Piezoelectricity and ferroelectricity
Phenomena and properties

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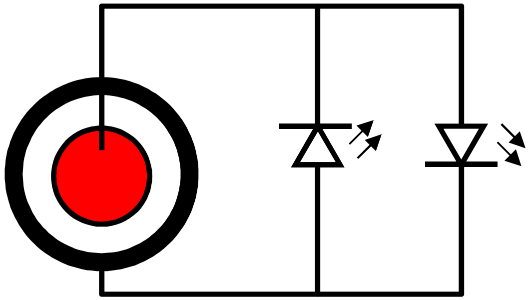
OP Vzdělávání pro konkurenceschopnost
INVESTICE DO ROZVOJE VZDĚLÁVÁNÍ

What is the phenomenon about?

- Metallic membrane with „something strange“
- LEDs flash, why?



J.Erhart: Demonstrujeme piezoelektrický jev, Matematika, fyzika, informatika **20** (2010) 106-109

History

1880, 1881 - Pierre a Jacques Curie, discovery of piezoelectricity, tourmaline and quartz

1917 – A.Langevin – ultrasound generation, sonar

1921 – ferroelectricity - J.Valasek: Piezoelectricity and allied phenomena in Salt, Phys.Rev. 17 (1921) 475

1926 – W.Cady – frequency stabilization of oscillator circuit by the quartz resonator

1944-1946 – USA, SSSR, Japonsko – $BaTiO_3$ ferroelectric ceramics

1954 – B.Jaffe et al – PZT ceramics

60. léta – $LiNbO_3$ and $LiTaO_3$ single crystals

70.léta – ferroelectric polymer PVDF

80.léta – piezoelectric composites

90.léta – domain engineering in PZN-PT, PMN-PT single crystals

Predecessors of piezoelectricity

Pyroelectricity

tourmaline = lapis electricus

$(Na,Ca)(Mg,Fe)_3B_3Al_6Si_6(O,OH,F)_{31}$

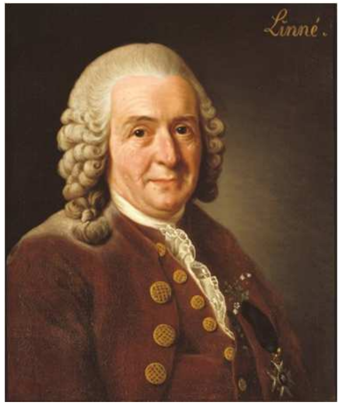
Franz Ulrich Theodor Aepinus

(1724 – 1802) – polar phenomenon

David Brewster – "pyroelectricity" 1824

pyros=oheň

Tourmaline crystal.



Carl Linnaeus
(1707 – 1778)

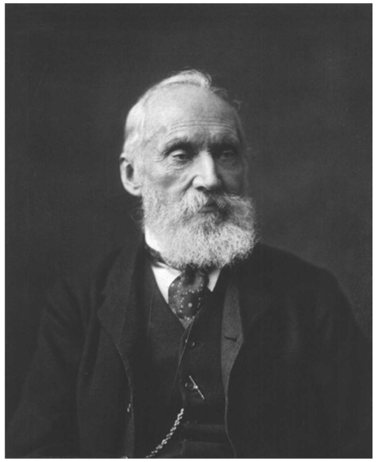
David Brewster
(1781 – 1868)

Pyroelectricity

A.C.Becquerel
first pyroelectricity measurement
1828

Antoine César
Becquerel
(1788 –1878)

William Thomson (Lord Kelvin)
first pyroelectricity theory
1878, 1893



William Thomson,
Lord Kelvin
(1824 – 1907)

$$\Delta P_S = p \cdot \Delta\theta$$

Piezoelectricity discovery

Tourmaline crystal, 1880

Pierre Curie
(1859 – 1906)

Paul-Jacques
Curie
(1856 – 1941)

8.4.1880
Société minéralogique de France

24.8.1880

Académie des Sciences

$$\Delta P = d \cdot T$$

Curie J, Curie P (1880) Développement, par pression, de l'électricité polaire dans les cristaux hémièdres à faces inclinées. Comptes rendus de l'Académie des Sciences 91: 294; 383.

Curie J, Curie P (1881) Contractions et dilatations produites par des tensions électriques dans les cristaux hémièdres à faces inclinées. Comptes rendus de l'Académie des Sciences 93: 1137-1140.

Phenomenon properties

- Linear phenomenon, charge is independent from crystal length, it depends on the electrode area
- Phenomenon exists due to crystal anisotropy (polar axes), amorphous materials are not piezoelectric

Curie's principle

Symmetry elements of external field must be included in the phenomenon symmetry.

Crystal under the influence of external field exhibits only symmetry elements common to the symmetry of crystal itself and of the external field.

Example: mechanical pressure along [111] direction exerted on the cubic crystal with m3m symmetry causes symmetry reduction of deformed crystal to 3m symmetry class

Piezoelectricity

Direct phenomenon — mechanical pressure → electrical charge

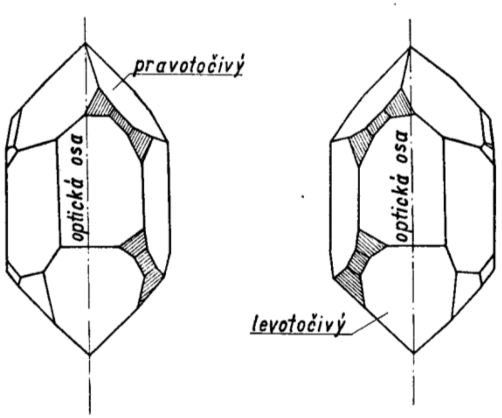
Converse phenomenon — electric field → mechanical deformation

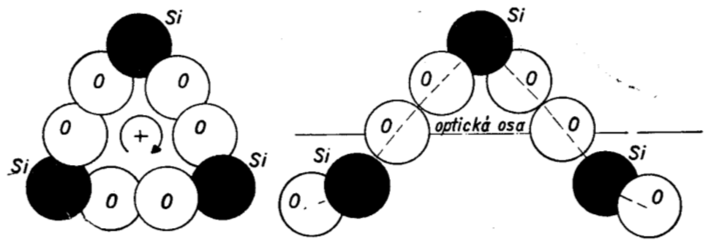
It is limited to certain crystallographic symmetry classes (20 from 32 classes)

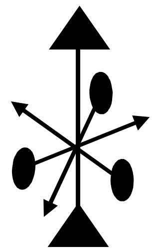
$1,\quad 2,\quad m,\quad 222,\quad mm2,\quad 4,\quad \bar{4},\quad 422,\quad 4mm,\quad \bar{4}2m,$

$3,\quad 32,\quad 3m,\quad 6,\quad \bar{6},\quad 622,\quad 6mm,\quad \bar{6}2m,\quad \bar{4}3m,\quad 23$

Example: SiO_2 (quartz) symmetry *32*





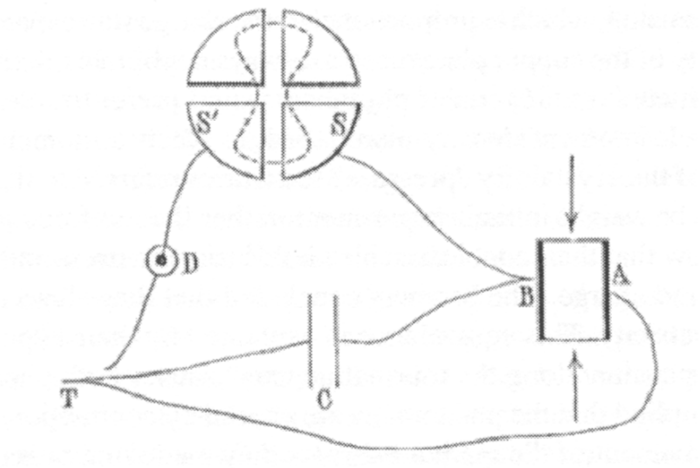


Measurement technique

Direct phenomenon (Brothers Curie)

- Thompson's electrometer
- Null method $\Delta Q = V \Delta C$

(Daniel's cell – 1.12V, 1836)



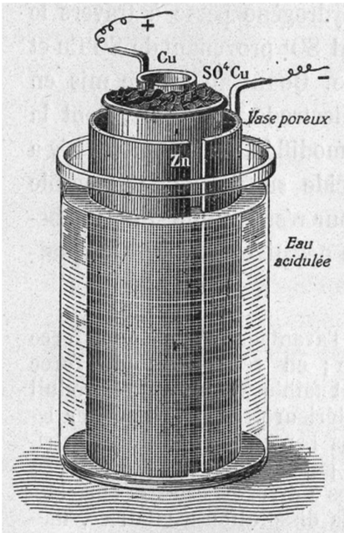
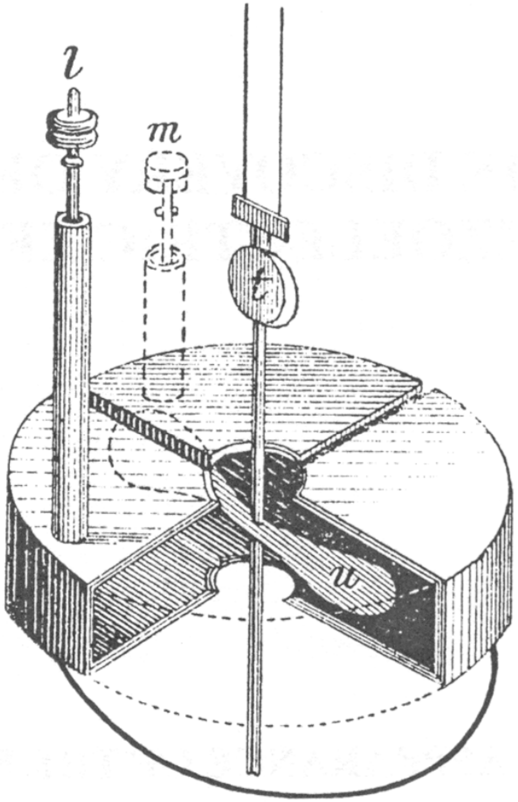


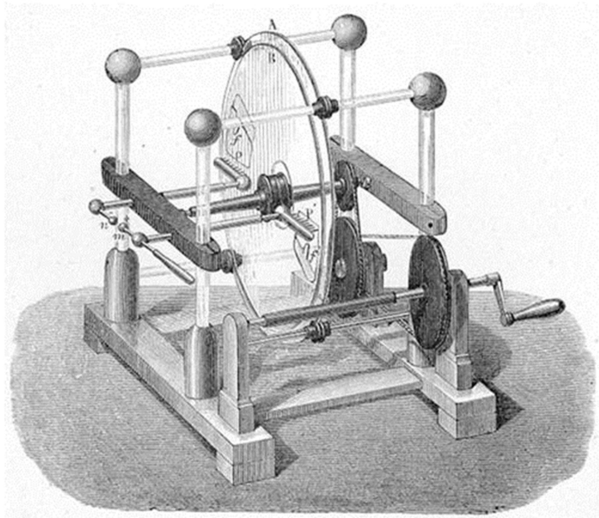
Fig. 284. — Élément Daniell.

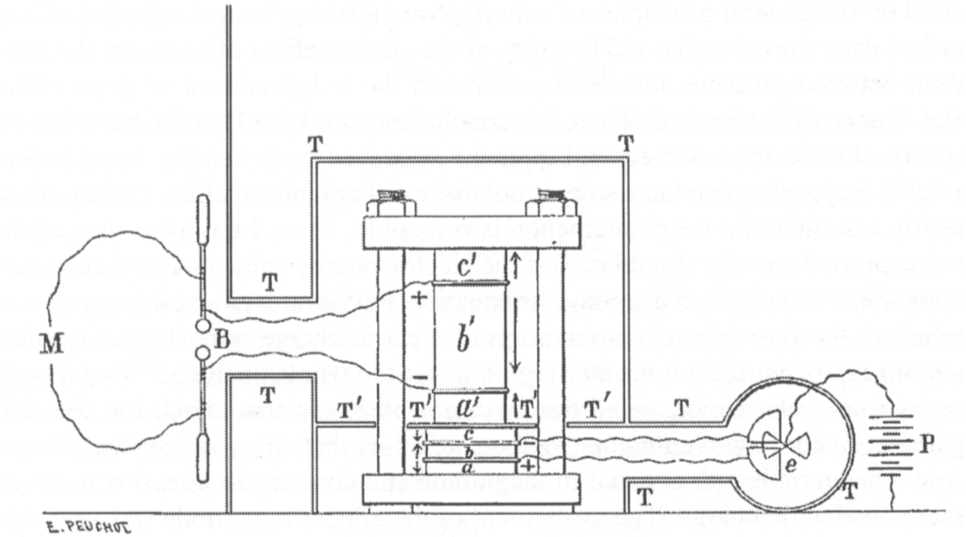


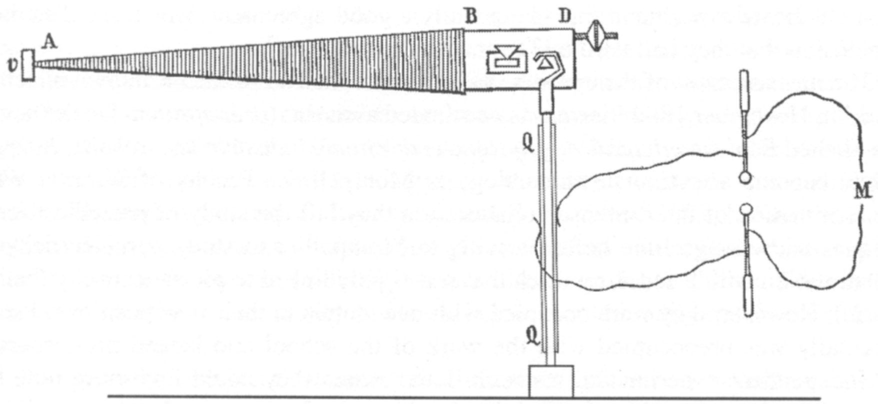
Measurement technique

Converse phenomenon (Brothers Curie)

- Holtz's machine (induction electricity, HV)
- Strain measured by the direct effect, optically

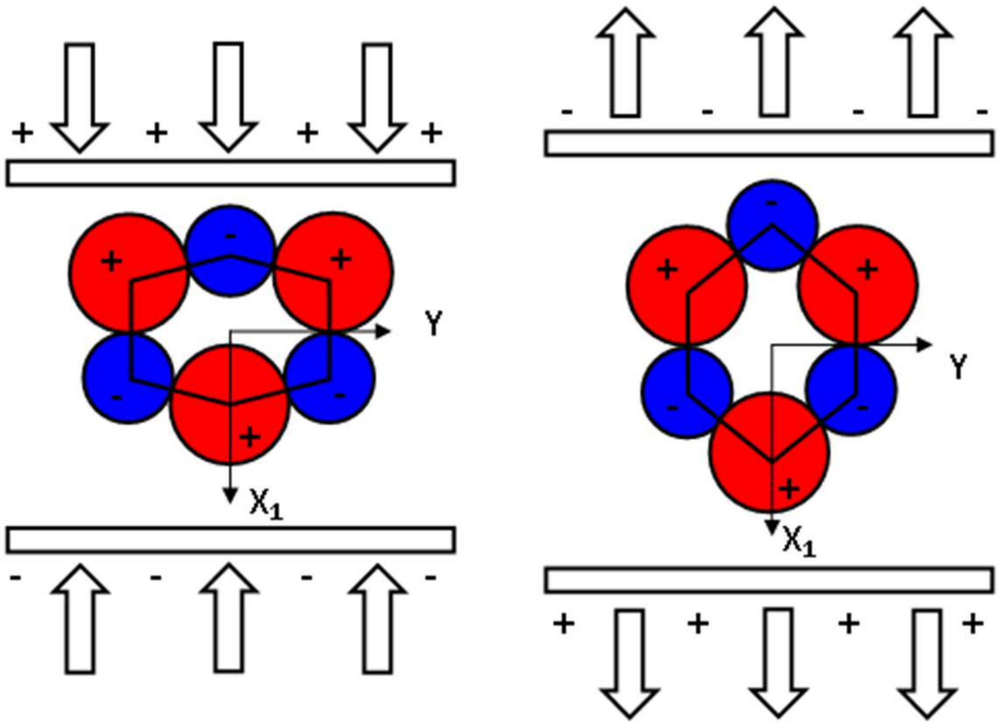






Piezoelectricity origin

Brothers Curie – molecular theory



Charges generated at the compression (a) and tension (b) of quartz crystal basic unit

First theory and application of piezoelectricity

Tensor analysis developed for the crystal anisotropy description 1890
Woldemar Voigt: Lehrbuch der Kristallographie, Teubner Verlag 1910

Woldemar Voigt
(1850 –1919)

First application – electrometer,
radioactivity study (Maria Skłodowska-Curie)

Curie's electrometer

Maria Skłodowska-Curie
(1867 – 1934)

Coupled field phenomena

Heckmann's diagram

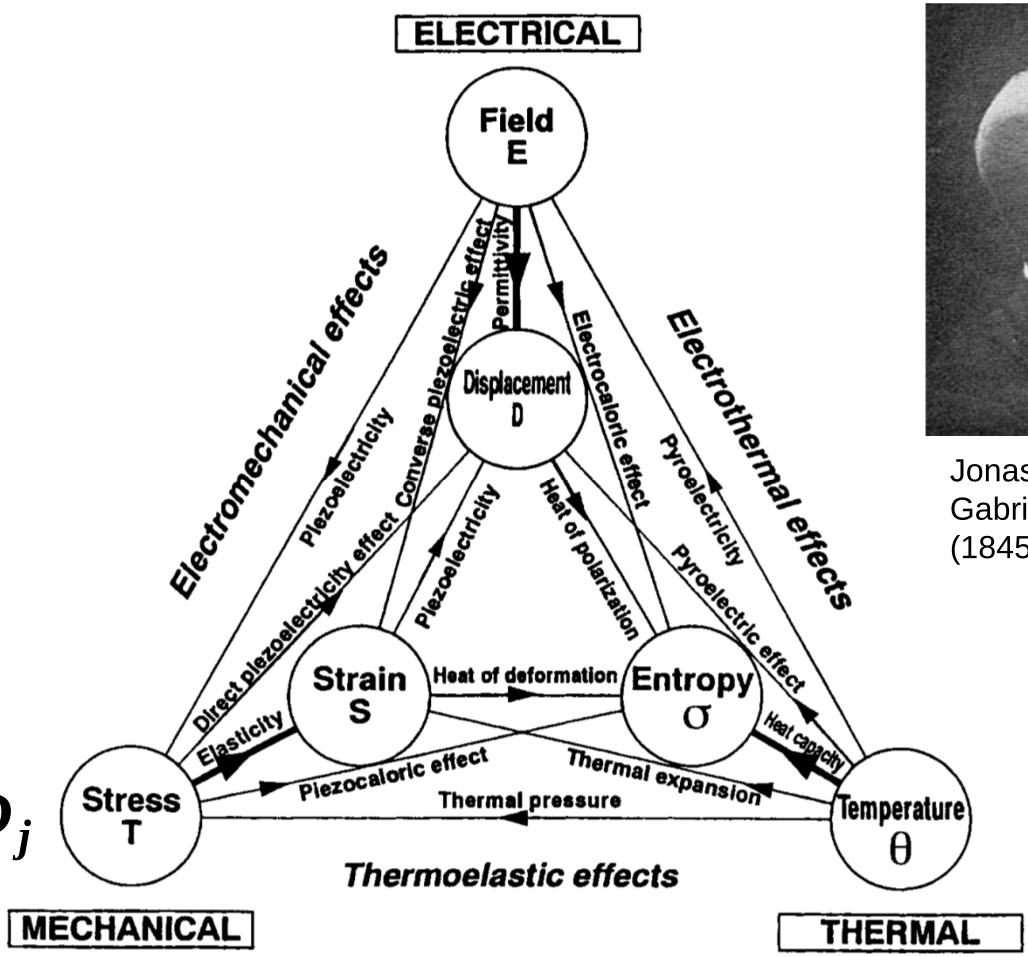
Piezoelectricity

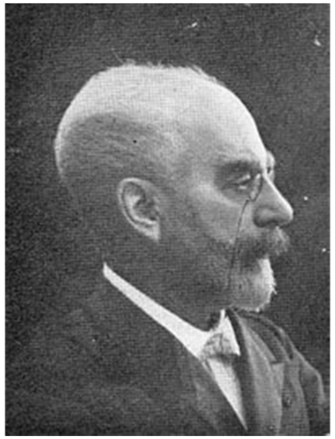
$$S_\lambda = s^E_{\lambda\mu} T_\mu + d_{i\lambda} E_i$$

$$D_i = d_{i\mu} T_\mu + \varepsilon^T_{ij} E_j$$

$$S_\lambda = s^D_{\lambda\mu} T_\mu + g_{i\lambda} D_i$$

$$E_i = -g_{i\mu} T_\mu + \beta^T_{ij} D_j$$



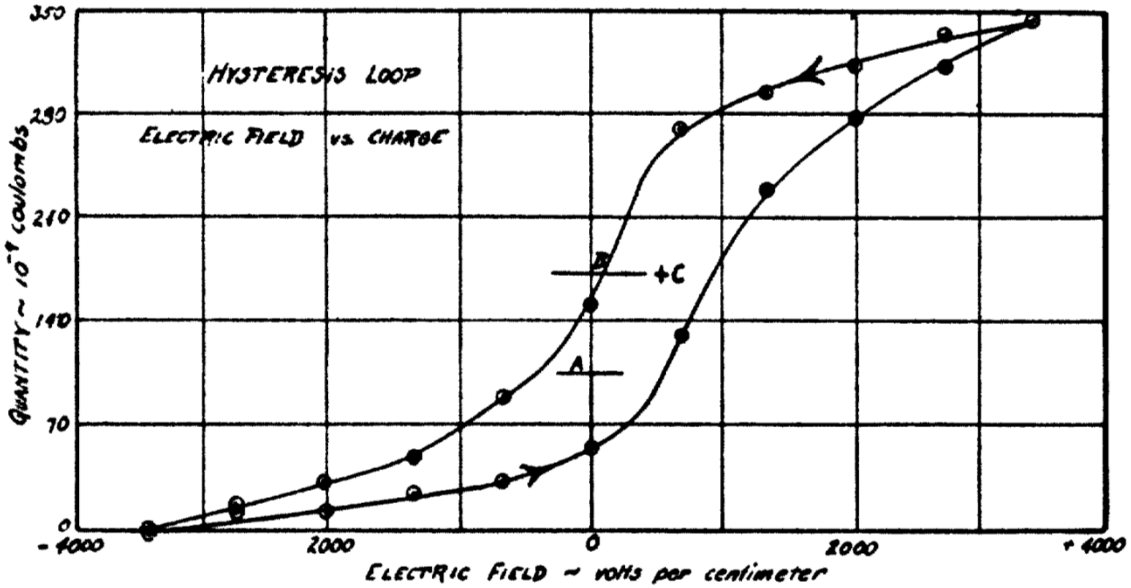


Jonas Ferdinand Gabriel Lippmann (1845 –1921)

Ferroelectricity

Joseph Valasek, 1920, Rochelle Salt $NaKC_4H_4O_6.4H_2O$

Joseph Valasek
(1897-1993)



Hysteresis loop of Rochelle Salt – dry crystal, $0^{o}C$

Valasek J (1921) Piezoelectricity and allied phenomena in Rochelle salt. Phys. Rev. 17: 475-481

Electromechanical phenomena

Direct conversion between mechanical and electrical energy

- Linear effects – Piezo- and Pyroelectricity
- Nonlinear effects - Ferroelectricity, Electrostriction

Analogy in magnetic materials

- Piezomagnetic properties, magnetostriction, magnetoelectric effect, etc.

Crystallographic constraints for piezoelectricity

Noncentrosymmetric classes (except of *432*)

20 piezoelectric crystallographic classes

- Polar classes (10) – singular polar axis

  1, 2, *m*, *mm*2, 4, 4*mm*, 3, 3*m*, 6, 6*mm*

- Polar-neutral classes (10) – multiple polar axes

  $222,\ \bar{4},\ 422,\ \bar{4}2m,\ 32,\ \bar{6},\ 622,\ \bar{6}m2,\ \bar{4}3m,\ 23$

Equations of state

Example: piezoelectricity

$$S_\mu = s^E_{\mu\nu} T_\nu + d_{k\mu} E_k \qquad S_\mu = s^D_{\mu\nu} T_\nu + g_{k\mu} D_k$$

$$D_i = d_{i\nu} T_\nu + \varepsilon^T_{ik} E_k \qquad E_i = -g_{i\nu} T_\nu + \beta^T_{ik} D_k$$

$$T_\mu = c^E_{\mu\nu} S_\nu - e_{k\mu} E_k \qquad T_\mu = c^D_{\mu\nu} S_\nu - h_{k\mu} D_k$$

$$D_i = e_{i\nu} S_\nu + \varepsilon^S_{ik} E_k \qquad E_i = -h_{i\nu} S_\nu + \beta^S_{ik} D_k$$

Piezoelectric coefficients

Different coefficients due to the choice of independent variables

$$d_{i\mu} = \frac{\partial D_i}{\partial T_\mu} = \frac{\partial S_\mu}{\partial E_i} \qquad \mathrm{C\,N^{-1}}$$

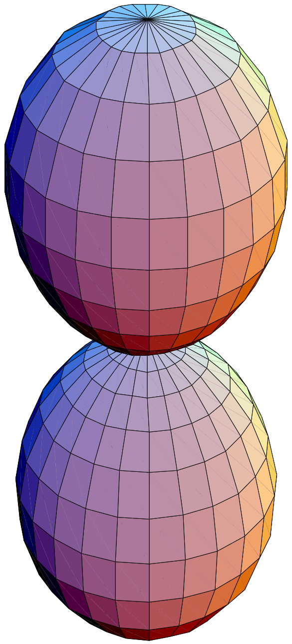
$$g_{i\mu} = -\frac{\partial E_i}{\partial T_\mu} = \frac{\partial S_\mu}{\partial D_i} \qquad \mathrm{C^{-1}\,m^2}$$

$$h_{i\mu} = -\frac{\partial E_i}{\partial S_\mu} = -\frac{\partial T_\mu}{\partial D_i} \qquad \mathrm{C^{-1}\,N}$$

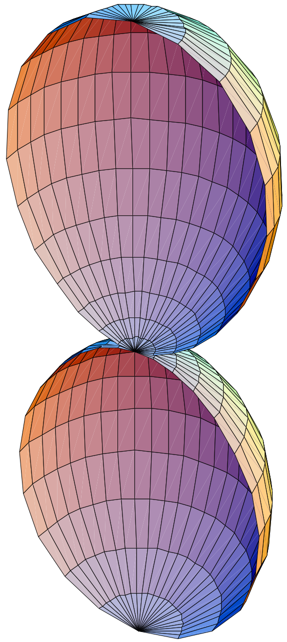
$$e_{i\mu} = \frac{\partial D_i}{\partial S_\mu} = -\frac{\partial T_\mu}{\partial E_i} \qquad \mathrm{C\,m^{-2}}$$

Material property anisotropy
d_{33} surface

$P_S[001]_T$



$PbTiO_3$ – 4*mm*



Maximum

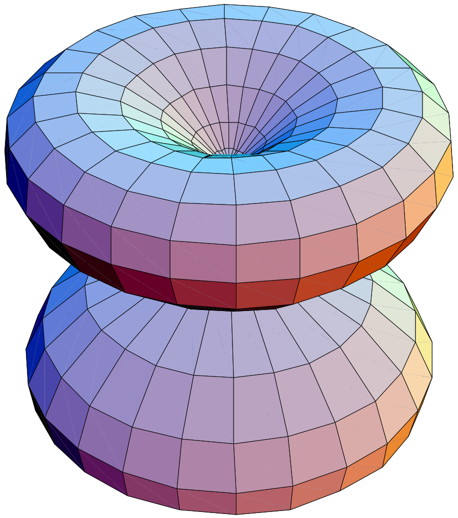
83.7pC/N for $\theta = 0^o$, i.e. $[001]_C$

$d_{33}l_3^3+(d_{31}+d_{15})l_3(l_1^2+l_2^2)$

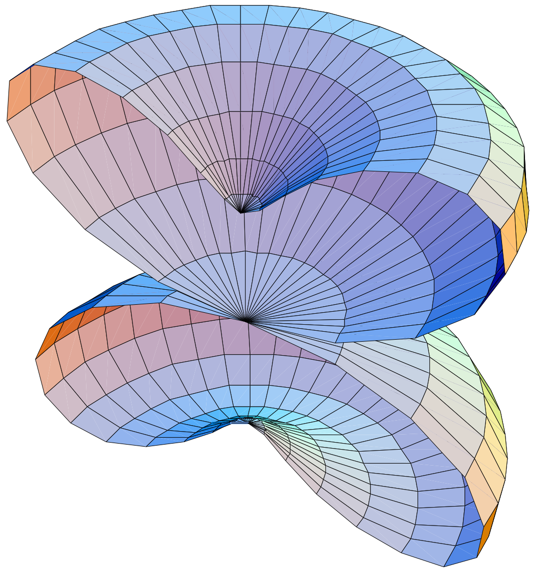
$l_1=\sin(\theta)\cos(\varphi)$, $l_2=\sin(\theta)\sin(\varphi)$, $l_3=\cos(\theta)$

d_{33}= 83.7, d_{31}= -27.2, d_{15}= 60.2 [pC/N]

Material property anisotropy
d_{33} surface



$P_S[001]_T$



BaTiO_3 – 4*mm*

$d_{33}l_3^3+(d_{31}+d_{15})l_3(l_1^2+l_2^2)$
$l_1=\sin(\theta)\cos(\varphi)$, $l_2=\sin(\theta)\sin(\varphi)$, $l_3=\cos(\theta)$
d_{33}= 90, d_{31}= -33.4, d_{15}= 564 [pC/N]

Maximum
224pC/N for $\theta = 51^o$
90pC/N for $[001]_C$
221pC/N for $[111]_C$

Pyroelectricity

Direct effect

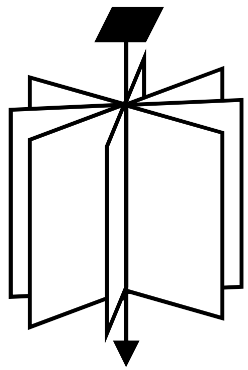
Temperature change → electric charge

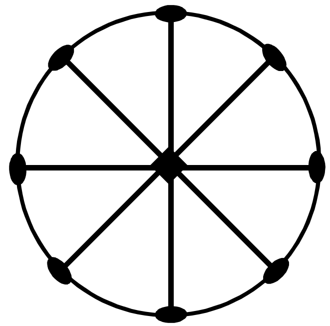
Converse effect (electrocaloric effect)

Electric field → heat generation or absorption

Anisotropy

Example:
Lithium tetraborate $Li_2B_4O_7$,
symmetry *4mm*





Crystallographic constraints for pyroelectricity

Polar symmetry classes (10) – singular polar axis

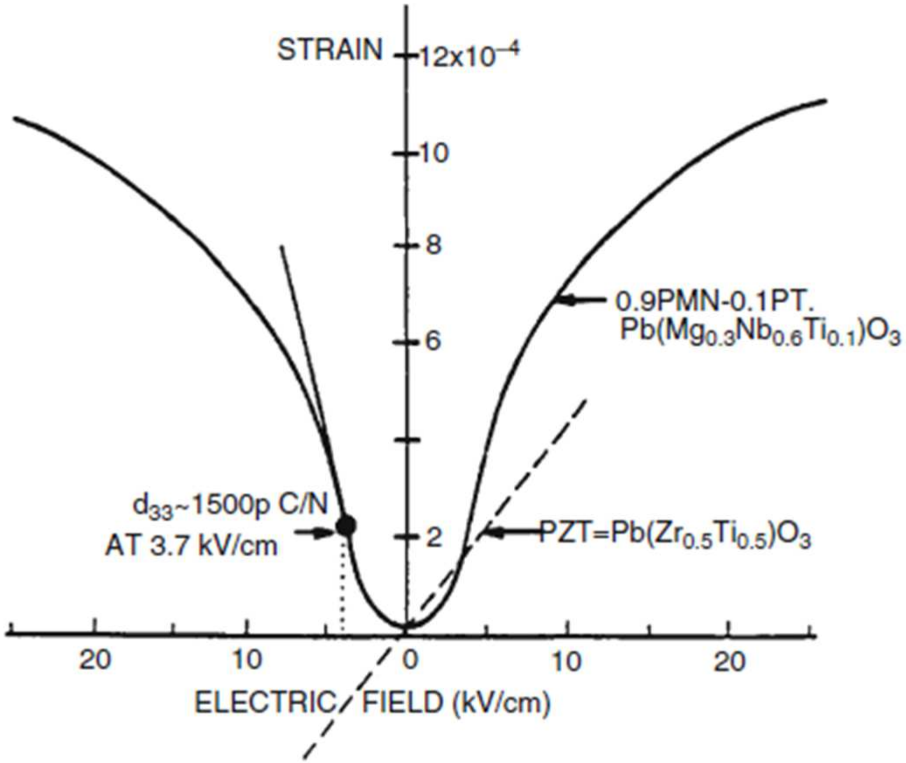
1, 2, *m*, *mm*2, 4, 4*mm*, 3, 3*m*, 6, 6*mm*

Pyroelectric polarization of material (dipole moment) – polar axis direction

Electrostriction

- Nonlinear effect

Deformation proportional to the square of electric field



- No constraints on the material symmetry, effect exists in all materials

Electrostriction

Nonlinear equations of state

$$S_{ij} = d_{kij} E_k + Q_{ijkl} P_k P_l$$

Without crystallographic limits!

All materials exhibit electrostrictive properties

Electrostriction in cubic materials

Electrostrictive coefficients Q_{11}, Q_{12}, Q_{44}

$$\begin{pmatrix} S_1 \\ S_2 \\ S_3 \\ S_4 \\ S_5 \\ S_6 \end{pmatrix} = \begin{pmatrix} Q_{11} & Q_{12} & Q_{12} & 0 & 0 & 0 \\ Q_{12} & Q_{11} & Q_{12} & 0 & 0 & 0 \\ Q_{12} & Q_{12} & Q_{11} & 0 & 0 & 0 \\ 0 & 0 & 0 & Q_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & Q_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & Q_{44} \end{pmatrix} \begin{pmatrix} P_1^2 \\ P_2^2 \\ P_3^2 \\ P_2 P_3 \\ P_1 P_3 \\ P_1 P_2 \end{pmatrix}$$

Ferroelectricity

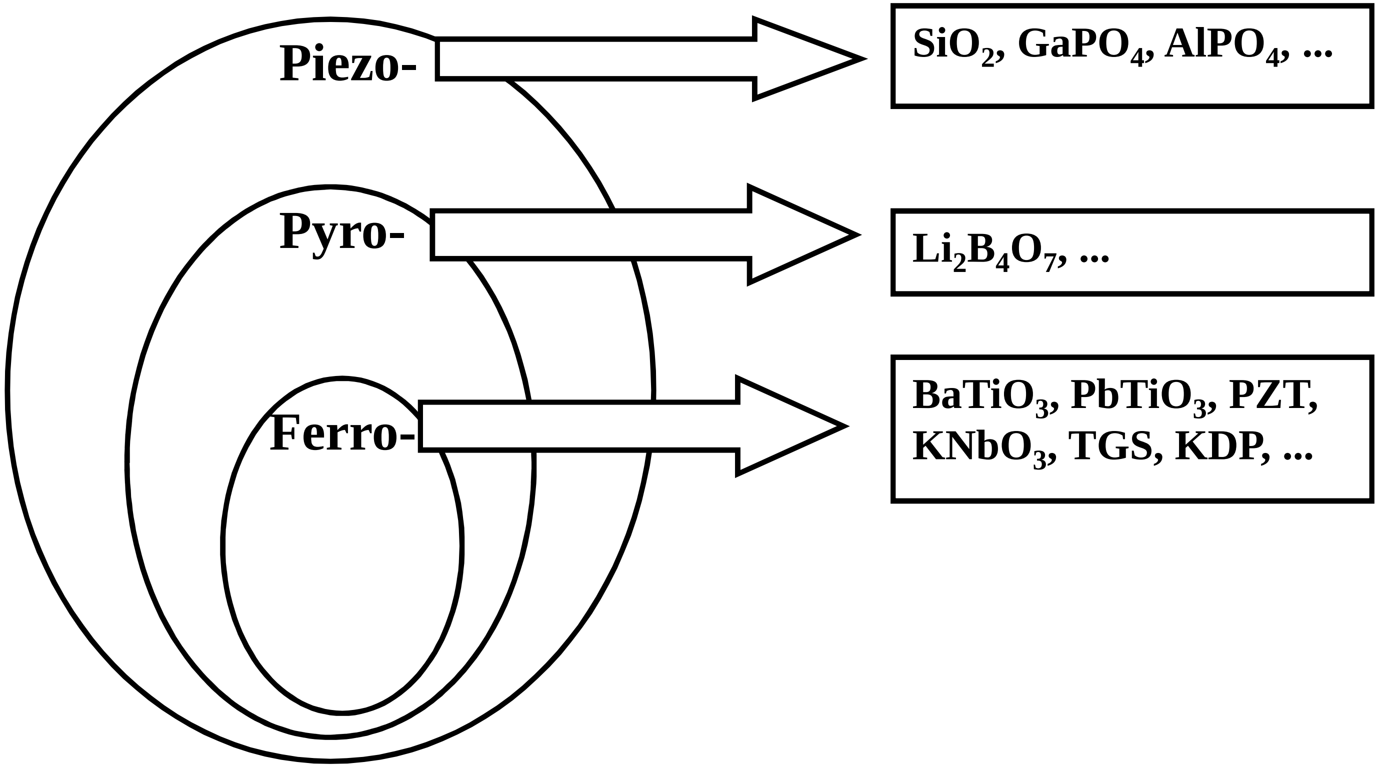
Spontaneous dipole moments = pyroelectricity with switchable polarization

Electric analogy of permanent magnets

Characteristic properties

- Ferroelectric domains and domain walls
- Hysteresis loop D-E (S-E)

Hierarchy of electromechanical phenomena



Ferroelectricity

Spontaneous existence of polarization (and spontaneous strain at the same time)

- structural phase transition

Ferroelectric/ferroelastic domains

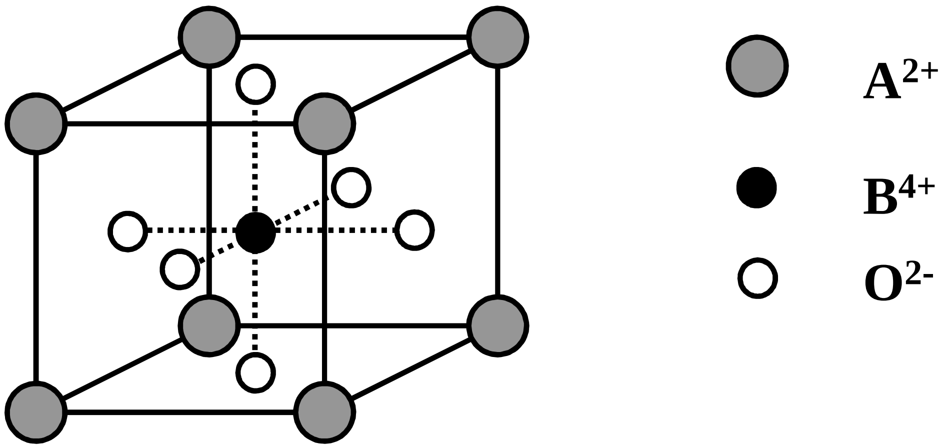
- orientation domain states

Domain walls

Hysteresis

$BaTiO_3$ – paraelectric phase

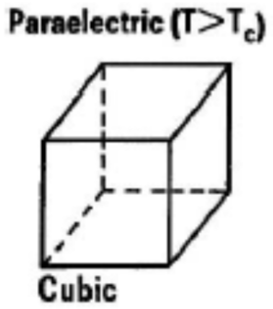
Perovskite structure



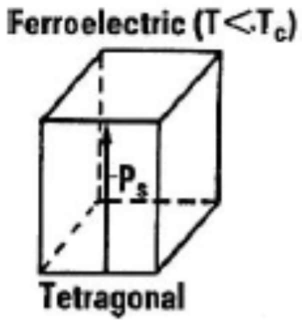
$BaTiO_3$ – ferroelectric phase

Several orientation states exist for the spontaneous polarization

$m\bar{3}m$

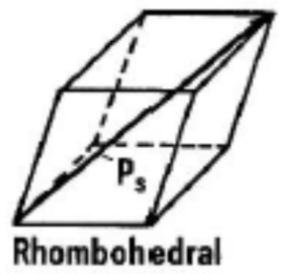


$4mm$



$mm2$

$3m$

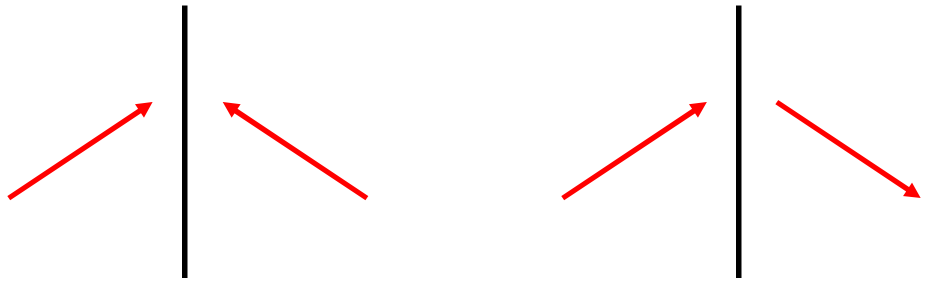


Domains, domain walls

Domain – space continuous region with the same orientation of the spontaneous dipole moment (polarization)

Domain walls – interfaces between domains

- Charged walls
- Neutral walls



Ferroelectric domains exist in ferroelectric phase

Phase transition – Curie temperature T_C

D-E a S-E hysteresis loops

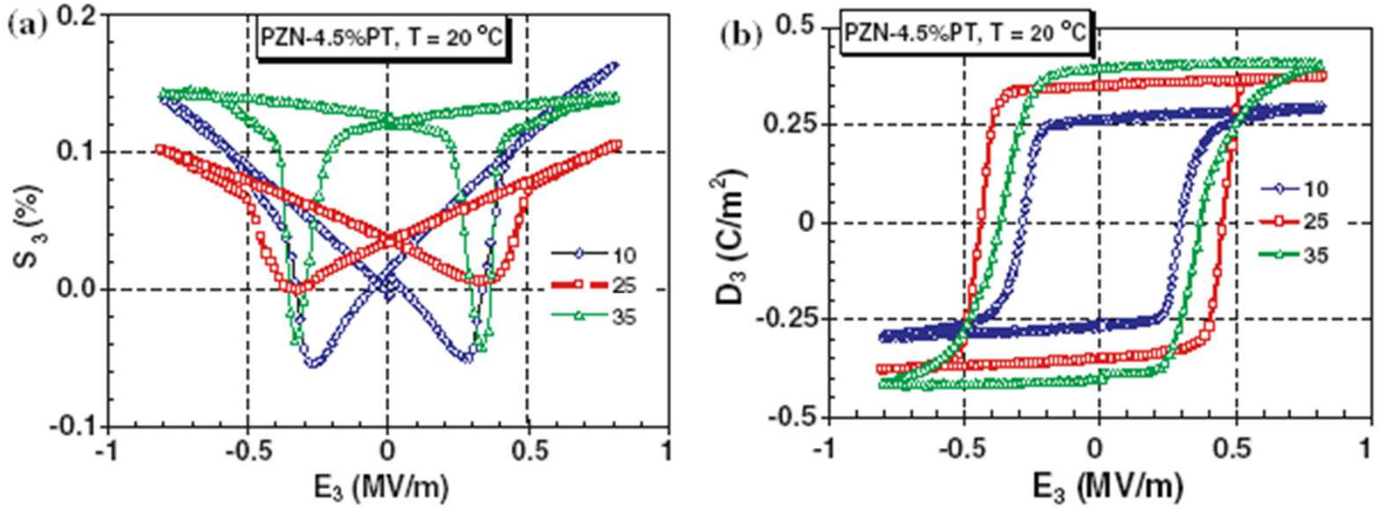
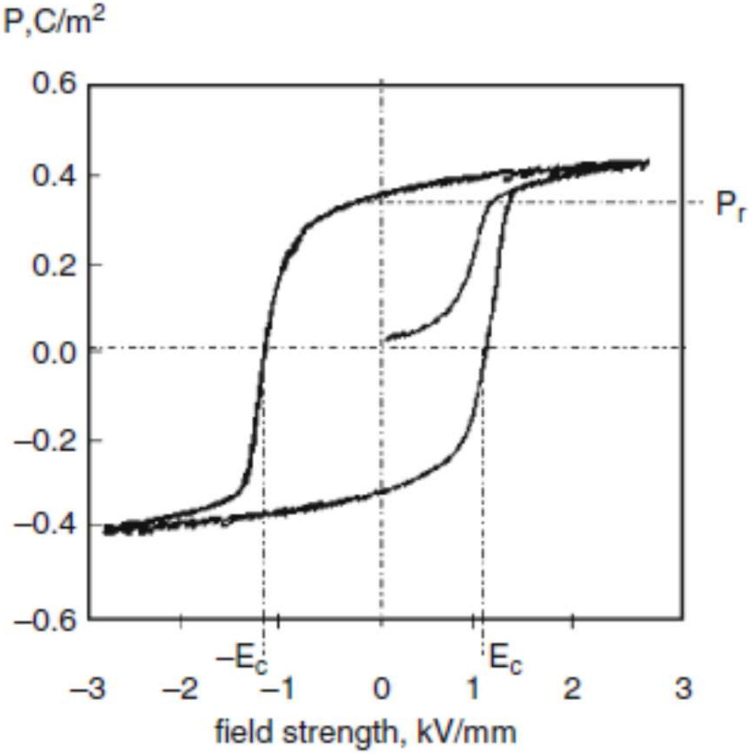


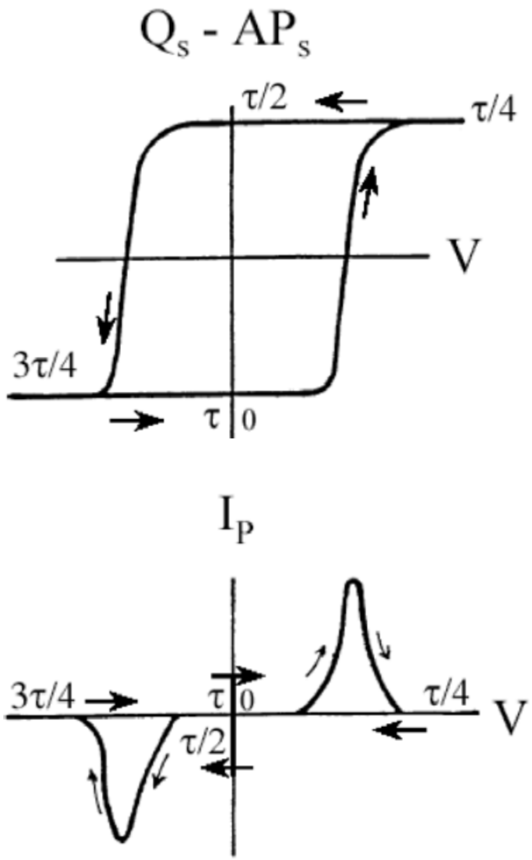
Fig. 5 Polarization and strain response of PZN-4.5%PT single crystals under electric field in directions 10°, 25°, and 35° off <001>. **a** Strain versus electric field; **b** electric displacement versus electric field

T. Liu, C. S. Lynch: Domain engineered relaxor ferroelectric single crystals
Continuum Mech. Thermodyn. **18** (2006) 119–135

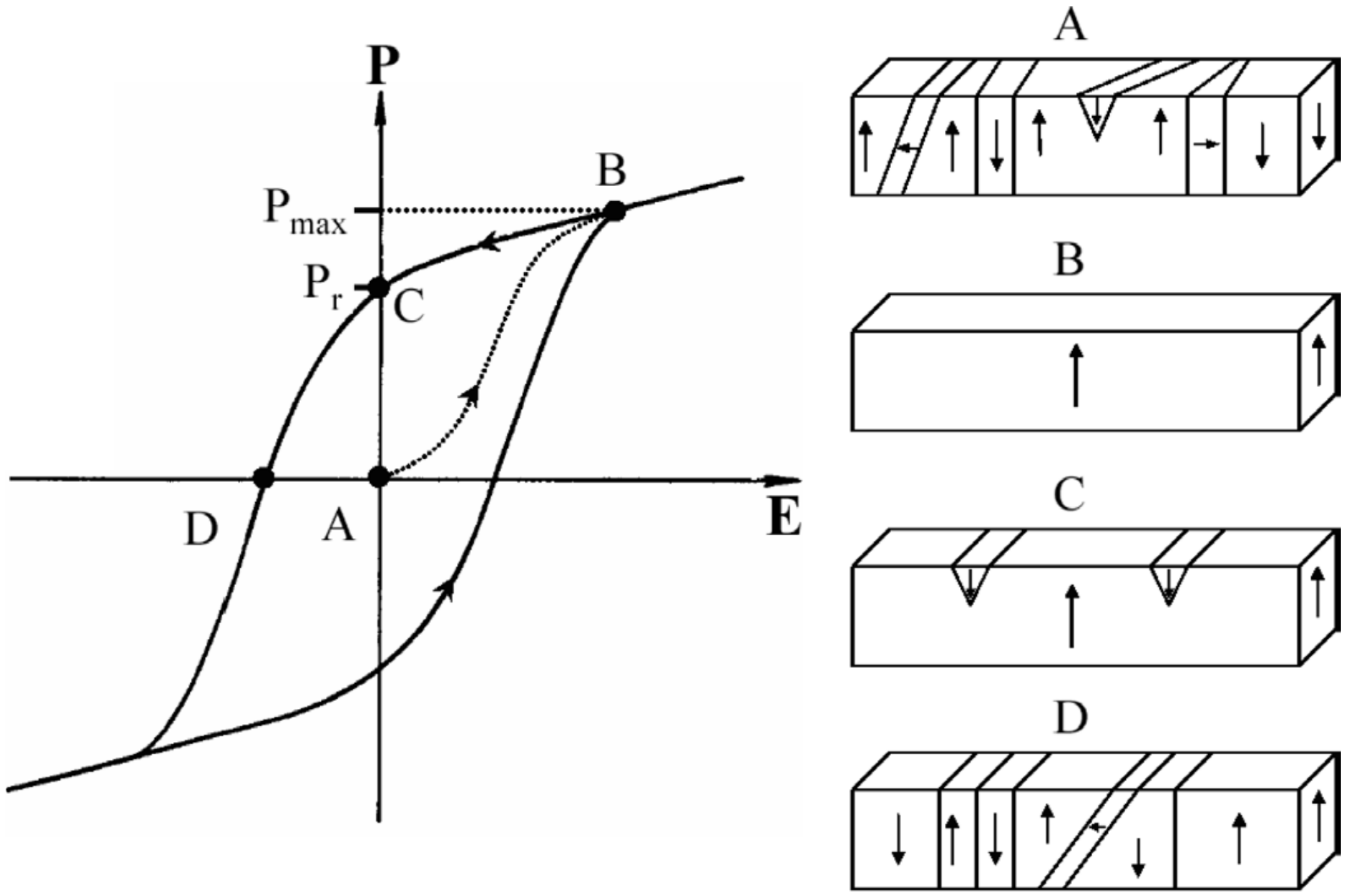
Remanent polarization

P_S, P_r, E_C coercive field





Mechanism of domain reorientation



Remanent deformation

Ferroelasticity in ferroelectric materials

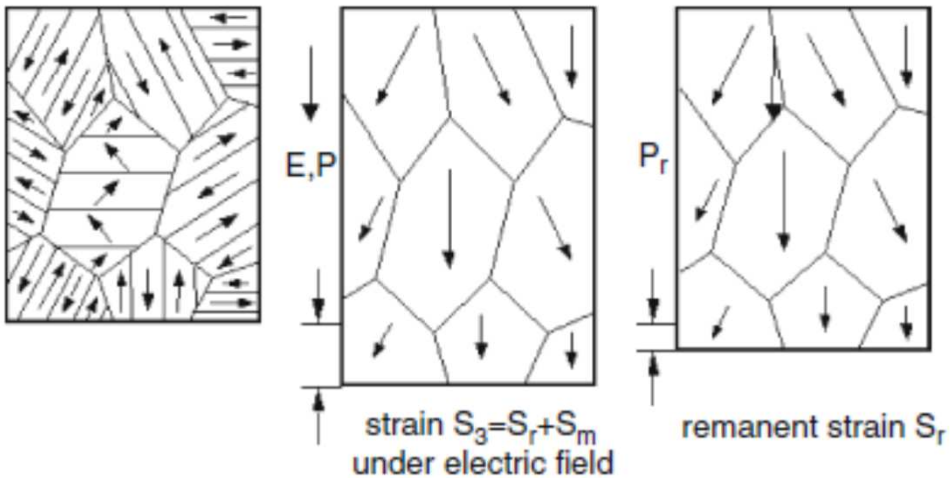
$$S_3^{pr} = 2\,Q_{33}^* P_r P_3 = g_{33} P_3,$$

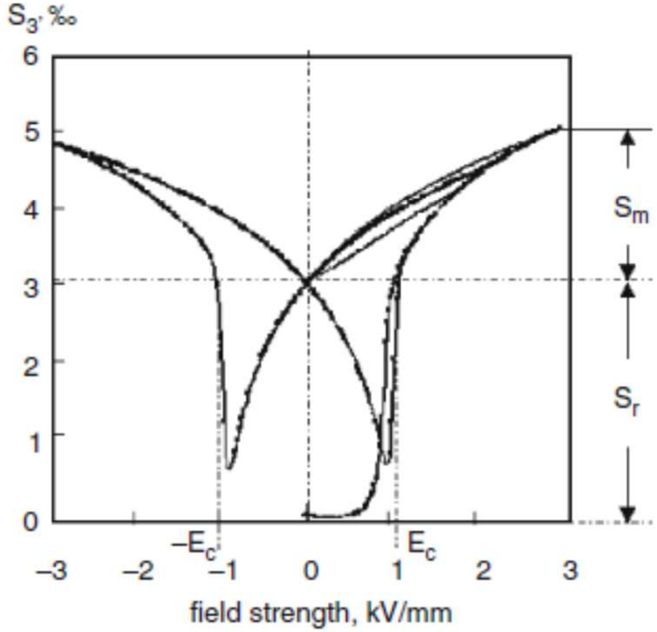
$$g_{33} = 2\,Q_{33}^* P_r,$$

S_3^{pr} = strain at constant remanent polarisation P_r.

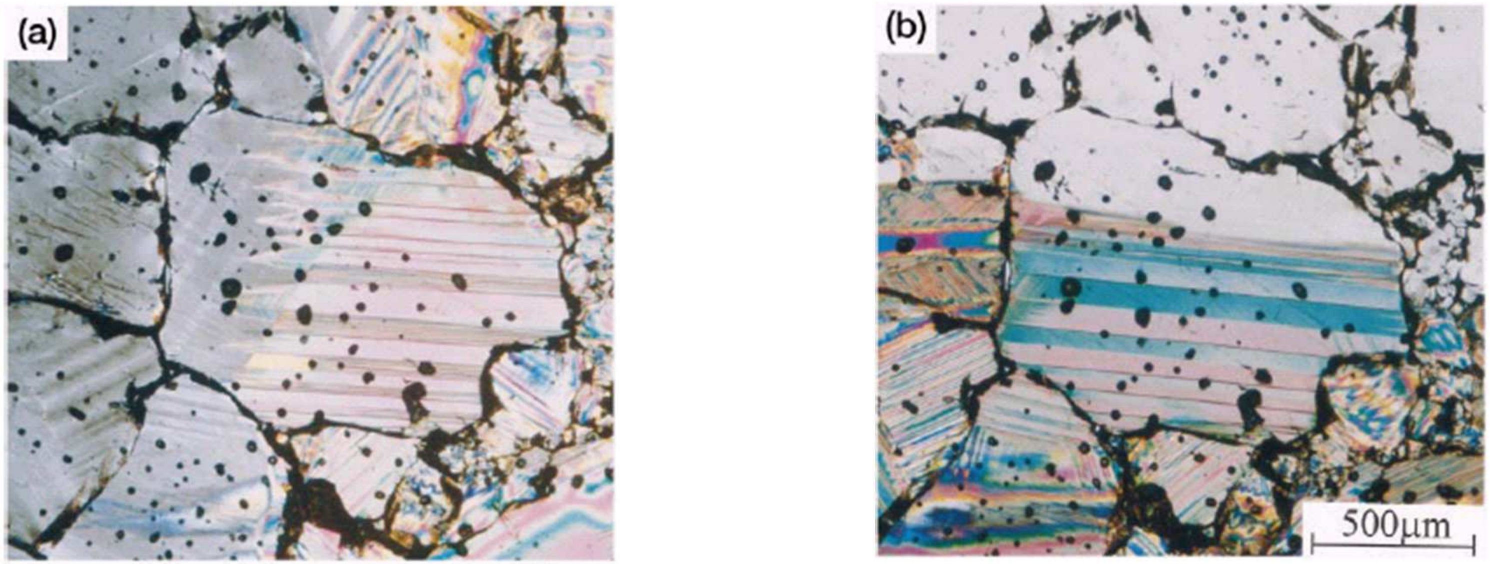
$$d_{33} = \varepsilon^T{}_{33}\, g_{33},$$

$$d_{33} = 2\,Q_{33}^* \varepsilon^T{}_{33} P_r.$$





Domains in $BaTiO_3$ ceramics $m\bar{3}m \rightarrow 4mm$



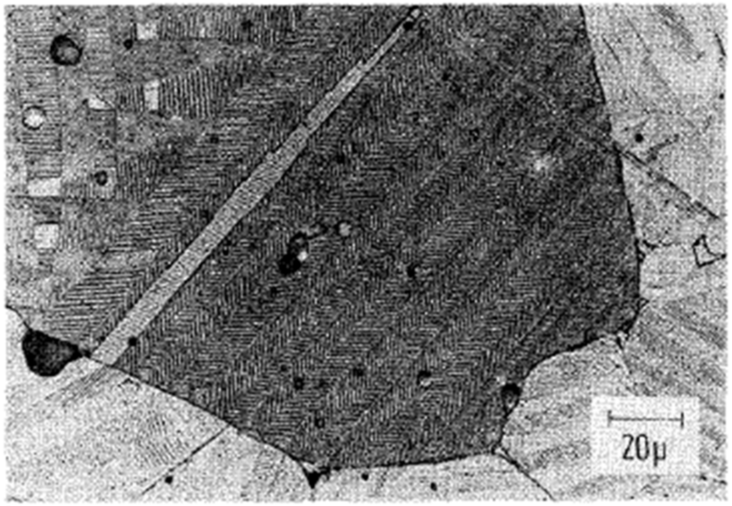
Domain structure evolution during heating of $BaTiO_3$ ceramics over the Curie temperature

(a) Temperature gradient parallel to the domain walls

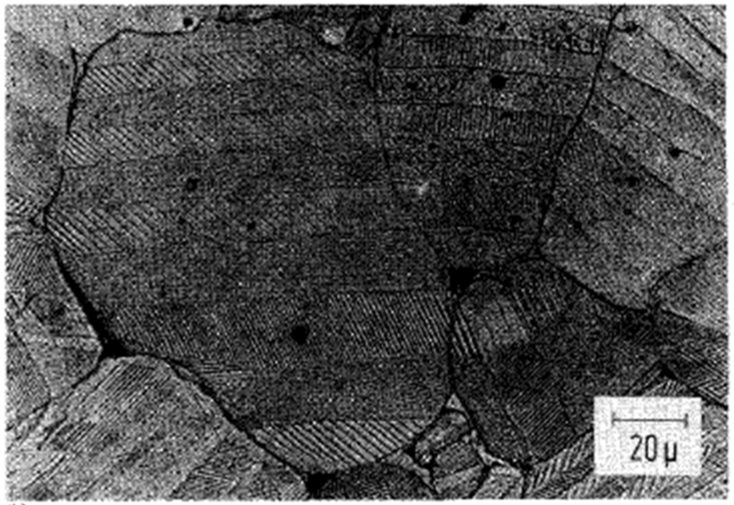
(b) Temperature gradient perpendicular to the domain walls

Sang-Beom Kim, Doh-Yeon Kim:*J. Am. Ceram. Soc.*, **83** [6] 1495–98 (2000)

Domains in $BaTiO_3$ ceramics $m\bar{3}m \rightarrow 4mm$



(a)



(b)

Etched surface of $BaTiO_3$ ceramics

a) Herringbone and chessboard pattern

b) Band structure of DW's, domains continuously cross the grain boundaries

G.Arlt, P.Sasko: J.Appl.Phys. **51** (1980) 4956-4960

Ferroic phases

Structural phase transitions

Parent phase (e.g. paraelectric) $\rightarrow$ ferroic phase

Feroelectrics

$LiNbO_3$ $\bar{3}m \rightarrow 3m$

$KNbO_3$ $m\bar{3}m \rightarrow mm2$

$BaTiO_3$ $m\bar{3}m \rightarrow 4mm$

$Pb_5Ge_3O_{11}$ $\bar{6} \rightarrow 3$

KIO_3 $3m \rightarrow m$

Ferroelastics

$AgNbO_3$

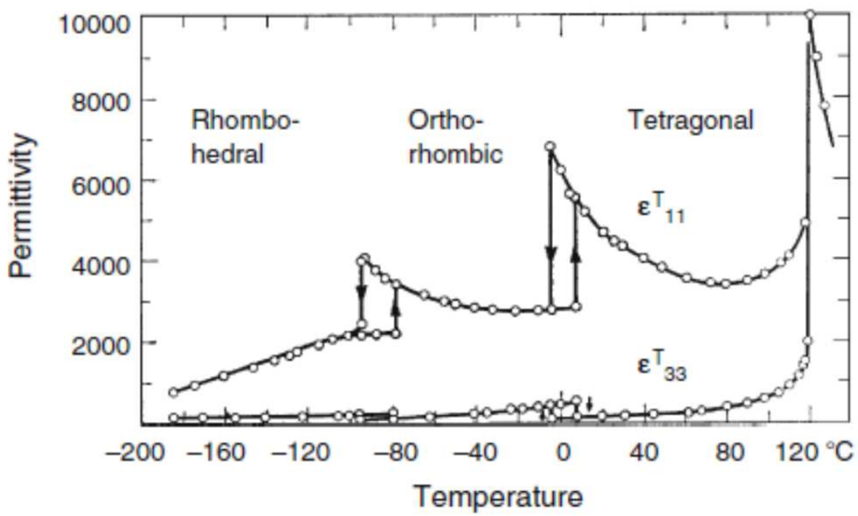
$NaNbO_3$ $m\bar{3}m \rightarrow mmm$

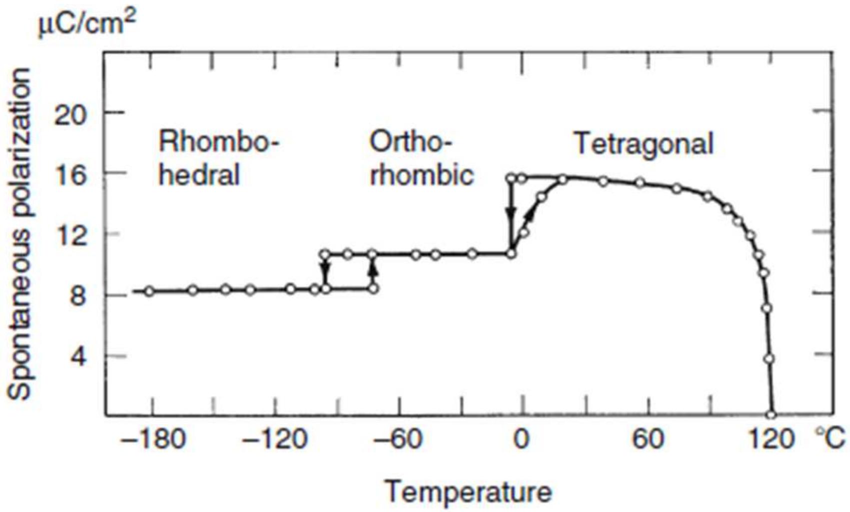
$Pb_3(PO_4)_2$ $\bar{3}m \rightarrow 2/m$

Material property anomalies

Dielectric permittivity, spontaneous polarization, etc.

$BaTiO_3$

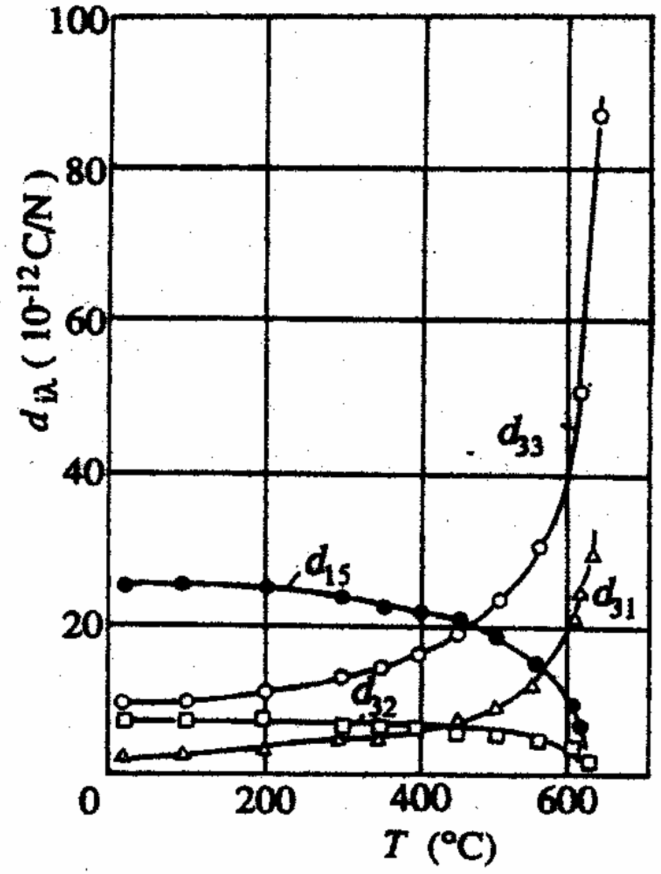




Material property anomalies

Piezoelectric coefficient

$LiTaO_3$



Landau-Ginzburg-Devonshire theory

Power expansion of thermodynamic potential

$$\boldsymbol{\Phi}(\boldsymbol{T},\eta) = \boldsymbol{\Phi}_0 + \frac{1}{2}\alpha(\boldsymbol{T} - \boldsymbol{T_C})\eta^2 + \frac{1}{4}\beta\eta^4$$

Equilibrium and stability

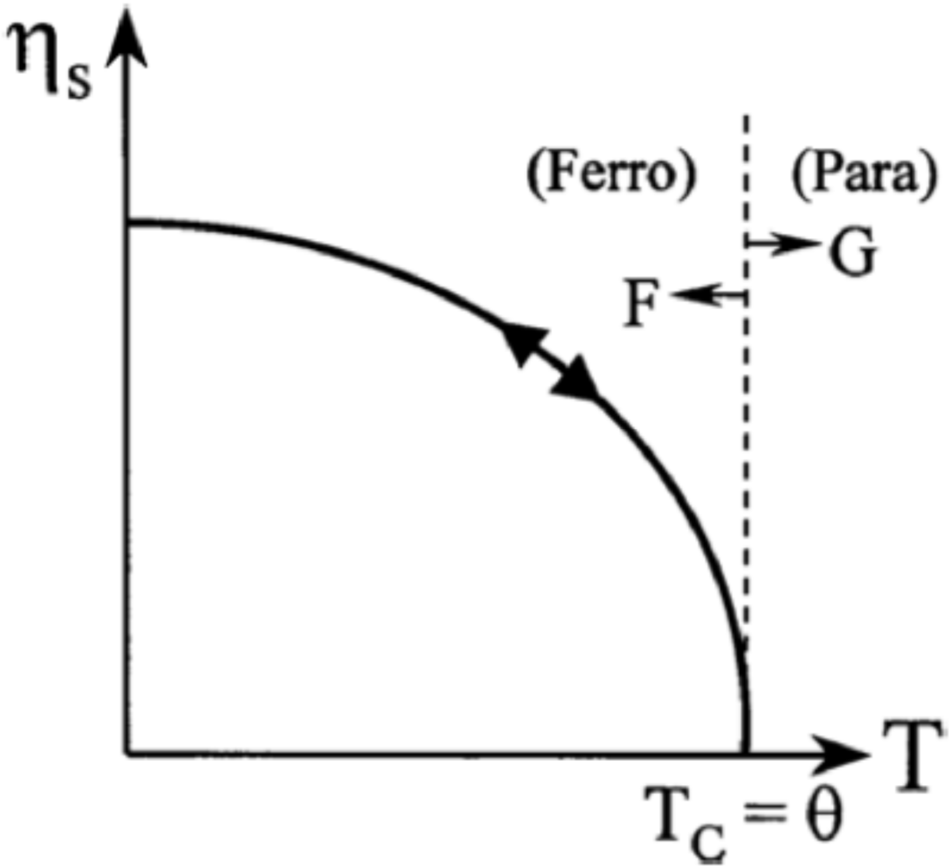
$$\frac{\partial \Phi}{\partial \eta} = 0, \frac{\partial^2 \Phi}{\partial \eta^2} > 0$$

Equilibrium phase transition parameter value

$$\frac{\partial \Phi}{\partial \eta} = \alpha(T - T_C)\eta_0 + \beta\eta_0^3 = 0 \qquad \eta_0 = \begin{cases} 0 & T > T_C \\ \pm\left(-\frac{\alpha(T - T_C)}{\beta}\right)^{1/2} & T < T_C \end{cases}$$

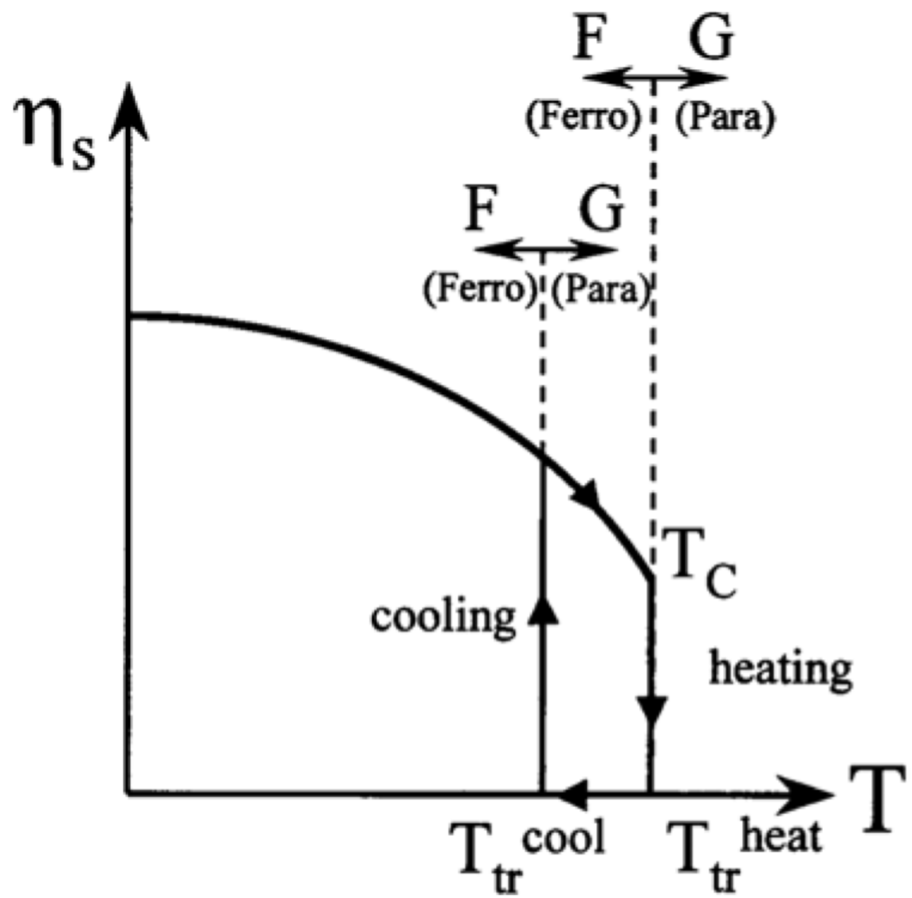
2^{nd} order phase transition

Without hysteresis



1st order phase transition

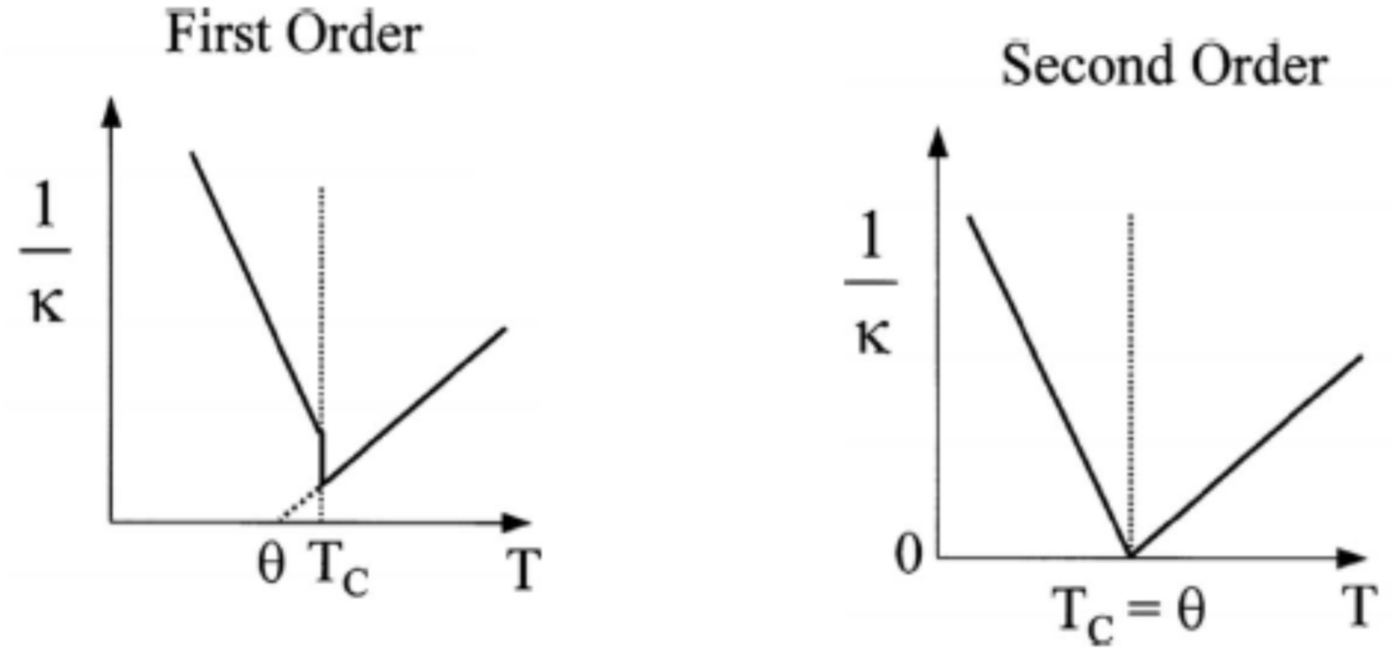
Hysteresis



Curie – Weiss law

Dielectric permittivity temperature dependence

$$\kappa = \frac{\varepsilon}{\varepsilon_0} = \frac{C}{T - \theta}$$



Spontaneous strain vs. spontaneous polarization

Generally (for normal ferroelectrics)

$$S_{kl} = Q_{ijkl} P_i P_j$$

Example for the ferroelectric species $4/mmm \rightarrow m_{xy}$

$$\begin{pmatrix} S_1 \\ S_2 \\ S_3 \\ S_4 \\ S_5 \\ S_6 \end{pmatrix} = \begin{pmatrix} Q_{11} & Q_{12} & Q_{13} & 0 & 0 & 0 \\ Q_{12} & Q_{11} & Q_{13} & 0 & 0 & 0 \\ Q_{31} & Q_{31} & Q_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & Q_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & Q_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & Q_{66} \end{pmatrix} \begin{pmatrix} P_{S1}^2 \\ P_{S1}^2 \\ P_{S3}^2 \\ P_{S1}P_{S3} \\ P_{S1}P_{S3} \\ P_{S1}^2 \end{pmatrix}$$

$$P_S = (P_{S1}, P_{S1}, P_{S3})$$

Experimental characterization of spontaneous deformation

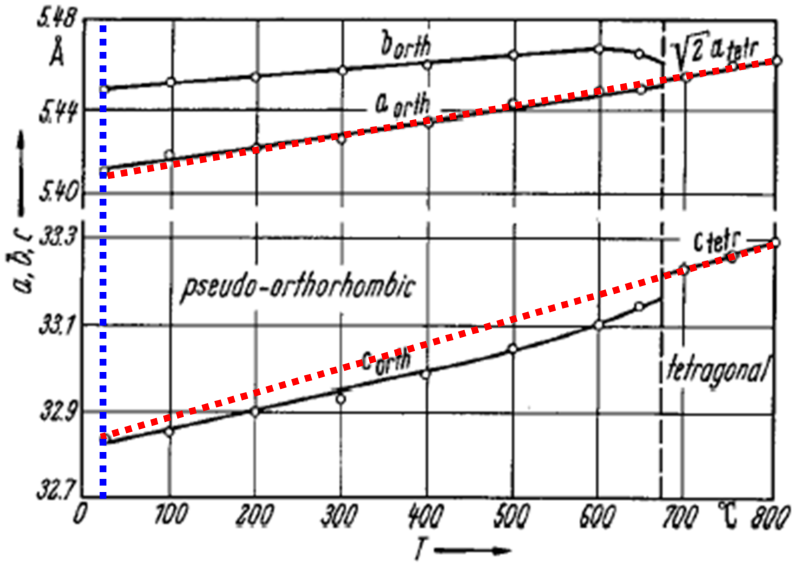


Fig. 1430. $Bi_4Ti_3O_{12}$. Lattice parameters vs. T [61 S 15].

LB Tables III/16a

Lattice constants measured by X-Ray diffraction

- In parent phase (extrapolation down to the ferroic phase)
- In ferroic phase

General formula for the components of strain tensor

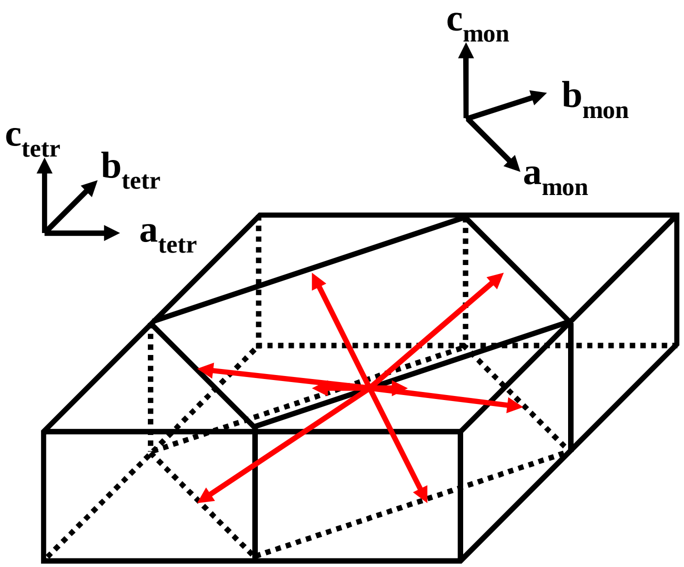
J.L.Schlenker, G.V.Gibbs, M.B.Boisen, Jr.: Acta Cryst. **A34** (1978) 52-54

$Bi_4Ti_3O_{12}$

$$4/mmm \rightarrow m_{xy}$$

8 ferroelectric DS
4 ferroelastic DS

$P_a >> P_c$



$Bi_4Ti_3O_{12}$

Spontaneous deformation/polarization (components in the parent phase coordinate system)

$$S^{(1)} = \begin{pmatrix} S_{11} & S_{12} & S_{13} \\ S_{12} & S_{11} & -S_{13} \\ S_{13} & -S_{13} & S_{33} \end{pmatrix} \quad P^{I} = (\frac{1}{\sqrt{2}}P_a, -\frac{1}{\sqrt{2}}P_a, P_c) \quad P^{II} = (-\frac{1}{\sqrt{2}}P_a, \frac{1}{\sqrt{2}}P_a, -P_c)$$

$$S^{(2)} = \begin{pmatrix} S_{11} & -S_{12} & S_{13} \\ -S_{12} & S_{11} & S_{13} \\ S_{13} & S_{13} & S_{33} \end{pmatrix} \quad P^{III} = (\frac{1}{\sqrt{2}}P_a, \frac{1}{\sqrt{2}}P_a, P_c) \quad P^{IV} = (-\frac{1}{\sqrt{2}}P_a, -\frac{1}{\sqrt{2}}P_a, -P_c)$$

$$S^{(3)} = \begin{pmatrix} S_{11} & S_{12} & -S_{13} \\ S_{12} & S_{11} & S_{13} \\ -S_{13} & S_{13} & S_{33} \end{pmatrix} \quad P^{V} = (-\frac{1}{\sqrt{2}}P_a, \frac{1}{\sqrt{2}}P_a, P_c) \quad P^{VI} = (\frac{1}{\sqrt{2}}P_a, -\frac{1}{\sqrt{2}}P_a, -P_c)$$

$$S^{(4)} = \begin{pmatrix} S_{11} & -S_{12} & -S_{13} \\ -S_{12} & S_{11} & -S_{13} \\ -S_{13} & -S_{13} & S_{33} \end{pmatrix} \quad P^{VII} = (-\frac{1}{\sqrt{2}}P_a, -\frac{1}{\sqrt{2}}P_a, P_c) \quad P^{VIII} = (\frac{1}{\sqrt{2}}P_a, \frac{1}{\sqrt{2}}P_a, -P_c)$$

Domain wall orientation

$$(ds^{(2)})^2 - (ds^{(1)})^2 = (S_{ij}^{(2)} - S_{ij}^{(1)}) ds_i ds_j = 0$$

Example for $Bi_4Ti_3O_{12}$

Domain state pair $S^{(1)}$ ($P^{(1)}$, $P^{(2)}$) and $S^{(2)}$ ($P^{(3)}$,$P^{(4)}$)

$$(ds_1 - \frac{S_{13}}{S_{12}} ds_3) ds_2 = 0$$

Two perpendicular domain walls

Charged wall (010) W-wall

Neutral wall (10K) S-wall

$$K = -\frac{S_{13}}{S_{12}}$$

Domain wall orientations in $Bi_4Ti_3O_{12}$

	$S^{(1)}$ $P^{(1)},P^{(2)}$	$S^{(2)}$ $P^{(3)},P^{(4)}$	$S^{(3)}$ $P^{(5)},P^{(6)}$	$S^{(4)}$ $P^{(7)},P^{(8)}$
$S^{(1)}$ $P^{(1)},P^{(2)}$	N/A	(010) (10-K)	(1-10) (001)	(100) (01K)
$S^{(2)}$ $P^{(3)},P^{(4)}$		N/A	(100) (01-K)	(110) (001)
$S^{(3)}$ $P^{(5)},P^{(6)}$			N/A	(010) (10K)
$S^{(4)}$ $P^{(7)},P^{(8)}$				N/A

Domain wall types

W-walls

- W_∞ - arbitrary wall orientation
- W_f – fixed crystallographic domain wall orientation

S-walls (“strange” walls, W’-walls)

- S_1 – P_S direction
- S_2 – b_{ijk} and Q_{ijkl}
- S_3 – P_S direction, b_{ijk} and Q_{ijkl}
- S_4 – P_S direction and magnitude, b_{ijk} and Q_{ijkl}
- S_5 – P_S magnitude, b_{ijk} and Q_{ijkl}

Permissible domain wall pairs

W_∞ - arbitrary domain wall orientation

$W_f W_f$ – fixed domain wall orientation

$W_f S$ – fixed and „strange“ walls

SS – pair of „strange“ walls

R – walls are not permissible

J.Fousek, V.Janovec: J.Appl.Phys. **40** (1969) 135

J.Sapriel: Phys.Rev. **B12** (1975) 5128

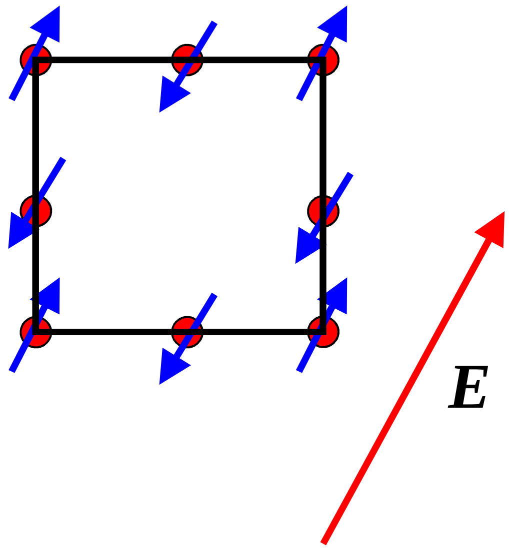
J.Erhart: Phase Transitions **77** (2004) 989-1074

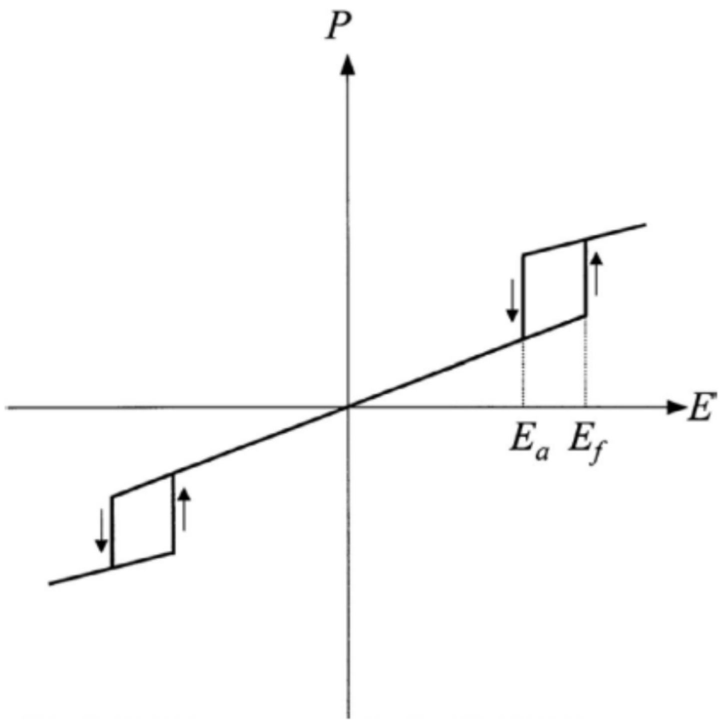
Antiferroelectricity

Dipole moments are partially compensated

Switching possible

at higher fields





Electromechanical coupling coefficient

Energy transfer efficiency between mechanical and electrical energy via piezoelectric effect

$$k^2 = \frac{\text{electrical energy converted to mechanical energy}}{\text{input electrical energy}}$$

$$k^2 = \frac{\text{mechanical energy converted to electrical energy}}{\text{input mechanical energy}}.$$

Electromechanical coupling

$$k_{33}^2 = \frac{d_{33}^2}{s_{33}^E \varepsilon_{33}^T}$$

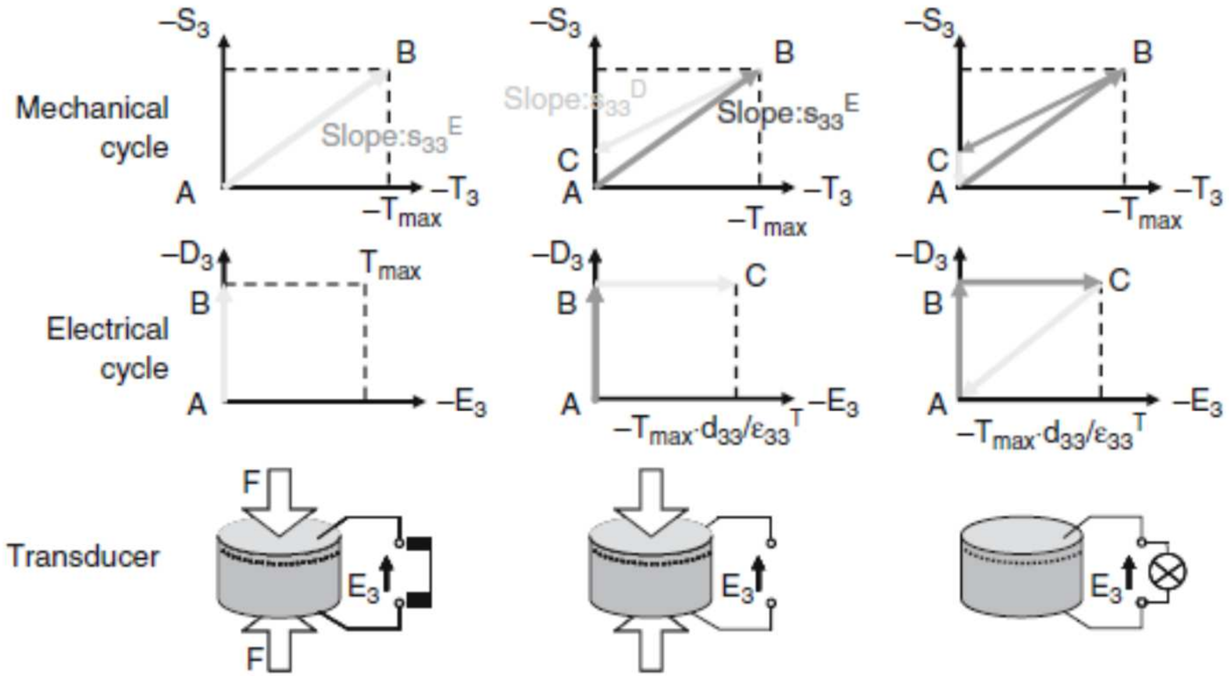


Fig. 18.2. Diagram showing a conversion cycle of mechanical to electrical energy to illustrate the relation between the electromechanical coupling factor k_{33} and d_{33}, $\varepsilon_{33}{}^{\mathrm{T}}$, and $s_{33}{}^{\mathrm{E}}$

W.Heywang, K.Lubitz, W.Wersing (edts.): Piezoelectricity, Springer Verlag 2008

Electromechanical coupling coefficients

Different modes

Transversal

$$k_{31}^2 = \frac{d_{31}^2}{s_{11}^E \varepsilon_{33}^T}$$

thickness-shear

$$k_{15}^2 = \frac{d_{15}^2}{s_{55}^E \varepsilon_{11}^T}$$

thickness

$$k_t^2 = \frac{e_{33}^2}{c_{33}^D \varepsilon_{33}^S}$$

radial

$$k_p^2 = \frac{2d_{31}^2}{\varepsilon_{33}^T (s_{11}^E + s_{12}^E)}$$

Dielectric losses

Permittivity – complex values

$$\varepsilon = \varepsilon' - j\varepsilon''$$

$$\tan(\delta) = \frac{\varepsilon''}{\varepsilon'}$$

Dissipated power = energy loss during 1s

$$P = U \cdot I = Z_C I^2 = \frac{C'' I^2}{\omega(C'^2 + C''^2)} - j\frac{C' I^2}{\omega(C'^2 + C''^2)}$$

$$C = (\varepsilon' - j\varepsilon'')\frac{S}{d} = C' - jC''$$

$$Z_C = \frac{1}{j\omega C} = \frac{C'' - jC'}{\omega(C'^2 + C''^2)}$$

Mechanical losses

Elastic properties – complex compliance

$$s = s' - js''$$

Mechanical quality Q_m

$$Q_m = \frac{1}{2\pi f_m |Z| C^T k_{eff}^2}$$

Young's modulus and elastic coefficients

Examples for 4mm symmetry

Mechanical pressure

$$T_{11} = c_{11}S_{11} + c_{12}S_{22} + c_{13}S_{33}$$
$$T_{22} = c_{12}S_{11} + c_{11}S_{22} + c_{13}S_{33}$$
$$T_{33} = c_{13}S_{11} + c_{13}S_{22} + c_{33}S_{33}$$

Mechanical deformation

$$S_{11} = \frac{1}{Y_{11}}T_{11} - \frac{\sigma_{11}}{Y_{11}}T_{22} - \frac{\sigma_{33}}{Y_{33}}T_{33}$$
$$S_{22} = \frac{1}{Y_{11}}T_{22} - \frac{\sigma_{11}}{Y_{11}}T_{11} - \frac{\sigma_{33}}{Y_{33}}T_{33}$$
$$S_{33} = \frac{1}{Y_{33}}T_{33} - \frac{\sigma_{11}}{Y_{11}}T_{11} - \frac{\sigma_{11}}{Y_{11}}T_{11}$$

Young's modulus

Matrix form

$c^{-1}=s$

$s_{11}=1/Y_{11}$

$s_{33}=1/Y_{33}$

$$Y_{33}=c_{33}-\frac{2c_{13}^2}{c_{11}+c_{12}}$$

$$Y_{11}=\frac{(c_{11}-c_{12})[c_{33}(c_{11}+c_{12})-2c_{13}^2]}{c_{11}c_{33}-c_{13}^2}$$

Thank you for your attention!