The astronomical units

Nicole Capitaine and Bernard Guinot

Observatoire de Paris / SYRTE , France

l'Observatoire de Paris — SYRTE

Systèmes de Référence Temps-Espace

Introduction

The IAU-1976 System of astronomical constants includes 3 astronomical units:

– *The astronomical unit of time, i.e. the day (D),*

- is related to the SI second by a defining number (*D*=86400 s),
- its role is to provide a unit of time of "convenient" size for astronomy, as is the Julian century of 36 525 days,
- the "day" appears in the SI Brochure (Table 6), along with the minute and the hour, as one of the "Non-SI units accepted for use with the International System of Units“.

– *The astronomical unit of length, ua, and the mass of the Sun,* M_{Sun}*,*

- are specific astronomical units for expressing distances and masses in the solar system,
- the best estimated values in SI of these astronomical units have been regularly improved in the successive lists of numerical standards,
- although these two astronomical units are still acknowledged as being appropriate for expressing distances and masses in the solar system, the current definition and use of the ua lead to some ambiguities and difficulties.

Aim of this presentation: discuss the status of the astronomical unit of length and mass within the modern context

The current definition of the astronomical unit of length (ua)

Definition of the ua in the IAU-1976 System of astronomical constants

The astronomical unit of length is that length (A) for which the Gaussian gravitational constant (k) takes the value of 0.017 202 098 95 when the units of measurements are the astronomical unit of length, mass and time. The dimensions of k^2 are those of the constant of gravitation (G), i.e., $L^3M^{-1}T^{-2}$. The term "unit distance" is also for the length A.

Definition of the of the ua in the SI brochure (intended to non-astronomers)

The astronomical unit is approximately equal to the mean Earth-Sun distance. It is the radius of an unperturbed circular Newtonian orbit about the Sun of a particle having infinitesimal mass, moving with a mean motion of 0.017 202 098 95 radians per day (known as the Gaussian constant).

The current definition of the ua is complicated and obscure for non-experts and very difficult to teach to students

The IAU-1976 gravitational constant related to the mass of the Sun

- The IAU 1976 heliocentric gravitational constant, GM_{Sun}, is a "derived constant“, which can be expressed as: $GM_{Sun} = (ua)^3 k^2 D^{-2}$,

  → the SI value of GM_{Sun} is derived from the SI values for the ua and the day.

- The estimation of the mass of the Sun in kg has to be derived from the SI values of (1) GM_{Sun} and (2) the gravitational constant G (current relative uncertainty~ 1×10^{-4}).

- The mass of the Sun in kg is provided in the numerical standards in astronomy, but is not provided in the SI brochure.

The role of the of the astronomical unit of length (ua) (simplified outline)

For historical reasons, the ua is defined by the value of the Gaussian gravitational constant **k** (with $k^2 = G$), called a “defining constant”, and the 3d Kepler’s law ($n^2 a^3 = GM_{Sun}$ for a planet of negligible mass).

Official situation (IAU-1976)

- GM_{Sun} is fixed by convention,
- for a fictitious planet with an infinitesimal mass, $n = n_0$ by convention,
- for any planet, n is measured in SI → a in ua.

➡ $a_0 = ua$

The precision of the time measurements is reported into the relative distances.

There is no special link to the SI.

Current practical situation

- for any planet, n is measured in SI,
- ua is measured in SI (c.f. Standish 1995),

➡ GM_{Sun} is estimated

- the ua appears in the current version of the SI Brochure (Table 7) as being one of the "Non-SI units whose values in SI units must be obtained experimentally“.

Changes in the status of the ua and GM_S since the adoption of the IAU-1976 System

The context of the recent IAU Resolutions on reference systems

The celestial reference systems have been defined in a GR framework (IAU Resolutions 1991: GR framework; IAU 2000: GCRS, BCRS, re-definition of TT; IAU 2006: re-definition of TDB).

The context of the modern observations in the solar system

High accuracy observations are mainly based on range and Doppler measurements, especially for terrestrial bodies (Moon, Mercury, Venus, Mars, …).

The context of the recent ephemerides

According to recent publications:

- the GM_i of the planets are actually determined in SI (TDB-compatible values) and then converted to values in astronomical units (Folkner et al. 2008),
- GM_{Sun} will be estimated in future version of the INPOP ephemerides (Fienga et al. 2008).

The status of the ua and GM_{Sun} should be reformed to be more in agreement with the modern context.

Discussion
(interpretation based on recent ephemerides publications)

- For the terrestrial bodies, for which there are very precise range and Doppler measurements, GM_{Sun} can be estimated with a very high precision.
  - **This defines the scale in SI for the distances in the solar system with high precision.**
- For the planets for which observations are mainly angular measurements (Jupiter, Saturn, etc.)
  - **the relative distances of the planets are determined as with old observations,**
  - **the scale in SI of the global solution is provided by the GM_{Sun} value determined by the terrestrial bodies (heigh weight) → distances in SI .**
- If very precise angular measurements of the external planets expressed in an unit linked to GM_{Sun} are available
  - **the relative distances can be determined with very high precision,**

→ the (absolute) distances in SI may be different, but are with the same ratio.

Suggested reform in the status of the astronomical units

- Several options can be considered in the GR context (Guinot 1995, Capitaine & Guinot 1995, Klioner 2007).

- There are two possibilities in the Newtonian context (compatible with GR):
  - GM_{Sun} fixed and ua estimated in SI,
  - GM_{Sun} estimated in SI and ua fixed in SI.

- **Estimating GM_{Sun} is the option that has the most physical meaning since it does not suppose that the mass of the Sun is constant.**

Conclusion

- A re-definition of the ua is necessary in the modern context in order to make the system of astronomical constants best compliant with modern dynamical astronomy.
- The ua should be re-defined as an astronomical unit of length defined trough a fixed relation to the SI meter by a defining number.
- From the point of view of the principles, the important point is the change of status for the astronomical unit of length (and not the value of its defining number).
  This would mean:
  - dropping the k constant (and implicitly GM_{Sun} =constant), and abandoning the experimental determination of the ua in SI unit,
  - determining experimentally GM_{Sun}, which would not be considered any more as being a "constant“,
  - limiting the role of the ua to that of a unit of length of "convenient" size for solar system applications.
- Such a change of status of the ua would:
  - **be a great simplification for the users of the astronomical constants (i.e. the ua would have a fixed numerical value in meters),**
  - **let appear directly the possible variation of the mass of the Sun, and/or of G.**

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1st system of fundamental astronomical constants

(Conférence internationale des étoiles fondamentales, Paris, 1896)

1° La *constante de la précession* .... $p = 50'',2564$;

2° La *constante de la nutation* .. $N_0 = 9'',21$;

3° *L'obliquité de l'écliptique* ... $\varepsilon = 23°27'8'',26$;

4° La *constante de l'aberration* .. $\varkappa = 20'',47$;

5° La *parallaxe solaire* .... $\varpi_0 = 8'',80$.

6° *L'aplatissement de l'ellipsoïde terrestre* .... $\alpha = 1/297$;

7° Le *rayon équatorial terrestre* .... $a_0 = 6\,378\,388$ m;

8° La *constante de la gravitation universelle* .... $k = 0,017\,202\,098\,95$.

IAU 1976 System of astronomical constants

Defining constants:

1.	Gaussian gravitational constant	$k = 0{\cdot}017\ 202\ 098\ 95$
2.	Speed of light	$c = 299\ 792\ 458\ \mathrm{m\ s^{-1}}$

Primary constants:

3.	Light-time for unit distance	$\tau_A = 499{\cdot}004\ 782\ \mathrm{s}$ [499·004 7838 . . .]
4.	Equatorial radius for Earth	$a_e = 6378\ 140\ \mathrm{m}$ [6378 137]
5.	Dynamical form-factor for Earth	$J_2 = 0{\cdot}001\ 082\ 63$ [0·001 082 626]
6.	Geocentric gravitational constant	$GE = 3{\cdot}986\ 005 \times 10^{14}\ \mathrm{m^3\ s^{-2}}$ [3·986 004 33 . . . × 10^{14}]
7.	Constant of gravitation	$G = 6{\cdot}672 \times 10^{-11}\ \mathrm{m^3\ kg^{-1}\ s^{-2}}$
8.	Ratio of mass of Moon to that of Earth	$\mu = 0{\cdot}012\ 300\ 02$ [0·012 300 038]
9.	General precession in longitude, per Julian century, at standard epoch 2000	$\rho = 5029''0966$
10.	Obliquity of the ecliptic, at standard epoch 2000	$\epsilon = 23^\circ\ 26'\ 21''448$

Derived constants:

11.	Constant of nutation, at standard epoch 2000	$N = 9''2025$
12.	Unit distance	$c\tau_A = A = 1{\cdot}495\ 978\ 70 \times 10^{11}\ \mathrm{m}$ [1·495 978 706 91 × 10^{11}]
13.	Solar parallax	$\arcsin(a_e/A) = \pi_\odot = 8''794\ 148$ [8″794 144]
14.	Constant of aberration, for standard epoch 2000	$\kappa = 20''49\ 552$
15.	Flattening factor for the Earth	$f = 0{\cdot}003\ 352\ 81 = 1/298{\cdot}257$
16.	Heliocentric gravitational constant	$A^3k^2/D^2 = GS = 1{\cdot}327\ 124\ 38 \times 10^{20}\ \mathrm{m^3\ s^{-2}}$ [1·327 124 40 . . . × 10^{20}]
17.	Ratio of mass of Sun to that of the Earth	$(GS)/(GE) = S/E = 332\ 946{\cdot}0$ [332 946·050 895 . . .]
18.	Ratio of mass of Sun to that of Earth + Moon	$(S/E)/(1+\mu) = 328\ 900{\cdot}5$ [328 900·561 400]
19.	Mass of the Sun	$(GS)/G = S = 1{\cdot}9891 \times 10^{30}\ \mathrm{kg}$

IERS Conventions 2003

Table 1.1 IERS Numerical Standards.

ITEM	VALUE	UNCERTAINTY	REF.	COMMENTS
c	$299792458 ms^{-1}$	Defining	[2]	Speed of light
L_B	$1.55051976772 \times 10^{-8}$	2×10^{-17}	[4]	Average value of 1-d(TT)/d(TCB)
L_C	$1.48082686741 \times 10^{-8}$	2×10^{-17}	[4]	Average value of 1-d(TCG)/d(TCB)
L_G	$6.969290134 \times 10^{-10}$	Defining	[4]	1-d(TT)/d(TCG)
G	$6.673 \times 10^{-11} m^3 kg^{-1} s^{-2}$	$1 \times 10^{-13} m^3 kg^{-1} s^{-2}$	[2]	Constant of gravitation
$GM_{\odot}$	$1.32712442076 \times 10^{20} m^3 s^{-2}$	$5 \times 10^{10} m^3 s^{-2}$	[from 3]	Heliocentric gravitational constant
$\tau_A{}^{\dagger}$	$499.0047838061 s$	$0.00000002 s$	[3]	Astronomical unit in seconds
$c\tau_A{}^{\dagger}$	$149597870691 m$	$6 m$	[3]	Astronomical unit in meters
$\psi_1{}^{\dagger}$	$5038.47875''/c$	$0.00040''/c$	[6]	IAU(1976) value of precession of the equator at J2000.0 corrected by $-0.29965''$. See Chapter 5.
ϵ_0	$84381.4059''$	$0.0003''$	[5]	Obliquity of the ecliptic at J2000.0. See Chapter 5 for value used in IAU precession-nutation model.
$J_{2\odot}$	2×10^{-7}	(adopted for DE405)		Dynamical form-factor of the Sun
μ	0.0123000383	5×10^{-10}	[3]	Moon-Earth mass ratio
$GM_{\oplus}$	$3.986004418 \times 10^{14} m^3 s^{-2}$	$8 \times 10^5 m^3 s^{-2}$	[1]	Geocentric gravitational constant (EGM96 value)
$a_E{}^{\ddagger}$	$6378136.6 m$	$0.10 m$	[1]	Equatorial radius of the Earth
$1/f^{\ddagger}$	298.25642	0.00001	[1]	Flattening factor of the Earth
$J_{2\oplus}{}^{\ddagger}$	1.0826359×10^{-3}	1.0×10^{-10}	[1]	Dynamical form-factor
ω	$7.292115 \times 10^{-5} rad s^{-1}$	variable	[1]	Nominal mean angular velocity of the Earth
$g_e{}^{\ddagger}$	$9.7803278 ms^{-2}$	$1 \times 10^{-6} ms^{-2}$	[1]	Mean equatorial gravity
W_0	$62636856.0 m^2 s^{-2}$	$0.5 m^2 s^{-2}$	[1]	Potential of the geoid
$R_0{}^{\dagger\dagger}$	$6363672.6 m$	$0.1 m$	[1]	Geopotential scale factor

† The values for τ_A, $c\tau_A$, and ψ_1 are given in "TDB" units (see discussion above).

‡ The values for a_E, $1/f$, $J_{2\oplus}$ and g_E are "zero tide" values (see the discussion in section 1.1 above). Values according to other conventions may be found from reference [1].

†† $R_0 = GM_{\oplus}/W_0$

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Table 7. Non-SI units whose values in SI units must be obtained experimentally

Quantity	Name of unit	Symbol for unit	Value in SI units (a)
Units accepted for use with the SI			
energy	electronvolt (b)	eV	1 eV = 1.602 176 53 (14) x 10^{-19} J
mass	dalton, (c)	Da	1 Da = 1.660 538 86 (28) x 10^{-27} kg
	unified atomic mass unit	u	1 u = 1 Da
length	astronomical unit (d)	ua	1 ua = 1.495 978 706 91 (6) x 10^{11} m
Natural units (n.u.)			
speed	n.u. of speed (speed of light in vacuum)	c_0	299 792 458 m/s (exact)
action	n.u. of action (reduced Planck constant)	$\hbar$	1.054 571 68 (18) x 10^{-34} J s
mass	n.u. of mass (electron mass)	m_e	9.109 3826 (16) x 10^{-31} kg
time	n.u. of time	$\hbar/(m_e c_0^2)$	1.288 088 6677 (86) x 10^{-21} s
Atomic units (a.u.)			
charge	a.u. of charge (elementary charge)	e	1.602 176 53 (14) x 10^{-19} C
mass	a.u. of mass (electron mass)	m_e	9.109 3826 (16) x 10^{-31} kg
action	a.u. of action (reduced Planck constant)	$\hbar$	1.054 571 68 (18) x 10^{-34} J s
length	a.u. of length, bohr (Bohr radius)	a_0	0.529 177 2108 (18) x 10^{-10} m
energy	a.u. of energy, hartree (Hartree energy)	E_h	4.359 744 17 (75) x 10^{-18} J
time	a.u. of time	$\hbar/E_h$	2.418 884 326 505 (16) x 10^{-17} s

(d) The astronomical unit is approximately equal to the mean Earth-Sun distance. It is the radius of an unperturbed circular Newtonian orbit about the Sun of a particle having infinitesimal mass, moving with a mean motion of 0.017 202 098 95 radians per day (known as the Gaussian constant). The value given for the astronomical unit is quoted from the IERS Conventions 2003 (D.D. McCarthy and G. Petit eds., *IERS Technical Note 32*, Frankfurt am Main: Verlag des Bundesamts für Kartographie und Geodäsie, 2004, 12). The value of the astronomical unit in metres comes from the JPL ephemerides DE403 (Standish E.M., Report of the IAU WGAS Sub-Group on Numerical Standards, *Highlights of Astronomy*, Appenzeller ed., Dordrecht: Kluwer Academic Publishers, 1995, 180-184).