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# THE DYNAMICS OF COLLIDING AND OSCILLATING DROPS*

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## INTRODUCTION

In order to study the oscillations, fusions, and fissions of liquid drops, we are developing a hydrodynamic computer code which simulates these processes for viscous, incompressible drops endowed with a body charge and surface tension. Although our interest lies in the field of nuclear hydrodynamics, the application of this code to classical liquid drops is general. Our code traces the dynamic evolution of an axially symmetric system as a function of time. It utilizes a free surface that requires no parameterization and its velocity field can be rotational or irrotational as required. When completed, the present version will yield the following information about the classical dynamics of liquid drops: (a) free surface shape and energy, (b) free-flow velocity field, (c) kinetic energy distribution, (d) Coulomb energy, (e) free-flow moment of inertia, and (f) free-flow electric quadrupole moment. These quantities can be studied as a function of viscosity, initial shape, and energy.

We present here some studies of the oscillations of charged and neutral drops as a function of initial shape, charge, and viscosity. These studies are not restricted to small amplitudes. We also present some preliminary simulations of fissions and fusions of viscous charged drops, with some comments about the possible role of nuclear viscosity in the creation of "superheavy" elements in heavy ion accelerator reactions.

## NUCLEAR HYDRODYNAMICS

The general problem of liquid drop dynamics, important as it is to many scientific investigations, has never been fully solved due to the complexity of the hydrodynamic equations involved. Even the linear problem of viscous small amplitude oscillations is described by complex Bessel functions (1). Large amplitude oscillations and distortions, including the fission and fusion of liquid drops, cannot be described analytically in complete detail.

In the particular field of nuclear hydrodynamics, it has become of critical importance to be able to follow in some way the dynamic path of those large amplitude motions leading to nuclear fusion and fission. Some studies of this problem using parameterized surfaces and irrotational flow have been carried out by Sierk and Nix (2). Other efforts have concentrated on solutions of Hamilton's equations using parameterized shapes (3). Disruptive processes such as fusion have not been studied dynamically because they do not lend themselves easily to shape parameterization.

While the atomic nucleus usually displays a quantum mechanical nature, the nuclei of the heavier elements, which contain hundreds of nucleons,

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begin to show properties that are characteristic of classical fluid flow. The best example of this is the success of the celebrated liquid drop model of nuclear fission proposed by Bohr and Wheeler (4). Another example is that the moment of inertia of rotating nuclei lies somewhere between that of a rigid body and of a drop with irrotational flow. At low temperatures the nuclear liquid can become a superfluid and a classical description is probably not meaningful. However most accelerator reactions between nuclei take place at elevated temperatures where classical behavior, if not well established, becomes a reasonable assumption, at least for the heavier nuclei. In Figure 1 we have attempted to show where the physics of heavy nuclei fits in the transition from the classical to the quantum mechanical regimes. The transition point is taken to be at the temperature where the de Broglie wavelength equals the inter-nucleon spacing.

In the spirit of this classical assumption we have undertaken to simulate by computer methods the dynamics of colliding and oscillating viscous liquid drops in a very general way. The only previous attempt at such simulations was performed by Hill and Wheeler (5) for inviscid, irrotational fissions at a time when computers were not fast enough to handle the general problem. The overriding interest today is in the problem of fusing two charged liquid drops to make a large composite drop. This is the basic way in which the new transuranic elements are being produced: two nuclei are made to collide in an accelerator and the fused compound system becomes the new element.

Recent theoretical predictions by Nilsson and others (6) of the possibility of using fusion reactions such as this to produce the so-called "superheavy" elements with atomic number around 114 have not been substantiated by experiments (7). We now suspect that this disparity may be due to a finite nuclear viscosity at elevated temperatures. The effect of this viscosity would be to convert the energy of the forward motion of the fusion process into frictional heating, making fusion impossible due to the repulsive Coulomb forces that are always ready to pull distorted shapes apart.

Therefore we are developing a computer code, called SQUISH, that we hope will have sufficient accuracy to predict the correct bulk motion of charged viscous liquid drops with axial symmetry. This program is quite general. It can be used for any liquid drops, nuclear or otherwise, that are subject to a calculable external force. The code is still in development and although we are presently studying fusions the results are still preliminary, so we present in this paper mostly some simulations of the dynamics of large-amplitude motions of single drops.

## THE COMPUTER CODE

Without going into details, we describe in Figure 2 how the code works. SQUISH is an example of the two-dimensional hydrodynamic finite-difference codes that are becoming very popular because of the increased capability of modern computers to handle them. SQUISH uses a basic hydrodynamic technique developed at Los Alamos by Harlow and Amsden (8) for low Reynolds number flow, the SMAC or Simplified Marker And Cell method. In this scheme, the drop is contained in a fixed Eulerian mesh and movable Lagrangian particles are injected

into the cells of the mesh. A special set of surface markers keeps track of the free surface. Typical full and surface cells are shown in the insets. The forces and pressures are calculated at the centers of the cells, and velocities are deposited on the cell boundaries. The computer is then instructed to move the particles according to Newton's laws, which in this case become the Navier-Stokes equation for a viscous and incompressible fluid. In our case the forces include surface tension and Coulomb forces. The method of Foote (9) has been used for the application of surface tension. The particles are moved by applying these equations in finite-difference form, and the whole system is adjusted to conserve volume according to $\nabla \cdot U = 0$. The program, after moving the particles, increments the time, recalculates the new forces and pressures, and proceeds in this cyclical manner to follow the dynamical paths of all the particles that make up the drop.

The advantages of this method are that it contains (a) a free surface that requires no parameterization, (b) a free-flow velocity calculation that can be rotational or irrotational, and (c) provisions for including any calculable external force, or any bulk property such as viscosity or elasticity. The limitations are that (a) the code must be axially symmetric if it is not to become extremely costly to run, (b) some accuracy must be surrendered by the choice of a reasonably large mesh size, and (c) the physics is purely classical, although it may be possible to simulate some quantum mechanical behavior in a pseudo-classical manner.

At present SQUISH is capable of calculating at any time step the following properties of a system of viscous charged liquid drops: (a) free surface shape and energy, (b) free-flow velocity field (rotational or irrotational), (c) kinetic energy distribution, (d) center of mass kinetic energy, position, and moments, (e) Coulomb energy, (f) free-flow moment of inertia, and (g) free-flow electric quadrupole moment. These quantities can be studied as a function of viscosity, initial shape, and initial energy. It should be pointed out that not all of these quantities will have analytical accuracy. The size of finite difference mesh employed sets a limit on the accuracy of the free surface shape, upon which most of these quantities depend. Computer simulations of this sort should be regarded as quasi-analytical experiments with definite errors associated with the variables. However the bulk motion should be correct, and the accuracy can be made as good as necessary, should the need merit the expense.

## SIMULATIONS OF DROP DYNAMICS

As an illustration of the versatility of such a code, let us examine the motion of a particular drop as we change its size, viscosity, and charge. In the following figures the units are scaled to nuclear dimensions, but the dynamic evolution shown applies to any classical viscous drop by virtue of dynamic similarity. In Table 1 we present a unit conversion table for the benefit of non-nuclear scientists. The figures in this paper represent smoothed composites of motion picture sequences generated by the computer.

We begin with a study of the uncharged drop. Wong and Tang (1) have discussed the analytical solutions to small-amplitude oscillations of such drops. We shall now study large-amplitude oscillations, for which there is no analytical theory. The sequence shown in Figure 3 simulates the oscillations of a drop with an initial shape given by the second-order Legendre polynomial, with the coefficient $a_2 = 0.4$. The viscosity is of a medium value that results in a damped oscillatory motion. The drop is started at rest; the initial excess surface energy is then exchanged for kinetic energy as the drop oscillates from a prolate spheroid to a sphere (with minimum surface energy and maximum kinetic energy) and then back to an oblate spheroid (with minimum kinetic energy and maximum surface energy). The drop simulations at the top are from the film strip; the large dots are the surface markers and the small interior dots are the cell particles. There are about a thousand particles in this drop. The total energy is seen to decrease exponentially as the damping proceeds. The period of oscillation, $15.35 \times 10^{-22}$ seconds, is slightly longer than the theoretical period for small-amplitude oscillations, which is $13.38 \times 10^{-22}$ seconds. However the general motion exhibits the behavior described by Lord Rayleigh (10) for linear oscillations many years ago, even though the ratio of amplitude to drop radius is not very small. This result has been observed experimentally many times.

The simulation in Figure 4 shows this same drop except that the viscosity has been increased by a factor of ten. Now the motion is overdamped and the drop slowly damps out to a sphere without any oscillatory motion.

In Figure 5 the size of the drop is changed so that it now has an initial Legendre coefficient given by $a_2 = 1.0$. This is a very large-amplitude motion which no longer preserves the $P_2$ shape. In the least square fit of the free surface shown on the left, a substantial $a_4$ component is seen to grow with the motion, and this is also very evident in the simulated drop sequences at the top of the figure. The energy changes are shown on the right; at the end of this simulation we have been left with a sphere with a great deal of kinetic energy which will probably proceed into some complex oblate spheroidal type of shape, still following the general Rayleigh behavior. The extrapolated period for this oscillation is about twice the corresponding Rayleigh period. The Weber number for this simulation, defined by $We = \sigma d u^2/\gamma = 12\, E_{kin}/E_{sur}$ (where $\sigma$ is the density, d the drop diameter, u an average velocity, and $\gamma$ the surface tension coefficient), is $We = 1.37$, and there is no sign of extreme disruption in the simulation.

The addition of a full body charge to the drop involves a substantial increase in computer running time, and this tends to result in a loss of accuracy due to the necessity of using a larger mesh size. In Figure 6 we show the smoothed energy curves for the same drop that we discussed in Figure 3 except that now it has a body charge of 62 proton charges. The charge density, like the mass density, is assumed to be constant throughout the drop. This particular charge represents a $^{152}Sm$ nucleus, which is known to be stable against fission even for fairly large distortions. In this simulation the surface forces still predominate the motion, but now the surface

energy is exchanged for Coulomb energy as well as kinetic energy, resulting in a longer period for the oscillation. The Coulomb energy and kinetic energy are at a maximum for the spherical shape, while the surface energy is at a minimum. Toward the end of the simulation the motion has damped out to small amplitude oscillations about a sphere. The accuracy of these calculations is about one percent.

The motion of this same drop was studied as more charge was added to the drop. In Figure 7 we plot the kinetic energy of the drop after a fixed time interval as a function of the atomic charge Z of the drop. For small charges the motion is oscillatory. As the charge is increased, the Coulomb forces oppose the surface tension more and more, slowing down the motion until the two forces just balance at the minimum in the curve. This minimum represents a saddle point between oscillation and fission; for charge densities beyond this point the Coulomb forces predominate and the drop begins to fission with no oscillatory behavior. The saddle point charge predicted by SQUISH is in agreement with the theoretical saddle point charge of 76 predicted for this shape by Cohen and Swiatecki (11). In Figure 8 is shown the same drop of Figure 7 with charge Z = 110. Here the Coulomb energy plunges down as the fission proceeds, and the surface energy and kinetic energy move up almost equally in the initial stages of the motion.

The dimensionless parameter that is used to represent the relative strengths of the Coulomb force and the surface tension is the fissility, defined by $x = E_{coul}/(2 \cdot E_{surf})$ . In Figure 9 is shown a comparison of two simulations of a drop that was initialized with a shape corresponding to an $x = 0.9$ saddle shape, but with enough charge to make $x = 1.56$. This highly charged drop was then expected to fission very rapidly. The drop at the right was given a high viscosity that overdamps oscillations at lower fissilities, and the drop at the left was given a lower viscosity that would lead to damped oscillations at lower fissilities. Instead of rapidly breaking apart into two drops, we find that the highly charged drop becomes very elongated and eventually develops a long, thin neck that will not pinch off until the neck has extended to virtually no width. This is an unsettling result, especially for nuclear physics, for such a thin neck would contain less than one nucleon and we could not expect to apply a classical treatment under such conditions. This is not entirely surprising because nuclear fission is usually a low-temperature phenomenon, unlike accelerator-induced fusion. However it means that classical treatments of the fission dynamics, and possibly the statics, may be misleading. One might hypothesize that in real charged drops, whether classical or quantum mechanical, the pinch-off is actually initiated by a surface fluctuation in the thin neck. This is substantiated to some degree by our knowledge that nuclear fission is not necessarily symmetric; one fission fragment often is larger than the other, which would be possible if the long neck predicted by SQUISH were to be broken off at a random place along its length at some time by a random surface fluctuation.

Experimental studies of charged drops would be very helpful in this respect, but it would be extremely difficult to charge them sufficiently to represent

nuclear liquid drops, which are held together by the strong nuclear forces despite their high charge density. In Figure 10, nonetheless, we show an approximation to a real charged drop. These profiles are from a film made by two nuclear physicists at LBL. They suspended a drop of water containing trichlorethylene in a bath of transformer oil of about the same density as the drop. Then they placed the drop between two capacitor plates and filmed its subsequent motion as a function of the applied voltage. The sequence shown here was at one of their higher voltages. These drops, of course, are not body-charged like a nucleus, but rather polarized. However we see in their development a pronounced elongation followed by asymmetric fission, which lends some credence to the computer simulations that we have presented. Further support is found in the experimental films of pendant drops presented at this conference by A. A. Kovitz.

Besides these simulations of the fission process, which require further study, we are examining the problem of fusion. This has required special programming to handle the cusp-like neck at the initial stages of the fusion. Our fusion results are still preliminary and will not be presented in detail at this time. In Figure 11 is shown, by way of example, a simulation of the fusion of two liquid spheres. The Weber number for this simulation was around 5.5, so surface tension effects are negligible compared to inertial effects, and large distortions are expected. The general features of this viscous fusion are that the necking process involves a large amount of viscous friction which hinders the formation of the neck and hence the fusion. The implication of this for the fusion of heavy nuclei is quite serious, if nuclear viscosity does indeed exist (and we do not yet know that it does), for it implies that at high accelerator velocities the viscosity may hinder the formation of compound nuclei, while at low velocities the Coulomb repulsion also hinders the fusion. Therefore there may be only a narrow range of energies for which fusion experiments can be performed in the heavy nuclei. We plan to explore many of these speculations in the near future.

ACKNOWLEDGEMENT

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Table I - Unit Conversion Table

| Quantity | Nuclear Unit | | SI Unit |
|---|---|---|---|
| Time | 1 dsec | = | $10^{-22}$ sec |
| Length | 1 fm | = | $10^{-15}$ m |
| Energy | 1 Mev | = | $1.602 \times 10^{-13}$ j |
| Velocity | 1 fm/dsec | = | $10^{7}$ m/sec |
| Kinematic viscosity* | 1 $fm^2$/dsec | = | $10^{-8}$ $m^2$/sec |
| Surface tension** | 1 Mev/$fm^2$ | = | $1.602 \times 10^{17}$ nt/m |

* The nuclear density is 0.13 amu/$fm^3$ = $1.66 \times 10^{18}$ Kg/$m^3$. Thus a nuclear kinematic viscosity of 1 $fm^2$/dsec means a viscosity of $1.66 \times 10^{8}$ poise.

** The nuclear surface tension is approximately 1 Mev/$fm^2$.

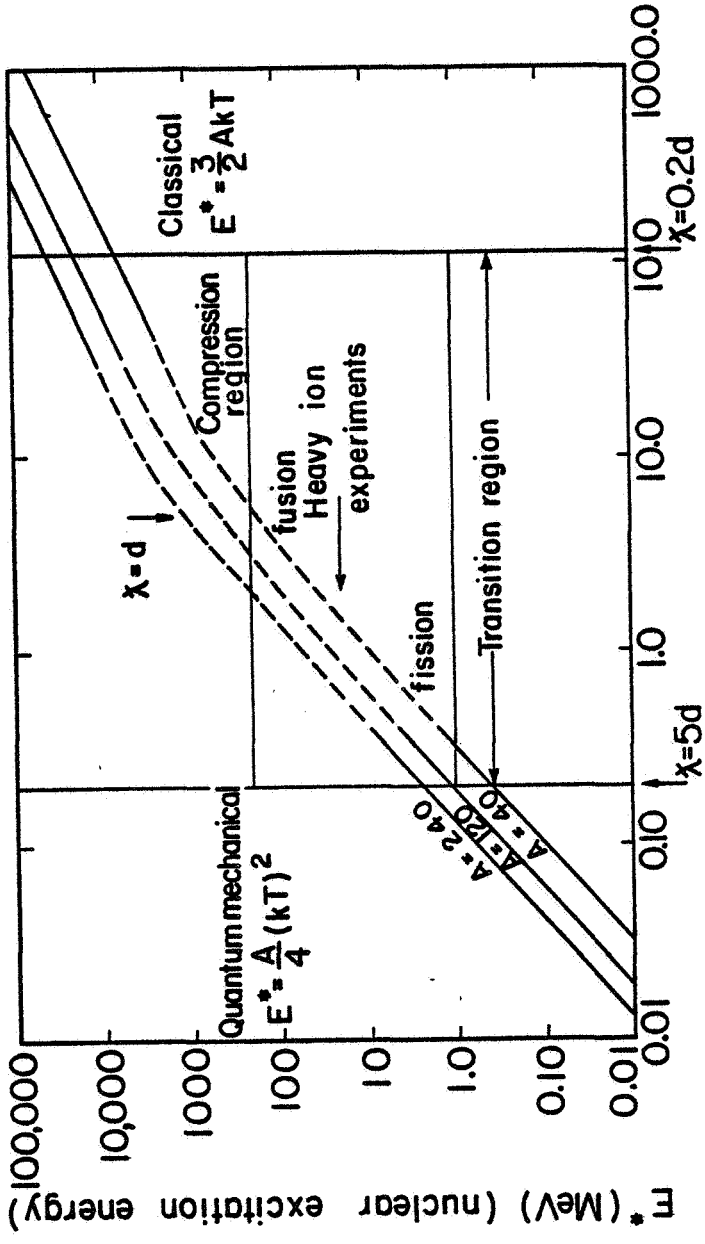
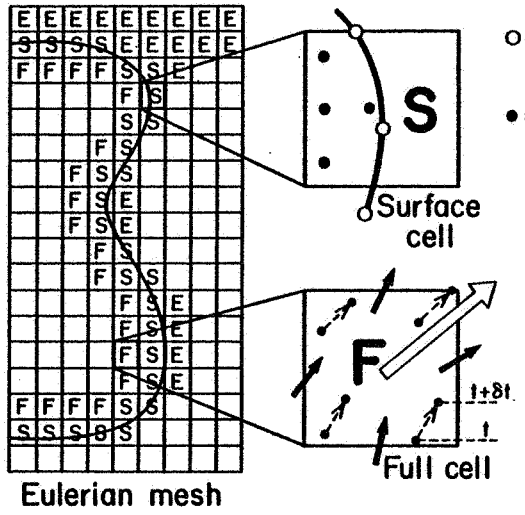



Figure 1. Transition from quantum mechanical to classical regime. Heavy ion accelerator experiments lie in the transition region where the de Broglie wavelength $\lambda$ is close to the inter-nucleon spacing d.

# SQUISH-HOW IT WORKS




o=Free surface marker

•=Lagrangion particle

Velocities stored on cell boundaries

Resultant force applied at cell center

1. Forces calculated from free-flow shape (surface tension and coulomb forces)
2. Velocities calculated from Navier-Stokes equation $\left(\frac{\delta}{\delta t}+\bar{u}\cdot\bar{\nabla}\right)\bar{u}=-\frac{1}{\rho}\bar{\nabla}P+\frac{\bar{F}}{m}+\frac{n}{\rho}\nabla^2\bar{u}$
3. Volume conserved from incompressibility $\bar{\nabla}\cdot\bar{u}=0$
4. Move cell particles and surface markers and advance time to $t+\delta t$

The SMAC method (Harlow and Amsden, Los Alamos)

Figure 2.

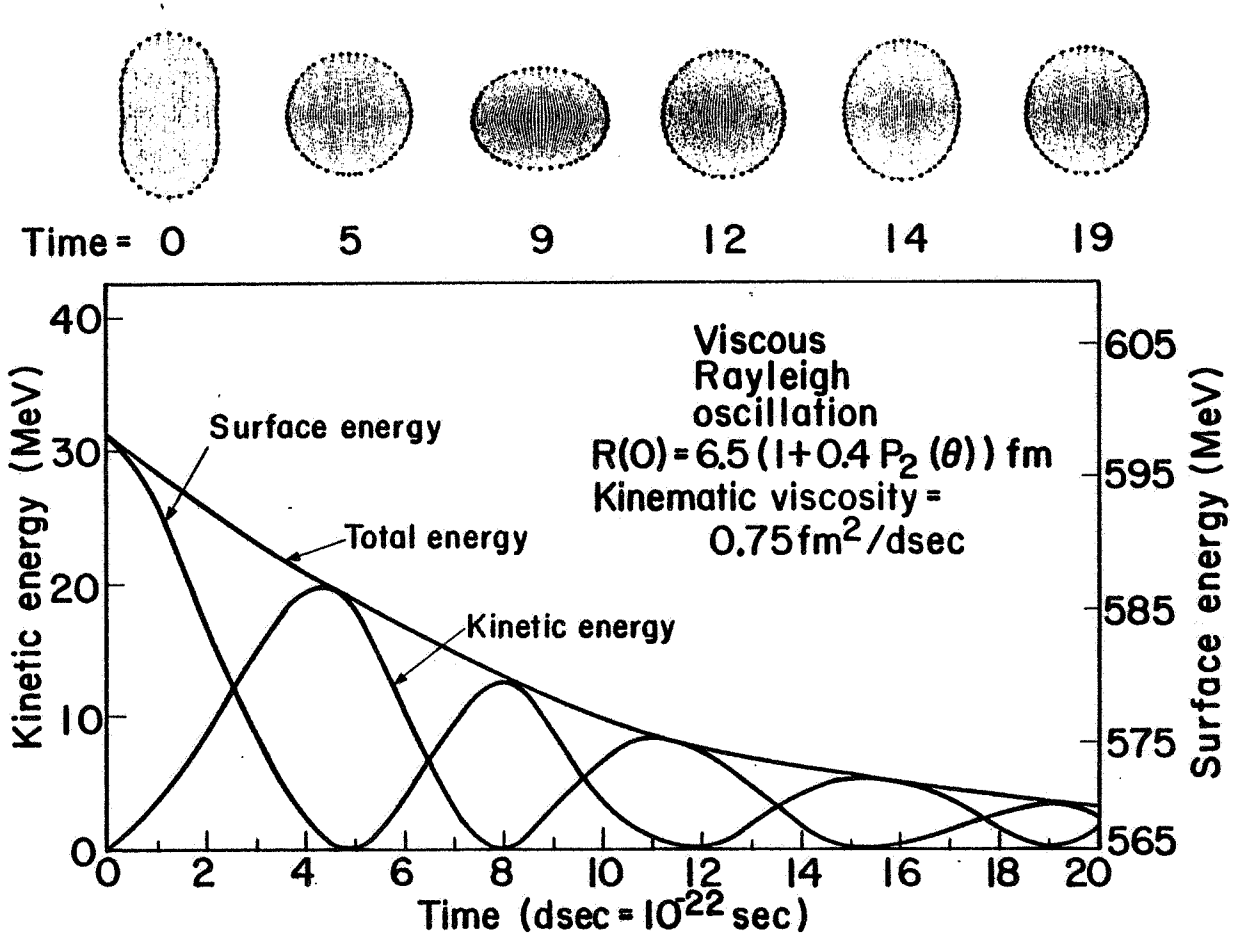

Time = 0
5
9
12
14
19
Viscous
Rayleigh
oscillation
R(0) = 6.5 (1+0.4 P2 (θ)) fm
Kinematic viscosity =
0.75 fm2/dsec
Surface energy
Total energy
Kinetic energy
Kinetic energy (MeV)
Surface energy (MeV)
Time (dsec = 10-22 sec)
0
10
20
30
40
565
575
585
595
605
0
2
4
6
8
10
12
14
16
18
20

Figure 3.

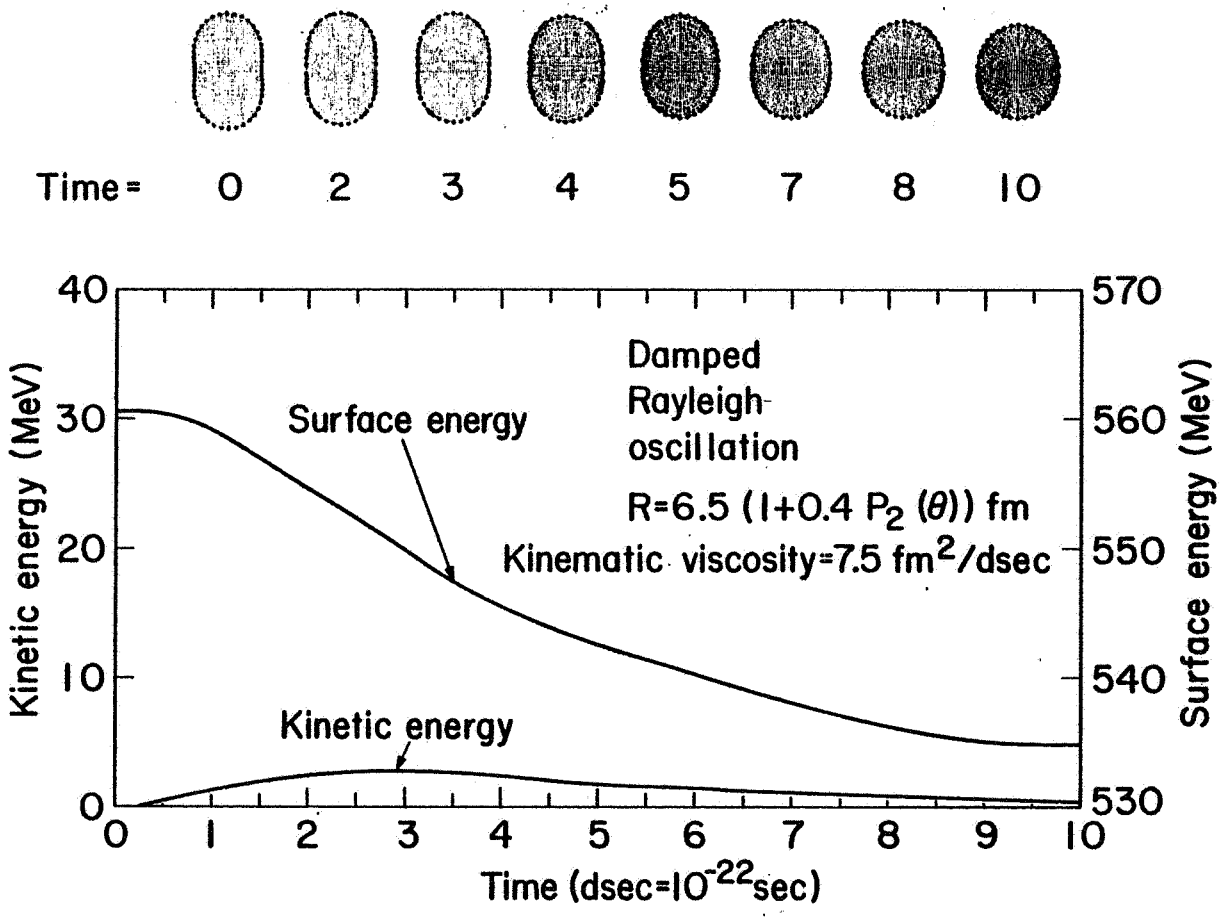

Time = 0 2 3 4 5 7 8 10
Damped
Rayleigh-
oscillation
R=6.5 (1+0.4 P2 (θ)) fm
Kinematic viscosity=7.5 fm2/dsec
Surface energy
Kinetic energy
Kinetic energy (MeV)
Surface energy (MeV)
Time (dsec=10-22sec)
0 1 2 3 4 5 6 7 8 9 10
0 10 20 30 40
530 540 550 560 570

Figure 4.

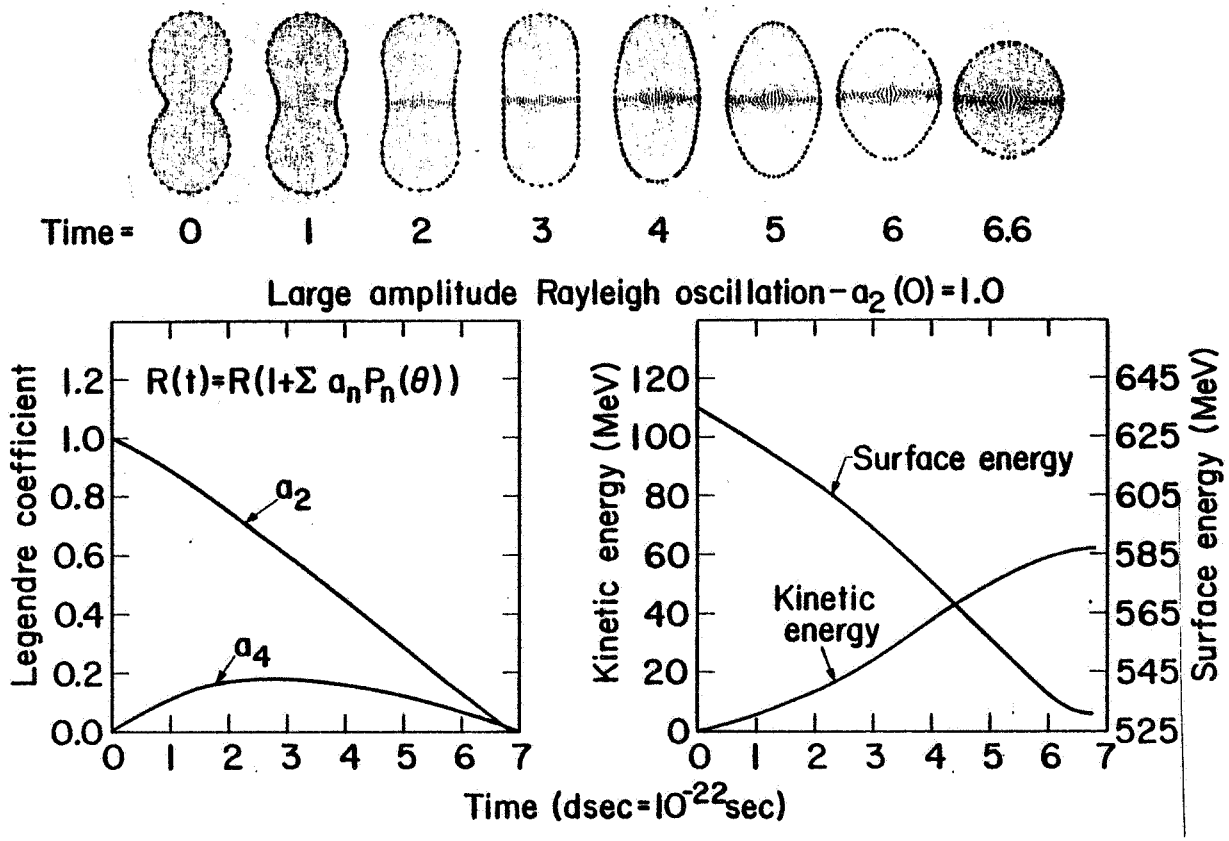

Time = 0 1 2 3 4 5 6 6.6
Large amplitude Rayleigh oscillation – $a_2(0) = 1.0$
$R(t) = R(1 + \Sigma a_n P_n(\theta))$
Legendre coefficient
1.2
1.0
0.8
0.6
0.4
0.2
0.0
$a_2$
$a_4$
0 1 2 3 4 5 6 7
Kinetic energy (MeV)
120
100
80
60
40
20
0
Surface energy
Kinetic energy
645
625
605
585
565
545
525
Surface energy (MeV)
Time (dsec = $10^{-22}$ sec)

Figure 5

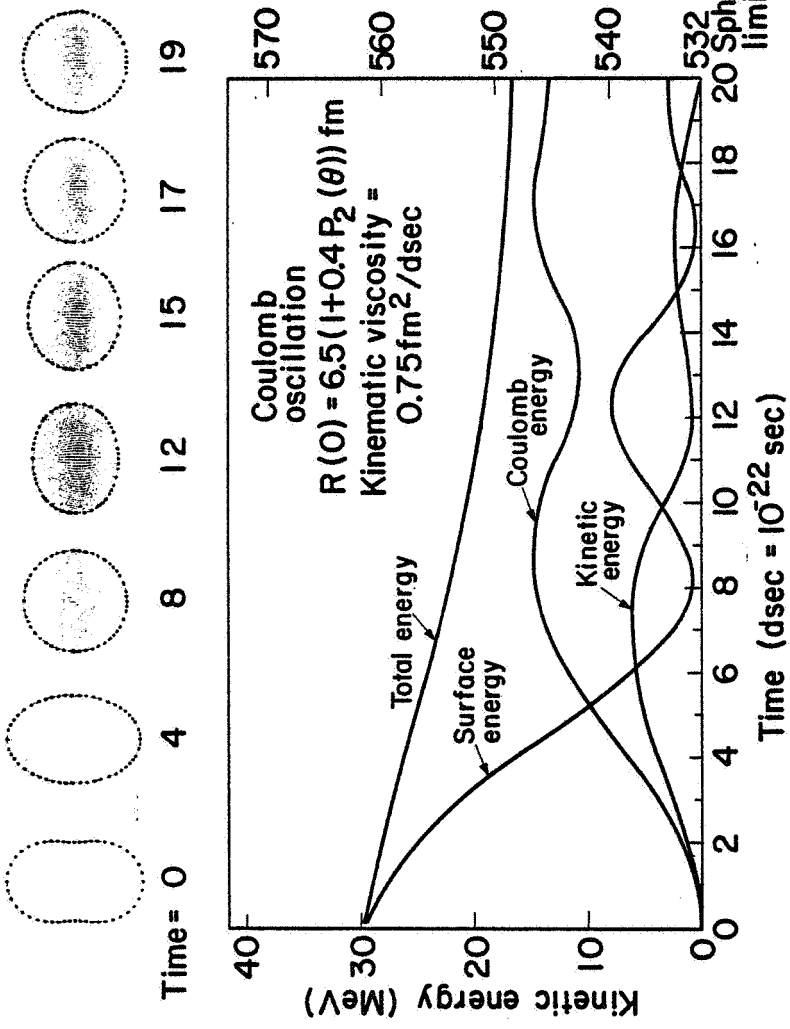

Time= 0
4
8
12
15
17
19
Coulomb oscillation
R(0) = 6.5(1+0.4 P2 (θ)) fm
Kinematic viscosity = 0.75fm2/dsec
Total energy
Surface energy
Coulomb energy
Kinetic energy
Kinetic energy (MeV)
Surface energy (MeV)
Coulomb energy (MeV)
Time (dsec = 10-22 sec)
Sphere limit
Sphere limit

Figure 6

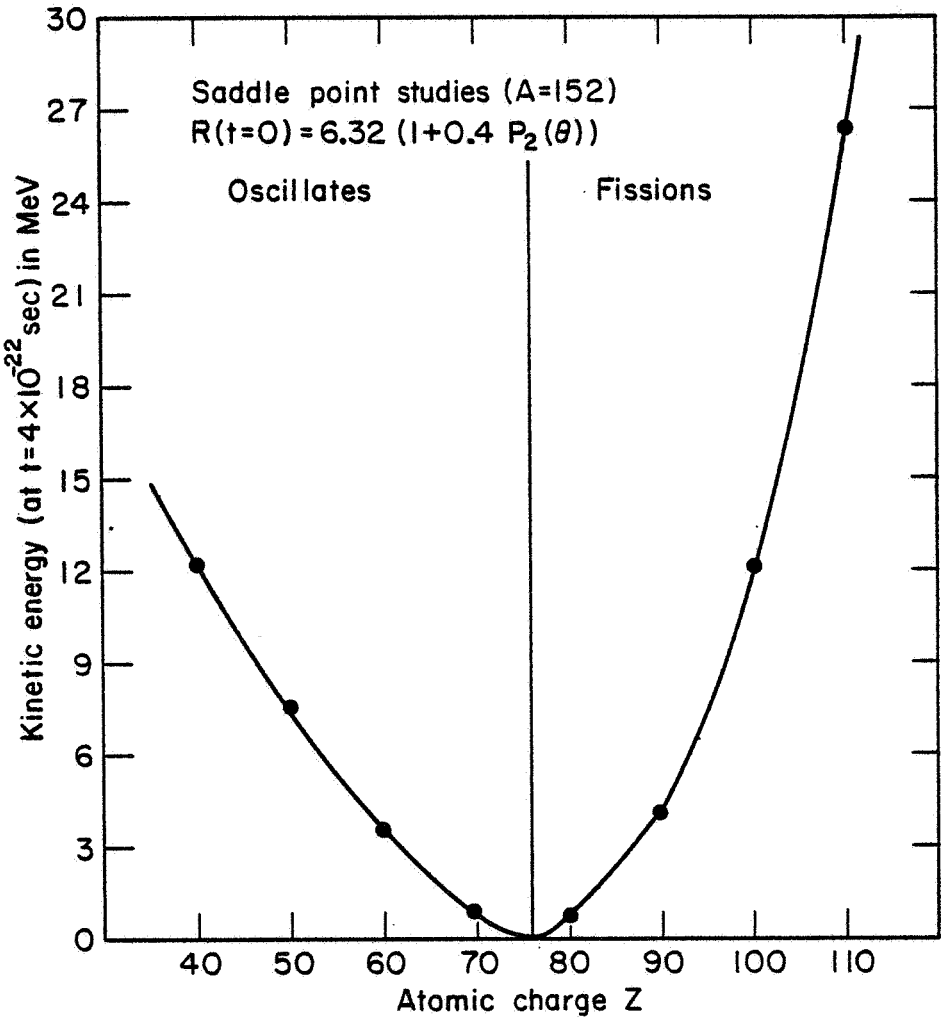

Saddle point studies (A=152)
R(t=0) = 6.32 (1+0.4 P2(θ))
Oscillates
Fissions
Kinetic energy (at t=4×10^-22 sec) in MeV
Atomic charge Z
0
3
6
9
12
15
18
21
24
27
30
40
50
60
70
80
90
100
110

Figure 7

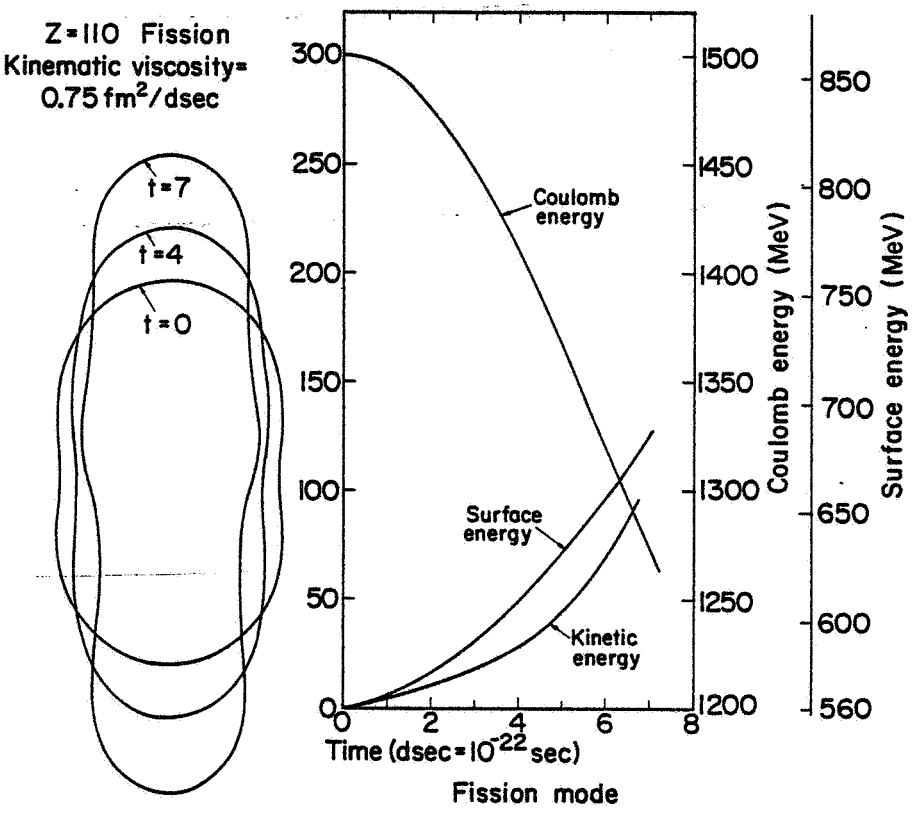

Z=110 Fission
Kinematic viscosity=
0.75 fm²/dsec
t=7
t=4
t=0
300
250
200
150
100
50
0
0
2
4
6
8
Time (dsec=10⁻²² sec)
Fission mode
Coulomb energy
Surface energy
Kinetic energy
1500
1450
1400
1350
1300
1250
1200
Coulomb energy (MeV)
850
800
750
700
650
600
560
Surface energy (MeV)

Figure 8

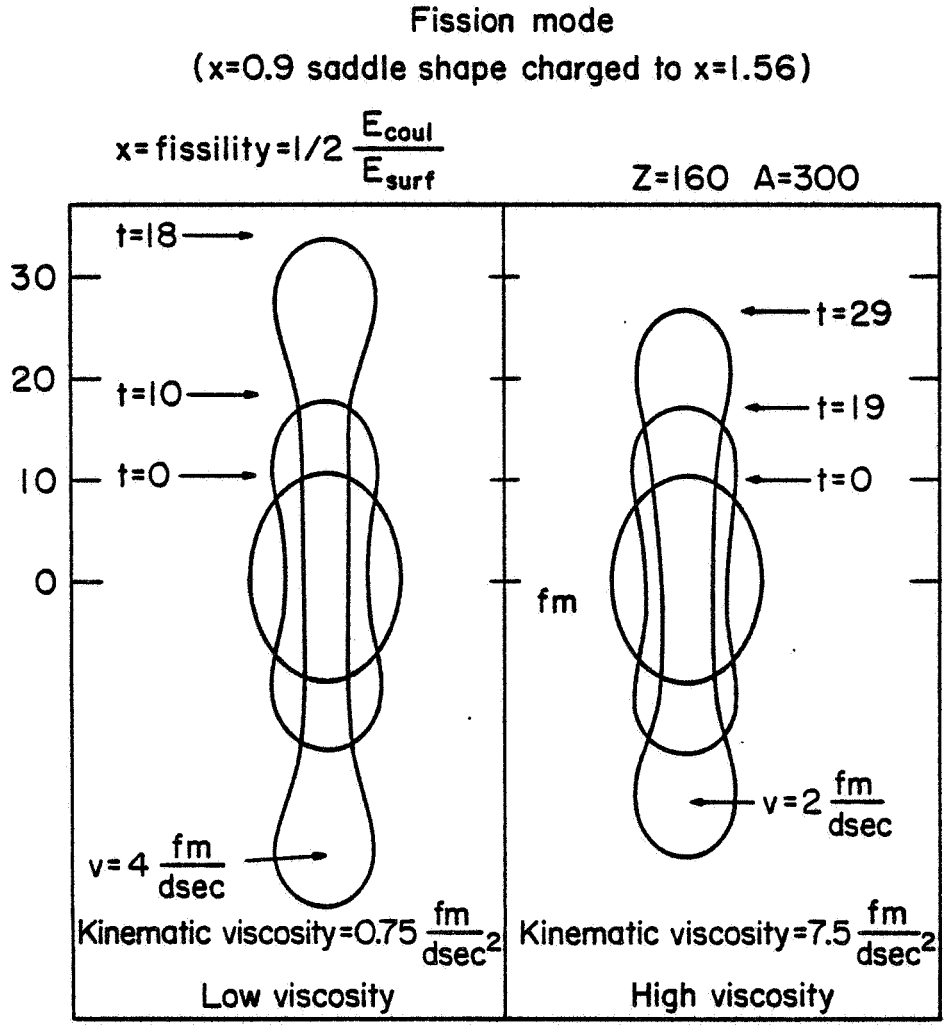

Fission mode
(x=0.9 saddle shape charged to x=1.56)
x= fissility = 1/2 Ecoul / Esurf
Z=160 A=300
t=18
t=10
t=0
30
20
10
0
fm
v = 4 fm/dsec
Kinematic viscosity = 0.75 fm/dsec²
Low viscosity
t=29
t=19
t=0
v = 2 fm/dsec
Kinematic viscosity = 7.5 fm/dsec²
High viscosity

Figure 9

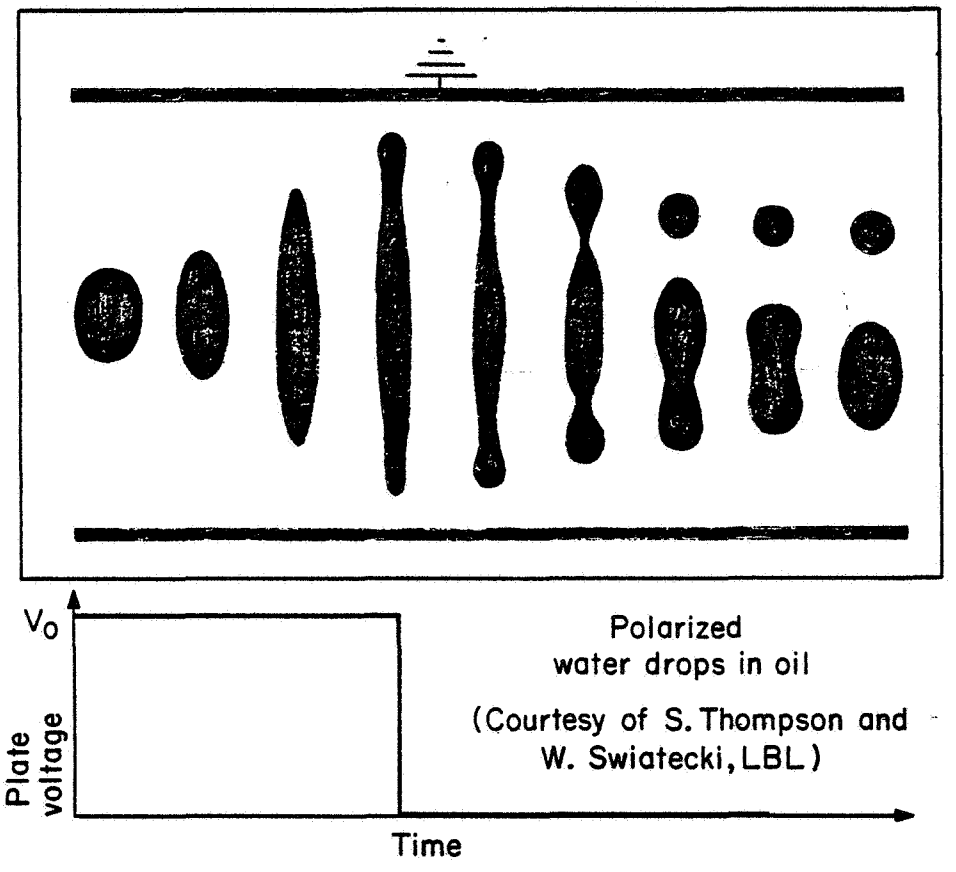

Vo
Plate voltage
Time
Polarized
water drops in oil
(Courtesy of S. Thompson and
W. Swiatecki, LBL)

Figure 10

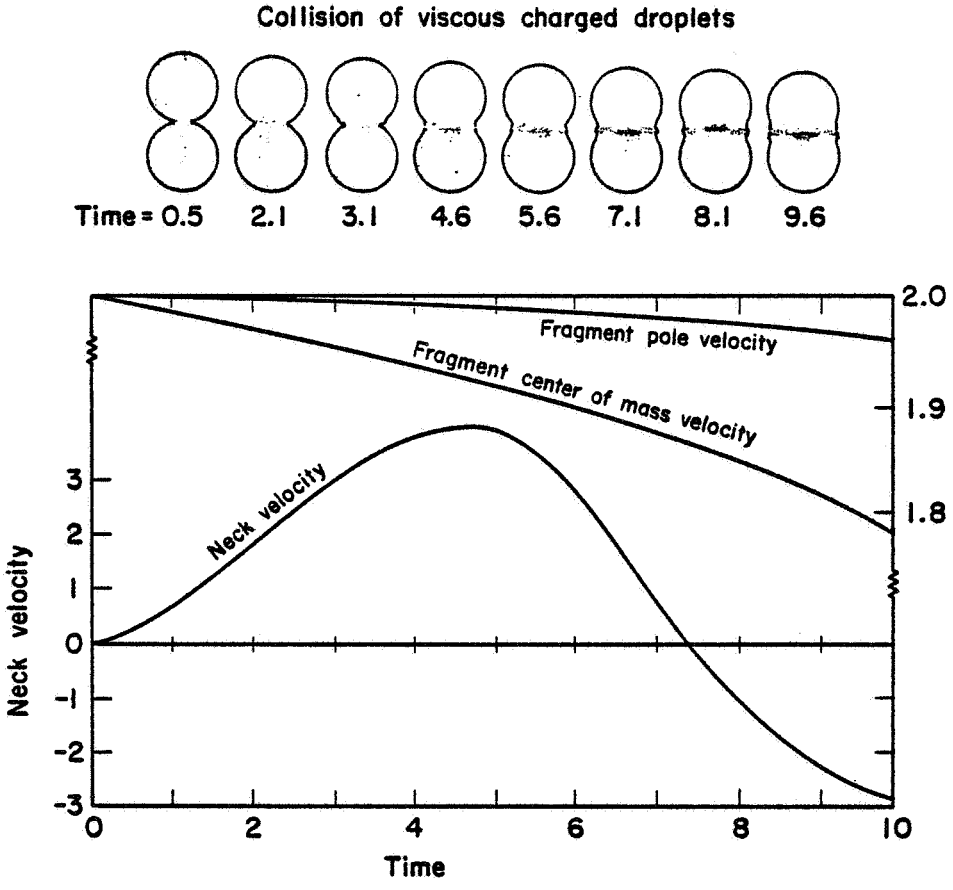

Collision of viscous charged droplets
Time = 0.5
2.1
3.1
4.6
5.6
7.1
8.1
9.6
Fragment pole velocity
Fragment center of mass velocity
Neck velocity
Neck velocity
Time
2.0
1.9
1.8
3
2
1
0
-1
-2
-3
0
2
4
6
8
10

Figure 11