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REPORT NO. 1371

THE PRODUCTION OF FIRING TABLES FOR CANNON ARTILLERY

by

Elizabeth R. Dickinson

November 1967

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BALLISTIC RESEARCH LABORATORIES

REPORT NO. 1371

NOVEMBER 1967

THE PRODUCTION OF FIRING TABLES
FOR
CANNON ARTILLERY

Elizabeth R. Dickinson

Computing Laboratory

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RDT&E Project No. 1T523801A287

ABERDEEN PROVING GROUND, MARYLAND

BALLISTIC RESEARCH LABORATORIES

REPORT NO. 1371

ERDickinson/sw
Aberdeen Proving Ground, Md.
November 1967

THE PRODUCTION OF FIRING TABLES
FOR
CANNON ARTILLERY

ABSTRACT

This report describes in detail the acquisition and analysis of firing data as well as the conversion of the measured data to printed firing tables.

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I. TABLE OF SYMBOLS AND ABBREVIATIONS

AC	Azimuth correction for wind
BRLESC	Ballistic Research Laboratories Electronic Scientific Computer
C	Ballistic coefficient
CDC	Combat Developments Command
CS	Complementary angle of site
D	Deflection, drift
E	$\frac{\rho V K_D}{C}$
F	Fork
FS	Fuze setting
FSC	Fuze setting correction for wind
G	Gâvre function
GFT	Graphical firing table
GST	Graphical site table
H	Heavy, height
ICAO	International Civil Aviation Organization
J	Jump
K_D	Drag coefficient
L	Latitude, lightweight, distance from the trunnion of the gun to the muzzle
M	Mach number (velocity of projectile/velocity of sound)
MDP	Meteorological datum plane
MFT	Mean firing time
MLW	Mean low water

1. TABLE OF SYMBOLS AND ABBREVIATIONS (Continued)

MRT	Mean release time
MSL	Mean sea level
N-Factor	Factor for correcting the velocity of a projectile of nonstandard weight
PE	Probable error
QMR	Qualitative materiel requirement
R	Range
RC	Range correction for wind
S	Spin
T	Observed time of flight, target
V	Velocity
VI	Vertical interval
W	Weight, wind, wind speed
W_h	Headwind
W_t	Tailwind
W_x	Range wind
W_z	Cross wind
Z	Deflection
a_x, a_y, a_z	Accelerations due to the rotation of the earth
d	Diameter
g	Acceleration due to gravity
h	Height of the trunnions of the gun above mean low water
n	Calibers per revolution, $1/n$ = twist in revolutions per caliber

I. TABLE OF SYMBOLS AND ABBREVIATIONS (Continued)

s	Standard deviation
t	Computed time of flight
x	Distance along the line of fire
y	Height
z	Distance to the right or left of the line of fire
α	Azimuth of the line of fire
Θ	Chart direction of the wind
Θ_1	Azimuth of the wind
$\lambda_{1,2,3}$	Components of the earth's angular velocity
ϕ	Angle of elevation, angle of departure
ρ	Air density as a function of height
Ω	Angular velocity of the earth
Mil	Mil
' , ''	First and second derivatives with respect to time
Subscripts	
o	Initial value
n	Nonstandard value
s	Standard value
w	Final value

II. INTRODUCTION

Ideally, a firing table enables the artilleryman to solve his fire problem and to hit the target with the first round fired. In the present state of the art, this goal is seldom achieved, except coincidentally. The use of one or more forward observers, in conjunction with the use of a firing table, enables the artilleryman to adjust his fire and hit the target with the third or fourth round fired.

It is, however, the ultimate goal of the Firing Tables Branch of the Ballistic Research Laboratories (BRL) to give the artilleryman the tool for achieving accurate, predicted fire.

This report is concerned with the steps involved in the production of a current artillery firing table, with principal emphasis on the conversion of measured data to printed tables. No report has been published on the computational procedures for producing firing tables since that of Gorn and Juncosa^{(1)*} in 1954. As a new high speed computer, with a much larger memory, the Ballistic Research Laboratories Electronic Scientific Computer (BRLESC), has been added to the computing facilities at BRL, the computation of firing tables has become more highly mechanized than it was in 1954.

No description has been published of the process by which the necessary data for a firing table are acquired. Thus, the purpose of this report is two-fold: to update the work of Gorn and Juncosa, and to trace the history of a firing table from the adoption of a new weapon or a new piece of ammunition to the publication of a table for the use of that weapon or ammunition in the field.

*Superscript numbers in parentheses denote references which may be found on page 90.

III. REQUIREMENT FOR A FIRING TABLE

Whenever a new combination of artillery weapon and ammunition goes into the field, a firing table must accompany the system. There is a rather lengthy chain of events leading up to the placing in the field of a new weapon or a new projectile. Initially, the requirement for such an item is written by the Combat Developments Command (CDC). The qualitative materiel requirement (QMR) set up by CDC is very detailed, listing weight, lethality, transportability and many other attributes as well as performance characteristics.

After the item has passed through the research and development stage, it is subjected to safety, engineering and service tests. When all tests are passed satisfactorily, the item is classified as standard and goes into production. As soon as the production item is available, it and its firing table are released to troops for training.

Thus as soon as a QMR is published, it is known that a firing table will be ultimately required. In addition, a provisional firing table will be needed to conduct the firings for the various tests of the item.

IV. ACQUISITION OF DATA FOR A FIRING TABLE

We acquire data for firing tables in several stages and for several purposes: to provide velocity zoning, to produce a provisional table and to produce a final table.

A. Data for Velocity Zoning

Velocity zoning can be defined as the establishment of a series of finite weights of propellant, called charges, which are to be used to deliver a projectile, by means of a given weapon system, to certain required ranges. Within each zone, or charge, an infinite variation of elevations permits the gunner to deliver the projectile at any range up to the maximum for that charge. Zoning gives flexibility to a weapon system.

For a given projectile, fired from a given weapon, by means of a given propellant, the range of that projectile is a function of the elevation of the tube and of the amount of propellant used. For a given amount of propellant, maximum range is reached with an elevation in the neighborhood of 45°. The maximum amount of propellant which can be used is governed by the chamber volume of the weapon and by the maximum gas pressure which the weapon can safely withstand.

It is the responsibility of the Firing Tables Branch of the Ballistic Research Laboratories to zone artillery weapons. The zoning is done in such a way as to meet the criteria established by the qualitative materiel requirement. THE QMR establishes (1) either a maximum range or a maximum muzzle velocity, (2) a minimum range, (3) a low or high elevation criterion, or a mask clearance criterion, and (4) the amount of range overlap. These criteria will be discussed on the following page.

The first step in the zoning process is to compute a series of trajectories,* all at an elevation of 45° , at various muzzle velocities up to and including the maximum velocity. From these data a plot is then made of muzzle velocity versus range. The next step is determined by the criterion set up by the QMR in (3), above, with its three possibilities. (a) If the weapon is a gun, trajectories may be computed for various muzzle velocities, all at an elevation of 15° . (b) If the weapon is a howitzer, trajectories may be computed at various muzzle velocities, all at an elevation of 70° . (c) For either type of weapon, however, a mask clearance criterion may be used. A mask clearance requirement might be stated as: 300 meters at 1000 meters. In other words, all trajectories must clear a height of 300 meters at a distance from the weapon of 1000 meters. If this is the criterion established by the QMR, a series of trajectories is computed at various muzzle velocities, all clearing a 300 meter mask at 1000 meters.

Thus, depending on the requirements for the weapon system (15° , 70° or mask clearance), data are obtained for a second velocity-range relationship. These values are plotted on the same graph with the 45° curve. By means of these two curves, zoning is now done graphically.

If the maximum range has been established by military requirements, the horizontal line is drawn, between the two curves, that intersects the 45° curve at this range (Figure 1). This is the velocity for the zone with greatest range (highest charge). If, however, the maximum velocity has been established, the horizontal line is drawn that

*The method by which trajectories are computed is described in Section V.

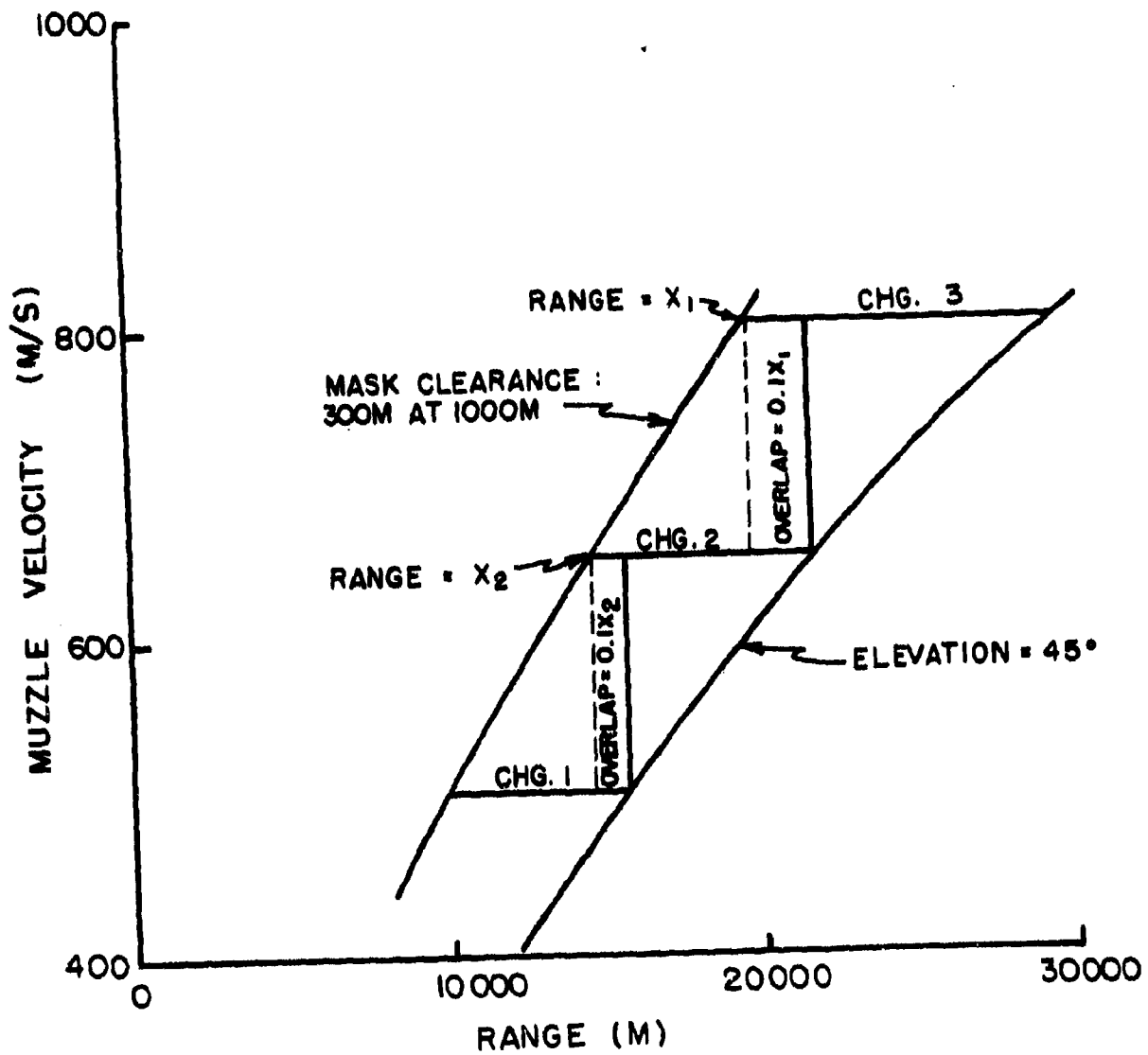


Figure 1. Velocity zoning for artillery weapons

intersects the two curves at this velocity. This line's intersection with the 45°-curve will show the maximum attainable range. Usually, the QMR states that zoning must provide for a 10 percent range overlap between successive zones. This overlap is determined by computing 10 percent of the minimum range on the horizontal line, adding this to the minimum range and dropping a perpendicular from this point to the 45°-curve. From this point of intersection, a horizontal line is drawn to the other curve, determining the next lower charge. This process is continued until the minimum range, set by military requirements, is reached. The number of charges for a typical gun system is three; for a typical howitzer system, seven.

The tests that are conducted during the research and development stage of the item's existence are all fired with the charges established by this zoning. If the item is so modified that the zoning no longer satisfies the QMR, a new velocity zoning is established. Thus, at the end of the research and development stage, final zoning has been established.

B. Data for Provisional Table

A firing table is basically a listing of range-elevation relationships for a given weapon firing a given projectile at a given velocity under arbitrarily chosen "standard" conditions. In addition, information is given for making corrections to firing data due to nonstandard conditions of weather and materiel.

This type of data is necessary for the conduct of safety, engineering and service tests of a weapon system, before it can be accepted and classified as standard. Hence a provisional firing table must be provided

for firing these tests. The data for a provisional table are gleaned from all available firings during the research and development phase. These data do not come from a series of firings designed to furnish firing table data, hence the provisional table is at best as accurate an estimate as can be made of the range-elevation relationship of the system, and of the effects of nonstandard conditions. This type of table is adequate, however, for conducting the tests.

C. Data for Final Table

A range firing program is drawn up and submitted, for inclusion in the engineering tests, to the agency that will be conducting those tests. The program is so designed (Table I) that the range-elevation relationship with both standard and nonstandard material can be determined. Only two nonstandard conditions of material are considered: projectile weight and propellant temperature. The arbitrarily chosen "standard" propellant temperature for United States artillery ammunition is 70° Fahrenheit.

At the time of firing, pertinent observed data are recorded and forwarded to the Firing Tables Branch (Table II). From the firing and meteorological data obtained, corrections can be determined for nonstandard conditions of weather: wind velocity and direction, air temperature and density.

Because of the mission of an illuminating projectile, less accuracy of fire is required than with an HE projectile. A range firing program for an illuminating projectile is, consequently, less extensive. A typical firing program is shown in Table III.

Table I. Typical Range Firing Program for HE Projectiles

105mm Howitzer M108
 firing
 HE Projectile M1

Elevation (degrees)	No. of Rounds	Condition
5	10	Std. wt. (33 lbs), std. temp. (70°F).
15	20	Std. wt., 10 rds. at 70°, alternating w/10 rds. at 125°.
25	20	Std. wt., 10 rds. at 70°, alternating w/10 rds. at 0°.
35	20	Std. wt., 10 rds. at 70°, alternating w/10 rds. at -40°.
45	10	Std. wt., std. temp.
55	20	Std. temp., 10 rds. of 32.5 lbs., alternating w/10 rds. of 33.5 lbs.
65	10	Std. wt., std. temp.
(Max. trail) -3	10	Std. wt., std. temp.

The above program of 120 rounds is to be fired for each of the seven charges with propellant M67.

Table II. Complete Range Data for Artillery and Mortar Firing Tables

A. General Information

1. Weapon

- a. Complete nomenclature
- b. Length of tube, defined as distance from trunnions to muzzle

2. Projectile

- a. Complete nomenclature
- b. Lot number

3. Fuze

- a. Complete nomenclature
- b. Lot number

4. Propellant

- a. Complete nomenclature
- b. Lot number

5. Impact data*

- a. Land: height of impact area above mean low water or mean sea level
- b. Water: tide readings every hour-bracketing times of firing

B. Metro**

- 1. Metro aloft every hour - bracketing times of firing
- 2. Altitude of met station above mean low water or mean sea level

*Impact data are not to be supplied for illuminating shell. Time to burst, range to burst and height of burst above MLW or MSL are required instead.

**Metro referenced to the line of fire.

Table II. Complete Range Data for Artillery and Mortar Firing Tables
(Continued)

C. Round-by-Round Data

1. Date
2. Time of firing
3. Test round number and tube round number
4. Azimuth of line of fire
5. Height of trunnions above mean low water
6. Angle of elevation (clinometer), before and after each group
7. Charge number
8. Propellant temperature
9. Fuze Temperature (when applicable)
10. Projectile temperature (when applicable)
11. Projectile weight
12. Slant distance from gun muzzle to first coil, and between coils
13. Coil time
14. Time of Flight
15. Range
16. Deflection*
17. Fuze setting (when applicable)

* *Not required for illuminating shell.*

Table III. Typical Range Firing for Illuminating Projectiles*

105mm Howitzer M108
firing
Illuminating Projectile M314

Charge	Elevation (degrees)					
	20	25	30	35	40	45
1					X	X
2					X	X
3				X	X	X
4			X	X	X	X
5			X	X	X	X
6		X		X	X	X
7	X		X		X	X

10 rounds are to be fired under each condition.

*All charges are not fired at all elevations, because low charges will not fire the projectile to the requisite height (750 meters) at low elevations.

V. ANALYSIS OF RAW DATA

We must select the appropriate equations of motion and the appropriate drag coefficient in order to reduce the range firing data.

A. Equations of Motion

The equations of motion, incorporating all six degrees of freedom of a body in free flight, have been programmed for the BRLESC and are used for the burning phase of rocket trajectories. (2) The procedure is a very lengthy one, however; even on the very high speed BRLESC, average computing time is approximately 4 seconds per second of time of flight.* For cannon artillery tables, this computing time would be prohibitive. In preparing a firing table for a howitzer, we compute about 200,000 trajectories having an average time of flight of about 50 seconds. This would mean approximately 10,000 hours of computer time!

In contrast, the equations of motion for the particle theory, which are currently used for computing firing tables for cannon artillery, use far less computer time: approximately 1 second per 160 seconds of time of flight. For the same howitzer table used as an example above, approximately 20 hours of computer time are required.

Although the trajectory computed by the particle theory does not yield an exact match along an actual trajectory, it does match the end points. For present purposes, this theory provides the requisite degree of accuracy for artillery firing tables.**

*This long computing time is for spinning projectiles. Machine time for nonspinning or slowly spinning rockets and missiles is in the ratio of approximately 100 seconds per 60 seconds of time of flight.

** At the time of writing, a new procedure is evolving. Because it is expected that greater accuracy will be required in the future, the equations of motion, for three degrees of freedom, in a modified form have been developed. (3) When this new method has been fully programmed for BRLESC, all firing table work will be computed with the new equations.

The accelerations, velocities and positions necessary to describe the particle theory are referenced to a ground-fixed, right hand, coordinate system. The equations of motion which are used in the machine reduction of the firing data are:

$$\ddot{x} = - \frac{\rho V K_D}{C} (\dot{x} - W_x) + a_x$$

$$\ddot{y} = - \frac{\rho V K_D}{C} \dot{y} - g + a_y$$

$$\ddot{z} = - \frac{\rho V K_D}{C} (\dot{z} - W_z) + a_z$$

where the dots indicate differentiation with respect to time,

x, y and z = distances along the x, y and z axes,

ρ = air density as a function of height,

V = velocity,

K_D = drag coefficient,

C = ballistic coefficient,

W_x = range wind

W_z = cross wind

g = acceleration due to gravity

and a_x , a_y and a_z are accelerations due to the rotation of the earth.

For a given projectile, K_D varies with Mach number and with angle of attack. The ballistic coefficient, C, defined as weight over diameter squared (W/d^2) is a constant. However, for convenience in handling data along any given trajectory, K_D is allowed to vary only with Mach number, and C becomes a variable. In other words, the K_D used is that for zero angle of attack. In actual flight, drag increases with an

increase in angle of departure, due to large summital yaws at high angles. Thus, if K_D is not allowed to increase with increasing angle, C will decrease in order to maintain the correct K_D/C ratio. Up to an angle of departure of 45° , however, summital yaws are so small that C is usually a constant for any given muzzle velocity.

B. Drag Coefficient

Before any trajectories can be computed, the drag coefficient of the projectile must be determined. In the past, one of the Gâvre functions, (G_1 through G_8) with an appropriate form factor, was used. Today, most of the firing table computations are made with the drag coefficient for the specific projectile under consideration.

If a completely new configuration is in question, the drag coefficient is usually determined by means of firings in one of the free-flight ranges^(4,5) of the Exterior Ballistics Laboratory. The drag data thus obtained are then fitted in a form suitable for use with the BRLESC⁽⁶⁾

Usually, the drag coefficient is determined rather early in the research stage. If the configuration is modified only slightly during its development, differential corrections can be applied to the original drag coefficient. For example, the effects of slight changes in head shape, overall length, boattail length or boattail angle can be quite accurately estimated, as these effects have been determined experimentally. (See references 7-14.) If a major modification is made to a projectile, such as a deep body undercut, a new drag curve must be determined experimentally.

C. Reduction of Range Firing Data

All of the rounds fired with the same charge (muzzle velocity) are handled at one time. The following description of the reduction of data applies to one charge. Each charge is handled successively in exactly the same manner. Because most of the parameters determined by the reduction of data are functions of muzzle velocity, judgment must be used in correlating the values obtained for each charge, before the final computations are made for the printed firing table.

The following discussion is divided into two main sections: the first describes computations preparatory to the reduction of data for the ballistic coefficient; the second describes the reduction for the ballistic coefficient.

1. Preparation for Reduction for Ballistic Coefficient.

a. Formation of Points. As shown in Table I (page 17), similar rounds, under similar conditions, are fired in groups of ten. Each group is designated as a point. As soon as raw data are received, the instrument velocity of each round in a given point (supplied by the agency conducting the range firings) is plotted against its range. Any inconsistent rounds are investigated and, if a legitimate reason for doing so is discovered, are eliminated from any further computations.* The remaining rounds constitute the point.

All of the computations concerned with the reduction of the observed data are machine computations. The only handwork is the plotting of rounds described here.

*Possible reasons for eliminating a round include: incorrect charge, weight, or temperature.

b. Meteorological Data⁽¹⁵⁾. Metro at the surface is computed for each round for use in the computation of muzzle velocity for the given round. Metro aloft is computed for each point for use in the computation of ballistic coefficient.

To compute metro aloft for any given point, linearly interpolated values, for even increments of altitude, are determined from the observed data. A linear interpolation factor is then computed, based on the mean release time of the given balloon runs and the mean firing time for the given point. This factor is used to weight the observed metro data at the given intervals to obtain the metro structure to be used with the given point.

In Figure 2:

MRT = mean release time, (release time + 1/2 ascension time)

MFT = mean firing time, (mean of all rounds in the point)

y = altitude,

W_x = range wind,

W_z = cross wind,

T = air temperature,

ρ = air density.

Surface metro is computed in a similar manner, the linear factor being determined by using the release time of the given balloon runs and the firing time of each given round.

c. Muzzle Velocity. The coordinates of the muzzle are the initial coordinates of each trajectory. At Aberdeen Proving Ground, the origin of the co-ordinate system is the intersection of a vertical

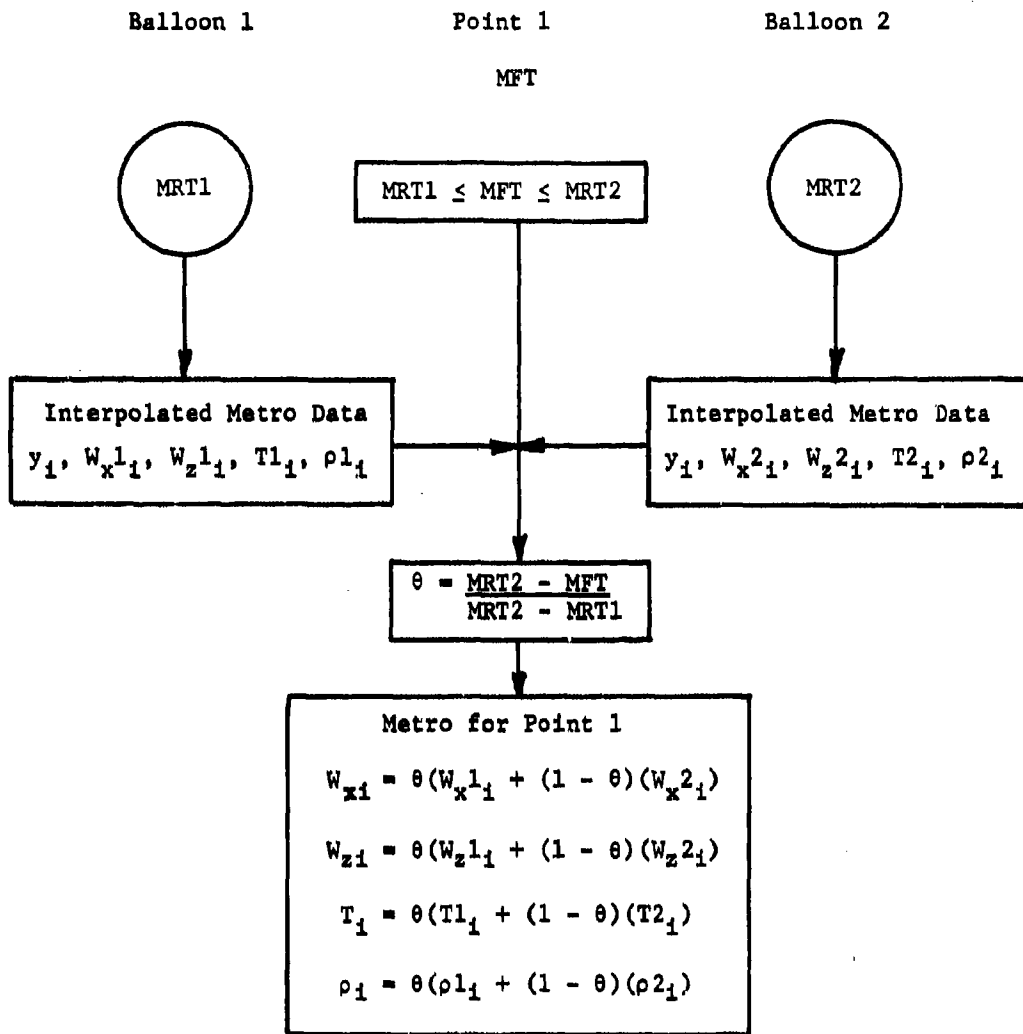


Figure 2. Metro structure for use with a given point

line through the trunnions of the gun and a horizontal line established by mean low water. The coordinates of the muzzle are, therefore:

$$x_0 = L \cos \phi_0$$

$$y_0 = h + L \sin \phi_0$$

where L = distance from the trunnions of the gun to the muzzle,

ϕ_0 = angle of elevation of the gun,

h = height of trunnions above mean low water.

After leaving the muzzle of the gun, each round (magnetized just prior to firing) passes through a pair of solenoid coils. These coils are placed as close to the muzzle as possible without danger of their being triggered by blast or unburned powder. The time lapse as the projectile travels from the first coil to the second is measured by a chronograph; the distance from the muzzle to the first coil, and the distance between coils are known. Using the equations of motion, an estimated $C = W/d^2$ (of the individual round) and the observed metro, we can simulate the flight of the projectile. By an iteration process, a muzzle velocity is determined for each round such that the time-distance relationship given by the computed trajectory matches that measured by the chronograph.

d. Means and Probable Errors, per Point. For each point, the mean and the probable error are computed for: range, muzzle velocity, deflection, time of flight, height of impact, weight, time to burst (when appropriate) and fuze setting (when appropriate). For any given set of data $\{x_i\}$ ($i = 1, 2, 3 \dots n$), the mean,

$$\bar{x} = \sum_{i=1}^n \frac{x_i}{n} .$$

Probable errors are computed in two ways, by the root-mean-square method and by successive differences. Both probable errors are printed, followed by the preferred (smaller) probable error.

Probable error by the root-mean-square method:

$$PE = S \text{ (Kent-factor) (see Table IV)}$$

where

$$S = \sqrt{\frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n}}$$

Probable error by successive differences:

$$PE = .4769 \sqrt{\frac{\sum_{i=1}^{n-1} \delta_i^2}{n-1}}$$

where $\delta_i = x_{i+1} - x_i$ ($i = 1, 2, 3 \dots n-1$).

e. N-Factor. One of the conditions that affect the range of a projectile is its weight. Hence it must be possible to correct for a projectile of nonstandard weight.

Given some standard velocity, V_s , and some standard weight, W_s , and assuming all deviations from standard weight to be small, we can compute a number, N , such that

$$\frac{\Delta V}{\Delta W} = N \frac{V_s}{W_s}$$

where $\Delta V = V_s - V_n$

$$\Delta W = W_s - W_n$$

Subscript s = standard

n = nonstandard

Table IV. Kent Factors

Computation of Probable Error (PE)
by Standard Deviation (s) and Successive Differences (δ)

$$s = \sqrt{\frac{\sum \Delta^2}{n}}$$

where Δ = deviations from arithmetic mean, n = number of observations

Let s be the observed standard deviation, then the best estimate for the PE, i.e., such that average PE → true PE, is given by

$$PE = \frac{.6745 \beta \left(\frac{n-1}{2}, \frac{1}{2}\right)}{(2\pi/n)^{1/2}} s$$

See Deming and Birge, Reviews of Modern Physics Vol. 6, No. 3, July 1934, P. 128

<u>n</u>	<u>PE/s</u>	<u>n</u>	<u>PE/s</u>	<u>n</u>	<u>PE/s</u>	<u>n</u>	<u>PE/s</u>
2	1.19550	21	.69984	40	.68747	59	.68322
3	.93213	22	.69863	41	.68715	60	.68307
4	.84535	23	.69753	42	.68684	61	.68293
5	.80225	24	.69652	43	.68655	62	.68279
6	.77650	25	.69560	44	.68627	63	.68265
7	.75939	26	.69476	45	.68600	64	.68252
8	.74719	27	.69398	46	.68574	65	.68240
9	.73805	28	.69325	47	.68550	66	.68228
10	.73096	29	.69258	48	.68527	67	.68216
11	.72529	30	.69196	49	.68504	68	.68204
12	.72065	31	.69138	50	.68482	69	.68193
13	.71679	32	.69083	51	.68462	70	.68183
14	.71353	33	.69032	52	.68442	71	.68172
15	.71073	34	.68984	53	.68423	72	.68162
16	.70831	35	.68939	54	.68404	73	.68152
17	.70619	36	.68896	55	.68386	74	.68142
18	.70432	37	.68856	56	.68369	75	.68133
19	.70266	38	.68818	57	.68353	∞	.67449
20	.70117	39	.68782	58	.68337		

$$PE \text{ by successive differences} = \frac{.6745}{\sqrt{2}} \sqrt{\frac{\sum \delta^2}{n-1}} \quad .4769 \sqrt{\frac{\sum \delta^2}{n-1}}$$

where δ = differences between successive observation, n = number of observations

In order to determine N, 20 rounds (see Table I) at standard temperature are fired at the same elevation and muzzle velocity. Ten of these rounds, weighing less than standard, are fired alternately with 10 rounds weighing more than standard. The N-factor is computed from these data using the following equation:

$$N = \frac{\sum_{i=1}^k \Delta V_i \Delta W_i}{\sum_{i=1}^k \Delta W_i^2} \cdot \frac{W_s}{V_s}$$

where, $\Delta V_i = V_{H_i} - V_{L_i}$

$$\Delta W_i = W_{H_i} - W_{L_i}$$

and, V_{H_i} = velocity of the ith heavy round

$$V_{L_i}$$
 = velocity of the ith light round

$$W_{H_i}$$
 = weight of the ith heavy round

$$W_{L_i}$$
 = weight of the ith light round

The equation for computing N-factor was determined by a least squares technique as follows:

let, $G(N) = \sum_{i=1}^n \left\{ \Delta V_i - N \frac{V_s}{W_s} \Delta W_i \right\}^2$

$$\frac{\partial G(N)}{\partial N} = 2 \sum_{i=1}^n \left[\left\{ \Delta V_i - N \frac{V_s}{W_s} \Delta W_i \right\} \frac{V_s}{W_s} \Delta W_i \right] = 0$$

thus, $N = \frac{\sum_{i=1}^n \Delta V_i \Delta W_i}{\sum_{i=1}^n \Delta W_i^2} \cdot \frac{W_s}{V_s}$

f. Change in Velocity for a Change in Propellant Temperature.

As shown in Table I, range firings are conducted with rounds conditioned to various nonstandard propellant temperatures (-40°F, 0°F, 125°F) alternated with rounds conditioned to the standard propellant temperature (70°F). The following information can be obtained for a given pair of points:

$$\begin{aligned} V_{70^\circ}, W_{70^\circ}; V_{-40^\circ}, W_{-40^\circ} \\ V_{70^\circ}, W_{70^\circ}; V_{0^\circ}, W_{0^\circ} \\ V_{70^\circ}, W_{70^\circ}; V_{125^\circ}, W_{125^\circ} \end{aligned}$$

where, V = velocity

W = weight.

The V_{70° 's and W_{70° 's for all three firings are not necessarily equivalent.

Since the weight for each point is not necessarily standard, the N-factor for each charge is used to strip out the effects of projectile weight. Given some velocity V_{70° , some weight W_{70° , and some standard weight W , the standard V is computed.

$$\text{Since, } \Delta V = N \frac{V}{W} \Delta W,$$

$$\text{where, } \Delta V = V_{70^\circ} - V$$

$$\Delta W = W_{70^\circ} - W,$$

$$\text{then, } V_{70^\circ} - V = N \frac{V}{W} (W_{70^\circ} - W),$$

$$\text{and } V = V_{70^\circ} / \left[1 - \frac{N}{W} (W - W_{70^\circ}) \right].$$

Thus all six points must be corrected for nonstandard projectile weight. Then,

$$\Delta V_{-40^\circ} = \frac{V_{-40^\circ}}{1 - \left(\frac{N}{W}\right)(W - W_{-40^\circ})} - \frac{V_{70^\circ}}{1 - \left(\frac{N}{W}\right)(W - W_{70^\circ})}$$

By the same process, a ΔV is computed for the 0°F point and the 125°F point. A least squares technique is then used to compute a function

$$\Delta V = a_1 (T - 70^\circ) + a_2 (T - 70^\circ)^2$$

using the three given data points at -40°F, 0°F and 125°F.

g. Compensation for Rotation of Earth. The final computations to be made in preparation for determining the ballistic coefficient are those to determine the coefficients used in the equations of motion to compensate for the rotation of the earth.

$$\lambda_1 = 2 \Omega \cos L \sin \alpha$$

$$\lambda_2 = 2 \Omega \sin L$$

$$\lambda_3 = 2 \Omega \cos L \cos \alpha$$

where, Ω = angular velocity of the earth in radians/second

$$2\Omega = .0001458424$$

L = latitude

α = azimuth of line of fire, measured clockwise from North

In the equations of motions given on page 22:

$$a_x = -\lambda_2 \dot{y}$$

$$a_y = \lambda_1 \dot{x}$$

$$a_z = \lambda_2 \dot{x} - \lambda_3 \dot{y}$$

2. Reduction for Ballistic Coefficient.

a. Ballistic Coefficient and Jump. The raw data, including metro, are by now processed; all parameters except the ballistic coefficient are now available for solving the equations of motion. The available data are: elevation (ϕ), drag coefficient (K_D vs M), standard weight and velocity, initial coordinates of the trajectory (x_0, y_0, z_0),

meteorological conditions [range wind (W_x), crosswind (W_z), air density and air temperature at even intervals of height], means and probable errors ($x_w, y_w, z_w, PE_x, PE_y, PE_D, PE_T$), time of flight, and the rotational forces ($\lambda_1, \lambda_2, \lambda_3$).

An iterative process is used to determine the ballistic coefficient. As a first approximation, C is set equal to weight/diameter². By computing a trajectory to ground ($y_w = 0$), the equations of motion are solved for each point, using all the available data for the given point. The computed range is then compared with the observed mean range for the point. If the ranges do not match*, C is adjusted by plus or minus C/16 (plus if the computed range is less than the observed range, minus if it is greater) and another trajectory is computed. If this new range does not match the observed range, another C is computed either by interpolation (if the two computed ranges bracket the observed range) or by extrapolation (if the two computed ranges do not bracket the observed range). This process is repeated, always using the last two computed C's for the interpolation (or extrapolation), until the computed range matches the observed range with the required tolerance. The final C is called the ballistic coefficient from reduction (C_{red}).

As part of the same program, a change in range for a unit change in C is computed for use in future computations. To obtain this value, $\Delta X/\Delta C$, a trajectory is computed with

$$C = C_{red} + 2^{-8} C_{red}$$

resulting in a range value, X. Another trajectory is computed, with

*The present BRLESC program requires the ranges to match to 1×10^{-8} meters.

$$C = C_{\text{red}} - 2^{-8} C_{\text{red}}$$

resulting in a range value, X_2 .

$$\text{thus } \frac{\Delta X}{\Delta C} = \left(\frac{X_1 - X_2}{2^{-7} C_{\text{red}}} \right).$$

As stated earlier, the C used in the muzzle velocity computations was merely $C = W/d^2$. If this C is in error by a significant amount, the muzzle velocities computed using this C would also be in error.

At the time muzzle velocity was computed, two C 's were used for each round: $C_1 = W/d^2$ and $C_2 = 1.1 W/d^2$. Hence two muzzle velocities were obtained for each round: V_1 and V_2 . Thus it was possible to compute a change in velocity for a one percent change in ballistic coefficient.

$$d \frac{(\Delta V)}{dC} = \frac{\Delta V}{.1 C_1}$$

where ΔV is the mean of the absolute values: $|V_1 - V_2|$,

V_1 and V_2 are mean values for all rounds in the point, and C is corrected to standard weight.

The change in C for a one mil change in elevation, $\Delta C/\Delta \phi$, is also computed in the reduction program. The C_{red} corresponds to the elevation, ϕ , of the particular data point. This elevation is increased by one mil and the point is then reduced again, for a new ballistic coefficient, C' .

thus:

$$\frac{\Delta C}{\Delta \phi} = \frac{C' - C_{\text{red}}}{1}$$

The foregoing procedure is used for obtaining $\Delta X/\Delta C$ for all the data points and $\Delta C/\Delta \phi$ for each data point whose elevation was equal to or less than 45° .

Because final computations for firing table entries make use of the ballistic coefficient for a projectile of standard weight, all C's must now be corrected to standard weight.

$$C_{\text{corrected}} = C_{\text{red}} \frac{W_{\text{standard}}}{W_{\text{observed}}}$$

By comparing the ballistic coefficient used for determining muzzle velocity (C_{vel}) with the corrected ballistic coefficient determined from the reduction (C_{red}) we can judge the significance of any error in the C_{vel} .

$$\Delta V = \frac{d(\Delta V)}{dC} \cdot \Delta C$$

$$\Delta C = |C_{\text{vel}} - C_{\text{corr}}|$$

If this ΔV , the velocity error due to using an incorrect C_{vel} , is greater than 0.5 meters per second, muzzle velocity must be recomputed using the ballistic coefficient obtained from the reduction. In actual practice, it is seldom, if ever, necessary to recompute muzzle velocity.

ϕ , up to this point in the computations, has been defined as the angle of elevation of the gun. The angles listed in a firing table, however, are angles of departure of the projectile. Although the difference between the angle of elevation of the gun and the angle of departure of the projectile is small, it does exist and is defined as vertical jump. The shock of firing causes a momentary vertical and rotational movement of the tube prior to the ejection of the projectile. This

motion changes the angle of departure of the projectile from the angle of elevation of the static gun. Because jump depends mainly on the eccentricity of the center of gravity of the recoiling parts of the gun with respect to the axis of the bore, it varies from weapon to weapon and from occasion to occasion. In modern weapons, vertical jump is usually small. For this reason, being only a minor contributing factor to range dispersion, jump is not considered in the field gunnery problem.

It is considered desirable, however, when computing data for a firing table, to compute jump for the particular weapon, fired on the specific occasion for obtaining firing table data. Thus the firing table data will be published for true angle of departure, and the jump of a weapon in the field will not be added to the jump of the weapon used for computing the table.

As mentioned earlier, the ballistic coefficient, considered as a function of angle of departure, is a constant up to an angle of 45° ; a variable, due to large summital yaws, for angles greater than 45° . Having determined the corrected ballistic coefficients for all points, we now determine the best value to use for the constant C for $\phi \leq 45^\circ$ (\bar{C}).^{*} The desired \bar{C} is a function of angle of departure, not of elevation. Hence, it is necessary to take into account the effect of jump (J) on \bar{C} . The \bar{C} and J obtained will be used to compute or will influence, the range listed in the firing table. In the computation for \bar{C} and J , the objective being to match observed data, range error (ΔX) must be minimized.

^{*} All rounds fired with nonstandard propellant temperature are excluded from the computations for the fits of ballistic coefficient and jump.

A least squares method is used for the function, $f = \sum(\Delta X)^2$, to minimize the residuals. The error in C will be

$$\Delta C = \bar{C} - C_{\text{corr. for } J}$$

where

$$C_{\text{corr. for } J} = C^*_{\text{corr}} + (\Delta C / \Delta \phi) J.$$

The product of $\Delta X / \Delta C$, obtained above, and ΔC gives the amount of error in range due to selecting a given value for \bar{C} and J. Combining this information:

$$f = \sum_{i=1}^n \left\{ \left[\bar{C} - \left(C_{\text{corr. } i} + \frac{\Delta C}{\Delta \phi_i} \cdot J \right) \right] \frac{\Delta X}{\Delta C_i} \right\}^2,$$

where n = number of data points.

To minimize the residuals, the function is differentiated first with respect to \bar{C} and then with respect to J. Equating these partial derivatives to zero, we have two equations with two unknowns which can be solved simultaneously for \bar{C} and J.

$$(1) \left[\sum (\Delta X / \Delta C)^2 \right] \bar{C} - \left[\sum \left(\frac{\Delta C}{\Delta \phi} \right) \left(\frac{\Delta X}{\Delta C} \right)^2 \right] J = \sum C_{\text{corr}} (\Delta X / \Delta C)^2$$

$$(2) - \left[\sum (\Delta C / \Delta \phi) (\Delta X / \Delta C)^2 \right] \bar{C} + \left[(\Delta C / \Delta \phi)^2 (\Delta X / \Delta C)^2 \right] J = - \sum C_{\text{corr}} (\Delta X / \Delta C)^2 (\Delta C / \Delta \phi)$$

The value of \bar{C} , thus obtained, will be used in all computations where $\phi \leq 45^\circ$. Therefore, a check must be made for any significant difference between this value and the value used for computing muzzle velocity, C_{vel} , just as was done for C_{red} and C_{vel} .

It is assumed that the value for jump is constant for all elevations. Using this value plus the observed angle of elevation as ϕ , the angle of departure, the high angle points are now reduced for ballistic coefficient, C_{red} . The process for obtaining C_{red} , and other factors,

* $C_{\text{corr}} = C_{\text{red}} \left(\frac{Wt_{\text{standard}}}{Wt_{\text{observed}}} \right)$ as described on page 35.

for each high angle point, is the same as that for low angle points (of course eliminating the \bar{C} and J computations). C_{corr} is then computed for each point:

$$C_{\text{corr}} = C_{\text{red}} (W_{\text{standard}}/W_{\text{observed}}).$$

The next step is to fit these values of C_{corr} as a function of φ , where $\varphi \geq 45^\circ$, satisfying the following three conditions.

- (1) The equation must be a quadratic of the form:

$$\hat{C} = a_0 + a_1 \varphi + a_2 \varphi^2.$$

- (2) The value of $C = f(\varphi)$ must equal \bar{C} at the point of juncture (φ^*).

$$\bar{C} = a_0 + a_1 \varphi^* + a_2 \varphi^{*2}.$$

- (3) The slope of $C : f(\varphi)$ must be zero at the point of juncture.

$$0 = a_1 + 2 a_2 \varphi^*.$$

From (3), $a_1 = -2 a_2 \varphi^*$. Substituting this in (2), $a_0 = \bar{C} + a_2 \varphi^{*2}$.

Substituting both of these in (1), $\hat{C} = \bar{C} + a_2 (\varphi^{*2} - 2 \varphi^* \varphi + \varphi^2)$

The error (ΔC) between the observed value (C_{corr}) and the computed value (\hat{C}) must now be minimized. A least squares technique is again used, with the function

$$F = \sum \left\{ C_{\text{corr}} - \left[\bar{C} + a_2 (\varphi^{*2} - 2 \varphi^* \varphi + \varphi^2) \right] \right\}^2$$

The partial derivative of F with respect to a_2 is equated to zero. Thus a_2 can be obtained, hence a_0 and a_1 from the previous equations. In this manner, the equation for ballistic coefficient for high angle fire is obtained.

$$C = f(\varphi) = a_0 + a_1 \varphi + a_2 \varphi^2$$

A computation is now made to determine the magnitude of the range error due to using angle of elevation plus jump, and the appropriate ballistic coefficient. For each low angle point, the range is computed using $(\varphi_0 + J)$ and \bar{C} . Then,

$$|x_{[\text{observed}]} - x_{[\varphi + J, \bar{C}]}| - PE_R = \text{Range Error}$$

For each high angle point, the range is computed using $(\varphi_0 + J)$ and $C = f(\varphi)$. Then,

$$|x_{[\text{observed}]} - x_{[\varphi + J, C = f(\varphi)]}| - PE_R = \text{Range Error}$$

A tabular summary, of the foregoing explanation of the determination of ballistic coefficient, may be helpful in understanding this procedure.

Steps in the Computation of Final Values for Ballistic Coefficient

1. First approximation: $C = W/d^2$

By an iterative process, one obtains:

2. C_{red} (for all points)

In the same portion of the program, one obtains:

$\Delta X/\Delta C$ (for all points)

$\Delta C/\Delta \varphi$ (for points: $\varphi \leq 45^\circ$)

3. $C_{\text{corr}} = C_{\text{red}} (W_{\text{standard}}/W_{\text{observed}})$

4. \bar{C} and J (for points: $\varphi \leq 45^\circ$)

Using: $\Delta X/\Delta C$, $\Delta C/\Delta \varphi$ and C_{corr} obtained above.

In the same portion of the program, one obtains:

comparison of C_{vel} and \bar{C} (using \bar{C} , $\varphi + J$)

5. New C_{red} (for points: $\varphi > 45^\circ$)

Using: $\varphi = (\text{observed angle of elevation}) + J$

6. C_{corr} (for points: $\varphi > 45^\circ$)

7. Fit of $C_{corr} = f(\varphi) = \hat{C}$

By minimizing $\Delta C (= C_{corr} - \hat{C})$

8. $C = f(\varphi)$ (for points: $\varphi > 45^\circ$)

9. Determination of range error due to using \bar{C} and $C = f(\varphi)$;

$\varphi_0 + J$.

b. Change in Range for Change in Elevation, Velocity and Ballistic Coefficient. For use in later computations, three other quantities are computed in the reduction program: a change in range for a one mil change in elevation, a change in range for a one meter per second change in muzzle velocity and a change in range for a one percent change in ballistic coefficient. For each of these quantities, a plus and a minus change is made to the standard value for each point ($\varphi' = \varphi \pm 1\text{mil}$, $V' = V_0 \pm 10 \text{ f/s}$, $C' = C_0 \pm 2^{-8} C$) and trajectories are computed. The difference in range (e.g. $x_{\varphi+1} - x_{\varphi-1}$) divided by the change in standard (e.g. 2 mils) yields $\Delta X/\Delta\varphi$, $\Delta X/\Delta V$ and $\Delta X/\Delta C$.

c. Drift. In addition to ballistic coefficient, deflection is obtained from the reduction program. Total deflection of the projectile, or lateral displacement from the line of fire, is caused by cross wind, rotation of the earth and drift. Drift, defined as deflection due to the spin of the projectile, is to the right of the line of fire because of the right hand twist of the rifling of the gun tube. Observed deflection is made up of all three components of total deflection; computed deflection, of only cross wind and rotation of the earth.

Therefore,

$$\text{Drift (meter)} = \text{Deflection}_{\text{observed}} - \text{Deflection}_{\text{computed}}$$

$$\text{Drift (m)} = \sin^{-1} (\text{Drift}/X_w) 6400/2\pi$$

Drift, in mils, is fitted, by a least squares technique as a function of angle of departure. The values for drift and angle are those of the points for the given charge, fired with standard propellant temperature and standard projectile weight. Several functions are fitted, and that function yielding a small root-mean-square error and a physically logical curve through the data points is the function accepted for use in the computation of the table. The functions used in the various fits follow, where D = drift, and ϕ = angle of departure.

$$D = a \tan \phi$$

$$D = a_1 \tan \phi + a_2 \tan^2 \phi$$

$$D = a \phi / (\phi + b)$$

d. Time. Still another output of the reduction program is the time of flight of the projectile to the terminal point: time to impact on the target with a point detonating fuze, time to burst with a time fuze. Due to the fallibility of the mathematical model used (particle theory), the computed time of flight (t) is less than the observed time of flight (T). Hence, for later computations, this difference in time is determined.

$$\Delta t = T - t$$

Δt is then fitted, by least squares, as a function of t. As with drift, several functions are fitted; and the best one accepted for use in the computation of the table. (Figure 3). The functions used in the various fits are:

(1) $\Delta t = a t$

Linear fits are computed to five different terminal points: the time for the elevations of 25°, 35°, 45°, 55° and 65°. With the data remaining beyond the terminal points for 25°, 35° and 45°:

$$\Delta t = a_0 + a_1 t + a_2 t^2$$

with the value of Δt and its first derivative, equal at the point of juncture.

(2) $\Delta t = a (t - t_0)^3$

where t_0 is set equal to the time corresponding to elevation of 25°, 35° and 45°; and Δt is zero below this value of t_0 .

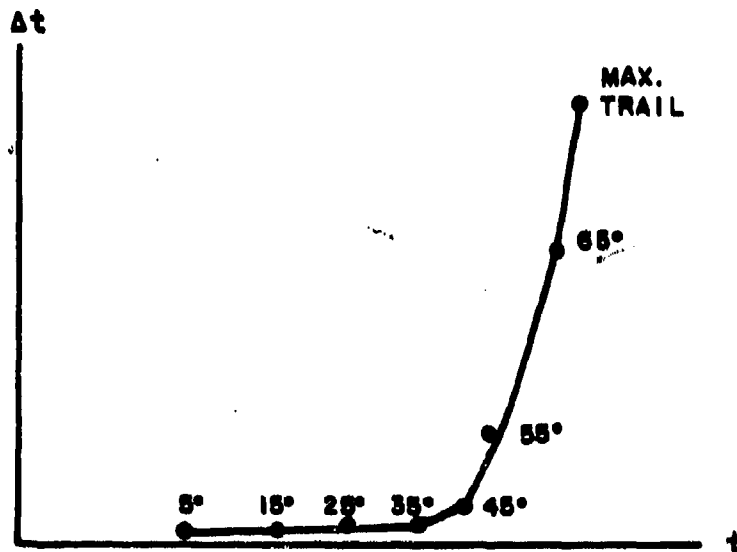


Figure 3. Difference between observed and computed times of flight as a function of computed time

3. Probable Errors, per Charge

a. Probable Error in Range to Impact. The expression used for the determination of the probable error in range to impact is:

$$(PE_R)^2 = (PE_V)^2 (\Delta X/\Delta V)^2 + (PE_\phi)^2 (\Delta X/\Delta\phi)^2 + (PE_C)^2 (\Delta X/\Delta C)^2 \quad (16)$$

where

- PE_R = probable error in range, in meters
- PE_V = probable error in velocity, in meters per second
- $\Delta X/\Delta V$ = change in range in meters for a change in velocity on one meter per second
- PE_ϕ = probable error in angle of departure, in mils
- $\Delta X/\Delta\phi$ = change in range in meters for change in angle of departure of one mil
- PE_C = probable error in ballistic coefficient, in percent
- $\Delta X/\Delta C$ = change in range in meters for change in ballistic coefficient of one percent

For the determination of PE_R , all points are used (except maximum trail points) that have been fired with standard-temperature propellant. PE_R and PE_V^* were determined in the means and probable error computations. $\Delta X/\Delta V$, $\Delta X/\Delta\phi$ and $\Delta X/\Delta C$ are computed by the following equations.

$$(1) \frac{\Delta X}{\Delta V} = \frac{x_1 - x_2}{6.096}$$

where

- x_1 = range for a velocity = $V + 3.048$ m/s
- x_2 = range for a velocity = $V - 3.048$ m/s
- (3.048 m/s = 10 f/s)

$$(2) \Delta X/\Delta\phi = \frac{x_1 - x_2}{2}$$

where

- x_1 = range for elevation = $\phi + 1$ mil
- x_2 = range for elevation = $\phi - 1$ mil

*The value of PE_V that is used in the computations is a pooled value for all the points in a given charge (see page 27).

$$(3) \Delta X/\Delta C = \frac{x_1 - x_2}{2^{-7} C}$$

where x_1 = range for ballistic coefficient = $C + 2^{-8} C$

x_2 = range for ballistic coefficient = $C - 2^{-8} C$

Therefore, in the expression for PE_R , all of the quantities are known except PE_ϕ and PE_C . These quantities are determined by a least squares technique.

Unfortunately, the values for PE_R are sometimes such that the quantity $[(PE_R)^2 - (PE_V)^2 (\Delta X/\Delta V)^2]$ is negative. Whenever this situation arises, the above quantity is set equal to zero and the least squares solution is computed. Occasionally, even this expedient results in a negative value for $(PE_\phi)^2$ or $(PE_C)^2$. When this occurs, the Powell method⁽¹⁷⁾ is used for obtaining the coefficients for a minimum of the function:

$$[(PE_R)^2 - (PE_V)^2 (\Delta X/\Delta V)^2] - (PE_\phi)^2 (\Delta X/\Delta \phi)^2 + (PE_C)^2 (\Delta X/\Delta C),$$

forcing $(PE_\phi)^2$ and $(PE_C)^2$ to be positive.

b. Probable Error in Deflection. Probable error in deflection is a function of both range and angle of departure. The least squares solution of the following function is accepted for use in the computation of the table.

$$PE_D = a x / \cos \phi$$

where PE_D = probable error in deflection, in meters

a = a constant, different for each charge

x = range, in meters

ϕ = angle of departure, in mils

For each charge, all points are used that have been fired with standard-weight projectiles and standard-temperature propellant.

4. Fuze Data.

a. Fuze Setting. When the firing mission requires a time fuze, a large proportion of modern artillery weapons use the M520 fuze. Some years ago, an exhaustive study was made of all available data on this fuze. It was found that fuze setting for a graze burst* was a function of time of flight and spin. The following relationship holds for this fuze, regardless of the shell on which it is flown.

$$FS = T + (0.00129442 + 0.00021137T) S$$

where

FS = fuze setting

T = time of flight, in seconds

S = spin, revolutions per second = $V_0/n d$

V_0 = muzzle velocity, in meters per second

$1/n$ = twist, in revolutions per caliber

d = diameter, in meters

The fuze setting for time fuzes other than the M520 is assumed to be a function of the time of flight of the particular projectile under consideration. A least squares solution of the data is computed.

$$FS - T = \Delta T = a_0 + a_1 T + a_2 T^2$$

where FS = fuze setting

T = time of flight, in seconds

a_0, a_1, a_2 = constants, different for each charge.

*Graze burst is defined as burst on impact.

b. Probable Error in Time to Burst. Each time an angle is computed to hit a given target, not only are the angle, range and height of the target computed, but also:

\dot{x}_ω = the horizontal component of velocity

\dot{y}_ω = the vertical component of velocity

t = the computed time of flight.

From these data we can compute true, or observed, time of flight, T (page 41); probable error in time to burst, PE_{TB} ; probable error in range to burst, PE_{RB} ; and probable error in height of burst, PE_{HB} .

Because the ballistic coefficient has been determined from impact data, not time data, the time of flight of the projectile has an associated probable error. When a time fuze is to be employed for a graze burst, there is an associated probable error in fuze running time. Thus the probable error in time to burst is a function of the probable errors of both time of flight and fuze running time.

From the same study discussed in connection with the fuze setting for the M520 fuze, an expression for probable error in time to burst was derived.

$$PE_{TB} = \left[(PE_T)^2 + (PE_F)^2 \right]^{1/2}$$

where

PE_{TB} = probable error in time to burst

PE_T = probable error in time to impact

PE_F = probable error in fuze running time

$$= 0.065 + 0.00220 \left(1 + \frac{100}{S} \right) FS$$

S = spin, as on page 45

FS = fuze setting

For each charge, probable error in time to impact (PE_T) is fitted, by least squares, as both a linear and a quadratic function of time of flight.

$$PE_T = a T$$

$$PE_T = a_1 T + a_2 T^2.$$

The better fit is used later in the computation of the table.

The probable error in time to burst for time fuzes other than the M520 is computed by means of the same basic equation:

$$PE_{TB} = \left[(PE_T)^2 + (PE_F)^2 \right]^{\frac{1}{2}}$$

For these fuzes, however, both PE_T and PE_F are fitted as functions of time to impact.

$$PE_1 = a T$$

$$PE_1 = a_1 T + a_2 T^2.$$

Again, the better fit for each parameter is used in the computation of the table.

c. Probable Error in Height of Burst and Probable Error in Range to Burst. By using the same parameters that were used for determining PE_{TB} , probable error in height of burst (PE_{HB}), and probable error in range to burst (PE_{RB}) can be determined.

$$PE_{HB} = \left[(PE_T)^2 + (PE_F)^2 \right]^{\frac{1}{2}} \dot{y}$$

$$PE_{RB} = \left[(PE_T)^2 + (PE_F)^2 \right]^{\frac{1}{2}} \dot{x}$$

5. Illuminating Projectile.

The main portion of the average firing table is in two parts. Data on the primary, or high explosive, projectile are in the first part; data on the illuminating projectile are in the second. Because

the mission of the latter projectile is to illuminate a relatively large area, precision firing is not as essential as for the primary projectile. A less sophisticated technique provides adequate solution of the fire problem. No computations are made for means and probable errors, N-factor, change in velocity for a change in propellant temperature, compensation for the rotation of the earth, jump, drift nor change in range for a change in elevation, velocity or ballistic coefficient.

Unlike the primary (HE) rounds, which are grouped into points of ten rounds each, the illuminating projectiles are handled round by round. Although each projectile of the ten round group of illuminating projectiles is fired with the same charge, elevation and fuze setting, the heights of burst differ significantly. Hence, round by round, rather than point, reduction of the data is necessary.

For the range firing program, standard-weight projectiles and standard-temperature propellant are used for firing all angles of elevation. Meteorological data are processed in the manner previously described. All muzzle velocities for a given charge, with the exception of any obvious mavericks, are averaged. This average value is established as the standard muzzle velocity for the given charge.

Ballistic coefficients are computed as previously described, with two exceptions: the data are processed round by round, and the jump computation is omitted. Whereas the terminal point of the basic trajectories for the primary shell is on the target, the terminal point

(burst point) of the trajectories for the illuminating shell is at some optimum height above the target: usually 750 meters. Trajectories are, however, computed to impact as well as to burst height.

Because illuminating projectiles are not fired at elevations greater than 45° , and because, for a given charge, the ballistic coefficient is constant up to 45° , all individual values of the ballistic coefficient for a given charge are averaged to establish the ballistic coefficient for that charge.

Time corrections, for the difference between observed and computed time, are determined in the same manner as for the primary shell.

VI. COMPUTATIONS FOR THE TABULAR FIRING TABLE

The tabular portion of an artillery firing table contains data based on standard and nonstandard trajectories for a given weapon and combination of projectile, fuze and propelling charge. This information is essential to the successful firing of the projectile on the target. The table also presents certain other information computed from uncorrected firing data. The whole table is divided into smaller tables, some of which are used in the preparation of fire, some of which present information useful to the artilleryman in other phases of his work. The basic parameter in the solution of the fire problem is the range. As the artilleryman uses a firing table, he makes all corrections (due to meteorological conditions, projectile weight and propellant temperature) to range, not to quadrant elevation. Having determined a hypothetical range, which under standard conditions would correspond to the true range and height of the target under nonstandard conditions, he then finds in the table the quadrant elevation necessary to hit the target at that range. Thus all computations for entries in the firing table are made to enable the artilleryman to determine the requisite hypothetical range.

Following is a detailed discussion of each of the individual tables. In order to clarify the content of each table, some explanation must occasionally be made of the method by which the artilleryman solves a fire problem.

A. Table A: Line Number (page 93)

This table lists the line number of a meteorological message as a function of the quadrant elevation of the weapon system. A line number,

which represents a preselected standard height (Table V), is related to the maximum ordinate of the trajectory of the projectile in flight.

An understanding of the content and use of a meteorological message will aid in an understanding of the necessity for most of the information contained in a firing table. A NATO meteorological message is divided into two parts: the introduction containing, primarily, identification information; and the body of the message, containing meteorological information. The introduction consists of two lines broken into four groups of letters and numbers; the body of the message consists of a sequence of up to sixteen lines, each broken into two groups of six digit numbers. The various parts of a message are explained below.

SAMPLE METEOROLOGICAL MESSAGE

METS :	344983	}	Introduction
121410	037013		
002109	945071	}	Body of Message
012215	937079		
022318	933082		
032419	926084		
042620	941075		
052822	949065		
063123	960051		

MET indicates that the transmission is a meteorological message. The S indicates that the message is applicable to surface fire; an A would indicate that it was applicable to antiaircraft fire. The 3 indicates by numerical code the weapon system to which the message is applicable.

Table v. Line Numbers

Number	Standard Height (meters)
00	0
01	200
02	500
03	1000
04	1500
05	2000
06	3000
07	4000
08	5000
09	6000
10	8000
11	10000
12	12000
13	14000
14	16000
15	18000

The 1 indicates by numerical code the octant of the globe in which the message is applicable. The second group indicates the latitude and longitude of the center of the area of applicability. In this sample message: latitude $34^{\circ} 40'$, longitude $98^{\circ} 30'$. The third group indicates the period of validity of the message. In this sample: the 12th day of the month from 1400 to 1600 hours Greenwich mean time. The fourth group indicates the altitude, in tens of meters, of the meteorological datum plane (MDP) and the atmospheric pressure at the MDP. In this sample: the MDP is 370 meters above mean sea level, the atmospheric pressure is 101.3 percent of standard atmospheric pressure at sea level.

All sixteen lines of the body of the message have the same form. The initial line is identified by the first pair of digits (00) and deals with surface meteorological conditions. Each subsequent line furnishes information applicable to firings for which the maximum ordinate of the trajectory is equal to the standard height associated with the first pair of digits of the line (Table V). Because all of the lines in the body of the message have the same form, an explanation of any one line will serve. Assume that the appropriate line number is 04. The full line of the sample is: 042620 941075. The first group of numbers tells that this is line 4 applicable to a trajectory whose maximum ordinate is 1500 meters, that the ballistic wind is blowing from 2600 mils (measured clockwise from geographic North) at a speed of 20 knots. The second group tells that the ballistic air temperature is 94.1 percent of standard, and that the ballistic air density is 107.5 percent of standard.

As indicated above, each line of the body of the meteorological message contains the ballistic wind, ballistic air temperature and ballistic air density at the indicated height. When this height is zero, these quantities are the actual wind, air temperature and air density at the MDP. For other heights, there are certain effective mean values of the actual atmospheric structure which are used in conjunction with the data given in the firing table to determine the effects of the actual atmospheric structure. These mean values are computed, at the meteorological station, to apply to a trajectory having a maximum ordinate exactly equal to a particular standard height. For firings where the maximum ordinate is not equal to one of the standard heights, it has been found to be sufficiently accurate to use the ballistic wind, temperature and density computed for that standard height which is nearest to the maximum ordinate of the firing. A projectile following a trajectory whose maximum ordinate is equal to some particular standard height passes through layers of the atmosphere where winds are blowing in various directions and at various speeds. The ballistic wind for this standard height is that wind which is constant in speed and direction and which produces the same effect on the range, height and deflection of the projectile as the actual wind⁽¹⁵⁾. Definitions of ballistic air density and ballistic air temperature are essentially the same as that of ballistic wind, but differ in that there are, in these cases, no deflection effects.

Thus, if for a given firing, the quadrant elevation is known or can be reasonably inferred, Table A is used to obtain the line number, hence the ballistic atmosphere through which the projectile will fly. An

exception should be noted, however. When a projectile impacts the target on the ascending branch of its trajectory, height of target rather than maximum ordinate must be used in obtaining line number. (If neither quadrant elevation, nor height of target on an ascending trajectory is known, line number can be obtained from Table B, as a function of range and height of target.)

The entries for Table A are obtained from computing standard trajectories at discreet intervals of quadrant elevation to obtain maximum ordinates. In order to obtain the most accurate maximum ordinates, these trajectories are computed with the appropriate constant ballistic coefficient, not the fitted ballistic coefficient. Use of the fitted C gives the most accurate results for the terminal values; use of the constant C gives the most accurate results for the maximum ordinate. By means of interpolation, quadrant elevation is then listed against exact line number. For example, if the quadrant elevation for charge 7 for the 105mm projectile M1 fired from the M108 howitzer is 467m, the line number is 4⁽¹⁸⁾.

B. Table B: Complementary Range and Line Number (pages 94-95)

As mentioned above, line number is given in Table B as a function of range and height of the target. Primarily, however, this table lists range corrections corresponding to the complementary angle of site, tabulated as a function of range and height of the target above the gun.

In Figure 4, the angle to hit a range, x, on the ground is designated ϕ_0 . The angle of site to the target above the ground, T, is indicated by the dotted line. The angle resulting from adding the

angle of site to ϕ_0 is insufficient to bring about impact of the projectile on the target, T. The relatively small angular correction required to place the projectile on the target is called complementary angle of site, CS.

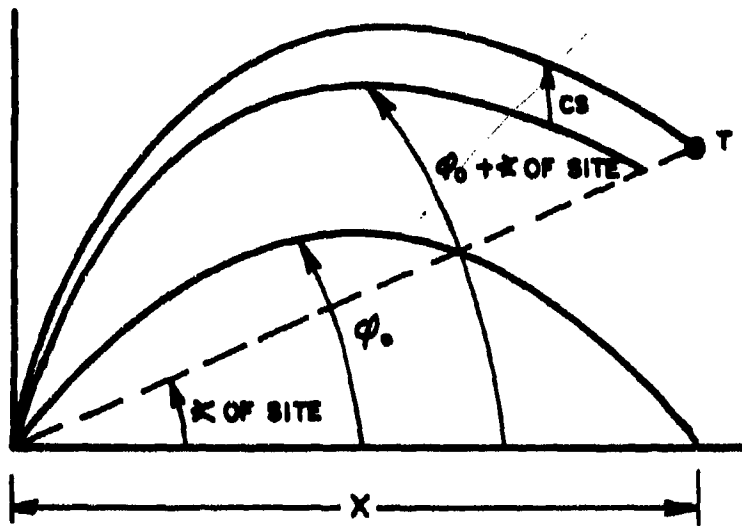


Figure 4. Complementary site. $\phi_T = \phi_0 + \text{angle of Site} + CS$

The computations for complementary angle of site are based on two-dimensional trajectories, computed using the equations of motion (without the rotation terms) and the ballistic data corresponding to the weapon system and charge. The basic equations are:

$$\ddot{x} = \frac{-\rho V K_D \dot{x}}{C}$$

$$\ddot{y} = \frac{-\rho V K_D \dot{y}}{C} - g$$

The necessary ballistic data are.

$$C = f(\omega); V_0; K_D = f(M); T = f(t)$$

and the standard ICAO atmosphere (U.S.

Standard Atmosphere, 1962)

The computation of complementary angle of site is an iterative process. Trajectories are computed with various angles until an angle is found that will result in a trajectory whose terminal point is at a given range and height. Trajectories are run to fifteen heights (from -400 meters to +1000 meters, at 100-meter intervals) for each range (at 100-meter intervals) up to maximum and back to the range at the maximum elevation of the system. Thus for any given point in space, T, ϕ_T has been computed; ϕ_0 (for T at $x_\omega, 0$) has been computed; and angle of site is known

$$\left(\tan^{-1} \frac{y_\omega}{x_\omega} \right).$$

As explained earlier, the fire problem is solved on the basis of a hypothetical range. For this reason, complementary range, not complementary angle of site, is listed in the firing table. In order to compute complementary range, complementary angle of site is added to ϕ_0 and a trajectory is run to $y = 0$.

The difference in range between the trajectory with an elevation of $\phi_0 + CS$ and that with an elevation of ϕ_0 is the complementary range (Figure 5). Thus, in Table B, the change in range to correct for the complementary angle of site is tabulated as a function of range and height of target above or below the gun.

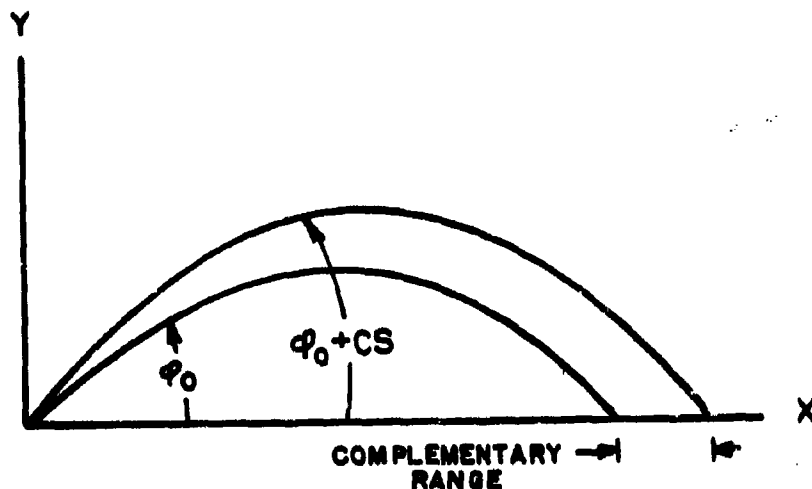


Figure 5. Complementary range

C. Table C: Wind Components (page 96)

This table resolves a wind of one knot, blowing from any chart direction, into its cross wind and range wind components. Chart direction is defined as the azimuth of the wind direction minus the azimuth of the direction of fire. Chart direction is listed from 0 mils to 6400 mils by 100-mil intervals. The cross wind component is designated right or left; the range wind, head or tail. The determination of the wind components is a simple trigonometric computation; it was done once and does not need to be repeated. Table C is the same in all firing tables.

D. Table D: Air Temperature and Density Corrections (page 97)

In this table are listed corrections which are to be added to the ballistic air temperature and the ballistic air density, obtained from the meteorological message, in order to compensate for the difference

in altitude between the firing battery and the meteorological station. The computations for this table are based on the standard ICAO atmosphere; these computations were performed once and do not need to be repeated. Table D is the same in all firing tables.

E. Table E: Propellant Temperature (page 97)

This table lists corrections to muzzle velocity as a function of propellant temperature in degrees Fahrenheit and Centigrade. The function $\Delta V = f(T - 70^\circ\text{F})$, determined as described on page 31 is evaluated for temperatures from -50°F to 130°F , at 10° intervals. Also listed in this table are the Centigrade equivalents, to the nearest 0.1° , of the Fahrenheit temperatures.

F. Table F: Ground Data and Correction Factors (pages 98-99)

1. Ground Data (page 98). This portion of Table F is divided into nine columns: range, elevation, change in elevation for 100 meters change in range, fuze setting for graze burst (for a specific time fuze), change in range for 1 mil change in elevation, fork (the change in the angle of elevation necessary to produce a change in range, at the level point, equivalent to four probable errors in range), time of flight, and two columns for azimuth corrections: drift and a cross wind of 1 knot.

The first column lists range, by 100-meter intervals, from zero to maximum range and back to the range for the maximum angle of elevation of the weapon system. In order to obtain the entries for the remaining columns, a series of trajectories is computed using the equations of motion with the appropriate values determined according to the computa-

tions described in section V, above. These trajectories are computed to find the angle of elevation to hit each range (100, 200, 300 ...) under standard conditions. From these trajectories we obtain range, elevation and time of flight (columns 1, 2 and 7). Elevation is printed in the table to the nearest 0.1 mil; time of flight, to the nearest 0.1 second. In order to compute the change in elevation for 100 meters change in range, for a given range ($x_i - 100$) is subtracted from that of the succeeding range ($x_i + 100$) and the result is divided by two:

$$(\Delta\phi/\Delta x)_i = (\phi_{(x_i + 100)} - \phi_{(x_i - 100)})/2 = A$$

The change in range for a one-mil change in elevation is derived directly from the above:

$$\Delta X/\Delta\phi = 100/A = B$$

These two entries (columns 3 and 5) are computed before the elevation is rounded to the nearest tenth of a mil. The change in elevation for a 100-meter change in range is rounded to the nearest tenth of a mil; the change in range for a 1-mil change in elevation is rounded to the nearest whole meter.

Fork (column 6) is computed from the above value and from the probable error in range, determined according to the method of section V.

$$\text{Fork} = PE_R/B \text{ (above)}$$

The fuze setting for a graze burst (column 4) is computed for the time of flight listed in column 7, by using the equations determined according to the method of section V, above.

Drift entries (column 8) are obtained by evaluating the function determined according to the method of section V at the angles of eleva-

tion listed in column 2. Computations are made before the pertinent parameters are rounded off.

Azimuth corrections due to cross wind (column 9) are computed on the basis of the following equations:

$$z_w = \frac{1017.87 W_z (T_w - x_w / \dot{x}_o)}{x_w}$$

where z_w = deflection due to cross wind (mils)
 W_z = cross wind
 T_w = time of flight
 x_w = range
 \dot{x}_o = range component of muzzle velocity

Cross wind is converted from meters to mils by the constant: 1017.87*

2. Correction Factors (page 99). This portion of Table F lists, as a function of range, the corrections to range which must be made to account for nonstandard conditions of muzzle velocity, range wind, air temperature and density, and projectile weight. Muzzle velocity corrections are tabulated for a decrease (column 10) and an increase (column 11) of one meter per second; range wind, for a decrease (column 12) and an increase (column 13) of one knot; air temperature for a decrease (column 14) and an increase (column 15) of one percent; air density for a decrease (column 16) and an increase (column 17) of one

* Radians $\times \frac{6400}{2\pi} = \text{mils}$

$\frac{6400}{2} \times (\text{factor to convert tangent to angle} = 1017.87)$

percent; and projectile weight for a decrease (column 18) and an increase (column 19) of one square.*

A series of trajectories is computed to find the elevation needed to hit each range (100, 200, 300 ...) under each of the ten nonstandard conditions. For these computations, the values of the nonstandard conditions are: ± 15 meters/second, ± 50 knots, $\pm 10\%$ air temperature, $\pm 10\%$ air density and \pm one square (or some other appropriate weight change).

Using the angles of elevation determined above, standard trajectories are next computed, and the unit range correction then determined.

$$\frac{\Delta X}{1 \text{ unit (nonstandard)}} = \frac{x(\text{standard}) - x(\text{nonstandard})}{\text{No. of units (nonstandard)}}$$

This unit range correction is listed against nonstandard range.

G. Table G: Supplementary Data (page 100)

This table lists probable error information and certain trajectory elements as functions of range. These additional data are not necessary to the solution of a given fire problem, but are useful to the artilleryman in other aspects of his work. Range is in column 1, usually listed at 1000-meter intervals.

*The weight of most artillery projectiles is indicated by the number of squares (☐) stamped on the body of the projectile. Standard weight is usually represented by two, three or four squares. This does not mean, however, one square of a standard three-square weight represents one-third of the weight. For 105mm projectiles, one square represents 0.6 pounds. Thus, two squares being standard (33.0 pounds), three squares denotes 33.6 pounds. For 175mm projectiles, one square represents 1.1 pounds. Thus, three squares being standard (147.8 pounds), two squares denote 146.7 pounds. Occasionally, the actual weight in pounds is stamped on the projectile. For projectiles of this type, firing tables are computed for pound, rather than square, increases and decreases.

Column 2 lists the elevation necessary to achieve the range listed in column 1. Columns 3 and 4 list, as functions of range, probable error in range to impact and probable error in deflection at impact. The method of computing these values was described on pages 43 through 45. Columns 5 through 7 list as functions of range, probable errors for a mechanical time fuze: probable error in height of burst, probable error in time to burst and probable error in range to burst. The method of computing these values was described on pages 46 and 47.

The angle of fall, ω (column 8), the cotangent of the angle of fall, $\text{Cot } \omega$ (column 9), and terminal velocity, V_ω (column 10) are computed from the information obtained in standard trajectories.

$$\omega = \tan^{-1} \dot{y}_\omega / \dot{x}_\omega$$

$$\text{Cot } \omega = \dot{x}_\omega / \dot{y}_\omega$$

$$V_\omega = \left[(\dot{x}_\omega)^2 + (\dot{y}_\omega)^2 \right]^{1/2}$$

Maximum ordinate, MO (column 11), is a function of ballistic coefficient, angle of departure and velocity. To determine this value, a trajectory is computed with \bar{C} , the given ϕ and V_0 ; maximum ordinate is that height where the vertical component of the velocity (\dot{y}) is zero.

$$\text{MO} = f(\bar{C}, \phi, V_0, \dot{y} = 0)$$

Angle of fall, terminal velocity and maximum ordinate are standard outputs of all trajectory computations.

The last two columns in Table G (11 and 12) list the complementary angle of site for an increase and a decrease of one mil angle of site. The computations for these values are made for an angle of site of 50

mile. An angle, φ_o , is computed to hit a target $(x_w, 0)$. Another angle φ_t , is computed to hit a target $[x_w, y = (\tan 50m) x_w]$.

$$\text{Since, } \varphi_T = \varphi_o + CS + \text{\% of site,}$$

$$\text{then, } +CS = \frac{\varphi_T - \varphi_o - 50}{50}$$

To compute $-CS$, φ_T is determined for a target $[x_w, y = -(\tan 50m) x_w]$.

$$\text{Then, } -CS = \frac{\varphi_T - \varphi_o + 50}{50}$$

Although the fire problem, in the field, is solved by determining a hypothetical range, the complementary angles of site listed in Table G make it possible to make corrections to angles. Assume the target is at a distance of 8,000 meters and a height of 500 meters. The angle of site is, therefore, 63.6m. If FT 105-AS-2 is the firing table being used, φ_o is 336.3m; $CS = (.096)(63.6) = 6.1m$. Then:

$$\varphi_T = \varphi_o + CS + \text{\% of site} = 406m$$

H. Table H: Rotation (Corrections to Range) (page 101)

Corrections to range, in meters, to compensate for the rotation of the earth, are tabulated as functions of azimuth (in mils) and range (in meters) to the target. The main body of the table is for latitude 0° . Below the main table are tabulated constants by which the corrections are to be multiplied for latitudes from 10° to 70° , by 10° intervals. These constants, cosine functions, were computed once, and do not need to be recomputed; they are the same in all tables, for all charges.

The equations used to compute the corrections to compensate for rotation are:

$$\begin{aligned} \ddot{x} &= -E \dot{x} - \lambda_1 \dot{y} \\ \ddot{y} &= -E \dot{y} - g + \lambda_1 \dot{x} \\ \ddot{z} &= -E \dot{z} + \lambda_2 \dot{x} - \lambda_3 \dot{y} \end{aligned}$$

where λ_1 , λ_2 and λ_3 = values defined on page (32),

$$E = \frac{\rho V K_D}{C}.$$

Thus for each charge, the appropriate muzzle velocity and ballistic coefficient must be used in the computations. λ_1 , λ_2 and λ_3 are computed for each of the following conditions:

1. Assume $L = 0^\circ$, $\alpha = 1600\text{m}$
then $\lambda_1 = 2 \Omega$, $\lambda_2 = 0$, $\lambda_3 = 0$
2. Assume $L = 90^\circ\text{N}$, $\alpha = 6400\text{m}$
then $\lambda_1 = 0$, $\lambda_2 = 2 \Omega$, $\lambda_3 = 0$
3. Assume $L = 0^\circ$, $\alpha = 0\text{m}$
then $\lambda_1 = 0$, $\lambda_2 = 0$, $\lambda_3 = 2 \Omega$

Trajectories are then computed for each of the above three conditions* for all of the following angles:

$\phi_1 = 2, 4, 6, 8, 10, 15 \dots 30, 40, 50, 75, 100, 125 \dots 1275, 1300$ mils.

Under condition 1, above, the range for a given angle, ϕ_1 , is designated $x1_1$. ($z1_1 = 0$, since $\lambda_2 = 0$, $\lambda_3 = 0$ and λ_1 does not affect z .)

Under condition 2, above, the range for a given angle, ϕ_1 , is designated $x2_1$; z is designated $z2_1$. [Since $\lambda_1 = 0$, $x3_1 = x_1$ (standard x).]

* The results of the computations under the third set of conditions are used for Table I, not Table H.

Under condition 3, above, the range for a given angle, α_1 , is designated $x3_1$; z is designated $z3_1$. [Since $\lambda_1 = 0$, $x3_1 = x_1$ (standard x).]

Then Δx_1 is set equal to $x2_1 - x1_1$. Interpolations are then performed in $x1_1$ to determine the Δx_1 for even intervals in range. This interval is 1000 meters for 155mm weapons and larger, 500 meters for weapons smaller than 155mm. This Δx_1 , $\Delta x1_1$ relationship gives the values for the 1600m column in Table H, for latitude 0° . The value for any given azimuth is computed as follows:

$$\Delta x = \Delta x_1 (\sin \alpha).$$

The corrections are rounded to the nearest whole meter.

I. Table I: Rotation (Corrections to Azimuth) (page 102)

Corrections to azimuth, in mils, to compensate for the rotation of the earth, are tabulated as functions of azimuth (in mils) and range (in meters) to the target. A separate table is given for each latitude from 0° to 70° , by 10° intervals. The computations described in the explanation of Table H are used for the entries in Table I.

Interpolations are performed in $x2_1$ and $x3_1$ (for 500- or 1000-meter intervals) to determine the corresponding values of $z2_1$ and $z3_1$. For a given $x2_1$ (standard range) the angular deflection is computed as follows:

$$z = z2_1 (\sin L) + z3_1 (\cos L) (\sin \alpha)$$

$$\Delta \alpha = \tan^{-1} (z/x)$$

If $\Delta \alpha$ is greater than zero, the angular deflection is from left to right; if less than zero, from right to left.

J. Table J: Fuze Setting Factors (page 103)

In this table are listed fuze setting changes, as a function of fuze setting, to compensate for the effects of the same nonstandard conditions as are given in Table F, correction factors. Fuze setting is listed, in whole numbers, from zero to the maximum setting of the given fuze.

A series of trajectories is computed to find the elevation needed to hit a given range (100, 200, 300 ...) under standard conditions. These trajectories are computed with true, or observed, time of flight (T).

$$T = t + \Delta t \quad [\Delta t = f(t), \text{ see pages 41 and 42}]$$

The time of flight corresponding to a given fuze setting, in whole numbers, is computed by means of the equations given on page 45. Interpolations are then performed in the ϕ vs T data obtained from the standard trajectories to find the angle that corresponds to the time of flight. Then, by iteration, the exact angle is determined. With this exact angle, trajectories are computed with each of the nonstandard conditions. By means of the fuze equation, the fuze setting corresponding to the time of flight of these nonstandard trajectories is computed. Thus the unit fuze setting change can be computed:

$$\frac{\Delta FS}{1 \text{ unit}} = \frac{FS(\text{nonstandard}) - FS(\text{standard})}{\text{No. of units (nonstandard)}}$$

This ΔFS is tabulated against FS.

K. Table K: ΔR , ΔH (Elevation) (page 104)

In this table are listed the change in range (ΔR) and height (ΔH), in meters, for an increase of 10 mils in elevation (Figure 6). A given

value, tabulated as a function of range and of height of the target above the gun, is the difference in the terminal coordinates of two standard trajectories having initial elevations 10 mils apart and terminating at the same time of flight.

The angle, ϕ_T , to hit a given target, was determined in the computation of Table B. ϕ_T is increased by ten mils and a second trajectory is computed which terminates with the same time of flight as the first. Thus ΔR and ΔH are differences in the terminal values of the two trajectories.

ΔR and ΔH are tabulated for every 500 or 1000 meters in range, from zero to maximum range and back to the range for the maximum angle of elevation of the weapon system, and for every 200 meters in height, from -400 to +1000 meters.

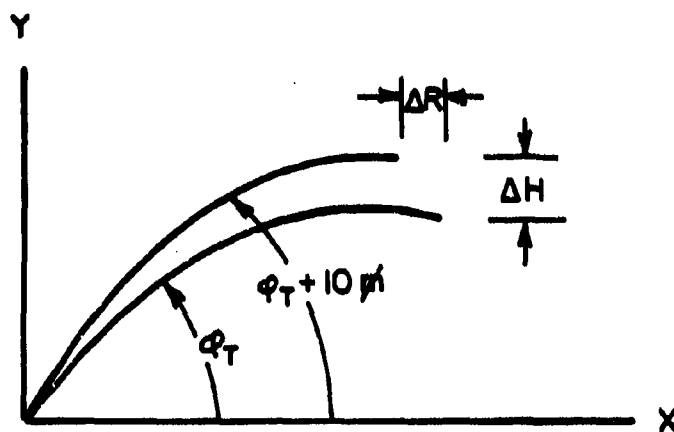


Figure 6. Change in range and height for a 10-mil increase in elevation

L. Table L: $\Delta R, \Delta H$ (Time) (page 105)

In this table are listed the change in range (ΔR) and height (ΔH), in meters, for an increase of one second in time of flight. A given value, tabulated as a function of range and of the height of the target above the gun, is the difference in the terminal coordinates between two points along a single standard trajectory at times of flight differing by one second (Figure 7).

The actual computation of this table is no more than an additional print from the iteration trajectories.

$$\Delta R = \dot{x}_\omega \text{ and } \Delta H = \dot{y}_\omega.$$

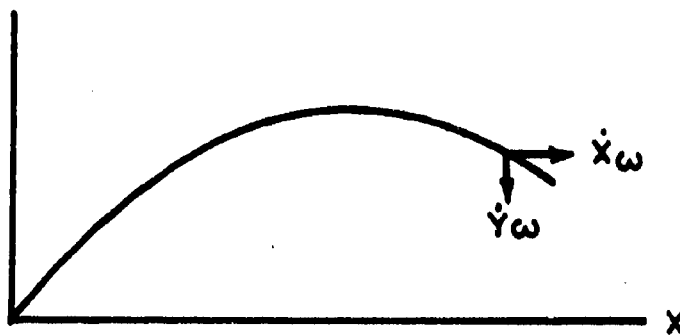


Figure 7. Change in range and height for a 1-second increase in time of flight

\dot{x}_ω and \dot{y}_ω are the horizontal and vertical components, respectively, of the velocity.

M. Table M: Fuze Setting (page 106)

As previously described on pages 59 and 60, data on fuze setting are included in Table F. The data given there are for the time fuze most often used with the systems for which the firing table has been produced. Most systems, however, are capable of functioning equally well with an alternative fuze. Data on this alternate fuze are given in Table M, which lists the amount to be added to or subtracted from the time of flight (column 7, Table F) to obtain the correct fuze setting. Corrections are listed, to the nearest 0.1 of a fuze setting opposite the time span to which they are applicable. The fuze setting-time relationship is computed as explained on page 45.

N. Illuminating Projectile (page 107)

Only one table per charge is printed for an illuminating projectile. Elevation (column 2), in mils to the nearest 0.1, and fuze setting (column 3), to the nearest 0.1 are tabulated as functions of range to burst (column 1). These relationships, which are for standard conditions, are computed in the same manner as similar computations for the primary shell.

Ranges are listed for 100-meter intervals. The shortest is that range achieved by firing at an elevation of 45° to the optimum burst height on the ascending branch of the trajectory. The longest range is that achieved by firing at an elevation of 45° to the optimum burst height on the descending branch of the trajectory (Figure 8).

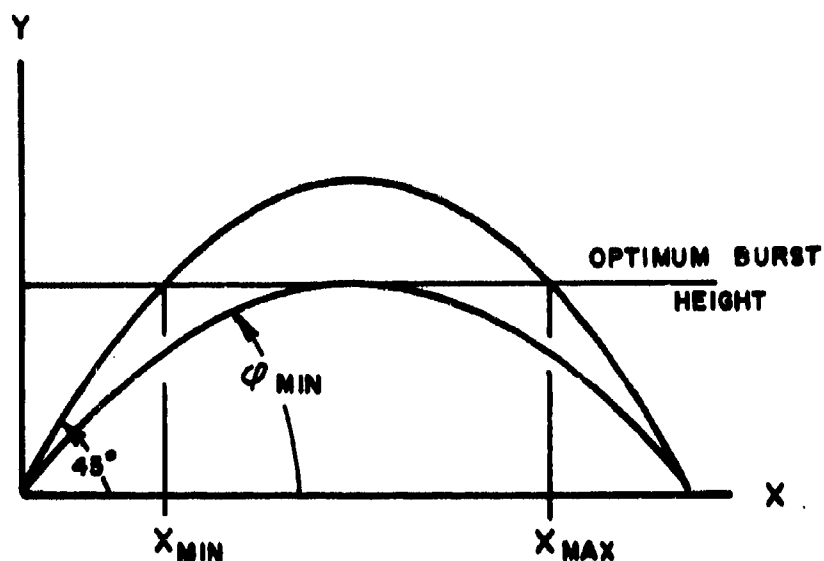


Figure 8. Trajectories of illuminating projectiles:
range-elevation relationship

Trajectories are computed to determine the minimum angle (ϕ_{\min}) to hit the optimum height. By an iteration process, angles are then computed to hit the optimum height at even intervals of range up to an angle of 800μ . The first and last entries in the table are for those angles nearest, but not greater than, 800μ which will hit an even hundred meters in range. Similarly, the minimum elevation listed will be the one nearest, but not less than, ϕ_{\min} which will hit an even hundred meters in range. Times of flight, for the listed entry ranges, are obtained from the same trajectory computations. Corrected time is determined as for the primary shell (page 41):

$$T = t + \Delta t$$

Fuze setting, as a function of time of flight, is computed from observed data. This function is then evaluated for the appropriate entries in the table.

If for some reason an illuminating projectile fails to burst, it is of concern to the artilleryman to know the point of impact of the projectile. Hence, in column 6 of this table, the range to impact is listed as a function of the range to burst. These data are computed by running the above trajectories beyond optimum burst height to zero height.

Column 4 in the table lists change in elevation for a 50-meter increase in height of burst; column 5, change in fuze setting for a 50-meter increase in height of burst. Computations for these data are made in exactly the same way as those for columns 2 and 3 except that the burst height is increased by 50 meters. The tabular entries are merely differences:

$$\Delta \phi = \phi_{H+50} - \phi_H,$$
$$\Delta FS = FS_{H+50} - FS_H,$$

Where H = optimum burst height

O. Trajectory Charts (page 108)

Following the main body of a firing table are appendices containing trajectory charts for the primary projectile for which the table has been produced. In each of these appendices, trajectories are shown for a given propelling charge. Altitude, in meters, is plotted against range, in meters, for every 100 mls of elevation, up to maximum

elevation. Time of flight, by 5-second intervals, is marked on each trajectory. For these charts, standard trajectories are computed with the requisite angle of elevation, with a print-out for every second.

VII. PUBLICATION OF THE TABULAR FIRING TABLE

A program has been written for BRLESC to interpret the coding on the output cards of the firing table computations. Thus, after the computations have been spot checked, the cards are fed back into the BRLESC. The computer spaces the numbers, pages, puts headings in the proper columns et cetera. The output goes on the IBM 1401 which tabulates the final manuscript. Transparent overlays are made for the ruling on standard pages. Pages of nonstandard length are ruled by hand. The trajectory charts described above are drawn by a Magnetic Tape Dataplotter into which are fed the output cards of the BRLESC computations.

In addition, the final manuscript contains an introduction giving an explanation of the various tables, and sample fire problems and their solutions. This introduction is punched on IBM cards, the BRLESC automatically composes the material⁽¹⁹⁾ and it is then printed in final manuscript form on the IBM 1401.

The completed manuscript is sent to the Government Printing Office for publication and distribution.

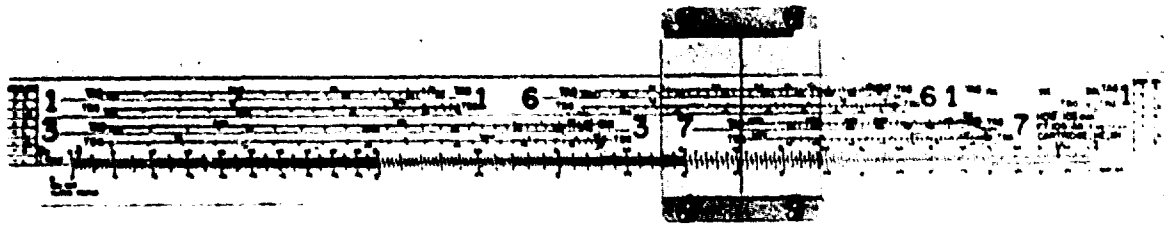
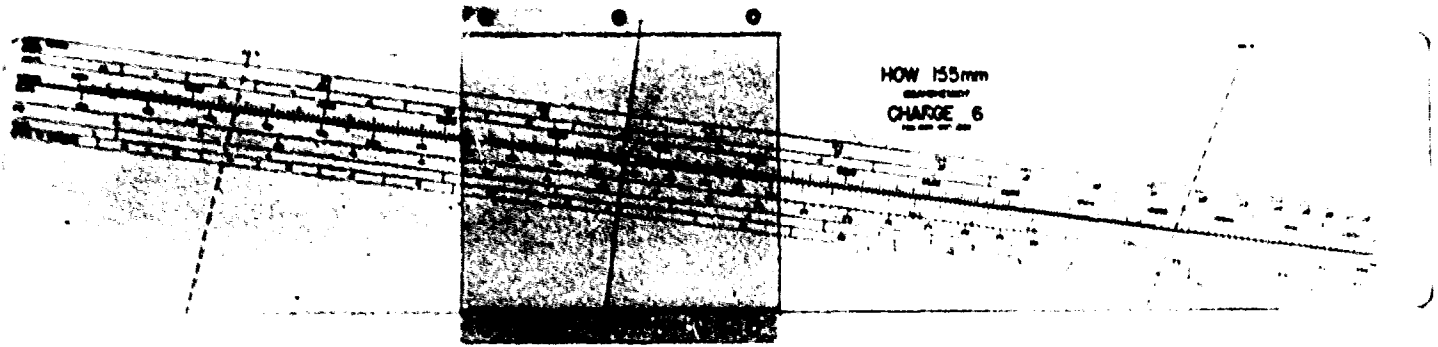
VIII. ADDITIONAL AIMING DATA

A. Graphical Equipment

The tabular firing table, described in the preceding sections, is the basic source of firing data for a given weapon system. In the field, however, the determination of aiming data is greatly simplified by the use of "graphical" equipment: a graphical firing table (GFT), a graphical site table (GST), and wind cards (Figure 9). The so-called graphical tables are sticks similar in appearance and operation to a slide rule. Wind cards, on the other hand, consist of tabular information printed on plastic cards.

Essentially, the same information is in these tables (in simplified form) as is in the tabular tables. The information is, however, in a somewhat different relationship. Hence, some additional computations are necessary. Although the Ballistic Research Laboratories are responsible for the preparation of the actual manuscript for the tabular firing table and the wind cards, they are not responsible for the design of the graphical sticks, only for the computations. The necessary numbers are sent to Frankford Arsenal (the responsible agency) and to the Artillery School, Fort Sill, for the design and manufacture of the equipment.

1. Graphical Firing Table. The graphical firing table is used principally for determining the elevation corresponding to the range to a target. Each table consists of one or more rules and a cursor, with a hairline, which slides on the rule. The range scale is the basic scale on the rule, all others being printed with reference to it. Above



HOW 155mm
CHARGE 6

EXPLANATION OF ABBREVIATIONS

LETTER	MEANING
A	ANGLE OF ELEVATION
B	BURSTING POINT
C	CHARGE
D	DRIFT
E	ELEVATION
F	FIRE
G	GRADE
H	HORIZONTAL RANGE
I	INTERVAL
J	JOB
K	KIND OF TARGET
L	LOAD
M	METHOD OF FIRING
N	NUMBER OF SHELLS
O	ORDER OF FIRING
P	POINT OF IMPACT
Q	QUANTITY OF SHELLS
R	RANGE
S	SIGHT
T	TARGET
U	UNIT
V	VARIABLE
W	WIND
X	EXTRA
Y	YARD
Z	ZERO

HOW 155mm
CHARGE 6

EXPLANATION OF ABBREVIATIONS

LETTER	MEANING
A	ANGLE OF ELEVATION
B	BURSTING POINT
C	CHARGE
D	DRIFT
E	ELEVATION
F	FIRE
G	GRADE
H	HORIZONTAL RANGE
I	INTERVAL
J	JOB
K	KIND OF TARGET
L	LOAD
M	METHOD OF FIRING
N	NUMBER OF SHELLS
O	ORDER OF FIRING
P	POINT OF IMPACT
Q	QUANTITY OF SHELLS
R	RANGE
S	SIGHT
T	TARGET
U	UNIT
V	VARIABLE
W	WIND
X	EXTRA
Y	YARD
Z	ZERO

Figure 9. Graphical firing table, graphical site table and wind cards

Best Available Copy

the range scale are a drift/deflection scale and a scale marked 100/R. This scale (100/range), representing the tangent of the angle of site for a target 100 meters above the gun, is associated with the angle-of-elevation scale.

Beneath the range scale are three additional scales: elevation, fork and fuze setting. These six scales are repeated, with the appropriate relationship, for each charge. One rule can have as many as four charges printed on it: two on each side. In addition to these six scales, other information is shown on a graphical firing table. A segment of a line between the elevation and fork scales indicates the normal range limits for the given charge. On this same line segment, two marks indicate the optimum range limits for computing meteorological corrections. On a heavy line below the fork scale, the dividing line between the data for two different charges, are two fuze setting gage points. The one to the right indicates the fuze setting at which the probable error in height of burst is 15 meters. The gage point to the left indicates the range at which the probable error in height of burst for the next lower charge is 15 meters.

The rules described above are affixed to the stick on a slant, so that the hairline of the cursor does not intersect the scales at a right angle. The slope of the rules is determined at BRL.

The foregoing description applies to a graphical firing table for low angle fire. The table for high angle fire differs somewhat. The top scale is 100/R, with below this the range scale. A heavy dividing line separates these two scales from the remainder of the rule. Below

this are scales for two or three different charges: elevation, 10 mil site, drift and time of flight. (The Fork scale is dropped, a 10 mil site scale is added, and time of flight substituted for fuze setting.) The scales on this stick are parallel to the edge of the stick.

Computations for the graphical firing table are five-point, reverse interpolations of the data computed for the tabular firing table. For low angle fire, drift is determined for each mil, at the half mil, and the corresponding range printed; for high angle fire, each mil at the whole mil. Similarly, interpolations are made for each 10 mils in elevation, for each mil at the half mil for Fork, each whole number for fuze setting, each whole second for time of flight and the nearest whole number for fuze setting gage point. $100/R$ was computed once, for each mil at the half mil, and does not need to be recomputed; it is the same, for a given range, on all graphical firing tables.

The 10 mil site computation is similar to that for complementary angle of site described on pages 63 and 64. A trajectory with a given angle, ϕ_0 , will hit a target with coordinates (x_w, o) . A trajectory with an angle ϕ_{+50} , will hit a target $[x = x_w, y = \tan 50 (x_w)]$. Thus:

$$10 \text{ mil site} = \frac{\phi_{+50} - \phi_0}{5}$$

where site = CS + $\frac{1}{2}$ of site

2. Graphical Site Table. The graphical site table is used to facilitate the computation of angle of site or of site*. It can also be used to determine the vertical interval between the gun and the tar-

*By definition, site = angle of site + complementary angle of site (see pages 55 and 56).

get (or burst point) when site, or vertical angle, and range are known. The graphical site table consists of the base, on which is printed the D scale (site and vertical interval); the slide, on which is printed yard and meter gage (index) points, the C scale (range, which can be read in yards or meters), and site-range scales, for various charges, in meters; and the cursor, with a vertical hairline. The C and D scales are identical to those on any slide rule, and are used to determine angles of site, or of site, of 100 mils or less, or to determine the vertical interval when angle of site is known. (For angles of site greater than 100 mils, the vertical angle must be determined by means of the tangent functions.)

For each charge there are two site-range scales on the slide: one in black for a target above the gun (TAG) and one in red for a target below the gun (TBG). The scales are so printed with respect to the C scale (range) that when the vertical interval (D scale) is divided by the range on the site-range scale, complementary angle of site is included in the result (site). The meter gage point (when the vertical interval is in meters), or the yard gage point (when the vertical interval is in yards), rather than the normal index point, is used for multiplication or division of the result by 1.0186 and, in effect expresses the formula ϕ (mils) = 1.0186 y/x which is more precise than the formula ϕ (mils) = y/x. This factor is 1.0186 rather than 1018.6 (6400/2 π) because the range is expressed in thousands of meters to the nearest hundred meters; e.g., 4060 meters is expressed as 4.1.

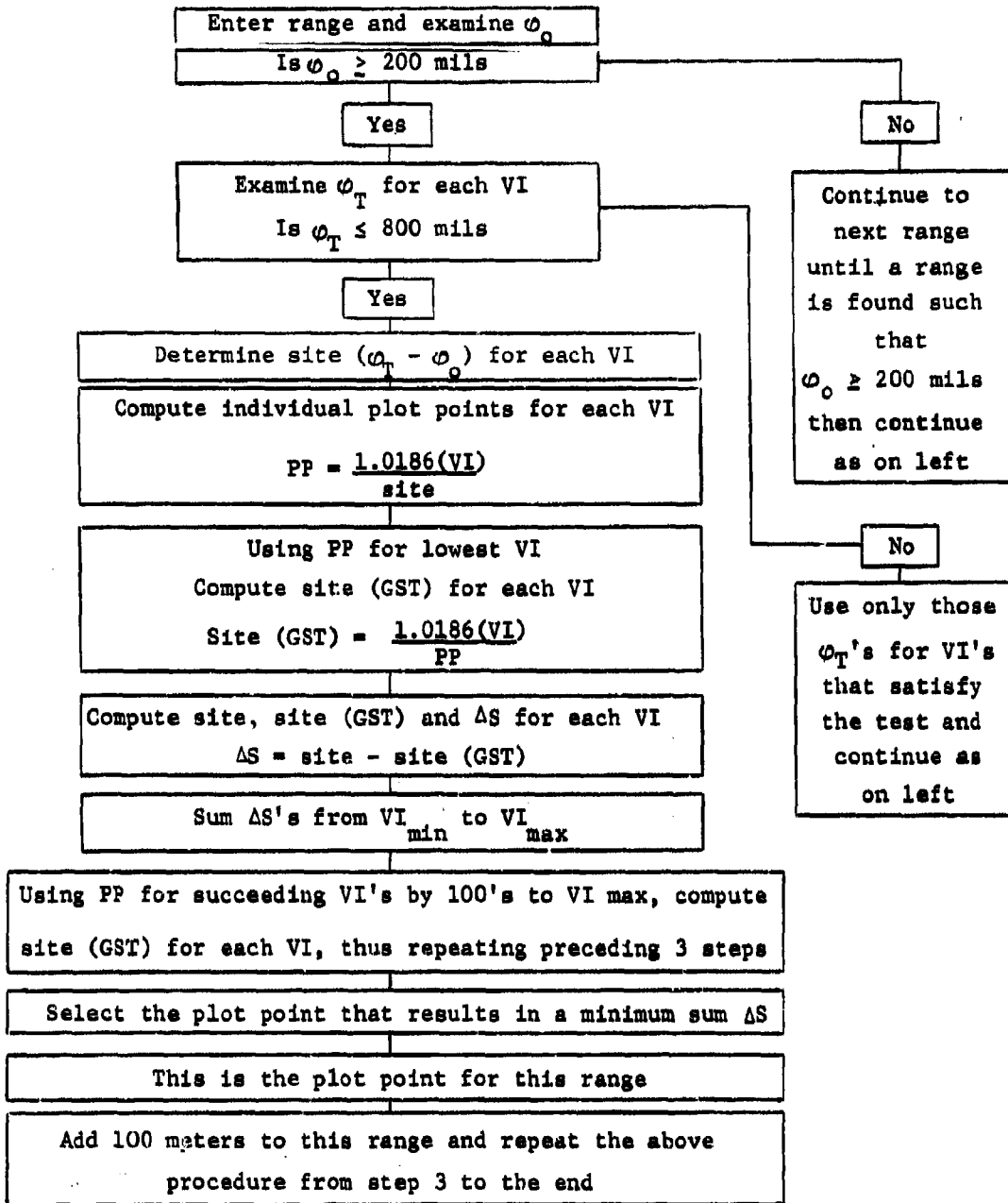
The input data for the computation of a graphical site table are the output data of the double entry program used to compute Table B in the tabular firing table. These output data consist of the angles to hit targets at given ranges and at given heights (vertical intervals) below or above the gun.

Graphical site table computations are made for low-angle fire for each 100-meter interval of range up to maximum range. Computations begin with that range, at the level point, that can be reached by an angle equal to, or greater than, 200 mils. Computations are made for angles equal to, or less than, 800 mils.

The outputs of the computations are plot points: those values on the C-scale of the stick that align with the appropriate range on the TBG (target below gun) or TAG (target above gun) scale. The computational procedure is shown in the flow chart on page 81. Computations are identical, but made independently, for targets below and above the gun.

3. Wind Cards. Wind cards enable the artilleryman to transfer fire, without lateral limitations, from one target to another. The accuracy of lateral transfers is dependent on the rotation of the earth, wind direction and wind speed. A correction for the rotation of the earth is omitted from the graphical solution of the fire problem. Wind cards, however, provide a rapid means of determining corrections to deflection, fuze setting and range when the chart direction of the wind, and the range and direction of the new target are known.

For a given firing table, there is one wind card for each charge: one side of the card is designated Wind Card A; the other, Wind Card B.



The cards contain corrections for a one-knot wind, blowing from the chart direction, divided into two components - the unit deflection correction (Wind Card A) perpendicular to the line of fire, and the unit range correction (Wind Card B) parallel to the line of fire. Corrections to fuze setting are also included on Wind Card B. Corrections to deflection and range are tabulated as functions of range and wind direction; corrections to fuze setting are tabulated as functions of fuze setting and wind direction. Line number is also listed on both Wind Card A and Wind Card B, as a function of range, so that the artilleryman can select the appropriate line of the meteorological message containing wind direction and speed.

Wind card data are computed by using the range, azimuth and fuze setting corrections for wind from the tabular firing table data and applying appropriate trigonometric functions for the chart direction of the wind. Given a wind speed, W , with some azimuth, Θ_1 , and an azimuth of fire, α , (Figure 10) range, azimuth and fuze setting corrections for the wind can be computed from the tabular firing table data. For a given range, the correction to range for a one-knot head wind ($\Delta x/W_h$), the correction to range for a one-knot tail wind ($\Delta x/W_t$) and the change in azimuth per one-knot cross wind ($\Delta \alpha/W_z$) are obtained. For a given fuze setting, the correction to fuze setting for a one-knot head wind ($\Delta FS/W_h$) and the correction to fuze setting for a one-knot tail wind ($\Delta FS/W_t$) are obtained.

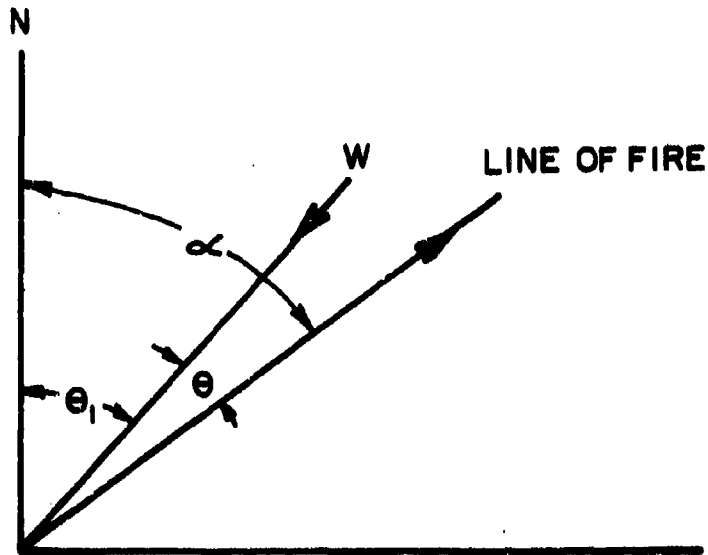


Figure 10. Relation between azimuth of fire and azimuth of wind

The necessary computations are made in the following sequence:

$$1) \quad \Theta = \Theta_1 - \alpha$$

$$2) \quad W_x = W \cdot \cos \Theta \qquad W_z = W \cdot \sin \Theta$$

$$3) \quad \frac{\Delta x}{W_x} = .5 \left(\frac{\Delta x}{W_h} - \frac{\Delta x}{W_t} \right)$$

$$4) \quad RC = \frac{\Delta x}{W_x} \cdot W_x \qquad AC = \frac{\Delta \alpha}{W_z} \cdot W_z$$

$$5) \quad RC = \frac{\Delta x}{W_x} (W \cdot \cos \Theta) \qquad AC = \frac{\Delta \alpha}{W_z} (W \cdot \sin \Theta)$$

$$6) \quad RC = W \left(\frac{\Delta x}{W_x} \cdot \cos \Theta \right) \qquad AC = W \left(\frac{\Delta \alpha}{W_z} \cdot \sin \Theta \right)$$

$$7) \quad \frac{\Delta FS}{W_x} = .5 \left(\frac{\Delta FS}{W_h} - \frac{\Delta FS}{W_t} \right)$$

$$8) \quad FSC = \frac{\Delta FS}{W_x} \cdot W_x$$

$$9) \text{ FSC} = \frac{\Delta \text{FS}}{W_x} (W \cdot \cos \Theta)$$

$$10) \text{ FSC} = W \left(\frac{\Delta \text{FS}}{W_x} \cdot \cos \Theta \right)$$

The following are those numbers which are tabulated on the wind cards:

$$11) \frac{\Delta x}{W_x} \cdot \cos \Theta$$

$$12) \frac{\Delta \alpha}{W_z} \cdot \sin \Theta$$

$$13) \frac{\Delta \text{FS}}{W_x} \cdot \cos \Theta$$

where $\Theta = 0, 200, 400 \dots 6400$.

For the definition of terms, see Table of Symbols and Abbreviations, pages 7-9.

4. Graphical Equipment for Illuminating Projectiles. The graphical firing table for the illuminating shell is a stick with a cursor and hairline, one rule per charge. (Figure 11). As on the GFT for HE projectiles, the basic scale is the range. Above this is the 100/R scale. Below the range scale, and on the same base line, are angles of elevation for impact at any given range on the range scale. This combination scale serves the same purpose as column 6 (Range to Impact) in the tabular firing table. It is essentially zero height of burst on the descending branch of the trajectory.

The major portion of the rule consists of nine horizontal scales, representing heights of burst from 600 meters to 1000 meters at 50 meter intervals. On each horizontal scale, angles of elevation are marked off up to 800 mils. At the bottom of the rule is fuze setting, in whole numbers. From each fuze setting, a curved line is printed to intersect the nine horizontal scales at the appropriate elevation.

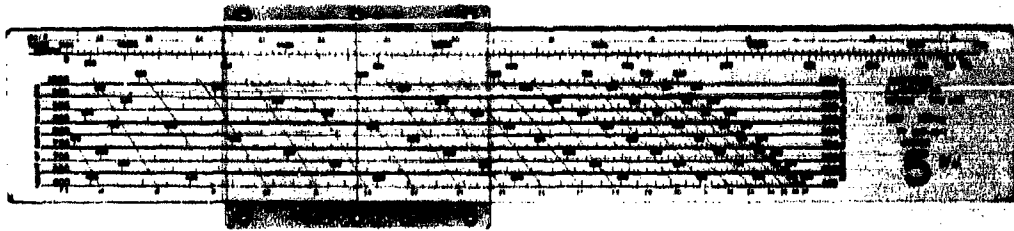


Figure 11. Graphical firing table for illuminating projectile

For each of the heights of burst (600, 650, 700...1000 meters), trajectories are computed for quadrant elevations, at 10-mil intervals, from 800 mils down to the lowest elevation that will hit the desired height of burst (800, 790, 780 ...). For each angle of elevation, a range and fuze setting are printed, for the given height, on both the ascending and descending branch of the trajectory (Figure 12).

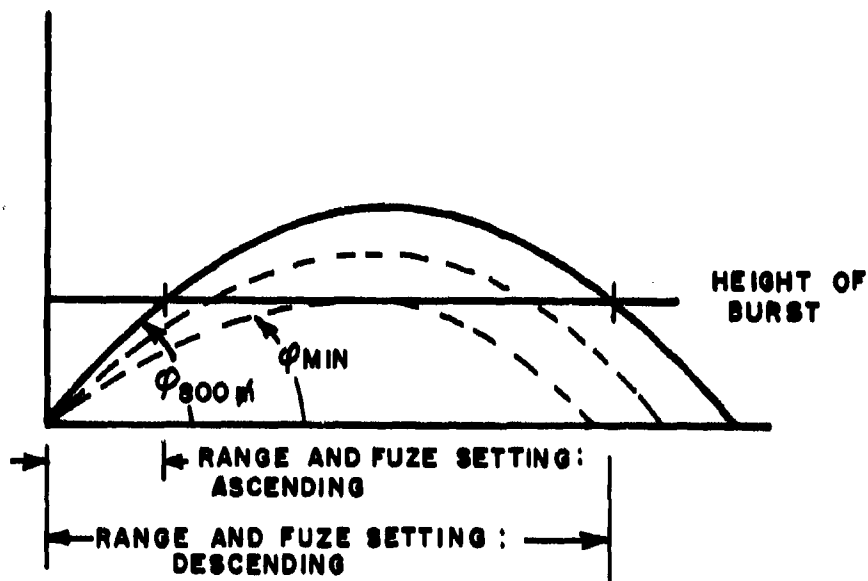


Figure 12. Trajectories of illuminating projectiles: range-fuze relationship

From these data, a fuze setting is interpolated for each whole number and printed with its corresponding range to burst. When an illuminating projectile is to be fired in the field, the lowest possible charge is used in order to reduce the possibility of ripping the parachute when the flare is ejected from the shell. Consequently, when the scales are printed for the illuminating projectile, the values for the ascending branch of the trajectory, for the high charges, are omitted. This is because the same range and height can be reached by a lower charge on the descending branch of the trajectory.

For zero height of burst data (range to impact), trajectories are computed to $y_w = 0$ for quadrant elevations up to 800 mils by 10-mil intervals.

B. Reticles and Aiming Data Charts

A ballistic reticle is a direct-fire aiming device, incorporated into a telescope which is mounted as an integral unit with the gun (Figure 13).

A line, approximately vertical, on the reticle represents the azimuth of fire. The slight deviation from the vertical is a correction for drift. Horizontal lines on the reticle represent distances to the target in meters or yards. In use, the weapon is traversed, and elevated or depressed so that the vertical line and the appropriate range line are on the target.

As in the case of the graphical firing table and the graphical site table, BRL is responsible only for the computation of the data for reticles, not for their design. Thus the data computed for the tabular firing table is all that is required. Range (at 50-meter

90 AP M318A1

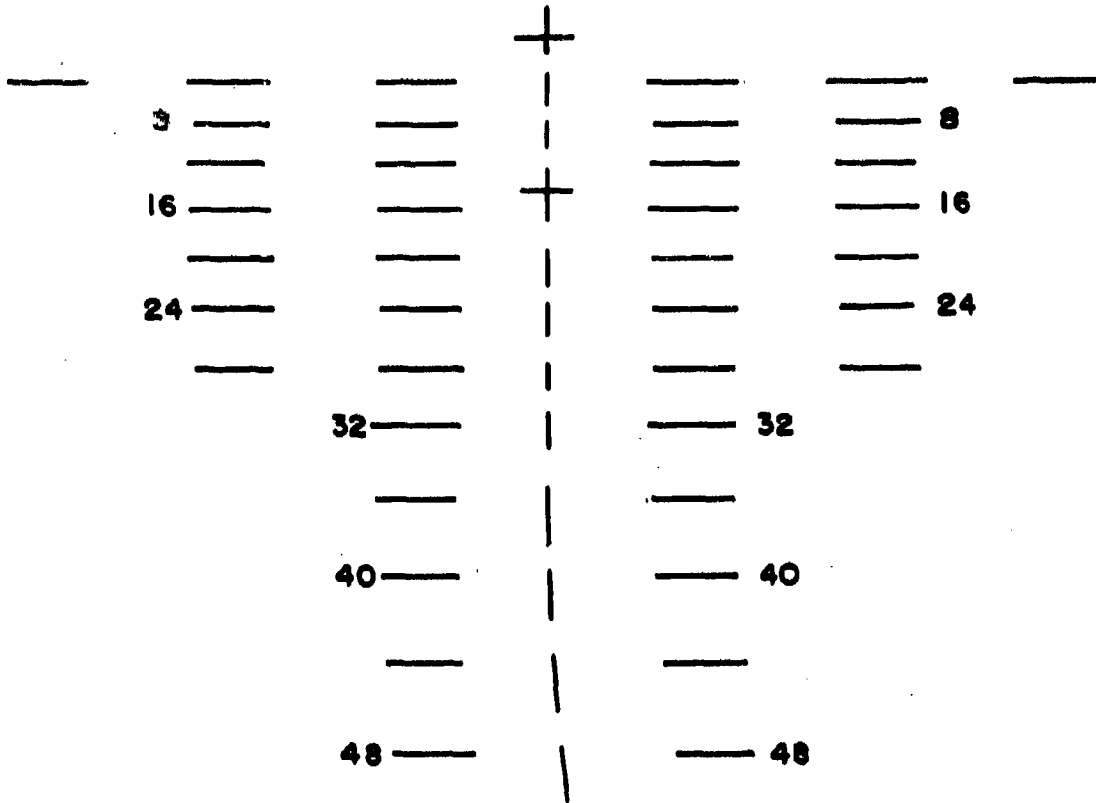


Figure 13. Reticle

intervals), elevation and drift values (up to a range designated by Frankford Arsenal) are sent to Frankford Arsenal (the agency responsible for the design and manufacture of reticles).

Reticles are graduated for use with a particular weapon and ammunition. Compensation must be made for differences in other projectiles fired from the same weapon. Hence, an aiming data chart is provided for making the necessary corrections. On an aiming data chart may be listed two or more projectiles which can be fired from the given weapon. Listed under each projectile are a series of ranges. Also listed are the ranges etched on the reticle. In use, the gunner finds the type of ammunition to be fired, reads down the column to the range at which he will fire, moves across to the reticle column to find the corresponding standard setting. This point in the telescope is used to aim the weapon on the target.

Aiming data charts (Figure 14) are provided by BRL in final manuscript form. For each range and mid-range (half-way between range marks) on the reticle there is a corresponding elevation. Trajectories are computed at these elevations for all of the projectiles to be included in the aiming data chart. The resulting ranges are rounded to the nearest 100 meters and listed opposite a range or mid-range point of the reticle.

TANK, 90MM GUN, M48 SERIES
 W/CANNON, M41 AND
 TELESCOPE, M105

AP-T, M318A1 and
 HEAT, T300E56 IN METERS
 ON RETICLE

ADC 90-Y-1

HE, M71	M318A1 RETICLE PATTERN	HE-T, M71A1
WP, M313		WP, M313C
2		1
3		3
5		4
7	8	5
8		7
10		8
12		9
13	16	11
15		12
17		13
19		15
20	24	16
22		18
24		19
26		21
27	32	22
29		24
31		25
33		27
35	40	29
37		30
39		32

CAL.0.30 MACHINE GUN	M318A1 RETICLE PATTERN	ELEV. MILS
1		1.2
3		2.5
4		3.7
	8	5.1
5		6.4
6		7.8
		9.3
7	16	10.8
		12.3
8		13.9
		15.6
9	24	17.3
		19.1
10		20.9
		22.8
11	32	24.8
		26.8
		28.9
12		31.1
	40	33.4
13		35.8
		38.4

The reticle pattern for Cartridge, AP-T, M318A1 may also be used to fire Cartridge, TP-T, M353.

Numbers and lines under the reticle column are those which appear on the reticle for Cartridge, AP-T, M318A1

- To use chart:
- (1) Find the type of ammunition to be fired.
 - (2) Read down to the range at which you will fire.
 - (3) Move left (or right) and find the corresponding sight setting under the reticle column.
 - (4) Use this point in the telescope to lay and fire.

Figure 14. Aiming data chart

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APPENDIX

The following sample pages are from FT 105-AS-2. charge 7.

FT 105-AS-2

TABLE A

CHARGE

CTG, HE, NI
FUZE, PD, M557

LINE NUMBER

7

LINE NUMBERS OF METEOROLOGICAL MESSAGE

QUADRANT ELEVATION MILS	LINE NUMBER
0.0- 100.1	0
100.2- 201.1	1
201.2- 313.0	2
313.1- 423.5	3
423.6- 518.5	4
518.6- 646.0	5
646.1- 803.4	6
803.5- 959.7	7
959.8-1131.2	8
1131.3-1333.0	9

NOTE - WHEN THE PROJECTILE MUST HIT THE TARGET ON THE ASCENDING BRANCH OF ITS TRAJECTORY, USE HEIGHT OF TARGET IN METERS TO ENTER THE TABLE ON PAGE XXVI TO DETERMINE LINE NUMBER.

CHARGE
7

TABLE B
COMPLEMENTARY RANGE
LINE NUMBER

FT 105-AS-2
CTG, HE, M1
FUZE, PD, M557

CHANGE IN RANGE, IN METERS,
TO CORRECT FOR COMPLEMENTARY ANGLE OF SITE

LINE NUMBERS OF METEOROLOGICAL MESSAGE

LINE NO.	RANGE METERS	HEIGHT OF TARGET ABOVE GUN - METERS							
		-400	-300	-200	-100	0	100	200	300
4	10500	-128	-99	-67	-34	0	36	75	116
	10600	-133	-102	-70	-36	0	38	78	121
	10700	-138	-106	-73	-37	0	39	82	127
	10800	-143	-110	-76	-39	0	41	85	132
	10900	-149	-115	-79	-41	0	43	90	139
5	11000	-155	-120	-82	-43	0	45	94	147
	11100	-162	-125	-86	-45	0	48	100	157
	11200	-170	-131	-90	-47	0	51	107	169
	11300	-178	-138	-95	-50	0	54	116	188
	11400	-187	-146	-101	-53	0	59	131	
	11500	-198	-155	-108	-57	0	71		

7	11500	-442	-318	-202	-96	0	75		
	11400	-464	-336	-215	-103	0	91	163	
	11300	-484	-352	-226	-109	0	99	186	255
	11200	-503	-367	-237	-114	0	106	201	284
	11100	-522	-381	-247	-120	0	111	214	305
8	11000	-539	-395	-256	-124	0	117	225	324
	10900	-556	-408	-265	-129	0	122	235	341
	10800	-573	-420	-274	-134	0	128	245	356
	10700	-589	-433	-282	-138	0	131	255	371
	10600	-606	-445	-291	-142	0	135	264	385
	10500	-622	-457	-299	-146	0	140	273	398
	10400	-638	-469	-307	-150	0	144	281	411
	10300	-653	-481	-315	-154	0	148	289	424
	10200	-669	-493	-323	-158	0	152	298	436
	10100	-684	-505	-331	-162	0	156	306	449
	10000	-700	-516	-339	-166	0	160	314	461
	9900	-715	-528	-346	-170	0	164	322	473
	9800	-730	-540	-354	-174	0	168	330	485
	9700	-746	-551	-362	-178	0	172	338	497
	9600	-761	-563	-370	-182	0	176	346	510
9500	-777	-575	-378	-186	0	180	354	522	
9400	-792	-586	-385	-190	0	184	362	533	
9300	-806	-598	-393	-194	0	188	370	545	
9200	-824	-610	-401	-198	0	192	378	557	
9100	-840	-622	-409	-202	0	196	386	569	
9	9000	-856	-634	-417	-206	0	200	394	582
			9					8	

FT 105-AS-2

TABLE B

CHARGE
7

CTG, HC, MI
FUZE, PD, M557

COMPLEMENTARY RANGE
LINE NUMBER

CHANGE IN RANGE, IN METERS,
TO CORRECT FOR COMPLEMENTARY ANGLE OF SITE
LINE NUMBERS OF METEOROLOGICAL MESSAGE

HEIGHT OF TARGET ABOVE GUN - METERS							RANGE	LINE	
400	500	600	700	800	900	1000	METERS	NO.	
159	206	255	309	366	429	499	10500	6	
167	215	268	324	387	456	534	10600		
174	226	282	343	411	489	582	10700		
183	238	298	365	442	535	691	10800		
193	253	319	395	488			10900		
206	270	346	441				11000		
221	295	393					11100	7	
243	355						11200		
							11300		
							11400		
							11500		
*****									11500
							11400		
							11300		
							11200		
							11100		
349	366						11000		
385	447	476					10900		
412	487	546	576				10800		
436	521	592	647	675			10700		
458	550	630	698	749	775	728	10600		
478	576	665	742	805	852	875	10500		
497	601	696	781	854	913	956	10400		
516	625	726	817	898	967	1022	10300		
534	648	754	851	939	1015	1080	10200		
551	670	782	884	977	1061	1135	10100		
568	692	808	916	1014	1104	1183	10000		
585	713	834	946	1050	1145	1230	9900		
601	734	859	977	1085	1185	1276	9800	8	
618	755	884	1006	1120	1225	1321	9700		
634	775	909	1035	1153	1263	1364	9600		
650	795	934	1064	1187	1301	1407	9500		
666	816	958	1093	1220	1338	1448	9400		
682	836	982	1121	1252	1375	1490	9300		
698	856	1007	1150	1285	1412	1531	9200		
714	876	1031	1178	1317	1449	1572	9100		
730	896	1055	1206	1349	1485	1612	9000		
746	916	1079	1234	1382	1521	1653	8900		
762	937	1103	1263	1414	1558	1693	8800		
8									

CHARGE

TABLE C

FT 105-AS-2

WIND COMPONENTS

CTG, ME, M1
FUZE, PD, M557

CORRECTION COMPONENTS OF A ONE KNOT WIND

CHART DIRECTION OF WIND MIL	CROSS WIND KNOT	RANGE WIND KNOT	CHART DIRECTION OF WIND MIL	CROSS WIND KNOT	RANGE WIND KNOT
0	0	H1.00	3200	0	T1.00
100	R.10	H.99	3300	L.10	T.99
200	R.20	H.98	3400	L.20	T.98
300	R.29	H.96	3500	L.29	T.96
400	R.38	H.92	3600	L.38	T.92
500	R.47	H.88	3700	L.47	T.88
600	R.56	H.83	3800	L.56	T.83
700	R.63	H.77	3900	L.63	T.77
800	R.71	H.71	4000	L.71	T.71
900	R.77	H.63	4100	L.77	T.63
1000	R.83	H.56	4200	L.83	T.56
1100	R.88	H.47	4300	L.88	T.47
1200	R.92	H.38	4400	L.92	T.38
1300	R.96	H.29	4500	L.96	T.29
1400	R.98	H.20	4600	L.98	T.20
1500	R.99	H.10	4700	L.99	T.10
1600	R1.00	0	4800	L1.00	0
1700	R.99	T.10	4900	L.99	H.10
1800	R.98	T.20	5000	L.98	H.20
1900	R.96	T.29	5100	L.96	H.29
2000	R.92	T.38	5200	L.92	H.38
2100	R.88	T.47	5300	L.88	H.47
2200	R.83	T.56	5400	L.83	H.56
2300	R.77	T.63	5500	L.77	H.63
2400	R.71	T.71	5600	L.71	H.71
2500	R.63	T.77	5700	L.63	H.77
2600	R.56	T.83	5800	L.56	H.83
2700	R.47	T.88	5900	L.47	H.88
2800	R.38	T.92	6000	L.38	H.92
2900	R.29	T.96	6100	L.29	H.96
3000	R.20	T.98	6200	L.20	H.98
3100	R.10	T.99	6300	L.10	H.99
3200	0	T1.00	6400	0	H1.00

NOTE - FOR A COMPLETE EXPLANATION OF THE USE OF THIS TABLE, SEE PAGE LVII.

CTG, HE, M1
FUZE, PD, M557

TEMPERATURE AND DENSITY CORRECTIONS

7

CORRECTIONS TO TEMPERATURE (DT) AND DENSITY (DD), IN PERCENT,
TO COMPENSATE FOR THE DIFFERENCE IN ALTITUDE,
IN METERS, BETWEEN THE BATTERY AND THE MDP

DH		0	+10-	+20-	+30-	+40-	+50-	+60-	+70-	+80-	+90-
0	DT	0.0	0.0	0.0	-0.1+	-0.1+	-0.1+	-0.1+	-0.2+	-0.2+	-0.2+
	DD	0.0	-0.1+	-0.2+	-0.3+	-0.4+	-0.5+	-0.6+	-0.7+	-0.8+	-0.9+
+100-	DT	-0.2+	-0.2+	-0.2+	-0.3+	-0.3+	-0.3+	-0.3+	-0.4+	-0.4+	-0.4+
	DD	-1.0+	-1.1+	-1.2+	-1.3+	-1.4+	-1.5+	-1.6+	-1.7+	-1.8+	-1.9+
+200-	DT	-0.5+	-0.5+	-0.5+	-0.6+	-0.6+	-0.6+	-0.6+	-0.7+	-0.7+	-0.7+
	DD	-2.0+	-2.1+	-2.2+	-2.3+	-2.4+	-2.5+	-2.6+	-2.7+	-2.8+	-2.9+
+300-	DT	-0.7+	-0.7+	-0.7+	-0.8+	-0.8+	-0.8+	-0.8+	-0.9+	-0.9+	-0.9+
	DD	-3.0+	-3.1+	-3.2+	-3.3+	-3.4+	-3.5+	-3.6+	-3.7+	-3.8+	-3.9+

- NOTES - 1. DH IS BATTERY HEIGHT ABOVE OR BELOW THE MDP.
2. IF ABOVE THE MDP, USE THE SIGN BEFORE THE NUMBER.
3. IF BELOW THE MDP, USE THE SIGN AFTER THE NUMBER.

TABLE E

PROPELLANT TEMPERATURE

VARIATIONS IN MUZZLE VELOCITY DUE TO PROPELLANT TEMPERATURE

TEMPERATURE OF PROPELLANT DEGREES F	VARIATION IN VELOCITY M/S	TEMPERATURE OF PROPELLANT DEGREES C
-40	-13.1	-40.0
-30	-11.6	-34.4
-20	-10.2	-28.9
-10	-8.9	-23.3
0	-7.7	-17.8
10	-6.6	-12.2
20	-5.5	-6.7
30	-4.4	-1.1
40	-3.4	4.4
50	-2.3	10.0
60	-1.2	15.6
70	0.0	21.1
80	1.2	26.7
90	2.6	32.2
100	4.1	37.8
110	5.7	43.3
120	7.4	48.9
130	9.4	54.4

CHARGE
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TABLE F
BASIC DATA

FT 105-AS-2
CTG, HE, M1
FUZE, PD, M557

1 R A N G E	2 E L E V	3 D ELEV PER 100 M DR	4 FS FOR GRAZE BURST FUZE M564	5 DR PER 1 MIL D ELEV	6 F O R K	7 TIME OF FLIGHT	8 AZIMUTH CORRECTIONS	
							DRIFT (CORR TO L)	CW OF 1 KNOT
							M	MIL
10500	548.2	11.7	40.0	9	8	38.8	14.0	0.70
10600	560.1	12.2	40.8	8	9	39.5	14.5	0.71
10700	572.6	12.8	41.5	8	9	40.2	15.0	0.72
10800	585.8	13.6	42.3	7	10	41.0	15.5	0.72
10900	599.9	14.6	43.1	7	10	41.8	16.1	0.73
11000	615.0	15.7	44.0	6	11	42.6	16.8	0.74
11100	631.3	17.3	45.0	6	13	43.6	17.5	0.75
11200	649.5	19.4	46.0	5	14	44.6	18.3	0.77
11300	670.1	22.7	47.2	4	17	45.7	19.3	0.78
11400	694.9	29.1	48.6	3	22	47.1	20.6	0.80
11500	728.4		50.5			48.9	22.4	0.82

11500	850.9		57.1			55.3	30.2	0.94
11400	881.1	26.2	58.7	4	22	56.8	32.6	0.98
11300	903.3	20.3	59.8	5	17	57.9	34.4	1.00
11200	921.7	17.2	60.7	6	14	58.7	36.0	1.03
11100	937.8	15.3	61.4	7	13	59.5	37.5	1.06
11000	952.3	13.9	62.1	7	11	60.1	38.9	1.08
10900	965.5	12.8	62.7	8	10	60.7	40.2	1.10
10800	977.9	11.9	63.3	8	10	61.3	41.5	1.12
10700	989.4	11.3	63.8	9	9	61.8	42.8	1.15
10600	1000.4	10.7	64.3	9	8	62.2	44.0	1.17
10500	1010.8	10.2	64.7	10	8	62.7	45.2	1.19
10400	1020.7	9.8	65.2	10	8	63.1	46.4	1.21
10300	1030.3	9.4	65.6	11	7	63.5	47.6	1.23
10200	1039.5	9.0	65.9	11	7	63.8	48.8	1.25
10100	1048.4	8.7	66.3	11	7	64.2	49.9	1.27
10000	1057.0	8.5	66.6	12	6	64.5	51.1	1.29
9900	1065.3	8.2	67.0	12	6	64.8	52.3	1.32
9800	1073.4	8.0	67.3	12	6	65.1	53.4	1.34
9700	1081.4	7.8	67.6	13	6	65.4	54.6	1.36
9600	1089.1	7.6	67.9	13	6	65.7	55.8	1.38
9500	1096.6	7.5	68.2	13	5	66.0	57.0	1.40

FT 105-A5-2

TABLE F

CHARGE
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CTG, HE, M1
FUZE, PD, M557

CORRECTION FACTORS

1	10	11	12	13	14	15	16	17	18	19
R A N G E	RANGE CORRECTIONS FOR									
	MUZZLE VELOCITY 1 M/S		RANGE WIND 1 KNOT		AIR TEMP 1 PCT		AIR DENSITY 1 PCT		PROJ WT OF 1 SQ 2 SQ STD	
	DEC	INC	HEAD	TAIL	DEC	INC	DEC	INC	DEC	INC
M	M	M	M	M	M	M	M	M	M	M
10500	18.2	-17.9	18.9	-15.1	26.3	-26.2	-36.6	37.0	23	-19
10600	18.3	-18.0		-15.3	26.6	-26.5	-37.1	37.7	23	-19
10700	18.4	-18.1		-15.6	26.8	-26.8	-37.6	38.4	24	-20
10800	18.6	-18.2		-15.8	27.1	-27.0	-38.2	39.2	25	-21
10900	18.7	-18.3		-16.0	27.3	-27.3	-38.8	40.0	26	-22
11000	18.8	-18.4		-16.2	27.5	-27.5	-39.3	40.9	27	-23
11100	18.9	-18.6		-16.4	27.6	-27.8	-40.0	42.0	28	-24
11200	19.1	-18.7		-16.6	27.7	-28.0	-40.6		29	-24
11300	19.3	-18.8		-16.9	27.5	-28.2	-41.2		31	-26
11400		-18.9		-17.1		-28.4	-41.9		32	-27
11500		-19.0		-17.3		-28.6	-42.6		33	-28

11900		-19.2		-19.0		-27.2	-48.2		36	-32
11400		-19.1		-18.9		-26.9	-47.9		37	-32
11300	19.4	-18.9		-18.9	27.4	-26.6	-47.6		37	-33
11200	19.3	-18.8		-18.8	26.8	-26.4	-47.4		37	-33
11100	19.2	-18.6		-18.7	26.4	-26.1	-47.1	43.6	37	-33
11000	19.0	-18.4		-18.6	26.0	-25.9	-46.8	43.8	37	-33
10900	18.9	-18.3		-18.6	25.7	-25.6	-46.5	43.7	37	-33
10800	18.7	-18.1		-18.5	25.4	-25.4	-46.1	43.6	37	-33
10700	18.5	-18.0		-18.4	25.1	-25.1	-45.8	43.4	37	-33
10600	18.4	-17.8		-18.3	24.9	-24.9	-45.5	43.2	37	-32
10500	18.2	-17.6	19.8	-18.3	24.6	-24.7	-45.1	43.0	37	-32
10400	18.0	-17.4	19.8	-18.2	24.4	-24.5	-44.8	42.7	36	-32
10300	17.8	-17.3	19.8	-18.1	24.1	-24.2	-44.4	42.4	36	-32
10200	17.7	-17.1	19.7	-18.0	23.9	-24.0	-44.1	42.1	36	-32
10100	17.5	-16.9	19.7	-18.0	23.7	-23.8	-43.7	41.9	36	-32
10000	17.3	-16.8	19.6	-17.9	23.4	-23.6	-43.4	41.5	36	-32
9900	17.1	-16.6	19.6	-17.8	23.2	-23.4	-43.0	41.2	36	-31
9800	17.0	-16.4	19.5	-17.8	23.0	-23.1	-42.6	40.9	35	-31
9700	16.8	-16.2	19.4	-17.7	22.7	-22.9	-42.3	40.6	35	-31
9600	16.6	-16.0	19.4	-17.6	22.5	-22.7	-41.9	40.2	35	-31
9500	16.4	-15.9	19.3	-17.6	22.3	-22.5	-41.5	39.9	35	-31

CHARGE
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TABLE G
SUPPLEMENTARY DATA

FT 105-AS-2
CTG. HE. M1
FUZE, PD, M557

1	2	3	4	5	6	7	8	9	10	11	12	13
R A N G E	E L E V	PROBABLE ERRORS					A N G L E O F F A L L	C O T A N G L E O F F A L L	T M L V E L	M O	C O M P S I T E F O R	
		R	D	F U Z E M 5 6 4							+1 MIL S I T E	-1 MIL S I T E
				M	M	M						
M	MIL	M	M	M	SEC	M	MIL		M/S	M	MIL	MIL
0	0.0	10	0	0	0.05	25	0		494	0	0.000	0.000
1000	22.4	9	0	1	0.06	24	24	41.8	433	6	0.001	0.000
2000	49.3	8	0	1	0.06	24	59	17.3	378	26	0.002	-0.002
3000	82.0	9	0	3	0.07	24	107	9.5	333	69	0.006	-0.005
4000	121.3	9	1	4	0.08	26	166	6.1	312	142	0.012	-0.010
5000	166.6	10	1	6	0.09	28	231	4.3	300	251	0.021	-0.017
6000	217.2	11	1	9	0.11	30	301	3.3	292	400	0.034	-0.028
7000	273.4	13	1	12	0.12	31	377	2.6	285	595	0.057	-0.047
8000	336.3	14	2	16	0.13	33	459	2.1	281	847	0.096	-0.079
9000	408.4	15	2	21	0.15	35	550	1.7	278	1176	0.169	-0.136
10000	495.1	16	2	27	0.16	37	653	1.3	279	1622	0.331	-0.251
11000	615.0	18	3	37	0.19	38	785	1.0	283	2311	1.101	-0.587
11500	728.4	50	4	46	0.21	38	899	0.8	289	3018		-1.432

11500	850.9	56	4	56	0.23	37	1011	0.7	296	3807		2.445
11000	952.3	20	5	65	0.24	35	1098	0.5	301	4454	-2.135	1.657
10000	1057.0	19	5	72	0.26	31	1185	0.4	305	5087	-1.402	1.327
9000	1132.2	17	6	77	0.27	28	1245	0.4	306	5505	-1.234	1.205
8000	1195.4	16	6	80	0.27	25	1295	0.3	306	5824	-1.152	1.137
7000	1251.9	14	7	82	0.28	21	1338	0.3	305	6081	-1.102	1.094
6000	1304.2	12	7	83	0.28	18	1379	0.2	304	6291	-1.068	1.064
5000	1354.0	10	7	83	0.28	15	1417	0.2	302	6463	-1.045	1.043

CIG, ME, MI
FUZE, PD, M557

ROTATION - RANGE

CORRECTIONS TO RANGE, IN METERS, TO COMPENSATE
FOR THE ROTATION OF THE EARTH

RANGE METERS	AZIMUTH OF TARGET - MILS									
	0 3200	200 3000	400 2800	600 2600	800 2400	1000 2200	1200 2000	1400 1800	1600 1600	
1000	0	-1+	-2+	-4+	-5+	-5+	-6+	-6+	-6+	
2000	0	-2+	-4+	-6+	-8+	-9+	-10+	-11+	-11+	
3000	0	-3+	-6+	-8+	-10+	-12+	-13+	-14+	-15+	
4000	0	-3+	-7+	-10+	-12+	-14+	-16+	-17+	-17+	
5000	0	-4+	-8+	-11+	-14+	-17+	-19+	-20+	-20+	
6000	0	-4+	-9+	-13+	-16+	-19+	-21+	-23+	-23+	
7000	0	-5+	-10+	-14+	-18+	-21+	-24+	-25+	-26+	
8000	0	-5+	-11+	-15+	-20+	-23+	-26+	-27+	-28+	
9000	0	-6+	-11+	-16+	-21+	-24+	-27+	-29+	-29+	
10000	0	-6+	-11+	-16+	-21+	-25+	-27+	-29+	-30+	
11000	0	-5+	-11+	-15+	-20+	-23+	-26+	-27+	-28+	
11500	0	-5+	-9+	-13+	-17+	-20+	-22+	-23+	-24+	

11500	0	-3+	-6+	-8+	-11+	-13+	-14+	-15+	-15+	
11000	0	-1+	-3+	-4+	-5+	-6+	-7+	-7+	-7+	
10000	0	0	+1-	+1-	+2-	+2-	+2-	+2-	+2-	
9000	0	+2-	+3-	+5-	+6-	+8-	+8-	+9-	+9-	
8000	0	+3-	+6-	+8-	+10-	+12-	+13-	+14-	+15-	
7000	0	+4-	+7-	+11-	+13-	+16-	+18-	+19-	+19-	
6000	0	+4-	+9-	+13-	+16-	+19-	+21-	+22-	+23-	
5000	0	+5-	+10-	+14-	+18-	+21-	+23-	+25-	+25-	
	3200	3400	3600	3800	4000	4200	4400	4600	4800	
	6400	6200	6000	5800	5600	5400	5200	5000	4800	
AZIMUTH OF TARGET - MILS										

- NOTES - 1. WHEN ENTERING FROM THE TOP USE THE SIGN BEFORE THE NUMBER.
 2. WHEN ENTERING FROM THE BOTTOM USE THE SIGN AFTER THE NUMBER.
 3. AZIMUTH IS MEASURED CLOCKWISE FROM NORTH.
 4. CORRECTIONS ARE FOR 0 DEGREES LATITUDE. FOR OTHER LATITUDES
 MULTIPLY CORRECTIONS BY THE FACTOR GIVEN BELOW.

LATITUDE (DEG)	10	20	30	40	50	60	70
MULTIPLY BY	.98	.94	.87	.77	.64	.50	.34

LIG, ME, MI
FUZE, PD, M557

ROTATION - AZIMUTH

CORRECTIONS TO AZIMUTH, IN MILS, TO COMPENSATE
FOR THE ROTATION OF THE EARTH

30 DEGREES NORTH LATITUDE

RANGE METERS	AZIMUTH OF TARGET - MILS								
	0 6400	400 6000	800 5600	1200 5200	1600 4800	2000 4400	2400 4000	2800 3600	3200 3200
1000	LO.1R	LO.1R	LO.1R	LO.1R	LO.1R	LO.1R	LO.1R	LO.1R	LO.1R
2000	LO.2R	LO.2R	LO.2R	LO.2R	LO.2R	LO.2R	LO.2R	LO.2R	LO.2R
3000	LO.2R	LO.2R	LO.3R	LO.3R	LO.3R	LO.3R	LO.3R	LO.3R	LO.3R
4000	LO.3R	LO.3R	LO.3R	LO.4R	LO.4R	LO.4R	LO.4R	LO.4R	LO.4R
5000	LO.4R	LO.4R	LO.4R	LO.5R	LO.5R	LO.5R	LO.5R	LO.5R	LO.5R
6000	LO.5R	LO.5R	LO.5R	LO.6R	LO.6R	LO.6R	LO.6R	LO.7R	LO.7R
7000	LO.6R	LO.6R	LO.6R	LO.7R	LO.7R	LO.8R	LO.8R	LO.8R	LO.8R
8000	LO.7R	LO.7R	LO.7R	LO.8R	LO.9R	LO.9R	L1.0R	L1.0R	L1.0R
9000	LO.8R	LO.8R	LO.8R	LO.9R	L1.0R	L1.1R	L1.2R	L1.2R	L1.3R
10000	LO.8R	LO.9R	LO.9R	L1.1R	L1.2R	L1.3R	L1.5R	L1.5R	L1.6R
11000	LO.9R	LO.9R	L1.0R	L1.2R	L1.4R	L1.7R	L1.8R	L2.0R	L2.0R
11500	LO.8R	LO.9R	L1.1R	L1.3R	L1.7R	L2.0R	L2.2R	L2.4R	L2.5R

11500	LO.7R	LO.8R	L1.0R	L1.4R	L1.9R	L2.3R	L2.7R	L3.0R	L3.0R
11000	LO.5R	LO.6R	LO.9R	L1.4R	L2.0R	L2.6R	L3.1R	L3.5R	L3.6R
10000	LO.1R	LO.2R	LO.7R	L1.4R	L2.2R	L3.0R	L3.6R	L4.1R	L4.3R
9000	RO.3L	RO.1L	LO.4R	L1.3R	L2.2R	L3.2R	L4.1R	L4.6R	L4.8R
8000	RO.8L	RO.6L	LO.1R	L1.1R	L2.3R	L3.5R	L4.5R	L5.2R	L5.4R
7000	R1.4L	R1.1L	RO.3L	LO.9R	L2.3R	L3.8R	L5.0R	L5.8R	L6.1R
6000	R2.1L	R1.8L	RO.8L	LO.6R	L2.4R	L4.1R	L5.5R	L6.5R	L6.9R
5000	R3.1L	R2.7L	R1.5L	LO.3R	L2.4R	L4.5R	L6.2R	L7.4R	L7.9R
	3200	2800	2400	2000	1600	1200	800	400	0
	3200	3600	4000	4400	4800	5200	5600	6000	6400

AZIMUTH OF TARGET - MILS

30 DEGREES SOUTH LATITUDE

- NOTES - 1. WHEN ENTERING FROM THE TOP USE THE SIGN BEFORE THE NUMBER.
 2. WHEN ENTERING FROM THE BOTTOM USE THE SIGN AFTER THE NUMBER.
 3. R DENOTES CORRECTION TO THE RIGHT, L TO THE LEFT.
 4. AZIMUTH IS MEASURED CLOCKWISE FROM THE NORTH.

FT 105-AS-2

TABLE J

CHARGE

CTG, HE, M1
FUZE, MTSQ, M564

FUZE SETTING FACTORS

7

1	2	3	4	5	6	7	8	9	10	11
FS	FUZE SETTING CHANGES FOR									
	MUZZLE VELOCITY 1 M/S		RANGE WIND 1 KNOT		AIR TEMP 1 PCT		AIR DENSITY 1 PCT		PROJ WT OF 1 SQ 2 SQ STD	
	DEC	INC	HEAD	TAIL	DEC	INC	DEC	INC	DEC	INC
	35	-.039	.039	-.011	.012	-.026	.033	.052	-.047	.007
36	-.040	.040	-.011	.013	-.028	.035	.054	-.049	.006	-.011
37	-.041	.041	-.012	.014	-.030	.036	.056	-.051	.004	-.010
38	-.042	.042	-.012	.014	-.032	.038	.058	-.052	.003	-.009
39	-.042	.042	-.013	.015	-.033	.040	.059	-.054	.002	-.008
40	-.043	.043	-.013	.015	-.035	.042	.061	-.055	.000	-.006
41	-.044	.044	-.014	.015	-.037	.043	.063	-.057	-.001	-.005
42	-.044	.044	-.014	.016	-.039	.045	.065	-.059	-.002	-.004
43	-.045	.045	-.015	.016	-.040	.047	.067	-.060	-.004	-.003
44	-.046	.046	-.015	.017	-.042	.048	.069	-.062	-.005	-.001
45	-.047	.047	-.016	.017	-.044	.050	.071	-.064	-.007	.000
46	-.047	.047	-.016	.018	-.045	.052	.073	-.066	-.008	.001
47	-.048	.048	-.016	.018	-.047	.053	.075	-.067	-.010	.002
48	-.049	.049	-.017	.018	-.049	.055	.077	-.069	-.011	.004
49	-.050	.050	-.017	.019	-.050	.057	.079	-.072	-.013	.005
50	-.051	.051	-.018	.019	-.052	.058	.081	-.074	-.014	.006
51	-.052	.052	-.018	.019	-.054	.060	.084	-.076	-.016	.008
52	-.053	.054	-.018	.020	-.055	.062	.086	-.078	-.017	.009
53	-.055	.055	-.019	.020	-.057	.063	.089	-.080	-.019	.010
54	-.056	.056	-.019	.020	-.058	.065	.092	-.083	-.021	.012
55	-.057	.057	-.019	.021	-.060	.067	.095	-.085	-.022	.013
56	-.058	.059	-.020	.021	-.061	.068	.098	-.088	-.024	.015
57	-.060	.060	-.020	.021	-.063	.070	.101	-.091	-.025	.016
58	-.061	.061	-.020	.021	-.065	.072	.104	-.093	-.027	.018
59	-.062	.063	-.020	.021	-.066	.074	.107	-.096	-.029	.020
60	-.064	.064	-.021	.021	-.068	.075	.110	-.099	-.032	.021
61	-.065	.066	-.021	.022	-.070	.077	.113	-.102	-.034	.023
62	-.067	.067	-.021	.022	-.072	.079	.117	-.105	-.037	.025
63	-.068	.069	-.021	.022	-.073	.081	.120	-.108	-.039	.027
64	-.070	.070	-.021	.022	-.075	.084	.124	-.112	-.042	.029
65	-.071	.072	-.021	.021	-.077	.086	.128	-.115	-.044	.032
66	-.073	.073	-.021	.021	-.079	.088	.132	-.118	-.047	.034
67	-.074	.075	-.021	.021	-.082	.090	.136	-.122	-.050	.037
68	-.076	.076	-.021	.021	-.084	.093	.141	-.126	-.054	.041
69	-.077	.078	-.021	.020	-.086	.096	.145	-.130	-.059	.044
70	-.079	.079	-.020	.019	-.089	.099	.150	-.134	-.063	.049

CHARGE
7

TABLE K
DR, DH (ELEVATION)

FT 105-AS-2
CTG, HE, M)
FUZE, PD, M557

CHANGE IN RANGE (DR) AND HEIGHT (DH), IN METERS,
FOR AN INCREASE OF TEN MILS IN ELEVATION

RANGE METERS	HEIGHT OF TARGET ABOVE GUN - METERS								
	-300	-200	0	200	400	600	800	1000	
7500	DR	-8.1	-10.4	-12.6	-15.2	-17.7	-20.2	-22.8	-25.5
	DH	73.8	74.1	74.4	74.6	74.9	75.2	75.5	75.7
8000	DR	-10.6	-13.1	-15.5	-18.1	-20.6	-23.3	-26.0	-28.9
	DH	78.9	79.2	79.5	79.8	80.1	80.4	80.7	81.0
8500	DR	-13.6	-16.2	-18.8	-21.4	-24.2	-27.1	-30.0	-33.0
	DH	84.1	84.4	84.8	85.1	85.4	85.7	86.1	86.4
9000	DR	-17.2	-19.9	-22.7	-25.5	-28.5	-31.6	-34.8	-38.1
	DH	89.3	89.7	90.0	90.4	90.7	91.1	91.4	91.8
9500	DR	-21.3	-24.5	-27.4	-30.5	-33.8	-37.2	-40.9	-44.8
	DH	94.6	94.9	95.3	95.7	96.1	96.5	96.9	97.2
10000	DR	-26.0	-30.0	-33.3	-36.9	-40.6	-44.6	-49.0	-53.8
	DH	99.0	100.3	100.7	101.1	101.6	102.0	102.4	102.8
10500	DR	-33.4	-37.1	-41.0	-45.3	-50.0	-55.2	-60.8	-67.7
	DH	105.2	105.7	106.2	106.7	107.1	107.6	108.1	108.6
11000	DR	-42.4	-47.0	-52.2	-58.0	-64.9	-74.1		
	DH	110.7	111.2	111.8	112.3	112.8	113.4		
11500	DR	-56.4	-63.7	-74.0					
	DH	115.2	116.9	117.5					

11500	DR	-120.9	-113.9	-103.5					
	DH	113.0	114.4	116.2					
11000	DR	-139.9	-135.9	-131.3	-126.0	-119.5	-110.4		
	DH	104.6	105.7	106.8	108.1	109.4	111.1		
10500	DR	-153.4	-150.2	-146.8	-143.1	-139.1	-134.5	-129.2	-122.7
	DH	96.5	97.6	98.7	99.7	100.7	101.9	103.1	104.4
10000	DR	-164.1	-161.5	-158.5	-155.5	-152.3	-148.9	-145.1	-140.9
	DH	88.6	89.5	90.6	91.6	92.6	93.5	94.5	95.7

FT 105-AS-2

TABLE L

CHARGE

CTG, HE, M1
FUZE, PD, M957

DR, DH (TIME)

7

CHANGE IN RANGE (DR) AND HEIGHT (DH), IN METERS,
FOR AN INCREASE OF ONE SECOND IN TIME OF FLIGHT

RANGE METERS	HEIGHT OF TARGET ABOVE GUN - METERS							
	-400	-200	0	200	400	600	800	1000
7500 DR	259.3	259.5	259.3	258.9	258.0	256.9	255.4	253.6
DH	-125.0	-118.7	-112.5	-106.5	-100.6	-95.0	-89.6	-84.5
8000 DR	253.0	253.0	252.7	252.1	251.2	250.0	248.4	246.5
DH	-133.3	-127.7	-122.3	-117.0	-111.9	-107.0	-102.4	-98.0
8500 DR	246.5	246.3	245.9	245.2	244.1	242.8	241.1	239.2
DH	-142.0	-137.1	-132.4	-127.8	-123.5	-119.3	-115.5	-111.9
9000 DR	239.8	239.5	238.9	237.9	236.7	235.2	233.4	231.3
DH	-151.3	-147.1	-143.1	-139.2	-135.6	-132.3	-129.2	-126.4
9500 DR	232.8	232.2	231.4	230.3	228.8	227.1	225.0	222.5
DH	-161.3	-157.7	-154.4	-151.3	-148.5	-145.9	-143.8	-142.0
10000 DR	225.3	224.5	223.3	221.9	220.1	217.9	215.2	212.0
DH	-172.0	-169.3	-166.7	-164.5	-162.5	-161.0	-160.0	-159.6
10500 DR	217.0	215.8	214.2	212.2	209.7	206.5	202.6	197.5
DH	-184.0	-182.1	-180.5	-179.3	-178.7	-178.7	-179.8	-182.3
11000 DR	207.4	205.4	202.9	199.6	195.1	188.5		
DH	-197.7	-197.0	-197.0	-197.8	-199.9	-204.7		
11500 DR	194.5	190.3	183.5					
DH	-215.4	-217.6	-223.0					

11500 DR	147.5	153.5	161.8					
DH	-267.2	-259.2	-248.1					
11000 DR	134.1	138.1	142.4	147.2	153.0	160.6		
DH	-277.2	-271.7	-265.4	-258.2	-249.6	-238.3		
10500 DR	123.8	127.1	130.5	134.1	138.0	142.2	147.1	152.9
DH	-283.5	-279.0	-273.9	-268.2	-262.0	-254.9	-246.9	-237.2
10000 DR	115.1	117.9	120.8	123.8	126.9	130.3	133.9	137.7
DH	-288.2	-284.1	-279.7	-274.8	-269.5	-263.6	-257.2	-250.0

FY 105-AS-2
CTG. HE. M1
FUZE, MTSQ, M520A1

TABLE M
FUZE SETTING

CHARGE
7

CORRECTIONS TO FUZE SETTING OF FUZE, MTSQ, M564 FOR
FUZE, MTSQ, M520A1

FUZE SETTING FUZE M564		CORRECTIONS
FROM	TO	
2.0	2.4	0.4
2.5	7.3	0.5
7.4	12.1	0.6
12.2	17.0	0.7
17.1	21.8	0.8
21.9	26.7	0.9
26.8	31.5	1.0
31.6	36.4	1.1
36.5	41.2	1.2
41.3	46.1	1.3
46.2	50.9	1.4
51.0	55.8	1.5
55.9	60.6	1.6
60.7	65.5	1.7
65.6	70.4	1.8
70.5	73.8	1.9

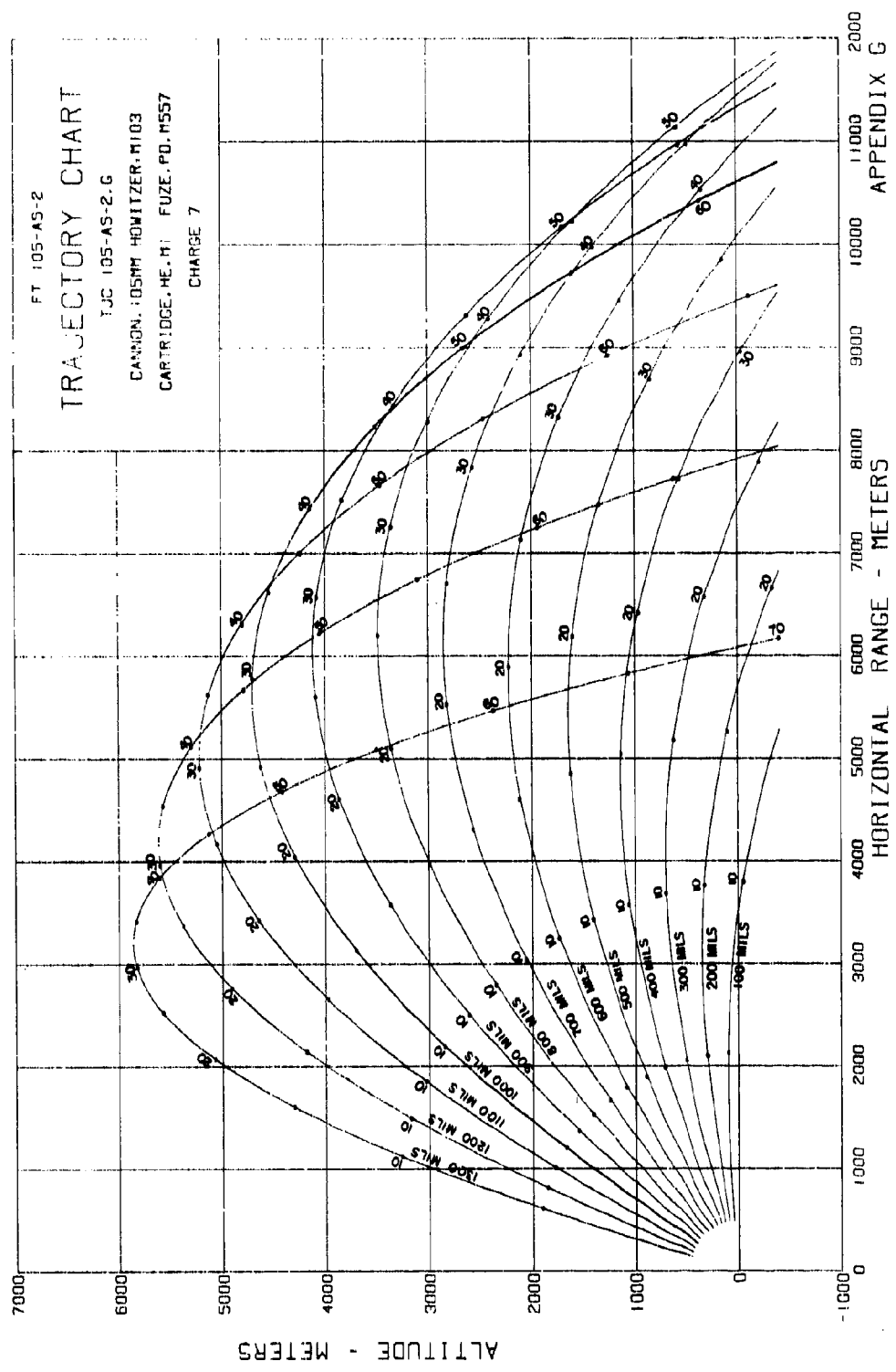
CHARGE
7

FT 105-AS-2

CTG, ILLUMINATING, M314A2E1
FUZE, MT, M565

1	2	3	4	5	6
RANGE TO BURST	E L E V	FS FOR FUZE M565	CHANGE IN ELEV FS FOR 50 M INCREASE IN HEIGHT OF BURST FOR FUZE, M565		RANGE TO IMPACT
M	MIL	FS	MIL	FS	M
800	788.2	2.6	33.0	0.10	9551
900	731.5	2.8	32.6	0.09	9541
1000	682.2	3.0	32.0	0.09	9459
1100	639.7	3.3	31.1	0.08	9332
1200	601.7	3.5	30.1	0.08	9180
1300	568.8	3.7	29.1	0.08	9015
1400	539.9	4.0	28.0	0.08	8845
1500	514.5	4.3	27.0	0.07	8677
1600	492.0	4.5	26.0	0.07	8514
1700	472.1	4.8	25.0	0.07	8358
1800	454.5	5.1	24.1	0.07	8211
1900	438.8	5.4	23.2	0.07	8073
2000	425.0	5.7	22.4	0.06	7944
2100	412.6	6.0	21.6	0.06	7825
2200	401.6	6.3	20.8	0.06	7716
2300	391.9	6.6	20.1	0.06	7616
2400	383.3	6.9	19.5	0.06	7525
2500	375.6	7.2	18.8	0.06	7443
2600	368.9	7.5	18.2	0.06	7369
2700	363.0	7.8	17.7	0.06	7303
2800	357.8	8.2	17.2	0.06	7244
2900	353.3	8.5	16.7	0.06	7193
3000	349.4	8.8	16.2	0.05	7148
3100	346.1	9.2	15.7	0.05	7109
3200	343.4	9.5	15.3	0.05	7077
3300	341.1	9.9	14.9	0.05	7050
3400	339.3	10.2	14.5	0.05	7028
3500	337.9	10.6	14.2	0.05	7012
3600	337.0	10.9	13.8	0.05	7000
3700	336.4	11.3	13.5	0.05	6993
3800	336.1	11.7	13.2	0.05	6990
3900	336.3	12.0	12.9	0.05	6991
4000	336.7	12.4	12.6	0.05	6997
4100	337.4	12.8	12.4	0.05	7005
4200	338.4	13.2	12.1	0.05	7018
4300	339.7	13.6	11.9	0.05	7033
4400	341.3	13.9	11.6	0.06	7052
4500	343.1	14.3	11.4	0.06	7073

FT. 105-AS-2
TRAJECTORY CHART
 TJC 105-AS-2-G
 CANNON, 105MM HOWITZER-M103
 CARTRIDGE, HE. M1 FUZE, PD. M557
 CHARGE 7



APPENDIX G

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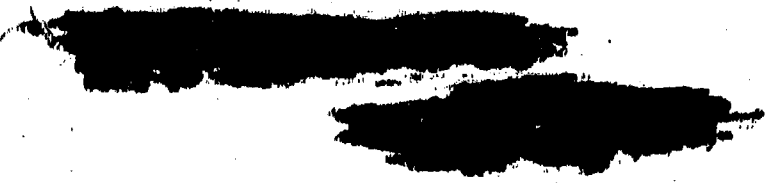
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2. Page 44, 5th line from bottom of page - should read

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