

## CHAPTER 5

# COORDINATE SYSTEMS

### 501. REFERENCE SYSTEMS

To define an orbit in space or a position in an orbit a reference system is necessary. All positions being relative, the common practice is to relate a position (in space) to a convenient coordinate system that moves with the observer. In defining such a coordinate system an origin and a reference plane, the  $XY$  plane, are selected.

### 502. GALACTIC SYSTEM

If the origin lies at the center of the galaxy and the galactic plane is used as the reference plane with an arbitrary direction chosen to orient it, the system is called a galactic system. This system, however, has no practical application to space navigation at present.

### 503. HELIOCENTRIC ECLIPTIC SYSTEM

If the center of the Sun lies at the origin, the system is called heliocentric. And if the plane of the Earth's orbit defines the reference plane, it is called heliocentric ecliptic. The illustration (figure 503) shows how a body's position is defined by means of heliocentric longitude  $\ell$  measured eastward from the vernal equinox, heliocentric latitude  $b$  and radius  $r$  from the origin.

### 504. GEOCENTRIC SYSTEMS

With the origin lying at the center of the Earth, as would be used for close-in terrestrial satellites, a geocentric system is obtained. Two geocentric coordinate systems are called equatorial or ecliptic depending on whether the plane of the equator or the plane of the ecliptic is used as the respective reference plane. Right ascension  $\alpha$  and declination  $\delta$  are the two angles used in the geocentric equatorial system for locating, in space, a line of bearing from the origin. Sidereal hour angle (SHA) is used in place of  $\alpha$  for navigation purposes. They are shown in figure 504A. It can be seen that

right ascension is measured eastward (SHA westward) from the vernal equinox and declination measured north or south of the equatorial plane. The third component, the distance from the origin, remains  $r$ .

When the ecliptic plane is the reference plane in a geocentric system the three coordinates become longitude  $\lambda$  measured eastward from the vernal equinox, latitude  $\beta$  measured from the ecliptic plane and  $r$ , the distance from the center.

### 505. HORIZON SYSTEMS

Another coordinate system deserves mention. The topocentric horizon system (observer is at the origin of the coordinate system) used for tracking purposes, is illustrated in figure 505. Here the coordinates become altitude  $H$  above the horizon, azimuth  $A_z$  measured from true north and  $\rho$ , distance from the observer. A horizon system with the center of the Earth as the origin is called a geocentric horizon system.

### 506. TRANSFORMATIONS

To define an orbital plane in space relative to any coordinate system, the longitude of the ascending node  $\Omega$  and the inclination  $i$  are used; to orient the orbit in the orbital plane, longitude of periapsis  $\omega$  is needed.

In order to transform from one coordinate system to another, translations and rotations are performed in sequence. Rectangular coordinates simplify the translation operation, while polar coordinates reduce the messiness of a rotation. A transformation from an arbitrary  $XYZ$  system to a  $PLP'$  system (figure 506), where the origin remains identical (no translation), is defined by

$$\begin{aligned}
 P &= X \cos \omega \cos \Omega - X \cos i \sin \Omega \sin \omega + Y \cos \omega \sin \Omega \\
 &+ Y \cos i \cos \Omega \sin \omega + Z \sin \omega \sin i
 \end{aligned}
 \tag{1}$$

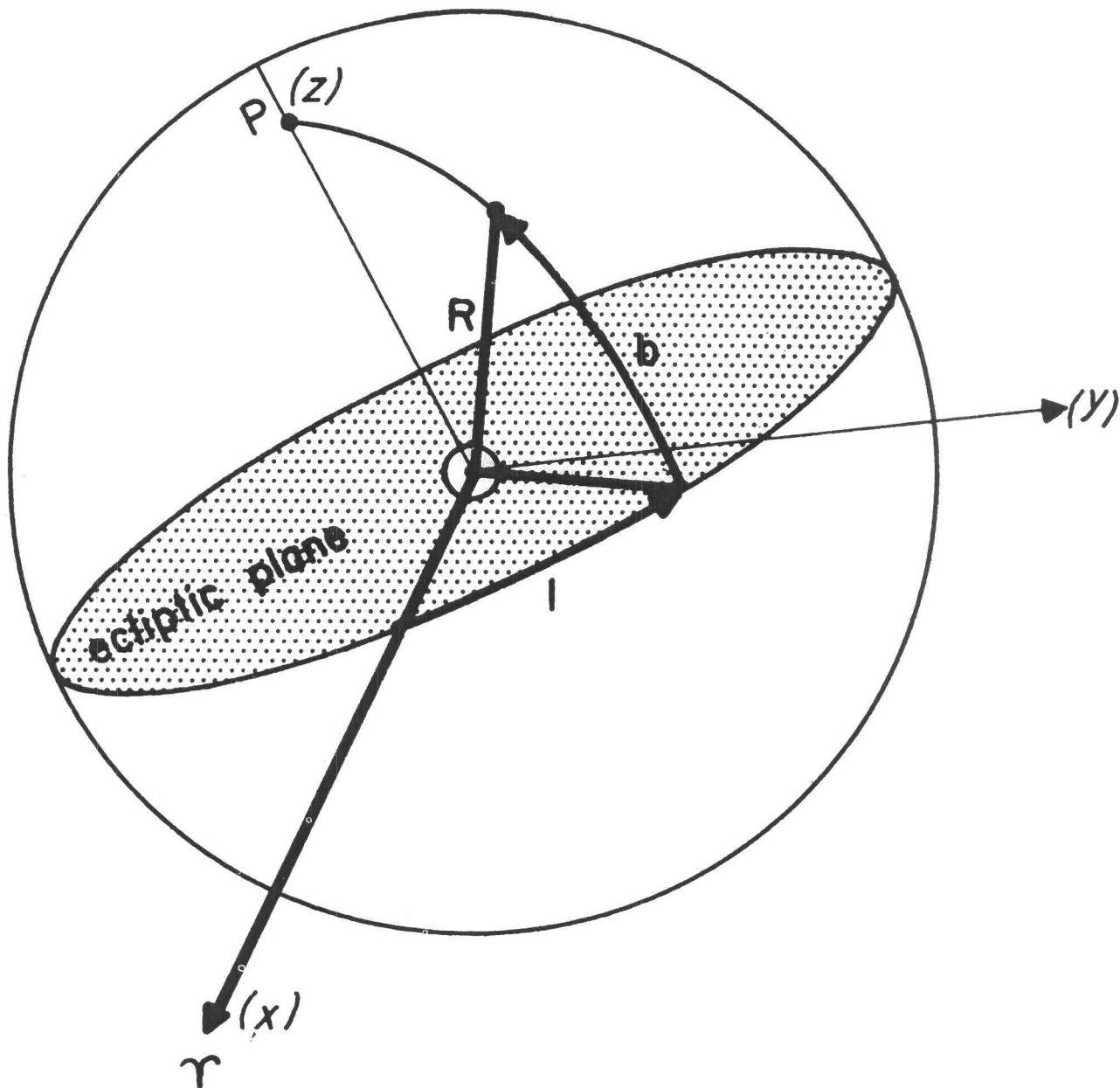


Figure 503.—The Heliocentric Ecliptic Coordinate System.

$$L = -X \sin \omega \cos \Omega - X \cos i \sin \Omega \cos \omega - Y \sin \omega \sin \Omega \\ + Y \cos i \cos \Omega \cos \omega + Z \cos \omega \sin i \quad (2)$$

$$P' = X \sin i \sin \Omega - Y \sin i \cos \Omega + Z \cos i. \quad (3)$$

These equations come about by performing in succession three separate rotations about the Z axis, about the new P axis and about the new P' axis.

#### 507. TRANSFORMATION FROM HELIOCENTRIC ECLIPTIC TO GEOCENTRIC ECLIPTIC

Since this transformation involves only a translation of the origin from the center of the Sun to the center of the Earth, polar coordinates are changed to rectangular coordinates by

$$X = R \cos b \cos \ell, \quad Y = R \cos b \sin \ell, \quad Z = R \sin b.$$

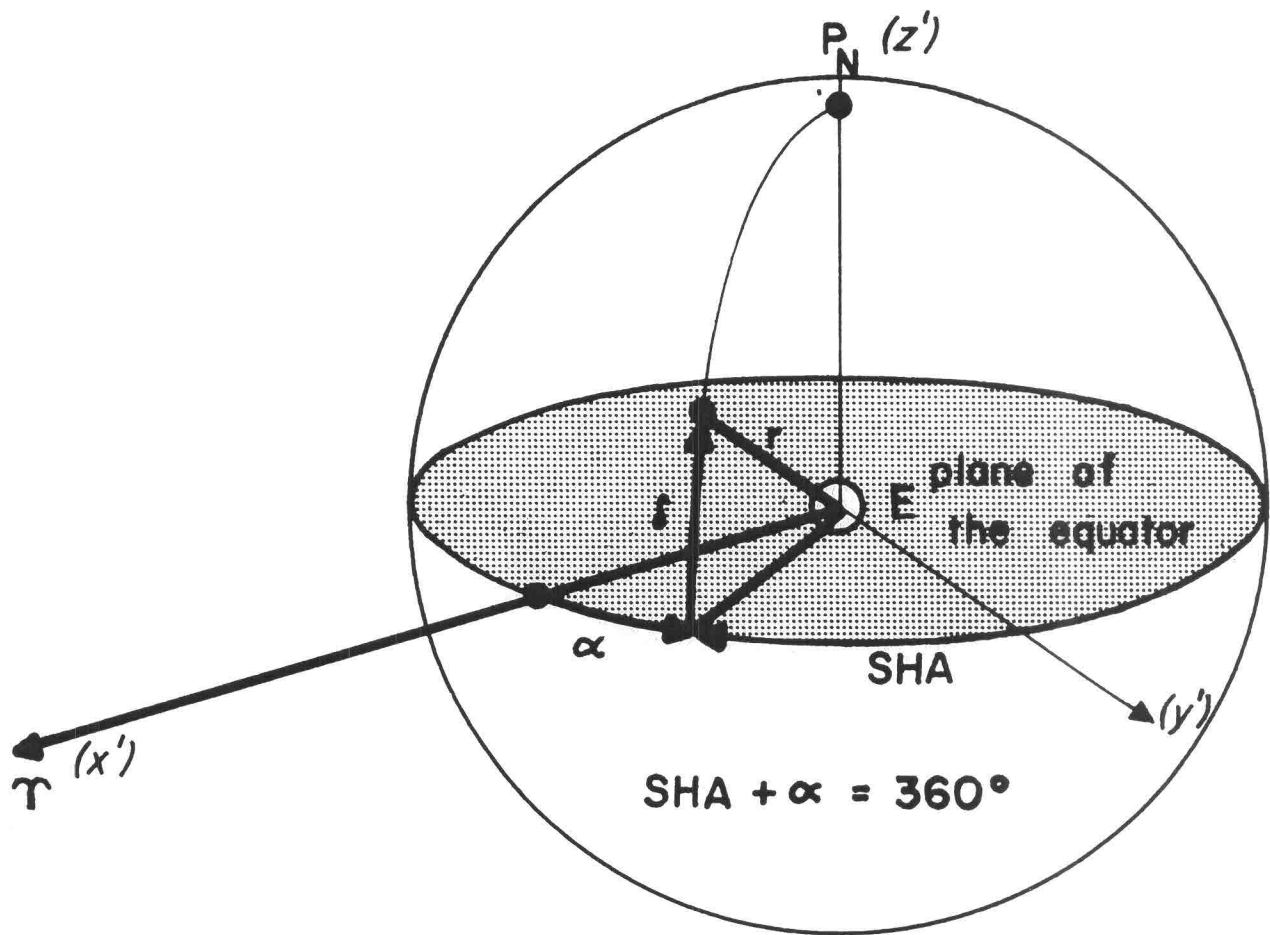


Figure 504A.—The Geocentric Equatorial Coordinate System.

The heliocentric coordinates of the Earth at the particular time of transformation are  $X_e$ ,  $Y_e$ , and  $Z_e^*$  and those of a body are  $X$ ,  $Y$ , and  $Z$ . To get the coordinates in the geocentric coordinates  $x$ ,  $y$  and  $z$  one writes

$$x = X - X_e$$

$$y = Y - Y_e$$

$$z = Z - Z_e$$

where

$$x = r \cos \beta \cos \lambda$$

$$y = r \cos \beta \sin \lambda$$

$$z = r \sin \beta$$

\*It should be noted that  $b_e$  is rarely more than one second of arc and therefore  $Z_e \sim 0$  which would make  $z \sim Z$ .

thus

$$x = R \cos b \cos l - R_e \cos b_e \cos l_e$$

$$y = R \cos b \sin l - R_e \cos b_e \sin l_e$$

$$z = R \sin b - R_e \sin b_e$$

$$r = \sqrt{x^2 + y^2 + z^2}$$

$$\beta = \tan^{-1} \left[ \frac{z}{\sqrt{x^2 + y^2}} \right]$$

$$\lambda = \tan^{-1} \frac{y}{x}$$

#### 508. TRANSFORMATION FROM GEOCENTRIC ECLIPTIC TO GEOCENTRIC EQUATORIAL

Let  $x$ ,  $y$  and  $z$  be the geocentric ecliptic coordinate of a body as before and  $x'$ ,  $y'$  and  $z'$

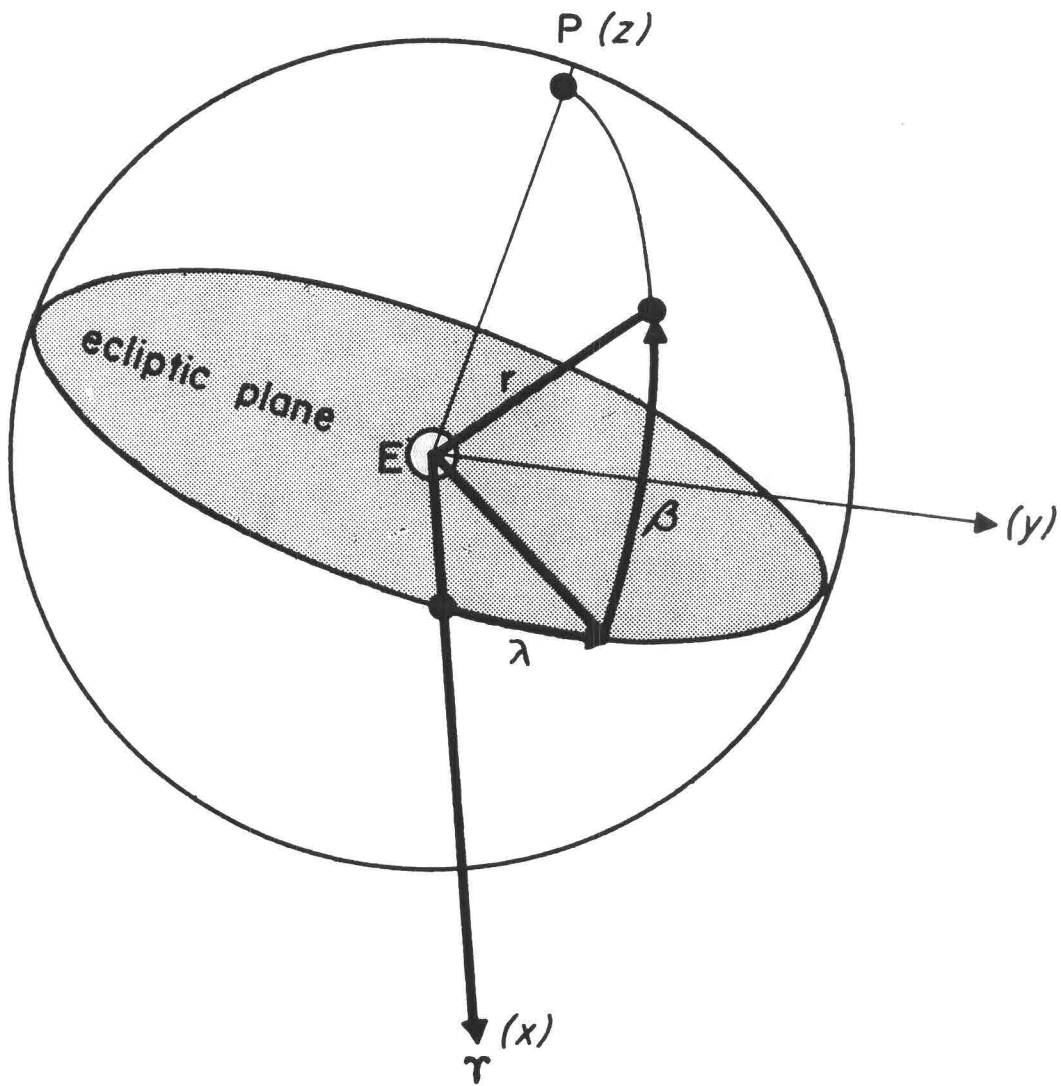


Figure 504B.—The Geocentric Ecliptic Coordinate System.

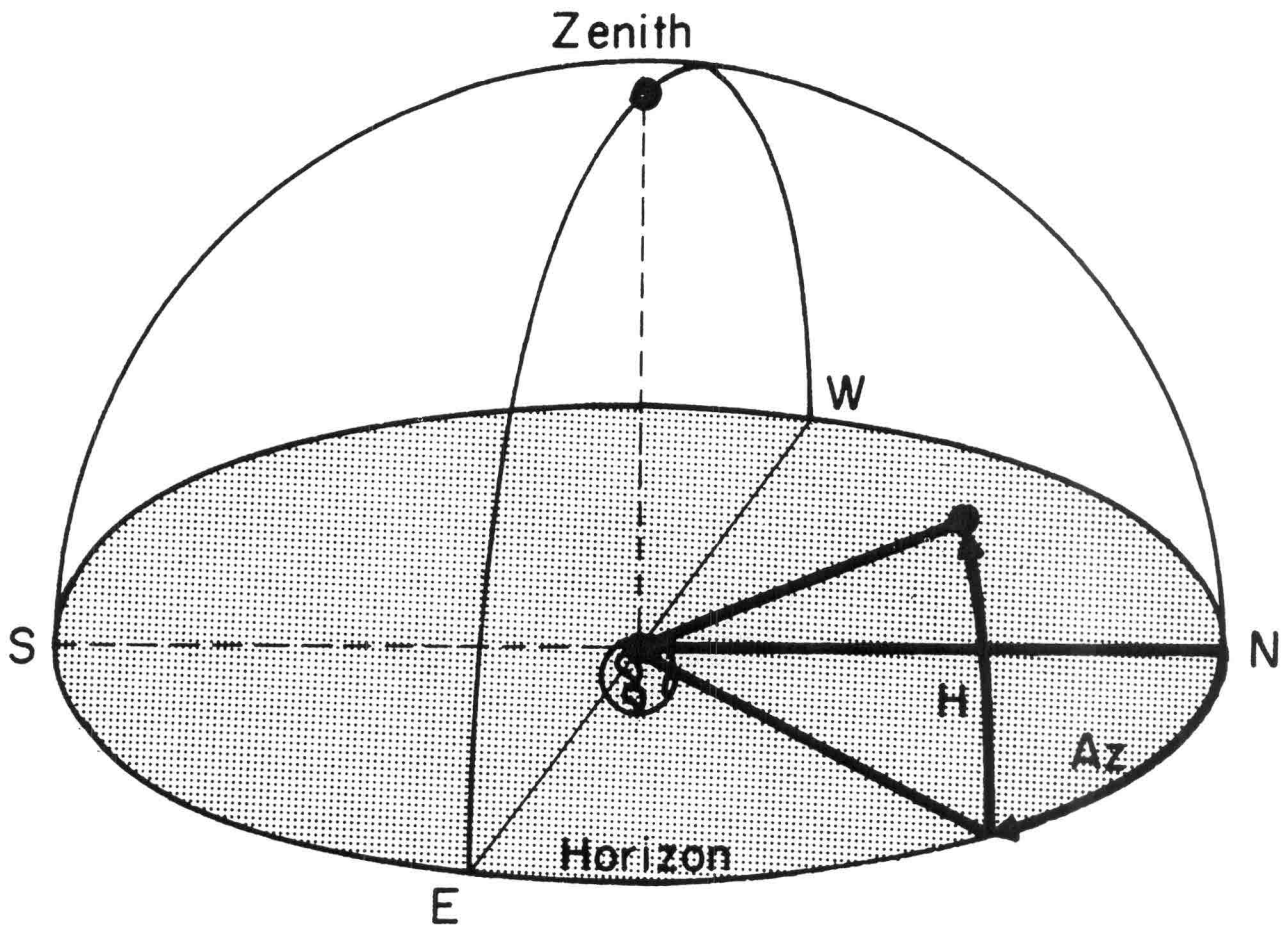


Figure 505.—The Topocentric Horizon Coordinate System.

the geocentric equatorial coordinates of the body. Using the relations (1), (2) and (3) with  $\Omega = 0$ ,  $\omega = 0$  and  $i = \epsilon = 23^\circ 26' 59''$ , the angle between the ecliptic and equatorial planes, one has

$$x' = x$$

$$y' = y \cos \epsilon + z \sin \epsilon$$

$$z' = y \sin \epsilon + z \cos \epsilon$$

and in polar coordinates

$$x' = r \cos \beta \cos \lambda$$

$$y' = r \cos \beta \sin \lambda \cos \epsilon + r \sin \beta \sin \epsilon$$

$$z' = -r \cos \beta \sin \lambda \sin \epsilon + r \sin \beta \cos \epsilon.$$

Therefore

$$r = \sqrt{x'^2 + y'^2 + z'^2} = \sqrt{x^2 + y^2 + z^2}$$

$$\delta = \tan^{-1} \left[ \frac{z'}{\sqrt{x'^2 + y'^2}} \right]$$

$$a = \tan^{-1} \frac{y'}{x'}.$$

#### 509. TRANSFORMATION FROM GEOCENTRIC EQUATORIAL TO GEOCENTRIC HORIZON

Again we use only equations (1), (2), and (3) because the origin is not shifted. For the rectangular coordinates in the geocentric horizon system use  $N_e$ ,  $W_e$ , and  $Z_e$ . Hence

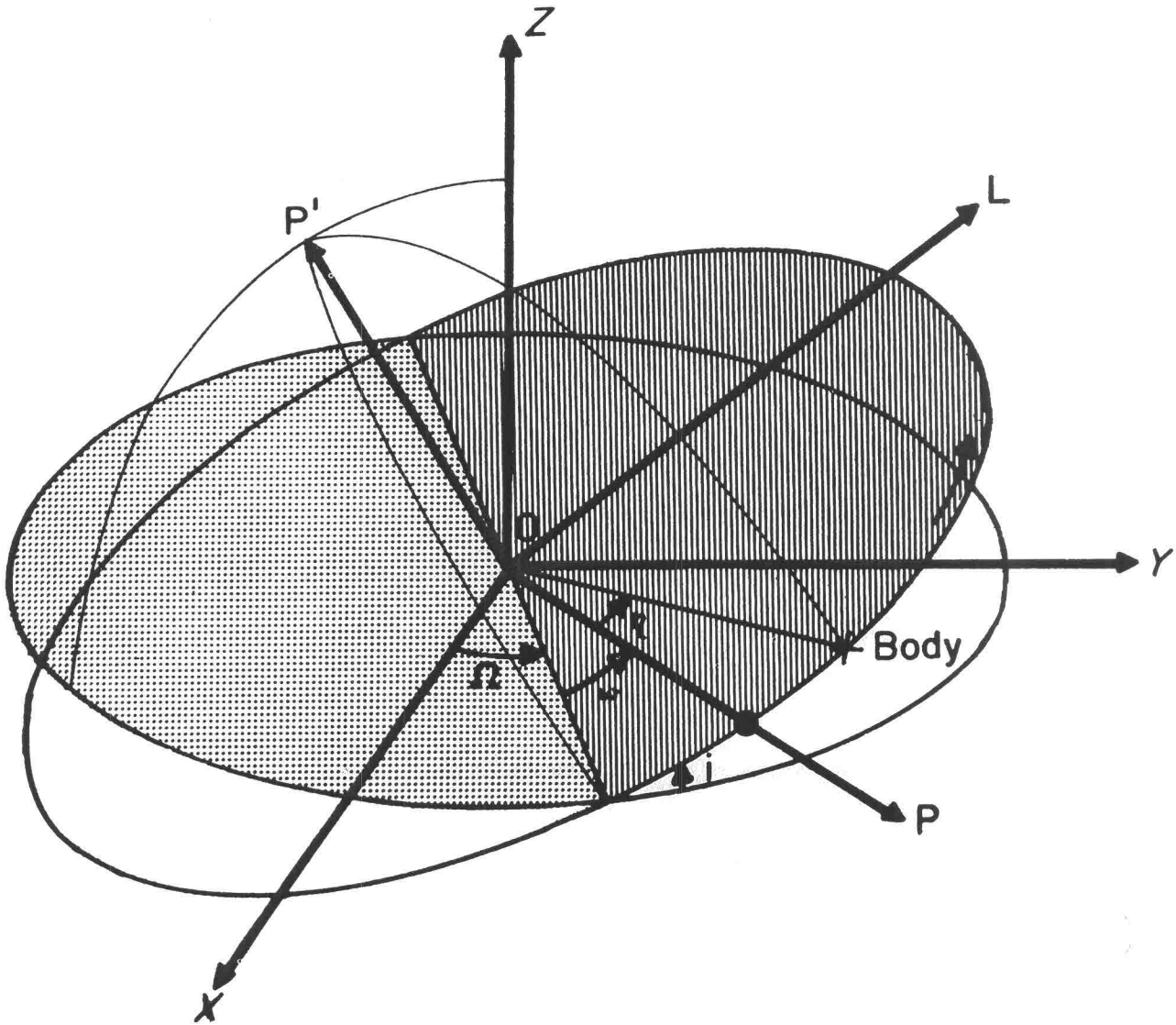


Figure 506.—Rotations of Coordinate System.

$$N_e = -x' \cos (90 - L) \sin (\text{GHA } T + 90 - \text{Long}) \\ + y' \cos (90 - L) \cos (\text{GHA } T + 90 - \text{Long}) \\ + z' \sin (90 - L)$$

$$W_e = -x' \cos (\text{GHA } T + 90 - \text{Long}) \\ - y' \sin (\text{GHA } T + 90 - \text{Long})$$

$$Z_e = x' \sin (90 - L) \sin (\text{GHA } T + 90 - \text{Long}) \\ - y' \sin (90 - L) \cos (\text{GHA } T + 90 - \text{Long}) \\ + z' \cos (90 - L)$$

where  $L$  is latitude (north positive, south negative) and Long is longitude (east negative, west positive). Substituting into polar form

$$N_e = -(r \cos \delta \cos \alpha) \cos (90 - L) \sin (\text{GHA } T + 90 - \text{Long}) \\ + (r \cos \delta \sin \alpha) \cos (90 - L) \cos (\text{GHA } T + 90 - \text{Long}) \\ + (r \sin \delta) \sin (90 - L)$$

$$W_e = -(r \cos \delta \cos \alpha) \cos (\text{GHA } T + 90 - \text{Long}) \\ - (r \cos \delta \sin \alpha) \sin (\text{GHA } T + 90 - \text{Long})$$

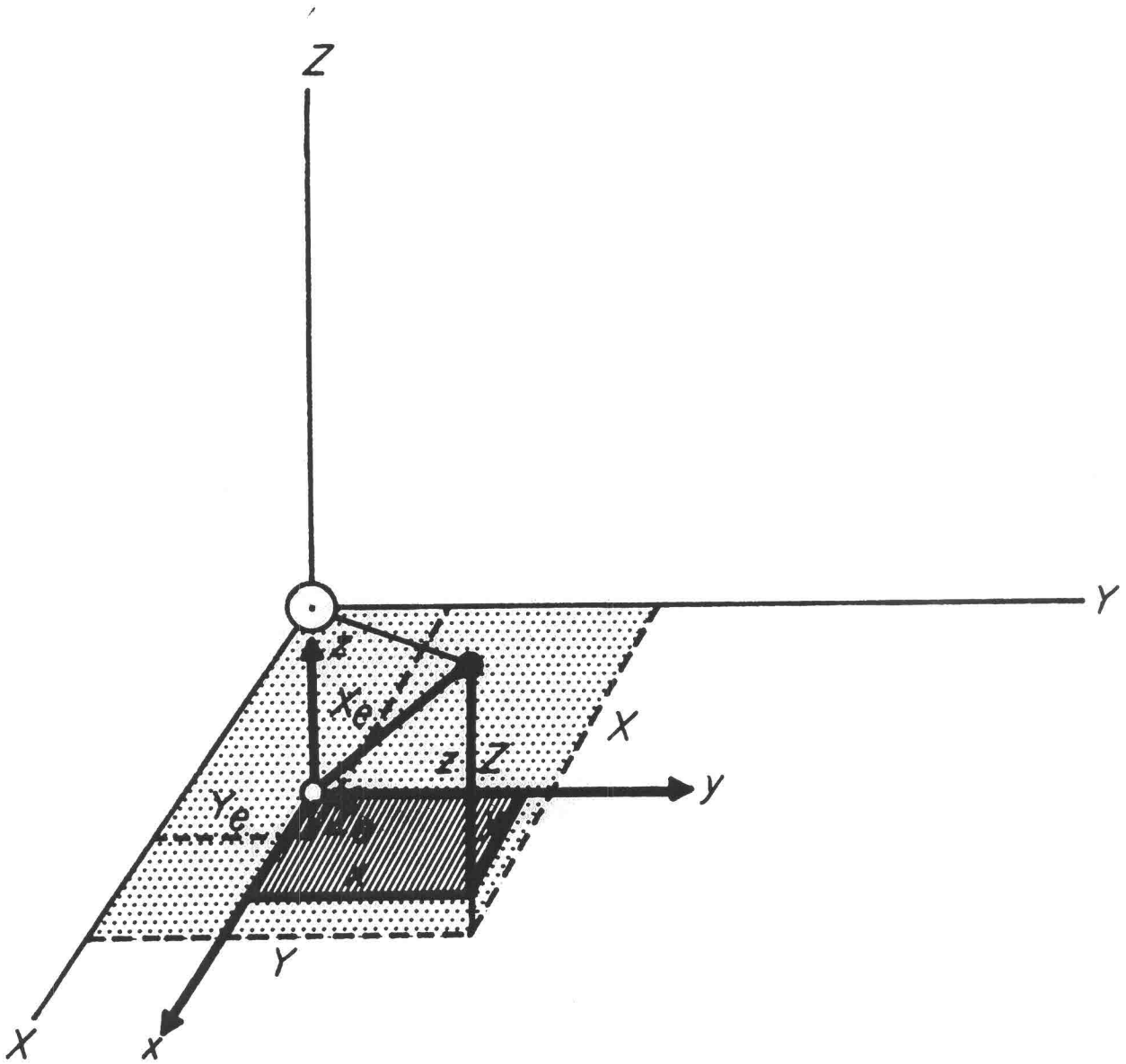


Figure 507.—Transformation from Heliocentric to Geocentric Ecliptic Coordinate System.

$$Z_e = (r \cos \delta \cos \alpha) \sin(90 - L) \sin(\text{GHA } T + 90 - \text{Long})$$

$$- (r \cos \delta \sin \alpha) \sin(90 - L) \cos(\text{GHA } T + 90 - \text{Long})$$

$$+ (r \sin \delta) \cos(90 - L).$$

$$Az_e \text{ (Azimuth)} = \tan^{-1} \frac{W_e}{N_e}.$$

It follows that

$$r = r \text{ (naturally)}$$

$$H_e \text{ (Altitude)} = \tan^{-1} \left[ \frac{Z_e}{\sqrt{N_e^2 + W_e^2}} \right]$$

#### 510. TRANSFORMATION FROM TOPOCENTRIC HORIZON TO GEOCENTRIC HORIZON

Altitude angle  $H$  and range  $\rho$  measured from the observer's position will be different than the geocentric horizon coordinates  $H_e$  and  $r$  because of a translation in the  $Z$  direction as given by the following relations:

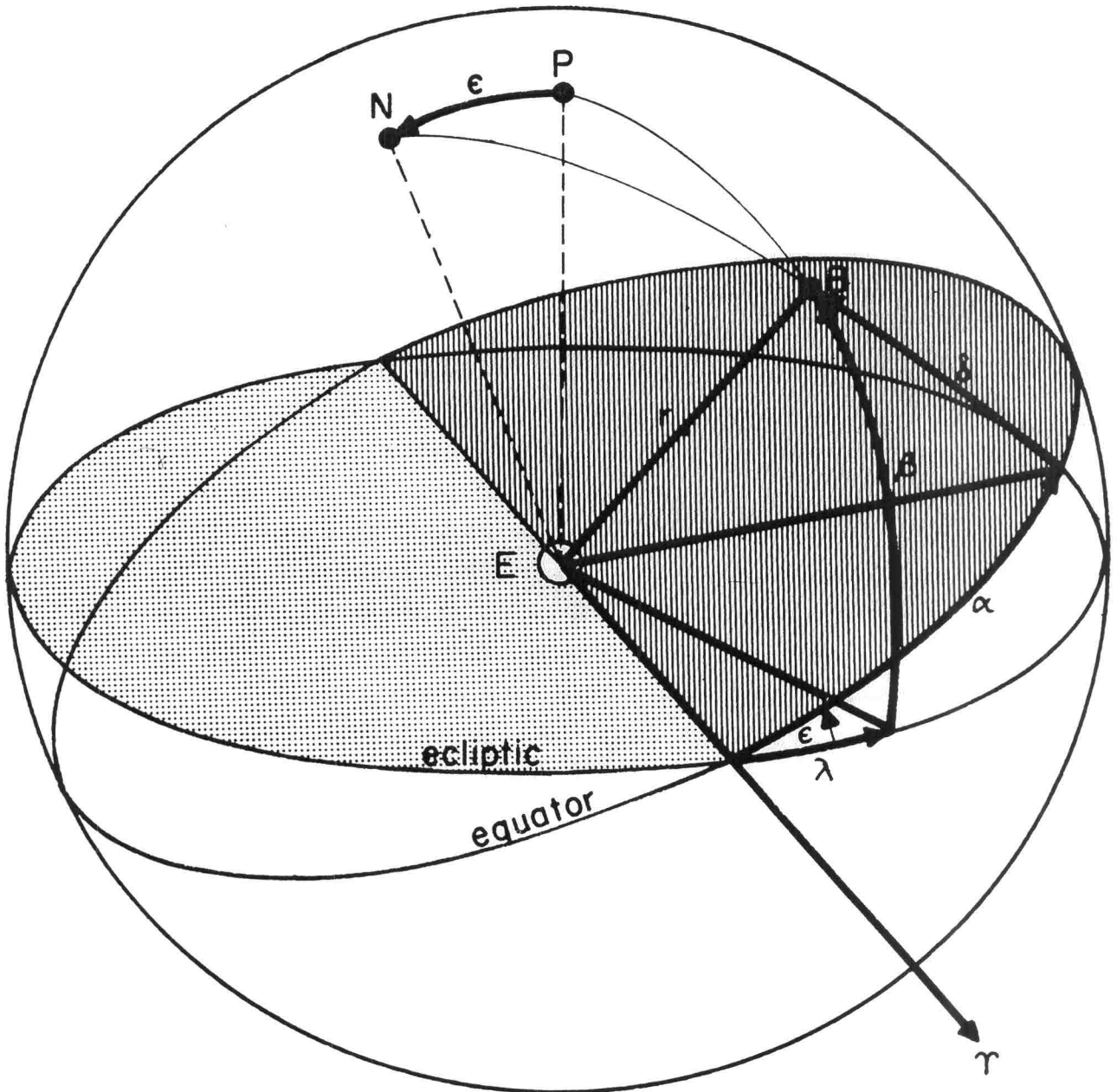


Figure 508.—Transformation from Geocentric Ecliptic to Geocentric Equatorial.

$$N_e = N = \rho \cos H \cos Az$$

$$W_e = W = \rho \cos H \sin Az$$

$$Z_e = Z + r_e = \rho \sin H + r_e$$

$$r = \sqrt{N_e^2 + W_e^2 + Z_e^2}$$

$$H_e = \tan^{-1} \left[ \frac{Z_e}{\sqrt{N_e^2 + W_e^2}} \right]$$

where  $r_e$  is the radius of the Earth at the observer's location. Then

$$Az_e = Az = \tan^{-1} \frac{W_e}{N_e}.$$



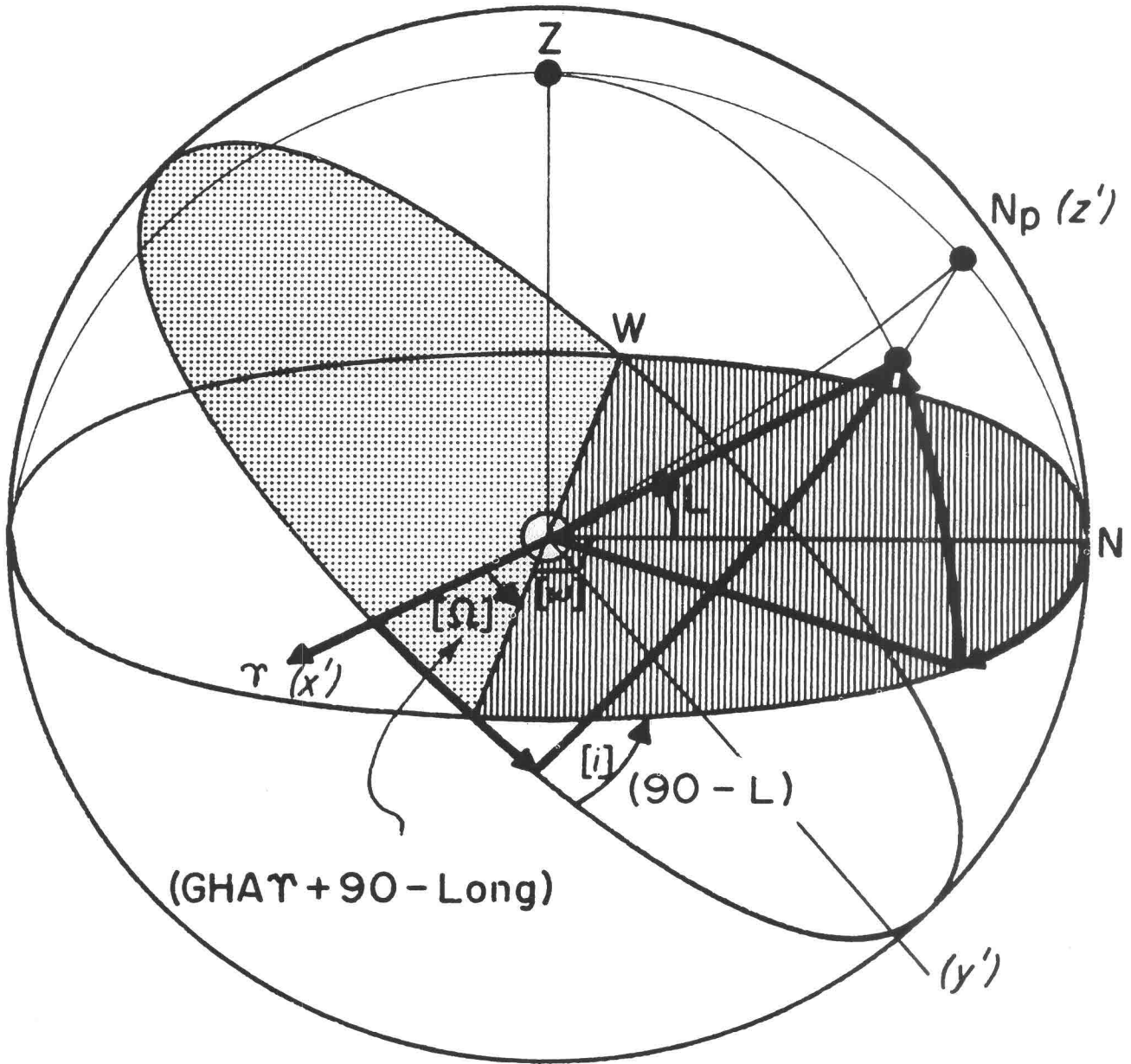


Figure 509.—Transformation from Geocentric Equatorial to Geocentric Horizon Coordinate System.

### 511. PARALLAX

Angle  $p$  in figures 510, 511A, and 511B is called parallax and is the difference in direction of a heavenly body as seen by an observer and as seen from the origin of a reference system (the center of the Earth).

It can also be defined as the angle at the body made by lines drawn from the body to the observer and to the center of the Earth. This last definition defines geocentric parallax. The

geocentric parallax is accounted for by a transformation of topocentric coordinates into geocentric coordinates. In figure 511B the parallax  $p$  of body  $B$  is the angle  $OBC$  and

$$OBC = AOB = ZOB - ZCB.$$

From the law of sines and  $\triangle OCB$

$$\frac{OC}{CB} = \frac{\sin OBC}{\sin ZOB}$$

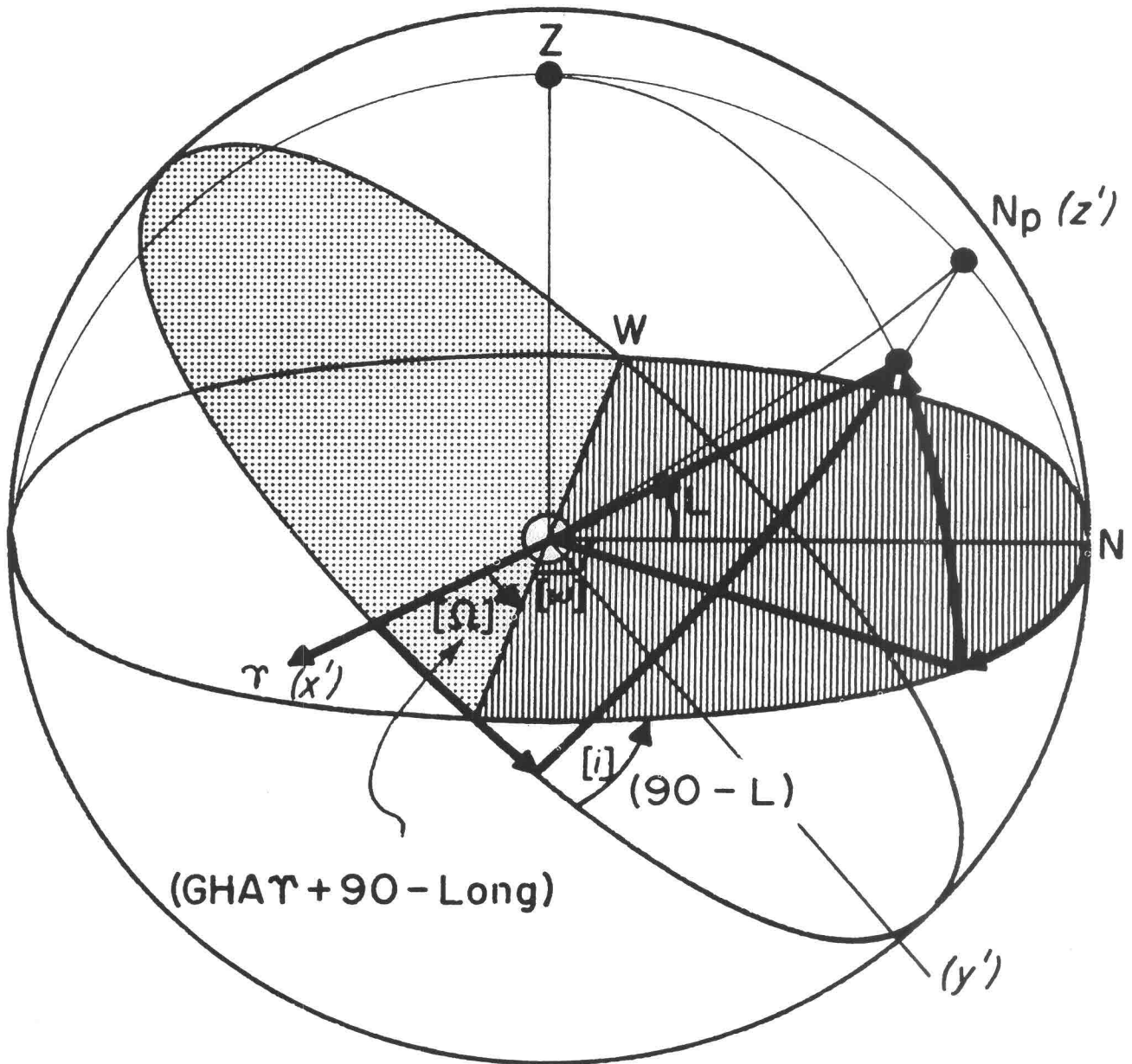


Figure 509.—Transformation from Geocentric Equatorial to Geocentric Horizon Coordinate System.

### 511. PARALLAX

Angle  $p$  in figures 510, 511A, and 511B is called parallax and is the difference in direction of a heavenly body as seen by an observer and as seen from the origin of a reference system (the center of the Earth).

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From the law of sines and  $\triangle OCB$

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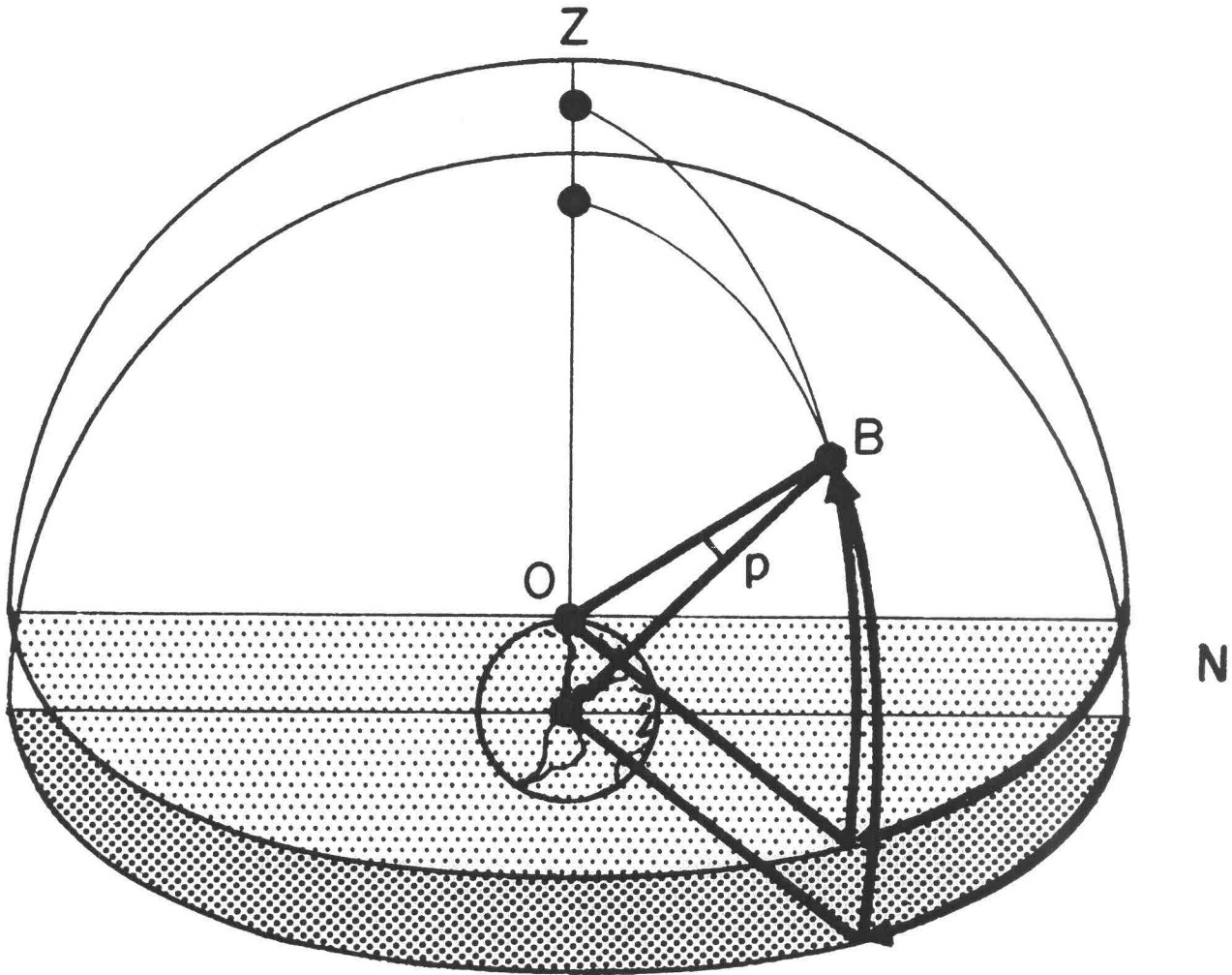


Figure 510.—Transformation from Topocentric Horizon to Geocentric Horizon Coordinate System.

or 
$$\frac{r_e}{r} = \frac{\sin p}{\sin ZOB}$$

whence 
$$p = \sin^{-1} \left[ \frac{r_e}{r} \sin (ZOB) \right]$$

By knowing parallax  $p$  we can find the distance  $\rho$  or  $r$  and vice versa.

## 512. SUMMARY

These coordinate systems and the transformations from one to another have a variety

of established uses, and are often necessary in the reduction of astronomical data to locate objects in the heavens. There is no single coordinate system which is best suited to all applications in space position determination.

The material is included here basically for reference purposes, and to illustrate the complexity of the problem of coordinate transformations. In the chapter on dead reckoning in space a method of predicting a body's position will be discussed which will require only one transformation, and that by graphical methods.

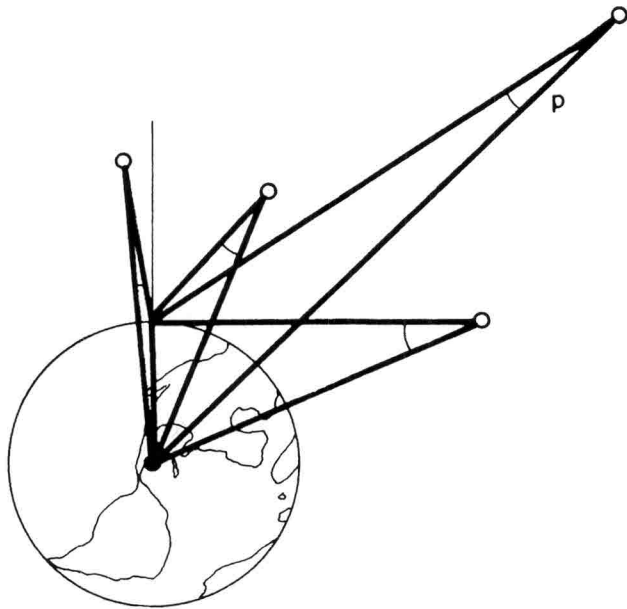


Figure 511A.—Geocentric Parallax.

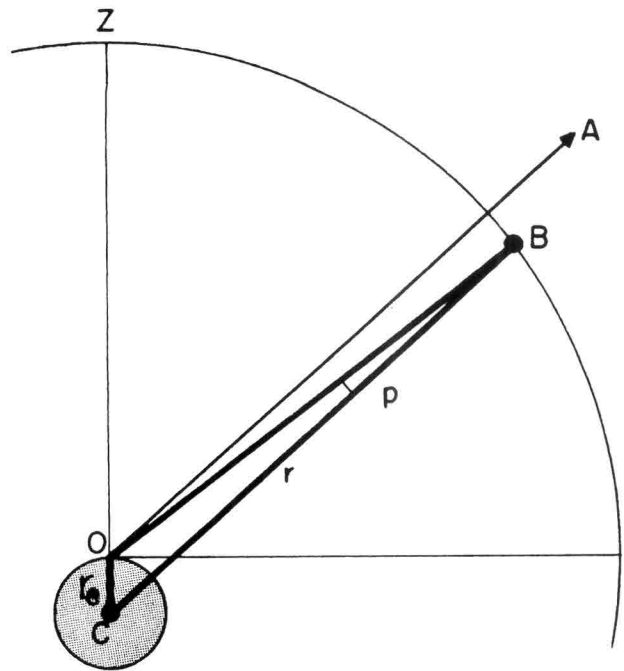


Figure 511B.—Parallax.

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2. Goldstein, Herbert. Classical Mechanics. Reading, Massachusetts, Addison-Wesley, 1959.