Problems for the Web

P4W8: Magic Triangles and Other Figures

Curriculum Areas: Number Sense, Computation, Problem Solving

Introduction:

P4W8 problems are similar to those that students may have met on P4W1: Magic Squares. In a Magic Triangle (or other figure) there will be designated rows of numbers with a common sum, sometimes called the "Magic Constant".

If students have not had experience with Magic Squares, you may wish to retrieve P4W1 from the archives and use it as an introduction to P4W8.

To save students frustration, you may wish to suggest that they write the relevant numbers on small pieces of paper and move them about on the diagrams. This avoids having to erase incorrect placements of numbers.

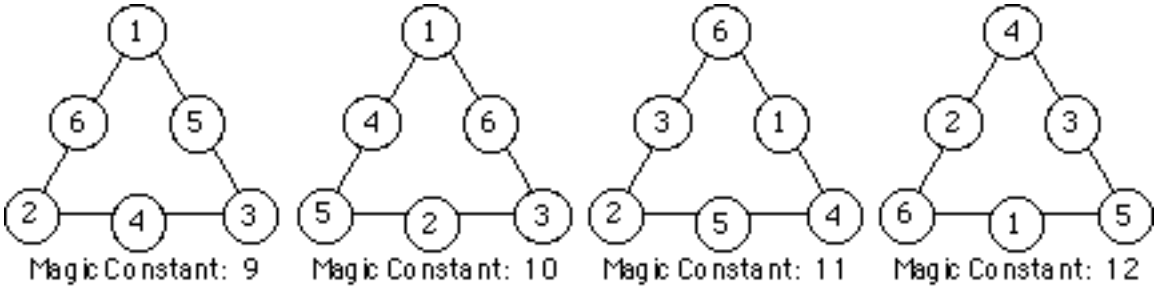
For the Teacher:

P4W8(a): Magic Triangles

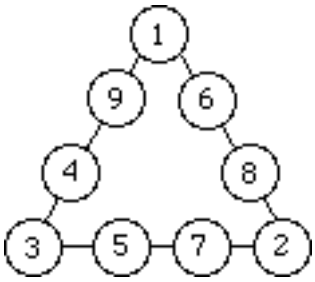
Extensive notes are given here for completing Magic Triangles because the strategies used are quite different from those usually used to solve Magic Squares.

#1: There are four possible solutions for this problem. Place 1, 2, and 3 in the corners and then place 4, 5, and 6 along the sides of the triangle to give a Magic Constant of 9, the minimum value of the Magic Constant. Alternatively, place 4, 5, and 6 at the corners, and 1, 2, and 3 along the sides to give a Magic Constant of 12, the maximum value of the Magic Constant. Ask students to compare the two solutions. Ask them if they think there are other solutions and to try to explain why they decided "yes" or "no".

Challenge students to determine the other two solutions. All solutions are given below.



#2. If the Magic Constant is 17, then the sum of all three sides of the triangle will be 51. However, this counts each corner number twice. The sum of all number used



(1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9) is 45. The difference is 51 - 45, or 6. The only three different numbers with a sum of 6 are 1, 2, and 3. Therefore they are placed in the corners. The other two numbers on each side should give totals of 12, 13, and 14. An examination of the numbers remaining shows the pairs to be 5 + 7, 4 + 9, 6 + 8._

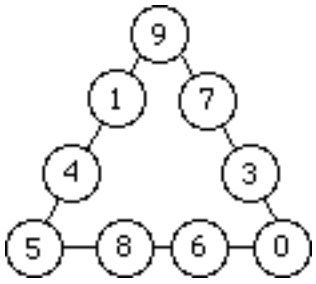
The minimum value for a Magic Constant for this problem is 17. The maximum value is 23. Students could be challenged to determine as many other solutions as possible.

Analysing the problem as described above, the possible Magic Constants between 17 and 23 can be listed. The corner sum can be determined. The third column in the table gives the number of combinations of 3 numbers that might be the corner numbers. Finally, the number of solutions for each possible Magic Constant is given. Notice the symmetry in the last two columns.

Possible Magic Constant	Sum of Corner Numbers	Number of Triads with that sum	Number of Solutions
17	6	1	1
18	9	3	0
19	12	7	2
20	15	8	4
21	18	7	2
22	21	3	0
23	24	1	1

Most students will not be interested in pursuing the theory of Magic Squares this far. The information is provided for the few students who become involved in analysing the problems.

#3. This problem can be analysed in the same way as #2, but students will have to try eliminating different numbers. One way to eliminate unnecessary work would be to assign a deleted number to each group. For example, group 1 could try eliminating the 1; group 2 could try to solve the problem by eliminating the 2, and so on.



One solution is given. For this solution, the number '2' was deleted.

P4W8(b): Magic in Other Figures

#1 should be fairly easy for students after they have dealt with the Magic Triangles.

As in the Magic Squares, the 'middle number', 5, is placed in the centre cell. The remaining numbers are paired to give totals of 10: 1 + 9, 2 + 8, 3 + 7, 5 + 6. The numbers in each pair are placed on the same line, but on opposite sides of the centre cell.

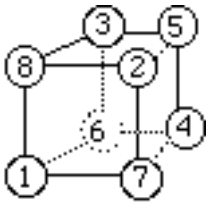
#2 can be solved with a similar strategy. Place '6' in the centre cell and pairs with the sum of 12 on opposite sides of the centre.

To get a Magic Constant of 14, the centre number should be 1.

To get a Magic Constant of 22, the centre number should be 11.

P4W8(c): Magic in 3D

The solution given here uses the numbers 1 to 8, and a Magic Constant of 18.



Students should be encouraged to describe their strategies for solving this problem.

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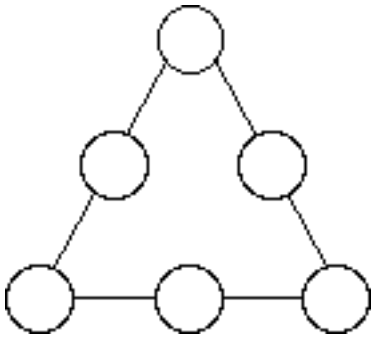
For the Students:

P4W8(a): Magic Triangles

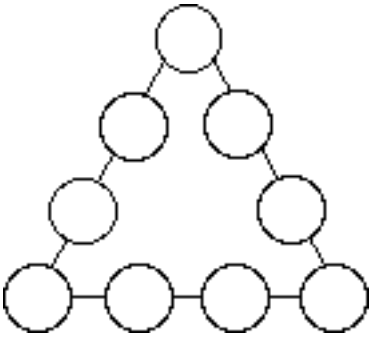
1. Place the numbers 1,2,3,4,5,6 in the circles so that each side of the triangle has the same sum.



What is the Magic Constant?

Is there more than one solution?

What Magic Constants are possible?



2. Place the numbers 1,2,3,4,5,6,7,8,9 in the circles so that each side of the triangle has a sum of 17.

Is there more than one solution?

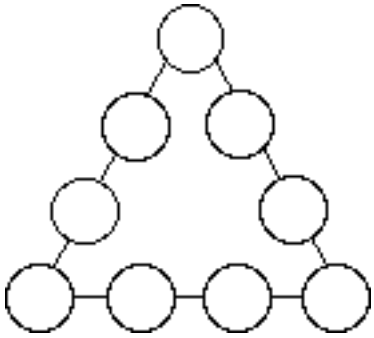
Is there a solution with a different Magic Constant?

How many different solutions (with different Magic Constants) are possible?

3. Use the numbers

0,1,2,3,4,5,6,7,8,9

to complete this Magic Triangle so that the Magic Constant is 19.



One of the numbers must be omitted. Which one did you omit?

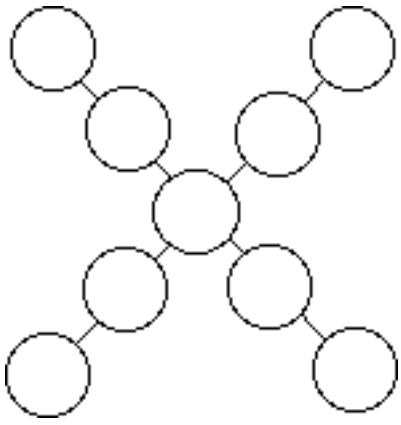
Find a solution with another number omitted.

How many such solutions are there?

P4W8(b): Magic in Other Figures

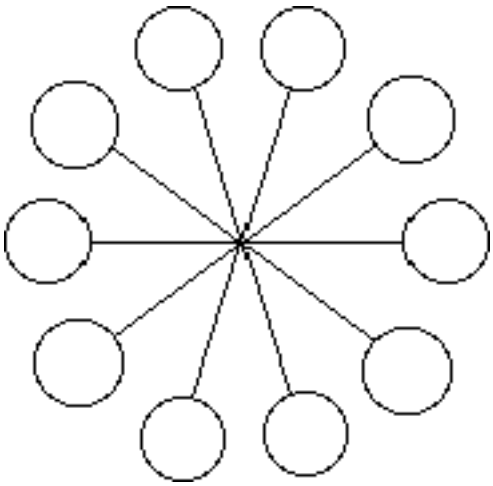
1. Place the numbers 1,2,3,4,

5,6,7,8,9 in the circles, so that each line of three circles has the same sum.



2. Place the numbers 1,2,3,4,5,

6,7,8,9,10,11in the circles so that each line of three circles has a sum of 18.



Now try to get a Magic Sum

of 14, of 22.

P4W8(c): Magic in 3D

Place eight different numbers in the circles so that each face of the cube has the same sum.

Is there more than one solution?