Review of Collective Effects

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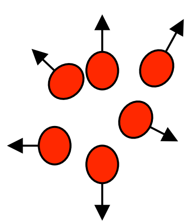
Collective Effects

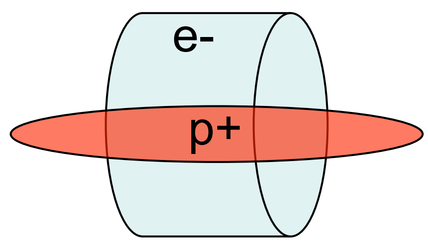
So far, we have treated only the forces of external magnets and fields on a particle. Collective effects take into account the effects of the beam's own Coulomb force field on itself and on it's environment.

In a very general sense, we can break collective effects down into three catagories: Beam-self, beam-beam, and beam-environment.

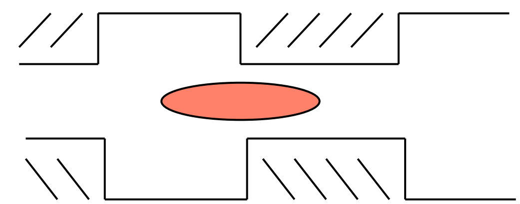
Beam-self: beam interacts with itself through space charge.



Beam-beam: colliding beams in colliders or ambient electron clouds (e-p instability).



Beam-environment: beam interacts with machine (impedance-related instabilities).

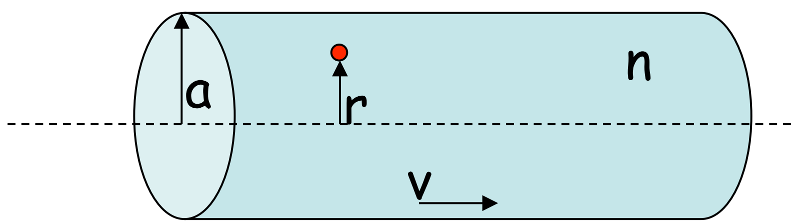


Collective effects and their instabilities constitute an active and extensive field of research. Here we only briefly review some of the more common effects.

Space Charge Fields

One of the most studied collective effects is that of the beam's own Coulomb field on particles in the beam (beam-self interaction). In accelerator jargon, this is called the "space charge" field, or the "space charge" effect.

A simple model treats the beam as a long cylinder. Consider the total force (E + B fields) felt on a "test particle" within this beam:



(**Derivation**)

$$F_{total} = F_E + F_B = \frac{ne^2}{2\pi\varepsilon_o a^2 \gamma^2} r$$

a = radius of the beam
n = Number particles / length

Space Charge Force

It is a straightforward matter to plug this force in to our Hill's equation of motion, and to assess the effect:

$$x'' + K(s) = \frac{F_{\text{space charge}}}{m\gamma\beta^2 c^2}$$

$$x'' + \left(K(s) - \frac{2 r_o n}{\beta^2 \gamma^3 a^2} \right) x = 0 \qquad r_o = \frac{e^2}{4\pi\varepsilon_o m c^2}$$

Notice that the space charge term is defocusing!

Also,the more we focus the beam, the higher the particle density and the larger the Coulomb field repulsion, i.e., the term a^2 in the denominator decreases.

Space Charge Effect

Since the space charge term scales as x, we can treat it as a quadrupole error term.

$$\Delta\nu = \frac{1}{4\pi}\oint \beta(s)\Delta K(s)_{\text{space charge}}\,ds$$

For a constant density cylindrical beam, we get

$$\boxed{\Delta\nu = -\frac{n r_o R}{\varepsilon_N \beta\gamma^2}} \quad \text{where} \quad \varepsilon_N = \frac{a^2(\gamma\beta)}{\beta_{\text{twiss}}}$$

while for a Gaussian beam of RMS width σ

$$\boxed{\Delta\nu = -\frac{\pi n r_o R}{2\varepsilon_N \beta\gamma^2}} \quad \text{where} \quad \varepsilon_N = \frac{\pi\sigma^2(\gamma\beta)}{\beta_{\text{twiss}}}$$

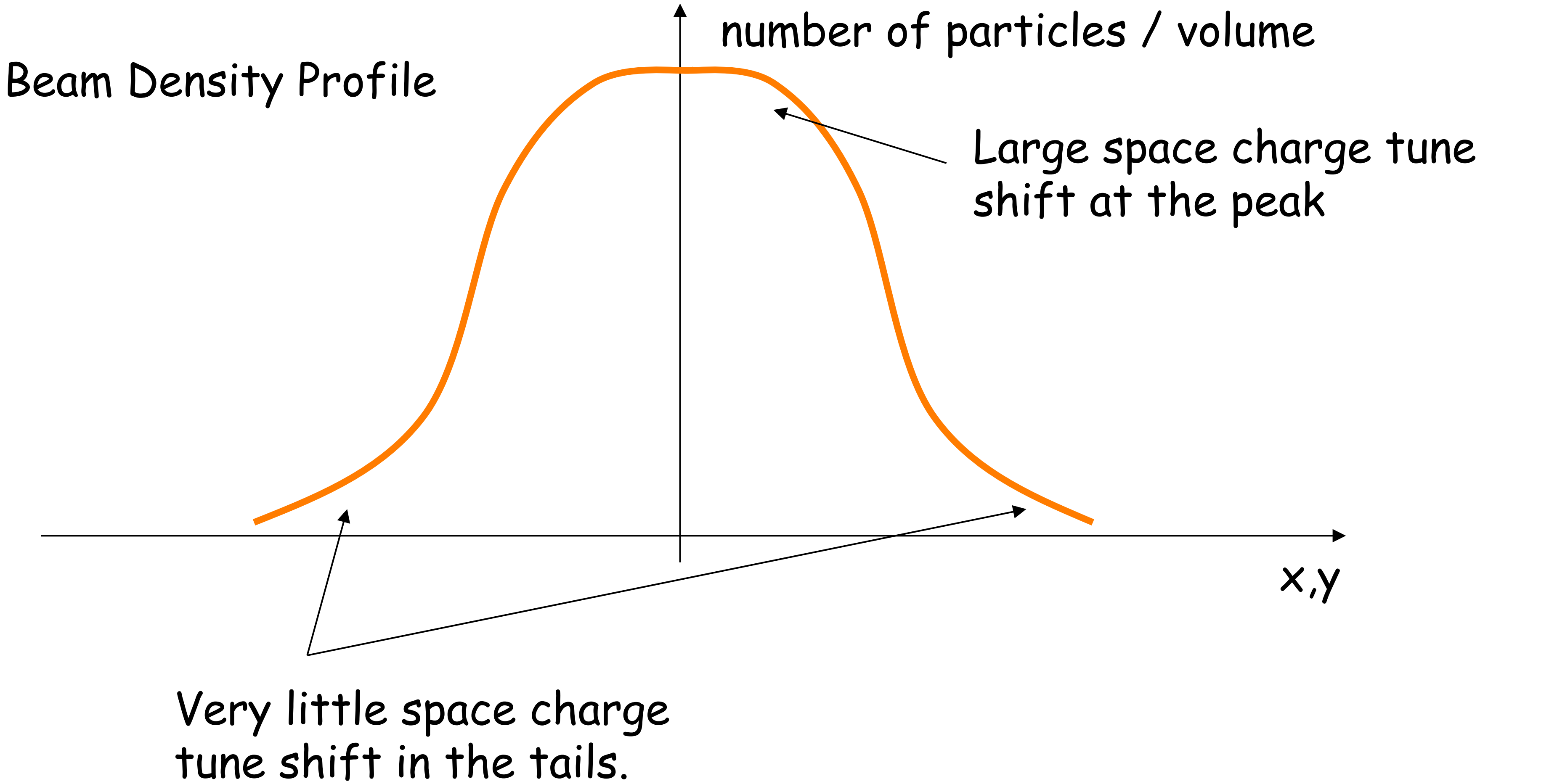
<u>Example:</u>

$$\left.\begin{array}{l} R=75\text{m}, \\ E=200\ \text{MeV}, \\ \varepsilon_N=3\ \text{mm mrad}, \\ N=6\times10^9\ \text{proton/m} \end{array}\right\} \Delta\nu=-0.4$$

The design lattice tune is shifted down by this amount! “Space charge tune depression”

Space Charge Effect

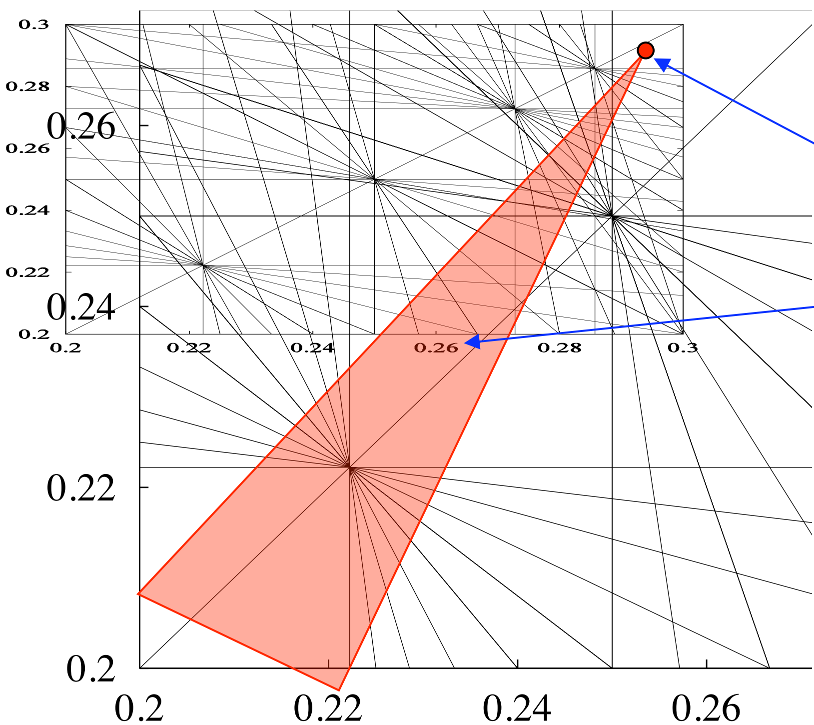
A real beam has a variation in particle density, and therefore, the space charge tune shift varies across the bunch (both in the transverse and in the longitudinal directions).



Space Charge Tune Shift

A real beam is neither round nor uniform in particle density. Particles in high density regions will be "tune-depressed" more than particles in low-density regions, giving rise to a distribution of particle tunes.

The overall effect is a *tune spread*, leading to a "tune-footprint" in the tune diagram:



Nominal tune point

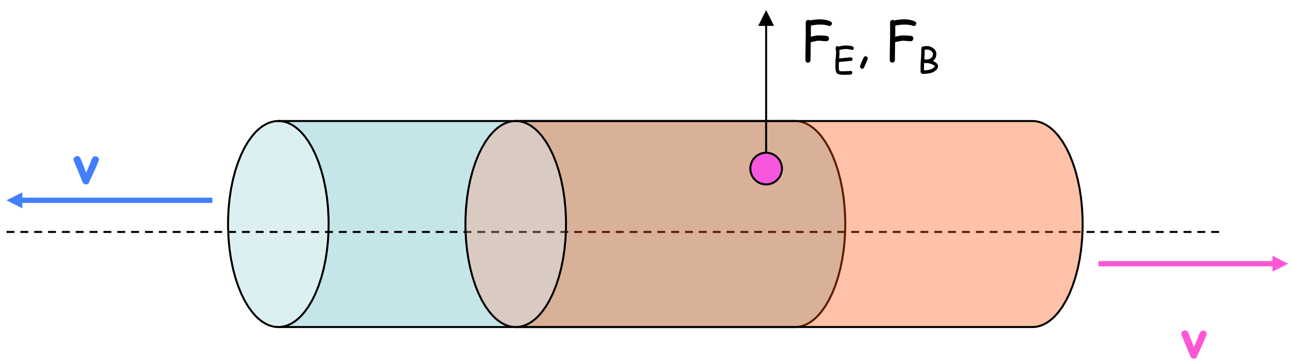
Tune spread due to space charge.
Max tune spread here is,
$\Delta\nu_{max} \cong 0.1$

Now it becomes much harder to avoid resonance!

Beam-Beam Tune Shift

In a colliding beam accelerator, two beams are circulating in opposite directions and pass through each other at certain interaction points.

During this time, the particles in one beam feel the electric and magnet forces of the particles in the other beam.



In this case, the force vectors on a test particle in one beam, due to the fields in the other beam, are in the same direction. Both the E and B forces are defocusing.

Beam-Beam Tune Shift

Using the same procedure as in the space charge case, we find that the beam-beam tune shift of beam 2 due to beam 1 is given by:

$$\boxed{\Delta\nu_{\text{beam-beam}} = -\frac{N_1 r_o (1+\beta_1\beta_2)\gamma_1\beta_1}{2\pi\varepsilon_{N1}\gamma_2\beta_2(\beta_1+\beta_2)} = -\frac{N_1 r_o}{2\pi\varepsilon_{N1}} \quad (\text{when } \gamma_1 = \gamma_2 \text{ and } \beta_1 = \beta_2)}$$

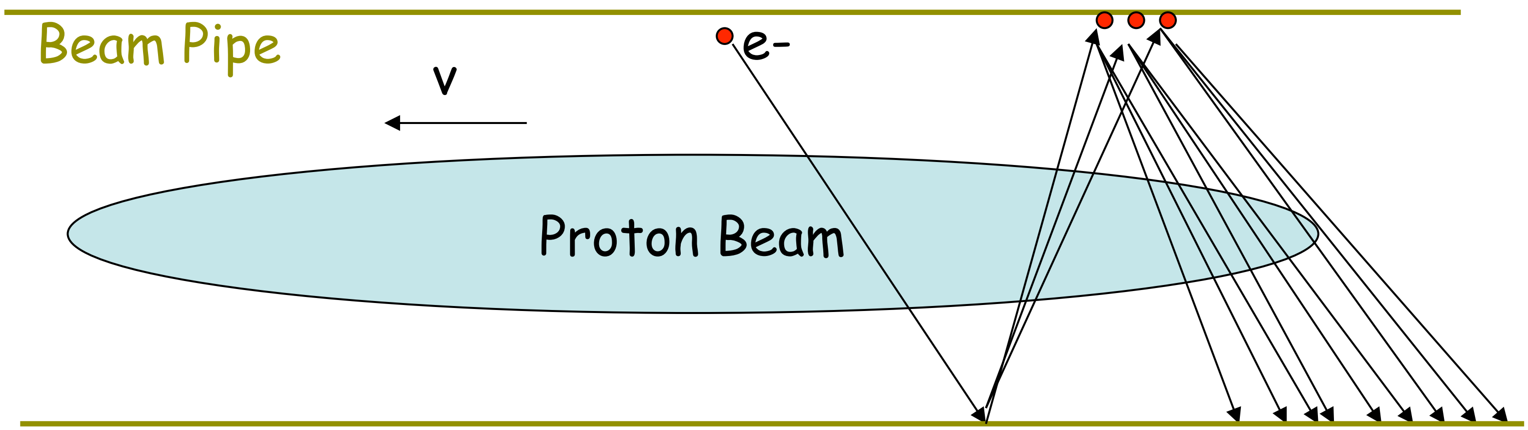
where, $\displaystyle \varepsilon_{N1} = \frac{a_1^2\gamma_1\beta_1}{\beta_{\text{twiss}}}$

and N_1 is the total number of particles in beam 1.

Because the beams only overlap and “feel” each other for a short time, this tune shift is much smaller than the space charge tune shift.

e-P Instability

An "electron-Proton" instability can be generated when the proton beam interacts with ambient electrons in the vacuum chamber.

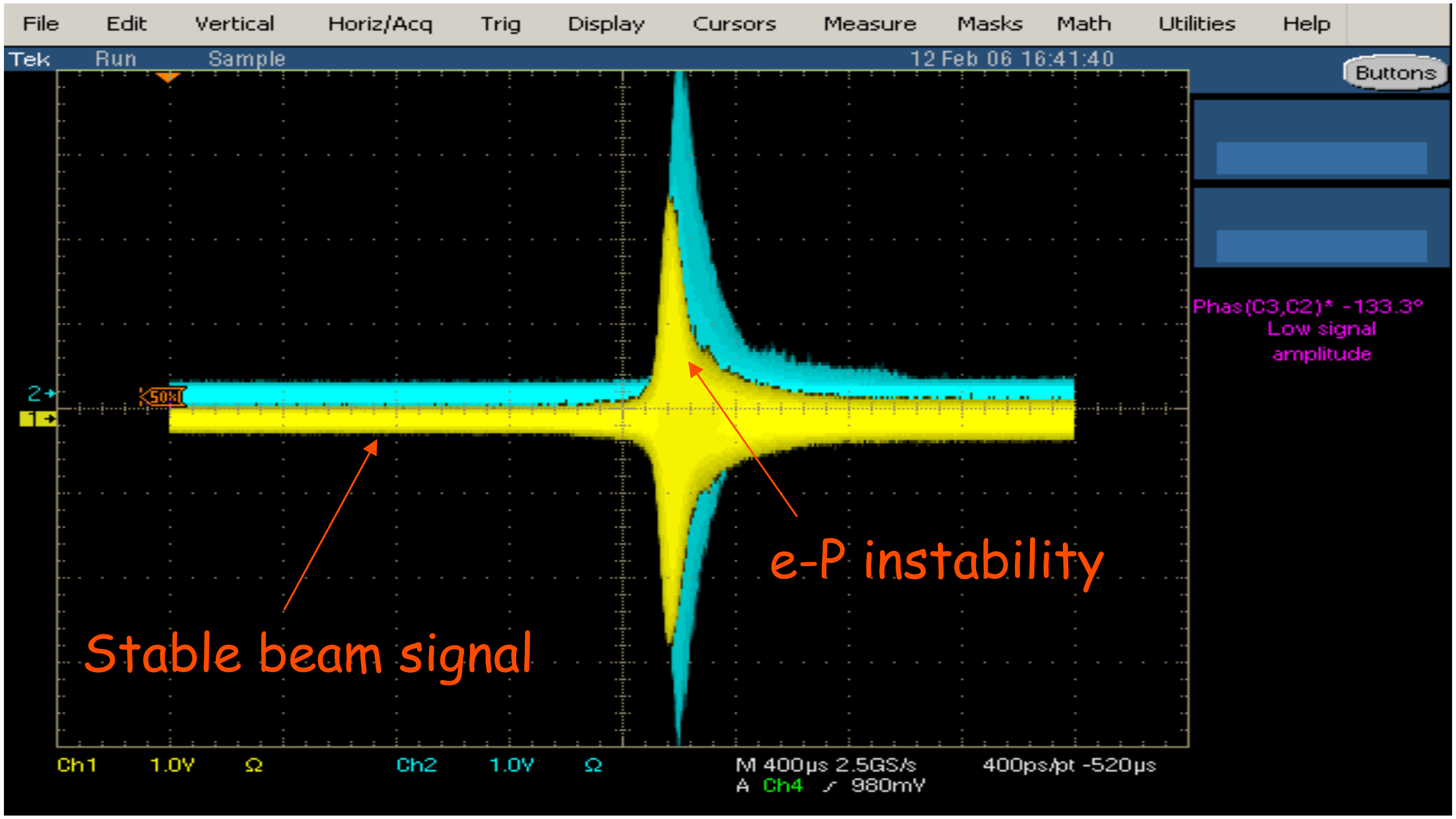


Scenario (simplified):

1) Ambient electron is accelerated through beam potential
2) Electron strikes the wall on the opposite side and ejects more electrons
3) These electrons are accelerated through the beam and strike the opposite side wall, ejecting more electrons.
4) If electrons live until the beam returns on the next pass, the "electron cloud" grows until it causes an instability in the proton beam.

Observed e-P Instability at SNS

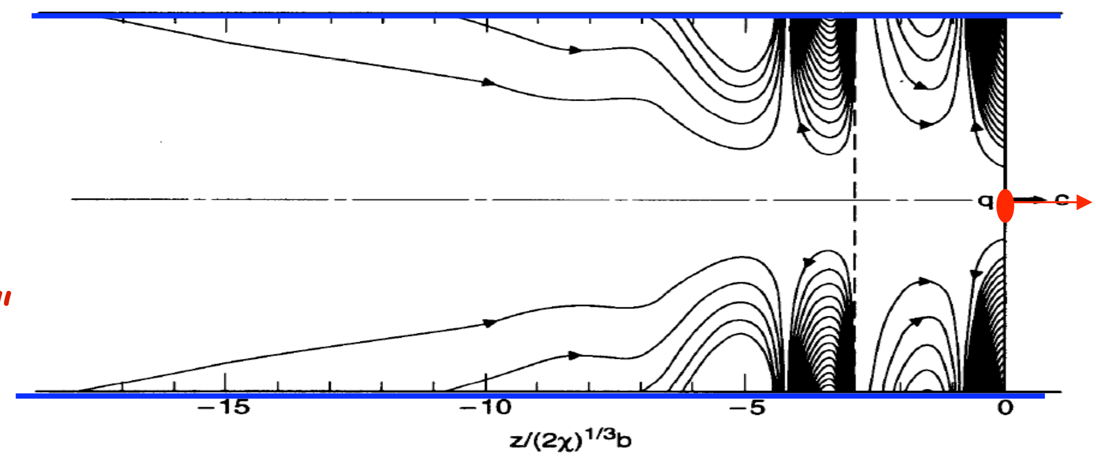
- Instability was *fast*: 20 – 200 turns.
- Instability was observed in both planes – vertical plane was stronger.



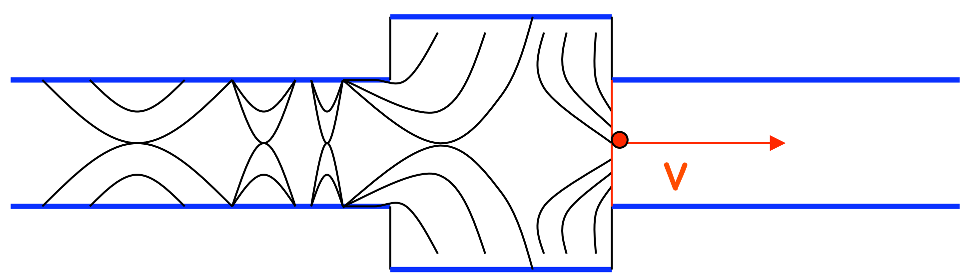
Wakefields and Impedances

Since particles travel in the accelerator environment, with beam pipes and magnets, etc, they induce fields in the accelerator structures. These fields can act back on a trailing particle.

Wakefields are generated in a smooth pipe of constant radius if it has finite resistance: "Resistive Wall Impedance"



Wakefields are also generated in a conducting pipe near the intersection of a geometry change.



Wakefields: The Force from an Annulus of Charge

Trailing test particle   Multipolar charge annulus Q_m.

We can write down the radial force, F_r , on the test particle from the upstream annulus of charge:

$$F_r = eQ_m m r^{m-1} \cos(m\theta) W_m(s)$$

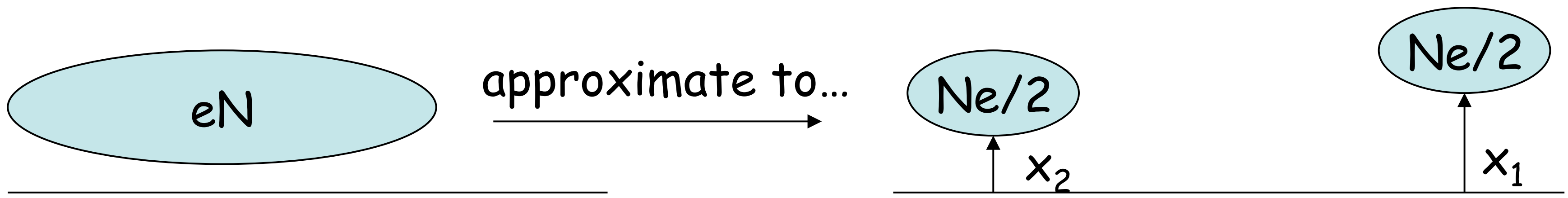
$$Q_m = \int \rho r^m \cos(m\theta) r dr d\theta dz$$

The m's represent different possible charge distributions in the annulus.

Q_m is the multipole factor for the charge distribution, and $W_m(s)$, is the wakefield of the annulus, it represents the response of the vacuum chamber to the beam, and is found by solving Maxwell's equations, with boundary conditions matched for the particular environment.

Example: Macroparticle Model

Consider a simplified example where we approximate a beam as two macroparticles, each with half of the beam charge, Ne/2, and separated by a distance, s, equal to the length of the beam:



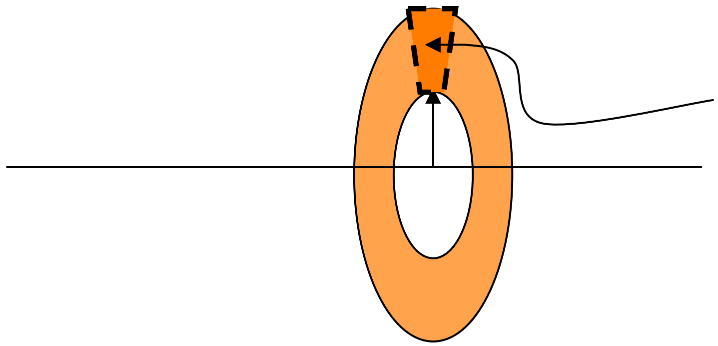
We let the particles have different offsets from the axis, and we assume that they propagate in a uniform focusing channel (also known as the smooth approximation), which leads to equal betatron frequencies.

The equation of motion in the absence of wakefields, and in the time domain is given by:

$$\ddot{x} + \omega_\beta^2 x = 0$$

Example (continued)…: Macroparticle Multipole

The leading particle is a small chunk of a uniform annulus of charge and, as such it induces moments of all m. We consider the case m=1, at the radius r = x_1, here.



Our leading particle is like a little chunk of the annulus, at $r=x_1$

$$Q_m = \int \rho r^m \cos(m\theta) r dr d\theta dz \rightarrow Q_1 = \int \rho r \cos(\theta) r dr d\theta dz$$

Translating this into Cartesian coordinates, $x=r\cos\theta$, and using the δ function operator to identify the location of the macroparticle, we have:

$$Q_1 = \int \rho \delta(x - x_1)\delta(y)\delta(z - z_0) x \, dxdydz$$

This charge is upstream a distance of $z_o=ct_0$ ahead of the second particle, where we have the force.

$$Q_1 = \frac{Ne}{2} x_1$$

Example (continued)...: The Force

Using the m=1 term and our charge multipole, we can write down the force on the second particle due to the wake fields generated by the first particle.

$$F_r = eQ_m m r^{m-1}\cos(m\theta)W_m(s) \rightarrow F_r = eQ_1 W_1 \cos(\theta)\Big|_{\theta=0}$$

$$F_x = eQ_1 W_1 = \frac{Ne^2}{2} W_1 x_1$$

But x_1 obeys the betatron equation of motion with constant frequency:

$$\ddot{x}_1 + \omega_\beta^2 x_1 = 0 \longrightarrow x_1 = a_1 \cos(\omega_\beta t)$$

And finally, the force on the trailing particle is:

$$\frac{F_x}{\gamma m} = \frac{Ne^2}{2\gamma m} W_1 a_1 \cos(\omega_\beta t)$$

Example (continued)…: The Eq. of Motion

The second particle obeys the betatron equation of motion with frequency ω_β, but it also experiences the force from the leading particle.

Combining both of these into one equation, we arrive at the equation of motion for the second particle:

$$\ddot{x}_2 + \omega_\beta^2 x_2 = \frac{Ne^2 W_1 a_1}{2\gamma m} \cos(\omega_\beta t)$$

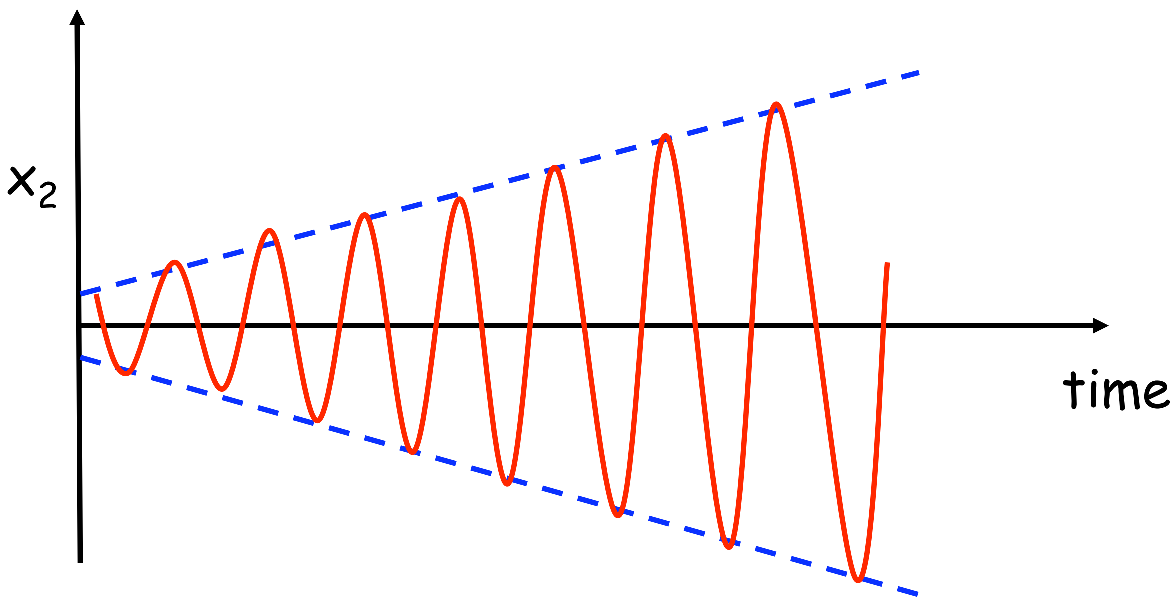
This is the equation for a driven harmonic oscillator. And we are driving right on the resonance frequency!!

Example (continued)...: The Instability

Solving the driven harmonic oscillator equation of motion, we finally arrive at:

$$x_2 = a_2 \cos(\omega_\beta t) + a_1 \frac{Ne^2 W_1(s)}{4\omega_\beta \gamma m} t \sin(\omega_\beta t)$$

Linear growth with time!



Though many approximations were used in this example, the basic principles carry over to a real machine and lead to a phenomenon called "Beam break up".

Wakefields → Impedances

In practice, in can be very difficult to calculate the wakefield for real accelerator beams and vacuum chamber geometries.

It's often easier to work with the Fourier transform of the wakefield, namely the Impedance, which we can separate into pieces that are transverse and parallel to beam motion. The impedance per unit length is written:

$$\frac{Z_o^{\perp}}{L} = \frac{1}{ic}\int e^{i\omega s/c} W(s)\,ds = \frac{c}{\omega}\frac{Z_o^{\parallel}}{L}$$

The impedance is the frequency domain representation of the wakefield. Once we have calculated or measured one quantity (wakefield or impedance) we get the other almost for free (just a Fourier transform).

More on Impedance

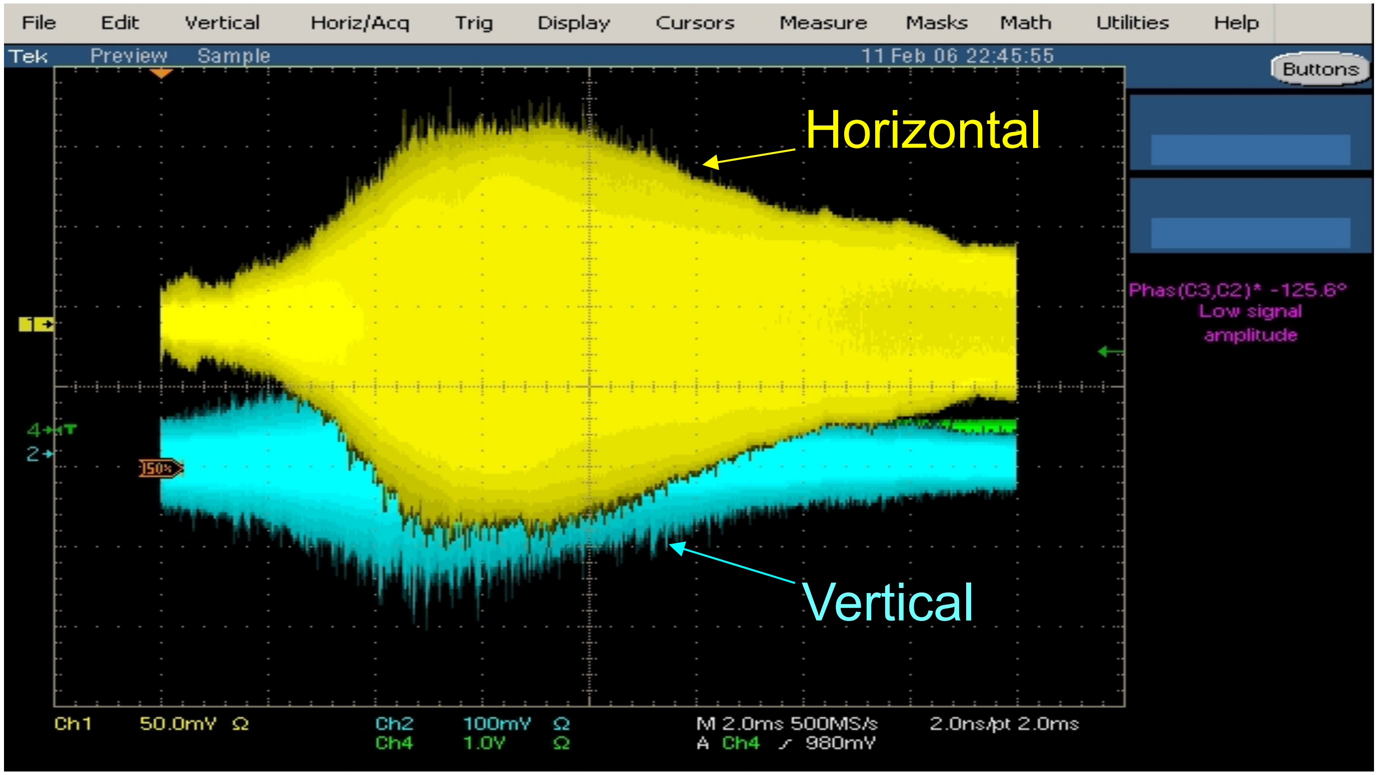
The impedance is a complex resistance, so a beam with current I_{beam} will induce a voltage proportional to the impedance:

$$V(\omega) = -I_{beam}(\omega)Z(\omega)$$

Because of this, we are often able to measure the impedance of an accelerator structure in the lab, using a probe and a network analyzer. In some cases we can derive analytic formulas for the impedance; calculations of wakefields in the time domain can be a more laborious task.

One step in the design of any accelerator involves the creation of an "impedance budget", in which the sum of the impedances of the individual machine components is compared to the threshold value for instability. If the impedance budget yields instability, the machine design must be modified accordingly.

SNS Extraction Kicker Instability



This was an observed transverse impedance instability in the SNS ring. This is caused by a high transverse impedance in the ring extraction kickers.