Ferdinand George Frobenius

German algebraist, analyst, and group theorist **Ferdinand Georg Frobenius** (October 26, 1849 – August 3, 1917) formulated the concept of the abstract group, the first abstract structure of "new" mathematics. Groups are invaluable for algebraically simplifying the physical analysis of systems possessing some degree of symmetry. Frobenius' research into groups and complex number systems proved extremely useful in the development of quantum mechanics.

Frobenius was born in Charlottenburg, now a district in Berlin. Following a semester at Göttingen, attending lectures in analysis and physics, he went to the University of Berlin, where at the age of 21 he was awarded a doctor's degree with distinction. His thesis was on the representation of analytical functions in one variable by infinite series, supervised by Karl Weierstrass. For the next few years, Frobenius taught at the secondary school level. In 1874, he was appointed an extraordinary professor of mathematics at the University of Berlin. Due to the influence of Weierstrass, who considered him one of his most gifted students, Frobenius was able to avoid the customary habilitation period. After one year at Berlin, he was appointed an ordinary professor of mathematics at the Eidgenössiche Polytechnikum [Federal Polytechnic] in Zurich. There in 1876 he met and married Christiane Ellis. For the next seventeen years he published important papers on a wide variety of topics. When Leopold Kronecker died Frobenius was recalled to Berlin to replace him, staying there for the remainder of his career.

Frobenius made major contributions in numerous fields, including: the development of analytic

functions in series, algebraic solutions of differential equations, the theory of linear differential equations, the theory of elliptic and Jacobi functions, the theory of biquadratic forms, and the theory of surfaces. However it is his work in group theory that is most notable. Frobenius' findings in abstract group theory were published in his paper "Über Gruppen von vertauschbaren Elementen" (Concerning Groups of Permutable Elements) in collaboration with Ludwig Stickelberger, a colleague at Zurich. Over the next several years Frobenius established the foundation of all group representation theory. Working with one of his 19 doctoral students, Issai Schur, he developed group theory by means of the theory of finite groups of linear substitutions. Frobenius created the theory of group characters and group representations, which are fundamental tools for studying the structure of groups.

In mathematics, it is more challenging to tell the story of the most recent mathematics than it is to relate what was developed in the distant past. Mathematics is hierarchical and it is difficult to understand later work without a strong grasp of what came earlier. Group theory is no exception, as there have been so many innovative investigators who have contributed to its expanding growth. In the process, the vocabulary for the field has exploded as well. It's not appropriate in this article to attempt to lay out any more than a hint of its fundamental importance and prodigious fertility, not only in mathematics, but in an ever growing number of real world applications in fields as varied as: tensegrity (a contraction of tensional integrity, the design of "strut-and-cable" constructions), telephone network problems, control theory, robotics, radar design, coding theory, mechanics, models from elementary particles, quantum mechanics, spectroscopy, crystallography, knot theory (with applications to modeling DNA), circuit design, and so forth. In a certain sense all groups arise from the collection of symmetries of some object preserving some of its structure. It took mathematicians more than 2000 years to achieve a formulation of the group concept. Lagrange, Cauchy, Abel, Galois, and others used permutation groups in their work on the solution of algebraic equations using radicals. A permutation is an arrangement of all or part of a set of things.

Once the concept of an abstract group was established there were two approaches to studying the concept. One is to approach groups directly in their abstract form using combinatorial methods, that is, to work with the group without giving any interpretation to the elements of the underlying set. The group and binary operation were abstract items to be manipulated according to the rules of the game of group theory. The other approach is to utilize concrete realizations of groups for investigations. For instance, every finite group can be represented by a permutation group or by a group of matrices. This means that from the point of view of group theory, all three representations are the same group. They may appear different, but the difference is only in appearance not in the structure or the behavior of the elements.

Although a brilliant mathematician, Frobenius did not get along with his colleagues who found him "quarrelsome and given to invectives." He was often cranky and moody, especially in his old age. He had exceedingly high standards for himself and the University of Berlin, and he felt that the mathematics faculty should concern themselves only with pure mathematics, leaving the applied to the technical schools. He believed the Ministry of Education was trying to dilute standards in order to attract more students. To an extent he was correct. Over the years the number of doctoral students declined while the number of other students increased. Frobenius did not respect the intellects of non-doctoral students. His views on cooperative efforts in mathematics are as follows: "In mathematics … organizing talent plays a most subordinate role. Here weight is carried out by the individual. The slightest idea of a Riemann or a Weierstrass is worth more than all organizational endeavors. To be sure, such endeavors have pushed to take center stage in recent years, but they are exclusively pursued by people who have nothing, or nothing more, to offer in scientific matters. There is no royal road to mathematics."

Quotation of the Day: "A group has been compared to the grin that remains when Lewis Carroll's

Cheshire cat fades away." – George A. W. Boehm