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Mini-course

Recent Developments in Behavioral System Theory

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The aim of this mini-course is to give a self contained tutorial introduction to the behavioral approach to systems and control. Since it is assumed that most of the participants have had some previous exposure to the basic underlying ideas, the emphasis will be on recent developments. The behavioral approach is motivated by the aim of obtaining a framework for analysis and synthesis that respects the underlying physics and sets up the appropriate mathematical concepts from there. This aspect will be the leitmotiv of the lectures. The mini-course is divided in 6 modules of approximately 50 minutes each. The following is a tentative outline of the lectures.

Basics The purpose of this introductory lecture is to familiarize the audience with the basic concepts of behavioral system theory. Among these are the notion of a model and the definition of a system in terms of its behavior, the role of latent variables, structural properties of systems such as controllability and observability, and various types of system representations by means of differential equations, as kernel, image, and latent variable representations. A central result here is the equivalence of a linear time-invariant differential system and its module of annihilators.

State and model reduction A first principles model of a physical system invariably involves auxiliary (latent) variables, besides the variables the model aims at, and usually consists of many differential equations of higher order and algebraic constraints. For important classes of systems, these auxiliary variables may be eliminated. The elimination theorem and algorithms for elimination will be discussed. State variables are a special case of latent variable models. While they are not a natural end point of system modeling, they are of utmost importance in simulation and control. An efficient way of obtaining a state model is through a state map, a differential operator that acts on the variables of a model and induces a state trajectory and a state model from there. This leads to algorithms for state construction and to model reduction procedures.

Interconnections and control The usual procedure for obtaining mathematical models of complex systems proceeds by tearing and zooming: a system is viewed as an interconnection of subsystems, leading to a combination of models for the subsystems and the interconnection laws. This procedure is usually executed hierarchically. The central idea of interconnection is hence sharing variables, following the physics, with input-to-output assignment as an important but special case. From this point of view, the central problem of control is the design in an interconnected system of a subsystem that interacts with the plant through control variables, and thus imposes additional laws on the system variables in order to obtain a desired controlled behavior. Stabilization will be discussed from this point of view, using system representations in terms of polynomial models as well as (stable) rational functions.

Quadratic differential forms The concept of bilinear- and quadratic differential forms has been developed in the behavioral framework in order to study the interplay of dynamics and quadratic functionals in the system variables. Two-variable polynomial matrices yield efficient representations and a convenient calculus for these differential forms. Examples of application that will be discussed are Lyapunov theory for high order differential equations, the theory of dissipative systems and the construction of storage functions, LQ control, and stability of interconnected systems.

Multidimensional systems The behavioral language is readily generalized to distributed systems with many independent variables. These are typically models in terms of PDE's, involving both time and space variables. While the main research issues addressed in this context are similar to those arising in ODE's, there are very essential differences between the two areas in the mathematical tools needed. The main problems discussed in this lecture is the elimination of latent variables in systems described by PDE's, controllability algorithms and potential representations, and the use of multivariable quadratic differential forms for studying dissipative distributed systems.

System identification Three types of SYSID algorithms will be discussed: exact, approximate, and stochastic identification. Exact SYSID centers around the concept of Most Powerful Unfalsified Model (MPUM), that is, a model that explains the given data and as little else as possible. Algorithms will be given to compute the MPUM and some of its representations for linear, time-invariant complete systems from observed time-series. Especially effective algorithms are obtained that compute a state trajectory directly from the observed vector time-series and deduce a state model from there. These algorithms lend themselves very well for approximate implementation and approximate SYSID. Often these algorithms are referred to as subspace identification methods, and readily lead to stochastic generalizations.