

SYDNEY CATEGORY SEMINAR ABSTRACTS

1986

This document consists of abstracts of most of the talks given at the Sydney Category Seminar during 1986. They were compiled by Michael Zaks, following a suggestion of Bob Walters. They are arranged in two parts – February to July, and August to December.

R.F.C. Walters
1 August 1989

ABSTRACTS OF THE SYDNEY CATEGORY SEMINAR 1986, 5
February - 9 July

Compiled by M. Zaks

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This document is a record of seminars given at the regular Wednesday afternoon category seminars held alternately at Macquarie and Sydney Universities. Each afternoon visitors or regular participants give reports on recent research or work in progress. Usually there is time for three one hour sessions starting at two p.m. through to half past five. The actual abstracts are meant as a record for those who have been unable to attend to find out what has been going on. As such they vary from brief titles to detailed abstracts depending on the speaker.

January 14, 1987

ABSTRACTS OF THE SYDNEY CATEGORY SEMINAR 1986, 5
February - 9 July

Compiled by M. Zaks

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ABSTRACTS OF THE SYDNEY CATEGORY SEMINAR.

DATE: 5/February/1986 TIME: 3PM

PLACE: Sydney University

SPEAKER: M. Johnson

TITLE: Many Sorted Theories Derived from not so Many Sorted Theories

ABSTRACT: The Moore-category on a space X , is an important "homming into X " construction but cannot be classically explained (Moore's domains are not a co-category object in Top). We present an analysis of the Moore construction.

Suppose \mathbf{T} is a finite limit theory, G a \mathbf{T} -algebra then $\text{el}G$ is a theory called the many sorted theory of \mathbf{T} with algebra of sorts G . $\text{El}G$ -algebras correspond (via el left adjoint to Fam) to \mathbf{T} -algebras in $\text{Fam}C$ whose projection onto C is G ; similarly $\text{el}G$ -coalgebras (co-many-sorted \mathbf{T} -algebras) correspond to \mathbf{T} -algebras in $\text{Fam}(C^{op})$. Every \mathbf{T} -algebra is trivially an $\text{el}G$ -algebra and any $\text{el}G$ -algebra K yields a \mathbf{T} -algebra F via Kan extension which is given on objects $T \in \mathbf{T}$ by $F(T) = \sum Kx \ x \in GT$ (forget that the G -Indexed sorts are different).

Moore's domains are a co-many-sorted category in Top , so homming into X gives a many sorted category whose associated ordinary category is the Moore category on X . Similarly the free monoid on a set and the free category on a directed graph result from homming out of co-many-sorted monoid and category objects in Set and Grph respectively, and the collection of well formed simplicial complexes form a co-many-sorted ω -category in the category of simplicial sets.

ABSTRACTS OF THE SYDNEY CATEGORY SEMINAR.

DATE: 26/February/1986 TIME: 2PM

PLACE: Sydney University

SPEAKER: R. Paré

TITLE: Preservation of Finite Connected Limits

ABSTRACT: Proposition 1. A category has finite connected limits iff it has pullbacks and equalizers. Proposition 2. A functor between finitely complete categories preserves finite connected limits if it preserves pullbacks. Example. Categories A and B have finite connected limits and $F: A \rightarrow B$ preserves pullbacks but doesn't preserve equalizers. (Let $f: H \rightarrow G$ be a non one-one group homomorphism. Build a category G out of G by considering it as a one object category and formally adjoining an initial object. Do the same for H and define a functor in the obvious way. This last mentioned functor is an example of the above.) Question: Why are the finite limits needed in proposition 2? Or rather, what is the A and B have limits of finite connected diagrams which admit a cocone, then F preserves these iff F preserves pullbacks.

ABSTRACTS OF THE SYDNEY CATEGORY SEMINAR.

DATE: 26/February/1986 TIME: 3.15PM

PLACE: Sydney University

SPEAKER: G. Munro

TITLE: Logic and Factorization Systems I.

ABSTRACT: Start with a category with finite limits and a factorization system (E, M) satisfying; Property E: an arbitrary pullback of a morphism in E is again in E , and, Property M: M contains the monomorphisms. Call such a category an EM -category. These have a natural logic, a restricted predicate calculus with "and", "there exists", $=$ as logical operators. The interpretation is as usual, but with predicates interpreted as arrows in M . Leading example: an elementary topos with a topology; E consists of epimorphisms composed with j -dense monomorphisms, M consists of j -closed monomorphisms.

ABSTRACTS OF THE SYDNEY CATEGORY SEMINAR.

DATE: 5/March/1986 TIME: 2PM

PLACE: Macquarie University

SPEAKER: Iain Aitchinson

TITLE: The Finer Structure of Cubes

ABSTRACT: An inductive geometric definition of k -dimensional source and target for an n -cube is described. Viewed as functions $\mathbf{n} = \{1, 2, \dots, n\} \rightarrow \Lambda = \{-, 0, +\}$, the subcubes of the n -cube can be interpreted as words of length n in $\{-, 0, +\}$. The sources and targets of the n -cube arise as particular " k -blocks in Λ^n ", where a " 0 -block" is an element of Λ^n , and we inductively define a k -block, k odd (resp. even) is a column (resp. row) of $(k-1)$ -blocks. The sources $\theta_k(n)$, $\tau_k(n)$ of dimensions k of the n -cube can be defined inductively by the maps $\lambda_k, \nu_k, \mu_k : B_n^k \rightarrow B_{n+1}^k, B_{n+1}^k, B_{n+1}^{k+1}$ respectively, where B_n^k is the set of k -blocks in Λ^n . Cocycle conditions for an n -category structure can be defined which correspond naturally to the cocycles discovered by Street in his work on orientals.

ABSTRACTS OF THE SYDNEY CATEGORY SEMINAR.

DATE: 5/March/1986 TIME: 3.15PM

SPEAKER: R. Walters

TITLE: Report on Work done while on Leave.

ABSTRACT: While I was on leave (June 85-Aug 85 in USA, England and Italy; Oct 85-Jan 86 in Italy) I wrote two papers: 1) An Axiomatics for Bicategories of Modules (with A. Carboni and S. Kasangian), and 2) On Completeness of Locally-Internal Categories (with R. Betti). Both are to appear in JPAA. In the lecture I gave a brief discussion of the contents. I also suggested that we should produce "Abstracts of the Sydney Category Seminar". There was discussion resulting in general agreement. Michael offered to set this up.

ABSTRACTS OF THE SYDNEY CATEGORY SEMINAR.

DATE: 12/March/1986 TIME: 2PM

PLACE: Sydney University

SPEAKER: B. Jay

TITLE: Banach Algebras as Enriched Categories

ABSTRACT: A normed abelian group $(A, \|\cdot\|)$ is an abelian group A together with a monoidal functor $\|\cdot\| : A \rightarrow R^+$ (see Lawvere's Metric Spaces paper) (ie $\|a+b\| \leq \|a\| + \|b\|$, $\|0\| \leq 0$) satisfying further $\|-a\| = \|a\|$. Morphisms of these are norm-reducing group homomorphisms. The category is called Nab . Theorem 1: There is a forgetful functor $U : Nab \rightarrow Met$ (=Metric spaces) which is monadic. Proof: $F(x,d) = (F'X, \|\cdot\|)$ where $F'X$ is the free abelian group in X . The norm is generated by $\|x-y\| = d(x,y)$. \square A normed R -algebra is an R -algebra R together with an abelian group norm satisfying $\|xy\| \leq \|x\| \|y\|$, $\|rx\| \leq \|r\| \|x\|$ $r \in R$, $\|1\| \leq 1$. The morphisms are norm-reducing R -algebra homomorphisms; the category is NR^+alg . Theorem 2: $NR^+alg \rightarrow Met$ is monadic. By cauchy-completing we get the corollary that Banach algebras are monadic over complete metric spaces.

ABSTRACTS OF THE SYDNEY CATEGORY SEMINAR.

DATE: 5/March/1986 TIME: 4.10PM

PLACE: Sydney University

SPEAKER: R. Paré

TITLE: Introduction To Accessible Categories.

ABSTRACT: Some basic facts on κ -filtered colimits were reviewed. Right κ -filtered profunctors were introduced, and a number of their properties given. A κ -accessible category was defined to be a category for which there existed a small set of κ -presentable objects of which every object was a κ -filtered colimit. Homework: show that every small category with split idempotents is κ -accessible for some κ .

ABSTRACTS OF THE SYDNEY CATEGORY SEMINAR.

DATE: 12/March/1986 TIME: 3PM

PLACE: Sydney University

SPEAKER: R. Paré

TITLE: Accessible Categories (ct'd)

ABSTRACT: It was shown that the 2-category of κ -accessible categories, κ -accessible functors and natural transformations is biequivalent to the bicategory of small functors, right κ -flat profunctors, and natural transformations. This followed from the fact that a κ -accessible category is

equivalent to the category of flat functors from a small category into Set. Also, last week's homework problem was worked out in detail.

ABSTRACTS OF THE SYDNEY CATEGORY SEMINAR.

DATE: 12/March/1986 TIME: 4.15PM

PLACE: Sydney University

SPEAKER: G. Monro

TITLE: Logic and Factorization Systems II.

ABSTRACT: If T is a theory based on the restricted logic, we construct a category C_T whose objects are formulae and whose arrows are (equivalence classes of) n -tuples of **terms** in the theory. (This is different from the construction of Makkai and Reyes.) C_T is an EM-category and can be used to show that if a sequent is true in all EM-categories which are models of T , then the sequent is a theorem of T (completeness theorem). A proof using Boolean-valued models shows that the classical predicate calculus is a conservative extension of the restricted logic. A **relation** in an EM-category A is $R \rightarrow AB$, where the arrow is in M . A **functional relation** is one satisfying $r(a,b) \& r(a,b') \implies b=b'$ and $b \text{ r}(a,b)$. There is a category A_{fr} , with arrows functional relations, which is regular, and a left exact functor $\Delta: A \rightarrow A_{fr}$. Define an object A in an EM-category to be **separated** if the diagonal $\Delta: A \rightarrow AA$ is in M and **complete** if for every functional relation $r: B \rightarrow A$ there is a unique morphism $f: B \rightarrow A$ such that $\Delta(f) = r$. Separated (resp. complete) objects enjoy some (but not all) of the properties that separated presheaves (resp. sheaves) have in a category of presheaves.

ABSTRACTS OF THE SYDNEY CATEGORY SEMINAR.

DATE: 19/March/1986 TIME: 2PM

PLACE: Macquarie University

SPEAKER: R. Paré

TITLE: Sketches and Accessible Categories

ABSTRACT: The definition of sketch (Ehresmann) was reviewed and it was shown that categories of models of sketches in Set are exactly the accessible categories. This was then used to prove that $\text{Acc} \subset \text{Set}$ is closed under weighted limits of retract type.

ABSTRACTS OF THE SYDNEY CATEGORY SEMINAR.

DATE: 19/March/1986 TIME: 3PM

PLACE: Macquarie University

SPEAKER: R. Street

TITLE: Some Parametrized Categorical Concepts I.

ABSTRACT: A category parametrized by a category C was taken to merely be a functor $\partial: A \rightarrow C$. Categorical matters to do with limits were claimed to do with cartesian cones in A . The existence of enough cartesian cones includes the fibration condition and completeness of fibres with respect to stable limits. Monics also have to do with limits and so we get a notion of parametric monic in A . This leads to a natural notion of wellpoweredness for parametrized categories.

ABSTRACTS OF THE SYDNEY CATEGORY SEMINAR.

DATE: 19/March/1986 TIME: 5PM

PLACE: Macquarie University

SPEAKER: C. B. Jay

TITLE: A Tensor Product of Symmetric Monoidal Categories.

ABSTRACT: A tensor product of symmetric monoidal categories. A tensor product of symmetric monoidal categories $A \square B$ is introduced with the universal property $strict : A \square B \rightarrow C$ is in bijection with $A \rightarrow SMon(B,C)$ where $F=(F,F',F'')$ is strict if F' and F'' are identities and $SMon(B,C)$ is the (symmetric monoidal) category of symmetric monoidal functors from A to C . More formally, there is an adjunction $- \square B$ left adjoint to $SMon(B,-) : SMon_{strict} \rightarrow SMon$. A related adjunction is $- \square B$ left adjoint to $SMon_{strongmaps}(B,-) : SMon_{strict} \rightarrow SMon_{strong}$. Replacing strict morphisms by strong morphisms we get an equivalence between $SMon_{strong}(A \square B, C)$ and $SMon_{strong}(A, SMon_{strong}(B,C))$. This \square gives a monoidal structure for $SMon_{strong}$. A semi-ring is a symmetric monoidal cat V with a strong map $V \square V \rightarrow V$ satisfying the appropriate conditions. Then V may be used to replace R^+ in the construction of Banach algebras as in last weeks talk.

ABSTRACTS OF THE SYDNEY CATEGORY SEMINAR.

DATE: 2/April/1986 TIME: 2PM

PLACE: Sydney University

SPEAKER: R. Street

TITLE: A Letter From Max To Ross, McGill 15/3/86

ABSTRACT: The letter discusses: 1) Wood's work on proarrows. 2) Resemblance of $i : T\text{-Alg} \rightarrow T\text{-Alg}$ to Wood's situation which suggests the question of the adjoint to i . 3) Kan arrows in Street-Walters sense. 4) Does W left adjoint to Z left adjoint to Y imply A isomorphic to $[B^{Op}, Set]$ where Y is yoneda for A . (Remark: R. Walters believes locales give a counterexample to this.)

ABSTRACTS OF THE SYDNEY CATEGORY SEMINAR

DATE: 2/April/1986 TIME: 3.15PM

PLACE: Sydney University

SPEAKER: M. Johnson

TITLE: Many Sorted Algebras and the Moore Construction.

ABSTRACT: Section titles: 1) Motivation 2) Many sorted algebras 3) Theory of many sorted algebras 3) Examples.

ABSTRACTS OF THE SYDNEY CATEGORY SEMINAR.

DATE: 9/April/1986 TIME: 4.30PM

PLACE: Macquarie University

SPEAKER: G. Monro

TITLE: Logic and Factorization Systems III.

ABSTRACT: Let A be an *EM-category* and A an object of A . A power object of A in A is an object PA together with a relation $\epsilon_A : PA \rightarrow A$ such that for every relation $r : X \rightarrow A$ there is a unique morphism $f : X \rightarrow PA$ such that $r = \epsilon_A \wedge (f)$. If every object has a power object we call A a *near-topos*. The full higher-order logic of topoi can be interpreted in a near-topos, every PA is complete (proof uses higher-order logic); the

complete objects form a reflective subcategory with left exact reflector and the complete objects form an elementary topos. Examples: (i) Any quasitopos, with $(E, M) = (\text{Epis}, \text{Strong Monos})$ (ii) Category of topological spaces, with $(\text{Epis}, \text{Subspace inclusions})$ (iii) Elementary topos with topology j ; $E =$ composites epi with j -dense mono, $M = j$ -closed monos. Here the complete objects are the j -sheaves, so the result above generalizes the sheafification theorem for elementary topoi.

ABSTRACTS OF THE SYDNEY CATEGORY SEMINAR.

DATE: 9/April/1986 TIME: 2PM

PLACE: Macquarie University

SPEAKER: M. Adelman

TITLE: Reading Between the Lines of Paré's Talks

ABSTRACT: What does it mean for a category to have ultra-powers? Paré, Makkai and myself seem to have independently come to the same conclusion that having ultra-powers is part of the axiomatics of a category of models of a theory written in first order logic. Preferably we should be able to state it as a diagonal functor having an adjoint as is the case with having limits and colimits. If X is a set and U is an ultrafilter on X , form a site X as follows: The underlying category is 2^X and the covers of I are finite families $\{J_i\}$ so that $I = \cup J_i$. Then the characteristic function is a point of the site inducing an adjoint pair p_* left adjoint to $p^* : \text{Shv}(X) \rightarrow \text{Set}$. If we view p_* as a diagonal; saying p^* exists is saying the U -ultrapower exists. If T is a coherent theory, p_* restricts to $\text{Mod}(T) \rightarrow \text{Mod}(T, \text{Shv}(X))$; but not in theories with negation (eg. the theory of sets with at least two elements). We seem to need $\text{Shv}(X)/U$ for this. We loose adjoints in this construction; however perhaps this is the right thing interpreted over a different base topos other than sets, or viewed as a fibred category with a "non-standard" fibration.

ABSTRACTS OF THE SYDNEY CATEGORY SEMINAR

DATE: 9/April/1986 TIME: 3.15PM

PLACE: Macquarie University

SPEAKER: R. Walters

TITLE: Bicategorical Aspects of Representation Theory.

ABSTRACT: Many examples exist of the following phenomenon: a bicategory B (distributive and with further structure), an additive category A , and a "cardinality functor" $\# : B \rightarrow A$. The cardinality of a finite set, the dimension of a vector space, the cardinality of a species, all fit into this pattern. I discussed the following important example. Let B be the bicategory whose objects are finite groups, arrows from G to H are finite dimensional left G , right H modules. Let A be the additive category with objects same as B , but arrows from G to H are class functions $G \times H \rightarrow C$. Then the **character** of a representation provides a cardinality functor. (Which preserves biclosed structure, global tensor product, etc.) The fact that it preserves composition is a fundamental fact not usually stressed in standard books on group representation.

ABSTRACTS OF THE SYDNEY CATEGORY SEMINAR.

DATE: 16/April/1986 TIME: 2PM

PLACE: Sydney University

SPEAKER: R. Street

TITLE: Some Parametrized Categorical Concepts II.

ABSTRACT: Suppose $\partial : A \rightarrow C$ is a fibration, A, C are finitely complete and ∂ is left exact. Various categories over C are defined: $Mon_C(A)$, $Hom_C(A, B)$, $Idem_C(A)$. Write A_{gpd} for the category with the same objects as A and cartesian arrows. Call A wellpowered when each $Mon_C(A)_{gpd}$ has a terminal object. Say A has small homs when each $Hom_C(A, B)$ has a terminal object. Say A has small idempotency when each $Idem_C(A)$ has a terminal object. Theorems: (i) Wellpowered implies $Mon_C(A)$ is essentially small. (ii) Small homs iff small idempotency.

ABSTRACTS OF THE SYDNEY CATEGORY SEMINAR.

DATE: 16/April/1986 TIME: 3.30PM

PLACE: Sydney University

SPEAKER: M. Adelman

TITLE: Standard and Nonstandard Parametrized Categories

ABSTRACT: (This results from a discussion with R. Pare while he was in Sydney) If A is any category define $T(C)=[C, A]$ to be the standard parametrization of A . If T is a parametrized category, say T is good if $T(f)$ has both adjoints for all $f: C \rightarrow D$. The standard parametrization is good iff A is complete and cocomplete. If H denotes the homotopy category the we know it is not complete. Heller has introduced an alternative parametrization. For any small category C , we form Top^C . This has a quillen model structure which gives a class of weak equivalences. We invert these and call the result $H(C)$. Theorem (Heller): H is a good parametrization of H . Similarly: If U is a filter on a set X , we can define $N(C)$ for each C by $N(C) = Set^C / U$ (filter-power of Set^C). Theorem: N is a good parametrization of Set^X / U . Both of these, N and H are non-standard parametrizations. Are they part of a general process to make a parametrization good? One feels that for A complete and cocomplete one needn't go beyond the standard parametrization. The standard parametrization of Set^X / U is not good (Adelman-Johnstone).

ABSTRACTS OF THE SYDNEY CATEGORY SEMINAR.

DATE: 23/April/1986 TIME: 2.00PM

PLACE: Sydney University

SPEAKER: B. Jay

TITLE: $A \square B$

ABSTRACT: Following a previous talk, a new proof is given of the existence of $A \square B$, for symmetric monoidal categories A and B . Proposition 1: $SMon(A, SMon(B, C)) \simeq SMon(B, SMon(A, C))$ 2-naturally in all variables \square . Note that by replacing $SMon$ by $SMon_{strong}$ or $SMon_{strict}$ ie replace "all monoidal functors $B \rightarrow C$ " by "all strong (resp. strict)..." many parallel theorems result. Proposition 2: Let α be a 2-cell in $SMon$ then $SMon(\alpha, -)$ is defined. Theorem 3: $SMon(B, -): SMon_{strict} \rightarrow SMon$ has a left 2-adjoint $-\square B$ which is 2-natural in B . Hence $\square: SMon \xrightarrow{2} SMon$ is a 2-functor. Proposition 4: $i: SMon_{strict}(A \square B, C) \rightarrow SMon(A \square B, C)$ has a right adjoint S . Proof. S is

constructed on the generators of $A \square B$ given in the previous lecture. Corollary 5: There are 2-natural strict monoidal functors a and a' between $(A \square B) \square C$ and $A \square (B \square C)$ which both satisfy the pentagon law for monoidal categories. Note 1) a and a' are not inverse. Note 2) Prettier calculations may be made with strong monoidal functors (eg. a becomes an equivalence) but a 'norm' is not strong in general. Note 3) There is a third version of the theorem for bicategories and homomorphisms. For morphisms of bicategories the naturality conditions break down.

ABSTRACTS OF THE SYDNEY CATEGORY SEMINAR.

DATE: 23/April/1986 TIME: 3.20PM

PLACE: Sydney University

SPEAKER: R. Street

TITLE: Some Parametrized Concepts III.

ABSTRACT: Generators and strong generators for a parametrized category were defined. If A is fibred over C and has small homs we have: C^2 admits monic parts and A has a generator implies A admits monic parts. C^2 admits invertible parts and A has a strong generator implies A admits invertible parts.

ABSTRACTS OF THE SYDNEY CATEGORY SEMINAR.

DATE: 23/April/1986 TIME: 4.50PM

PLACE: Sydney University

SPEAKER: M. Adelman

TITLE: Linear Finiteness

ABSTRACT: What follows arises from conversations with A. Kock. Let B be a cartesian closed category and let $Vect_R(B)$ denote the category of R -vector spaces (=modules) in B , where R is a ring in B . The restricted double dual of an object B of B , denoted by $D(B)$ is defined as $D(B) = Hom_R(R^B, R)$ where Hom_R is the object of homomorphisms and R^B has its pointwise vector space structure. Evaluating gives a map $k : B \rightarrow Hom_R(R^B, B)$. Now assume that free vector spaces exist, that is, a left adjoint to the forgetful functor $Vect_R(B)$. The map k above induces a homomorphism $h : F(B) \rightarrow D(B)$. We call B linearly finite when h is an isomorphism. If $B = Set$ and R is any field then linearly finite is equivalent to finite. This needs decidability to prove. It is even interesting to ask when h is a mono. This situation doesn't cover the motivating example ie. where $D(B)$ is distributions on B , B is a manifold, and the statement that h is an iso is Walbroek's theorem. The problem is we cannot use $Vect$ but some form of complete vector spaces.

ABSTRACTS OF THE SYDNEY CATEGORY SEMINAR.

DATE: 7/May/86 TIME: 4.30PM

PLACE: Sydney University

SPEAKER: B. Jay

TITLE: $A \square B$ II.

ABSTRACT: (I) The right-hand associativity for \square , $r : A \rightarrow A \square A$ is natural in A and satisfies the triangle law. There is a strict symmetric monoidal functor $r' : a \square \rightarrow A$ but it is not natural. (II) On generalization of the notion "monoid" is "category". Another is "monoidal category". In the same way "abelian group" becomes "additive category" or "symmetric monoidal closed category". There is a full embedding $Ab \rightarrow SMon$ mapping A tensor B

to $A \square B$. (III) Definition. Let A be a SMC and V be SMon. Then a V -norm for A is a symmetric monoidal functor $||: A \rightarrow V$ satisfying the following condition, $||A(A,B) \simeq A(B,A)||$. $||A||$ is the V -cat with the same objects as A but with $||A(A,B) = ||A(A,B)||$ etc. Say $||$ is cauchy complete iff $||A||$ is.

ABSTRACTS OF THE SYDNEY CATEGORY SEMINAR.

DATE: 21/May/1986 TIME: 3.45PM

PLACE: Macquarie University

SPEAKER: I. Aitchinson.

TITLE: String Diagrams.

ABSTRACT: Not recieved.

ABSTRACTS OF THE SYDNEY CATEGORY SEMINAR.

DATE: 28/May/1986 TIME: 2PM

PLACE: Sydney University

SPEAKER: R. Street

TITLE: Braided Monoidal Categories

ABSTRACT: Crossed modules give rise to monoidal categories of a special kind (groups in Cat). Braidings on these monoidal categories amount to bracket operations (like abstract commutators) on crossed modules. These arise as Samelson brackets in homotopy theory $\pi_2 X \times \pi_2 X \rightarrow \pi_3 X$. In preparation for characterizing braided monoidal groupoids with each A_q - an equivalence, aspects of group cohomology were discussed.

ABSTRACTS OF THE SYDNEY CATEGORY SEMINAR.

DATE: 11/June/1986 TIME: 2PM

PLACE: Macquarie University

SPEAKER: R. Walters

TITLE: The Foundations of Homotopy Theory

ABSTRACT: I gave some motivation for my work with Mike Johnson. Our first idea is that one should study paths first and then homotopy. (Following Lavwere) we begin with a topos E . We expect that paths in an object X should have a co-category structure (paths of all dimensions). The first axiom should be that paths are represented. In examples (Sets, simplicial sets, etc.) we find that paths are in fact represented by a family of objects $I \rightarrow N$. The paper we are currently completing deals with the difficult task of defining paths (of all dimensions) in a simplicial set and establishing the ω -category structure on the paths.

ABSTRACTS OF THE SYDNEY CATEGORY SEMINAR.

DATE: 11/June/1986 TIME: 3.00PM

PLACE: Macquarie University

SPEAKER: M. Johnson

TITLE: Well-formed Simplicial Sets.

ABSTRACT: This talk presented an alternative description of Street's *orientals*. The description is geometric. The elements of the orientals turn out to be certain simplicial sets. A finite n -dimensional simplicial set A is called **compatible** if for $x, y \in X_n$, $x \partial_i = y \partial_j$, $i+j$ even, implies $x=y$. A compatible simplicial set is "sensibly oriented" at its top dimension. To analyze its lower dimensions we define the domain and codomain of a simplicial set. If A is an n -dimensional simplicial set, $x \in A$ is called an end of A if there exists $y \in A_n$, $\{a_0, a_1, \dots, a_k\}$ even integers $a_0 < a_1 < \dots < a_k$ with $x = y \partial_{a_k} \partial_{a_{k-1}} \dots \partial_{a_0}$ (composing a la Eilenberg). The graded set of ends

of A is denoted by $E(A)$. The domain of A is by definition, $A-E(A)$. Similarly for the codomain of A (odd integers). **Well formed** simplicial sets are defined inductively: A zero dimensional well formed simplicial set is a singleton; an n -dimensional simplicial set is well formed if it's compatible and its domain and codomain are well formed simplicial sets. The collection of well formed simplicial sets form an ω -category (with union as composition).

ABSTRACTS OF THE SYDNEY CATEGORY SEMINAR.

DATE: 11/June/1986 TIME: 4.15PM

PLACE: Macquarie University

SPEAKER: B. Jay

TITLE: A New Notion of Field.

ABSTRACT: None.

ABSTRACTS OF THE SYDNEY CATEGORY SEMINAR.

DATE: 18/June/1986 TIME: 2.10PM

PLACE: Macquarie University

SPEAKER: B. Day

TITLE: Convolution For The Working Mathematician.

ABSTRACT: Not recieved.

ABSTRACTS OF THE SYDNEY CATEGORY SEMINAR.

DATE: 18/June/1986 TIME: 3.15PM

PLACE: Sydney University

SPEAKER: R. Street

TITLE: Some Remarks on Group Cohomology.

ABSTRACT: The monoidal structure on $H^*(G,-)$ was described. Artin-Tate periodicity was discussed with applications to the Eilenberg-Mac Lane cohomology of abelian groups.

ABSTRACTS OF THE SYDNEY CATEGORY SEMINAR.

DATE: 25/June/1986 TIME: 2PM

PLACE: Macquarie University

SPEAKER: M. Zaks

TITLE: Nerve of n -categories I.

ABSTRACT: Elementary introduction to simplicial sets and the nerve of a category. I showed that simplicial sets that are nerves of categories satisfy simple extension properties and that this characterizes them.

ABSTRACTS OF THE SYDNEY CATEGORY SEMINAR.

DATE: 25/June/1986 TIME: 4.50PM

PLACE: Macquarie University

SPEAKER: R. Walters

TITLE: Paths in Simplicial Sets.

ABSTRACT: Any geometric subject should begin with the correct abstract operations. To begin, we should have a category with finite limits, subobject classifiers, exponentiation. For homotopy theory we should have an object I to parametrize paths; ie paths should be a representable functor into ω -categories. The idea is to first study this structure and later study homotopy classes of functions. In fact, work with Michael Johnson has lead to considering **many sorted co- ω -category objects** (where the sorts form an ω -category) to parametrize paths. The Moore-path category is obtained from a co-category with sorts being real intervals. The free monoid on a set

arises from a co-monoid with sorts being natural numbers. In simplicial sets, Street's orientals yield a many-sorted co- ω -category to parametrize paths (Johnson).

ABSTRACTS OF THE SYDNEY CATEGORY SEMINAR.

DATE: 2/July/1986 TIME: 2PM

PLACE: Macquarie University

SPEAKER: M. Zaks

TITLE: Nerve of n-categories II.

ABSTRACT: A description of the nerve of the free omega category on 2 [Street, oriented simplices] was given. I used the notion of a shift on a simplicial set.

ABSTRACTS OF THE SYDNEY CATEGORY SEMINAR

DATE: 2/July/1986 TIME: 3.00PM

PLACE: Macquarie University

SPEAKER: R. Walters

TITLE: Paths and Omega Categories

ABSTRACTS: More on paths in simplicial sets. The new aspect of the talk was that we (Johnson, Walters) have discovered that the higher identity laws arise from the orientals - against our expectations that the identity laws have a different geometry than the associative laws.

ABSTRACTS OF THE SYDNEY CATEGORY SEMINAR.

DATE: 2/July/1986 TIME: 4.00PM

PLACE: Macquarie University

SPEAKER: B. Jay

TITLE: Algebraic Structure on Enriched Categories.

ABSTRACT: A summary of the series of talks given on normed abelian groups using enriched category theory. (No abstract; comment is by editor, me!)

ABSTRACTS OF THE SYDNEY CATEGORY SEMINAR.

DATE: 9/July/1986 TIME: 2PM

PLACE: Sydney University

SPEAKER: R. Walters

TITLE: Paths and Omega Categories

ABSTRACT: Continuation of previous lecture (11/June/1986).

ABSTRACTS OF THE SYDNEY CATEGORY SEMINAR.

DATE: 9/July/1986 TIME: 3PM

PLACE: Sydney University

SPEAKER: S. Eilenberg

TITLE: Cellular Spaces

ABSTRACT: No abstract recieved.

ABSTRACTS OF THE SYDNEY CATEGORY SEMINAR.

DATE: 9/July/1986 TIME: 4.20PM

PLACE: Sydney University

SPEAKER: R. Street

TITLE: Why There Are No Loops.

ABSTRACT: Let M be the endomorphism monoid of 2 in Cat . Then the category of right M -sets is the category of graphs (directed, with chosen

endo-edges at each vertex). Let $\omega = \{0, 1, 2, \dots\}$ as an ordered set. Call $\alpha : \omega \rightarrow \omega$ in Cat **eventually consecutive** when there exists k such that $\alpha(i+1) = \alpha(i) + 1$ for all $i > k$. Let Δ denote the monoid of eventually constant $\alpha : \omega \rightarrow \omega$ under composition. The category of right Δ -sets is essentially the category of simplicial sets : there can be some infinite dimension cells. (This was *not* the main point of the lecture.) The remainder of the lecture described O_n giving some technicalities as to why no loops occur.

**ABSTRACTS OF THE CATEGORY SEMINAR. August-December
1986**

Compiled by M. Zaks.

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This document is a record of seminars given at the regular Wednesday afternoon category seminars held alternately at Macquarie and Sydney Universities. Each afternoon visitors or regular participants give reports on recent research or work in progress. Usually there is time for three one hour sessions starting at two p.m. through to half past five. I expect the abstracts will be useful as a record of the talks for those who have been unable to attend or are trying to recall the remarks of a speaker. As such they vary from brief titles to detailed abstracts depending on the speaker.

February 4, 1987

PARTICIPANTS:

A/Prof R. Street
Dr A. Scedrov
Prof. S. Eilenberg
Prof. G. Kelly
C. Foot
M. Zaks
Dr G. Monro
M. Johnson
Dr R. Moore
Dr R. Walters
M. Shum
Dr J. Powers
Dr B. Day
T. Smit
Dr M. Adelman
Dr A. Pitts
Dr S. Kasangian
Dr D. Lever

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ABSTRACTS OF THE SYDNEY CATEGORY SEMINAR.

DATE: 6/August/1986 TIME: 1.50PM

PLACE: Macquarie University

SPEAKER: A. Pitts

TITLE: Lax Descent For Essential Surjections

ABSTRACT: Theorem: Let $f: F \rightarrow E$ be a geometric morphism between Grothendieck toposes. Form the comma square and pullback square respec-

tively in Grothendieck toposes as follows.

$$\begin{array}{ccc}
 \mathbb{F}/\varepsilon \mathbb{F} & \xrightarrow{p_1} & \mathbb{F} \\
 \downarrow \beta & \Rightarrow & \downarrow f \\
 \mathbb{F} & \xrightarrow{f} & \mathbb{E}
 \end{array}
 \qquad
 \begin{array}{ccc}
 \mathbb{F}/\varepsilon \mathbb{F}/\varepsilon \mathbb{F} & \longrightarrow & \mathbb{F}/\varepsilon \mathbb{F} \\
 \downarrow & \cong & \downarrow \\
 \mathbb{F}/\varepsilon \mathbb{F} & \longrightarrow & \mathbb{F}
 \end{array}$$

One gets a "weak" category object

$$C_f : \mathbb{F}/\varepsilon \mathbb{F}/\varepsilon \mathbb{F} \rightrightarrows \mathbb{F}/\varepsilon \mathbb{F} \rightrightarrows \mathbb{F}$$

in Grothendieck toposes. Let $\text{Des}(C_f)$ be the category of descent data for this and $K : \mathbb{E} \rightarrow \text{Des}(C_f)$ the comparison functor. Then K is an equivalence when f is essential and surjective. [Objects of $\text{Des}(C_f)$: (Y, y) where $Y \in \mathbb{F}$ and $y : p_0^* Y \rightarrow p_1^* Y$ satisfy counit and cocycle conditions.]

ABSTRACTS OF THE SYDNEY CATEGORY SEMINAR.

DATE: 6/August/1986 TIME: 3PM

PLACE: Macquarie University

SPEAKER: A. Scedrov

TITLE: Sheaf Semantics and a Decision Procedure

ABSTRACT: Let $C(X)$ be the ring of continuous real functions on a compact Hausdorff space X . We characterize those spaces X for which various normal form and decomposition theorems of real linear algebra may be lifted to analogous theorems on real vector bundles over X and on locally finitely generated $C(X)$ -modules. Furthermore, we give a decision procedure for certain ring-theoretic conditions of a local nature on $C(X)$. These conditions are closely related to analogous conditions on the reals. Our results will be stated in the real case; similar results may be obtained in the complex and the symplectic cases.

Three classes of spaces naturally occur in this context. The *sub-stonean* spaces, or *F-spaces*, are those in which any two disjoint open F_σ sets have disjoint closures. The decomposition properties of $C(X)$ -modules considered here force X to be 0-dimensional. Such a sub-stonean space is a *U-space* and $C(X)$ is a *U-ring* : for each f there is a unit u such that $f=uf$. A second, smaller class of spaces to appear in our discussion is the class of *Rickart spaces* or *basically disconnected* spaces, in which every open F_σ set has an open closure. An equivalent condition on $C(X)$ is that every non-decreasing bounded sequence has a supremum (σ -completeness). A third, still smaller class consists of the *P-spaces*, in which every F_σ set is closed; that is, $C(X)$ is a von Neumann regular ring. A compact *P-space* is finite.

ABSTRACTS OF THE SYDNEY CATEGORY SEMINAR.

DATE: 6/August/1986 TIME: 4PM

PLACE: Macquarie University

SPEAKER: G. Kelly

TITLE: Categories with Essentially Algebraic Structure I.

ABSTRACT: Given a (strict) 2-monad T on Cat (or on Cat^X etc.), enriched category theory tells us about the 2-category $T\text{-Alg}_*$ of T -algebras and *strict* maps; but we want to consider the 2-category $T\text{-Alg}$ of T -algebras and *pseudo* maps. The inclusion $T\text{-Alg}_* \rightarrow T\text{-Alg}$ has a left adjoint Q' ; retracts in $T\text{-Alg}_*$ of objects of the form A' are called *flexible*. Using this notion and enriched category theory, we construct a bi-adjoint to the alge-

ABSTRACTS OF THE SYDNEY CATEGORY SEMINAR.

DATE: 20/August/1986 TIME: 2PM

PLACE: Sydney University

SPEAKER: A. Pitts

TITLE: A note on locally cartesian closed categories with a generic family of objects.

ABSTRACT: Proposition. If C is locally cartesian closed and possesses a generic family of objects $\tau : T \rightarrow V$, then C is degenerate, in the sense that each object of C is isomorphic to the terminal object I .

ABSTRACTS OF THE SYDNEY CATEGORY SEMINAR.

DATE: 20/August/1986 TIME: 3.15PM

PLACE: Sydney University

SPEAKER: A. Scedrov

TITLE: Lambda Calculus.

ABSTRACT: Typed lambda-calculus, Gerry-Howard isomorphism.

ABSTRACTS OF THE SYDNEY CATEGORY SEMINAR.

DATE: 20/August/1986 TIME: 4.15PM

PLACE: Sydney University

SPEAKER: R. Street

TITLE: Higher Order Deductions and Higher Order Categories.

ABSTRACT: This was the first talk in the seminar on my joint work with Samuel Eilenberg. Free n -categories were discussed from three points of view: a) their universal property, b) their construction expressed in terms of normal forms for derivations in rewrite systems, and c) their internal characterization (e.g. a free category has every arrow a unique composite of indecomposables). The n -category O_n as in my paper "The Algebra of Oriented Simplexes" JPAA (to appear) can now be characterized as the unique solution to a universal problem: O_n is the free n -category generated by the finite non-empty subsets $x=(x_0, x_1, \dots, x_k)$ of $(012\dots n)$ such that the $(k-1)$ -source of x has the form $d_1 *_{k-2} d_3 *_{k-2} d_5 \dots$ and the $(k-1)$ -target of x has the form $\dots d_4 *_{k-2} d_2 *_{k-2} d_0$ where d is the i -th $(k-1)$ -dimensional face of x suitably "whiskered" by $(k-2)$ -cells (the whiskers are uniquely determined).

ABSTRACTS OF THE SYDNEY CATEGORY SEMINAR.

DATE: 27/August/1986 TIME: 2.00PM

PLACE: Macquarie University

SPEAKER: M. Johnson

TITLE: Towards Homotopy Via Co-m.s.categories.

ABSTRACT: This lecture noted the analogous use of n -categories in the recent work of Scedrov, Eilenberg, Street, Walters and Johnson, with most attention devoted to describing the Walters-Johnson approach to homotopy theory. We viewed the theory of **many sorted categories** (see abstracts: April 2, 1986) and applied it to define the path ω -category of a topological space. This structure contains all the information needed for homotopy theory. Our analysis suggests a generalization of homotopy theory. An **abstract homotopy theory** should be a topos with a co-many-sorted- ω -category object (domains of paths) which satisfies certain extra properties. Suggestions for the extra properties were discussed.

ABSTRACTS OF THE SYDNEY CATEGORY SEMINAR.

DATE: 27/August/1986 TIME: 3.15PM

PLACE: Macquarie University

SPEAKER: A. Pitts

TITLE: First Order Categorical Model Theory I.

ABSTRACT: References and introductory chat. Definition of logos and logos morphism. Many-sorted first order language, L ; L -terms and L -formulas. Definition of an L -structure in a category with finite limits ; definition of a homomorphism of L -structures.

ABSTRACTS OF THE SYDNEY CATEGORY SEMINAR.

DATE: 27/August/1986 TIME: 4.15PM

PLACE: Macquarie University

SPEAKER: A. Scedrov

TITLE: Examples Of Polymorphic Types.

ABSTRACT: No abstract.

ABSTRACTS OF THE SYDNEY CATEGORY SEMINAR.

DATE: 3/September/1986 TIME: 2PM

PLACE: Sydney University

SPEAKER: A. Pitts

TITLE: First Order Categorical Model Theory II.

ABSTRACT: Interpretation of terms and formulas in an L -structure in a logos. Soundness theorem. Theories in IPC ; category of models and iso's/elementary embeddings $Mod_{\underline{}}(T,C)/Mod_e(T,C)$. The 2-functor $Mod_{\underline{}}(T,-) : LOG \rightarrow CAT$, existence of classifying logos and generic model for a theory in IPC ; construction of $C(T)$.

ABSTRACTS OF THE SYDNEY CATEGORY SEMINAR.

DATE: 10/September/1986 TIME: 2.15PM

PLACE: Macquarie University

SPEAKER: R. Street

TITLE: 3-dimensional Cohomology Of Groups.

ABSTRACT: This talk reports a development in the joint work of Joyal and the speaker on braided monoidal categories. We have found an improved form of the interpretation of $H^3(G,A)$ where G is a group and A is a ZG -module. In fact, there is a 2-category H^3 whose objects are (G,A,l) where l is a normalized 3-cocycle for G with coefficients in A , and, arrows $(g,p,r) : (G,A,l) \rightarrow (G',A',l')$ have $g : G \rightarrow G'$ a group homomorphism, $p : A \rightarrow A'$ a G -module homomorphism and $r : G^2 \rightarrow A'$ a function whose coboundary is the difference between pl and $l'g$. Let CMG_n denote the 2-category of compact monoidal (normalized) groupoids and monoidal functors (normal). Let CM denote the 2-category of crossed modules (in the sense of J.H.C. Whitehead) regarded as a full sub-2-category of CMG_n by regarding crossed modules as groups in Cat . **Theorem.** There are biequivalences of 2-categories

ABSTRACTS OF THE SYDNEY CATEGORY SEMINAR.

DATE: 10/September/1986 TIME: 3.30PM

PLACE: Macquarie University

SPEAKER: A. Pitts

TITLE: First Order Categorical Model Theory III.

ABSTRACT: Disjoint coproducts and effective coequalizers of equivalence relations. Heyting pretoposes : completion and its characterization. Classifying Hpt of a theory in IPC. Interpretations between theories as Hp functors. Interpolation theorem for IPC, interpolation property of bipushouts in Hpt (iff stability of conservative morphisms under bipushout).

ABSTRACTS OF THE SYDNEY CATEGORY SEMINAR.

DATE: 10/September/1986 TIME: 4.50PM

PLACE: Macquarie University

SPEAKER: R. Walters

TITLE: Many Sorted Equivalence Relations.

ABSTRACT: The principal operations of geometry are finite limits and glueing. Usually glueing is expressed by colimits, but an alternative notion is that of "many-sorted equivalence relations and quotients of such". To express this in an abstract category C we need first that the bicategory of spans in C has a reflection (Day) into relations. This implies that C is regular. Further, we need stable sups of relations so that morphisms of many-sorted relations can be composed. The existence of quotients then is exactly the requirement that C be an infinitary pretopos. If every object can be glued out of a fixed small set then C is a Grothendieck topos.

ABSTRACTS OF THE SYDNEY CATEGORY SEMINAR.

DATE: 18/September/1986 TIME: 2.05PM

PLACE: Sydney University

SPEAKER: R. Walters

TITLE: The Cauchy-Completion Of A Sup-Lattice Category.

ABSTRACT: Consider a heyting algebra as a sup-lattice category with one object. The cauchy-completion is formed by adding absolute indexed colimits. As usual, the splitting of idempotents is such a colimit, but our work with distributive bicategories shows that collages are also preserved by any sup-lattice functor. In fact, the symmetric cauchy completion of a heyting algebra is $\text{Rel}(\text{Shv}(H))$.

ABSTRACTS OF THE SYDNEY CATEGORY SEMINAR.

DATE: 18/September/1986 TIME: 3.15PM

PLACE: Sydney University

SPEAKER: R. Street

TITLE: When Are Locally Presentable Categories Regular, Toposes, etc.?

ABSTRACT: When Barr was in Australia he outlined a new proof of his old embedding theorem for regular categories. This began with the observation that C^{OP} regular implies $\text{Lex}(C^{OP}, \text{Set})^{OP}$ regular. In joint work with Brian Day, we use similar techniques to show that C regular and finitely cocomplete implies $\text{Lex}(C^{OP}, \text{Set})$ is regular. This result is used in our work on enriched sheaf theory.

ABSTRACTS OF THE SYDNEY CATEGORY SEMINAR.

DATE: 18/September/1986 TIME: 4PM

PLACE: Sydney University

SPEAKER: J. Powers

TITLE: Bicategories and Bilimits

ABSTRACT: We show that every bicategory with bilimits is biequivalent to a 2-category with flexible limits. First, we use the well-known result that every bicategory is biequivalent to a 2-category. Then, we show that a 2-category C with bilimits is biequivalent to the pseudo-limit closure of the full subcategory of $\text{Ps}[C^{OP}, \text{Cat}]$ on $\text{ob}C$. Finally, we show that this new 2-category is biequivalent to its closure in $[D^{OP}, \text{Cat}]$ under flexible limits, giving the result.

ABSTRACTS OF THE SYDNEY CATEGORY SEMINAR.

DATE: 18/September/1986 TIME: 4.40PM

PLACE: Sydney University

SPEAKER: A. Pitts

TITLE: First Order Categorical Model Theory IV.

ABSTRACT: Conservative-quotient factorization in HPT. Completeness ; "sufficient" classes of HPT's for small Hpt's. Statement of conceptual completeness theorem.

ABSTRACTS OF THE SYDNEY CATEGORY SEMINAR.

DATE: 24/September/1986 TIME: 2.00PM

PLACE: Macquarie University

SPEAKER: A. Pitts

TITLE: First Order Categorical Model Theory V.

ABSTRACT: Proof of conceptual completeness theorem using "topos of filters" construction plus properties of open geometric morphisms.

ABSTRACTS OF THE SYDNEY CATEGORY SEMINAR.

DATE: 24/September/1986 TIME: 3.30PM

PLACE: Macquarie University

SPEAKER: R. Street

TITLE: Cohomology of Groups and Abelian Groups.

ABSTRACT: Two weeks ago I described a 2-category H^3 whose objects (G, M, h) consisted of a G -module M and a normalized 3-cocycle $h : G^3 \rightarrow M$. I also gave a 2-functor $T : H^3 \rightarrow \text{CMG}$ into the 2-category of compact monoidal groupoids and another one $S : H^3 \rightarrow \text{CMG}$ whose values are groups in Cat . Without requirements of normality, T is a biequivalence, and, $S \simeq T$ and hence S too is a biequivalence. Eilenberg-MacLane, in work appearing in 1950, developed cohomology of abelian groups. For abelian groups G, M , an abelian 3-cocycle is a pair (h, c) where h is a 3-cocycle (in the group sense with G acting on M via $x\mu = \mu$) and c precisely amounts to a braiding on $T(G, M, h)$. Define a 2-category Hab^3 whose objects are (G, M, h, c) . The T gives $T : \text{Hab}^3 \rightarrow \text{CBMG}$ into the 2-category of compact braided monoidal groupoids. This T too is a biequivalence. There is a 2-functor $\text{trace } \text{tr} : H^3 \rightarrow \text{Quad}$, into the category of quadratic functions $t : G \rightarrow M$ between abelian groups, given by $\text{tr}(G, M, h, c) = t$ where $t(x) = c(x, x)$. This tr is not a biequivalence since it is not faithful on arrows. However it does provide a "complete invariant" 2-functor $K : \text{CBMG} \rightarrow \text{Quad}$ with the following

properties: (i) KV isomorphic to KV' implies V is equivalent to V' , (ii) each $t \in \text{Quad}$ is in the image of K , (iii) The fibres of $K : \text{CBMG}(V, V') \rightarrow \text{Quad}(KV, KV')$ are all equivalent to $\text{Ext}(G, M')$ where $T(G, M, h, c) \simeq V$, $T(G', M', h', c') \simeq V'$.

ABSTRACTS OF THE SYDNEY CATEGORY SEMINAR.

DATE: 24/September/1986 TIME: 4.45PM

PLACE: Macquarie University

SPEAKER: G. Kelly

TITLE: Remarks.

ABSTRACT: In connection with complete functors.

ABSTRACTS OF THE SYDNEY CATEGORY SEMINAR.

DATE: 1/October/1986 TIME: 2.15PM

PLACE: Sydney University

SPEAKER: G. M. Kelly

TITLE: The Ordered Set Of Reflective Subcategories.

ABSTRACT: Given a category A , we consider the (often large) set $\text{Ref } A$ of its reflective (full, replete) subcategories, ordered by inclusion. It is known that, even when A is complete and cocomplete, wellpowered and cowellpowered, the intersection of two reflective subcategories need not be reflective. Supposing that A admits (i) small limits and (ii) arbitrary (even large) intersections of strong subobjects, we prove that an infimum of $\{C_i\}$ in $\text{Ref } A$ must necessarily be the intersection $\bigcap_i C_i$. Accordingly $\text{Ref } A$ is not in general, even for good A , a complete lattice. We show, however, under the same conditions on A , that $\text{Ref } A$ does admit small suprema of $\{C_i\}$, given by the closure in A of the union $\bigcup_i c_i$ under the limits of types (i) and (ii) above.

ABSTRACTS OF THE SYDNEY CATEGORY SEMINAR.

DATE: 1/October/1986 TIME: 3.45PM

PLACE: Sydney University

SPEAKER: A. Pitts

TITLE: Makkai's example of a false-localic topos.

ABSTRACT: If E is localic, then $\text{GTOP}(F, E) \simeq \text{poset}$, all $F \in \text{GTOP}$. Converse is false, and Makkai has given an example.

ABSTRACTS OF THE SYDNEY CATEGORY SEMINAR.

DATE: 1/October/1986 TIME: 4.19PM

PLACE: Sydney University

SPEAKER: R. Street

TITLE: Free 2-Categories

ABSTRACT: (with Eilenberg) Graphs, categories, free categories, and 2-graphs were discussed. Intrinsic characterizations of free categories and free 2-categories were proposed.

ABSTRACTS OF THE SYDNEY CATEGORY SEMINAR.

DATE: 8/October/1986 TIME: 2.15PM

PLACE: Macquarie University

SPEAKER: R. Walters

TITLE: The Cauchy Completion Of A Locale As A Suplattice Category.

ABSTRACT: An object of the cauchy completion of a V-category A is a module $M : I \rightarrow A$ with a right adjoint; that is, a module M and a module map

$$?: I \longrightarrow \text{Hom}_A(M, A)_{A \otimes A M}$$

such that

$$(E \otimes 1)(1 \otimes ?) = 1$$

When V is Ab, and A is a ring, this amounts to an element $\sum \sigma_i \otimes e_i$ $i \in \mathbb{N}$ of $\text{Hom}_A(A, M)_{A \otimes A M}$ satisfying $\sum \sigma_i(m) e_i = m$ for all $m \in M$.

From this data we obtain an idempotent matrix $E = (e_{ij})$ where $e_{ij} = \sigma_j(e_i)$

and M is the splitting of $E : A^n \rightarrow A^n$, and hence is a finitely generated projective module. The suplattice case is closely analogous. Let A be a locale considered as a suplattice category. Then a symmetric idempotent matrix with entries in A is just an A-valued set. The symmetric cauchy completion of A is $\text{Rel}(\text{Shv}(A))$.

ABSTRACTS OF THE SYDNEY CATEGORY SEMINAR.

DATE: 8/October/1986 TIME: 3.15PM

PLACE: Macquarie University

SPEAKER: S. Kasangian

TITLE: Discussion Of Recent Research.

ABSTRACT: I want to give a brief outline of the research I am presently involved in to do with categorical computer science. 1) On the line of (tree) automata as enriched categories (see Betti-Kasangian), a paper has been published jointly with Bob Roseburgh: "On the decomposition of tree automata" and work has been done in the direction of a new characterization of fuzzy automata in the topos of "fuzzy sets a la Born". 2) I reported about work done by Labelle & Pettorossi on synchronization of processes (a la Milner?) in a categorical setting that is reminiscent of Lawvere's dynamical systems. In particular I mentioned work in progress with Labelle involving the exploitation of the Conduche property of the labelling functors (& others) which give meaningful computer-scientific interpretations. 3) The enriched categorical setting, where Labelle's enriched. This gives a ??? setting to their interpretation and justifies their result for general reasons. I made also some suggestions about possible links with λ -calculus.

ABSTRACTS OF THE SYDNEY CATEGORY SEMINAR.

DATE: 15/October/1986 TIME: 4.30PM

PLACE: Sydney University

SPEAKER: R. Street

TITLE: Conduché Functors.

ABSTRACT: A functor $p : E \rightarrow B$ is called Conduché when the pullback along p, as a functor $p^* : \text{Cat}/B \rightarrow \text{Cat}/E$ has a right adjoint. We looked at what a right adjoint to p^* means. Any functor $p : E \rightarrow B$ gives a normal morphism of bicategories $B \rightarrow \text{Mod}$ [Bénabou]. But p is Conduché iff the morphism is a homomorphism. There is also a characterization in terms of lifting of factorizations of arrows in the image of p which are unique up to an undirected path.

ABSTRACTS OF THE SYDNEY CATEGORY SEMINAR.

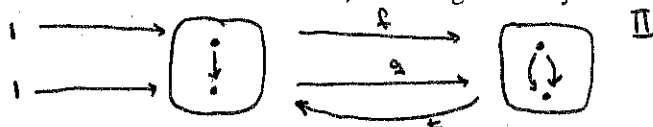
DATE: 15/October/1986 TIME: 3.15PM
 PLACE: Sydney University
 SPEAKER: M. Johnson
 TITLE: The 2-category Pasting Theorem.

ABSTRACT: We describe **loop free pasting diagrams**. These are the appropriate domains of parametrization for diagrams in ω -categories. The **well formed** loop free pasting diagrams are the domains of parametrization for **composite** diagrams in ω -categories. The collection of well formed sub pasting diagrams of a loop free pasting diagram form a free ω -category with union as the compositions. As applications of all this we obtain (again) a proof that the **well formed simplicial sets** form the free ω -category on the ω -simplex and a precise statement and proof of the 2-category pasting theorem. The proof of the 2-category pasting theorem generalizes easily to the n-category pasting theorem (which is important for the analysis of coherence).

ABSTRACTS OF THE SYDNEY CATEGORY SEMINAR.

DATE: 15/October/1986 TIME: 2.05PM
 PLACE: Sydney University
 SPEAKER: J. Powers
 TITLE: Bilimits And Coherence.

ABSTRACT: A counterexample was given to the following conjecture: for any 2-category C with a terminal object t , every 2-functor $F : C \rightarrow \text{Cat}$ sending t to a biterminal object is equivalent (in $\text{Ps}[C, \text{Cat}]$) to a 2-functor G sending t to 1 . If I is the free living iso, and Π has two objects and two isos between them, F is given by



with f, g different.

ABSTRACTS OF THE SYDNEY CATEGORY SEMINAR.

DATE: 29/October/1986 TIME: 2.10PM
 PLACE: Sydney University
 SPEAKER: R. Street
 TITLE: Localizations Of Locally Finitely Presentable Categories.

ABSTRACT: [with Brian Day] Grothendieck style topologies on enriched categories were discussed and used to prove two results: one concerns when localizations of $\text{Lex}(C^{op}, V)$ are all of the form $\text{LexSh}(C^{op}, V)$ for some topology on C ; and, the other is a characterization of categories equivalent to $\text{LexSh}(C^{op}, V)$.

ABSTRACTS OF THE SYDNEY CATEGORY SEMINAR.

DATE: 29/October/1986 TIME: 3.30PM
 PLACE: Sydney University
 SPEAKER: G. Kelly
 TITLE: Monoidal Adjunctions Continued.

ABSTRACT: Let V and V' monoidal [symmetric monoidal, symmetric monoidal closed] categories. Various equivalent conditions are given for the enrichability of an adjunction $\eta, \varepsilon : \psi \rightarrow \phi : V_0 \rightarrow V'_0$ to an adjunction $\eta, \varepsilon : \psi \rightarrow \phi$

: $V \rightarrow V'$ in the 2-category of monoidal [symmetric monoidal] categories. This is further related to the existence of a left adjoint to the V' -functor $\Phi : \Phi_* V \rightarrow V'$.

ABSTRACTS OF THE SYDNEY CATEGORY SEMINAR.

DATE: 5/November/1986 TIME: 2.10PM

PLACE: Macquarie University

SPEAKER: B. Day

TITLE: A Criterion For Total Completeness.

ABSTRACT: If A is a V -category, where V is symmetric monoidal closed small cocomplete and small complete admitting arbitrary intersections of monos, call A total if the yoneda embedding has a left adjoint. Then we have the following theorem; A V -category A is total if and only if it is complete and admits arbitrary intersections of monos and there exists a functor $r : [A^{op}, V] \rightarrow A$ with a natural monic $\mu : id(A) \rightarrow ry$, where y is the yoneda functor.

ABSTRACTS OF THE SYDNEY CATEGORY SEMINAR.

DATE: 5/November/1986 TIME: 3.30PM

PLACE: Macquarie University

SPEAKER: D. Lever

TITLE: Comprehension In Indexed Categories.

ABSTRACT: The logic in a B -indexed category A is constructed by interpreting first order logic in A and second order logic in B . To get the logic going we assume B has finite limits and has "for all", "&", and "implication", and assume A has "there exists" and A is well-powered over B . Then all of the first order logic of A can be interpreted. As an application of one of the interpretations we show A has homs in B .

ABSTRACTS OF THE SYDNEY CATEGORY SEMINAR.

DATE: 5/November/1986 TIME: 4.48PM

PLACE: Macquarie University

SPEAKER: M. Adelman

TITLE: Background On Ultrafilters For Next Weeks Talk.

ABSTRACT: See next week.

ABSTRACTS OF THE SYDNEY CATEGORY SEMINAR.

DATE: 12/November/1986 TIME: 2.15PM

PLACE: Sydney University

SPEAKER: M. Adelman

TITLE: Non-Standard Category Theory.

ABSTRACT: We know how to say that a category A has α -ary powers. We just ask that the diagonal functor $A \rightarrow A^\alpha$ has a right adjoint. Can we say a category has ultra-powers in a similar manner? Let A be the category of models of the theory which has there exists some $x=y$. This is the simplest non-geometric first order theory that I can think of. Let X be any set and let U be an ultra-filter on X . Denote by $C^{(U)}$ the ultra-power of a category C . Let $A(X)$ be the category of sheaves on PX with the finite cover topology. Then the ultra-powers of objects of A exist iff the diagonal $A^{(U)} \rightarrow A(X)^{(U)}$ exists. The same is true if A is a category of models of any coherent theory. It somehow seems we are using the fibred category whose fibre over S is $A(S)^U$ instead of A^S as for standard

products. This idea arose from conversations with Bob Paré when he was in Sydney.

ABSTRACTS OF THE SYDNEY CATEGORY SEMINAR.

DATE: 12/November/1986 TIME: 3.45PM

PLACE: Sydney University

SPEAKER: D. Lever

TITLE: Calculus And Manifolds Over a Base Space I.

ABSTRACT: The real line $R_B \in \text{Top}/B$ is taken to be $\text{proj} : R \times B \rightarrow B$. Closed intervals are characterized as subspaces of R_B which are connected, compact, nonsingular and open as spaces over B.

ABSTRACTS OF THE SYDNEY CATEGORY SEMINAR.

DATE: 12/November/1986 TIME: 4.15PM

PLACE: Sydney University

SPEAKER: G. Kelly

TITLE: Strict Pseudo Maps Of Algebras For A 2-monad.

ABSTRACT: let T be a 2-monad on a cocomplete 2-category K. Besides the strict morphisms of T-algebras (commutativity on the nose) we have the pseudo or mere morphisms (commutativity to within coherent isomorphisms) and the lax ones (the isomorphisms replaced by general 2-cells). Hence three 2-categories $T\text{-Alg}_s \rightarrow T\text{-Alg} \rightarrow T\text{-Alg}_l$. It is important that the inclusions $T\text{-Alg}_s \rightarrow T\text{-Alg}$ and $T\text{-Alg}_s \rightarrow T\text{-Alg}_l$ have left adjoints. We outline the technique of proof.

ABSTRACTS OF THE SYDNEY CATEGORY SEMINAR.

DATE: 19/November/1986 TIME: 2.00PM

PLACE: Macquarie University

SPEAKER: G. Kelly

TITLE: Blackwell Thesis Result (continued).

ABSTRACT: Continuing from Kelly's 12/Nov/86 talk, we give the details of the proof that $T\text{-Alg}_s \rightarrow T\text{-Alg}$ and $T\text{-Alg}_l \rightarrow T\text{-Alg}$ have left adjoints as 2-functors.

ABSTRACTS OF THE SYDNEY CATEGORY SEMINAR.

DATE: 19/November/1986 TIME: 2.15PM

PLACE: Macquarie University

SPEAKER: D. Lever

TITLE: Topology Over A Base.

ABSTRACT: Differential calculus is introduced to Top/B and a generalization of the implicit function theorem is established. (Street has aptly renamed this result the implicit point theorem).

ABSTRACTS OF THE SYDNEY CATEGORY SEMINAR.

DATE: 19/November/1986 TIME: 4.15PM

PLACE: Macquarie University

SPEAKER: M. Adelman

TITLE: Fields And Flat Functors.

ABSTRACT:

ABSTRACTS OF THE SYDNEY CATEGORY SEMINAR.
DATE: 26/November/1986. No seminar.

ABSTRACTS OF THE SYDNEY CATEGORY SEMINAR.
DATE: 3/December/1986. No seminar.

ABSTRACTS OF THE SYDNEY CATEGORY SEMINAR.
DATE: 10/December/1986 TIME: 2.10PM
PLACE: Sydney University
SPEAKER: M. Johnson
TITLE: Coherence and Orientals. I : Theory.

ABSTRACT: Coherence Theorems have been described (eg Categories for the working mathematician) as assertions that all of a certain class of diagrams commute (but see also Kelly: coherence theorems for lex algebras). This description needs at least two qualifications: in the applications the diagrams asserted to commute are actually diagrams of natural transformations (not their components), and they are *formally described diagrams* independent of particular *realizations*. We take account of these qualifications by investigating formally described diagrams (pasting diagrams) in n -categories (where natural transformations live as 2- or even 3-cells). A **situation** is a collection of k -dimensional pasting diagrams (vertices), a collection of $k+1$ -dimensional pasting diagrams (edges), and a realization of all this in an n -category. Inspired by Eilenberg's "only check overlaps" slogan, but in quite different context (Sammy didn't use realizations) we obtain two *coherence lemmas*. The lemmas extend respectively ideas used by Laplaza and Mac Lane to prove coherence (although the ideas are much older) and have the form: In a suitable situation, if all "forks" can be "commutatively completed" in a particular way, then all diagrams formed from the vertices and edges above commute in the n -category.

ABSTRACTS OF THE SYDNEY CATEGORY SEMINAR.
DATE: 10/December/1986 TIME: 4.05PM
PLACE: Sydney University
SPEAKER: D. Lever
TITLE: Calculus and Manifolds over a base space.

ABSTRACT: Manifolds are defined over a base space and the Grassmanian of a vector bundle is constructed as an example of one of these.

ABSTRACTS OF THE SYDNEY CATEGORY SEMINAR.
DATE: 17/December/1986 TIME: 2.15PM
PLACE: Macquarie University
SPEAKER: D. Lever
TITLE: Fixed Points of Functors of $\text{Man} \rightarrow \text{Man}$.

ABSTRACT: A functor $F : \text{Man} \rightarrow \text{Man}$ is called continuous if it has a Top indexed extension. Such a functor preserves open inclusions, immersions between manifolds of the same dimension and open covers. If the empty manifold is admitted to Man , then F stabilizes in dimension after one iteration and F has an initial fixed point. Also F preserves homotopy so any construction which is defined up to homotopy equivalence will be preserved as well. Thinking of a path $I \rightarrow \text{Man}$ as an object of Man/I allows us to define a generalized cohomology theory with test space Man (cf J. T. Schwartz, "Differential Geometry and Topology").

ABSTRACTS OF THE SYDNEY CATEGORY SEMINAR.

DATE: 17/December/1986 TIME: 3.30PM

PLACE: Macquarie University

SPEAKER: M. Johnson

TITLE: Coherence and Orientals II : Applications.

ABSTRACT: As easy applications of the coherence ands (C & O, I) we (re)prove the general associative and identity laws, the coherence theorem for associativity and identity isomorphisms (Mac Lane), and the coherence theorem for an associativity non-isomorphism (Laplaza). More importantly, we prove that Street's orientals do embody all the "higher coherence conditions" for associativity and identity isomorphisms, and explain how we have made the geometry of associative and identity laws the same (cf Abstracts July 2, 1986).

ABSTRACTS OF THE SYDNEY CATEGORY SEMINAR.

DATE: 17/December/1986 TIME: 4.25PM

PLACE: Macquarie University

SPEAKER: R. Street

TITLE: Computability

ABSTRACT: Stable vertices in a graph are those with no non-trivial elements out of them. A stabilizer of a vertex is a path to a stable element. The graph is perfect when each vertex u has a stabilizer $p : u \rightarrow v$ with v uniquely determined by u . The following result is implicit in many works (& possibly explicit in some): A graph is perfect iff it satisfies the chain condition, is locally finite, and, any two elements with the same source can be "closed" by paths into a common vertex. The first two of these follow if we can find a graph morphism, which reflects vertices, into a graph which has them (a *rank* function). A derivation scheme D is a 2-graph with a category structure on the graph of 1-vertices. Passing through derivation schemes $D_\varepsilon, D_\partial$ of elementary derivations, derivations (respectively), we derive a 2-category D^* such that morphisms of derivation schemes $D \rightarrow A$ into a 2-category A are in bijection with 2-functors $D^* \rightarrow A$.