AN EMPIRICAL COMPARISON OF PRIORITY-QUEUE AND EVENT-SET IMPLEMENTATIONS

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Basic Priority Queue Operations

- Enqueue (Insert)
  - Places an item in the priority queue
- Dequeue (Delete-min)
  - Removes and returns the highest priority item from queue

Relation to Simulation

- Priorities represent event times in discrete event simulation
  - Enqueue Schedules Events
  - Dequeue Finds Next Pending Event (lowest numbered time)

Measuring Performance

- Hold Method
  - Based on simple discrete-event simulation
  - All events cause scheduling of one new event
    - Keeps constant queue size
    - Direct measure of $\frac{queuesize}{performance}$
    - Random priority value, like next-event simulation
  - Repeatedly dequeue and enqueue items
  - Divide by total time by number of trials

Measuring Performance (Cont.)

- 5 priority increment distributions were used

- Measurements based on 1000 trials

Distribution	Expression to compute random values
1. Exponential	$-ln(u)$
2. Uniform 0.0-2.0	$2 * u$
3. Biased 0.9-1.1	$0.9 + 0.2 * u$
4. Bimodal	$0.95238 * u +$ if $u < 0.1$
	then 9.5238 else 0
5. Triangular	$1.5 * u^{0.5}$

u is a Uniform(0,1) call

Implementations

- Linear List
- Implicit Heaps
- Leftist Trees
- Two List
- Henriksen's

More Implementations

- Binomial Queues
- Pagodas
- Skew Heaps
- Splay Trees
- Pairing Heaps

Linear List

- Singly linked list searching from the head at insertion
- Favors LIFO behavior
- Minimizes storage requirements
  - Only one pointer per item
- $O(n)$ sequential search for enqueue, $0(1)$ dequeue
- Best implementation for 10 or less item queues

Implicit Heaps

- O(log n) performance
- Fast, but many newer queue implementations faster
- Represented as binary tree with heap invariant
- Any item has higher priority than its children
- Stored as an array
  - Location 1 is root
  - $2i$ and $2i+1$ are children of location i

Implicit Heaps (Cont.)

- Enqueue operation
  - Search begins from leaf at upper bound of heap
  - Search toward root
  - Passed items are demoted to make space for new item

- Dequeue operation
  - Returns the root
  - Promotes other items while searching for new place for the most distant leaf.

Leftist Trees

- Heap structure explicitly represented with pointers from parents to their children

- Enqueue operation
  - Item initialized as one node tree
  - Then merged with original tree

- Dequeue operation
  - Root returned
  - Right and Left subtrees then merged

Leftist Trees (cont.)

- Merge operation
  - Merge rightmost branches of the 2 trees
  - Distance to the nearest leaf is recorded for each item
  - 2 children sorted so that path to nearest leaf is always through the right child
  - This guarantees $O(\log n)$ bound
- About 30% slower than implicit heaps in tests

Queues Favoring Discrete-Event Simulation

- Two List and Henricksen’s implementations

- Stable queue behavior

  – 2 events scheduled to occur at same time are FIFO

- Most other priority queues cannot guarantee this

Two List

- One short sorted list of items near the head of the queue
- One long unsorted list of more distant events
- Enqueued item compared with a threshold priority to determine correct list to put it in
- Dequeued items just removed from sorted list
- When sorted list is empty
  - Advance threshold and search unsorted list for items to move to sorted list
  - Keeps an average of $n^{0.5}$ items in sorted list

Two List (cont.)

- Average enqueue time of $O(n^{0.5})$
- Worst-case dequeue $O(n)$, but most are done in $O(1)$ time
- Average dequeue of $0(n^{0.5})$
- Good performance for queues up to a few hundred items
- Very poor with Bimodal distribution

Henriksen's

- Uses Simple linear list

- Auxiliary array of pointers into list

- Allows $O(\log n)$ binary search to find range of entries where enqueued items should be placed

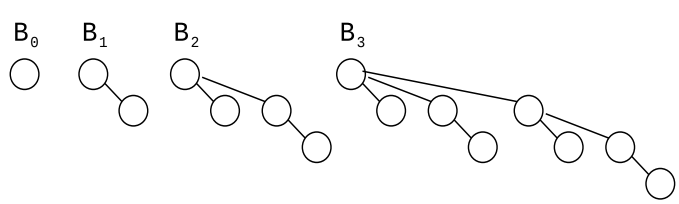
- Significant cost of maintaining array and searching subsection of list pointed to by array entry

- Average performance bounded by $O(n^{0.5})$

- Performed well comparatively

Binomial Queues

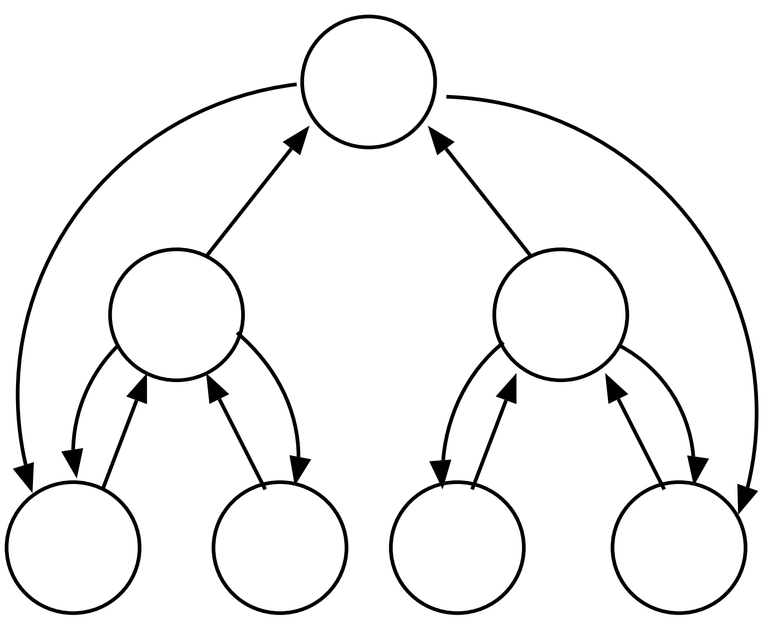
- A forest of binomial trees where the number of elements in each tree is an exact power of 2
- Height n Binomial Tree
  - Root has $n-1$ children
  - Children are binomial trees with heights $n-1$, $n-2$, ..., 0
- Performs extremely well
- Varies for small queue size changes based on binary representation of size



Binomial trees of heights 0, 1, 2, and 3.

Pagodas

- Based on heap ordered binary trees
- Primary pointers lead from leaves toward root
- Secondary pointers point down to item's left- and rightmost descendants



Pagodas (cont.)

- Enqueue and dequeue operations
  - Merge the right branch of one pagoda with left branch of another
- Insertions occur in constant time
- No balancing effort made, resulting in infinite sequences of $O(n)$ per operation
- Arbitrary deletions occur in $O(\log n)$ time
  - All branches circularly linked
- Performs about as well as Binomial Queues

Skew Heaps

- Similar to leftist tree, but no record of path length to nearest leaf

- Children of each item visited on the merge path are exchanged to randomize the tree structure

- Per operation cost never exceeds $O(\log n)$ over a sufficiently long sequence of operations

- Performs faster than implicit heaps

Splay Trees

- Set up as binary search trees
  - All items in left subtree smaller than root
  - All items in right subtree larger than root

- Dequeue operation simply removes the leftmost item

- Blindly performs pointer rotations
  - The basic balancing operation
  - Avoids keeping and testing balancing records
  - Causes increased number of rotations

Splay Trees (cont.)

- Stable - Equal priority items are FIFO

- Like Henriksen's performed exceptionally well for the biased distribution

- Overall faster than Henriksen's implementation

- In a sense optimal

Pairing Heaps

- Heap-ordered tree
- Constant time Enqueue
  - Can make new item root
  - Or adds new item as additional child of root
- Dequeue returns root then searches for new root
- Key to pairing heaps is method of finding new root
- Link successive children of old root in pairs, then link each pair to the last pair produced

Pairing Heaps (cont.)

- Combining two pairing heaps
  - Adds heap with lower priority root as child of other heap

- Performed about the same as bottom-up skew heap

- Ran especially well on the biased distribution

Conclusions

- Linked list is best implementation for < 10 items
- Two-list performs well up to a couple hundred items except for some distributions
- Leftist trees don't perform well enough for any application
- Henricksen's acceptable for all queue sizes
- Splay trees challenge it where stable behavior is required

Conclusions (cont.)

- Implicit Heaps one of worst for less than 20 items

- Binomial queues are erratic and most complex to code

- Skew heaps, pairing heaps, and pagodas all almost as good as splay trees

- Top-down skew heap is very simple

- When other operations are needed like arbitrary deletions or priority changes

  – Bottom-up skew heaps, splay trees, and pairing heaps are best alternatives

Summary

Implementation	Relative Speed
Linked list	11
Implicit heap	8
Leftist tree	9 – 10
Two List	9 – 10
Henriksen's	1 – 7
Binomial Queue	1 – 7
Pagoda	4 – 8
Skew heap	4 – 7
Splay Tree	1 – 3
Pairing Heap	3 – 6

1 is fastest; 11 is slowest