Édouard Lucas

French mathematician **François Édouard Anatole Lucas** (April 4, 1842 – October 3, 1891) was a pioneer in number theory. He specialized in Fibonacci numbers, which he named after the series introduced by Leonardo of Pisa (ca. 1180 – 1250), also known as Fibonacci. Lucas a devised a test to decide whether or not a number is prime without the necessity of checking all the primes less than the square root of the number as possible divisors of the number. The method was later refined by Dirk H. ("Dick") Lehmer (1905 – 1991) in the 1930s and is now known as the Lucas-Lehmer primality test. The test for deciding if a number n is prime or not requires knowing the prime factors of $n - 1$. If there exists some a less than n and greater than 1 such that $a^{n-1}$is congruent to 1 (mod n) and $a^{(n-1)/q}$ is not congruent to 1 (mod n) for all factors q of $n - 1$, then n is prime. If there is no such a, then n is a composite number.

Lucas was born in Amiens and educated at the École Normale Supérieure of that city. His first job was as an assistant to the astronomer Urbain Jean Joseph Le Verrier (1811-77) at the Paris Observatory. The latter is renowned for predicting the existence of an undiscovered planet, to be named Neptune when later found by Johann Gottfried Galle (1812-1910) of the Berlin Observatory, by observing disturbances in the motions of planets. During the Franco-Prussian War (1870-71), Lucas served as an artillery officer. After France's defeat, he studied mathematics at the Lycée Saint-Louis and the Lycée Charlemagne, both in Paris. He later became a professor of mathematics in Paris. Lucas had a reputation as an entertaining teacher, amusing and challenging his students with mathematical puzzles that required considerable ingenuity and mathematical insight to solve.

Lucas' serious mathematics is especially centered on primes and factorization. He studied the Fibonacci sequence 0,1,1,2,3,5,8,13,21,34,55 … in which each term after the first is the sum of its two predecessors. Lucas gave a closed formula for the Fibonacci numbers. They are generated by the recurrence rules,

$$F_0 = 0, \quad F_1 = 1, F_{n+1} = F_{n-1} + F_n.$$

In addition Lucas explored properties of the series of numbers: 2,1,3,4,7,11,18,29,47,76,123, which are today called the Lucas Numbers in his honor. The Lucas series is generated by a modification of the above rules, starting with 2 and 1 rather than 0 and 1, as did Fibonacci. The series is given by the rules,

$$L_0 = 2, L_1 = 1, L_n = L_{n\text{-}1} + L_{\text{n-2}} \text{ for } n > 1.$$

Lucas used the two series to devise the modern method of testing the primality of Mersenne's numbers. These numbers, named for the French monk Marin Mersenne (1588 – 1648) are numbers of the form $2^p - 1$, where p is a prime. Lucas proved the following theorem: The Mersenne number $M_p = 2^p - 1$, in which p is a prime greater than 2, is itself prime if and only if M_p divides S_p, with S_p generated by the rules $S_2 = 4$, $S_3 = 14$, with S_n for $n > 2$, inductively given by $S_n = (S_{n\text{-}1})^2 - 2$. Using his theorem, in 1876, Lucas was able to show that $2^{127} - 1$ is a prime. This was the first new Mersenne prime discovered in over a century, and the largest one ever checked without electronic assistance. It would be three-quarters of a century before a larger Mersenne prime was found. In expanded form M_{127} is

170,141,183,460,469,231,731,687,303,715,884,105,727.

M_{127} is large, but S_{127} is incredibly large. Fortunately Lucas didn't have to calculate it. Instead he showed that it is divisible by M_{127}.

Lucas was also interested in recreational mathematics and is remembered for inventing the Tower of Hanoi puzzle. When it first appeared in 1883, it was attributed to one N. Claus de Siam, which is an anagram of Lucas d′Amiens. The puzzle is described in his classic four-volume *Récréations mathematiques* (1892-94). It consists of three pegs fastened to a stand and of eight circular disks, each having a hole in the center. The disks, all of different radii, are initially placed on one of the pegs, with the largest disk on the bottom and the smallest on top; no disks rest upon one smaller than itself. The challenge is to transfer the individual disks from one peg to another so that no disk ever rests on one smaller than itself, and, finally, to transfer the tower; that is, all the disks in their proper order, from their original peg to one of the other pegs. The puzzle was a popular toy in France in the late 19^{th} century.

Supposedly the Tower of Hanoi puzzle was inspired by the following Hindu legend. “In the great temple of Benares under the dome that marks the center of the world, three diamond needles, a cubit high and as thick as the body of a bee, stand on a copper base. At creation God strung 64 disks of pure gold on one needle, known as the Tower of Brahma, the largest disk at the bottom and the others ever smaller on top of each other. Priests of the temple were given the task of continuously moving the plates until they will be set in the same configuration on another needle. The rule for moving the disks is simple: only one disk at a time, and never a larger one on a smaller one. When the priests complete the transfer of all the gold discs from one needle to another in the manner prescribed in the, the tower, the temple and everything on the earth will crumble into dust and with a thunderclap and the Universe will cease to be.” One should not worry too much about the imminent end of the world. Assuming that the priests never made a mistake and moved one disc a second non-stop, the number of transfers necessary would be $2^{64} – 1$, or 18,446,744,073,709,552,615 and would take 590,000,000,000 years to complete. Today it is believed that Lucas invented the legend as well as the puzzle.

Lucas also wrote on the history of mechanical aids to calculation, did some highly original work with the arithmetization of elliptic functions, and claimed to have made important progress in the proof of Fermat's last theorem. He died as a result of a tragically absurd accident at the banquet of the annual congress of *Association française pour l'advancement des sciences*. A plate was dropped, shattered, causing a piece of it to fly up and gash his cheek. A few days later he died of erysipelas, an acute infectious disease of the skin or mucous membranes caused by a streptococcus.

Quotation of the Day: "The problem of distinguishing prime numbers from composite numbers and resolving the latter into their prime factors is known to be one of the most important and useful in arithmetic…. The dignity of the science itself seems to require that every possible means be explored for the solution of a problem so elegant and so celebrated. – Carl Friedrich Gauss