Philipp Otto Runge's Color Sphere

R o l f G. K u e h n i

Philipp Otto Runge's Color Sphere

A translation, with related materials
and an essay

Contents

Preface

Philipp Otto Runge's *Farben-Kugel* (Color sphere) of 1810 was a seminal contribution to the development of three-dimensional color order systems. Within the year of its publication Runge, at age 33, succumbed to tuberculosis. The model of the color sphere, based on the interaction of three chromatic (yellow, red, and blue) and two achromatic (white and black) fundamentals represents the next step in three-dimensional color order after Tobias Mayer's double tetrahedral solid of 1756 and Johann Heinrich Lambert's single tetrahedron of 1772.

Runge was a founder and important member of the German Romantic School of painting. The intuitive value of the color sphere and his reputation as a painter helped to keep it in public view, at least in the Germanic world. The text of *Farben-Kugel* appears not to have been translated into English up to now.
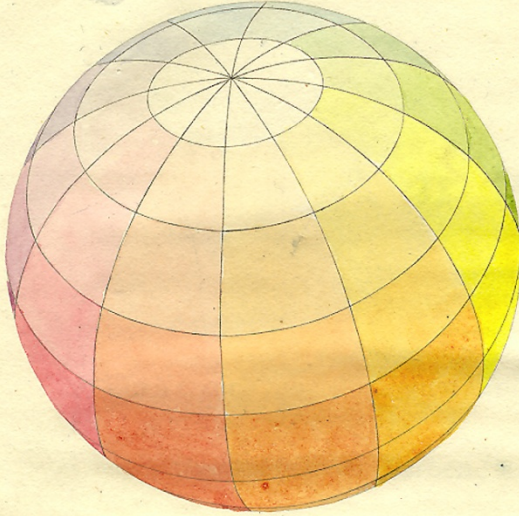
In the present publication, the original published text is presented side-by-side with the translation. In addition to Runge's appendix in *Farben-Kugel* on the relation of the sphere to color harmony, it includes four appendices with translations of important documents related to the development of the color sphere. In addition, there is an essay by the translator about the place of Runge's effort in the history of color order and the validity of the concept from today's point of view. Biographical information about Runge's life concludes the volume.

Runge's style of writing in *Farben-Kugel* is typical for its time, but also influenced to a degree by his limited formal education. Indicative of the former is that the first paragraph of the introduction consists of two long sentences only. To aid comprehensibility, most of the sentences with clauses within clauses have in the translation been split into multiple sentences, while care has been taken to translate the essential information accurately. Explanatory words have been added in angled brackets.

I would like to express my appreciation to Sarah Lowenstein for helpful comments on my essay.

Farbenkugel.

Ansicht des weissen Pols.   *Ansicht des schwarzen Pols.*
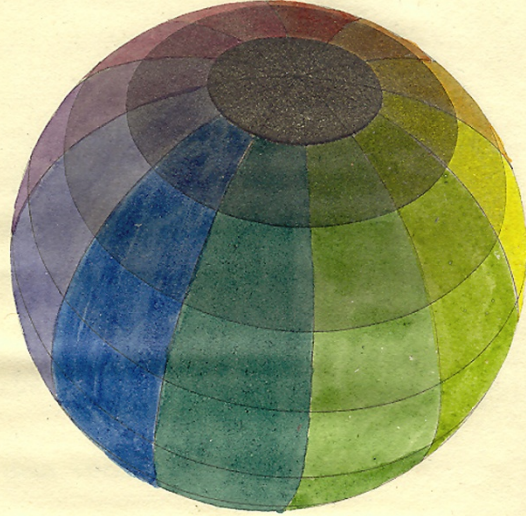
Durchschnitt
durch den Aequator.
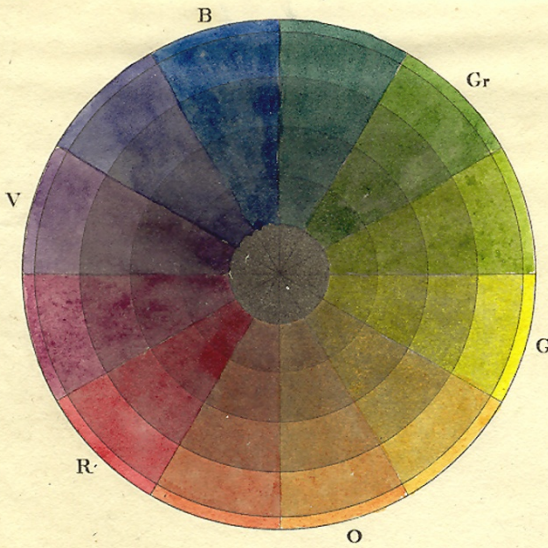
Durchschnitt
durch die beyden Pole.

7
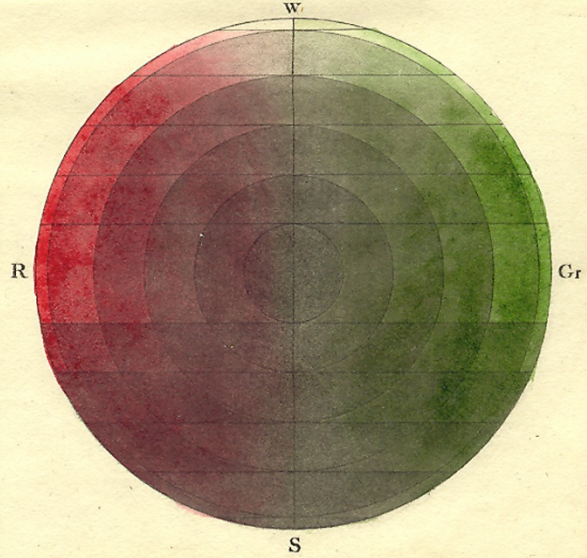
FARBEN-KUGEL

oder

Construction des Verhältnisses aller Mischungen der Farben zu einander, und ihrer vollständigen Affinität,

mit angehängtem

Versuch einer Ableitung der Harmonie in den Zusammenstellungen der Farben.

Von

Philipp Otto Runge, Mahler.

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Nebst einer Abhandlung

über die Bedeutung der Farben in der Natur,

von Hrn. Prof. Henrik Steffens in Halle.

Mit einem Kupfer, und einer beygelegten Farbentafel.

8

**Color Sphere, or construction of the relationship between all mixtures of colors and in their complete affinity,
with appended 'Attempt to bring the sensory impressions from compositions of different colors into agreement with the previously developed color scheme.'
By Philipp Otto Runge, Painter.
With a copper engraving and an enclosed color chart.**

[In the following pages the translation text per page has been made to coincide with the original, however, without braking sentences.]

Introduction

The figures in this small book have the purpose of making the construction of the spherical relationship clear and end in the sphere itself. The sphere is pictured in colored engravings of two perspective views and two cross sections. The purpose of the engravings and all other figures is to aid the reader's comprehension of the subject matter. The engravings should not be expected to represent with perfection the mixtures mentioned in the text. A more perfect execution of the illustrations, had it been possible, would only have delayed publication of the work and made it more expensive. Applying colors in their mixtures and nuances according to the presented principles to a real sphere and to different sphere sections would have been of considerable help to the comprehension of the relationships. But the included engravings will make clear the meaning of my concept.

The color combinations on the second chart were intentionally executed with opaque paints, even though higher brightness could have been achieved by other means. Material differences should be disregarded and color perception alone considered. This would not have been possible if material differences of colorants would have influenced the results. The demonstrated phenomena can be experienced more intensely if taffetas or atlas textile ribbons are used in place of colored paper.

Vorbericht.

Die Figuren in diesem Büchlein, welche die Construction des Kugelverhältnisses anschaulich machen sollen, endigen mit der Kugel selbst, welche, colorirt, nach zwey perspectivischen Aufrissen, und mit zwey Durchschnitten, auf der Kupfertafel dargestellt ist. Von dieser Abbildung, da sie so wie die übrigen Figuren nur der Vorstellung zu Hülfe kommen soll, wird man nicht verlangen, daß alle Mischungen so bestimmt und klar erscheinen, als davon in der wörtlichen Construction die Rede ist; auch hätte eine sorgfältigere Ausführung der Illumination, wenn sie nicht gar hier am Orte unmöglich gewesen wäre, nur die Herausgabe des Werkes verspätet und vertheuert; und obschon die Vorstellung des Verhältnisses für jedermann sehr an Klarheit gewinnen müßte, wenn an einer wirklichen Kugel, und verschiedenen Kugelabschnitten, die Farben in ihren Mischungen und Nüancen nach dieser Construction aufgetragen würden, so wird man doch auch schon nach gegenwärtiger Kupfertafel deutlicher begreifen können, was gemeynt ist.

Bey den Farben-Zusammenstellungen auf der anderen Tafel, sind mit Vorbedacht lauter undurchsichtige oder Deckfarben angewandt, wenn man selbige gleich auf andere Weise brillanter hätte haben können; es sollte vom Unterschied des Materials ganz abgesehen, und bloß das Verhältniß des Farbeneindrucks an und für sich in Betracht gezogen werden, dieses konnte nicht so abgesondert statt finden, wenn die Verschiedenartigkeit des Materials mit in

Collision getreten wäre. Will man jedoch die Effecte etwas lebhafter empfinden, so könnte man statt des gefärbten Papieres Taffent- oder Atlasbänder wählen.

Anstatt diese Zusammenstellungen, eine jede an ihren Ort, im Contexte meiner Schrift selbst einzuschalten, schien es den Vorzug zu verdienen, sie insgesammt in gegenwärtiger Folge auf einer Tafel darzustellen, weil, was in den verschiedenen Abschnitten erörtert wird, itzo dem Blicke wieder in einer Figur anspricht. Das Störende aber, was aus der Zusammenstellung des Ganzen für die Betrachtung eines einzelnen Effectes entstehen muß, wird dadurch gehoben, daß die Tafel nicht angeheftet ist, und man das Buch selbst dazu anwenden kann, die störenden Zusammenstellungen zuzudecken.

Der Abhandlung meines Freundes Steffens verdanken wir den Blick in eine Fülle der herrlichsten Erscheinungen in der Natur; und ich würde glauben ein erfreuliches Ziel erreicht zu haben, wenn mein kleines Werk zur ruhigen Ueberschauung aller dieser interessanten Phänomene einiges beyzutragen im Stande gewesen wäre.

P. O. Runge.

——•••◦◦◦‹◦›◦◦◦•••——

It seemed advantageous to have the combinations of the second chart in sequential order in one place rather than where they are discussed in the text. In this way, what is mentioned in different sections of the text can be viewed at once on the chart. On the other hand, viewing the total chart may be disadvantageous in regard to the perception of individual phenomena. By publishing the chart on a separate sheet this has been eliminated: the book itself can be used to cover the disruptive sequences on the chart.[1]

The included essay by my friend Steffens provides a view of the abundance of most beautiful color phenomena in nature.[2] My brief essay will have achieved its goal if it contributes some insights to the appreciation of all these interesting phenomena.

[1] In extant copies of the separate chart with harmonious and non-harmonious color combinations colorants are more or less deteriorated. Interpretations of the color combinations found on this chart have been included in the text of the translation.

[2] Steffens's essay is not included in this translation.

So natürlich, ja unumgänglich es scheint, die regelmäfsigen Resultate, welche beym Vermischen färbender Materialien uns in die Augen fallen, an den Theorieen des Lichts, oder der Entstehung der Farben, zu vergleichen und zu prüfen, und eine Lehre, einen wissenschaftlichen Unterricht für den Mahler, von den Theoremen oder Hypothesen herzuleiten, aus welchem demnächst fruchtbare Regeln erwachsen könnten, so ist doch bekannt, wie hülflos den Künstler die aufgestellte Wissenschaft gelassen hat, wenn die bestehenden Verhältnisse farbiger Substanzen Wirkungen erzeugten, die aus der blofsen Brechung des Lichtstrahles nicht zu erklären waren.

Wenn erwogen wird, wie neben einer richtigen Erkenntnifs der Formen des menschlichen Cörpers, und ihrer Maafsverhältnisse, dem Mahler auch die Einsicht in die Perspectiv vonnöthen ist, wodurch Gröfse und Ort in Hinsicht ihrer Erscheinung den Gestalten bestimmt wird; nicht weniger die Kenntnifs von der Richtung der Lichtstrahlen, so wie ihrer Brechung und Zurückwirkung, damit es möglich werde, die Gegenstände rund, und in einem räumlichen Verhältnifs erscheinend, darzustellen: so gesellet sich

A

13

[P1]
It appears natural, even necessary, to examine and compare the usual results of mixing coloring materials with the theories of light or the origin of colors, and to derive from theorems or hypotheses a theory, or scientific instruction, for painters. Fruitful rules might soon develop out of such instruction. But it is well-known that scientific schemes of this kind have left the artist helpless because the existing relationships of colored materials produce results not explainable from the refraction of the light beam only.

It is evident that, in addition to truthful cognition of the forms of the human body and their metric relationships, a painter also requires insight into perspective that, depending on their appearance, determines size and location of figures. No less required is knowledge of the direction of light beams, as well as their refraction and reflection, so that objects can be represented in a manner that they appear to be solids having a spatial relationship.

unmittelbar die Betrachtung hinzu, dafs alle Dinge auch ihre Farben haben, und die Farben in manchen Zusammenstellungen einen angenehmen, in andern aber einen widrigen Eindruck machen, endlich, dafs dieselben durch Vermischung, entweder andere erzeugen, oder sich auflösen.

Beruhet aber die Wissenschaft der Zeichnung, in welcher sich die Kenntnifs von der Form, der Proportion, von den perspectivischen Verhältnissen, und der Beleuchtung der Gegenstände vereinigen, wesentlich auf Entdeckung der Gesetze, nach welchen die Gegenstände dem Auge sichtbar werden, mit nichten aber auf Erkenntnifs der Körper oder ihrer Formen an und für sich; so möchten wir, wenn unsere Aufmerksamkeit sich nun auf die Farben lenkt, auf ähnliche Weise streben, die Verhältnisse der gegebenen Farben zu einander, sowohl in ihrer Reinheit als nach dem Gesetze wornach ihre Mischungen vorzugehen scheinen, zu erforschen, um die Eindrücke, welche ihre Zusammenstellungen auf uns machten, und die veränderten Erscheinungen, welche aus ihren Mischungen entstehen, bestimmt ausfinden, und jedesmahl mit unserem Material wiedergeben zu können.

Diese Erkenntnifs kann daher angesehen werden als ganz abgesondert von der Wissenschaft, wie durch das Licht die Farben entstehen; indem wir vielmehr die Farbe als eine gegebene ja selbstständige Erscheinung, und in Verhältnissen zum Licht und zur Finsternifs, zu hell und dunkel, zu weifs und schwarz, betrachten und so begreifen möchten. Gelangten wir auf diesem practischen Wege, von einem so entgegengesetzten Standpuncte, endlich auf einerley Resultat mit dem Lehrer der Theorie des Lichts, so würde es nur desto gewinnvoller seyn.

[P2]

At the same time, it is evident that all objects also have *colors*, and that the colors in some combinations produce a pleasant impression, in others an unpleasant one; finally, that colors, through mixture, create new ones, or neutralize themselves.

The science of *drawing*, in which knowledge of form, proportion, conditions of perspective, and illumination of objects is combined, is in essence based on the discovery of laws according to which objects become visible to the eye, rather than on the cognition of the solids or their forms alone. When shifting attention to *colors*, we would like to attempt in a similar manner to research the relationships of given colors among themselves, in their pure state as well as according to the law that appears to guide the result of their mixtures. The purpose is to be able to determine with certainty the impressions that their combinations create in us, and the changed appearances that result from their combinations, so that we can reproduce them every time with our materials.

Discovered knowledge of this kind can be seen as entirely different from the science that describes how colors are generated from light. We would like to view and understand color as a given, even independent, phenomenon, standing in certain relationships to light and darkness, to white and black. Should we on this practical path, from an entirely opposite point of view, in the end have the same results as those of the teacher of the theory of light [Newton], it would only be the more satisfying.

Es ist einleuchtend, daſs alle reinen Farben, unter und von welchen eine Zusammenstellung möglich ist, auch die Gesammtzahl der Elemente aller und jeder Mischungen ausmachen müssen. Dieser Elemente sind fünfe: weiſs, schwarz, blau, gelb, roth; auſser welchen nicht möglich ist, sich eine völlig unvermischte Tinctur vorzustellen.

Wir sondern aber weiſs und schwarz von den andern drey Farben (welche wir überhaupt nur Farben nennen) aus, und stellen sie in eine verschiedene, den Farben wie entgegengesetzte Classe; weil nämlich weiſs und schwarz einen bestimmten Gegensatz (den von hell und dunkel, oder Licht und Finsterniſs) nicht nur für sich allein in unserer Vorstellung bezeichnen, sondern auch in ihrer mehreren oder minderen Vermischung sowohl mit den Farben als mit allen farbigen Mischungen, das hellere oder dunklere überhaupt, durch mehr oder weniger weiſslich oder schwärzlich, vorstellen: mithin auch als hell und dunkel überhaupt, in einem allgemeinen und andern Verhältniſs zu den Farben stehen, als diese gegen sich unter einander beweisen.

Es haben öfters Bestrebungen, wiewohl nur als Versuche statt gefunden, in einer tabellarischen Form das Verhältniſs aller Mischungen zu einander darzustellen. Die Figur nun, durch welche der ganze Zusammenhang aller Verhältnisse ausgedrückt werden soll, kann nichts willkührliches, sie muſs vielmehr das Verhältniſs selbst seyn, indem solche nothwendig aus der natürlichen Neigung so wie Feindschaft, welche die Elemente zu einander äuſsern, hervorgehen muſs.

Wenn wir uns nun die drey Farben, blau, gelb, roth, eine

A 2

[P3]

It is plausible that all pure colors, among and from which *combinations* are possible, must also be the total number of elements of any and all possible *mixtures*. There are five such elements: *white, black, blue, yellow, red.* Completely unmixed tinctures other than these are impossible to conceive.

However, I separate *white* and *black* from the other three colors (I give the name *colors* only to those three), and place them in a separate class, as if they were opposite to the class of colors. The reason is that the kind of opposition between white and black (that of *lightness* and *darkness*) exists in our perceptual experience not only for those two alone but also in their weaker or stronger co-mixtures with the primary colors and with all colored mixtures. By imparting more or less whiteness or blackness they represent lightness or darkness as such, thus making the colors lighter or darker. In this way, they represent light and dark in general. Their relationship to colors is of a general nature and different from the relationship of colors among themselves.

There have been frequent efforts, even though only at the level of attempts, to represent the relationships between all mixtures in tabular form. The geometric *figure* that is to represent the complete connection of all relationships cannot be arbitrary. Rather, it must represent the relationship itself and must necessarily arise out of the natural inclination or disinclination between the colors, as expressed by the elements.

jede in ihrem völlig reinen Zustande vorstellen; so verlangen wir, dafs das blaue weder von gelb noch von roth den geringsten Zusatz habe; so wie von dem gelben, dafs es nicht im mindesten weder ins blaue noch ins rothe spiele; auch von dem rothen, dafs es weder gelblich noch bläulich schillere. Da nun vielleicht kein vorhandenes Farbenmateriale in der gesetzten völligen Abwesenheit von aller Beymischung da ist; wenigstens aber es der Theorie zukömmt, wenn wir in den vorhandenen Farben noch eine Mischung und Mehrheit erkennen, von solcher zu abstrahiren, und jedes reine Element als eine absolute Einheit anzunehmen, so beweisen diese so gesetzten ganz mischungsfreyen Farbenpuncte eine Analogie mit dem dimensionslosen mathematischen Puncte. Und da die Qualität einer jeden der drey Farben völlig individuell, und gesondert von jeder Qualität der beyden andern ist, ich also die Differenz derselben in gleicher Gröfse setze, so formiren die drey Puncte: **blau**, **gelb**, **roth**, wenn ich die gleiche Differenz durch gleiche Linien ausdrücke, ein gleichseitiges Dreyeck, als den (nicht unbekannten) figürlichen Ausdruck für das Verhältnifs unter diesen drey reinen Naturkräften.

Fig. I.
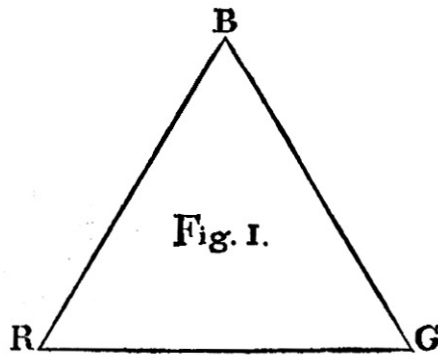
[P4]
When considering the three colors, *blue, yellow*, and *red*, each in its completely pure state, we require blue not to have the slightest addition of yellow or of red. Of yellow we require that it is tinged neither toward blue nor toward red, and of red that it shimmers neither yellowish nor bluish. However, no colorant may exist with the required complete absence of admixture of other colors. The theory remains valid if we mentally separate main component and perceived admixtures of colorants, so that each pure element can be taken as an absolute unity. In analogous manner, such posited colors, ideal and free of admixture, can be represented by dimensionless, geometric points. And, because the quality of each of the three colors is of a completely individual nature, different from all qualitative aspects of the other two, I equate the distance between them. As a result, the three points representing *blue, yellow*, and *red*, their distance expressed by lines of equal length, form an equilateral triangle. It is the (not unknown) figural expression for the relationship between these three pure, natural forces.[3]

[Fig.1]

[3] Runge alludes to T. Mayer's (1775) or J. H. Lambert's (1772) color order systems that are based on equilateral triangles.

Note: It is important for the reader to keep in mind that G (for gelb) stands for yellow, when considering all figures and when reading the text. As will be seen, green is identified as Gr.

Bekannt ist, dafs durch die Vermischung von blauem und gelbem, grün; von gelbem und rothem, orange, und von rothem und blauem, violett erzeugt werden, dafs aber auch, wenn z. B. in grünem das blaue stärker wirkt als gelb, sich das grüne in blau, und wenn gelb stärker darin wirkt, es sich ins gelbe abstufft oder neigt, und sich zuletzt völlig darin verliehrt. Das übereinstimmende ist mit orange der Fall, welches sich in gelb und roth neigt und verliehrt, so wie violett in roth und blau. Diese Beweglichkeit von grün, orange und violett, würde nun im Gegensatz von den drey reinen isolirten Farbenpuncten B. G. R. wenn wir uns diese als gegen einander wirkend vorstellen, als ihre Neigung von einem Puncte zum andern, durch die drey Seiten des Dreyecks ausgedrückt werden.

B

violett grün

Fig. 2.

R G
orange
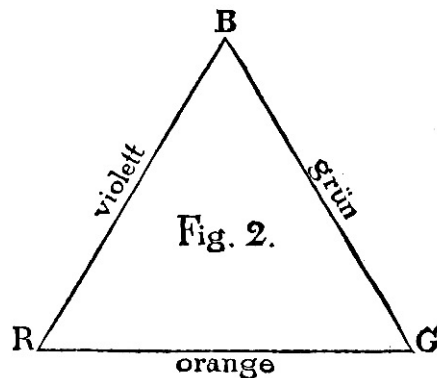
Obgleich nun, im Gegensatz von der Einheit jeder der drey Puncte B. G. R. die drey Mischungen: grün, orange, violett, jede eine Mehrheit sind, und in unzählichen Stuffen zwischen je zweyen Farben sich befinden, so wird doch, wenn zum Beyspiel B. und G. in gleicher Kraft zusammen wirken, oder sich vermischen, in dem Mittelpuncte der Linie BG grün ebensowohl

[P5]

It is known that mixture of blue and yellow produces *green*, yellow and red produce *orange*, and red and blue *violet*. It is also known that when, for example, blueness in green exceeds yellowness, green inclines or graduates toward blue. When yellow is stronger, green inclines toward yellow. In either case it eventually loses itself completely in the other [elementary color]. A comparable situation applies in the case of orange inclining toward or losing itself in yellow and red, as well as violet in red and blue. This *flexibility* of green, orange, and violet, contrary to the three pure, isolated color points B, G, and R imagined as acting against each other, can be expressed as inclination of one point towards another in the three sides of the triangle.

[Fig. 2]

Contrary to the *unitary* nature of each of the three points B, G, and R, each of the three kinds of mixtures, green, orange, and violet, are *multiples*, existing in countless grades between the two colors. Nevertheless, when for example B and G operate jointly, or are mixed, green will appear as a *singular* color at the center point of the line BG, equally inclining toward blue and yellow and having the same distance from the two (the differences, in this specific ratio, turning into indifferences).

als eine eigene Farbe erscheinen, die zu blau und zu gelb in gleicher Neigung und gleicher Differenz (welche in diesem besondern Verhältnifs Indifferenz wird) steht. Ebenso verhält es sich mit orange, und wiederum mit violett. Weil nun grün, orange und violett in diesen Mittel- oder abstracten Puncten mit B. G. und R. in gleicher Differenz stehen, und auf den Seiten des Dreyecks auch in gleiche Entfernung von denselben zu setzen sind, so werden sie auch in ihrem Verhältnisse unter sich in gleicher Differenz stehen, und ein gleichseitiges Dreyeck formiren, welches in dem ersteren mitten inne läge.

als eine eigene Farbe erscheinen, die zu blau und zu gelb in gleicher
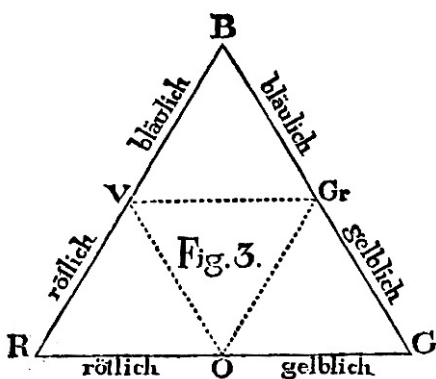
Da aber alle drey reinen Mischungspuncte Gr. O. V. sowohl, als alle sich von Gr. in B. und G. von O. in G. und R. und von V. in R. und in B. neigende Mischungen, nur aus der Zusammenwirkung je zwey er reinen Farben hervorgegangen sind, so sind sie von aller Neigung, zu jeder dritten Farbe sowohl als irgend einer sonstigen Tinctur, völlig frey.

[P6]

The same applies to orange, as well as to violet. Because in these conceptual median positions green, orange, and violet are equally distant from B, G, or R and require placement on the sides of the triangle in equal distance to them they, among themselves, will be equally different, forming also an equilateral triangle, located in the interior of the original one.

[Fig. 3]

[*rötlich* means reddish, *gelblich* yellowish, *bläulich* bluish]

All three mixture points Gr, O, and V, just as all intervening mixtures between green and blue or yellow, between orange and yellow or red, and between violet and red or blue, are in each case generated only from interaction of two pure colors. As a result, they are completely free of the respective third [elementary] color or any other tincture.

Es ist aber vorher bestimmt worden, daſs alle Farben und reinfarbige Mischungen, zu weiſs und schwarz (zu weiſs als einer Erhellung und Schwächung, zu schwarz als einer Verdunkelung oder Trübung) in einem allgemeinen Verhältniſs stehen, und der Einwirkung derselben empfänglich sind. Es sind also die drey Puncte Gr. O. V. sowohl, als alle zwischen ihnen und den Puncten B. G. R. liegenden einfachen Mischungen, mit dem Puncte weiſs nach der einen, und schwarz nach der anderen Seite, (als zwey vollkommenen Gegensätzen) in derselben Differenz, und mithin alle in dieselbe Entfernung von weiſs wie von schwarz zu setzen, in welcher die drey Puncte B. G. R. von ebendenselben (nämlich von weiſs und von schwarz) stehen; da wir gleiche Differenz unter Naturkräften durch gleiche Linien (Entfernungen) auszudrücken zur Regel angenommen haben.

Diese allgemein gleiche Entfernung aber von zwey verschiedenen Puncten, können wir unter keiner andern Figur uns vorstellen, als wenn wir die Totalität aller reinen Farben und ihrer einfachen Mischungen (nämlich die drey Puncte B. G. R. sowohl, als Gr. O. V. mit ihrer ganzen Neigung in die einfachen Farben,) eine vollkommene Creislinie bildend annehmen; innerhalb welcher die beyden gleichseitigen Dreyecke BGR und GrOV zusammen ein gleichseitiges Sechseck ausmachen, und zu welcher weiſs und schwarz, oder die zwey Puncte W. und S. wie auſserhalb der Creisfläche liegende Pole sich verhalten, deren Entfernung von einander WS als eine Linie (Achse) anzunehmen ist, welche durch das Centrum des Creises geht.

[P7]

Earlier, it was determined that all colors and pure mixtures relate to *white* and *black* in a *general* manner (to white by becoming lighter and weaker, to black by becoming darker or dimmer). In this manner they are receptive of the influence of white and black. Therefore, the three points Gr, O, and V, as well as all simple mixtures located between them, and the points B, G, and R all have the same difference from the *white* point in one direction and the *black* point in the other (those being two perfect opposites). This is the consequence of the rule that equal differences between natural forces are represented by equal lengths of lines (distances).[4]

The only figure I can think of representing this uniformly equal distance between two points is that of a perfect circle onto which the totality of all pure colors and their simple mixtures is placed (that is, the three points B, G, and R, as well as Gr, O, and V, together with their intervening mixtures). Within this circle, the two equilateral triangles BGR and GrOV form an equilateral hexagon. W*hite* and *black*, or the two points W and S, relate to the circle like polar points with the distance between them to be taken as a line (axis) passing though the center of the circle.

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[4]As a consequence, the distance between pairs of chromatic elementary colors is, in Runge's system, geometrically slightly smaller than the distance between white and black.

Fig. 4.
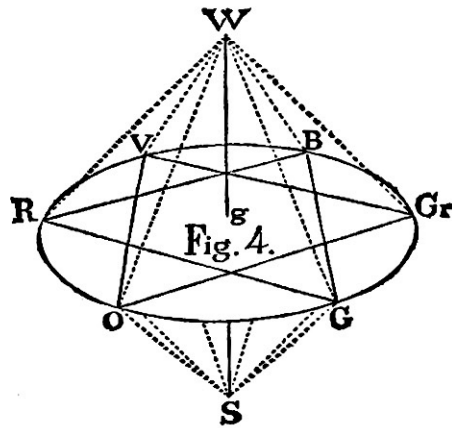
Es ist daher das zweyte Dreyeck GrOV eben so grofs wie das erstere BGR anzunehmen, und man wird sich izt die Totalität aller grünen, orangen und violetten Mischungen in ihrer wahren Richtung so vorstellen können, als wenn das Dreyeck GrOV sich um die Achse WS zwischen den Puncten B.G.R. hin und her bewegte, und so den ganzen Creis bildete.

Fig. 5.
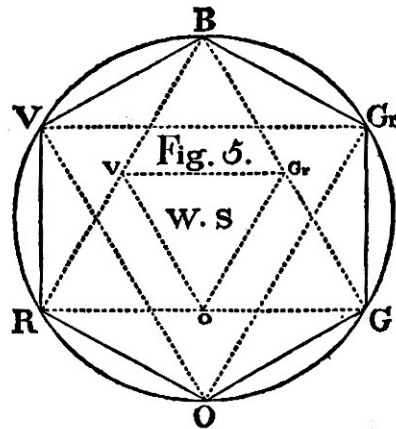
Beyde Dreyecke, oder das vorhin (Fig. 4.) aufgestellte gleichseitige

[P8]

[Fig. 4]

The second triangle GrOV, therefore, is to be assumed as of size equal to that of the first one, BGR. As a result, the true direction of the totality of all *green, orange,* and *violet* mixtures can be imagined as being the result of rotation of the triangle GrOV around the axis WS, thereby forming the complete circle.[5]

[Fig. 5]

[5]The implication is that all pure mixtures of pairs of the three elementary colors are of equal saturation than the elemental colors themselves. In pigment mixture this is never the case. It is also not the case in light mixture, as J. J. von Görres pointed out to Runge after publication of *Farben-Kugel* (see section "Early comments on *Farben-Kugel*" in the translator's essay).

Sechseck, enthalten, in der Folge: blau, grün, gelb, orange, roth, violett, die sogenannten sieben Farben des Regenbogens; wenn man violett in bläuliches und röthliches an beyden Seiten des Regenbogens zertrennt, annimmt. Und so enthält der Übergang und Umfang des ganzen Creises, alle reinfarbigen Mischungen, und die reinen Farben selbst.

Wie grün durch die Vermischung von blau und gelb erzeugt wird, so entsteht durch die Vermischung von weifsem und schwarzem, grau; welches sich in weifslicher und schwärzlicher Neigung auf der Linie zwischen diesen beyden Puncten abstufft, und auf der einen Seite in weifs, wie auf der anderen Seite in schwarz, sich verliehrt. Im Mittel aber, wo die beyden Kräfte in gleicher Stärke gegen einander wirken, wird der Punct seyn, wo dasselbe als ein völlig gleichgültiges grau, in gleicher Differenz und gleicher Neigung zu schwarz wie zu weifs steht; welcher Punct, unserer Configuration gemäfs, eben derselbe ist, auf welchem die Linie WS die Fläche des Farbencreises berührt und schneidet.

In dem Farbencreise sind, wie wir gezeigt haben, die drey abstracten Puncte des grünen, orangen und violetten, welche das Dreyeck GrOV bilden, die Producte von je zweyen reinen Elementarfarben, welche sich in diesen Puncten in gleicher Kraft innigst vereinigt und durchdrungen haben. Wenn wir aber zu dem reinen grün, als dem Producte aus gelb und blau, die dritte Farbe, roth, im geringsten zumischen, so erfahren wir, dafs diese den heitern Schein des grünen blofs zerstört und beschmutzt, ohne solchem einen rothen Schein mitzutheilen. Es wird also grün durch eine stärkere Beymischung von roth, in einen völlig farblosen Schmutz, oder

B

[P9]

The sequence of colors in both triangles, or the previously (Fig. 4) shown equilateral hexagon, is *blue, green, yellow, orange, red, violet,* the so-called seven colors of the rainbow if one assumes violet to be separated at the two ends of the rainbow into a *bluish* and *reddish* one, respectively.[6] In this manner, the transitions and the perimeter of the complete circle contain all pure-color mixtures, as well as the pure colors themselves.

Just as green is generated by intermixture of blue and yellow, mixture of white and black generates *gray,* graded in the directions of white and black along the line between these two points and losing itself in white in one direction and in black in the other. In the *center,* the two forces balance each other in equal strength. It is the point occupied by a *completely indifferent gray,* equally different from, and related to, black and white. In my configuration, this point is the same as the one where the line WS intersects the plane of the color circle.

As has been shown, the three conceptual points of green, orange, and violet, forming the triangle GrOV, represent the product of always *two* pure elementary colors, completely united and fused in equal strength. But when we mix into pure *green,* being the product of *yellow* and *blue,* the smallest amount of the third elemental color, *red,* we find that such addition merely destroys or muddies the pure appearance of green, without introducing the appearance of redness. A stronger admixture of red changes green into colorless mud, or *gray,* taking on a reddish cast only after even higher admixture.

[6] See Runge's original description of the idea of the agreement between his six colors of the hue circle with the seven categorical colors of Newton's circle in a ca. 1807/08 letter to an unknown recipient (Appendix C).

in grau, aufgelöset; welches nur erst durch noch stärkere Beymischung einen rothen Schein annimmt. Diese Auflösung aller farbigen Erscheinung ist die Folge von der gleich starken Zusammenwirkung aller drey reinen Farben. Denn es werde blau mit orange vermischt, so lösen beyde sich eben sowohl in dasselbe farblose grau auf; so wie auch gelb mit violett. Wie wir uns denn auch ein röthliches grün, ein bläulich orange, oder ein gelblich violett, so wenig vorstellen können, als einen östlichen West, oder einen südlichen Nord. Da nun die drey reinen individuellen Qualitäten B.G.R. wenn sie in gleicher Kraft zusammen wirken, alle Individualität völlig verliehren, und in eine absolute Allgemeinheit aufgelöset werden; die Individualitäten von B.G.R. aber, in allen einfachen Mischungen des ganzen Farbencreises in vollkommener Wirksamkeit erscheinen: so sind diese einfachen Mischungen sowohl, als die drey reinen Farben, in gleicher Differenz mit der absoluten Allgemeinheit des farblosen Punctes; welcher daher, in gleicher Entfernung von jedem Puncte des ganzen Umcreises stehend, der Mittelpunct des Creises ist. In demselben lösen sich auch alle diametral entgegenstehenden Farben und Mischungen auf; indem in jedem Diameter des Creises alle drey reinen Farben gleich wirkend sind. Denn wenn (Fig. 6.) der Punct Gr. näher an G. gerückt, und auf der gegenüber liegenden Seite roth (R.) sich in ein röthliches violett (oder zu B.) neigt, so ist B. ins rothe um eben soviel hineingerückt, als Gr. dem Blauen entzogen wurde.

[P10]
Such dissolution of all colored appearance is the result of equally strong interaction of *all three* pure colors. If blue is combined with orange, the two also dissolve into the same colorless *gray*, just as yellow with violet. Similarly, a *reddish green,* a *bluish orange,* or a *yellowish violet* are just as unimaginable as *eastern West* or *southern North.*

The three pure, individual qualities B, G, and R, acting together with equal force, completely lose their individuality. At the same time, the individuality of B, G, and R appears in its full effectiveness in any simple mixture of the complete color circle. These simple mixtures, as well as the pure simple colors, are equally different from the common *colorless point.* As a result this point, located at equal distance from each point on the circle, is the *center point* of the circle. This same point is also the location in which all diametrically opposite colors and mixtures dissolve due to the fact that in this point all three pure colors are equally active. Moving (in Fig. 6) point Gr closer to point G means that on the opposite side red (R) is shifted into a reddish violet (or toward B). Thereby, B moved toward red to the same degree as Gr has distanced itself from blue.
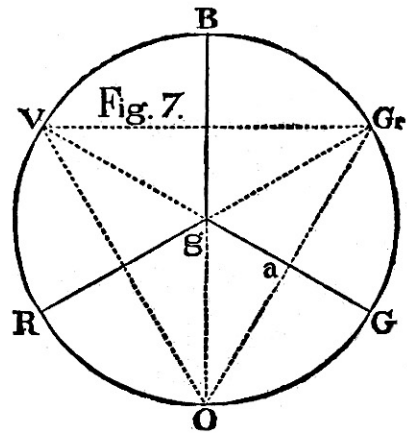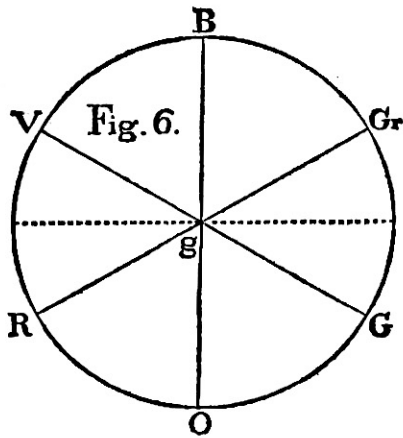
Zugleich ist hier noch anzumerken, dafs in demselben Verhältnisse des gleichseitigen Dreyeckes, welches B. G. und R. gegen einander beweisen, und wie diese dreye sich im Mittelpuncte auflösen, sich auch alle, in dem ganzen Umcreise, in einem gleichseitigen Dreyeck gegen einander stehenden Mischungen auf dieselbe Weise gegen einander verhalten. Denn Gr. und O. werden sich, da in beyden G. sowohl mit B. als mit R. zu gleichen Theilen wirkt, durch ihre Vermischung in ein gelbliches grau verwandeln, welches sich zu gelb (G.) verhalten wird, wie der Punct a. (Fig. 7.) zu dem Mittelpunct g. Welcher Punct a. eben sowohl das Mittel der Linie Gg ist, als sich daselbst die Qualität G. in der Vermischung von Gr. und O. in doppelter Quantität oder Kraft befunden hat, wie B. und R. jedes einzeln. Es wird also, wenn zu Gr. und O. noch V. hinzukommt, das Gleichgewicht von B. G. und R. wiederhergestellt. Eben so verhält es sich mit jedem gleichseitigen Dreyecke, welches der Peripherie anzulegen

B 2

33

[P11]

[Figs. 6 and 7]

At the same time it should be mentioned that just as B, G, and R, standing in an equilateral triangle relationship, dissolve in the center, so do all other mixtures in the color circle that stand in an equilateral triangle relationship, thereby behaving identically. But when mixing Gr and O the result is a *yellowish gray*, having a distance from yellow (G) identical to the distance of point *a* (Fig 7) from the central point *g*. Point *a* is the middle point of line Gg. In this point, due to the mixture of Gr and O, quality G is present in twice the quantity or force of either B or R. When V is added to Gr and O, the equilibrium of B, G, and R is re-established. The same applies to any other equilateral triangle that can be placed with points falling on the periphery of the color circle.

möglich ist; das Product desselben wird immer die totale Auflösung aller farbigen Erscheinung seyn.

Wir schliefsen nun: da weifs (W.) in gleicher Differenz mit jeder der drey Farben B.G.R. und in gleicher Neigung zu allen dreyen stehet, und da schwarz (S.) in gleichem Verhältnifs sich befindet: so sey irgend ein Punct der Neigung beyder Pole zu einander auf der ganzen Linie WS, und unter diesen auch der Mittelpunct g. eben dieser Linie, für sich ebenfalls in gleicher Differenz mit jedem der drey Farbenpuncte B.G.R. und in gleicher Neigung zu allen dreyen zu setzen.

Da ferner die drey Farben B.G.R. in gleicher Differenz mit W. und S. und in gleicher Neigung zu eben diesen stehen; so mufs auch der Mittelpunct g. der Farbenscheibe, in welchem jene dreye ihre Individualitäten durch gleiche Wirksamkeit eingebüfst haben, in gleicher Neigung zu W. wie zu S. und in gleicher Differenz mit eben diesen stehen. Folglich, da diese beyden Puncte g. (der Mittelpunct von W. und S. und der Mittelpunct des Dreyecks BGR) schon mathematisch angesehen in eins zusammenfielen, gehet izt, dafs beyde nur einer und derselbe seyn können, auch aus der gleichen Neigung in demselben zu allen fünf Elementen, durch die gleichmäfsige Wirksamkeit derselben in diesem Puncte, hervor; so wie aus der gleichen Differenz eine vollkommene Indifferenz; in welcher alle individuelle Qualitäten sich aufgelöset haben, und also nur die blofsen Quantitäten ihrer materialen Substanz, in einer Summe übrigbleiben können.

[P12]
The result of mixture of any such triangle is always the complete dissolution of all colored appearance.

We conclude: because white (W) is equally distant from each of the three colors B, G, and R and thereby in equal relationship to the three, and the same for black (S), therefore any point on line WS, among these also the *central point g of this line*, is in the same relationship to the three color points B, G, and R, and thereby equally related to them.

In addition, because the three colors B, G, and R have the same distance from W and S and stand in the same relationship to them, therefore also the *central point g of the color disk*, in which the three have lost their individualities as a result of equal activity, must be in equal relationship and of equal difference to W and S. As a result, the two geometrically coincident points *g* (the center point of line WS and the center point of triangle BGR) must be *identical*, standing in the same relationship to all five elements, due to identical activity of these elements at this point. Identical difference is changed into perfect *indifference*. At this point all individual qualities are dissolved, with only the sum of *quantities* of their material substance remaining.

Dieser Punct ist also, da er in gleicher Differenz mit allen **fünf** Elementen steht, als der allgemeine Mittelpunct von allen anzusehen.

Fig. 8.
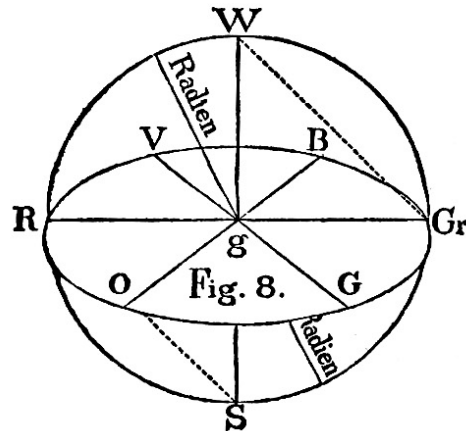
Alle Mischungen, welche aus der Neigung irgend eines Punctes von dem ganzen Farbencreise in weiſs oder in schwarz hervorgehen, (eine Neigung die allen diesen Puncten gemein ist) werden sich in allmähligen Abstuffungen nach W. und nach S. verliehren; und müssen, (da alle nur das Product je zweyer reiner Farben sind, und sich als solche bloſs zu weiſs oder zu schwarz neigen) als ganz frey von Zumischung einer dritten Farbe gedacht werden. Sie sind also in jedem Puncte ihrer Neigung in derselben Differenz von dem Mittelpuncte g. als der Zusammenwirkung dreyer Farben, (oder vielmehr als der Nichterscheinung aller Individualität der Elemente, im Gegensatze von der deutlichen Zusammenwirkung und Erscheinung in den ebengedachten Mischungen) und bilden mithin, da die Differenzen aller Puncte ihrer Neigungen (zu W. oder S.) mit dem Mittelpuncte g. Radien ausmachen, lauter in die Pole W. und S. ablaufende Bogen-

(P13)
This point, because it has equal distance from all *five* elements, must be seen as the *general center* of all points.

[Fig. 8]
[*Radien* means radii]

All mixtures resulting from inclination of any point on the color circle toward *white* or *black* (an inclination that is common for all of them) will in steps slowly lose themselves toward W and S. These mixtures must be taken as completely free of admixture of the third color (because they are only the product of *two* pure colors and as such only incline toward *white* or *black*). Therefore, in each point of their inclination the mixtures are equally distant from point *g*, the latter caused by joint action of *three* colors (or, rather, by the lack of appearance of the individuality of all elements, contrary to the obvious cooperation and visual presence of the colors in the just mentioned mixtures). The points [of the mixtures] form curved lines or *quadrants,* ending in the poles W and S. The reason is that the lines connecting all points of inclination (to W or S) with the central point *g* form radii.

linien oder Quadranten. Wodurch denn das ganze Verhältnifs aller fünf
Elemente zu einander, durch ihre Differenzen und durch ihre Neigungen,
die vollkommene Kugelfigur formirt; deren Oberfläche alle fünf Elemente,
und diejenigen Mischungen derselben enthält, welche in freundlicher
Neigung der Qualitäten zu einander erzeugt werden; und nach deren Mittel-
puncte zu, alle Nüancen der Oberfläche in gleicher Stuffenfolge sich in ein
völlig gleichgültiges grau auflösen: in Verhältnissen, wie ferne sie mit
gleicher oder ungleicher Wirksamkeit der gesammten Elemente sich berührt
haben. So wie überhaupt in jeder Bildung, die Gröfse aus der Differenz,
und die Form aus der Neigung der Elemente zu einander, hervorgeht.

Man wird izt, wenn man sich die Farbenkugel (wobey die
gedoppelt beygefügte Abbildung, von dem weifsen, wie von dem schwarzen
Pole herabgesehen, zur Vergleichung dienen möge) von der Oberfläche bis
zum Mittelpuncte in gleichmäfsiger Wirksamkeit durchdrungen vorstellt,
die gleichfalls hiebey abgebildeten beyden Scheiben, die eine als einen
Durchschnitt im Aequator (als die Farbenscheibe,) die andere aber
durch beyde Pole geführt (in der Richtung, dafs im Aequator roth und
grün (R. und Gr.) die beyden Endepuncte des Diameters ausmachen) zu
erkennen im Stande seyn. Wie ich denn auch nicht zweifle, dafs man
nach diesem Schema, sich die auf willkührliche Weise zwölffach ein-
getheilte Oberfläche leicht als einen völligen Übergang wird denken
können.

Leicht ist nun einzusehen, dafs auf gleiche Weise jeder Ab-
schnitt, welcher parallel mit dem Aequator geführt würde, in dem-

[P14]
In this way, all relations between the five elements, their differences, and inclinations, form a perfect *sphere*. Its surface contains all five elements and those mixtures generated in *favorable* reciprocal inclination of their qualities. Toward the center, all nuances of the surface dissolve by an identical number of grades into totally indifferent gray: in ratios that depend on the degree of activity in the total sum of elements. Just as for any geometric form, the size of the structure develops from the differences between the elements and its form from the reciprocal inclination of the elements.

One can consider the *color sphere* (the two attached images, views toward the top as well as toward the bottom poles, can be used for comparison) to be constituted, from surface to center, of layers of uniform activity of the elements. The additional two *cross-sections,* one through the equator (the *color disk*) [figure below right], the other through the two poles (in the direction making red and green (R and Gr) the endpoints at the equator) support comprehension [of the sphere model]. I have no doubt that it will be easy for the reader to understand that the arbitrary twelve-fold division of the sphere surface in the colored engraving [figure below left] is, in reality, a *completely* continuous progression.

It can easily be imagined that, in the same manner, any *cross-section* parallel to the equator has a gray center point that is *blackish* to the degree it is approaching the black pole of the sphere, and identically for *whiteness* of the gray in direction of the white pole.
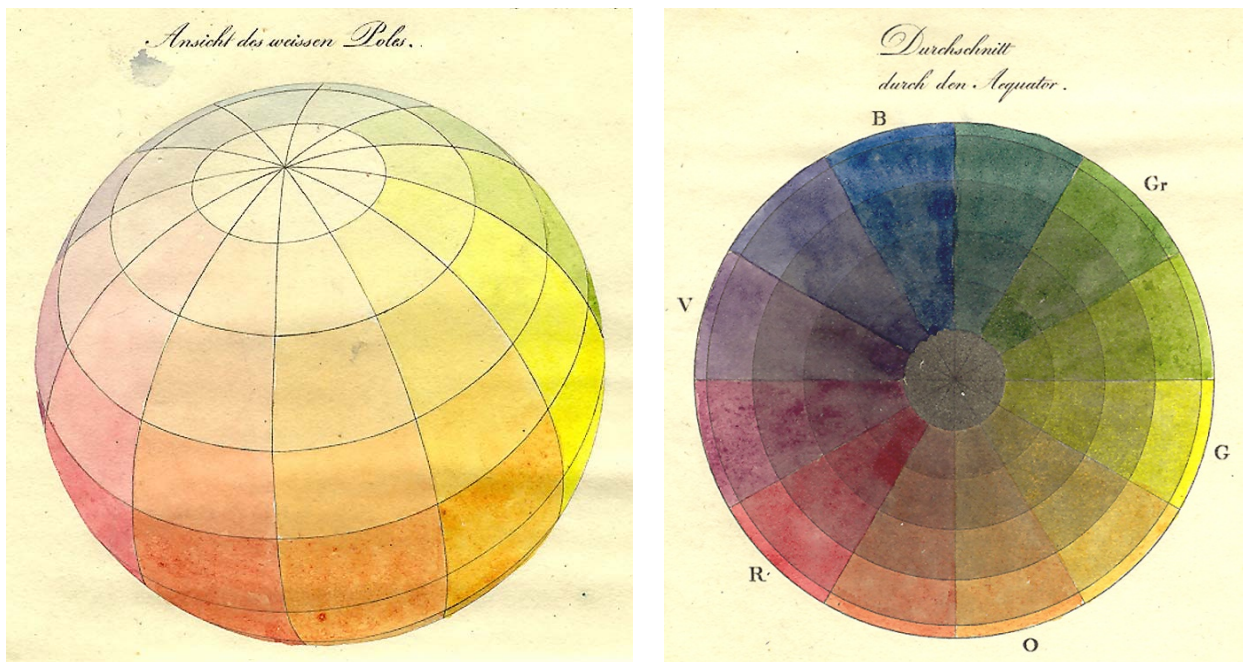
[Two figures from the hand-illuminated engraving of the color sphere]

selben Verhältnifs einen schwarzgrauen Mittelpunct zeigen müfste, wie derselbe nach dem schwarzen, so wie einen weifsgrauen, wie er nach dem weifsen Pole hin geschähe.

So würden auch in allen Durchschnitten durch die Pole, welche im Aequator die Richtung eines verschiedenen Diameters zeigten, auf die gleiche Weise sich die Farben beym Zutreffen auf die Linie WS in grau zerstören.

Man wird sich nun eben so wenig irgend eine Nüance, welche, durch Vermischung, aus den fünf Elementen hervorgegangen wäre, denken können, welche nicht in diesem Verhältnifs berührt oder enthalten wäre, als man sich eine andere richtige und vollständige Figur für das Ganze dieses Verhältnisses wird vorstellen können. Und da jede Nüance zugleich in ihr richtiges Verhältnifs, zu allen reinen Elementen wie zu allen Mischungen gestellt ist, so ist diese Kugel als eine General-tabelle zu betrachten, wodurch derjenige, welcher zu seinem Geschäfte verschiedener Tabellen bedürfte, sich immer wieder in den Zusammenhang des Ganzen aller Farben, zurechtfinden könnte. Wie es denn izt dem Aufmerksamen einleuchten mufs, dafs sich auf einer ebenen Fläche keine Figur zu einer vollständigen Tabelle aller Mischungen finden könne; indem sich das Verhältnifs nur cubisch nachweisen läfst.

[P15]

Further, in all cross-sections through the poles, identified by different diameters on the equator, the colors of the perimeter will fade out into gray when approaching the central line WS.

It is impossible to think of a color nuance generated by mixture from the five elements that is not contained in this scheme, neither can one think of another true and complete figure for the totality of the relationship. With every color nuance placed according to its correct relationship to the pure elements as well as to all mixtures, the sphere can be taken to be a *general table* with which those requiring various tables in their occupations can, at any time, orient themselves in the totality of colors. The attentive reader will now understand that no planar representation can contain the total table of all mixtures, because the relationships can only be represented in cubical form [three-dimensionally].

Anhang.

———•••◎◈◉◉••———

Ein Versuch, die sinnlichen Eindrücke aus den Zusammenstellungen

verschiedener Farben, mit dem vorhin entwickelten

Schema zu reimen.

Appendix

An attempt to bring the sensory impressions resulting from arrangements of different colors into agreement with the previously developed color scheme.

Vorzüglich bey Betrachtung der Scheibe, welche den Durchschnitt der Farbenkugel im Aequator darstellt, und indem man sich erinnert, dafs alle einander auf derselben grade gegenüber liegenden Farben, als Kräfte anzunehmen sind, welche einander entgegenstehen, und sich durch ihre Vermischung zerstören in grau, wird man bemerken müssen, dafs, wenn man diese sich entgegengesetzten Farben auf einer Fläche neben einander hinstellte, solche eben daher die allerlebhaftesten Contraste bilden werden. Zugleich aber macht diese Gegeneinanderstellung einen sehr angenehmen Eindruck. Man vergleiche auf unserer beygelegten Farbentafel Fig. 1. blau mit orange, 2. gelb mit violett, 3. roth mit grün.

<div align="center">2.</div>

Der Eindruck aber wird sehr verschieden, wenn man wie Fig. 4. blau mit gelb, 5. gelb mit roth, und 6. roth mit blau, zusammenstellt. Diese Zusammenstellung wird das Auge mehr reizen und auffordern, als demselben Vergnügen gewähren.

<div align="center">3.</div>

Würde man nun roth mit violett, violett mit blau u. s. w. paaren, oder die Farben alle so neben einander stellen, wie sie an der Scheibe (im Farbencreise, oder auch im Regenbogen) auf einander folgen, (Fig. 7.)

<div align="center">C 2</div>

[P19]
1. When viewing the disk that shows the equatorial plane of the color sphere and remembering that all colors opposite from each other on it are to be considered forces opposing each other and that, in mixture, they destroy each other forming gray, the reader will notice that as a result of placing these opposing colors next to each other on a plane, they will form the highest possible contrast. At the same time, the impression created by these pairings is very pleasant. Compare *blue* with *orange* (Fig. 1), *yellow* with *violet* (Fig. 2), or *red* with *green* (Fig. 3).[7]
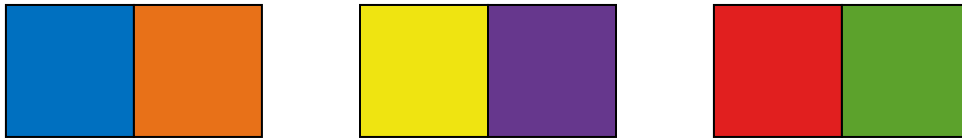
Figs. 1, 2, and 3 Harmonious effect, consisting of the direct contrasts of the three pure colors.

2. A distinct change in impression results when opposing *blue* against *yellow* as in Fig. 4, *yellow* against *red* in Fig. 5, and *red* against *blue* in Fig. 6. These combinations will irritate and provoke the eye rather than offer it pleasure.
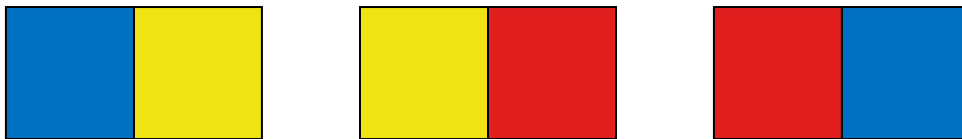
Figs. 4, 5, and 6 Disharmonious effect, the result of combinations of pure colors.

[7]Colors on extant copies of Runge's separate chart of harmonious and disharmonious color combinations are more or less degraded. What is shown here are approximate reproductions. The color fields represent hues of saturation achievable with paints.

so entsteht, auch bey der schönsten Lebhaftigkeit der Farbe, eine Eintönigkeit.

<div align="center">4.</div>

Die erstere Zusammenstellung, von entgegengesetzten Farben, ist harmonisch zu. nennen.

<div align="center">5.</div>

Die zweyte Zusammenstellung, von den drey reinen Farben, disharmonisch.

<div align="center">6.</div>

Die dritte Zusammenstellung, von den Farben in der Folge welche sich an der Farbenscheibe, oder im Regenbogen befindet, monoton.

<div align="center">7.</div>

In dem ersten Falle mufs eine Beziehung liegen auf das, mit welchem alle Farben in Beziehung stehen; und diese Beziehung zweyer Farben, auf das eine zu welchem das Verhältnifs allen gemein ist, ist die Harmonie.

<div align="center">8.</div>

Im zweyten Fall mufs eine individuelle Wirksamkeit von zwey völlig verschiedenen Kräften auf einander statt finden; welches Disharmonie ist.

<div align="center">9.</div>

Und im dritten Falle müssen blofs die beyden neben einander gestellten Farben mit einander in Beziehung stehen, ohne die allgemeine Beziehung; welches Monotonie ist.

<div align="center">10.</div>

Wenn man drey Farben oder gefärbte Felder, so auf einander folgen

[P20]

3.   When placing in pairs red and violet, violet and blue etc., or when all colors are placed next to each other as they follow each other on the disk (in the color circle, or in the rainbow (Fig. 7) the result is monotony, even if the colors are of the most beautiful liveliness.

Fig. 7 Monotonous sequence of the colors of the circle or the rainbow

4.   The first kind of arrangement, of opposing colors, should be termed harmonious.

5.   The second kind of arrangement, of the three pure, simple colors, is disharmonious.

6.   The third arrangement, where colors are in sequence of the color disk or the rainbow, is *monotonous*.

7.   In the first case, there must be a relationship with the property by which all colors are related, and this relationship of two colors with the property by which the relationship is common to all of them is *harmony*.

8.   In the second case there must be an individual effectiveness of two totally different forces against each other, which is *disharmony*.

9.   In the third case two juxtaposed colors must be neighbors [on the color disk], without having a more general relationship, which is *monotony*.

läfst, wie Fig. 8. blau, grau, roth; so ist grau als ein Zwischensatz zu betrachten, welcher die beyden Gegensätze blau und roth verbindet, und beruhigt; indem grau der Punct ist, zu welchem alle Farben des ganzen Creises in gleicher Beziehung stehen.

11.

Wenn man aber, wie Fig. 9. blau, gelb, roth auf einander folgen läfst, so steht gelb, als Zwischensatz oder Verbindung betrachtet, eben so isolirt in seiner individuellen Wirksamkeit, als blau und roth. Ja man möchte sagen, eine jede von diesen drey Kräften sucht den Uebergang, durch welchen sie sich mit der benachbarten verbinden möchte; der Streit wird also nur vermehrt, und es bleibt ein disharmonischer Effect.

12.

Und wenn man die Folge Fig. 10. von blau, violett, roth, hinstellt, so bezieht sich zwar blau, wie auch roth, auf den Zwischensatz, indem violett beyde in sich vereinigt. Allein violett ist nur der Beziehungspunct dieser beyden, nicht aller übrigen Farben, und zieht solche, anstatt den allgemeinen Beziehungspunct ahnden zu lassen, blofs in sich zusammen; daher ist die Wirkung monoton.

13.

Man erinnere sich, dafs zwey neben einander gestellte Farben, wenn sie vermischt werden, entweder feindselig auf einander wirken, oder sich freundschaftlich zu einander neigen; oder drittens, sie vereinigen sich productiv, und verliehren sich beyde in ihrem Producte.

10. If three colors or colored areas are juxtaposed in sequence, as are *blue, gray,* and *red* in Fig. 8, then gray is to be viewed as intermediate, connecting and calming the two opposites blue and red, because gray is the point to which all colors of the color circle have the same relationship.
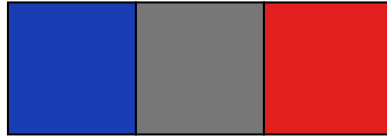
Fig. 8   Calming or separation of disharmony as the result of indifference

11. However, if *blue, yellow,* and *red* in Fig. 9 are juxtaposed in sequence then yellow, considered as connecting link, is as isolated in its individual activity, as are blue and red. One might say that each of the three forces searches for the transition path through which it would like to connect with its neighbor. This only increases the dissonance, the resulting effect being disharmonic.
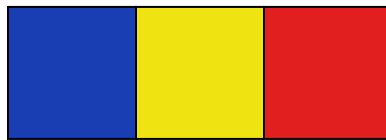
Fig. 9 Increase of disharmony by addition of the third color

12. In the sequence *blue, violet,* and *red* in Fig. 10, violet relates to both blue and red and unites the two. But violet is only the intermediate of these two colors, not the general point of relation for all colors [gray]. It merely pulls the two together without providing a hint of the general point of relation, with the result of monotony.

Fig. 10 Weakening of disharmony via a transition or product.

13. Remember that two juxtaposed colors, when mixed, either act hostile or incline in a friendly manner toward each other. The third case is that they unite productively and loose themselves both in their product.

14.

Das erste ist der Fall mit roth und grün, welche sich durch ihre Vereinigung vernichten in grau.

15.

Das zweyte mit roth und orange, welche sich in einander ziehen und neigen.

16.

Das dritte mit roth und gelb, welche durch ihre Vermischung orange erzeugen, und in demselben ihre Individualitäten vereinigen.

17.

Durch einen Zwischensatz nun von grau, da es der Gegensatz aller Individualität, und die eigentliche Allgemeinheit ist, wird insoferne eine harmonische Verbindung zu wege gebracht werden, da die Individualität einer jeden reinen Farbe oder Mischung mit derselben im Contraste stehet; die Individualität also stärker und beruhigter hervortritt, und zugleich doch alle in gleicher Beziehung zur Allgemeinheit stehen.

18.

Wenn man hingegen roth mit blau durch violett verbindet, so erscheint beydes, roth wie blau, nur als die beyden Seiten des violetten, indem ja roth wie blau, mit violett nicht blofs wie mit grau in Beziehung stehen, sondern im violetten vereint wirksam sind, und auch so erscheinen. Roth und blau werden also durch die Zwischenstellung von violett an ihrer individuellen Erscheinung und Kraft einbüfsen.

[P22]

14. The first situation applies to the case of red with green. They destroy each other in the unification, turning gray.

15. The second applies to red and orange which incline toward each other, each pulling the other into a joint location.

16. An example of the third case is red and yellow, producing in their mixture orange, in which they unite their individualities.

17. Gray in the middle, because it is the opposite of all individuality and of true generality, results in a harmonious connection. The reason is that the individuality of every pure color or mixture contrasts with it. As a result, the individuality of these colors shows more noticeably and calmly, while at the same time they all stand in equal relationship to the common gray.

18. But if red is combined with blue via violet, both red as well as blue are merely the two components of violet. Red and blue do not stand in the same *relationship* to violet as they do to gray, but rather are both equally active in violet and are *perceived* in this way. Red and blue lose some of their individual appearance and force when violet is placed between them.

19.

Ein jeder wird die Bemerkung gemacht haben, daſs zwey hart an einander abschneidende Farbenflächen, wenn wir sie aus einiger Entfernung ansehen, auf der Gränze etwas in einander flieſsen. Am besten wird man diese Erfahrung sich zu eigen machen bey Mosaicbildern, oder gewirkten Tapeten, wo die Mischungen durch neben einander isolirt stehende Puncte oder Linien hervorgebracht werden, die durch Entfernung in einander flieſsen. (Ob dieses nun durch die zwischentretende Luft geschieht, oder dadurch, daſs die von den verschiedenen Farben in unser Auge dringenden Strahlen, sich in demselben creuzen, davon ist hier die Rede nicht.)

20.

Durch dieses Ineinanderflieſsen aber entsteht ein Zwischensatz von selbst; und leicht ist einzusehen, daſs wenn ein blaues Feld an einem gelben abschneidet, sich durch das Ineinanderflieſsen auf der Gränze ein grüner Rand zeigen wird.

21.

Stellte man nun grün und roth zusammen, so wird grau auf der Gränze bemerkbar werden. (Man kann dieses am deutlichsten darthun, wenn die Flächen sich in Winkeln gegen einander neigen, so daſs die eine Farbe an die andere reflectiret. Wenn ein Gewand grün und roth changeant ist, und die beleuchteten Stellen etwa alle roth erscheinen, die Schatten aber grün, so wird die eine erleuchtete Falte in dem Schatten der andern graue Reflexe zu wege bringen.)

19. Every reader will have noticed that two color fields juxtaposed without separation will, when viewed from some distance, appear in the border region to flow to a degree into each other. This experience is best exemplified when viewing mosaic images or woven tapestries, where mixture is obtained from isolated points or lines that, when viewed from a distance, appear to flow into each other. (If this is the result of the intervening air layer or of rays coming from different colors crossing each other in the eye is a matter beyond this essay.)[8]

20. This flowing together is itself the cause of intermediary colors. It is easy to see that when viewing a blue field juxtaposed with a yellow one, a green border area will appear along the edge as the result of flowing together.

21. When juxtaposing green and red, gray will appear at the borderline. (This is most clearly apparent if the two color fields are inclined toward each other at an angle so that one color reflects onto the other. In a garment made of green and red shot fabric [cangiante] where the illuminated areas appear red and those in the shade green, an illuminated crease will produce gray reflection in a shaded one.)[9]

[8] Runge refers to mixture of color stimuli reflected from neighboring small areas of color that, at a distance, can no longer be distinguished and are seen as the sum of both areas (partitive mixture).

[9] Here, the reference is to a colored surface reflecting light onto another surface of a different color. The appearance of the second surface changes as a result of the spectral composition of light falling on it and being reflected by it.

22.

Da nun grau, welches sich zwischen roth und grün zeigt, keine Individualität, sondern die allgemeine Auflösung entgegengesetzter Kräfte ist, so liegt in dem Streite zweyer entgegengesetzten Farben schon von selbst die Harmonie, nämlich die Beziehung auf die Allgemeinheit.

23.

Hingegen der zwischen blau und gelb eintretende grüne Uebergang, stört, als eine neue Individualität, die Wirkung des blauen wie des gelben, indem die ganze Individualität derselben für ihr Product in Anspruch genommen wird. Es muß also, da grün (auf welches gelb und blau mit ihrer ganzen Kraft dringen) nicht bestimmt erscheint, eine Unruhe in den beyden reinen Farben nothwendig erfolgen; und die Unruhe in dieser Zusammenstellung ist wirklich eine Dissonanz, welche durch einen bestimmten Zwischensatz aufzulösen ist. (Auch hat man, im Gefühl dieses Verhältnisses, eine solche disharmonische Zusammenstellung immer gewählt, wo das Auge mehr gereizt und aufmerksam gemacht, als vergnügt werden sollte, z. B. bey Monturen, Flaggen, Wappen, Spielcharten u. s. w.)

24.

Ueberlegt man, daß alle Farben, welche vermischt sich in ein völliges grau auflösen, einen lebhaften und harmonischen Contrast bilden; daß die reinen Farben durch ihre Zusammenstellung, als eine Dissonanz das Auge reizen; die monotonen Uebergänge im Regenbogen den Sinn am ruhigsten lassen; so wird man sich vorstellen können, daß eine verständig gewählte Zusammenstellung von lauter brillanten Farben, ohne daß es nöthig wäre

[P24]

22. Because gray, placed between red and green, is without individuality, but rather the general dissolution of opposing forces, harmony in this case is already implicit in the quarrel between two opposing colors because of their relationship to the generality of gray.

23. However, the green transition color between blue and yellow, having a different individuality, disturbs the effect of blue as well as of yellow, because both their individuality is fully consumed in their product. A certain agitation necessarily appears in the two pure colors because green (the result combined of full force action of blue and yellow) is not perceived in a determined manner. The agitation in this combination is, in truth, a dissonance, but it can be eliminated with the placement of an intermediate color. (In the past, as the result of intuitive knowledge of this relationship, this kind of disharmonic combination was used whenever the eye was to be stimulated and made attentive, rather than pleased, for example, in uniforms, flags, coats of arms, playing cards, etc.)

24. Consider the following facts: all those colors which, when mixed, dissolve into gray, produce lively and harmonic contrasts; pure colors in their juxtaposition stimulate the eye as a dissonance; the monotonous transitions in the rainbow calm the visual sense most strongly. In consequence, it is imaginable that a sensibly selected combination of only brilliant colors, without a need for interrupting their sequence with gray or muddy colors is, due to these properties, capable of enhancing the significance of, and impression created by, a work of art, just as musical sounds can do so for the sense and spirit of a poem.

die Folge derselben durch graue und schmutzige zu unterbrechen, wegen eben dieser Eigenschaften geschickt ist, in die Bedeutsamkeit und den Eindruck eines Kunstwerks einzugreifen; wie die Töne der Music in den Sinn und den Geist eines Gedichts.

<div style="text-align:center">25.</div>

So wie man die Gröfse der harmonischen Contraste auch noch durch eine Neigung beyder Theile, des einen ins dunkle, des andern ins helle vermehren kann, und solche dennoch immer in Beziehung auf den Mittelpunct (grau) bey ihrer Wirkung auf einander bleiben, so giebt es auch in diesen Contrasten Uebergänge, wo die Beziehung auf den Mittelpunct sich in irgend eine Farbe neigt. Wie Fig. 11. o r a n g e mit g r ü n; oder 12. mit v i o l e t t; oder auch 13. v i o l e t t mit g r ü n: indem orange mit grün vermischt ein g e l b l i c h e s g r a u geben würde; orange mit violett ein r ö t h l i c h e s; und violett mit grün ein b l ä u l i c h e s. Wie durch die siebente Figur bey der Construction der Farbenkugel bewiesen wurde.

<div style="text-align:center">26.</div>

Wenn man nun zwey reine Farben durch einen grauen Zwischensatz gewissermaafsen verbindet oder beruhiget, indem dieser als das allgemeine der Farbe mit der Individualität jener im Contrast stehet, und sie also in ihrer ganzen Wirksamkeit erhält; so füllt der Zwischensatz zwar eine Lücke aus, und trennt die beyden Farben; bringt aber keine eigentliche harmonische Verbindung zu stande, da in ihm die Individualität völlig aufgehoben ist, also auch alle active Erscheinung.

<div style="text-align:center">D</div>

25. The magnitude of the harmonious contrast can be increased by making one color darker and the other lighter, with their relationship to the center point (gray) remaining the same in regard to their effect on each other. But also in these contrasts there are transitions where the path does not proceed through the central point, the product being colored. Examples are *orange* with *green* in Fig. 11, or with *violet* in Fig. 12, or *violet* with *green* in Fig. 13: orange mixed with green results in a *yellowish gray*, orange with violet in a *reddish* one, and violet with green in a *bluish* one. This was previously demonstrated in Fig. 7 of the essay on the color sphere.
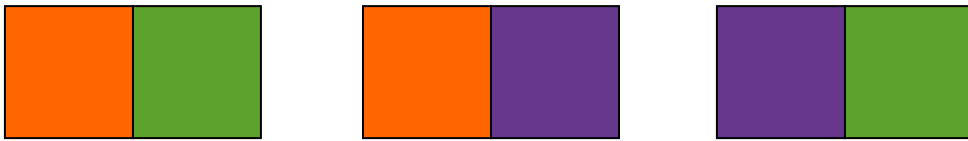
Figs. 11, 12, and 13 Indirect harmonic contrasts of two mixtures

26. When, so to speak, linking or calming down two pure colors with an intermediary gray, contrasting the latter that represents the universal in colors with the individuality of the former, the gray color fills a gap and separates the two colors. It brings about a *true* harmony because individuality is completely eliminated in gray, and with it all active appearance.

27.

Hingegen, weil orange und grün bey einander einen harmonischen Contrast bilden, so wird man in der Folge Fig. 14. von blau, orange, grün, roth, zwey reine Farben durch den Zwischensatz von einem harmonischen Contraste (orange und grün) mit einander zu eigentlicher Harmonie verbinden können, wenn grün neben roth, und orange neben blau zu stehen kommt. Dieser Accord enthält die volle individuelle Wirksamkeit der drey Farben; die Dissonanz ist aufgelöset, und die Eintönigkeit vermieden. Dasselbe erfolgt, wenn Fig. 15. gelb, violett, orange, blau; und Fig. 16. roth, grün, violett, gelb, abwechseln.

28.

Wenn man bey Betrachtung dieser drey Folgen, auf die siebente Figur zur Construction der Farbenkugel zurückgeht, wird man nicht ohne Vergnügen gewahr werden, wie die Ordnung in welcher hier je zwey Farben und zwey Mischungen stehen, ein regelmäßiges Resultat aus dem gesammten Verhältniß auf der Scheibe ist. Denn wir haben hier zwey reine Farben, (z. B. Fig. 14. blau und roth) und der Contrast durch welchen diese verbunden sind (orange und grün) erweckt die Ahndung der dritten. Es würde aus der Vermischung von orange und grün ein gelbliches grau (d. h. die Neigung des allgemeinen Mittelpunctes zur dritten Farbe, gelb) entstehen; und so läßt auch der bloße Anblick uns auf gelb, als den gemeinschaftlichen Character von orange und grün verfallen.

29.

Wer da weiß, wie Dissonanz, Harmonie, und Monotonie, in einem

27. However, because orange and green form by themselves a harmonious contrast, it is possible to link in the sequence *blue, orange, green, red* (Fig. 14) two pure colors in a true harmony by insertion of a harmonious contrast (orange and green), as long as green is placed next to red and orange next to blue. This accord contains the complete individual activity of the three [fundamental] colors, dissonance is dissolved and monotony is avoided. The same takes place in the sequences (Fig. 15) *yellow, orange, violet, blue* and (Fig. 16) *red, green, violet, yellow.*

Fig. 14

Fig. 15

Fig. 16

Figs. 14, 15, and 16 Dissolution of disharmonious effects through indirect harmonic contrast of two mixtures, resulting in a harmonious accord.

28. When, in this connection, referring back to Fig. 7 of the construction of the color sphere, the reader will be pleased to note that the order in which the two colors and their mixtures are arranged in this example is a regular result of the complete arrangement of the disk. There are two pure colors (e.g. in Fig. 14 blue and red) and the contrast through which they are connected (orange and green) induces the idea of the third. Mixture of orange and green produces a yellowish gray (that is, the inclination of the common central point toward the third color, yellow). Just looking at orange and green induces in us the idea of yellow as the common character of orange and green.

Kunstwerk dahin gehören, wo sie durch den Sinn der Composition erfor-
derlich sind, der wird es diesen wenigen Bemerkungen ansehen, dafs ich
durch dieselben nur einen Anknüpfungspunct suchte, um zu zeigen, wie
die nothwendige Construction der Farbenkugel, dieses und noch viele andere
Verhältnisse an die Hand giebt. So wie die scheinbare Trivialität solcher
Bemerkungen, nur bey der Prätension bestehen könnte, als sollte hier eine
vollständige Theorie der mahlerischen Harmonie gegeben werden; welches
doch so wenig der Fall ist, als ich meinen Aufsatz überhaupt für eine neue
Farbentheorie auszugeben gemeynt bin.

Da die Kugel aber die nothwendige Figur ist, welche die Con-
struction des Verhältnisses der fünf materiellen Elemente: weifs,
schwarz, blau, gelb, roth, zu einander, umfasst, so möchten sich
durch diese gefundene Figur, in der Folge vielleicht die reinen Einsichten
in die innere Natur dieser Erscheinung bestimmter ausdrücken lassen.

29. Whoever knows the proper place for dissonance, harmony, and monotony in a work of art, because the idea of the composition requires them, will note that in these few remarks I merely attempted to find a point of connection with the logically required construction of the color sphere. The sphere offers these and many other relationships to hand. The obvious triviality of these comments indicates that I do not presume to offer a complete theory of harmony in painting. The purpose of my main essay is that of announcing a new color theory.

The *sphere* is the logically required figure to encompass the construction of the relationships of the five material elements: *white, black, blue, yellow*, and *red*. With the aid of the sphere it may in the future be possible to express, with more certainty, pure insights into the inner nature of these phenomena.

Appendix A

Excerpt of letter of July 3, 1806 from Runge to Goethe

"It is my belief that the German artists of old, had they given thought to the subject of form, would have lost the immediacy and naturalness of expression in their figures until they reached a certain degree of knowledge in this science. — Many people, without being grounded in the relevant science, built bridges, hanging works, and all sorts of artificial things. This may work for a while; but when such a person reaches a certain level of knowledge and begins to draw mathematical conclusions by himself, his talent will be lost until he comes to a point of freedom again by working through the relevant science. Similarly, after having been astonished by the special phenomena of the mixture of the three colors, it was impossible for me to remain calm until I had established a certain point of view of the totality of the color world, one large enough to contain all such transformations and phenomena.

It is a completely natural idea for a painter to want to know, when viewing a beautiful landscape or being spoken to in some manner by an effect of nature, what material mixtures can be used to reproduce the effect. It is this situation that has led me to study the individual properties of colors, and to make an attempt to determine if it is possible to sufficiently investigate their powers so as to fully understand what they are capable of, or what is achievable with them, and what moves them. — I hope that you will look benevolently on this little essay of mine. I only write it down to make my views on this matter clear to you, even though I believe that this subject needs to be discussed in its totality. I do not think that to view colors in this way will be of no value in regard to the art of painting, or that one can do without. My viewpoint will neither contradict the results of optical experiments performed so as to learn the facts of color in their completeness, nor will it make them unnecessary. As I cannot present you with incontrovertible proofs, proofs that must be based on complete knowledge, I ask you to use your own feelings when attempting to understand what I mean when saying that a painter is not involved with any other elements than those that you will find mentioned here.

1. There are only three colors, yellow, red, and blue, as is known. If we take them at full power and limited within a circle, they form three transitions, orange, violet, and green (I give the name orange to every color that falls between yellow and red, or that transitions from yellow toward red or vice versa). These three, in their intermediate positions, are of highest brilliance and are the pure mixtures of the [three elemental] colors.
2. To think of a bluish orange, a reddish green, or a yellowish violet is like thinking of a southwesterly north wind, and so on. But I will attempt to show below how one can explain a warm violet color and similar effects of nature.
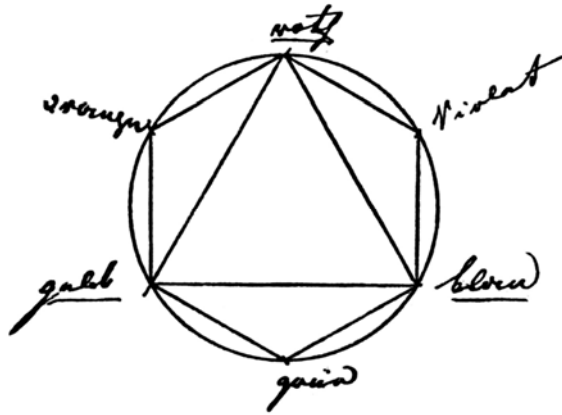
3. Two pure colors, such as yellow and red, produce a pure mixture: orange. When blue is added to it, orange is dirtied, and when the three components of the mixture are in equal parts, all color is canceled, resulting in dull gray. Two pure colors can be mixed [without loss of saturation]; but two intermediate colors cancel or muddy each other because the third [elemental] color is now also present in the mixture. — The three pure colors cancel each other in gray. The three intermediate colors, orange, violet, and green, do the same when mixed in equal parts because now all three [elemental] colors are again equally strong. Only the pure transitions between [pairs of] the three colors are on this complete circle and mixture of all three only adds gray to them. But outside of the circle there is also white and black, multiplying the possibilities for colors.
4. Addition of white renders all colors fainter and even though they become lighter, they lose clarity and fire.
5. Black makes all colors dirty and, becoming darker, they lose purity and clarity.
6. White and black mixed together result in gray.
7. However, we soon discover that the impressions of nature received through our eyes are, in their elements, not exhausted in the interaction between the three [elemental] colors as well as with white and black. Addition of white renders colors fainter and addition of black makes them muddy and in consequence we can assume that there is also an effect of lightness and darkness. The following observations will show to what extent this needs to be kept in consideration.
8. Aside from the previously mentioned difference of lighter and darker in pure colors, there is in nature another important and conspicuous one. When, for example, we compare a red fabric, paper, taffetas, silk, or velvet, all of the same lightness and purity, with the red of the sunset, or a red transparent glass, we will find that all of them are different in a manner that resides in the transparency or opacity of the matter involved.

9. If the three opaque colors red, blue, and yellow, are mixed, a gray results that can also be mixed from white and black.

10. When mixing the three colors in their transparent form so that none is predominant, a kind of darkness is obtained that cannot be generated otherwise.

11. White and black are opaque or physical. One should not get upset about a phrase like 'white glass,' because it simply means clear glass. White water that is pure is impossible, as is clear milk. Because black renders colors darker it might be clear. However, it cannot be clear because it muddies them.

12. Opaque colors are located between white and black: they can never be as light as white or as dark as black.

13. Transparent colors are infinite in their brightness or darkness. We can see this in the examples of fire for brightness and water for darkness.

14. Gray, the product of the three opaque colors, cannot be rendered pure again by light, neither can it be purified by mixture; it either bleaches out to white or chars to black.

15. When three glass filters of the three pure, transparent colors are superimposed, it generates a darkness that is deeper than that of each color alone. The cause is the following: Three transparent colors together result in a colorless darkness that is deeper than any of the three colors themselves. Yellow, for example, is the brightest and most luminous of the three colors. And yet, when one adds enough yellow to very dark violet so that the two neutralize each other, the darkness is increased to a considerable extent.

16. If one places side by side onto a white background material a dark transparent glass, such as an optical glass, and a slice of polished hard coal of half the thickness, the glass will appear lighter. If one doubles both, the darkness of the coal remains the same because it is opaque. The glass will darken without limit, even though in the end it will no longer be visible. Single colors in transparent form can result in comparable darkness. When compared with them, black will have the appearance of a dirty spot.[1]

17. When diluting a transparent product of the three transparent colors and pass light through it, the resulting appearance will be a kind of gray. However, this gray is much different from the mixture of the three opaque colors.[2]

[1] Glass that absorbs some light across the spectrum and is of infinite thickness will not return any light impacting on it, except for any reflected mirror-like off the surface. Depending on the surround, it will have as dark an appearance as is possible in the specific circumstances. An opaque material painted with black paint will still reflect some light and, therefore, in comparison appear grayish.

[2] Runge refers to the difference in appearance of a gray filter compared with a gray painted object.

18.   The brightness at sunrise around the sun while the sky is clear, or from the sun, is of a magnitude that we can barely tolerate. Assuming that the colorless clarity of that light is the product of three colors, they would have to be of such brightness, and so far removed from our powers of experience that they would be for us the same secret as those colors that disappeared in darkness.

19.   We now notice that brightness or darkness do not stand in the same relationship to the transparent colors as black and white do to the opaque colors. Brightness and darkness is much more *one property*, and united with clarity and color. Consider a pure ruby, as thin or as thick as you like. The resulting red is always one and the same and it is only a lighter or darker transparent red, depending to what degree it is visited by light. Light activates the product of this color in the same way when the color is deep and raises it to luminous clarity capable of revealing every color. Colors can be illuminated, the light incandescing them to ever higher burning. Such illumination often surges around us unnoticed and shows objects in a thousand different appearances, impossible to achieve with simple mixture. In such light everything remains clear and it even increases clarity. In this way one can often perceive the most common objects as having an allure that usually is more the result of brightening of the air between observer and the object rather than of illumination of its forms.

20.   If considered in depth, the relationship between light and transparent colors is infinitely stimulating. The ignition of colors and the merging into one another, the regeneration and disappearance, is like breathing with long intermissions from eternity to eternity, from the highest light to the lonesome and everlasting quietness in the deepest tones.

21.   Opaque colors are like earthly flowers, not daring to measure themselves against heaven. Yet they are involved on one side with weakness, white, and on the other with evil, black.

22.   Though, when mixed with neither white nor black but thinly applied over either of them, they are capable of such delightful variations and can produce such natural effects that practical use of the ideas must restrict itself to those. Transparent colors, in the end, only play like ghosts over them, their only purpose being to raise them and to increase them in their power.

True believe in a certain spiritual connection between the elements can in the end provide the painter with the kind of consolation and cheerfulness not obtainable by any other means. His own life is lost in his work to such a degree, and matter, means, and goals finally produce the kind of perfection that, generated through diligent and faithful efforts, will not be without beneficial effect on others.

When I consider the materials with which I work and I apply the same yardstick of quality to them, I know clearly where and how I can apply them, because no material used in practical work can be completely pure. It is not possible for me here to expound on the practice of painting because this subject is too broad and in this letter my idea was merely to demonstrate to you the point of view from which I regard colors. I also freely confess that much in it is still unsettled. At least you will recognize what my endeavors are in this matter. ..."

Source: Maltzahn, H. 1940. *Philipp Otto Runge's Briefwechsel mit Goethe*, Weimar: Verlag der Goethe-Gesellschaft.

Appendix B   ca. 1806/7

Elements of color, or to about what number of items all colors and their tonal variations can be reduced, and how the elements relate to each other   (A Fragment)

When imitating nature in some way, or wanting to create appearances similar to nature's, we have to make an effort to discover and isolate the simple parts in the endless diversity and flexibility existing in it, those from which the diversity is generated. We need to determine the order or the rhythm in which the parts that cause the phenomena exist. By such means, with similar materials we have to hand, it may be possible to create comparable effects. In this manner, a painting, for example, can exist in nature like a second creation of its own, its degree of perfection the higher, the deeper the painter investigates the elements of the natural phenomena. The simpler the nature of the recognized elements, the truer the order in which we place our material means may be, and the closer the internal order in a painting created by these means.

However, so as to reproduce nature in a painting, it is not enough that we investigate the phenomena and can explain their existence in nature. It is equally necessary to investigate the nature of the materials or means with which we want to reproduce the phenomena, and to know what degree of similarity our means have with those which create the phenomena in nature.

By searching for what is useful, we cannot avoid being attracted by the close relationship in which all things exist relative to each other. We dwell with pleasure on the idea of a larger and more intimate relationship between our materials and nature. In this manner, rather than just having a correct science, we can be assured that our materials have the same living powers as are active in nature, and that they have a constituent order that must generate the same effects. That is how the way of generating becomes one with what is generated; the spirit has transcended the materials and art becomes a second nature.

To discover this special order and the complete relationship between means and nature requires a deeper science than that of which I am capable. We may only succeed if those who have investigated the issue from various points of view communicate their findings to each other. I do not place a larger value on the following ideas than what can be afforded to isolated notions But it would please me if others would not only agree with me in the ideas and intention, but also may have had experiences of their own and made experiments that make clearer what I have merely given hints of, and better express that which I am saying with awkwardness.

"There are only three colors and from these, with addition of white and black, all mixtures are generated."

The unending multiplicity of colors, light, dark, brilliant, subdued, etc., can easily be reduced to the individual parts, from mixture of which they are generated. These parts, in their elementary properties can, without doubt, be numbered five. Their mixture results in an unending number of nuances, representing a large portion of the totality of effects we obtain through vision. These five parts or elements are white and black, and the three colors red, yellow, and blue. Mixture of white and black results in gray and is the transition from black to white, or vice versa. Similarly, blue and yellow transition into green, yellow and red into orange, and red and blue into violet. It is apparent that, in the transition from red to blue, red changes initially very little and then more and more; in violet first red predominates and later blue. On this path there is a point where red and blue are in balance and the resulting mixture is a color of its own. Contrary to the more reddish or bluish mixtures, this color can be named pure violet. Pure green and pure orange can be defined in a similar manner, as can be pure gray (even though in a different manner, as will be shown below). Just as there are multiple steps between the colors and white and black, there are also multiple steps between any undiluted mixture from two pure colors in their transitions toward white and black, or in the direction of the pure gray. Any color or pure mixture of two into which white has been mixed will turn pale; admixture of black will make them dark. This situation applies to any mixture falling between two pure colors; any color or mixture in transition toward gray is muddied. Earlier I used the term pure for the three mixtures green, orange, and violet, located at the point where the fundamental colors blue, yellow, and red are in equilibrium. But it might be better to use the term pure for all colors of mixtures located between blue, yellow, and red, in contrast to those which contain gray. Because if, for example, on the scale halfway between blue and yellow the mixture transitions toward gray, the result remains green, but a green that the closer it gets to gray, the muddier it is until it finally disappears in gray.

Attempting to develop a kind of count of the number of nuances obtained from mixture of the five elements and for this purpose arbitrarily dividing each of the mentioned scales into six parts (that is, placing six mixtures between each of the indicated nine points blue, yellow, red, - green, orange, violet, - black, white, - gray) one arrives at:

Pure colors	3
Black and white	2
Pure mixtures of the three colors	3
Pure gray	1
Six grades toward black each between the three colors and the three pure mixtures	36
Comparably toward white	36
All grades between the three colors and the three pure mixtures add up to	36
Between gray and black	6

Between gray and white   6
All grades between gray and the three pure colors,
   in three primary and 36 secondary mixtures   252
These 252 repeat themselves six times along the
   scale from gray to white and six times from gray
   to black according to the 42 grades among the
   three pure colors   3024 [1]
                                               3405

It is apparent that scaling with six intermediate grades does not cover all directions in which new mixtures can be produced and that, in this manner, we are still proceeding blindly without … conceiving a figurative model, one that demonstrates to us the general rule behind any and all possible mixtures.

Source: Runge, P. O. 1840/41. *Hinterlassene Schriften*, 2 vols., D. Runge, ed., Hamburg: Perthes.

[1] Obviously, at this point Runge has not yet settled on the form of the solid resulting from spatial arrangement of these colors. On the central circular plane there are a total of 295 defined colors, 42 pure hues, 258 tonal colors, and central gray. Between white and the six main pure hues there are six tint colors. The same applies to six shade colors each between black and the six main pure hues. Such an arrangement, if extended to all 42 pure hues results in a spherical or double-cone spatial arrangement. However, in the last two entries of the table Runge defines six planes identical to the central plane between it and white as well as black. The corresponding arrangement is cylindrical, with 13 horizontal planes of identical construction, but differing in lightness, from near white to near black.

     Unlike his contemporary, the Munich painter Matthias Klotz (A), Runge seemingly did not attempt extensive and 'quantitative' coloring of the central plane. As a result he did not experience the fact that with a hue circle of 42 colors and eight grades from the full color to (and including) gray, the tonal differences along a diagonal would have been perceptually only about half the size of the hue differences because of what is known as the hue-superimportance effect. Similarly, hue differences between the 42 neighboring colors along the hue circle nearest to gray would be imperceptible.

A. See entry *Klotz* in R. Kuehni and A. Schwarz, *Color Ordered*, New York: Oxford University Press, 2008.

Appendix C

Excerpt from a letter by Runge from 1807 or 1808 to an unknown recipient concerning the relationship between his ideas on color and the findings of Isaac Newton

"A survey of the complete subject of the five parts [elemental colors, white and black] provides only a faint idea of the totality of impressions received by our eyes from all phenomena... . By various means, Newton found a path to these phenomena and attempted to clarify their living origin by means of refraction of light beams with the help of a prism, resulting in a glow, separated into seven parts or colors:

1	2	3	4	5	6	7
Purple	*Red*	Orange	*Yellow*	Green	*Blue*	Violet

He then collected this beam, separated into seven parts, by means of a convex lens and in this manner obtained again the simple white beam and concluded: All colors together are white; light is white and, if refracted, separates into these seven colors and they can be combined again into white.[1]

When considering these seven parts we soon note that they are [identical to] our circle. Taking the two end colors, purple and violet, each as one half, what remains are the three [elemental] colors and their intermediates, those that according to my point of view produce gray. In Newton's case they produce white or, rather, a white beam of light. ... [T]his discovery has contributed nothing to clearer understanding in the open field of natural phenomena; rather, Newton's understanding has quite certainly only created confusion."

Source: Runge, P. O. 1840/41. *Hinterlassene Schriften*, 2 vols. D. Runge, ed., Hamburg: Perthes.

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[1] This is a description of Newton's *experimentum crucis* (crucial experiment) in which he showed the spatial separation of the light spectrum into its differently appearing components with the help of a prism and the recombination of the components into colorless light.

Appendix D

On the dual nature of color (A fragment)   ca. 1809

It was necessary to abstract color from all material attachments in order to obtain a clear concept of how they are ordered. This has resulted in a degree of certainty concerning the relationships in which the elements stand, requiring a particular form of representation. Nevertheless, we all experience difficulty in bringing this insight into agreement with what we do and require every day. As a result, it is necessary to raise the issue of abstraction over and over again.

In the sphere model, the five parts are assumed to be the elements of the total perception, not including the influence of light. But if one wants to imitate the phenomena of nature it becomes obvious that the five parts cannot be the final elements, neither in the phenomena themselves nor in the materials used in painting. We go past their limits and powers both in vision as well as in the work of painting. It is necessary to look for further clarification of the elements of the total impression that is supplied by the eyes. Only after the process of cognition has reached a degree of certainty will we simultaneously be able to understand how the materials [of painting] relate [to the total impression].

If, in the relationships in the sphere, pure red, blue, or yellow are defined as transitioning neither to another color, nor to black and white, the implication is that they have a specific lightness. But it is also apparent that [the three colors] can exist in very high purity and power in form of very light or very dark colors without addition of white or black; that therefore pure color is flexible within itself. Even if considered to have identical lightness, color would still be capable of modification depending on the nature of the material in which it appears (glass, paper, silk fabric, wool fabric, cloud, rock etc.). Independent of form, color identifies the material to which it is bound; they [color and material] are actually one and the same. Such changeability might induce us to ask: why the abstraction and the relationship among the different [elements] if we are not able to define an element itself? — However, we soon will recognize the cause of this variability to be an orderly one, once we recognize the dual nature of color: *transparent* and *opaque*. When the material is completely opaque, it shows the quality of its color only on its surface. In a transparent material, the quantity as well as the quality are recognizable, thus one can distinguish without form the quality of the material to which the color is bound. A completely opaque material, showing its color only on the surface, will show it in a given light as unchanging. On the other hand, transparent color is changing as the result of the quantity of the material. Constancy of opaque colors (based on the opacity of the material) can be seen as the abstract entity of the element, as has been assumed in the color sphere, where also black and white, with which an elementary color is in a certain relationship, imply a comparable opacity.

White is impossible to be seen as transparent. Even though the term 'white light' (as a beam) is in common usage, light really implies transparency.

When attempting to discuss a subject matter with clarity, one should not use the same term for two different phenomena. The difference between blackness and darkness, by its very nature, cannot be as striking as the difference in the case of milk and water between whiteness on the one hand and clarity and colorlessness on the other, because both blackness and darkness mean less visibility. But recalling the earlier mentioned [see July 3, 1806 letter to Goethe, Appendix A] example between a sliver of polished coal and a thick transparent glass, the latter with increasing thickness exceeding the coal in darkness, one recognizes, even though nothing would be discernible in either, that the specific difference between blackness and clearness or transparency is as comprehensible as the difference between whiteness and transparency. White and black, therefore, are only related to opaque colors, as is shown in the sphere.

Transparent colors relate to brightness and darkness according to their quantity and quality. When attempting to compare the five parts of opaque colors with the elements of transparent color in regard to lightness, darkness and the three colors, we find that the opaque colors fall, in regard to their lightness, into the following sequence:

1	2	3	4	5
White	Yellow	Red	Blue	Black

In the matter of brightness and darkness, each transparent element exceeds the five opaque elements. For example: a transparent crystal is completely penetrated by light, not so white [material]. Even though the sun can make white appear brilliant to the point that it blinds the eye, light can only be reflected by it and not penetrate it. In parallel fashion, clear water at infinite depth is of darkness equal to the blackness of a piece of coal, as already mentioned. In other words: a dark, transparent material, absorbing all light as an infinitely large room would, but not absorbing the light beam on its surface as opaque black does, will exceed black in darkness. Comparably, a beam of light passing through a transparent material will exceed white as well as any other opaque color in power and force.[1] Thinking of a dark ruby or a red garnet of the size of a fist, its deep glow of color will appear darker than black. If polished to the thickness of paper its color, though still the same red, would be illuminated by light in a manner noticeably exceeding white. A beam of light reflected in the glowing depth of a ruby will have such burning power that all opaque colors will appear dull in comparison. The same would be the case with a yellow or a blue crystal. It may appear strange at first when I say that yellow

[1] Here Runge attempts a discussion of what today would be called luminous power. Depending on the luminous power of the light beam, the spectral reflectance of the white material and the spectral transmittance of the transparent material the amount of light reaching the viewer's eye can be the same for both materials, higher for the reflecting material, or higher for the transmitting material. The resulting appearance depends also on the surround in which the reflecting or transmitting materials are viewed.

can exceed black in darkness.[2] Let me remind you that I am here not talking about the sensory appearance but about the relationship between the appearance and its inner nature, a nature that escapes our senses but can nevertheless be comprehended. I represented the necessary relationships of the five parts in spherical form because the transition from black to white relates to the relationship of the three colors as the axis to the equator. So as to demonstrate the difference between the opaque and the transparent colors, or the relationship between form and essence, I want to present in parallel the sequence of the five [opaque] parts according to lightness

White, Yellow, Red, Blue, Black

and that of the comparable five parts in transparency:

Light, Yellow, Red, Blue, Dark.

The three opaque colors transition via violet, green, and orange. — The same applies to the transparent colors.

When the three opaque colors are mixed in equal parts and equal power, they dissolve into gray. This color forms the line between white and black and thereby is the axis point of the circle formed by the three colors. — When the three transparent colors are mixed in equal parts so that they neutralize each other, they result in a transparent darkness three times as deep as each color.

Because: The gray color produced by the opaque colors is, in regard to lightness, the average of that of the three colors,[3] their qualities of lightness mixing but their character of color being destroyed, with nothing remaining on the surface but the quality of lightness (all of it in a form). — In case of transparent colors we not only see the quality but also the quantity and even though the colors are neutralized also in this case, the total quantity of all three qualities remains visible. It forms the colorless, transparent darkness taken, so as to be able to form a sphere, to be a pole of the sphere. But, according to the nature of its color, it is also the central point [of the sphere]. In the opaque sphere, colors that neutralize each other, such as red and green, oppose each other. In case of a transparent sphere, all of it would be eliminated

[2] Perhaps Runge did not have sufficient experimental material at hand, but a yellow transparent material, recognizable as yellow in color, cannot exceed a black opaque material in darkness. Modern research has indicated that a given reflecting material can have many different levels of apparent darkness (or lightness), depending on how it is illuminated and surrounded (see e.g. A. Gilchrist, *Seeing Black and White*, New York: Oxford University Press, 2006).

[3] This is not a factual statement. Admixture of opaque colors is subtractive in nature and the mixture appears darker than each of the three components.

because one can see the two antipodes simultaneously. There would be no differentiation and only one living quality would remain. Or what would remain is the essence in which the relationship rests, but without an appearance. — The [geometric] figure representing the relationships can only be valid for opaque colors because they also stand in a relationship to white and black, being the figure of light and darkness, just as the opaque color is the figure of the transparent color. As a result, the abstract relationship of opaque colors has little in common with the material substrate, because all substrates are of different quality. It is necessary to realize that the relationships [of colors] in the [spherical] figure are as certain and infinite as the living properties of color, because the perfect can only be revealed in a perfect form. Essence cannot reveal itself without form. Equally impossible is it for the [spherical] figure to have a qualitative existence without (the essence of an) appearance, because the life of the picture is in the picture, and the life of essence is in essence. Nothing can exceed itself.

The poles (black and white) and the three colors are the figure (form). If they penetrate each other, they neutralize into gray, comparable to death.

The essence, however, is the transparent quality in which light ignites the colors. Transparency carries the capability of being ignited by light and it is the quantity of the three qualities of color. — The three colors, individually capable of appearing in stronger or weaker color, can be taken as capable of concentrating their quantity into quality. When the three colors, in saturated quantity, flow into each other, the power of the total combined quantity will, under the influence of light, be ignited in the colorless substance of their qualities.

Source: Runge, P. O. 1840/41. *Hinterlassene Schriften*, 2 vols., D. Runge, ed., Hamburg: Perthes.

Rolf G. Kuehni

Philipp Otto Runge's *Farben-Kugel* and its place in the development of color order

The idea of three primary chromatic colors and its effect on color order

This idea dates back to Aristotle who, in his *Metereologia*, wrote that there are three main colors in the rainbow, scarlet (*phoinikoun*), green (*prasinon*), and violet (*halourgon*), and that these are "… almost the only ones that painters cannot manufacture, for they produce some colours by mixture of others …" (Barnes 1984).[1] Today, these three colors are seen as representing the beginning, middle, and end of the visible spectrum of light. The idea that the chromatic primaries are yellow, red, and blue developed from experiences of painters.

In the 13th century the English philosopher Roger Bacon applied the 2nd century AD Greek philosopher Porphyri's system of predicable categories (genus, species, difference, property, and accident) to colors. Bacon's system reduced the eight basic color categories described by Aristotle in *De sensu et sensato* to five genuses: whiteness (*albedo*), yellowness (*glaucitas*), redness (*rubedo*), blue-greenness (*viriditas*), and blackness (*nigredo*) (Parkhurst 1990),[2] Each contains multiple species: for example in glaucitas there are *lividus* (ivory or oatmeal color), *glaucus* (yellow), *flavus* (pale yellow), *ceruleus* (between *glaucus* and *citrinus*, color of beeswax), *pallidus* (between citrinus and ceruleus), and *citrinus* (orange). Bacon used the term *gradus* (gradation, step) to indicate Porphyrian steps of difference.

Five color genuses and the results of their intermixture were given graphical expression in 1613 by François d'Aguilon. D'Aguilon's diagram followed a form introduced by the 6th century Roman philosopher Boethius to express the relationships in the musical compass and the logic of categories (Fig. 1). D'Aguilon applied the term species to the five primaries only. There is an achromatic lightness scale between white and black in the graph. There are also mixture scales between the three chromatic primaries (yellow, red, and blue) that, properly arranged together with his intermediates golden (*aureus*), green, and purple, can be seen to form a hue circle. Tint (toward white) and shade (toward black) scales between the three chromatic primaries and white and black are indicated by lines. Similar scales of the three intermediates would have been a logical addition but would have made the diagram much less comprehensible. The result of combining all three chromatic primaries was at the time described as "ghastly and lurid." D'Aguilon mentions examples of

[1] Aristotle's is a statement that struck and strikes painters as strange: clearly green can be reproduced with a mixture of yellow and blue pigments and violet from red and blue pigments.

[2] In the seven basic color categories for which Aristotle is known, yellow is bundled with white and gray with black. Eight categories result from making yellow a category of its own.

friendly associations between black and white and the three chromatic species as well as their intermediates.

In 1664, the English natural scientist Robert Boyle, in his book *Experiments and Considerations Touching Colour*, wrote: "I have not yet found, that to exhibit [the] strange Variety they [painters] need imploy any more than *White*, and *Black*, and *Red*, and *Blew*, and *Yellow*; these *five*, Variously ompounded, … being sufficient to exhibit a Variety and Number of Colours, such as those that are altogether strangers to the Painters Pallets, can hardly imagine …" His compatriot, the physician Francis Glisson, described in 1677 a color specification system consisting of scales of yellow, red, blue, and black, considered by him to be sufficient to define with scale values the color of any object.
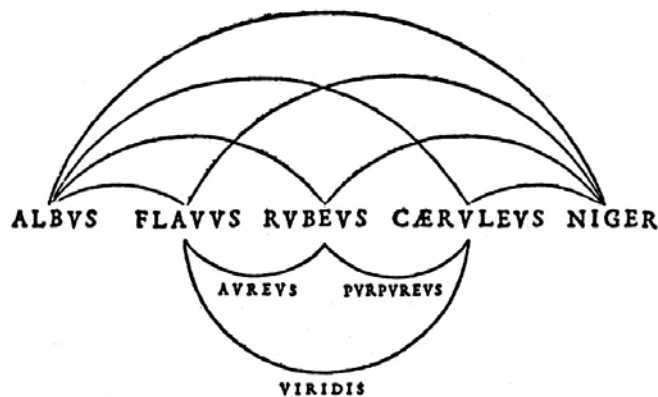

Fig. 1 François d'Aguilon's color mixture diagram based on the chromatic primaries yellow, red, and blue, as well as white and black (1664).

Consensus of opinion on the subject was disrupted in 1704 when Newton published his *Opticks.* In this book he described the spectrum as consisting of an endless number of colors that he divided into seven parts or categories. His circular color diagram (Fig. 2) treated colors, represented by specific components of the spectrum of daylight, in a revolutionary fashion as linear forces. Such an approach made it possible to predict the result of mixtures by vector addition, the vectors representing the amount and direction in the diagram of the force of each color. That mixtures of lights and mixtures of colorants produce different results was not yet understood. Newton's experimentally supported results derived from lights led to much confusion when some people erroneously interpreted his writings as having posed seven chromatic primary colors rather than three.
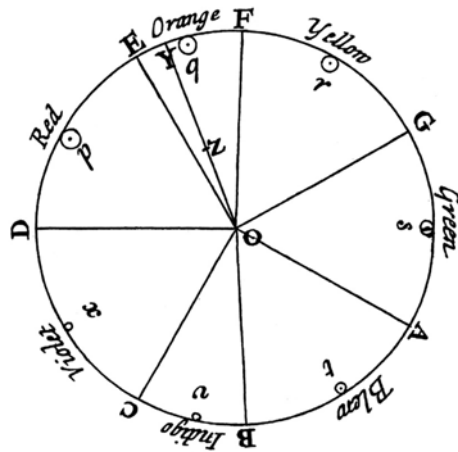
Fig. 2 Isaac Newton's circle of the spectrum colors, with seven major color categories red, orange, yellow, green, blue, indigo, and violet (1704).

This led to the conclusion that his claim – the mixture of appropriately selected colors (lights) in appropriate amounts produces colorless light appearing white – must be wrong: the mixture of pigment powders at best produced a medium gray.

The anonymous painter who wrote the chapter on pastel painting in the 1708 Dutch edition of *Traité de la Peinture en Mignature* (Treatise on miniature
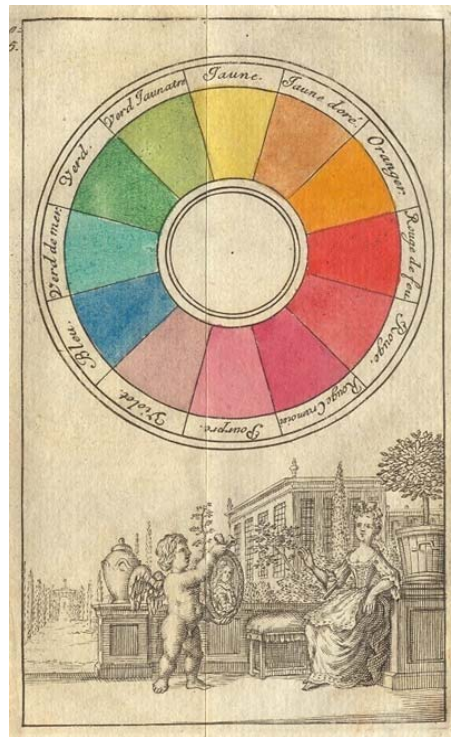

Fig. 3 Hue circle of the anonymous author of the chapter on pastel painting in *Traité de la Peinture en Mignature* (1708).

painting) included the first colored versions of a hue circle to show the results of mixtures of three primary paints, yellow, red, and blue (Fig. 3). The primaries are placed equidistant from each other in the circle, implying an equidistant triangle. The author's description of the circle reads as follows: "Here [is a figure] by which one will be able to see how the primitive colors yellow, red … and blue generate the other colors and which one might call the *Encyclopedia of Colors*."

The idea of three primary chromatic colors was strengthened by other practical results. German engraver Johann Christoph Le Blon produced life-like colored images using three printing plates to print with yellow, red, and blue pigments (later adding a fourth plate with black ink). Le Blon described the process his book, *Coloritto*, published in 1725. French chemist Charles François de Cisternay du Fay described in 1737 in a paper the applicability of the idea of three primary colorants to dyeing technology.

Tobias Mayer, a German cartographer, astronomer, and physicist was the first to place the three chromatic primaries yellow, red, and blue at the corners of an equilateral triangle, with their mixtures filling the sides and interior. He proposed this first at a public lecture in 1758. The lecture was published posthumously (1775) and contains the description of a color order solid he believed would contain all possible colors. (Like his predecessors, Mayer did not understand the difference between colors of lights and those of colorants.) Mayer justified the choice of his primary colors yellow, red, and blue as follows: "There are three simple and basic colors and no more than that, from mixture of which all others can be generated, but which themselves cannot be generated from others, in whatever ratio they might be mixed: red, yellow and blue. We see them in rainbows, but even more distinctly in rays of the sun captured by a glass prism, though there they are accompanied and surrounded by secondary colors."

Mayer understood colors to be sensory entities and recognized that three geometrical dimensions were required to order them in their completeness. The solid he proposed for this purpose is a triangular double pyramid (Fig. 4). Its central triangle has the three primaries in the corners and 11 intermediate grades between two primaries. The primaries, considered of equal power, change in regular fashion throughout the triangle (Fig. 5). The system envisages 11 additional triangular levels, each successive one smaller in size, above and below the central plane, the upper ending in white and the lower in black. The central color in the central plane, $r^4g^4b^4$, was to be a neutral gray, the result of mixture of equal parts of the three primaries. The central plane contains 91 and the total system 819 defined colors. Mayer made attempts at coloring the whole system. His colleague and editor of his posthumously published papers, G. C. Lichtenberg, developed a coloration of an abbreviated central plane, included in the book. Mayer realized that the choice of the colorants for the primary colors was critical for success. Three colorants that he identified with the index 12 (representing "pure" colors) were orpiment (King's yellow), vermilion, and *Bergblau* (azurite). These are the same chromatic

pigments Francis Glisson used in his color identifier scales. Mayer's premature death (age 39) prevented more detailed efforts in coloring the solid.
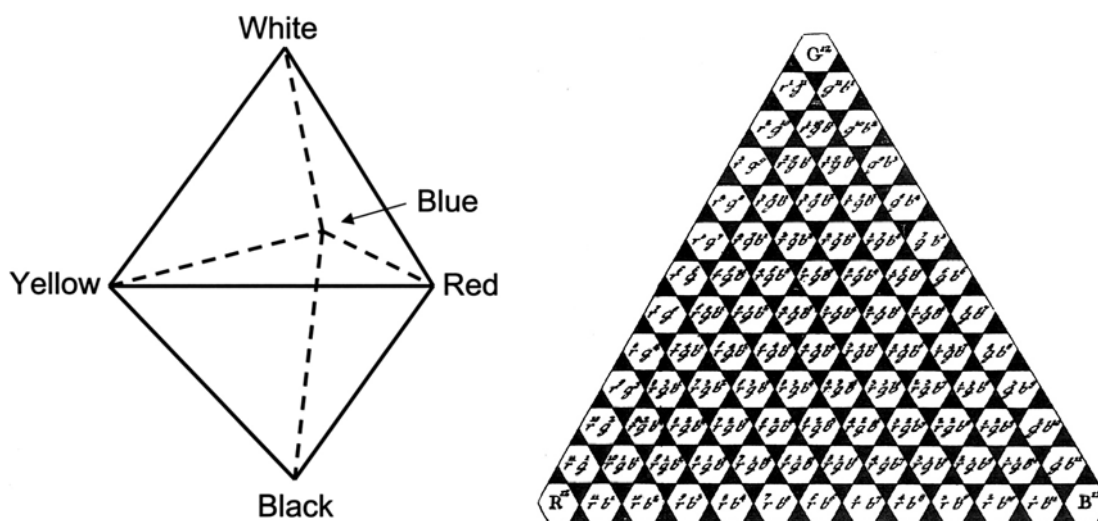

Fig. 4 (left) Conceptual sketch of the Tobias Mayer's triangular double pyramid color order system.

Fig. 5 (right) Mayer's depiction of the central triangle of the double pyramid, with the primary colors in the corners and intermediate colors changing in regular intervals of perceptual units of the primary colors (1775).

In 1764 the Swiss mathematician, physicist, and astronomer Johann Heinrich Lambert was named a member of the Academy of Sciences in Berlin. While there he began, together with the painter Benjamin Calau, a project of coloring Mayer's color solid. His source of information was a newspaper report about Mayer's public lecture of 1758. Lambert certainly knew Newton's work on color but decided to pursue the idea of an object color solid. The result of these efforts was published in 1772, three years before publication of Mayer's formal paper in his posthumous writings. After some experimentation to find colorants representing pure primary colors, Lambert and Calau settled on gamboge, carmine, and Prussian blue as primaries. When combining them in equal amounts in pairs, they found the three colorants to have different coloring power.[3] They assessed the power of each by making binary mixtures that in color appeared to be half way between those of the pure pigments. However, mixing all three in the resulting ratios did not produce a pure middle gray as envisaged by Mayer, but a bluish black. As a result, Lambert decided to discard the lower triangular pyramid of Mayer's double pyramid because black was already obtained in the basis triangle. Lambert reduced the spacing of the

[3] From today's point of view this is not surprising, but it was then a new practical finding.

basis triangle to seven intermediate grades (compared to Mayer's 11). In his depiction of the pyramid he showed only seven of the nine implicit lightness levels (Fig. 6), leaving two out so that all the color samples of the basis triangle and the third triangle could be shown.
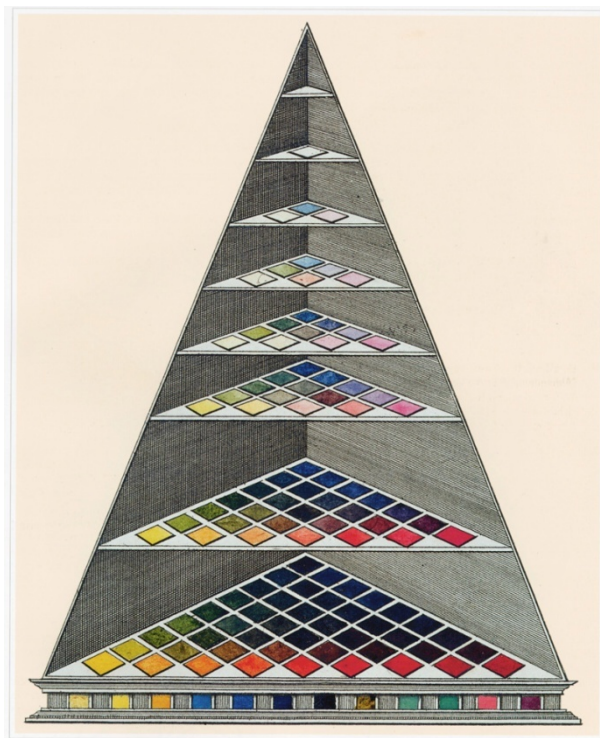

Fig. 6 Lambert's triangular pyramid color order system based on the chromatic primaries yellow, red, and blue in the corners and white at the top. Intermediate colors were meant to change regularly in the components (1772).

As a result of its construction, the Lambert pyramid does not have a central gray scale and, despite the 'calibration' of the pigments, the least chromatic colors in the basis plane are found closer to the blue primary than to the other two. It is evident that the problems encountered by Lambert and Calau are due to the nonlinear nature of the relationship between colorant concentration and color appearance, and that they considered reaching intermediate appearance between primary colors along the periphery as more important than balance in the center of the triangle.

Against this background, Runge began work toward a color order system in about 1805. It is not known in detail what he knew of the efforts of his predecessors. He mentioned Lambert's work in a letter to the poet and natural scientist Johann Wolfgang von Goethe (1749-1832) dated 1807. It is likely that he also knew of Mayer's work. He obliquely refers to the unilateral color triangle on page 4 of *Farben-Kugel*, where he parenthetically describes it as "nicht unbekannt" (not unknown). A likely source for Runge of such

information was his good friend Henrik Steffens[4] who taught natural philosophy at Halle at that time and authored an essay appended to the *Farben-Kugel* text.

Runge and colors

Raised in a ship-building merchant family in Wolgast on the Baltic Sea Runge, known to family and friends as Otto, learned the value of measures and numbers at an early age. At the same time he developed a strong affinity for the *Zeitgeist*, the *Sturm und Drang*, led by Goethe, Schiller, and Herder in poetry and Kant in philosophy. He was raised in the protestant religion and was considerably attracted by the natural mysticism of the early 17th century Lutheran mystic and shoemaker Jakob Böhme. In his youth, Böhme had a mystic experience while viewing the beauty of the play of sunlight reflected in a pewter dish. His first manuscript, written in 1610, was titled *Aurora, oder die Morgenröthe im Aufgang* (Aurora, or morning redness of the sunrise). This and most of his other writings on Christian theology were in his time considered heretical.[5]

Runge believed chromatic colors to have Christian symbolic power which, when properly applied, permitted humans to interpret God's creation. He considered blue, yellow, and red to be symbolic of the trinity and equated blue with God and the night, red with morning, evening, and Jesus, and yellow with the Holy Spirit (Runge 1841, I, p. 17). These ideas are particularly evident in the four representations of the stages of the day, Morning, Midday, Evening, and Night, that between 1802 and the end of his life consumed much of his artistic energy. He had plans to execute these in large wall paintings but did not progress beyond engravings that became popular and the well-known small (43 x 35") oil painting of 'Morning' (1808).

Runge hoped his color system would contribute to clearer understanding of the fundamental organization of all color perceptions, something particularly useful for artists. He also hoped, as other artists before him, to develop a rational system for selecting harmonious color combinations.

Despite his early death and limited number of paintings Runge, together with his northern German compatriot Caspar David Friedrich (1774-1840), rapidly became known as one of the chief representatives of German Romanticist painting, a style compatible with the Romanticist poetry and writing of the time.

[4]Henrik Steffens (1773-1845), Norway-born natural philosopher, geologist, and physicist. (Petersen 1884)

[5] Böhme's thought influenced many people down to the present. He was, for example, quoted frequently in the work of the psychologist Carl Gustav Jung.

Runge's relationship to Goethe[6]

Born far from the cultural centers of Germany, and with little formal education owing to frequent bouts of sickness in his youth, Runge was able to build a surprisingly wide range of friends and acquaintances. Perhaps the most unlikely friendship, at the same time a very fruitful one, was with Goethe, then at the height of his powers and influence. In 1798 Goethe and his friend Johann Heinrich Meyer founded a cultural and art theory periodical they named *Propyläen,* after an entryway on the Acropolis in Athens. Lasting only three years, *Propyläen* offered every year a competition for artists. The editorial team posed a subject, typically out of Greek mythology, the entries were rated and prizes were given for the top entry. In 1801 Runge, then a 24-year old student at the Academy of Arts in Copenhagen, decided to enter the competition. Its subject was the battle between Achilles and the river gods from Homer's *Iliad*. Runge made a pen and ink drawing and sent it, together with a lengthy letter, to Goethe. The judgment of his work, published in a literary journal in 1802, was quite negative. Runge concluded that to continue making art in the old, classical style was a mistake and that he would devote all future efforts to a new style of representational art.

In 1803 Runge and a friend, the poet and novelist Johann Ludwig Tieck (1773-1853),[7] traveled from Dresden to Hamburg. They made a side trip to Weimar, where Goethe resided, to visit an acquaintance from Dresden named Voigt. One day, coincidentally and unannounced, Goethe arrived. He and Runge began a conversation that must have impressed Goethe as he invite both Tieck and Runge to lunch at his house the following day, as well as the day after. Runge reported in a letter: "... after the meal, Goethe conversed at length with me, asked me about my views on many subjects, what I thought about his institutions in Weimar, explained what their meaning was, and was very complimentary about my views in all respects. The [next] morning I left Weimar." (Maltzahn 1940, p. 33) Goethe was comparably brief about the visits in his diary, only mentioning that on the two days Messrs. Tieck and Runge were guests in his house.

Three years later, Runge, now married and a father, had the drawings of the four stages of the day engraved in large format. He was then living with his parents in Wolgast because his brother Daniel could no longer support him in Hamburg. Runge decided to send copies of the engravings to Goethe together with a letter recalling his visit. Goethe responded some four weeks later, indicating a degree of puzzlement about the engravings but at the same time

[6] Goethe's first essay on the subject of chromatics was published in 1791. During the time of his interactions with Runge he was working on his extended theory of colors, *Farbenlehre* (Color theory), published in 1810.

[7] Together with Novalis and the brothers Schlegel, Tieck was a leader of the German Romantic School of writing. Both he and Runge were collectors of folk fairy tales.

his appreciation. That they had discussions on the subject of color in Weimar three years earlier seems implicit in Goethe's request: "Please let me know if you have [hand] illuminated or illustrated in color, not painted, one or the other of these [engravings]. That might give us an opportunity to exchange views about color and its meaning." (Maltzahn 1940, p. 39). Runge responded at length a month later. His letter informed Goethe, in 22 statements, of his views on the nature of color (see Appendix A for a lengthy excerpt). This letter impressed Goethe sufficiently to reprint most of it as an appendix in his book *Farbenlehre* of 1810. At the request of Goethe, Runge continued to send examples of his work, including a self-portrait and scissor cut flowers. At one point Goethe planned to install a Runge-Room in his house in Weimar, with artwork exclusively by Runge.

Runge wrote a total of thirteen letters to Goethe. In 1807 they exchanged three letters, the key aspect of all being Runge's ideas about and experiments with disk mixture (see section below). In 1809 Runge arranged to have a copy of the manuscript of *Farben-Kugel* transmitted to Goethe by Henrik Steffens who had known Goethe since 1798. It was important to Runge that his work did not contain any claims disagreeable to Goethe. Runge also had hoped to be able to read Goethe's *Farbenlehre* before the publication of his own work. Two months after sending the manuscript he became anxious because he had not yet heard from Goethe and his publisher was pressing him to submit the final version. Goethe's response is dated mid-October 1809 and indicates that he did not have a chance to study the manuscript in all its details but that "The work does not contain anything that is not in agreement with my work or does not connect in one way or another with my own work." *Farben-Kugel* was published in January of 1810 and Runge arranged to have a copy of the book sent to Goethe together with his last letter to him. Goethe responded in March and indicated that Runge could expect a copy of *Farbenlehre* sometime in spring of the same year. He expressed his hope that they could get together again soon. By then Runge was seriously ill and could no longer concentrate on reading *Farbenlehre* or even painting. After Otto's death, his brother Daniel expressed in a letter to Goethe the former's high opinion and appreciation of the poet.

In an essay titled "Two different worlds — Runge, Goethe and the sphere of colour," the eminent art historian John Gage points out (1999) that despite the personal relationship and mutual expression of admiration there was, in connection with color and art, little agreement between the two. Their key interests and arguments are in different areas of the field of color. Runge's concerns were the similarities and differences between transparent and opaque colors and creation of a color order system that was better than those existing at the time. Goethe's interests were the nature of colors existing between light and dark and the rejection of Newton's views on color. An important aspect in Goethe's thinking is the intensification of the basic colors yellow and blue toward red being the apex of power. Red occupies the top position in Runge's color circle in his 1806 letter to Goethe (Appendix A). In the sketch of the first description of the color sphere sent to Goethe (see figure in the next section)

orange was moved into the top position. In the published edition of *Farben-Kugel* the circle has been turned further to place blue on top. Runge's revision is in agreement with Ignaz Schiffermüller who, in his hue circle of 1771, placed blue on top because it is the only fundamental color that does not change its basic appearance between light and dark coloration. This development seems indicative of Runge's path from full acceptance of Goethe's views to an independent viewpoint.

The alliance between Goethe and Runge appears to have been one of convenience rather than deep agreement about details. The former's final view of the latter's work as an artist was contradictory. He found Runge's pieces very beautiful, but their creator touched with a degree of madness: "one who stands as precariously on a ridge as he did either has to die or go mad." (Maltzahn 1940, p. 117)

Development of the *Farben-Kugel*

As a painter, Runge was interested in the practical aspects and rules of paint mixture. He described the starting point of these efforts to have been difficulties he encountered when painting the second (1804/5) version of "The nightingale's lesson." (1841, II, p.497) As mentioned, in a long letter to Goethe, dated July 1806, he summarized his ideas on color in 22 statements and included a hue circle (Appendix A). Goethe reprinted most of that letter in an appendix to his *Farbenlehre* of 1810. In "Elements of color," an essay fragment from ca. 1806/7 (Appendix B), Runge included a table systematizing the number of colors that can be achieved from mixture of the three chromatic primaries, white and black. No mention of a sphere is made in this essay, but it appears in a letter to Goethe dated November 1807.[8] In that letter, Runge summarized the sphere concept as follows:

> "The relationship between the three [fundamental chromatic] colors, as well as with black and white, can be represented very well in form of a globe, as follows: I separate the equator into six parts, the three [fundamental] colors in a triangle, intersected by the triangle of the pure, intermediate mixtures. The North Pole is white, the South Pole black (without implying any mystical meaning). The equator represents the brilliant attribute of color; it transitions in its mixtures toward north to white and toward south to black. When bisecting the sphere from the North to the South Pole, white and black combine in the center point to gray. When bisecting it along the equator, the colors combine in the center point to form the same gray. Section of the sphere along a parallel line 40° from the equator toward north will result in a whitish gray, toward south the result will be a blackish gray. I hope you will understand me when I say that such a sphere contains all transitions between opaque

[8] In the early fall of that year he had lengthy discussions with Steffens who was stranded in Hamburg as a result of the Napoleonic war.
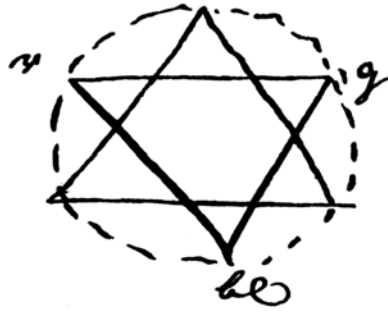
[Runge's sketch of the triangles of the fundamental colors with designations *bl* (blue), *g* (yellow), *r* (red) and the intersecting triangle of the pure intermediate mixtures.]

[fundamental] colors themselves and with white and black, as well as those between white and black. I believe it is possible to show this very clearly if the whole sphere is separated into sections: the six parts of the equator shown every 10° going north toward white and going south toward black, and the same in the comparable sequence [from the equator] toward the center point. Many different sections could be made to demonstrate the concept very clearly." (Maltzahn 1940, p. 71)

At the same time, as is also evident in letters to Goethe from 1807, Runge experimented with disk color mixture (see section on disk mixture below) in the hope of developing scientific support for the sphere model.

Sometime in 1807/08 Runge, in a letter to an unknown recipient, briefly addressed the question of the relationship between Newton's findings on the spectral composition of daylight and his color sphere of material colors (Appendix C). He made mention of this idea in *Farben-Kugel*. In the letter Runge concluded that Newton's findings did nothing but confuse the whole issue of color.

Encouraged by Steffens, Runge began work on a description of the color sphere in 1808 and completed it in 1809. His form of presentation has a narrow, functional focus on the sphere, its geometric structure, and logical content. Discussion of the relationship between the sphere and some of his ideas on color harmony is relegated to an appendix of the book. Runge submitted the manuscript in March 1809 to Steffens who would forward it to Goethe. Steffens supplied a separate essay on the significance of color in nature, published in the same volume, and apparently not seen by Runge before the book was printed. *Farben-Kugel* was published in January of 1810.

In ca.1809/10, presumably after completing the text on the sphere, Runge wrote the essay fragment "On the dual nature of color" in which he compared effects of transparent and opaque colors (Appendix D). There he conceded that his color sphere was only applicable to opaque colors.

Runge was a firm believer in the theory of three primary colors. He seemed to have accepted the findings of Newton in regard to the composition of light but considered it of little import in regard to color(ant) mixture and color perception. Runge's elemental colors are yellow, red, and blue, each of pure hue and highest saturation, without any admixture of neighboring elemental hues. At the time of Runge's writing, the difference between colorant mixture (dyes or pigments, subtractive mixture) and colored lights (additive mixture) was not understood. Runge assumed that 1:1 mixtures of the elemental colors (whether as perceptual primaries or colorants) would be of equal saturation as the elements themselves. He also did not realize that the red and blue colorants optimal for obtaining the largest gamut of mixed colors are not as defined by him, but a bright bluish red (magenta) and a bright greenish blue (cyan). It was the physicist James C. Maxwell who determined these facts some 60 years later.

Runge's premature passing was unfortunate in regard to the development of color order systems. Certain comments indicate that he was thinking of developing a more quantitatively illustrated version of the sphere.

Early comments on *Farben-Kugel*

Response to the publication of *Farben-Kugel* was largely positive. Readers and reviewers especially highlighted the simplicity of Runge's presentation. Examples of comments are:

Henrik Steffens: "Your essay must be printed as soon as possible. It is wonderful in every respect. In a few weeks I will pass it on to Goethe who loves you dearly and who will be much pleased by it. I myself will add a few pages on the importance of colors in nature, uninhibited and unpretentious as your essay. That is what is splendid about your work: it offers so much deep thought without any pretension; that you do not call it a theory but only a tale. … Your presentation is masterful and I have my doubts that I will be able to reach its level." (Runge 1841, Sep. 1809)

Johann Wolfgang von Goethe: "Very recently Philipp Otto Runge, of whose beautiful insights into color theory, from the viewpoint of the painter, we have offered a testimonial earlier, has presented the scaling of colors and their tinting toward light and shading toward darkness on a sphere, and, as we believe, has brought this kind of effort to complete conclusion." *Geschichte der Farbenlehre,* entry Johann Heinrich Lambert (1810).

Clemens Brentano (poet and novelist, 1778-1842): "I have read your essay on colors, just like a child. Given that I am the least scientific person the sun shines on, I was pleased to believe what you wrote." (Runge 1841, Jan. 1810)

Unidentified "friend of the author" from a lengthy review in the journal *Nordische Miscellen,* Hamburg, March 1810: "In addition, the included colored

figures (wherein the twelve-fold division along the equator and the meridians is arbitrary) to make clear how, on the surface of a sphere and with the help of cross-sections, the true and sufficient idea of a theory and table of color mixture can be clearly presented. The author did not name the various watercolor pigments he used in coloring the sphere, and this was clearly not necessary. … The only way to understand the author is to consider the posited colors, while material, as being ideal and one should not think of colorants such as cinnabar, Berlin blue and King's yellow. The author abstracts from all material properties, leaving only color and visibility." (Runge 1841)

Henrik Steffens: "Truly, dear Runge, it is a beautiful effort, and I have as yet not spoken with anybody, even not any Newtonians, that was not pleased by it and appreciated it in its own way. The sphere and the color compositions provide a good overview." (Runge 1841, Apr. 1810)

Johann Joseph von Görres (teacher of physics, author, publisher of an anti-Napoleonic journal, political pamphleteer, 1776-1848): "I received your *Farben-Kugel* quite some time ago and read it immediately with much pleasure. You have presented the whole matter in a well-rounded, pleasant, and geometrically charming manner. Quietly and innocently as a child, your idea moves forward toward its goal past dangers threatening from difficulties and contradictions. … In my view there is a contradiction between your construction of the colors in the triangle and the results of refraction. Refraction provides the specific weight of a color, and thereby also its intensity and saturation. … If you mix blue with red you obtain purple and violet but of a much reduced specific weight of colorfulness than what is shown in the color picture…. Your closure of the triangle is not natural but artificial; it applies to pigments but not to colors. Applicability to pigments is sufficient for you, but I find the issue in regard to colors distracting." (Runge 1841, Sep. 1810).

What is the validity of Runge's color sphere construct?

Runge's main (explicit and implicit) claims can be expressed as follows:

1. There are five elements that constitute all color experiences. Two are achromatic (not considered by him to be colors) and three are chromatic (blue, yellow, and red).
2. All five elements are unique, equivalent, and of equal force.
3. The geometric model for the chromatic elements in such a relationship is the equilateral triangle.
4. Binary mixtures of two of the chromatic elements are of force equal to that of the elements themselves and, therefore, fall on a circle passing through the apices of the elementary triangle.
5. Equal mixtures between two chromatic elements are complementary of the opposing element in the circle, that is, in equal mixture they neutralize themselves to achromatic gray. Mathematically: 0.5 parts red

+ 0.5 parts blue = 1 part violet; 1 part violet + 1 part yellow = 2 parts gray.

6. Equal binary mixtures of chromatic secondaries (orange, green, violet) lose power, become desaturated, and have the hue of the intermediate chromatic element, that is: 1 part orange and 1 part green = 2 parts grayish yellow. (The strictly arithmetical result should be 2 parts gray, because orange consists of 0.5 parts each of yellow and red and green of 0.5 parts each of yellow and blue, adding up to 1 part of yellow and 1 part of violet that would sum to 2 parts of gray.)

7. To represent all possible mixtures of the five elements requires a solid, a sphere.

8. Mixture of white and black in different ratios results in grays of different lightness. They form the vertical axis of the sphere between the poles white and black.

9. The central gray of that axis is identical to the central gray of the equatorial plane resulting from the neutralization of any color pair diametrically opposed on the equatorial periphery.

10.   Any two colors that are located at the ends of a diametrical axis of any direction in the sphere neutralize themselves when of equal force, that is equal distance from the center, to neutral mid gray. Such colors are located on spherical shells around the central mid-gray.

Runge's elementary colors are ideal entities of equal magnitude and force, and their mixture follow neither the additive principle of light mixture nor the subtractive mixture of colorants. It is not evident how he imagined mixture of these elementary colors, as well as black, and white would operate. When two colors "operate jointly, or … mix," are they being added together (such as in case of lights or colorants, though with different outcome) and thus one unit of each makes two units of the mix (light or paint)? Or, do they "flow into each other" without increase in volume or power? The process that would result most closely in Runge's construct is disk mixture with which he experimented (see below) but which is not mentioned in *Farben-Kugel*. Runge's key insight was that, given the assumed properties of the elementary colors and the fact that the median mixture between white and black and the mixture of equal parts of the chromatic elementary colors are the identical gray, and thereby the sphere is the proper representation for all possible combinations.

In the real world of colors, lights mix in an additive, vector-sum fashion. However, properly balanced amounts of "blue," "red," and "yellow" light will not form "mid-gray" light but rather a light in which a highly reflecting object appears to be white.[9] Similarly, a balanced mixture of "green" and "red" spectral lights will not form "mid-gray" but rather a highly saturated "yellow" light.

Matters are more complex in case of objects and colorants. The result of mixture of colorants is not additive but in a complex fashion non-linearly subtractive. Only in the 1930s did the two German physicists Kubelka and Munk develop a numerical model for the calculation of the results of mixtures

of dyes or paints on opaque substrates. Depending on the spectral reflectance functions of the pigments used, a balanced mixture of "elementary" blue, red, and yellow pigments will appear black or nearly so, as Lambert noticed. Pigments are never of pure (spectral) color but always more or less desaturated. The hue of colorations of a given pigment can change noticeably as a result of the concentration of the pigment in the paint. Balancing of blue, red, yellow, and white pigments to obtain mid-gray is a complex process requiring trial and error work or a good calculated starting formula.

Görres's objection (see section 'Early comments on *Farben-Kugel'* above) is valid. But also Görres did not fully understand the issues of light mixture vs. colorant mixture.[10]

Without making actual mixtures, the chromatic results of pigment mixture are best demonstrated in the CIE chromaticity diagram.[11] There the simplifying assumptions of an equal energy light source (uniform spectral power across the spectrum) and idealized reflectance curves have been made. The curves used represent colors that the average observer would perceive as pure (red neither bluish nor yellowish, and comparably for the other two). After weighting the curves with the color matching functions representing spectral sensitivity of the human eye and calculating the chromaticity coordinates, the locations of the colors represented by the reflectances are shown in Fig. 7. The intermediate mixtures orange *Or*, green *Gr* and violet *Vi* are also indicated. As Fig. 8 shows, reflectance functions of yellow and blue need to overlap so that their mixture produces a green. If they do not overlap, the mixtures will decline in color toward dark gray, thus be complementary, without forming green. In case of real colorants there is nearly always a smaller or larger overlap of the reflectance curves of yellow and blue. The exercise shows that, in reality, mixture of Runge's elementary and opposing secondary colors do not form an achromatic color. Combination of green and red results in a dark yellowish brown, of blue and orange in a dark desaturated purple, and yellow and violet form a dark reddish brown. None of the three pairs are complementary in the sense of losing all hue.

[9] The color names are in parentheses because, as already Newton pointed out, lights are not themselves colored but simply induce color experiences in the mind.

[10] When he received Görres's letter, Runge was gravely ill and less than three months away from death. He attempted to respond to Görres in early October, but did not complete the letter (1841, II p.420/1).

[11] In this diagram compensating colors are located on diametrical lines passing through the white point (E). This applies to lights and approximately to colorants.
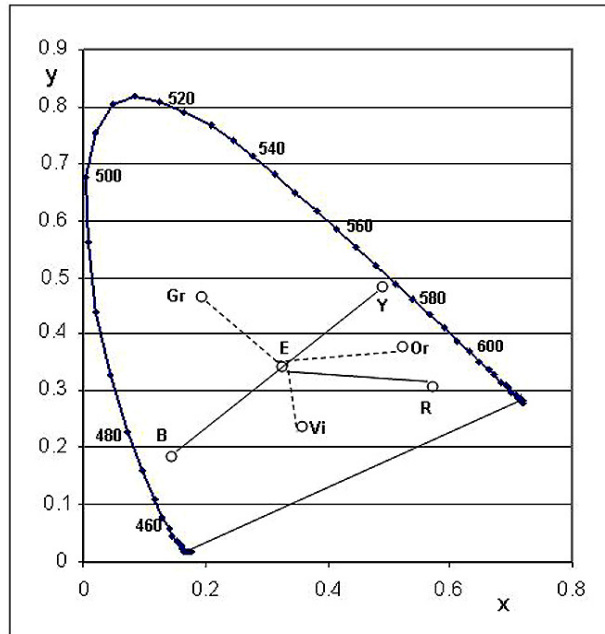
Fig. 7 CIE *x, y* chromaticity diagram with locations of spectral lights of some wavelengths identified by their wavelength in nanometers on the outline. Central point *E* is the location of the equal energy light source and of achromatic objects; *Y, R,* and *B* are the locations of the chromatic primaries with idealized reflectance curves. *Or* and *Vi* are the locations of the intermediate colors orange and violet. The location of the intermediate green depends on the degree, if any, of the overlap of the curves of yellow and blue, as illustrated in Fig. 8. Without any overlap *Y* and *B* are compensating, that is, fall on a line that passes through point E. The figure shows that none of the three pairs indicated by Runge as compensating (*Y* and *Vi, R* and *Gr, B* and *Or*) are compensating, thereby falling on a straight line passing through *E.*
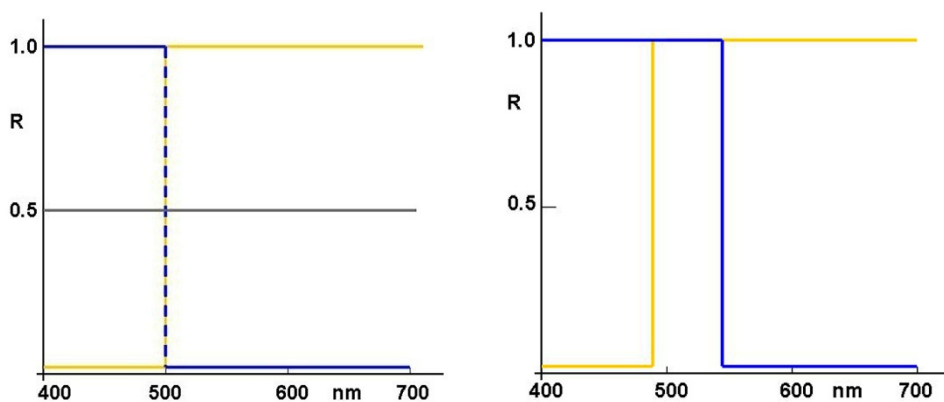

Fig. 8 (left) Idealized spectral reflectance curves of non-overlapping *Y* and *B* that are compensatory, one of their mixtures indicated by the central gray line. (right) Comparable curves of different *Y* and *B* that overlap and thus make reflection of "green" light in the center possible.

91

The essential differences of additive and subtractive color mixture were unknown when Runge developed his color sphere. Another 50 years passed until Hermann von Helmholtz clearly defined the difference. Spectral reflectance measurement had not yet become a reality and the complexities of pigment mixture were not understood for some 120 years. Runge's model nevertheless struck Goethe as plausible, as indicated by his statement: Runge "as we believe, has brought this kind of effort to complete conclusion" (see section 'Early comments on *Farben-Kugel*' above). Even Wilhelm Ostwald, some 110 years later, felt inspired to say "In all other respects, [Runge's] representation may be described as a nearly perfect solution of the problem." (1923, p. 13).

Runge's sphere cannot stand up to scientific scrutiny, but it is a wonderful example of a small step forward on the long path toward a useful, scientifically valid model of color order, a process not yet fully completed.

Runge's Thoughts and Experiments in Disk Mixture

Runge arrived at the idea of the color sphere sometime in the first half of 1807 and described it to Goethe in the letter of November 1807 (excerpted above). At the same time he gave thought to the desirability of objective support for the sphere concept. In the letter of October 1807 to Goethe he stated:

"I am now very active in finding an apparatus with which one can easily make experiments that might not only confirm my reasoning [about color order] in a tangible way and vividly demonstrate the matter to the eyes, but would also furnish proof of the statements made and counter-proof of erroneous ones. As soon as I have it in hand I will inform you of its construction ..."

The equipment he hinted at was a disk mixture apparatus (Fig. 9). It contains a disk painted in different color sectors. Spinning the disk rapidly results in additive mixture of the light stimuli reflected from it. At high speed of rotation, the eye can no longer distinguish the colors of the individual sectors but optically mixes them (termed partitive mixture by some). Disk mixture as a tool for objectively describing colors was mentioned in 1763 by the Italian naturalist G. A. Scopoli as a tool for matching and describing the colors of insects. Disk mixture experiments were also mentioned in 1771 by the Viennese Ignaz Schiffermüller, entomologist and developer of a color order system.
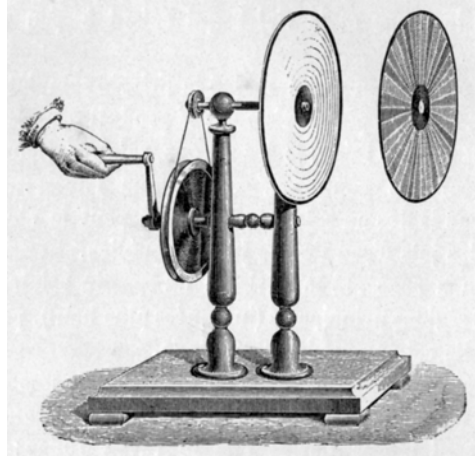
Fig. 9 Late 19th century disk mixture apparatus (Guignet 1889)

It is not clear where Runge got the idea for disk mixture. In his November 1807 letter to Goethe, Runge described his ideas concerning use of the disk mixture tool to provide support for the color sphere, but without yet having done any experimentation:

"The figure [sphere] presupposes a specific idea of color ...; it is necessary to make confirming experiments.

I have quite often heard mention of the spinning disk or vortex. If the seven [Newtonian] colors are, according to certain calculations, painted on the disk in sectors from the periphery to the center, they are supposed to result in white [when spinning the disk]. I have not seen this myself and claim: this is not true. What has been designated as colors in this case lacks sense and reason and what has been named white is gray, even if they [that claim this] all stand on their heads, and even if one of them should be you. If this is not so, I lack any knowledge and have to begin again to learn about vision and need to be told anew the meaning of black and white.

I have not done any experiments and only want to tell you how I imagine what the situation is. (I am at present having a machine constructed with which I can rapidly rotate a disk either vertically or horizontally.) I recently had a conversation with a gentleman who has done the experiment. He said that if one wants the result to be white, the colors painted on the disk have to be very light. I asked him how he lightens them. Answer: with white! — When the disk is properly lit and rapidly rotated, the colorless reflected beam is now supposed to be white. I also asked him if he tried the experiment with a white [continuous] strip painted on the disk and, when now spinning the disk, was the appearance of the strip and the areas covered with colored paints uniformly white?

[Runge's sketch in his letter to Goethe of the color arrangement of an experimental disk mixture disk. The middle, continuous strip is white.]

He had not attempted to do so because the appearance of the disk was sufficiently white. I then asked what the result would have been had he not made the colors very light. — I must try this, because in my opinion, the result will be gray.

I think that, according to the claim that spinning the colored disk sectors makes the colors on it white, we should at least be able to expect that, for example, cinnabar, an equally light blue, and a yellow painted in the proper ratios on a disk, when spun, appear white. If this is not the case, this special experiment that requires mixing the colors with much white before application so that the desired effect is obtained belongs into an entirely different experimental plan, for example, how much lighter colors appear when they are spun on a disk, and others of that kind. — It is very noteworthy that, for example, Lambert in his *Farben-Pyramide* [1772] clearly identified the pigments he used as fundamental colors. To obtain fundamental blue, he mixed Berlin blue with white. Where is the fundamental part in this? How did he select Berlin blue without noticing its depth of color? ..."

Early in 1808 Runge was in possession of a disk mixture apparatus and in April of that year he reported to Goethe:

"I have to tell you about a discovery that gives me much pleasure and confirms several things I had noticed before. I have made a number of experiments with my earlier mentioned machine and the results are as I had predicted. I am in the process of producing a disk with, on a circular strip, the three colors in a lightness that they are almost white. Next to them will be a white ring. More toward the center, a ring with black and white so that it produces, when spun, the same gray as the three colors do. This will be followed by a ring with the three colors applied in very dark color and next to that a ring of black and white that produces the same dark gray appearance. When complete, I will have a series of disks that experimentally confirm certain proofs.

In this connection I noticed that when I mix carbon black and white on the palette very little black is required to obtain middle gray. But in case of

the spinning disk much more black is required. In disk mixture white appears to be three times as strong as black.[12] When combining the three colors by disk mixture to obtain gray, it is essential that all three have the same effectiveness. If, for example, I combine a light yellow with a medium-dark blue, the result is a whitish green.

A striking example for me was that a brilliant orange (specifically orpiment [arsenic sulfide]) together with a pale, bluish violet produced in disk mixture a rather pure pink.

Many such results, odd when first seen, might be produced and might provide much information concerning the general representation and order of colors. But I am alone here and would need support from somebody willing to share time and expense required for such experiments. Only when doing this kind of experimental work does it become clear how best to construct the [disk mixture] equipment so that the results are as broad as possible. I want to tell you how I see this matter and how I explain these phenomena to myself and hope that you will agree with me. Disk mixture operates in a manner comparable to when I produce mixtures by superimposition of color, because they [layers] are all thin and the lightest and most opaque lies on the surface.

If I paint a thin layer of white on top of black the result is a very delicate bluish gray. It is the same as the blue of the pure sky because there the brightened air is superimposed over dark space.

Spinning the disk also generates gray by pulling a white sheen over the black, the gray being very bluish. This kind of appearance always results when spinning on a disk a pale opaque color with a dark one, frequently resulting in unforeseen effects.

Pink was generated from orange and violet in the following manner: The orange [pigment] is strong and of glowing color and at the same time completely opaque; the violet pigment is whitish and considerably bluish. Disregarding the difference in lightness, the result should be gray, because the three [fundamental] colors are represented nearly equally in the two pigments. But because the violet pigment is very weak, it compensates for only a very small amount of yellow in the orange pigment. The brilliance continues and the higher amounts of white in the violet glaze the brilliance. As a result, the brilliant redness [in orange] moderates itself toward blue, and pink is generated. ...”

[12]The effect Runge describes is due to the difference between subtractive mixture of pigments and partitive disk mixture. In the former case, because of the high light absorbance characteristic of black pigment, little black addition is required, for a medium gray 10-15% of the total mix. In disk mixture the ratio between the white and black sectors would have to be 1:1, that is, white appears to be much stronger.

The following comments may help gain a clearer understanding of the above:

a. Apparently, Runge made only limited experiments in disk mixture. He hints at possible deficiencies in his equipment. More importantly, he lacks the time and resources to expand on the experiments. Runge's life as an artist in Hamburg was supported by his father and, mainly, by his brother Daniel. In 1805, the Napoleonic wars caused much havoc in the Hamburg region. Runge and his wife moved in 1805 from Hamburg into his parental home in Wolgast where their first child was born. Daniel's firm went bankrupt in spring of 1806 as a result of the complete isolation of Hamburg by Napoleon's troops and he could no longer support his brother. Philipp Otto decided to give up his artistic career at least temporarily and work in a business to support himself and his family. He and his family moved back to Hamburg in 1807 where Daniel and Philipp Otto formed a new company in which the latter was a partner until the end of his life. As a result, in 1807 he had neither much time nor resources to experiment with disk mixture.

b. As an artist, Runge was fully aware of the different degrees of transparency and opacity of the then available artist's pigments. Transparency and opacity of pigments are not related to color but only to the chemical nature of the pigment and the properties of the crystals they form. Various effects can be obtained by glazing a painted background with more or less transparent pigments. However, these effects are not related to disk mixture effects. What is important in disk mixture is the amount of light reflected from the opaque surface of the disk. Disks are opaque even if the pigments used to paint them are transparent because the pigments are painted on paper, primed canvas, or metal background. Therefore, some of the light illuminating the disk is reflected, either directly from the opaque painted surface or indirectly from the background material after the light has passed through a transparent pigment. In the eye, the reflected light is optically mixed from all sectors of the spinning disk. The mixture of these lights is additive, but only in the ratio of the disk components. If the disk is evenly bisected, then the light arriving at the eye from the spinning disk consists of half of what would be reflected if the whole disk were painted (in Runge's example) bright orange, and half of what would be reflected if the whole disk were painted light violet. The appearance of the two halves together is pink. To clarify unambiguously what happens in disk mixture requires the ability to measure the reflectance of objects (painted disk sectors) and the spectral power of lights (illuminating the disk and reflected from it). Such capability was not generally available until the second half of the 20th century.

c. The assumption that disk mixture should produce Newton's "white" light is due to lack of understanding of the difference between additive (light), subtractive (colorant), and partitive (disk) mixture, something that was not understood until the mid-19th century. Newton showed that the "colorless" light of the sun can be broken into spectral parts that appear

in various strong colors. He clearly pointed out that the lights are not colored but merely cause color perception in the visual sense. When recombining these lights, the result is again colorless light. Combined proper amounts of, for example, "violet, green," and "red" light, results in colorless light. When mixing violet, green and red pigments in proper amounts together, the result, when painted, is black, because light is no longer reflected from the painted field or only in very small amounts. If the sectors are painted with appropriate violet, green, and red pigments in concentrations giving intense colorations, spinning the disk results in a grayness, the exact shade dependent on the lightness and size of the three disk sectors and the brightness of the surround of the spinning disk. The darker the surround, the lighter the appearance of the gray, and vice versa. The psychological results obtained from disk mixture are a complex mix of physical data (spectral power of the light reflected from each disk sector and the size of the sector) and the mind's interpretation of the result in its surround.

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Biographical Information

1777, July 23, birth of Philipp Otto Runge as the ninth of ten children of Daniel Nicolaus Runge (1737-1825) and Magdalena Dorothea Müller (1737-1818) in Wolgast in Western Pomerania, on the Baltic Sea. The family owned a shipping-building company. Phillip Otto is sick during much of his childhood and misses much schooling. His first artistic efforts, beginning at an early age, are in form of *Scherenschnitte* (scissor cuttings), an art taught him by his mother and one he practiced until the end of his life.

1789 Runge attends the city middle school in Wolgast. He is also taught geometry by a master carpenter.

1795 Runge travels with his oldest brother Daniel to Hamburg to begin a commercial apprenticeship in the firm where his brother is a partner. Runge is introduced to Daniel's circle of friends that includes the poet Matthias Claudius and the book publisher Friederich Perthes.

1797 First formal training in drawing. Runge decides, against his father's wishes, to become a painter, supported by Daniel and eventually also by his father.

1799 Travel to Copenhagen to attend the Academy of Arts where, in addition to drawing and painting, he also studies perspective and Euclidean geometry.

1800   In Copenhagen Runge meets the poet, writer, and salon hostess Friederike Brun. Her art collection and circle of friends, including the writers and poets Herder, Wieland, and Schiller, prove influential for Runge.

1801 Runge meets the painter Caspar David Friedrich on an extended trip to Dresden where he studies art in local collections on his own but also obtains further formal training in drawing and painting. He meets and falls in love with 15-year-old Pauline Bassenge, of French Huguenot background. He also meets the poet Ludwig Tieck, a key representative of German literary Romanticism, and the Norway-born natural philosopher Henrik Steffens. He intensively studies the work of the 17th c. mystic Jakob Böhme (whose first written work was titled *Aurora*) and the poet Novalis.

In the same year he enters a drawing based on Homer's *Iliad* in Goethe's art competition announced in the journal *Propyläen* and receives a scathing judgment. As an apparent result, Runge turns against classicism in art.

1803 Runge meets Goethe in Weimar, seemingly by accident, and is invited for dinner on two successive days.

Among several works, he creates the first drawings of four works titled *Die Zeiten* (The times).

1804 Runge marries Pauline Bassenge and moves with her to Hamburg.

1805 Otto and Pauline move into his parental home in Wolgast because of imminent war dangers. He visits and paints on the island of Rügen. Runge's son Sigismund is born in April. In the same year he begins a more intensive correspondence with Goethe, including an exchange of ideas on color theory.

Runge writes an essay fragment titled *Die Elemente der Farben* (Elements of color).

1807 The Napoleonic wars, causing the occupation of Hamburg in 1806, allow Runge to return to that city only in 1807. Together with his brother Daniel, he forms a new firm in which he is an active partner for the rest of his life. Birth of Runge's daughter Maria Dorothea. Henrik Steffens, unemployed due to the war, spends the fall of the year in Hamburg, visiting often with Runge and discussing the latter's color studies. Runge develops the construct of the *Farben-Kugel* (color sphere).

1808 Runge intensifies his color studies, including making disk mixture experiments. He publishes written versions of two local folk fairy tales "The fisherman and his wife" and "The almond tree," later included among the tales of the brothers Grimm. "The times" drawings, in engraved form, become popular and Runge begins work on a colored version.

1809 Runge completes the text of *Farben-Kugel* and forwards it to Steffens. Runge's second son Gustav Ludwig is born in April. Runge writes an essay fragment with the title *Von der Doppelheit der Farbe* (on the duplicity of color).

1810 *Farben-Kugel* is published in January by Runge's friend Friederich Perthes. Runge paints the self-portrait in the brown jacket and portraits of his wife and his brother Daniel. He contracts tuberculosis in March, improves somewhat during summer, but succumbs on December 2. His third son, Philipp Otto, is born a day later.

It is important to realize that the key years during which *Farben-Kugel* was created were overshadowed by the Napoleonic defeat of Prussia and its occupation in 1806-1807, lasting until 1812. Germany was freed only in 1812-1815 in the War of Liberation against Napoleon. In 1805 Runge and his wife moved back to Wolgast because of the war dangers in Hamburg and surrounding areas. In 1806 Napoleon personally closed down Halle University (apparently because the intelligentsia did not receive him with open arms) and Steffens, who was a professor there, lost his possessions and livelihood and was only able to return to Halle in 1808. In the meantime, Steffens had become an important agent of the resistance against Napoleon, constantly fearing arrest. The delay in forwarding Runge's manuscript to Goethe was due to his extended and secretive travels.