Fundamentals of Rendering - Reflectance Functions

Chapter 9 of "Physically Based Rendering"
by Pharr&Humphreys

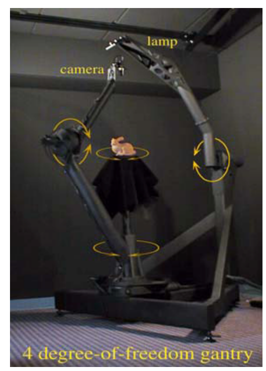
Chapter 9

9.0	Terms, etc.
9.1	PBRT Interface
9.2	Specular reflection and transmission Read about Snell's law and Fresnel reflection; we'll cover this after covering reflectance integrals
9.3-9.6	Specific models of reflection: Lambertian, microfacts, Lafortune, and Fresnel effects

Surface Reflectance

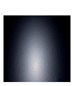
- Measured data
  - Gonioreflectometer (See the Cornell Lab)
- Phenomenological models
  - Intuitive parameters
  - Most of graphics
- Simulation
  - Know composition of some materials
  - simulate complicated reflection from simple basis
- Physical (wave) optics
  - Using Maxwell's equations
  - Computationally expensive
- Geometric optics
  - Use of geometric surface properties

Gonioreflectometer



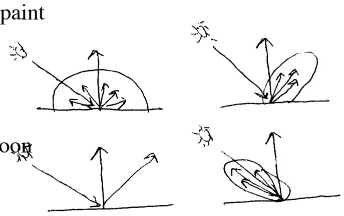
Surface Reflectance


Diffuse
- Scatter light equally in all directions
- E.g. dull chalkboards, matte paint


Glossy specular
- Preferred set of direction around reflected direction
- E.g. plastic, high-gloss paint

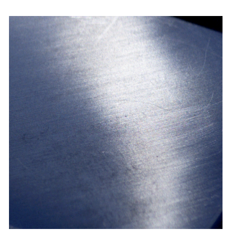

Perfect specular
- E.g. mirror, glass

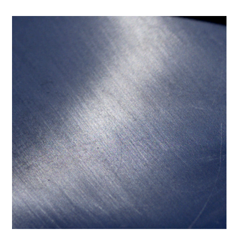
Retro-reflective
- E.g. velvet or earth's moon



Surface Reflectance

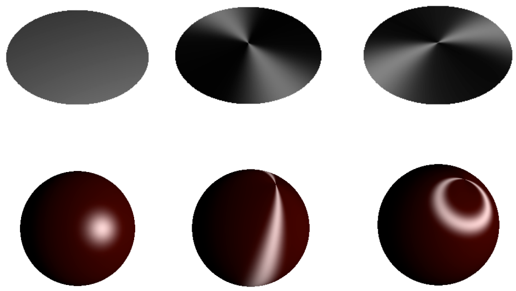
- Isotropic vs. anisotropic
  - If you turn an object around a point -> does the shading change?





Surface Reflectance

Phong (isotropic) | Banks (anisotropic) | Banks (anisotropic)



Surface Properties

- Reflected radiance is proportional to incoming flux and to irradiance (incident power per unit area).

$$dL_o(p,\omega_o) \propto dE(p,\omega_i)$$

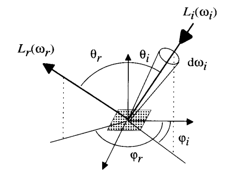


Figure 2.9: *Bidirectional reflection distribution function.*

The BSDF

- Bidirectional Scattering Distribution Function: $f(p,\omega_o,\omega_i)$
- Measures portion of incident irradiance (E_i) that is reflected as radiance (L_o)

$$f(p,\omega_o,\omega_i) = \frac{dL_o(p,\omega_o)}{dE(p,\omega_i)}$$

- Or the ratio between incident radiance (L_i) and reflected radiance (L_o)

$$f(p,\omega_o,\omega_i) = \frac{dL_o(p,\omega_o)}{dE(p,\omega_i)} = \frac{dL_o(p,\omega_o)}{L_i(p,\omega_i)\cos\theta_i d\omega_i}$$

The BRDF and the BTDF

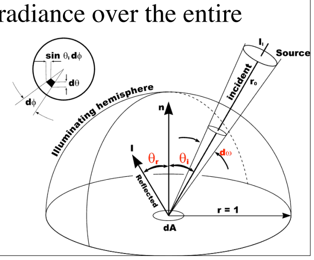
- Bidirectional Reflectance Distribution Function (BRDF)
  - Describes distribution of reflected light
- Bidirectional Transmittance Distribution Function (BTDF)
  - Describes distribution of transmitted light
- BSDF = BRDF + BTDF

Illumination via the BxDF

- The Reflectance Equation

$$L_o(p,\omega_o) = \int_{S^2} f(p,\omega_o,\omega_i) L_i(p,\omega_i) |\cos\theta_i| d\omega_i$$

- The reflected radiance is
  - the sum of the incident radiance over the entire (hemi)sphere
  - foreshortened
  - scaled by the BxDF



Parameterizations

- 6-D BRDF $f_r(p, \omega_o, \omega_i)$
  - Incident direction L_i
  - Reflected/Outgoing direction L_o
  - Surface position p: textured BxDF
- 4-D BRDF $f_r(\omega_o, \omega_i)$
  - Homogeneous material
  - Anisotropic, depends on incoming azimuth
  - e.g. hair, brushed metal, ornaments

Parameterizations

- 3-D BRDF $f_r(\theta_o, \theta_i, \phi_o - \phi_i)$
  - – Isotropic, independent of incoming azimuth
  - – e.g. Phong highlight
- 1-D BRDF $f_r(\theta_i)$
  - – Perfectly diffuse
  - – e.g. Lambertian

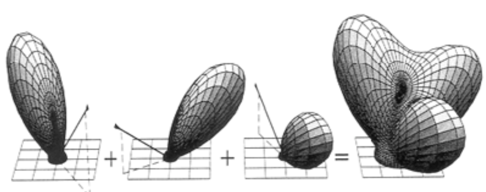
BxDF Property 0

- Ranges from 0 to ∞ (strictly positive)
- Infinite when radiance distribution from single incident ray

$$f_r(p, \omega_o, \omega_i) = \frac{dL_o(p, \omega_o)}{dE(p, \omega_i)} = \frac{dL_o(p, \omega_o)}{L_i(p, \omega_i)\cos\theta_i d\omega_i}$$

BRDF Property 1

- Linearity of functions



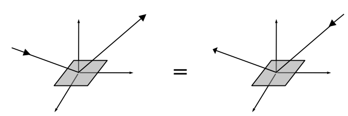
Sillion, Arvo, Westin, Greenberg

BRDF Property 2

Helmholtz Reciprocity

$$f_r(\omega_o, \omega_i) = f_r(\omega_i, \omega_o)$$

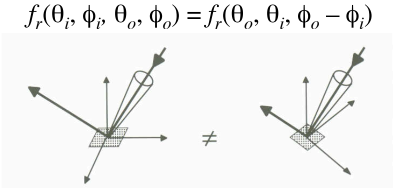
- – Materials are not a one-way street
- – Incoming to outgoing pathway same as outgoing to incoming pathway



BRDF Property 3

- Isotropic vs. anisotropic

$$f_r(\theta_i, \phi_i, \theta_o, \phi_o) = f_r(\theta_o, \theta_i, \phi_o - \phi_i)$$



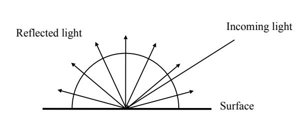
- Reciprocity and isotropy

$$f_r(\theta_o, \theta_i, \phi_o - \phi_i) = f_r(\theta_i, \theta_o, \phi_i - \phi_o) = f_r(\theta_o, \theta_i, |\phi_o - \phi_i|)$$

$$f_r(\omega_o, \omega_i, \phi_o - \phi_i) = f_r(\omega_i, \omega_o, \phi_i - \phi_o) = f_r(\omega_o, \omega_i, |\phi_o - \phi_i|)$$

BRDF Property 4

- Conservation of Energy
  - – Materials must not add energy (except for lights)
  - – Materials must absorb some amount of energy
  - – When integrated, must add to less than one



Reflectance

- Reflectance ratio of reflected to incident flux

$$\rho(p) = \frac{d\Phi_o(p)}{d\Phi_i(p)} = \frac{\int_{\Omega_o} L_o(p,\omega_o)\cos\theta_o d\omega_o}{\int_{\Omega_i} L_i(p,\omega_i)\cos\theta_i d\omega_i}$$

$$= \frac{\int_{\Omega_o}\int_{\Omega_i} f(p,\omega_i,\omega_o) L_i(p,\omega_i)\cos\theta_i\cos\theta_o d\omega_i d\omega_o}{\int_{\Omega_i} L_i(p,\omega_i)\cos\theta_i d\omega_i}$$

Reflectance between 0 and 1

Reflectance

- If incident distribution is uniform and isotropic

$$\rho(p) = \frac{\int_{\Omega_o}\int_{\Omega_i} f(p,\omega_i,\omega_o) L_i(p,\omega_i)\cos\theta_i\cos\theta_o d\omega_i d\omega_o}{\int_{\Omega_i} L_i(p,\omega_i)\cos\theta_i d\omega_i}$$

$$= \frac{\int_{\Omega_o}\int_{\Omega_i} f(p,\omega_i,\omega_o)\cos\theta_i\cos\theta_o d\omega_i d\omega_o}{\int_{\Omega_i}\cos\theta_i d\omega_i}$$

Relates reflectance to the BRDF

Reflectance

- Hemispherical-directional reflectance
  - Reflection in a given direction due to constant illumination over a hemisphere
  - Total reflection over hemisphere due to light from a given direction (reciprocity)
  - Also called albedo - incoming photon is reflected with probability less than one

$$\rho_{hd}(p,\omega_o) = \int_{H^2(n)} f_r(p,\omega_o\omega_i)|\cos\theta_i| d\omega_i$$

Reflectance

- Hemispherical-hemispherical reflectance
  - Constant spectral value that gives the fraction of incident light reflected by a surface when the incident light is the same from all directions

$$\rho_{hh}(p) = \frac{1}{\pi}\int_{H^2(n)}\int_{H^2(n)} f_r(p,\omega_o\omega_i)|\cos\theta_o\cos\theta_i| d\omega_o d\omega_i$$

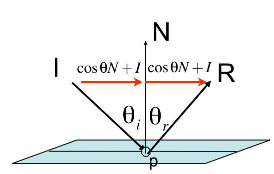
Representations

- Tabulated BRDF's
  - Require dense sampling and interpolation scheme
- Factorization
  - Into two 2D functions for data reduction (often after reparameterization)
- Basis Functions (Spherical Harmonics)
  - Loss of quality for high frequencies
- Analytical Models
  - Rough approximation only
  - Very compact
  - Most often represented as parametric equation (Phong, Cook-Torrance, etc.)

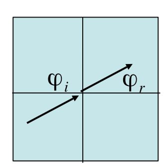
Law of Reflection

- Angle of reflectance = angle of incidence

$$R = -I + (\cos\theta N + I) + (\cos\theta N + I)$$
$$R = I - 2(I \cdot N)N$$
$$\omega_r = R(\omega_i, N)$$

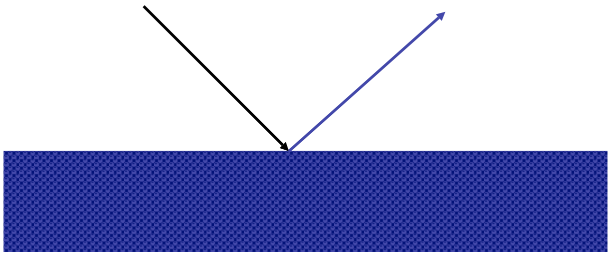


$$\theta_r = \theta_i$$



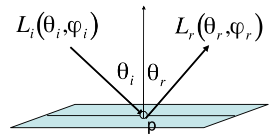
$$\varphi_r = (\varphi_i + \pi) \bmod 2\pi$$

Polished Metal



Ideal Reflection (Mirror)

- BRDF cast as a delta function

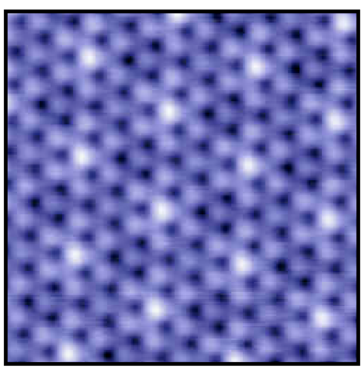


$$\boxed{L_{r,m}(\theta_o,\varphi_o) = L_i(\theta_r,\varphi_r \pm \pi)}$$

$$\boxed{f_{r,m}(\theta_i,\varphi_i,\theta_o,\varphi_o) = \frac{\delta(\cos\theta_i - \cos\theta_r)}{\cos\theta_i}\delta(\varphi_i - \varphi_r \pm \pi)}$$

$$L_{r,m}(\theta_o,\varphi_o) = \int f_{r,m}(\theta_i,\varphi_i,\theta_o,\varphi_o) L_i(\theta_i,\varphi_i)\cos\theta_i d\cos\theta_i d\varphi_i$$
$$= \int \frac{\delta(\cos\theta_i - \cos\theta_r)}{\cos\theta_i}\delta(\varphi_i - \varphi_r \pm \pi) L_i(\theta_i,\varphi_i)\cos\theta_i d\cos\theta_i d\varphi_i$$
$$= L_i(\theta_r,\varphi_r \pm \pi)$$

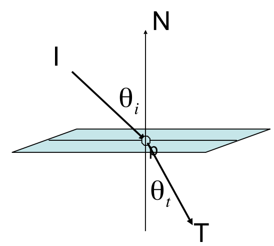
Mirror Surface

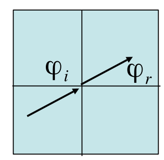


Snell's Law

- η_i, η_t indices of refraction (ratio of speed of light in vacuum to the speed of light i the medium)

$$\eta_i \sin\theta_i = \eta_t \sin\theta_t$$
$$\eta_i N \times I = \eta_t N \times T$$
$$\omega_t = T(\omega_i, N)$$





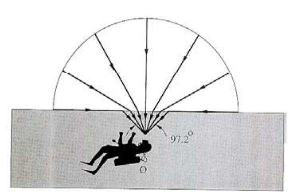
$$\varphi_r = (\varphi_i + \pi) \bmod 2\pi$$

Law of Refraction

- Starting at Snell's law: $\boxed{\frac{\eta_i}{\eta_t} N \times I = N \times T \quad N \times (T - \mu I) = 0}$
- We conclude that $T = \mu I + \gamma N$
- Assuming a normalized T: $\boxed{T^2 = 1 = \mu^2 + \gamma^2 + 2\mu\gamma(I \cdot N)}$
- Solving this quadratic equation: $\gamma = -\mu(I \cdot N) \pm \sqrt{1 - \mu^2(1 - (I \cdot N)^2)}$
- Leads to the total reflection condition: $\boxed{1 - \mu^2(1 - (I \cdot N)^2) \geq 0}$

Optical Manhole

- Total Internal Reflection
- For water $n_w = 4/3$



Livingston and Lynch

Fresnel Reflection

- At top layer interface
  - Some light is reflected,
  - Remainder is transmitted through
- Simple ray-tracers: just given as a constant
- Physically based - depends on
  - incident angle
  - Polarization of light
  - wavelength
- Solution of Maxwell's equations to smooth surfaces
- Dielectrics vs. conductors

Fresnel Reflection - Dielectrics

- Objects that don't conduct electricity (e.g. glass)
- Fresnel term F for a dielectric is proportion of reflection (e.g. glass, plastic)
  - grazing angles: 100% reflected (see the material well!)
  - normal angles: 5% reflected (almost mirror-like)

Fresnel Reflection - Dielectrics

- Polarized light:

$$r_{\parallel} = \frac{\eta_t \cos\theta_i - \eta_i \cos\theta_t}{\eta_t \cos\theta_i + \eta_i \cos\theta_t}$$

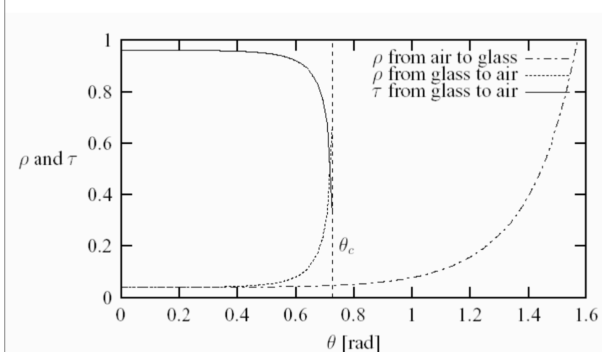
$$r_{\perp} = \frac{\eta_i \cos\theta_i - \eta_t \cos\theta_t}{\eta_i \cos\theta_i + \eta_t \cos\theta_t}$$

- Where ω_t is computed according to Snell's law
- Unpolarized light:

$$F_r(\omega_i) = \frac{1}{2}\left(r_{\parallel}^2 + r_{\perp}^2\right)$$

$$F_t(\omega_i) = \left(1 - F_r(\omega_i)\right)$$

Fresnel Reflection - Dielectrics



Fresnel Reflection - Conductor

- Typically metals
- No transmission
- Absorption coefficient k

Fresnel Reflection - Conductor

- Polarized light:

$$r_{\parallel}^2 = \frac{(\eta^2 + k^2)\cos^2\theta_i - 2\eta\cos\theta_i + 1}{(\eta^2 + k^2)\cos^2\theta_i + 2\eta\cos\theta_i + 1}$$

$$r_{\perp}^2 = \frac{(\eta^2 + k^2) - 2\eta\cos\theta_i + \cos^2\theta_i}{(\eta^2 + k^2) + 2\eta\cos\theta_i + \cos^2\theta_i}$$

- Unpolarized light:

$$F_r(\omega_i) = \frac{1}{2}\left(r_{\parallel}^2 + r_{\perp}^2\right)$$

Fresnel Reflection - Conductor

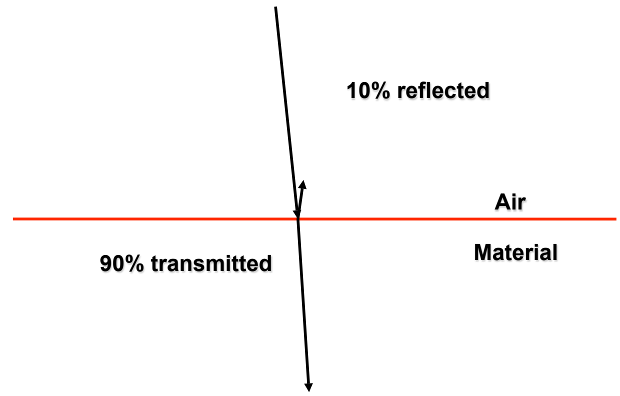
- How to determine k or η?
- Measure F_r for θ_i=0 (normal angle)
- 1. Assume k = 0

$$r_\perp^2 = r_\parallel^2 = \frac{(\eta - 1)^2}{(\eta + 1)^2} \qquad \eta = \frac{1 + \sqrt{F_r(0)}}{1 - \sqrt{F_r(0)}}$$

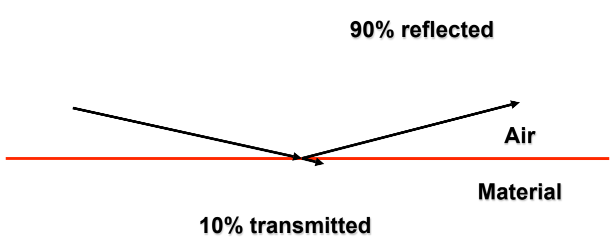
- 2. Assume η = 1

$$r_\perp^2 = r_\parallel^2 = \frac{k^2}{k^2 + 4} \qquad k = 2\sqrt{\frac{F_r(0)}{1 - F_r(0)}}$$

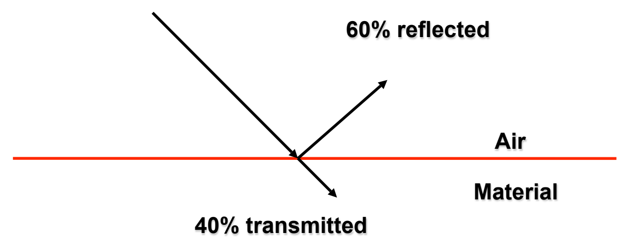
Fresnel Normal (Dielectric)



Fresnel Grazing (Dielectric)

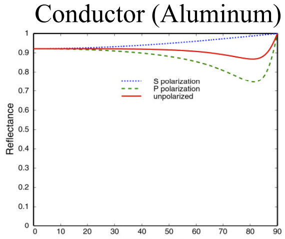


Fresnel Mid (Dielectric)

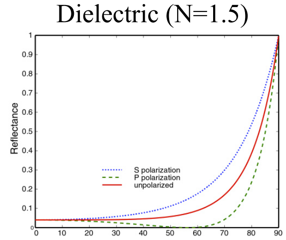


Fresnel Reflection

Conductor (Aluminum)



Dielectric (N=1.5)

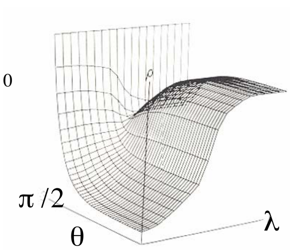


Schlick Approximation:

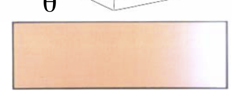
$$F(\theta) = F(0) + (1 - F(0))(1 - \cos\theta)^5$$

Fresnel Reflection

- Example - Copper
  - color shift as θ goes from 0 to $\pi/2$
  - at grazing, specular highlight is color of light



Measured Reflectance



Approximated Reflectance

Ideal Specular - Summary

- Reflection:

$$f_r(p,\omega_i,\omega_o) = F_r(\omega_i)\frac{\delta(\omega_i - R(\omega_o,N))}{|\cos\theta_i|}$$

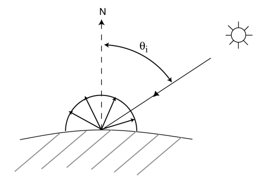
- Transmission:

$$f_t(p,\omega_i,\omega_o) = \frac{\eta_o^2}{\eta_i^2}(1 - F_r(\omega_i))\frac{\delta(\omega_o - T(\omega_i,N))}{|\cos\theta_i|}$$

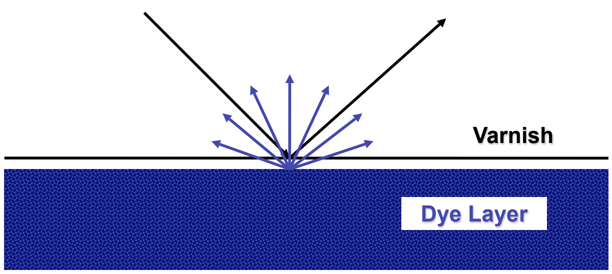
Ideal Diffuse Reflection

- Uniform
  - Sends equal amounts of light in all directions
  - Amount depends on angle of incidence
- Perfect
  - all incoming light reflected
  - no absorption

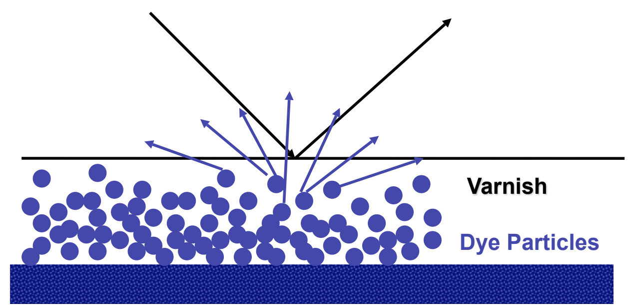
$$f_r(\omega_i,\omega_o) \propto k_d$$



Layered Surface



Layered Surface Larger



Ideal Diffuse Reflection

$$L_{o,d}(\omega_o) = \int_\Omega f_{r,d}(\omega_i,\omega_r) L_i(\omega_i)\cos\theta_i d\omega_i$$

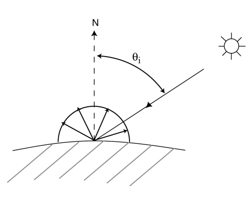
$$= f_{r,d}\int_\Omega L_i(\omega_i)\cos\theta_i d\omega_i$$

$$= f_{r,d}E$$

$$M = \int_\Omega L_{o,d}(\omega_o)\cos\theta_o d\omega_o$$

$$= L_{o,d}\int_\Omega \cos\theta_o d\omega_o$$

$$= L_{o,d}\pi$$



$$\rho_d = \frac{M}{E} = \frac{L_{o,d}\pi}{E} = \frac{f_{r,d}E\pi}{E} = f_{r,d}\pi$$

$$f_{r,d} = \frac{\rho_d}{\pi}$$

Lamberts Cosine Law: $M = \rho_d E = \rho_d E_s \cos\theta_s$

Diffuse

- Helmholtz reciprocity?
- Energy preserving?

$$\rho_d \leq 1$$

$$f_{r,d} = \frac{\rho_d}{\pi} \leq \frac{1}{\pi}$$

Reflectance Models

- Ideal
  - Diffuse
  - Specular
- Ad-hoc: Phong
  - Classical / Blinn
  - Modified
  - Ward
  - Lafortune
- Microfacets (Physically-based)
  - Torrance-Sparrow (Cook-Torrance)
  - Ashkhimin

Classical Phong Model

$$L_o(p,\omega_o) = (k_d(N \cdot \omega_i) + k_d(R(\omega_o,N) \cdot \omega_i)^e)L_i(p,\omega_i)$$

- Where $0<k_d$, $k_s<1$ and $e>0$
- Cast as a BRDF:

$$f_r(p,\omega_i,\omega_o) = k_d + k_s \frac{(R(\omega_o,N) \cdot \omega_i)^e}{(N \cdot \omega_i)}$$

- Not reciprocal
- Not energy-preserving
- Specifically, too reflective at glancing angles, but not specular enough
- But cosine lobe itself symmetrical in ω_i and ω_o

Blinn-Phong

- Like classical Phong, but based on half-way vector

$$f_r(p,\omega_i,\omega_o) = k_d + k_s \frac{(H(\omega_o,\omega_i) \cdot N)^e}{(N \cdot \omega_i)}$$

$$\omega_h = H(\omega_o,\omega_i) = norm(\omega_o + \omega_i)$$

- Implemented in OpenGL
- Not reciprocal
- Not energy-preserving
- Specifically, too reflective at glancing angles, but not specular enough
- But cosine lobe itself symmetrical in ω_i and ω_o

Modified Phong

$$f_r(p,\omega_i,\omega_o) = \frac{k_d}{\pi} + \frac{k_s(e+2)}{2\pi}(R(\omega_o,N) \cdot \omega_i)^e$$

- For energy conservation: $k_d + k_s < 1$ (sufficient, not necessary)
- Peak gets higher as it gets sharper, but same total reflectivity

Ward-Phong

- Based on Gaussians

$$f_r(p,\omega_i,\omega_o) = \frac{k_d}{\pi} + \frac{k_s}{\sqrt{\cos\theta_i \cos\theta_o}} \frac{\exp\left(-\tan^2\omega_h / \alpha^2\right)}{4\pi\alpha^2}$$

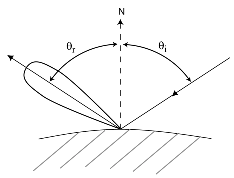
- α: surface roughness, or blur in specular component.

Lafortune Model

- Phong cosine lobes symmetrical (reciprocal), easy to compute
- Add more lobes in order to match with measured BRDF
- How to generalize to anisotropic BRDFs?
- weight dot product:

$$f_r(p,\omega_i,\omega_o) = \frac{k_d}{\pi} + \sum_{i=1}^{nlobes} (\omega_o R_i \omega_i)^{e_i}$$

Glossy



Physically-based Models

- Some basic principles common to many models:
  - Fresnel effect
  - Surface self-shadowing
  - Microfacets
- To really model well how surfaces reflect light, need to eventually move beyond BRDF
- Different physical models required for different kinds of materials
- Some kinds of materials don't have good models
- Remember that BRDF makes approximation of completely local surface reflectance!

Cook-Torrance Model

- Based in part on the earlier Torrance-Sparrow model
- Neglects multiple scattering

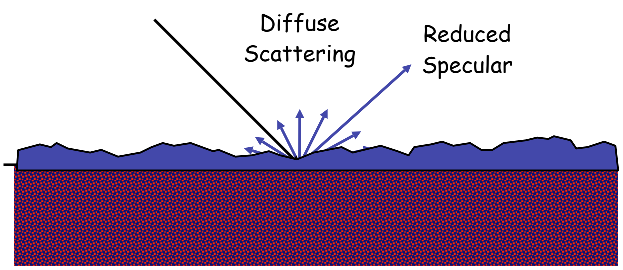
$$f_r(p,\omega_i,\omega_o) = \frac{F_r(\omega_h)D(\omega_h)G(\omega_o,\omega_i)}{4\cos\theta_i\cos\theta_o}$$

- D - Microfacet Distribution Function
  - how many "cracks" do we have that point in our (viewing) direction?
- G - Geometrical Attenuation Factor
  - light gets obscured by other "bumps"
- F - Fresnel Term

Microfacet Models

- Microscopically rough surface
- Specular facets oriented randomly
- measure of scattering due to variation in angle of microfacets
- a statistic approximation, I.e. need a statistic distribution function

Rough Surface



Microfacet Distribution Function D

- Blinn

$$D(\omega_h) = ce^{-(\frac{\omega_h \cdot N}{m})^2}$$

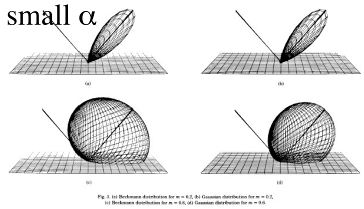
- where m is the root mean square slope of the facets (as an angle)
- Blinn says c is a arbitrary constant
- Really should be chosen to normalize BRDF. . .

Microfacet Distribution Function D

- Beckmann (most effective)

$$D(\omega_h) = \frac{1}{m^2 \cos^4 \alpha} e^{-(\frac{\tan\alpha}{m})^2}$$

- Represents a distribution of slopes
- But $\alpha = \tan\alpha$ for small α



Multiscale Distribution Function

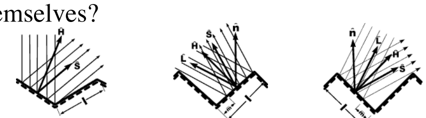
- May want to model multiple scales of roughness: $D(\omega_h) = \sum_j w_j D_j(\omega_h)$

$$\sum_j w_j = 1$$

- Bumps on bumps …

Self-Shadowing (V-Groove Model)

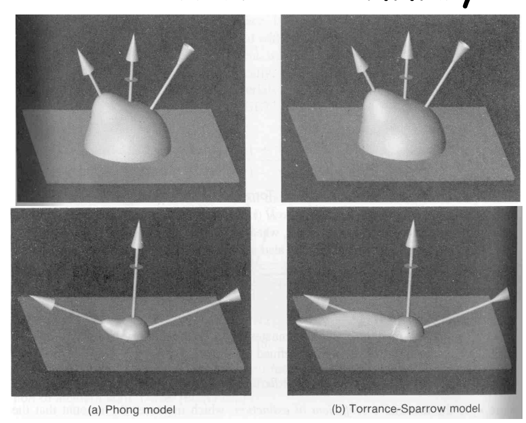
- Geometrical Attenuation Factor G
  - how much are the "cracks" obstructing themselves?



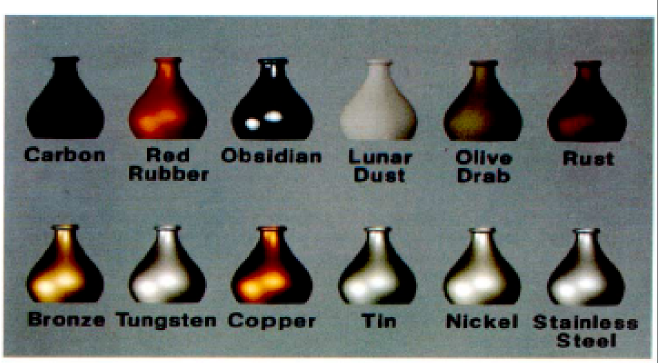
No interference   shadowing   masking

$$G = min[1, \frac{2(N\cdot\omega_h)(N\cdot\omega_o)}{(\omega_o\cdot\omega_h)}, \frac{2(N\cdot\omega_h)(N\cdot\omega_i)}{(\omega_o\cdot\omega_h)}]$$

Cook-Torrance - Summary



Cook-Torrance - Summary



Ashkhimin Model

- Modern Phong
- Phenomological, but:
  - Physically plausible
  - Anisotropic
- Good for both Monte-Carlo and HW implementation

Ashkhimin Model

- Weighted sum of diffuse and specular part:

$$f_r(p,\omega_i,\omega_o) = k_d(1-k_s)f_d(p,\omega_i,\omega_o) + k_s f_s(p,\omega_i,\omega_o)$$

- Dependence of diffuse weight on k_s decreases diffuse reflectance when specular reflectance is large
- Specular part f_s not an impulse, really just glossy
- Diffuse part f_d not constant: energy specularly reflected cannot be diffusely reflected
- For metals, $f_d = 0$

Ashkhimin Model

- k_s: Spectrum or color of specular reflectance at normal incidence.
- k_d: Spectrum or color of diffuse reflectance (away from the specular peak).
- q_u, q_v: Exponents to control shape of specular peak.
  - Similar effects to Blinn-Phong model
  - If an isotropic model is desired, use single value q
  - A larger value gives a sharper peak
  - Anisotropic model requires two tangent vectors u and v
  - The value q_u controls sharpness in the direction of u
  - The value q_v controls sharpness in the direction of v

Ashkhimin Model

- ϕ is the angle between u and ωh

$$D(\omega_h) = \sqrt{(q_u+1)(q_v+1)}(\omega_h \cdot N)^{(q_u \cos^2\phi + q_v \cos^2\phi)}$$

Ashkhimin Model

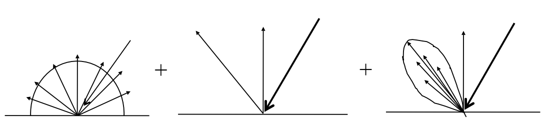
- Diffuse term given by:

$$f_d(p,\omega_i,\omega_o) = \frac{28}{23\pi}(1-(1-(\omega_o \cdot N))^5)(1-(1-(\omega_i \cdot N))^5)$$

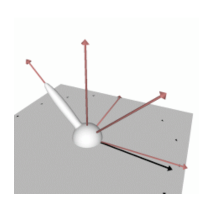
- Leading constant chosen to ensure energy conservation
- Form comes from Schlick approximation to Fresnel factor
- Diffuse reflection due to subsurface scattering: once in, once out

Complex BRDF

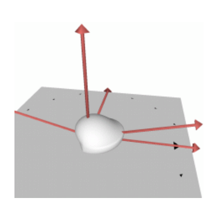
- Combination of the three.



BRDF illustrations



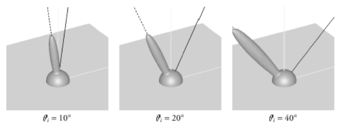
Phong Illumination



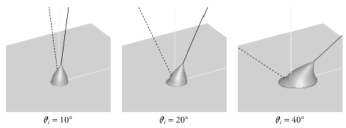
Oren-Nayar

BRDF illustrations

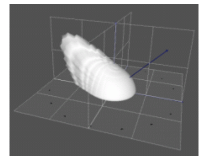
Cook-Torrance-Sparrow BRDF



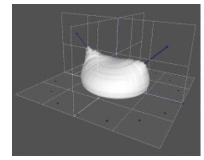
Hapke BRDF



BRDF illustrations

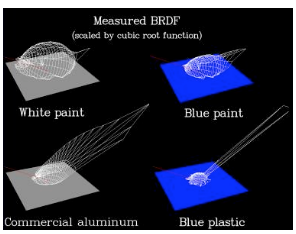


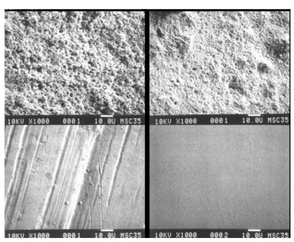
lumber



cement

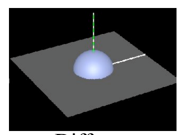
BRDF illustrations



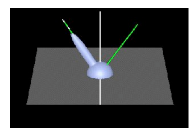


Surface microstructure

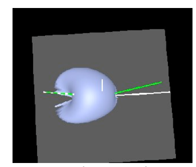
bv = Brdf Viewer



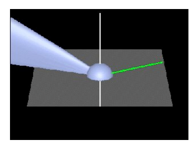
Diffuse



Torrance-Sparrow



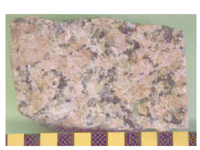
Anisotropic

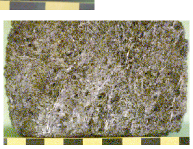


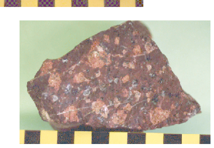
Szymon Rusinkiewicz
Princeton U.

BRDF cannot

Spatial variation of reflectance







BRDF cannot

Transparency and Translucency (depth)

Glass: transparent
Wax: translucent
BTDF

Opaque milk (rendered)

Translucent milk (rendered)

BSSRDF