Leonardo da Vinci's World Map

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Abstract

In addition to his better known artistic, scientific and engineering talents, Leonardo da Vinci has an extensive reputation as a cartographer, drawing maps for a wide range of hydro-engineering projects for the rulers of Florence, Milan, Arezzo and the Vatican, amongst others. However, he is not generally acknowledged as authoring a world map (or *mappamundi*) spanning the globe, which was the domain of a few specialized cartographers of the era. Nevertheless, there is a world map among his papers in the Royal Library, Windsor, which is one of the very first to name the Americas, and has the correct overall configuration of the continents, including an ocean at the north pole and a continent at the south pole. Moreover, it has a unique cartographic projection onto eight spherical-geometry triangles that provide close to isometric projection throughout the globe.

Although the authenticity of this world map has been questioned, there is an obscure page of his notebooks in the Codex Atlanticus containing a sketch of this precise form of global projection, tying him securely to its genesis. Moreover, the same notebook page contains sketches of eight other global projections known at that time (early C16th), from the Roman Ptolomaic conic section projection to Rossellli's [1] oval planispheric projection. This paper explores further remarkable aspects of the geometry and history of Da Vinci's unique *mappamundi*.

Global Awareness in Da Vinci's Youth

The contribution of Leonardo da Vinci[1] to many spheres of human endeavor are too well-known to need enumeration. However, one obscure aspect of his oeuvre is a complete world map, or *mappamundi*, to be found among his papers in the Royal Library, Windsor collection. With one exception, I have not seen this remarkable document mentioned in any of the large set of books on his works, including several authoritative surveys of his drawings and diagrams. And this despite extensive coverage of his cartographic output for the Renaissance rulers Ludovico Sforza in Milan, Piero Soderini in Florence, Cesare Borgia in the Marches around Arezzo, and Pope Leo X at the Vatican, almost all for a series of large-scale hydro-engineering projects that either never took place or could not be taken to completion due to unforeseen complications.

The sole exception to the lack of coverage of da Vinci's global cartography is the extensive treatment in Veltman's 'Studies on Leonardo da Vinci' [2] (a book that is no longer obtainable), which details his scattered efforts to capture the geometry of the sphere. Even here, however, the world map itself is mentioned only in passing, with no consideration of its place in the historical development of the cartographic representation of global geography.

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[1] A note on naming conventions. As opposed to the art-historical consensus, I will follow the convention of referring to him by his last name, in common with those of other intellects of the time, such as Columbus, Toscanelli, Bramante and so on. It could be argued that his contemporaries would have referred to him by his first name, but it is well documented that many used his full name, and it seems overfamiliar of present-day authors to associate themselves with the appellations of his contemporaries.

One can ask when da Vinci first became interested in global geography. Was it in the early 1500s, after he left his long-established position in Milan as a result of what must have been a flood of societal interest in the world geography following Columbus' discovery of the Americas? Or was it perhaps early in his life, as a result of early stirrings of interest in the nature of the globe when he first arrived in Florence? One item of interest in support of an early interest is the armillary sphere (or astrolabe) that he included in what is historically the first depiction of a perspective window for projecting accurate views of three-dimensional objects (Fig. 1). The drawing is generally dated to his first Florentine period, around 1480. The object chosen by da Vinci in this sketch is not a sculpture or a building, as might have been expected from his other output of this period, but a depiction of a the analytic device of adjustable circles forming an armillary sphere, although it cannot be determined whether it is a celestial or a terrestrial sphere that is depicted. Whichever is the case, this choice attests to da Vinci's interests in global matters beyond the Tuscan environs of Florence in which he grew up.

Figure 1. *Drawing of a man using a perspectograph to draw an armillary sphere. Leonardo da Vinci (Codex Atlanticus I, 5r; ~1480)*

The second link to a larger scale conception is the presence of Paolo di Pozzo Toscanelli in the Florence of da Vinci's youth. Toscanelli had been one of the progenitors of perspective geometry in discussions with Brunelleschi early in the century [3], then worked with Nicolas di Cusa to produce the classic Renaissance text on squaring the circle [4] and other philosophical matters before turning his attention to cartography. As a cartographer, he was in demand well beyond the shores of Italy, providing maps of the Mediterranean to rulers such as King John of Portugal in 1460. Moreover, as an astronomer, Toscanelli detailed his observations on what later became known as Halley's comet, three centuries before Edmund Halley's notation of it. Importantly, as an astronomer, he took the lead in installing the world's first astronomical gnomon in the Florence Duomo, a hole near the top of the dome that was incorporated into a *camera-obscura*-type installation to track the movements of the sun and determine the timing of movable festivals such as Easter. The unveiling of this remarkable intersection of science and religion took place to great public fanfare in 1468. As chief pupil of the workshop of Andrea Verrocchio, the heir to Donatello and Michelozzo as the principal artisanal provider to the ruling

Medici clan, da Vinci would certainly have been present as one of the supporting cast at this momentous event, and may even have performed musically to celebrate it (as he was famous in his youth for composing extempore songs at public events).

Da Vinci himself has a close connection with the Duomo at that time, when he was an impressionable young man in his late teens. In the same year, 1468, his master Verrocchio received a commission from the Medici to install a gilded copper sphere on top of the lantern surmounting the Duomo (Fig. 2). The sphere again represented a kind of global consciousness and weighed over 2 tonnes, requiring extensive use of hoisting machines that were still available from when Brunelleschi was erecting the dome itself. As Verrocchio's chief pupil at the time, da Vinci would have been intimately involved in every phase of the operation, particularly as Verrocchio was renowned for being highly collaborative with his team of pupils. Thus, he would have had the opportunity to inspect the aperture of Toscanelli's gnomon for himself while on the top of the dome. He would also have been vividly aware of the god-like view of Florence and the surrounding countryside from the 115 m height of this crow's nest location, which may have played a role in his lifelong obsession with cartography and with flight, both in terms of the analysis of the flight of birds and of the design of a human flying machine.

Figure 2. *Left: An oblique view of the Duomo of Santa Maria de Fiore, Florence, designed by Arnolfo di Cambio in 1294, with the dome built by Brunelleschi from 1418-1436; the marble cladding was designed by Emilio de Fabris in 1871 and completed in 1887. Right: the 22 m high lantern was completed by Michelozzo in 1461, and the 8 tonne gilded copper ball was added by Verrocchio and Leonardo da Vinci in 1469.*

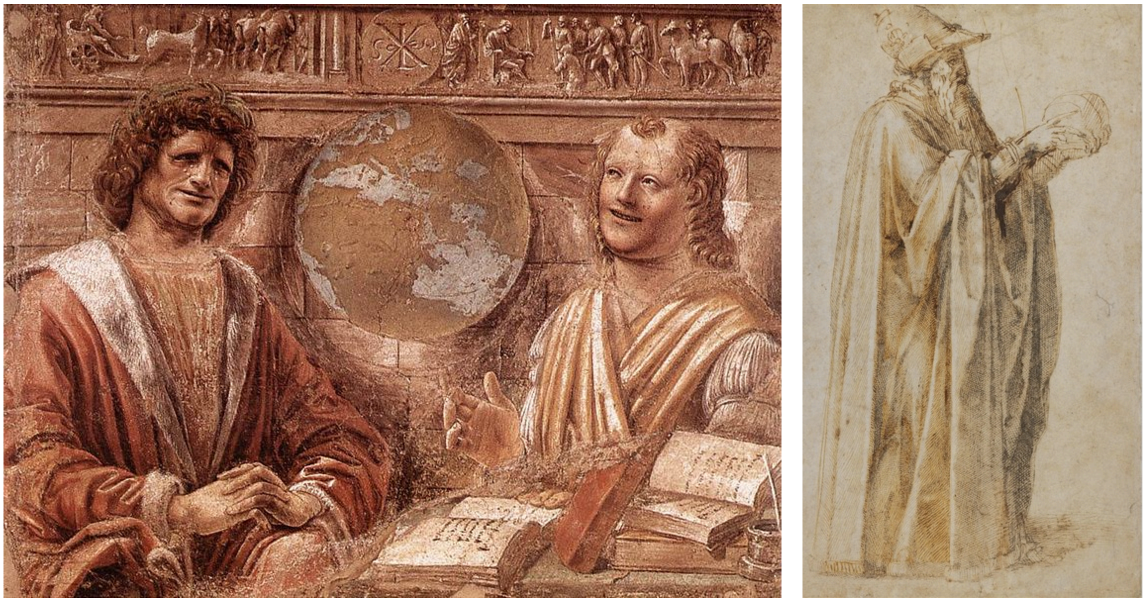

Figure 3. Left: '*Crying Heraclitus and Laughing Democritus' by Bramante (1483). Right: 'The Philosopher (?portrait of Leonardo da Vinci with a globe)' by Michelangelo (~1515).*

A further link of da Vinci to global cartography is found in a fresco by his friend Donato Bramante from his early years in Milan (Figure 3, left). The painting depicts the Greek philosophers Heraclitus and Democritus, representing pessimism and optimism, respectively, flanking a geographically detailed depiction of the globe. The two figures are generally accepted as depictions of da Vinci and Bramante himself, respectively. Their association with the globe seems to reflect a mutual interest in the large-scale conception of the land they inhabited, consistent with the idea that da Vinci brought the global interests that he had developed in Florence with him to Milan when he relocated there in 1482. The final association comes in a sketch by Michelangelo of a philosopher holding a globe (Figure 3, right) that is often considered to be a portrait of the aging Leonardo da Vinci (especially as it seems to be the model for Vasari's portrait of him in his *Vitae*). The fact that the philosopher is holding a globe has nevertheless been considered puzzling in relation to this attribution.

Da Vinci's World Map

Despite Leonardo da Vinci's renown as a Renaissance man, and as a cartographer in particular, he is not generally acknowledged as authoring a world map (or *mappamundi*) of the geography of the world [5]. Nevertheless, there is a world map among his papers in the Royal Library, Windsor (Figure 5), which is one of the very first maps to name the Americas, and has the correct overall configuration of the continents, including an ocean at the north pole and a continent at the south pole (at the centres of the left and right quartets, respectively). At that time (as subsequently), there was a wide variety of projective conventions attempting to depict the curved surface of the globe onto the planar surface of a sheet of paper. By breaking the surface into eight octant petals in two florets, da Vinci develops a unique projection that had by far the most isometric mapping geometry to that date (although at the cost of a set of crosscuts that split some of the continents).

Figure 4. *Da Vinci's octant world map projected onto Reuleaux triangles (~1508). Royal Library, Windsor.*

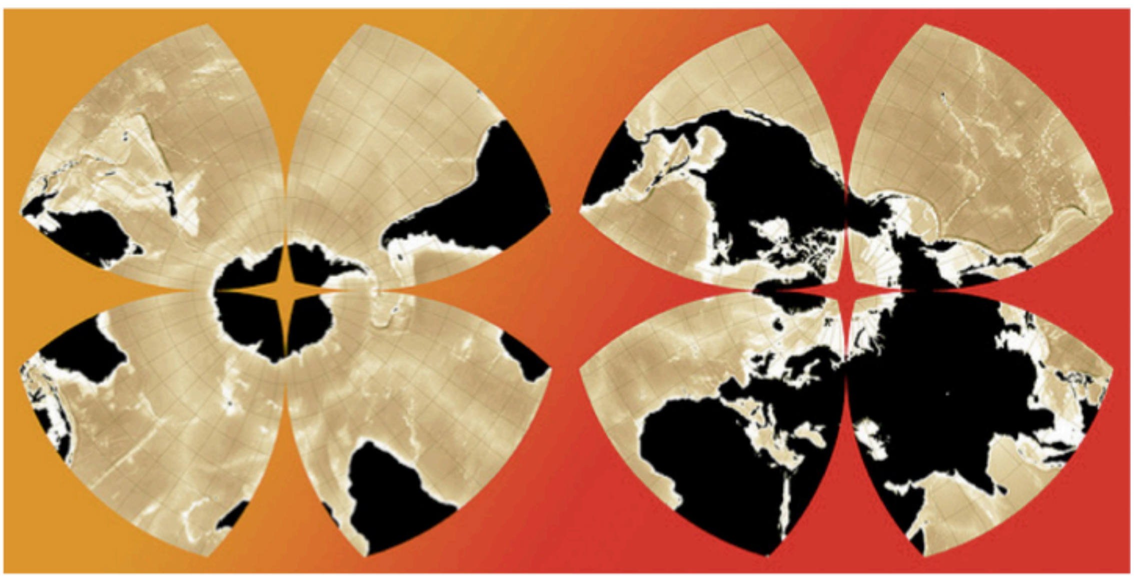

Figure 5. *Modern projection of the globe onto da Vinci's octant projection, based on a bathymetric image of the ocean depths from NASA Blue Marble [6].*

Da Vinci's projection may be compared with a modern mapping to the same projection (Figure 5). This mapping makes it clear that, although he has the right general idea, he has substantially exaggerated the size of Europe and has the Americas much too far to the west (somewhere near Hawaii). Nevertheless, he shows India, Malaysia, China, Japan and even the Russian Far East peninsula in approximately the right proportions, implying that he must have had access to some information from sailors of the oriental sea routes. Remarkably, also, he has approximately the correct dimensions for both the Arctic Ocean and the landmass of Antarctica (at the centres of the right and left florets, respectively), something achieved by no other cartographer of the era, or for the next two centuries! The Arctic is particularly surprising in view of the fact that it is continuously covered in a thick layer of ice.

Attribution to Leonardo da Vinci

Despite the fact that it was found in his collection of papers from those assembled by his pupil Francesco Melzi, the attribution of this *mappamundi* to Leonardo da Vinci has long been questioned by most da Vinci scholars [7, 8, 9], and has consequently been omitted from almost all books in his oeuvre (the sole exception being a passing mention in Veltman [2]). The line of the coastline is considered too curly and the notations too plain to match da Vinci's other works; thus, although it is occasionally acknowledged as his overall conception, the execution is commonly considered to be the work of a pupil. It should be noted, however, that all the features discussed to this point are those of conception, and thus should be accepted as relating to his autograph contributions to the design of the map, even if it is a copy of his original.

More recently, key evidence of da Vinci's ownership of the world map has emerged from references to it in the Codex Atlanticus [10]. He was a good friend of the rich merchant's son Giovanni di Amerigo di Benci, brother of the 17-year-old Ginevra di Benci whose portrait da Vinci had painted in 1473 (now in the National Gallery, Washington DC). Giovanni was a notable horseman who used to ride around Florence on horses costing as much as 600 florins, a scandalously extravagant sum at the time [11]. The two young men evidently had a close working relationship, because da Vinci has notes among his papers in the Codex Atlanticus about exchanging books, precious stones and supplies (such as "brass for eyeglasses") with Giovanni Benci. It was to Benci that da Vinci entrusted his most ambitious work of that period, the full-scale cartoon for the unfinished 'Adoration of the Magi', when he left Florence for Milan in 1482. Of particular relevance here is the scribbled reference in his notes to "my world map that is with Giovanni Benci", which is unequivocal evidence of an interest in global cartography at that time, and that his interests extended beyond the shores of Italy to the configuration of the lands of the entirety of the globe. There is a suggestion of consequentiality about the statement, as though it was more than a sketch on a folio sheet. One has the image of a notable document that was worth recording as having been loaned to a particular friend.

What remains ambiguous, however, is whether the "my" refers solely to ownership or implies that it was a map that he had drawn himself. Was da Vinci compiling world maps in his first Florentine period, before the age of 30? And did this indeed tie in with Toscanelli's cartography expertise of the previous decade? Or was it a world map that he happened to own somehow (as a gift from Toscanelli, for example)? It seems likely that the statement would have identified the source of the map in this case, as in "Ser Paolo's world map that I left with Giovanni Benci". Seen in this light, the statement does seem to imply that da Vinci was compiling a world map in this early period. Presumably this cannot have been have been the map in Figure 4 because that includes the American continent, whose existence was not suspected until after 1492, but an earlier attempt at a world map that has not survived. (An alternative interpretation is that the comments about exchanges with Benci were from da Vinci's later post-1500 period in Florence, and that the map was the one we now have, although evidence of exchange between the two during this later period is much less secure.)

With respect to his actual authorship, it is interesting to note that da Vinci's cartographic style is substantially different from his loose and flowing drawing style. His cartographic style can be seen in his map of the western Mediterranean (Figure 6), which is also one of the few of his maps known beyond the borders of Italy, helping to underline the global reach of his cartographic interests. This accepted autograph map helps to link the mappamundi to da Vinci in two ways. The line style can be seen to have the same curly character and the lettering to have a similar plain character to those in the mappamundi, although the letters are predominantly lower case as in most of his maps. The switch to upper case can be hypothesized to be indicative of an intention to provide the mappamundi to a sponsor of some kind, such a Cesare Borgia or Giuliano de Medici, both of who were appointed to the French Dukedoms (Borgia as Duc de Valentinois and Medici as Duc de Nemours), the French honorific titles reflecting the international range of their respective ambitions.

Figure 6. *Map of the eastern Mediterranean and drawing of the tower of the Palazzo della Signoria (Codex Atlanticus, 1106r, 1496).*

A third key link to da Vinci's works is a remarkable, but again little known, page from his notebooks containing an intensive exploration of what seem to be all the known cartographic projections of the globe at that time (reproduced in Figure 7). Each miniature sketch represents a different projection geometry, with several of them showing projection lines or shadows, clearly indicating that he understood them as geometric projections of the globe, not simply images from some maps that he may have had to hand. The particular relevance of this page for the present analysis is that, among this array of classic projection geometries, da Vinci includes (at lower right) a clear reference to the octant triangular projection, tightly coupling this evidently autograph sheet to the conception underlying the *mappamundi*. Taken together with Figure 6, this notebook page seems to remove any grounds for questioning da Vinci's authorship of the unique *mappamundi*. Even if it is in fact a copy by a pupil, the pupil is most unlikely to have the kind of knowledge required to make major modifications, so we are safe in assuming that all the characteristic details are reflective of da Vinci's own intentions.

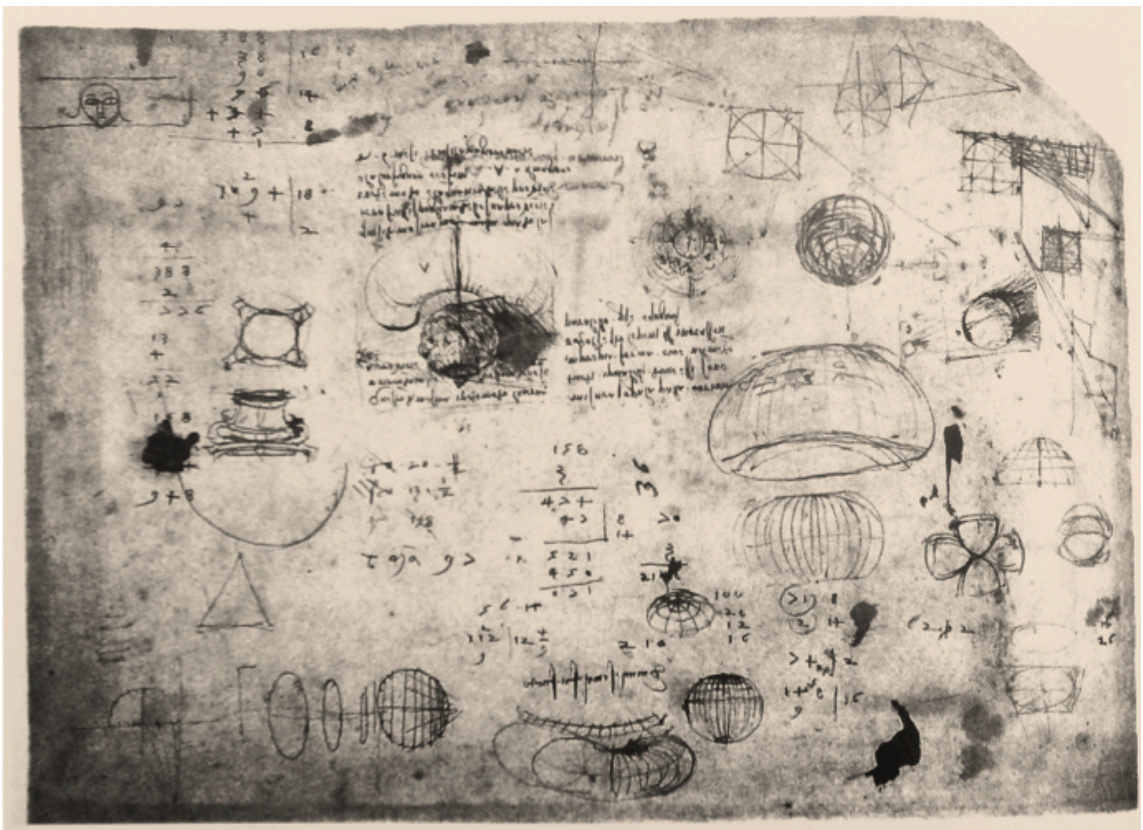

Figure 7. *A page of Leonardo da Vinci's cartographic notes (Codex Atlanticus, 521r, 1478-1519).*

To highlight the variety of projections considered in this notebook page, they have been isolated in Figure 8 and annotated with the originator of each form of projection (as far as they are known). Three of them date back to Roman times, as represented by the *Geographia* of Claudius Ptolemy, which had recently (in 1410) been translated and distributed by Jacopo da Scarperia, and of which da Vinci was known to have owned a printed copy with illustrations. He must have been among the many whose global conceptualizations were expanded and regularized by this systematic presentation of geographic information. Few, however, are likely to have gone into such depth in the analysis of the forms of projective geometry, and the scope of da Vinci's analysis is a further reason for assuming that he most likely sought out the counsel of an experienced cartographer like Toscanelli for discussions of these ideas.

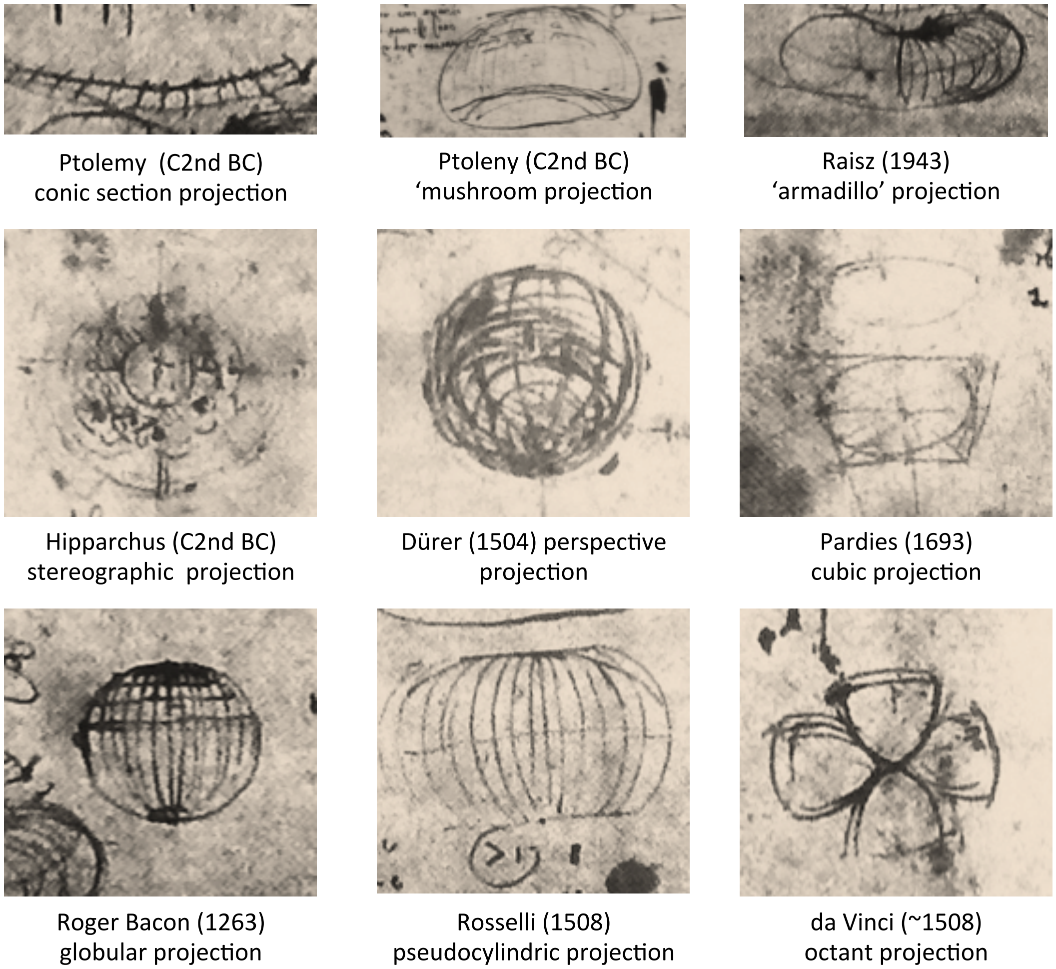

Figure 8. *Identification of the precursors or earliest known examples of nine different projections explored by Leonardo da Vinci in his notebook page. For those projections dated later than 1508, his drawings should be effectively considered the original precursors.*

The Reuleaux Triangle projection

It is noteworthy that da Vinci's world map has a unique cartographic projection onto 8 spherical-geometry triangles that provide close to isometric projection throughout the globe. These are triangles created from three arcs of circles with their centres at the apices of the triangle, and as such, were easily constructible with the compasses available in Renaissance times (Figure 9). The spherical triangle in this form is known as the Reuleaux triangle, after Franz Reuleaux, who realised that it would roll with a constant diameter. There is no evidence that da Vinci was aware of this property of its dynamics, despite his geometric construction of it from the three vertices of a Euclidean triangle with a compass set as a constant angle of its points. It seems appropriate, therefore, to say the da Vinci had anticipated the Reuleaux triangle rather than he invented it in this rolling sense, *per se*. Nevertheless, he was clearly fascinated with the geometric properties of this non-Euclidean form of a triangle and was certainly the first to develop it into a quasi-isometric mapping projection.

Figure 9. *Details of pages from Leonardo da Vinci's notebook developing the geometry of his octant triangle figure, which was later developed as the Reuleaux triangle [12] (Leonardo da Vinci, Paris Manuscript A, 15v; Codex Atlanticus, 923r).*

Da Vinci was evidently aware of the geometric significance of this figure, as he develops it as means of constructing a regular pentagon (Figure 9, right). If connected at one apex, equilateral triangles would form a hexagonal figure. To employ them to construct a pentagonal shape, da Vinci has subdivided the height into five subdivisions and indicated that the topmost division should be the centre of combination. This is a very good approximation to a valid construction, because the angle at the centre derived in this manner is 71.6° instead of the required 72°, which is accurate to 99.44%. Whether this pentagonal construction relates to other geometric figures that he developed, such as the Vitruvian Man, is unclear.

Geometric Development

Da Vinci is renowned for the range of geometric explorations in his notebooks. One such development that relates to his octant approach to world mapping is that of 'squaring the circle', which has already been noted as a key interest of Toscanelli [4]. Da Vinci's interest is often referred to his iconic illustration of the geometric relationships in human proportions developed by the Roman architect, Vitruvius, known as the Vitruvian Man. Since, however, those proportions and the configuration of the elements were indeed worked out by Vitruvius, it is not clear from this what level of understanding da Vinci needed to have had in order to construct this diagram. In this connection, therefore, it is of interest to find explicit analyses of the relationships between the circle and its circumscribing square in his notebook pages, such as that reproduced in Figure 10.

At lower left, he develops an octagonal analysis of the area of a circle inscribed in a square rhombus (regular diamond shape) by approximating it with the eight triangles making up an inscribed octagon, reconfigured as an 'accordion' figure of eight isosceles triangles (or, actually, seven isosceles triangles and two right triangles making up the eighth, labeled *abcdefghi*) together with a lenticular segment labeled *K* to account for the eight missing lenticular elements in the octagonal approximation to the circle. This large lenticular segment *K* should have an area eight times larger than that of each of the eight segments in the circular figure, giving it a linear extent of sqrt(8), or 2*sqrt(2). Deriving this ratio seems to be the motivation for the scaled figures of the semicircle, quadrant, octant and sector, which all fill the same area relative to the original circle, making it clear that the lenticular segment is in the correct ratio of sqrt(8)/4 = 0.707 to the width of the eight triangle figure *abcdefghi*.

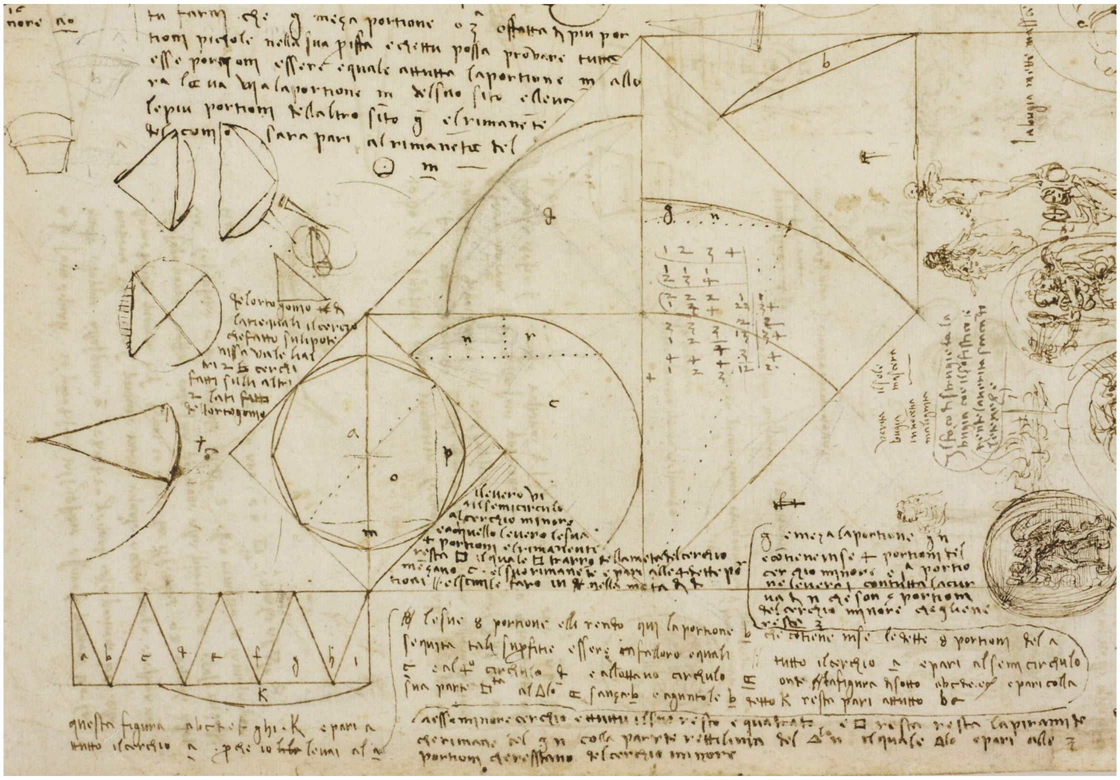

Figure 10. *Octagonal analysis of the area of a circle inscribed in a square rhombus (left-right reversed) in which Da Vinci is developing the idea that the circle, semicircle, quadrant, octant and sector all fill the same area relative to the enclosing rhombal square. (Leonardo da Vinci, Royal Library, Windsor, 12700v; detail).*

This analysis of the area of the circle does not solve the problem of squaring the circle, because there is apparently no indication how to derive the area of the lenticular segment, but it does show da Vinci thinking about the progressive fractional subdivisions of circles in a way that can be readily generalized to the solid geometry of spheres. It also links da Vinci directly with Toscanelli in the analysis of the problem of squaring the circle, which at one point he thought he had actually achieved: "On the night of St Andrew I reached the end [goal] of squaring the circle and at the end of candlelight, of the night and of the paper on which I was writing, it was completed." (Madrid Manuscript II, 112a).

A further analysis in which he took a similar approach to the spherical geometry *per se* is found in the Paris Manuscript (Figure 11). Here da Vinci subdivides a sphere into octants and explores different ways of segmenting (or digitizing!) the volume of an octant to quantify its volume. Indeed, this analysis may be considered a precursor to the differential calculus developed formally by Newton and Leibniz nearly two centuries later. Moreover, some of the approaches on these pages involve explicit visualization of the spherical octant as a Reuleaux-like spherical triangle, which may have been what suggested the Reuleaux triangle approach to the world map.

Figure 11. *Octahedral analysis of the volume of a sphere with various forms of segmentation (Leonardo da Vinci, Paris Manuscript E 24v,25r).*

Role in the Discovery of America

This paper began with the evidence that da Vinci had a global conceptualisation of the world in his youth, and would most likely have encountered Paolo Toscanelli in his role in the leading *bottega* in Florence working at the Duomo at the same time as Toscanelli's gnomon. Now, as a cartographer, Toscanelli was in communication with the authorities in Portugal and Spain, and was reputed to have developed the (erroneous) map that inspired Columbus to set out to discover the western route to the Indies (which were known from the overland route through Asia). Toscanelli lived to 1482 (about the time that da Vinci left Florence), so his map must have been drawn at least a decade before Columbus set sail, and in the period when he would have known da Vinci. This gives rise to the speculation that da Vinci may have been involved in the conversation about the extension of the Ptolemaic octant (for that is about how much of the globe was mapped in Ptolemy's recently popularized Geographia) to the remainder of the full globe.

Evidence that da Vinci had a larger conceptualisation of global exploration is found in an allegorical drawing of unknown date, but undoubted provenance (Figure 12). (Notice the characteristic Vinciesque rocks in the background at left.) This drawing shows a seafaring animal (apparently a wolf) using a compass-like device to target an eagle perched on a globe. The emblem of Isabella, the Queen of Spain in that era, has been inset for comparison, supporting the suggestion that the navigation target represents the Spanish throne (which was the actual patron of Columbus' voyages to the Indies). The suggested interpretation is that this image represents the conceptualisation of global exploration under the Spanish aegis, and that da Vinci was indeed somehow involved in that enterprise. One can imagine that, in enthusiastic discussions of global geography with Toscanelli, da Vinci was the one pushing to ask how the continents were located in relation to the Atlantic Ocean, with Toscanelli bringing his cartographic experience to bear on the details of the answer. Da Vinci was also an acquaintance of Amerigo Vespucci, whose voyages to the New World in the 1490s established the continents of the eponymous Americas, and according to Vasari had a sketched portrait of him among his notebook papers. Vespucci was a cousin of the Medici and also of Simonetta Vespucci, whose wedding to her cousin Marco Vespucci at the Medici Palace in that same year (1469) was the inspiration for Botticelli's famous paintings of the 'Birth of Venus' and 'Primavera'.

Figure 12. *Allegorical drawing of a voyage of discovery (Leonardo da Vinci, Royal Library, Windsor). Inset: emblem of Queen Isabella of Spain.*

Later influences

Although barely recognised in the canon of cartographic projections, da Vinci's octant projection can be seen to have influenced a line of cartographers to the present day. The first example of this influence was the diagrams of Oronce Fine (1494-1555), a French cartographer active in the years just after da Vinci died in France, who was notable for 'crossing swords' on academic matters with King François I, and who developed the gridlines of the spherical octant projection and elaborated it into several formats, including a whimsical heart-shaped projection (Figure 13). There was a further flurry of interest in octant maps the following century when Daniel Angelocrator (Engelhardt) produced several rigorous versions [13]. (Unlike da Vinci, all these successors had Antarctica dramatically too large, however.)

We may jump forward several centuries to the Pan-Pacific Exposition of 1915 announcing the recovery of San Francisco from the great earthquake of 1906, for which Bernard J. Cahill [14] developed a connected version of the da Vinci octant projection to map the challenge of a flight around the world (Figure 14). How da Vinci would have appreciated this combination of two of his lifelong interests! To provide more extended connection regions, Cahill slightly distorted the da Vinci octants into figures with each edge consisting of three straight segments, then connected the four pairs of north-south quasi-octants into a butterfly-shaped figure that allowed all the continents except Asia and Antarctica to appear as connected landmasses. Many of the continents are rotated to extreme angles relative to their natural north-south axes, however.

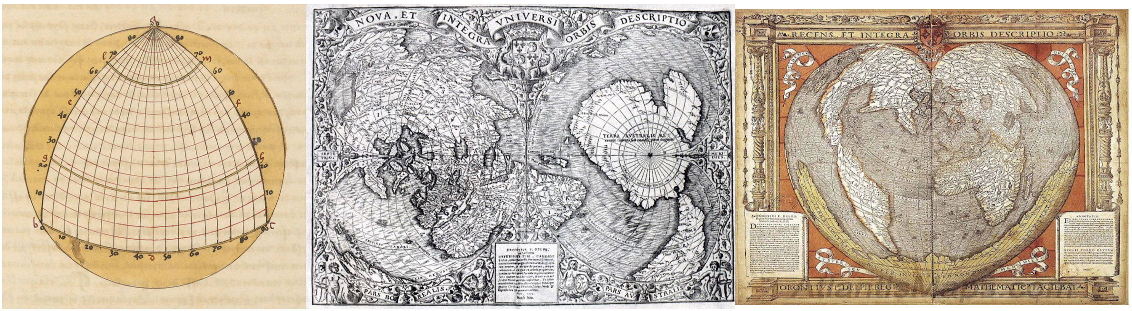

Figure 13. *Developments of the spherical octant projection by Oronce Fine (1542, 1531, 1536).*

Following Cahill, several cartographers developed further forms of segmented maps, by projection onto various regular polyhedral, notably the Buckminster Fuller octahedron (and the other regular polyhedra) and the Keyes projection that took a different configuration of the Cahill projection to avoid the cut through central Asia (but still lopped off the Russian Far East peninsula and segregated New Zealand from Australia, however). The Keyes projection also had continents such as Africa tilted at a rather severe angle.

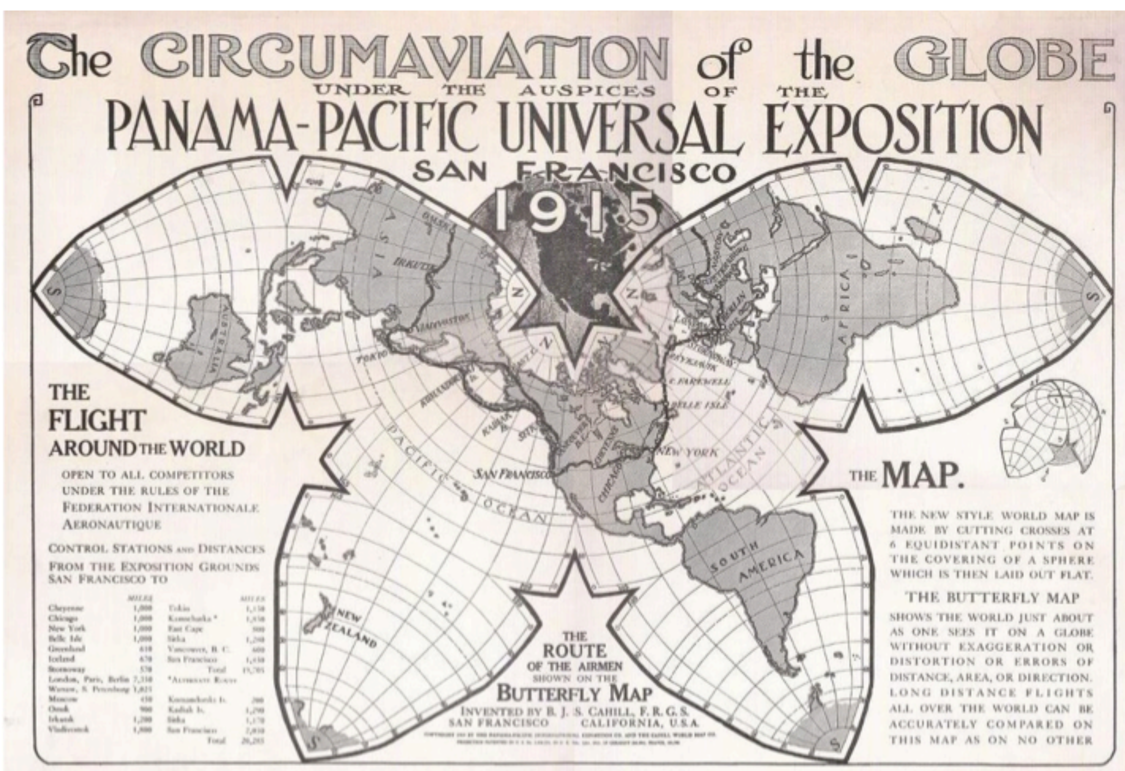

Figure 14. *Connected quasi-octant map by Bernard J. Cahill (1909).*

These deficiencies may be partially rectified with a Cahill-like projection that does not adhere to the principle of complete octants, but allows segments of the octants to be shifted to complete the continents and also to regularize the oceans somewhat. A final map taking this approach, developed by the author, is depicted in Figure 15. Part of the motivation for including this reconfigured projection is to illustrate how powerful the da Vinci approach was in providing for closer-to-isotropic projection of the relative shapes and sizes of all countries and continents throughout the globe. All other projections distort the shapes of at least one continent, and most projections vastly exaggerate the sizes of Greenland and Antarctica. Here one can clearly see the approximate equivalence in size of the United States, Brazil, China, Australia and Antarctica, for example. Distances, too are well represented by straight lines on the map within continents. The main sacrifice is of distances across oceans, which are heavily distorted in this projection. It may be possible, however, to derive an equivalent projection preserving the shapes and size of the oceans in the same way, although it is likely to be more challenging because they cover the majority of the earth's surface.

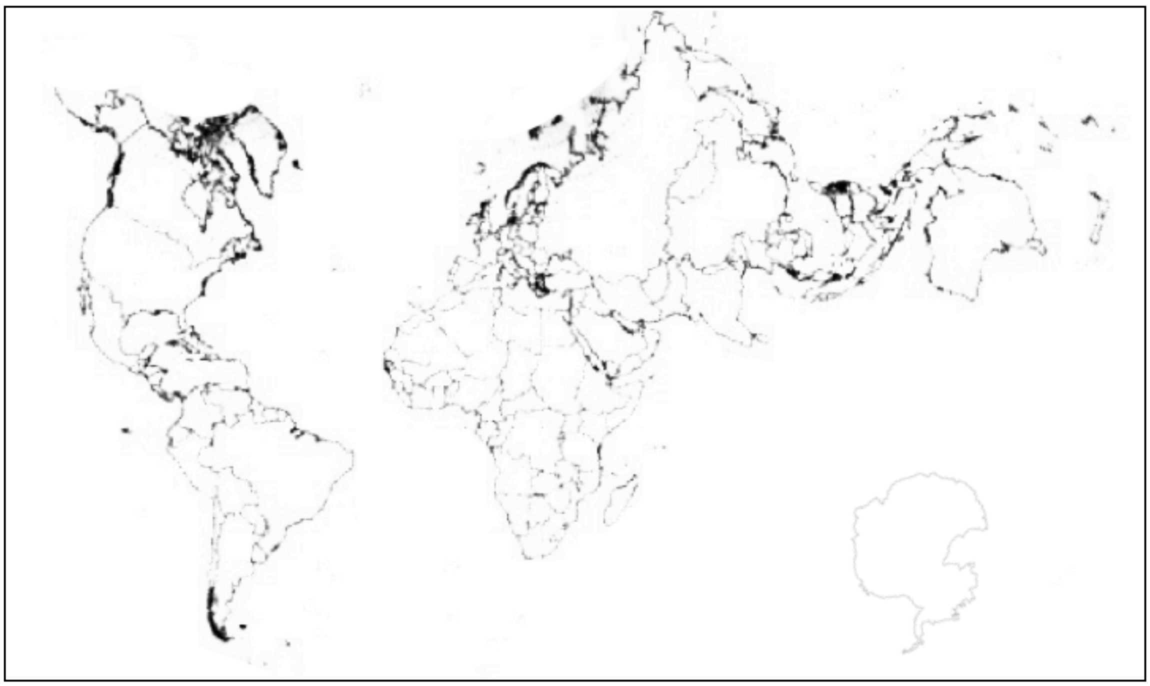

Figure 15. *Da Vinci-Cahill-Tyler quasi-isotropic world map, reconfigured from the Cahill projection to retain all continents as contiguous landmasses minimally distorted in shape and relative sizes.*

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References

[1] Rosselli F (1508) In: Almagià R. (1951) On the cartographic work of Francesco Rosselli. *Imago Mvndi*. International Society for the History of Cartography 8: 27-34.

[2] Veltman KH (1986) *Studies on Leonardo da Vinci I: Linear Perspective and the Visual Dimensions of Science and Art*. Deutscher Kunstverlag: Berlin.

[3] Vasari G 1550) *Le Vite de' piú Eccellenti Architetti, Pittori, et Scultori Italiani, da Cimabue Insino a' Tempi Nostri. Nell'edizione per i tipi di Lorenzo*. Torrentino:Firenze.

[4] di Cusa N (with Toscanelli) *Dialogus de circuli quadratura*. Manuscript, 1457.

[5] "In the Codice Atlantico we find sketches of a globe being unfolded that is not far from Waldseemüller's pseudo-Ptolemaic, cordiform projection (1507)." Veltman (1986).

[6] Burchill L (2007) *Leonardo's octant map*. https://www.flickr.com/photos/lloydb/529393602/

[7] Major RH (1865) *Memoir on a Mappemonde by Leonardo da Vinci, Being the Earliest Map Hitherto Known Containing the Name of America: Now in the Royal Collection at Windsor*. J.B. Nichols: London.

[8] Nordenskjöld AE (1889) *Facsimile-Atlas to the Early History of Cartography with Reproductions of the Most Important Maps Printed in the XV and XVI Centuries*. Trans. Johan Adolf Ekelöf (Stockholm, 1889; reprinted, New York: Dover, 1973).

[9] Snyder JP (1993) *Flattening the Earth: Two Thousand Years of Map Projection*. University of Chicago Press: Chicago.

[10] Cecchi A (2003) New light on Leonardo's Florentine patrons. In, Bambaugh C, Stern R, Manges A (eds) *Leonardo Da Vinci Master Draftsman*. Metropolitan Museum of Art: New York, 121-139.

[11] Möller E (1937-8) Leonardos Bildnis der Ginevra dei Benci. *Münchner Jahrbuch der bildenden Kunst*. n.s. 12, 185-209.

[12] Reuleaux F (1876) *The Kinematics of Machinery: Outlines of a Theory of Machines*. London: Macmillan.

[13] Angelocrator D (1628) *Doctrina de ponderibus, monetis, et mensuris per totum terrarum orbem usitatis*. Johann Berner: Frankfurt.

[14] Cahill BJS (1909) An account of a new land map of the world. *The Scottish Geographical Magazine,* 1909-09, 449-469.