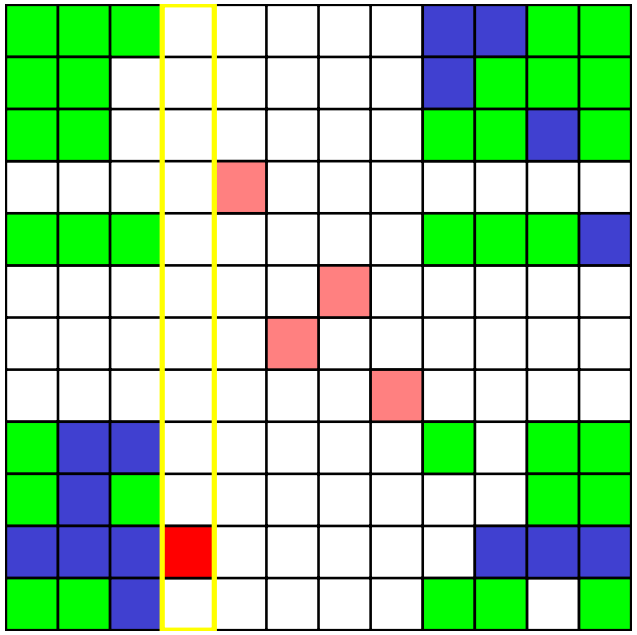

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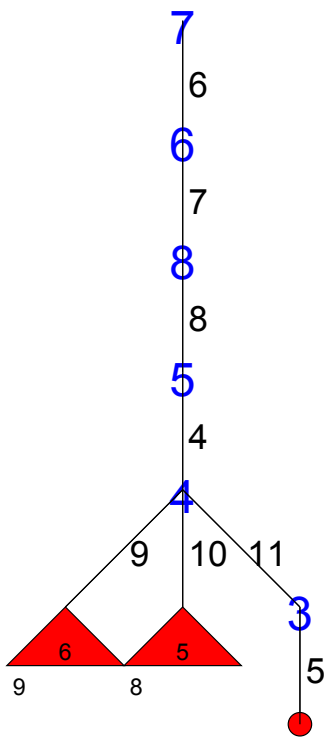

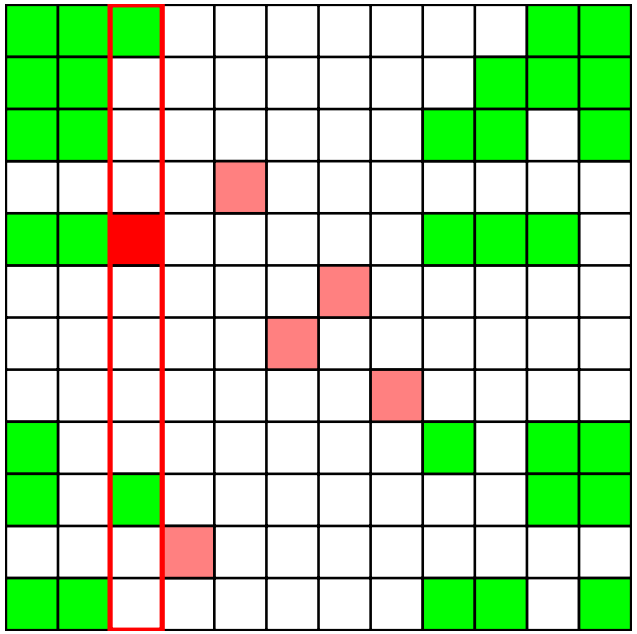

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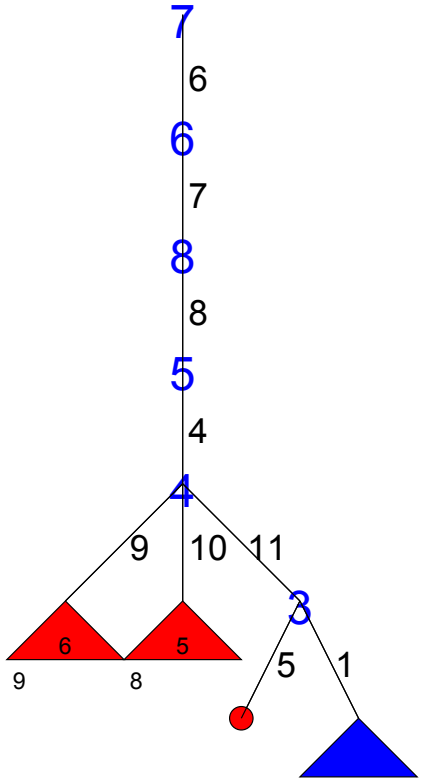

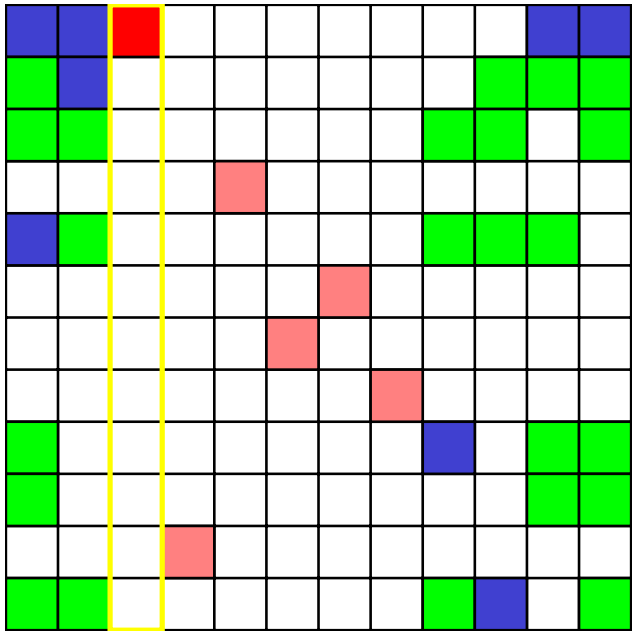

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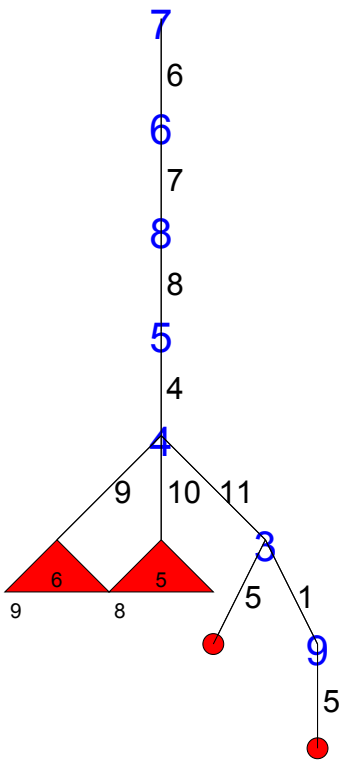

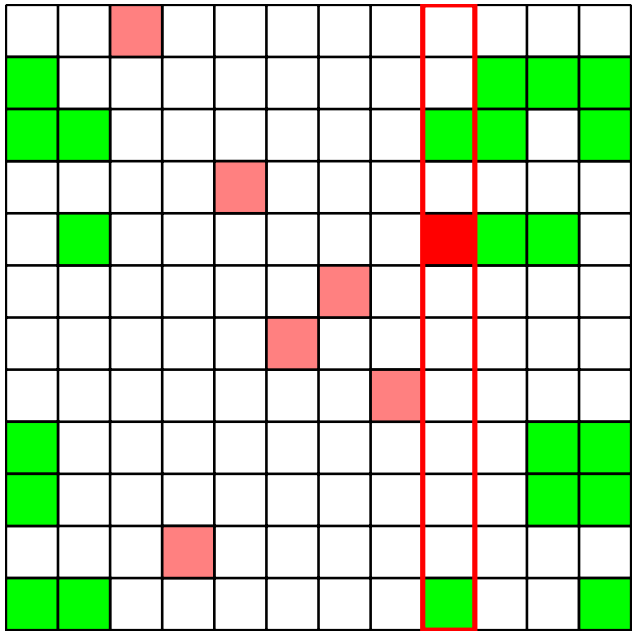

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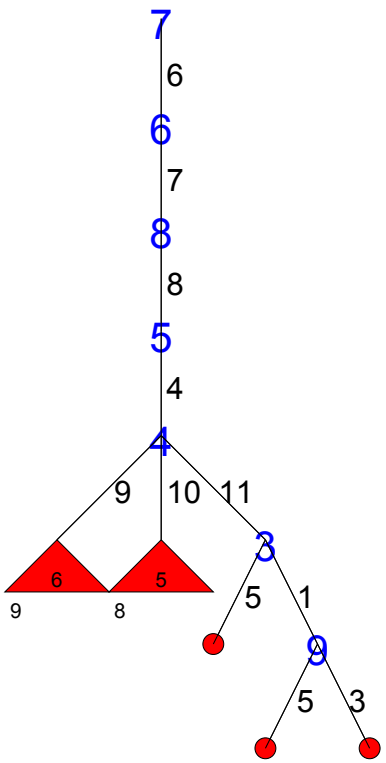

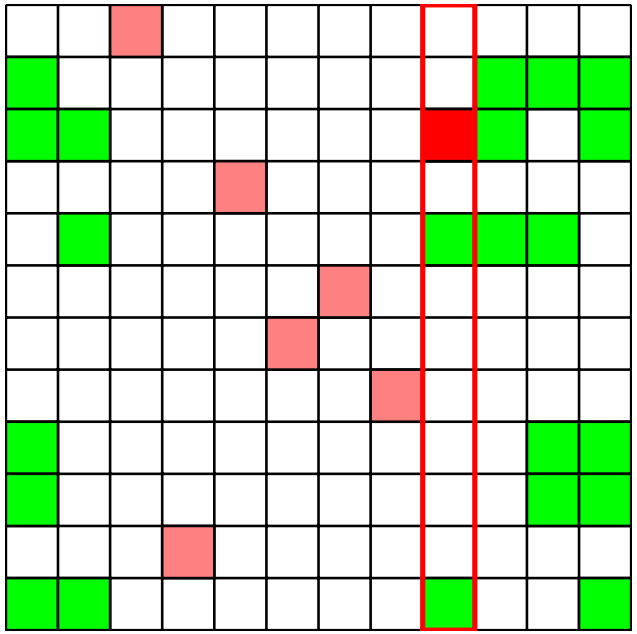

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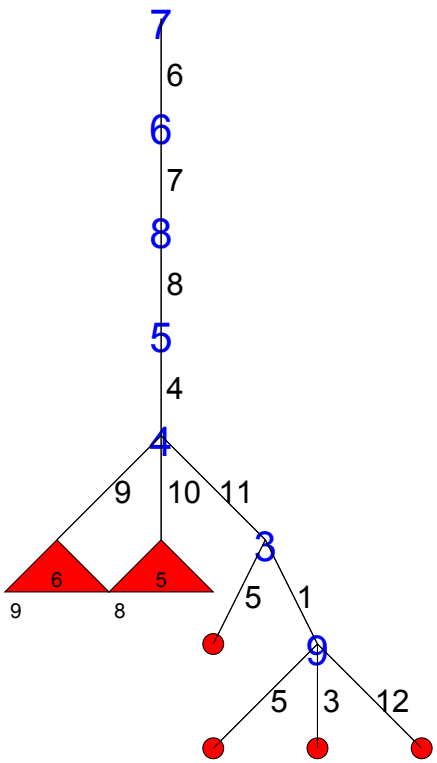

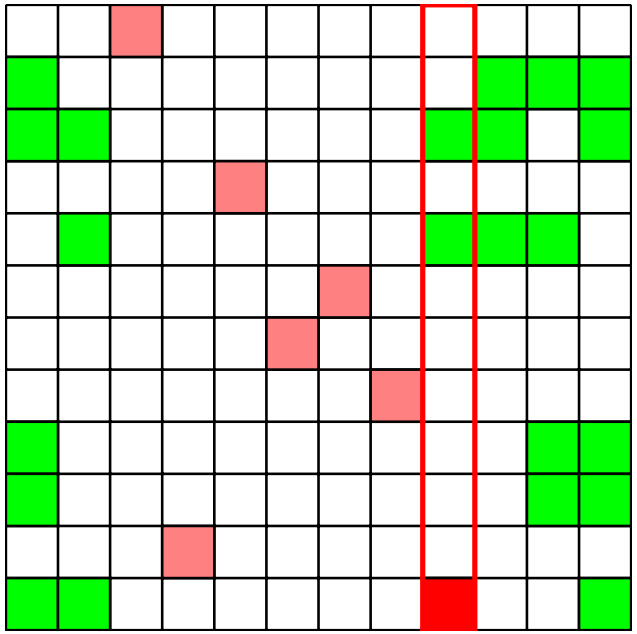

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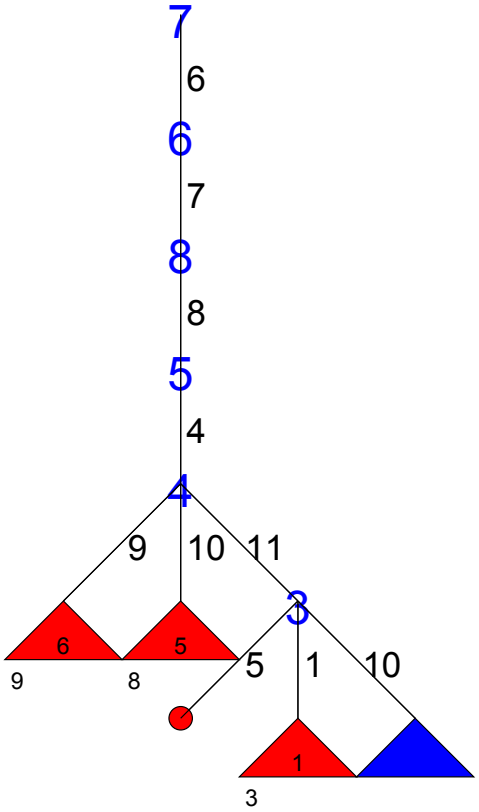

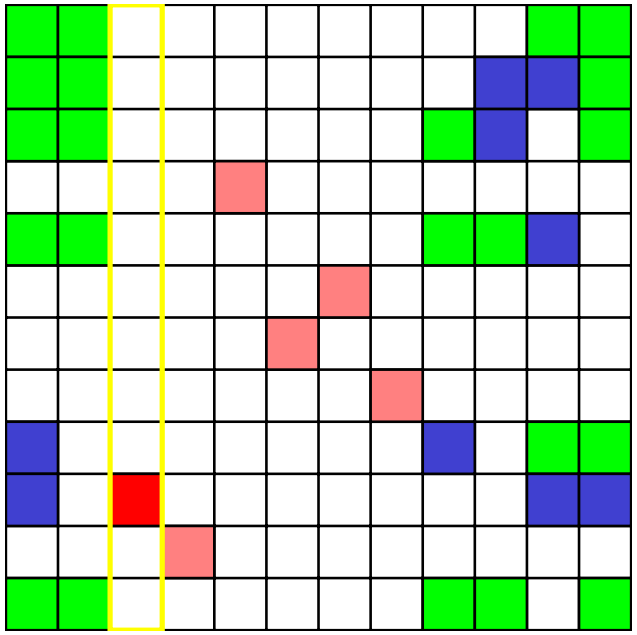

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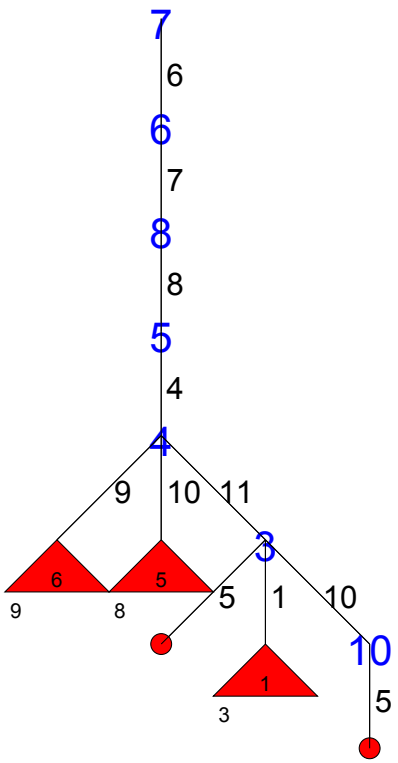

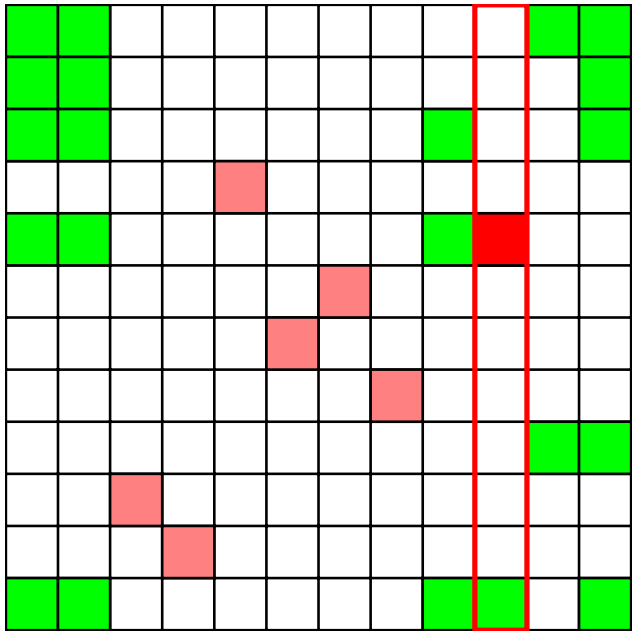

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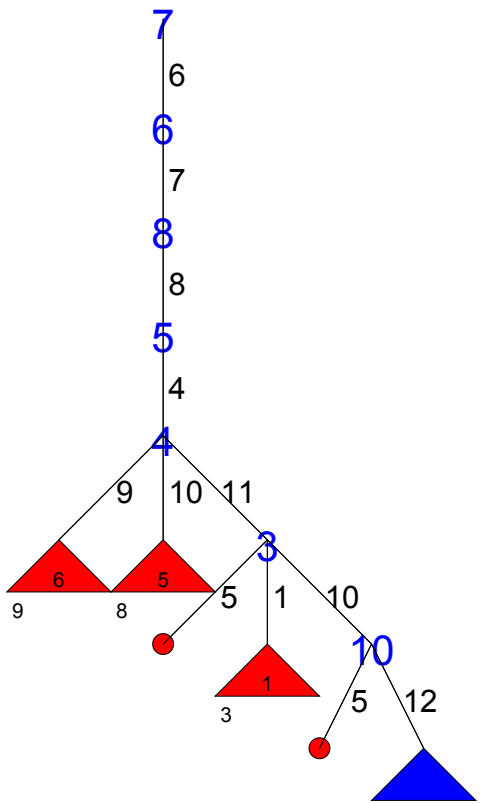

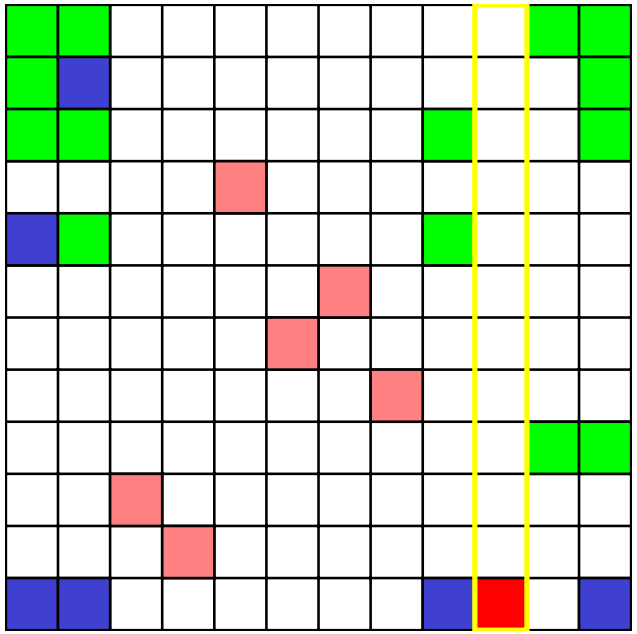

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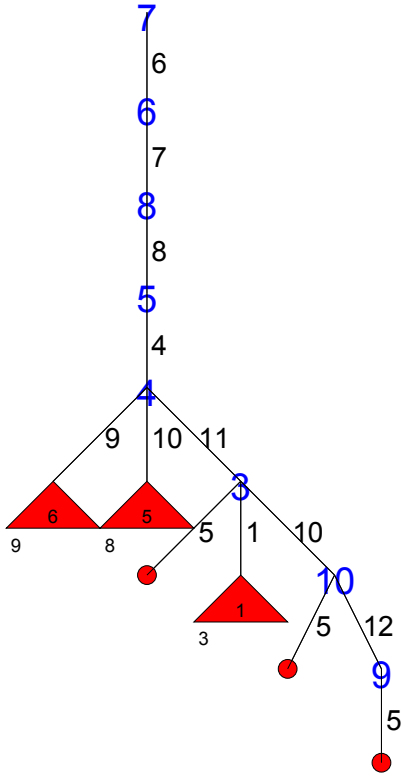

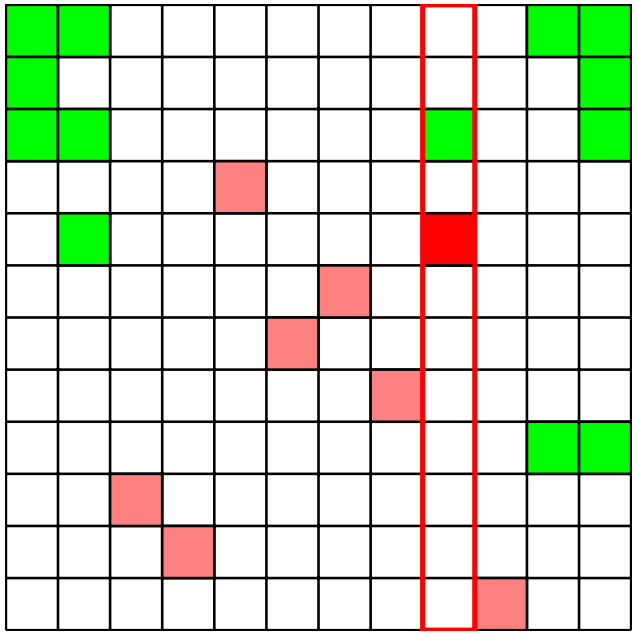

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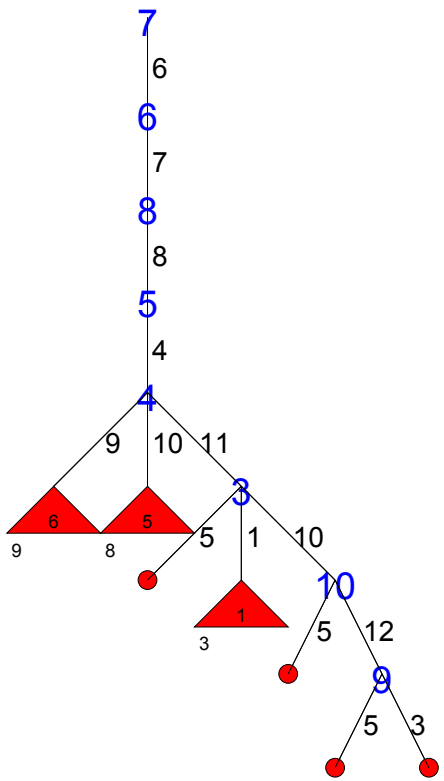

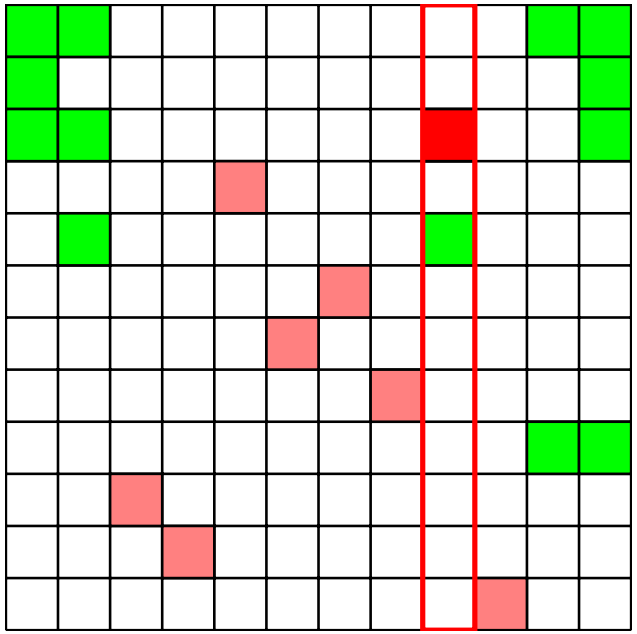

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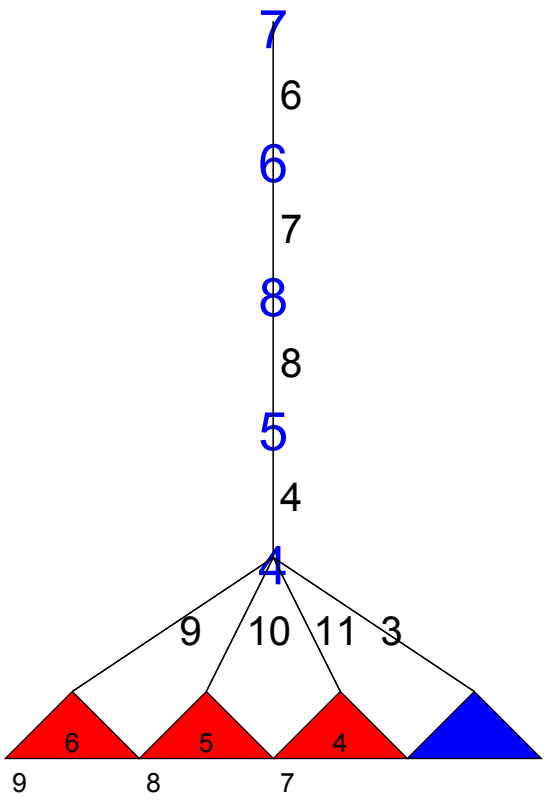

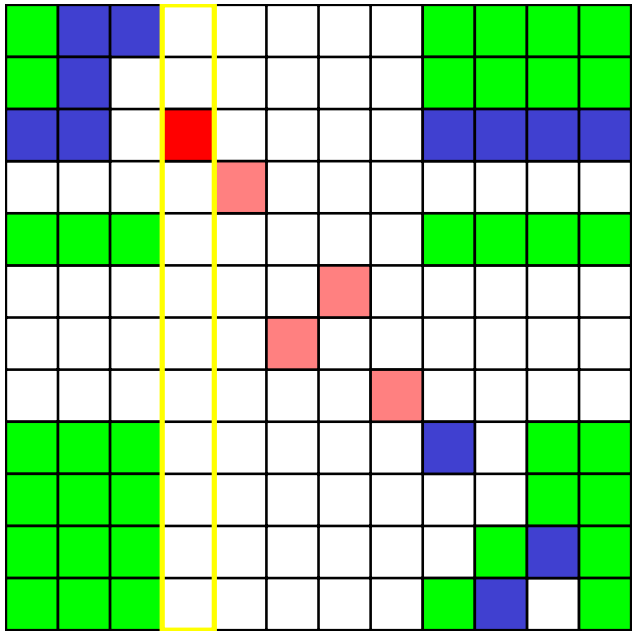

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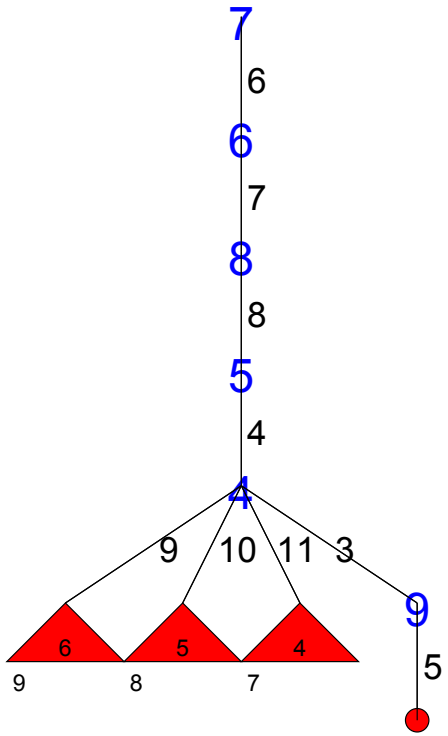

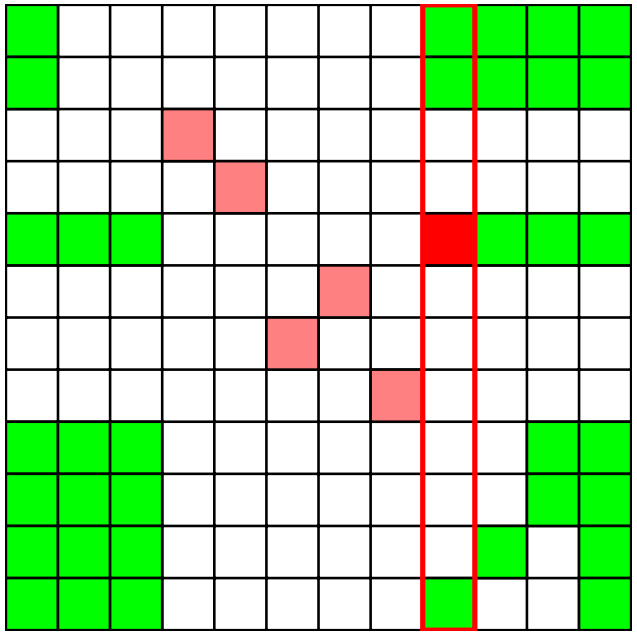

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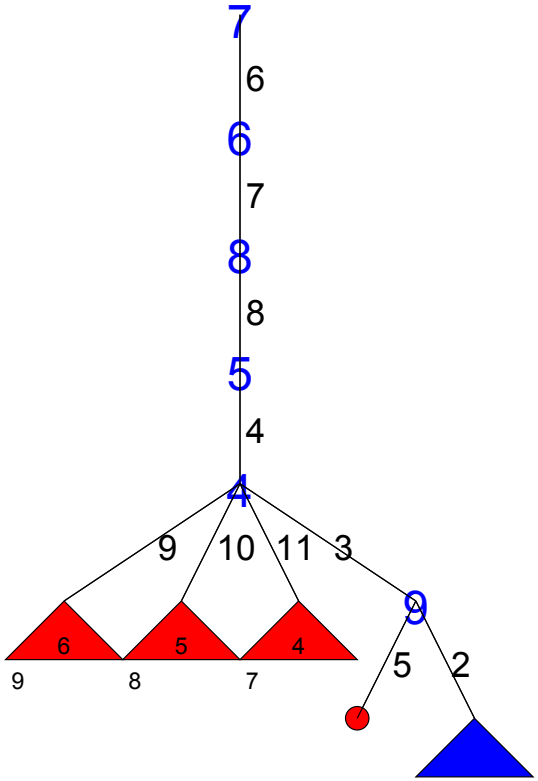

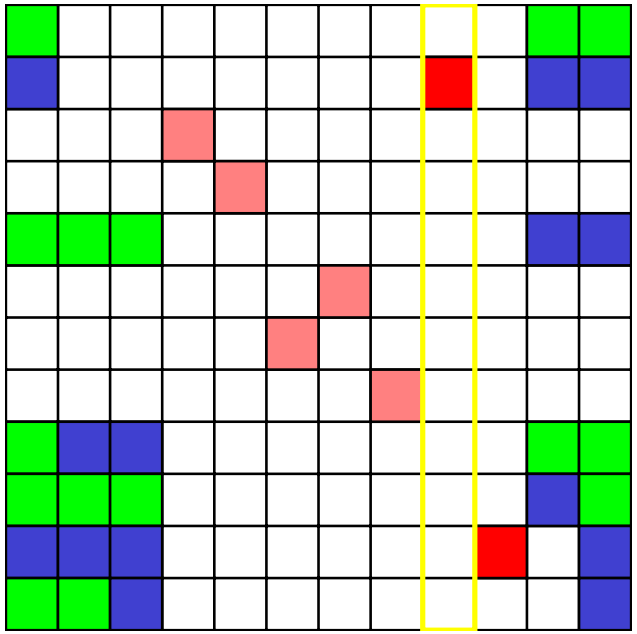

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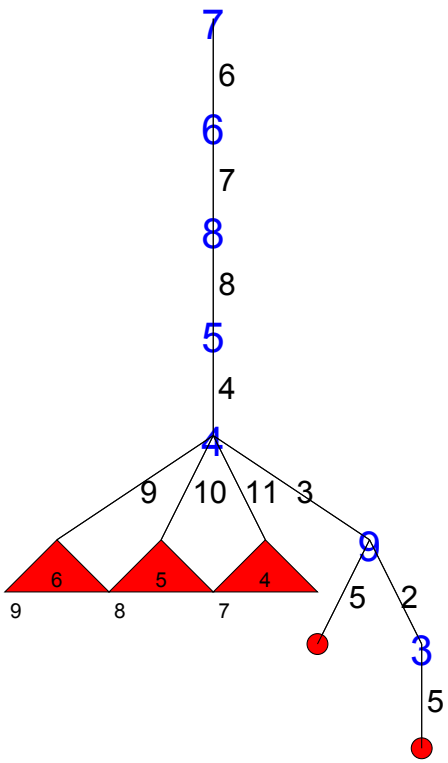

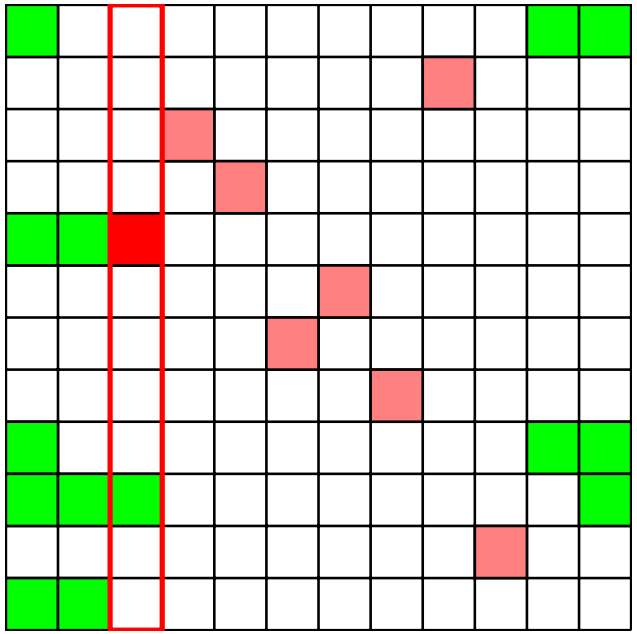

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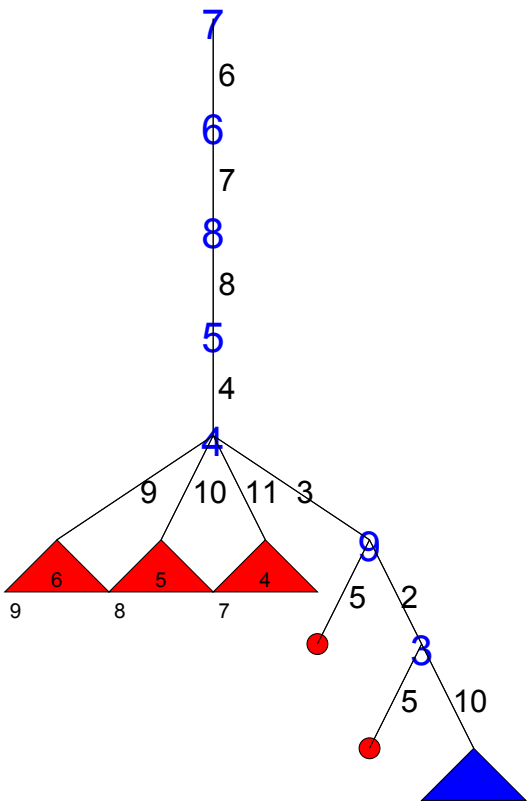

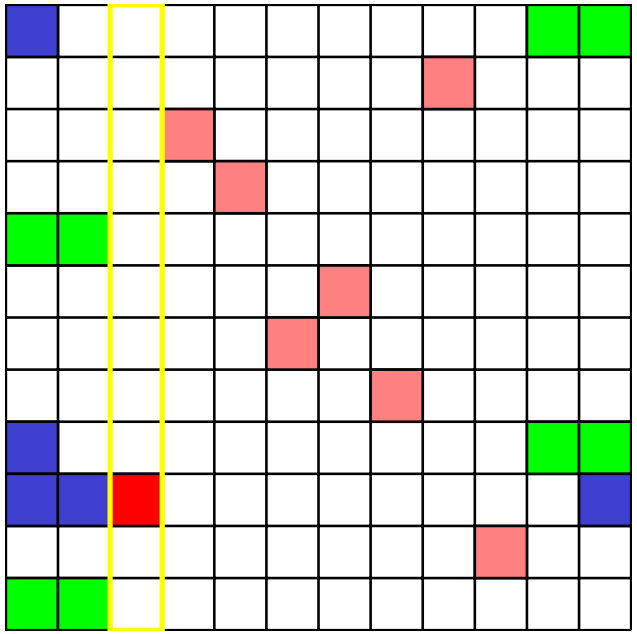

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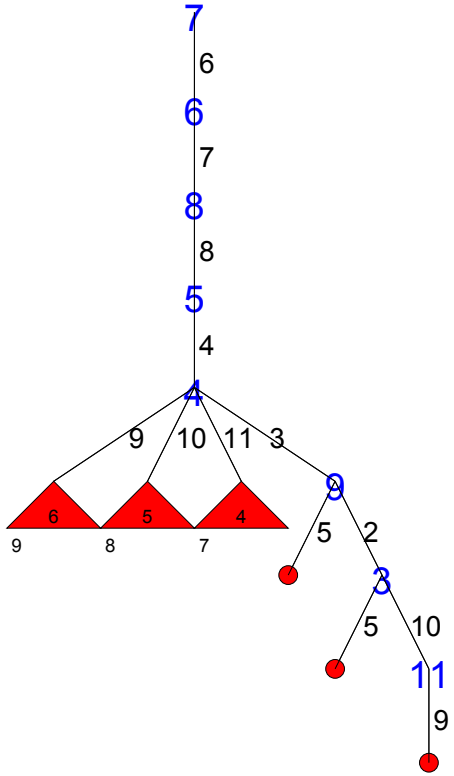

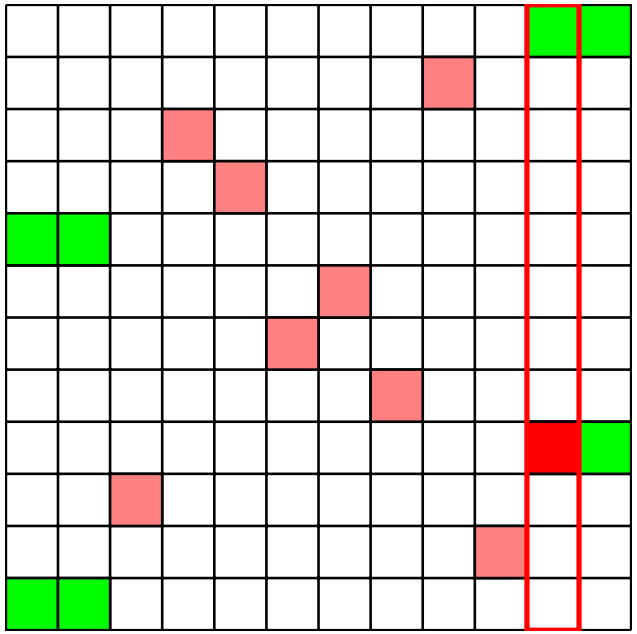

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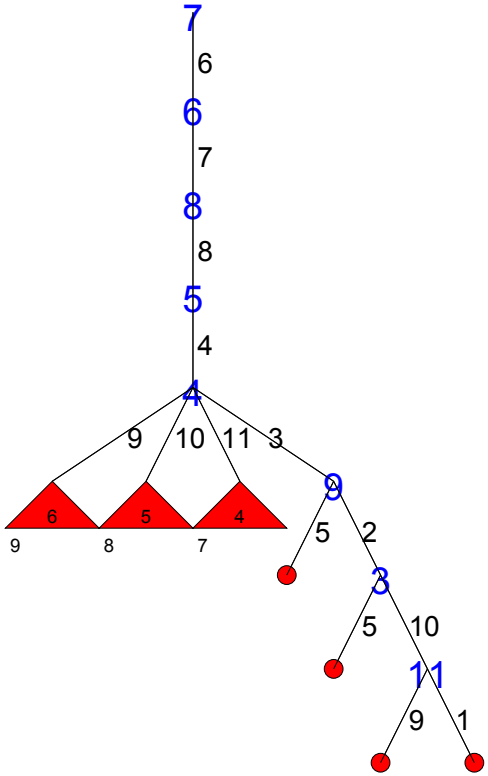

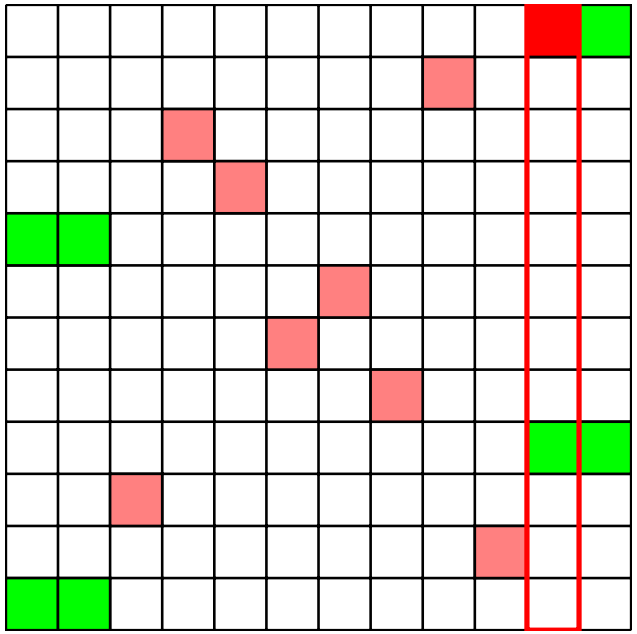

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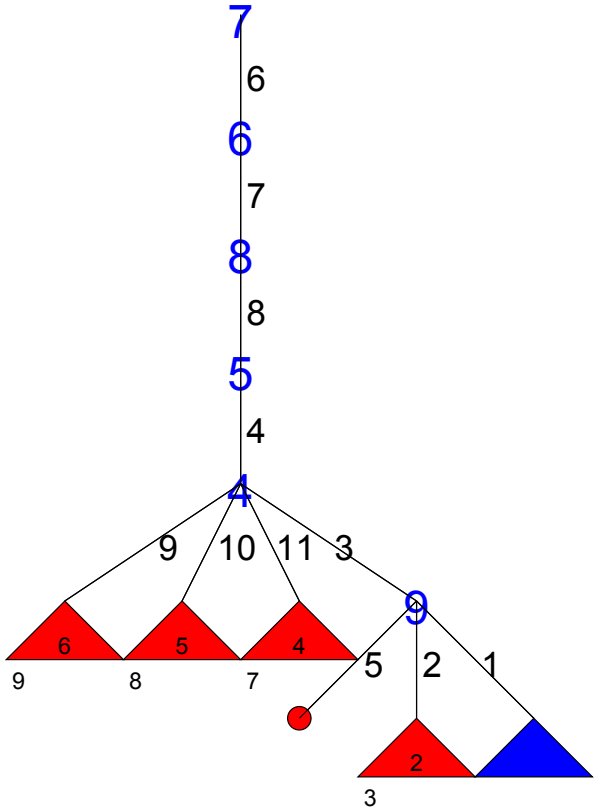

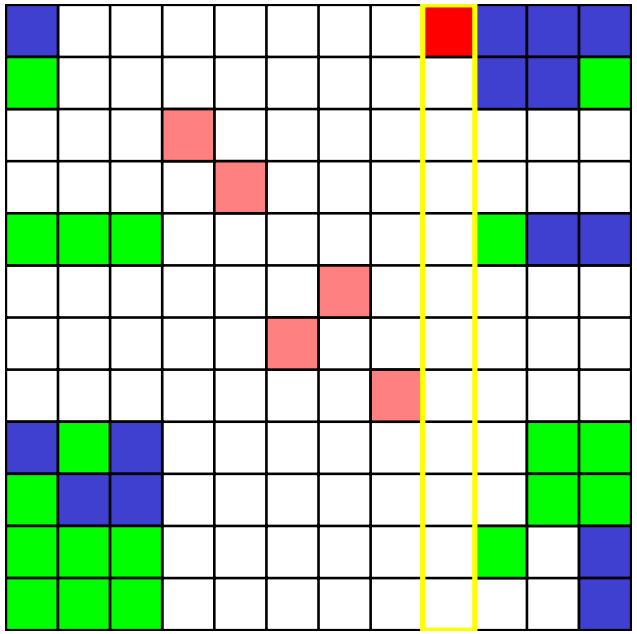

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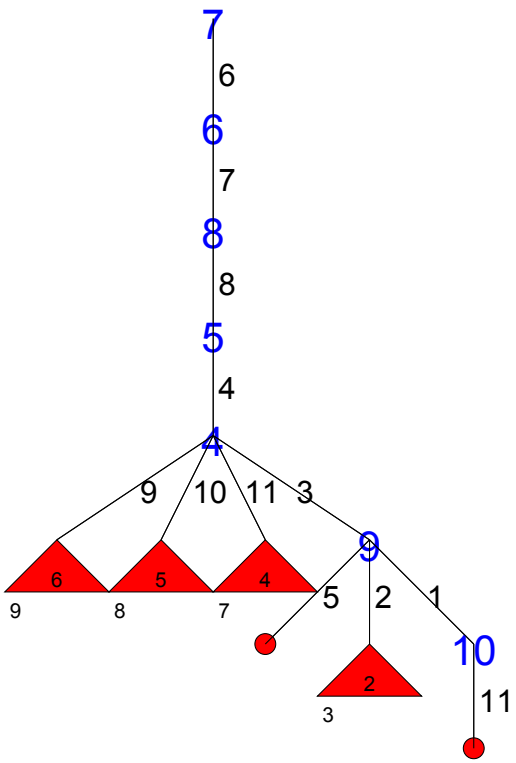

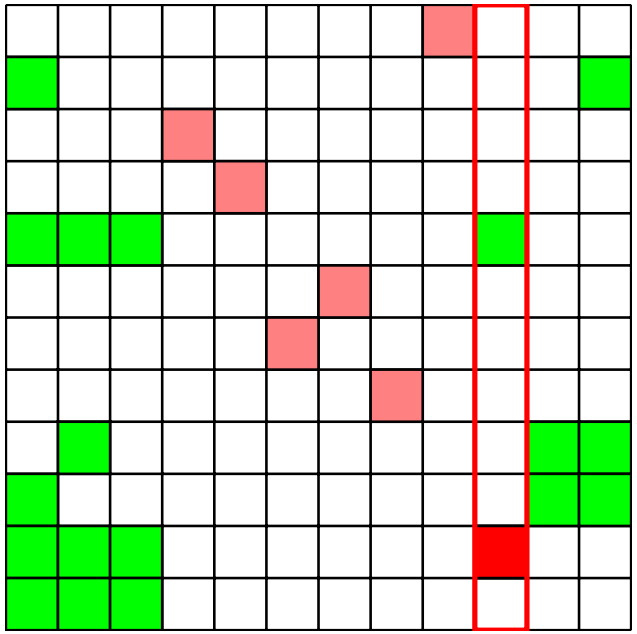

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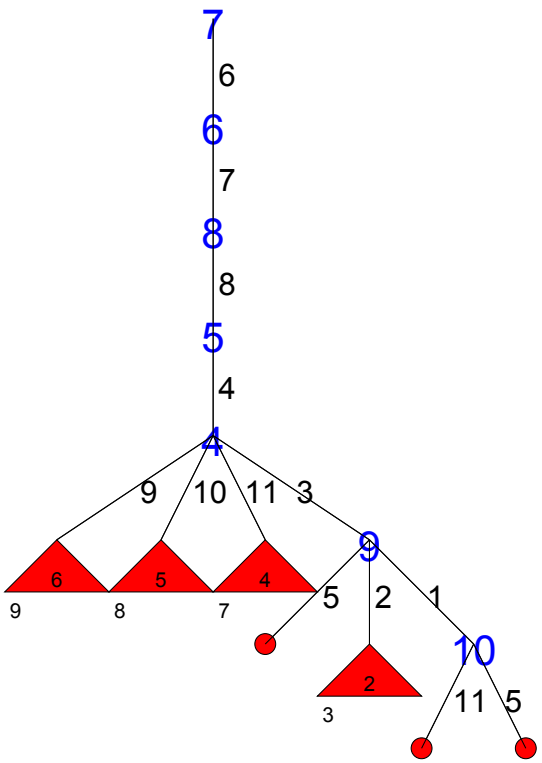

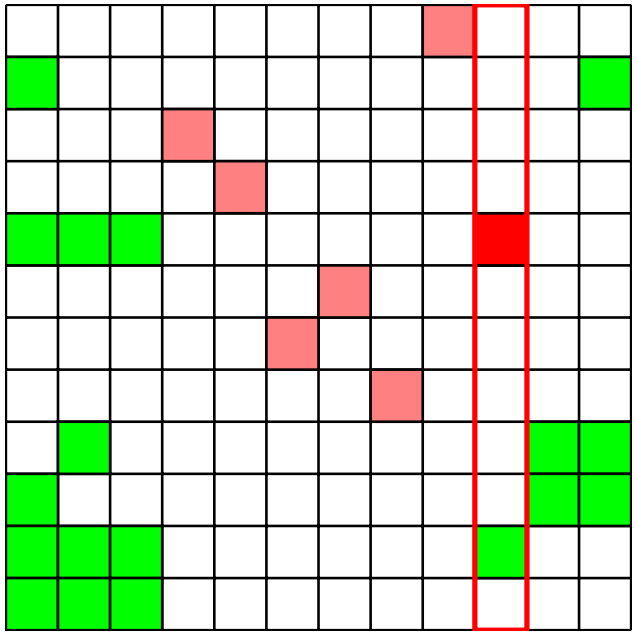

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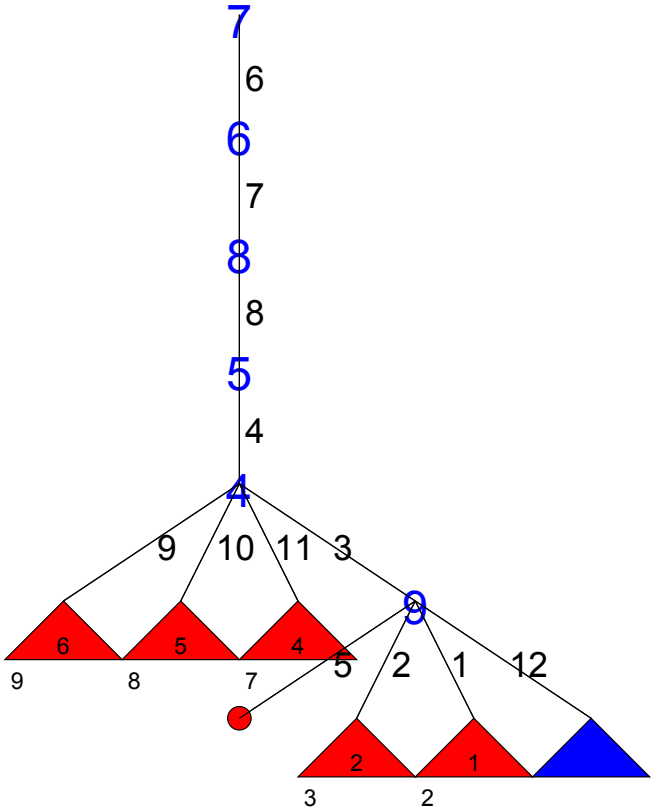

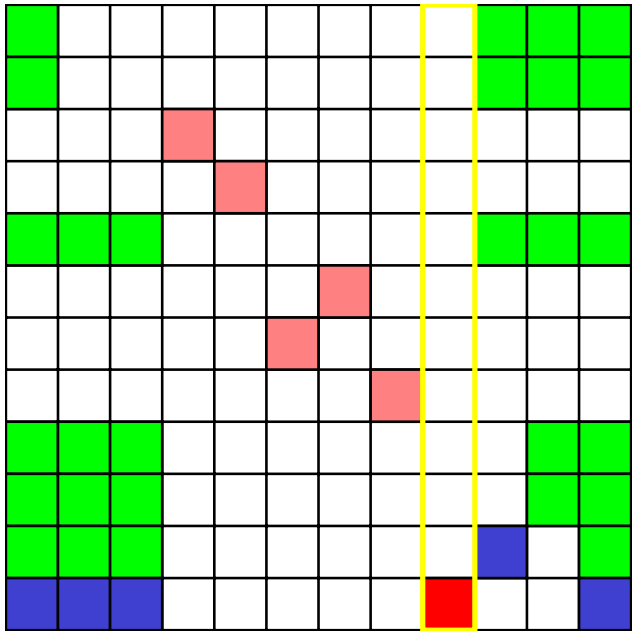

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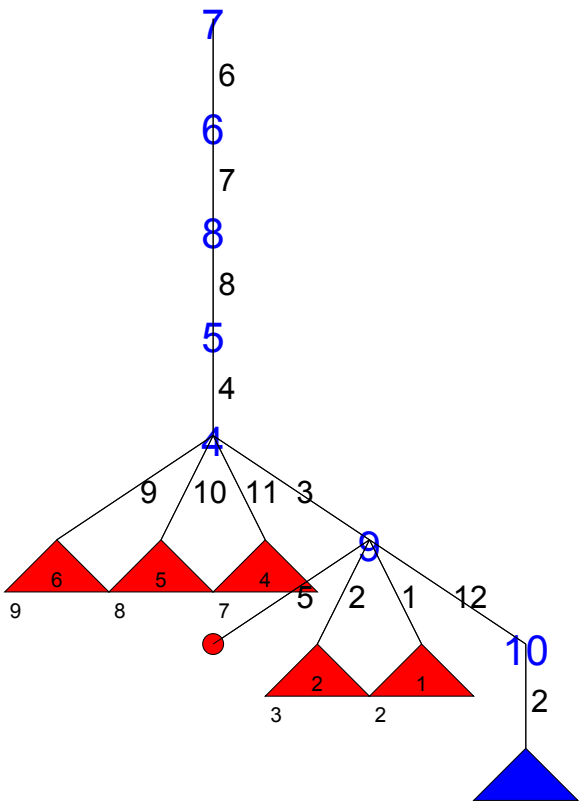

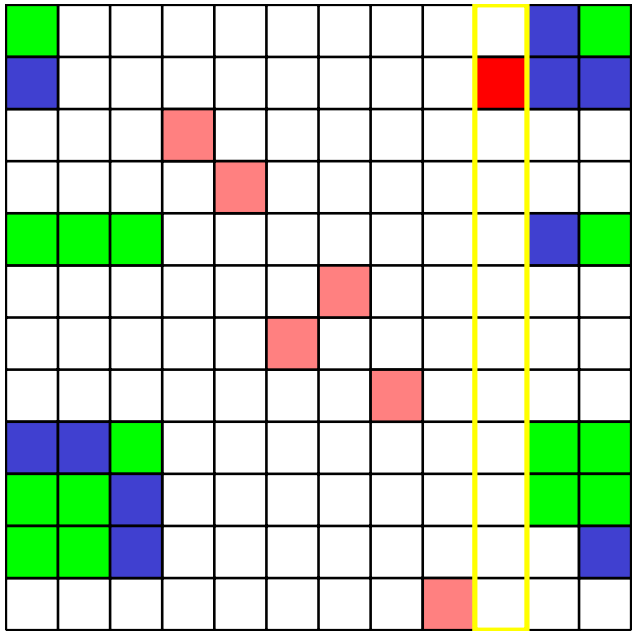

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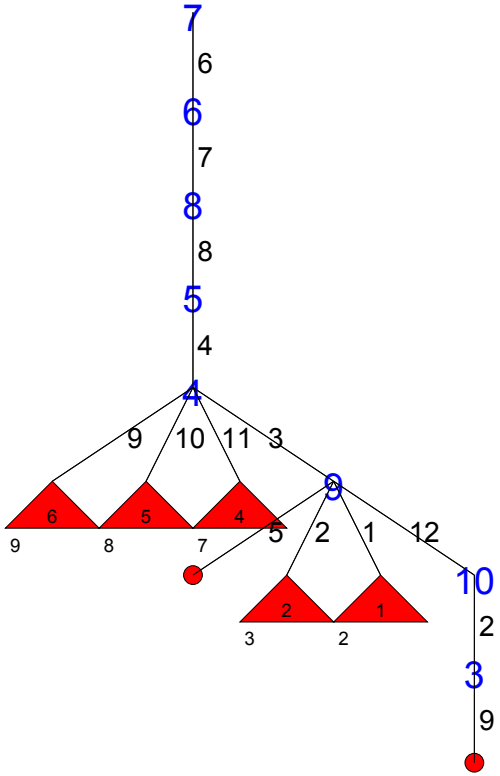

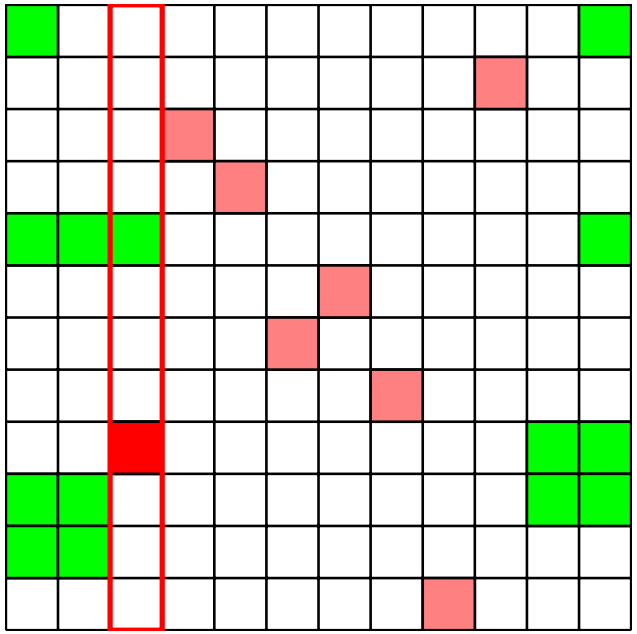

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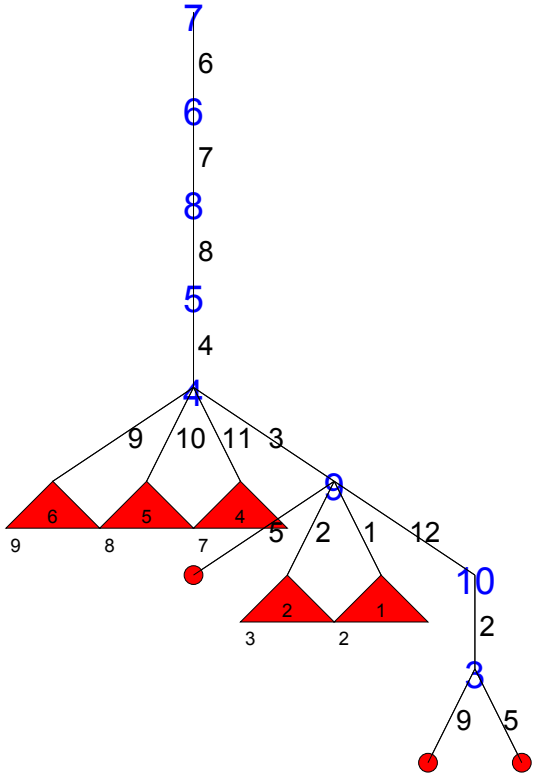

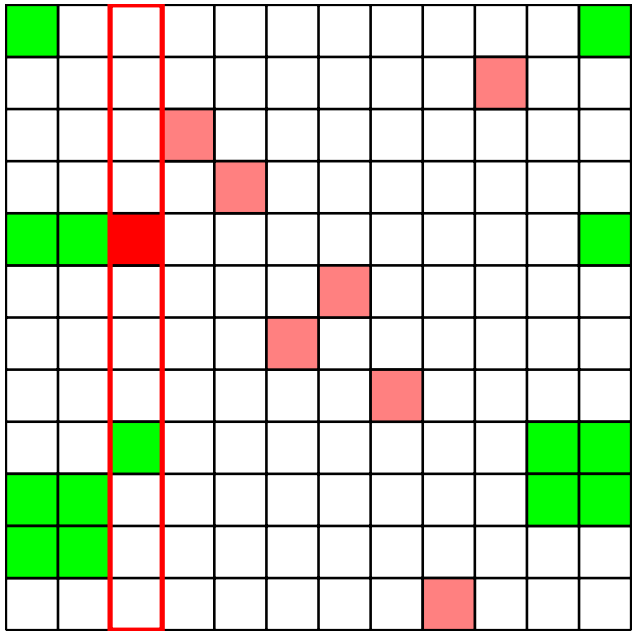

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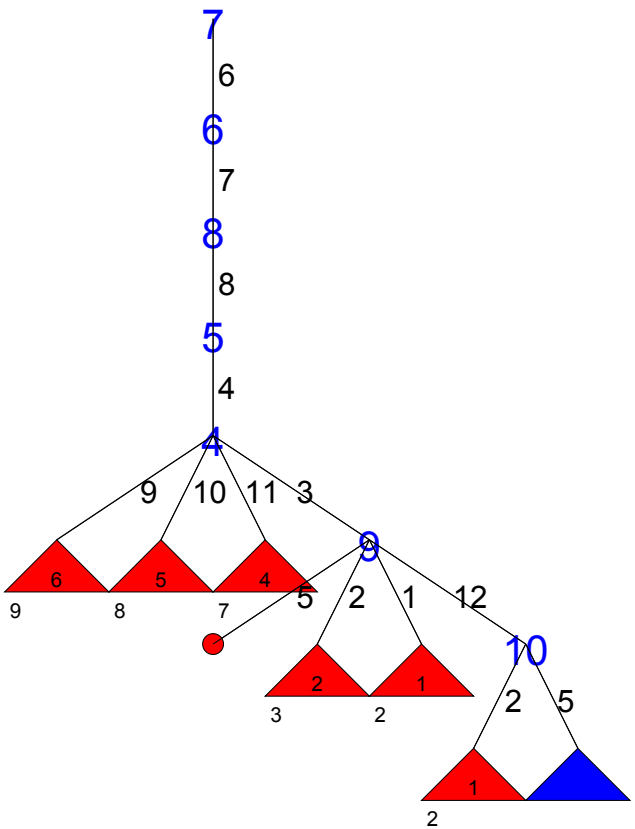

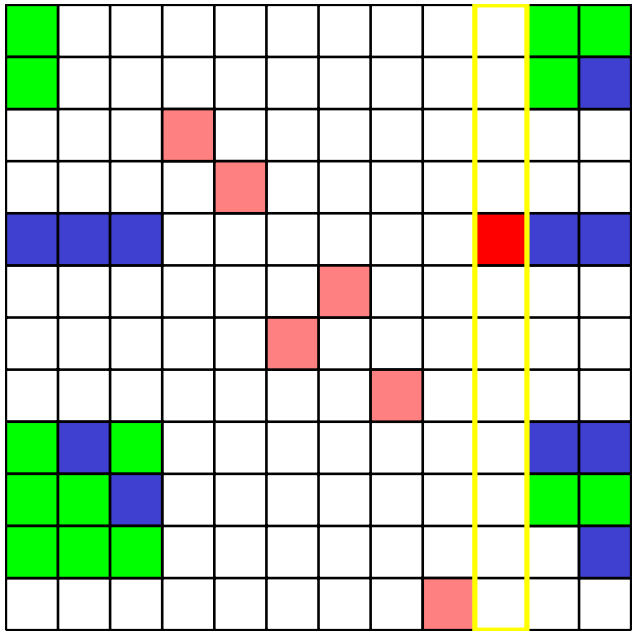

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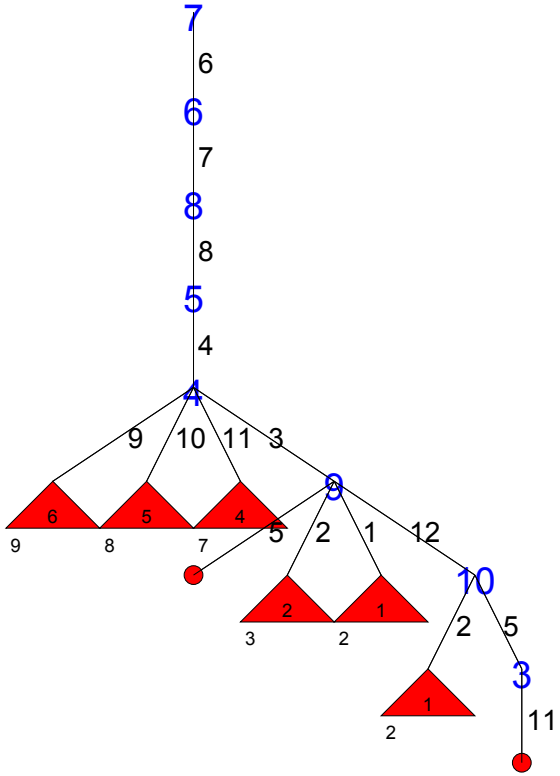

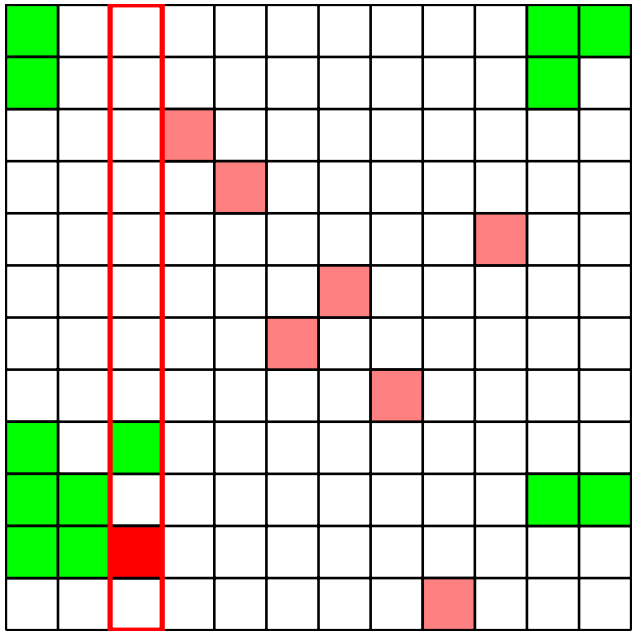

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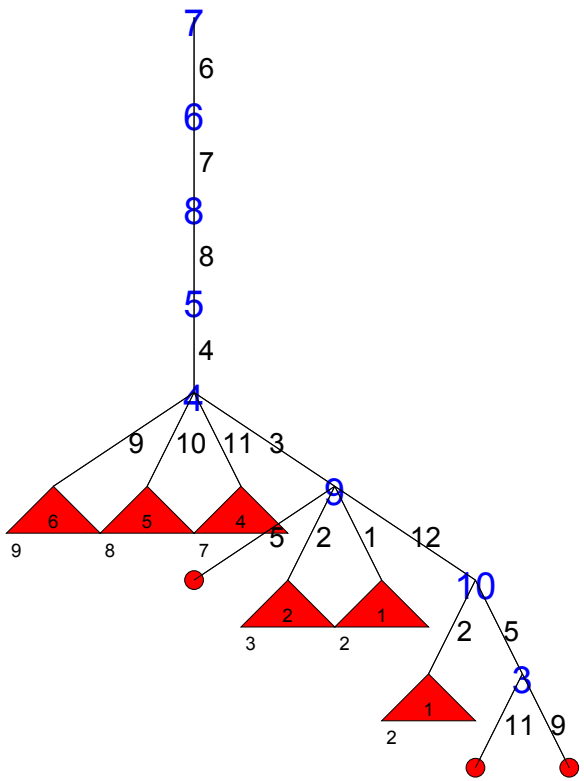

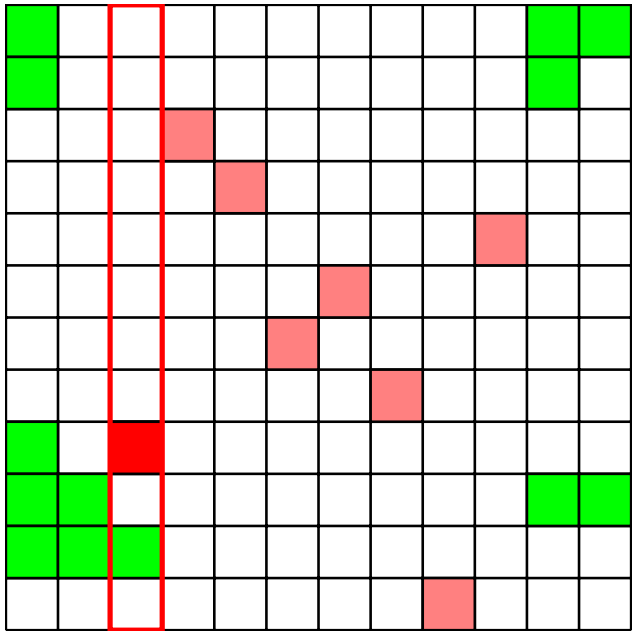

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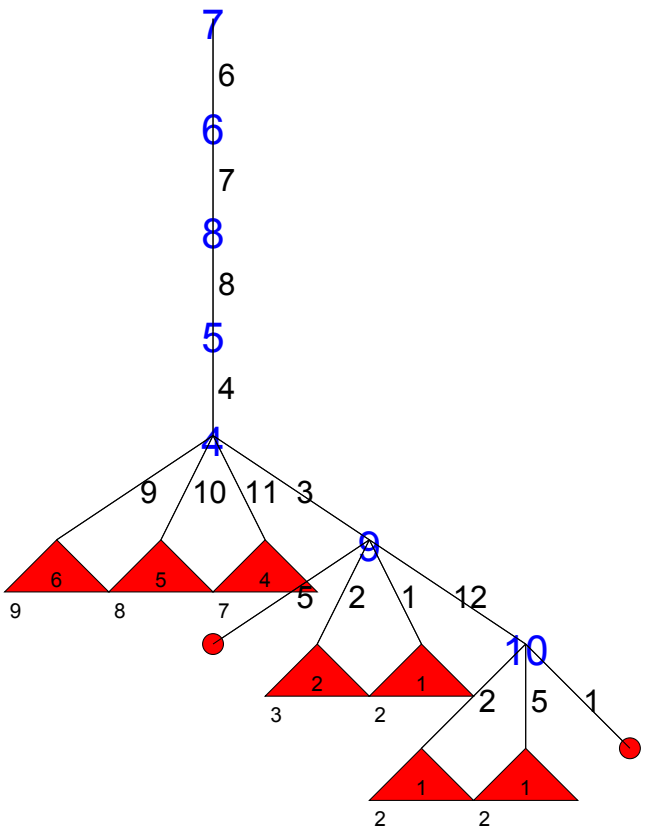

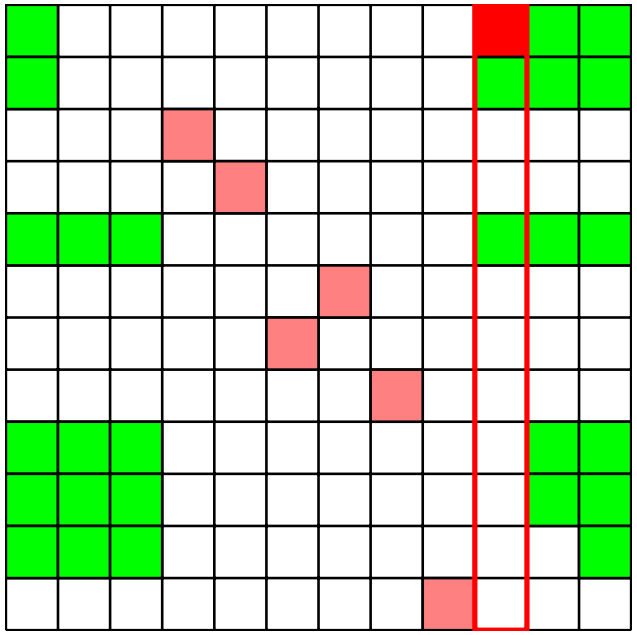

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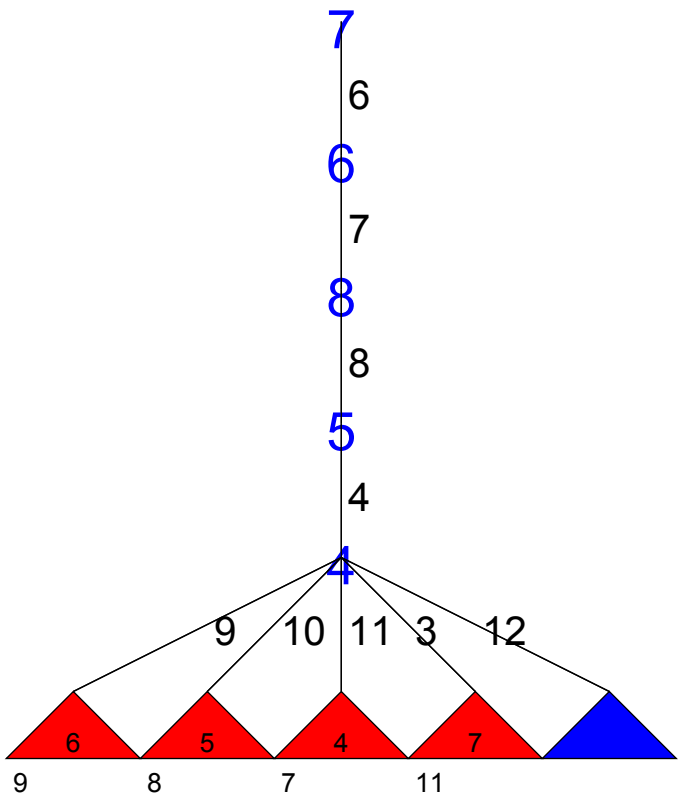

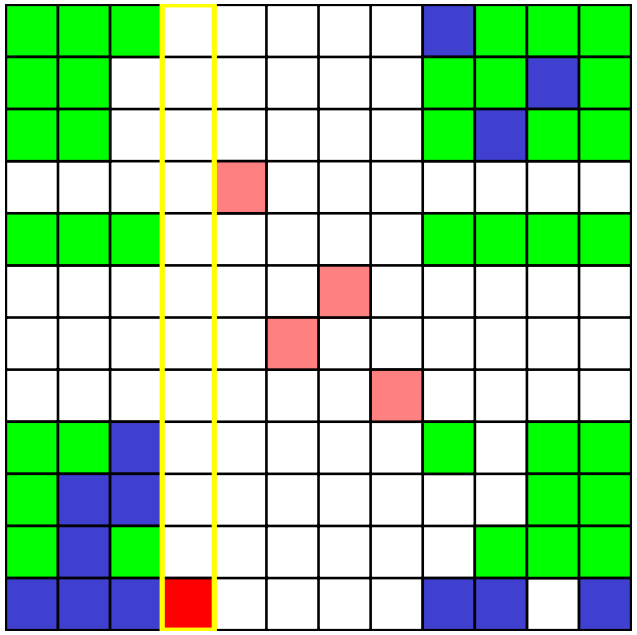

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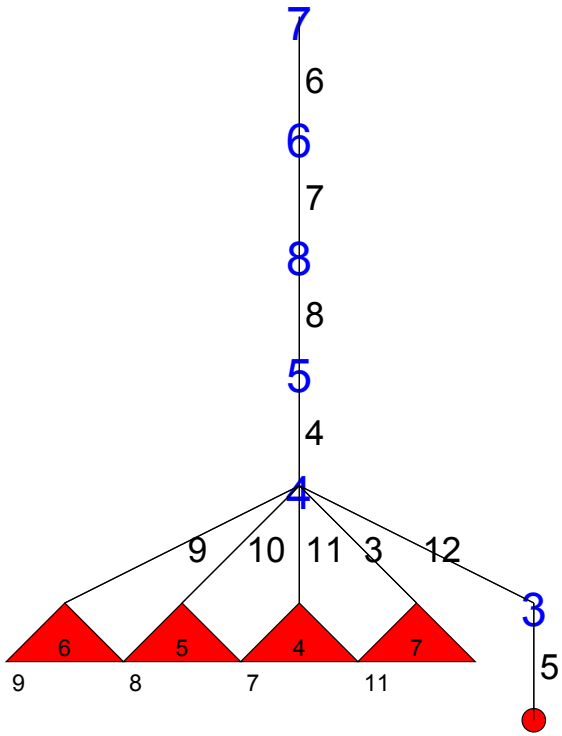

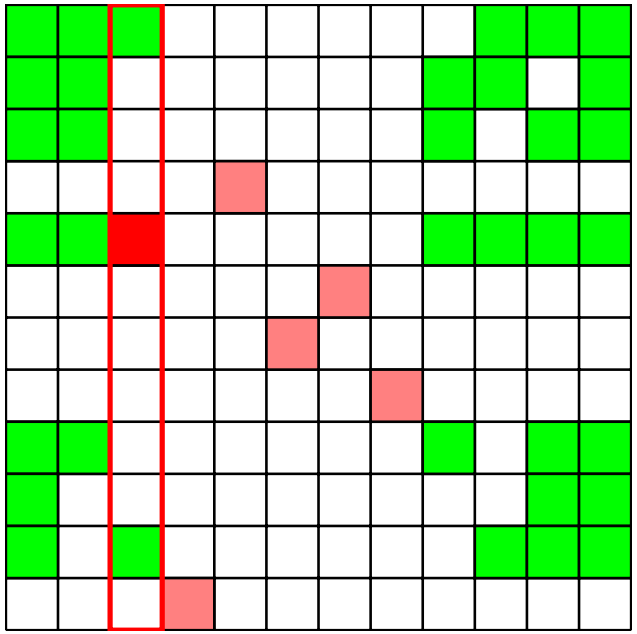

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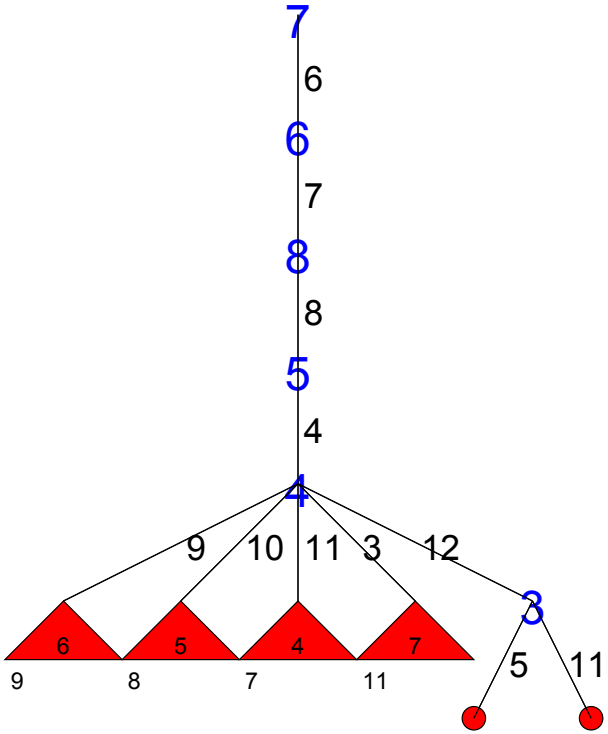

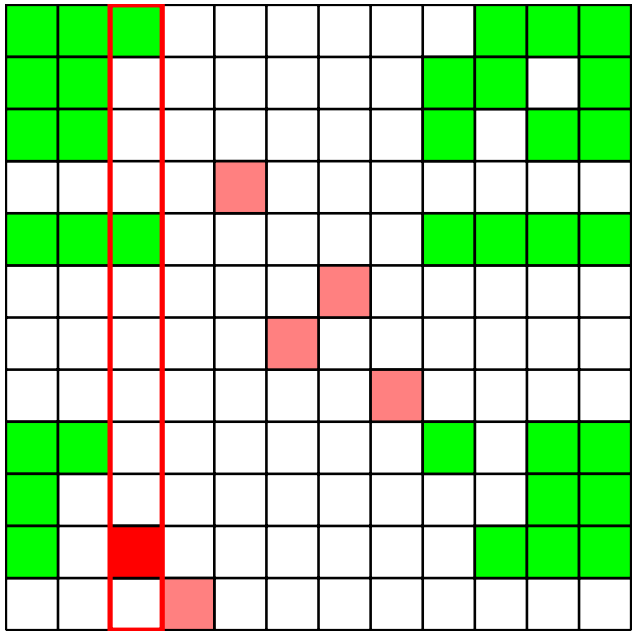

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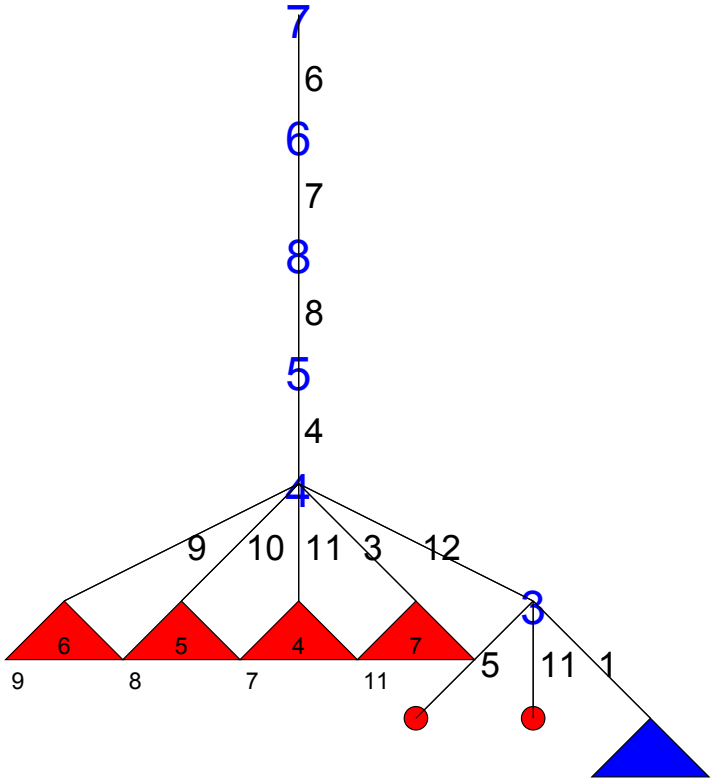

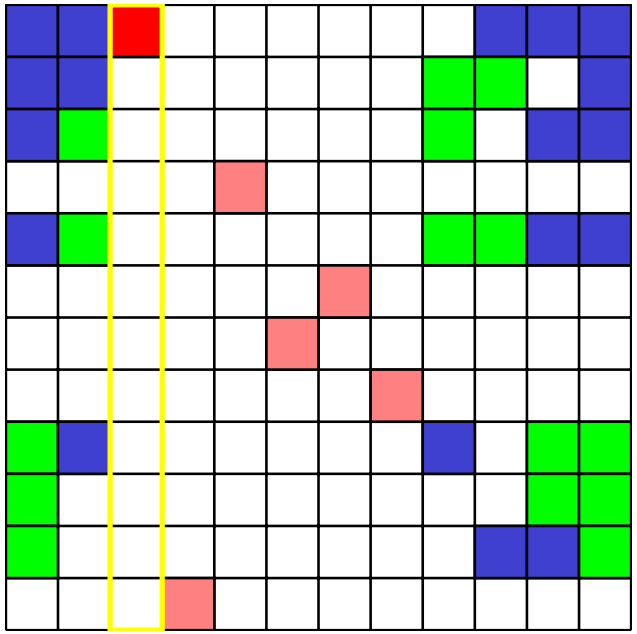

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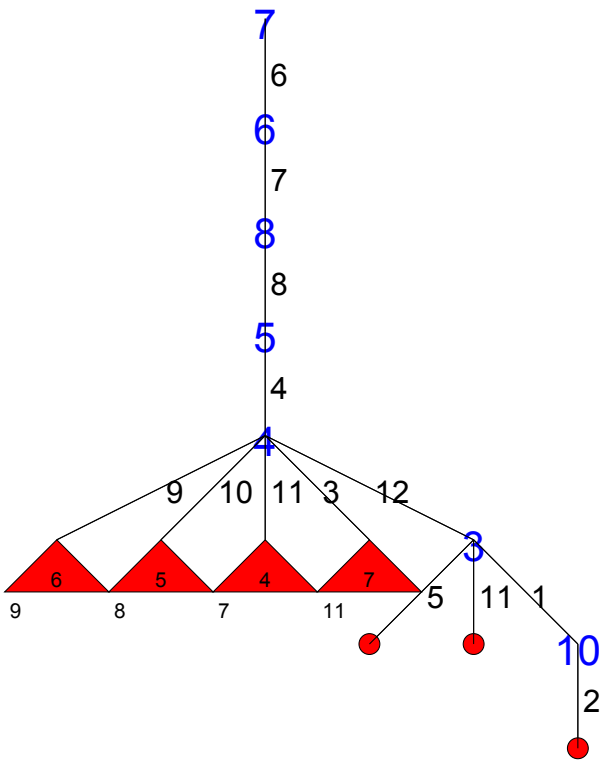

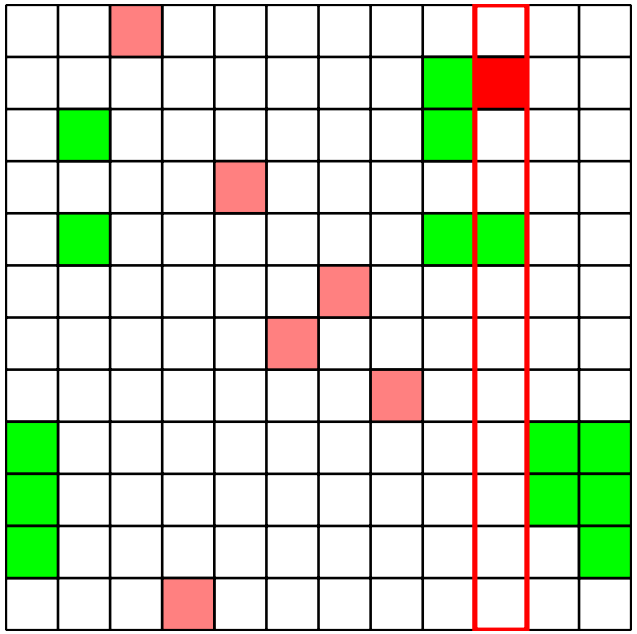

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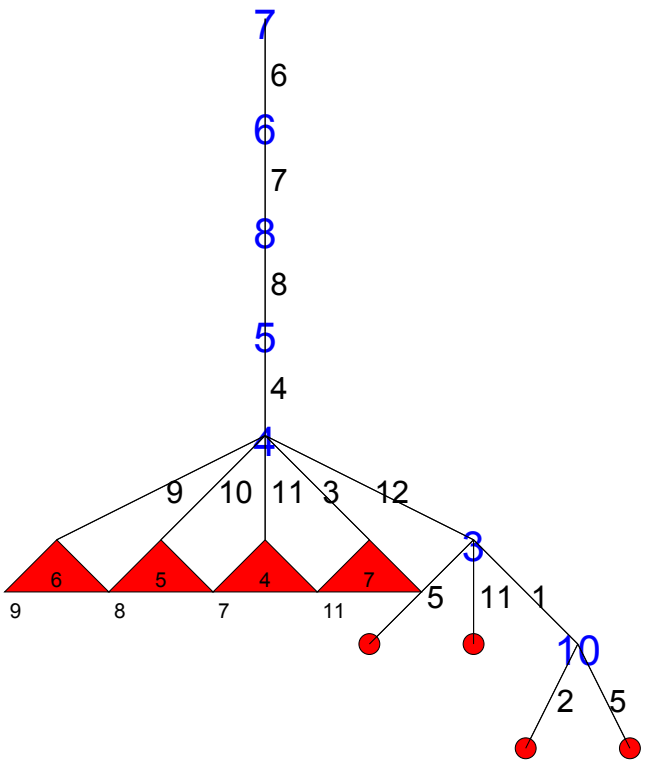

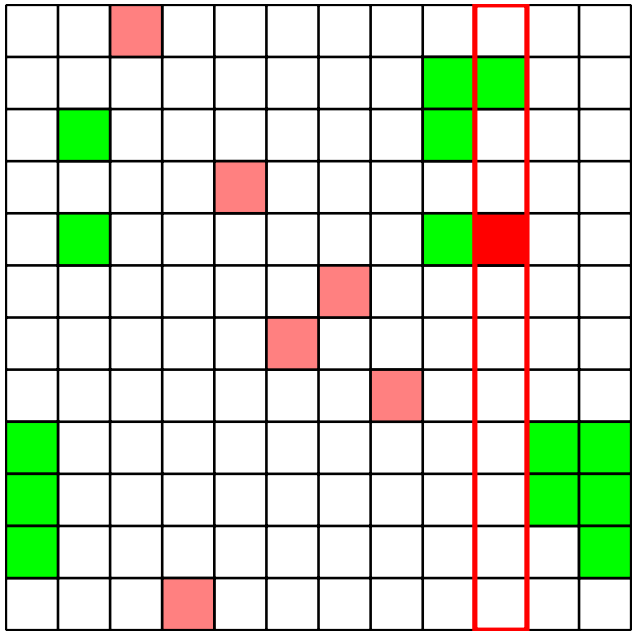

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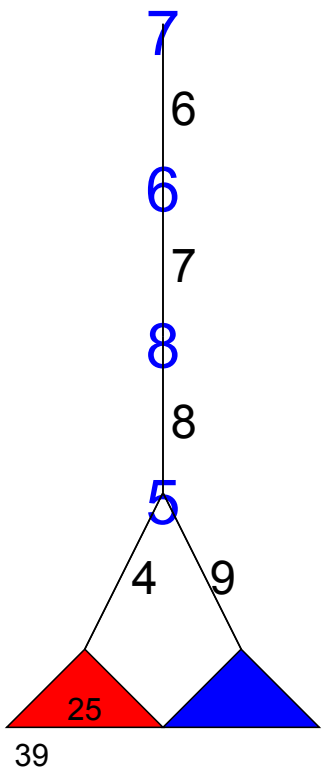

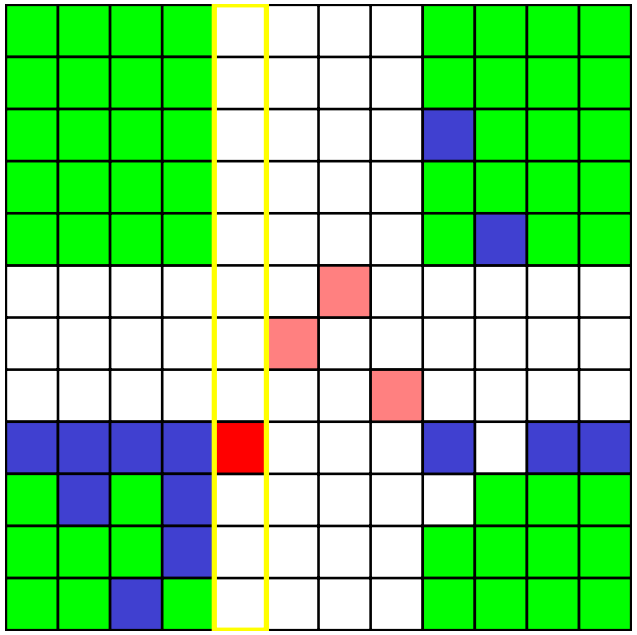

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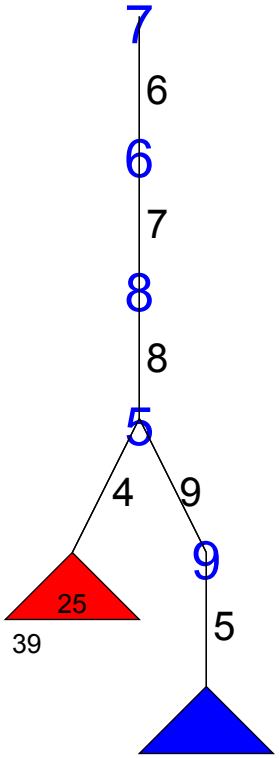

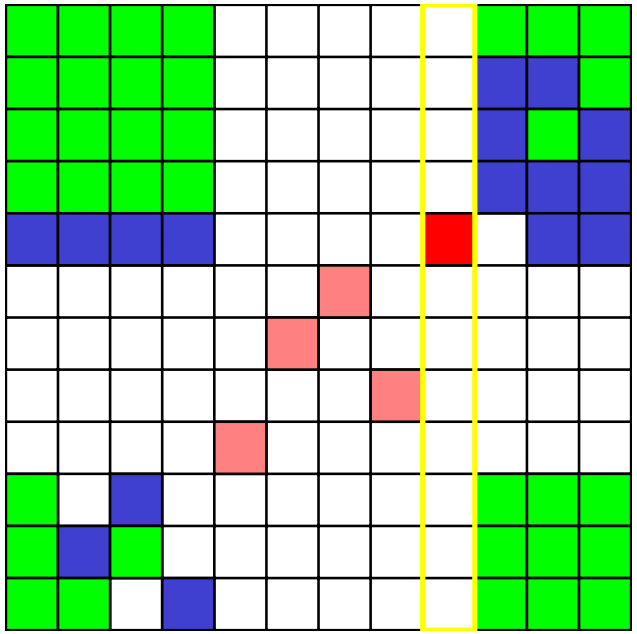

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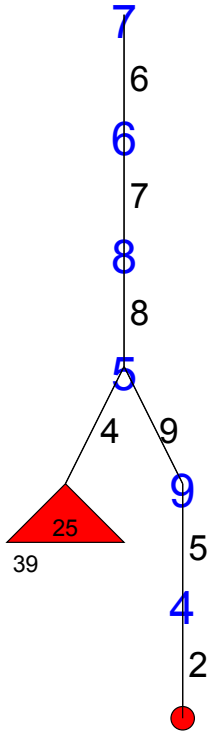

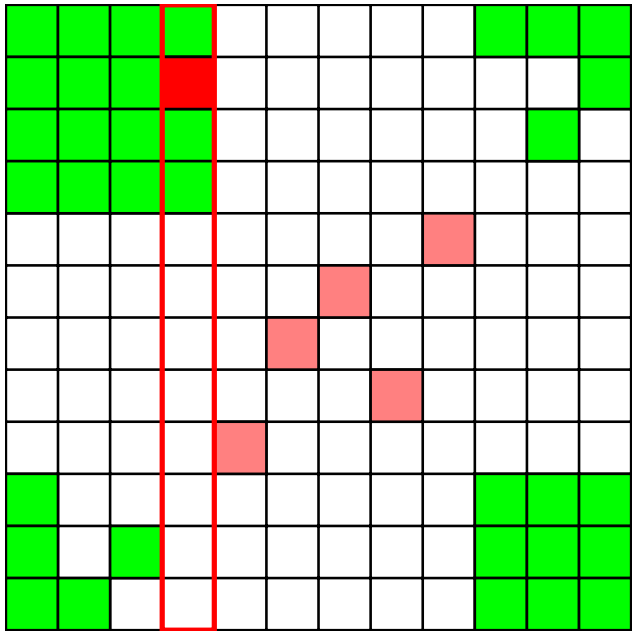

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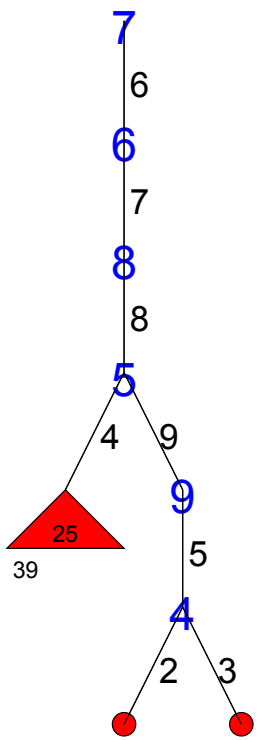

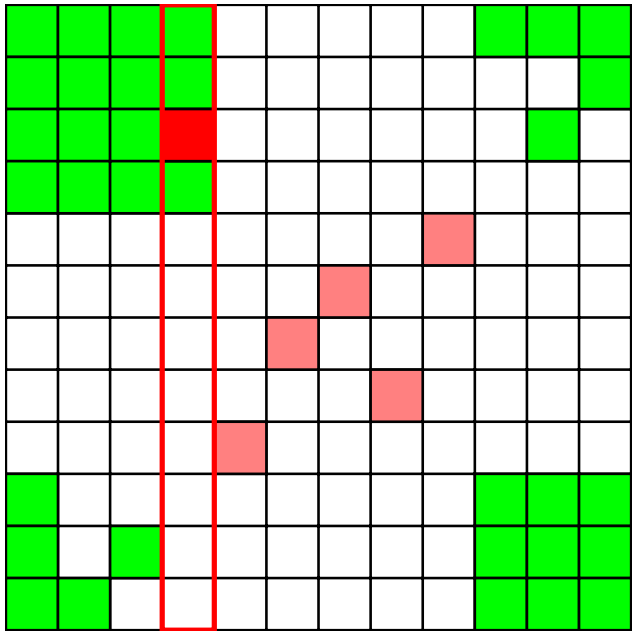

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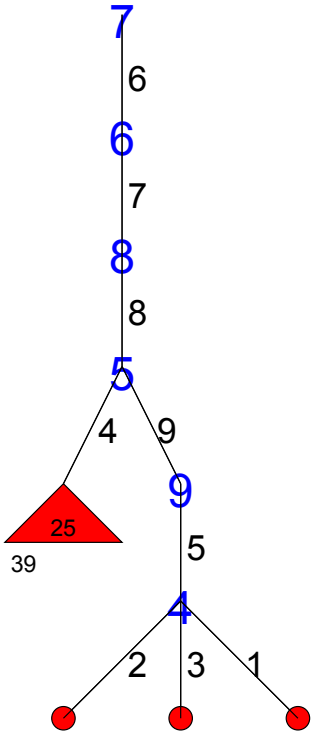

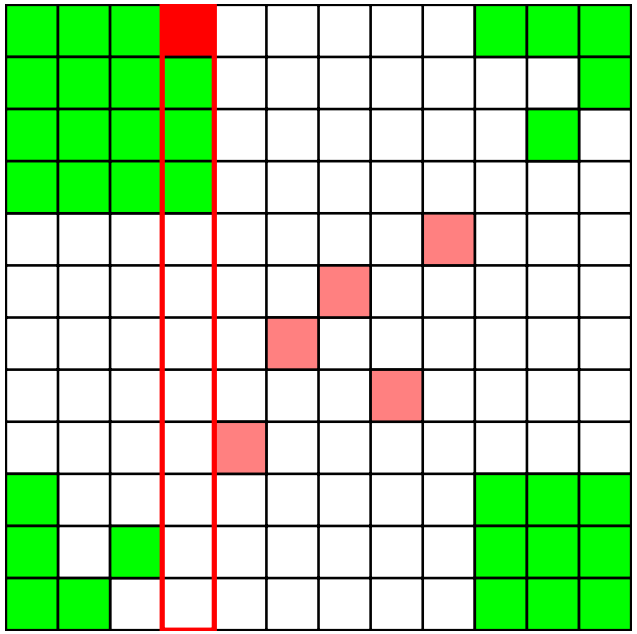

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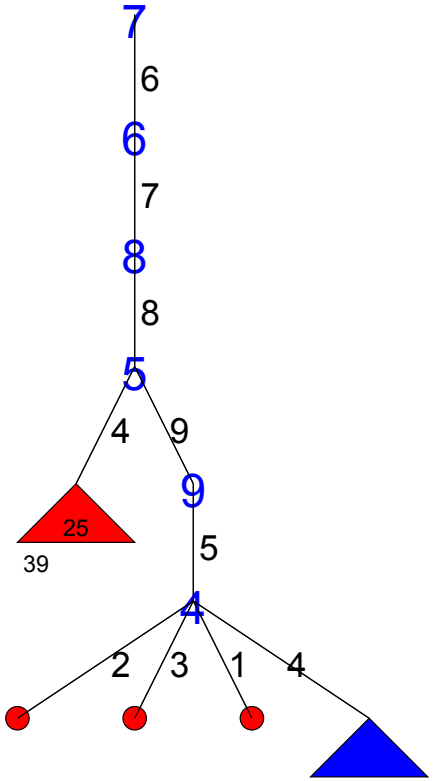

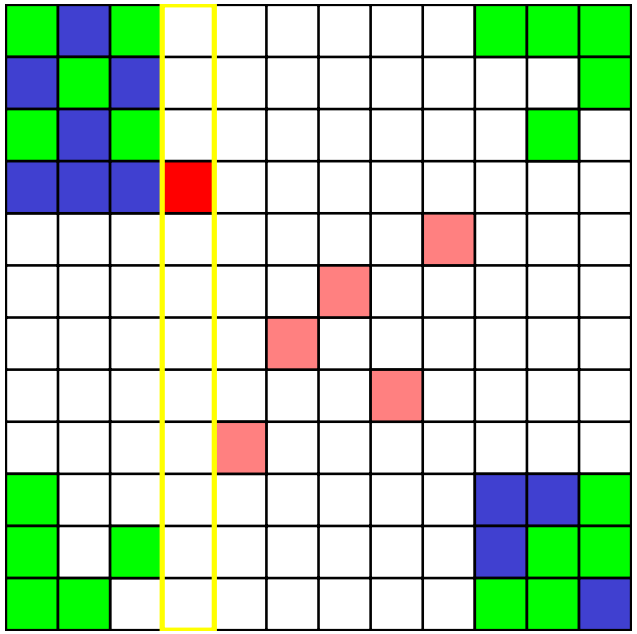

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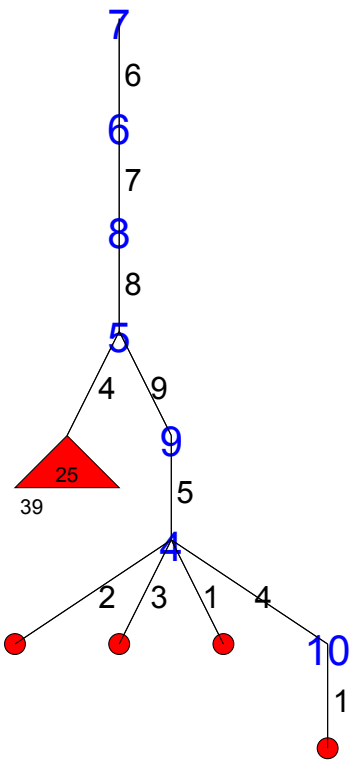

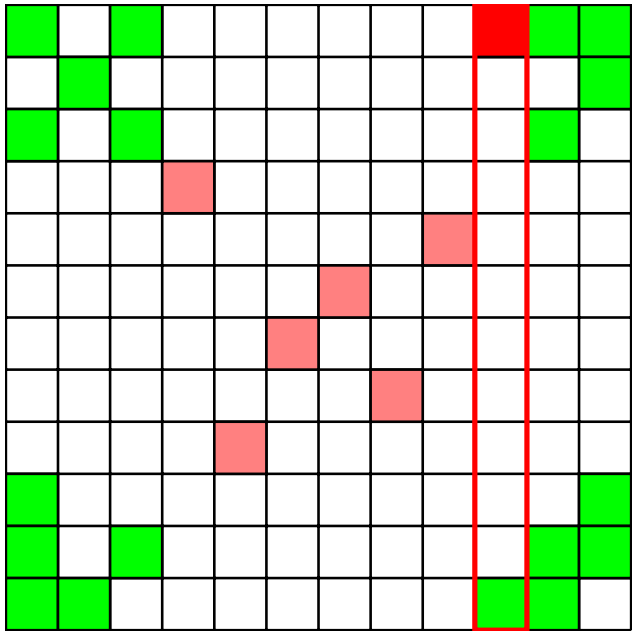

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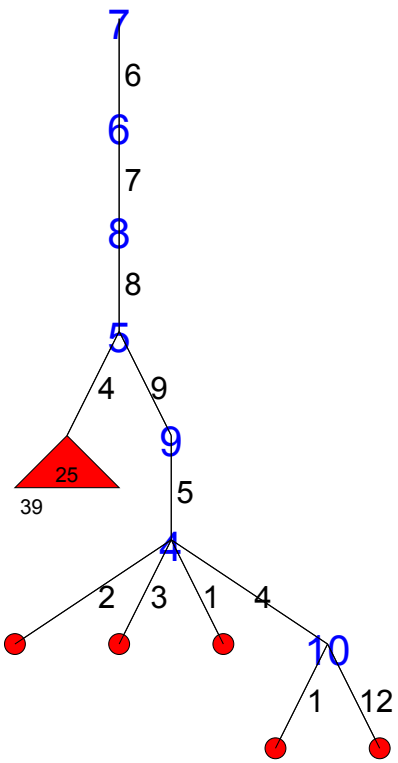

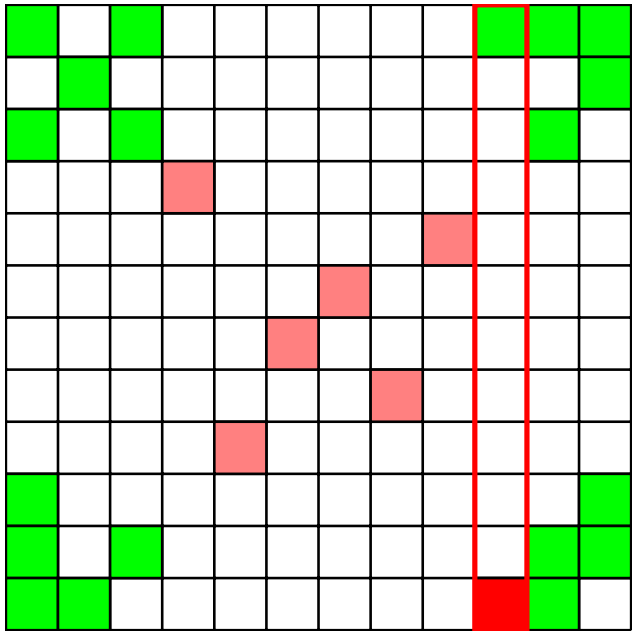

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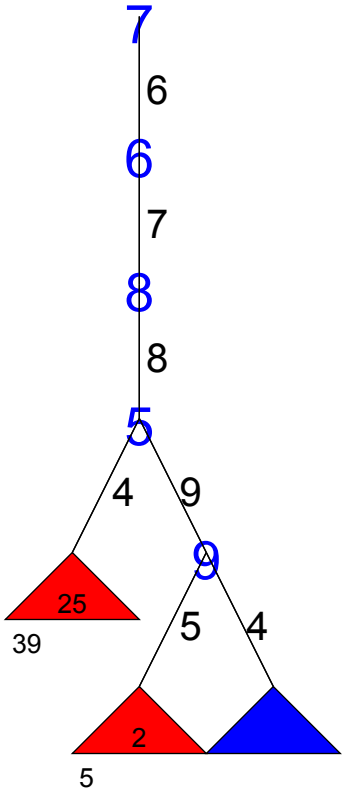

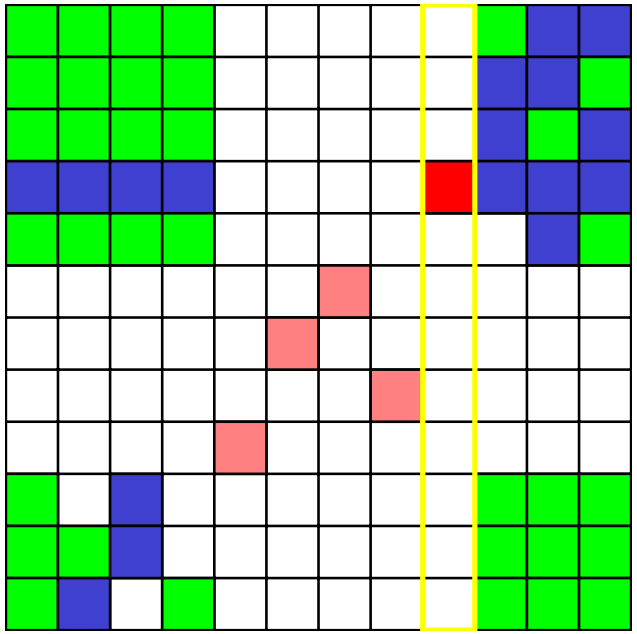

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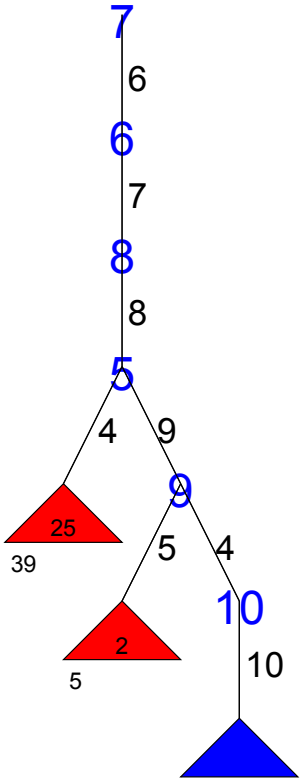

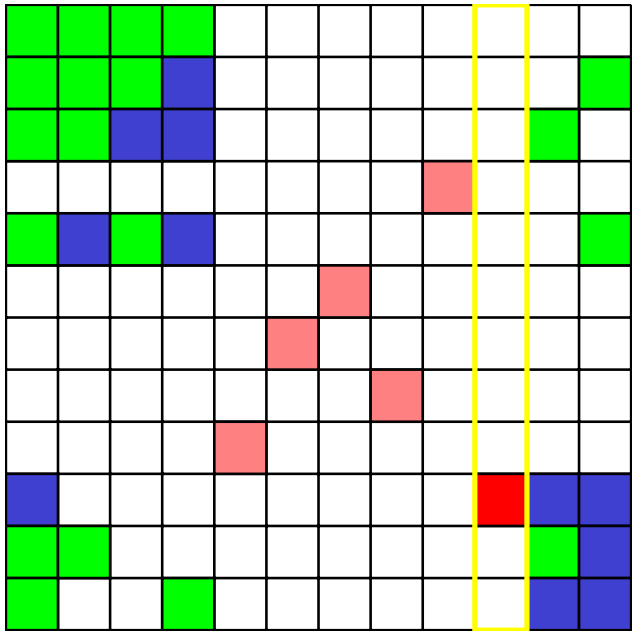

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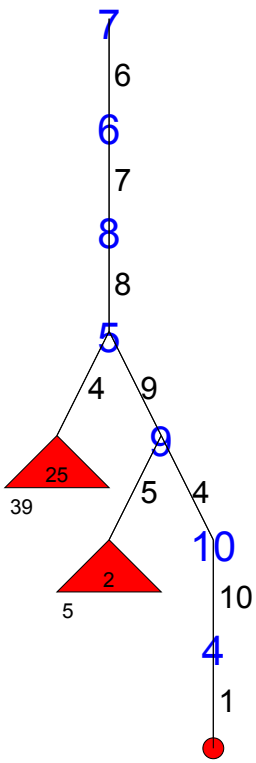

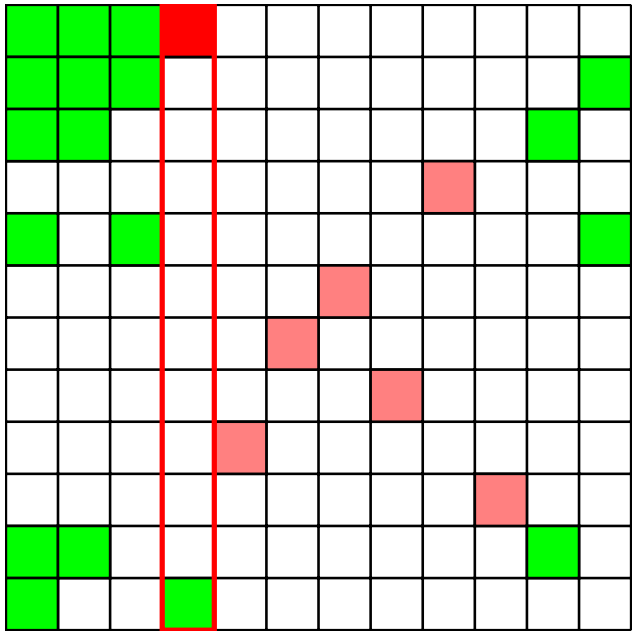

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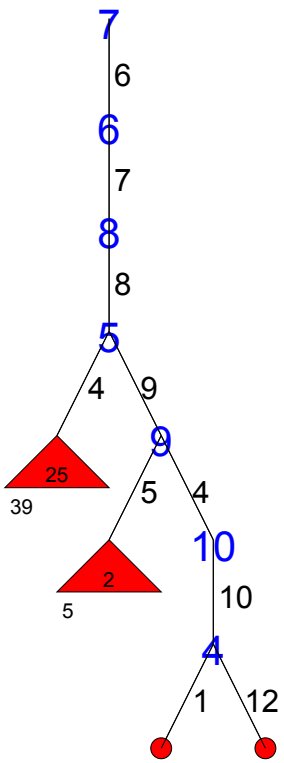

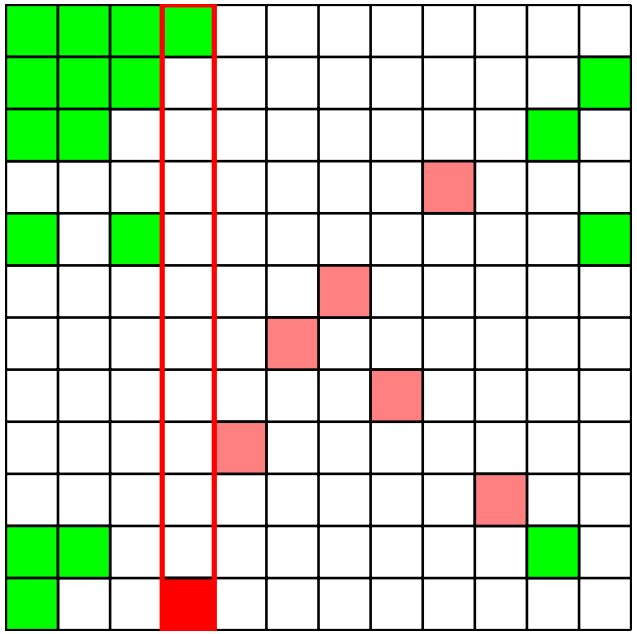

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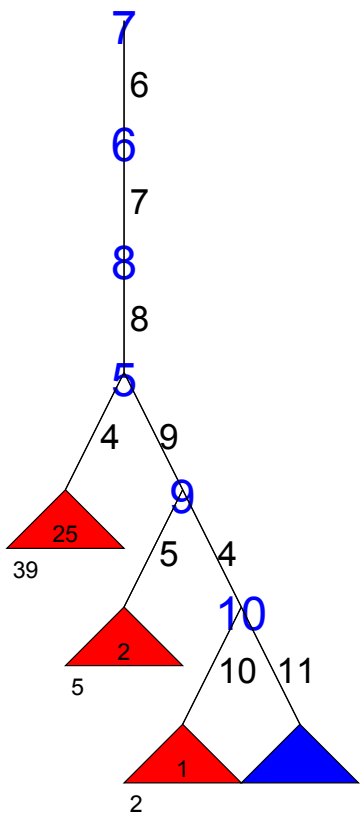

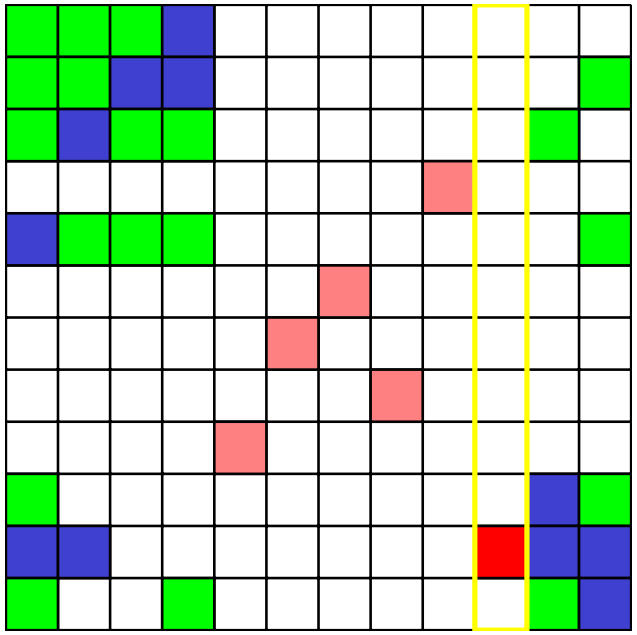

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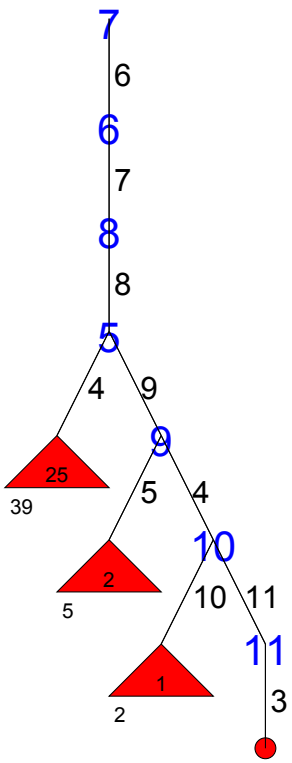

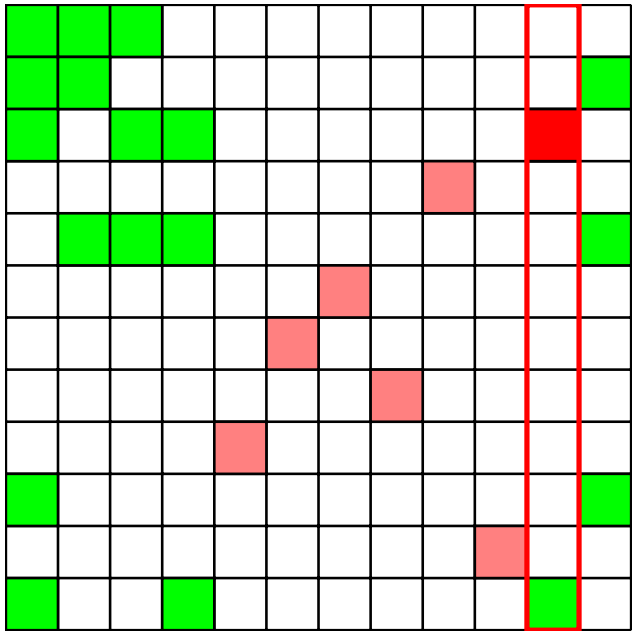

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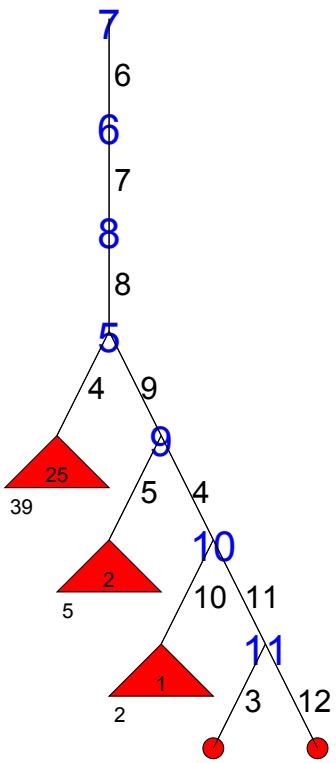

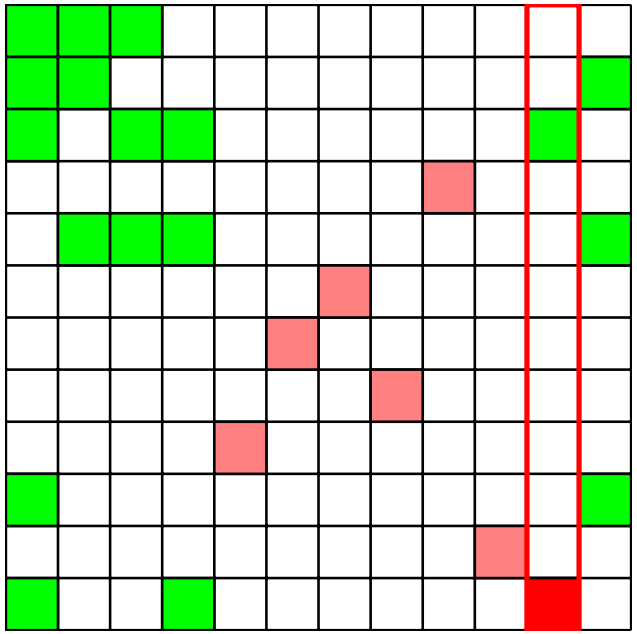

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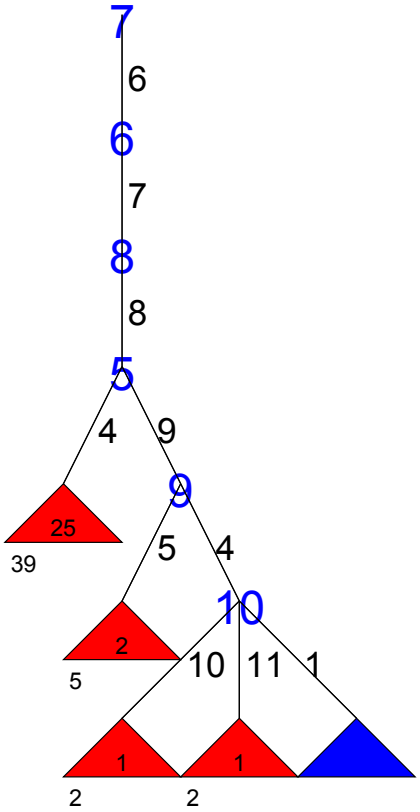

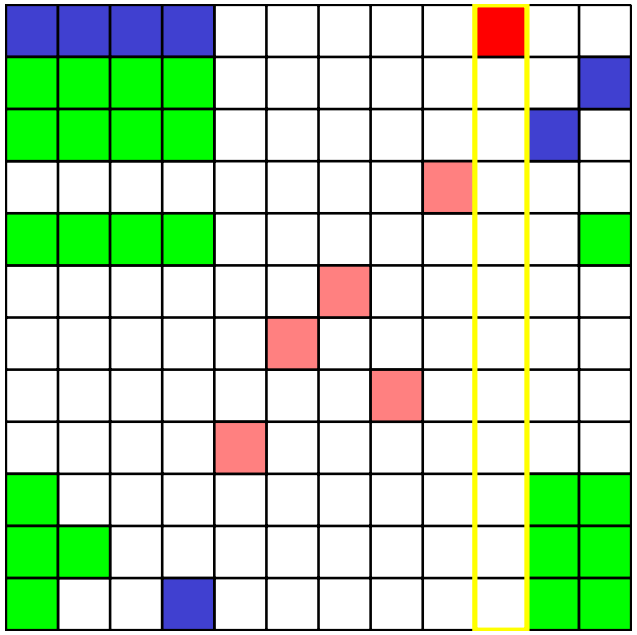

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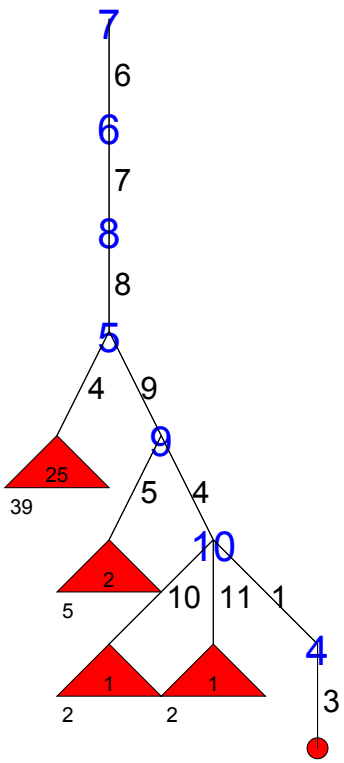

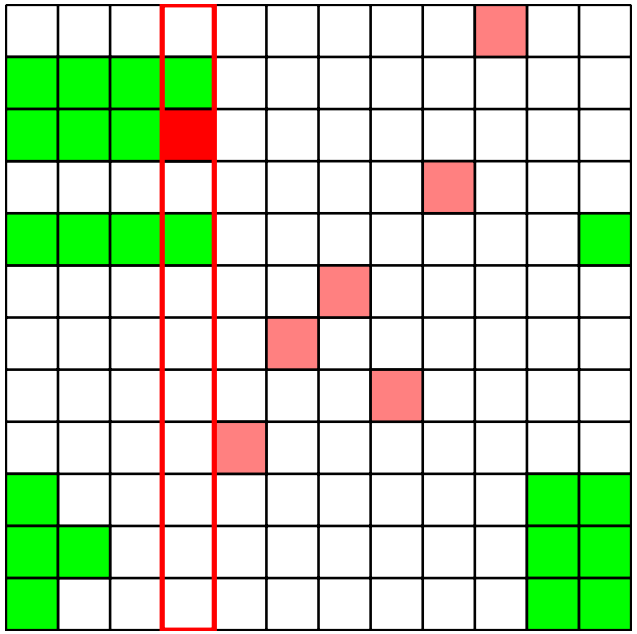

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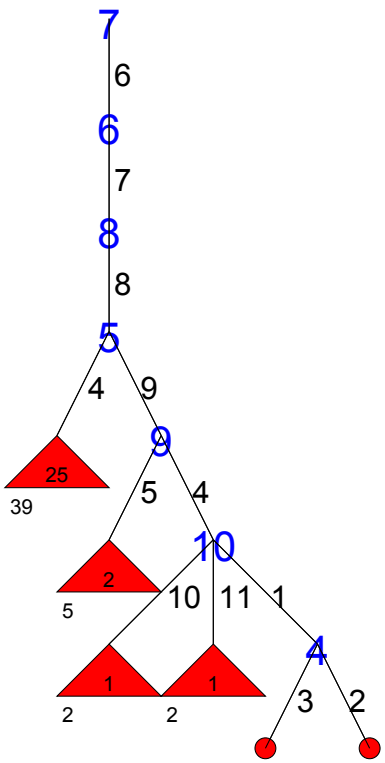

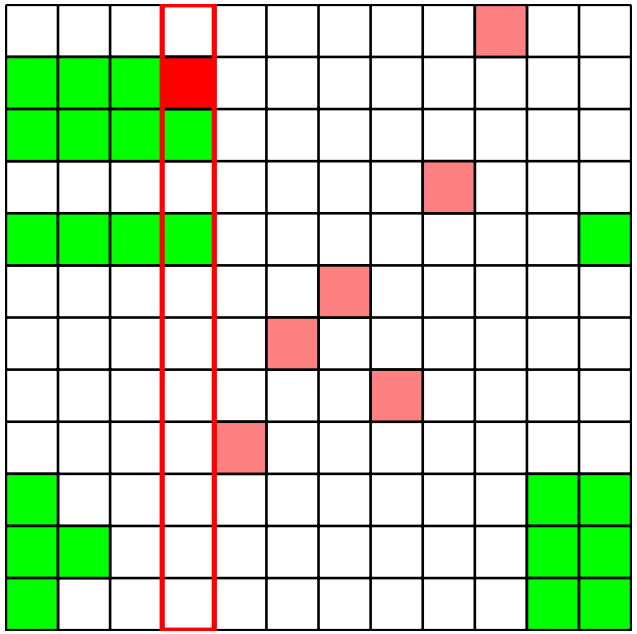

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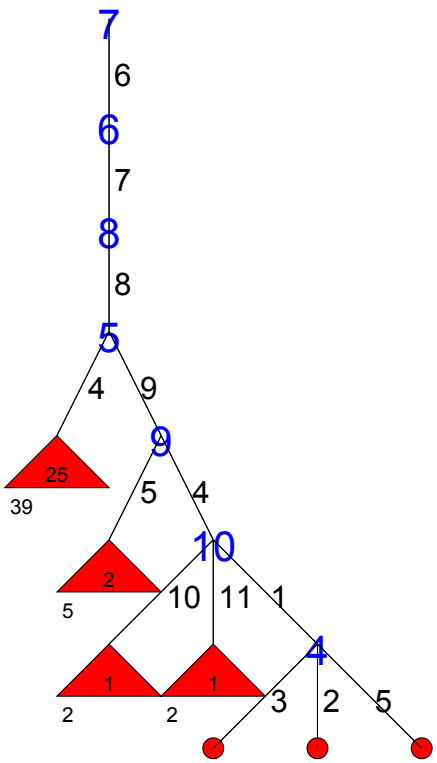

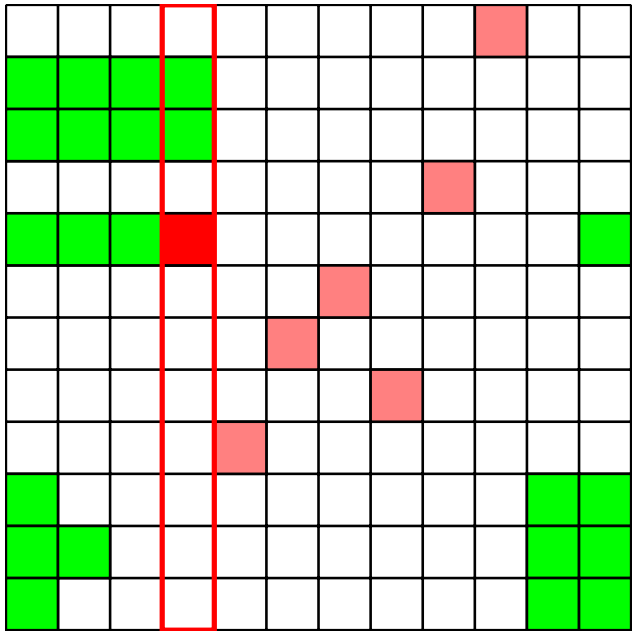

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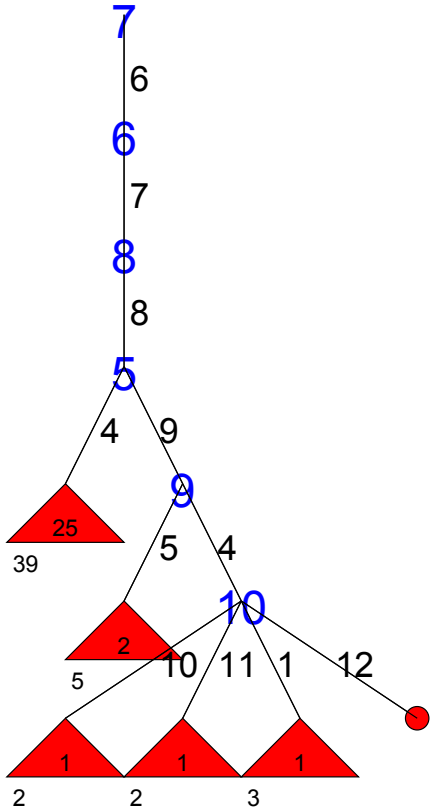

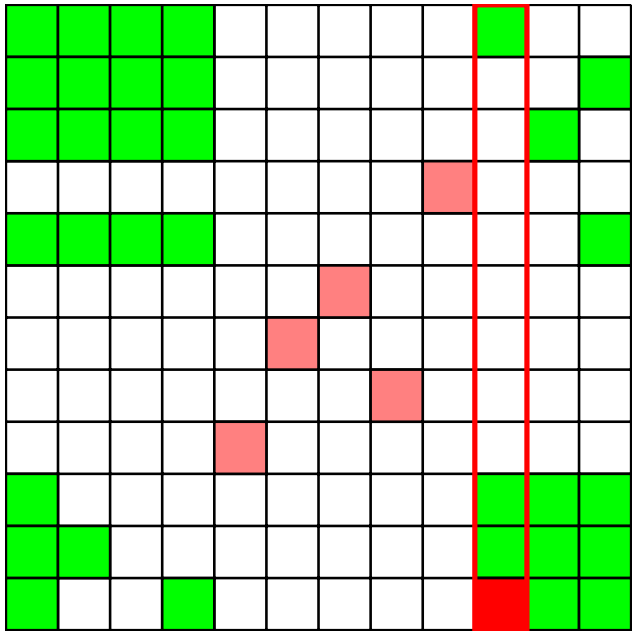

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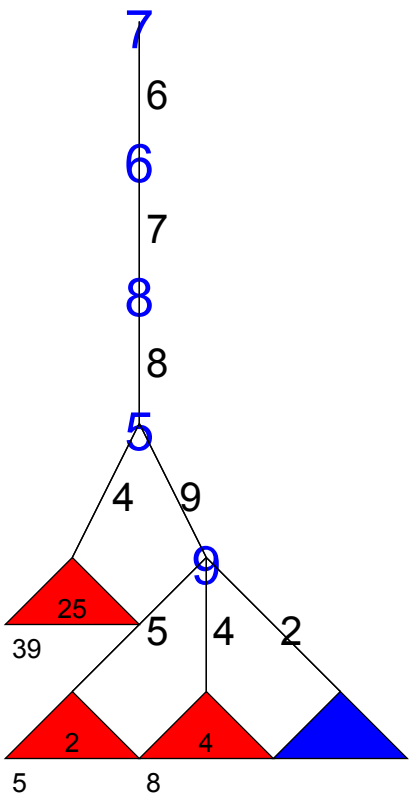

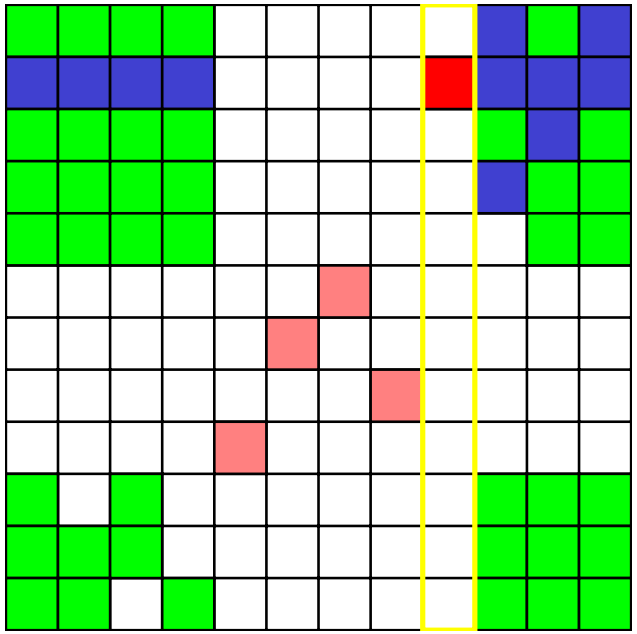

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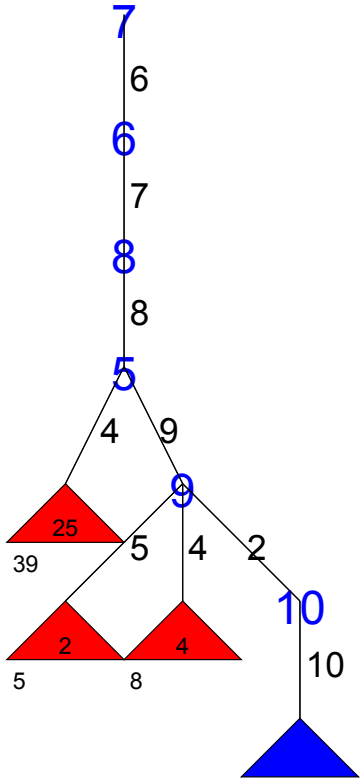

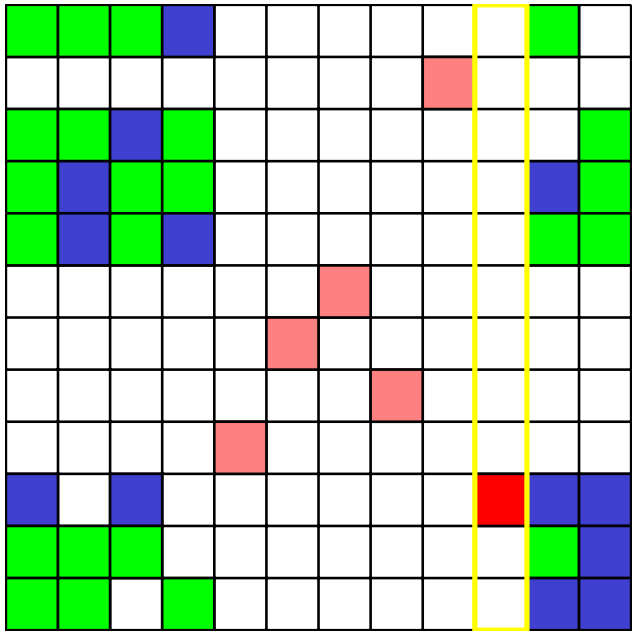

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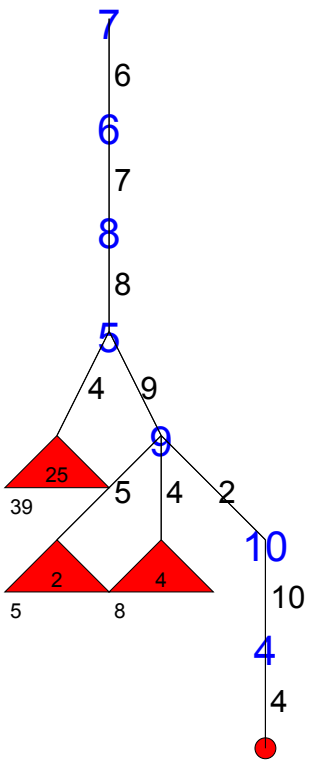

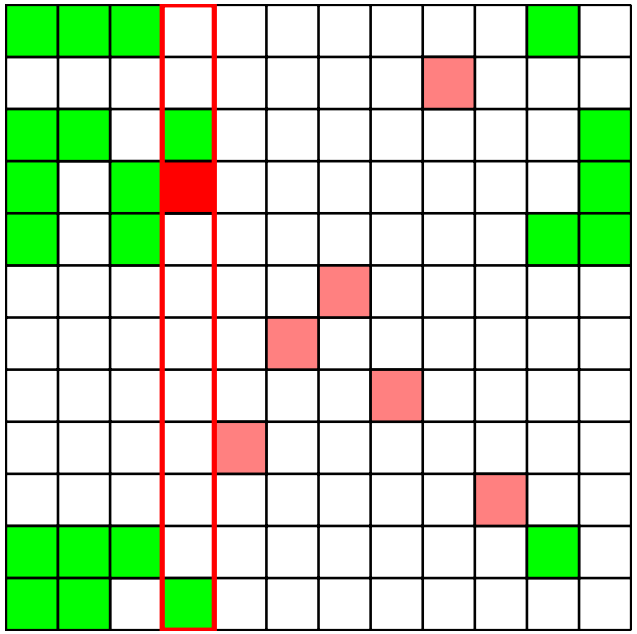

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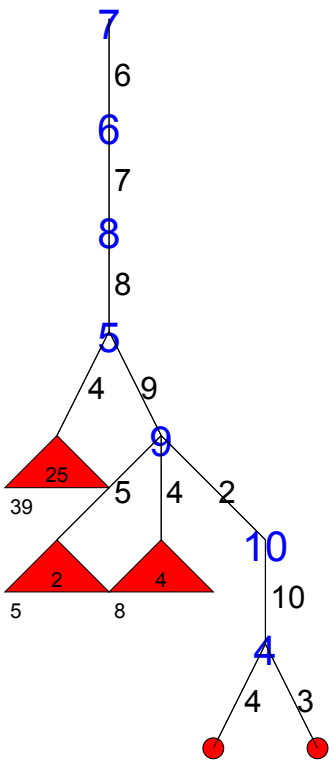

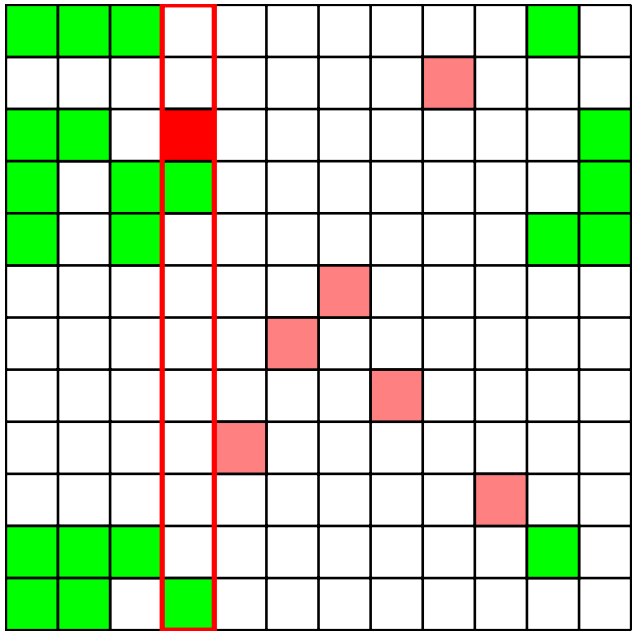

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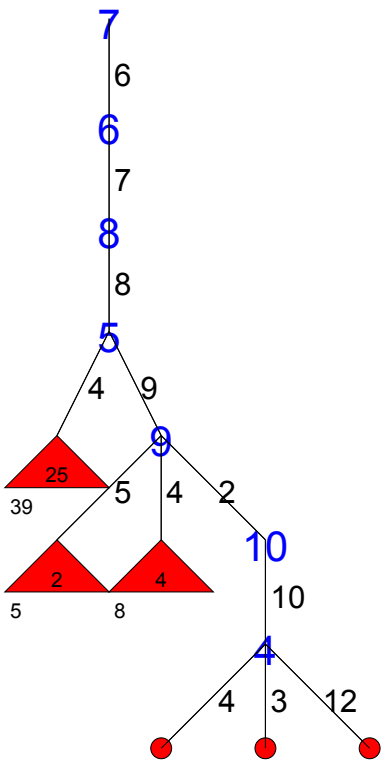

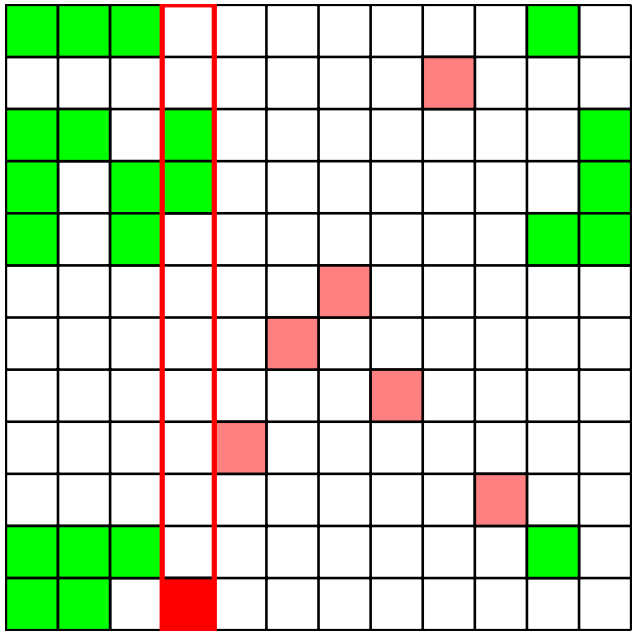

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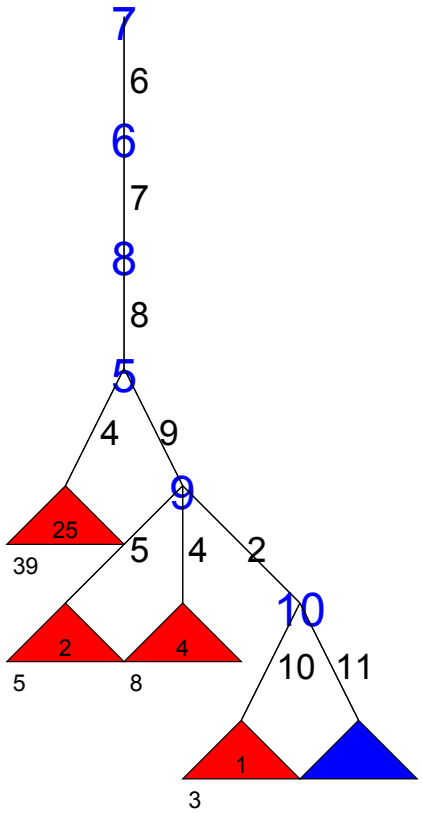

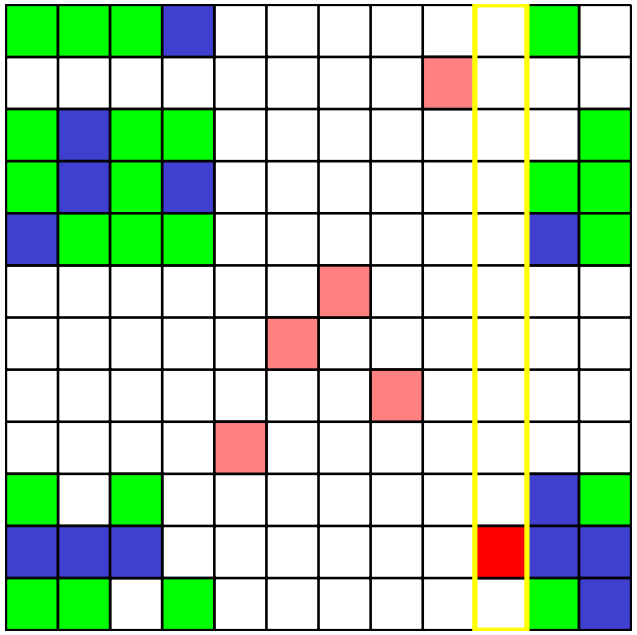

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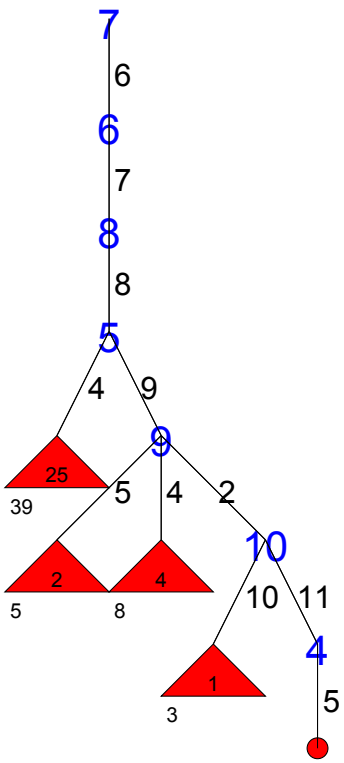

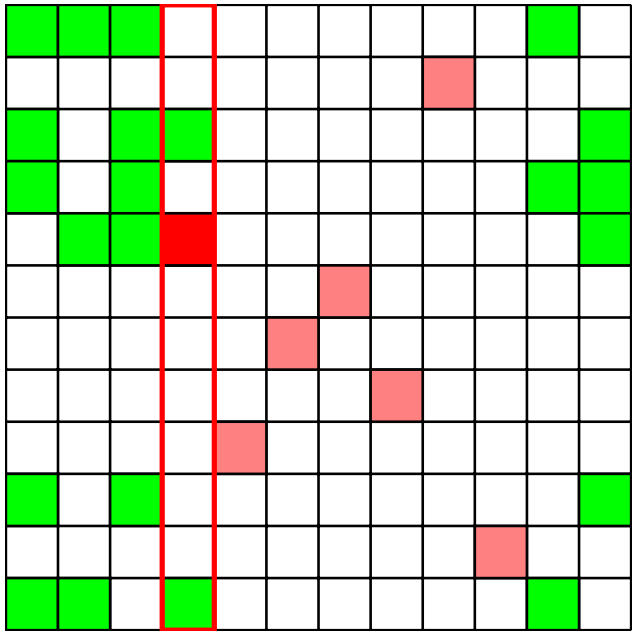

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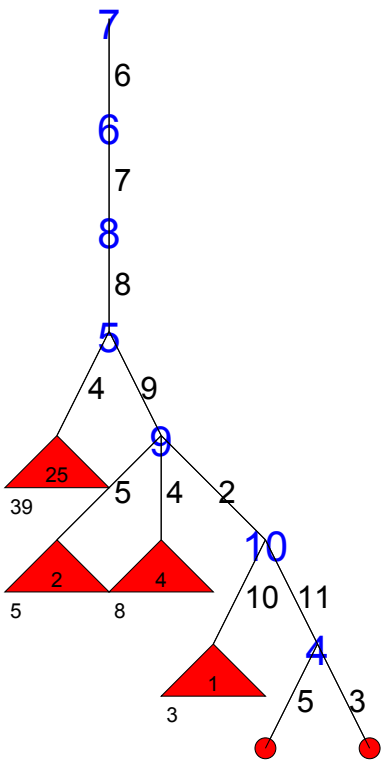

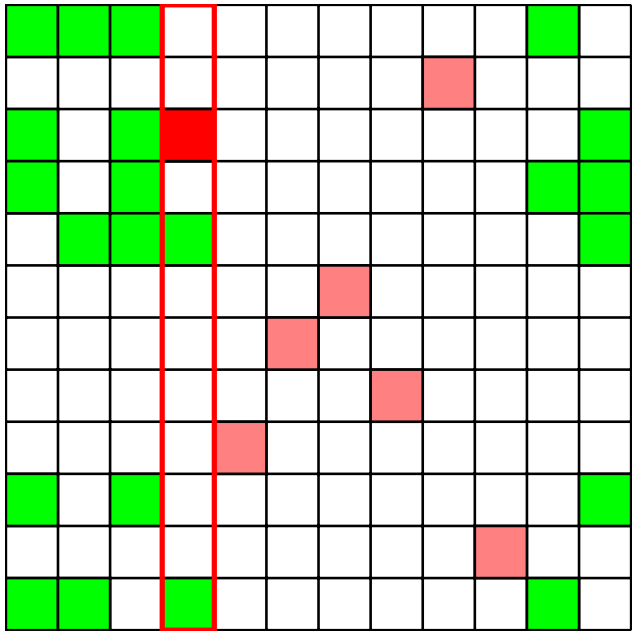

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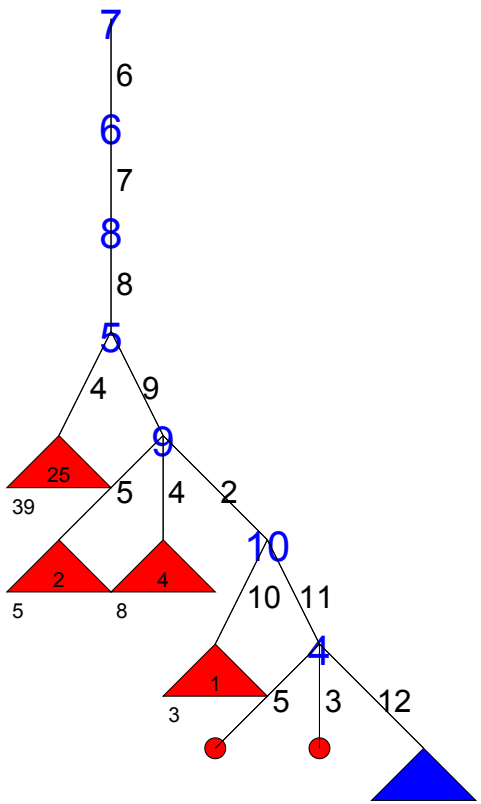

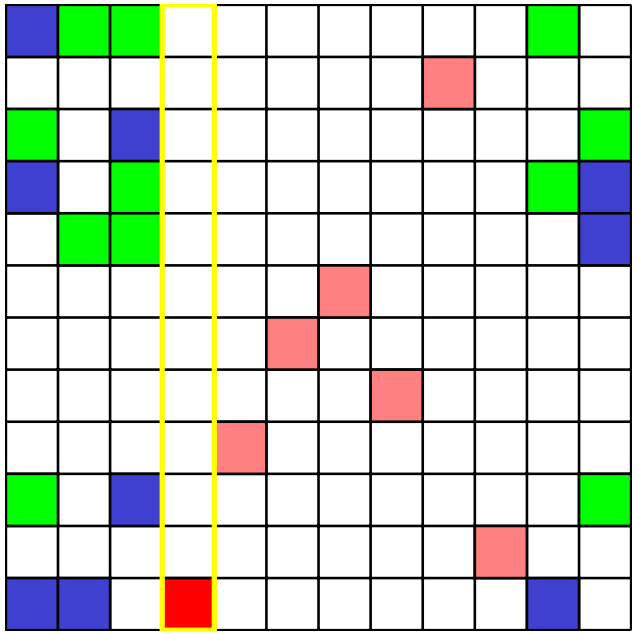

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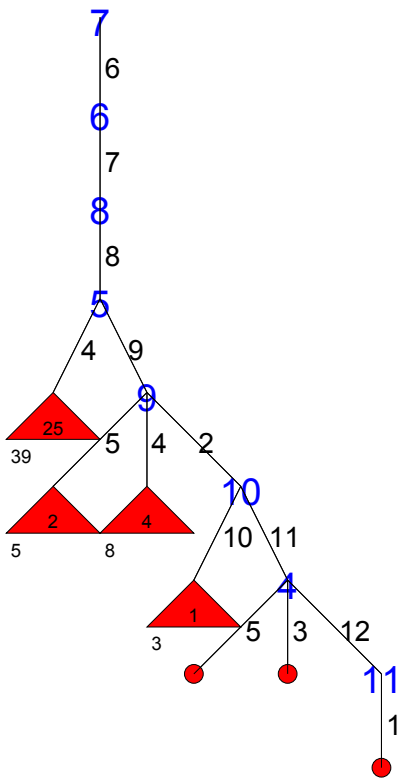

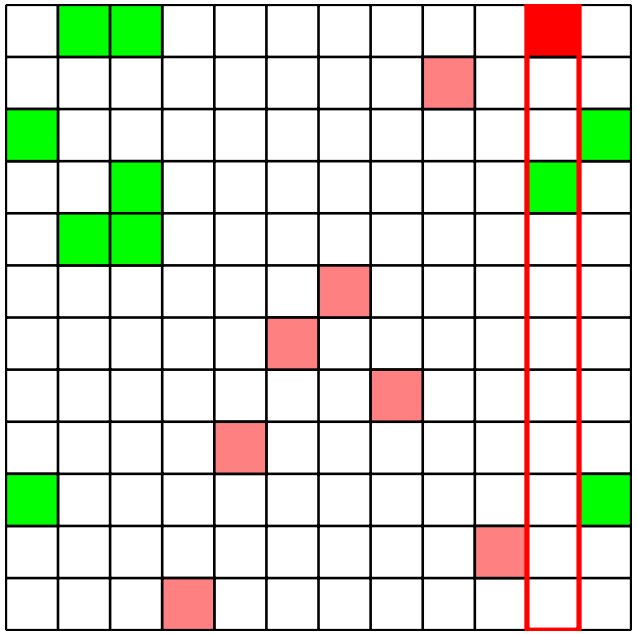

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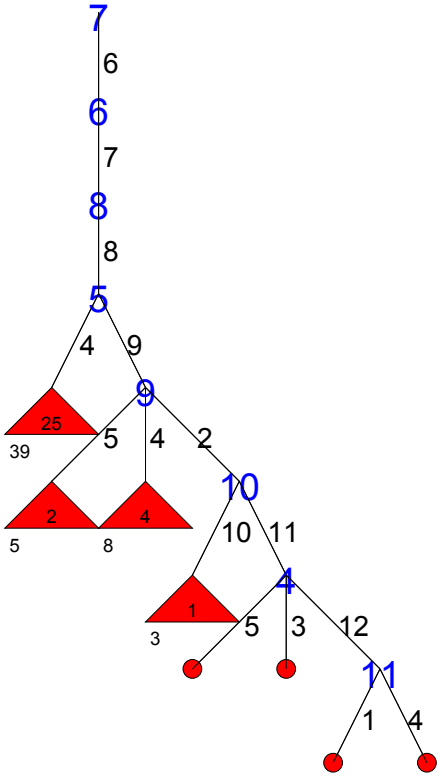

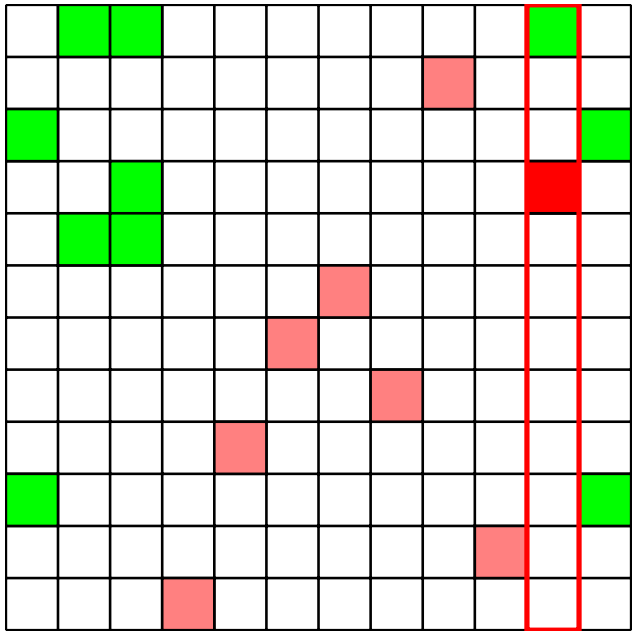

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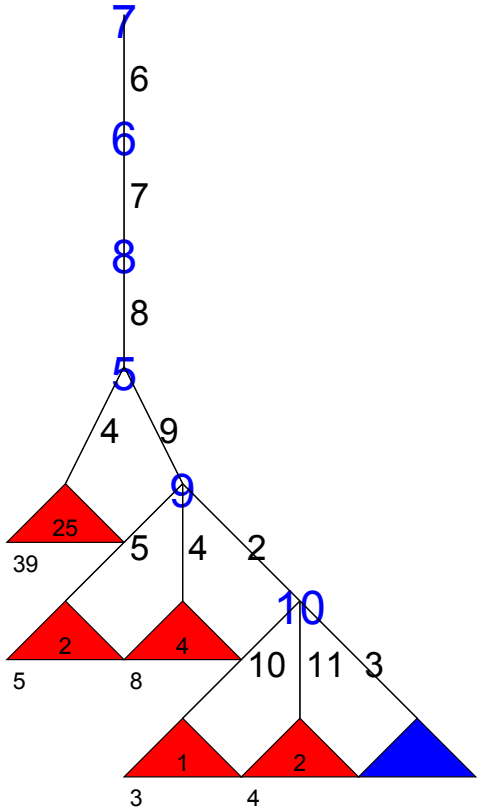

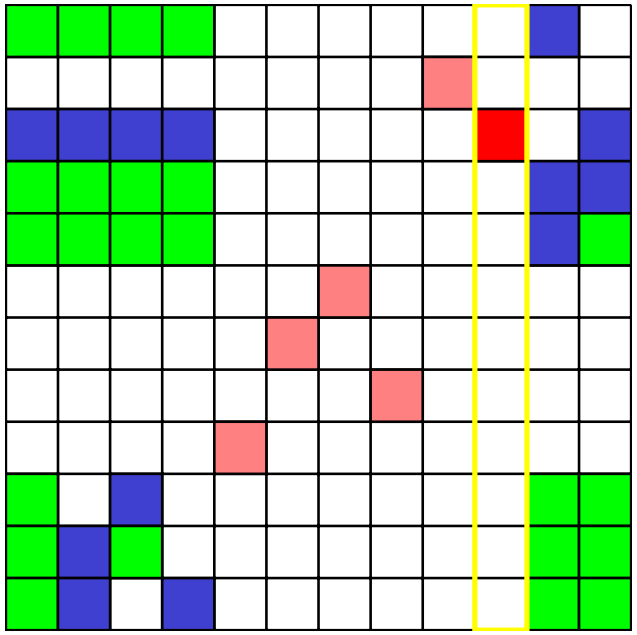

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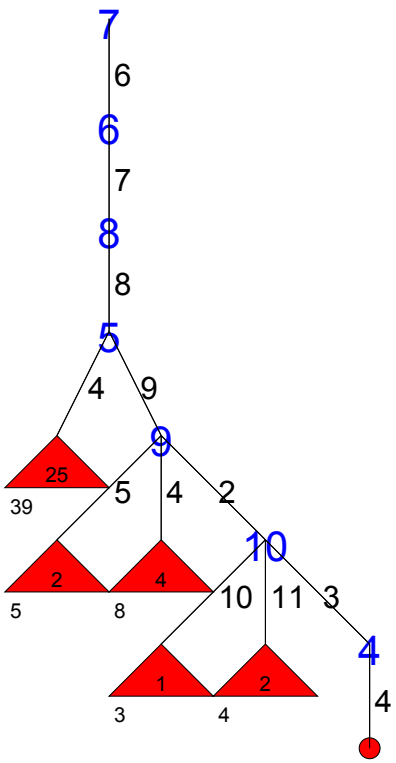

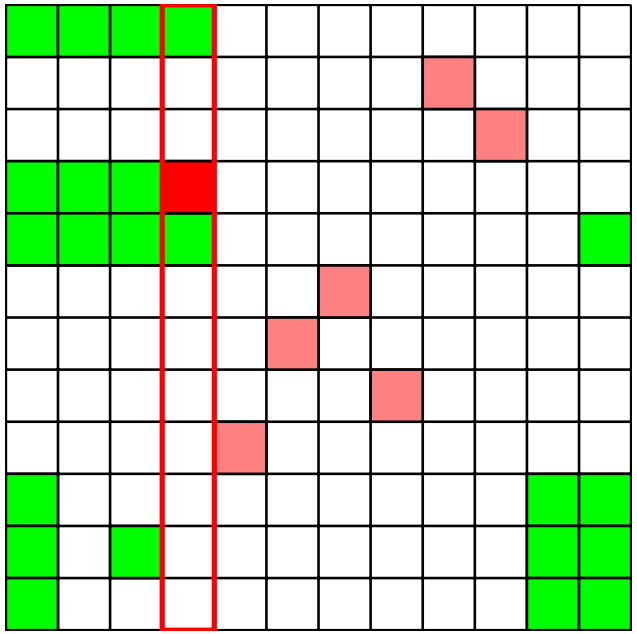

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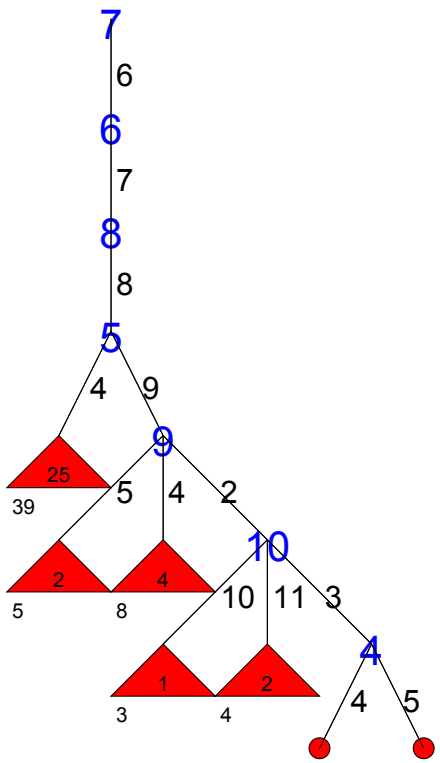

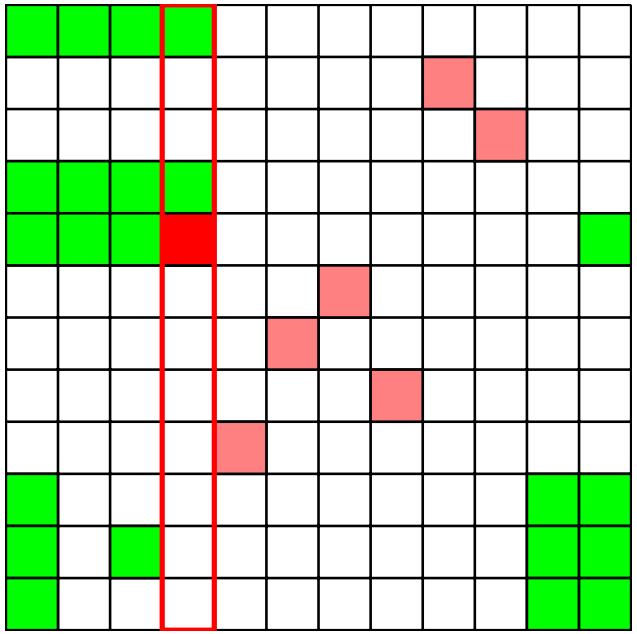

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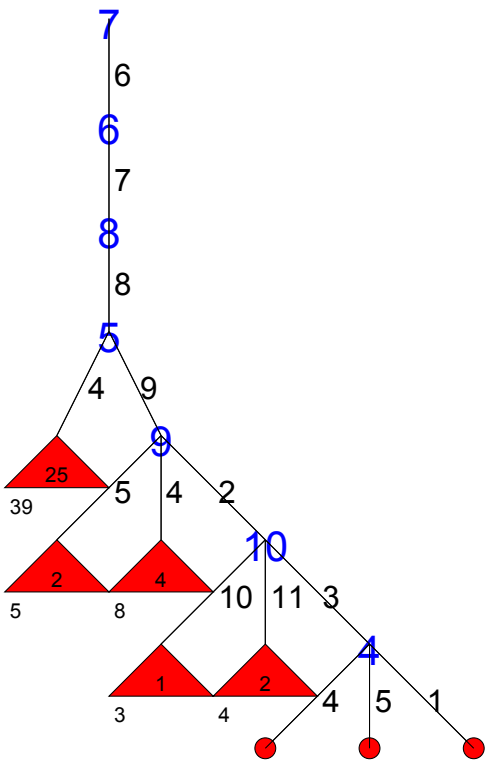

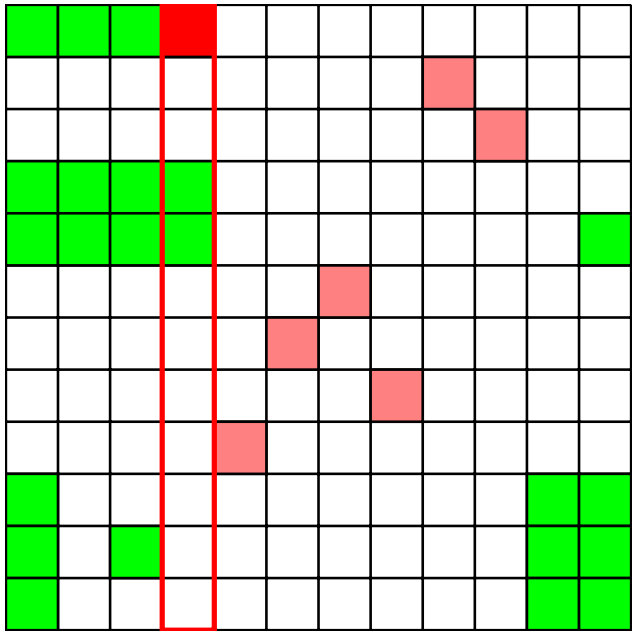

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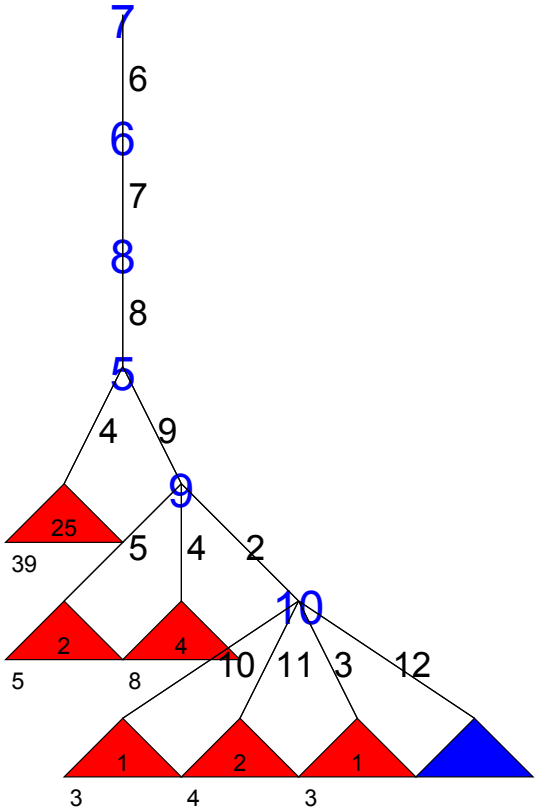

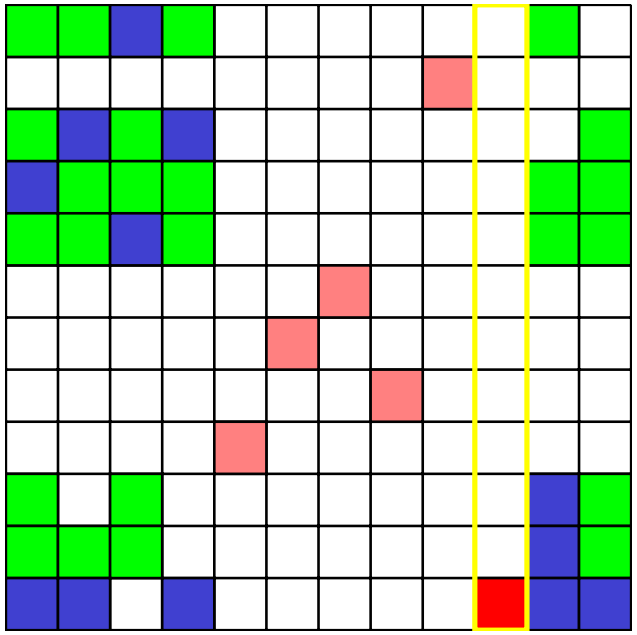

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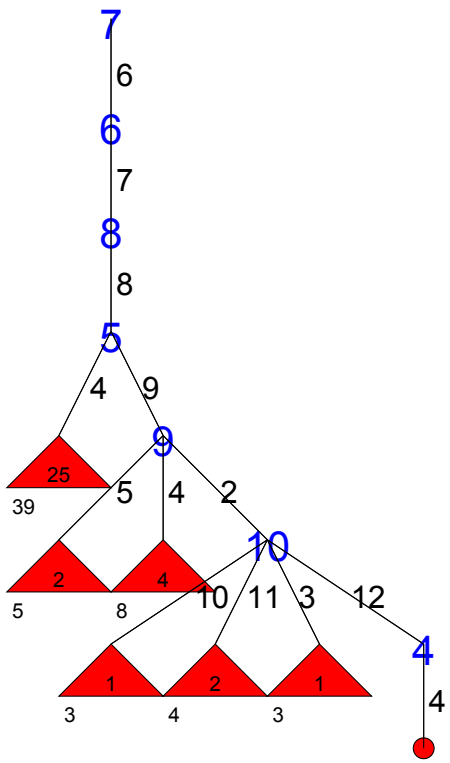

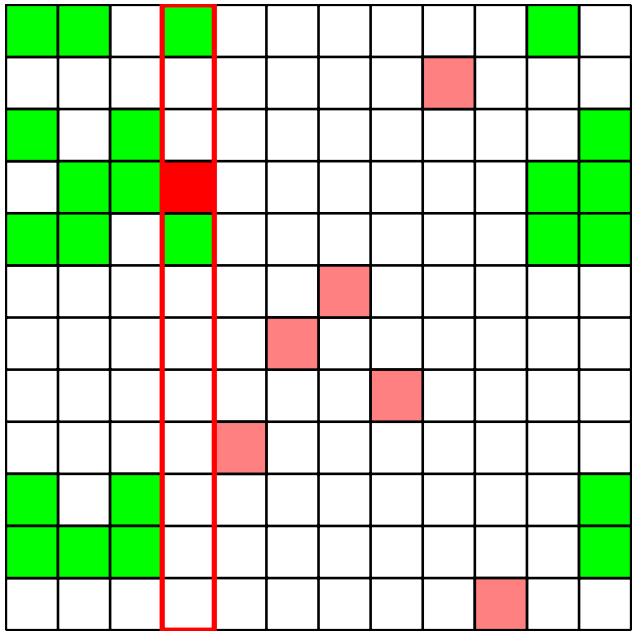

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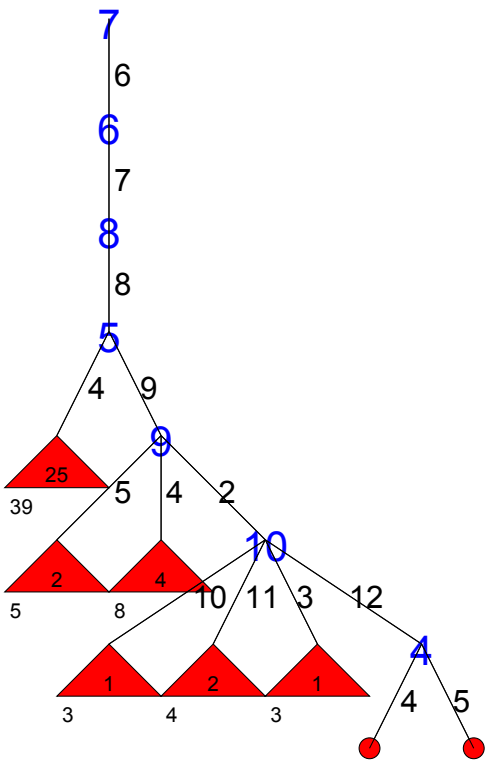

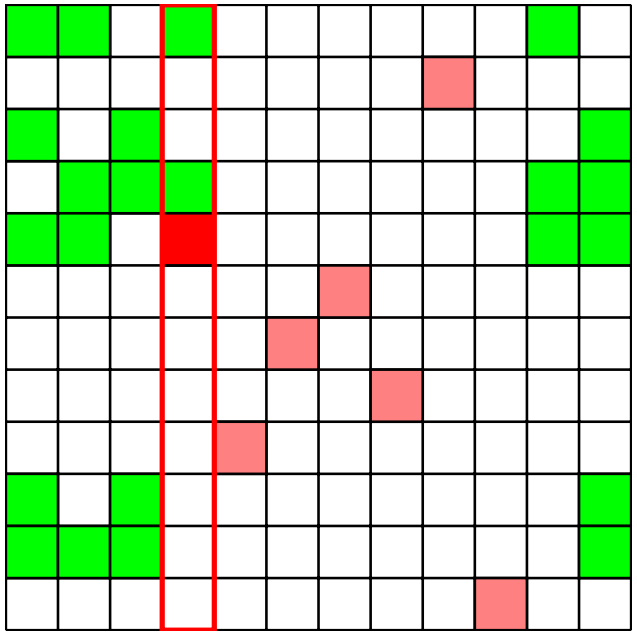

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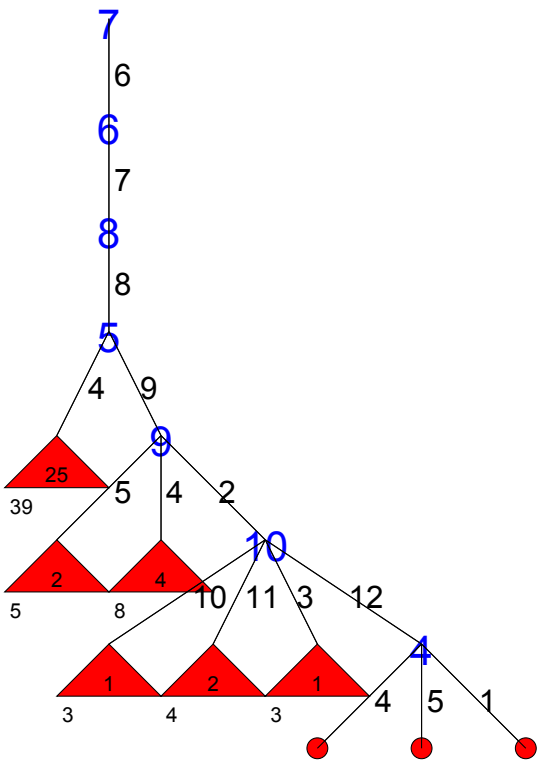

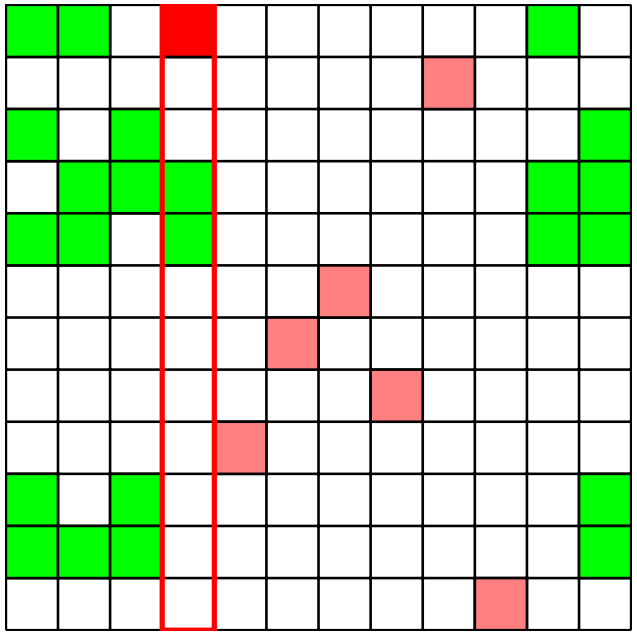

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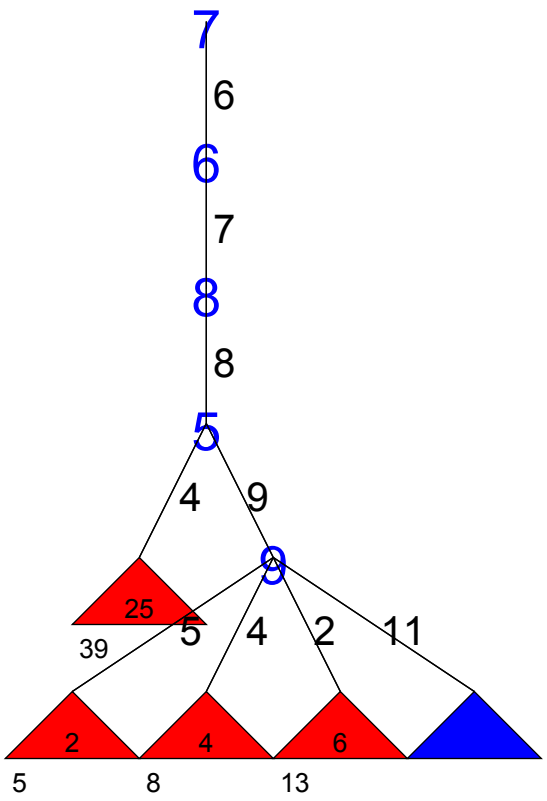

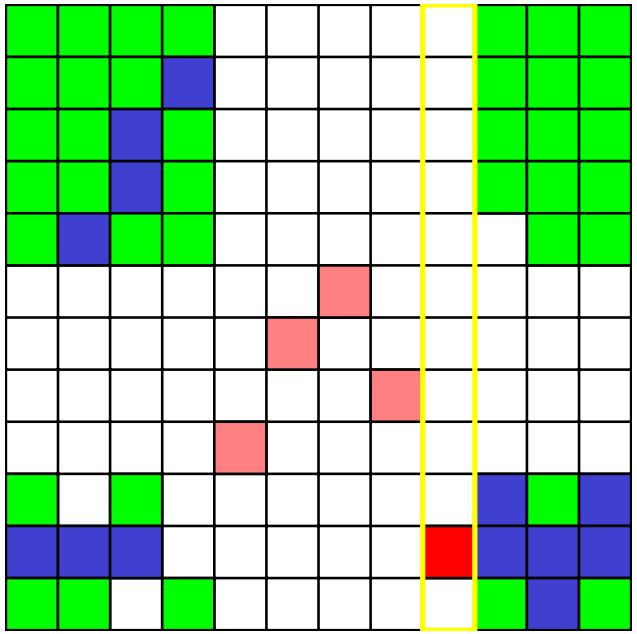

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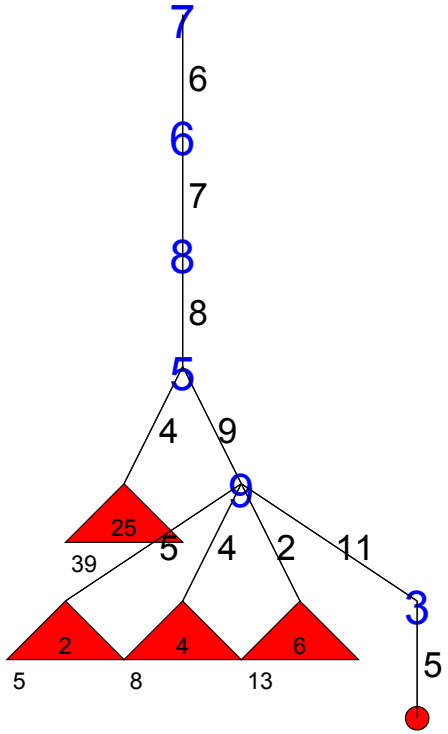

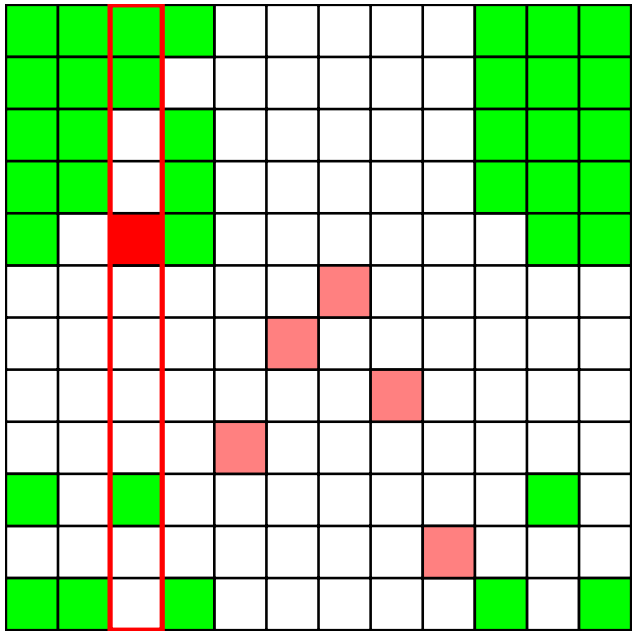

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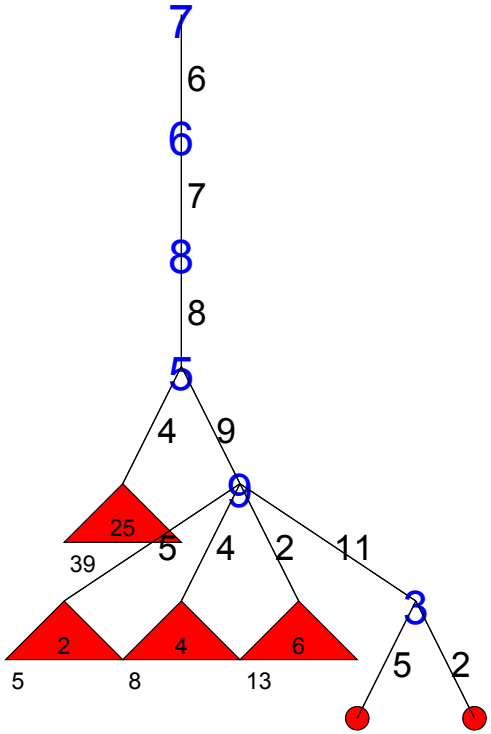

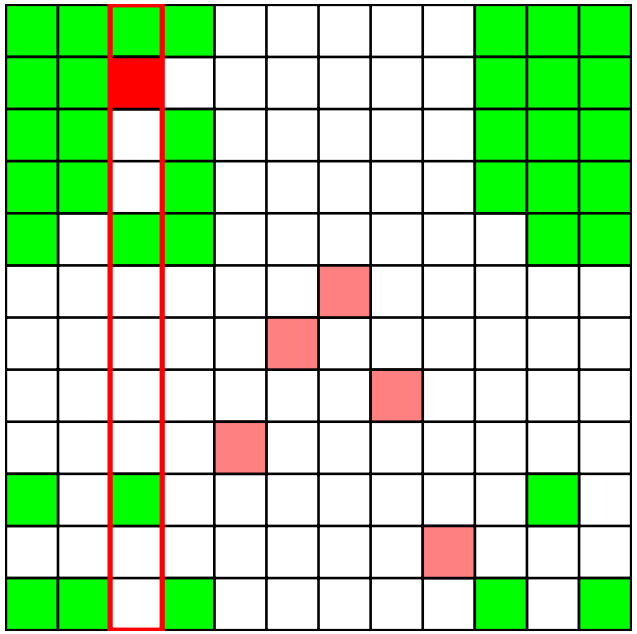

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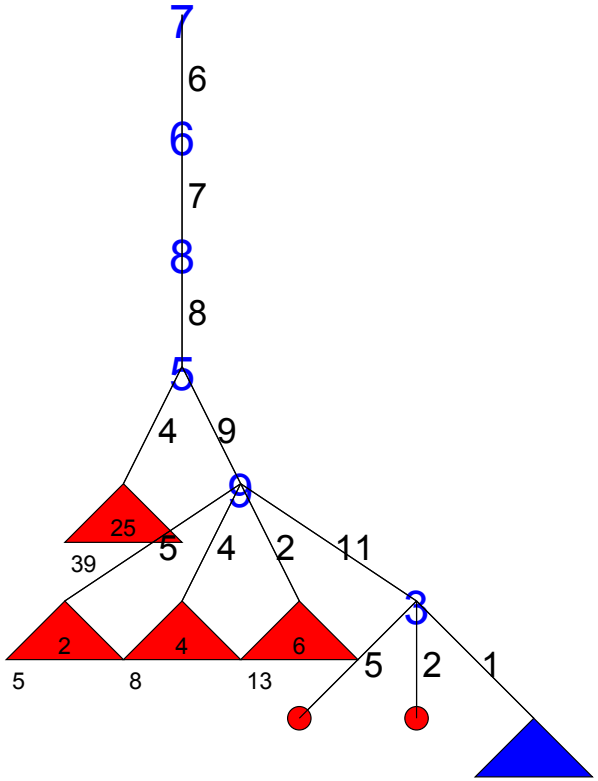

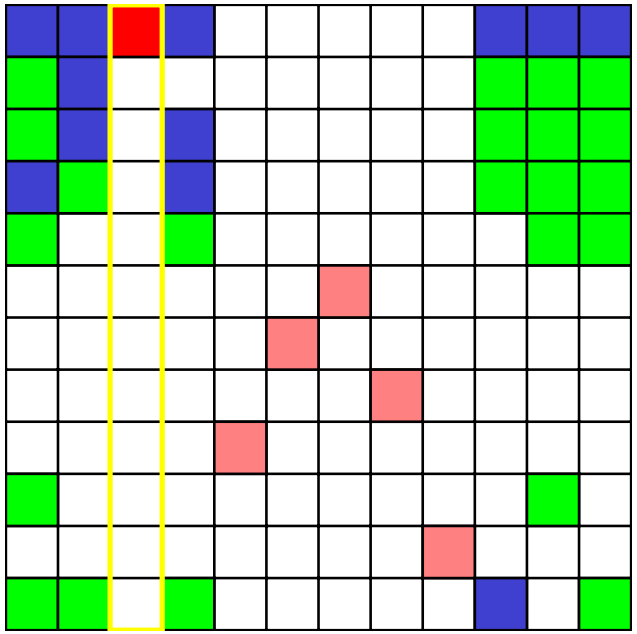

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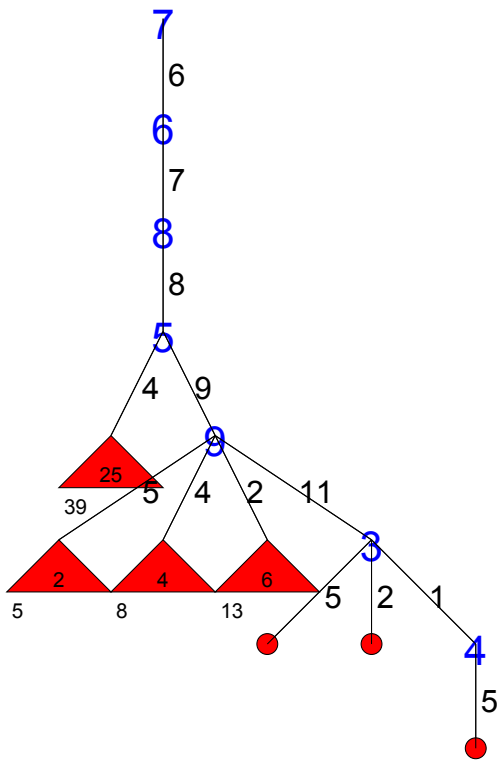

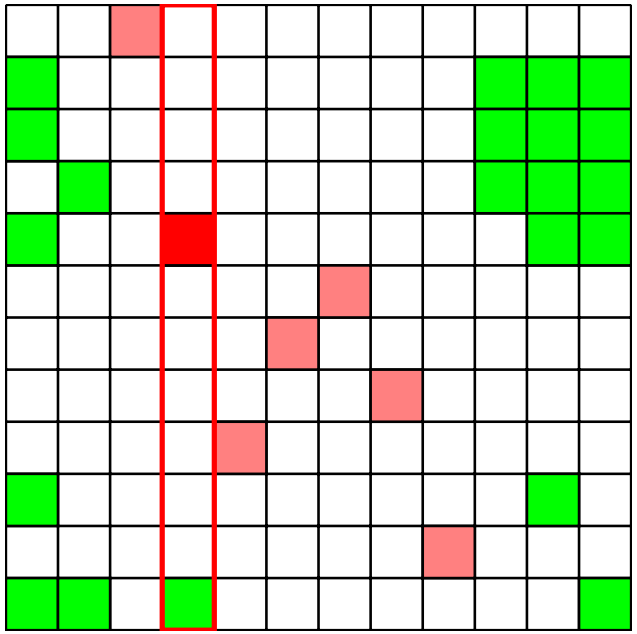

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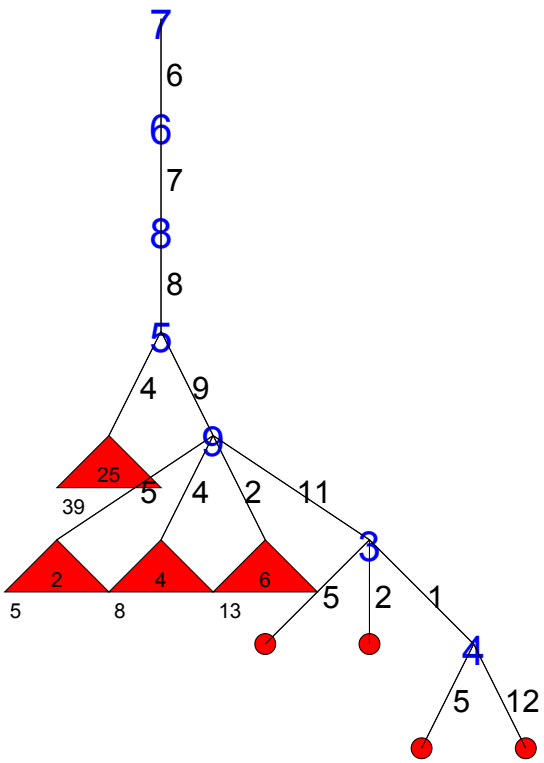

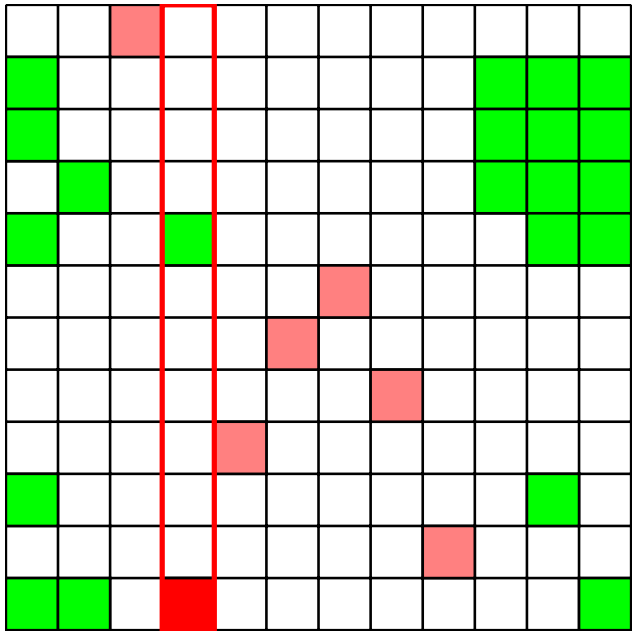

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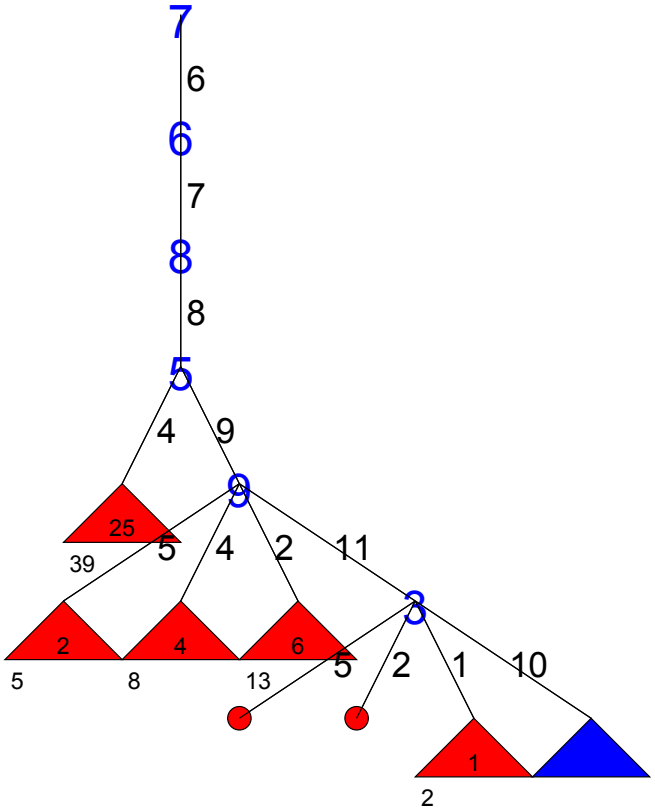

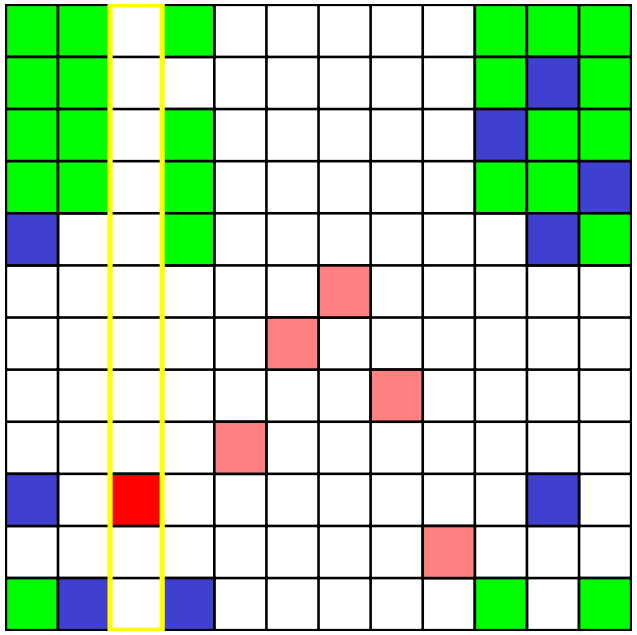

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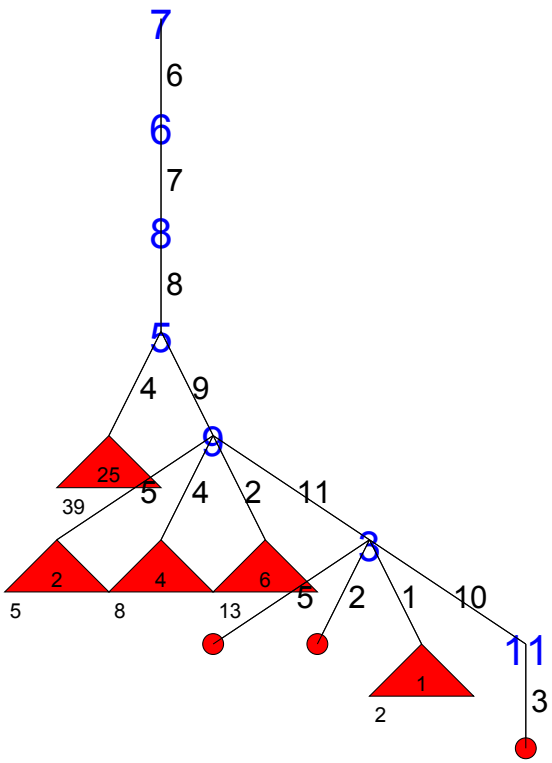

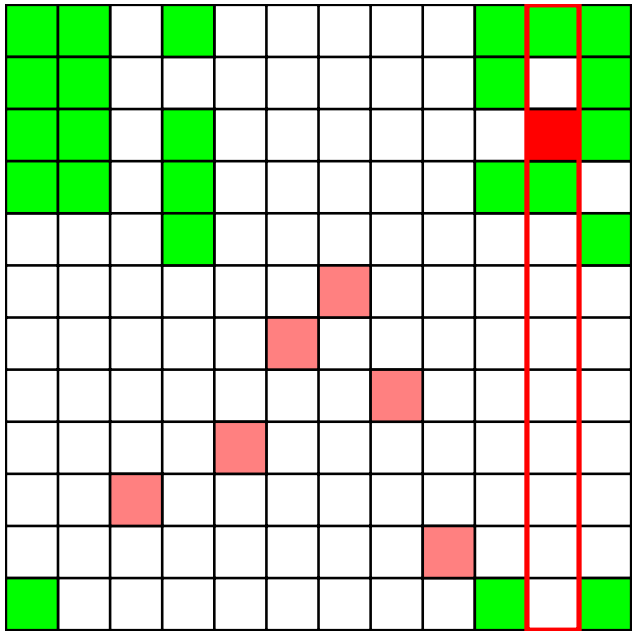

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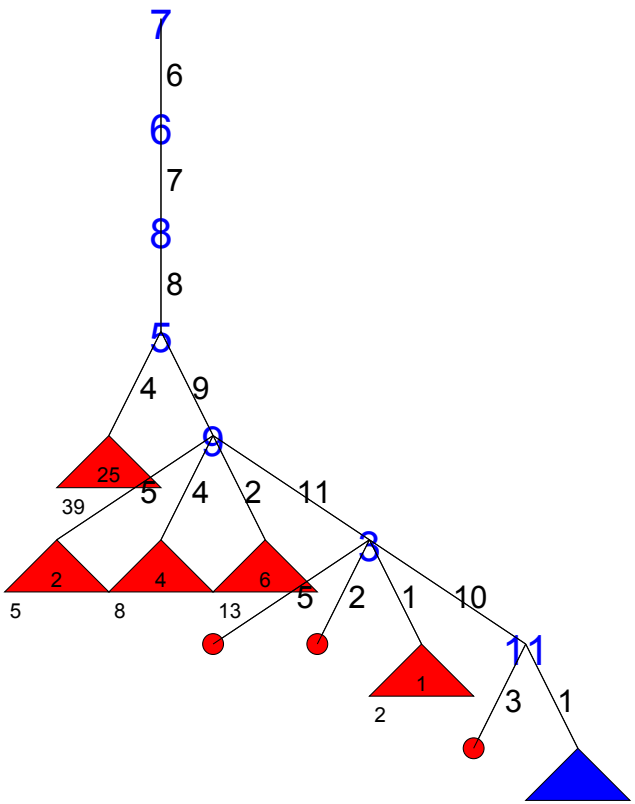

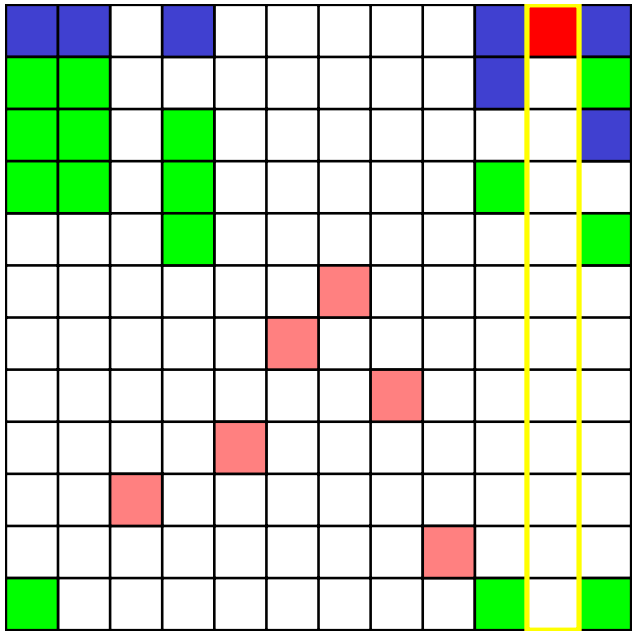

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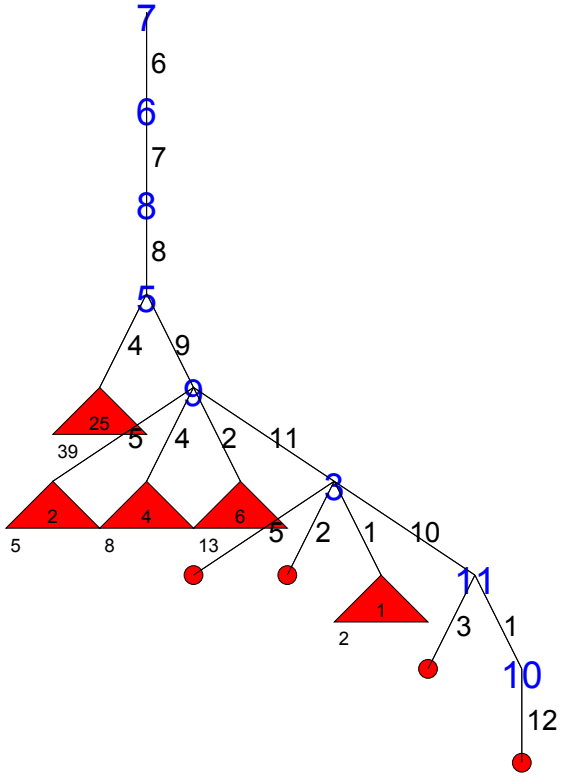

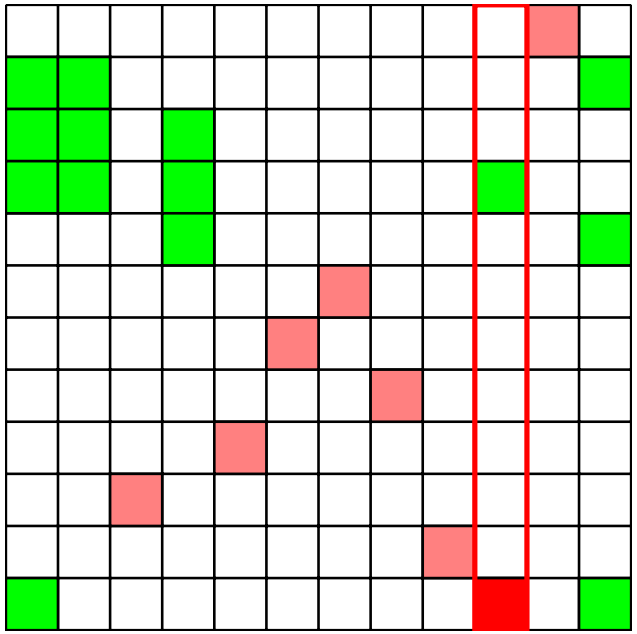

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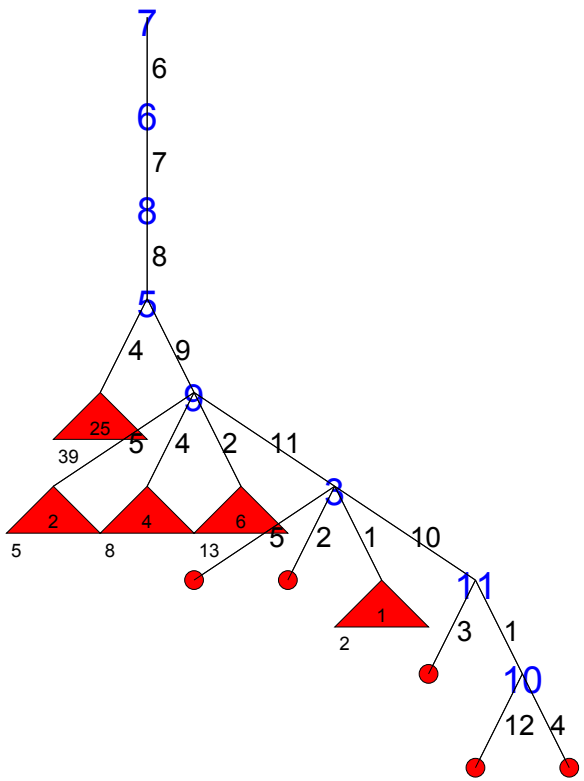

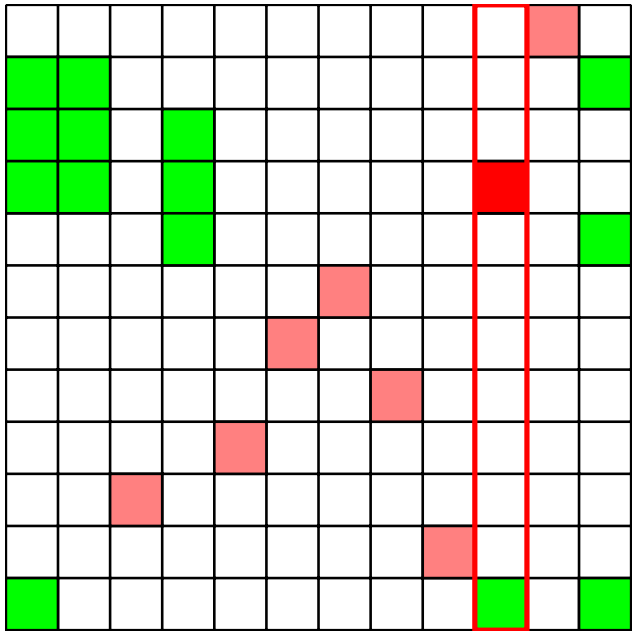

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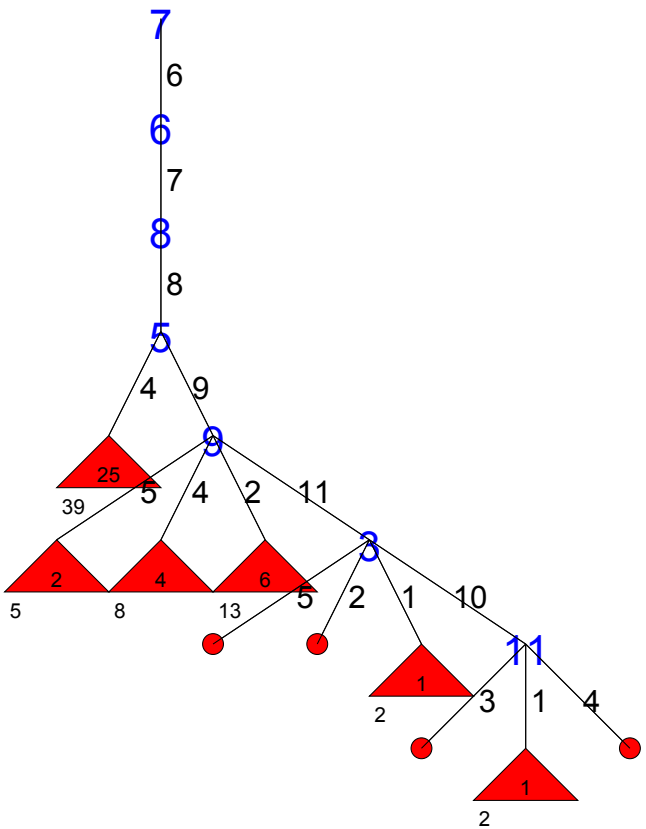

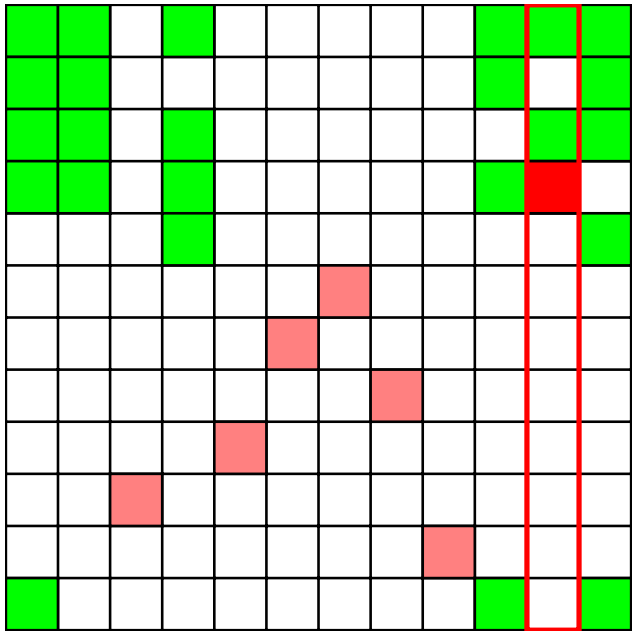

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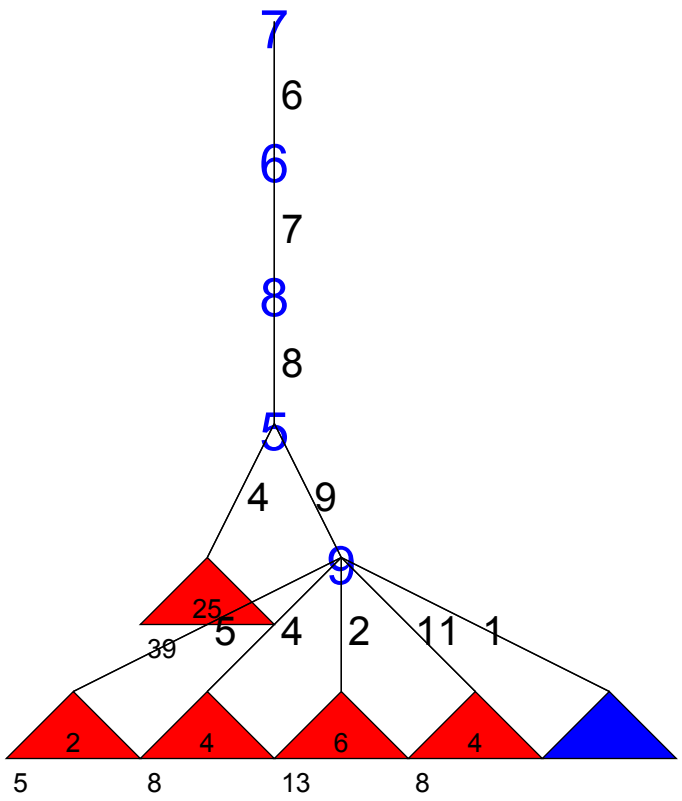

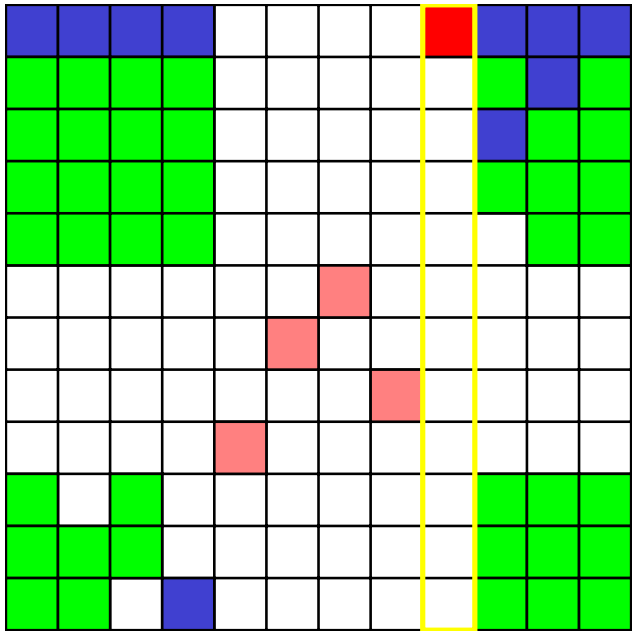

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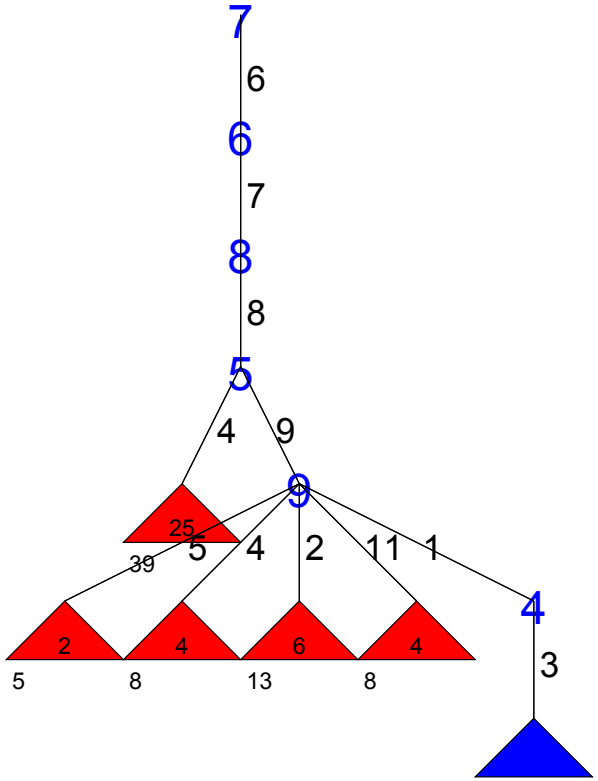

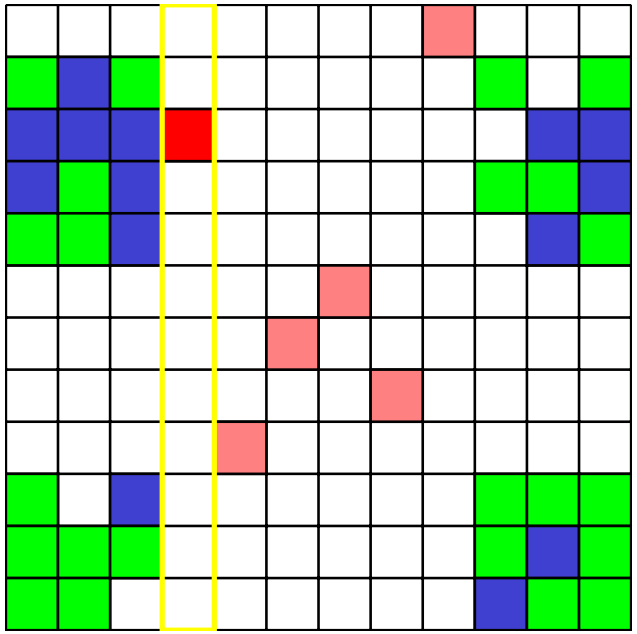

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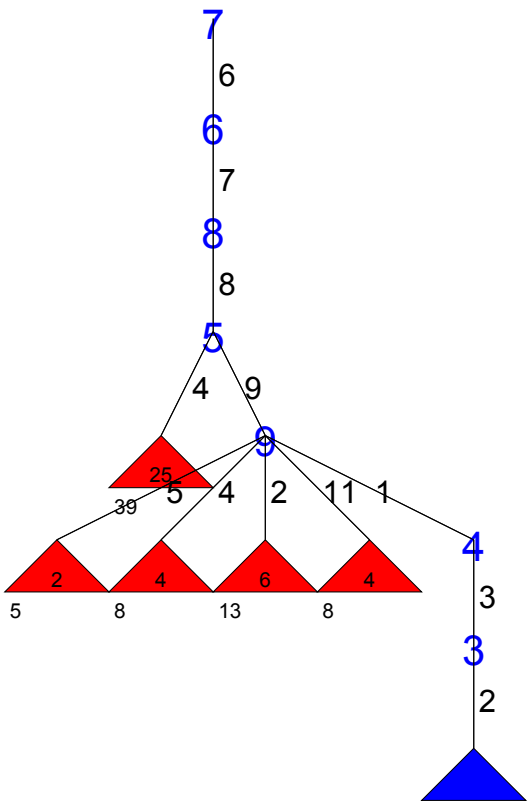

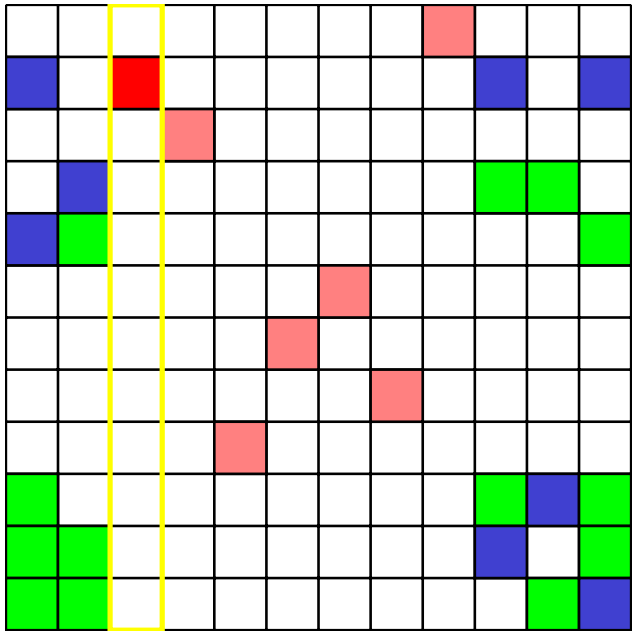

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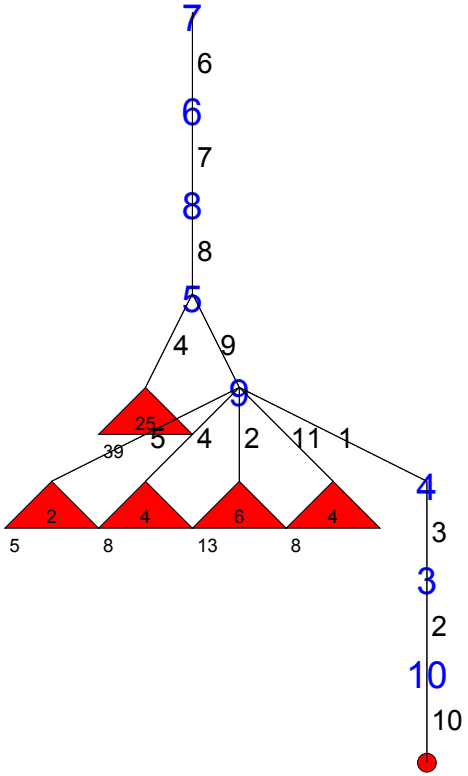

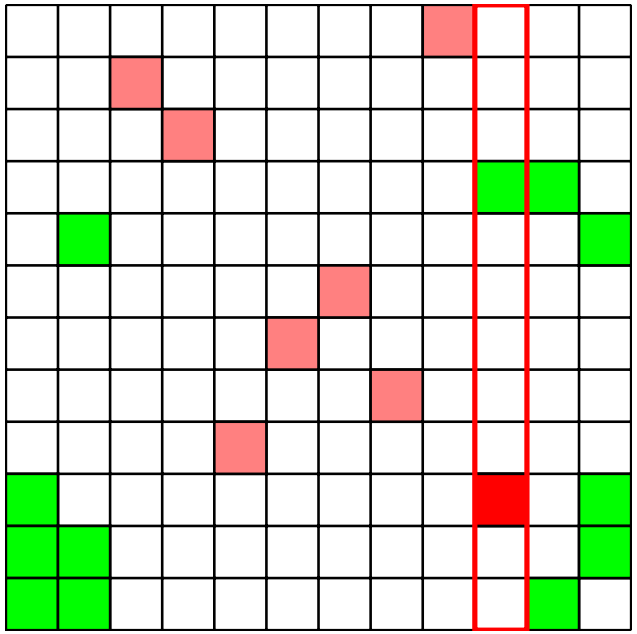

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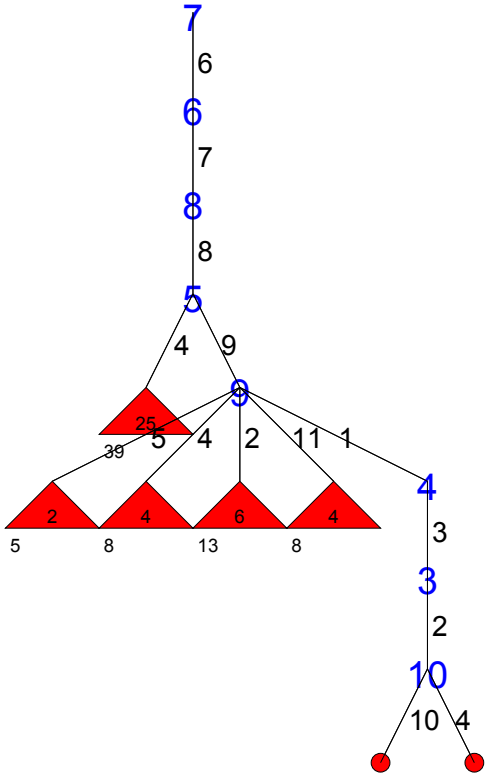

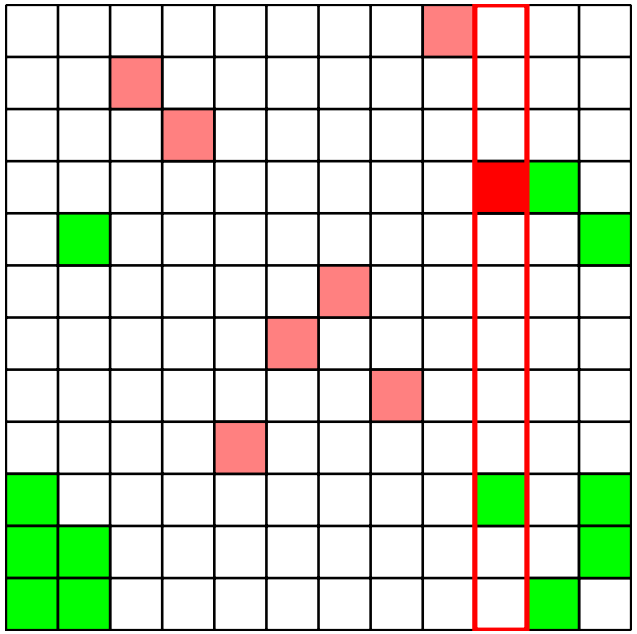

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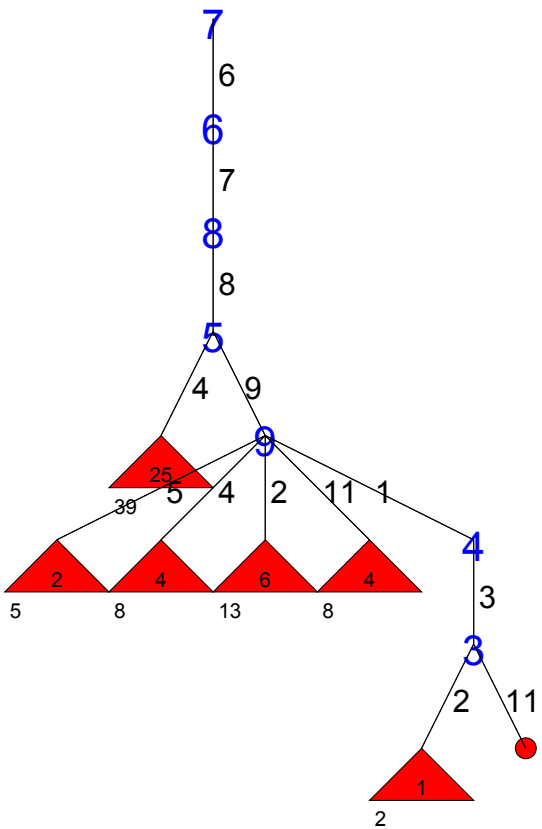

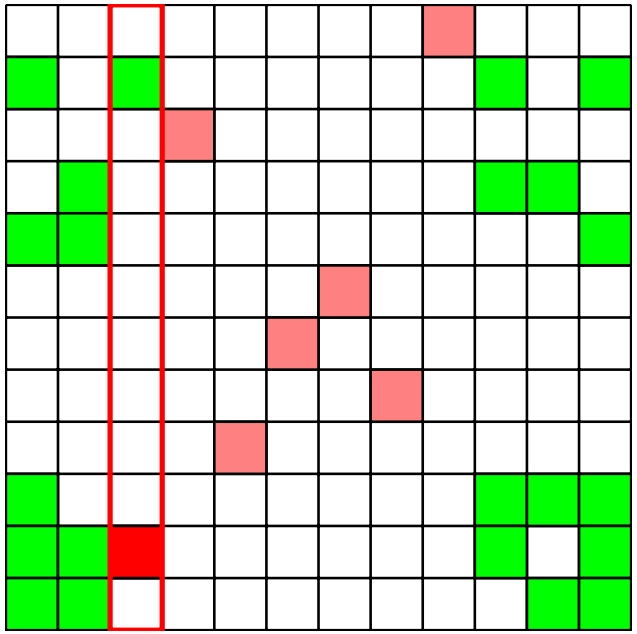

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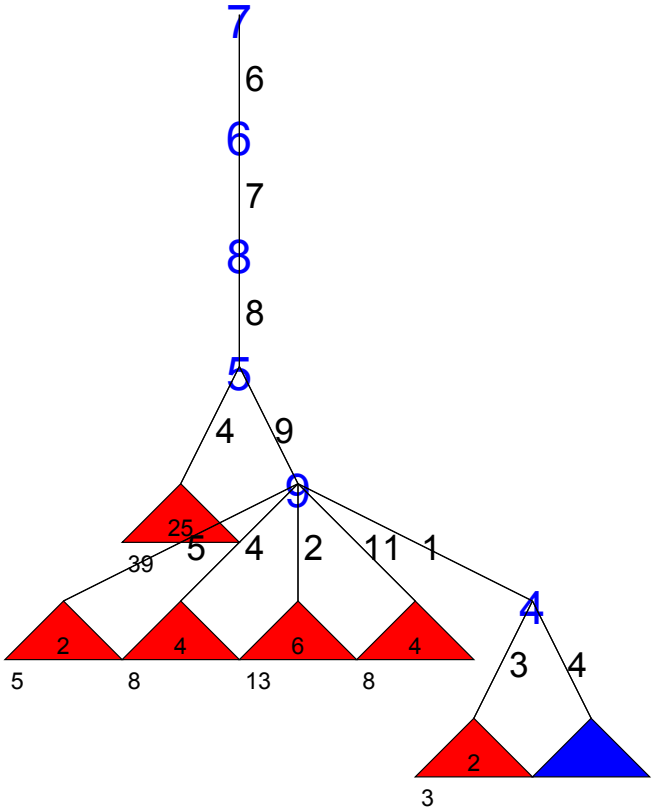

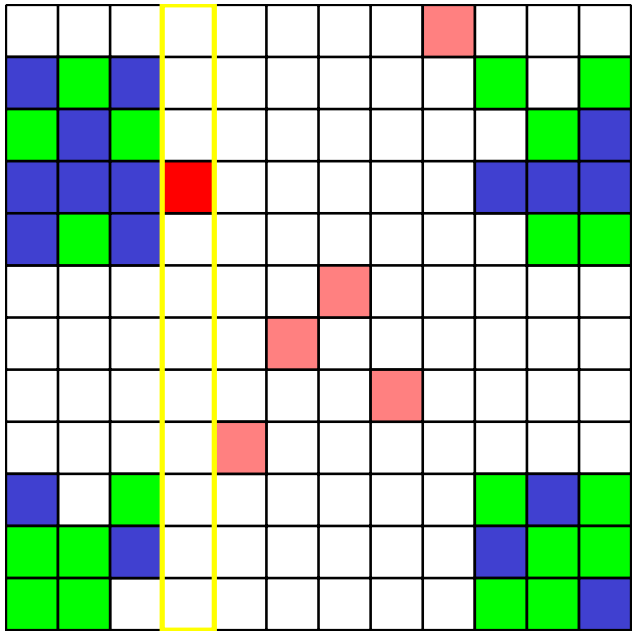

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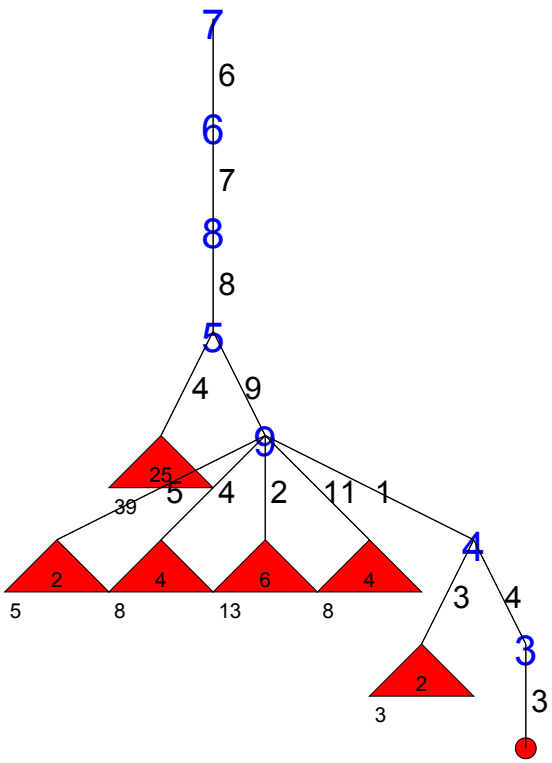

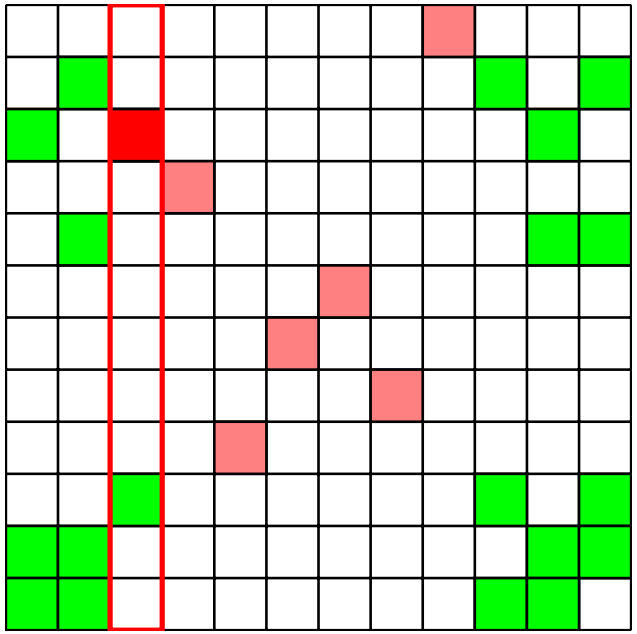

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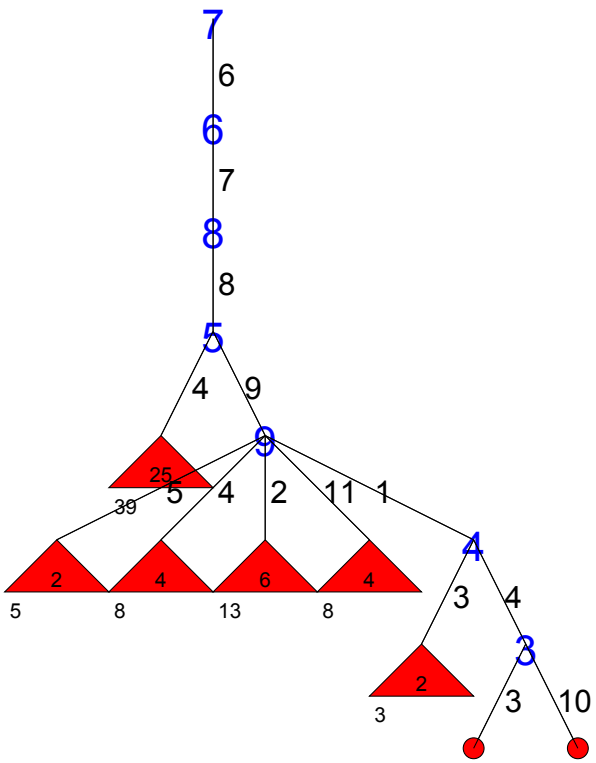

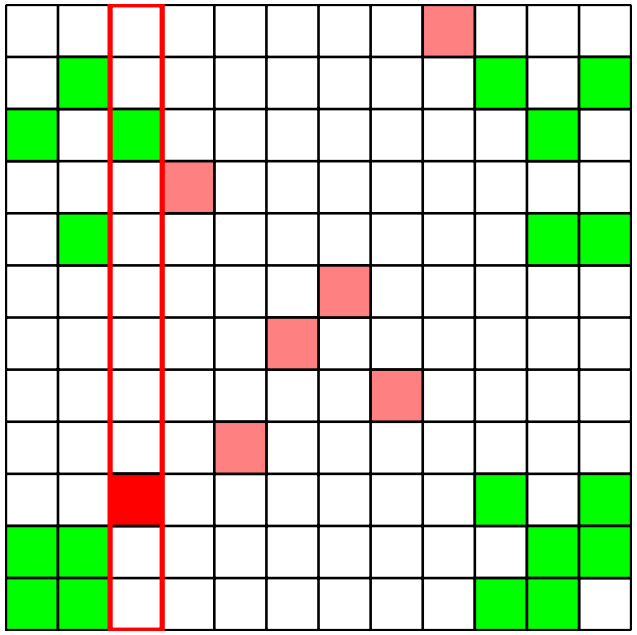

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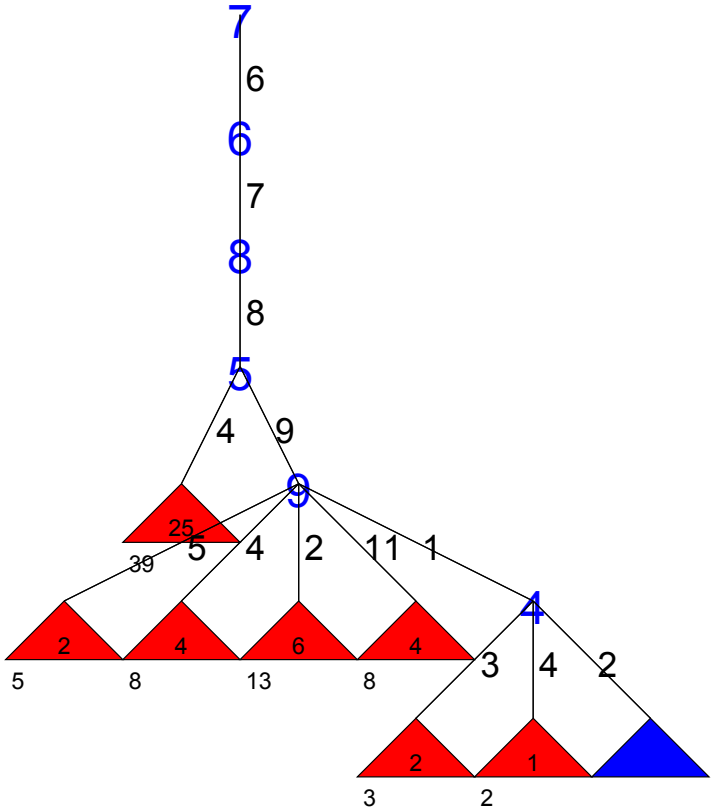

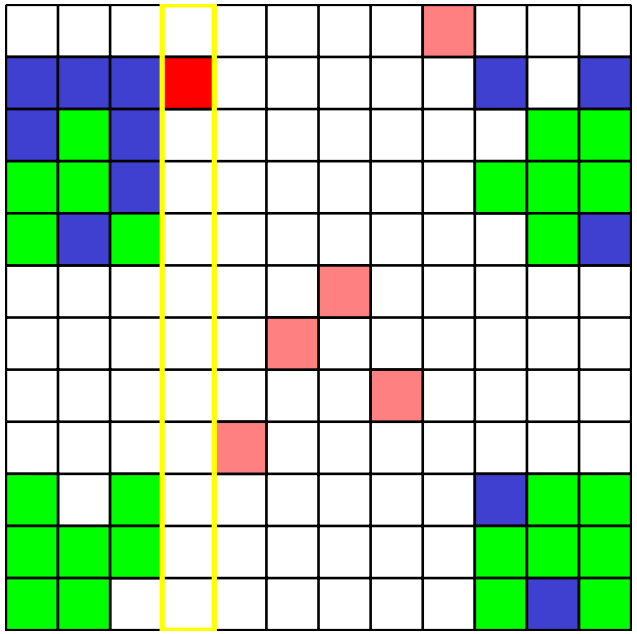

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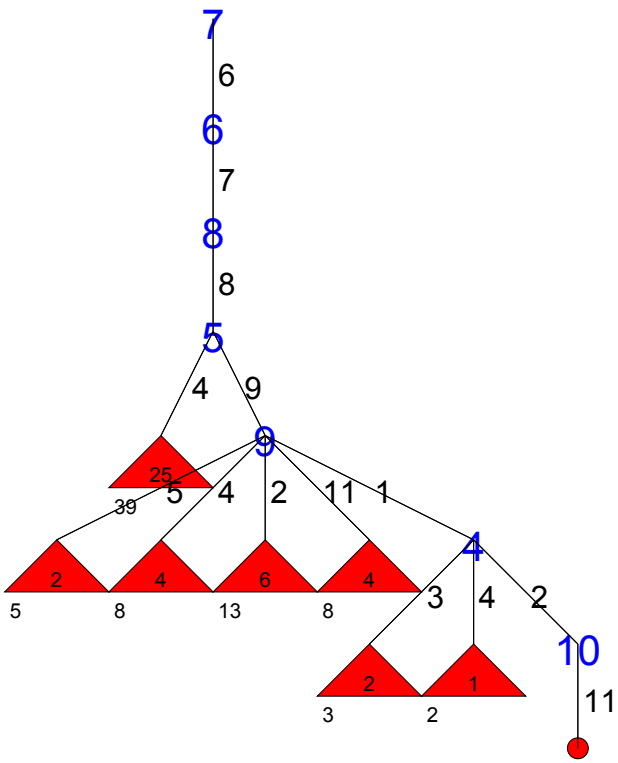

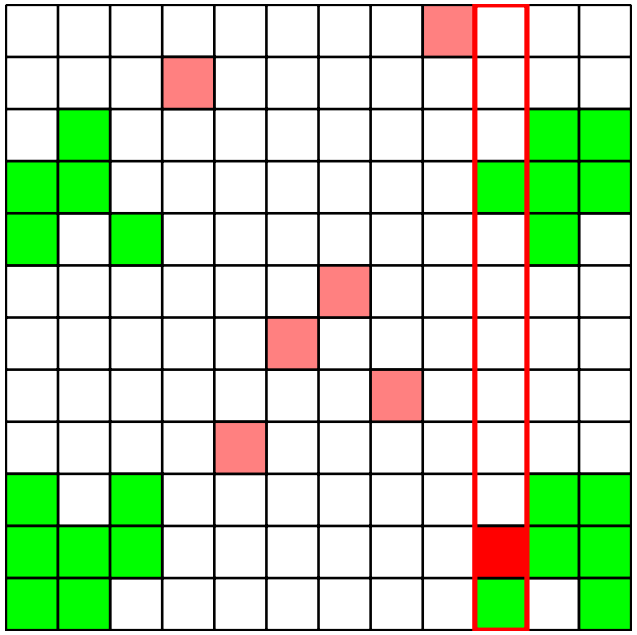

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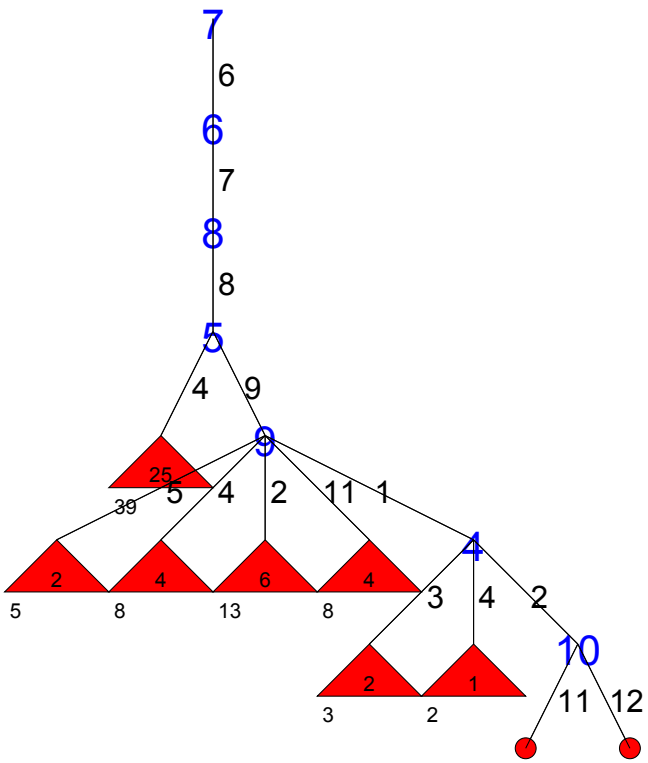

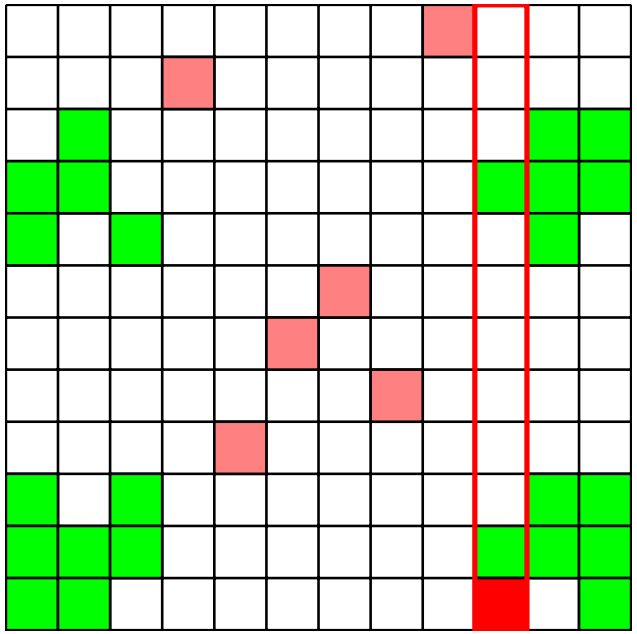

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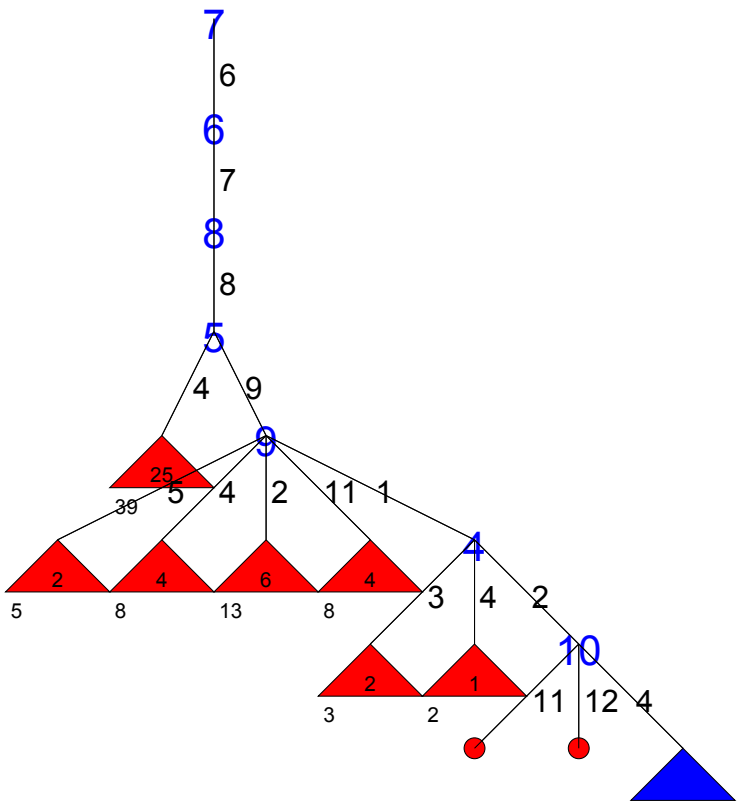

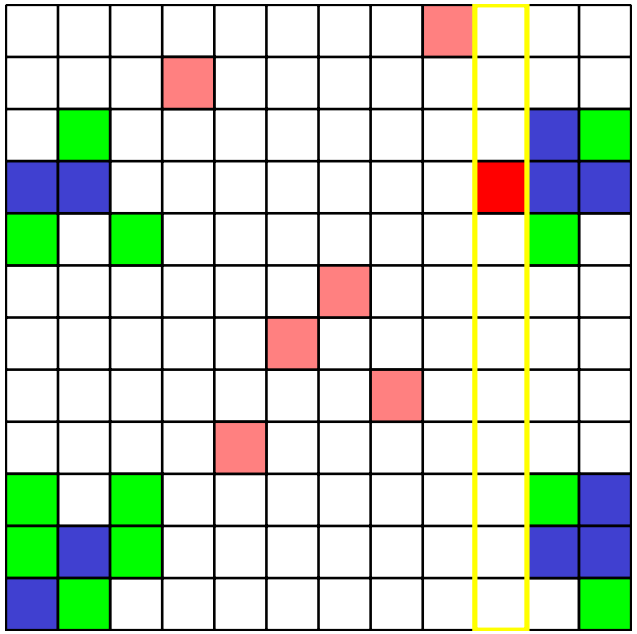

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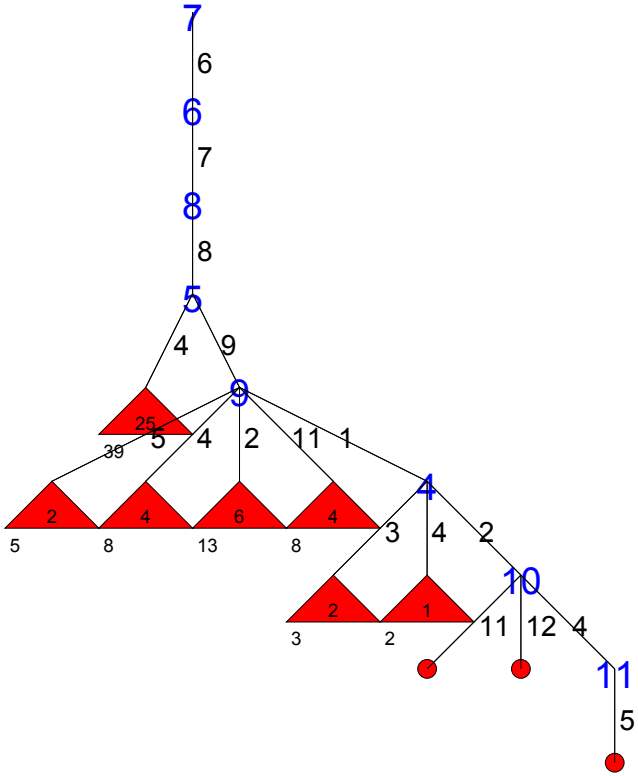

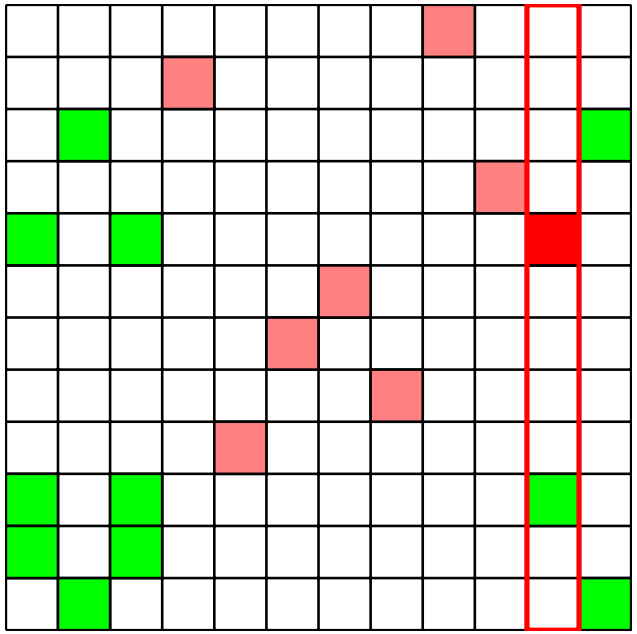

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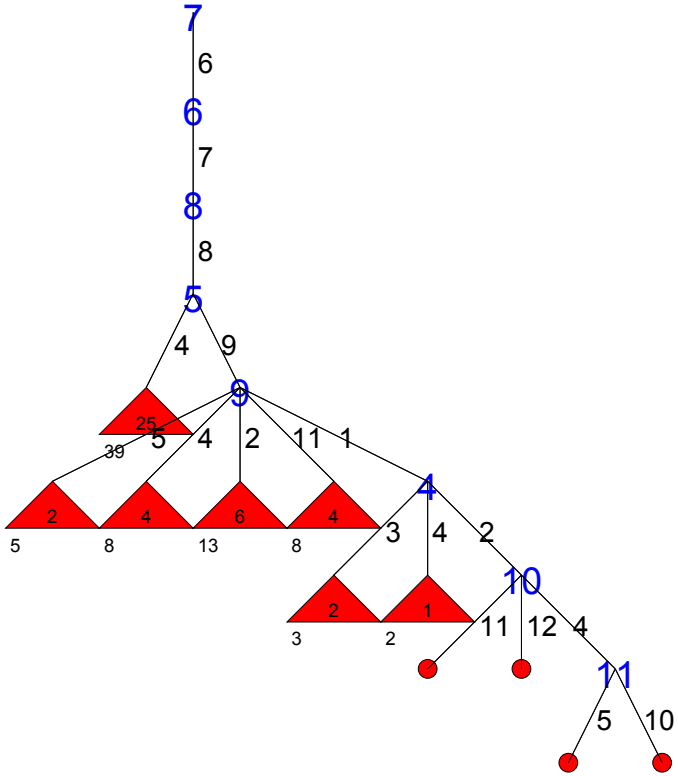

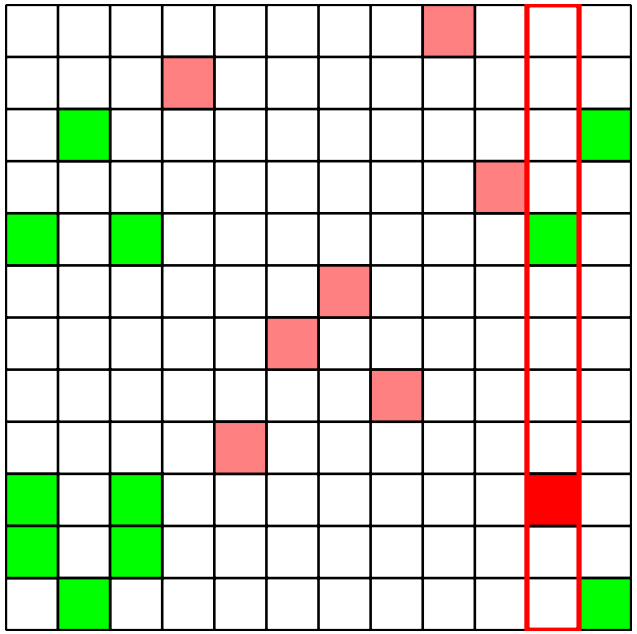

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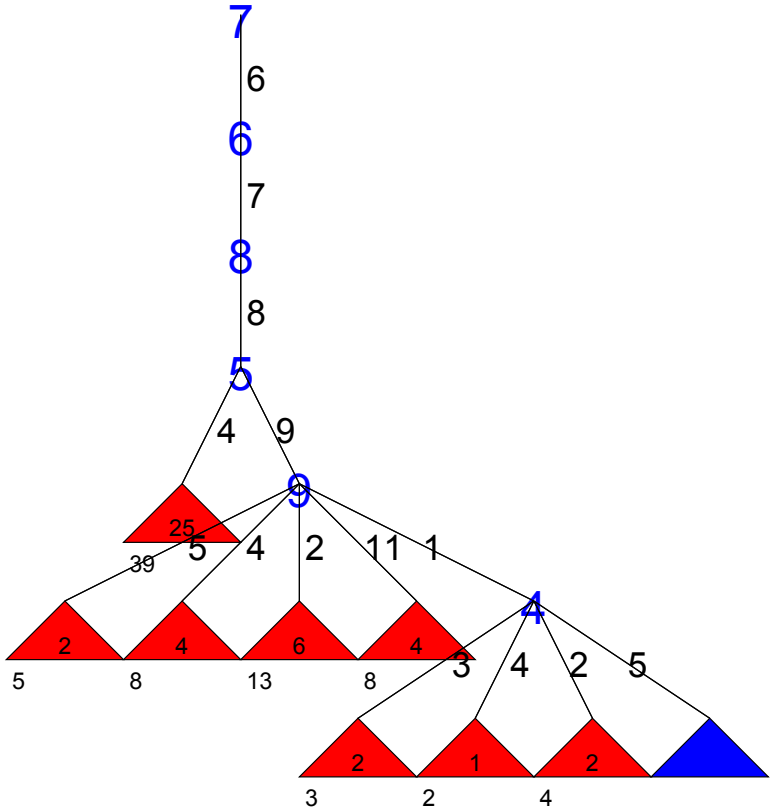

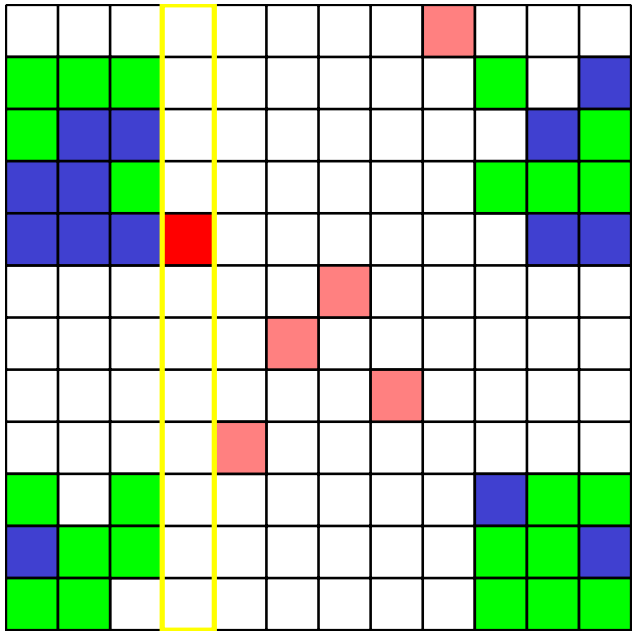

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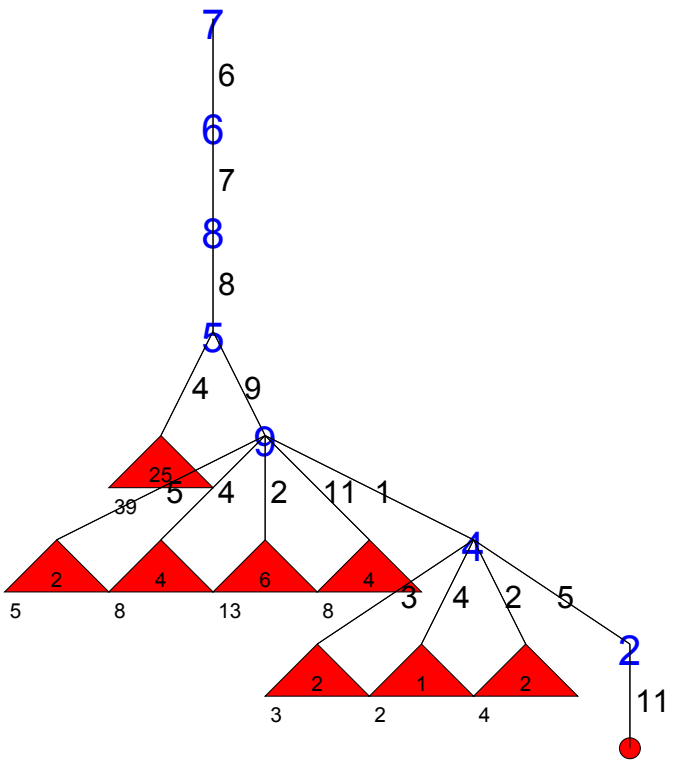

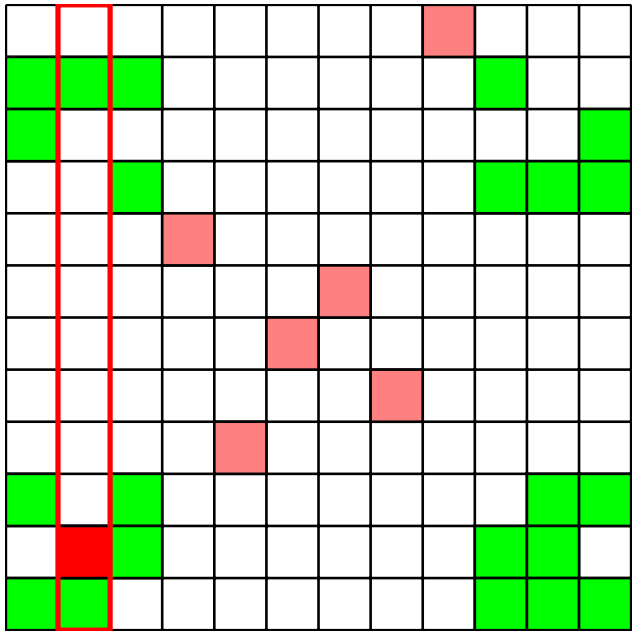

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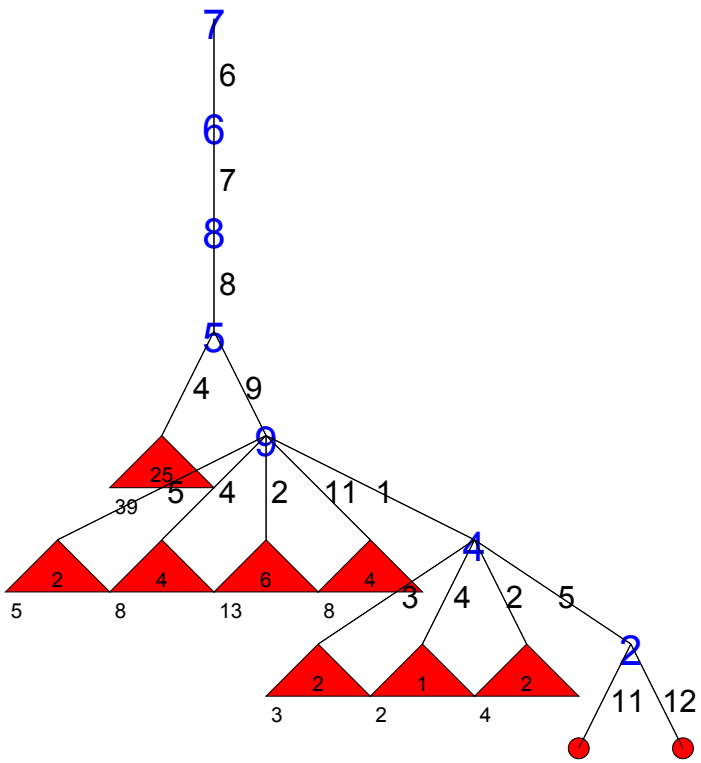

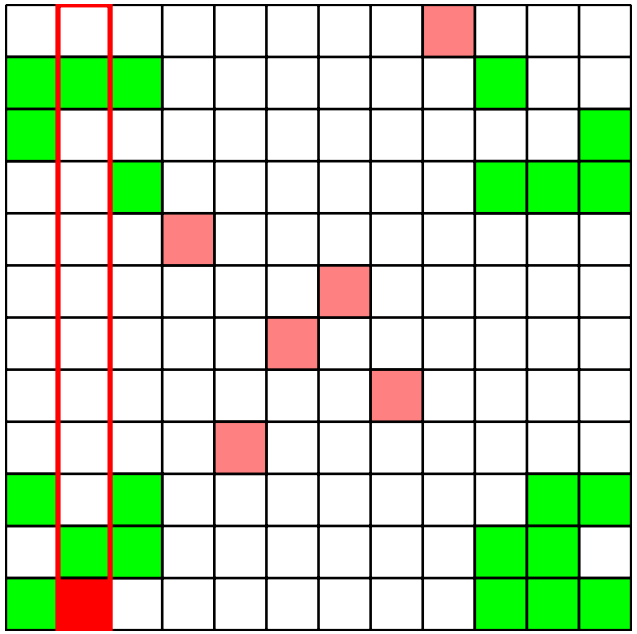

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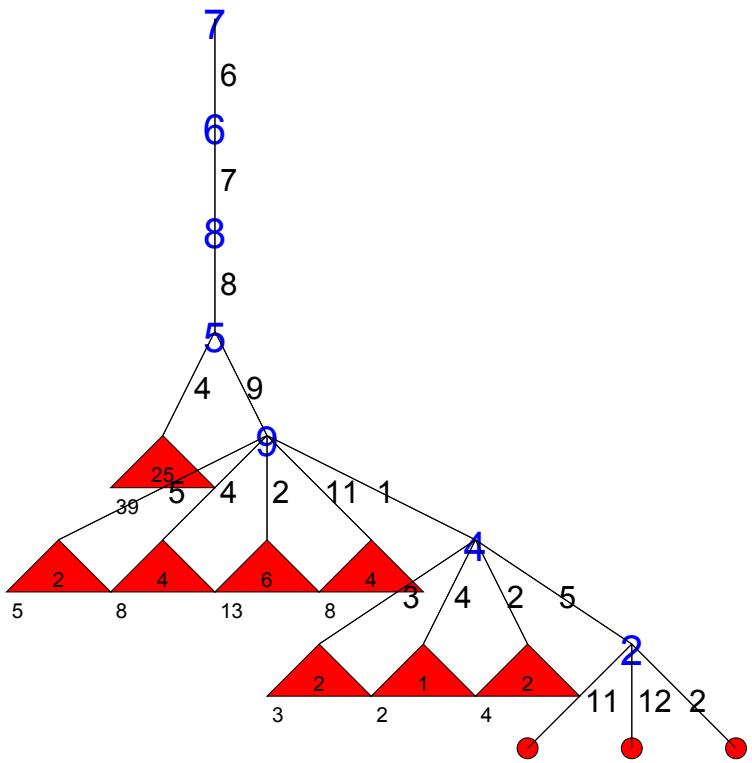

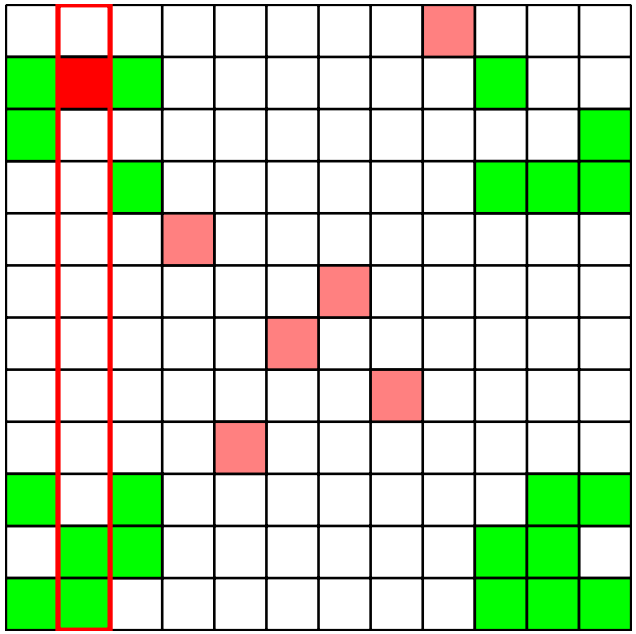

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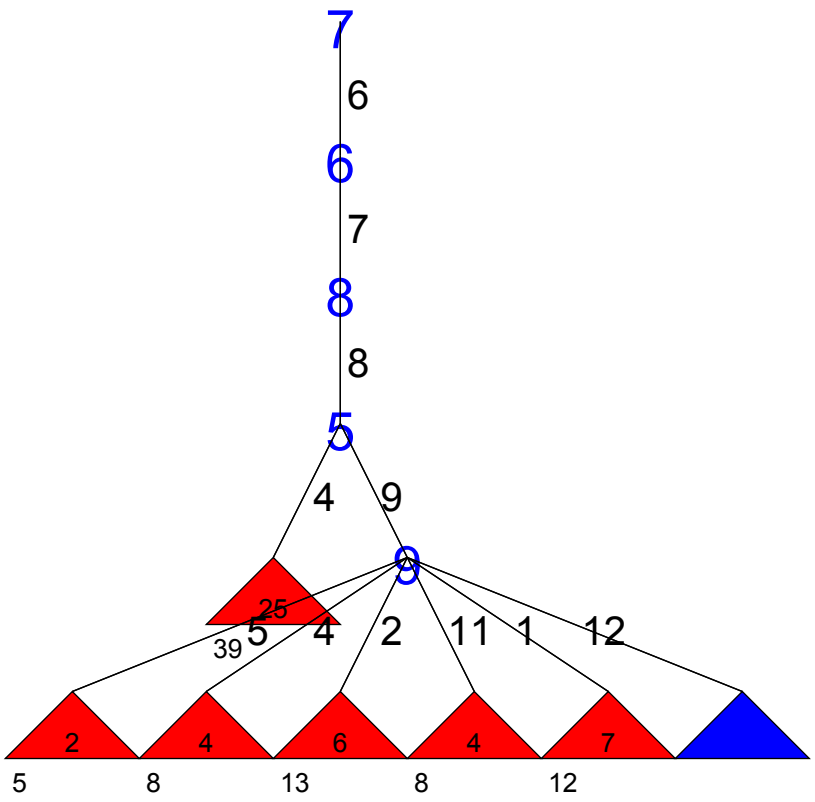

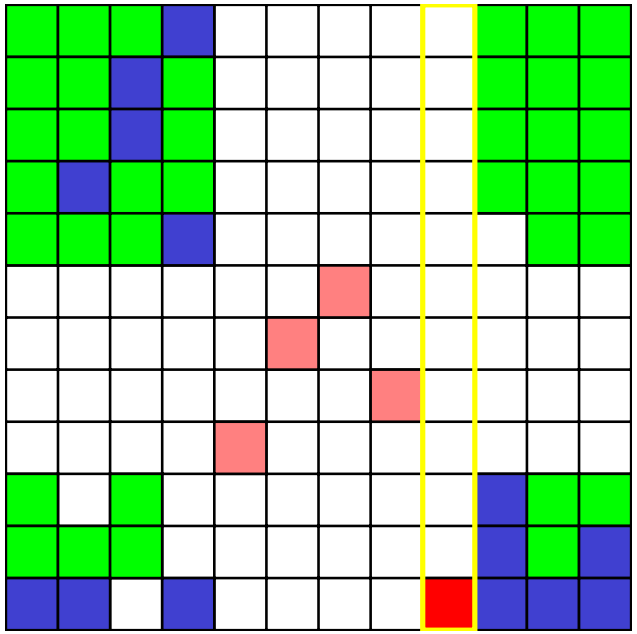

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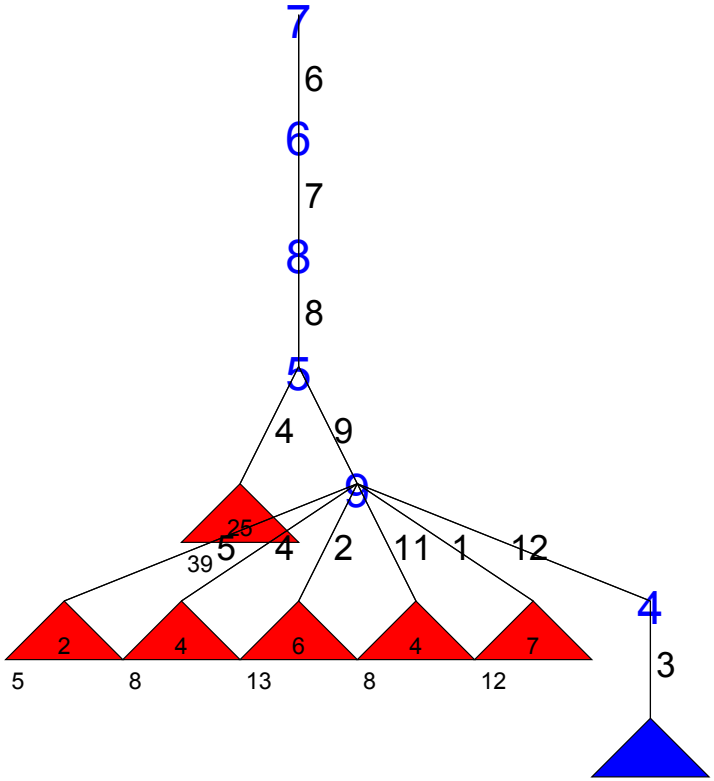

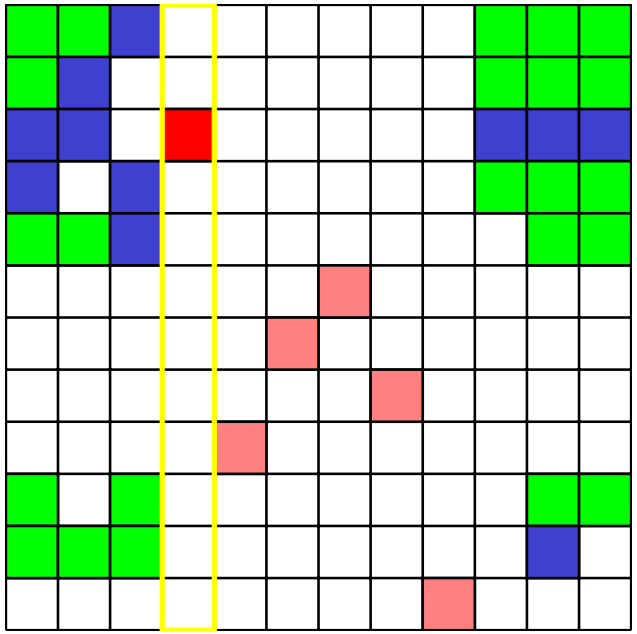

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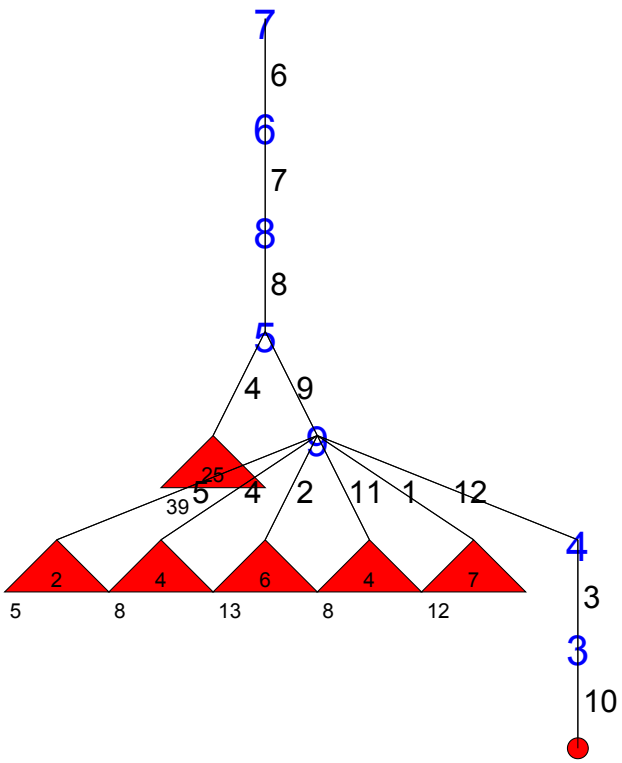

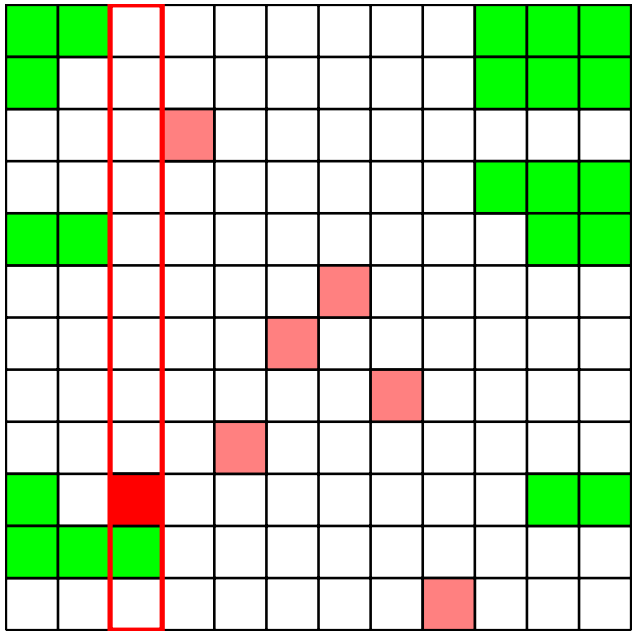

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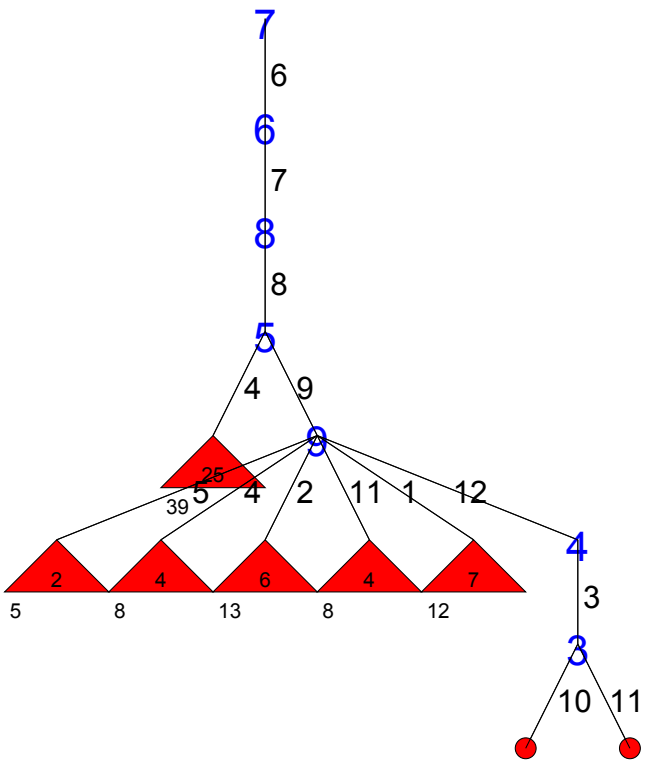

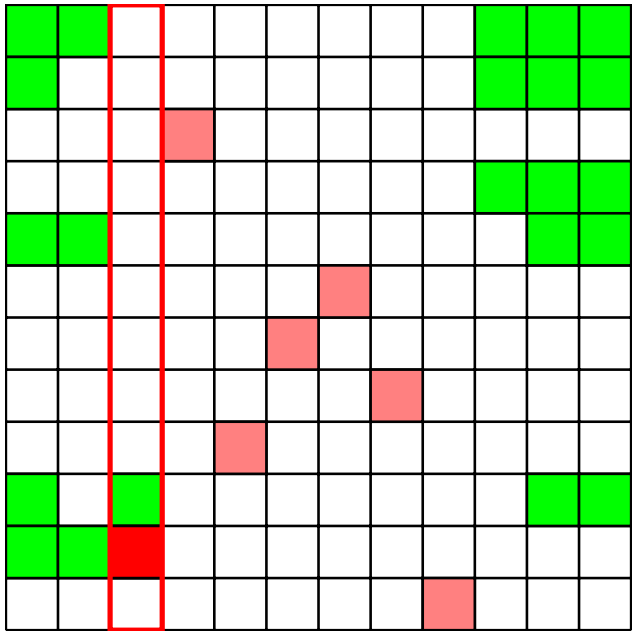

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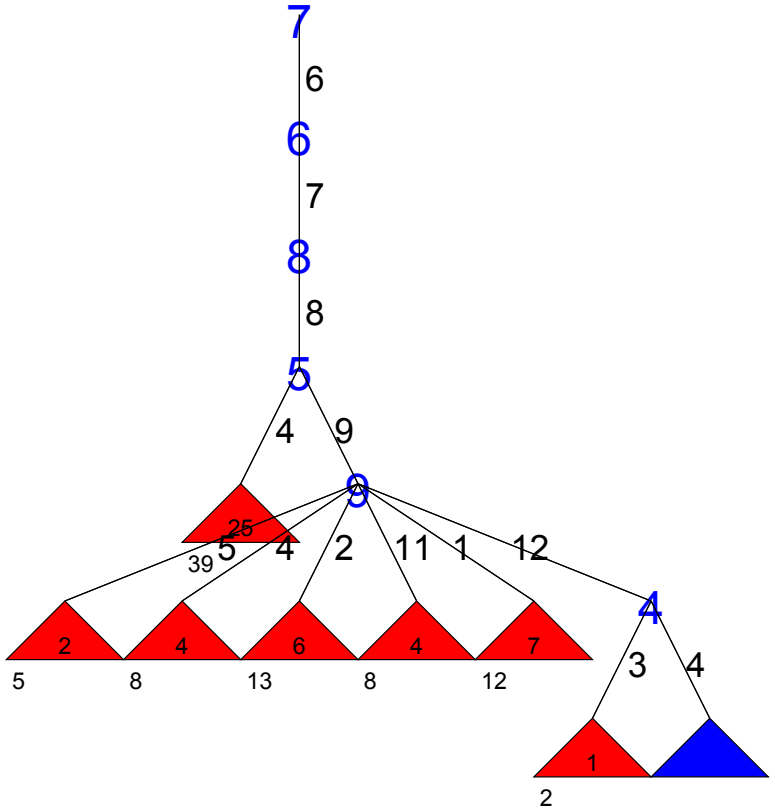

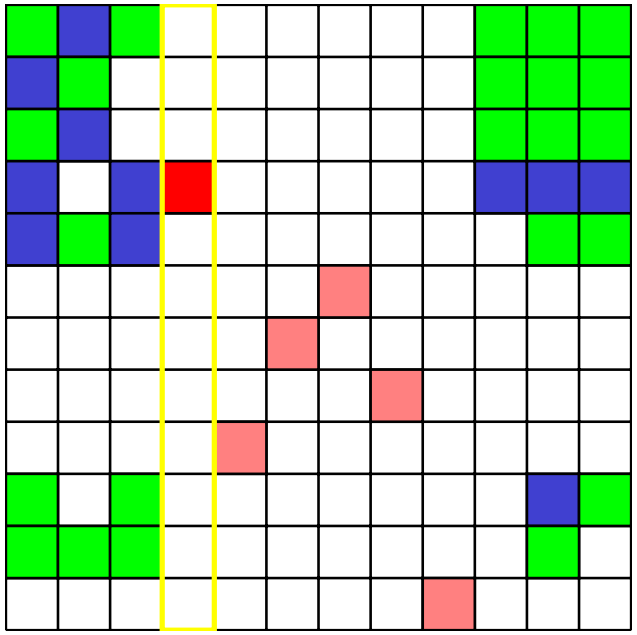

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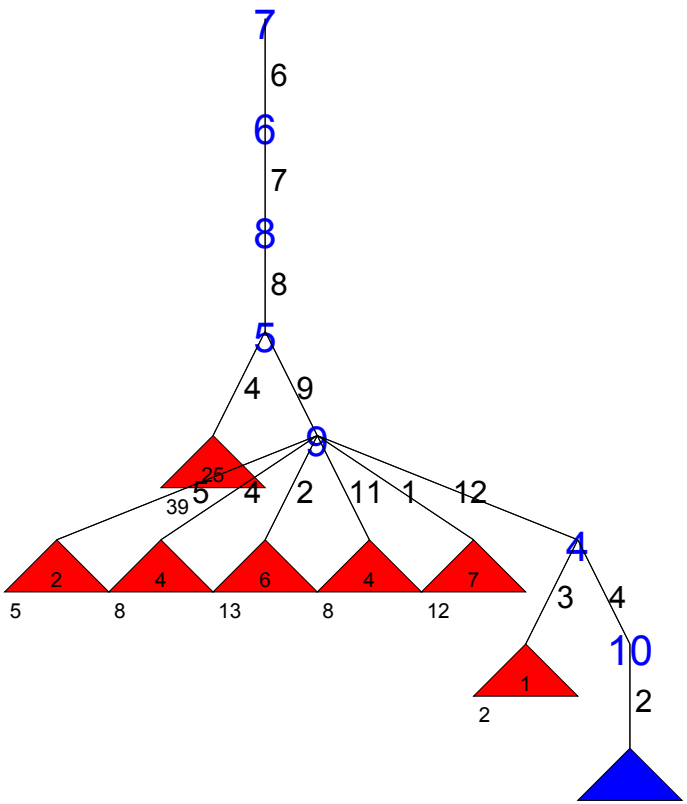

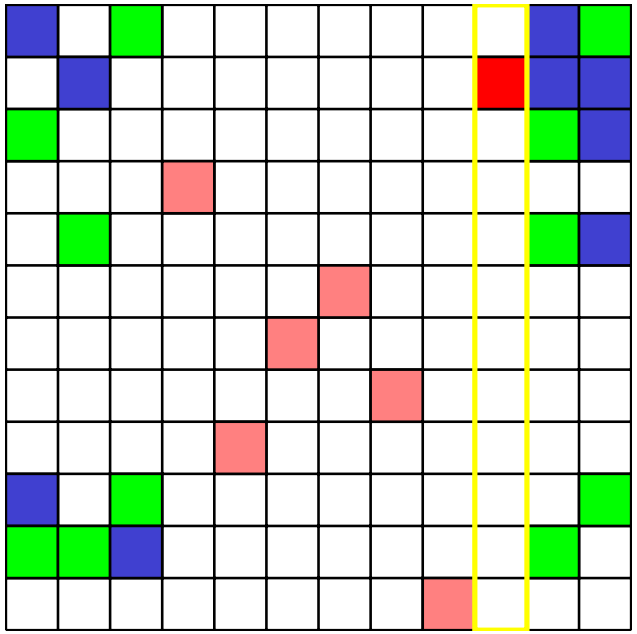

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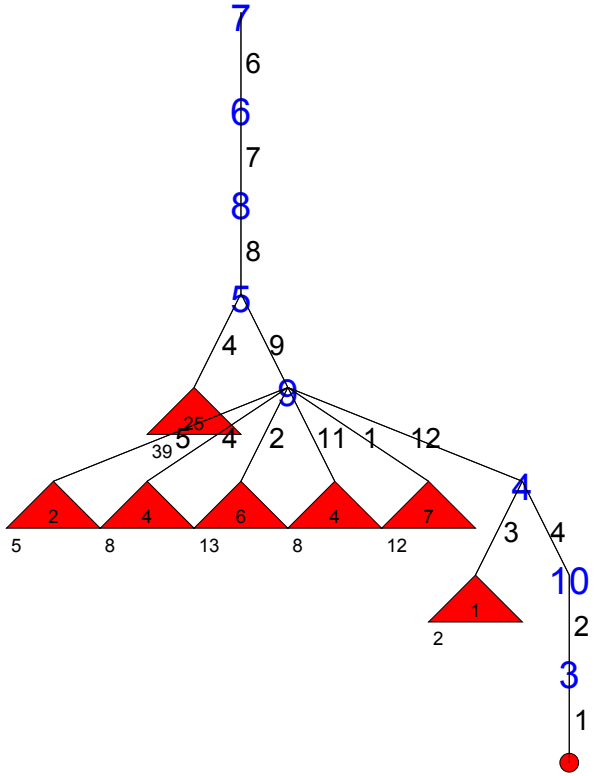

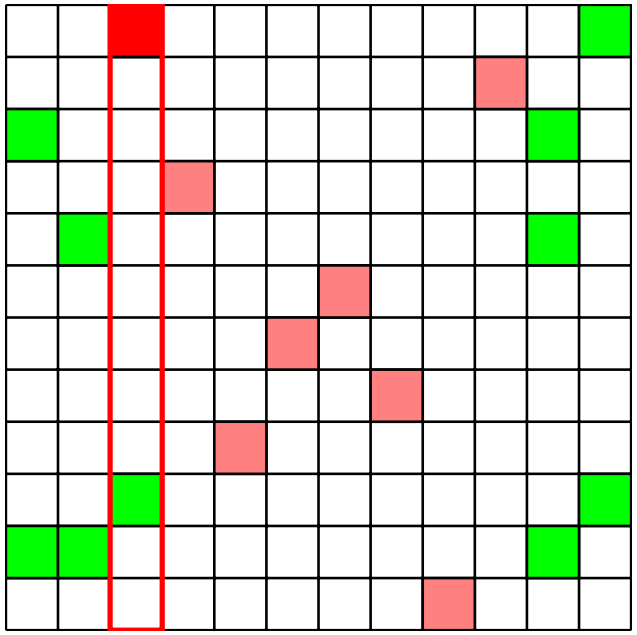

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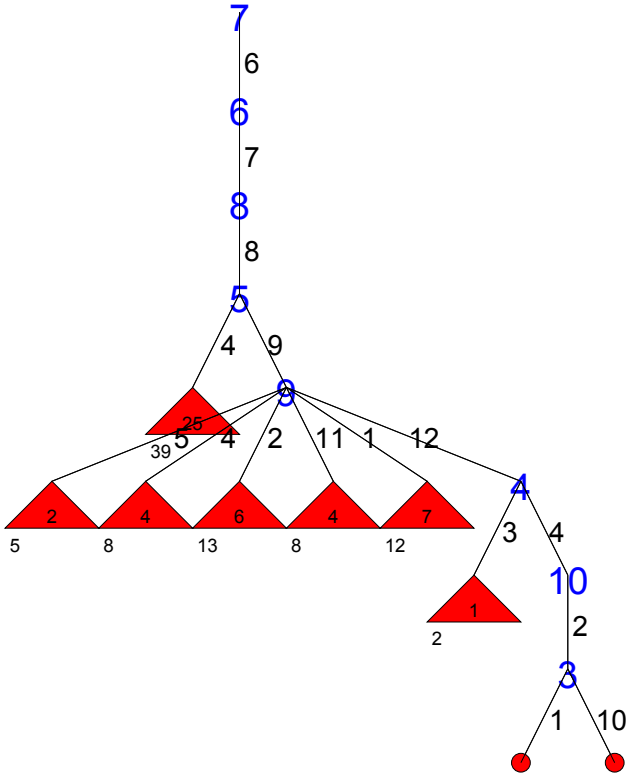

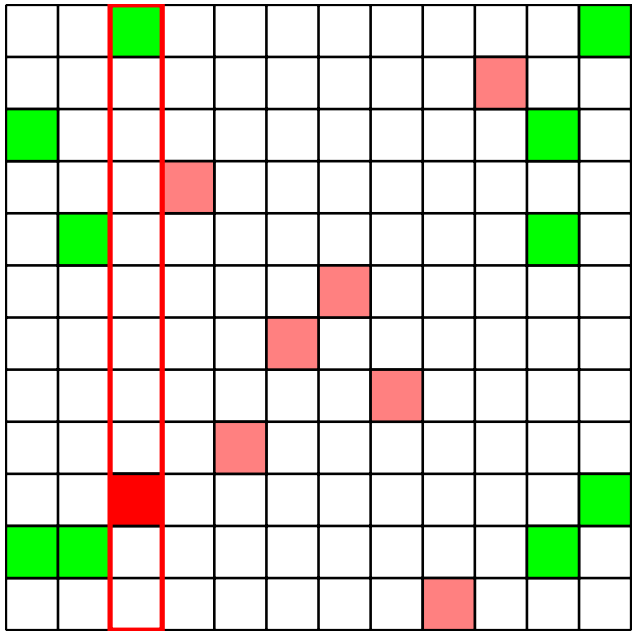

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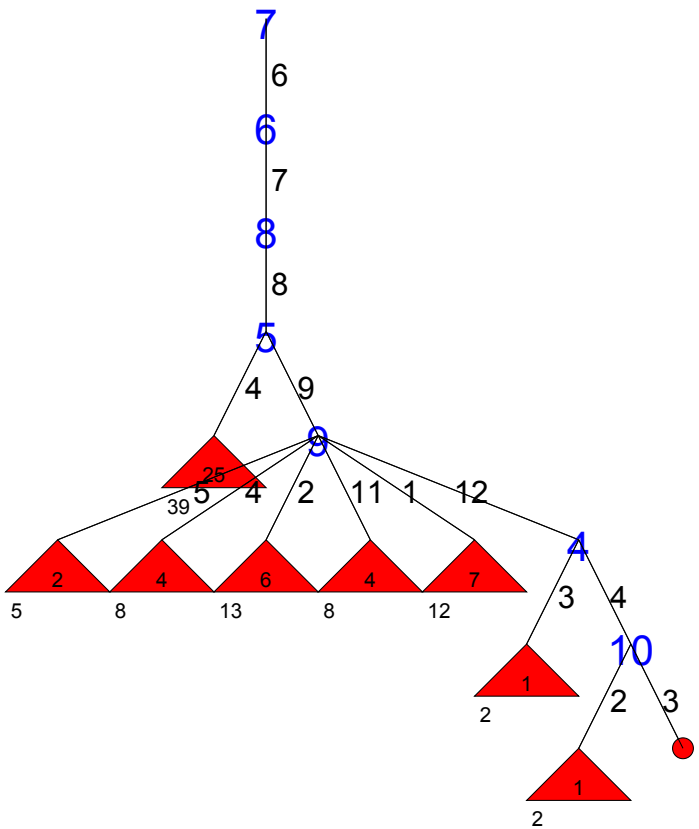

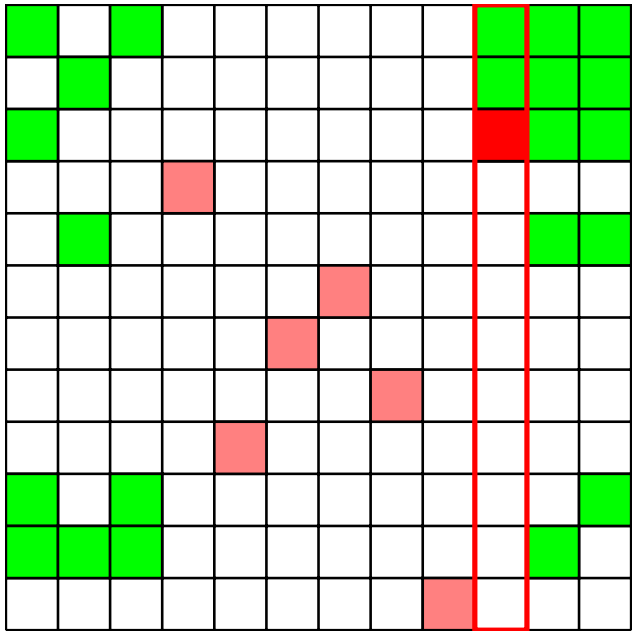

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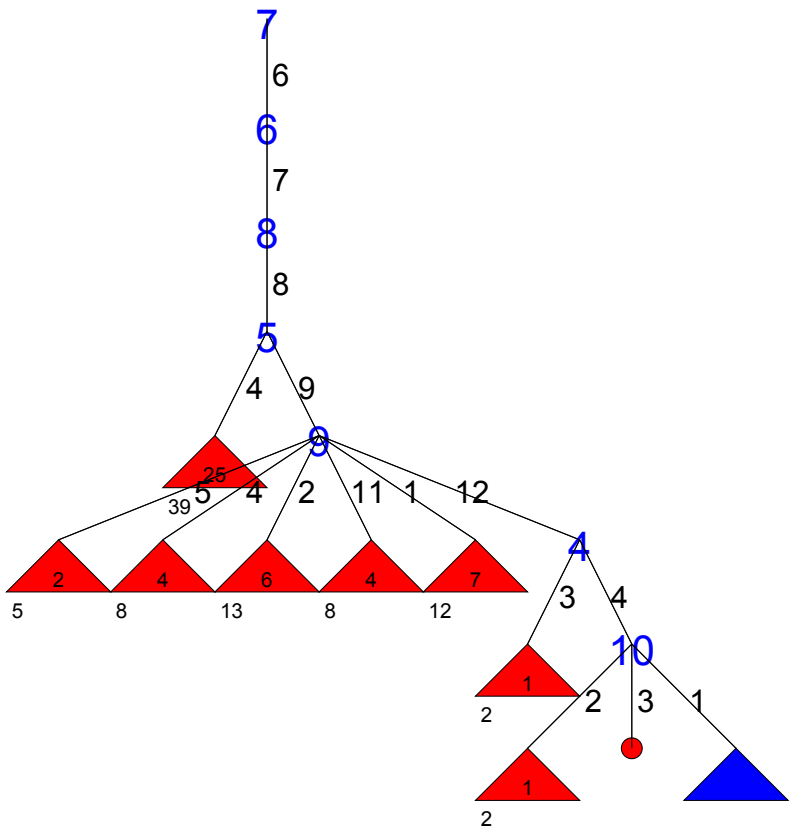

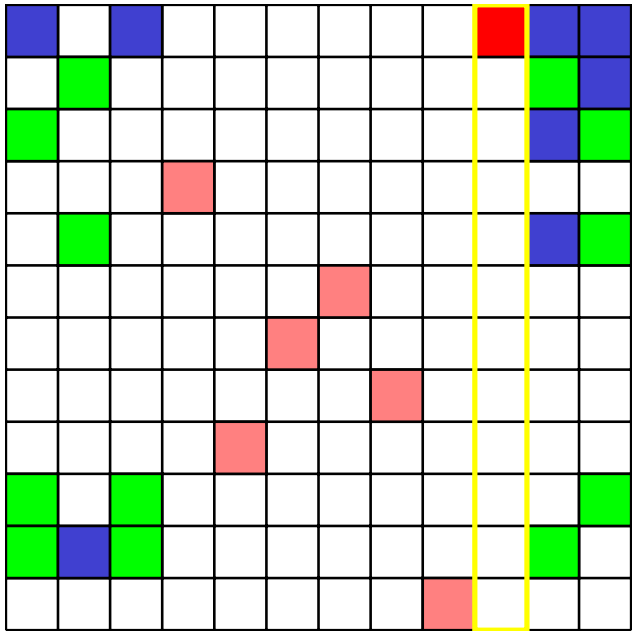

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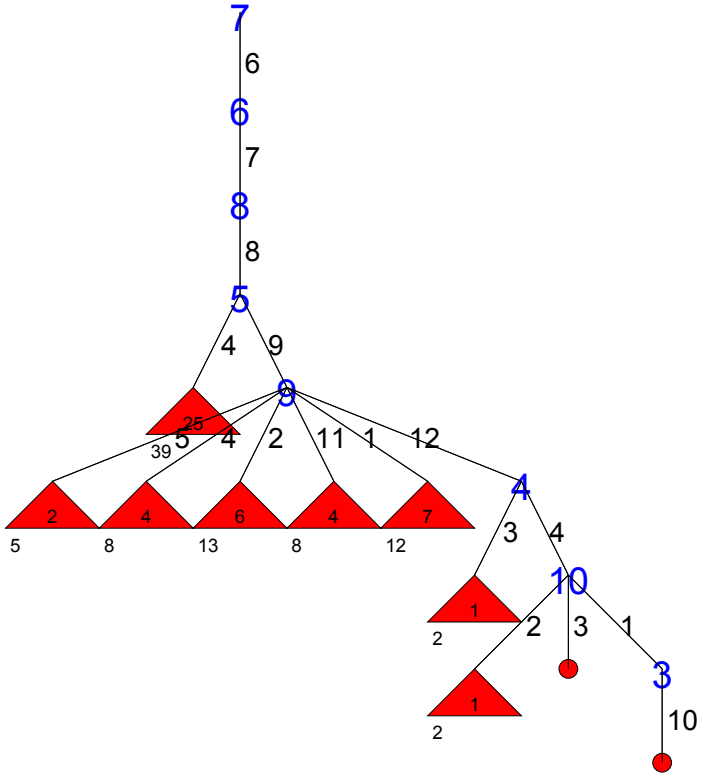

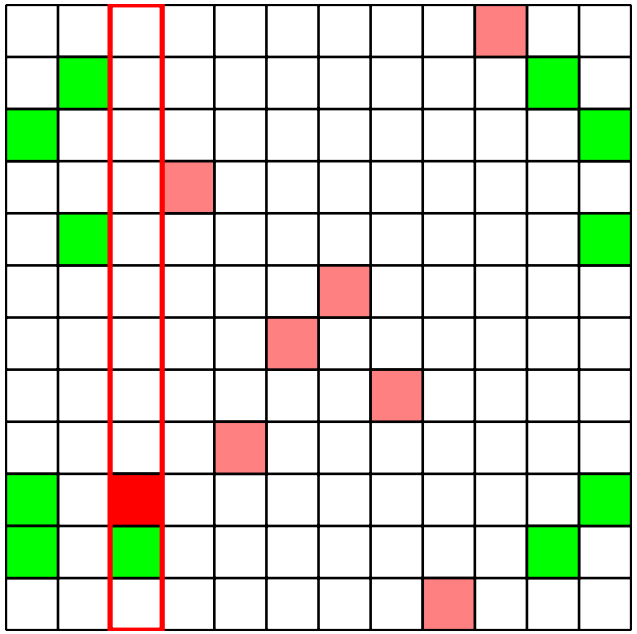

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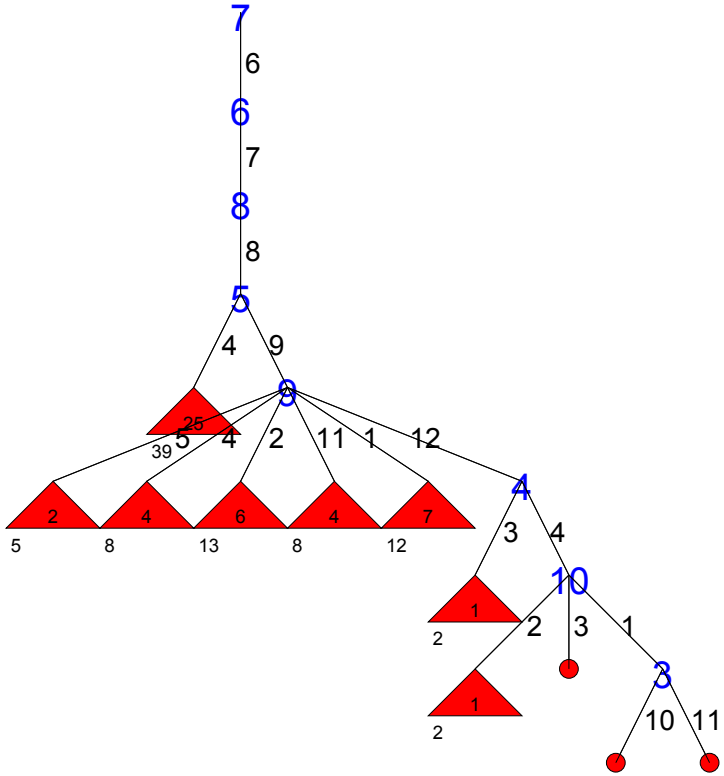

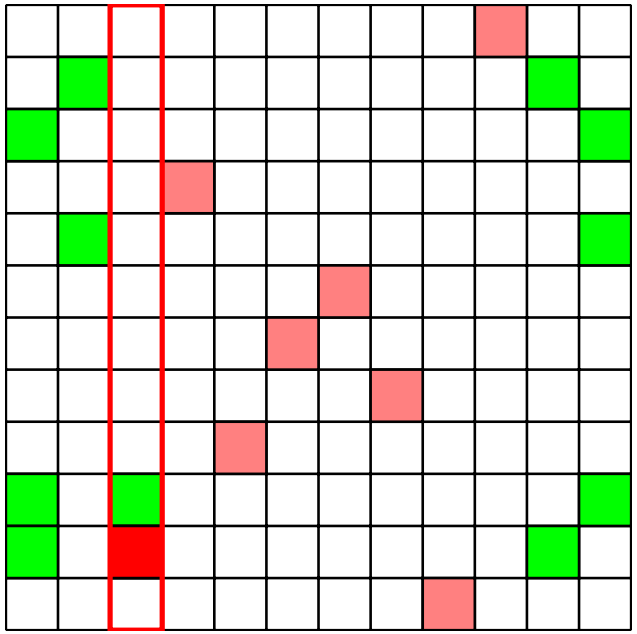

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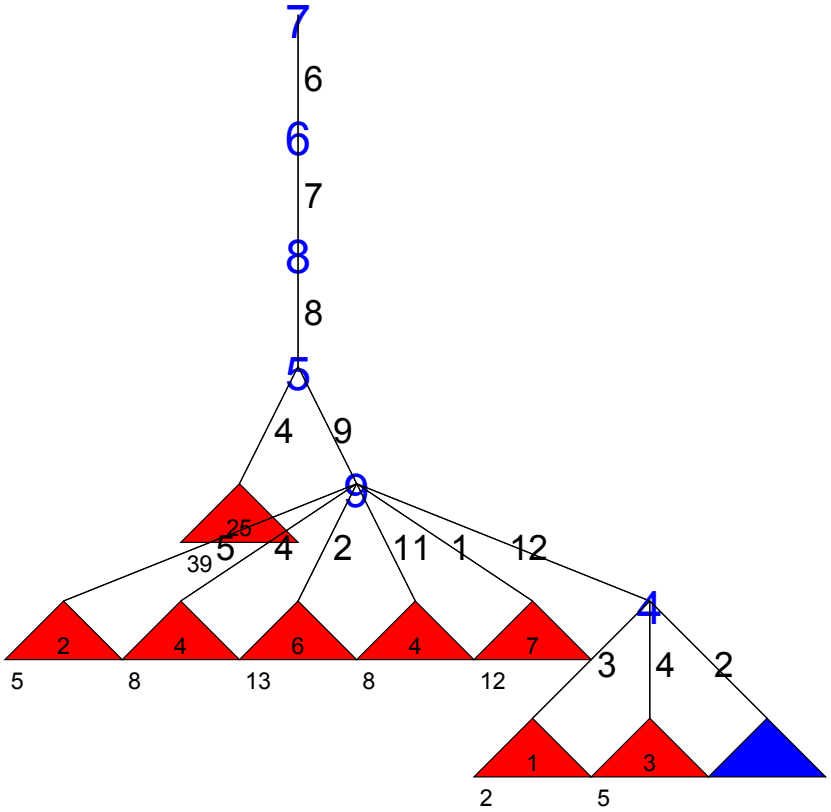

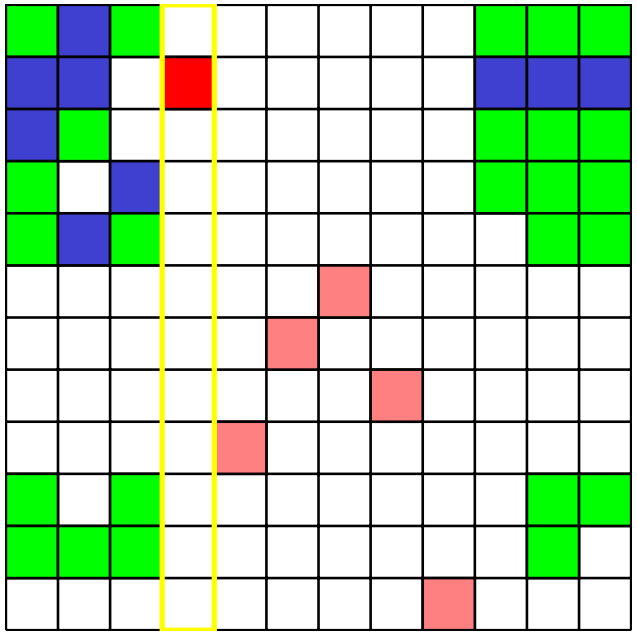

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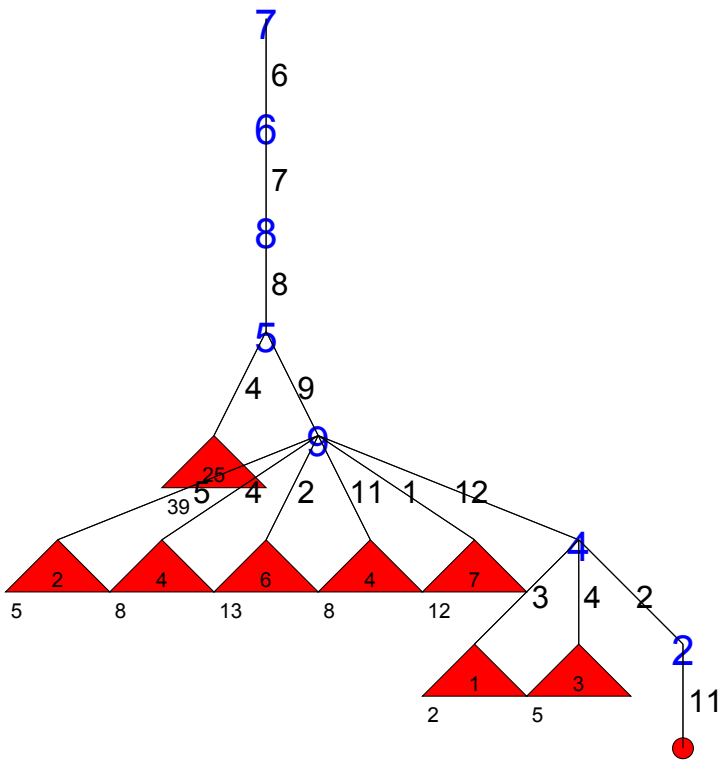

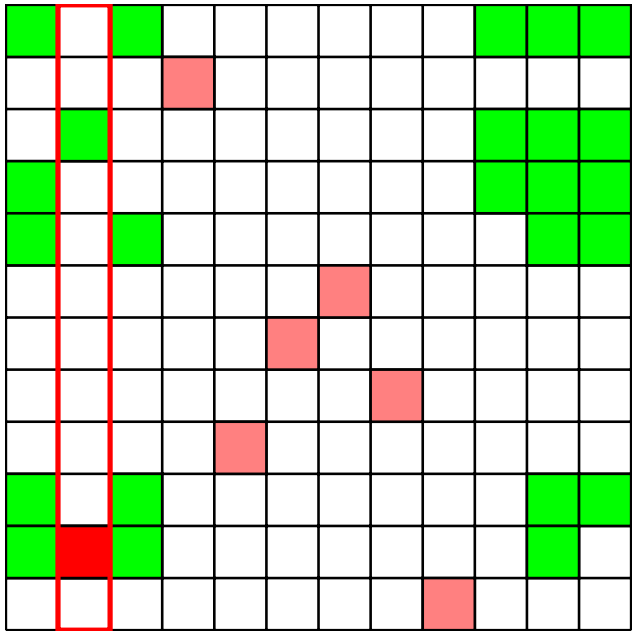

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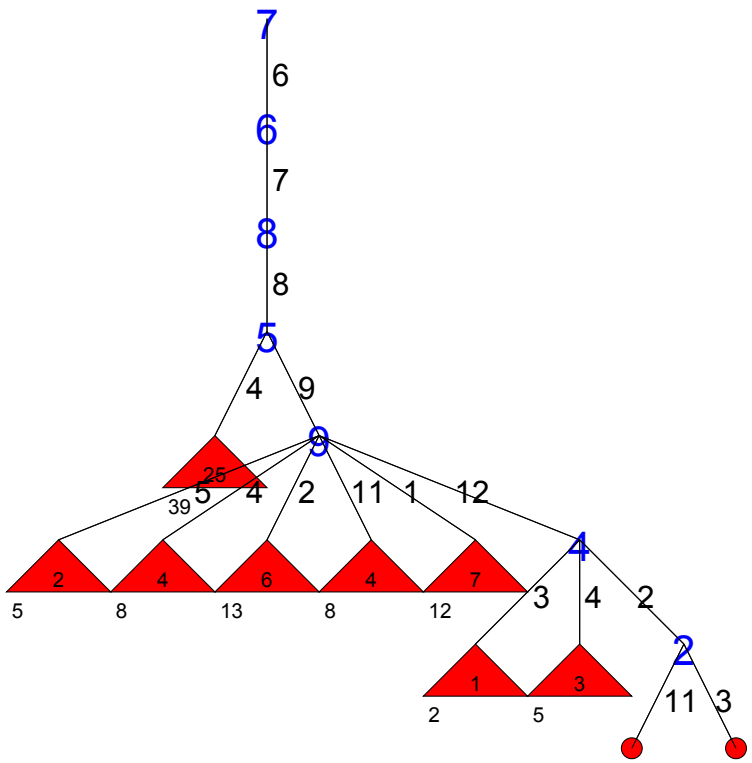

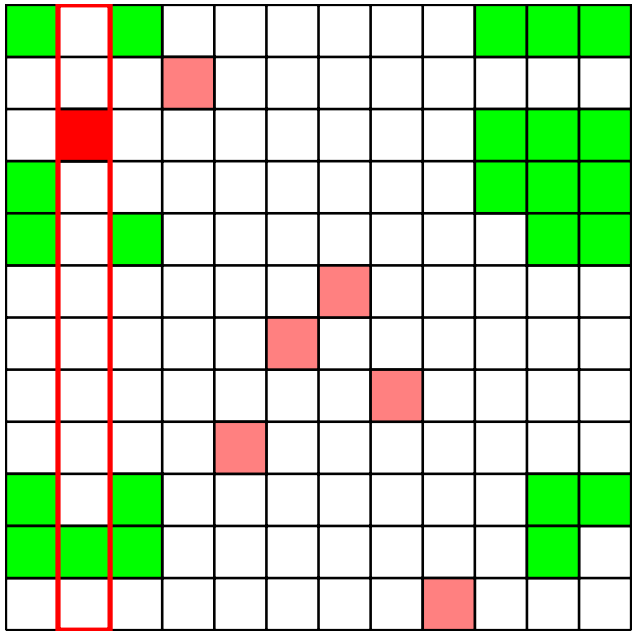

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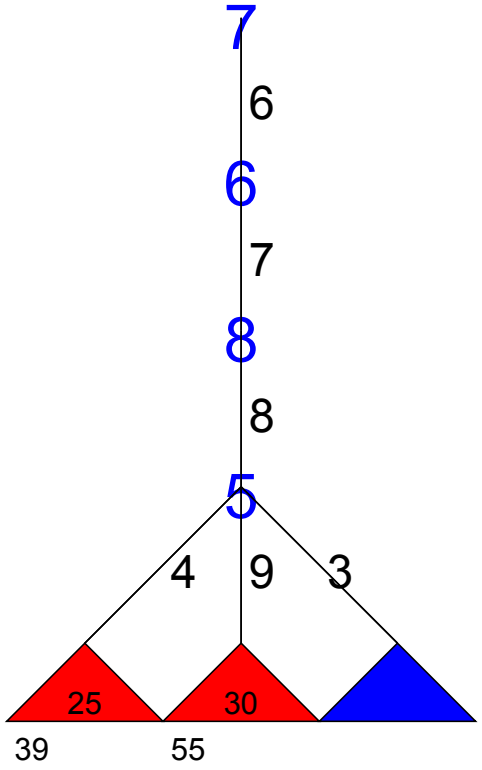

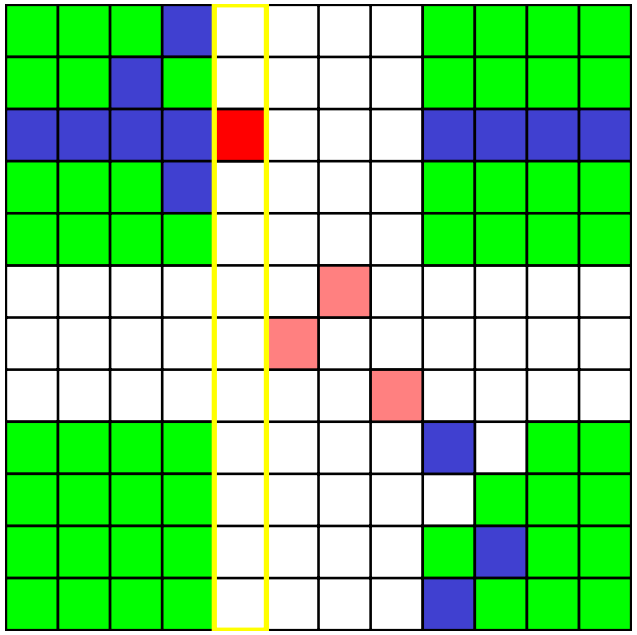

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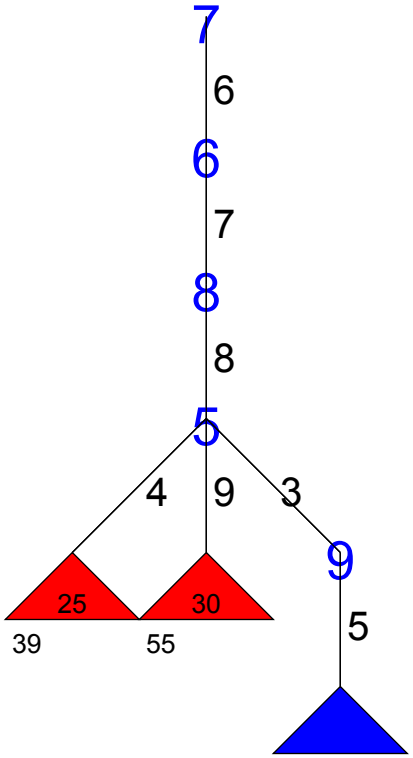

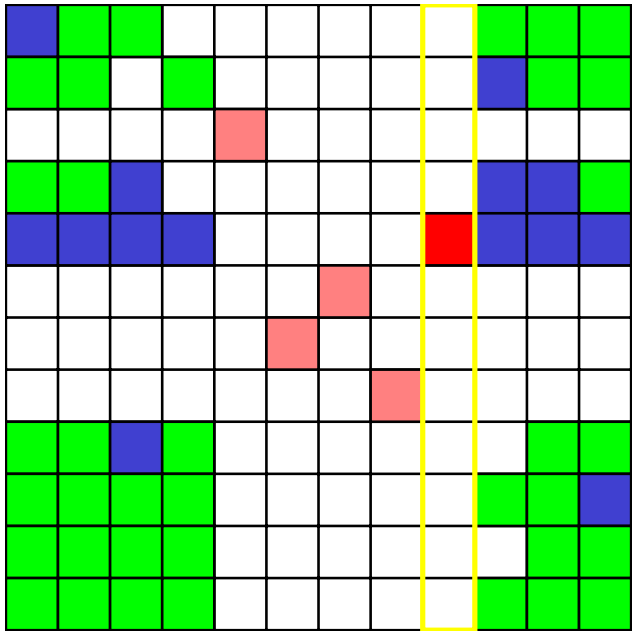

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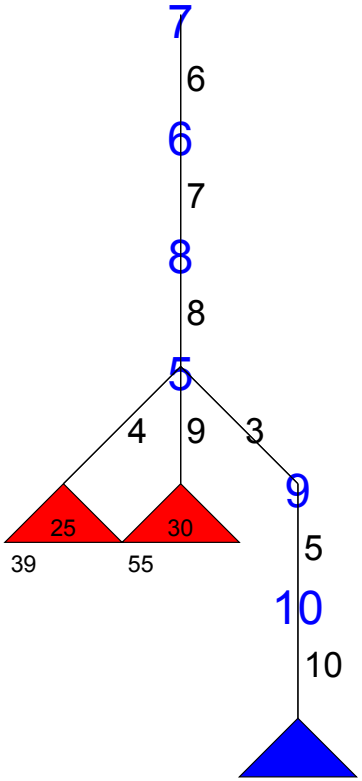

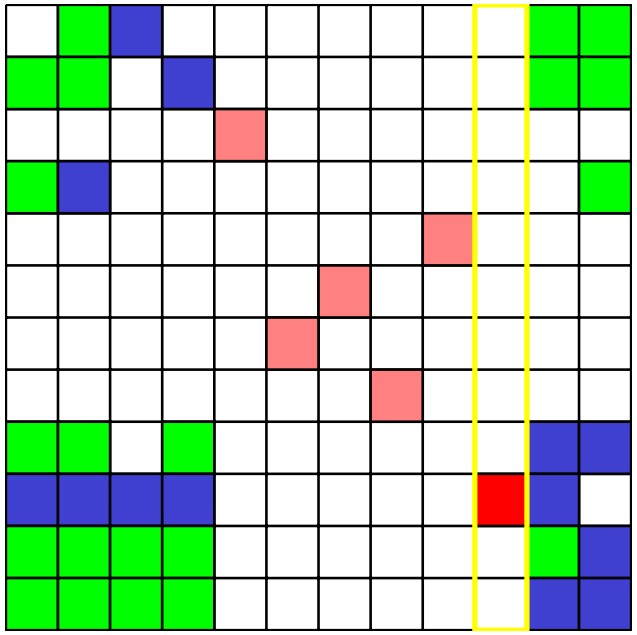

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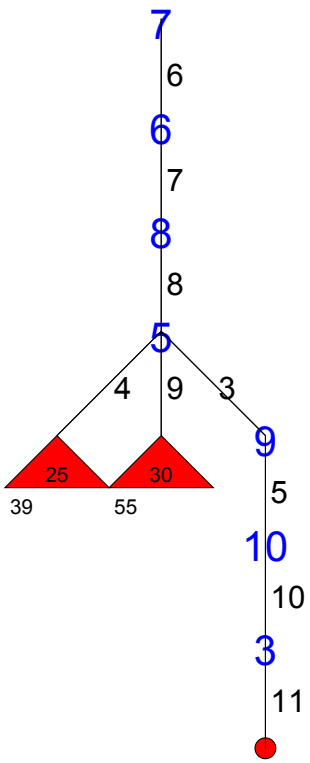

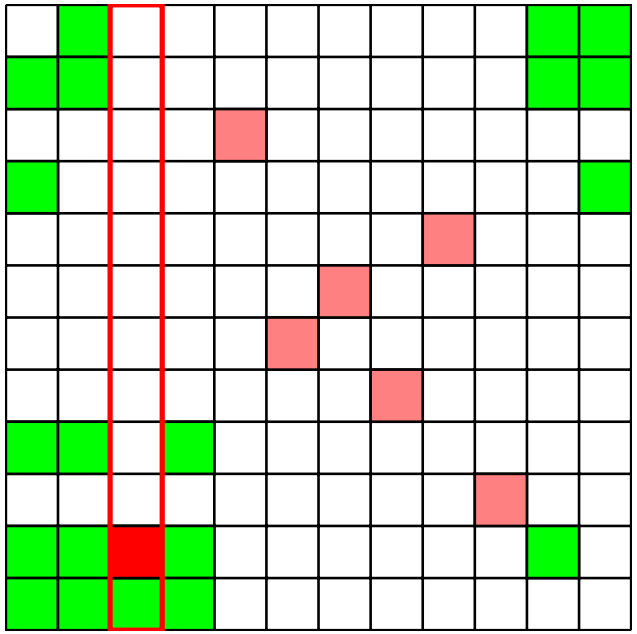

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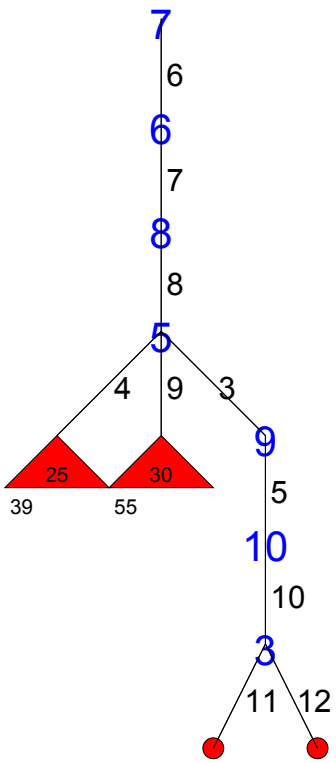

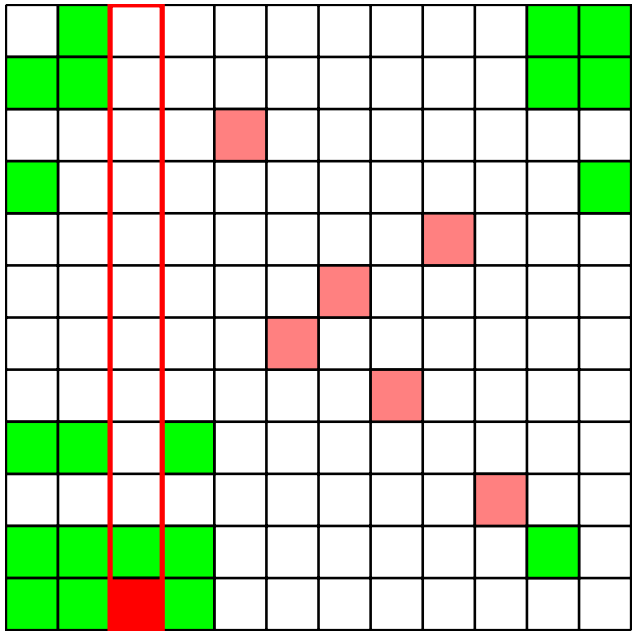

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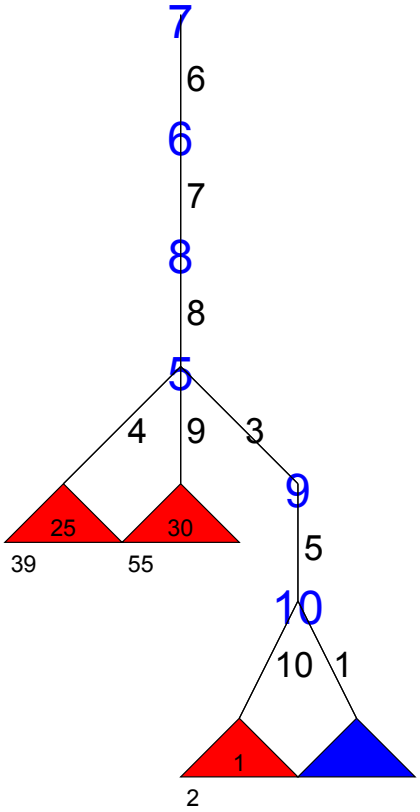

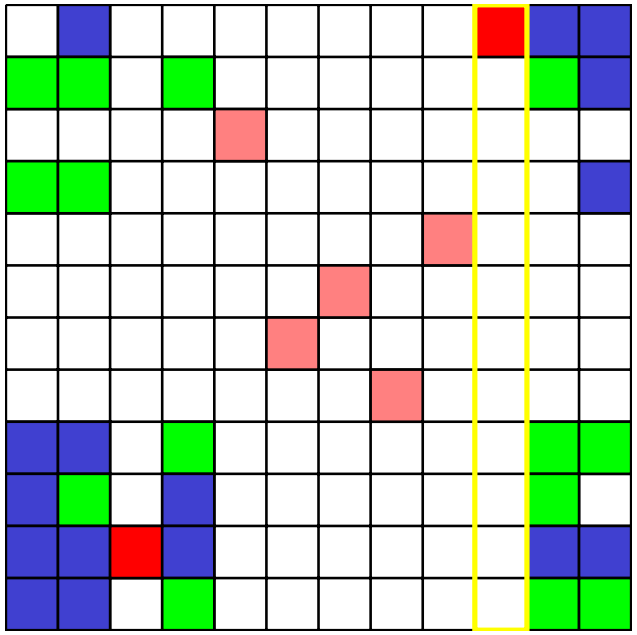

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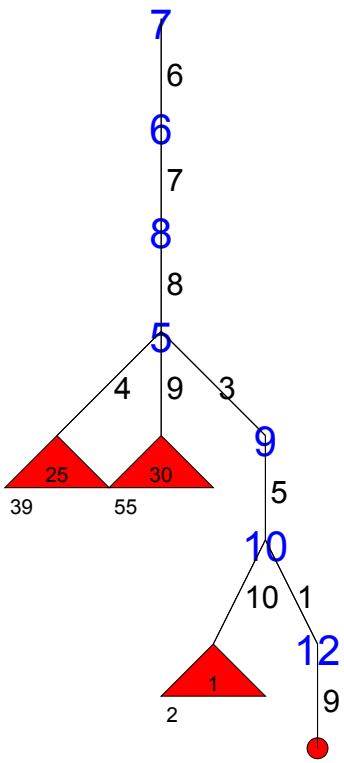

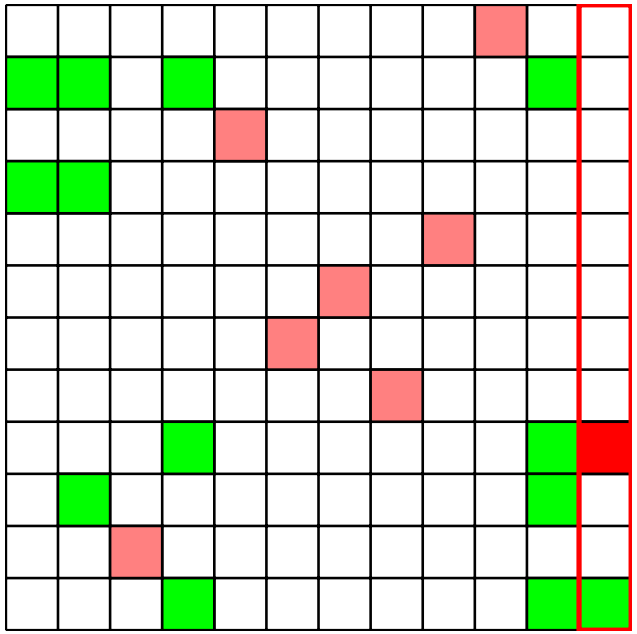

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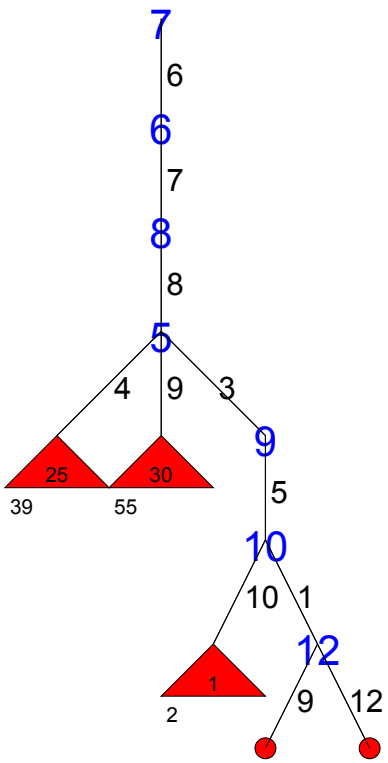

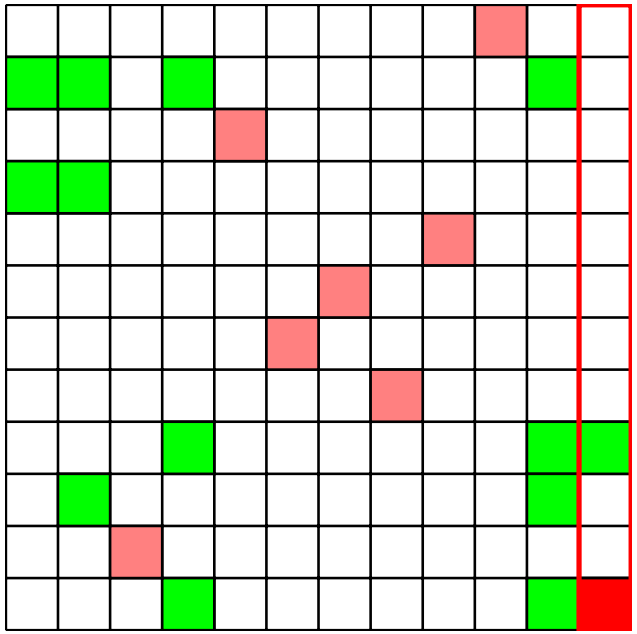

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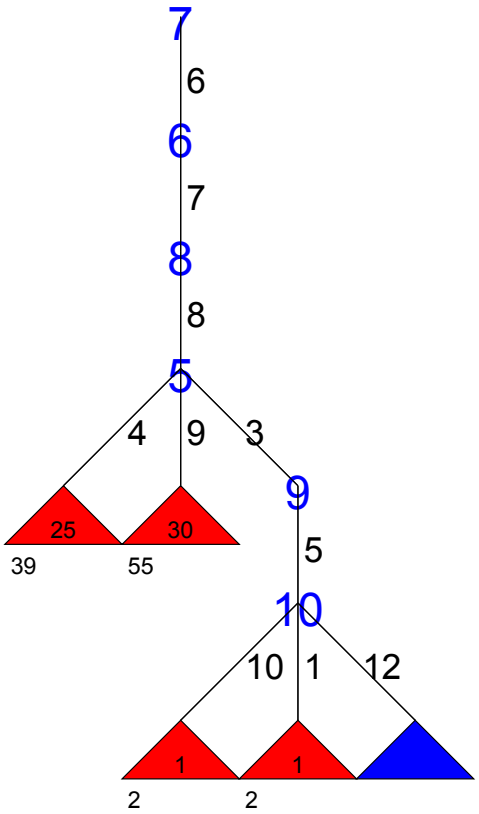

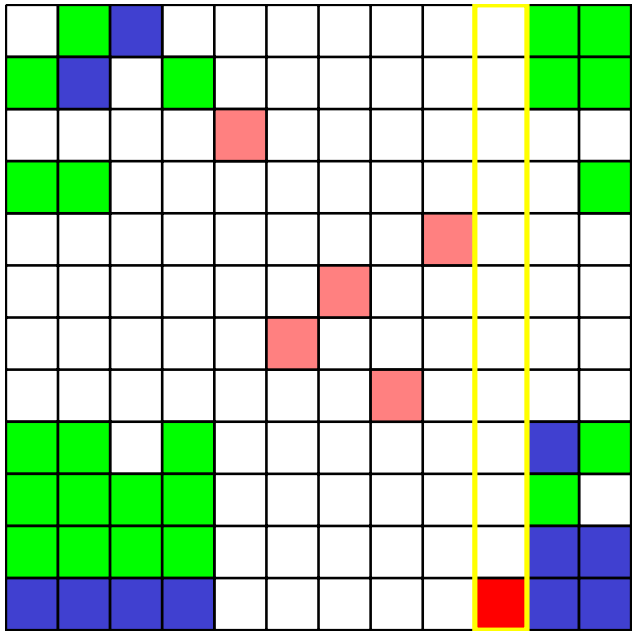

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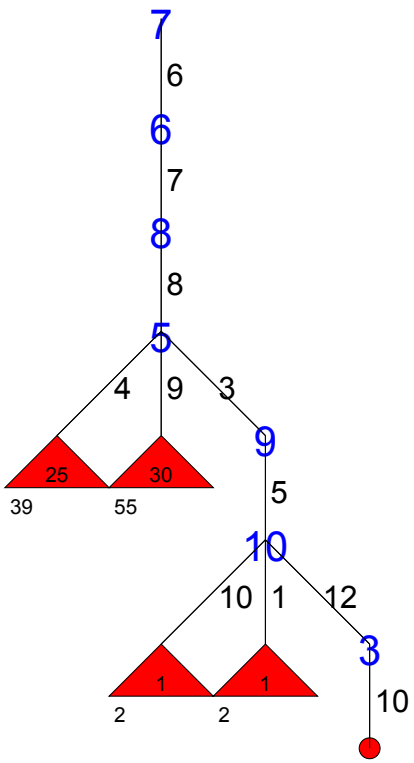

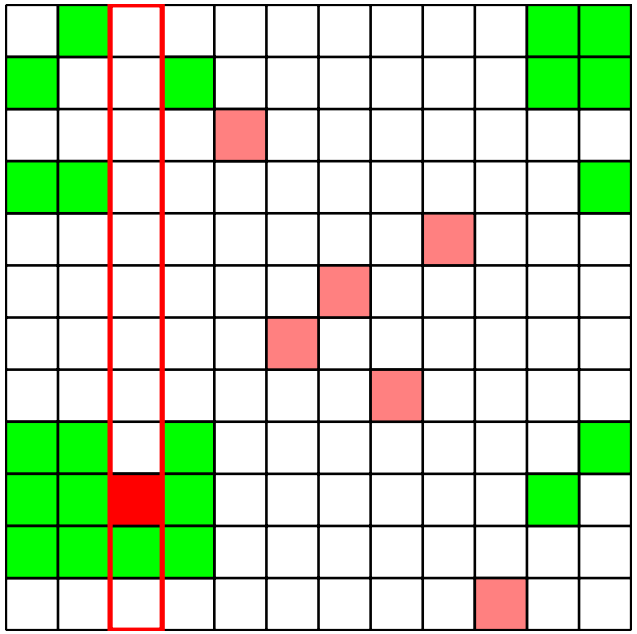

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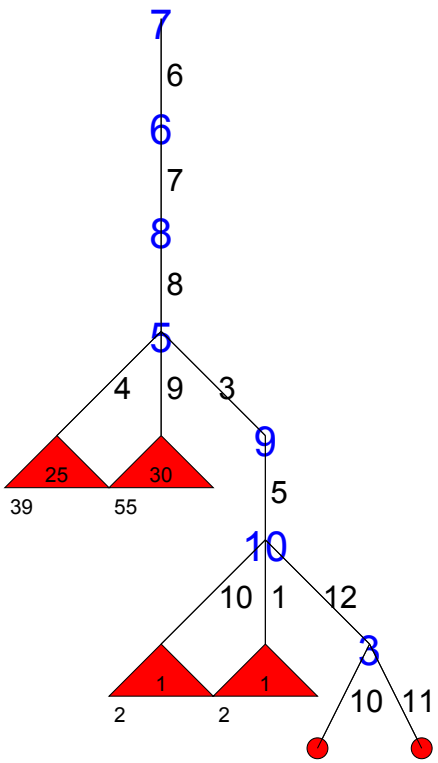

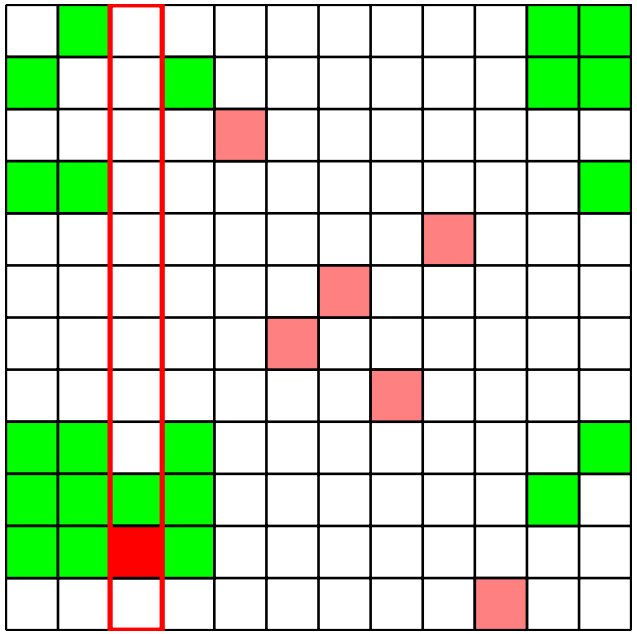

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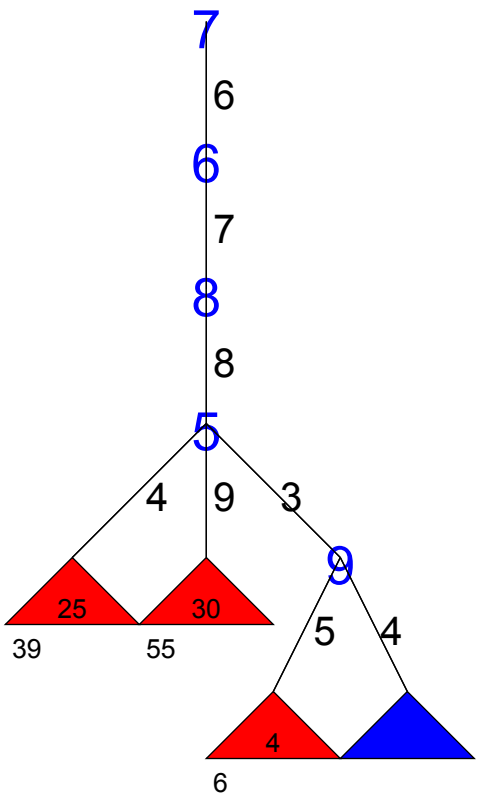

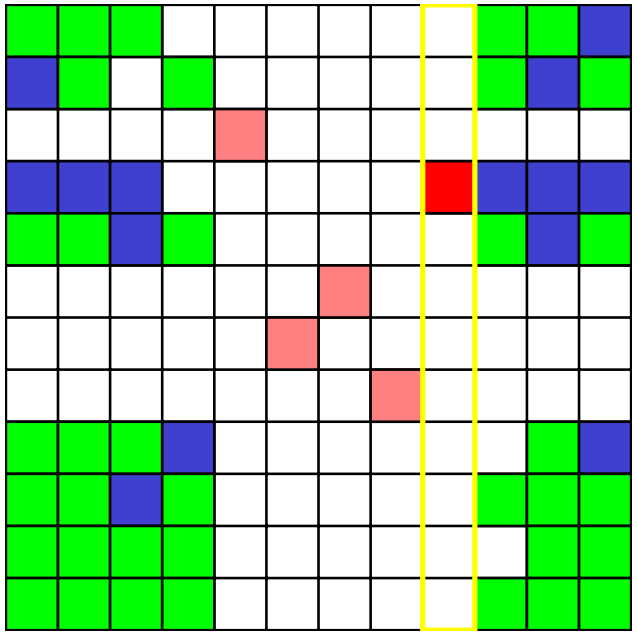

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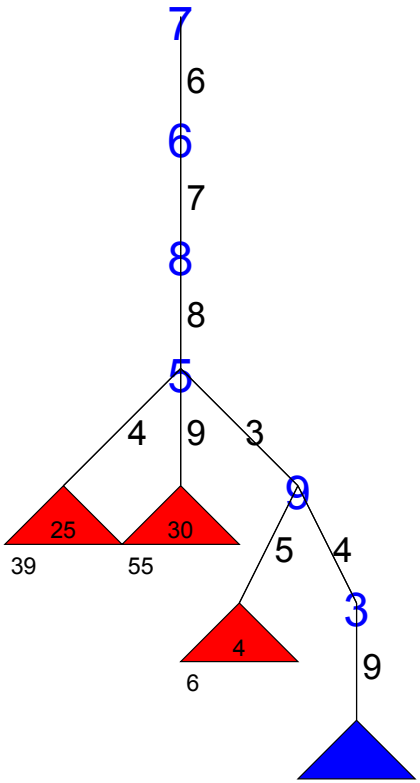

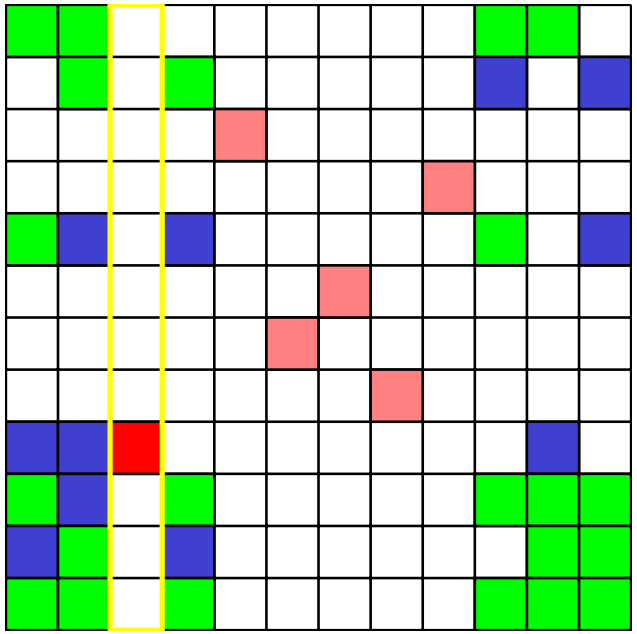

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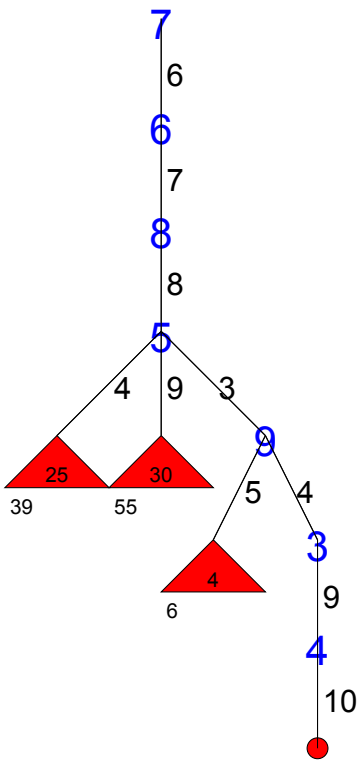

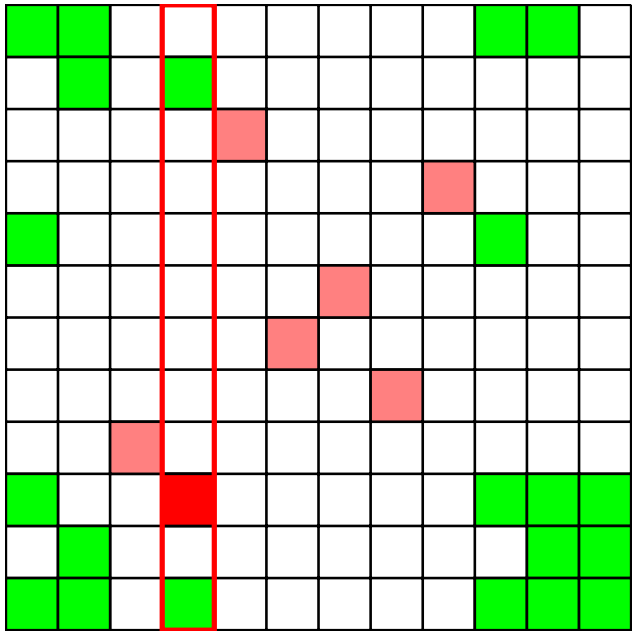

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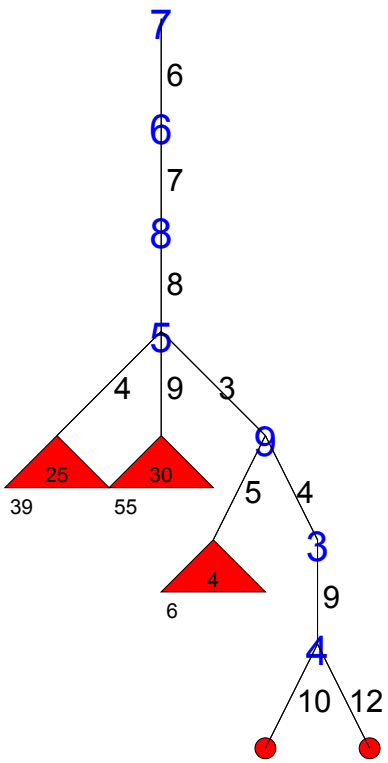

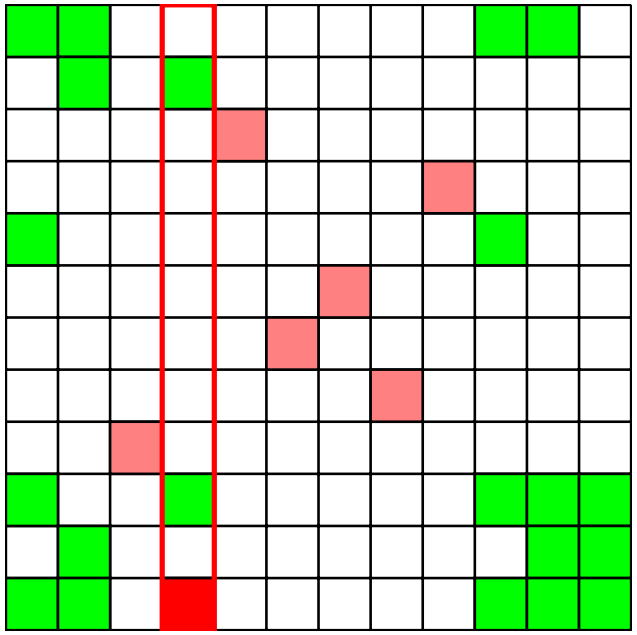

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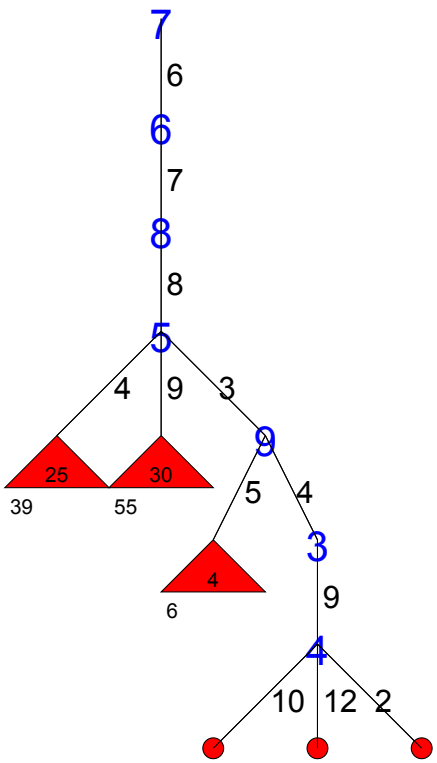

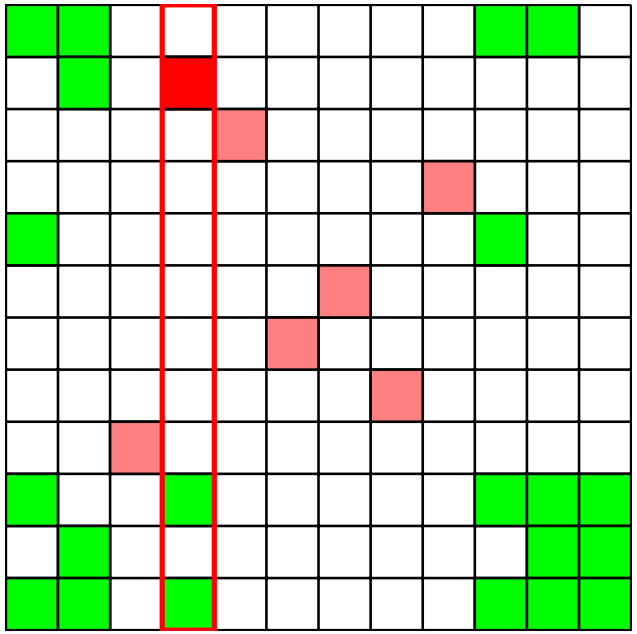

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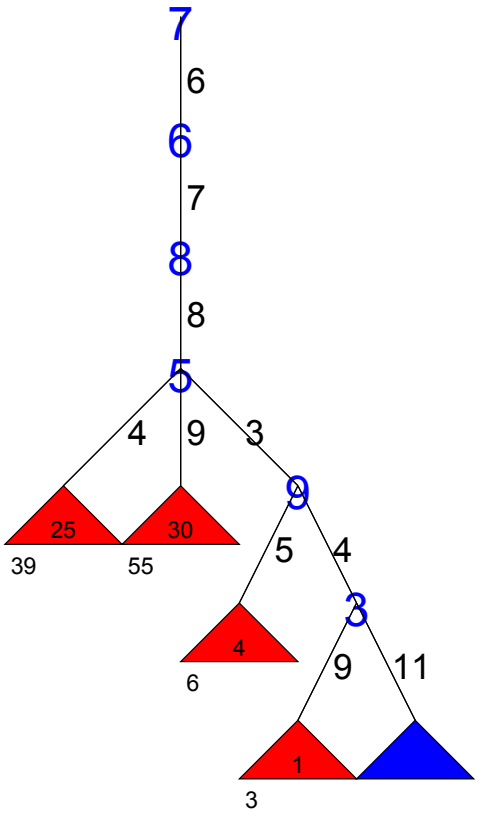

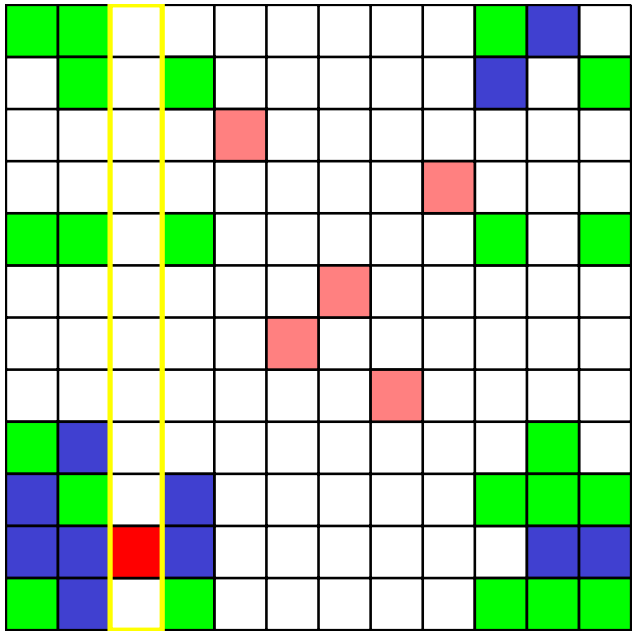

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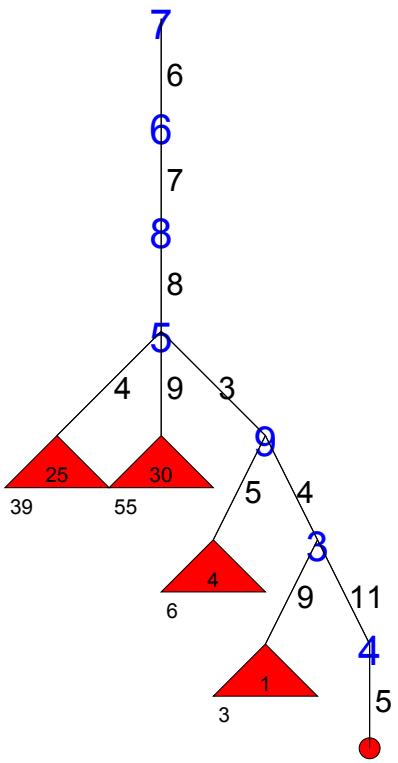

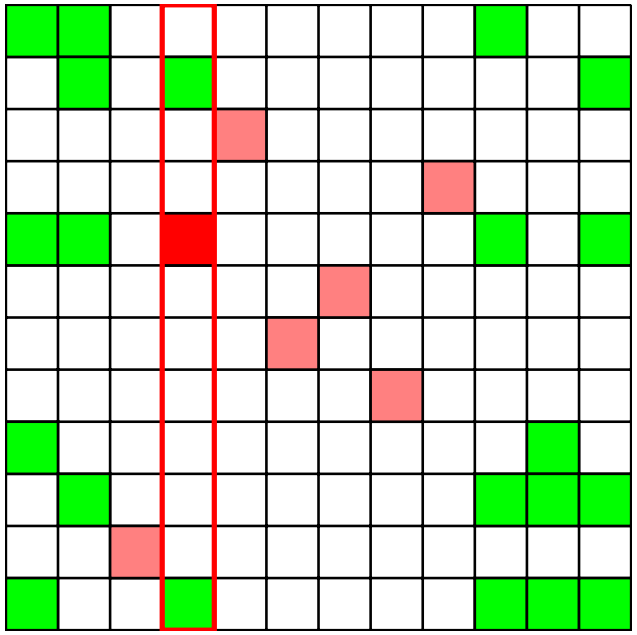

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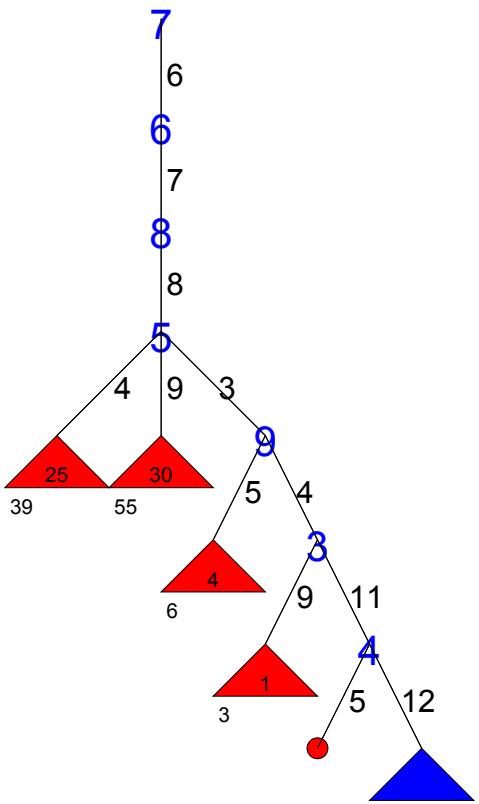

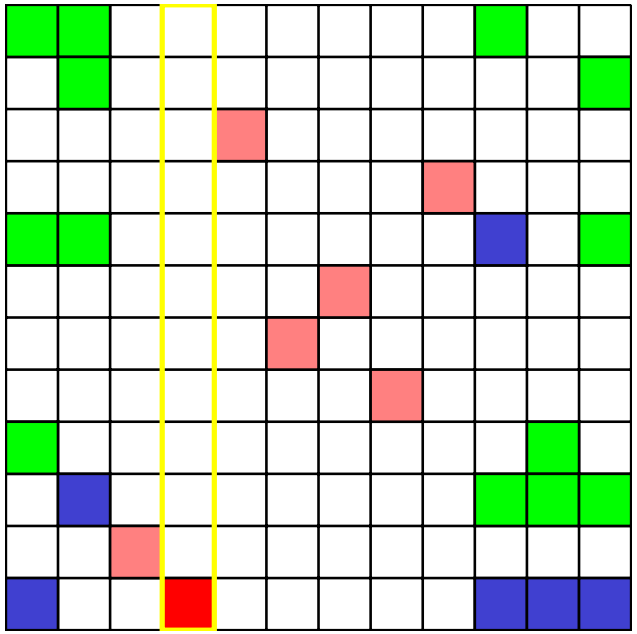

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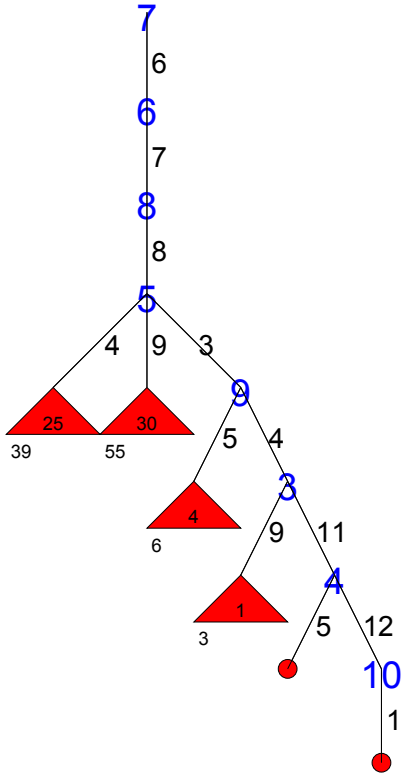

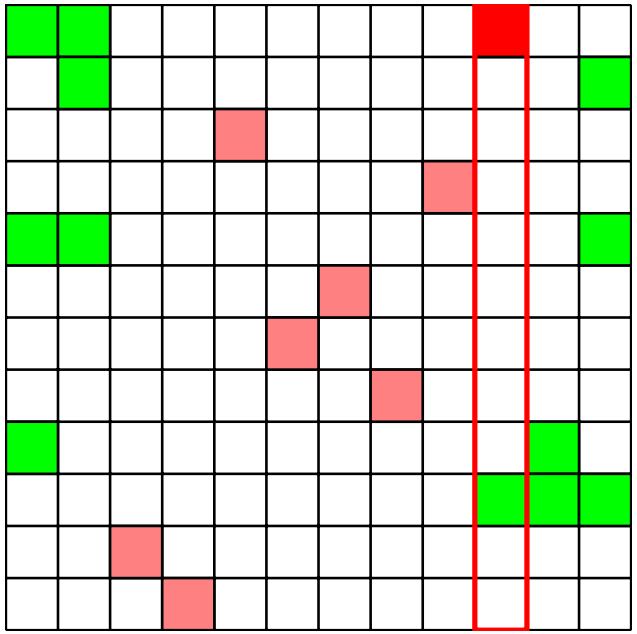

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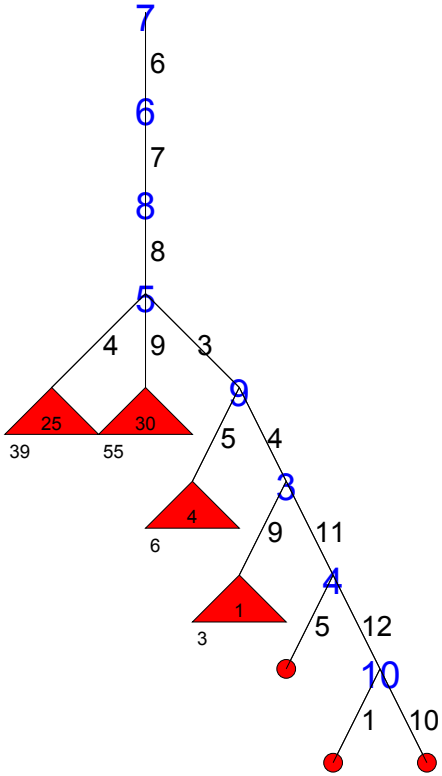

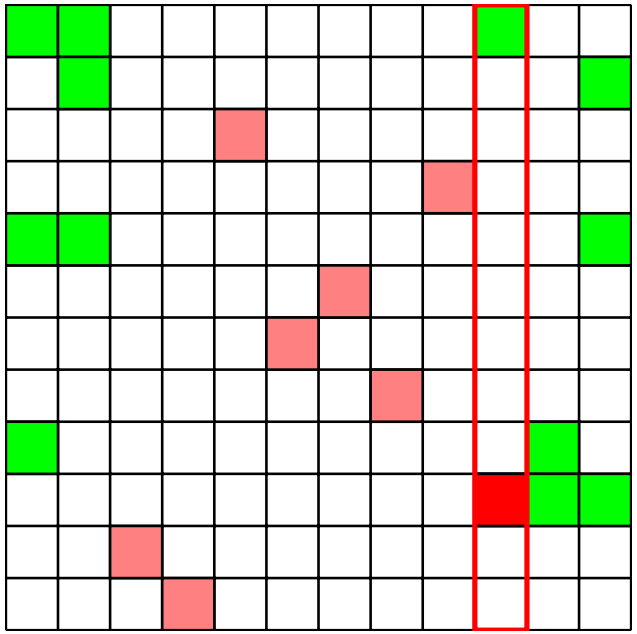

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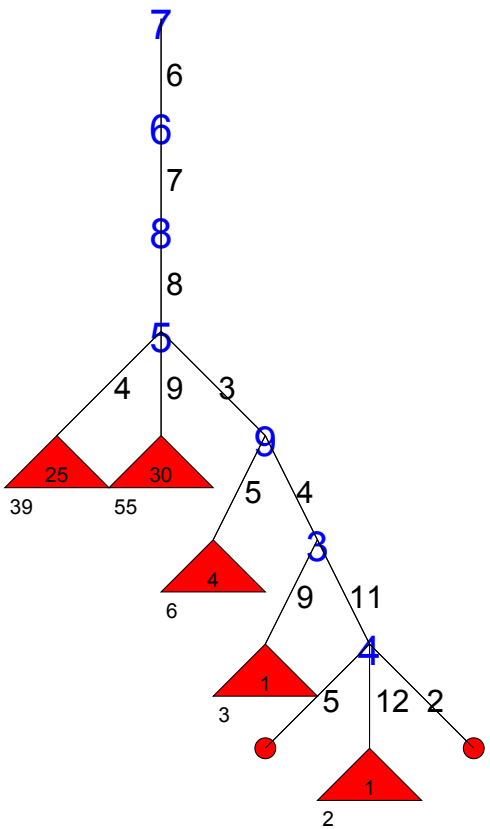

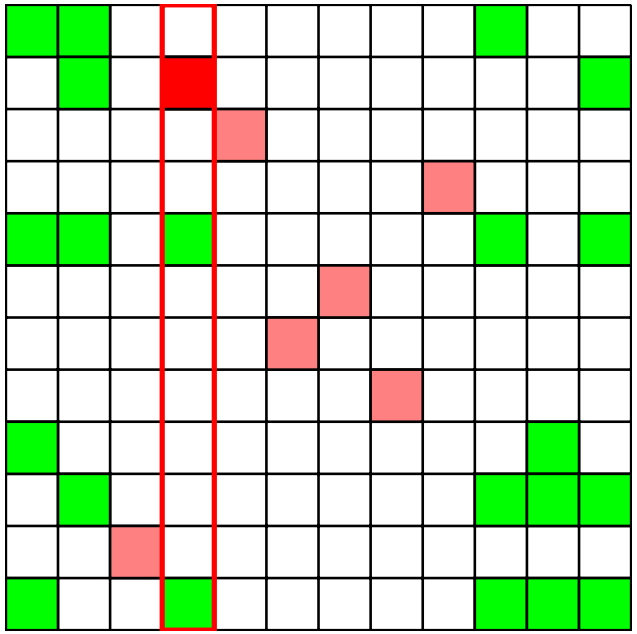

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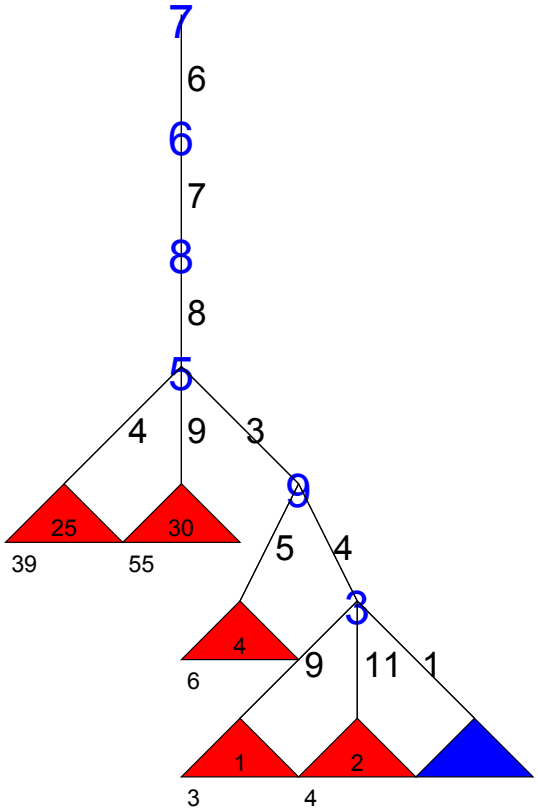

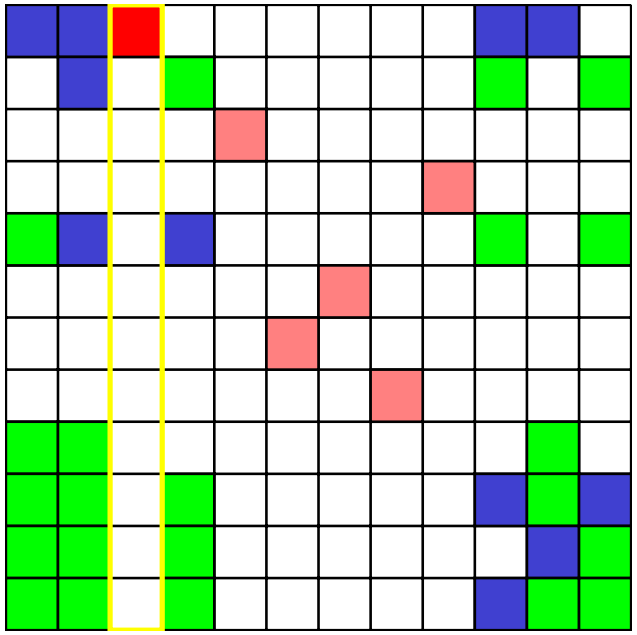

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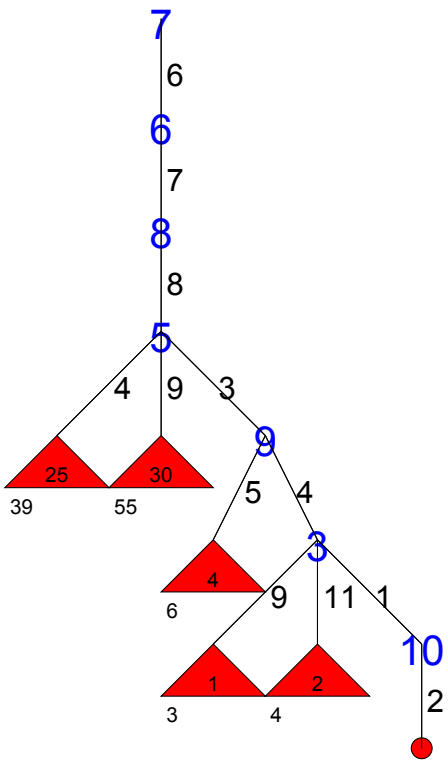

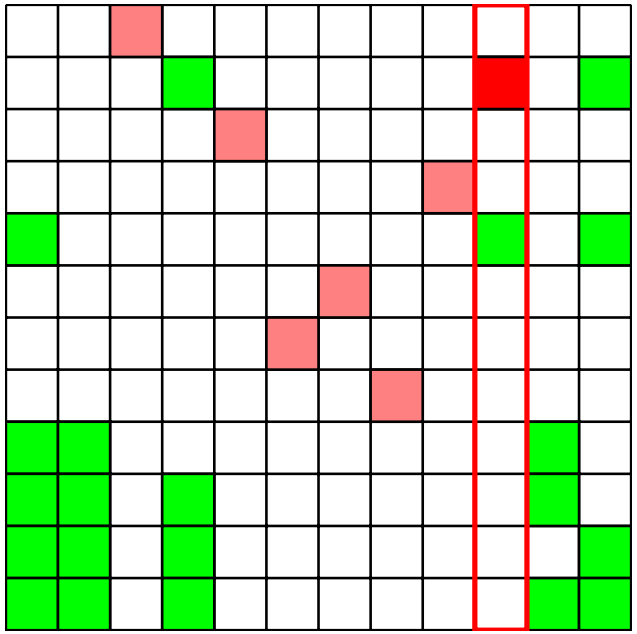

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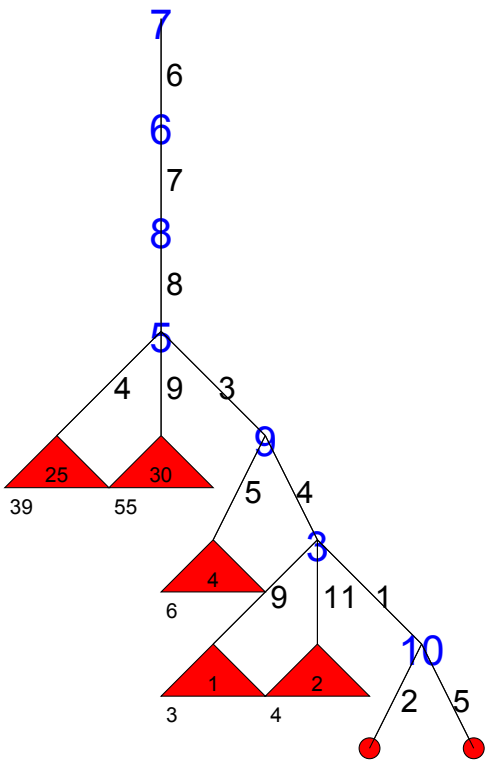

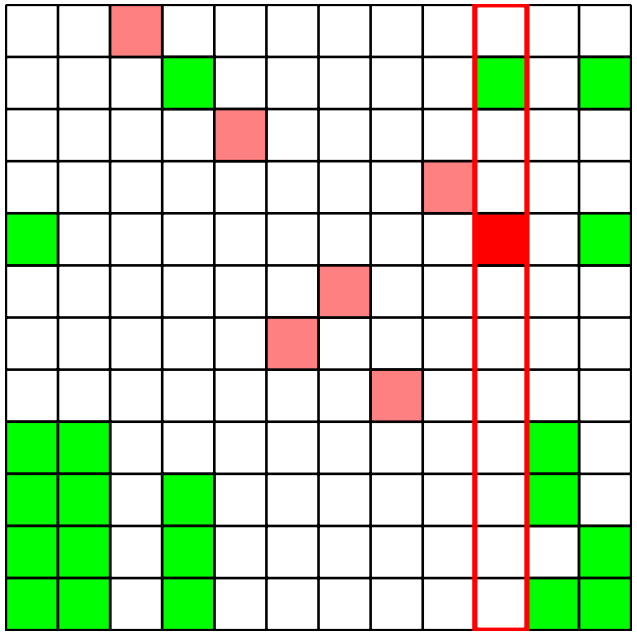

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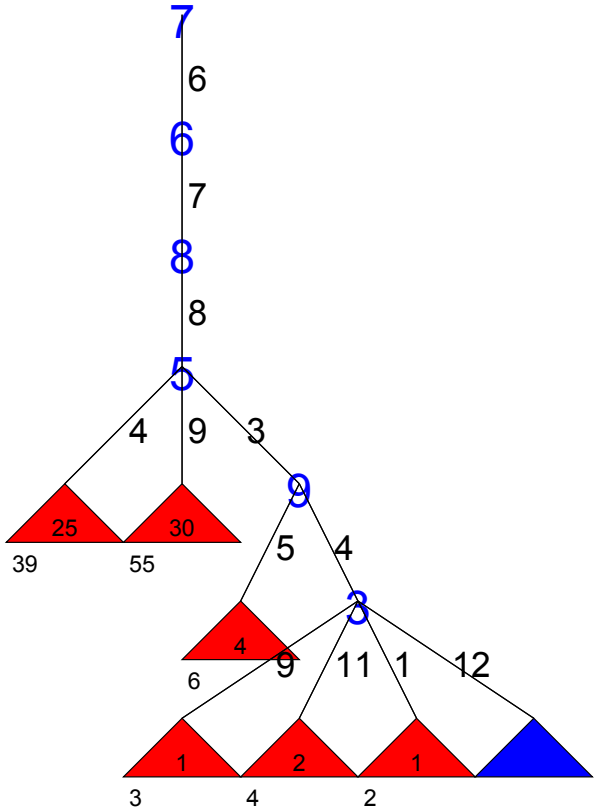

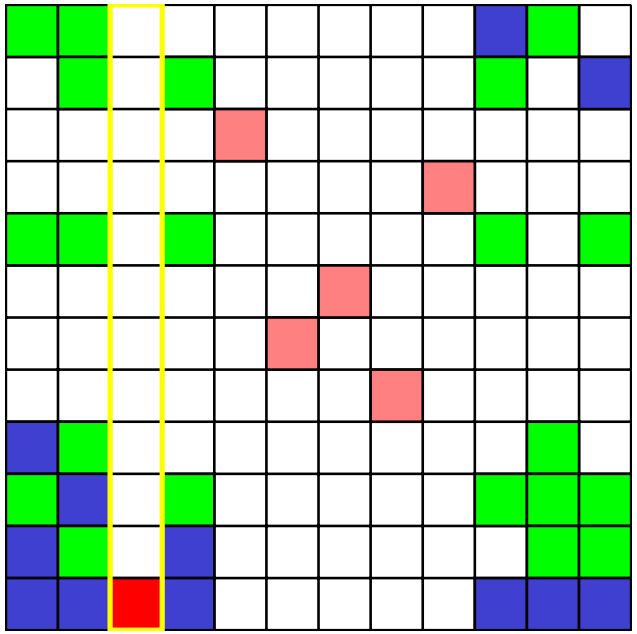

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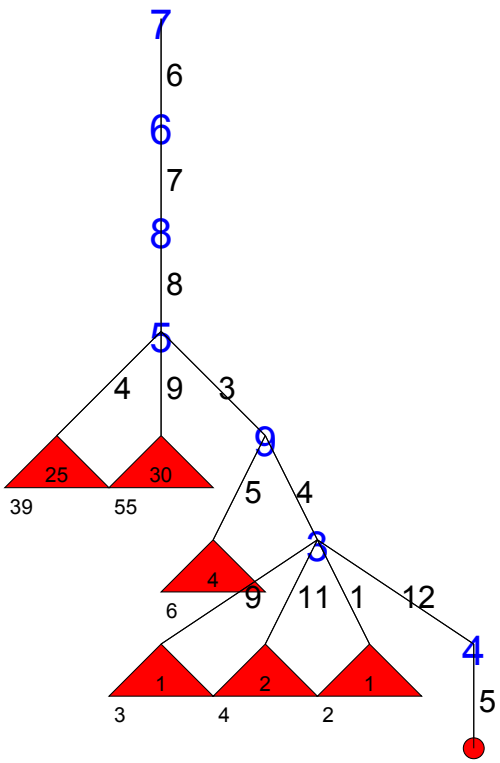

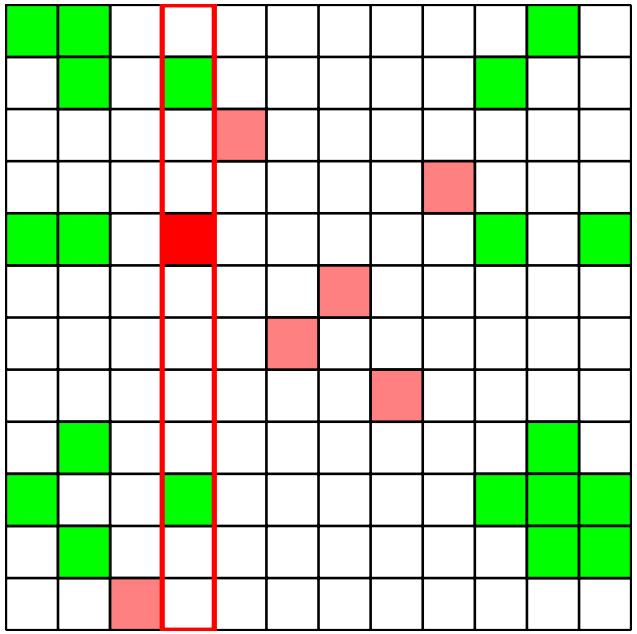

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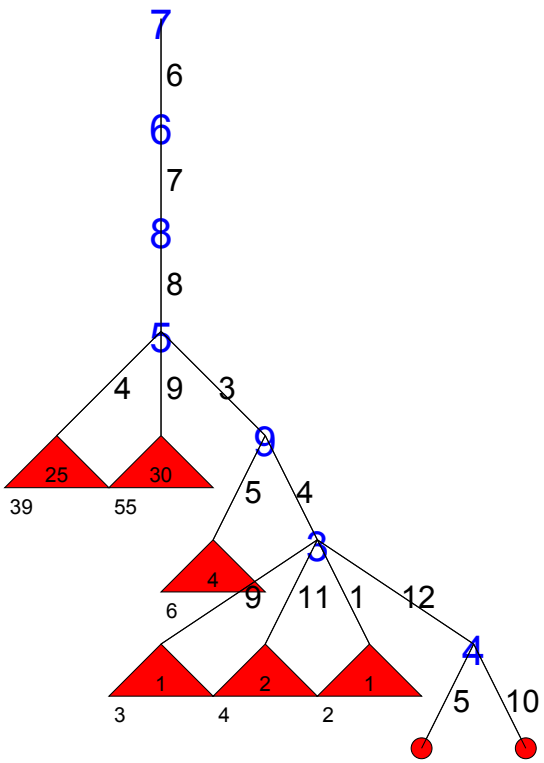

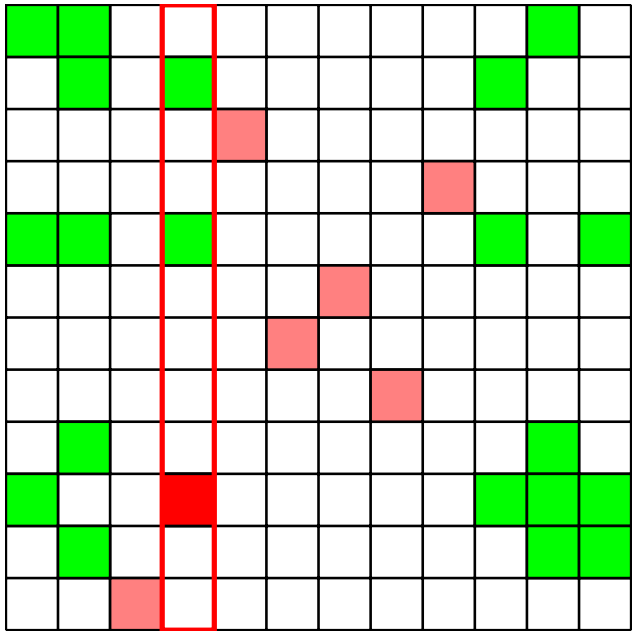

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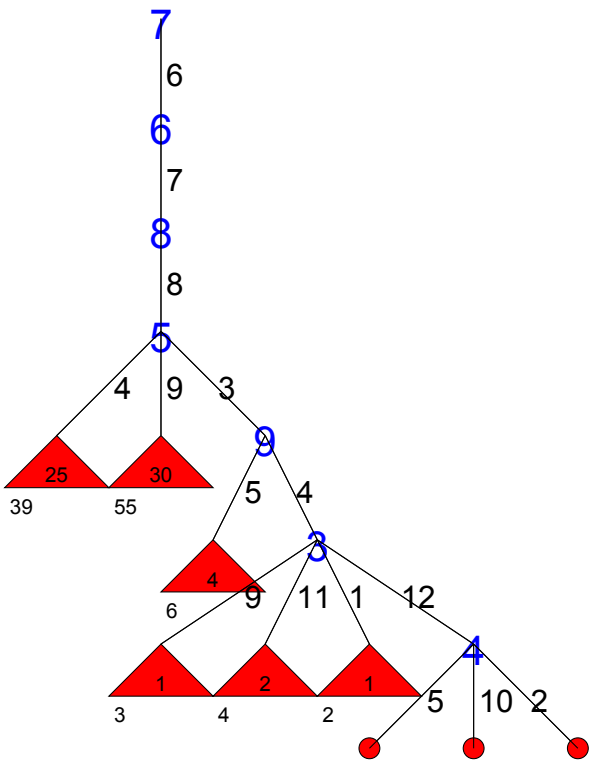

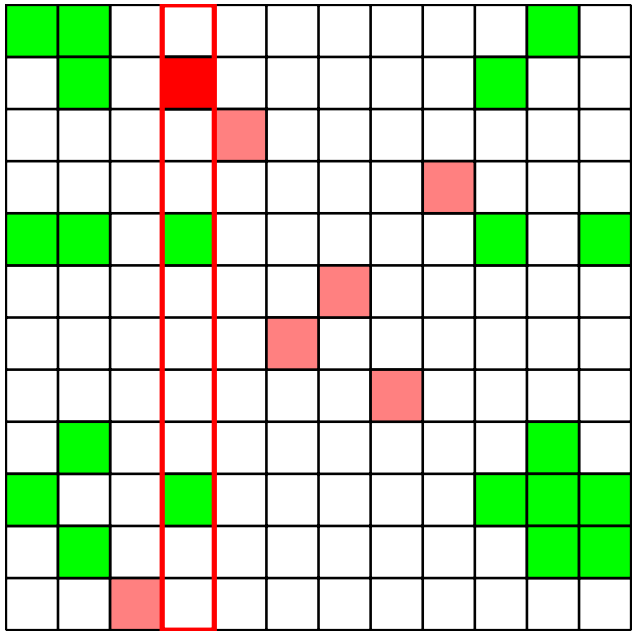

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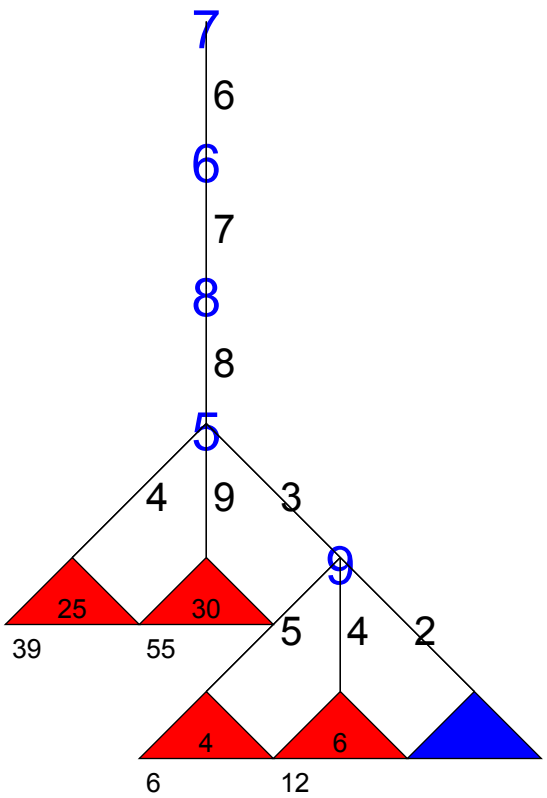

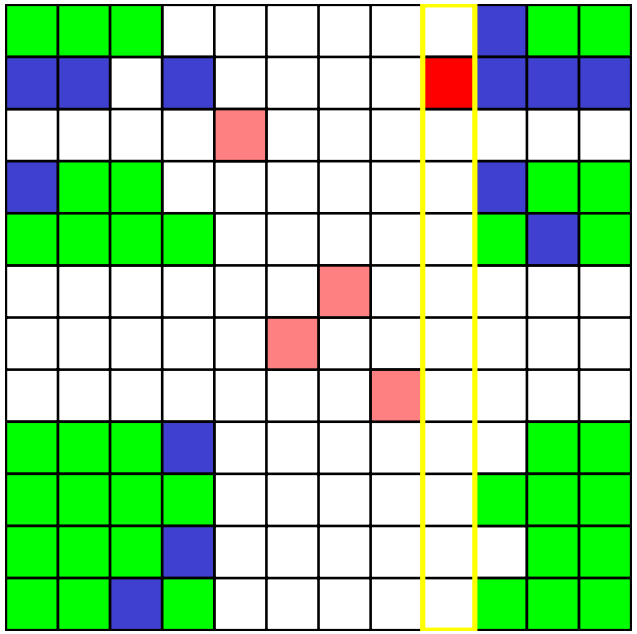

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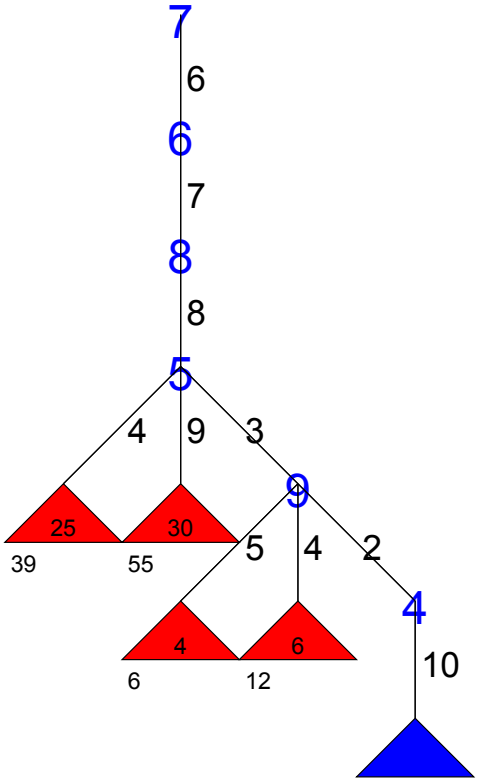

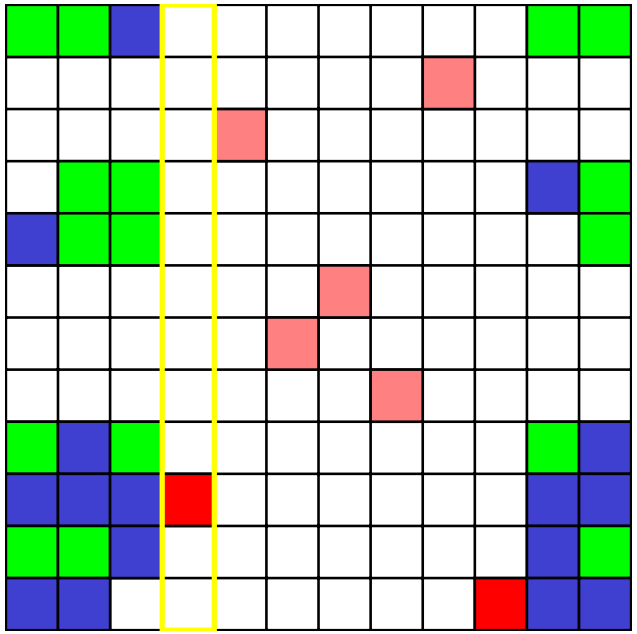

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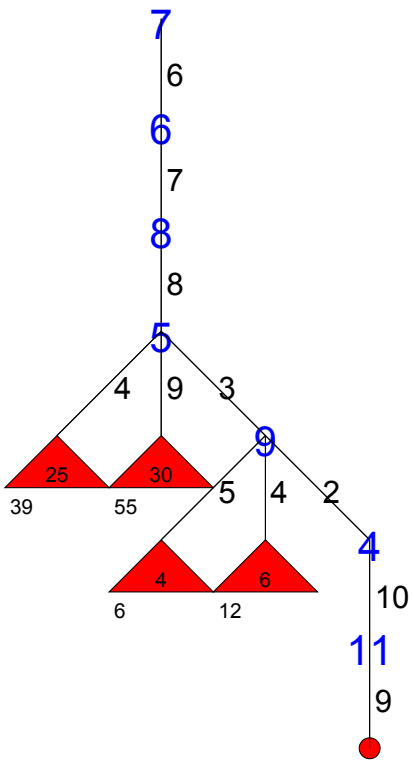

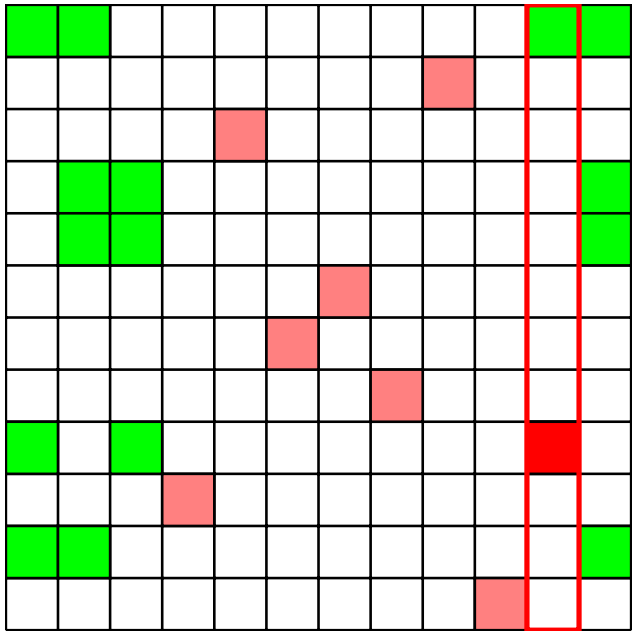

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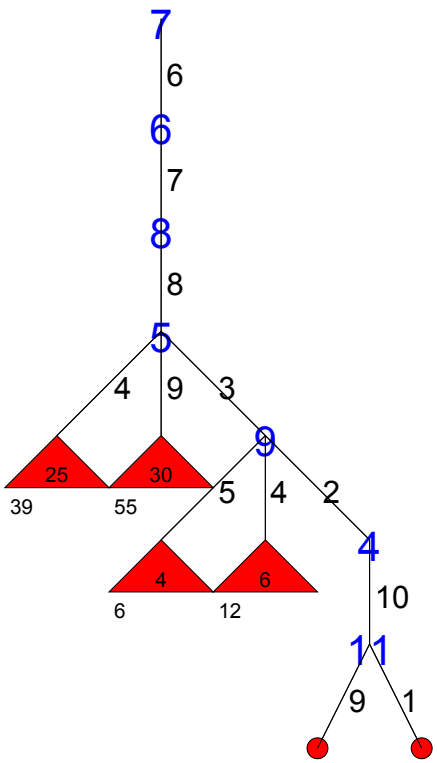

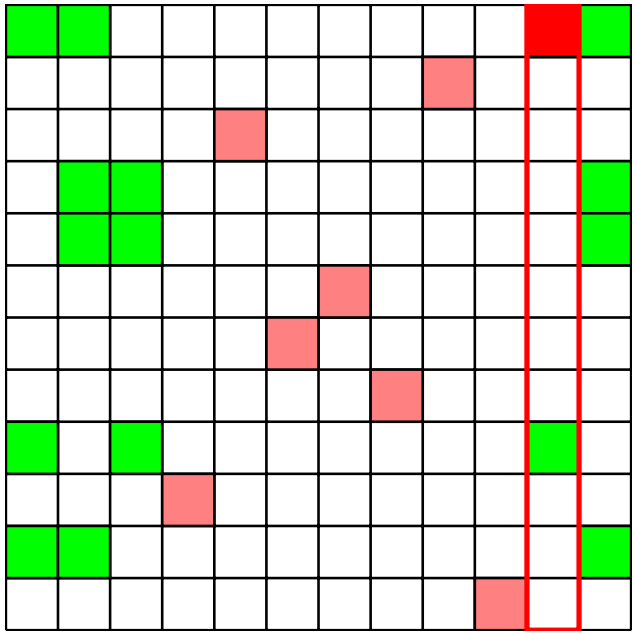

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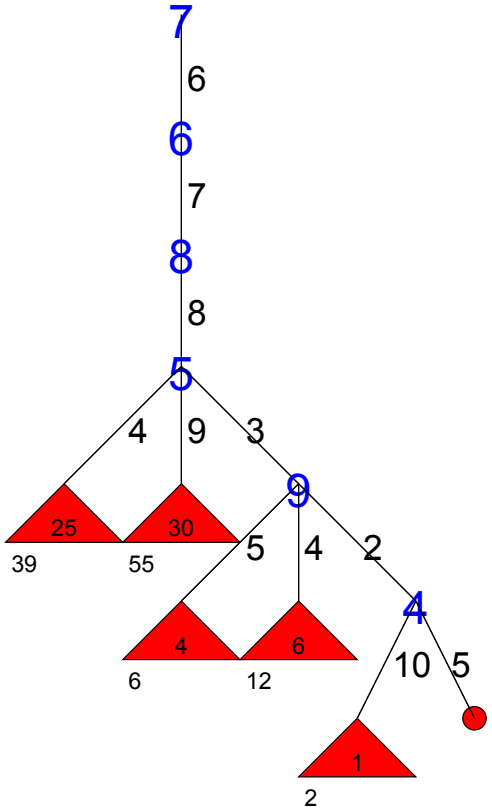

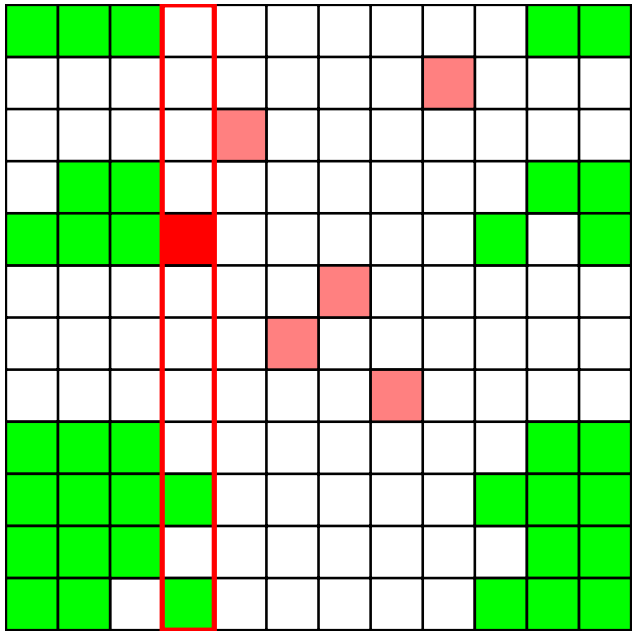

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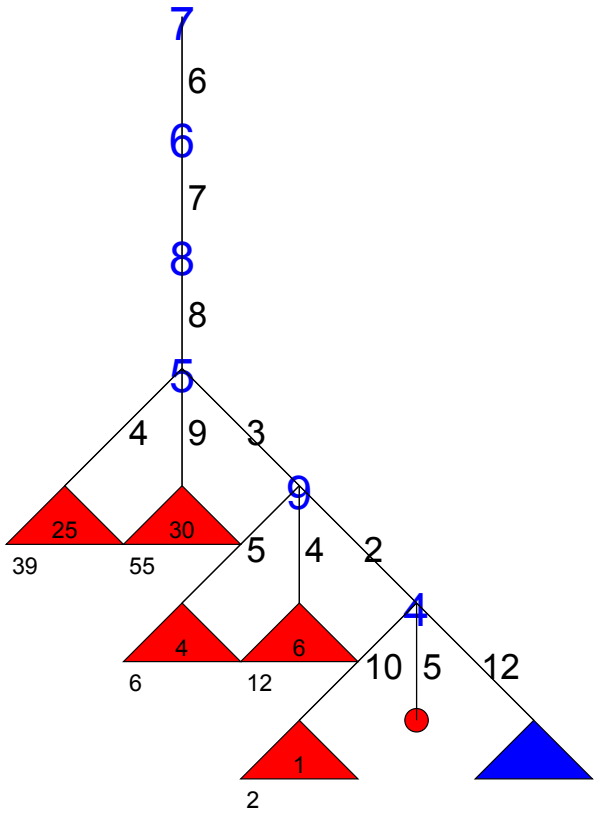

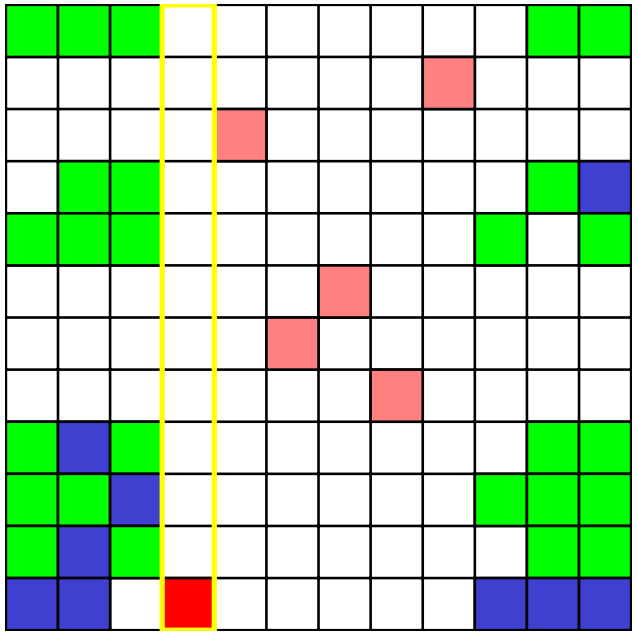

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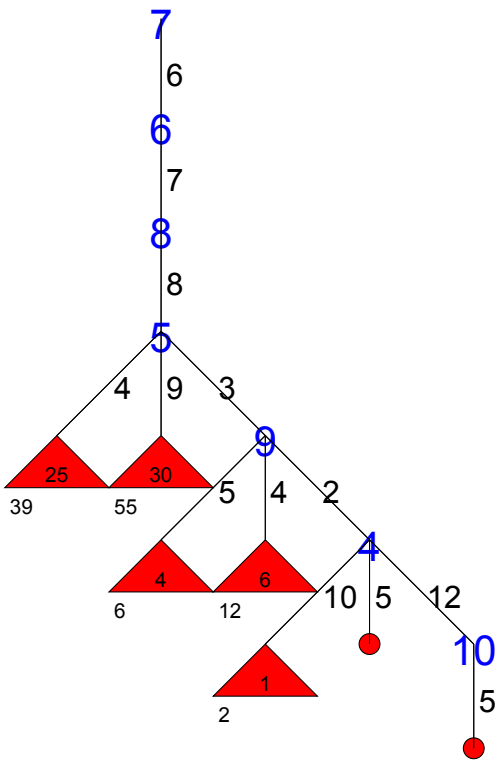

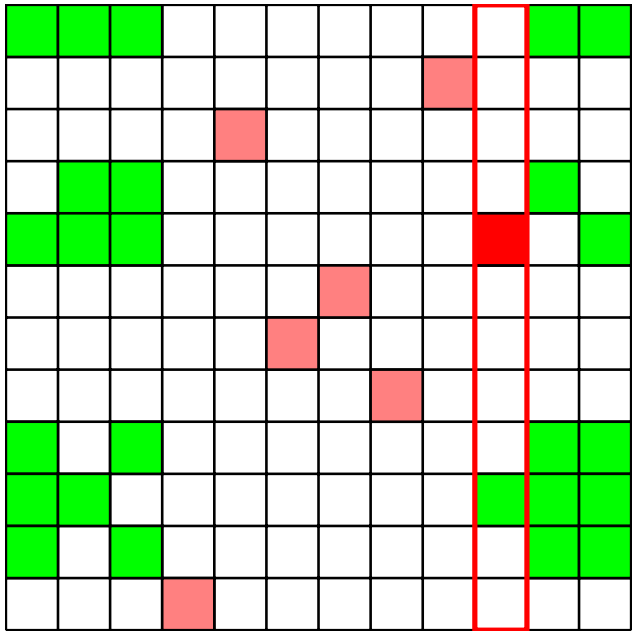

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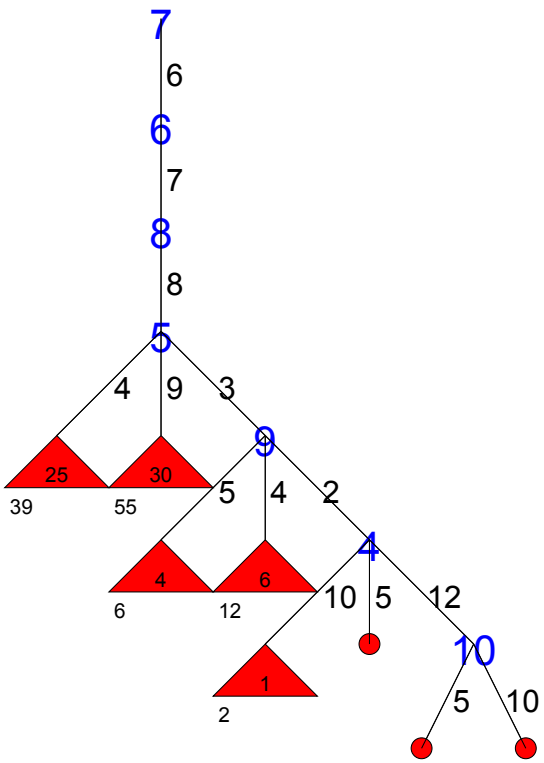

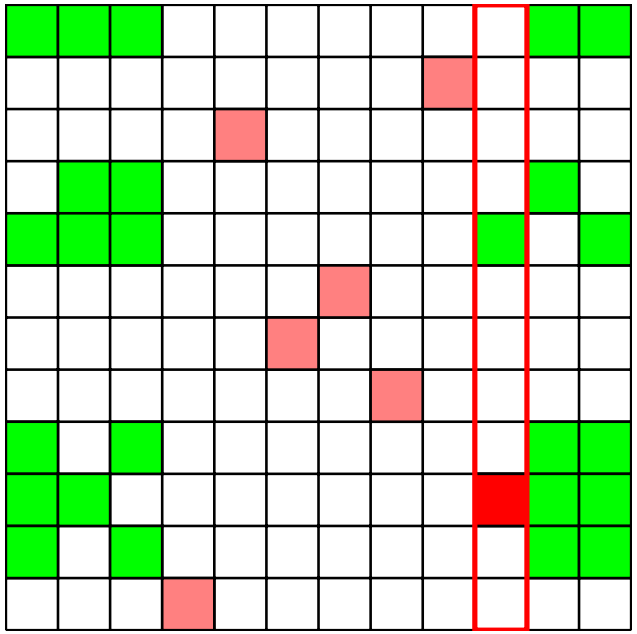

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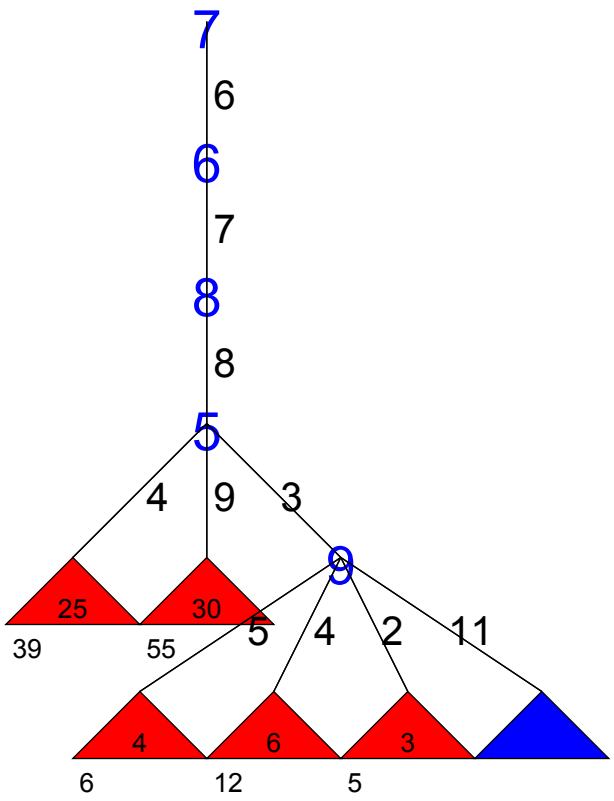

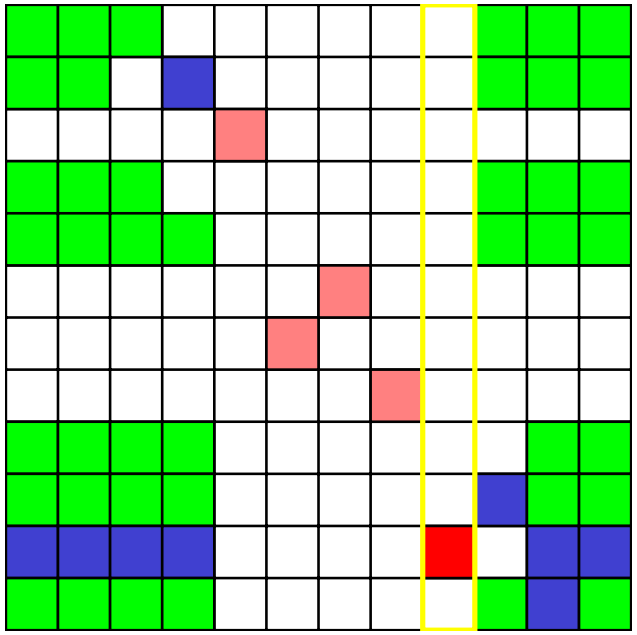

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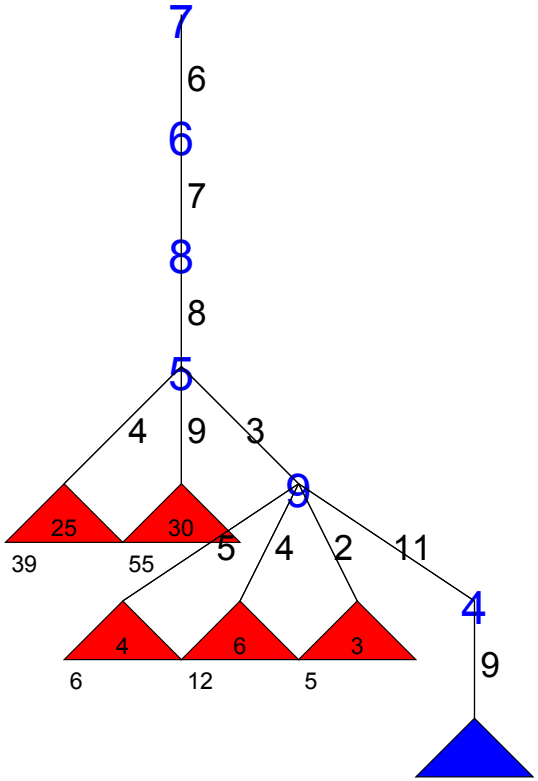

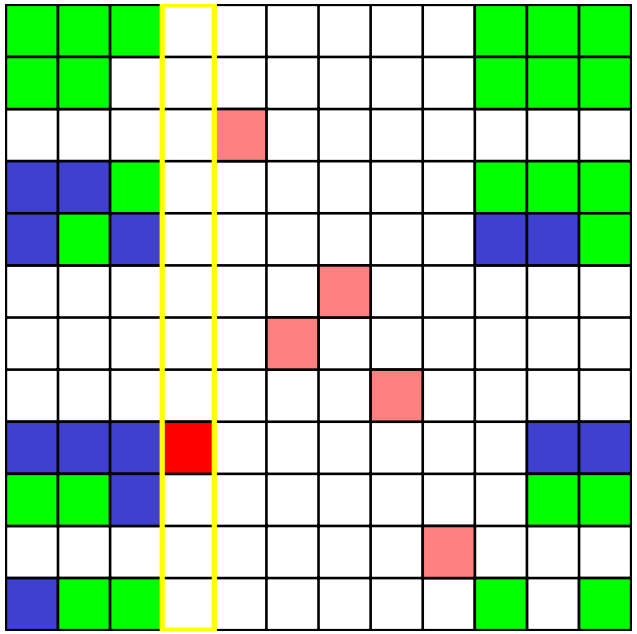

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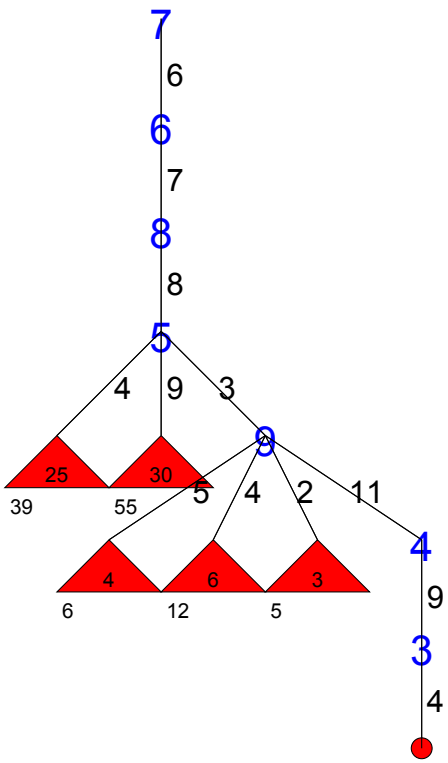

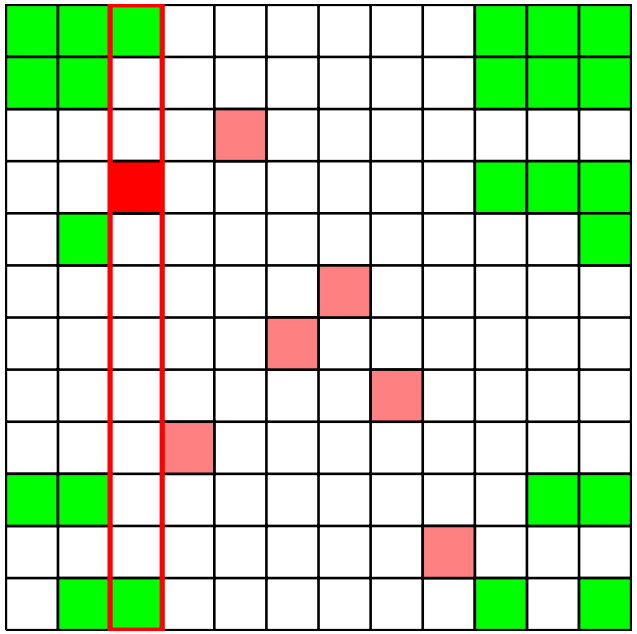

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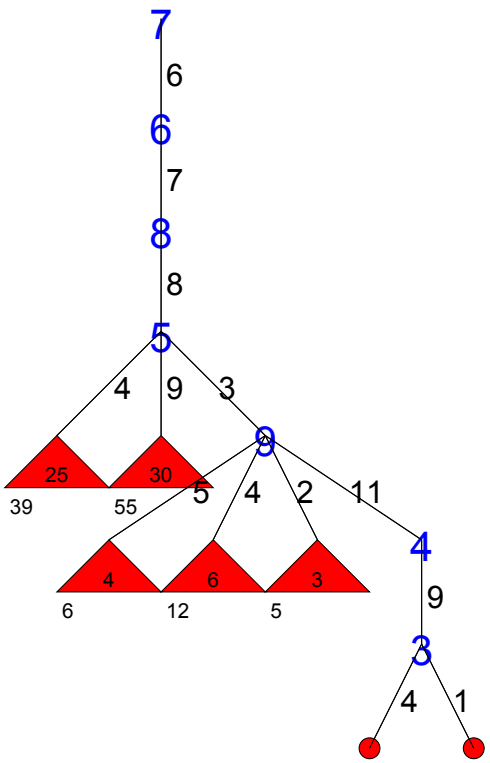

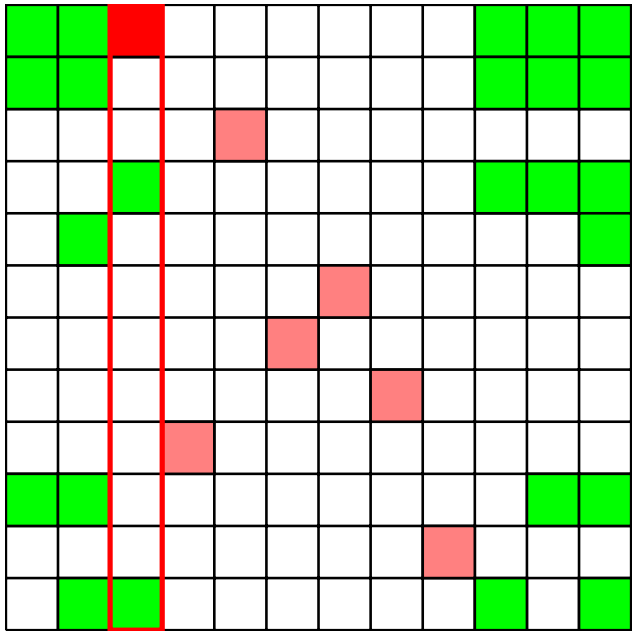

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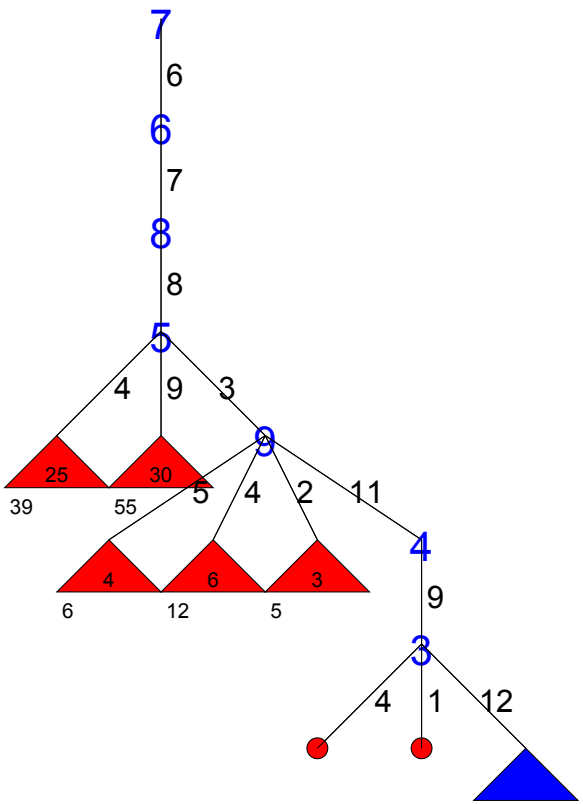

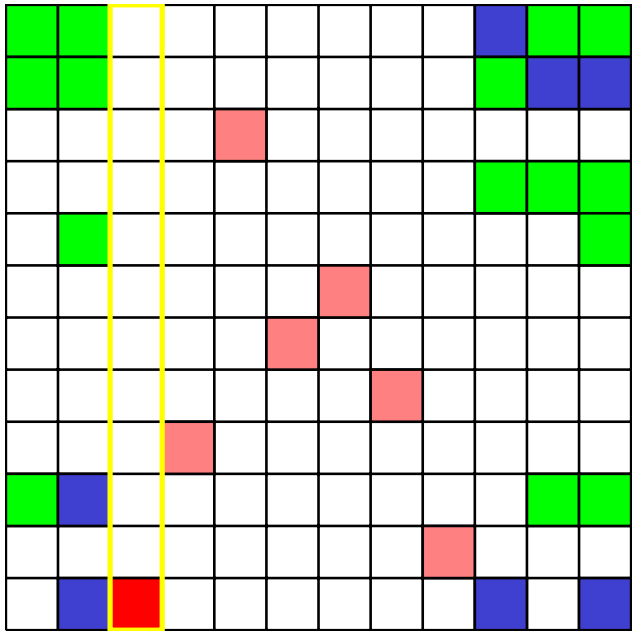

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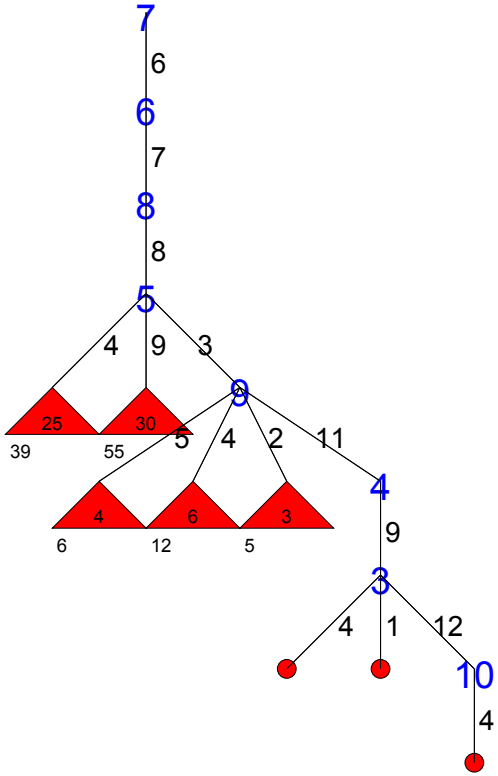

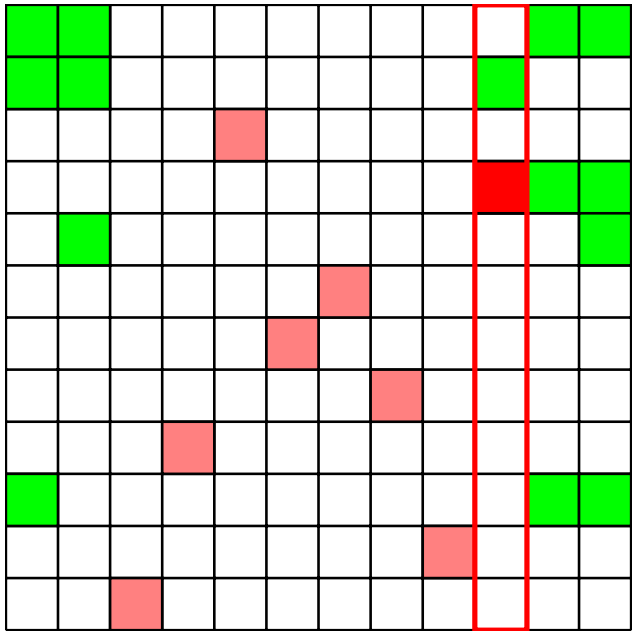

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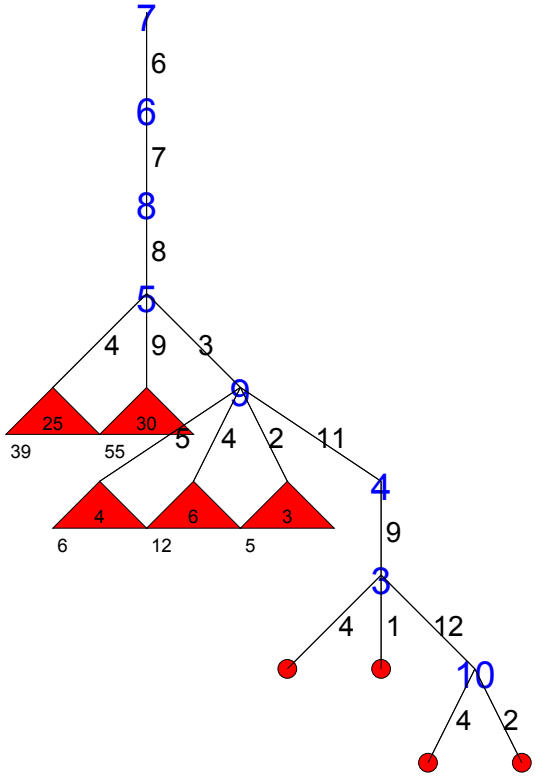

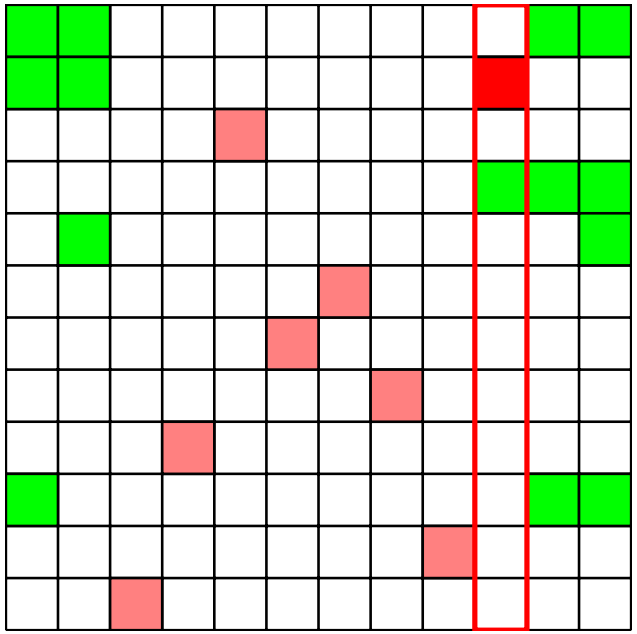

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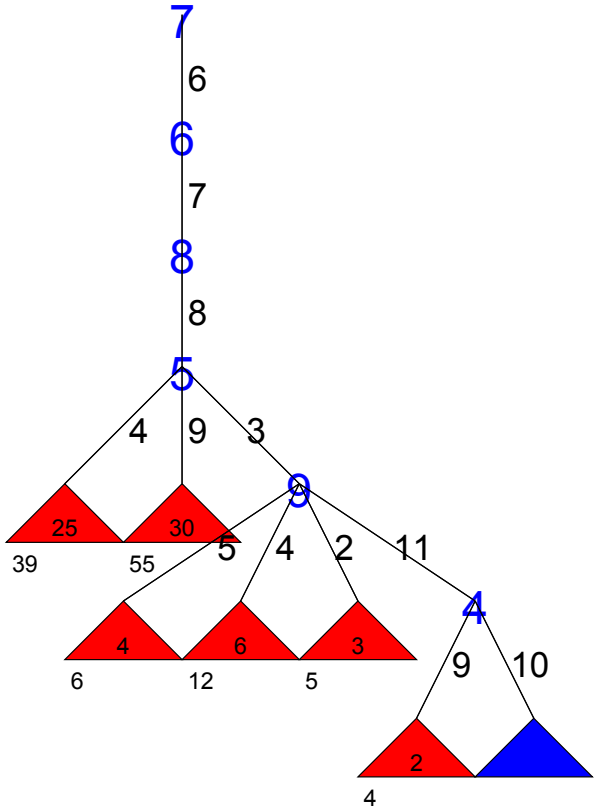

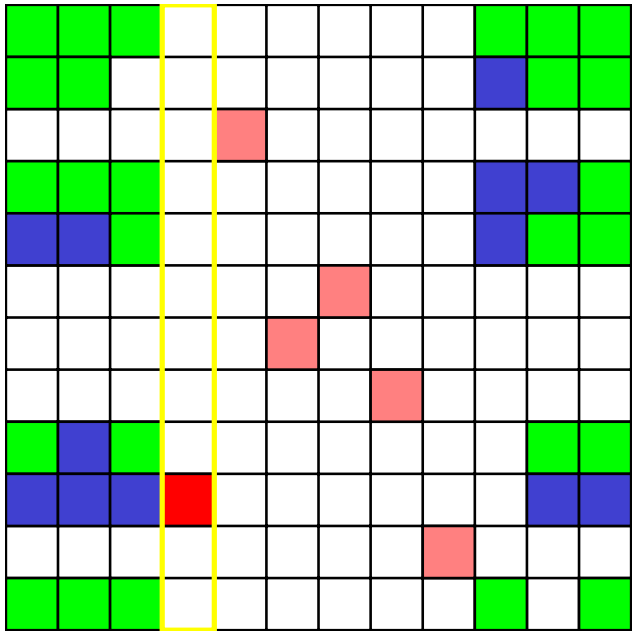

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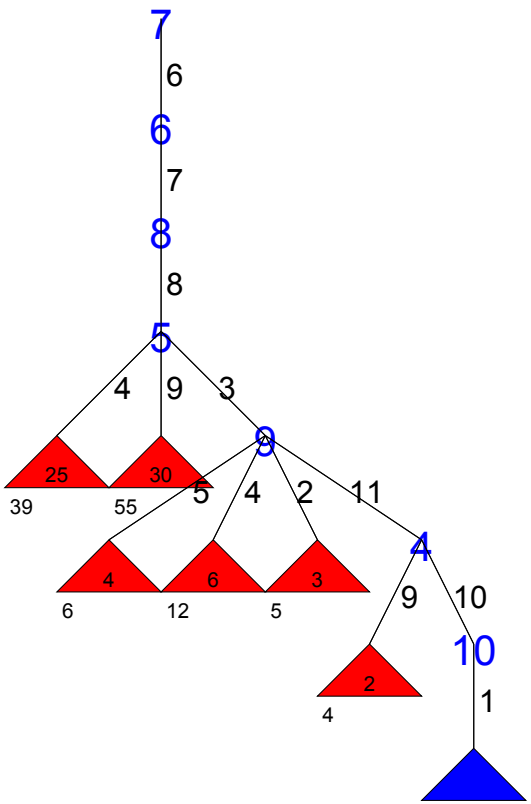

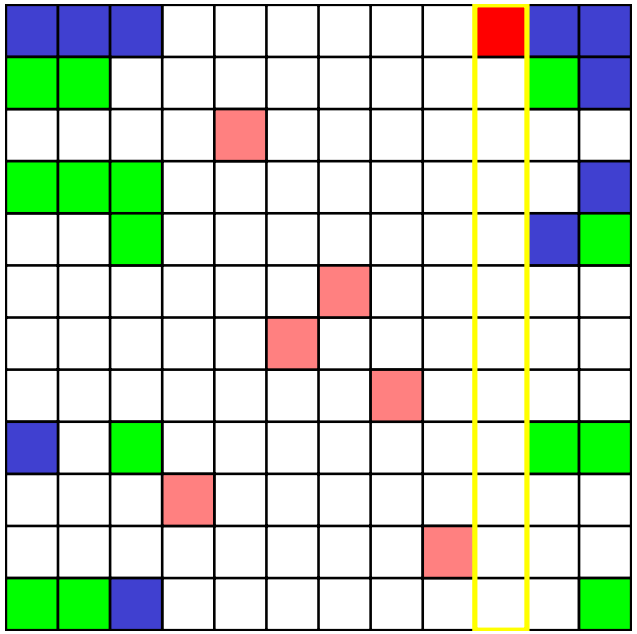

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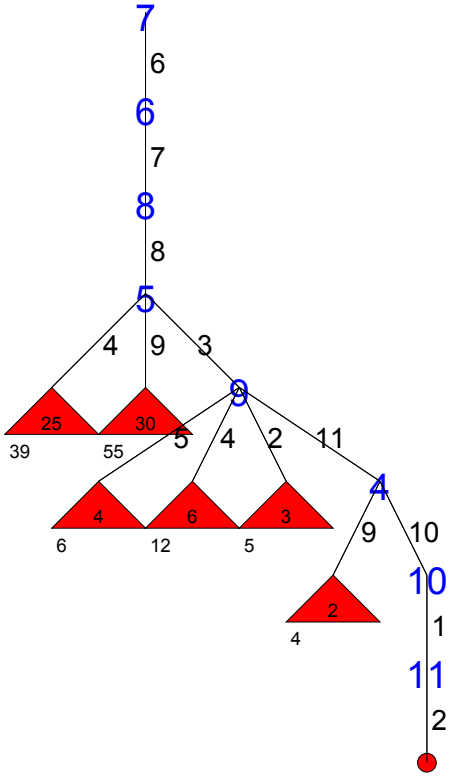

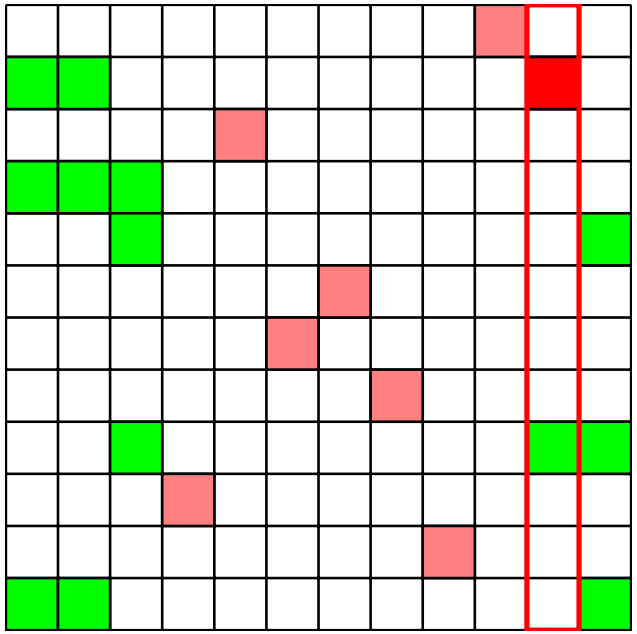

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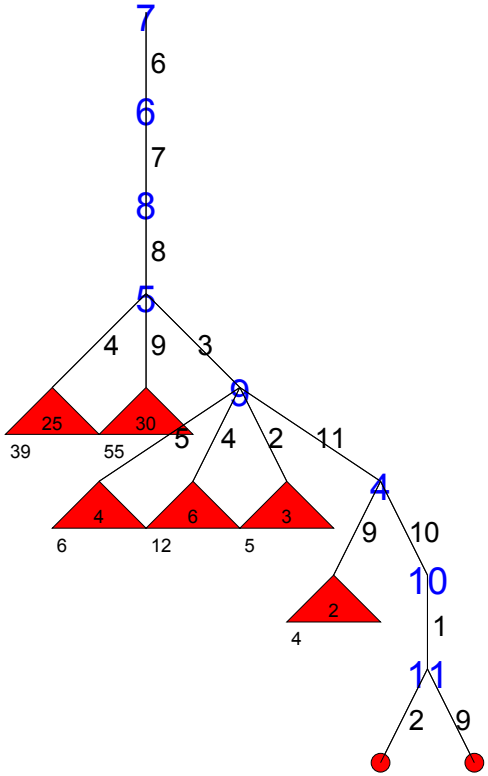

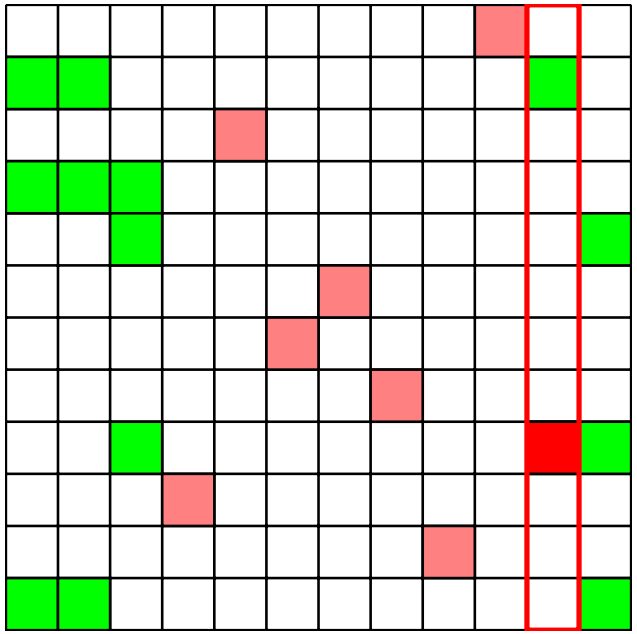

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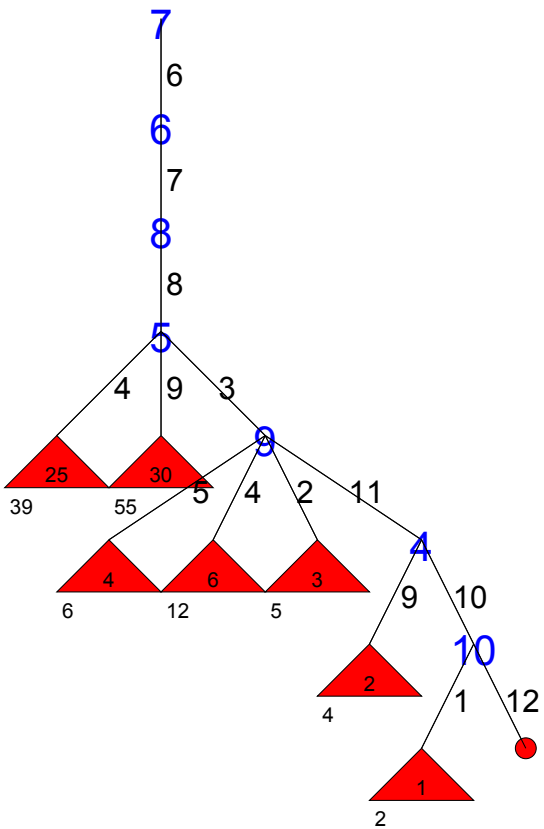

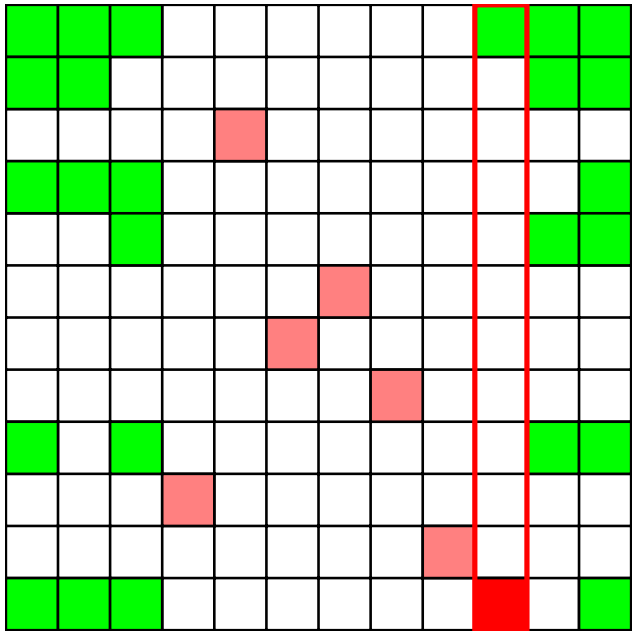

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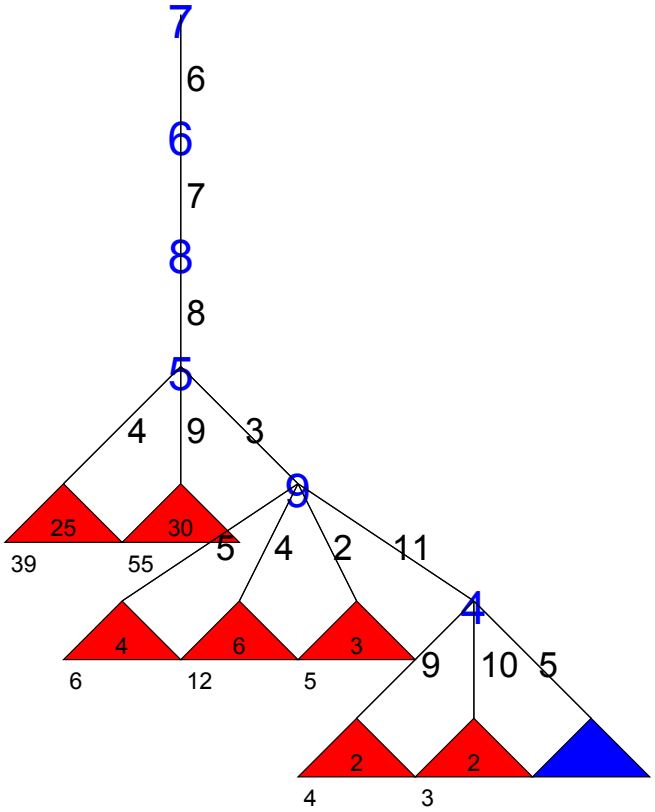

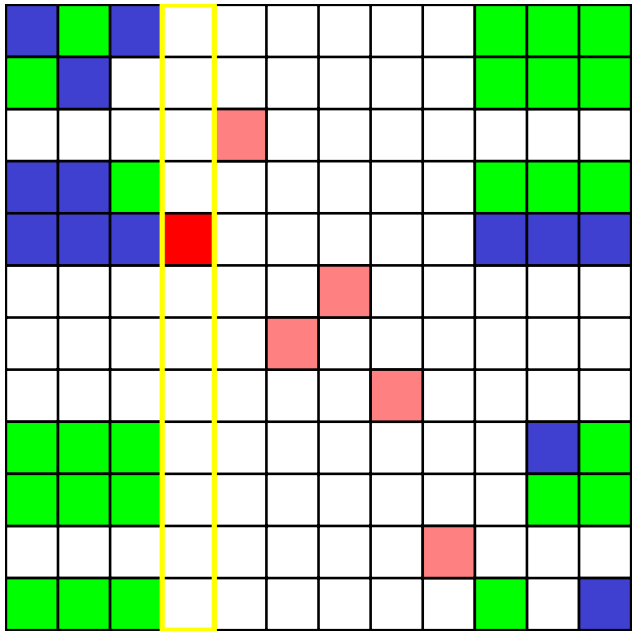

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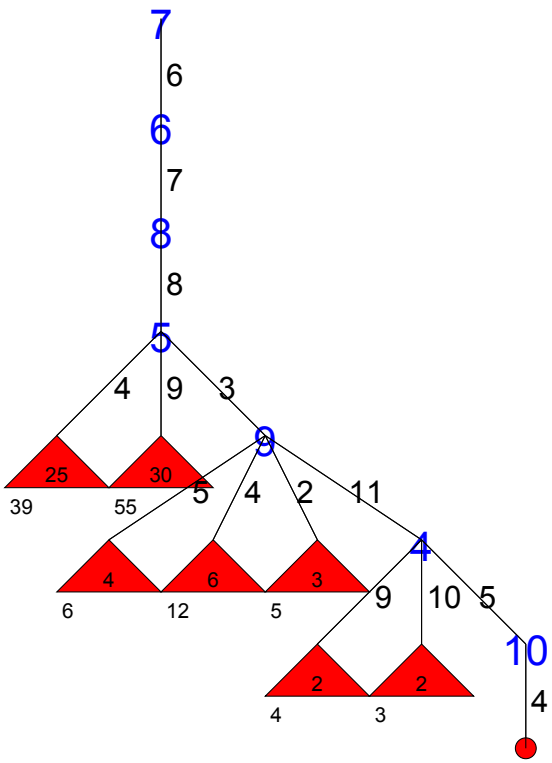

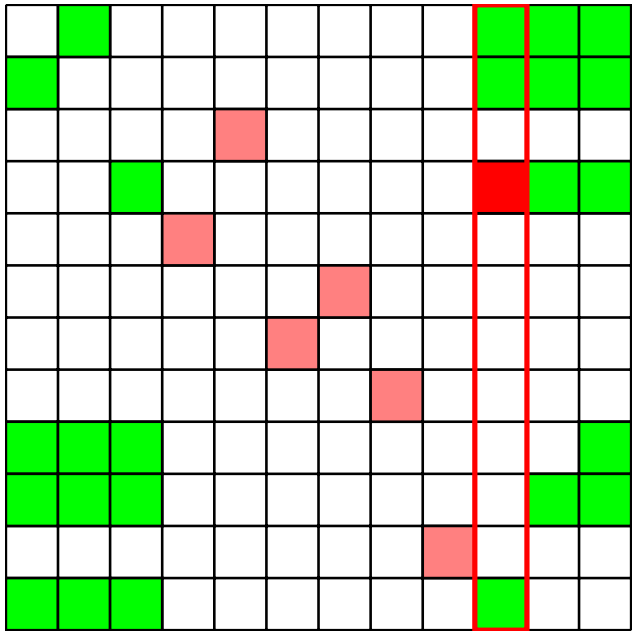

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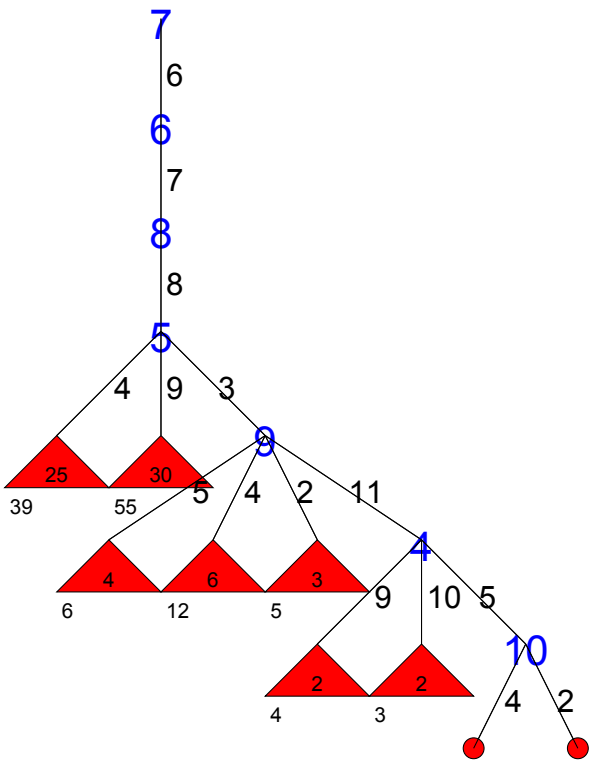

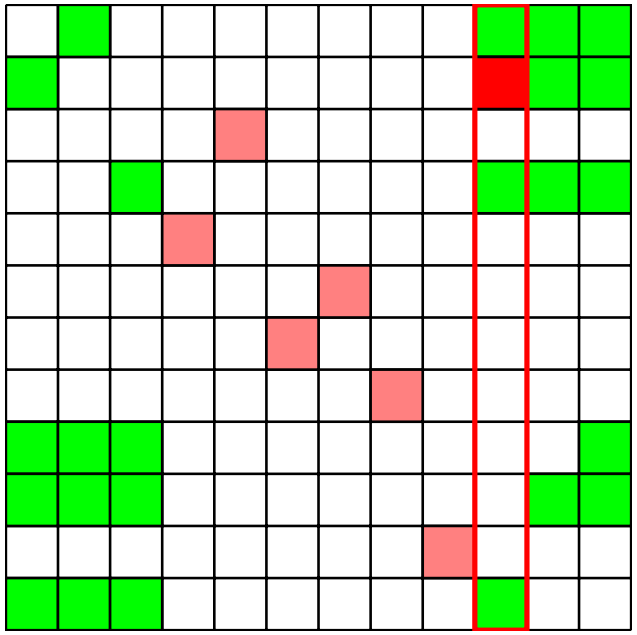

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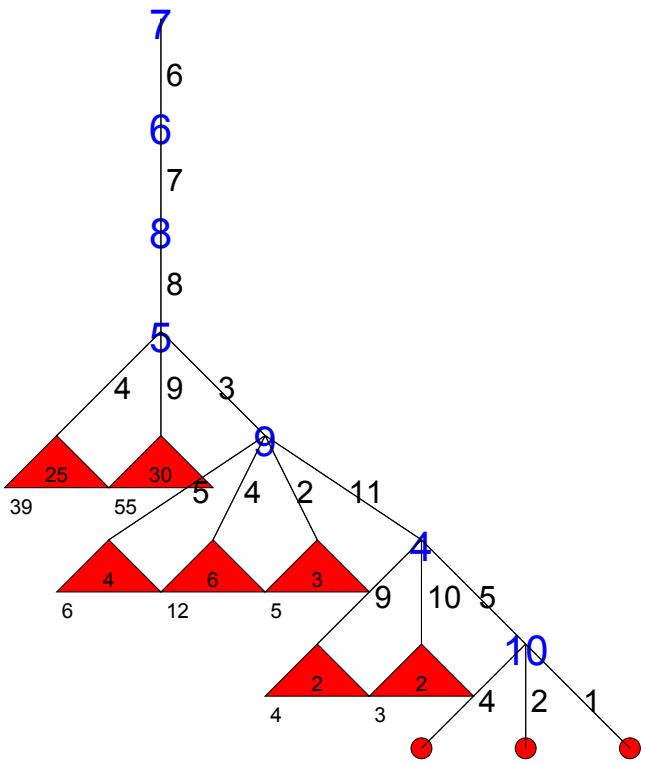

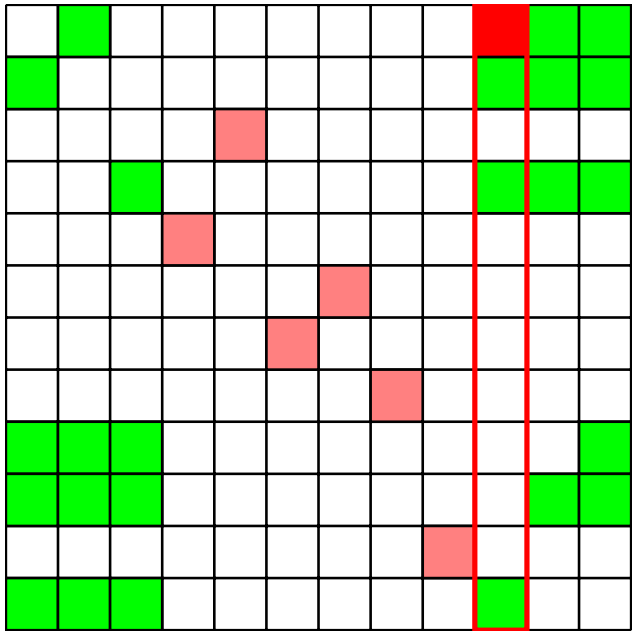

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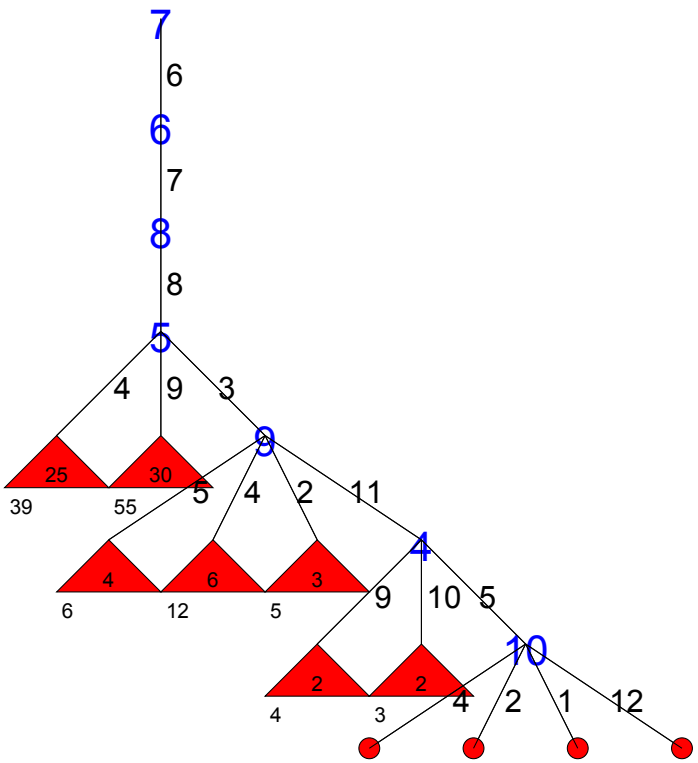

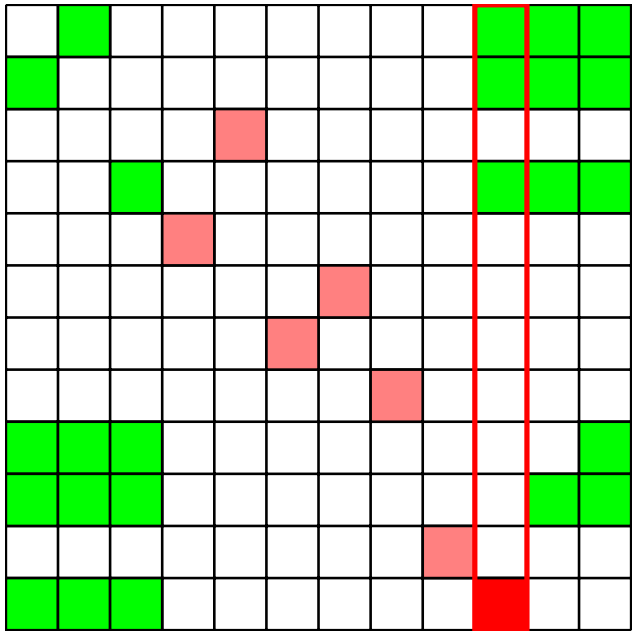

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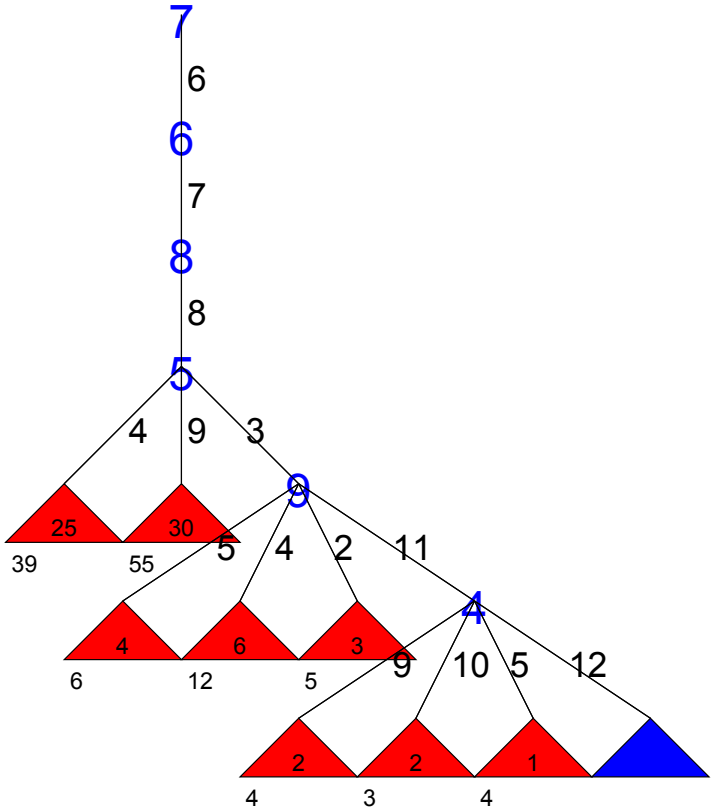

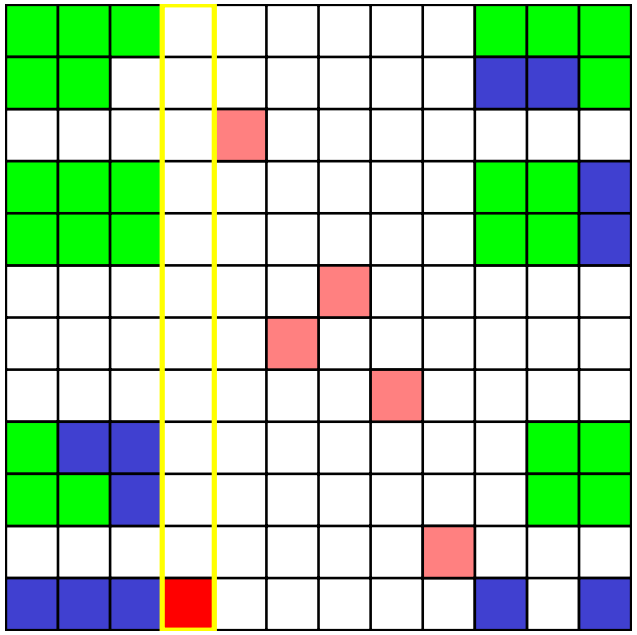

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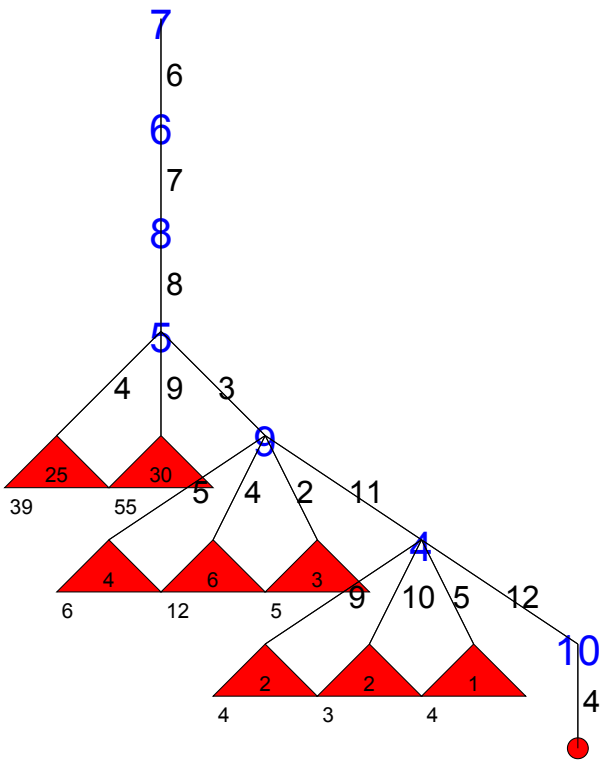

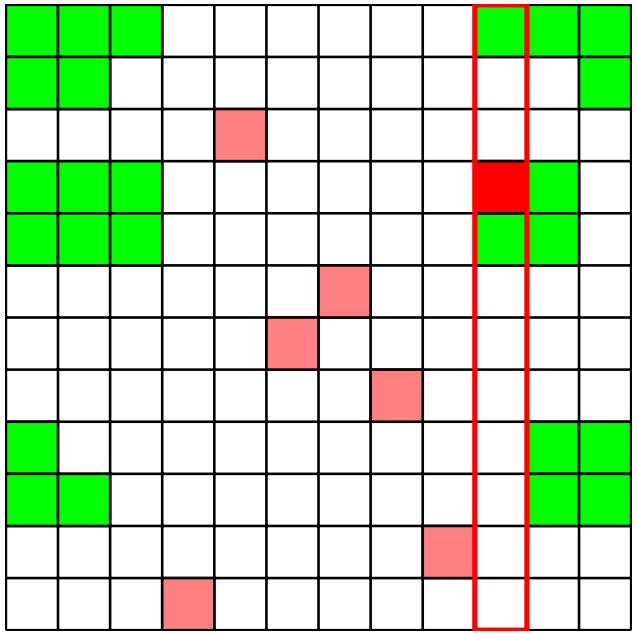

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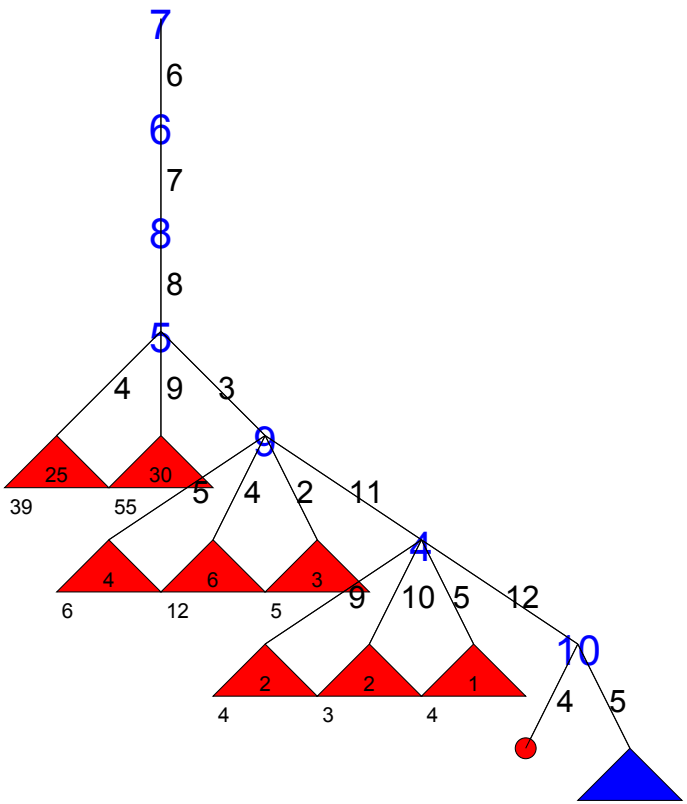

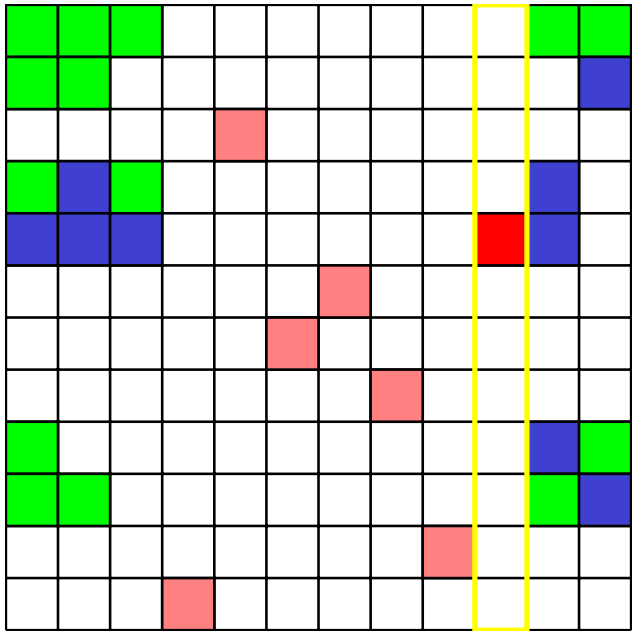

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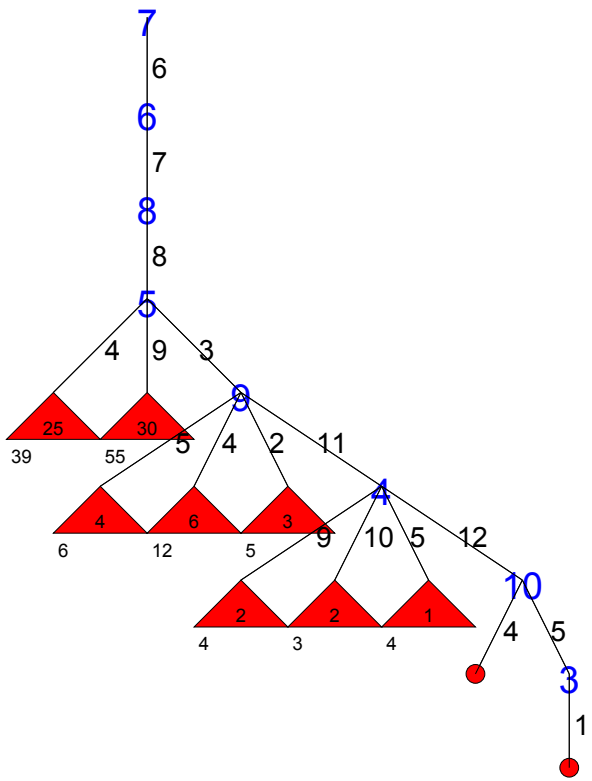

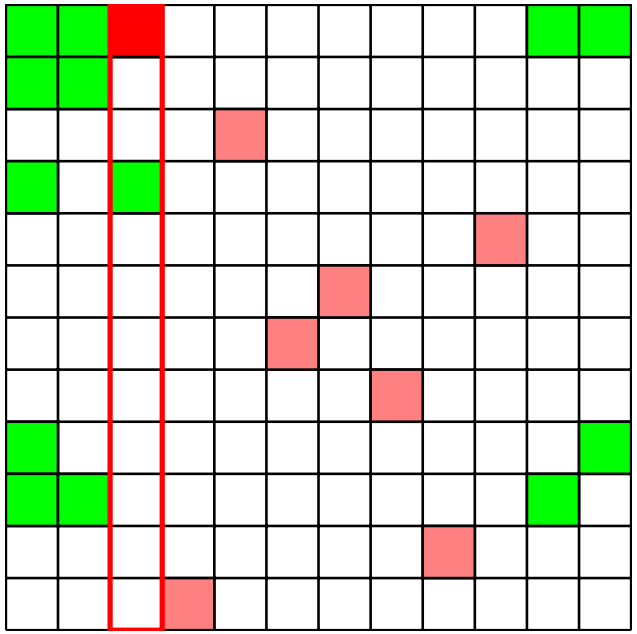

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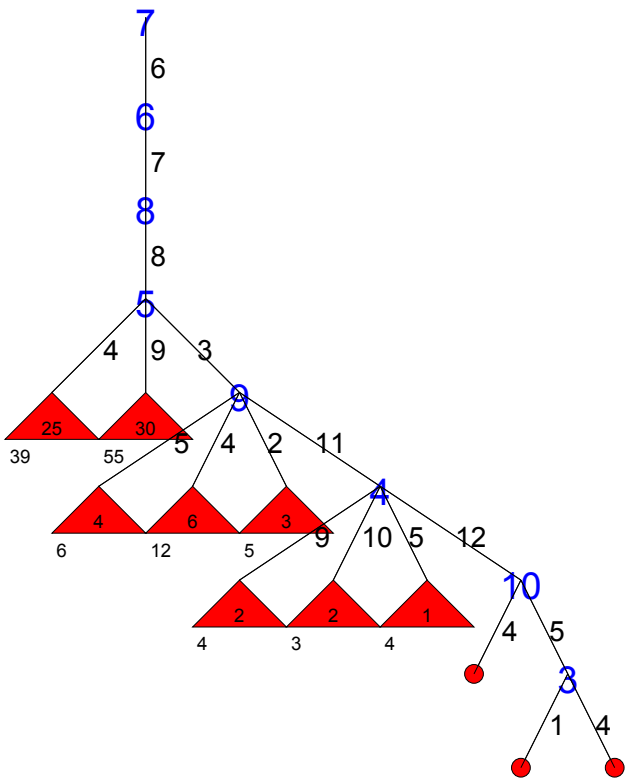

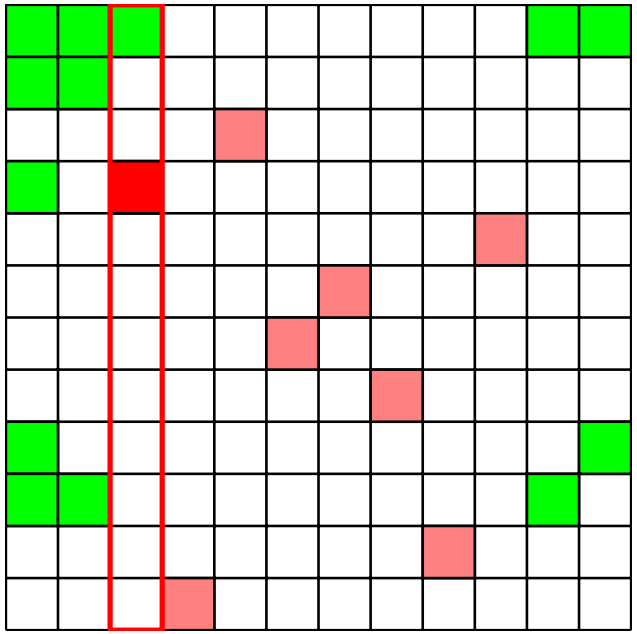

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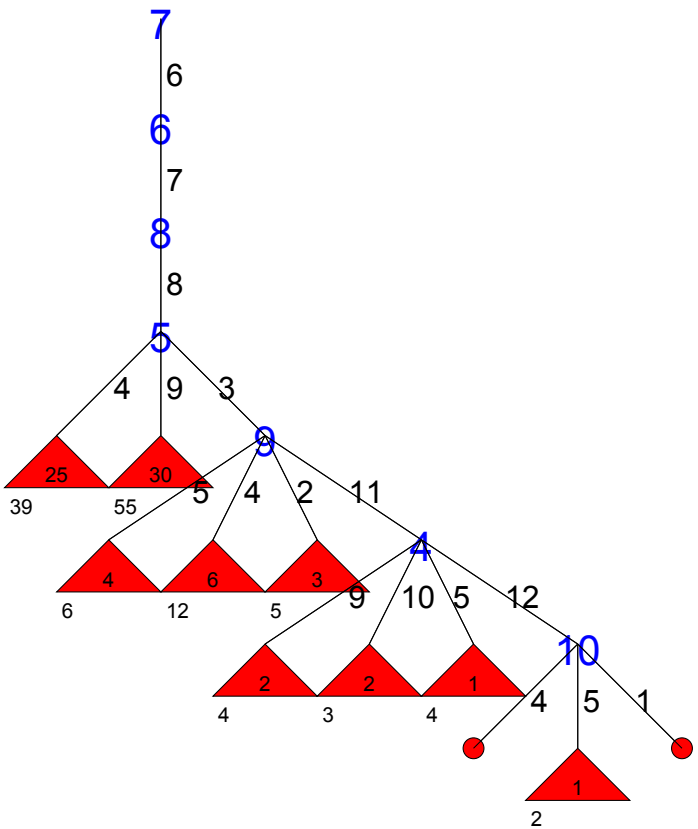

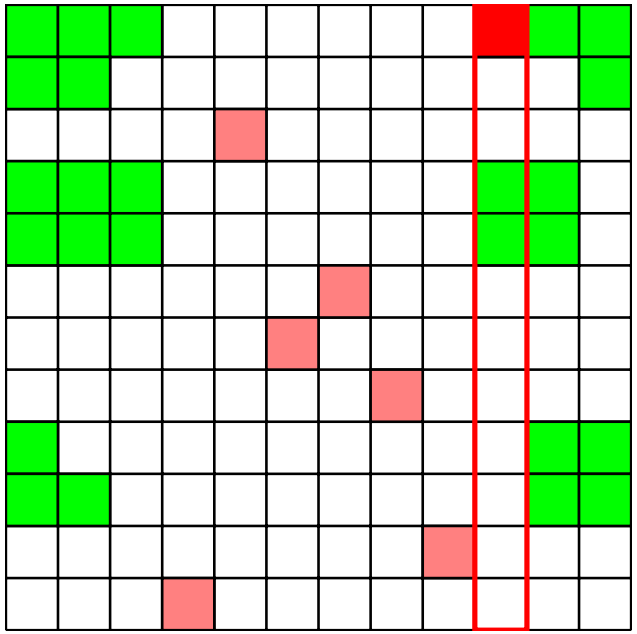

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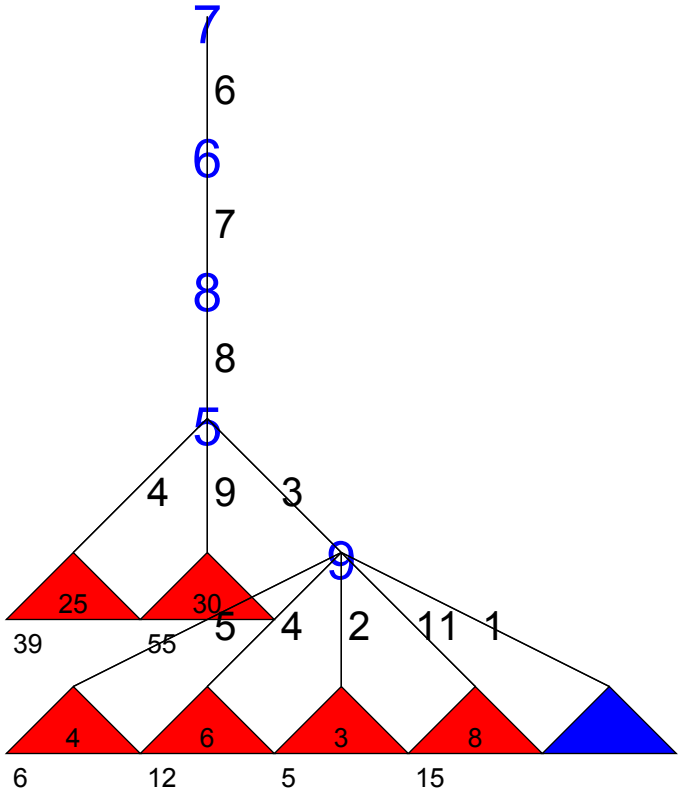

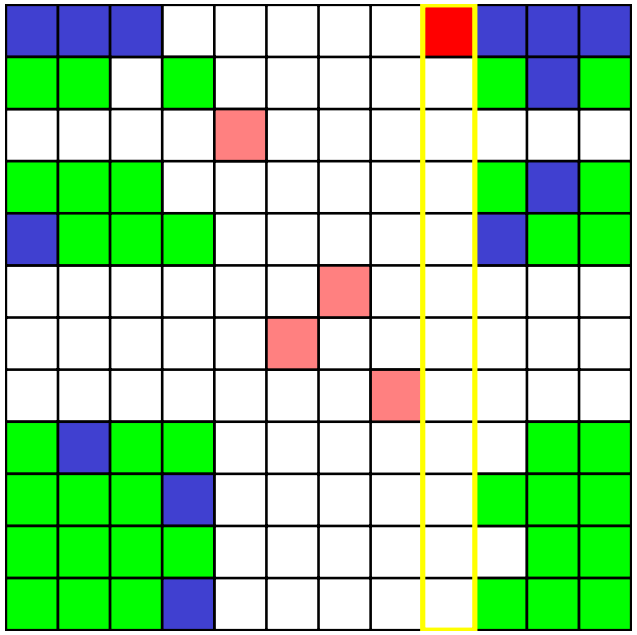

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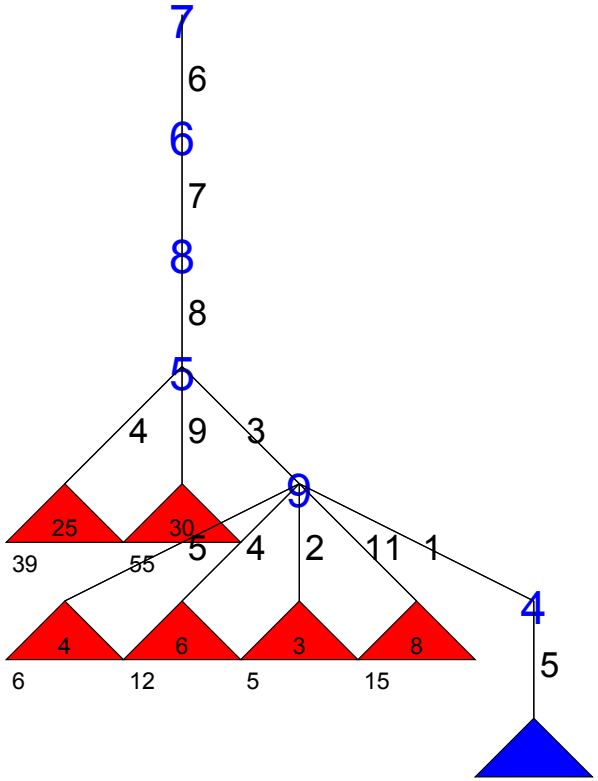

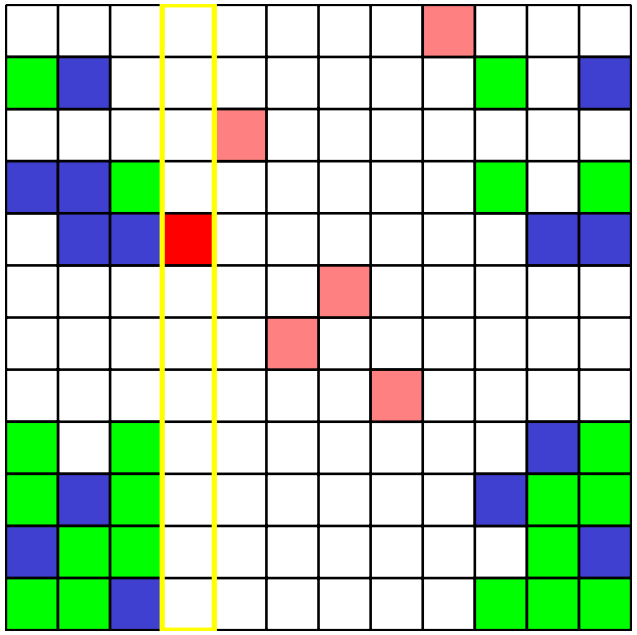

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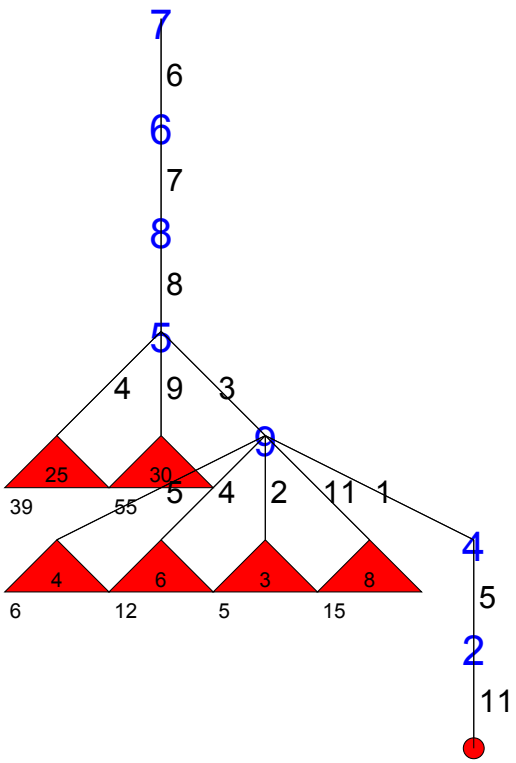

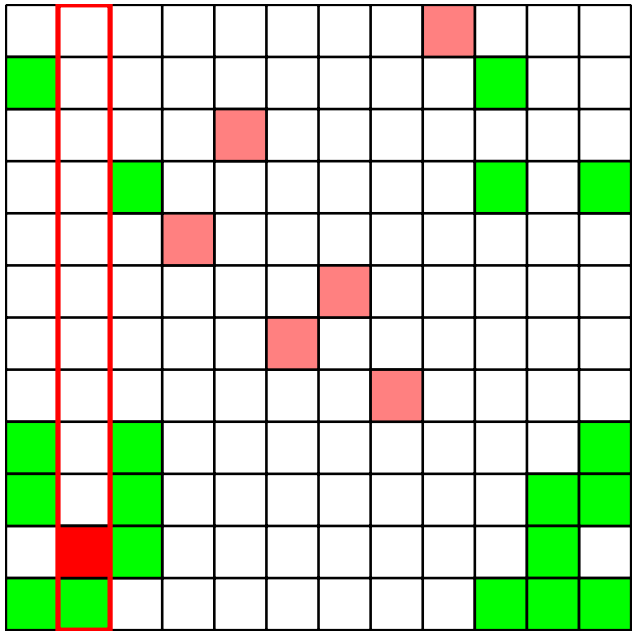

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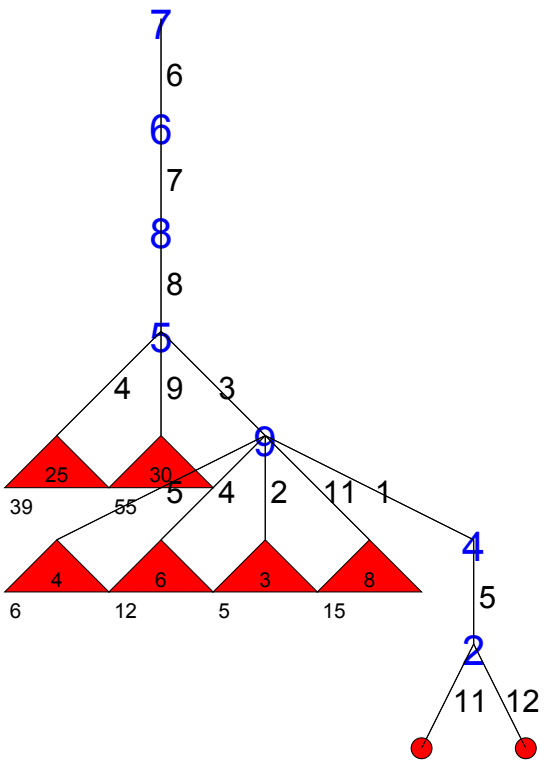

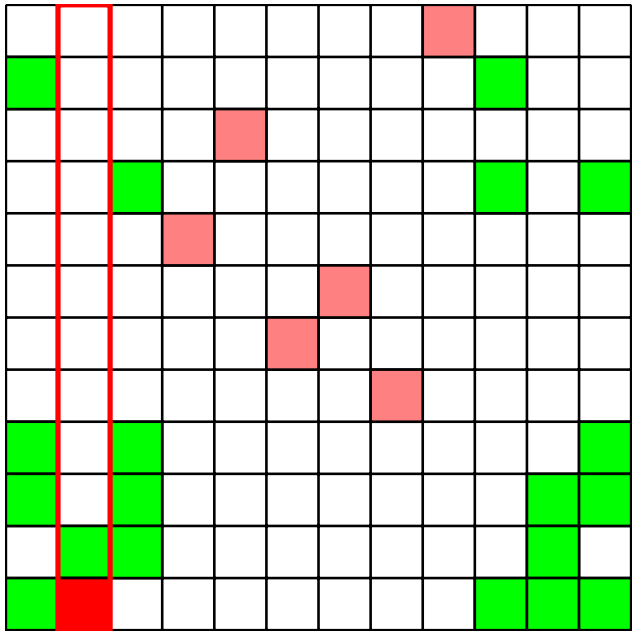

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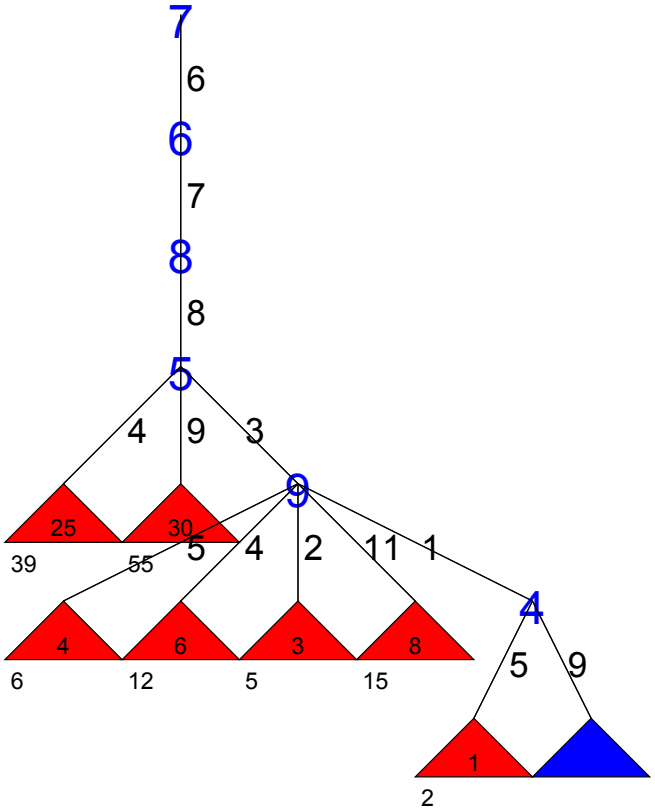

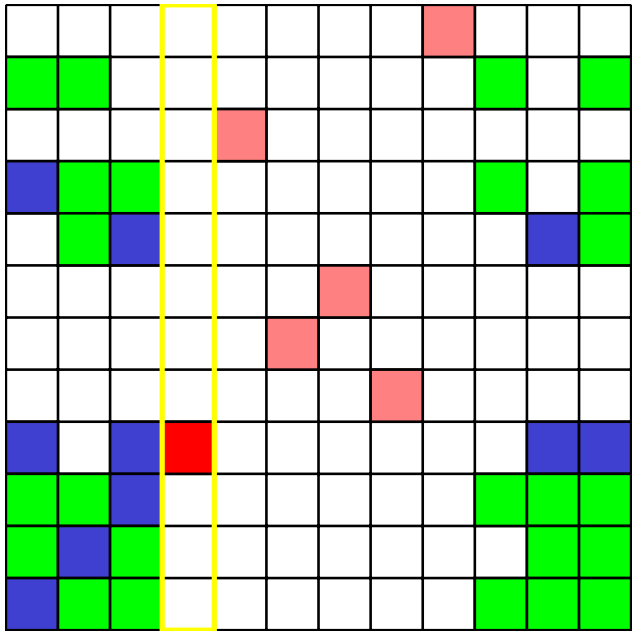

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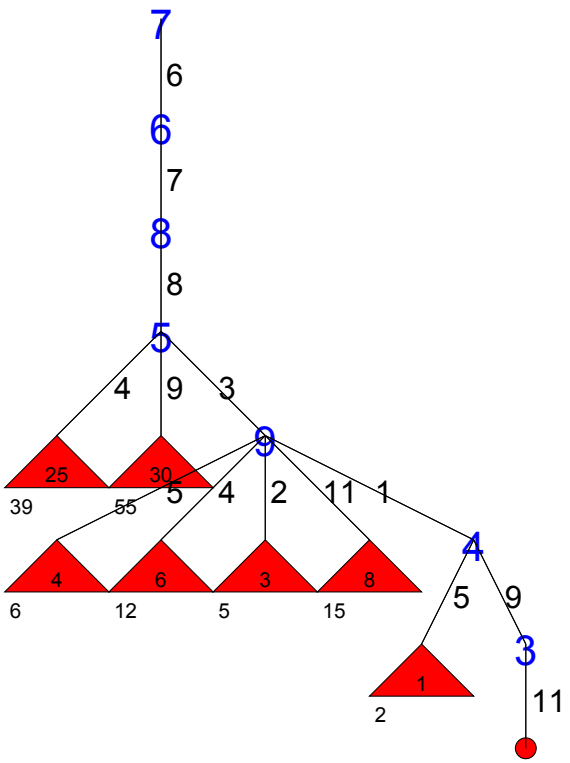

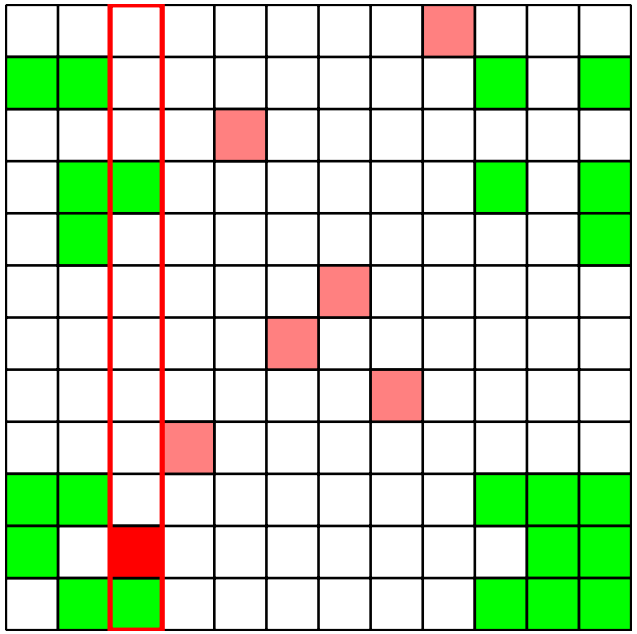

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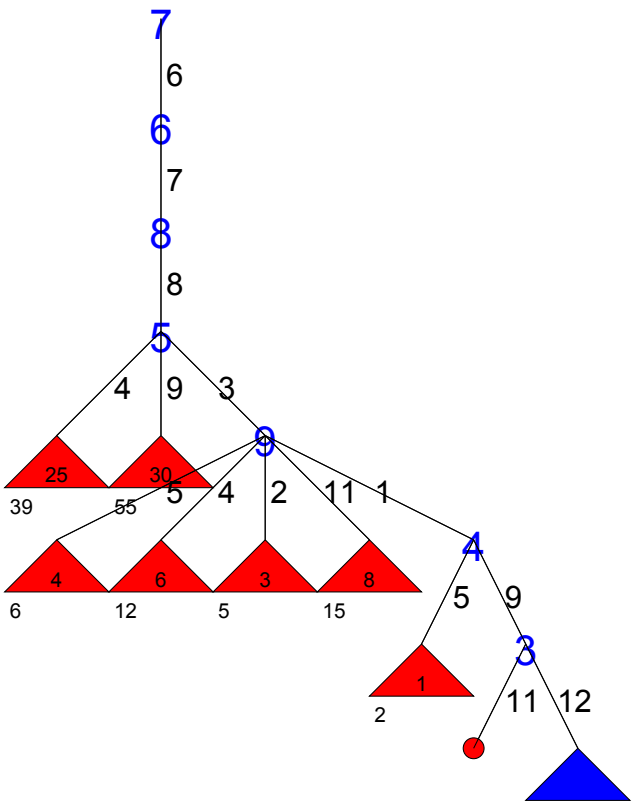

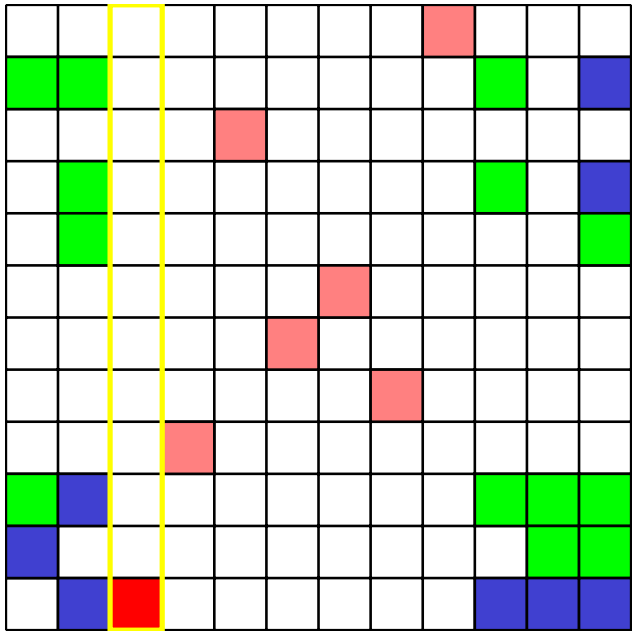

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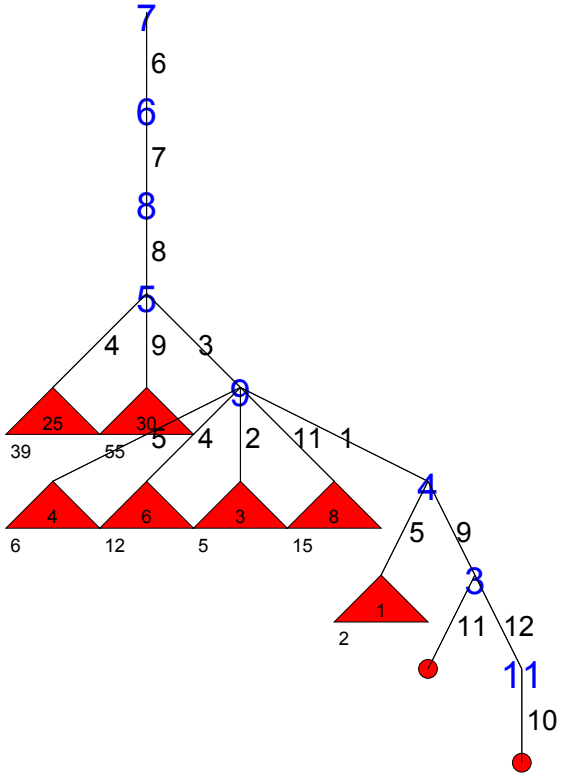

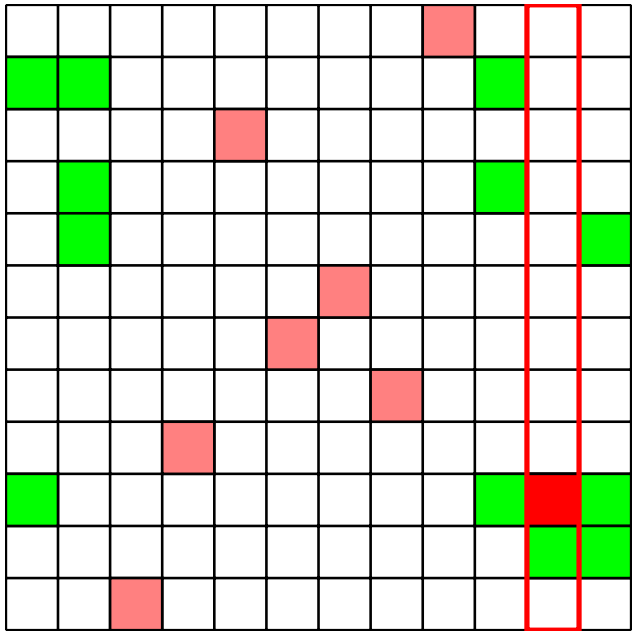

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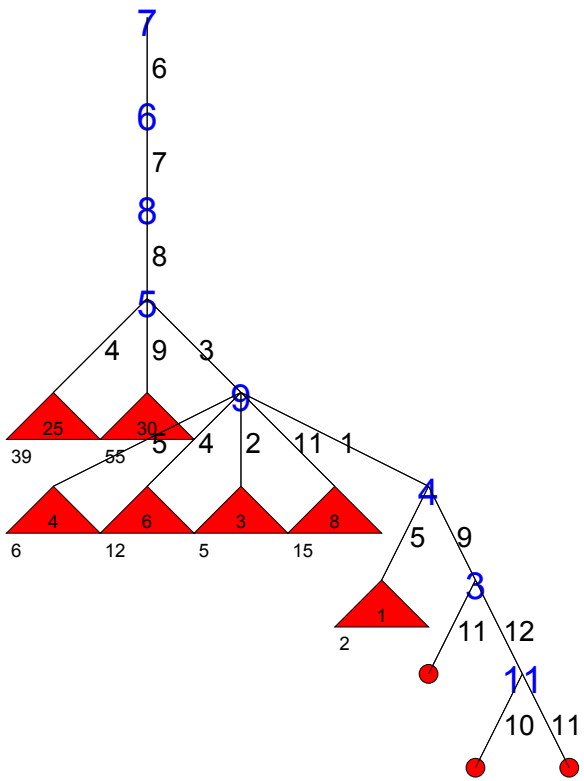

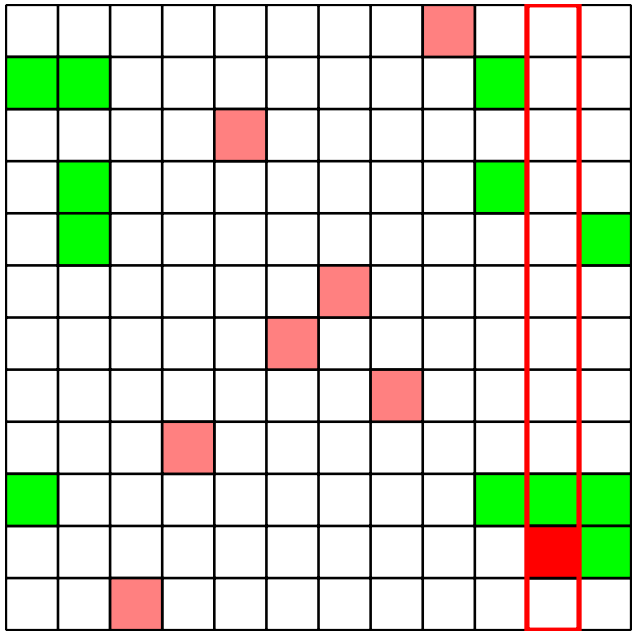

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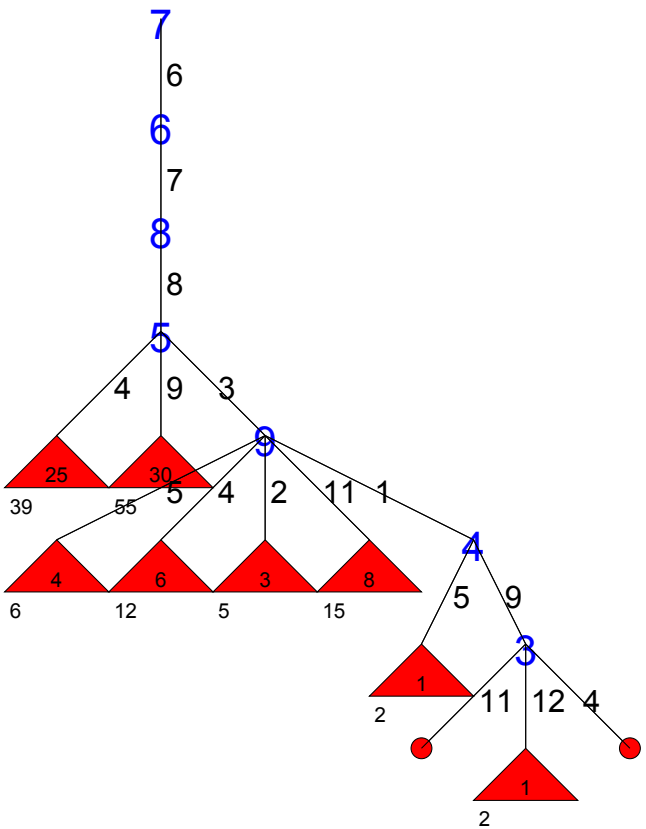

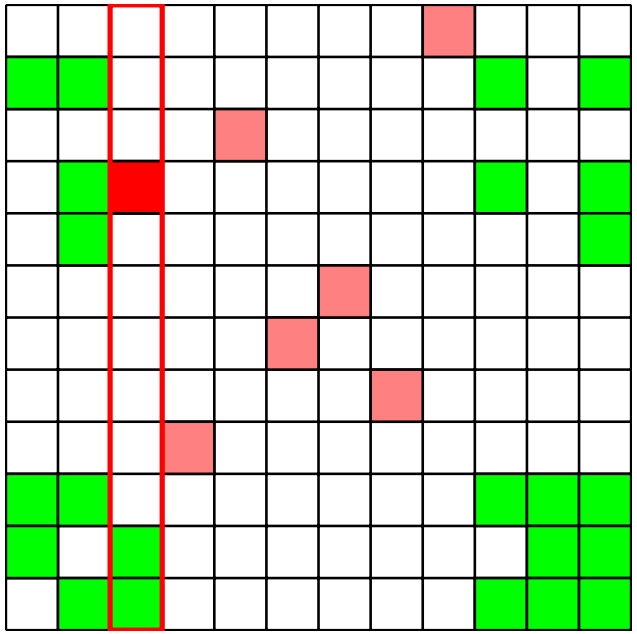

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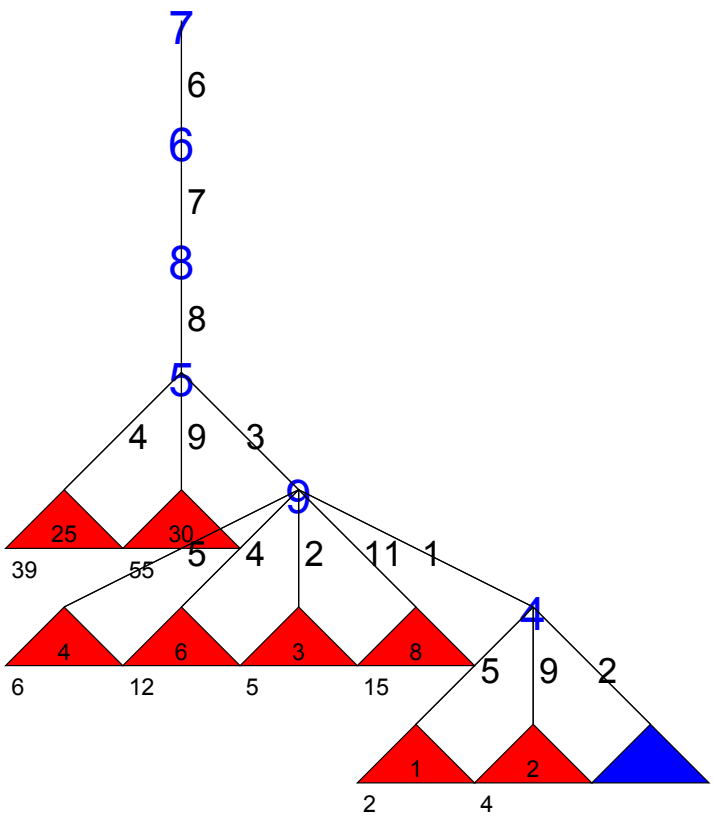

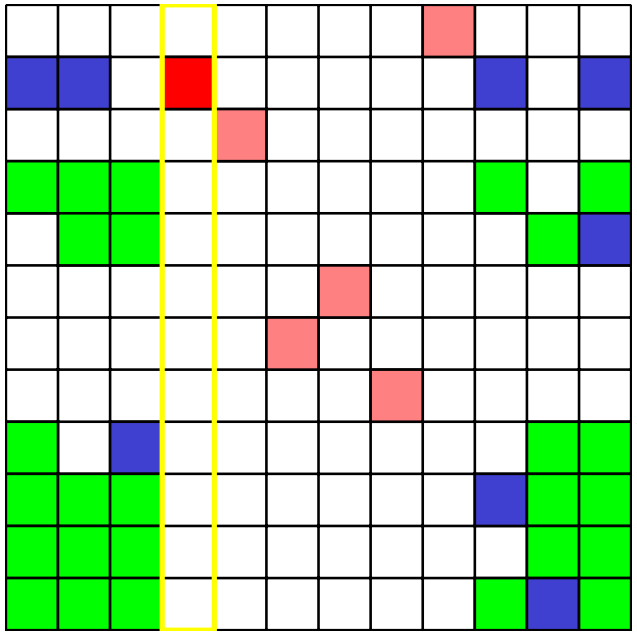

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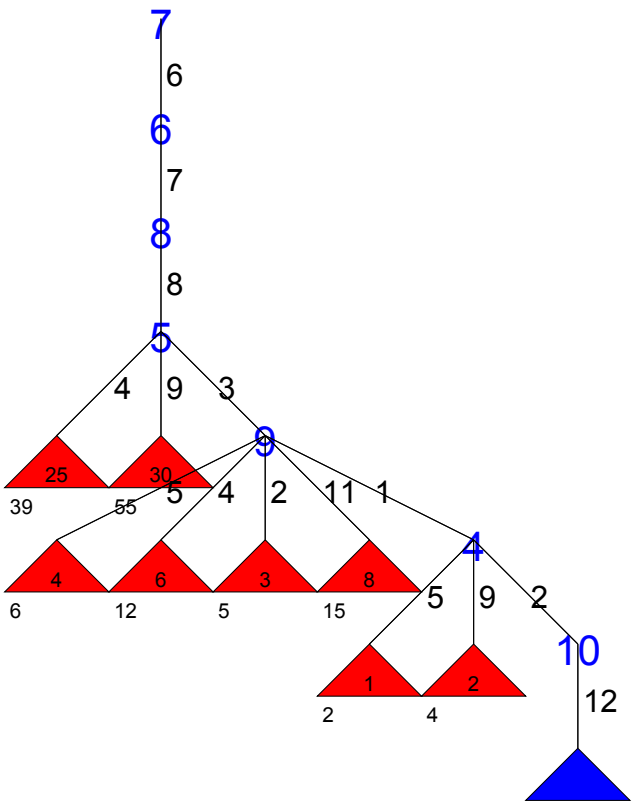

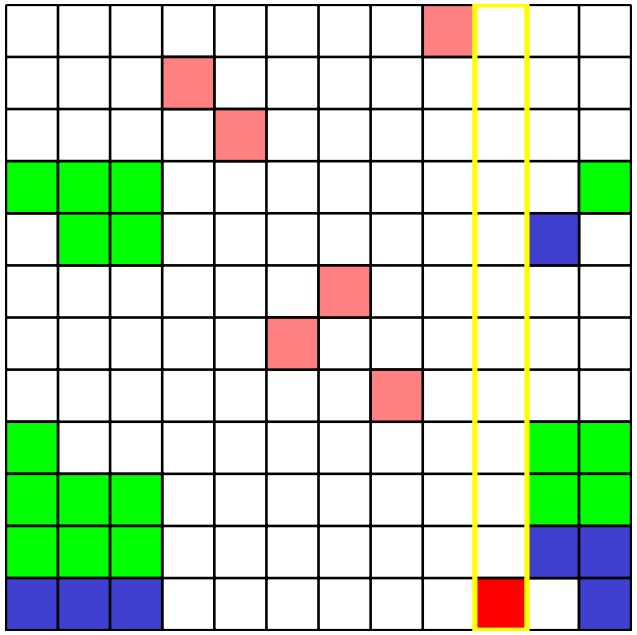

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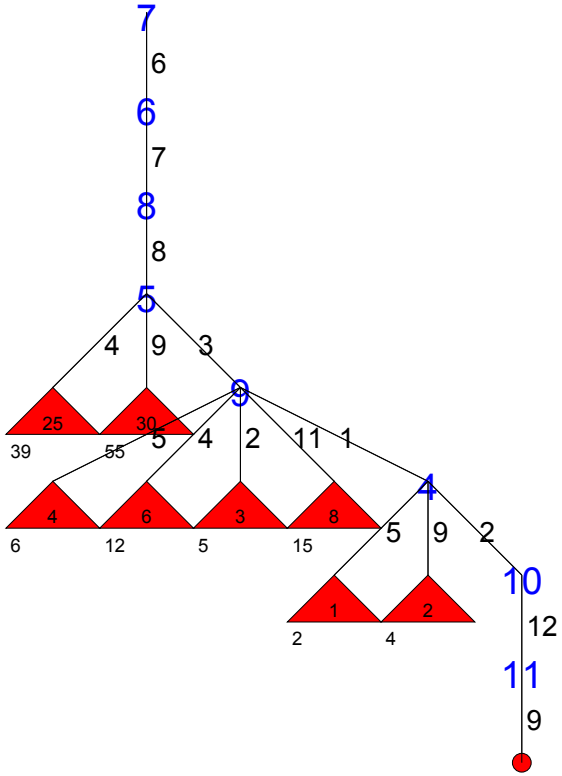

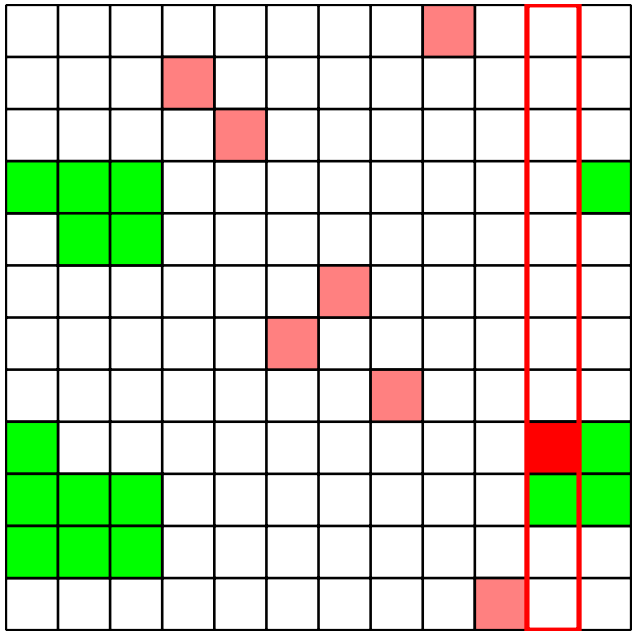

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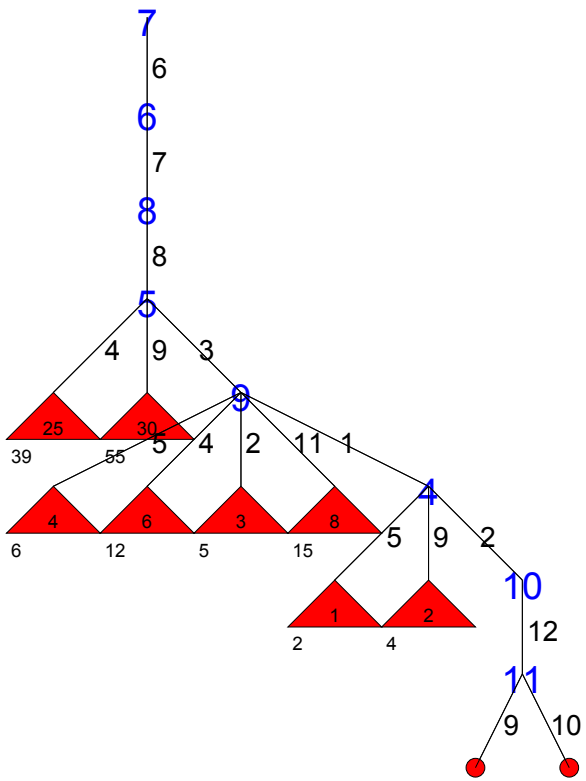

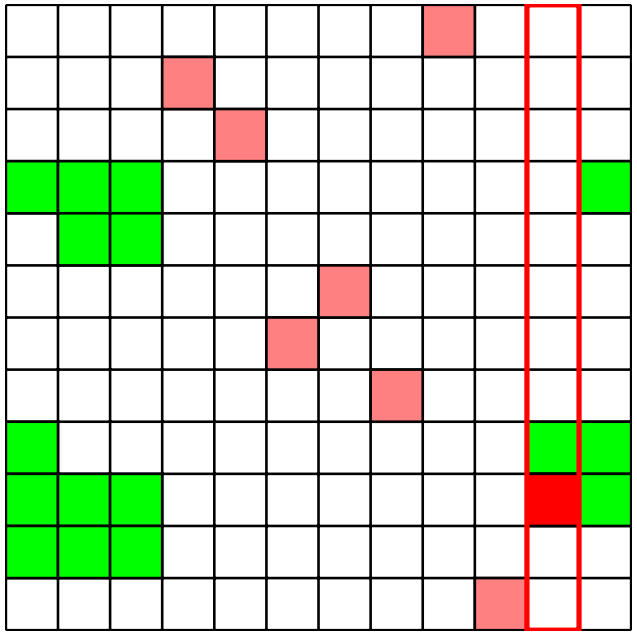

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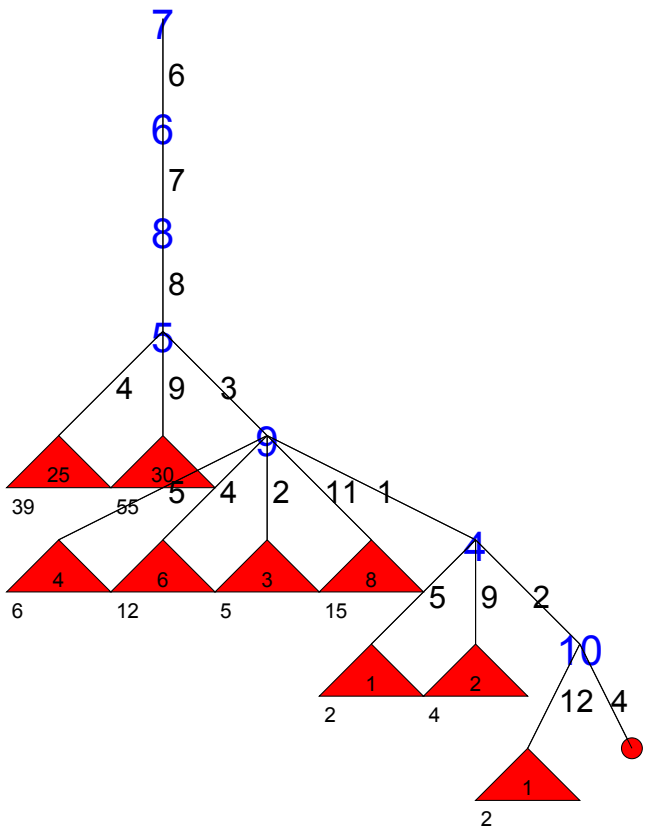

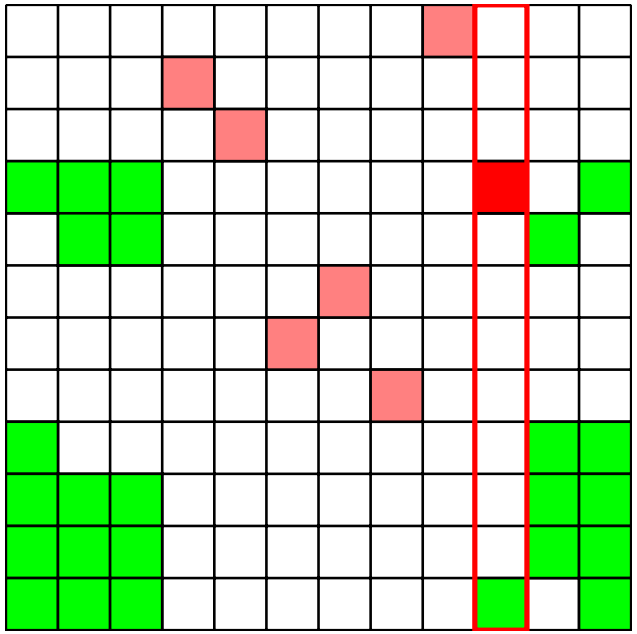

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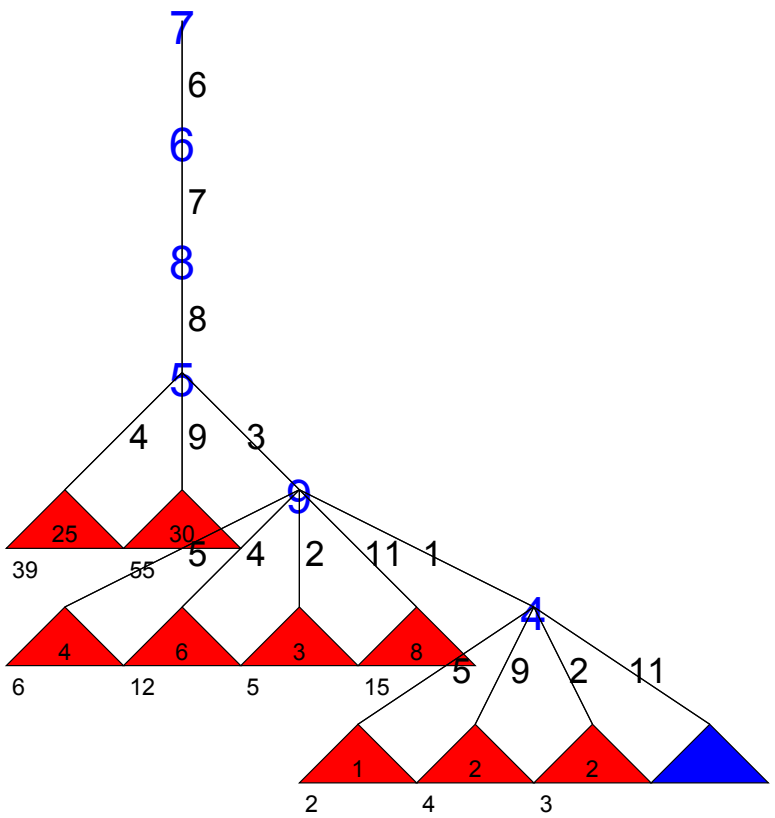

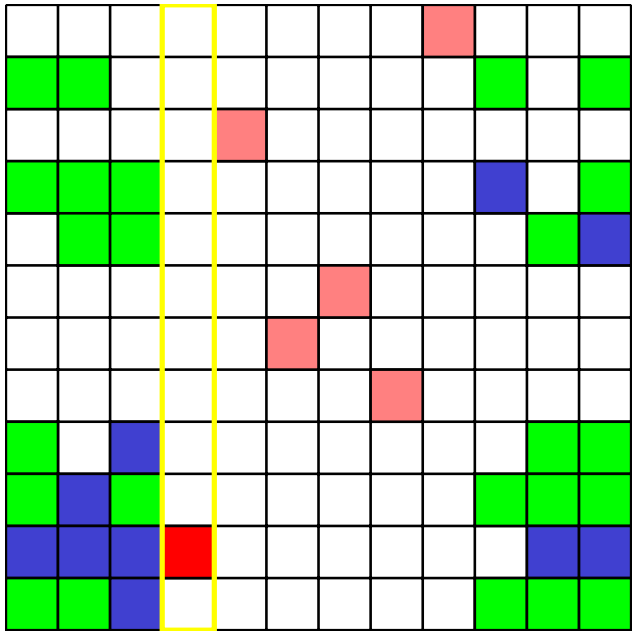

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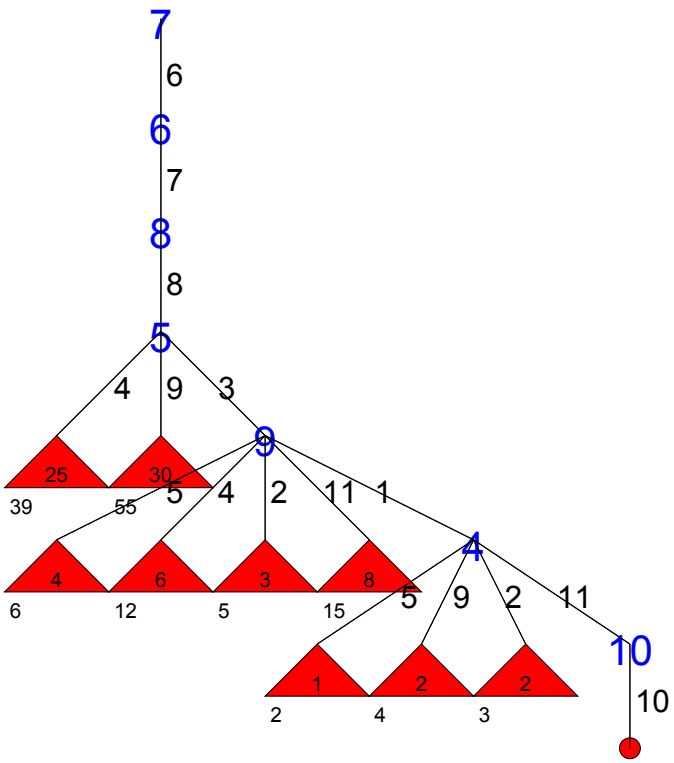

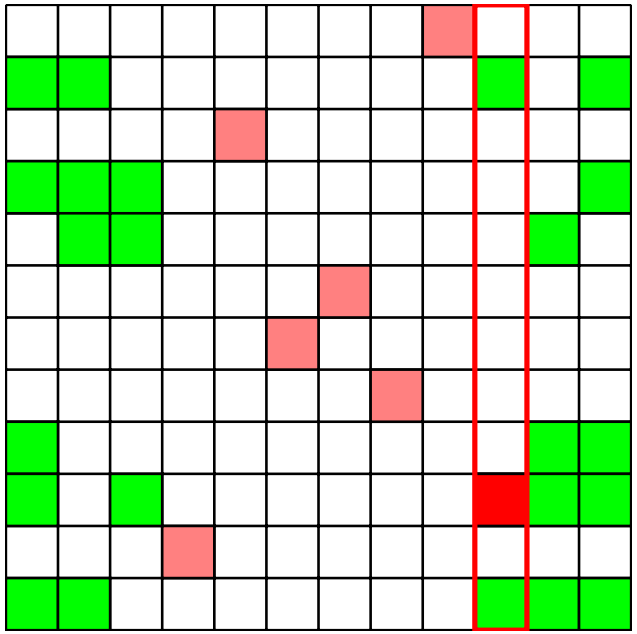

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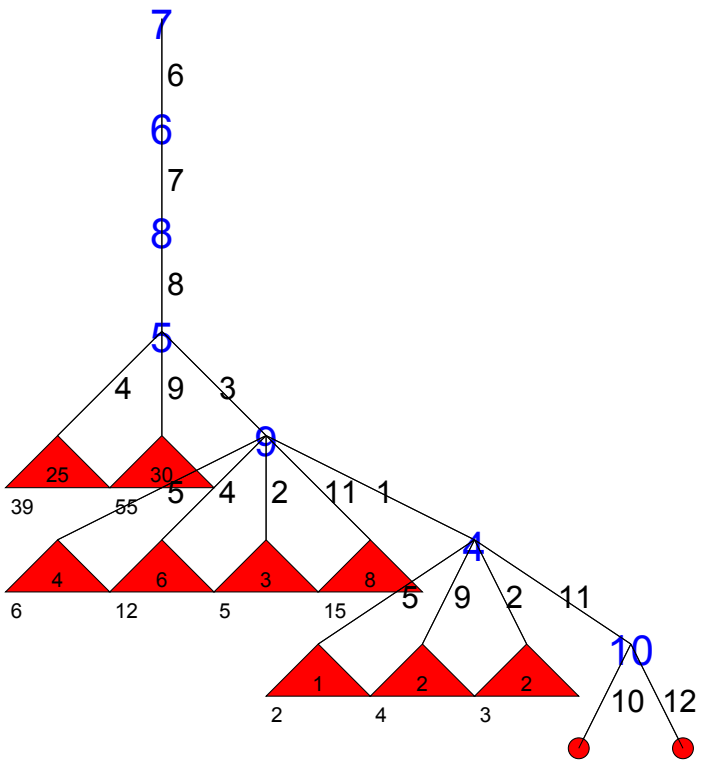

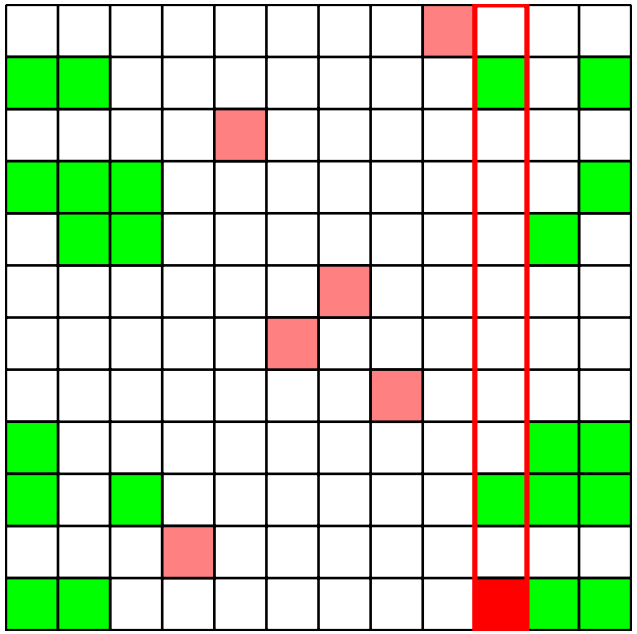

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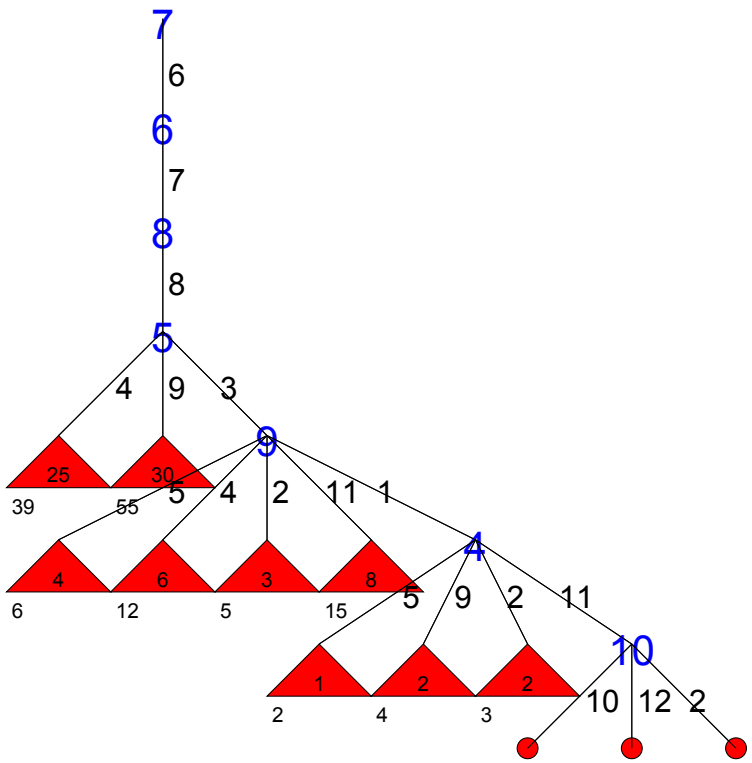

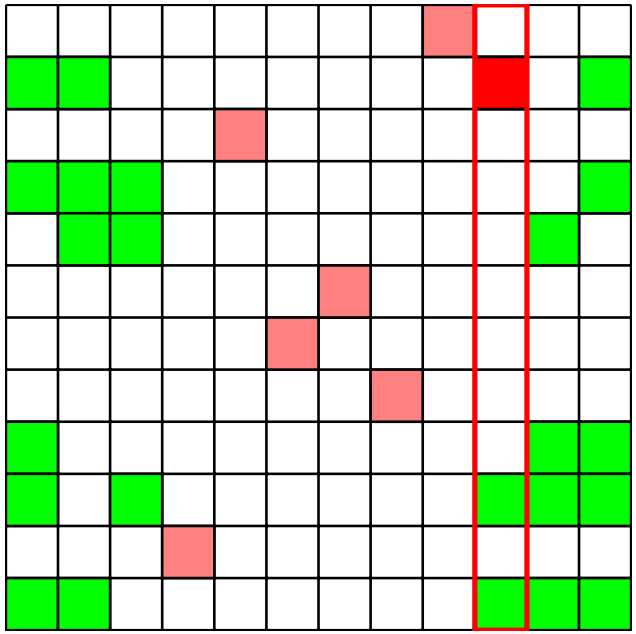

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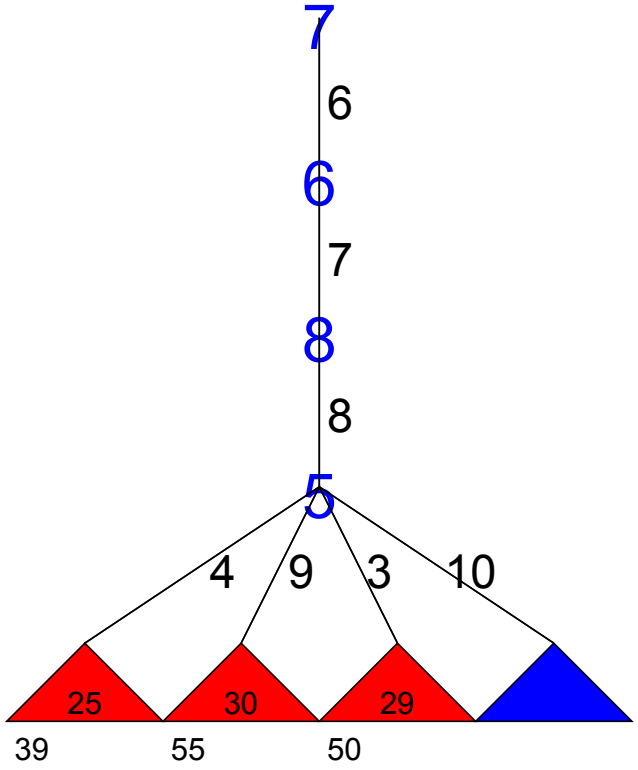

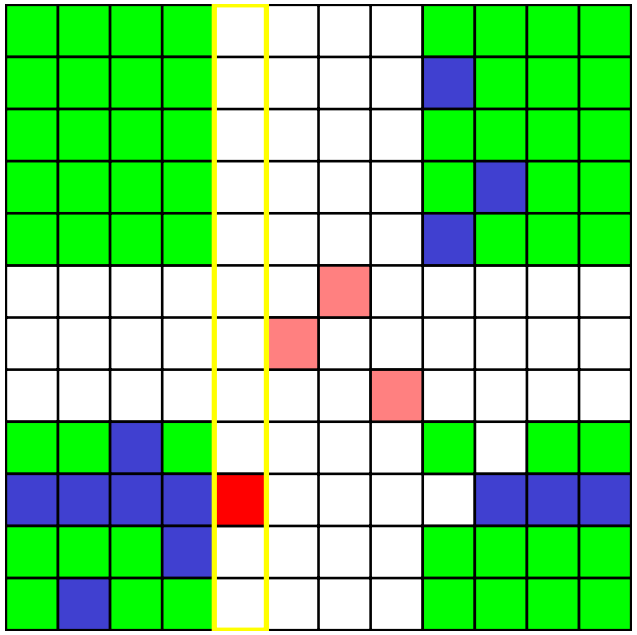

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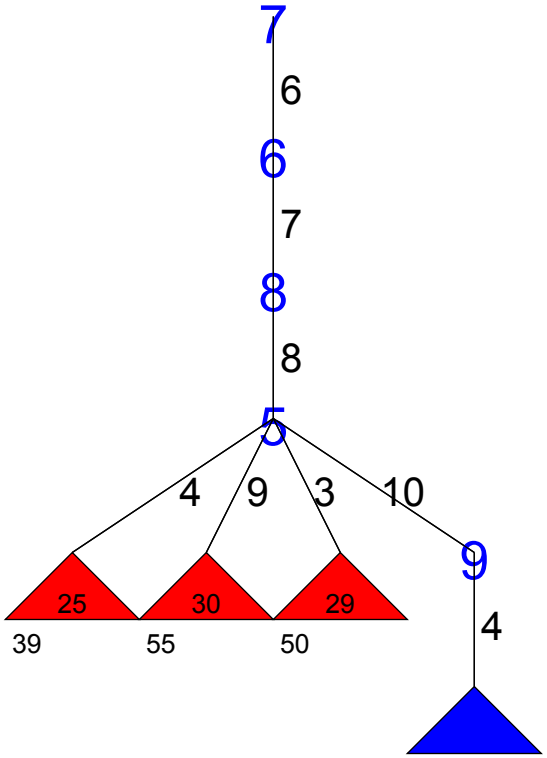

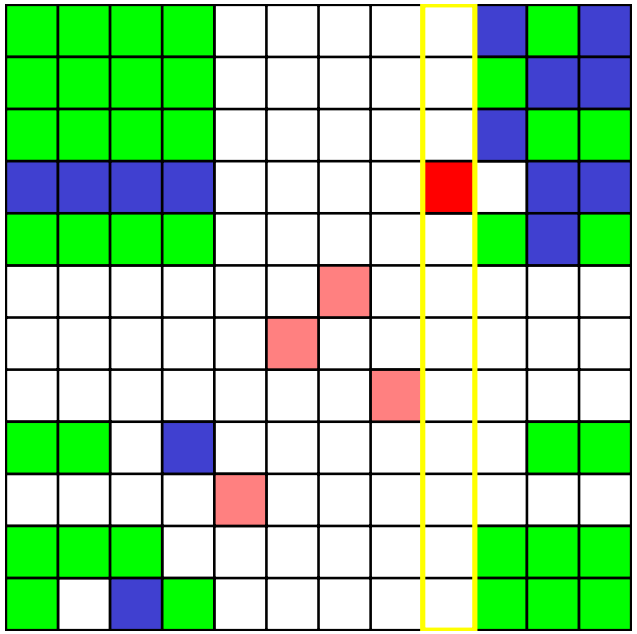

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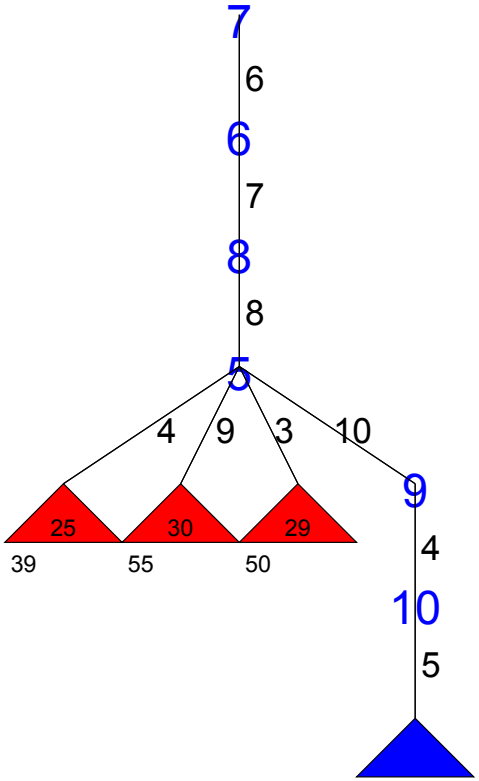

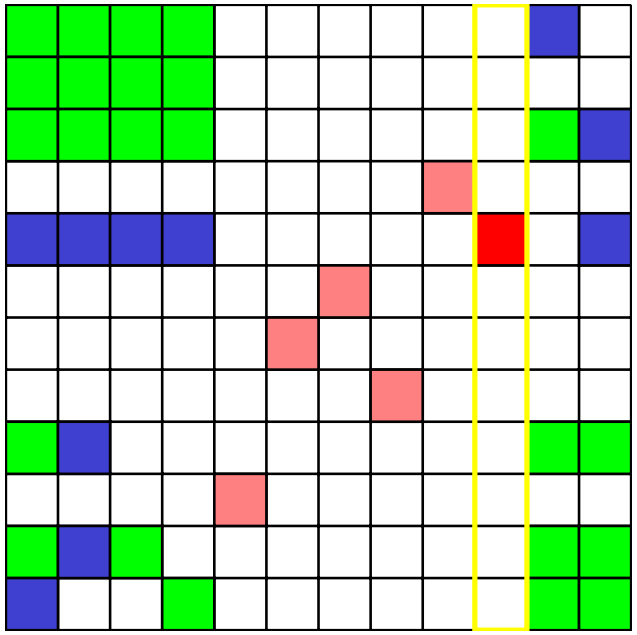

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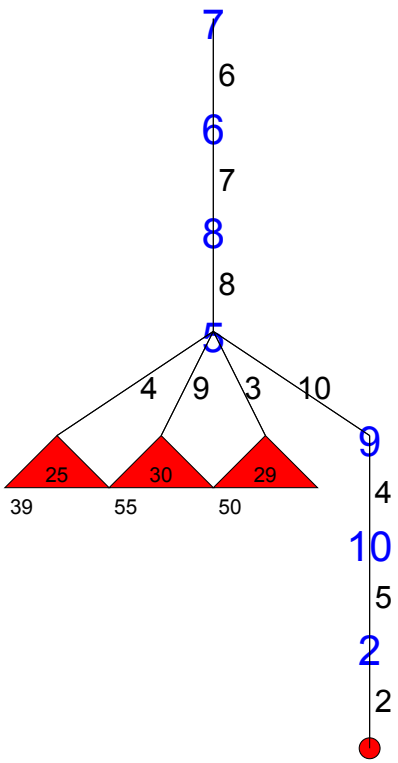

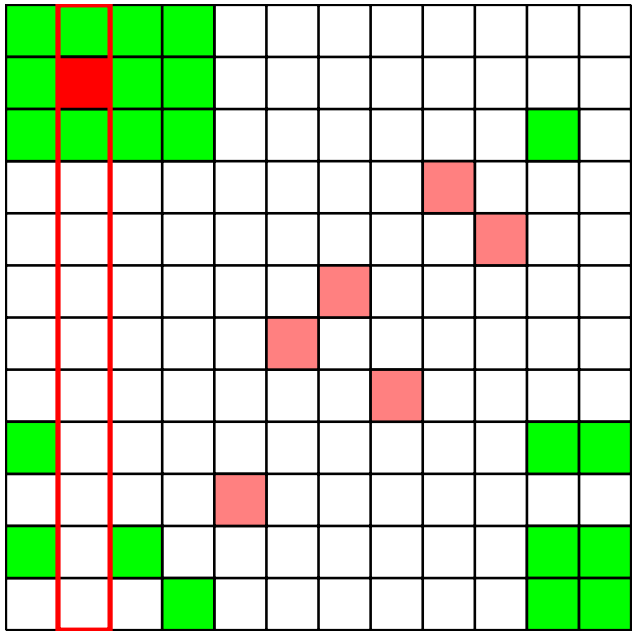

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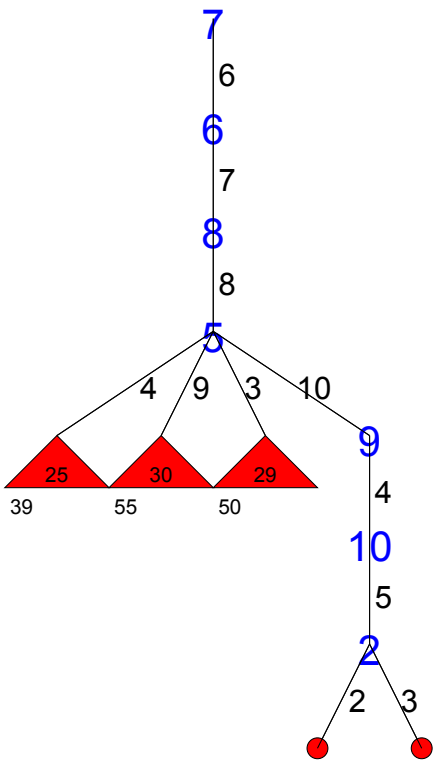

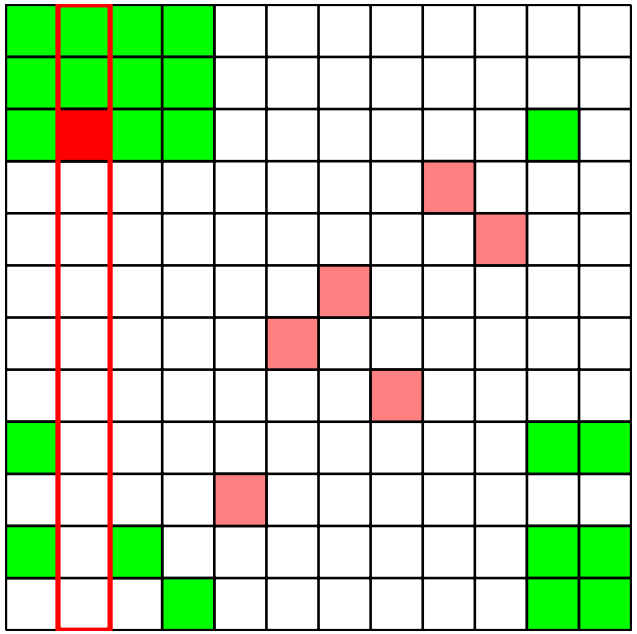

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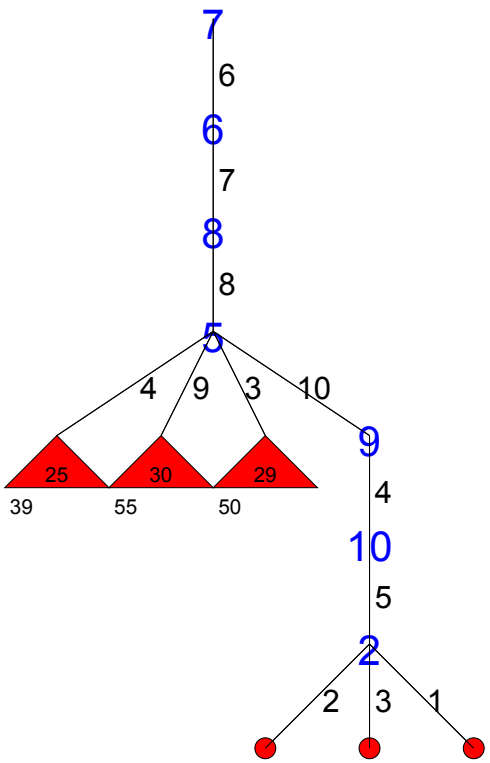

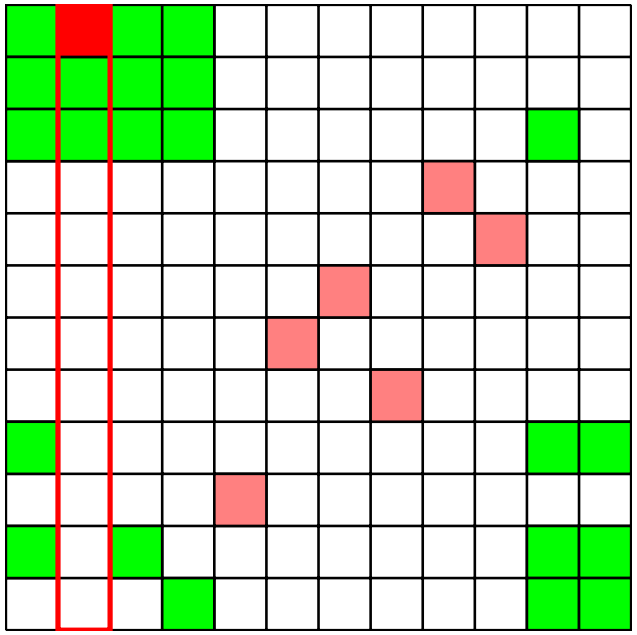

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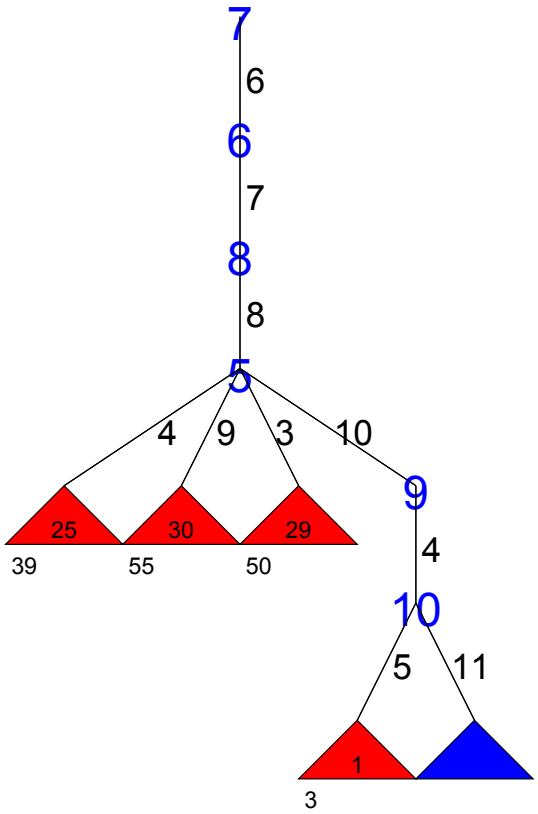

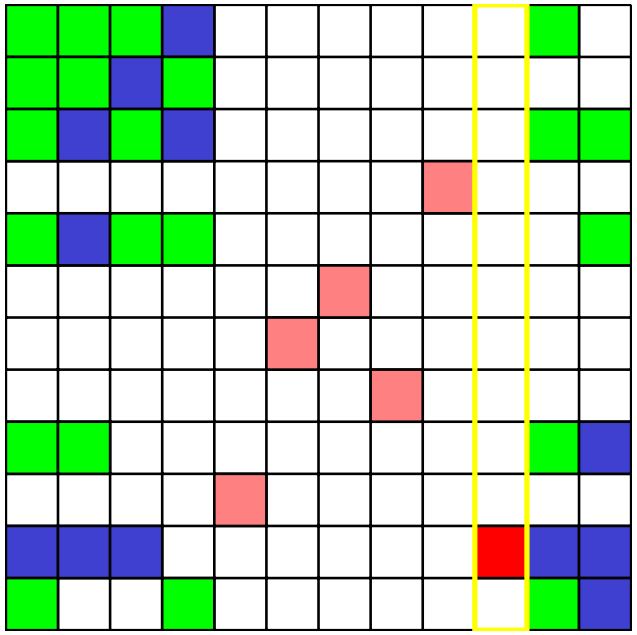

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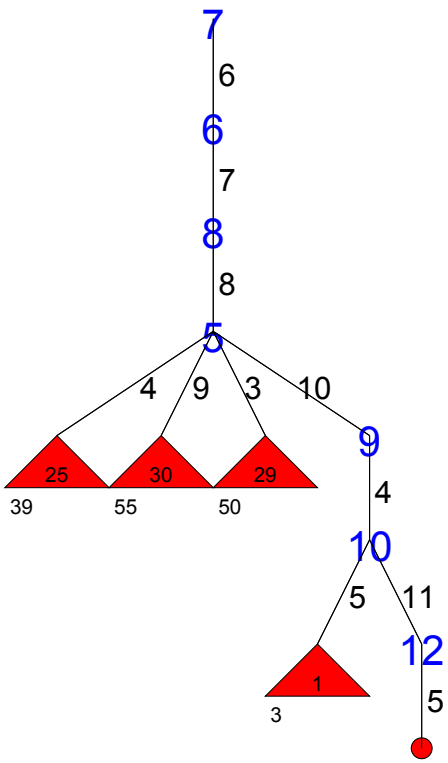

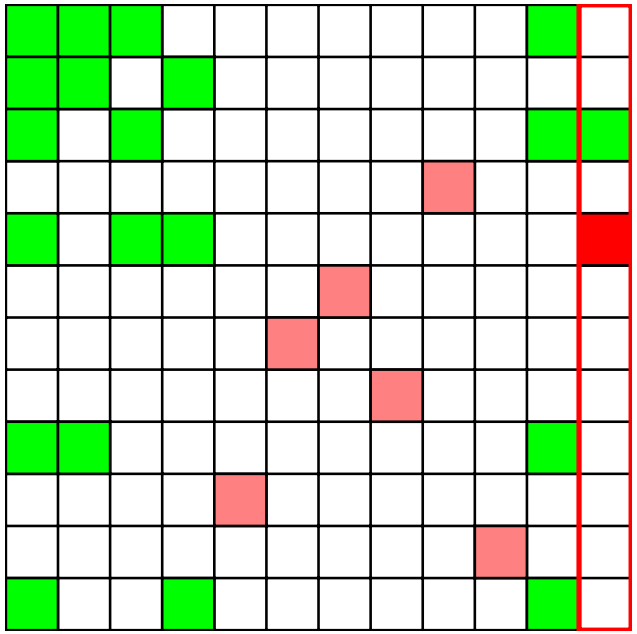

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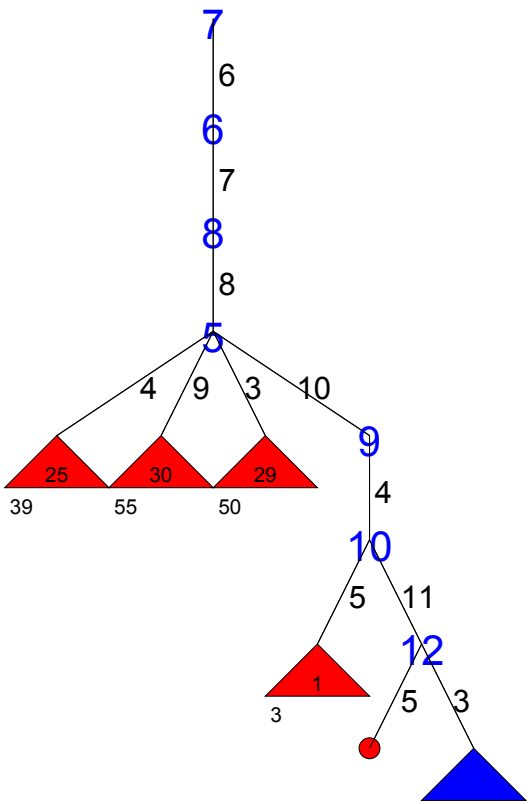

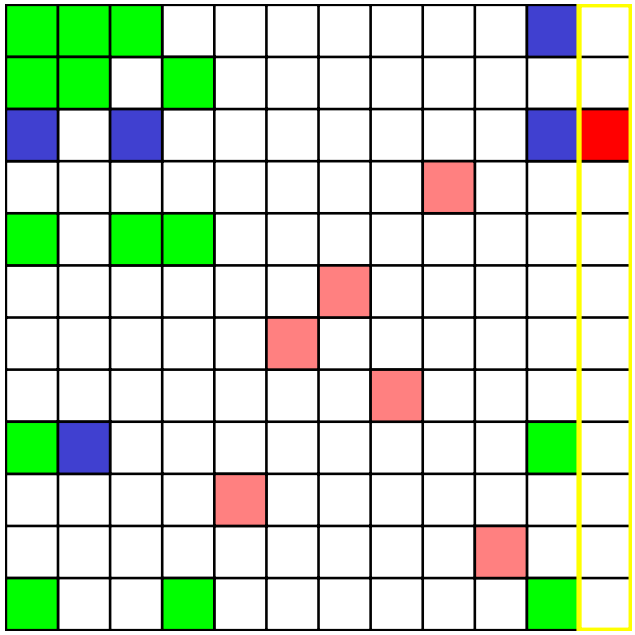

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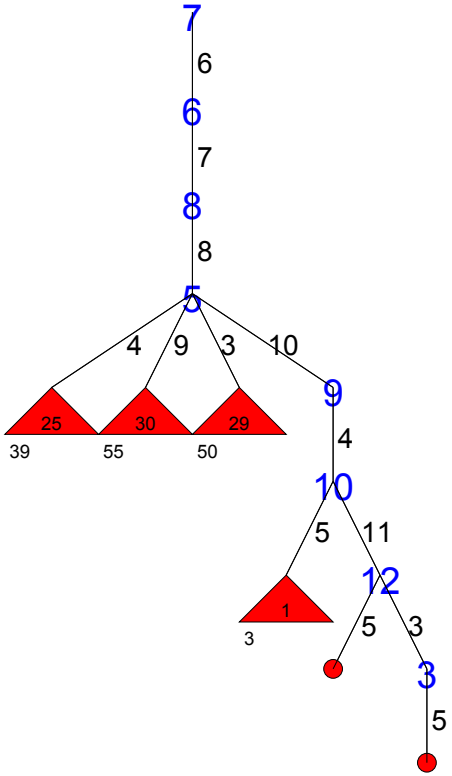

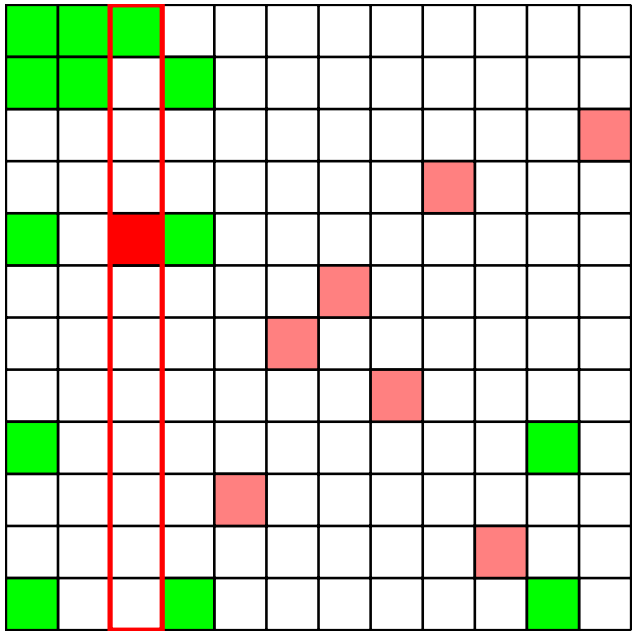

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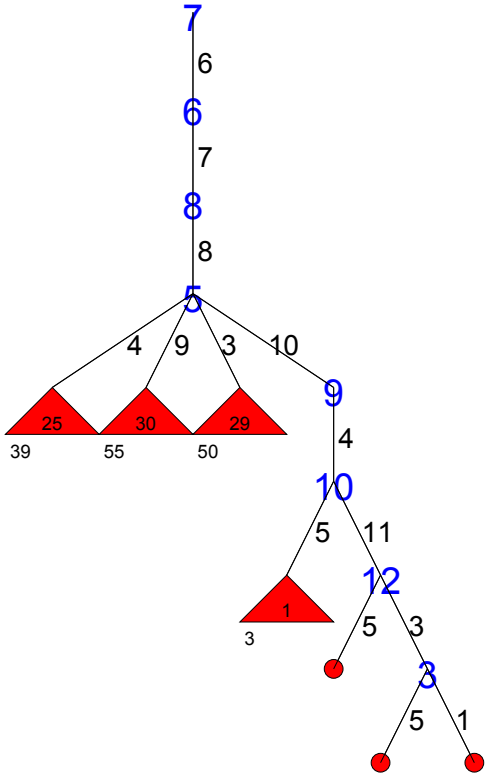

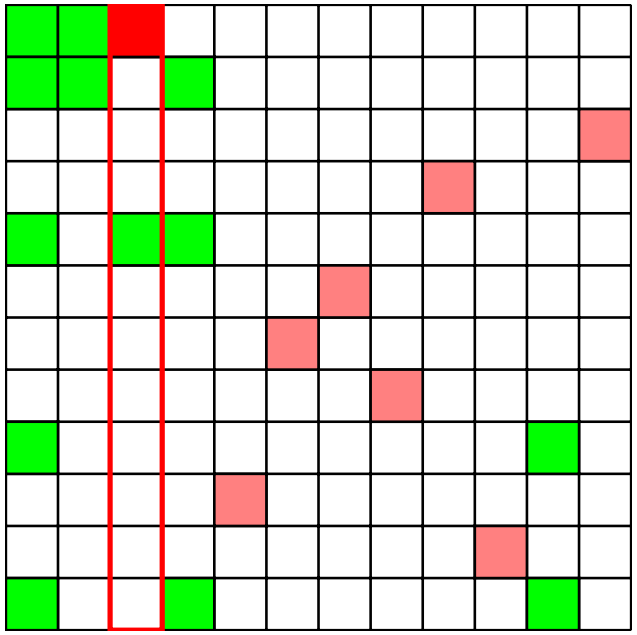

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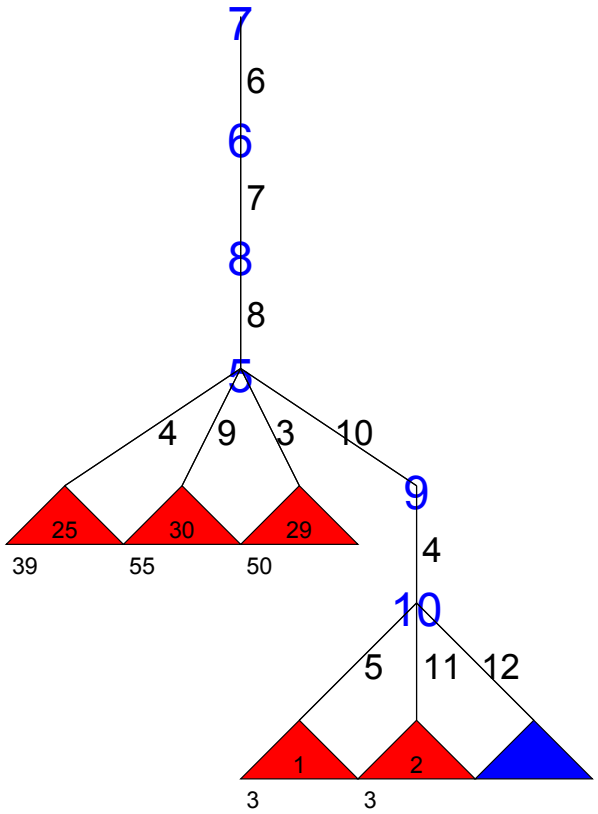

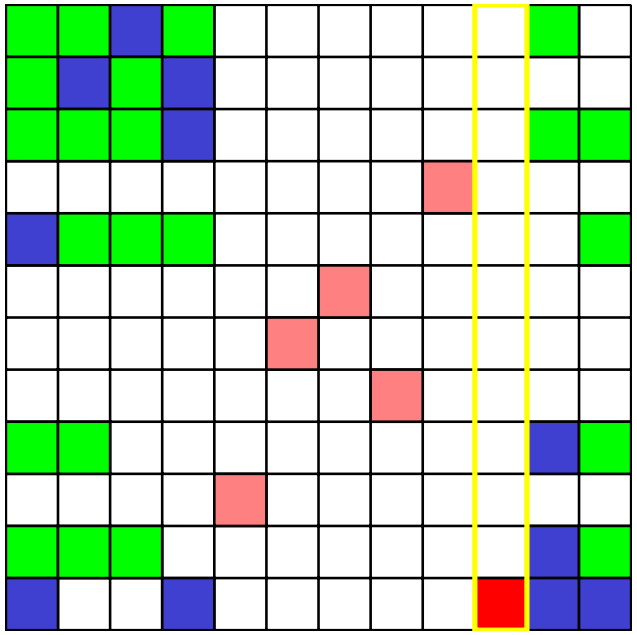

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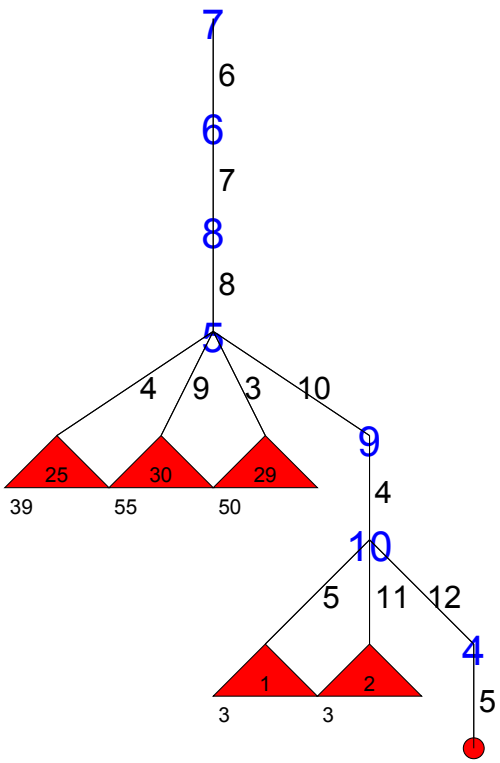

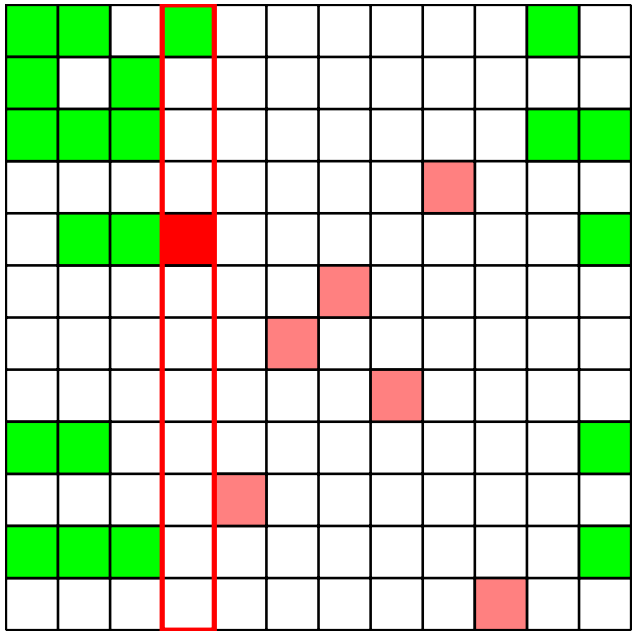

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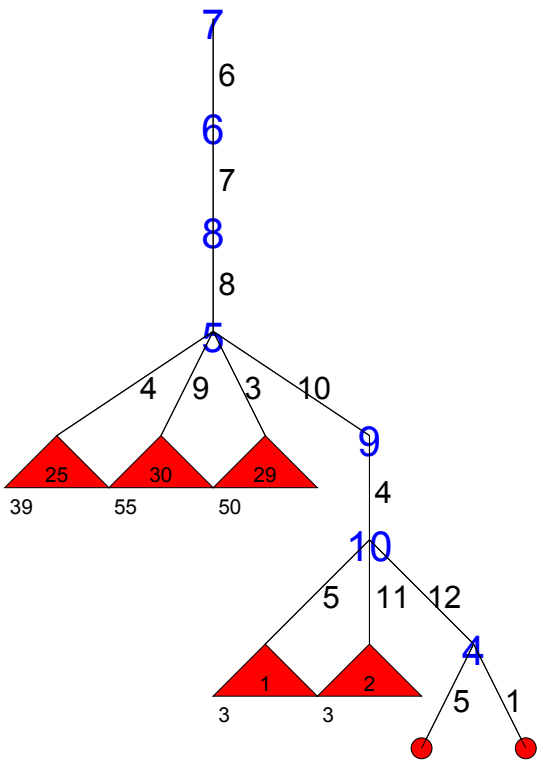

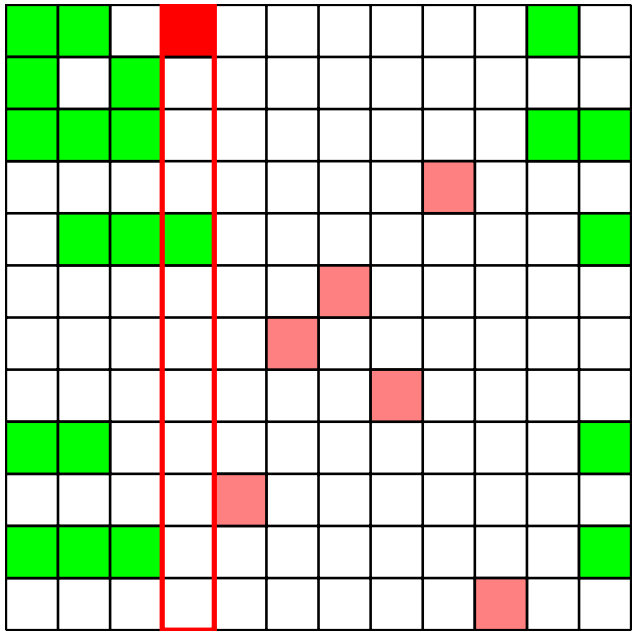

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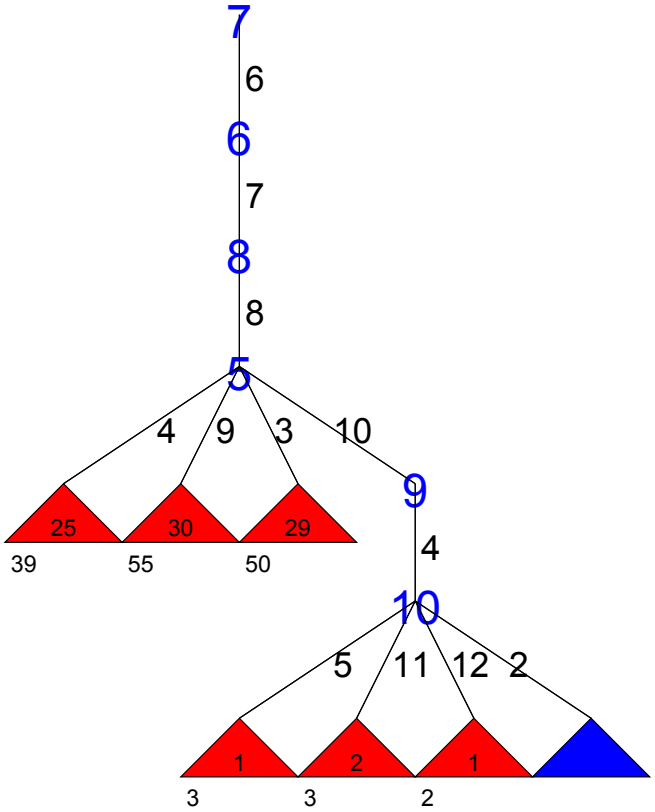

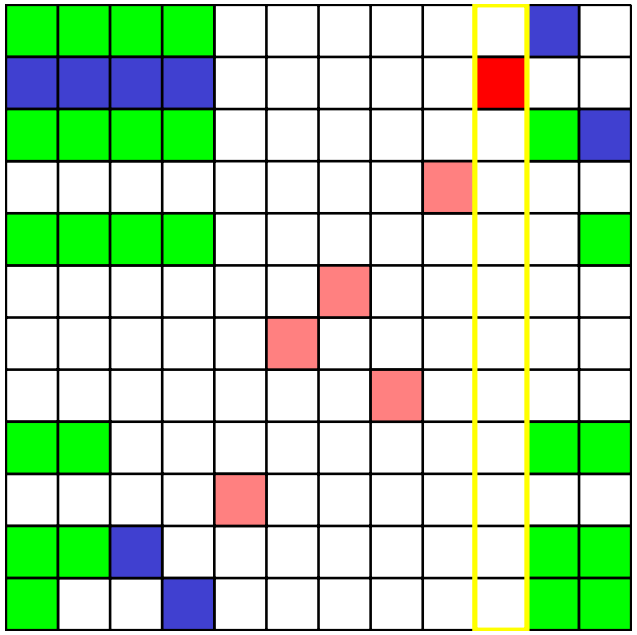

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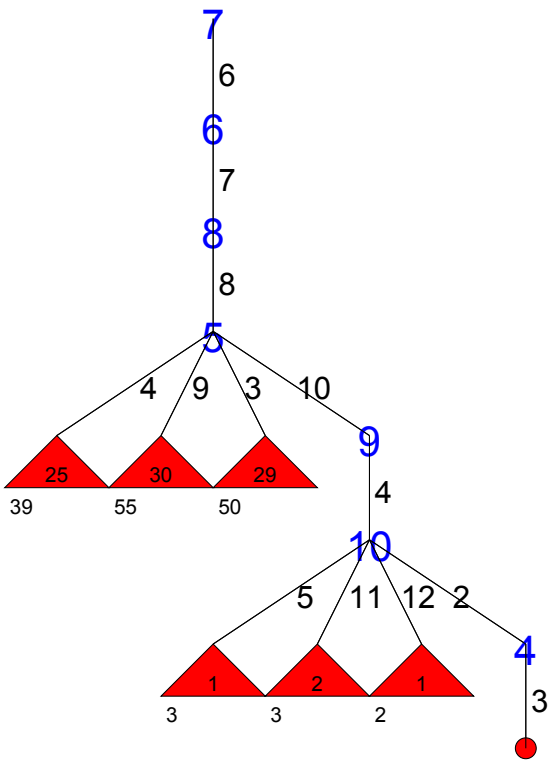

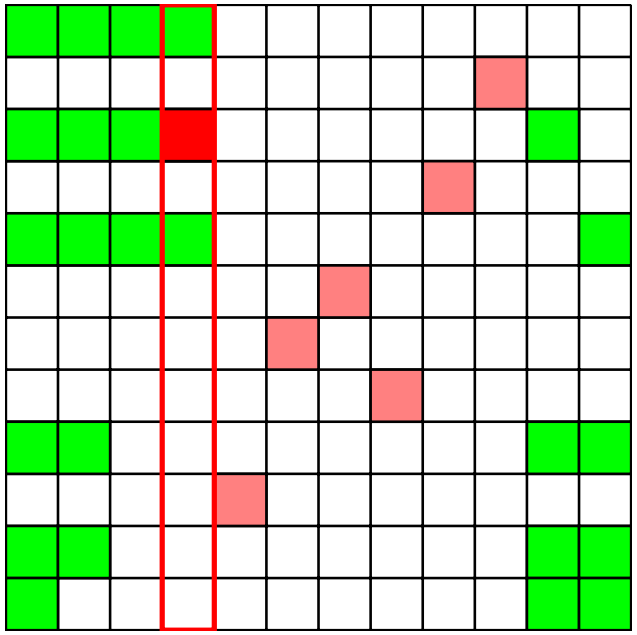

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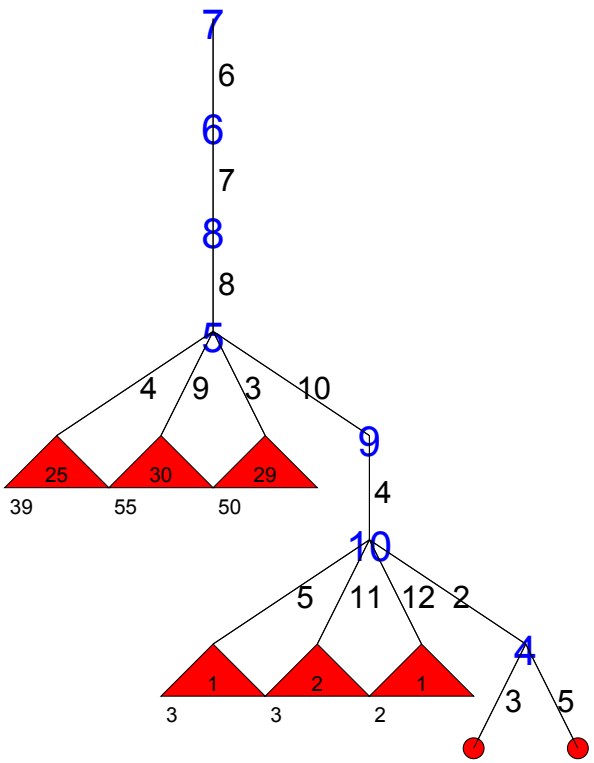

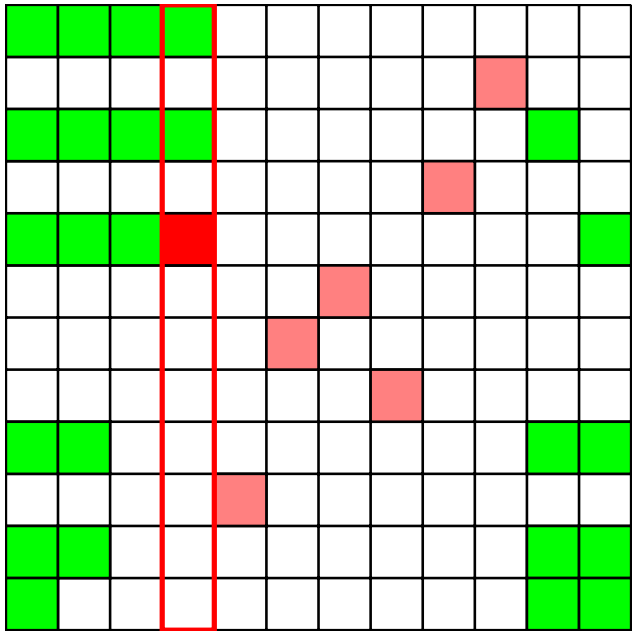

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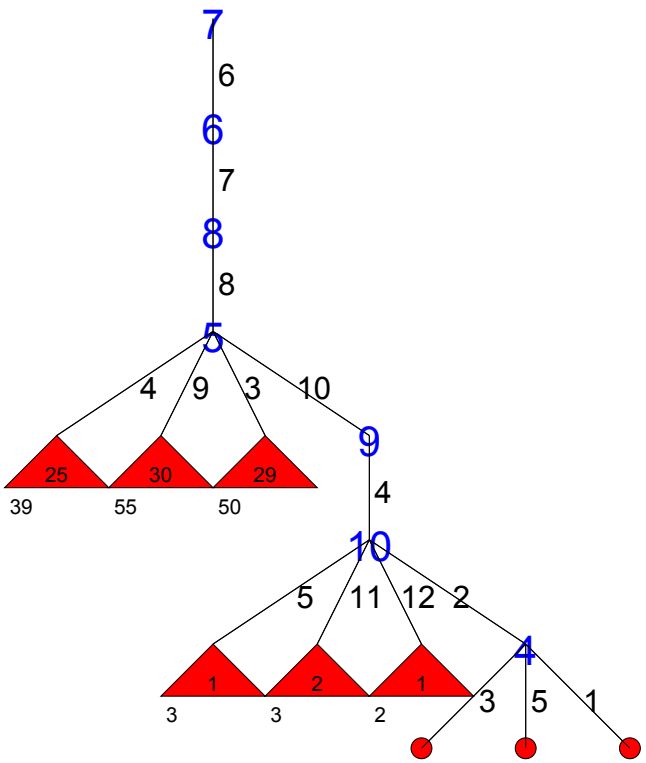

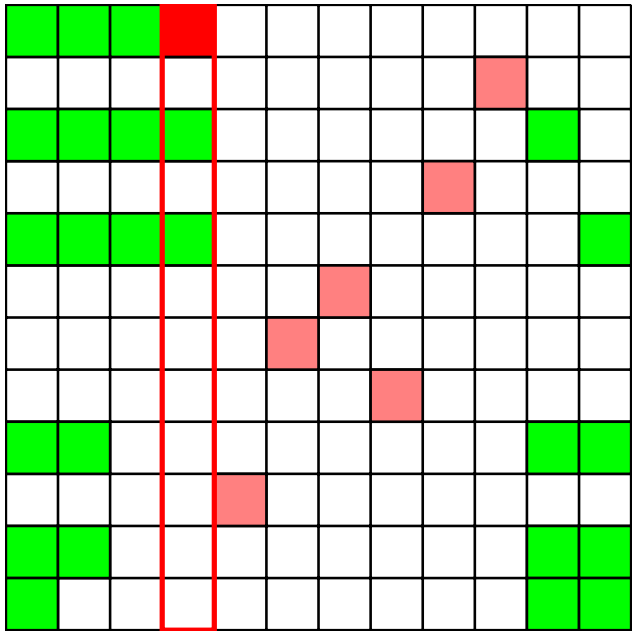

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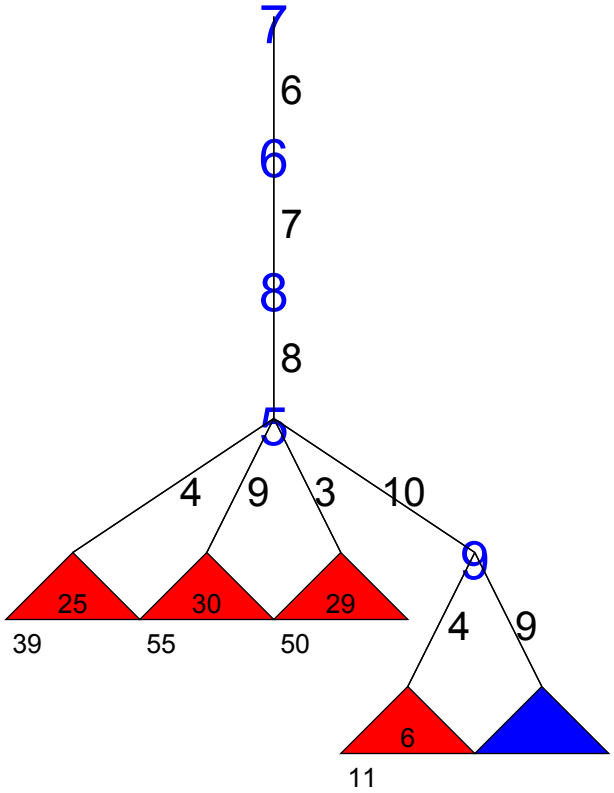

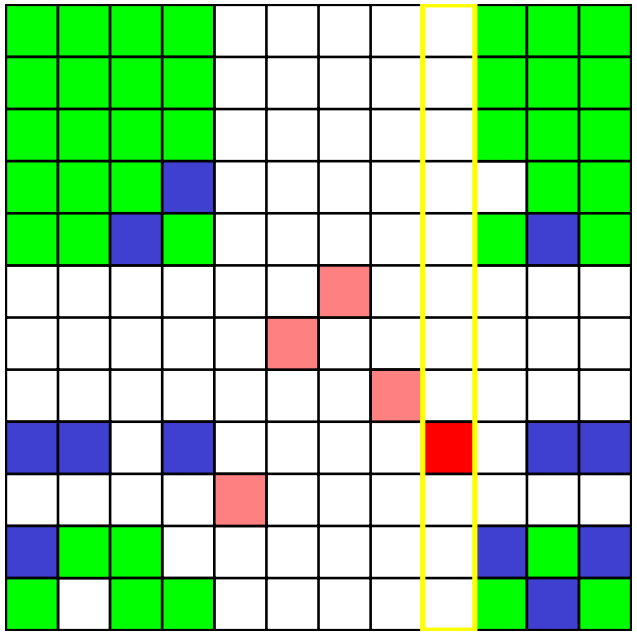

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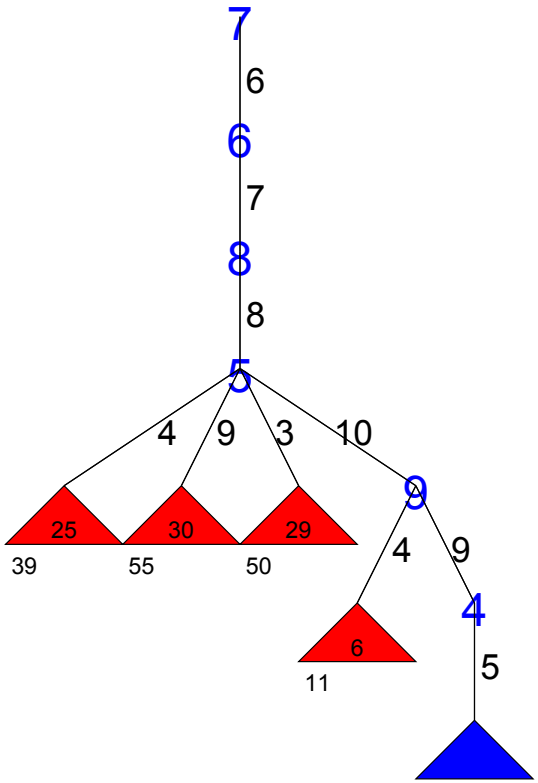

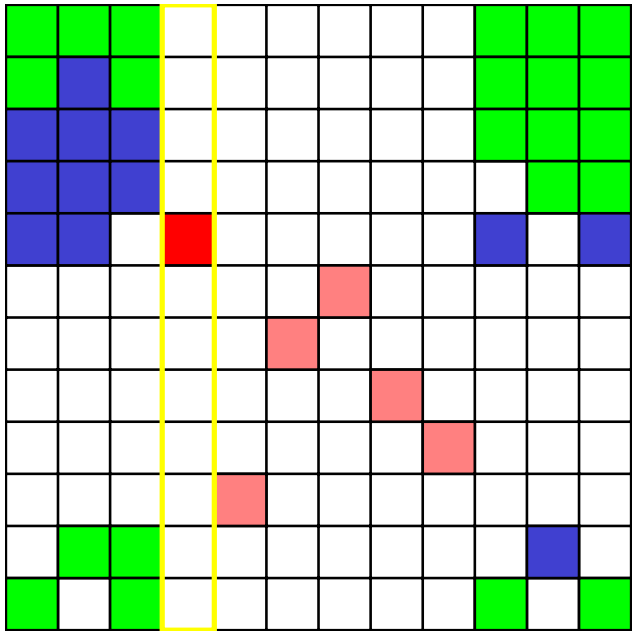

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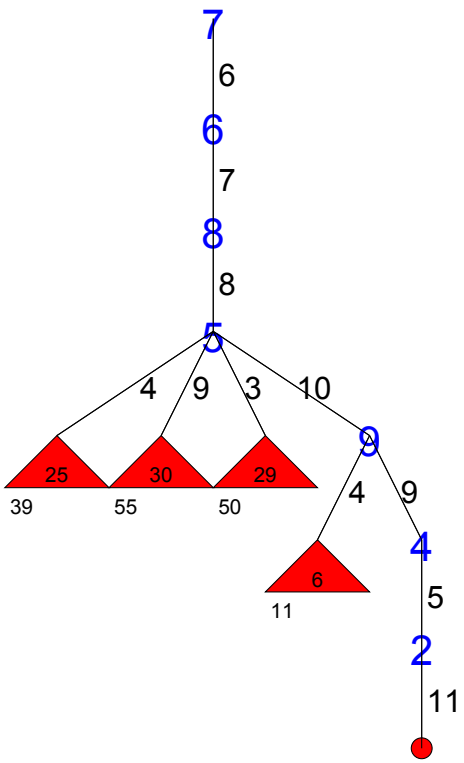

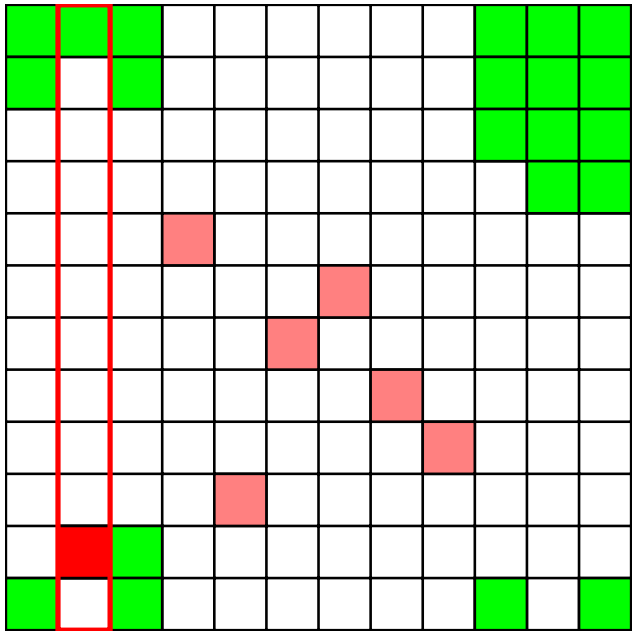

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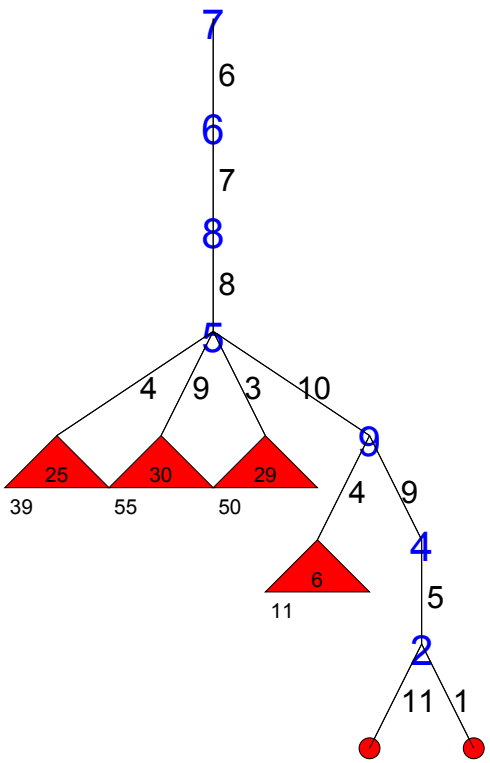

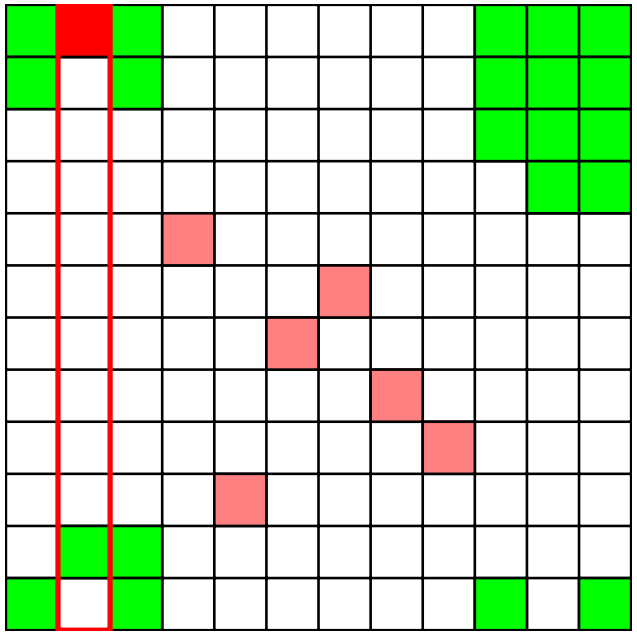

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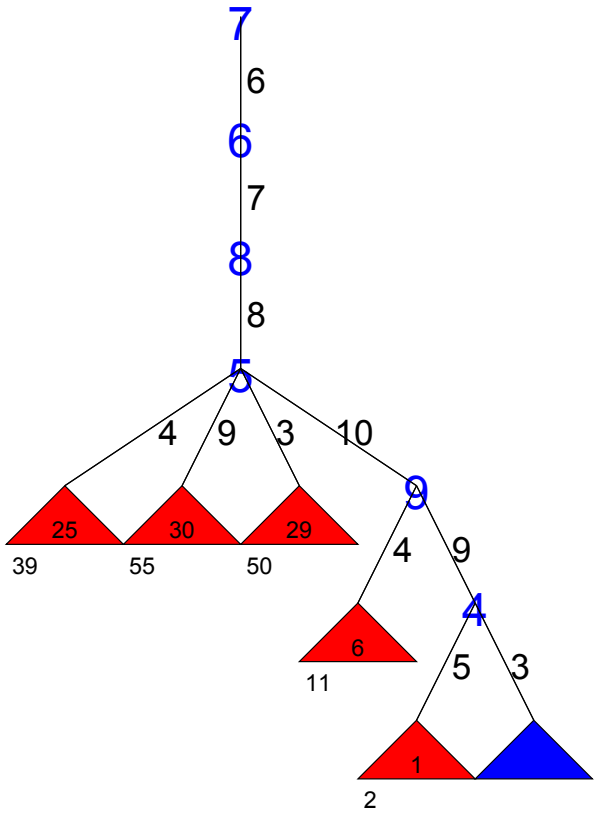

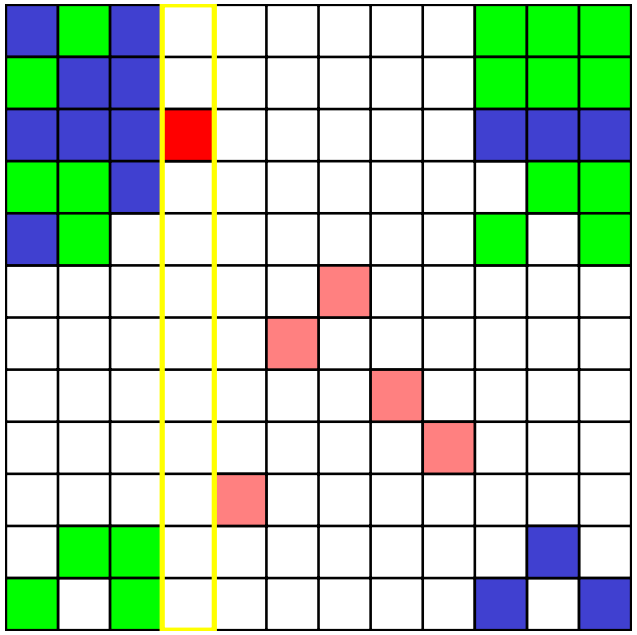

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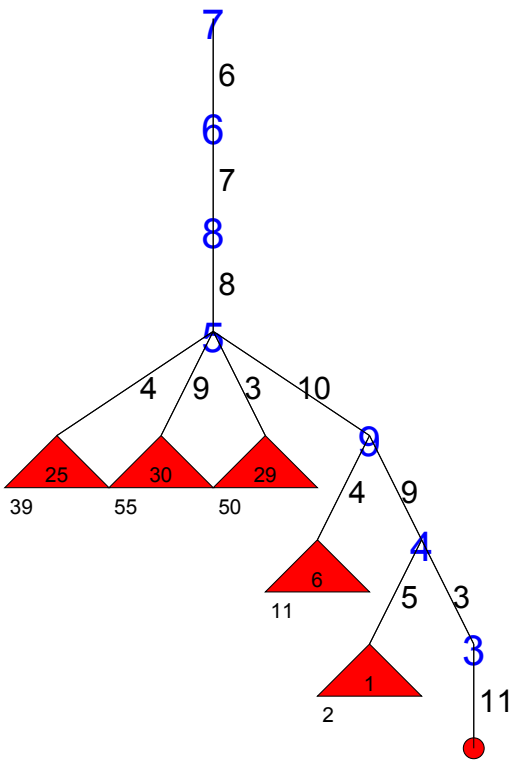

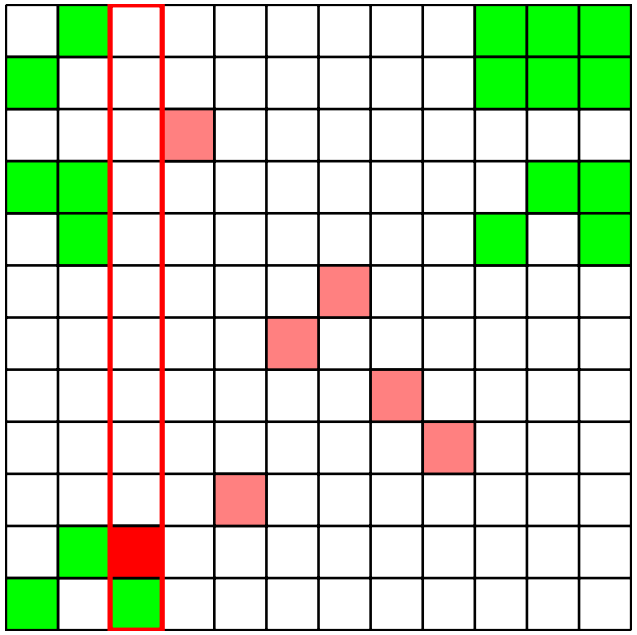

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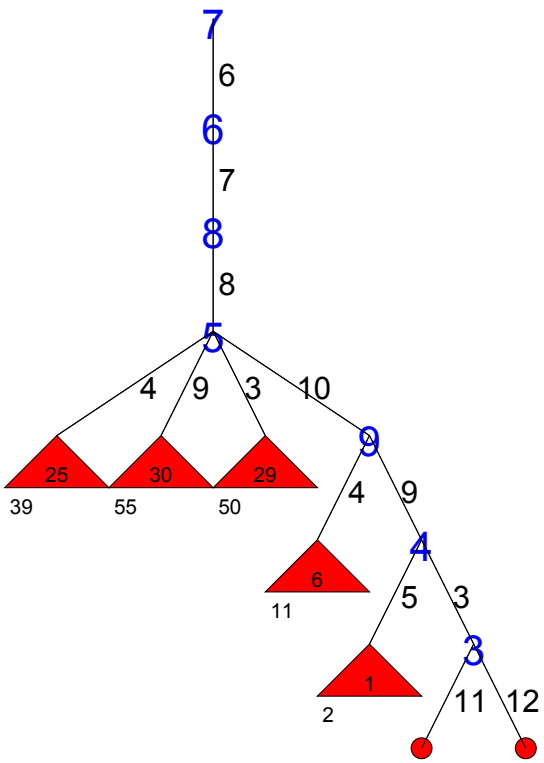

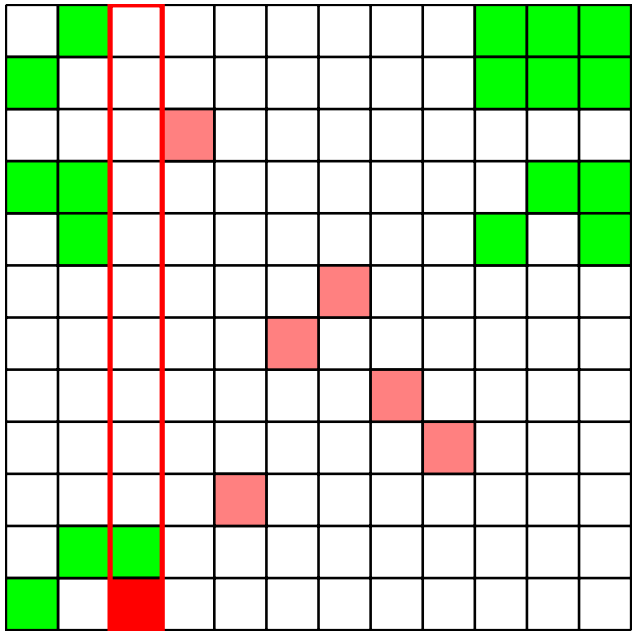

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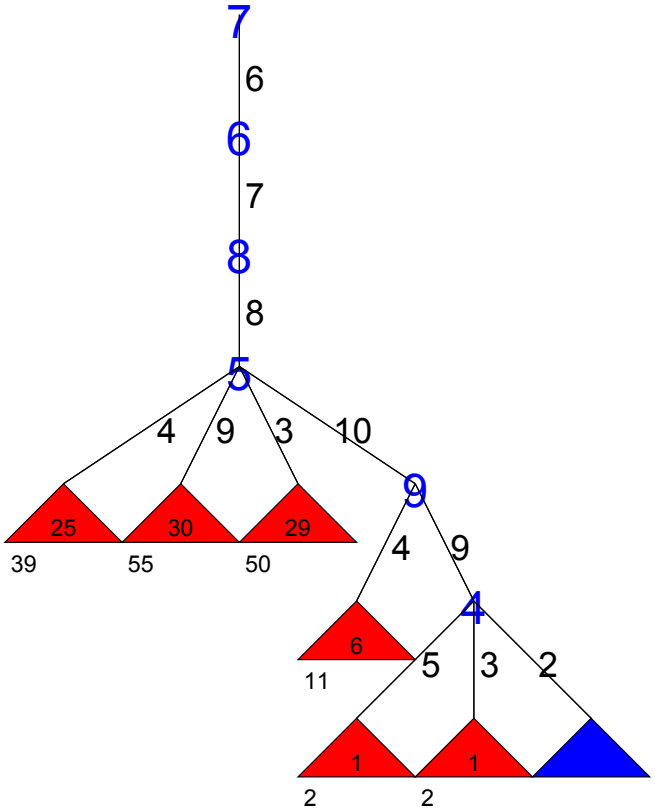

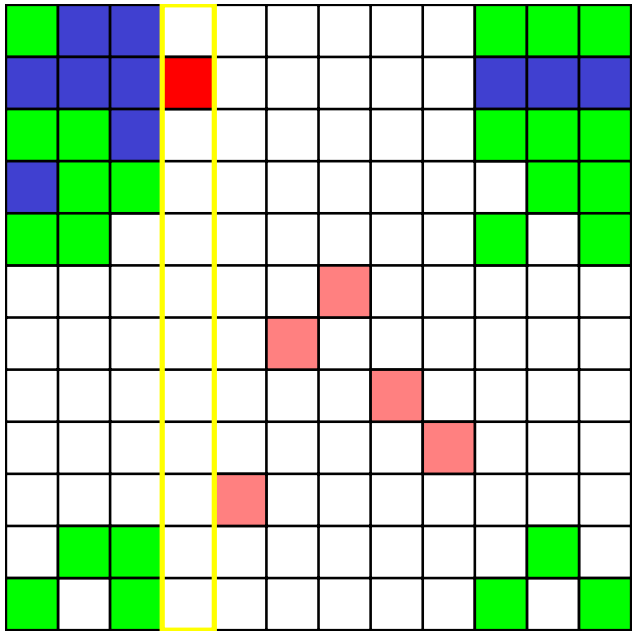

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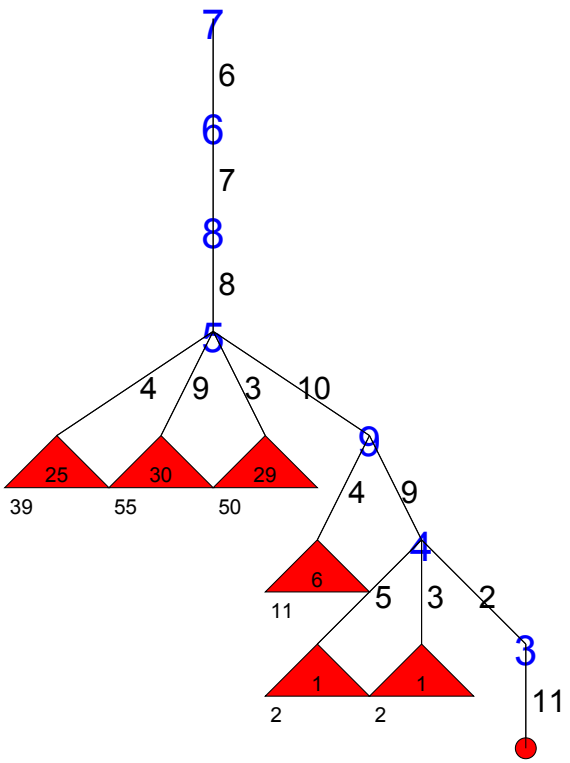

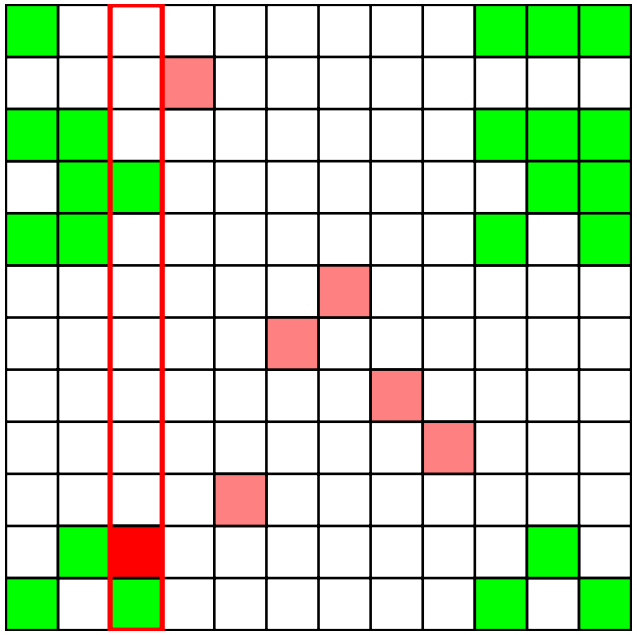

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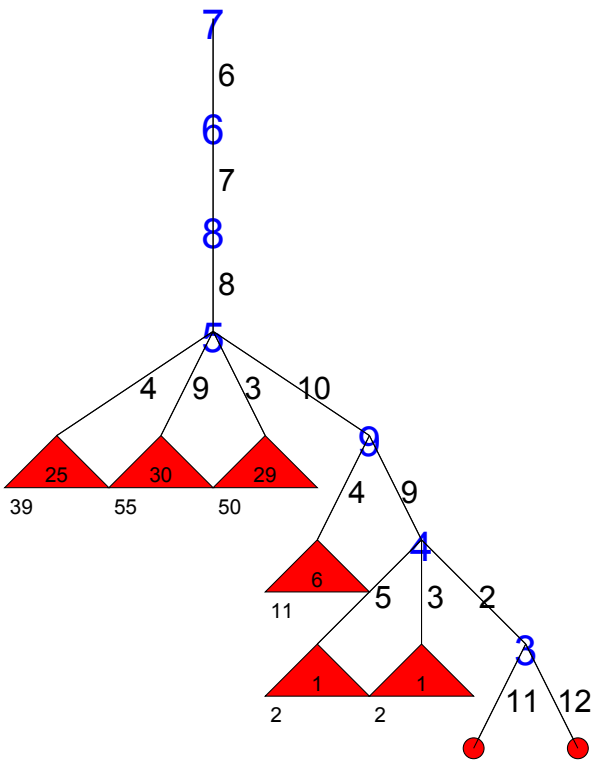

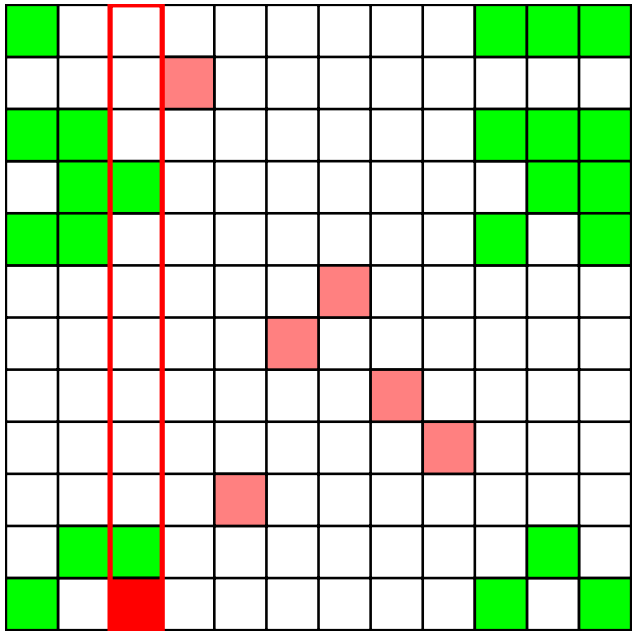

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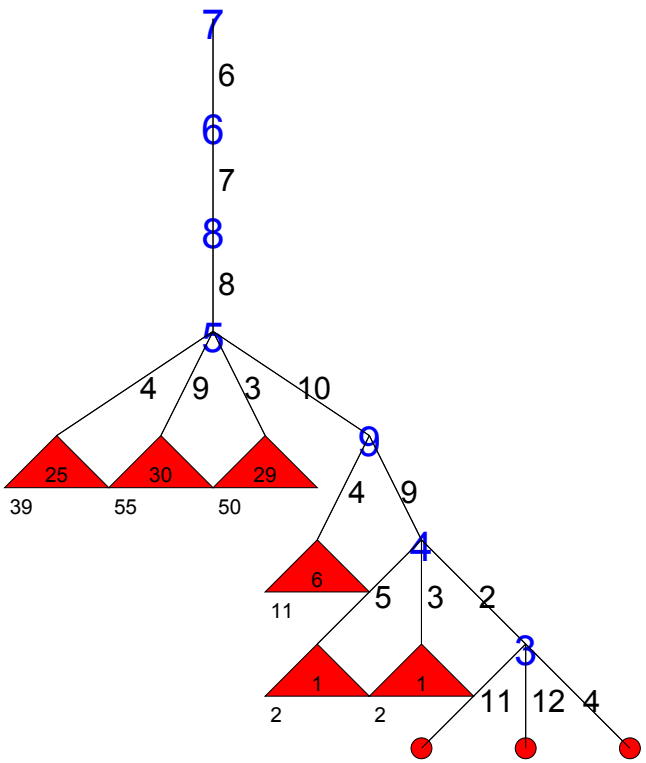

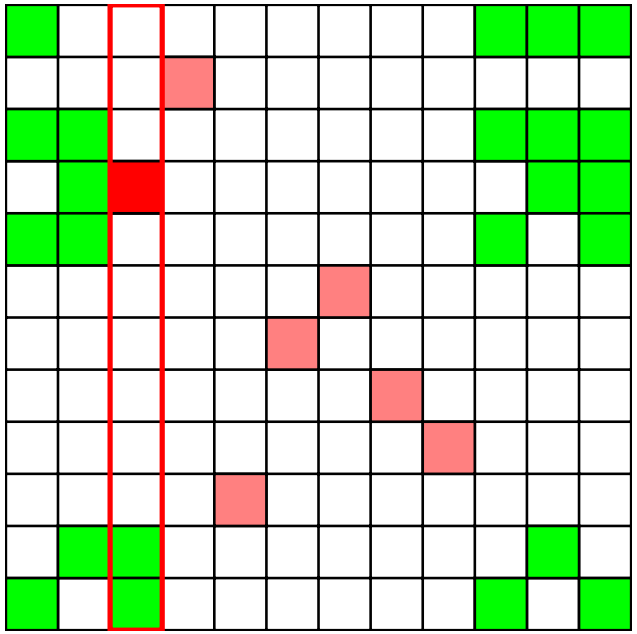

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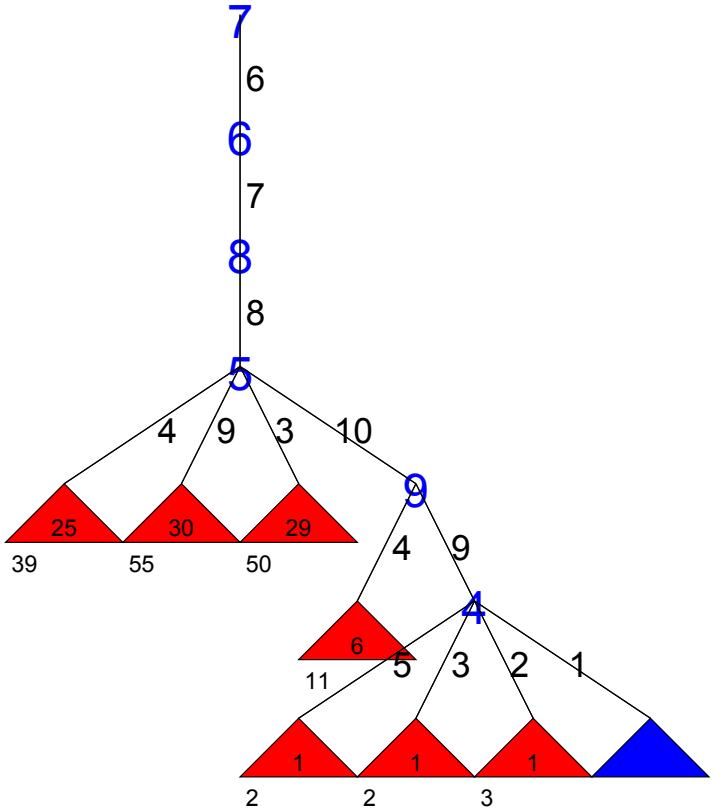

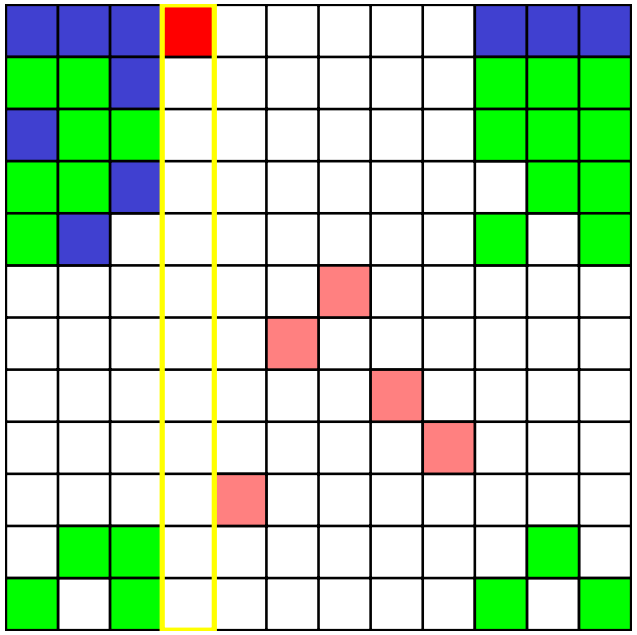

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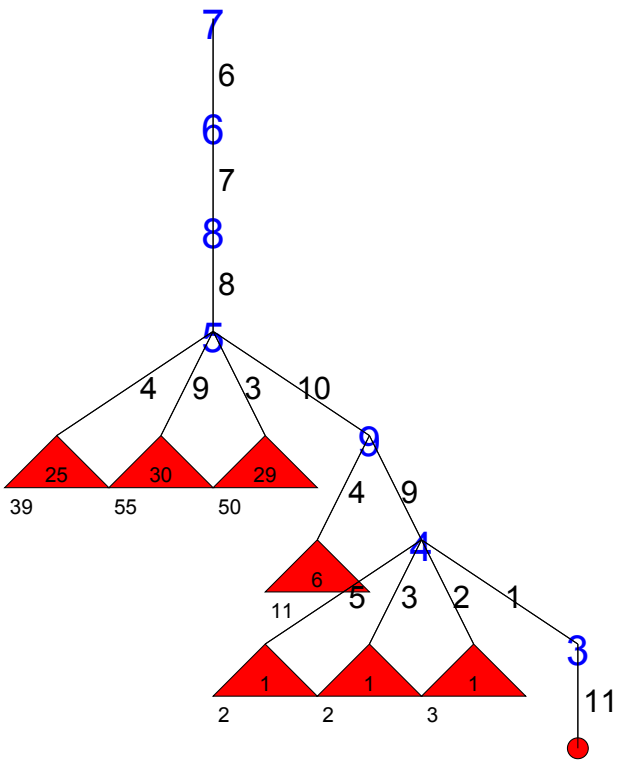

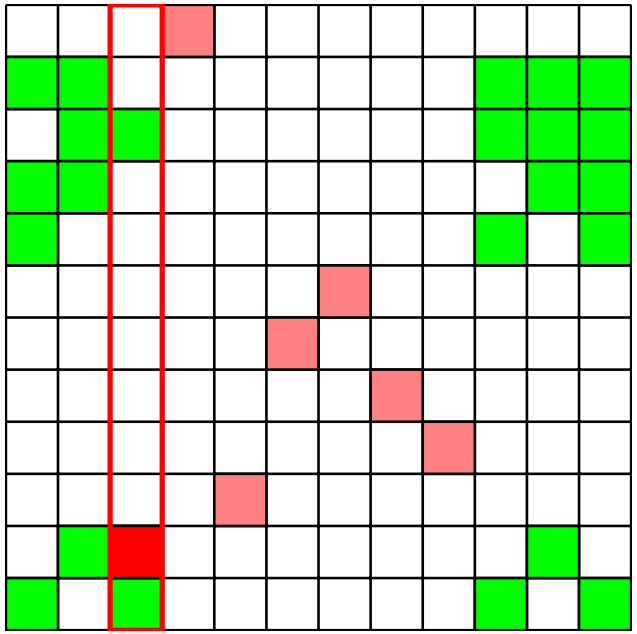

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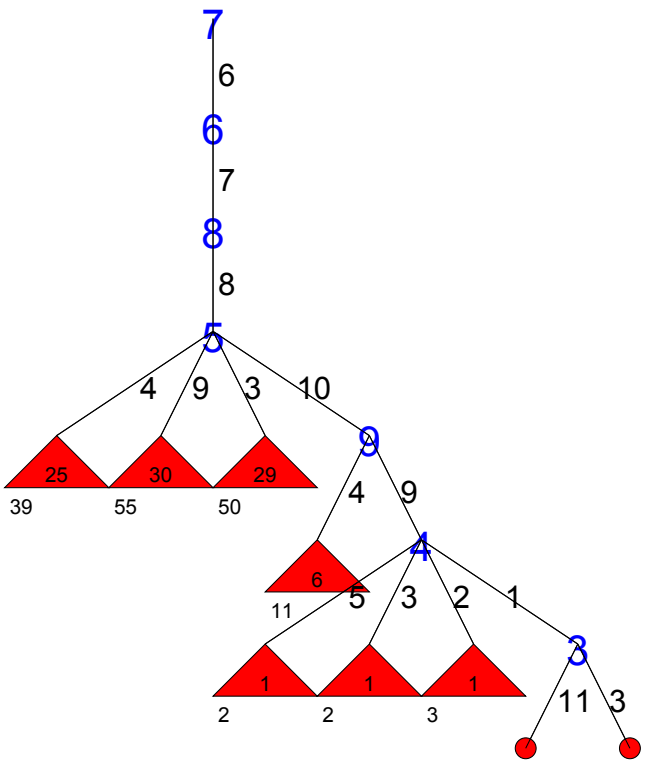

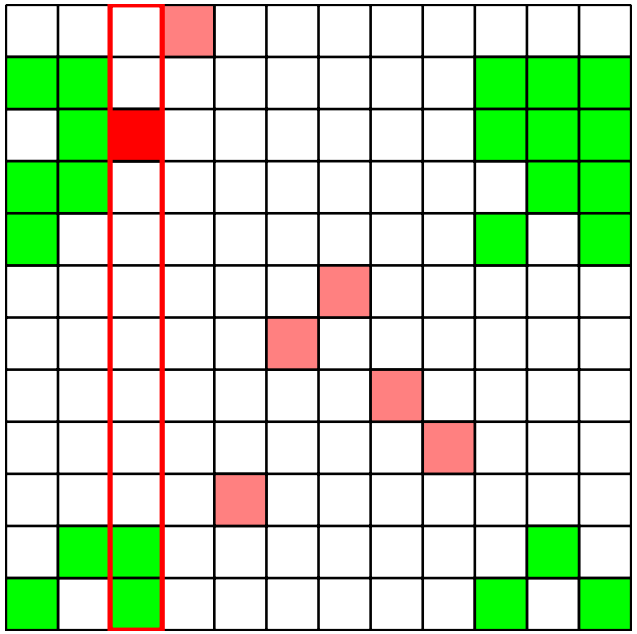

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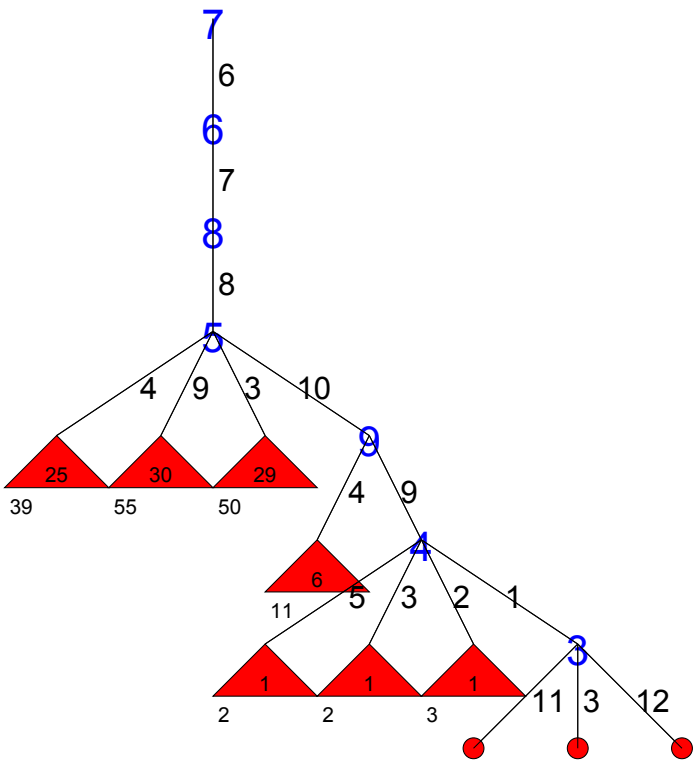

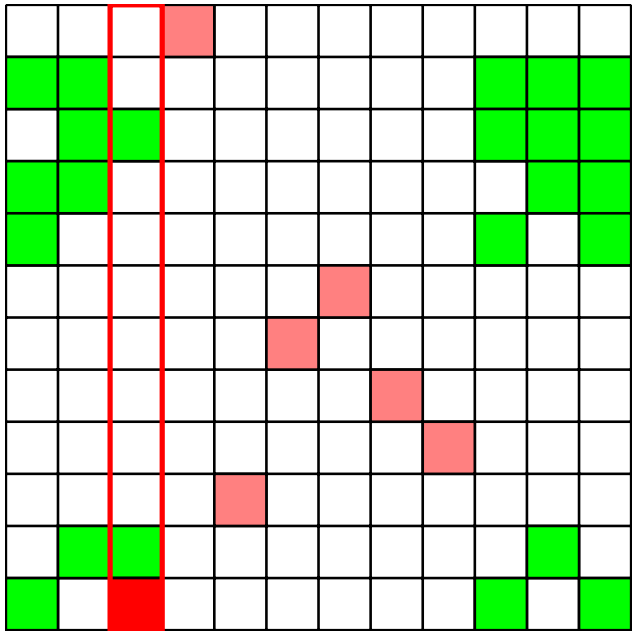

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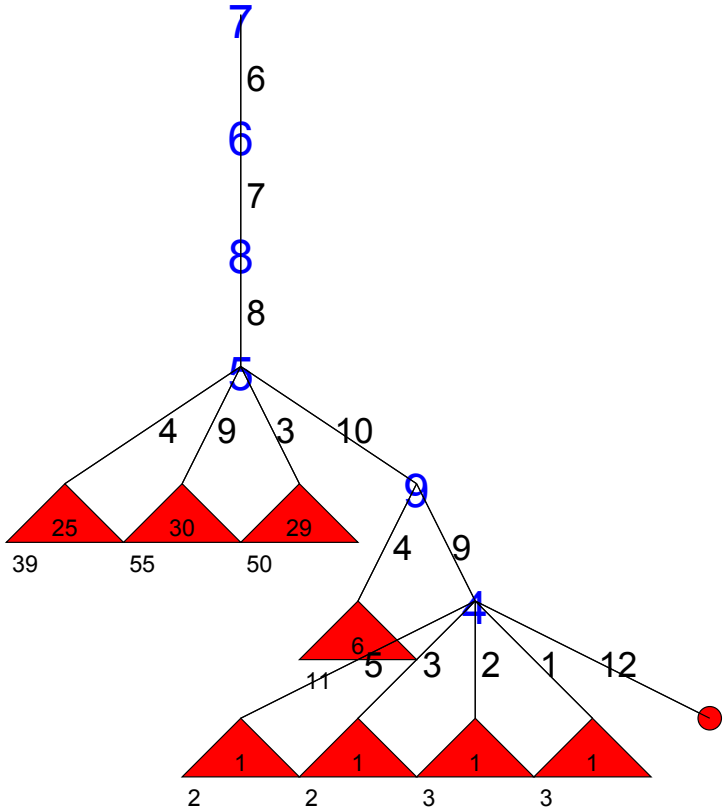

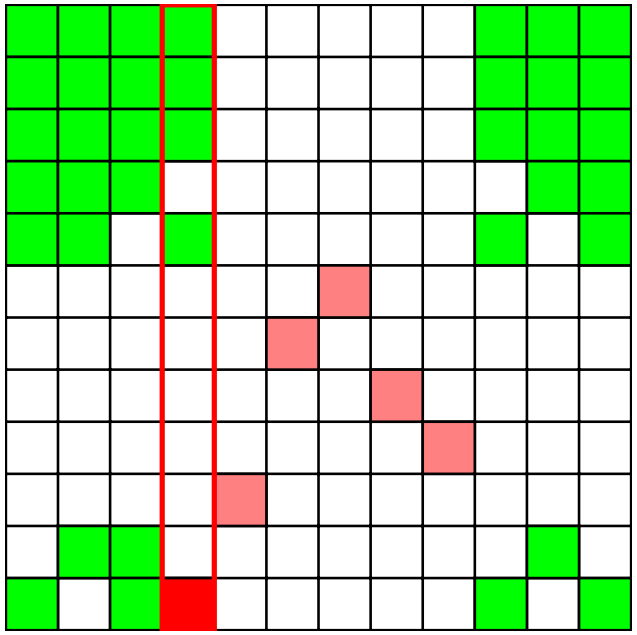

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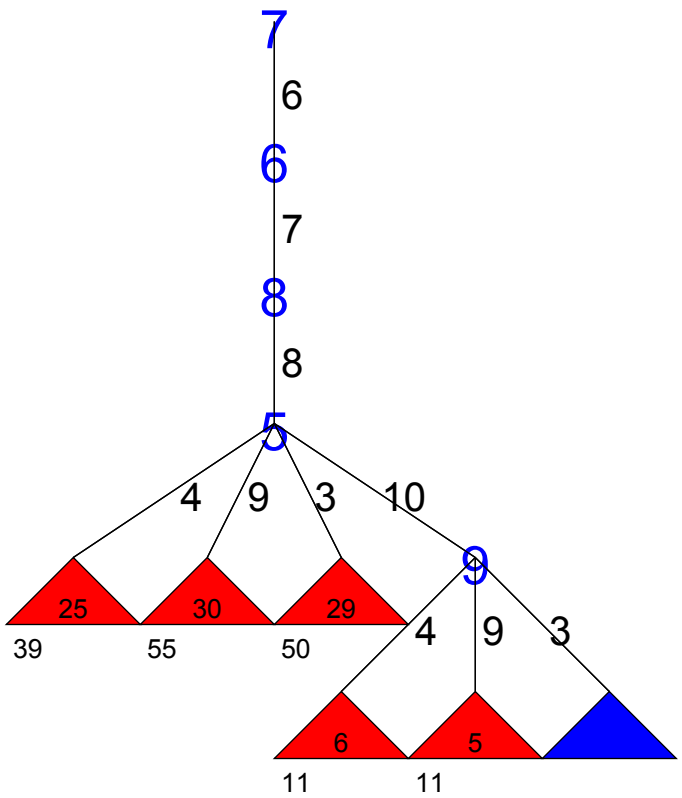

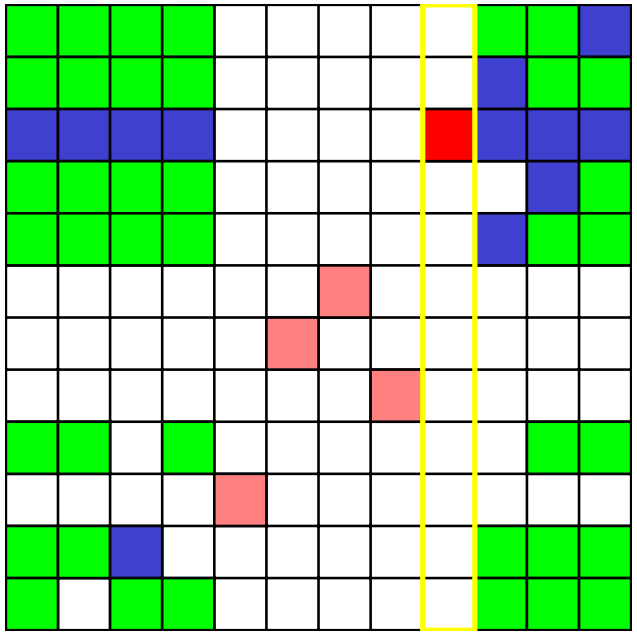

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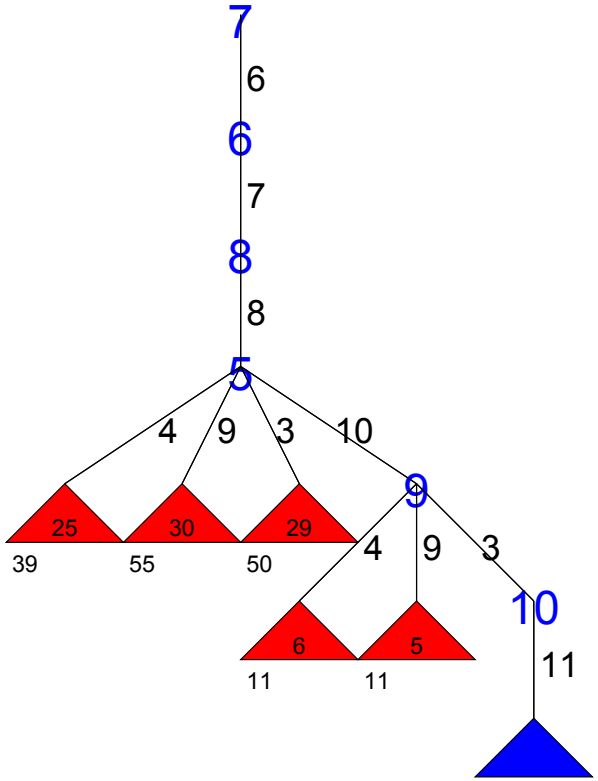

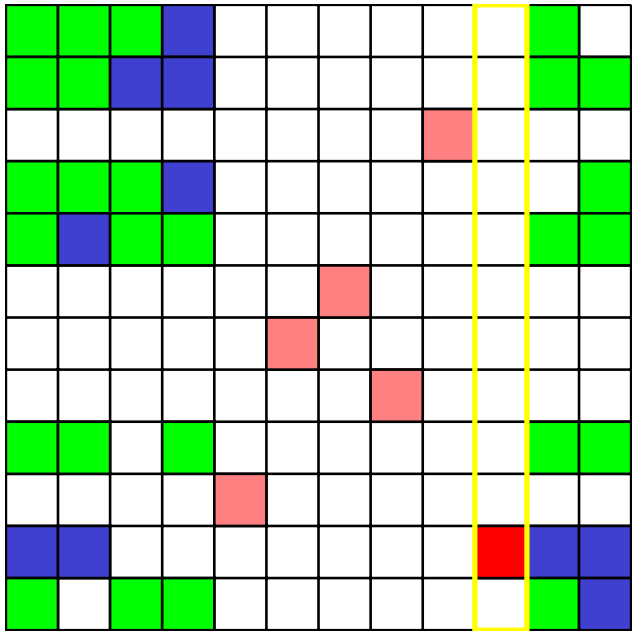

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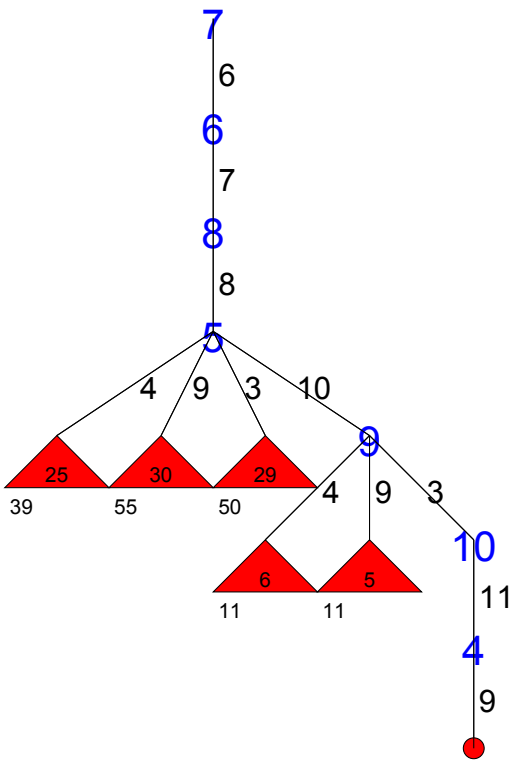

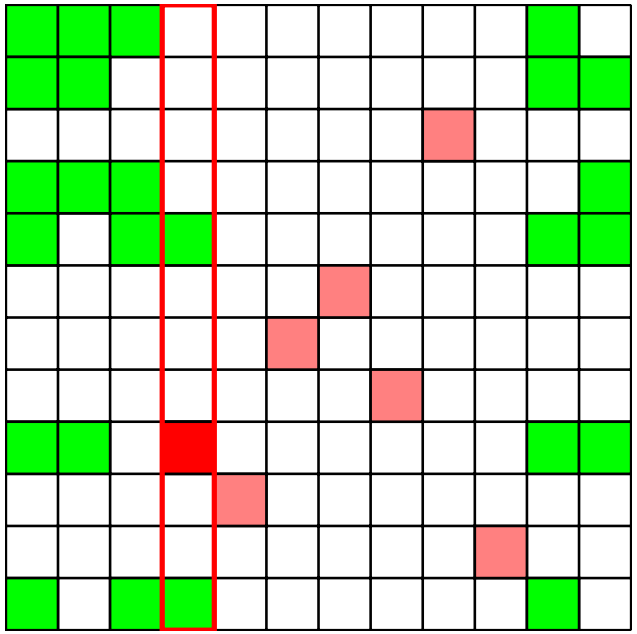

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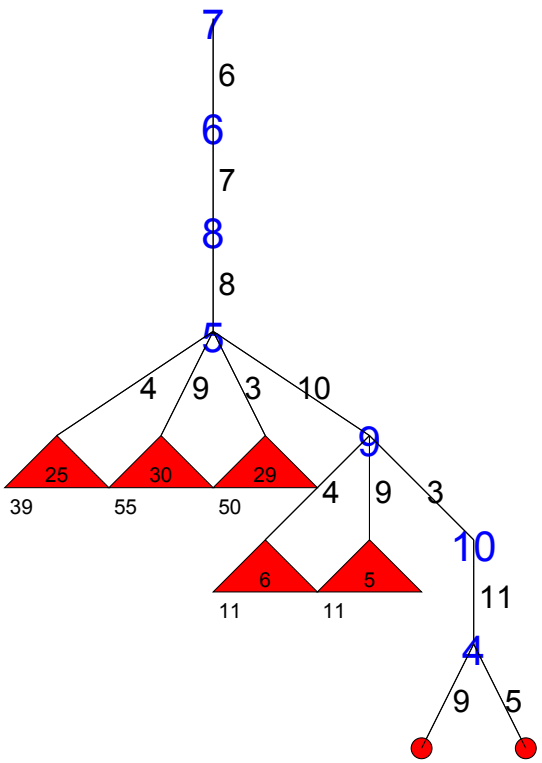

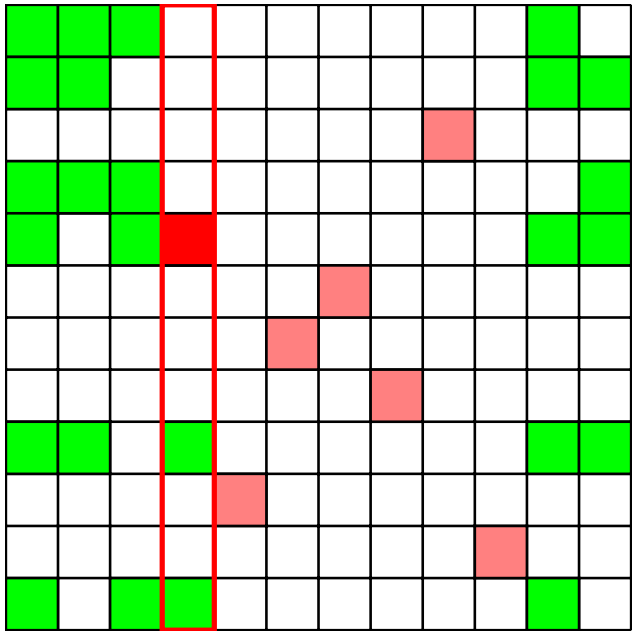

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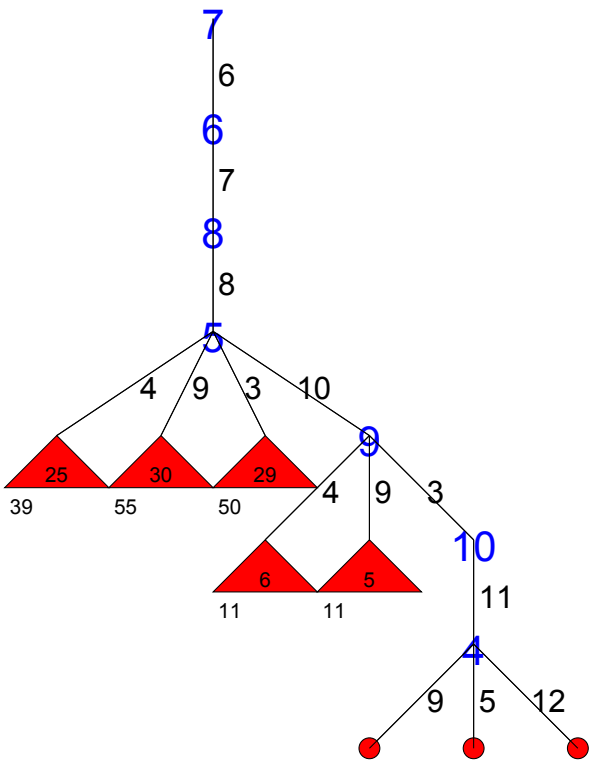

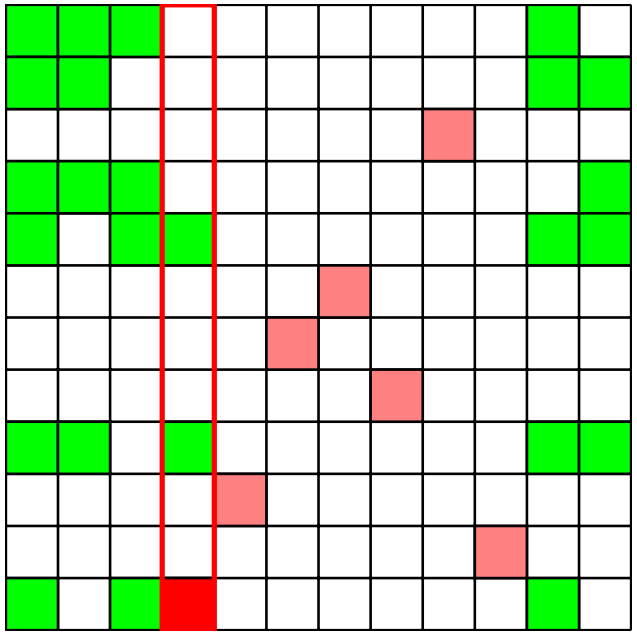

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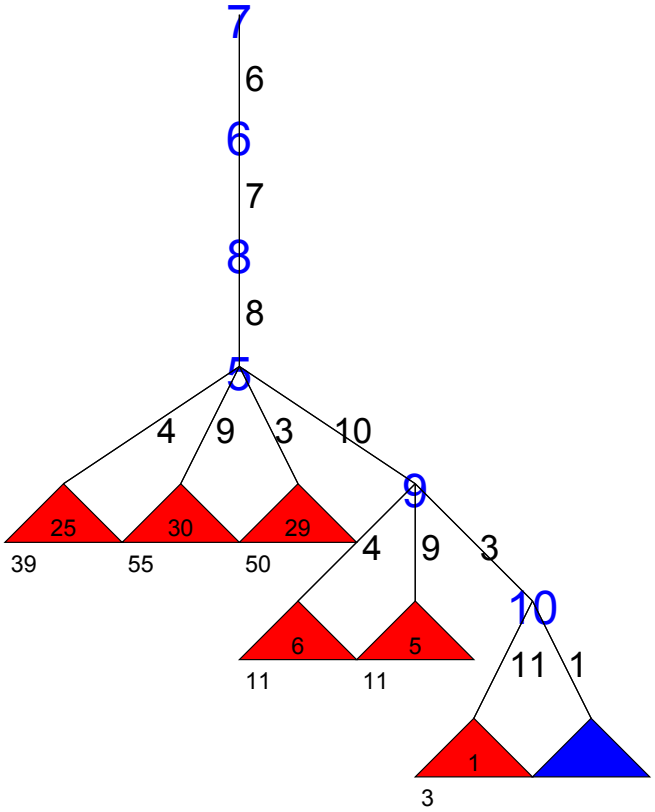

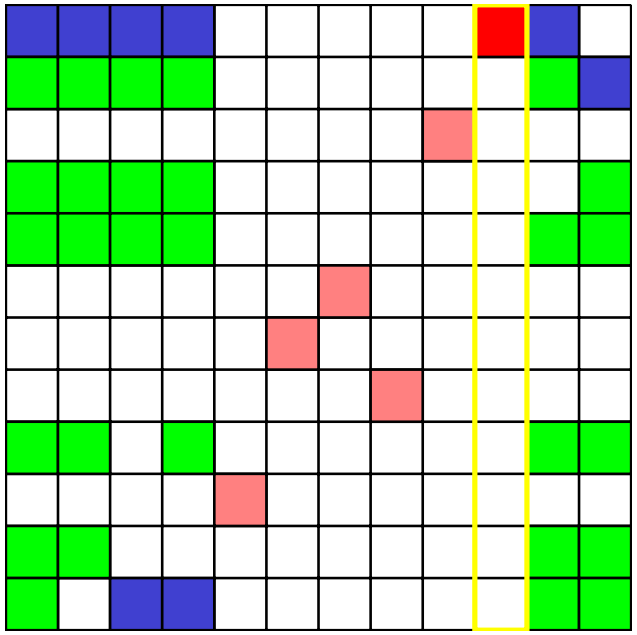

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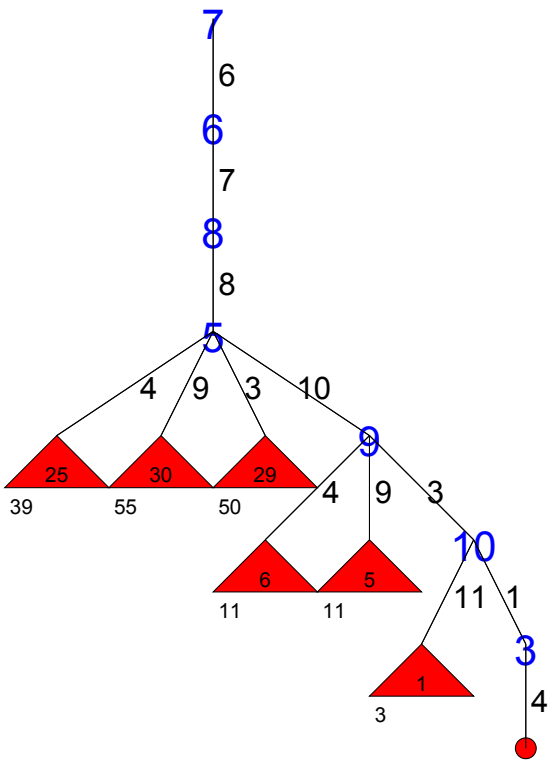

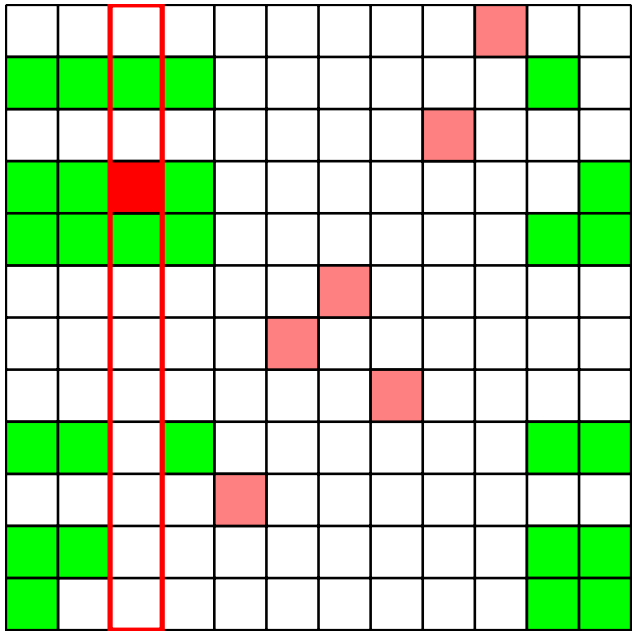

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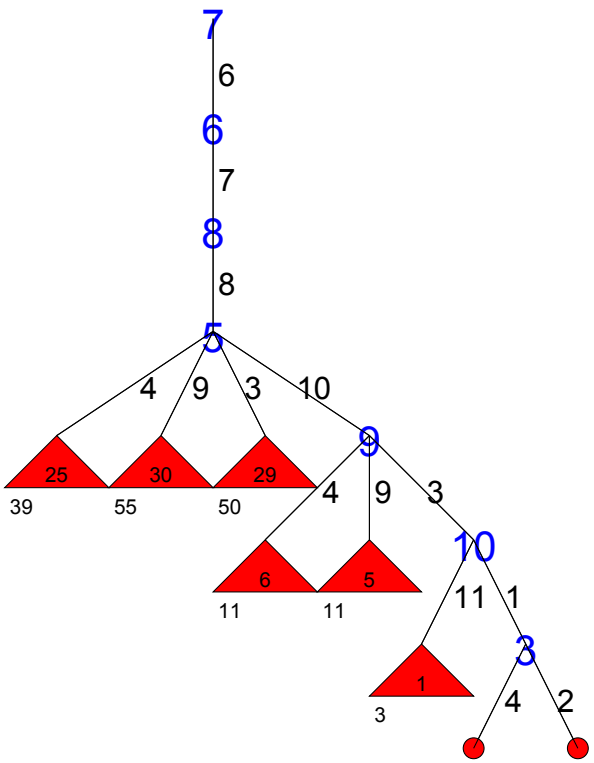

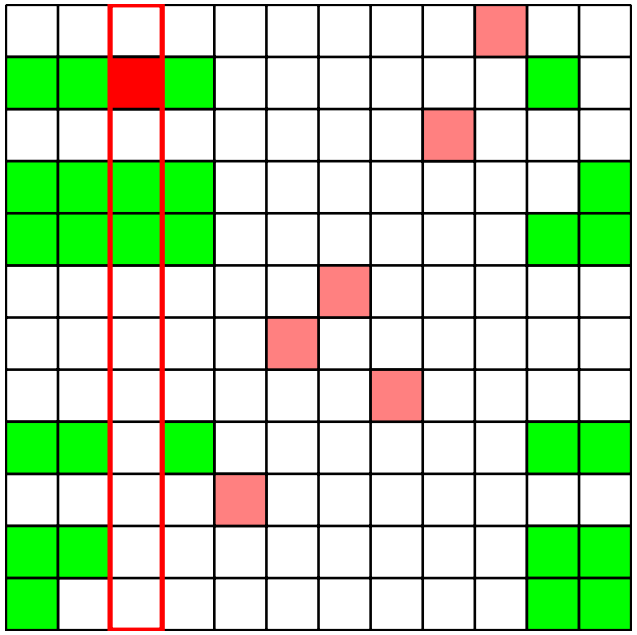

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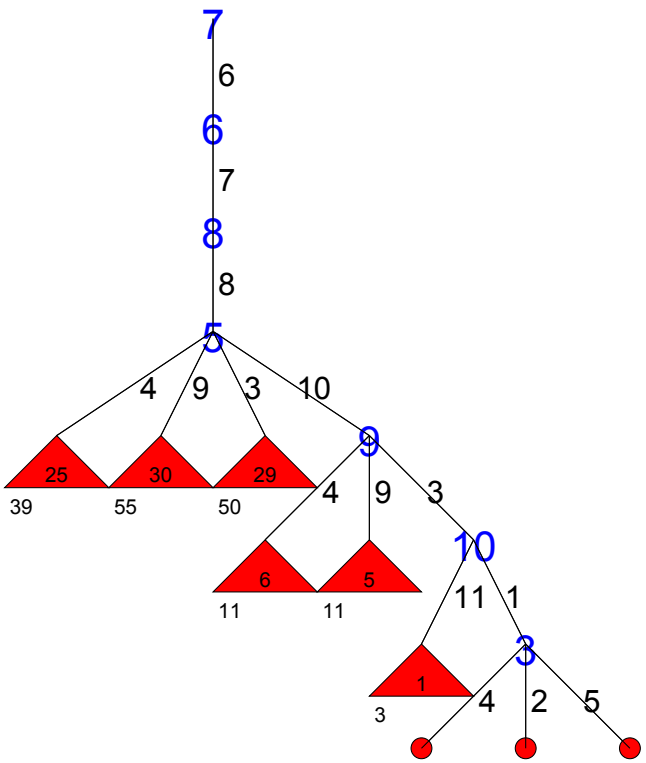

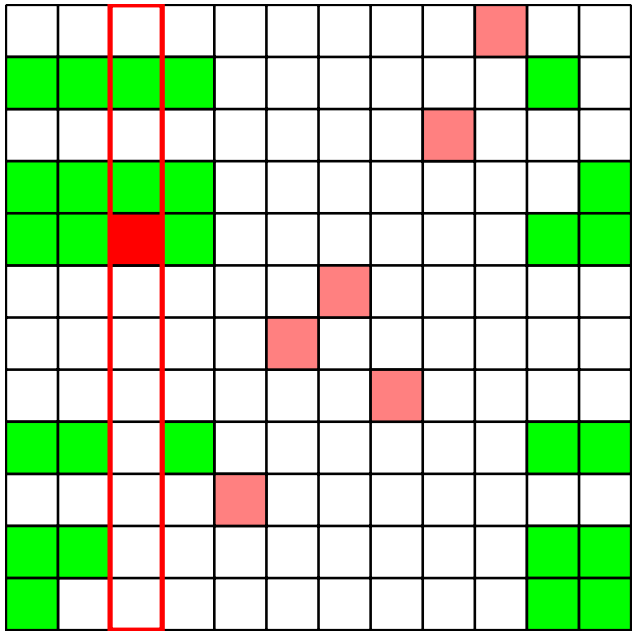

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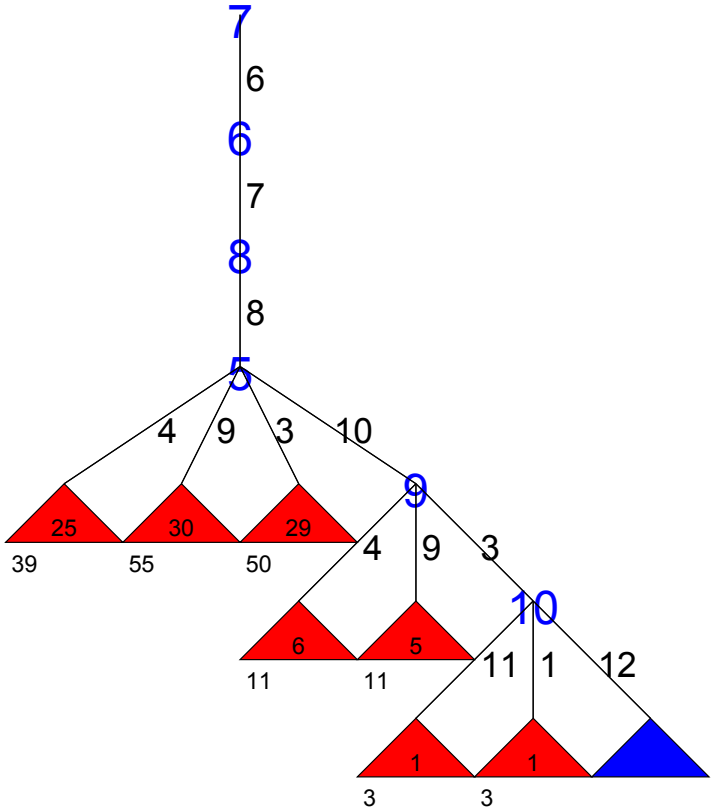

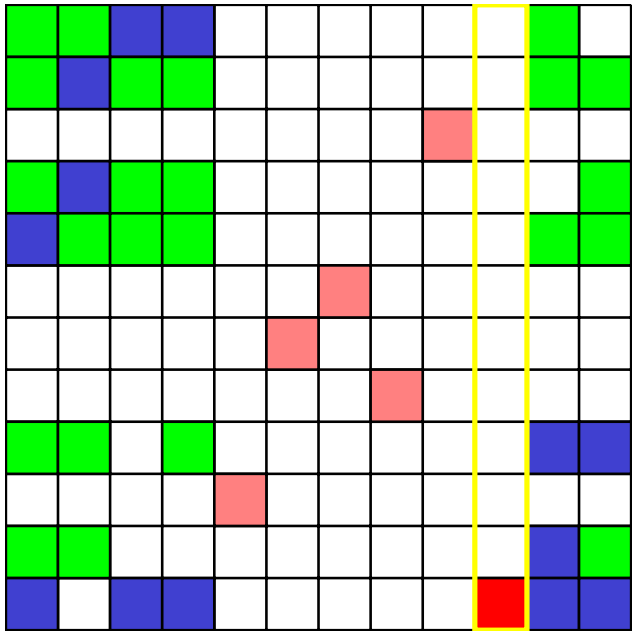

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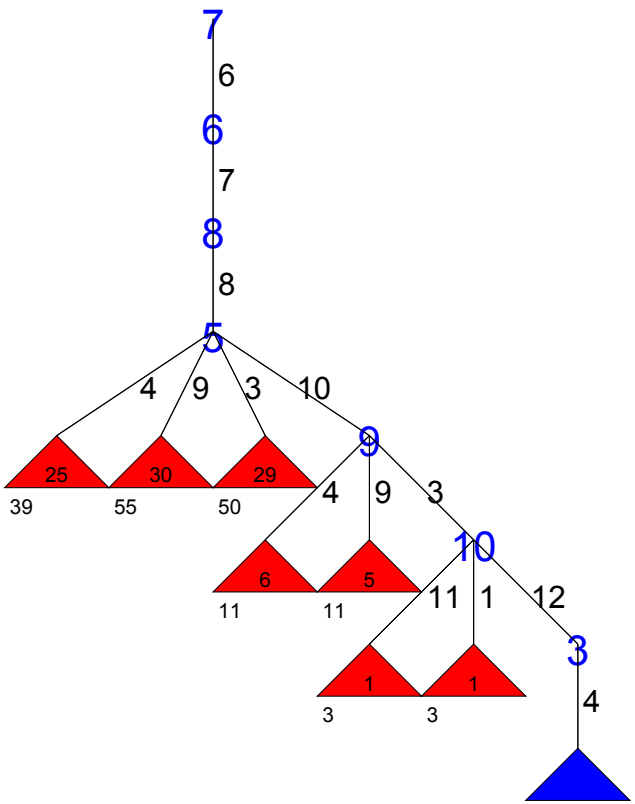

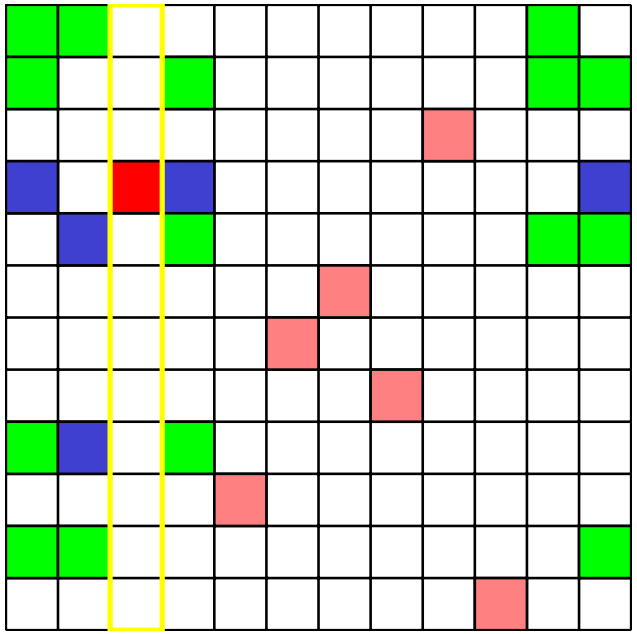

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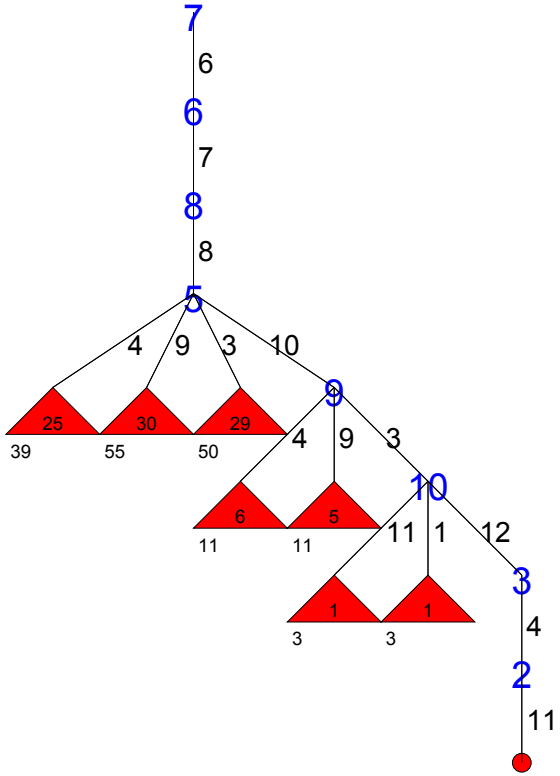

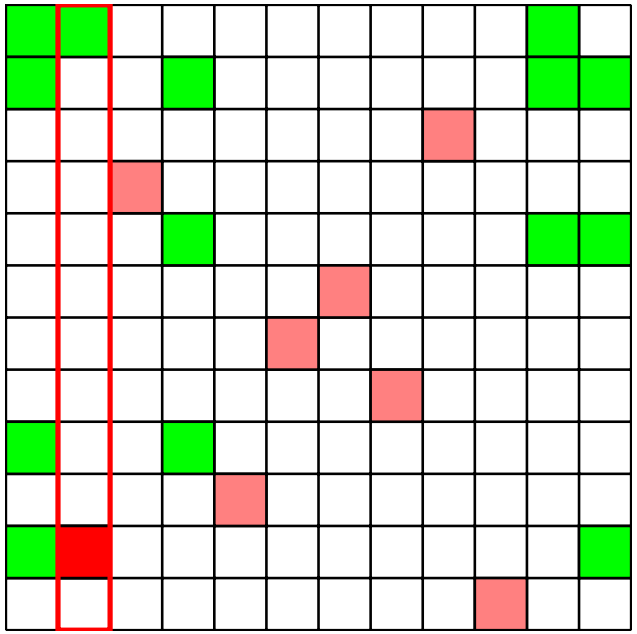

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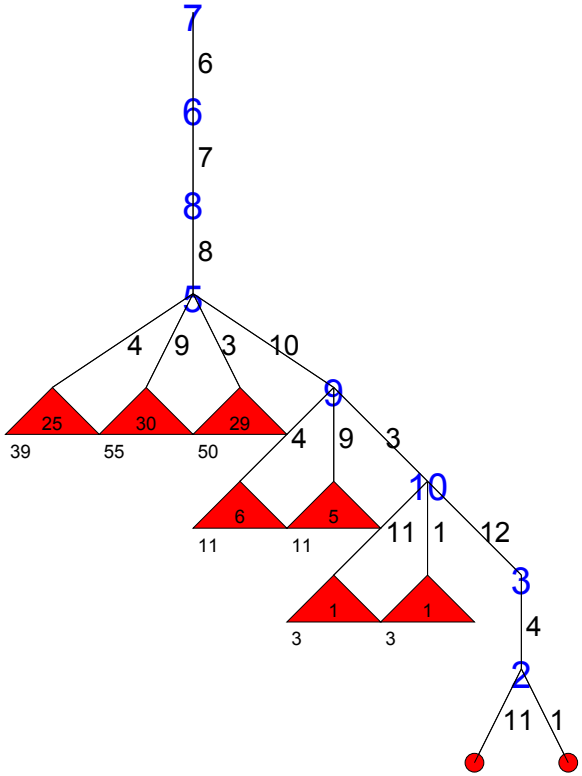

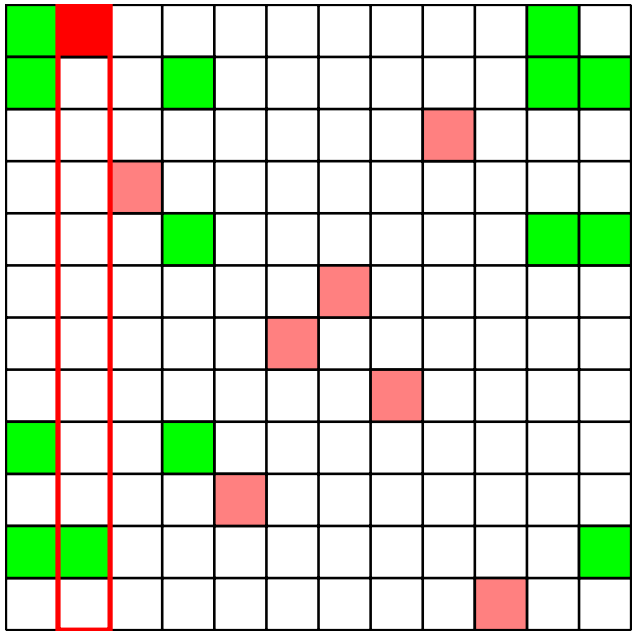

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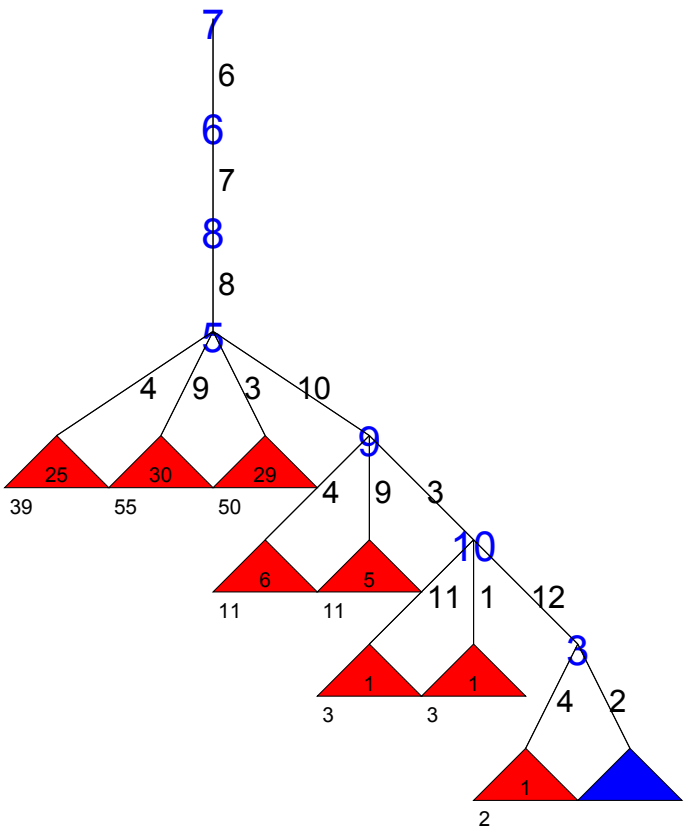

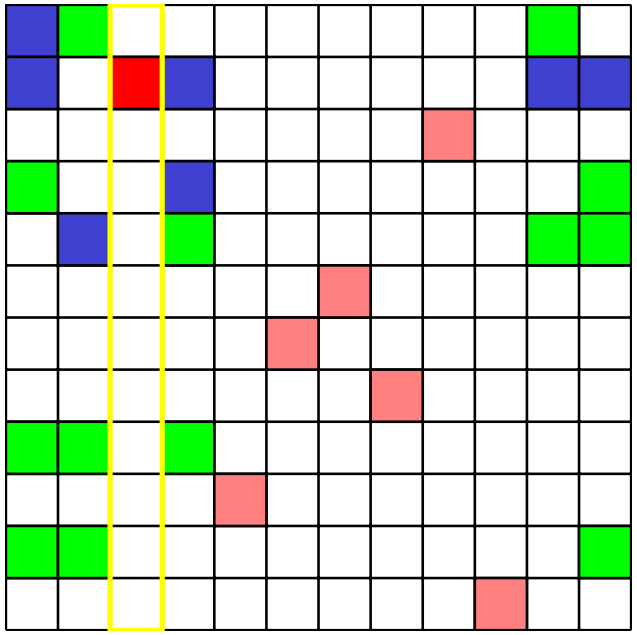

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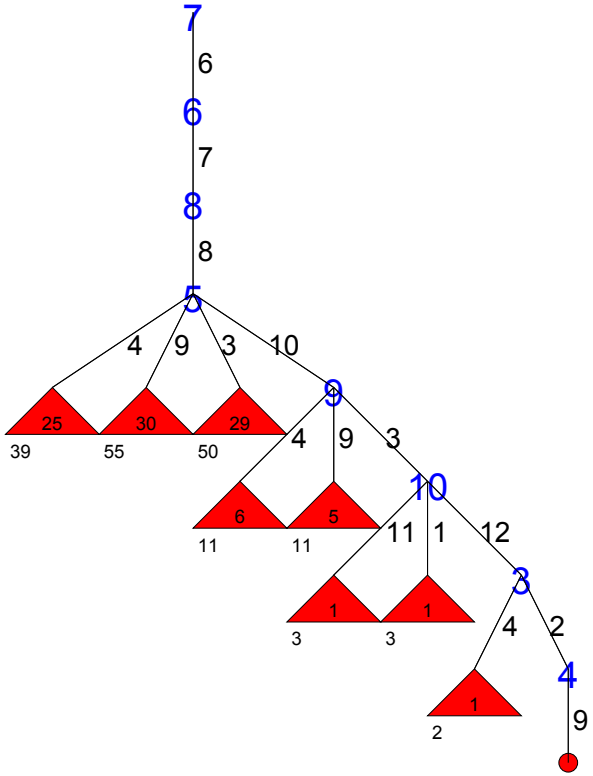

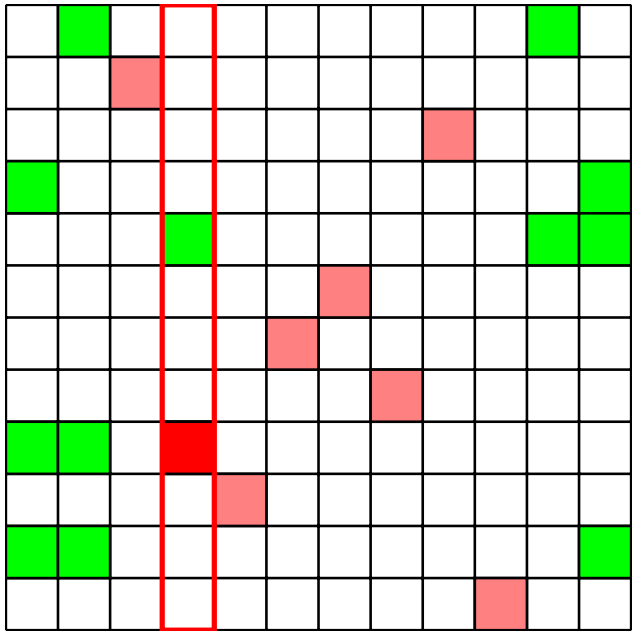

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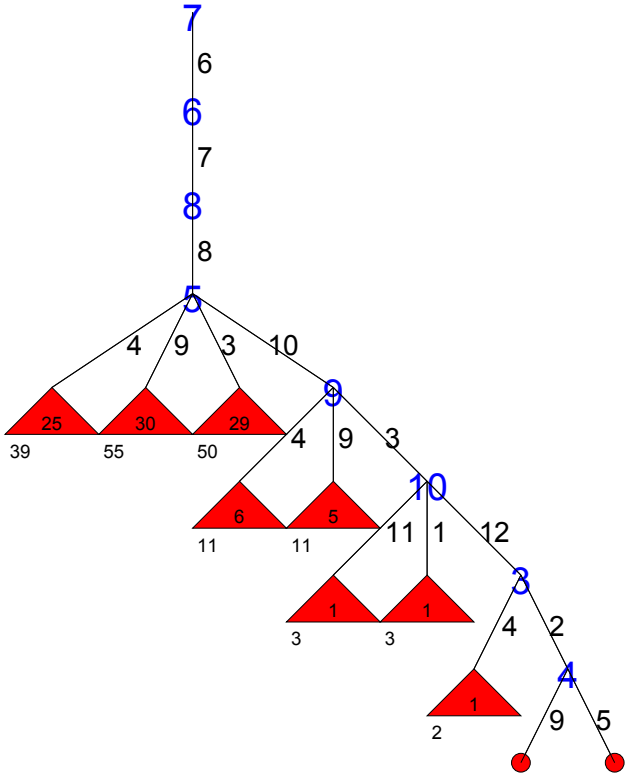

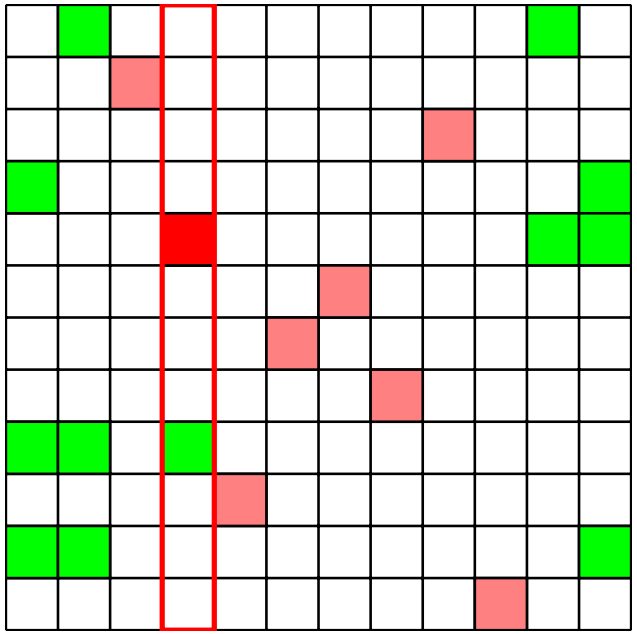

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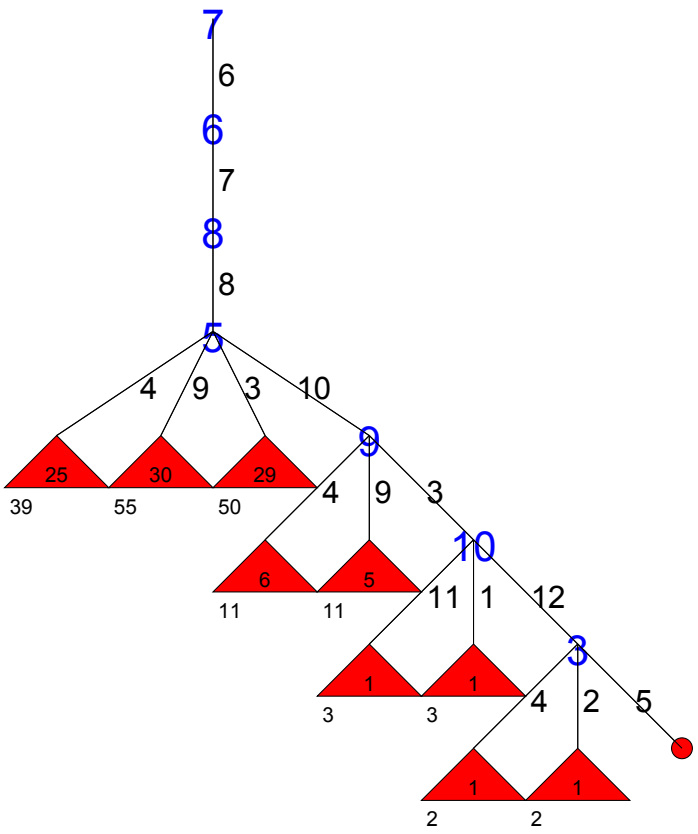

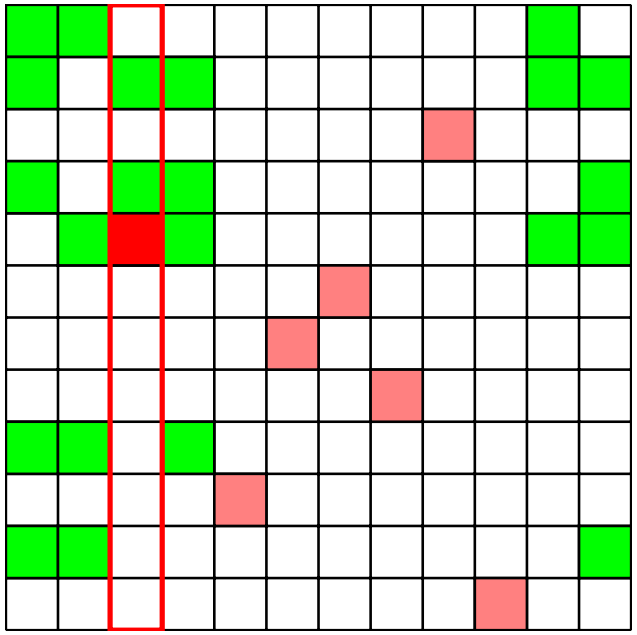

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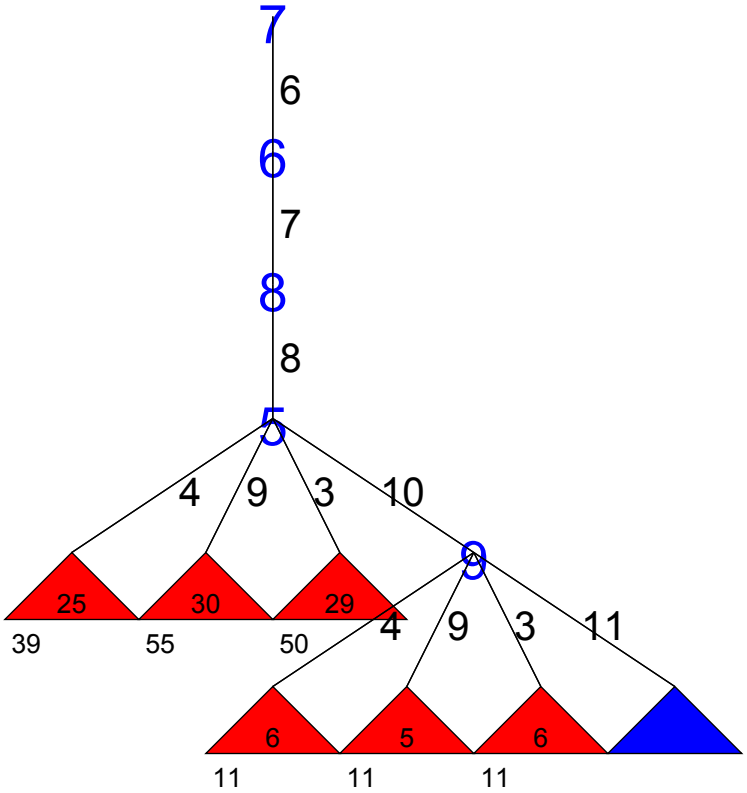

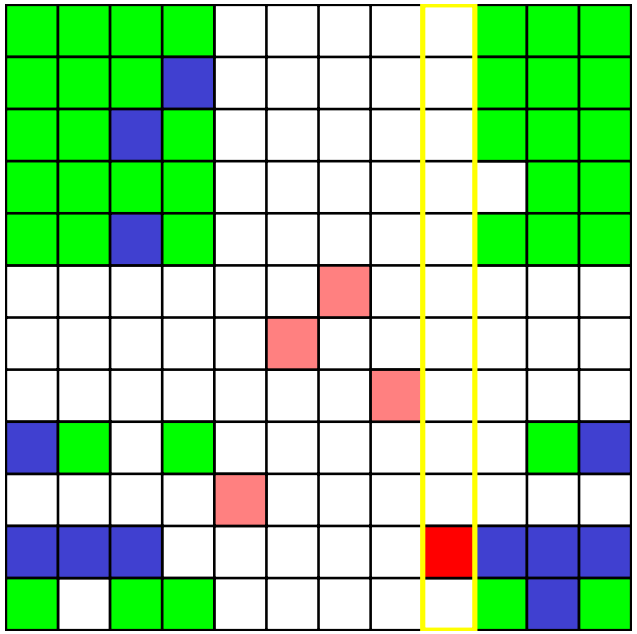

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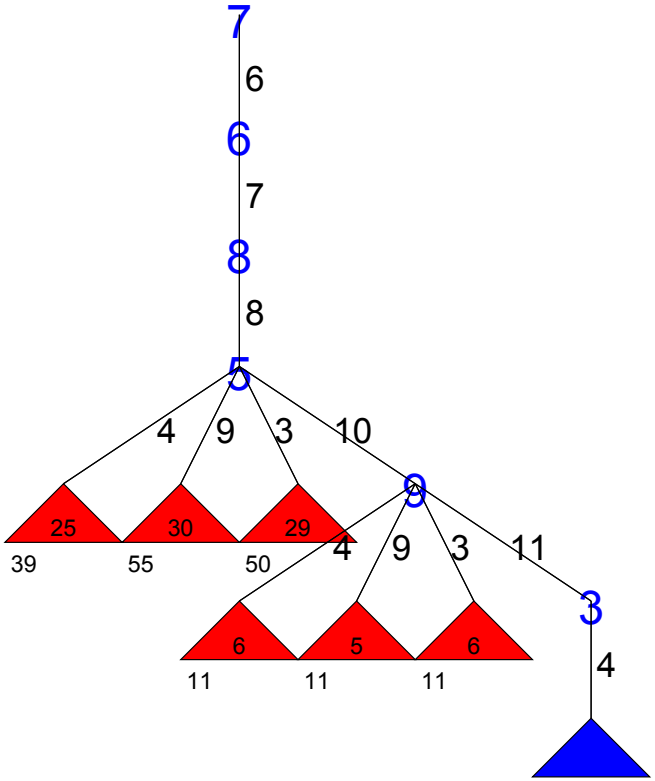

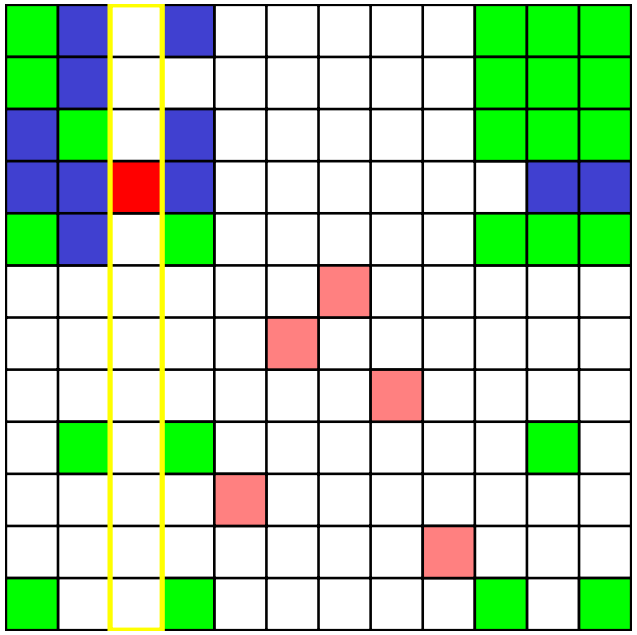

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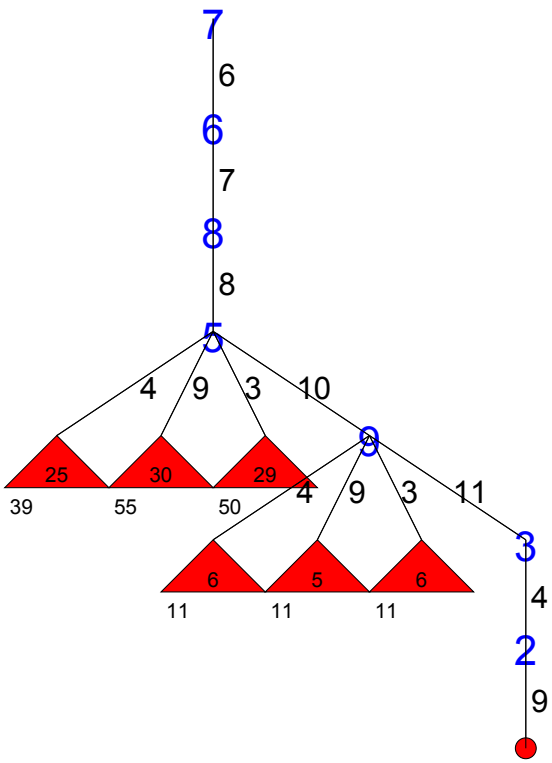

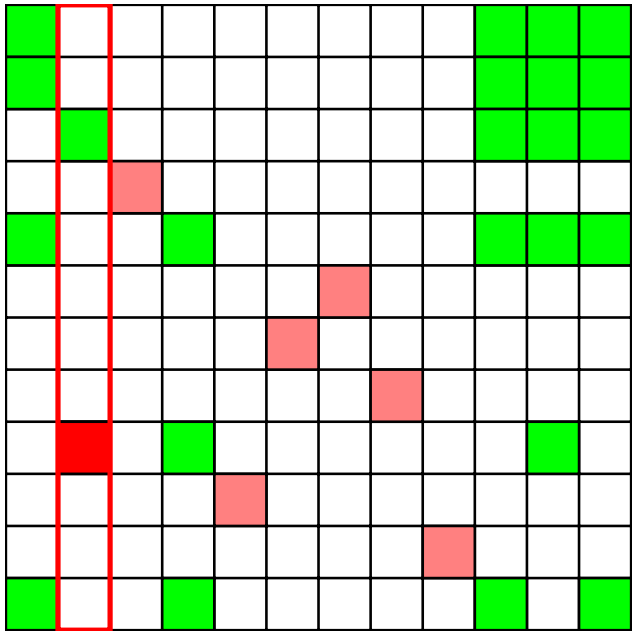

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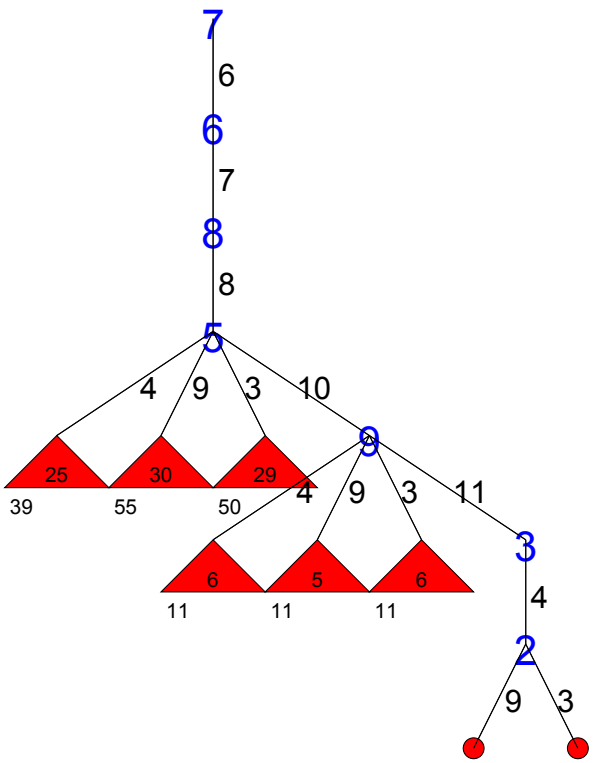

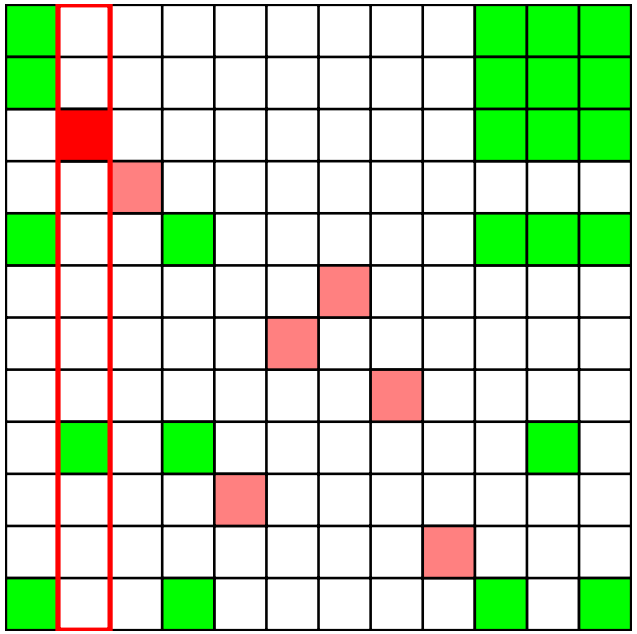

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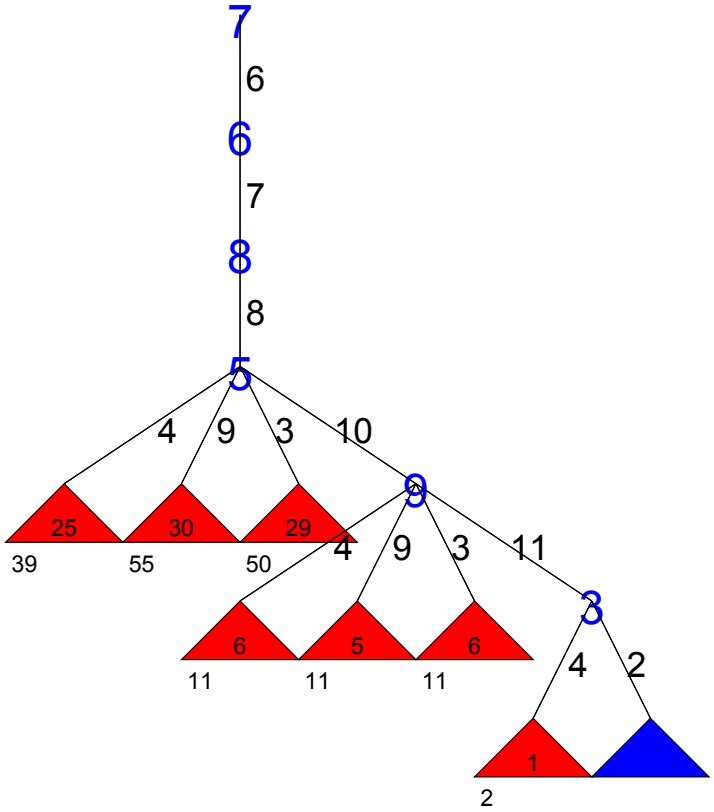

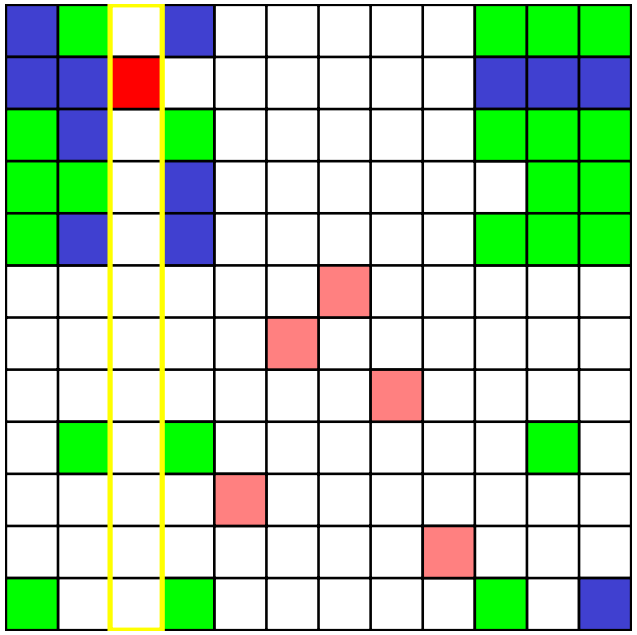

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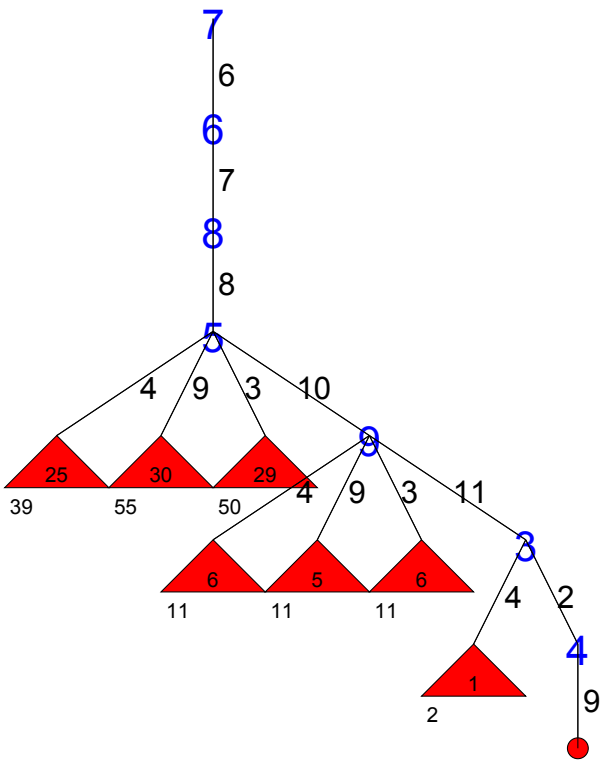

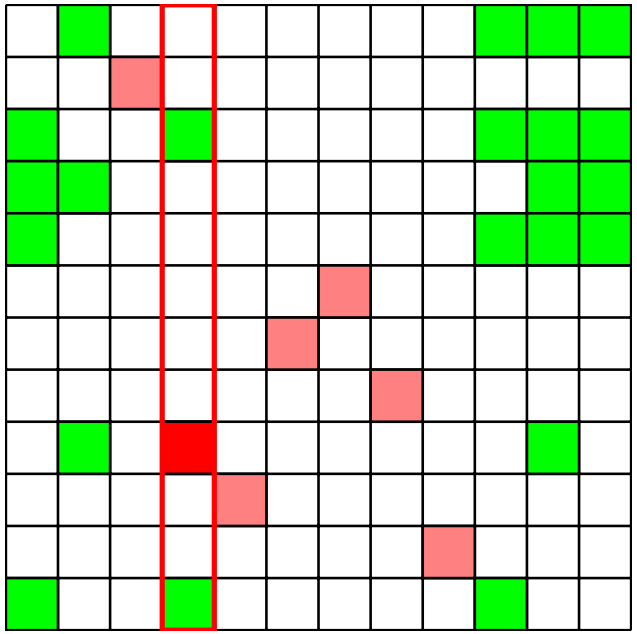

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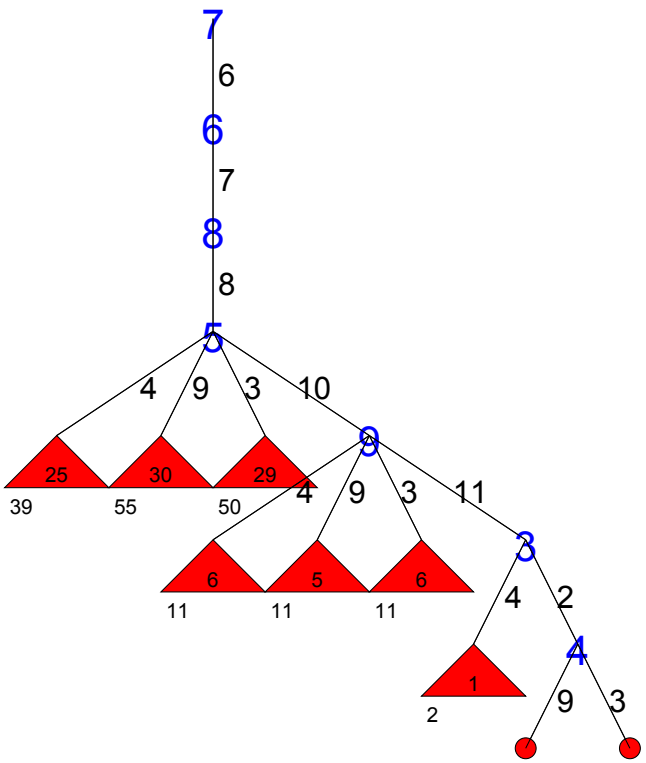

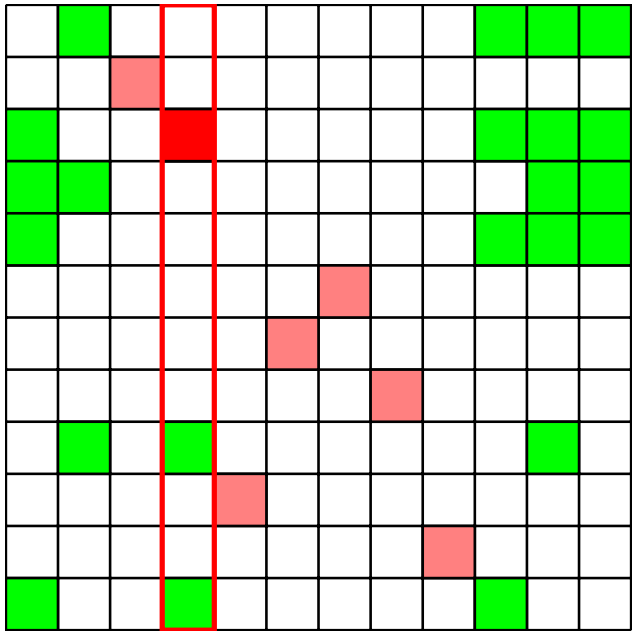

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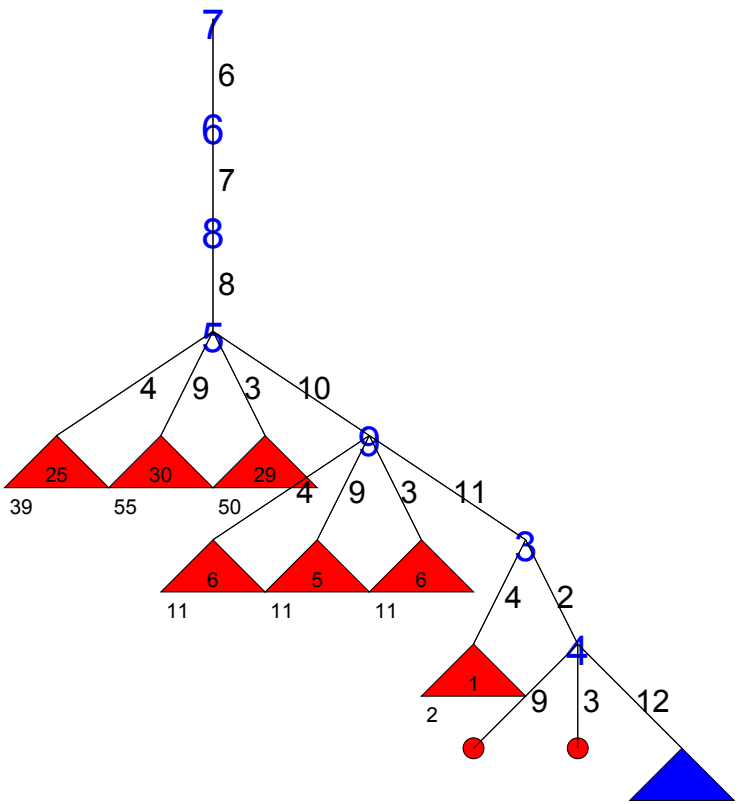

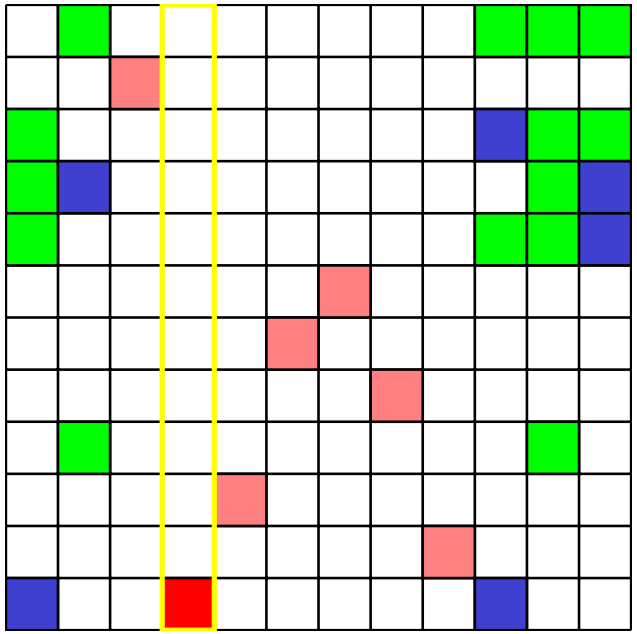

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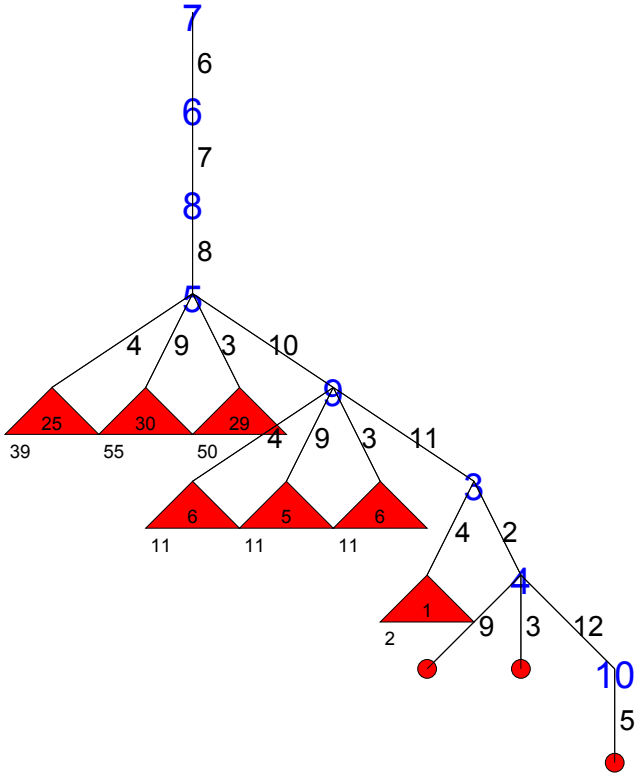

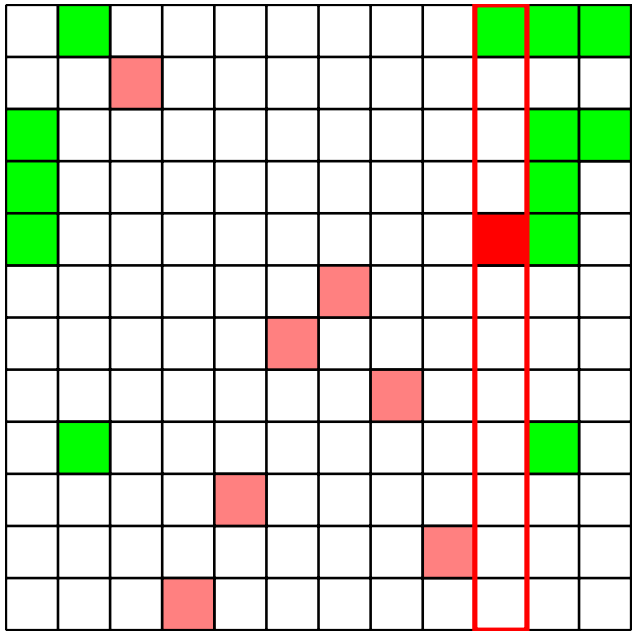

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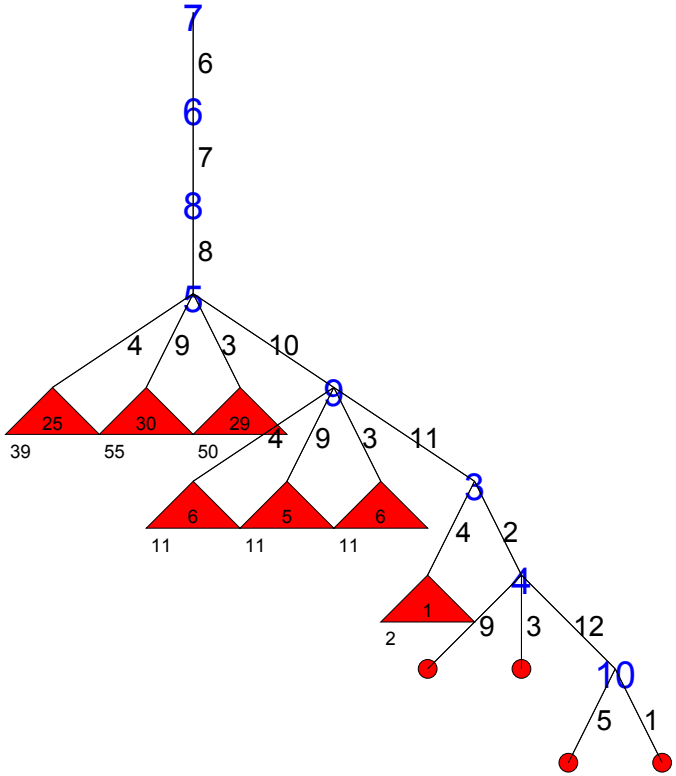

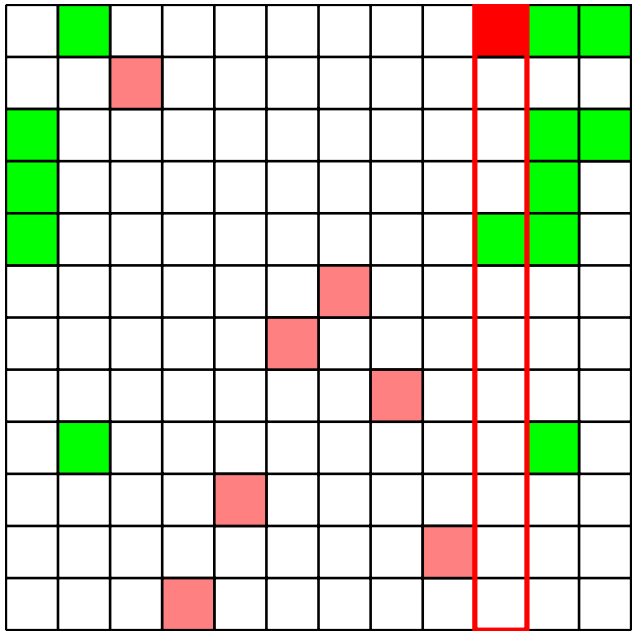

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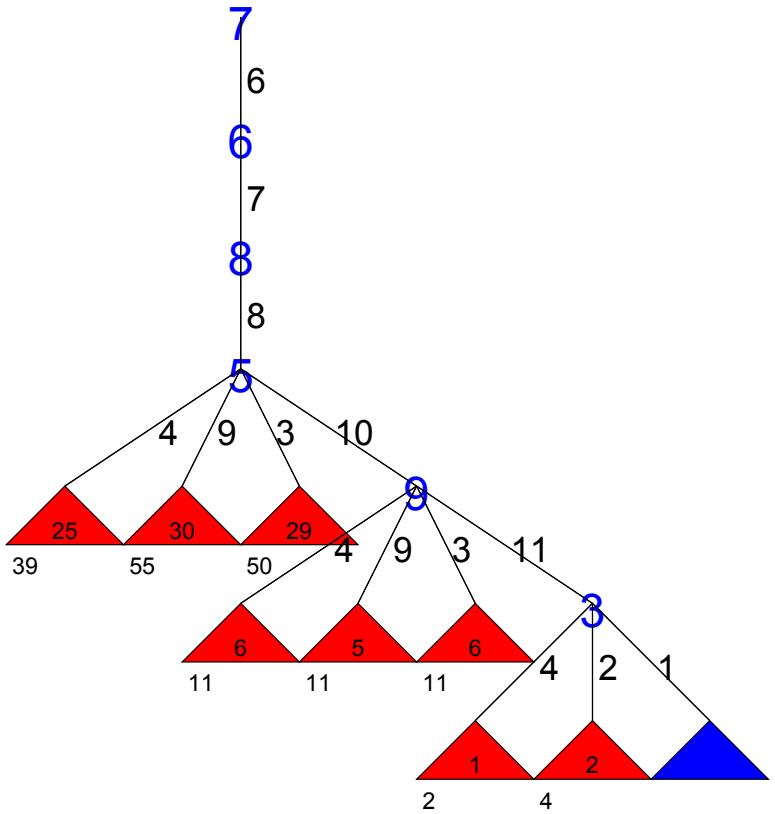

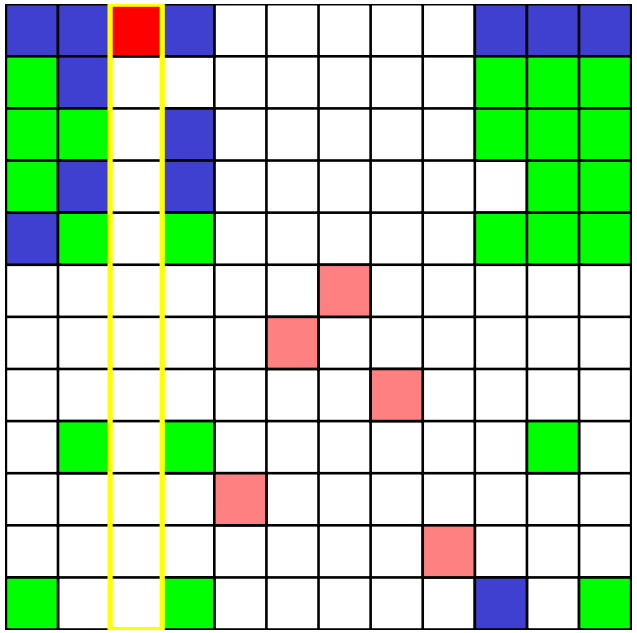

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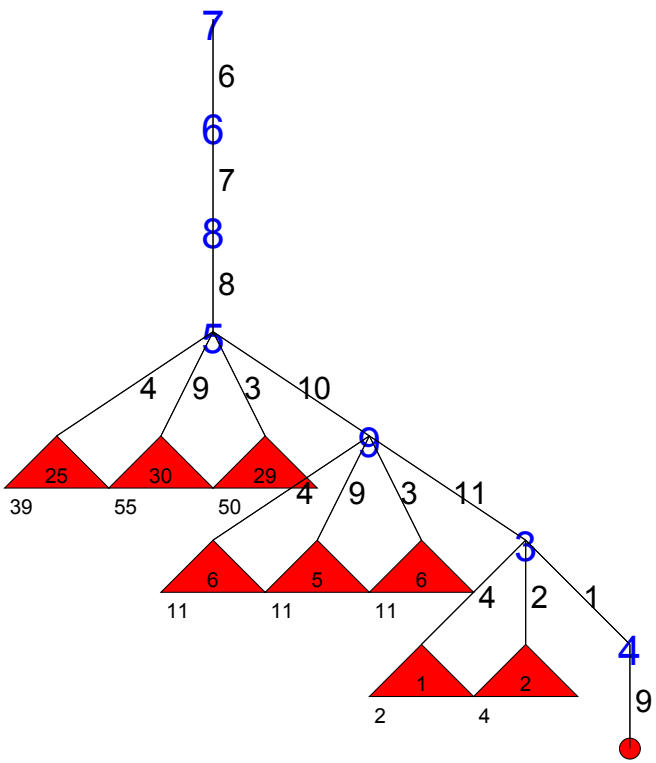

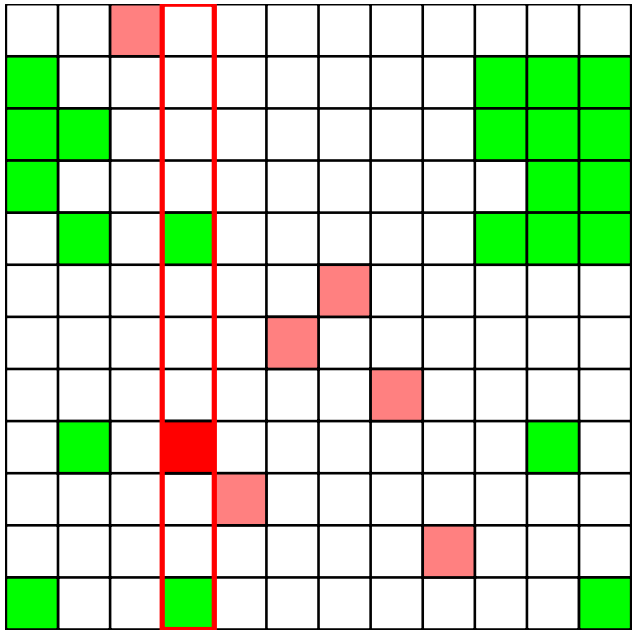

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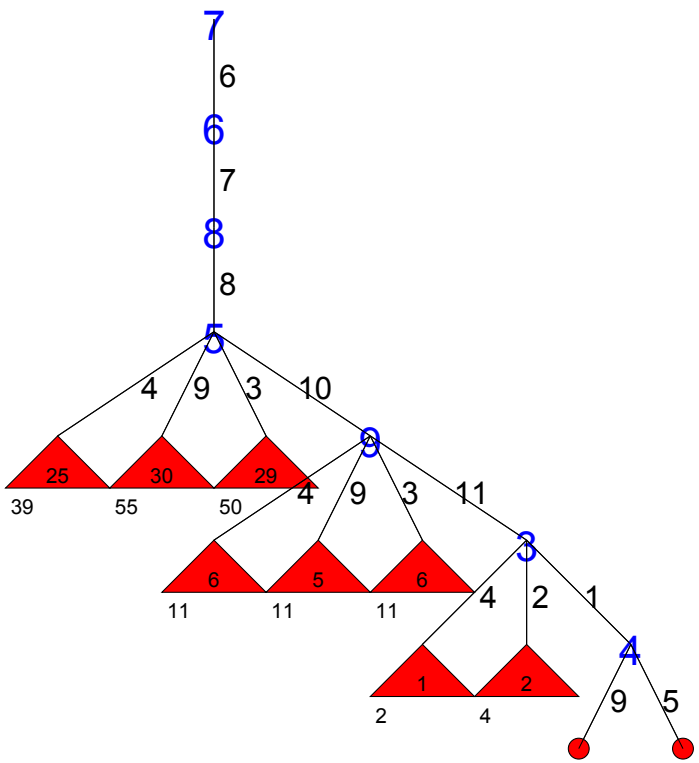

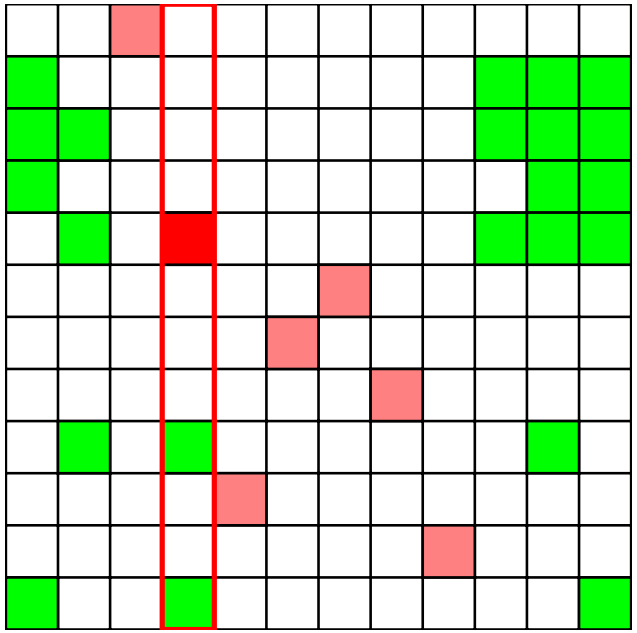

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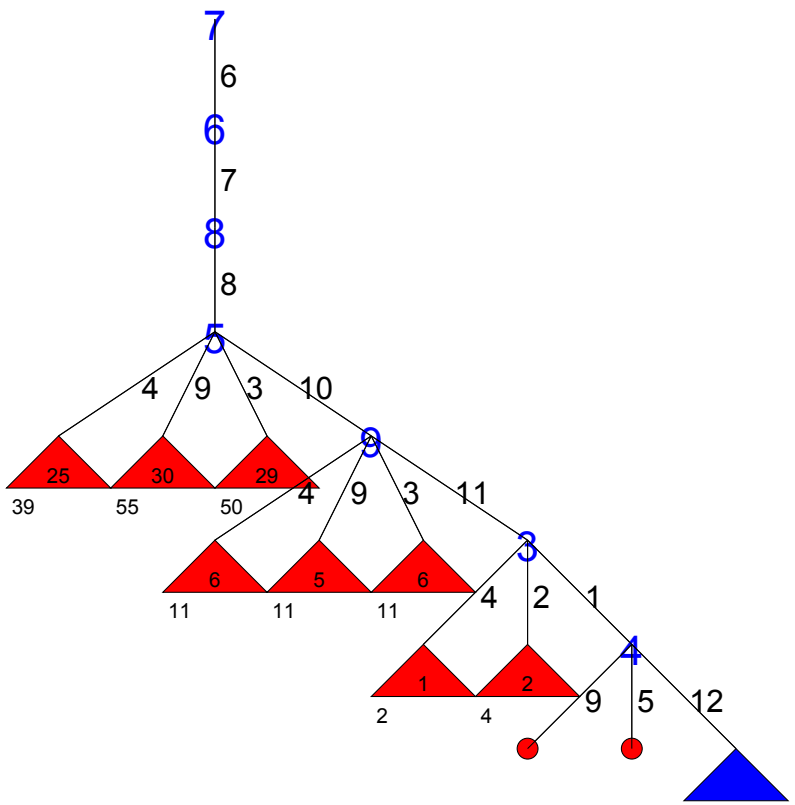

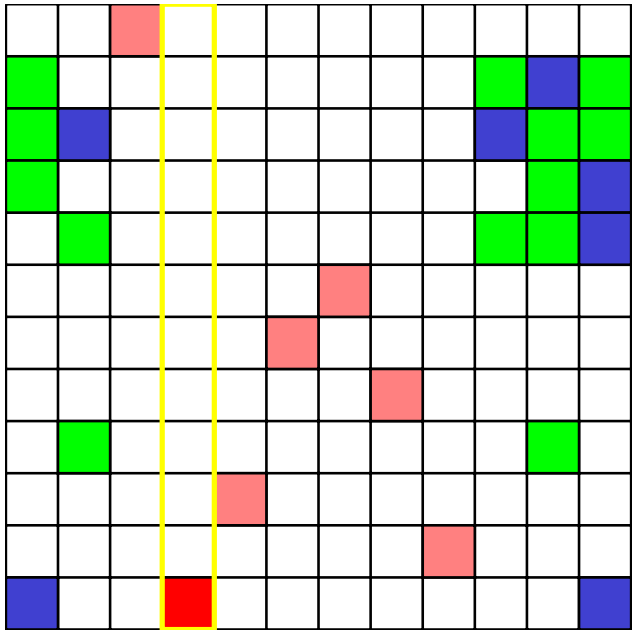

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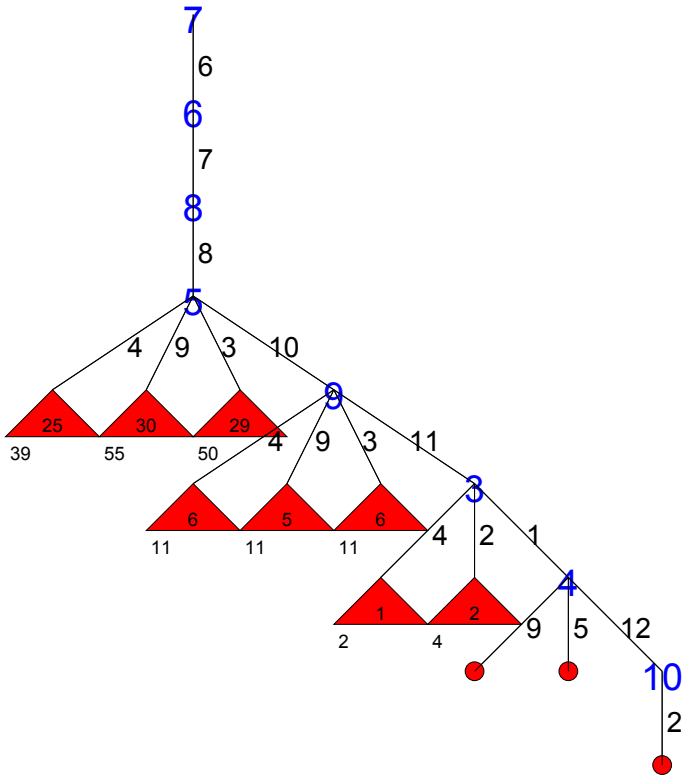

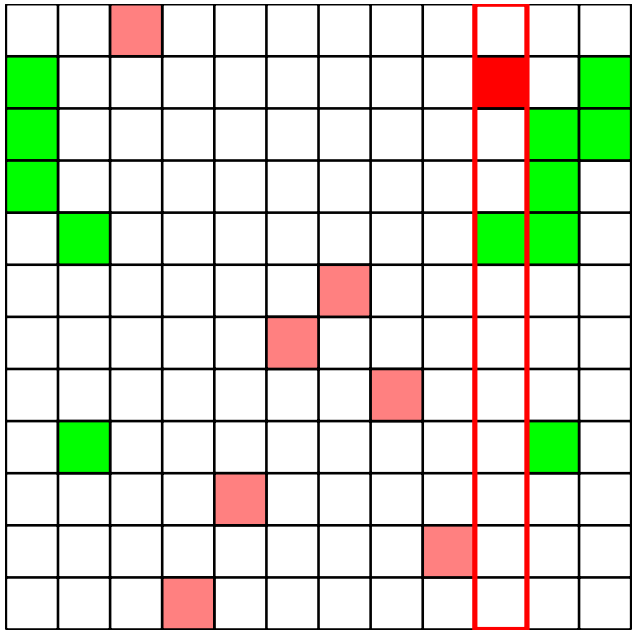

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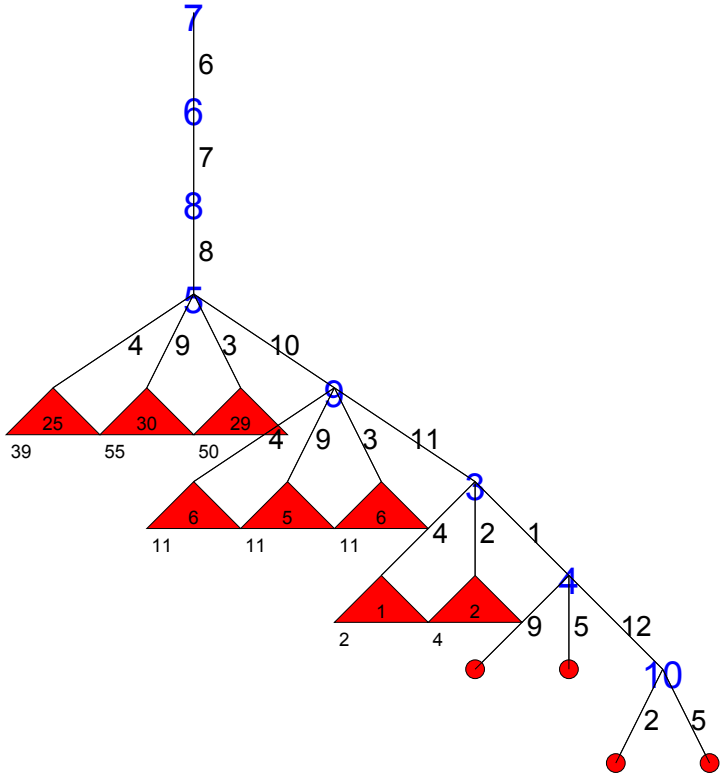

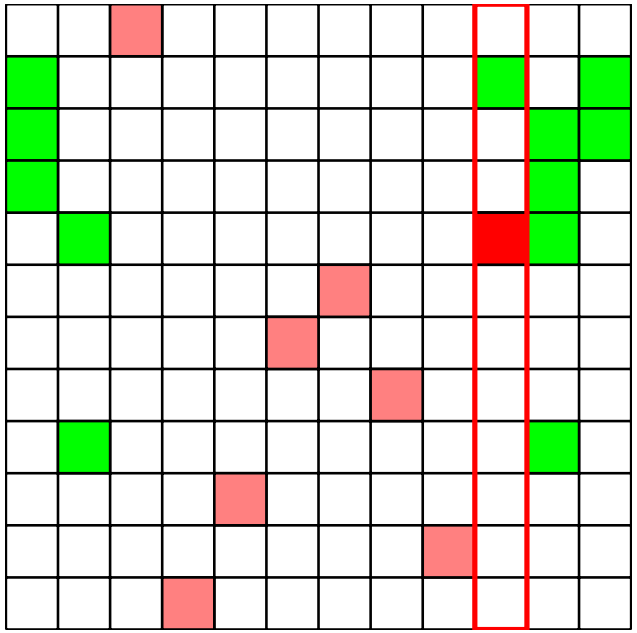

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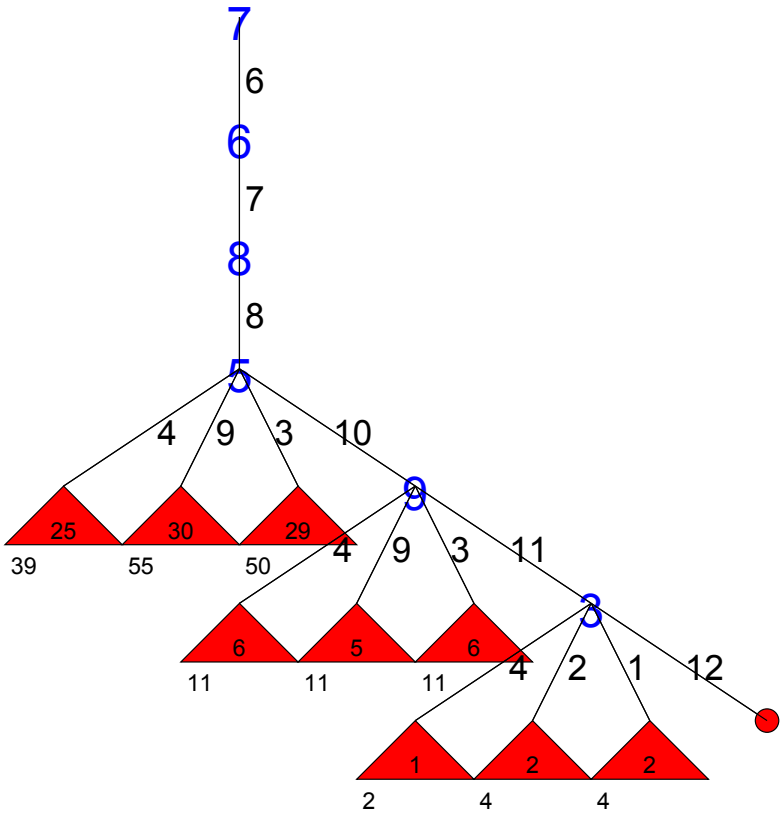

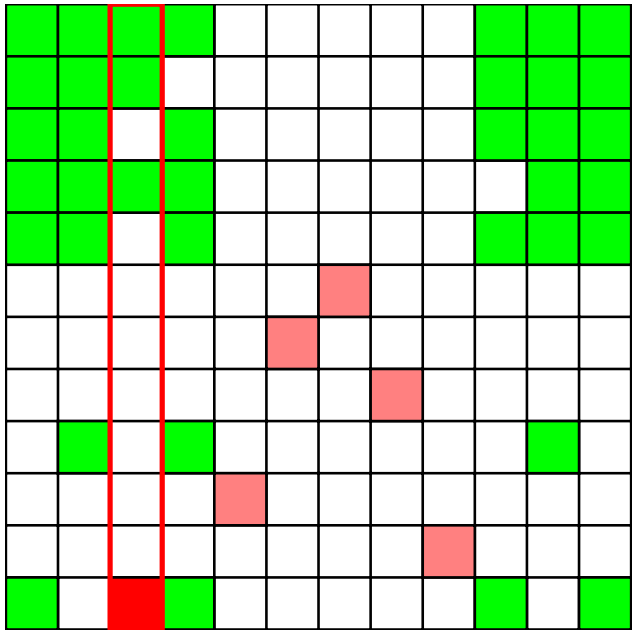

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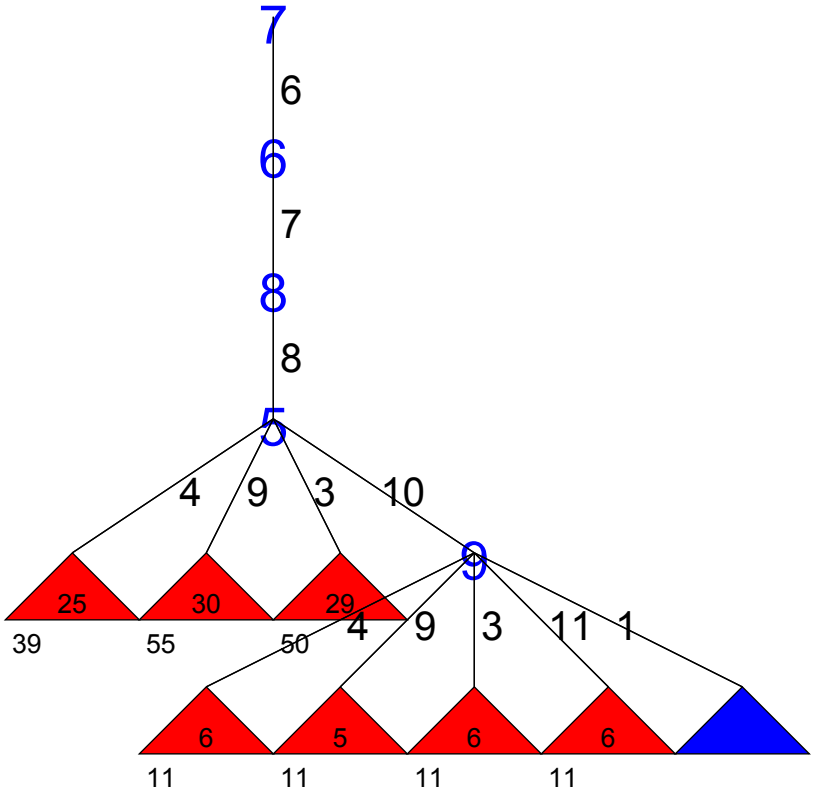

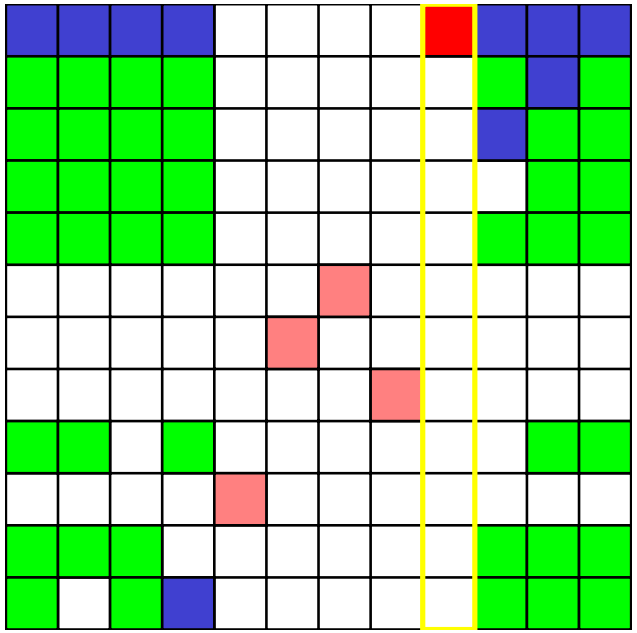

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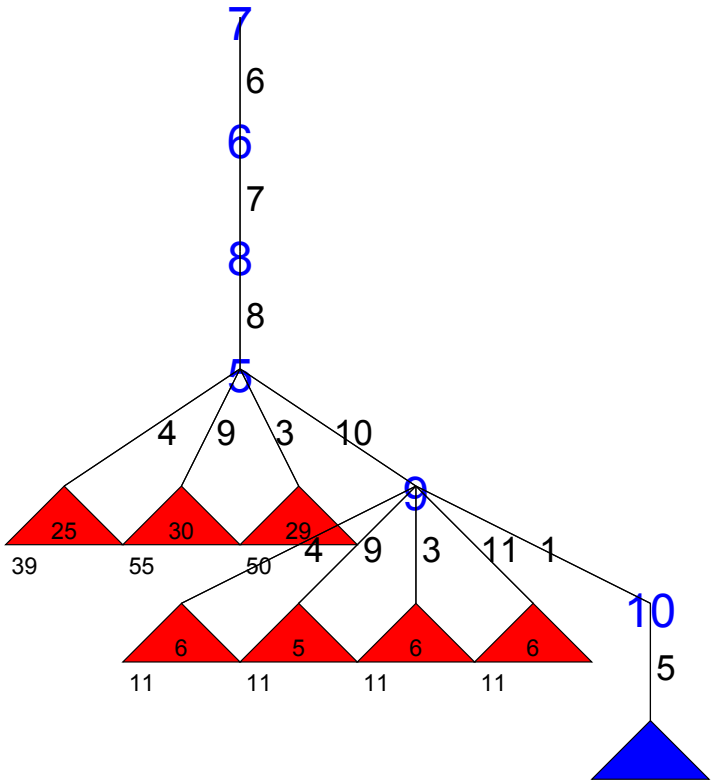

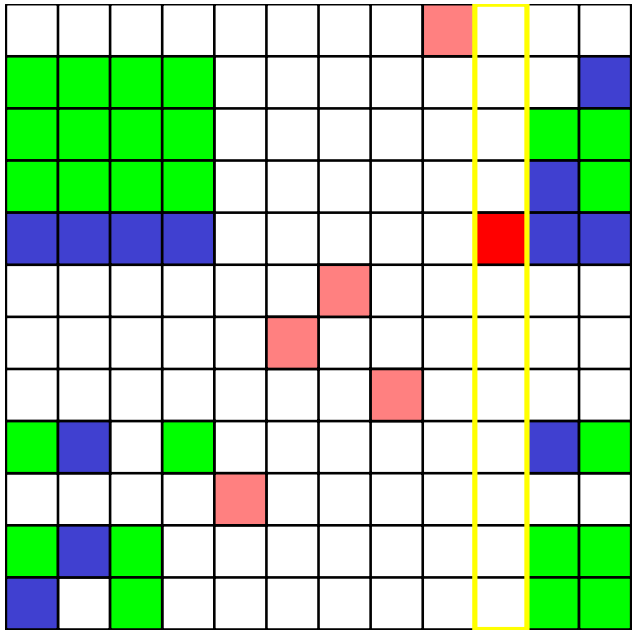

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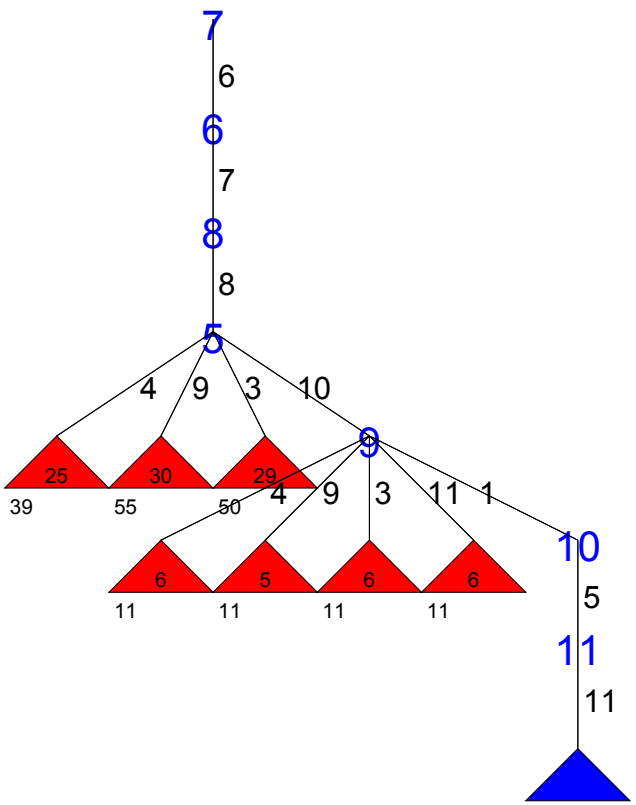

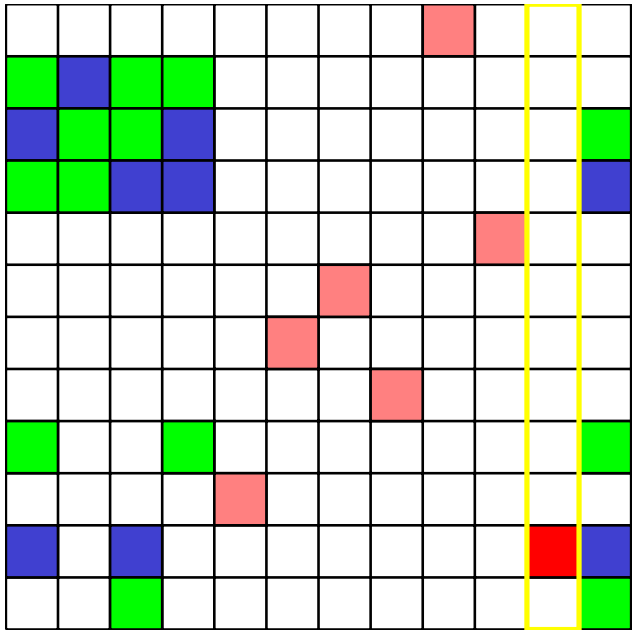

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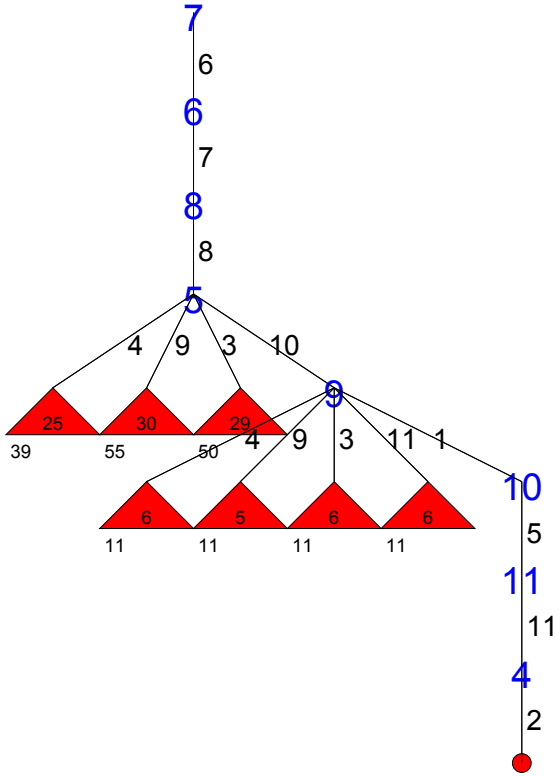

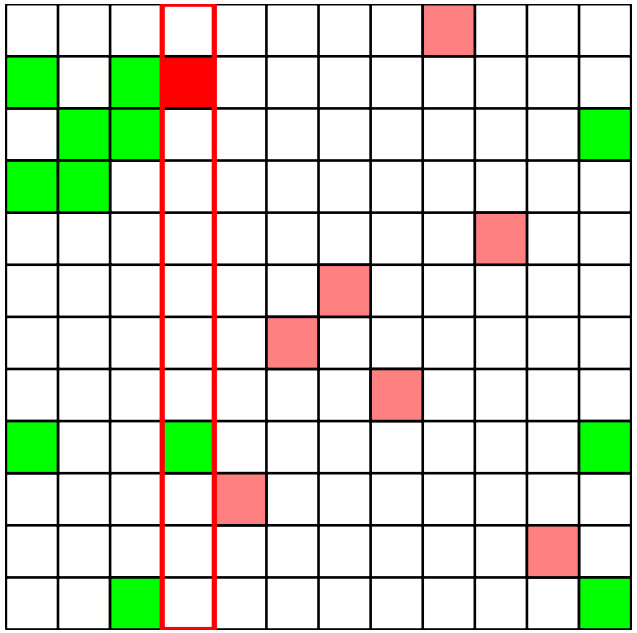

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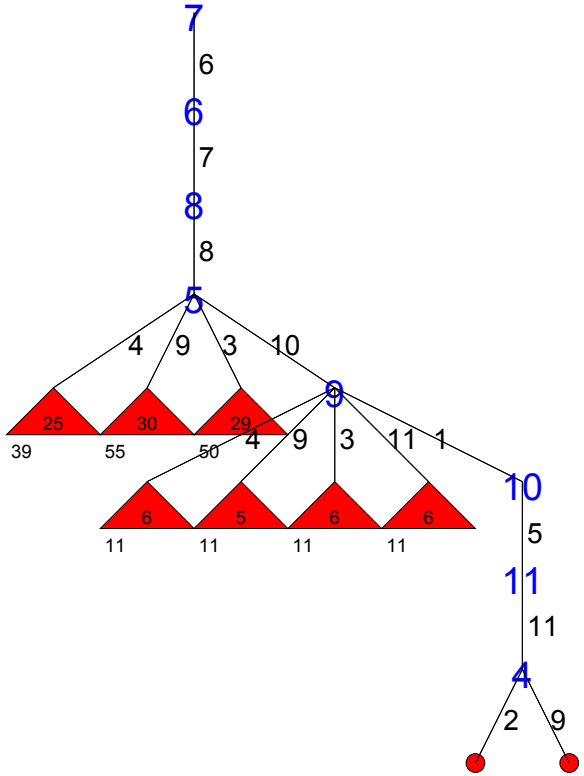

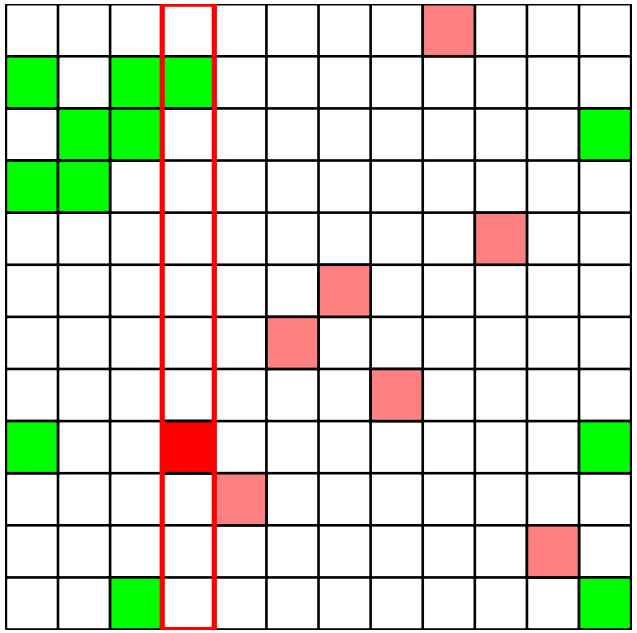

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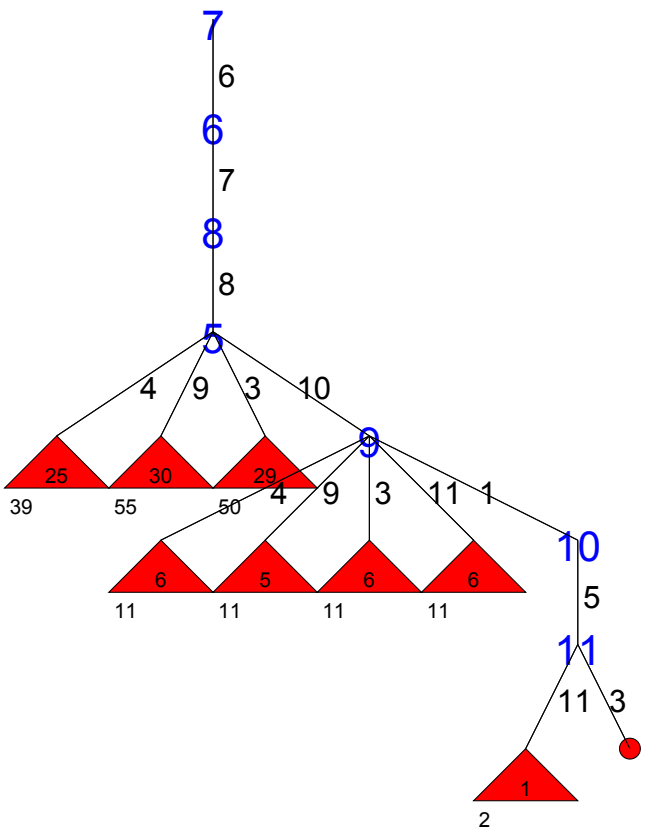

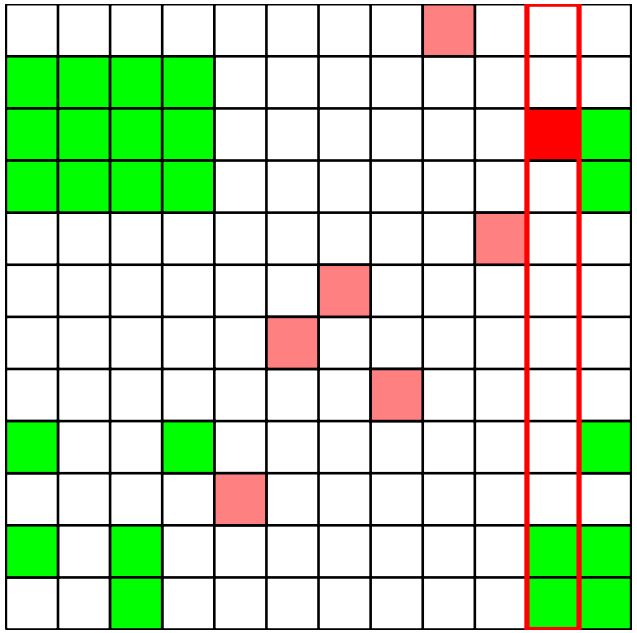

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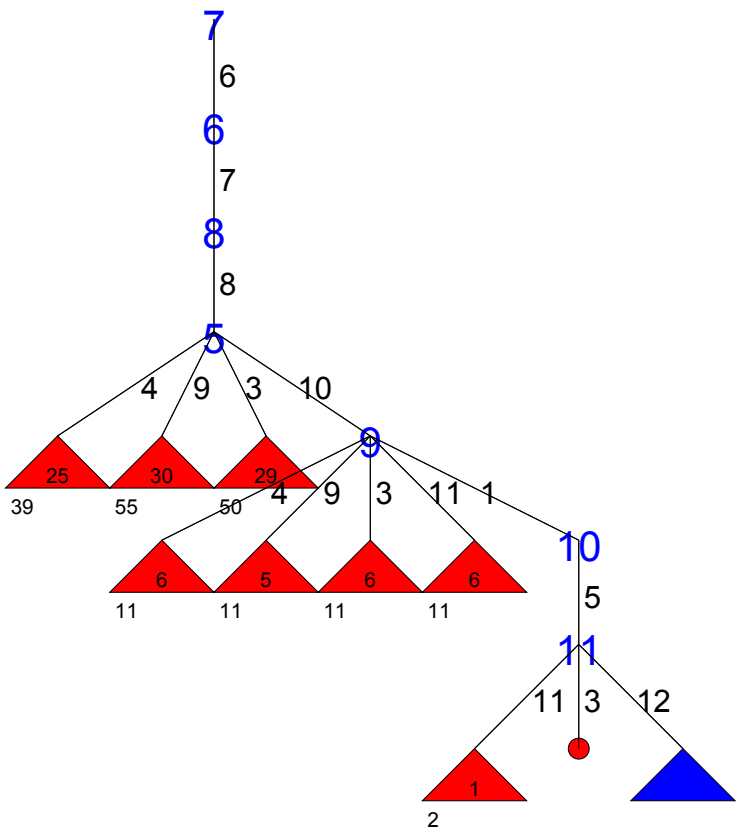

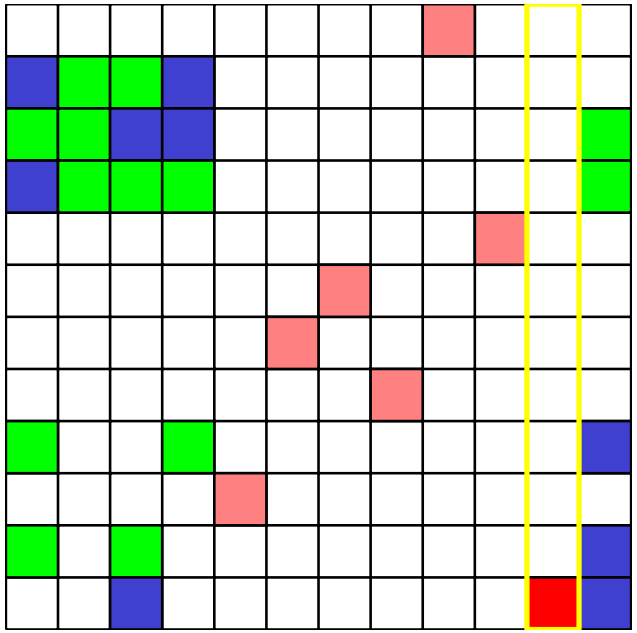

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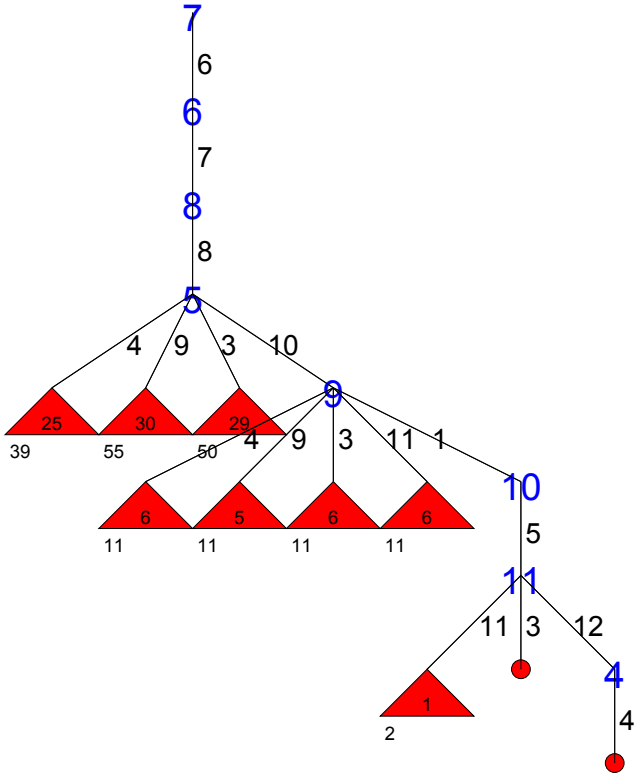

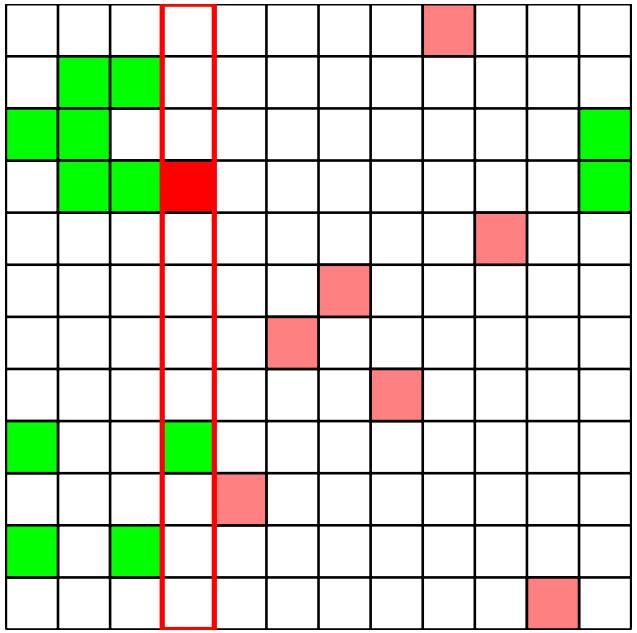

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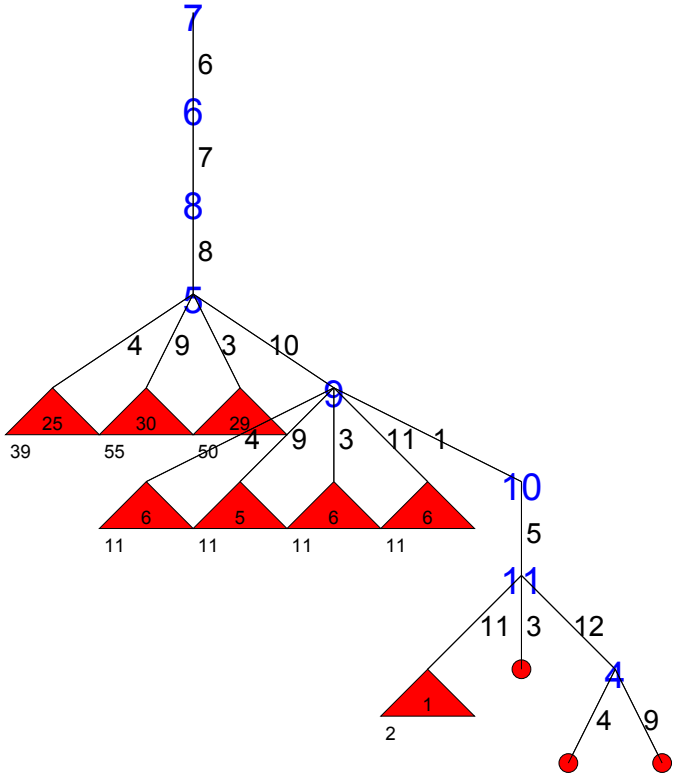

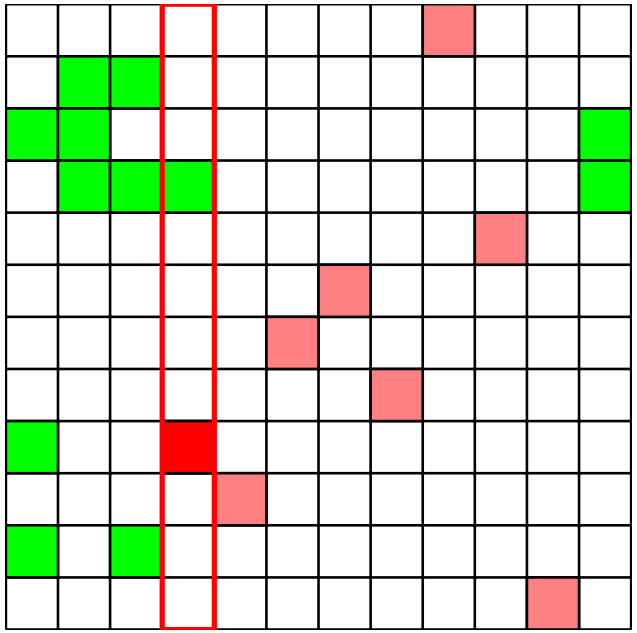

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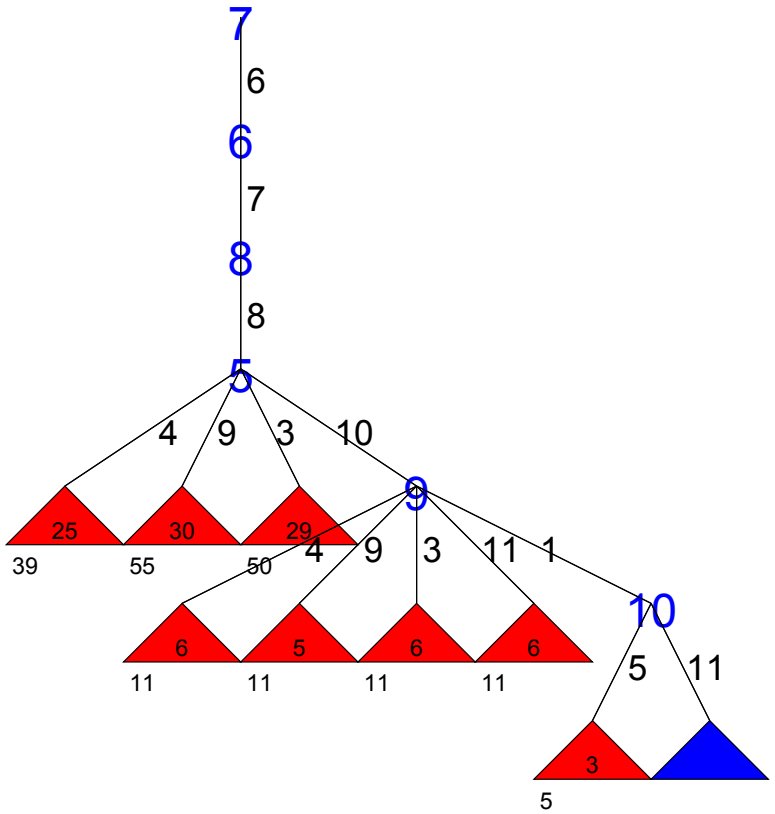

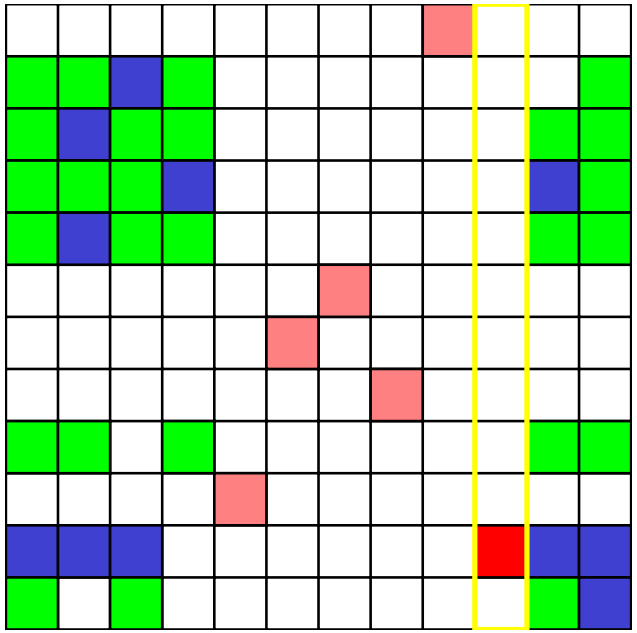

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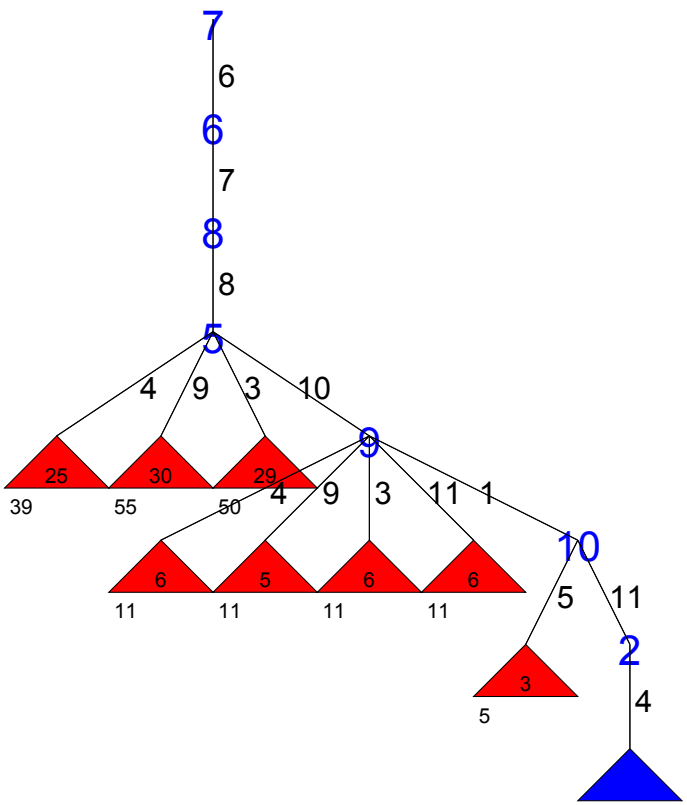

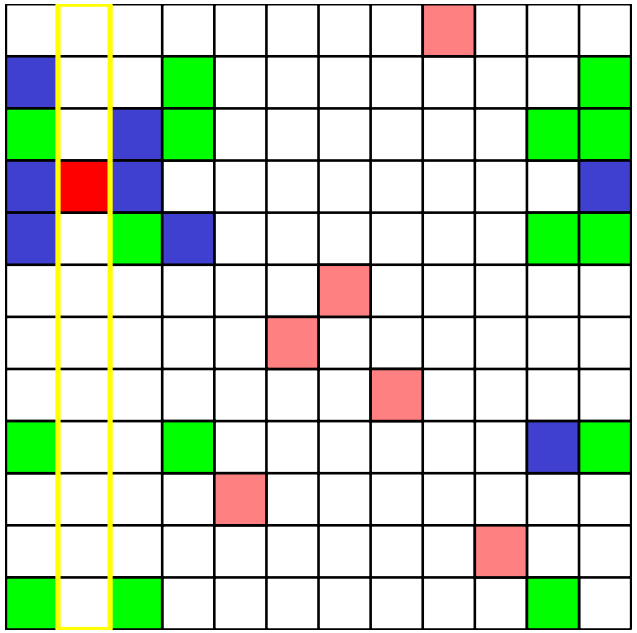

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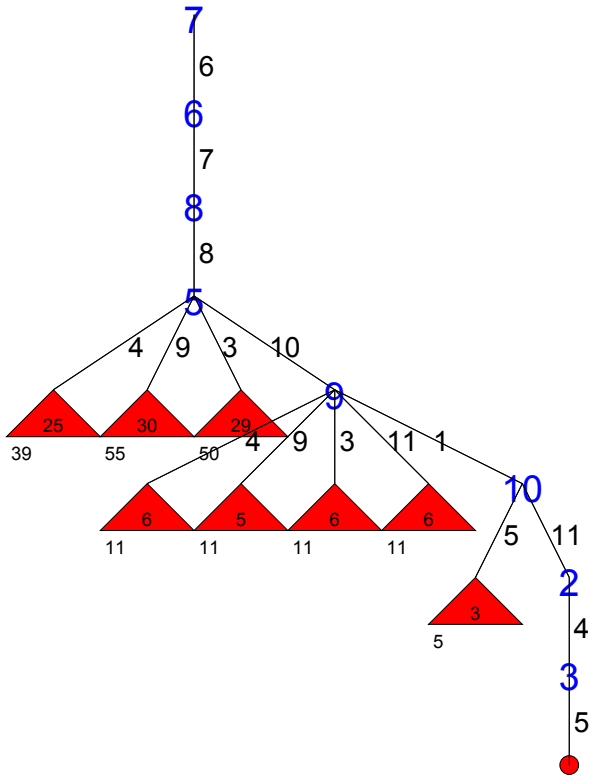

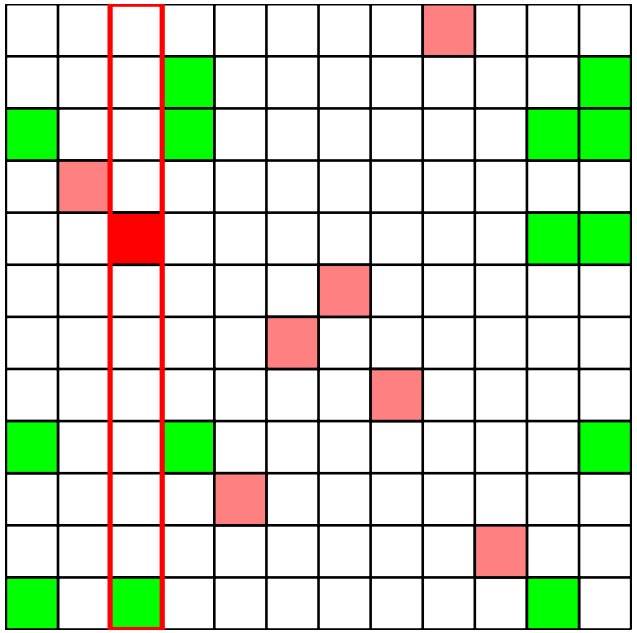

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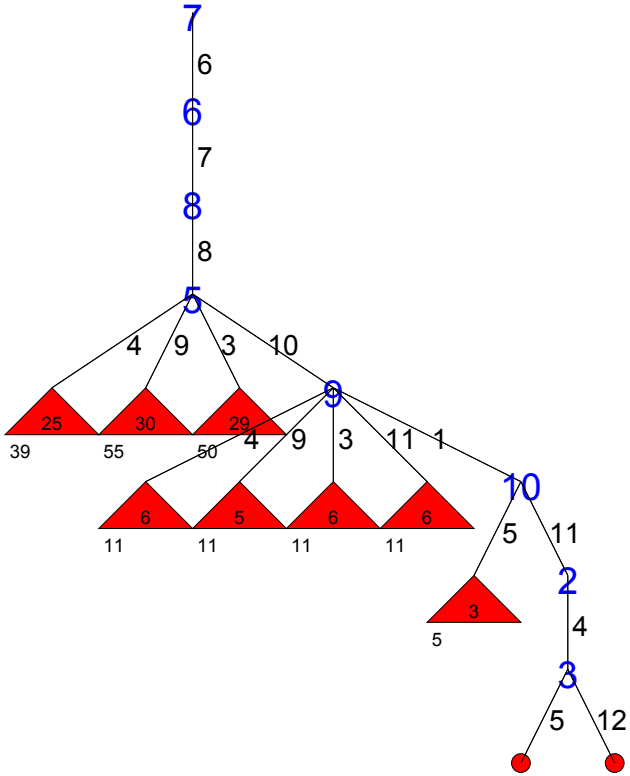

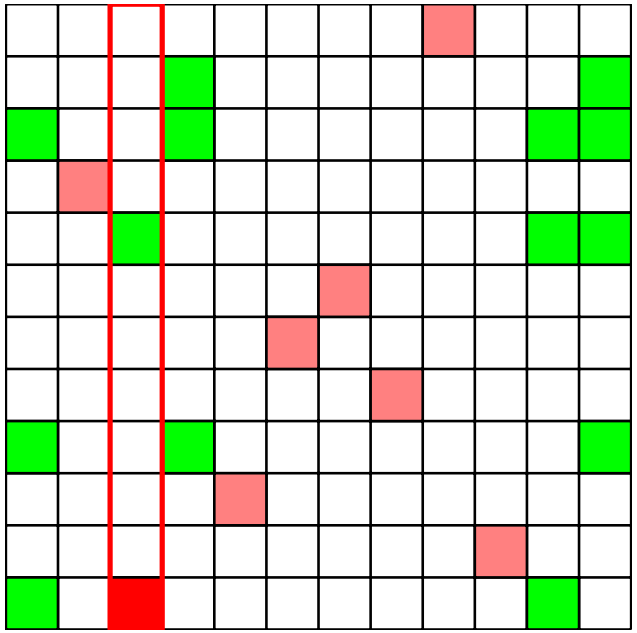

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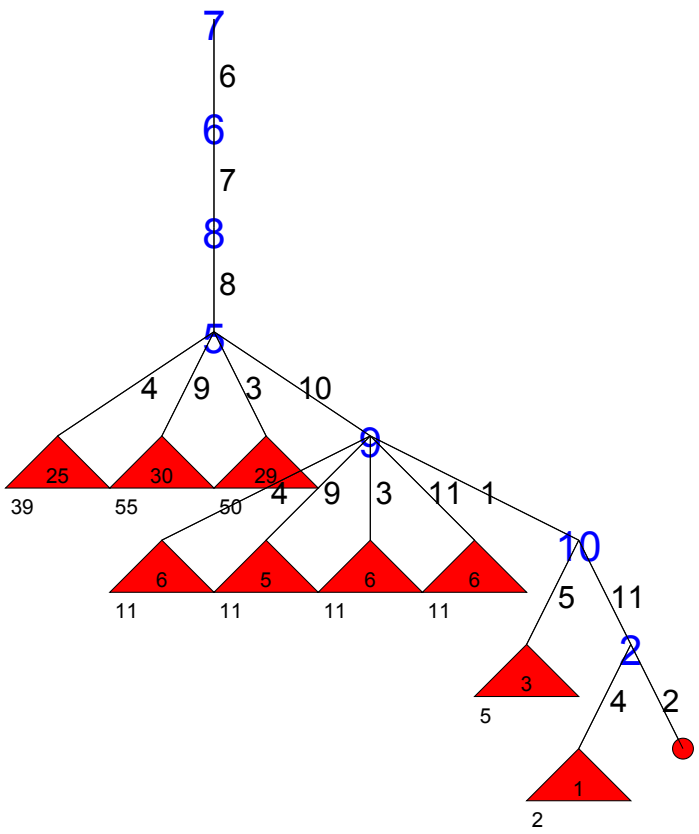

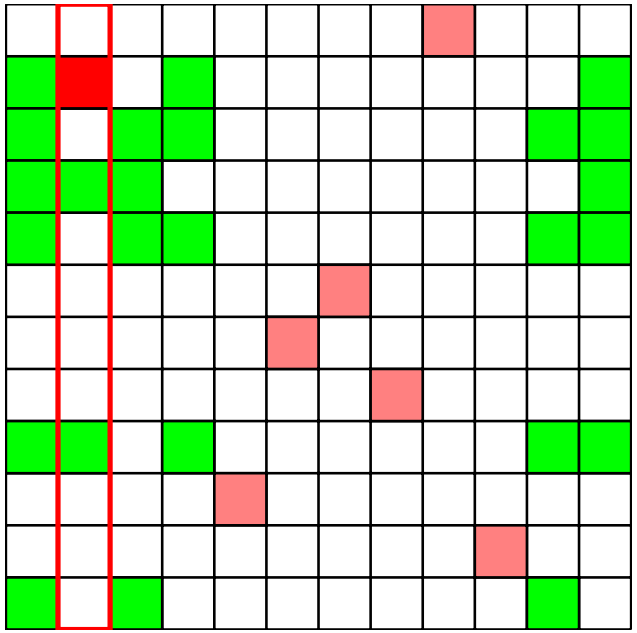

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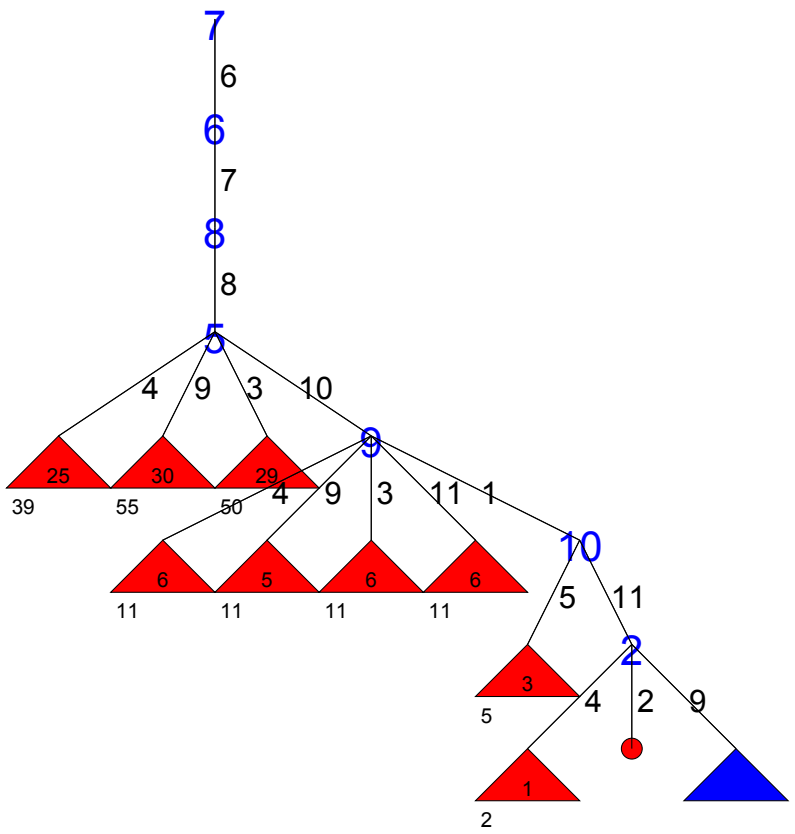

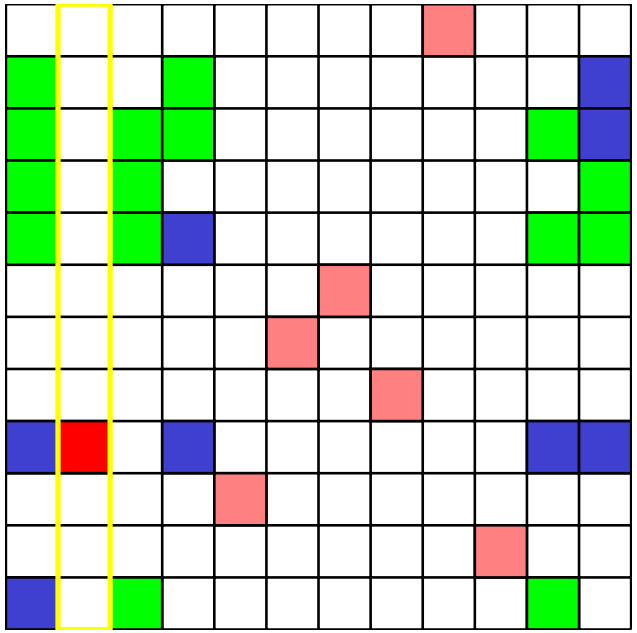

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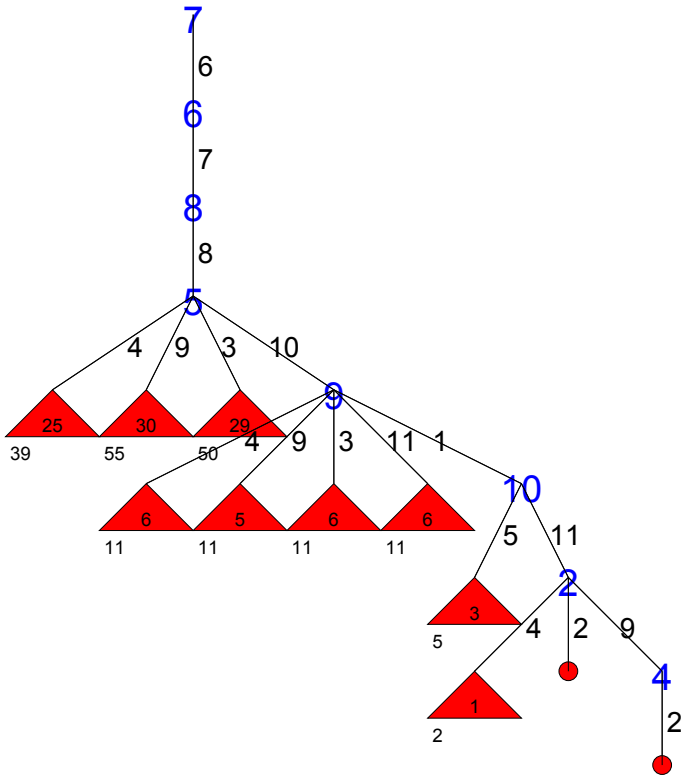

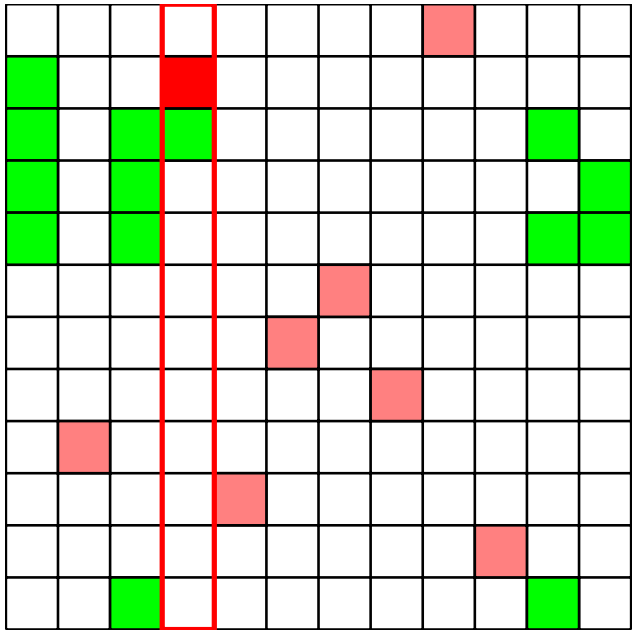

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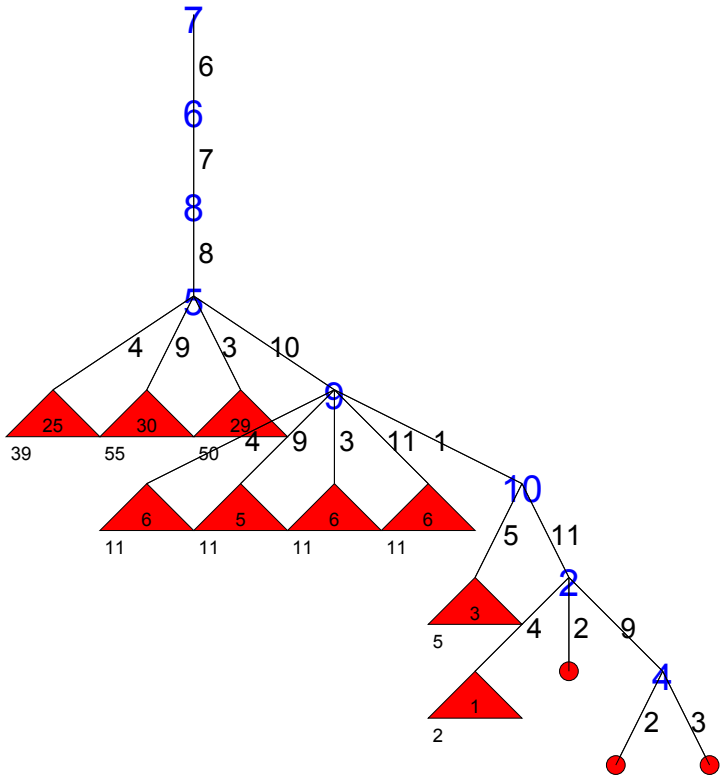

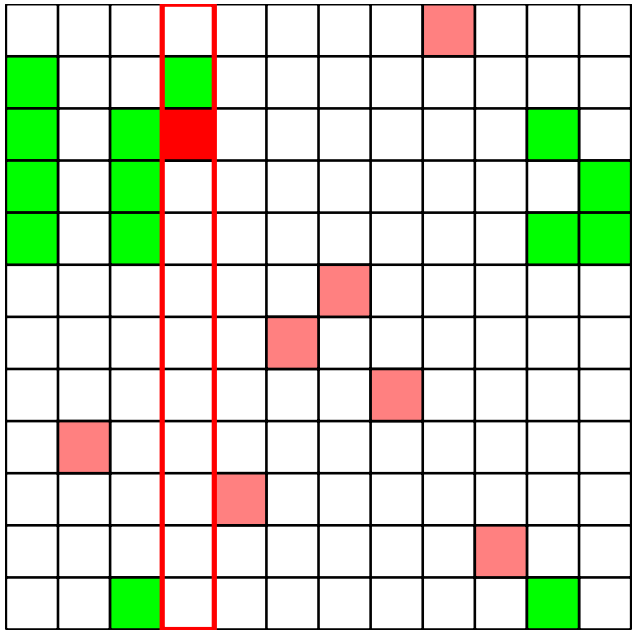

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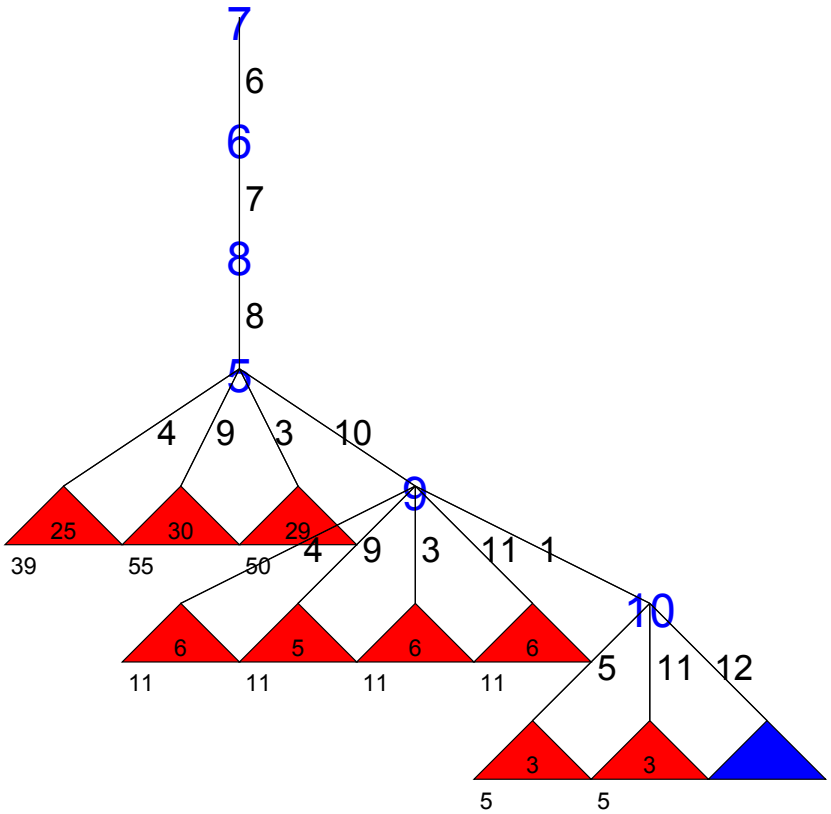

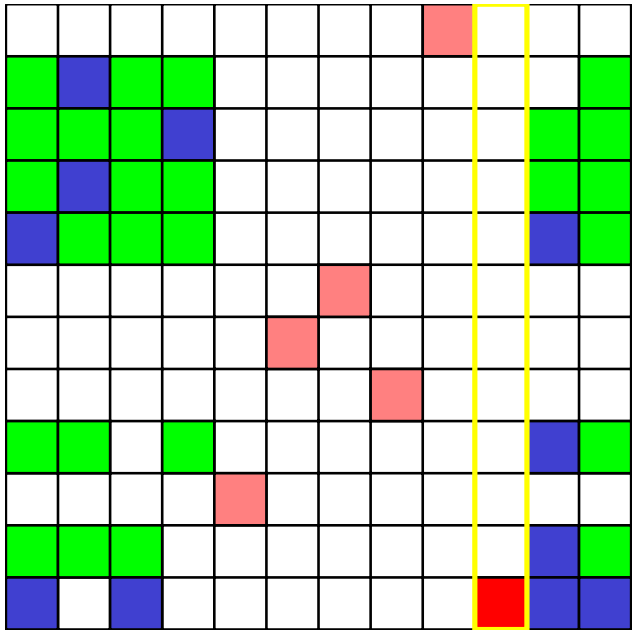

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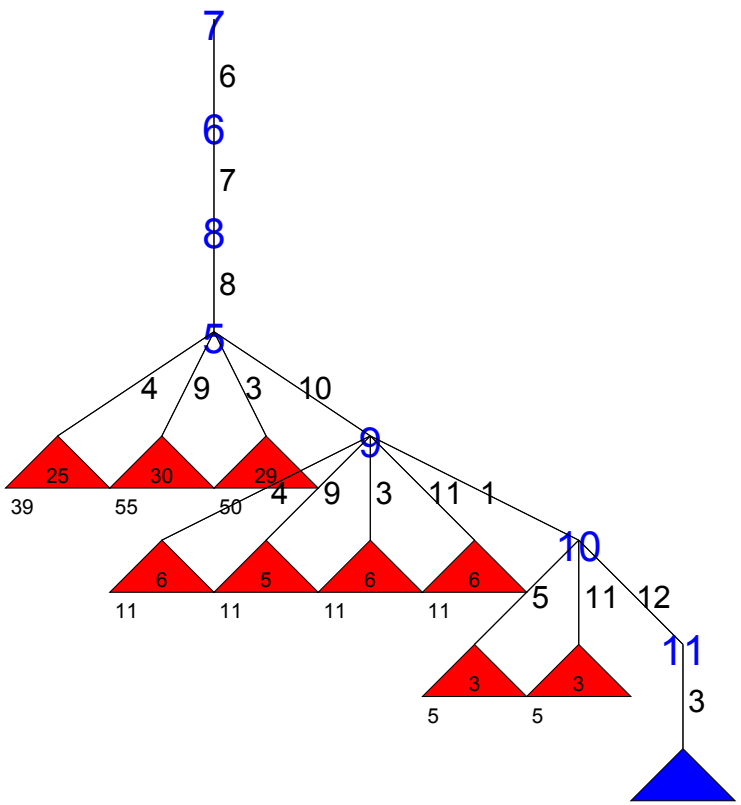

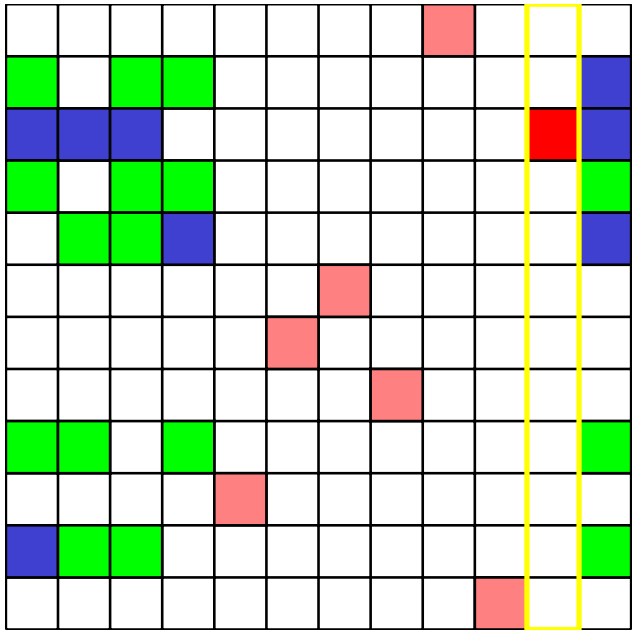

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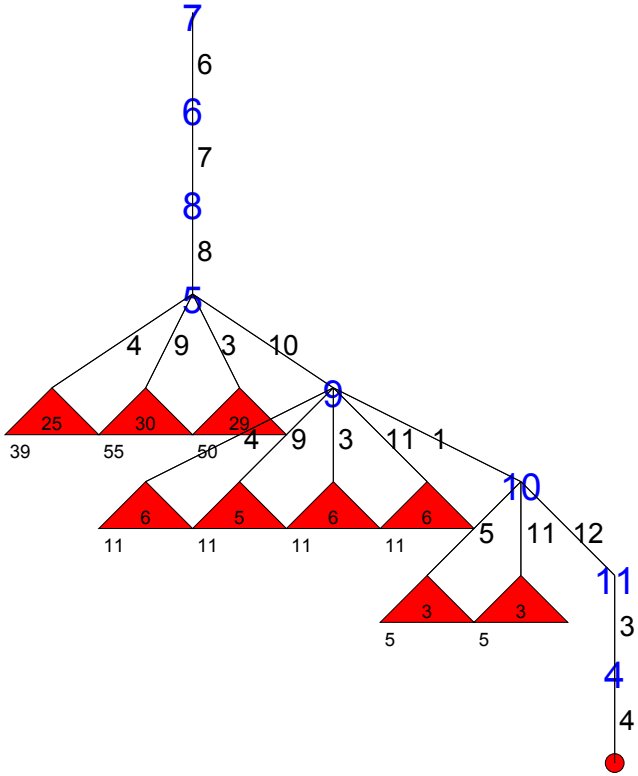

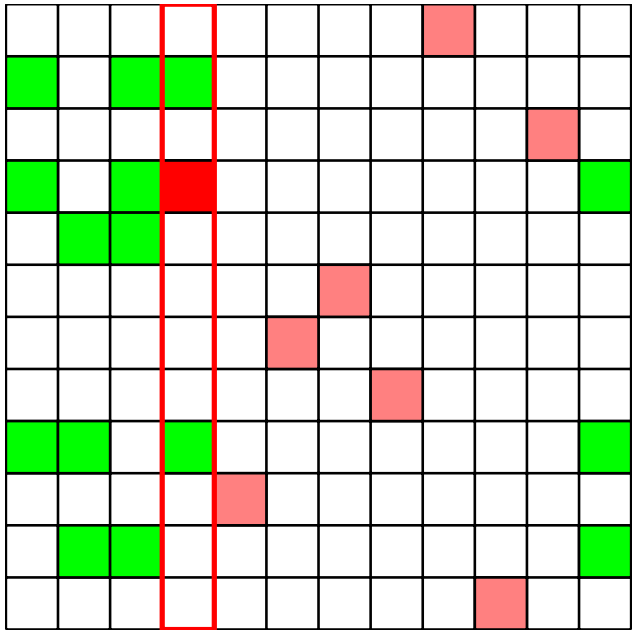

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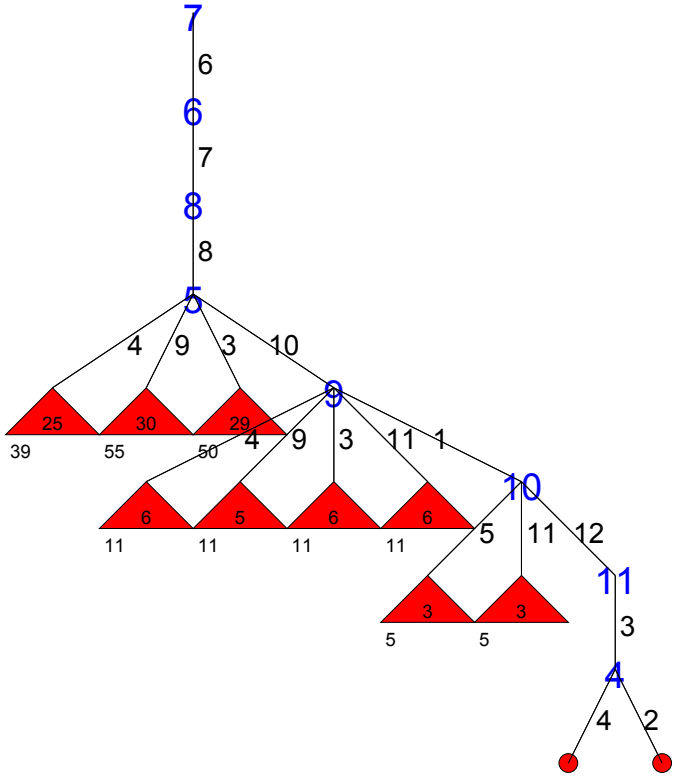

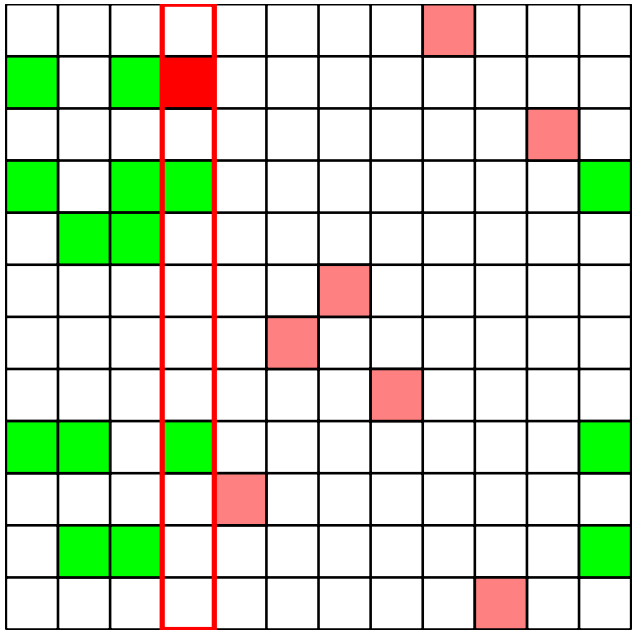

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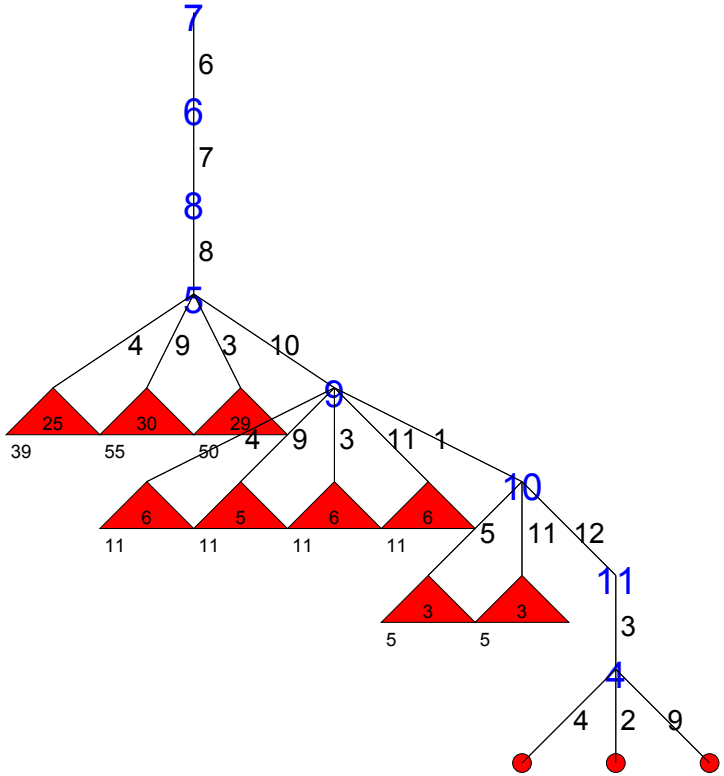

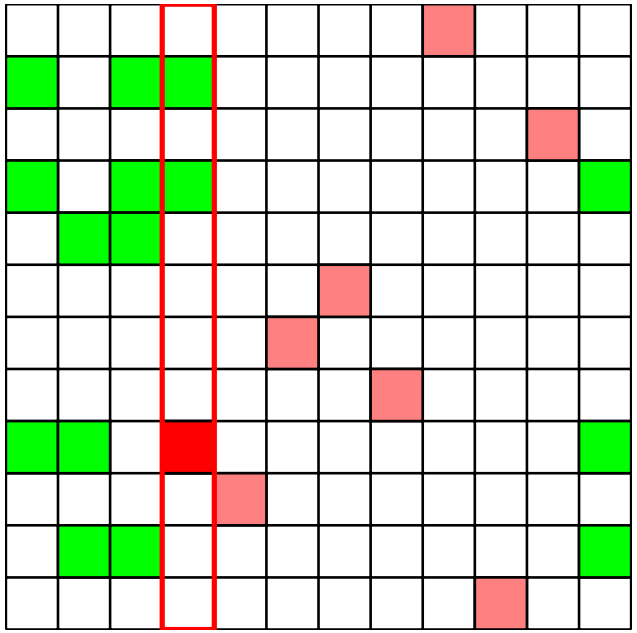

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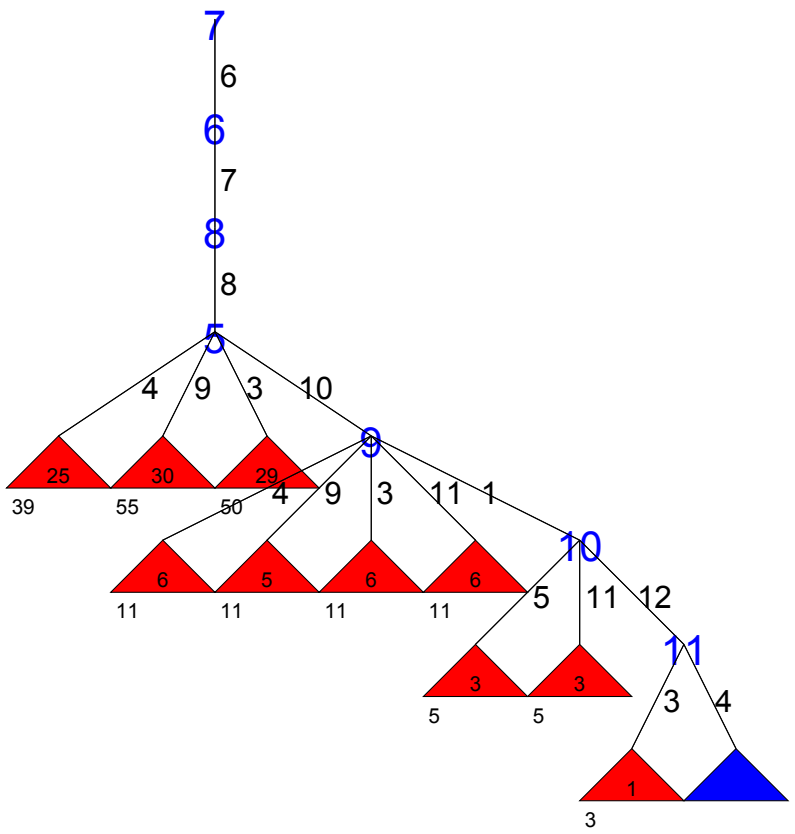

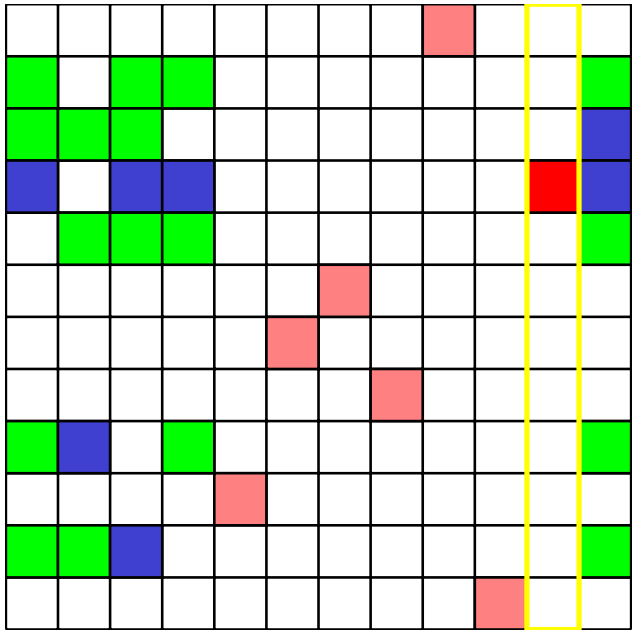

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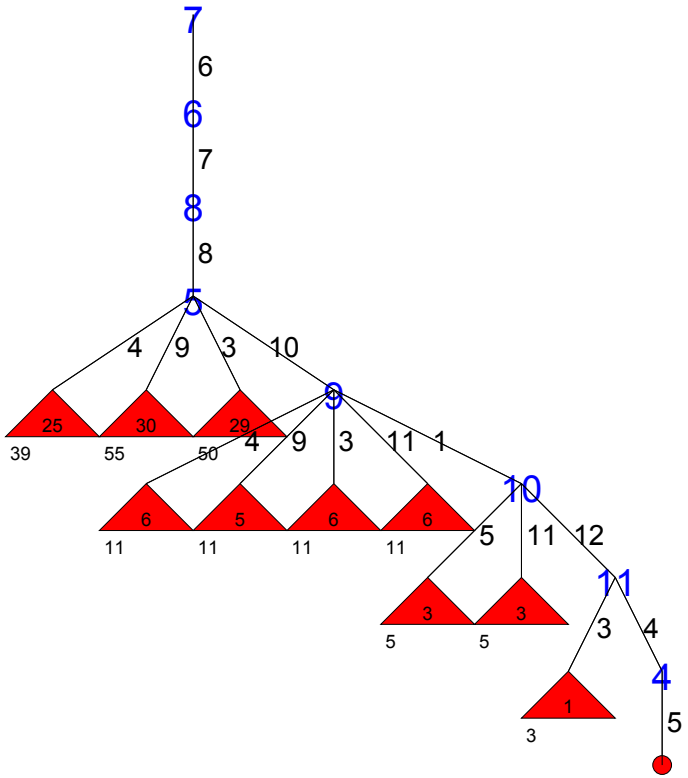

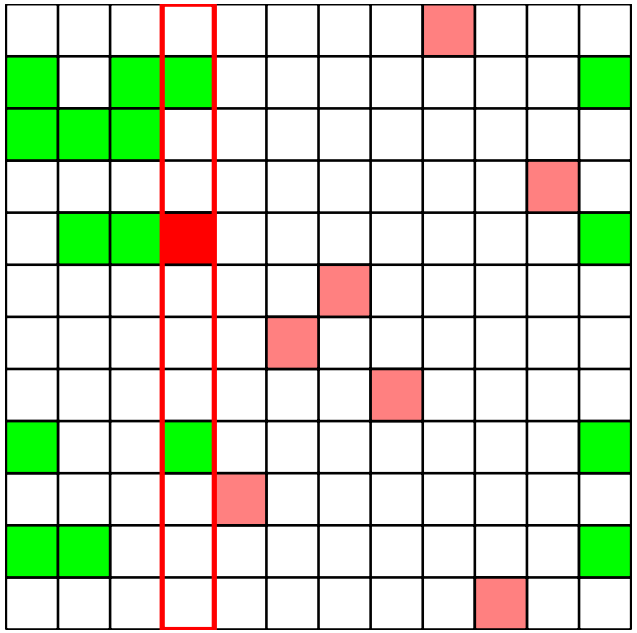

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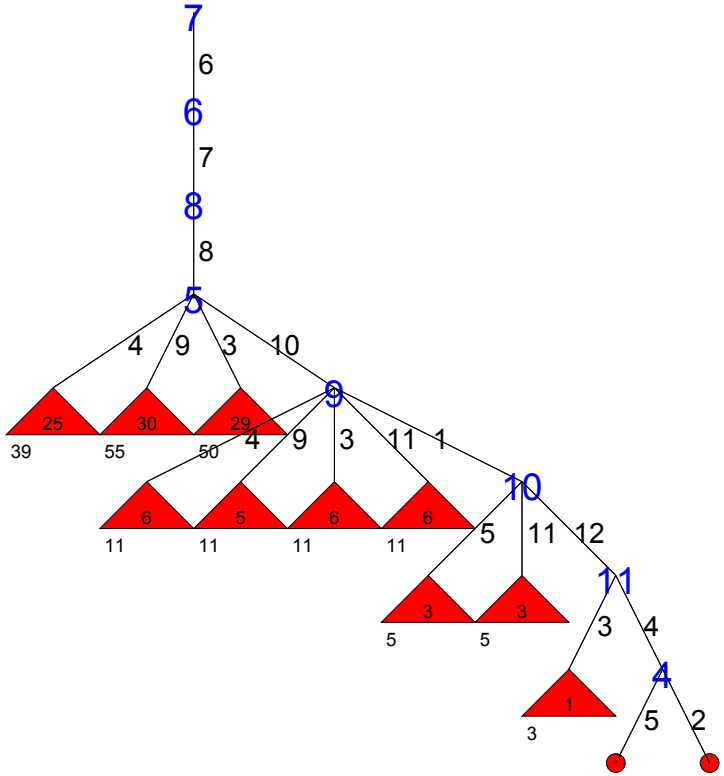

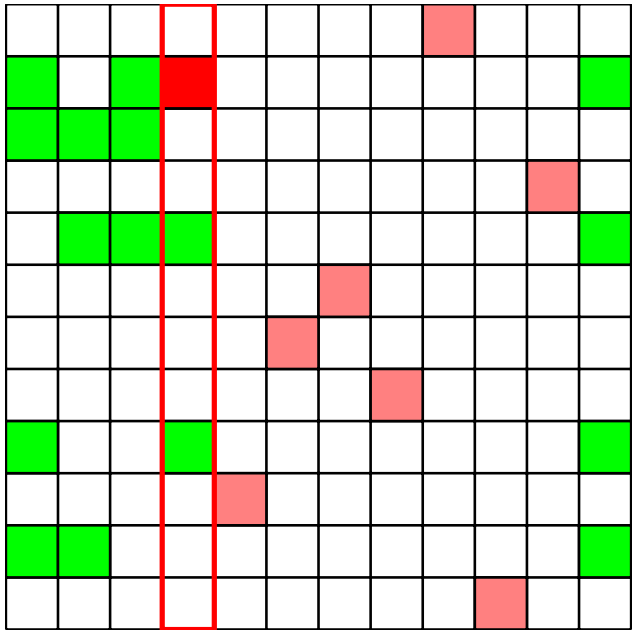

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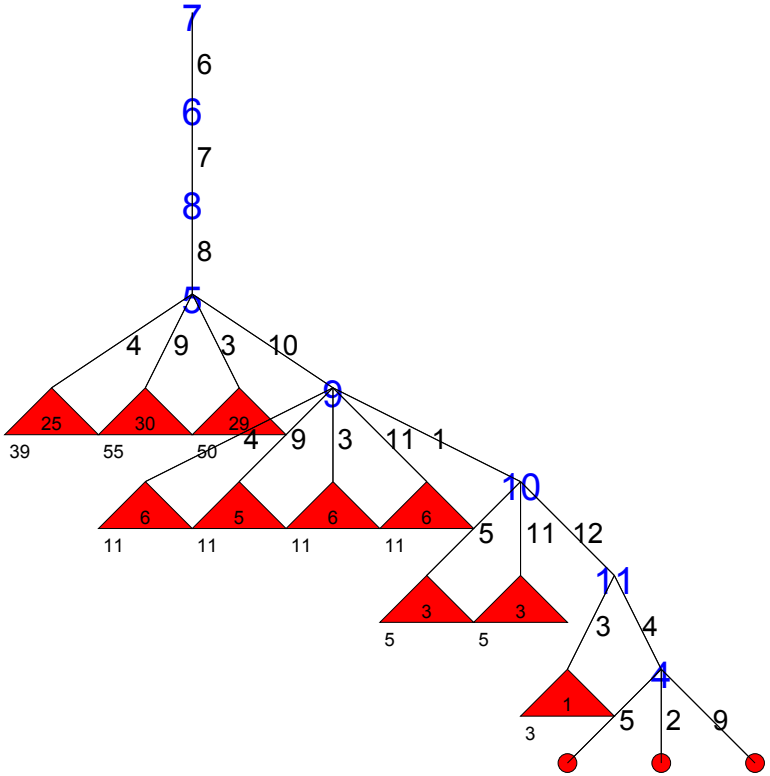

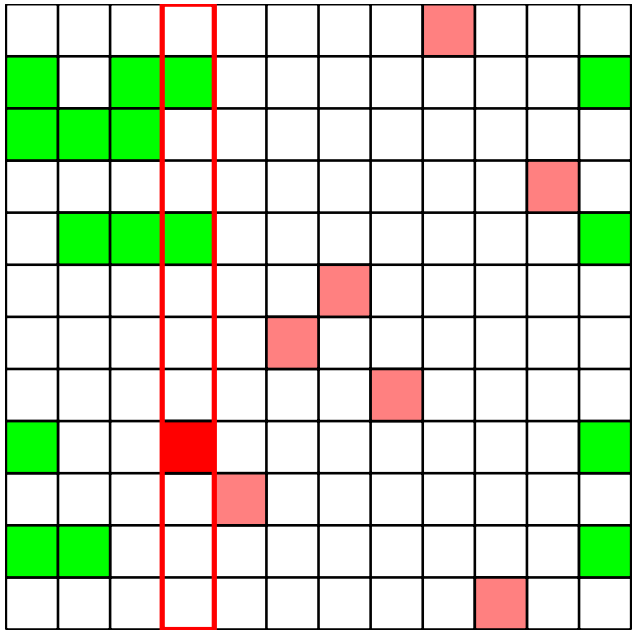

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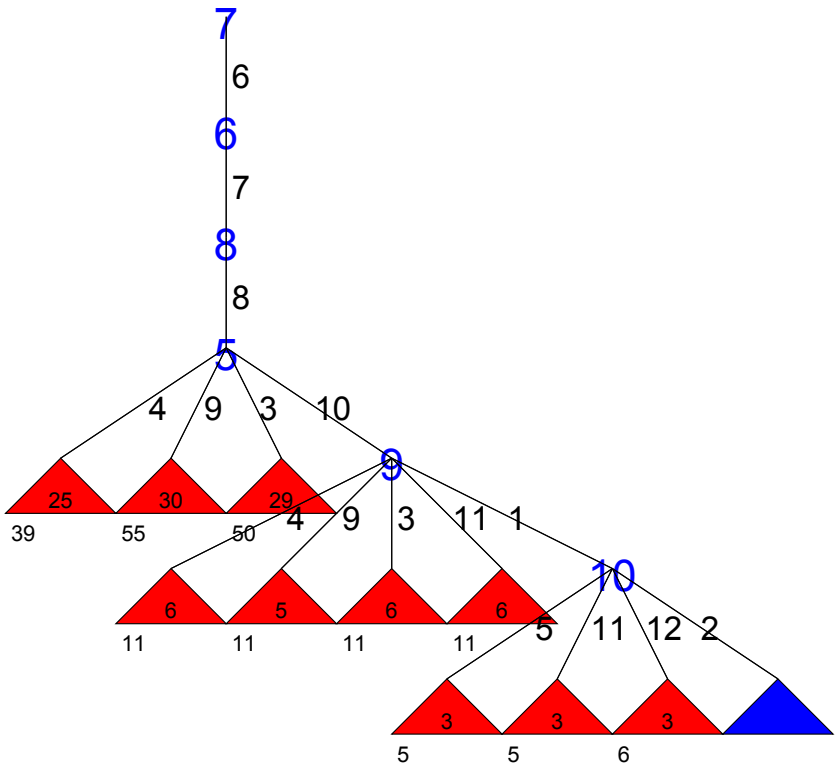

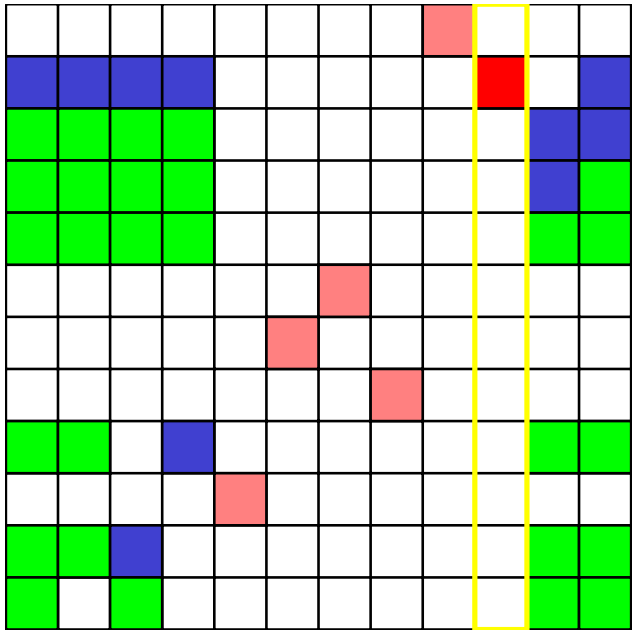

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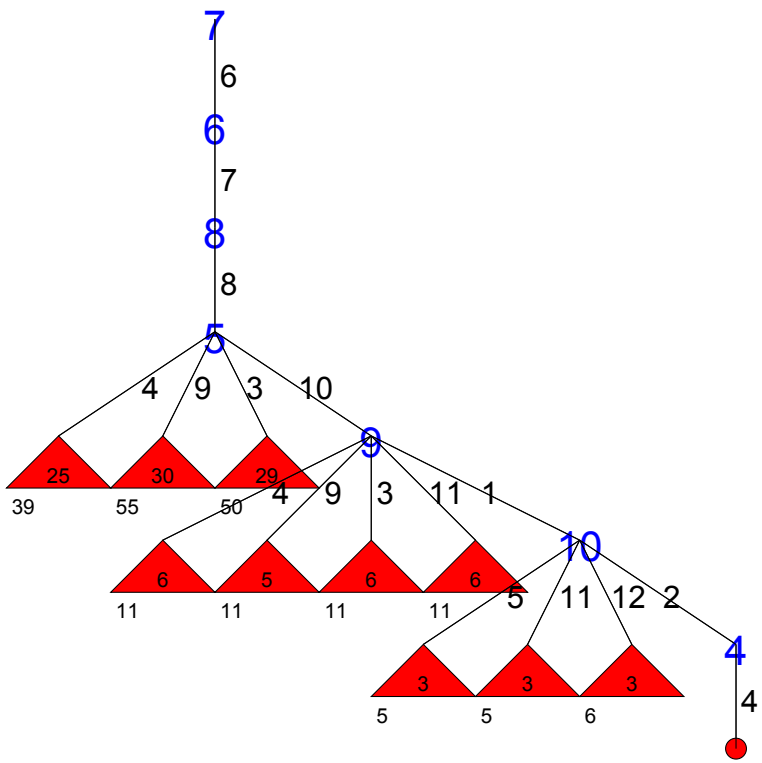

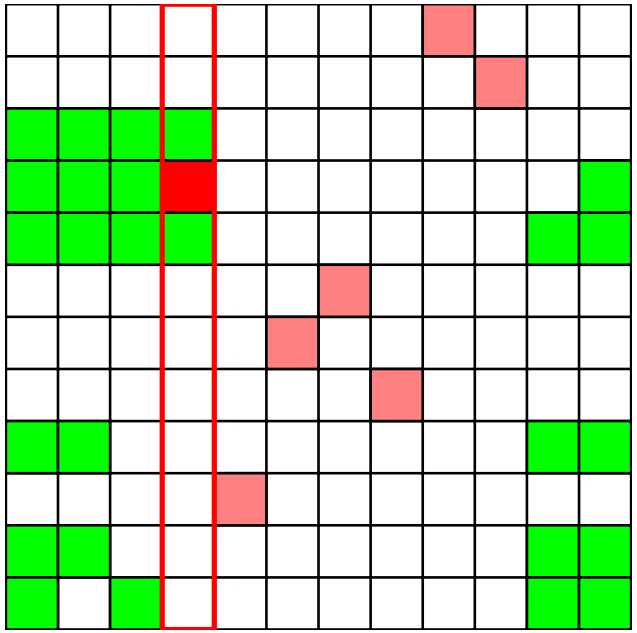

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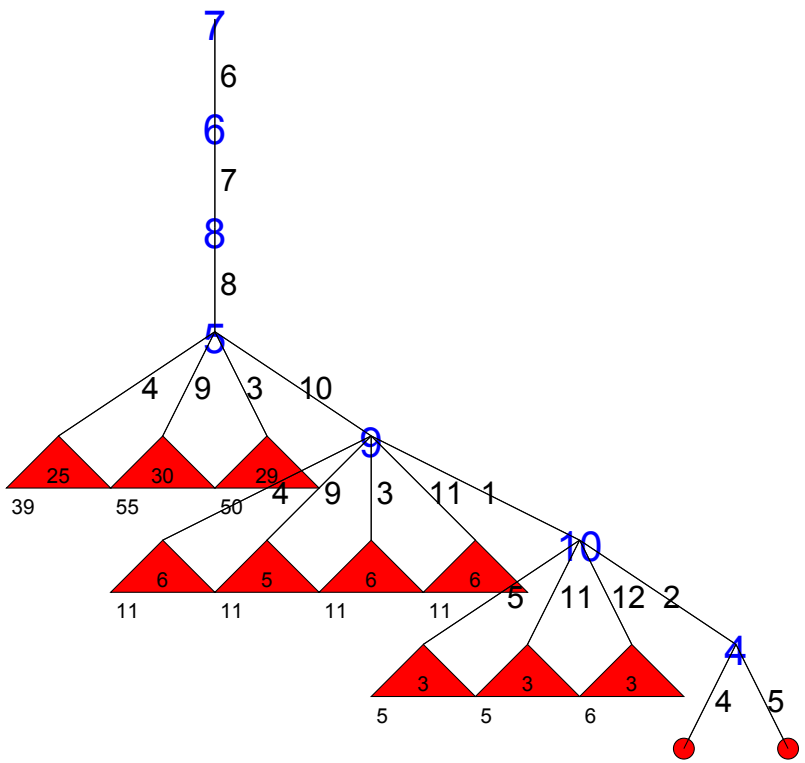

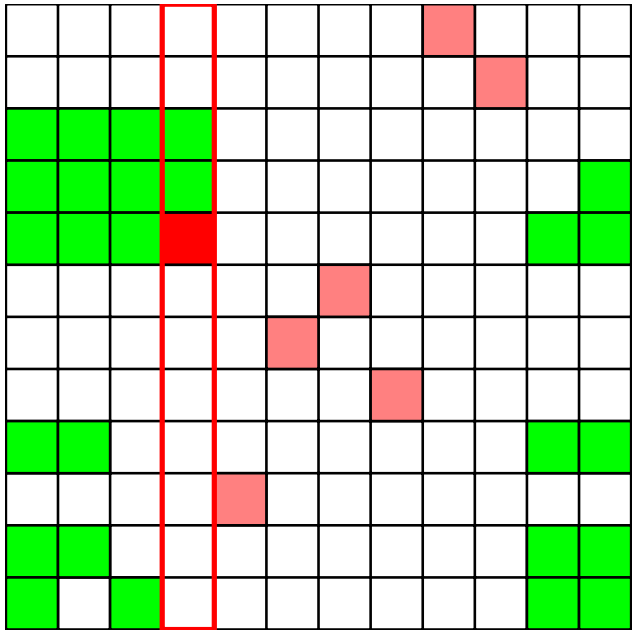

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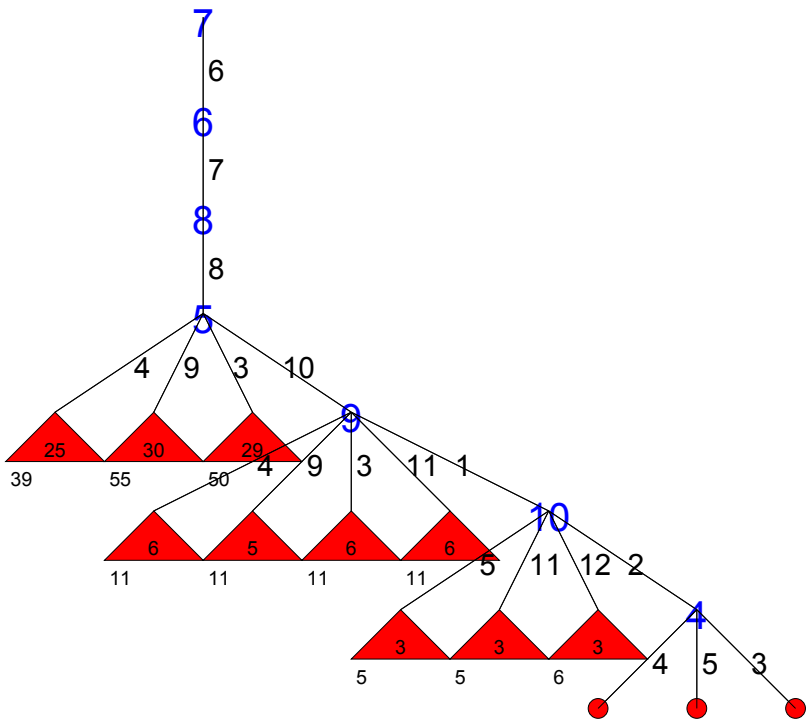

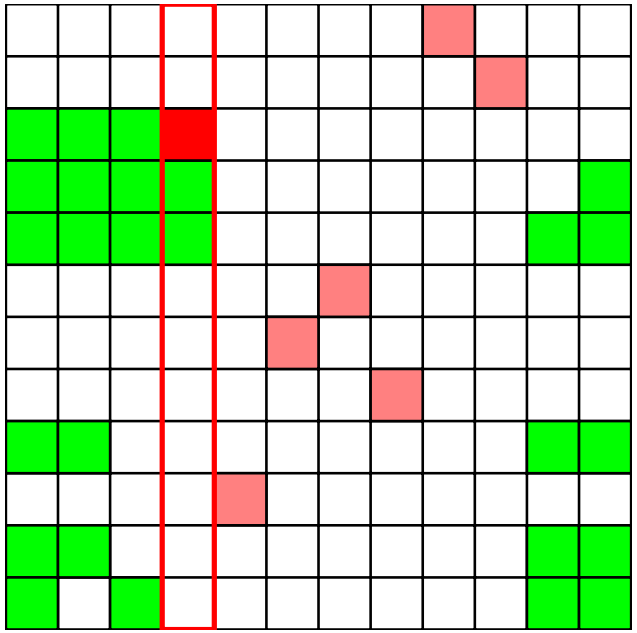

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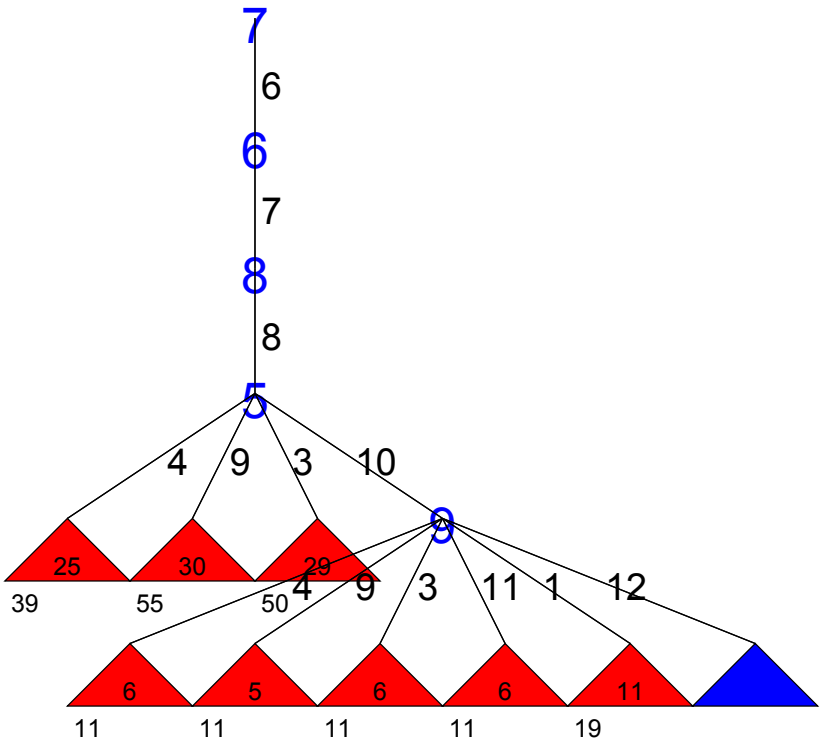

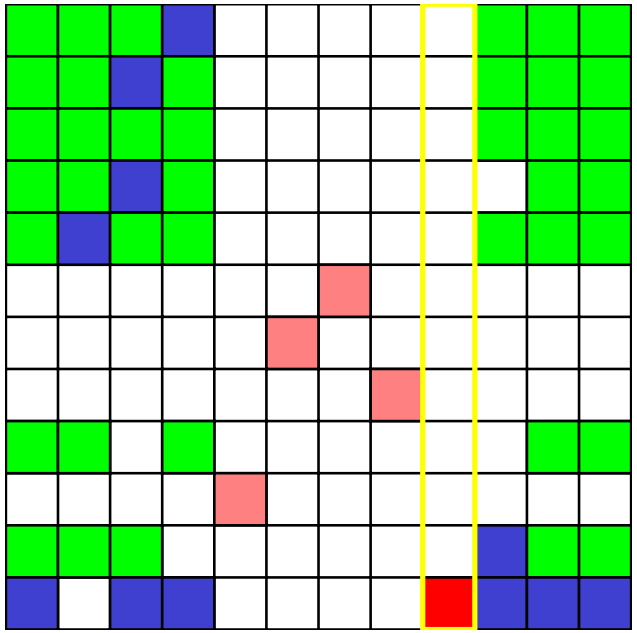

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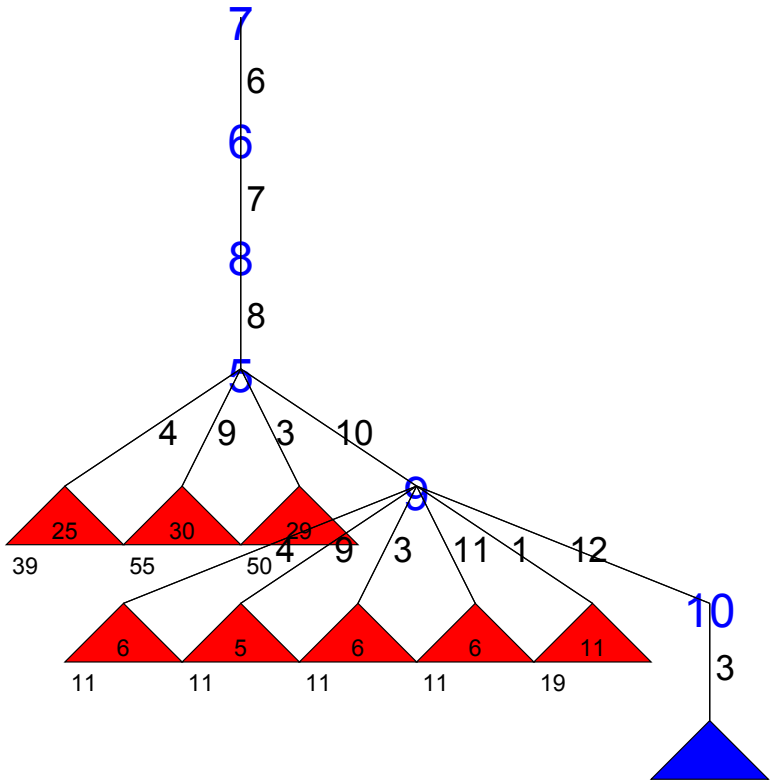

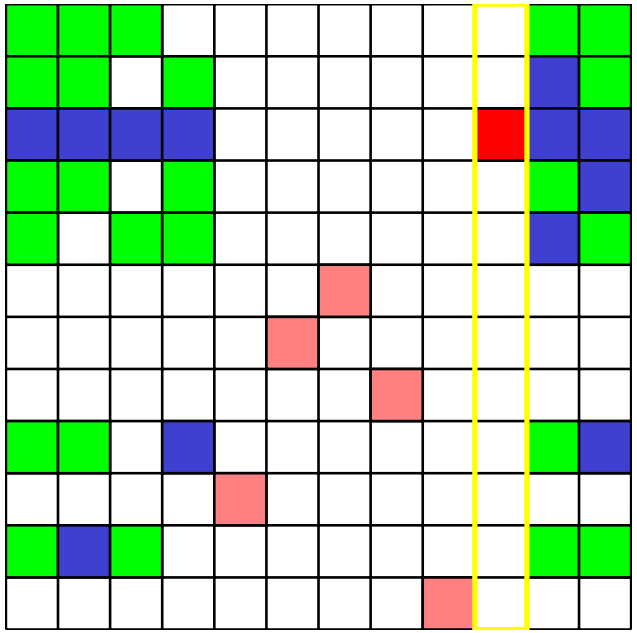

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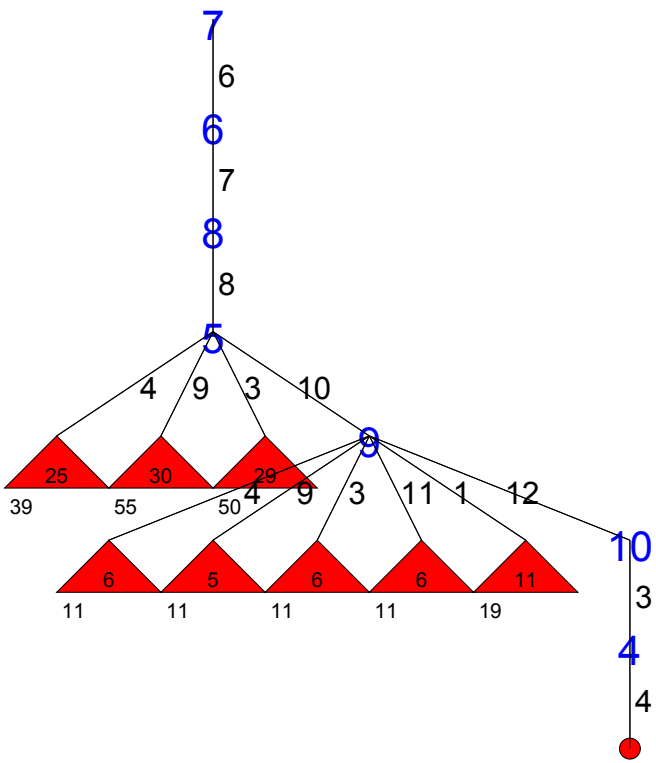

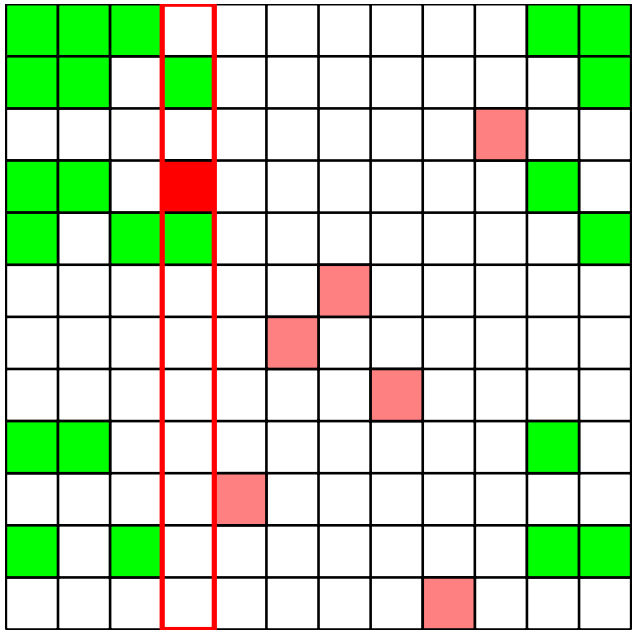

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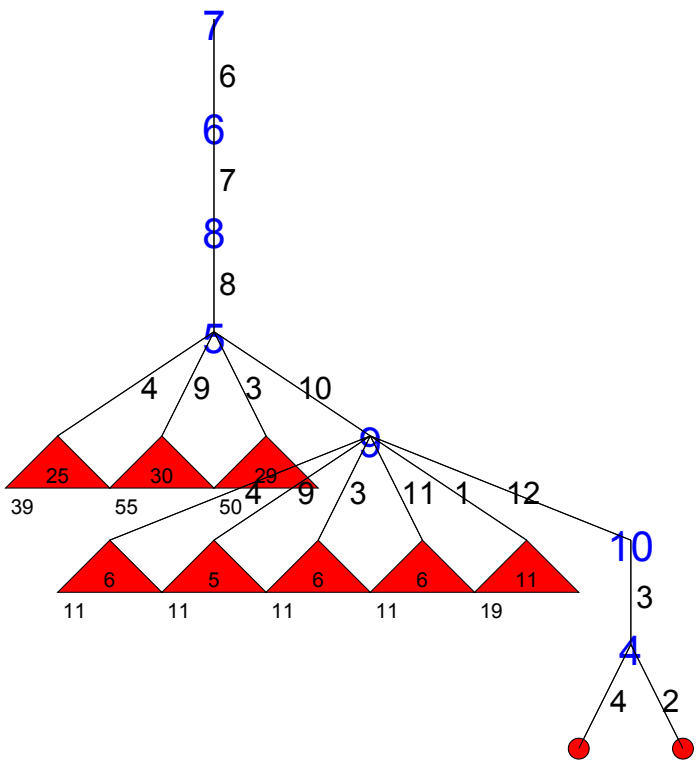

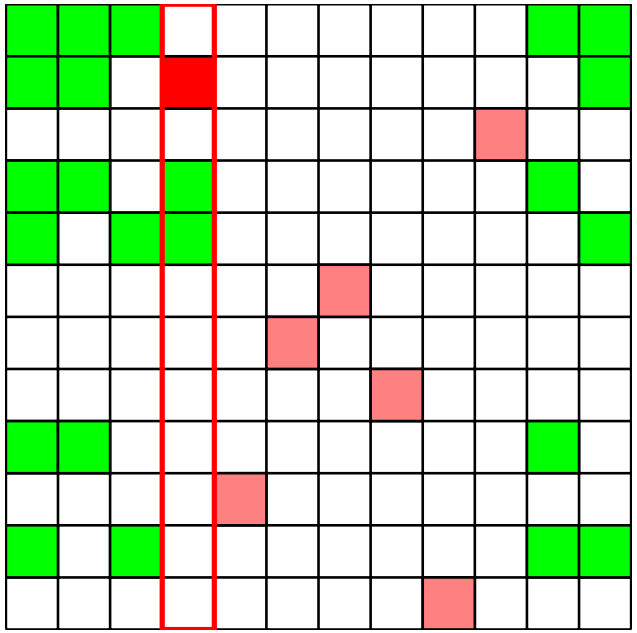

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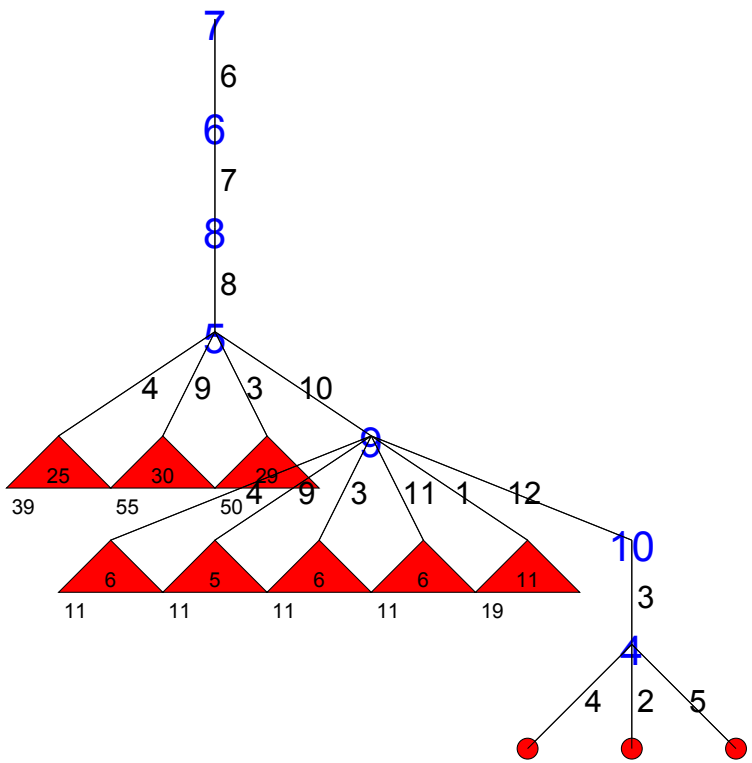

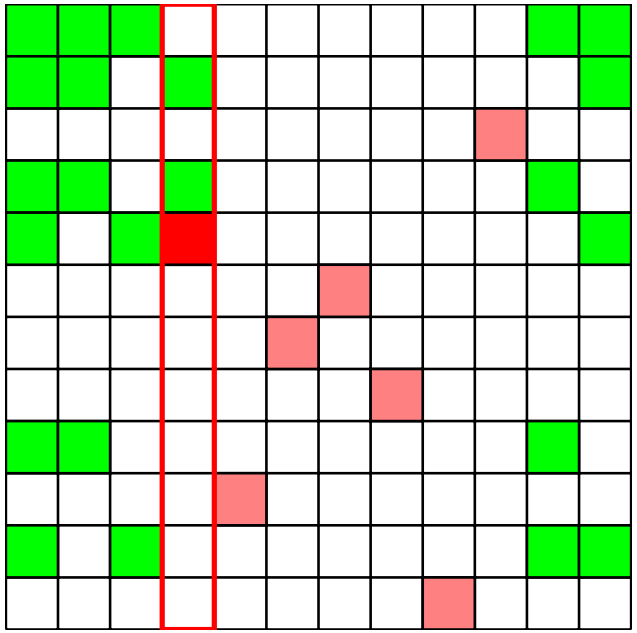

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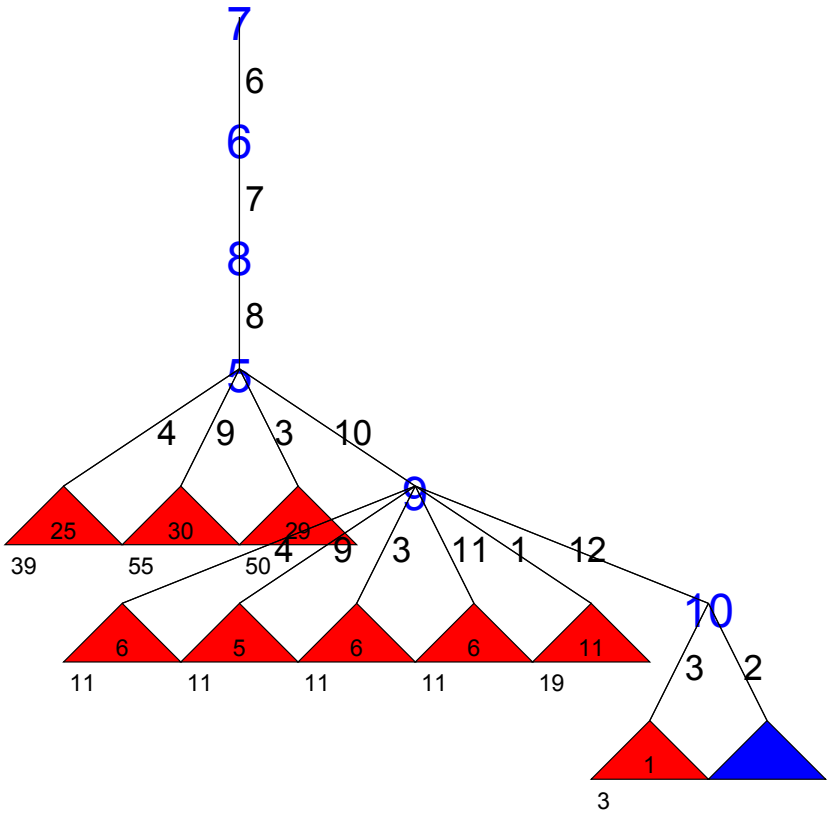

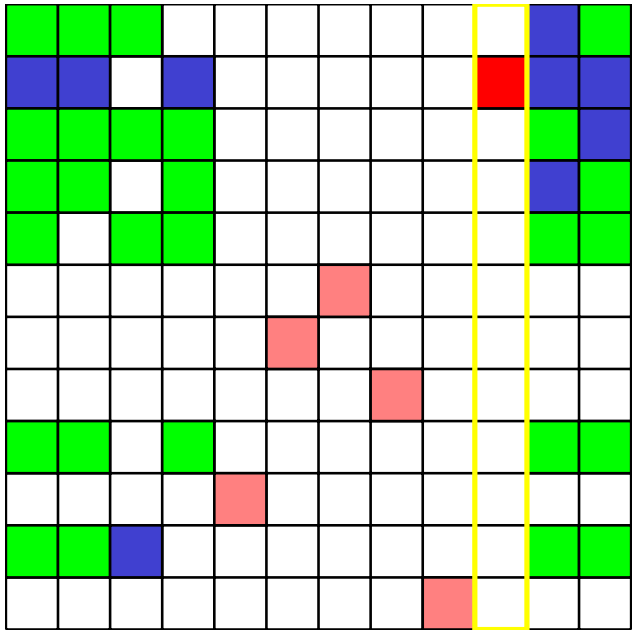

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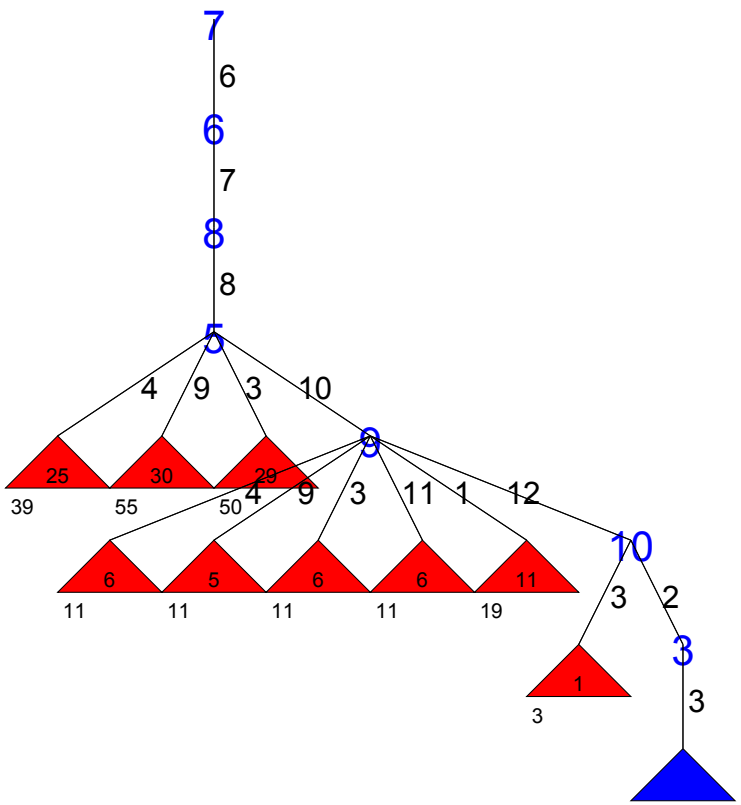

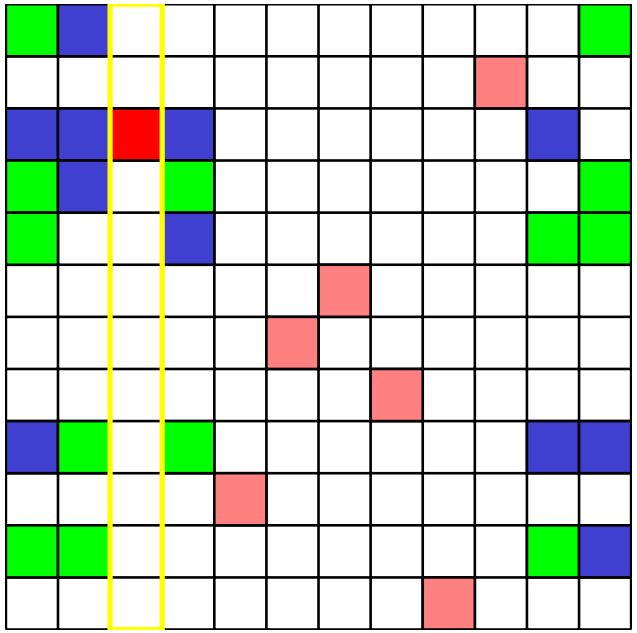

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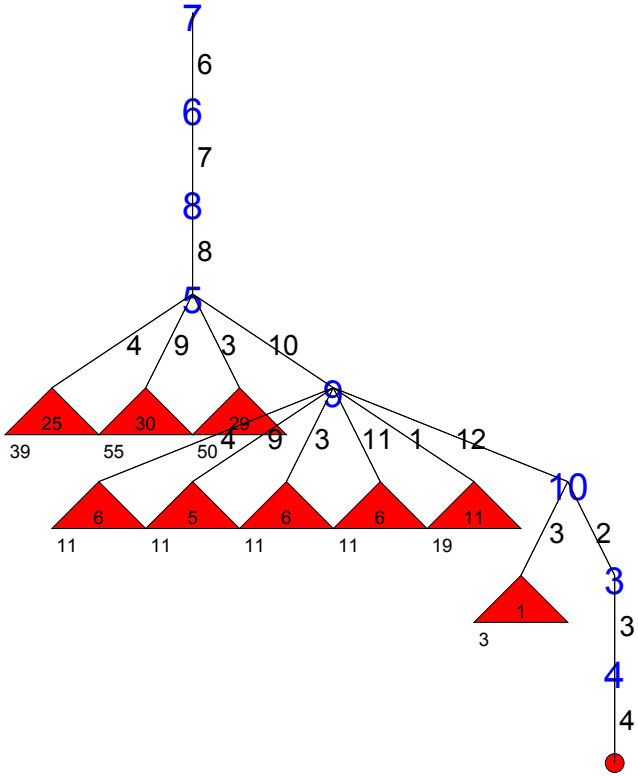

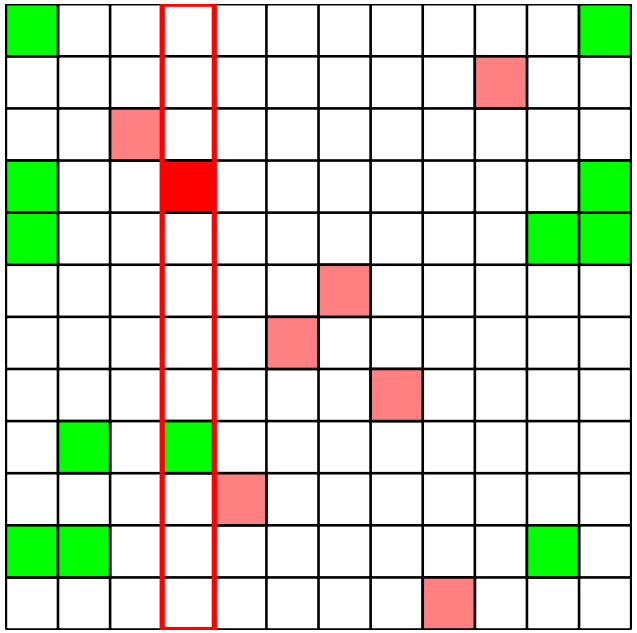

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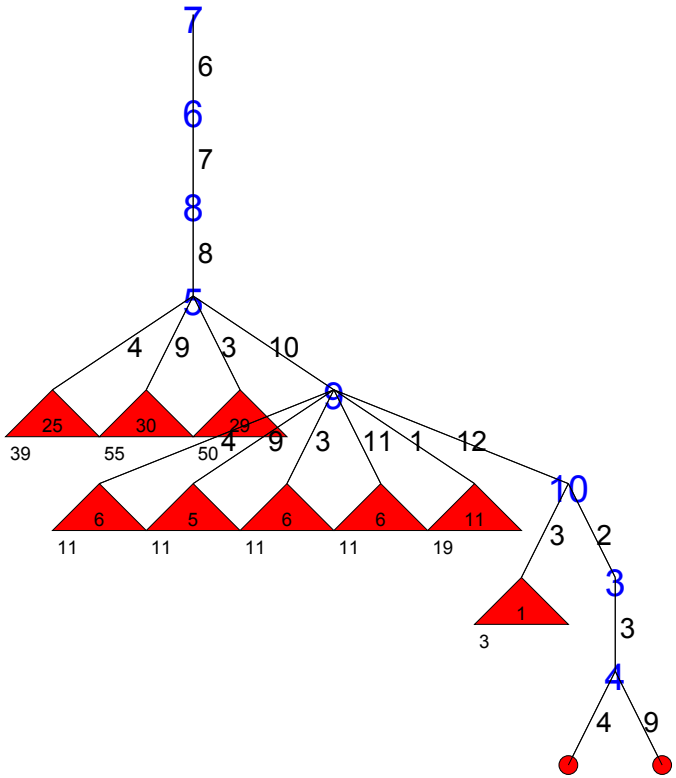

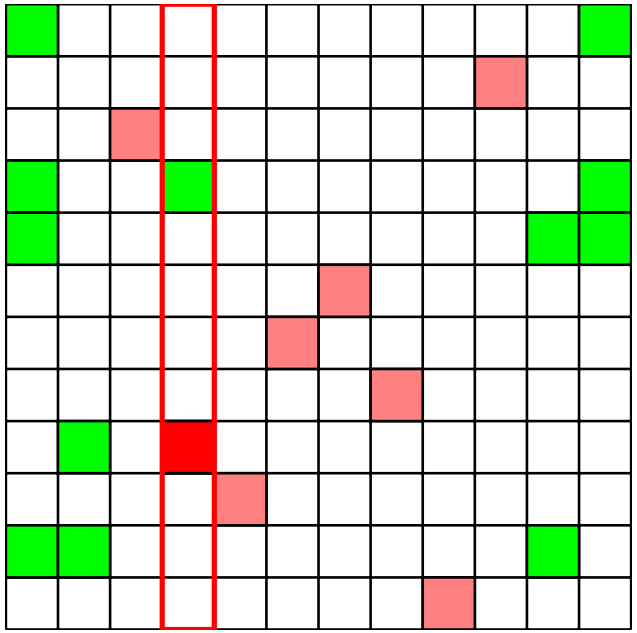

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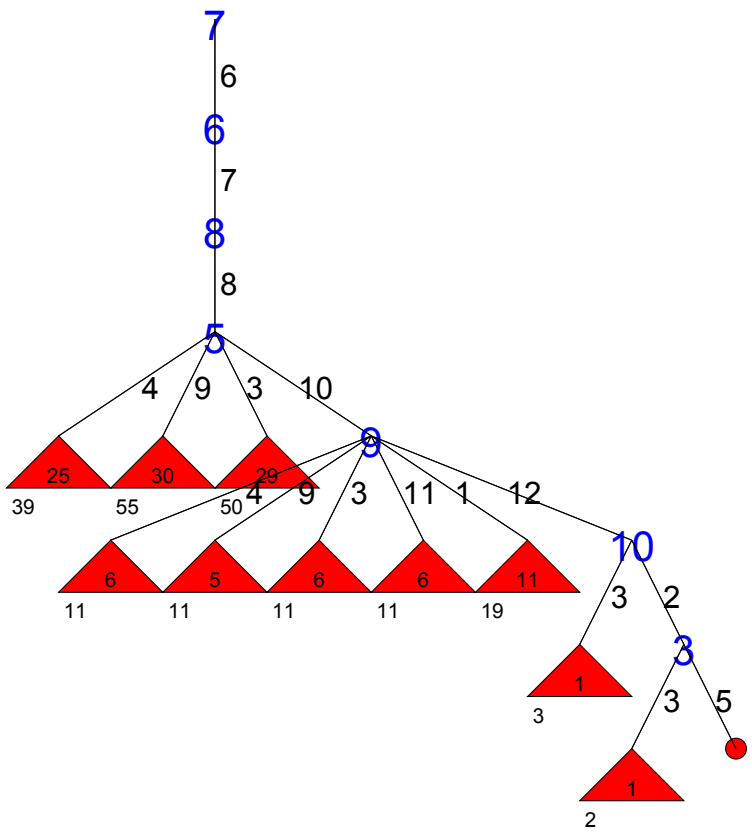

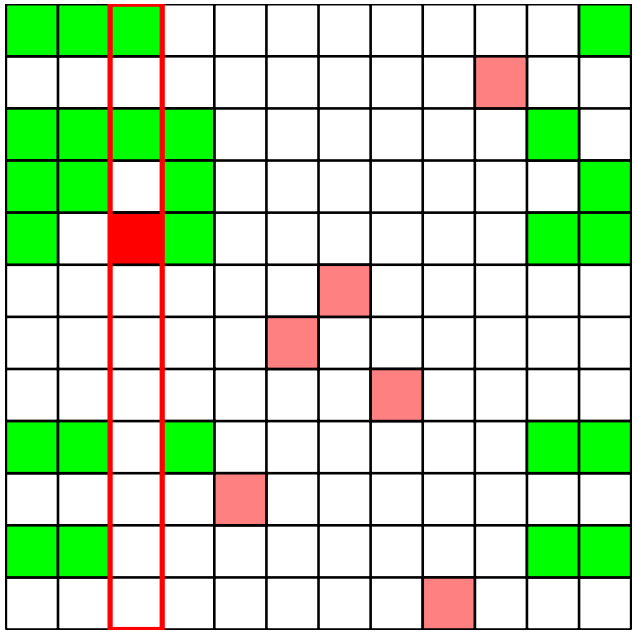

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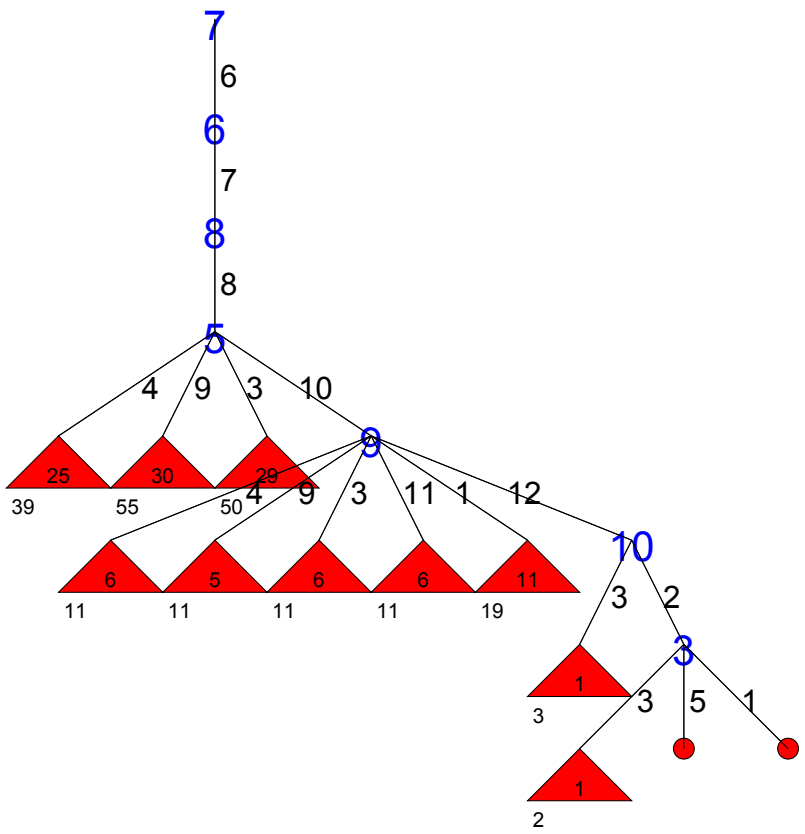

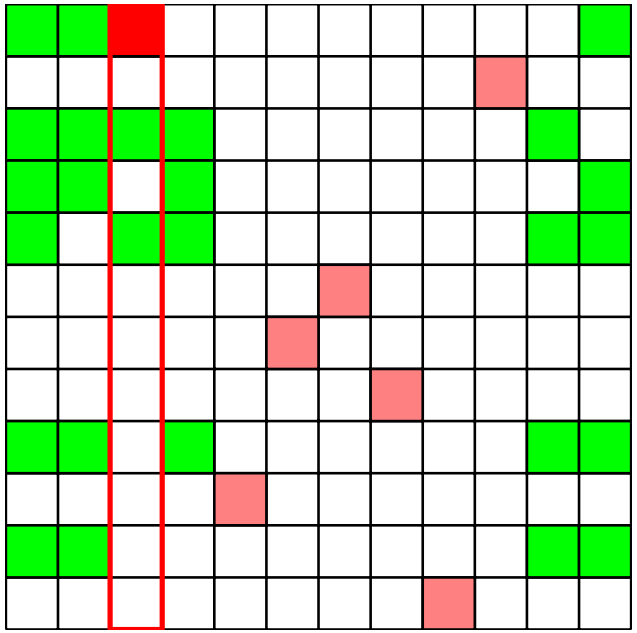

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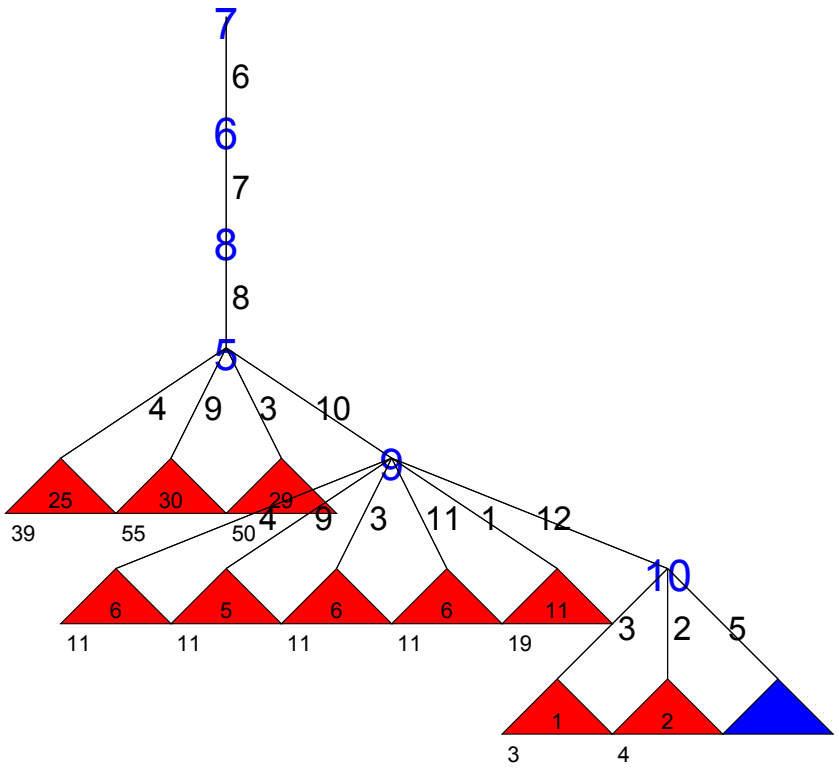

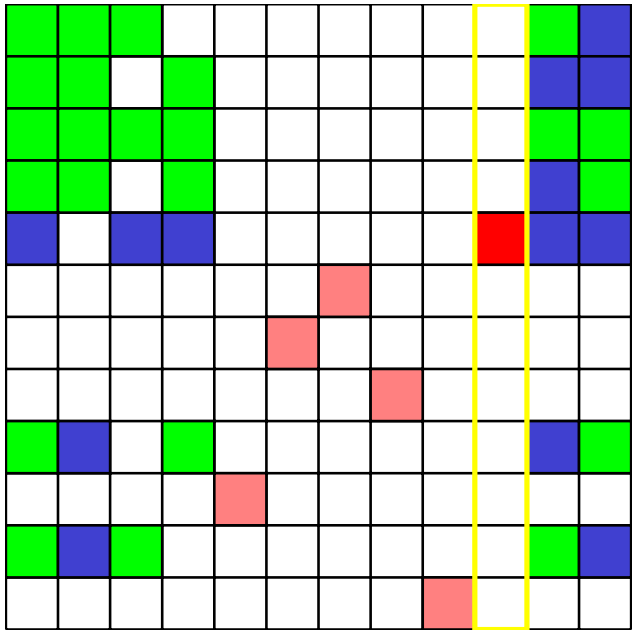

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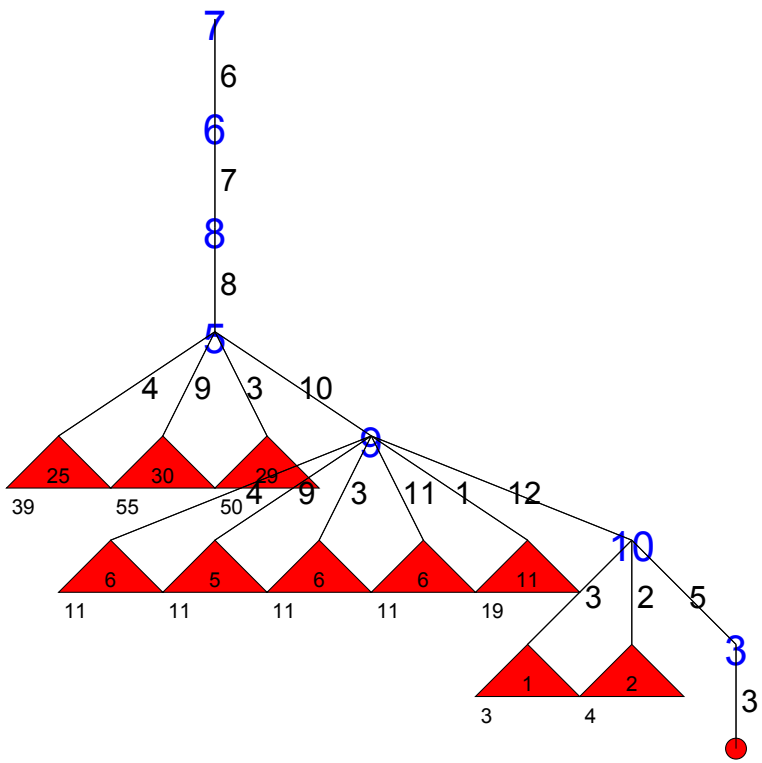

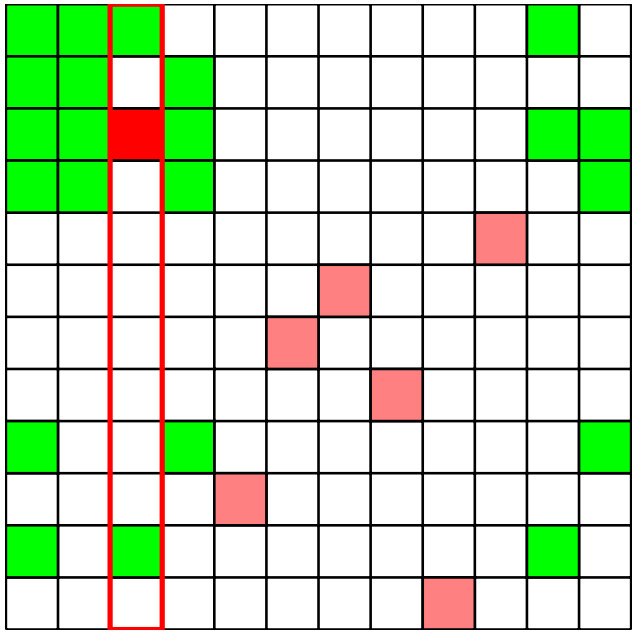

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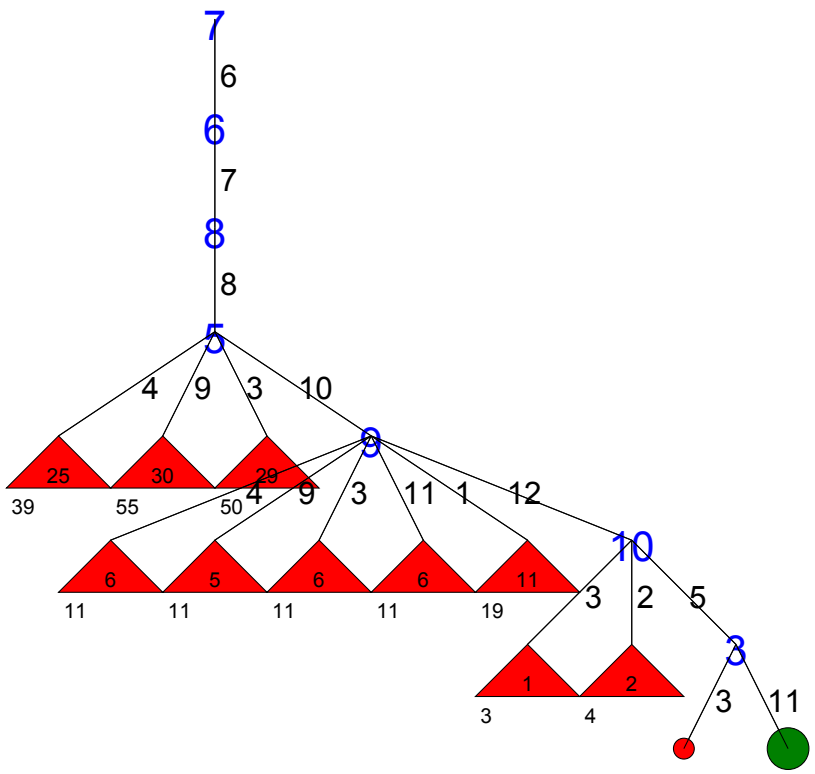

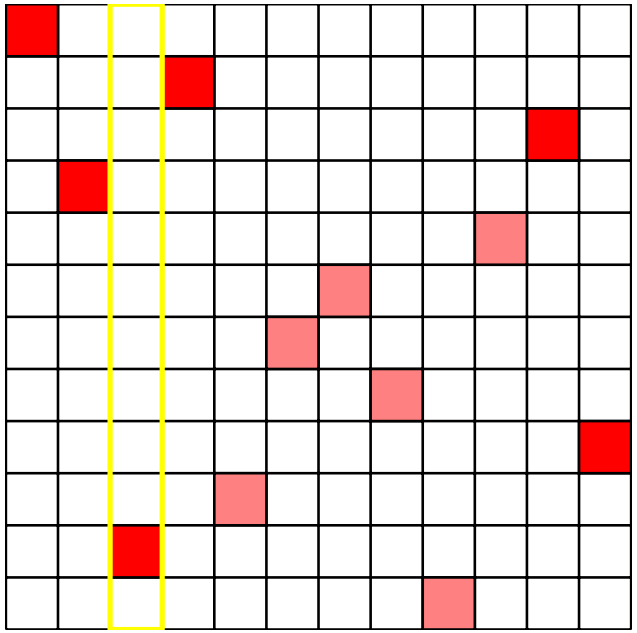

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Other Problem Sizes

	Improved Model		Improved Model, Middle	
Size	Backtrack	Time	Backtrack	Time
10	4	0.16	1	0.01
11	77	1.44	13	0.03
12	31	0.94	72	0.26
13	216	6.22	513	1.81
14	2875	95.94	589	2.37
15	25820	1046.75	7840	34.30
16	84161	4099.52	13158	63.91
17	825590	49371.02	56390	298.16
18	55102	4530.83	19750	115.64

Observation

- Big improvement in backtracks and time
- Not for all problem sizes
- Question: Do we need improvement of model for this to work?
- Experiment: Run changes search routine on basic model

Labeling Basic Model from Middle

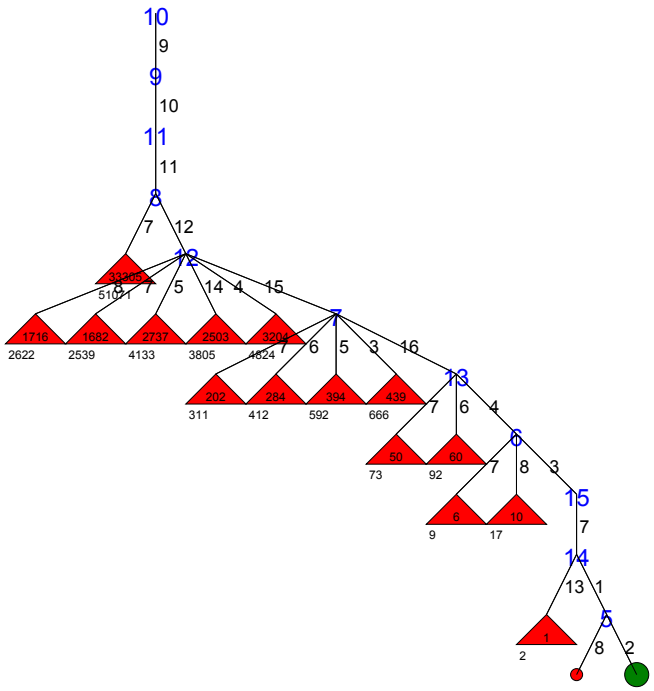
	Basic Model		Basic Model, Middle	
Size	Backtrack	Time	Backtrack	Time
10	4	0.00	1	0.00
11	118	0.08	17	0.01
12	50	0.05	97	0.09
13	335	0.36	644	0.74
14	5008	6.23	746	1.03
15	47332	68.92	10041	16.03
16	157773	271.22	17005	31.12
17	1641685	3278.19	73080	152.72
18	115745	283.97	28837	60.97

Comparison: Model Impact

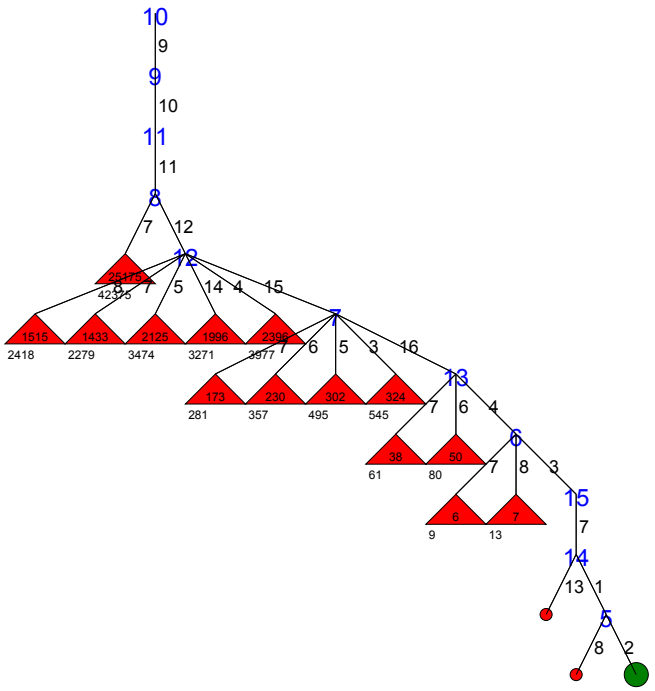
	Basic Model,Middle		Improved Model, Middle	
Size	Backtrack	Time	Backtrack	Time
10	1	0.00	1	0.01
11	17	0.01	13	0.03
12	97	0.09	72	0.26
13	644	0.74	513	1.81
14	746	1.03	589	2.37
15	10041	16.03	7840	34.30
16	17005	31.12	13158	63.91
17	73080	152.72	56390	298.16
18	28837	60.97	19750	115.64
19	1187618	3174.72	1044751	4474.56

Comparison (Search Tree, size 18)

Initial Model



Improved Model



Observation

- Search strategy does not depend on model
- Variable selection is the same!
- Basic model is about two times faster
- About 50% more backtrack steps
- Again, sometimes reasoning does not pay!
- Better search strategy pays off dramatically

Outline

5 0/1 Models

A Different Model

- Model shown is not the only way to express problem

0/1 Models

- SAT (Minisat)
- Pseudo Boolean (Minisat+)
- MIP (Coin-OR)

0/1 Models: Variables

- X_{iv}: Variable i takes value v
- D_{ijv}: Difference between variables i and j is v

MIP Model: Constraints

- alldifferent between variables
  - $\sum_i X_{iv} = 1$
  - $\sum_v X_{iv} = 1$
- alldifferent between differences
  - $\sum_v D_{ijv} = 1$
  - $\sum_{i-j=c} D_{ijv} \leq 1$
- link between variables and differences
  - $D_{ijv} = \sum_{v1=v2+v} X_{iv_1} X_{jv_2}$

More Information

- http://www.costasarrays.org/