SYDNEY CATEGORY SEMINAR ABSTRACTS

1988

This document consists of abstracts of some of the talks given at the Sydney Category Seminar in 1988.

R.F.C. Walters
26 July 1989

SYDNEY CATEGORY SEMINAR ABSTRACTS

Date: 13 January 1988
Time: 10:15 - 11:30 a.m.
Place: [illegible] University of Sydney
Speaker: WALTER THOLEN

Title: ON GENERATORS AND TOTALITY (joint work with REINHARD BÖRGER)

Abstract:

We discuss the $\mathcal{E}$-generalization of B. J. Day's Theorem "An $\mathcal{E}$-cocomplete category with an $\mathcal{E}$-generator is total"; here $\mathcal{E}$-cocomplete means that pullbacks of $\mathcal{E}$-morphisms along arbitrary morphisms exist and belong to $\mathcal{E}$, and that arbitrary co-intersections of $\mathcal{E}$-morphisms exist and belong to $\mathcal{E}$; a small set $\mathcal{G}$ of objects in $\mathcal{A}$ is an $\mathcal{E}$-generator if all coproducts $\coprod_{G\in\mathcal{G}} X_G \cdot G$ (all X_G' small sets) exist and if $\varepsilon_A : \coprod_{G\in\mathcal{G}} \mathcal{A}(G,A)\cdot G \to A$ is in $\mathcal{E}$ for all $A \in |\mathcal{A}|$. Our proof is based on the facts that (1) $\underline{Set}^{\mathcal{G}}$ is total, (2) every solid (= semi-topological) functor $U : \mathcal{A} \to \mathcal{X}$ lifts totality from $\mathcal{X}$ to $\mathcal{A}$.

We also discuss the validity of converse forms of the theorem; we have:

1) Every total category is $\mathcal{E}$-cocomplete for $\mathcal{E} = \{$regular epimorphisms in $\mathcal{A}\}$,

2) There is a total category $\mathcal{A}$ with a strong generator which fails to be $\mathcal{E}$-cocomplete for $\mathcal{E} = \{$strong epimorphisms in $\mathcal{A}\}$,

3) There is a total category $\mathcal{A}$ which is cowellpowered (hence $\mathcal{E}$-cocomplete for $\mathcal{E} = \{$all epimorphisms$\}$) but does not have a generator.

Signature

Walter Tholen

SYDNEY CATEGORY SEMINAR ABSTRACTS

Date: 13 Jan 1988
Time: 1:30 pm
Place: University of Sydney
Speaker: Max Kelly

Title: Enhanced factorization systems

Abstract: A factorization system $(\mathcal{E}, \mathcal{M})$ on a 2-category $\mathcal{K}$ is *enhanced* if, given

$$\begin{tikzcd} \bullet \arrow[r, "e"] \arrow[d, "u"'] \arrow[dr, phantom, "\Leftarrow \eta"] & \bullet \arrow[d, "v"] \\ \bullet \arrow[r, "m"'] & \bullet \end{tikzcd}$$

with $e \in \mathcal{E}$, $m \in \mathcal{M}$, and η invertible, there are a unique w and invertible

$$\begin{tikzcd} & \bullet \arrow[d, "v"] \arrow[dl, phantom, "\Leftarrow \zeta"] \\ \bullet \arrow[ur, "w"] \arrow[r, "m"'] & \bullet \end{tikzcd}$$

with $we = u$ and $\zeta e = \eta$; as well as the corresponding two-dimensional universal property. Example: $\mathcal{K} = \underline{Cat}$, $f \in \mathcal{E}$ if f is bijective on objects, $f \in \mathcal{M}$ if f is fully faithful.

Prop. 1. Let $\mathcal{K}$ admit the pseudo-limit of any map. Then a factorization system $(\mathcal{E}, \mathcal{M})$ on $\mathcal{K}$ is enhanced if & only if (i) $\alpha : f \cong m$ and $m \in \mathcal{M}$ implies $f \in \mathcal{M}$, and (ii) every equivalence is in $\mathcal{M}$.

Prop 2 Let $(\mathcal{E}, \mathcal{M})$ be an enhanced factorization system on $\mathcal{K}$ and T a 2-monad on $\mathcal{K}$ with $T\mathcal{E} \subset \mathcal{E}$. Then we get an enhanced f.s. $(\mathcal{F}, \mathcal{N})$ on T-Alg where $(f, \bar{f}) \in \mathcal{F}$ [resp. $\mathcal{N}$] iff $f \in \mathcal{E}$ [resp. $\mathcal{M}$].

Example If $\mathcal{K} = \underline{Cat}$ and $T = \mathcal{T}_0 -$ for a club $\mathcal{T}$, T preserves those functors that are bijective on objects.

SYDNEY CATEGORY SEMINAR ABSTRACTS

Date: 20 January 1988
Time: 11:45 – 12:30
Place: Macquarie University
Speaker: Walter THOLEN

Title: { Addendum to "Generators and totality"
Remarks on factorization systems

Abstract:

The generalized Day's Theorem ($\mathcal{A}$ $\mathcal{E}$-cocomplete with an $\mathcal{E}$-generator $\Rightarrow$ $\Rightarrow$ $\mathcal{A}$ total) is evaluated for $\mathcal{E}$ = regular epimorphisms (in Kelly's sense) to give:

Thm. TFAE for a category $\mathcal{A}$ with a regular generator:

(i) $\mathcal{A}$ is total,
(ii) $\mathcal{A}$ has pushouts and small co-intersections of regular epimorphisms,
(iii) $\mathcal{A}$ has coequalizers.

Cor. $\mathcal{A}$ with a regular generator and coequalizers, $F: \mathcal{A} \to \mathcal{B}$ preserves all small colimits $\Rightarrow$ F has a right adjoint.

Generalized factorization systems on a category are described as

- pointed endofunctors $\gamma : 1_{\mathcal{X}^2} \to C$ of $\mathcal{X}^2$,
- copointed endofunctors $\delta : D \to 1_{\mathcal{X}^2}$ of $\mathcal{X}^2$,
- Eilenberg-Moore algebras with respect to the canonical monad structure of $\mathcal{X} \mapsto \mathcal{X}^2$ on CAT,

all subject to certain extra conditions.

Walter Tholen

SYDNEY CATEGORY SEMINAR ABSTRACTS

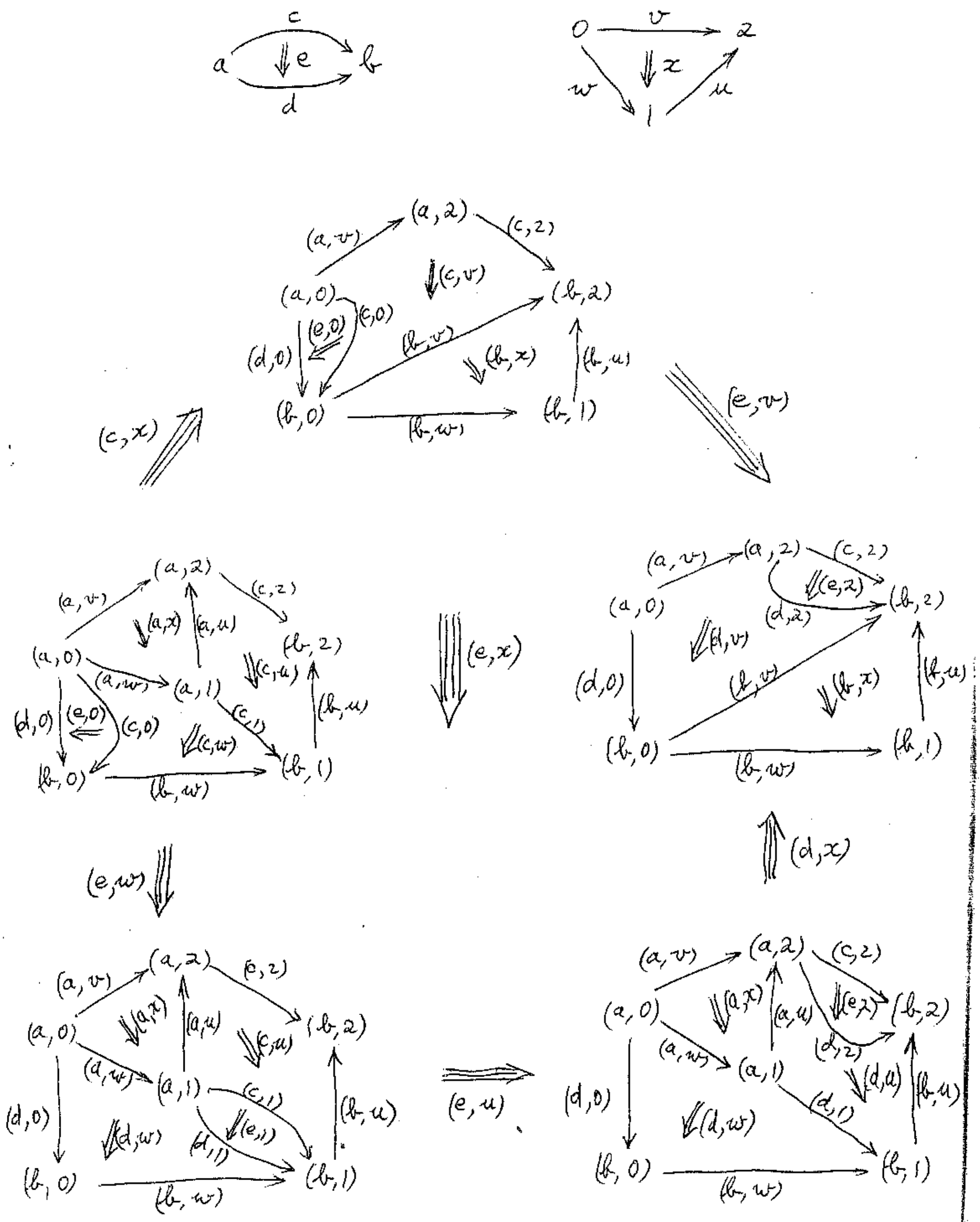
Date: 20 Jan 1988
Time: 1:30 pm
Place: Macquarie University
Speaker: Ross Street
Title: Parity complexes.

Abstract:

Some aspects of Macquarie Math Report No. 88-0015 Parity Complexes.

20 Jan 1988

$G[2] \times \Delta[2]$



SYDNEY CATEGORY SEMINAR ABSTRACTS

Date: 20 Jan 1988
Time: 4:00
Place: Macquarie University
Speaker: G. M. Kelly

Title: Gray's tensor product for 2-categories

Abstract: For 2-categories $\mathcal{B}$ and $\mathcal{C}$, $\mathrm{Lax}[\mathcal{B},\mathcal{C}]$ is the 2-category of 2-functors, lax natural transformations, & modifications. Changing the sense of the 2-cells in the lax n.t.'s gives $\mathrm{Oplax}[\mathcal{B},\mathcal{C}]$; making them invertible gives $\mathrm{Psd}[\mathcal{B},\mathcal{C}]$; making them identities gives $[\mathcal{B},\mathcal{C}]$. The mere category $2\text{-}\underset{\sim}{\mathrm{Cat}}_0$ has of course the cartesian monoidal closed structure, with internal-hom $[\mathcal{B},\mathcal{C}]$. It has a monoidal biclosed structure first observed by J. W. Gray: we have

$$2\text{-}\underset{\sim}{\mathrm{Cat}}_0(\mathcal{B}, \mathrm{Oplax}[\mathcal{A},\mathcal{C}]) \cong 2\text{-}\underset{\sim}{\mathrm{Cat}}_0(\mathcal{A}\otimes\mathcal{B},\mathcal{C}) \cong 2\text{-}\underset{\sim}{\mathrm{Cat}}_0(\mathcal{A}, \mathrm{Lax}[\mathcal{B},\mathcal{C}]),$$

and all the conditions are satisfied: see Gray, LNM 391, 1974 & Gray, Eilenberg memorial volume, 1976, 63-76; the latter uses the braid group to prove coherence for associativity of $\otimes$. Similarly

$$2\text{-}\underset{\sim}{\mathrm{Cat}}_0(\mathcal{B}, \mathrm{Oppsd}[\mathcal{A},\mathcal{C}]) \cong 2\text{-}\underset{\sim}{\mathrm{Cat}}_0(\mathcal{A}\,\square\,\mathcal{B},\mathcal{C}) \cong 2\text{-}\underset{\sim}{\mathrm{Cat}}_0(\mathcal{A}, \mathrm{Psd}[\mathcal{B},\mathcal{C}]);$$

but since $\mathrm{Oppsd}[\mathcal{A},\mathcal{C}] \cong \mathrm{Psd}[\mathcal{A},\mathcal{C}]$, we expect this to be a <u>symmetric</u> monoidal closed structure. Kelly posed the question of the neatest way to establish all of this; Gray's accounts are probably not "best possible", and he doesn't consider $\square$, where the two coherence conditions for symmetry need to be verified. Kelly suggested simplifying by using the strong generator $0 \overset{\Downarrow}{\rightrightarrows} 1$ of $2\text{-}\underset{\sim}{\mathrm{Cat}}_0$; Street made a good observation that should be followed up, namely that $\otimes$ can be got by Day's convolution from the dense subcategory $\underset{\sim}{1}, \underset{\sim}{2}, \underset{\sim}{2}\otimes\underset{\sim}{2}, \underset{\sim}{2}\otimes\underset{\sim}{2}\otimes\underset{\sim}{2}, \ldots$

SYDNEY CATEGORY SEMINAR ABSTRACTS

Date: 27 Jan 1988
Time: 4 pm
Place: University of Sydney
Speaker: Ross Street

Title: Parity complexes applied to Gray's tensor product of 2-categories.

Abstract: Let $\mathcal{D}$ denote the parity complex $+ \xrightarrow{0} -$. But $\mathbb{I}^n = \mathcal{O}(\mathcal{D} \times \cdots \times \mathcal{D})$, the free ω-category on the product of n copies of $\mathcal{D}$. Write $R : \omega\text{-Cat} \to 2\text{-Cat}$ for the left adjoint to the inclusion. Let $\mathcal{J}$ denote the full subcategory of 2-Cat consisting of the 2-categories $R\mathbb{I}^n$. Then $\mathcal{J}$ becomes a monoidal category with $\mathbb{I}^n \otimes \mathbb{I}^m = \mathbb{I}^{n+m}$ (I don't believe the definition on 2-functors is too hard). Using left Kan extension, we obtain a tensor product on 2-Cat which agrees with John Gray's. [To verify the denseness of $\mathcal{J}$ in 2-Cat and the agreement with Gray's tensor product, we must see that families of functions

$$\theta_{n_1,\ldots,n_r} : n_1\text{-cu}(A_1) \times \cdots \times n_r\text{-cu}(A_r) \longrightarrow (n_1 + \cdots + n_r)\text{-cu}(B),$$

where $n\text{-cu}(A) = \omega\text{-Cat}(\mathbb{I}^n, A)$ and $A_1, \ldots, A_r, B$ are 2-categories, natural in $R\mathbb{I}^{n_1}, \ldots, R\mathbb{I}^{n_r}$, are in natural bijection with quasi-functors $A_1 \times \cdots \times A_r \to B$ in r-variables.] The symmetry $A^{op} \otimes B^{op} \cong (B \otimes A)^{op}$ comes from the symmetry $XD \times YD \cong (Y \times X)D$ of parity complexes.

SYDNEY CATEGORY SEMINAR ABSTRACTS

Date: Feb 5, 1988
Time: 11:00 Am
Place: Macquarie University / University of Sydney
Speaker: Terrence Bisson (joint work with André Joyal)

Title: Geometric Models for Homology Operations

Abstract:

(Co)bordism theory (ignoring orientations) gives a geometric representation of the mod 2 (co)homology theory of smooth manifolds (see Quillen, "Some elementary proofs ...").

Then, suitable endofunctors of the category of spaces produce operations in (co)homology.

In particular, covering spaces give rise to certain "twisted power" constructions.

These twisting constructions are closed under addition, multiplication, & composition.

This determines an algebra, including substitution, of covering spaces.

In particular, it is interpreted as an algebra of external operations in (co)homology.

Pulling back along the diagonal gives internal operations in cohomology.

One gets internal homology operations on X if X is an infinite loop space.

The double covers $S^n \to \mathbb{R}P^n$ produce the usual Steenrod and Dyer-Lashof operations.

SYDNEY CATEGORY SEMINAR ABSTRACTS

Date: 3 Feb 88
Time: 4 p.m.?
Place: Macquarie University / ~~University of Sydney~~
Speaker: G. M. Kelly

Title: Splitting idempotents in a 2-category.

Abstract: Since idempotents do not split in the category of groups and conjugacy-classes of homomorphisms, it seemed that "T-algebra = category with an idempotent endomorphism" provided a counter-example to the conjecture "every pseudo-T-algebra is equivalent to a T-algebra". Yet a recent result of J. Power implies otherwise: the conjecture is true when T preserves b.o. (bijective on objects) functors, as it does here. The point is that a pseudo-T-algebra here is a category A with an endomorphism $e: A \to A$ and an isomorphism $\alpha: e^2 \to e$ which satisfies the <u>coherence axiom</u> $e\alpha = \alpha e$. The speaker painstakingly analyzed Power's proof in this special case, to see where the coherence axiom gets used. It turns out that there is some $C \simeq A$ and $f: C \to C$, which is a <u>strict idempotent</u> when the coherence condition is satisfied; and splitting f gives a "splitting" of e. The final result is particularly simple: If
$$\begin{tikzcd} A \arrow[rr, "e"] \arrow[dr, "s"'] & & A \\ & D \arrow[ur, "k"'] & \end{tikzcd}$$
is the factorization where s is b.o. and k is f.f. (fully faithful), then $sk \cong 1$.

SYDNEY CATEGORY SEMINAR ABSTRACTS

Date: 12 Feb. 88
Time: 10 am ?
Place: ~~Macquarie University~~ / University of Sydney
Speaker: G. M Kelly

Title: Splitting idempotents in a 2-category, continued.

Abstract: The speaker recalled, since many in the audience had not heard his January 1988 seminar, the concepts of factorization system and of <u>enhanced factorization system</u> in a 2-category. There was a general discussion of examples of EFS's; in particular, it was suggested that $(\mathcal{E}, M)$ might be an EFS on <u>Lex</u> if M = f.f. lex functors. The general question was raised: when do we get an EFS by taking M = the representably f.f. arrows? These are questions to be looked at later. The speaker tried, at the blackboard, to give a direct proof of the idempotent-splitting result of his Feb 88 talk, using <u>only</u> the existence of an EFS $(\mathcal{E}, M)$ with every M representably f.f.; but did not succeed (and hasn't done so in the days between the talk and this report).

SYDNEY CATEGORY SEMINAR ABSTRACTS

Date: 24 Feb 88
Time: 12.00 ?
Place: ~~Macquarie University~~ / University of Sydney
Speaker: G. M. Kelly

Title: On Peter May's operads.

Abstract:
An unpublished 1972 manuscript of Kelly analyzes Peter May's notion of operad in terms of a monoidal closed structure $\circ$ on the functor-category $[\mathbb{P}, \mathcal{V}]$, where $\mathbb{P}$ is the natural numbers and permutations, while $\mathcal{V}$ is a complete and cocomplete symmetric monoidal closed category; this $\circ$ is not the $\otimes$ given by Day's convolution process from the monoidal structure $+$ on $\mathbb{P}$. Kelly gave an explicit proof of the associativity of $\circ$ and considered also related structures on $[\mathbb{N}, \mathcal{V}]$ and $[\mathbb{S}, \mathcal{V}]$ — the last for $\mathbb{S}$ = finite sets and $\mathcal{V}$ cartesian closed; with connexions to props and theories. Lawvere gave an interpolation in Kelly's talk, on $[\mathbb{P}^n, \mathcal{V}]$ more generally, leading to a conceptual argument for the associativity of $\circ$, now exhibited as a composite of "operations" of a "theory".

SYDNEY CATEGORY SEMINAR ABSTRACTS

Date: 24 Feb 1988
Time: 4 pm
Place: University of Sydney
Speaker: Ross Street
Title: Group representation theory.

Abstract: This talk was to show that a category theorist knows the elements of group representation theory. In particular, I discussed the theorem characterizing induced representations and Mackey's decomposition theorem (as consequences of the calculus of Kan extensions). Characters were discussed:

$$\begin{tikzcd} H^Q \arrow[rr] \arrow[dr, "\varphi"'] & & G^Q \arrow[dl, "\varphi^G"] \\ & \mathbb{C} & \end{tikzcd} \qquad \varphi^G(g) = \frac{1}{\# H} \sum_{h \in H} \# G^Q(h,g) \cdot \varphi(h)$$

Ross Street.
25 Feb 1988.

SYDNEY CATEGORY SEMINAR ABSTRACTS

Date: Wednesday, 2nd March 1988
Time: 10.00 – 11.15 a.m.
Place: University of Sydney
Speaker: Albrecht DOLD

Title: Duality, Trace, and Transfer

Abstract:

Traces and transfers are discussed for certain endomorphisms $A \to A$ of objects in symmetric monoidal categories $\underline{C}$, where A is not necessarily finite (i.e. may not have an adjoint). Explicit adjunction maps (or "pre-adjunctions") are described if $\underline{C}$ is a stable homotopy category. Applications are to Alexander-Duality and to the Lefschetz-Hopf fixed point theorem.

Albrecht Dold

SYDNEY CATEGORY SEMINAR ABSTRACTS

Date: 2 Mar 88
Time: 12.00
Place: University of Sydney
Speaker: Max Kelly

Title: Limits for enriched categories

Abstract:

A purely expository lecture on "indexed" or "weighted" limits, with which not all of the audience were familiar, and cocompleteness relative to a class of weighting-types: finishing with examples of such limits in 2-categories, as background for my forthcoming talks on our work on two-dimensional universal algebra.

SYDNEY CATEGORY SEMINAR ABSTRACTS

Date: March 2, 1988
Time: 2.05 pm
Place: [illegible] University of Sydney
Speaker: IAIN AITCHISON

Title: The Calculus of Framed Links in S^3

Abstract: Every closed orientable 3-manifold can be represented by a link of n embedded circles in S^3, to which each is assigned an integer.

This representation is not unique. The lecture describes the two basic moves which generate the equivalence relation

$$L \sim L' \iff M_L \cong M_{L'}$$

where L, L' are framed links, and $M_L, M_{L'}$ are the corresponding 3-manifolds.

The fundamental result is due to R.C. Kirby, Invent. Math. 47 (1978).

[This is not original work]

SYDNEY CATEGORY SEMINAR ABSTRACTS

Date: 4:15 pm 2 March 1988
Time:
Place: University of Sydney
Speaker: Ross Street
Title: The Kan-Dold-Puppe Theorem and Euler characteristics.

Abstract: This was a report of my recollections of a talk by André Joyal on his first visit to Sydney (1979?). Newton recognized that a sequence $x_0, x_1, x_2, \ldots$ of quantities could be recaptured from its sequence of first differences $x_0, \Delta x_0, \Delta^2 x_0, \Delta^3 x_0, \ldots$ where $\Delta x_n = x_{n+1} - x_n$. To see this, we have $x_{n+1} = (1+\Delta)x_n$, so $x_n = (1+\Delta)^n x_0 = \sum_{k=0}^{\infty} \binom{n}{k} \Delta^k x_0$. Given a simplicial abelian group $X : \Delta^{op} \longrightarrow Ab$, consider $d' : XS \longrightarrow X$ where $S[n] = [n+1]$ & $d'_{[n]} = d_{n+1} : X_{n+1} \longrightarrow X_n$. Define $\Delta X \hookrightarrow XS$ to be the kernel of d'. We have $\partial = ((\Delta X)_0 \hookrightarrow X_1 \xrightarrow{d_0} X_0)$ and, by iteration, a chain complex

$$X_0 \xleftarrow{\partial} (\Delta X)_0 \xleftarrow{\partial} (\Delta^2 X)_0 \xleftarrow{\partial} (\Delta^3 X)_0 \longleftarrow \cdots .$$

Recapture X via $X_n = \sum_{k=0}^{\infty} \binom{n}{k} (\Delta^k X)_0$. This gives the Kan-Dold-Puppe equivalence $Ab^{\Delta^{op}} \simeq \partial\text{-}Ab$.

Using $\binom{-1}{k} = (-1)^k$, we obtain $X_{-1} = \sum_{k=0}^{\infty} (-1)^k (\Delta^k X)_0$ which is a <u>virtual</u> abelian group whose rank is the Euler characteristic of X.

Ross Street
2 March 1988.

SYDNEY CATEGORY SEMINAR ABSTRACTS

Date: 9-3-88
Time: 10:00 A.M.
Place: Macquarie University
Speaker: SK Johnson

Title: Small Cauchy Completions

Abstract: A proof is sketched that if the underlying category of $\mathcal{V}$ is locally presentable, then the Cauchy completion of a small $\mathcal{V}$-category is small. It is also noted that for $\mathcal{V}$ = complete lattices and sup-preserving maps (the example from Max's book producing a large Cauchy completion) the Cauchy completion of a $\mathcal{V}$ category A consists of the retracts of arbitrary products of representables.

SYDNEY CATEGORY SEMINAR ABSTRACTS

Date: 9 Mar 88
Time: 3 p.m.
Place: Macquarie University
Speaker: G. M. Kelly

Title: Two-dimensional monad theory, I.

Abstract: In last week's seminar, indexed limits had been discussed, for $\mathcal{V}$-categories in general. Now several important indexed limits for the case $\mathcal{V} = \underline{Cat}$ of 2-categories were discussed: inserters, equifiers, iso-inserters, cotensors, pseudo limits (conical or not) and lax limits; all of these exist if products, inserters, & equifiers do so.

Now let $\mathcal{K}$ be a complete & cocomplete 2-category, and T a 2-monad on $\mathcal{K}$. The Eilenberg-Moore 2-category $\mathcal{K}^T$ is denoted by $T\text{-}Alg_s$; that is, T-algebras with strict morphisms and the appropriate 2-cells. This is a locally-full sub-2-category of $T\text{-}Alg$, where the objects are still the T-algebras but the morphisms now preserve the structure only to within coherent isomorphisms. Many good properties of $T\text{-}Alg_s = \mathcal{K}^T$ are known: completeness and, if T has a rank, cocompleteness and left adjoints to algebraic functors; yet the morphisms of interest are those in $T\text{-}Alg$. Future lectures will deduce properties of $T\text{-}Alg$ from those of $T\text{-}Alg_s$.

In the rest of the present lecture, it was shown that $T\text{-}Alg$ admits products and all of the limits mentioned above, the generators of the "generalized limit-cone" being strict morphisms.

SYDNEY CATEGORY SEMINAR ABSTRACTS

Date: 16 March 1988
Time: 2:10 - 3:10
Place: University of Sydney
Speaker: Javad Tavakoli

Title: Vector spaces in Topoi

Abstract:

Let $\underline{E}$ be a topos with natural numbers object. If $\underline{E}$ satisfies Internal Axiom of Choice (IC), then it is boolean and so all definitions of fields will be equivalent.

We proved that in a topos with (IC), every vector space over a field is locally free. This means that for a vector space V in $\underline{E}$ there is $I \twoheadrightarrow 1$ and $X \in \underline{E}/I$ such that $F(X) \cong I^* V$, where F is the indexed free functor. As a consequence of this, we have: if $\underline{E}$ satisfies Axiom of Choice (AC), then every vector space is free. Also, every vector space in a topos with (IC) [(AC)] is locally [projective]. For example, if G is a group and K is a field in S^G, G-sets, then every vector space over K in S^G is locally free.

SYDNEY CATEGORY SEMINAR ABSTRACTS

Date: 18 March 1988
Time: 3:30 pm
Place: University of Sydney
Speaker: Ross STREET

Title: Representations of the finite general linear groups (cont.)

Abstract: Let $\mathbb{V}$ be the category whose objects are finite vector spaces over a finite fixed field F and whose arrows are linear isomorphisms. I wanted to make two points about the tensor product $M \otimes N$ on the category $[\mathbb{V}, \mathrm{Vect}]$ (where Vect is the usual category of finite dimensional vector spaces over the complex numbers) given by

$$(M \otimes N)C = \sum_{S \leq C} MS \otimes_{\mathbb{C}} N(C/S). \qquad (S \text{ subspace of } C)$$

Point 1. Connexion with multiplication of class functions given by J.A. Green Trans. AMS (1955). Let $\mathbb{V}'$ denote the category of pairs (C,S) where $S \leq C \in \mathbb{V}$ and with arrows $f : (C,S) \to (D,T)$ the arrows $f : C \to D$ in $\mathbb{V}$ with $f(S) = T$. Let $P : \mathbb{V}' \to \mathbb{V}$ be the discrete (op)fibration given by $P(C,S) = C$ and let $Q : \mathbb{V}' \to \mathbb{V} \times \mathbb{V}$ be given by $Q(C,S) = (S, C/S)$. Then the tensor product is the composite

$$[\mathbb{V}, \mathrm{Vect}] \times [\mathbb{V}, \mathrm{Vect}] \longrightarrow [\mathbb{V} \times \mathbb{V}, \mathrm{Vect}] \xrightarrow{[Q,1]} [\mathbb{V}', \mathrm{Vect}] \xrightarrow{\mathrm{Lan}_P} [\mathbb{V}, \mathrm{Vect}].$$

The clue to the connexion is then to realize skeletons of $\mathbb{V}$, $\mathbb{V}'$.

Point 2. Promonoidal structure on $\mathbb{V}$.

$$P : \mathbb{V}^{op} \times \mathbb{V}^{op} \times \mathbb{V} \longrightarrow \mathcal{S}_{fin}$$

$$(A,B,C) \longmapsto \{\text{short exact sequences } 0 \to A \xrightarrow{f} C \xrightarrow{g} B \to 0\}$$

I looked at the braiding in terms of this P.

Ross Street
18 March 1988.

SYDNEY CATEGORY SEMINAR ABSTRACTS

Date: 16 Mar 88
Time: ?
Place: University of Sydney
Speaker: G. M. Kelly

Title: Two-dimensional monad theory II.

Abstract: T is again a 2-monad on a complete & cocomplete 2-category K. It is classical that $T\text{-}Alg_s$ is complete, and it is easy to see that $T\text{-}Alg$ admits products, inserters, and equifiers, and hence all the limits implied by these, such as pseudo-limits. To go further we must suppose that T has a rank α, in the sense that it preserves α-filtered colimits. We use the notion of well-pointed endofunctor.

A pointed endofunctor on a category A is an $S : A \to A$ with a natural $\sigma : 1 \to S$; an S-algebra is an $A \in A$ with an "action" $a : SA \to A$ satisfying $a.\sigma A = 1$. We call (S, σ) well-pointed if $S\sigma = \sigma S$; then $S\text{-}Alg$ is a full subcategory of A, reflective if S has a rank.

Many operations on well-pointed endofunctors S_α produce a well-pointed S, and we can describe $S\text{-}Alg$ in terms of the S_α-alg. All of this works for 2-categories and 2-endofunctors.

We show that there is, in the situation of the first paragraph, a full embedding of $T\text{-}Alg_s$ into the comma-2-category T/K, such that $T\text{-}Alg_s$ is $S\text{-}Alg$ for a well-pointed S on T/K, which has a rank when S does so. Hence $T\text{-}Alg_s$ is reflective in T/K. Since T/K has evident good properties (it is cocomplete, and $T/K \to T'/K$ induced by a monad-map $T' \to T$ has a left adjoint), $T\text{-}Alg_s$ has similar properties.

SYDNEY CATEGORY SEMINAR ABSTRACTS

Date: 23 MARCH 1988
Time:
Place: Macquarie University / University of Sydney
Speaker: A. POLD

Title: The universal group UM of a monoid is also universal for polynomial maps.

Abstract: Let M be a commutative monoid and UM the abelian group it generates. By definition, UM represents the functor $\mathrm{Hom}(M,-)$ in the category of homomorphisms between abelian groups X; thus $\mathrm{Hom}(M,X) \cong \mathrm{Hom}(UM,X)$. A simple proof is given to show that UM also represents the functors $\mathrm{Map}_k(M,-)$; thus $\mathrm{Map}_k(M,X) \cong \mathrm{Map}_k(UM,X)$, where $\mathrm{Map}_k(M,X)$ is the group of maps $p: M \to X$ of degree $\leq k$, for all $k = 0, 1, \ldots$. $\mathrm{Map}_0(M,X)$ consists of all constant maps. For $k > 0$, the map p has degree $\leq k$ iff the function

$$p_2(x,y) = p(x+y) - p(x) - p(y)$$

has degree $< k$ in each variable.

SYDNEY CATEGORY SEMINAR ABSTRACTS

Date: 23 Mar 88
Time: 4:00
Place: Macquarie University
Speaker: G. M. Kelly

Title: Two-dimensional monad theory III

Abstract: For a 2-monad T with rank on the complete 2-category K, we have a full embedding $T\text{-}Alg_s \hookrightarrow T/K$, which admits a left adjoint $(\ )^{\circ}$ since $T\text{-}Alg_s$ consists of the algebras for a wellpointed endofunctor with rank on T/K. We now consider the (non-full) embedding $J : T\text{-}Alg_s \to T\text{-}Alg$, in the notation of recent abstracts. Using finite (indexed) colimits in K, we construct for each algebra A a map $A \to \bar{A}$ in T/K, composition with which induces an isomorphism of categories $T\text{-}Alg(A, B) \cong T/K(\bar{A}, B)$. This last category being isomorphic to $T\text{-}Alg_s(\bar{A}^{\circ}, B)$, we have exhibited a left adjoint $(\ )'$ to J, where $A' = \bar{A}^{\circ}$. If the unit is $p_A : A \to A'$, composition with it induces an isomorphism $T\text{-}Alg_s(A', B) \cong T\text{-}Alg(A, B)$. The counit $q_A : A' \to A$ is the strict morphism with $q_A p_A = 1$. Using the properties of pseudo-limits in $T\text{-}Alg$ discussed before, we see that $p_A q_A \cong 1$, so that the strict q_A is a surjective equivalence in $T\text{-}Alg$. When it is also a surjective equivalence in $T\text{-}Alg_s$, we call A flexible; A is flexible iff it is a retract of some B'. We call A semi-flexible if q_A is an equivalence, not necessarily surjective, of A in $T\text{-}Alg_s$; one of the conditions equivalent to semi-flexibility is (*) every morphism $A \to B$ of algebras is isomorphic to a strict one.

This leads to following: if $G : T\text{-}Alg \to \mathcal{C}$ is such that $GJ : T\text{-}Alg_s \to \mathcal{C}$ has a left adjoint H with unit $s : 1 \to GJH$, then G has JH as a left biadjoint, in the sense that s induces an equivalence (not an isomorphism) $\mathcal{C}(C, GA) \simeq T\text{-}Alg(JHC, A)$. The argument is easy: one shows that every HC is flexible, and uses (*).

SYDNEY CATEGORY SEMINAR ABSTRACTS

Date: 30 March 1988
Time: 4 pm
Place: University of Sydney
Speaker: Ross Street

Title: On the braiding for representations of the finite general linear groups.

Abstract: I showed before that the promonoidal version of the braiding was a matrix γ for given finite vector spaces A, B, C over F ($\# F = q$) with entries

$$\gamma_{(f,g),(h,k)} = \begin{cases} 1 & \text{for } (f,g),(h,k) \text{ a direct sum situation,} \\ 0 & \text{otherwise} \end{cases}$$

where (f,g) runs over short exact sequences

$$0 \longrightarrow A \xrightarrow{f} C \xrightarrow{g} B \longrightarrow 0$$

and (h,k) over s.e.s.

$$0 \longrightarrow B \xrightarrow{h} C \xrightarrow{k} A \longrightarrow 0.$$

This lecture examined the case $\dim A = \dim B = 1$.

[More recently (3 April) we have that the matrix $\gamma\gamma^t$ satisfies the polynomial $(x-1)(x-q)(x-q^2)$.]

Ross Street
14 April 1988

SYDNEY CATEGORY SEMINAR ABSTRACTS

Date: 30 Mar 88
Time:
Place: University of Sydney
Speaker: G. M. Kelly

Title: Two-dimensional monad theory IV.

Abstract: Consequences of the final theorem of last week's talk are: (a) T-Alg admits all (indexed) bicolimits, bicolim(ϕ, T) in T-Alg being psd-colim(ϕ, T') in T-Alg_s; (b) $\theta^{\#}: T\text{-}Alg \to S\text{-}Alg$ induced by a strict monad-map $\theta: S \to T$ admits a left biadjoint $\theta_{\#}$.

A special case of $T\text{-}Alg_s \xrightarrow{J} T\text{-}Alg \xrightarrow{K} T\text{-}Alg_l$ is $[A, Cat] \to Psd[A, Cat] \to Lax[A, Cat]$; the proof that J has a left adjoint $(\ )'$ is easily modified to show that KJ has a left adjoint $(\ )^{\dagger}$. The flexible algebras are now flexible indexing types $F: A \to Cat$ for limits. Since each G' is trivially flexible and each $G^{\dagger}$ is easily seen to be so by the theorem referred to above, the flexible limits include all pseudo limits and all lax limits, along with many others. They form a closed class of limits, and exist wherever products, inserters, and equifiers do so and idempotents split.

We write Cat_f for the 2-category of finitely-presentable categories. There is a finitary 2-monad M on $[Cat_f, Cat]$ whose algebras are the finitary ~~2~~-monads on Cat. The strict morphisms of algebras are the usual morphisms $\theta: S \to T$ of monads, while the general morphisms are the pseudo-morphisms of monads. We have $(\ )': M\text{-}Alg \to M\text{-}Alg_s$, and it follows that psd-T-Alg (given by pseudo actions) is just T'-Alg. When T is flexible, this is equivalent to T-Alg itself. A sufficient condition for flexibility of T is that it admit a presentation by operations and equations in which only morphisms, and never objects, are equated. Thus the T with T-Alg = strict monoidal categories is not flexible; while that with T-Alg = monoidal categories is so.

SYDNEY CATEGORY SEMINAR ABSTRACTS

Date: 13 April
Time: 3 pm
Place: Macquarie University
Speaker: Ross Street

Title: Localisations of locally presentable categories

Abstract: (Joint work with Brian Day).

This lectured reviewed results of our joint paper to appear in J.P.A.A. Also

Proposition. If $\mathcal{C}$ is finitely cocomplete, regular and finite coprods are disjoint & universal then $\mathrm{Lex}(\mathcal{C}^{op}, \mathcal{S})$ satisfies all Giraud's axioms for a Groth. topos except perhaps effective equivalence relations.

The distinction between $\mathrm{Lex}(\mathcal{C}^{op}, \mathcal{S})$ and $\mathrm{Sh}_J(\mathcal{C})$ (where J is the finite jointly strong epi cover topology) for $\mathcal{C}$ a pretopos was discussed.

Ross Street.
14 April 1988.

SYDNEY CATEGORY SEMINAR ABSTRACTS

Date: 27 Apr 88
Time: ?
Place: Macquarie University
Speaker: G. M. Kelly

Title: Finite (weighted) limits for 2-categories

Abstract: Kelly [Cahiers, 23 (1982), 3-42] called a weight $F: \mathcal{A} \to \underline{Cat}$ finite if (a) ob $\mathcal{A}$ is finite, (b) each category $\mathcal{A}(A,B)$ is finitely presentable, and (c) each category FA is finitely presentable; this gave a good generalization to 2-categories of the definitions & results for locally-finitely-presentable categories and their relation to finite-limit theories. If (Fin) denotes this class of weights, what is its closure $(\mathrm{Fin})^*$ in the Albert-Kelly sense?

It turns out that, for any small $\mathcal{A}$, a functor $F: \mathcal{A} \to \underline{Cat}$ is in $(\mathrm{Fin})^*$ if & only if it is finitely presentable in the 2-category $[\mathcal{A}, \underline{Cat}]$; which is to say that it is a finite colimit of representables (since finite limits of finite limits are finite limits), or that it is $\mathrm{Lan}_K H$ for some $K: \mathcal{B} \to \mathcal{A}$ and some finite $H: \mathcal{B} \to \underline{Set}$.

Various questions arise. Street [JPAA 8 (1976), 149-181] gives sufficient conditions for $F: \mathcal{A} \to \underline{Cat}$ to be in $(\mathrm{Fin})^*$; are they necessary? What is the relation between 2-categories satisfying (a) and (b) above and finitely-presentable 2-categories? The research students were invited to consider these and related questions.

SYDNEY CATEGORY SEMINAR ABSTRACTS

Date: 27 April 1988
Time: 4:15 pm
Place: Macquarie University
Speaker: Ross Street

Title: Hopf algebras.

Abstract: This was a report on my reading of

Milnor-Moore: On the structure of Hopf algebras; Annals of Math 81 (1965) 211-264, and

Zelevinski: Representations of Finite Classical Groups SLN 869. (1981)

Let Mat($\mathbb{Z}$) be the category of finite sets and integer matrices ($\simeq$ category of free abelian groups). Work in the category $\mathcal{V}$ of graded objects over Mat($\mathbb{Z}$) with obvious monoidal structure

$$(A \otimes B)_n = \sum_{r+s=n} A_r \times B_s$$

and the symmetry which does not introduce -1.

Theorem. Suppose R, $\mu : R \otimes R \to R$, $\eta : 1 \to R$ in $\mathcal{V}$ are such that $\eta_0 : 1 \to R_0$ is invertible and η is a unit for μ. If the transpose $\mu^t : R \to R \otimes R$ is a homomorphism for objects with binary multiplication and unit

---

then R, μ, η is a commutative monoid in $\mathcal{V}$. $\square$

This is a very general result implying commutativity and associativity reminiscent of, but apparently fundamentally different from, the Eckmann-Hilton result.

SYDNEY CATEGORY SEMINAR ABSTRACTS

Date: 11/5/88
Time:
Place: Macquarie University / University of Sydney
Speaker: Gordon Monro

Title: Distributive categories

Abstract:

The notion of distributive category used by Lawvere and Schanuel requires all finite limits. I propose a weakening, namely:

- finite products
- finite "disjoint", "universal" coproducts in the following sense:
  - i) strict initial object 0
  - ii) disjoint coproducts: $A \xrightarrow{\iota_1} A+B$ is monic and $\begin{tikzcd} 0 \arrow[r] \arrow[d] & A \arrow[d] \\ B \arrow[r] & A+B \end{tikzcd}$ is a pb.
  - iii) "universal" $\begin{tikzcd} X \arrow[r] \arrow[d] \arrow[dr, phantom, "pb"] & C \arrow[d] \arrow[dr, phantom, "pb"] & Y \arrow[l] \arrow[d] \\ A \arrow[r, tail, "\iota_1"] & A+B & B \arrow[l, tail, "\iota_2"'] \end{tikzcd}$ The two pullbacks are to exist and the top line is to be a coproduct diagram.

An example was given of a functor that preserves this data but does not preserve all finite limits. The functor is connected with the notion of "final model" in computer science

---

SYDNEY CATEGORY SEMINAR ABSTRACTS

Date: 25th May 1988
Time: 3-40 PM
Place: University of Sydney
Speaker: JOHN POWER

Title: A 2-categorical pasting theorem

Abstract: A 2-categorical pasting theorem was stated; and the list of definitions and propositions used in the proof was given. This differs from the Michael Johnson version primarily – in this one's heavy use of graph theory in the plane, heavily invoking our understanding of the geometry of the plane.

SYDNEY CATEGORY SEMINAR ABSTRACTS

Date: 1 June 1988
Time: 2:10 pm
Place: Macquarie University
Speaker: Ross Street

Title: Linearization of parity and pasting.

Abstract: Each pasting diagram (say in a 2-category) leads to a total order on the graded set of cells. I discovered this order when dealing with products and joins of parity complexes (called $\blacktriangleleft$ in the preprint "Parity complexes" Jan 1988), and proved it antisymmetric in a general situation. Looking back at this work with a view to extending it and improving it (since John Power pointed out that parity complexes in the technical sense of the preprint are not general enough to cover pasting diagrams), I am amazed that I overlooked the fact that the order is total (=linear). In the talk I presented arguments to suggest that parity complexes should possibly be graded finite sets with two linear orders subject to appropriate axioms. [The second linear order comes from the odd dual pasting diagram.]

SYDNEY CATEGORY SEMINAR ABSTRACTS

Date: 8 June 88
Time: 4:00
Place: [illegible] University of Sydney
Speaker: G. M. Kelly

Title: Practical applications of the Albert-Kelly result.

Abstract: The result in question asserts that the closure Φ^* of a class Φ of weights is given thus: $F: \mathcal{A} \to \underline{\mathrm{Cat}}$ lies in Φ^* iff $F \in [\mathcal{A}, \underline{\mathrm{Cat}}]$ belongs to the closure of the representables under Φ-colimits.

Flexible limits for 2-categories form a closed class Φ, various bases for which are known: (a) products, inserters, equifiers, and the splitting of idempotent equivalences, and (b) products, iso-inserters, cotensor products, and the splitting of idempotents. There are similar bases for <u>finite</u> flexible limits.

The result above was used to show that none of these bases is redundant. For example, cotensors $\{\underline{I}, -\}$ where $\underline{I}$ is the free isomorphism do not follow from products, equifiers, and the splitting of idempotents; for the closure of 1 in $\underline{\mathrm{Cat}}$ under the appropriate colimits consists purely of categories in which fg is never an identity unless f and g are identities; which excludes $\underline{I}$.

SYDNEY CATEGORY SEMINAR ABSTRACTS

Date: 15/6/88
Time: 2pm – 3pm
Place: Macquarie University
Speaker: JAVAD TAVAKOLI

Title: A Remark On Lawvere-Tierney Topology

Abstract:

It is well known that if F is an object in presheaf category, then there exists a unique largest topology in which F become a sheaf. In this talk we generalized this idea for any topos. In fact, we give a concrete formula for such topology.

For any object F in a topos $\mathcal{E}$, we give a simple way of constructing of an object which satisfies the existance part of sheaves axioms. We believe that, by using this construction, one might be able to construct the associated sheaf functor in a simple fashion.

SYDNEY CATEGORY SEMINAR ABSTRACTS

Date: 15 June 88
Time: 2 pm.
Place: Macquarie University
Speaker: G. M. Kelly

Title: Symmetric monoidal structures on $[n] = \{0, 1, \ldots, n\}$

Abstract: We consider symmetric monoidal structures on the totally-ordered finite set above; that is, commutative monoid structures for which $xy \le x'y'$ when $x \le x'$ and $y \le y'$. Such a structure is closed when $0x = 0$ for all x.

During Vaughan Pratt's visit earlier this year, he classified such structures in which every element is idempotent; there are 2^n such, of which half are closed.

We now drop this idempotence condition. It is still true that exactly half of the structures are closed; the rest are obtained by the isomorphism $[n] \cong [n]^{op}$. The idempotents form a submonoid, necessarily of the type discussed by Pratt. If e_r and e_{r+1} are successive idempotents less (say) than the identity, the subset $\{e_r, e_r + 1, \ldots, e_{r+1} - 1\}$ is a semi-group with zero with $xy \le x$ and with no idempotent except its zero e_r; one can make some progress on classifying these. All told we find three symmetric monoidal closed structures on [2] (Pratt's two and one other), and eleven on [3] (Pratt's four and one other).

SYDNEY CATEGORY SEMINAR ABSTRACTS

Date: 15 June 1988
Time: 4:30 pm
Place: Macquarie University
Speaker: Ross Street

Title: The length of a pasting diagram

Abstract: This is joint work with Steve Schanuel. A category A is free on a (directed) graph iff there exists a "length" functor $A \longrightarrow \mathbb{N}$ (where $\mathbb{N}$ is free on the terminal graph) with unique lifting of factorizations.

This talk discussed the generalization of this to 2-categories. The free 2-category $\mathbb{M}$ on the terminal computad was described. A 2-category A is free iff $\exists$ 2-functor $A \longrightarrow \mathbb{M}$ with unique lifting of factorizations. Actually, we have only proved the result restricted to computads whose 2-cells don't have sources of zero length, but believe we have the general case under control.

SYDNEY CATEGORY SEMINAR ABSTRACTS

Date: 22 June 88
Time: 2:00
Place: University of Sydney
Speaker: G. M. Kelly

Title: On the notion of club: the original concept

Abstract: The early coherence theorems of Mac Lane and Epstein are literally false as stated, since differently-formed functors $\mathcal{A}^n \to \mathcal{A}$ may fortuitously coincide for a particular model $\mathcal{A}$. Kelly–Mac Lane overcame this by replacing functors by "shapes", but still had concrete natural transformations.

Kelly [Lecture Notes 281] first gave a more abstract formulation, in the first instance for those cases where the structural natural transformations merely permute the variables. For categories $\mathcal{A}$ and $\mathcal{B}$, $\{\mathcal{A},\mathcal{B}\}$ has as objects pairs $\{n, T: \mathcal{A}^n \to \mathcal{B}\}$ and as morphisms $\alpha: TA^{\xi} \to S$ where ξ is a permutation of n. There is an augmentation $\Gamma: \{\mathcal{A},\mathcal{B}\} \to \mathbb{P}$, where $\mathbb{P}$ is the category of natural numbers & permutations. If $\mathcal{C} \in \underset{\sim}{Cat}/\mathbb{P}$, maps $\mathcal{C} \to \{\mathcal{A},\mathcal{B}\}$ in $\underset{\sim}{Cat}/\mathbb{P}$ correspond to maps $\mathcal{C} \circ \mathcal{A} \to \mathcal{B}$ in $\underset{\sim}{Cat}$ for a certain $\circ$. This extends to a tensor product $\circ$ on $\underset{\sim}{Cat}/\mathbb{P}$, with right-hom $\{\mathcal{A},\mathcal{B}\}$. Monoids for $\circ$ are ($\mathbb{P}$-)clubs; there are also $\mathbb{S}$ and $\mathbb{S}^{op}$-clubs, where $\mathbb{S}$ is natural numbers and functions; and $\mathbb{G}$-clubs, covering such mixed-variance $\mathbb{P}$-cases as symmetric monoidal categories.

For a club $K = (K, \Gamma)$ we have the monad $(K,\Gamma)\circ -$ whose algebras are the structures in question. Since the free structure on 1 is $(K,\Gamma)\circ 1 = K$, we have the strange situation that, in these "club" cases, the free $K \circ \mathcal{A}$ on $\mathcal{A}$ and hence the monad $K \circ -$ is fully determined by the free structure $K \circ 1 = K$ on 1, along with its augmentation Γ, which in practice can be read off trivially.

SYDNEY CATEGORY SEMINAR ABSTRACTS

Date: 29 June 88
Time: 2:00
Place: University of Sydney
Speaker: G. M. Kelly

Title: On the notion of club: the abstract concept.

Abstract: A monad on **Cat** is a monoid in the monoidal category $\mathcal{B} = [\mathbf{Cat}, \mathbf{Cat}]$ of endofunctors of **Cat**. If S is such a monad, we have a monoidal structure on $\mathcal{B}/S$, a monoid in which is a monad T together with a strict monad-map $T \to S$; considering these is considering monads *relative to* S. However $\mathcal{B}/S$ like $\mathcal{B}$ is not locally small. We have a localization $\mathcal{B} \to \mathbf{Cat}$ sending T to $T1$, giving rise to a local factorization system $(\mathcal{E}, \mathcal{M})$ on $\mathcal{B}$. The reflective full subcategory $\mathcal{M}/S$ of $\mathcal{B}/S$ is locally small, being equivalent to $\mathbf{Cat}/S1$. The subcategory $\mathcal{M}/S$ of $\mathcal{B}/S$ is closed under the tensor product on $\mathcal{B}/S$ iff the multiplication and unit of S lie in $\mathcal{M}$ and S preserves certain very-special pullbacks; in which case we call S a *club*. A monoid T in $\mathcal{M}/S$ is a monad T on **Cat** along with a monad-map $T \to S$ that lies in $\mathcal{M}$; and such a T is itself a club. The monoidal structure on $\mathcal{M}/S$ gives one on the equivalent $\mathbf{Cat}/S1$, a monoid in which is a *club over* S. Taking for S the monads for strict symmetric monoidal categories, categories with strictly-associative finite coproducts, and the same with strictly-associative finite products, we re-find the $\mathbb{P}$, $\mathbb{S}$, $\mathbb{S}^{op}$

---

clubs of the preceding lecture.

SYDNEY CATEGORY SEMINAR ABSTRACTS

Date: 3/8/88
Time: 2-3
Place: University of Sydney
Speaker: Javad Tavakoli

Title: Semi Continuous objects in a Topos

Abstract:

Let $\mathcal{A}$ be a poset in a topos $\mathcal{E}$. The object $P\mathcal{A}$ of downward closed subobjects of $\mathcal{A}$ is complete [Carboni - Street]. We look at $P\mathcal{A}$ as the object of internal 2-Valued presheaves on $\mathcal{A}$, $P\mathcal{A} = [\mathcal{A}^{op}, \Omega]$. We define canonical topology and we consider the object of internal sheaves $\mathcal{A}_\ell \xhookrightarrow{i} P\mathcal{A}$ as the object of lower-semi continuous on $\mathcal{A}$. We prove $\mathcal{A}_\ell$ is complete, consequency i has a left exact left adjoint. If we look at $P\mathcal{A}$ as the free locale generated by $\mathcal{A}$, then in fact $\mathcal{A}_\ell$ is a sublocale of $P\mathcal{A}$. If we replace $\mathcal{A}$ by $\mathcal{A}^{op}$, we get $\mathcal{A}_u$, upper-semi continous on $\mathcal{A}$.

SYDNEY CATEGORY SEMINAR ABSTRACTS

Date: 3 August 1988
Time: 4:15 pm
Place: University of Sydney
Speaker: Ross Street

Title: An appreciation of Hilbert's fifteenth

Abstract: This problem concerns enumerative geometry: justify Schubert's "principle of special position" or "argument by continuity". Refer to Semple–Roth ("Algebraic Geometry", Oxford) or Steven Kleiman's article in the AMS Hilbert volume. I attempted to use this setting as a lead up to presenting a result (essentially classical, but hard to find) which I recently learned from André Joyal. Let $\mathrm{Flag}_{\underline{d}}(V)$ denote the space of flags

$$\{0\} = A_0 \subseteq A_1 \subseteq \cdots \subseteq A_r = V$$

of subspaces of V where $\underline{d} : d_1 + \cdots + d_r = n$ is a partition of n and $\dim(A_i / A_{i-1}) = d_i$. Then $GL(V)$ acts on $\mathrm{Flag}_{\underline{d}}(V) \times \mathrm{Flag}_{\underline{e}}(V)$ is an obvious way and the result is that the orbits for this action can be identified with matrices (a_{ij}) of natural numbers satisfying $\sum_i a_{ij} = d_j$, $\sum_j a_{ij} = \delta_i$. Words like

---

Schubert cell, Bruhat order, Borel subgroup, Chern class, Jordan-Hölder Theorem have a concrete meaning in this setting.

Kind regards,

Ross.

SYDNEY CATEGORY SEMINAR ABSTRACTS

Date: 17 AUGUST 1988
Time: 2PM
Place: Macquarie University
Speaker: RFC. WALTERS

Title: DIGITAL CIRCUITS.

Abstract: A description was given of a digital circuit as a multigraph morphism to Sets, and the dynamics of the circuit as a morphism in an associated topos, $Sets^C$, from an object T (time) to E, the state space & actions of the circuit. The category C combines the active & passive dynamics. Different choices of T give rise to synchronous and asynchronous dynamics.

SYDNEY CATEGORY SEMINAR ABSTRACTS

Date: 17 AUGUST 1988
Time: 3PM
Place: Macquarie University
Speaker: TERRY BISSON

Title: INTENSIVE & EXTENSIVE QUANTITIES.

Abstract: Terry described work with Bob Walters in which they are looking at FULTON & MACPHERSON: American Math Soc Memoirs 243 & JOYAL: Homology of the symmetric groups, & attempting to find simple models. Terry described the intensive, extensive pair $\mathbb{C}/X$, $\mathbb{C}/X$ on a distributive category $\mathbb{C}$, together with the bivariate theory Span.

SYDNEY CATEGORY SEMINAR ABSTRACTS

Date: 17 August 1988
Time: 4 pm
Place: Macquarie University
Speaker: Ross Street

Title: Planar diagrams

Abstract: An attempt was made to find the most primitive connexion between monoidal categories with structure and planar topology. There is a close connection with Walters' circuit diagrams.

SYDNEY CATEGORY SEMINAR ABSTRACTS

Date: 24/8/88 and 21/9/88
Time: 3:30 - 4:30 4:30-5.00
Place: Macquarie University / University of Sydney
Speaker: Javad Tavakoli

Title: Semi Continuous Natural Numbers in a Topos

Abstract:

In the last talk, we introduce $\mathcal{A}_l$ and $\mathcal{A}_u$, lower and upper semicontinuous objects in a topos $\mathcal{E}$ for given poset $\mathcal{A}$.

If we let $\mathcal{A}$ to be natural numbers N in $\mathcal{E}$, then we prove that, there is addition and multiplication on PN, such that the yoneda functor $N \xrightarrow{y=\downarrow} PN$ preserves them.

Using left adjoint $j : PN \longrightarrow N_l$, we can define addition and multiplication on N_l and we prove that these operations are associative. We will use N_l and N_u to define lower (upper) semi dimension for a vector space in $\mathcal{E}$. We deduce a large number of applications.

SYDNEY CATEGORY SEMINAR ABSTRACTS

Date: Wed. 7 Sept. 1988
Time: 2pm.
Place: Macquarie University
Speaker: G. M. Kelly

Title: Recent joint work with Wood and Carboni, I.

Abstract: We define a 3-category $\mathcal{W}$: an object $\mathcal{A}$ is an $\underline{\mathrm{Ord}}$-enriched category together with a subcategory $\mathcal{A}_f$ of its arrows, each arrow g in $\mathcal{A}_f$ having a right adjoint g^* in $\mathcal{A}$; a morphism $T: \mathcal{A} \to \mathcal{B}$ is a lax functor with $T(\mathcal{A}_f) \subset \mathcal{B}_f$ and $Tg \dashv Tg^*$ for g in $\mathcal{A}_f$; a 2-cell $\alpha: T \to S: \mathcal{A} \to \mathcal{B}$ is a family $(\alpha_A : TA \to SA)$ in $\mathcal{B}_f$ with $\alpha_B . T\phi \le S\phi . \alpha_A$ for all $\phi \in \mathcal{A}(A,B)$; a 3-cell is just $\alpha \le \beta$, meaning $\alpha_A \le \beta_A$ for each A.

Adjunctions in $\mathcal{W}$ (seen as a 2-category) have the following properties. If $S \dashv T$ then (i) S is a 2-functor, (ii) there is a canonical "local adjunction" in $\underline{\mathrm{Ord}}$ between $\mathcal{A}(SB,A)$ and $\mathcal{B}(B,TA)$; (iii) each $T_{AB}: \mathcal{A}(A,B) \to \mathcal{B}(TA,TB)$ has a left adjoint in $\underline{\mathrm{Ord}}$; (iv) each $S_{CD}: \mathcal{B}(C,D) \to \mathcal{A}(SC,SD)$ has a right adjoint in $\underline{\mathrm{Ord}}$. An $S: \mathcal{B} \to \mathcal{A}$ in $\mathcal{W}$ has a right adjoint if & only if (a) S is a 2-functor, (b) the restriction $S_f: \mathcal{B}_f \to \mathcal{A}_f$ of S has a right adjoint, (c) each S_{CD} has a right adjoint, and (d) a certain relation holds between the right adjoint of S_{CD} and the counit of the adjunction in (c).

$\mathcal{W}$ has finite products. An object $\mathcal{A}$ is <u>cartesian</u> if the diagonal $\mathcal{A} \to \mathcal{A} \times \mathcal{A}$ and the unique $\mathcal{A} \to 1$ have right adjoints, $\otimes$ and I respectively. This happens precisely when $\mathcal{A}_f$ has finite products, each $\mathcal{A}(A,B)$ has finite infima, and the technical condition of (d) above is satisfied. It is automatic that $\otimes$ is associative with I as a unit and that Mac Lane's coherence conditions are satisfied; however $\otimes$ may be only a lax functor. When $\otimes$ is a 2-functor and $\mathcal{A}_f$ consists of <u>all</u> the g with right adjoints, we recapture the <u>cartesian bicategories</u> of Carboni and Walters. A morphism $T: \mathcal{A} \to \mathcal{B}$ between cartesian objects is <u>cartesian</u> if the canonical $TA \otimes TB \to T(A \otimes B)$ and $I \to TI$ are invertible.

SYDNEY CATEGORY SEMINAR ABSTRACTS

Date: 7 September 1988
Time: 4 pm
Place: Macquarie University
Speaker: Ross Street

Title: Quantum groups I

Abstract: This and the following talk were basically a report on five lectures I attended of Yu Manin at the University of Montreal (July 1988). I described the quantum version of GL(2). This involves matrices $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ in non-commutative elements subject to the relations

$$\begin{cases} ab = q^{-1}ba, \quad bd = q^{-1}db, \quad ac = q^{-1}ca, \quad cd = q^{-1}dc, \\ bc = cb \\ ad - da = (q^{-1} - q)bc. \end{cases}$$

A theorem of Kobyzev gives a "natural" reason for these relations in terms of geometric q-planes.

SYDNEY CATEGORY SEMINAR ABSTRACTS

Date: 14 Sept 1988
Time: 2:10 pm
Place: [illegible] University of Sydney
Speaker: Ross Street
Title: Quantum groups II

Abstract: Following on the talk from last week, I discussed Hopf algebras which are neither commutative nor cocommutative. The quantum $GL(2)$ of last week gives an example: $H = k\langle a, b, c, d\rangle / (\text{the quadratic relations})$. Yang-Baxter operators were introduced as an abstraction of the twist map $H \otimes H \longrightarrow H \otimes H$. I implied some connection with my work with Joyal to be explained in more detail later.

SYDNEY CATEGORY SEMINAR ABSTRACTS

Date: Wed. 14 Sept. 1988
Time: 3 p.m.
Place: University of Sydney
Speaker: G. M. Kelly

Title: Recent joint work with Wood & Carboni, II.

Abstract: See the lecture of 7 Sept. 88. Write $\mathcal{W}'$ for the sub-3-category of $\mathcal{W}$ whose morphisms are 2-functors, not just lax functors. Reg_ℓ is the 2-category of regular categories and left exact functors, and Reg is the sub-2-category given by the _regular_ functors — those that also preserve strong epimorphisms. There is a 2-functor $Rel : Reg_\ell \to \mathcal{W}$ sending $\mathcal{E}$ to $Rel\,\mathcal{E}$, where $(Rel\,\mathcal{E})_f = \mathcal{E}$; it restricts to $Rel : Reg \to \mathcal{W}'$; these 2-functors preserve products, and so — since $\mathcal{E}$ is cartesian in Reg — $Rel\,\mathcal{E}$ is cartesian not only in $\mathcal{W}$ but in $\mathcal{W}'$: so that here, the $\otimes$ of last week's lecture is a 2-functor. The _Ord_-categories of the form $Rel\,\mathcal{E}$ have been characterized by Carboni & Walters: they are the cartesian ones with $\otimes$ a 2-functor in which every object is _discrete_ and comonads admit Eilenberg-Moore objects. We now show that $T : Rel\,\mathcal{E} \to Rel\,\mathcal{F}$ is $Rel\,L$ for some $L : \mathcal{E} \to \mathcal{F}$ in Reg_ℓ if & only if T is cartesian and preserves the Eilenberg-Moore objects of comonads.

This extends to _geometric morphisms_, namely adjunctions $L \dashv M$ in Reg_ℓ; the conditions above are automatic for M, but must still be imposed for L.

There is another 2-functor $Idl : Reg_\ell \to \mathcal{W}$, $Reg \to \mathcal{W}'$, sending $\mathcal{E}$ to pre-ordered objects in $\mathcal{E}$ and _ideals_; those relations which respect the pre-order. Carboni-Walters

---

have characterized the _Ord_-categories $Idl\,\mathcal{E}$; we now characterize, much as above, the morphisms $Idl\,\mathcal{E} \to Idl\,\mathcal{F}$ in $\mathcal{W}$ of the form $Idl\,L$, at least when $\mathcal{E}, \mathcal{F}$ are _exact_. The point is that $Idl\,L$ restricts to the _discrete_ objects to give $Rel\,L$.

We hope to generalize all of the above to $Prof\,\mathcal{E}$ (profunctors) in place of $Idl\,\mathcal{E}$; this involves passing from _Ord_-enriched categories to general 2-categories.

SYDNEY CATEGORY SEMINAR ABSTRACTS

Date: 21st September 1988
Time: 2pm
Place: Macquarie University
Speaker: RFC. Walters.

Title: A remark on context free languages.

Abstract: A *regular grammar* is a morphism of reflexive graphs $\varphi: G \longrightarrow H$ where H is the graph with one object and non-identity arrows are the elements of the alphabet A. Consider the induced functor $\mathrm{Free}\,\varphi : \mathrm{Free}\,G \longrightarrow \mathrm{Free}\,H = A^*$. If I, J are two objects in G the image of $\mathrm{Free}\,G(I,J)$ under $\mathrm{Free}\,\varphi$ is a *regular language*.

A *context free grammar* is a morphism of multigraphs $\varphi: G \longrightarrow H$ where H is the generating multigraph for the theory of monoids with an alphabet A of additional constants (ie H has one arrow $M^n \to M$ for each n, together with an alphabet of constants $1 \to M$). Consider the product preserving functor $\mathrm{Free}_\times G \to \mathrm{Free}_\times G \longrightarrow$ Thy of monoids with constants. An arrow from 1 to S in $\mathrm{Free}_\times G$ is a *parsed expression of type S*. The image of f under the functor is the actual expression as an element of the free monoid on the alphabet. A set of expressions so obtained is a *context free language*.

Examples were given.

SYDNEY CATEGORY SEMINAR ABSTRACTS

Date: Wed. 21 Sept. 1988
Time: 2 pm.
Place: Macquarie University / ~~University of Sydney~~
Speaker: G.M. Kelly

Title: Recent work with Lawvere on essential localizations, I

Abstract: A localization of a category $\mathcal{A}$ is a full replete reflective subcategory $\mathcal{C}$ such that the reflexion $s : \mathcal{A} \to \mathcal{C}$ is left exact; the localization is essential if s has a left adjoint k; then the full replete image $\bar{\mathcal{C}}$ of k is an essential localization of $\mathcal{A}^{op}$. $\bar{\mathcal{C}}$ depends only on $\mathcal{C}$, not the choices of s and k; it is the class of objects co-orthogonal to the class $\mathcal{E}$ of maps orthogonal to $\mathcal{C}$ (which is also the class of maps inverted by s). So $\mathcal{C} \mapsto \bar{\mathcal{C}}$ gives a bijection between the ordered set $\mathrm{Ess}\,\mathcal{A}$ of essential localizations of $\mathcal{A}$, and $\mathrm{Ess}\,\mathcal{A}^{op}$. Of course $\mathrm{Ess}\,\mathcal{A}$ is an ordered subset of the ordered set $\mathrm{Loc}\,\mathcal{A}$ of all localizations of $\mathcal{A}$. We always suppose $\mathcal{A}$ locally small.

Theorem If $\mathcal{A}$ is complete with a strong cogenerating set, or equally if $\mathcal{A}^{op}$ is so, $\mathrm{Ess}\,\mathcal{A}$ is a small complete lattice with suprema formed as in $\mathrm{Loc}\,\mathcal{A}$.

Proof We refer to [B.-K.] = [Borceux-Kelly, On locales of localizations, JPAA 46(1987), 1-34]. Since, by the SAFT, the $\mathcal{A}$ above is also cocomplete, $\mathrm{Loc}\,\mathcal{A}^{op}$ is small by [B.-K. Thm. 6.4]; so $\mathrm{Ess}\,\mathcal{A} \cong \mathrm{Ess}\,\mathcal{A}^{op}$ is small. By [B.-K. Thm 3.1], $\mathrm{Loc}\,\mathcal{A}$ has small suprema, the $\mathcal{E}$ corresponding to $\mathcal{C} = \sup \mathcal{C}_\alpha$ being $\bigcap \mathcal{E}_\alpha$. When each $\mathcal{C}_\alpha$ is essential, each $\mathcal{E}_\alpha$ is closed under products; hence $\mathcal{E}$ is so, whence it follows that $s : \mathcal{A} \to \mathcal{C}$ preserves all small limits. Since $\mathcal{A}$ has a cogenerating set, s has a left adjoint, and $\mathcal{C}$ is indeed essential.

SYDNEY CATEGORY SEMINAR ABSTRACTS

Date: 21 September 1988
Time: 4:30 pm
Place: Macquarie University
Speaker: Ross Street

Title: Tensor transformations and duality.

Abstract: I announced the completion of a draft of work with André Joyal on tensor categories, planar graphs, and duality. This can be seen as a contribution to the algebra of tensor products by providing diagrams for computations, but just as importantly, as providing the algebra for a certain geometry of planar diagrams.

Since my talk was at the end of the day, I decided to present one purely categorical result which I discovered (probably rediscovered) on the weekend.

Proposition. *Suppose* $\alpha : F \to G$ *is a tensor transformation between tensor functors* $F, G : \mathcal{V} \to \mathcal{W}$. *If* $A \dashv B$ *in* $\mathcal{V}$ *then* α_A *is invertible*.

Corollary. *If* $\mathcal{V}$ *is an autonomous tensor category then* $\mathrm{Ten}(\mathcal{V}, \mathcal{W})$ *is a groupoid*.

SYDNEY CATEGORY SEMINAR ABSTRACTS

Date: Wed 28 Sept. 1988
Time: 2 pm
Place: ~~University of Sydney~~
Speaker: G. M. Kelly

Title: Recent work with Lawvere on essential localizations, II.

Abstract: This is a sequel to my lecture of 21 Sept. A category $\mathcal{A}$ is highly-presentable (h.p.) if it is cocomplete with a strongly generating set and has finite limits that commute with filtered colimits. It is a result of Day and Street [JPAA, Localisation of locally presentable categories, to appear] that the h.p. categories are exactly the localizations of the locally finitely presentable categories. They include in particular l.f.p. categories, presheaf categories, and sheaf categories. For an h.p. $\mathcal{A}$ it is shown in [B.-K.] that $\mathrm{Loc}\,\mathcal{A}$ is a small complete anti-Heyting-algebra, with inf b_α in $\mathrm{Loc}\,\mathcal{A}$ being the intersection $\bigcap b_\alpha$.

We now show that, unlike suprema in $\mathrm{Ess}\,\mathcal{A}$, infima (even binary ones) in $\mathrm{Ess}\,\mathcal{A}$ are not formed as in $\mathrm{Loc}\,\mathcal{A}$ by taking intersections, even when $\mathcal{A}$ is a presheaf category. We write $\mathcal{S}$ for the category of sets, and use bold-face for small categories.

If $\mathbb{C}$ is a full subcategory of $\mathbb{A}$ with inclusion i, the induced $i^*: \mathcal{S}^{\mathbb{A}} \to \mathcal{S}^{\mathbb{C}}$ has left and right Kan adjoints $i_!$ and i_*, each fully faithful. Hence the full replete image $\mathcal{C}$ of i_* is an essential localization of $\mathcal{A} = \mathcal{S}^{\mathbb{A}}$. Take for $\mathbb{A}$ the free category on the graph $0 \underset{g}{\overset{f}{\rightleftarrows}} 1$, with $\mathbb{C}$ and $\mathbb{D}$ its full subcategories $\{0\}$ and $\{1\}$. An object of $\mathcal{S}^{\mathbb{A}}$ is given by sets X and Y and functions $\phi: X \to Y$ and $\psi: Y \to X$. The essential localization $\mathcal{C}$ above consists of those objects with ψ invertible; similarly the essential localization $\mathcal{D}$ corresponding to $\mathbb{D}$ consists of those with ϕ invertible.

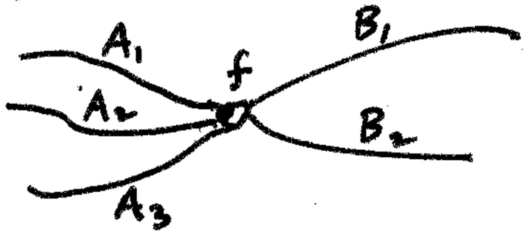
So $\mathcal{E} = \mathcal{C} \cap \mathcal{D}$, which by [B.-K.] is a localization, consists of those objects with both ϕ and ψ invertible. If we write $\mathbb{E}$ for the category generated by $0 \underset{v}{\overset{u}{\rightleftarrows}} 1$ subject to u and v being invertible, we have a projection $p: \mathbb{A} \to \mathbb{E}$, and the inclusion $\mathcal{E} \to \mathcal{A}$ is in effect $p^*: \mathcal{S}^{\mathbb{E}} \to \mathcal{S}^{\mathbb{A}}$, which has the left adjoint $q = \mathrm{Lan}_p$. We show in the next talk that $\mathcal{E}$ is not essential, by showing that q does not preserve infinite products.

SYDNEY CATEGORY SEMINAR ABSTRACTS

Date: 28 Sept 1988
Time: 4:30 pm
Place: University of Sydney
Speaker: Ross Street
Title: Tensor categories & planar diagrams

Abstract: (joint with André Joyal)

I gave the topological details of planar graphs and their deformations as needed for our work on tensor categories. For tensor categories (with no further properties or structure), special planar graphs, called recumbent, are required. A valuation v of a recumbent planar graph Γ is an assignment of morphisms to nodes and objects to edges such that locally we have



$$f : A_1 \otimes A_2 \otimes A_3 \longrightarrow B_1 \otimes B_2 .$$

Each Γ, v can be given a morphism $v(\Gamma)$ as value. The main theorem is that $v(\Gamma)$ is deformation invariant.

SYDNEY CATEGORY SEMINAR ABSTRACTS

Date: Wed. 5 Oct. 1988
Time: 2 p.m.
Place: ~~Macquarie University~~
Speaker: G. M. Kelly

Title: Recent work with Lawvere on essential localizations, III.

Abstract: See my talks of 21 & 28 Sept. We are to show that the reflexion of $\mathcal{A} = \mathcal{S}^{\mathbb{A}}$ into $\mathcal{E} = \mathcal{S}^{\mathbb{E}}$ does not preserve infinite products. Every object in $\mathcal{E}$ is isomorphic to one of the form $X \underset{1}{\overset{\phi}{\rightleftarrows}} X$ with ϕ invertible; these form a full but not replete subcategory $\mathcal{Z}$ of $\mathcal{E}$ isomorphic to $\mathcal{S}^{\mathbb{Z}}$. We can identify $\mathcal{S}^{\mathbb{N}}$ with the full subcategory $\mathcal{N}$ of $\mathcal{A}$ given by the objects $X \underset{1}{\overset{\phi}{\rightleftarrows}} X$, where now ϕ is arbitrary. A map in $\mathcal{A}$ from an object of $\mathcal{N}$ to one of $\mathcal{Z}$ is in effect a map in $\mathcal{S}^{\mathbb{N}}$. So we are reduced to proving that the left adjoint q of the inclusion $\mathcal{S}^{\mathbb{Z}} \to \mathcal{S}^{\mathbb{N}}$ induced by $\mathbb{N} \hookrightarrow \mathbb{Z}$ does not preserve infinite products. We treat an object of $\mathcal{S}^{\mathbb{N}}$ as (X, ϕ) where ϕ is an endomorphism of X. The reflexion in $\mathcal{S}^{\mathbb{Z}}$ of $(\mathbb{N}, s)$ where s is successor is clearly the inclusion $i : (\mathbb{N}, s) \to (\mathbb{Z}, s)$ where again s is successor. However the inclusion $i^{\mathbb{N}} : (\mathbb{N}, s)^{\mathbb{N}} \to (\mathbb{Z}, s)^{\mathbb{N}}$ is not the reflexion of $(\mathbb{Z}, s)^{\mathbb{N}}$. For let W be the subset of $\mathbb{Z}^{\mathbb{N}}$ given by those sequences $x \in \mathbb{Z}^{\mathbb{N}}$ such that, for some $k \in \mathbb{N}$, $x_i \geq -k$ for each i; & let $t : W \to W$ be given by $(tx)_i = x_i + 1$. Then t is invertible, so that (W, t) is a proper subobject in $\mathcal{S}^{\mathbb{Z}}$ of $(\mathbb{Z}, s)^{\mathbb{N}} = (\mathbb{Z}^{\mathbb{N}}, s^{\mathbb{N}})$; and the inclusion $i^{\mathbb{N}} : (\mathbb{N}^{\mathbb{N}}, s^{\mathbb{N}}) \to (\mathbb{Z}^{\mathbb{N}}, s^{\mathbb{N}})$ factorizes through (W, t), so that it cannot be the reflexion of $(\mathbb{N}^{\mathbb{N}}, s^{\mathbb{N}})$ into $\mathcal{S}^{\mathbb{Z}}$.

SYDNEY CATEGORY SEMINAR ABSTRACTS

Date: 5 October 1988
Time: 4:15 pm
Place: Macquarie University
Speaker: Ross Street

Title: Free tensor categories. Autonomous tensor categories.
(with André Joyal)

Abstract: A tensor scheme $\mathbb{D}$ is a directed graph whose set of vertices is a free monoid. Write $\mathrm{mor}\,\mathbb{D}$ for the set of edges & $\mathrm{ob}\,\mathbb{D}$ for the set of indecomposible vertices. For each tensor category $\mathbb{V}$, there is a category $[\mathbb{D}, \mathbb{V}]$ of $\mathbb{D}$-diagrams in $\mathbb{V}$. The free tensor category $\mathcal{F}(\mathbb{D})$ on $\mathbb{D}$ satisfies an equivalence of categories $\mathrm{Ten}(\mathcal{F}(\mathbb{D}), \mathbb{V}) \simeq [\mathbb{D}, \mathbb{V}]$. A concrete description of $\mathcal{F}(\mathbb{D})$ was provided in terms of deformation classes of centred recumbent diagrams in $\mathbb{D}$.

An autonomous tensor category is one for which each object A has both a left and a right dual: $A^* \dashv A \dashv A^\vee$. An example is the category $\mathrm{Pr}_k(A)$ of A-modules which are finitely generated and projective as k-modules, where k is a commutative ring and A is a (non-comm, non-cocomm) Hopf algebra (with invertible antipode).

SYDNEY CATEGORY SEMINAR ABSTRACTS

Date: Wed. 12 Oct. 1988
Time: 4 p.m.
Place: University of Sydney
Speaker: G. M. Kelly

Title: Recent work with Lawvere on essential localizations IV

Abstract: See my talks of the last three weeks. We saw in the last talk that $\mathcal{B} \cap \mathcal{D}$, which is the meet of $\mathcal{B}$ and $\mathcal{D}$ in Loc $\mathcal{A}$, is not essential. We now consider the meet $\mathcal{B} \wedge \mathcal{D}$ in Ess $\mathcal{A}$, which we know to exist. We shall see in the forthcoming talks that there is an order-preserving bijection between essential localizations of $\mathcal{A} = \mathcal{S}^{A}$ and idempotent ideals of A. An ideal I of A is a subset of its morphisms such that $fgh \in I$ whenever $g \in I$; it is idempotent if every $g \in I$ has the form $g = uv$ where $u, v \in I$. {Reason: in an evident notation, this says that $I \subset I \circ I$, while trivially $I \circ I \subset I$.} When, as in my recent talks, $\mathcal{B}$ is the full replete image of $i_* : \mathcal{S}^{C} \to \mathcal{S}^{A}$ where $i : C \hookrightarrow A$ is the inclusion of a full subcategory, I consists of those morphisms of A which factorize through an object of C.

Turning to the particular A, C, D of the last two talks, let the idempotent ideals I and J correspond to $\mathcal{B}$ and $\mathcal{D}$, and the idempotent ideal K to $\mathcal{B} \wedge \mathcal{D}$. Then I consists of all maps except 1_1 and J of all maps except 1_0. Since $K \subset I \cap J$, it consists of non-empty words in $f : 0 \to 1$ and $g : 1 \to 0$. If K is not empty, it contains a word of minimum length, which contradicts its idempotence. So K is empty, and accordingly $\mathcal{B} \wedge \mathcal{D}$ is the smallest localization of $\mathcal{A}$, consisting of the terminal object 1 of $\mathcal{A}$ and any isomorphs it may have.

Note that this provides an alternative — and simpler — proof that $\mathcal{B} \wedge \mathcal{D} \neq \mathcal{B} \cap \mathcal{D}$; although the proof in talk III that the reflexion onto $\mathcal{B} \cap \mathcal{D}$ does not preserve infinite products is still of sufficient interest to be included in the article.

SYDNEY CATEGORY SEMINAR ABSTRACTS

Date: Wed. 19 Oct. 88
Time: 2 p.m.
Place: Macquarie University
Speaker: G. M. Kelly

Title: Recent work with Lawvere on essential localizations V

Abstract: See the first four talks of this sequence, & in particular the last. Changing notation (the better to make comparison with classical notions) so that $\mathcal{A} = \mathcal{S}^{\mathbb{A}^{op}}$ we begin to study the bijection between essential localizations of $\mathcal{A}$ and idempotent ideals of $\mathbb{A}$. We identify $A \in \mathbb{A}$ with the representable $\mathbb{A}(-,A)$. If $\mathcal{E}$ is the class of maps in $\mathcal{A}$ inverted by the reflexion onto a localization $\mathcal{B}$, we call $\mathcal{D} = \mathcal{E} \cap \mathrm{Mono}$ the class of *dense monos*, write $\mathcal{J}$ for the class of maps in $\mathcal{D}$ with representable codomain, and $\mathcal{J}(A)$ for those with codomain A. It is a special case of a result of [B.-K.] that $\mathcal{B} = \mathcal{E}^{\perp} = \mathcal{D}^{\perp} = \mathcal{J}^{\perp}$. Localizations $\mathcal{B}$ of $\mathcal{A}$ are in order-preserving bijection with the classes $\mathcal{J}$ that arise thus, which are called *Grothendieck topologies*. It is classical that a class $\mathcal{J} = \Sigma_{A \in \mathbb{A}} \mathcal{J}(A)$ is a Grothendieck topology iff

GT1. $1_A : A \to A$ is in $\mathcal{J}(A)$

GT2. If $r : R \rightarrowtail A$ is in $\mathcal{J}(A)$ and $f : B \to A$ in $\mathbb{A}$ then $f^* r \in \mathcal{J}(B)$.

GT3. If $r : R \rightarrowtail A$ is in $\mathcal{J}(A)$ and $s : S \rightarrowtail A$ is such that for every $f : B \to R$ with $B \in \mathbb{A}$, $(rf)^* s \in \mathcal{J}(B)$, then $s \in \mathcal{J}(A)$.

If the localization is *essential*, $\mathcal{J}(A)$ is clearly closed under *arbitrary* intersections, & so has a least element $i_A : I_A \rightarrowtail A$; then $\mathcal{J}(A)$ consists of those $r : R \rightarrowtail A$ with $R \supseteq I_A$. Conversely, if each $\mathcal{J}(A)$ has a least element I_A, $\mathcal{J}(A)$ *is* closed under all intersections. Then the corresponding localization is indeed essential.

First, by [B.-K.], $f : F \rightarrowtail G$ is in $\mathcal{D}$ iff, for each $x : A \to G$ with $A \in \mathbb{A}$, $x^* f \in \mathcal{J}(A)$. If $f = \Pi f_i : \Pi F_i \to \Pi G_i$, $x^* f = \bigcap x_i^* f_i \in \mathcal{J}(A)$, so $\mathcal{D}$ is closed under products. Then $\mathcal{E}$ is closed under products by [B.-K. Prop. 2.4]. So the reflexion $s : \mathcal{A} \to \mathcal{B}$ preserves all limits. Since both $\mathcal{A}$ & $\mathcal{B}$ are locally presentable, s has a left adjoint by [Gabriel-Ulmer, LNM 221, Satz 14.6].

SYDNEY CATEGORY SEMINAR ABSTRACTS

Date: 19 October 1988
Time: 2 PM
Place: Macquarie University
Speaker: R.F.C. Walters

Title: The free category with products on a multigraph.

Abstract: Many considerations of free categories with structure have lacked either (i) a correct 2-categorical framework or (ii) an analysis of the kind of data on which free categories are constructed. It is urgent to get these matters cleared up, because of applications to computer science.

The example of the title fits into the following setting. Let Cat_{str} be a 2-category (or groupoid-enriched category of categories with structure). There is a topos $\mathbb{E}$ (in this case multigraphs) and a forgetful functor

$$Cat_{str} \xrightarrow{\mathcal{U}} Cat(\mathbb{E}).$$

The free structure on an object X of $\mathbb{E}$, is an object $\mathcal{F}X$ of Cat_{str} and an arrow $\eta : X \longrightarrow \mathcal{U}\mathcal{F}X$ in $Cat(\mathbb{E})$ which induces by composition an equivalence of categories

$$Cat_{str}(\mathcal{F}X, C) \xrightarrow{\simeq} Cat(\mathbb{E})(X, \mathcal{U}C),$$

(for each $C \in Cat_{str}$).

SYDNEY CATEGORY SEMINAR ABSTRACTS

Date: 19 October 1988
Time: 4:15 pm
Place: Macquarie University
Speaker: Ross Street

Title: Polarised diagrams and free autonomous tensor categories

---

Abstract: Autonomous tensor schemes were defined and the universal property of free autonomous tensor categories described. An explicit construction was given in terms of polarised diagrams.

Ross Street.

SYDNEY CATEGORY SEMINAR ABSTRACTS

Date: 26 October 1988
Time: 3 pm
Place: University of Sydney
Speaker: Ross Street

Title: Irreducible representations of the symmetric groups.

Abstract: I described a construction of these which André Joyal told me last June in Montréal. Given a partition $\lambda = (\lambda_1 \geq \lambda_2 \geq \dots \geq \lambda_r)$ of n, let E_λ be the set of surjections $f : [1, n] \longrightarrow [1, r] = \{1, 2, \dots, r\}$ for which $\# f^{-1}(i) = \lambda_i$. Then we can define

$$\langle \, , \rangle : E_\lambda \times E_{\lambda^t} \longrightarrow \{-1, 0, 1\} \subseteq \mathbb{C}$$

which preserves the obvious actions of $\mathfrak{S}_n$ (where $z\sigma = \mathrm{sgn}(\sigma) z$ for $\sigma \in \mathfrak{S}_n$, $z \in \mathbb{C}$). This gives

$$E_\lambda \longrightarrow \mathbb{C}^{E_{\lambda^t}}$$

mapping the $\mathfrak{S}_n$-set into the $\mathfrak{S}_n$-module. The subspace W_λ of $\mathbb{C}^{E_{\lambda^t}}$ spanned by the image of E_λ is the irreducible $\mathfrak{S}_n$-module sought. We get $W_{\lambda^t} \cong W_\lambda^*$.

$$\# E_\lambda = \frac{n!}{\lambda_1! \cdots \lambda_r!} \geq \dim W_\lambda = \frac{n!}{\left(\text{product of the hook lengths in Young diag. of } \lambda\right)}.$$

Ross Street

SYDNEY CATEGORY SEMINAR ABSTRACTS

Date: Wed. 26 Oct. 1988
Time: 4 pm.
Place: [illegible] University of Sydney
Speaker: G. M. Kelly

Title: Recent work with Lawvere on essential localizations VI

Abstract: See the preceding talks; we use the notation of the last. To give for each $A \in \mathbb{A}$ an $i_A : I_A \rightarrowtail A$ is to give for each $A, B \in \mathbb{A}$ a subset $I(B,A) = I_A(B)$ of $\mathbb{A}(B,A)$ which is a *right ideal*: if $g \in I(B,A)$ and $h \in \mathbb{A}(C,B)$ then $gh \in I(C,A)$. Given such I_A we *define* $\mathcal{J}(A)$ to consist of those $r : R \rightarrowtail A$ with $R \geq I_A$, and ask whether $\mathcal{J}$ satisfies GT1 – GT3 of the last talk. GT1 is trivially satisfied. GT2 reduces at once to the commutativity, for some t, of

$$\begin{tikzcd} I_B \arrow[r] \arrow[d, "t"'] & B \arrow[d, "f"] \\ I_A \arrow[r] & A \end{tikzcd}$$

; that is, that I be a subfunctor of the Yoneda embedding $Y : \mathbb{A} \to A$. Equivalently, I is an *ideal* of $\mathbb{A}$: for $g \in I(C,B)$ and $f : B \to A$ we have $fg \in I(C,A)$. It takes a little work to see that GT3 is equivalent to the idempotence of I. We first observe that it suffices to demand GT3 when r is $i_A : I_A \rightarrowtail A$. Then idempotence of I easily implies GT3, and the converse follows upon taking $s : S \rightarrowtail A$ to consist of those uv where $u, v \in I$.

It is easy to see that, when $\mathbb{C} \subset \mathbb{A}$ and $\mathcal{C}$ is the repletion of $\mathcal{S}^{\mathbb{C}^{op}} \to \mathcal{S}^{\mathbb{A}^{op}}$, I consists of those maps in $\mathbb{A}$ which factorize through an object of $\mathbb{C}$.

SYDNEY CATEGORY SEMINAR ABSTRACTS

Date: 2 November 1988
Time: 4 pm
Place: Macquarie University
Speaker: Ross Street

Title: Crossed modules of algebroids.

Abstract: An algebroid A is a small category enriched in R-modules (R comm ring). A crossed module of algebroids is an AA-bimodule M together with a bimodule homomorphism $\varepsilon : M \Longrightarrow A$. This simplifies certain approaches to their study.

Ross Street.
