

The Path to Equilibrium in Sequential and Simultaneous Games *

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June 2016

Abstract

We study in the laboratory three-, four- and six-player, dominance solvable games of complete information. We consider sequential and simultaneous versions of games that have the same equilibrium actions, and we use mousetracking to determine which payoffs subjects pay attention to. We find more equilibrium choices in the sequential version than in the simultaneous version, especially in the more complex treatments (six players and a random order display). Two intuitive attentional variables are extremely predictive of equilibrium behavior under both timings: looking at all the payoffs necessary to compute the Nash equilibrium and looking at payoffs in the order predicted by the sequential elimination of dominated strategies. Finally, the sequence of lookups reveals significantly different cognitive processes across timings, even among subjects who play the equilibrium strategy. Subjects have a harder time finding the player with a dominant strategy in the simultaneous timing than in the sequential timing. However, conditional on finding such player, the unraveling logic of iterated elimination of dominated strategies is performed (equally) fast and efficiently under both timings.

Keywords: laboratory experiment, sequential and simultaneous games, cognition, decision process, mousetracking.

JEL codes: C72, C92.

*Mousetracking was developed by Chris Crabbe as an extension to the Multistage program. We are grateful for his speed and enthusiasm. We thank the seminar audiences at UC Santa Barbara, USC and Caltech for comments. Send correspondence to: <brocas@usc.edu>.

1 Introduction

Our ability to strategize is highly contingent on the type of situations we face. Several studies (Camerer et al. (1993); Costa-Gomes et al. (2001)) have shown that decisions in games follow simple algorithms that implement a limited number of steps of reasoning. These theories build on the hypothesis that cognitive abilities are bounded. As the situation becomes more complex to evaluate, noticeable departures from theoretical predictions are observed. A striking example is timing. Individuals usually have a harder time making decisions when the order of play is simultaneous rather than sequential even when the equilibrium action are the same. This suggests that a simultaneous timing is perceived to be more complex than a sequential timing. It also indicates that the way information is processed varies across timings.

The idea that sequencing affects decision-making has been the object of research in a wide range of fields. Among others, it has been studied in computer science to determine whether vast amounts of information should be presented sequentially or simultaneously (Jacko and Salvendy (1996), Hochheiser and Shneiderman (2000)), in marketing to assess whether products of a new line should be introduced together or one at a time (Read et al. (2001), Mogilner et al. (2012)) and in criminology to compare the efficiency of sequential and simultaneous police lineups (Stebly et al. (2001), McQuiston et al. (2006)). Sequencing has this intuitive property of reducing the amount of information to consider in one batch and it is believed to ease the allocation of attentional resources (Just et al. (2001), Szameitat et al. (2002)). Little is known however about the underlying processes at play.

The goal of this paper is to study the differences in both *attention* and *decisions* between simultaneous and sequential formulations in the context of games. To this purpose, we conduct a controlled laboratory experiment where subjects play games that differ in the order of play but exhibit the same (unique) Nash equilibrium, and we track the information they attend to before making their decisions using the “mousetracking” method.

Isolating the effect of timing is key but non-trivial. The way we accomplish it is by constructing the following special class of t -player dominance solvable games. In our design, the payoff of each player depends on her action and the action of exactly one other player. Also, one (and only one) player has a dominant strategy, so that the unique Nash equilibrium can be deduced through iterated elimination of strictly dominated strategies. Importantly, in the sequential version of the game the player with a dominant strategy always moves last, and the payoff of a player depends on her action and the action of the player who moves *next*. As a result, observing the choice(s) of previous mover(s) does not provide any direct help in finding the equilibrium. In other words, independently of

the timing of the game, subjects need to identify the player with a dominant strategy and, from there, iteratively deduce the best response of the other players in order to find the Nash equilibrium. We also abstract from framing effects arising from normal-form vs. extensive-form representations. Instead, we provide the same formal representation: one matrix for each player that represents her payoff as a function of the actions of the two relevant players. We consider a baseline treatment and several additional treatments that are identical in essence but vary in complexity. In some games, more players are involved; in some other games, the presentation of the game is slightly altered to make more or less difficult to find the strategy that needs to be eliminated first. In all these games, the algorithm to find the equilibrium is exactly the same under the simultaneous and sequential timings. Finally, we obtain information about decision processes by hiding the payoffs in opaque cells. These are only revealed when the subject moves the computer mouse into the cell and clicks-and-holds the button down. By tracking the cells that are successively opened before making a decision, we obtain an indication of where attention is allocated and which reasoning is followed to make a decision. This information is key to understand differences in behavior in settings as ours, where none should be present.

We address three broad questions, with the second and third being the most novel ones. (1) Are choices different between the sequential and simultaneous versions of the games? Given that the games are equivalent and require the same algorithm to be solved, we should not observe any difference. If we do, this means that the timing of a game per se affects the way information is processed and choices are made. (2) Are decision processes different between equilibrium and non-equilibrium players and how does the timing interfere with those processes? If processes are different, then deviations from predicted behavior under both timings can be traced to the ability of the individual to understand the strategic implications of his and others' decisions. (3) Are decision processes conditional on equilibrium choices different between the simultaneous and the sequential timings? If Nash compliance differs across timings, it may be because one timing is cognitively more demanding than the other. Differences in cognitive difficulty can be assessed by determining whether subjects who play Nash exhibit different lookup patterns in sequential and simultaneous. Based on our experimental data, the answers to all three questions is 'yes', as we develop below.

First, equilibrium choice in the sequential timing is marginally higher than in the simultaneous timing for the baseline treatment and significantly higher for the more complex treatments. The result suggests that performing the elimination of dominated strategies is facilitated when subjects obtain a cue regarding the order of elimination, starting from last mover and proceeding backwards. Such help is more important when the game is more challenging, due either to an increased number of players or a more intricate presentation.

However, the cue is neither necessary nor sufficient to warrant equilibrium behavior.

Second, decision processes are tremendously different between equilibrium and non-equilibrium players in all treatments (baseline and complex). Two attentional variables are especially predictive of behavior. A measure of lookup occurrence, MIN (minimum information necessary), captures whether the subject has opened all the cells that are essential to compute her equilibrium action, independently of how many non-essential cells have also been opened. A measure of lookup transitions, COR (correct order of reasoning), captures whether at some point in the decision process the subject has looked at payoffs in the order predicted by sequential elimination of strategies, that is, from the matrix of the player with a dominant strategy all the way to the player’s own matrix. For the baseline treatment, equilibrium actions are 30 to 60 percentage points higher for subjects who look at MIN than for subjects who do not, and 40 to 70 percentage points higher for subjects who perform COR than for subjects who do not.¹ These differences in equilibrium behavior are even bigger (never below 50 and sometimes as high as 80 percentage points) in the more complex treatments. The conclusions are similar for the sequential and the simultaneous timings.

Third and perhaps most strikingly, the decision process *among equilibrium players* is very different depending on the timing. In the simultaneous timing, equilibrium players are erratic in their first few lookups, examining the payoff matrices of all players with no clear pattern. As a result, it takes them a long time to reach the matrix of the player with a dominant strategy. We call this behavior “wandering”. By contrast, in the sequential timing equilibrium players look directly at the matrix of the player moving last (which is the one with a dominant strategy). In both cases, however, once the matrix of that player is reached, the subsequent lookup transitions follow very closely the natural sequence of elimination of dominated strategies.

Our analysis has an interesting implication: even if behavior in both timings is reasonably similar when the setting is sufficiently simple, the reasoning process is not. Unveiling the logic of iterated elimination proves much harder for the simultaneous timing than for the sequential timing. It is therefore not surprising that as we increase the complexity of the game, Nash compliance decreases faster in simultaneous than in sequential.

Before turning to the analysis, we briefly review the most closely related literature. To our knowledge, we are the first to use attentional data to reveal the underlying reasons why timing per se affects choice. The earlier literature either focuses on attentional data but considers only one timing, or focuses on timing but only reports behavioral findings.

Limited use of iterative dominance is a well-known experimental result, which has

¹To put it in perspective, notice that in our two-action game going from random behavior to perfect Nash would imply an increase of 50 percentage points.

been analyzed in combination with attention. There are two seminal sets of studies that combine choice and information processing data. In one-shot games, Costa-Gomes et al. (2001) find that compliance with equilibrium is high when the game is solvable by one or two rounds of iterated dominance, but much lower when the game requires three rounds or more. Costa-Gomes and Crawford (2006) reach similar conclusions in two-person beauty contest games.² In dynamic games, there is also evidence of the limited predictive power of backward induction in alternating offer bargaining games (Camerer et al. (1993); Johnson et al. (2002)). Camerer and Johnson (2004) use attentional data to discriminate between backward and forward induction.³ These two sets of studies point to consistent violations of both iterated dominance and backward induction. They also show that attentional data can help understand the cognitive limitations of subjects and predict deviations from theory. Our paper borrows and extends the methodology of Costa-Gomes et al. (2001) (see section 2.2 for details). Our contribution is to combine static and dynamic games in the same study. This allows us to determine if decision processes are different under both timings (both unconditionally and conditional on equilibrium play) and if these differences can account for the deviations from equilibrium choices observed in the data. The measures of attention we use (MIN and COR) are very well adapted to our game and remarkably strong predictors of behavior.

Some earlier studies have compared behavior in sequential vs. simultaneous versions of games that predict the same equilibrium actions. Katok et al. (2002) analyze finitely repeated two-player coordination games and find that subjects apply only a limited number of iterations of dominance (simultaneous) and a limited number of steps of backward induction (sequential), with deviations being more prevalent in sequential than simultaneous. Carrillo and Palfrey (2009) study games of incomplete information and show that equilibrium actions are more frequent when subjects observe their rival's choice before acting (second player in sequential) than when they do not (simultaneous or first player in sequential). Our game is substantially simpler, in an attempt to isolate the effects of elimination of dominated strategies. Most significantly, by studying attentional data, our paper can unveil differences in cognitive reasoning between treatments.

Finally, some studies report systematic differences in behavior when a game is presented in extensive-form rather than in normal-form. Schotter et al. (1994) argue that differences occur because deductive arguments are more prominent in extensive-form representations. Rapoport (1997) suggests that knowing the order induces players to frame

²See also the eye-tracking studies by Knoepfle et al. (2009), Wang et al. (2010), Reutskaja et al. (2011), Devetag et al. (2015) and Polonio et al. (2015). Armel and Rangel (2008) and Armel et al. (2008) show that manipulation of visual attention can also affect choices.

³See also the directed cognition model of Gabaix et al. (2006) applied to individual choice problems.

the game as if it was sequential even if the actions of previous players are not observed. Cooper and Van Huyck (2003) argue that extensive-form induces players to choose the branch where the action of the other player has meaningful consequences.⁴ Our game focuses on the mirror problem since we propose different timings but the same formal representation.

The article is organized as follows. Section 2 presents the theoretical framework and the experimental design. Section 3 analyzes Nash compliance (question (1)). Section 4 studies attentional data in conjunction with behavior (question (2)). Section 5 analyzes the determinants of equilibrium play and the effect of timing (question (3)). Section 6 provides a cluster analysis to assess the heterogeneity in decision processes. Section 7 reports additional analyses and section 8 collects some final thoughts.

2 Theory and Design

2.1 The Game

Game structure. We consider the following T -player game of complete information. For reasons that will become obvious later, players are presented from left to right by role in decreasing order, from role T to role 1. Player in role $t \in \{T, \dots, 1\}$ has two possible actions $a_t \in \{X_t, Y_t\}$. Her payoff depends on her action and the action of exactly one other player. More precisely the payoff of role $t \in \{T, \dots, 2\}$ depends on the actions of role t and role $t - 1$ whereas the payoff of role 1 depends on the actions of role 1 and role T . Payoffs can be displayed in T matrices 2×2 , one for each role, where each cell in matrix t displays the payoff for role t from a particular pair of actions. Table 1 shows a generic representation of the game when $T = 4$ with one of the payoff structures used in the experiment.⁵

Dominance solvable. A key element of the game is that payoffs are chosen in a way that role 1 (and only role 1) has a dominant strategy. Since the payoffs of all roles depend on their action and that of one other player, this makes the game dominance-solvable which dramatically reduces the difficulty to compute the Nash equilibrium. For example, with the payoffs of Table 1 and applying iterated elimination of strictly dominated strategies it

⁴These experimental papers are related to the theoretical literature that studies whether the extensive form representation of a game is the same as the strategic form to which it corresponds (see e.g., Kohlberg and Mertens (1986) and Luce (1992)).

⁵Kneeland (2015) independently developed a very similar 4-player, 3-action game for a theoretical and experimental study of epistemic conditions of rationality, beliefs about others' rationality and belief consistency. Her experiment does not compare sequential vs. simultaneous games nor uses attentional data, the two key elements of our study.

Payoff of 4			Payoff of 3		Payoff of 2		Payoff of 1		
	X_3	Y_3		X_2	Y_2		X_1	Y_1	
X_4	15	25	X_3	38	18	X_2	14	36	
Y_4	30	14	Y_3	18	32	Y_2	30	10	
							X_1	34	26
							Y_1	20	12

Table 1: Example of the game with $T = 4$ players (shaded cell is Nash equilibrium).

is immediate to see that the equilibrium actions of roles 1, 2, 3 and 4 are X_1 , Y_2 , Y_3 and X_4 , respectively (shaded cells).

Role complexity. Even though the equilibrium is obtained through a simple elimination algorithm, the level of strategic sophistication required to compute the equilibrium is monotonically increasing as we move from the rightmost role to the leftmost role. Role 1 faces a trivial game with a dominant strategy and no need for strategic thinking. By contrast, in order to determine their equilibrium strategy, roles 2, 3 and 4 need to iteratively eliminate 1, 2 and 3 strategies of the rivals, respectively. The design thus gives significant variation across roles for a study of strategic thinking. Based on previous research with attentional data (Costa-Gomes et al. (2001); Costa-Gomes and Crawford (2006); Camerer et al. (1993); Johnson et al. (2002)), we expect more deviations from equilibrium behavior as the number of strategies that need to be iteratively eliminated increases.

Sequential vs. simultaneous timing. A main objective of the experiment is to compare cognition and behavior in sequential vs. simultaneous games that are as similar as possible. We therefore consider two orders of moves, from now on referred as “timings.” In the simultaneous version, all roles choose their actions concurrently. In the sequential version, role t chooses her action a_t after observing the actions of roles T to $t + 1$. Since the game is dominant solvable from role 1, the Nash equilibrium is unique and identical under both timings (that is, we do not need to worry about Nash equilibria of the sequential game that are not Subgame Perfect). This is key for a meaningful comparison. Moreover, observing the actions of roles T to $t + 1$ does not help the subject in role t find the equilibrium. Indeed, both in simultaneous and sequential timings, role t needs to sequentially determine the optimal actions of roles 1, 2, 3 all the way to t , with the choices of roles $t + 1$ to T being irrelevant. Finally, the formal display of the game is also identical under both timings, with a 2×2 payoff matrix for each role as depicted in Table 1.

Although the equilibrium, steps of reasoning to reach it and display of the game are the same under both timings, we conjecture that the mental processes likely to be employed by our subjects to approach the game may be different due to two effects. First, observing the actions of roles T to $t + 1$ in the sequential order reduces the set of feasible outcomes for

role t and thus the complexity of the analysis (even though, as we previously pointed out, these observed actions are irrelevant for decision-making). Second, even the reasoning of role T , the first mover, may be facilitated by the knowledge of a sequential order. Indeed, that subject may anticipate that her action will be observed by role 1, which will trigger a sequence of choices by roles 2, 3, etc. with predictable consequences. Interestingly, the behavioral theories typically considered in the literature (level k , cognitive hierarchy, steps of dominance, etc.) predict no differences in choices across timings.

Treatment complexity. To better assess how cognitive limitations apply in strategic settings, we consider games with different numbers of players and different displays. First, we study games with 3, 4 and 6 players with the same structure as the example in Table 1. As discussed above, the choice of role t in a T -player game is identical for all T : it depends on the choices of roles 1 to $t - 1$ and not on those of roles $t + 1$ to T (which is why we chose to label roles in decreasing order).⁶ This also means that, under sequential timing, the behavior in games with different number of players by subjects in the same role should be the same. However, we anticipate to find empirical differences. Second, we change the formal presentation of the different roles. Role 1 is always the player to move last (in sequential) and always the player with a dominant strategy (in both sequential and simultaneous). However, in some treatments role 1 is depicted in the rightmost matrix (as, for example, in Table 1) whereas in others it is presented in some other matrix. Again, this manipulation should not affect behavior but we anticipate it might, as it makes it more challenging to find the first strategy to be eliminated.

2.2 Non-choice data

As briefly explained in the introduction and following some of the recent literature, we analyze not only the choices made by subjects in the experiment but also the lookup patterns prior to the decision. To this purpose we use the same “mousetracking” technique as in Brocas et al. (2014), which is a variant of the methodology first introduced by Camerer et al. (1993) and further developed by Costa-Gomes et al. (2001), Johnson et al. (2002), Costa-Gomes and Crawford (2006) and others (see Crawford (2008) and Willemsen and Johnson (2011) for surveys of the literature and arguments for the use of non-choice data). During the experiment, information is hidden behind blank cells. The information can be revealed by moving a mouse into the payoff cell and clicking-and-holding the left button down. There is no restriction in the amount, sequence or duration of clicks and

⁶For example, role 2 in a 3-player, 4-player or 6-player game needs to perform the same iterative reasoning in order to find the equilibrium: starting from role 1 and ignoring roles 3 and above.

no cost associated to it, except for the subject’s effort which we argue is negligible.⁷ The mousetracking software records the sequence and duration of clicks.

The challenge with this method is the enormous amount of data it provides, which then needs to be classified and studied systematically. Costa-Gomes et al. (2001) pioneered a rigorous methodology to analyze attentional data. They focus on two main measures. First, whether a particular box (e.g., one that needs to be open by subjects with a certain behavior) has been looked at or not. They label it “occurrence.” Second, whether after opening a box, the next one to be opened can be rationalized by a certain reasoning process. They label it “adjacency.” This analysis of the information search process provides an imperfect yet cheap, simple and potentially informative way to measure which information people pay attention to. In the paper, we adapt this methodology to our game and search for different patterns related to lookup *occurrence* and lookup *transitions*.⁸ We analyze lookup data separately for each treatment, role, group size and timing.

2.3 Design and procedures

Baseline treatment. The Baseline treatment [B] consisted of 12 trials of 3-player games and 12 trials of 4-player games for a total of 24 paid trials. We ran 6 sessions with a sequential timing and 6 sessions with a simultaneous timing in the Los Angeles Behavioral Economics Laboratory (LABEL) at the University of Southern California so that the comparison of results across timings is performed between subjects.⁹ All participants were undergraduate students at USC. No subject participated in more than one session and each session consisted of exactly 12 subjects. All interactions between subjects were computerized using a mousetracking extension of the open source software ‘Multistage Games’ developed at Caltech.¹⁰

After each trial, subjects learned their payoff and the actions of the other participants in their group. This information was recorded in a “history” screen visible for the entire session. Subjects were then randomly reassigned to a new group (of three or four subjects depending on the game) and a new role. Before beginning the 24 paid trials, subjects had to pass a short comprehension quiz. They also played a practice round to ensure that they

⁷Earlier experiments have always run additional treatments with open boxes. They have typically found no significant behavioral differences between open and closed box treatments. Since we already ran 360 subjects in our experiment, we decided not to run the open box version.

⁸Some studies analyze also for how long has a cell been opened (lookup duration). In our preliminary data analysis, that measure did not add information to the other two, so we ignored it.

⁹We conjecture that learning may carry over, but differently, from sequential to simultaneous than from simultaneous to sequential. Although this is a fascinating possibility, it makes the data analysis substantially more complicated. We therefore opted to have subjects playing only one order.

¹⁰Documentation and instructions for downloading the software can be found at the website <http://multistage.ssel.caltech.edu>.

understood the rules and also to familiarize themselves with the click-and-hold method for revealing payoffs. A survey including questions about major, years at school, demographics and experience with game theory was administered at the end of each session. A sample of the instructions can be found in Appendix B1. Figure 1 provides screenshots of the computer interface used in the experiment for the simultaneous timing. The left screenshot shows the game the way our subjects saw it (close cells). For the purpose of comparison, the right screenshot shows the traditional version (open cells). It is important to notice that roles in the experiment are coded by colors rather than numerical indexes to avoid cuing subjects about the order of play or the order of elimination of strategies. At the same time, in treatment [B] we always provide the same display where the rightmost role is the one with the dominant strategy and the order of play in the sequential timing is from left to right.

For the simultaneous timing, subjects were instructed of their role and their two possible actions, as depicted in Figure 1 (left). They could open as few or as many payoff cells as they wanted. Whenever they picked an action, a “Please wait” screen appeared until all subjects in their group had locked their choice, at which point the trial ended. For the sequential timing, subjects saw a “Please wait” screen (without the possibility of looking at the game) until it was their turn to move, at which point they saw a screen similar to Figure 1 (left) except that they were instructed of the order of moves and the actions taken by their predecessors.¹¹ Again, once they chose an action, a “Please wait” screen appeared until all subjects in their group had locked their choice.

Since in the simultaneous timing all the subjects in a group could concurrently analyze the game whereas in the sequential timing subjects had to wait their turn to look at the game, simultaneous sessions were significantly shorter than sequential sessions (75 vs. 105 minutes on average). This, however, does not indicate that subjects spent more time analyzing the game under the latter timing.¹² Individual earnings (not including the \$5 show-up fee) averaged \$17.2 in the simultaneous and \$17.5 in the sequential timing, with a minimum of \$12.5 and a maximum of \$20.2.

Complex treatments. As we will develop below, our baseline treatment exhibits large differences in lookup patterns across timings but only small differences in behavior. Some readers suggested that if the simultaneous game is truly more difficult than the sequential game (as the attentional data suggests), then differences in equilibrium choices across timings should be exacerbated if the complexity of the game was accrued. To explore this

¹¹For example, the Orange role in Figure 1 would see the sentence “Red followed by Green followed by Orange followed by Blue” as well as the actions taken by Red (X_R or Y_R) and Green (X_G or Y_G).

¹²If anything, there is indirect evidence of the opposite: the average number of cells opened in the simultaneous and sequential timing is 26.6 and 22.0, respectively.

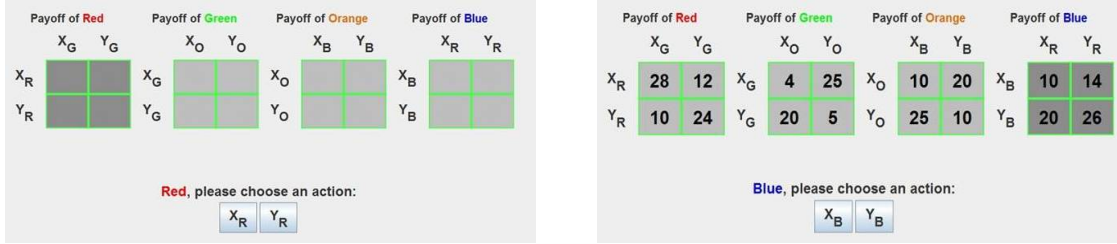


Figure 1 Sample screenshots of the game with close cells (left) and open cells (right)

possibility, we subsequently ran two treatments also at LABEL. Each treatment consisted of 3 sessions of sequential timing and 3 sessions of simultaneous timing with 12 participants per session playing 18 trials.

The Scrambled treatment [S] was identical to the baseline treatment [B] except for the display. Contrary to [B], the order of play for the sequential timing of [S] was *not* displayed in the computer screen from left (first mover) to right (last mover). Instead, it was chosen randomly by the computer.¹³ However, the game was identical in that it was always the role playing last in the sequential timing (role 1) who had the dominant strategy. The reason for such variant was the possibility that after a few trials, a savvy subject in [B] may realize both in the sequential and the simultaneous timing that the rightmost player has the dominant strategy and look immediately at that matrix to start the iterated elimination of dominated strategies.

The Expanded treatment [E] was identical to [B] except that participants played 6- rather than 4-player games. Since, as we will see below, compliance with equilibrium was lower for players in higher roles, we introduced this variant to study the effect of an increase in the cognitive requirements (roles 5 and 6) on both attention and choice.

Additional treatment. Finally and at the request of some reader, we also ran a Random treatment [R] at the same time as the Complex treatments [S] and [E]. The Random treatment [R] was identical to [B] except that the player with a dominant strategy could be in any role. This means that [R] and [S] were identical for the simultaneous timing. By contrast, the game in [R] was significantly simpler under sequential timing than under simultaneous timing since, in the former, the level of strategic sophistication needed to play the equilibrium was lower whenever the player with a dominant strategy was not the last mover.¹⁴ The objective of this treatment was to check whether subjects understood

¹³Of course, the order of play in the sequential timing was again clearly stated.

¹⁴For example, suppose that role 3 has the dominant strategy. In sequential, roles 1 and 3 do not need to eliminate any strategy of other players to find the equilibrium and roles 2 and 4 need to eliminate one

the basics of the game and played the equilibrium strategy whenever it required no or minimal strategic reasoning independently of the role and the display.

Overall, we ran 30 sessions with 12 subjects each for a total of 360 subjects. A summary of the differences across treatments is provided in Table 2.

Treatment	Order	Display	Group size	Dominant strategy	sessions	subjects	trials
Baseline							
Baseline [B]	SEQ	Left to Right	3 and 4	Last	6	12	24
Baseline [B]	SIM	same as above	3 and 4	—	6	12	24
Complex							
Scrambled [S]	SEQ	Scrambled	4	Last	3	12	18
Scrambled [S]	SIM	same as above	4	—	3	12	18
Expanded [E]	SEQ	Left to Right	6	Last	3	12	18
Expanded [E]	SIM	same as above	6	—	3	12	18
Additional							
Random [R]	SEQ	Left to Right	4	Random	3	12	18
Random [R]	SIM	same as above	4	—	3	12	18

Table 2: Summary of treatments

In Appendix B2 we present all the payoffs variants in all the treatments. These payoff matrices *are not* chosen randomly. Indeed, we know from previous research (Costa-Gomes et al. (2001), Brocas et al. (2014) and others) that (i) subjects with a dominant strategy or who need to perform only one step of dominance (roles 1 and 2 in our game) very often play the Nash equilibrium; and (ii) one common (though by no means universal) heuristic for subjects who need to perform two or more steps of dominance is to best respond to random behavior of the opponent (level 1 in the level k model). Therefore, in order to differentiate as much as possible Nash compliance due to equilibrium reasoning from Nash compliance due to that particular heuristic reasoning, we chose payoffs in a way that roles 3 and above would never play the equilibrium strategy in any of the games if they best responded to random behavior of the opponent. At the same time, all the payoff amounts and combinations for roles 3 and above were reasonably similar in all games.

dominated strategy of other players to find the equilibrium. In simultaneous, role 3 needs to eliminate no dominated strategy of other players, role 4 needs to eliminate one, role 1 needs to eliminate two and role 2 needs to eliminate three dominated strategies.

3 Behavior across roles, treatments and timings

Our first objective is to describe the behavior of our participants and to assess whether role complexity, treatment complexity and timing affect equilibrium compliance.

Remember that for treatment **[R]** the sequential timing is significantly simpler than the simultaneous timing (in the former, most choices require no or minimum strategic reasoning). Given that the two timings are not comparable with each other and the object of that control treatment is to check whether subjects understand the basics of the game (which they do), we relegate the analysis of the 72 subjects in treatment **[R]** to Appendix A1. For the main analysis, we will focus exclusively on the 288 subjects who play treatments **[B]**, **[S]** and **[E]**.

Table 3 reports for each treatment the aggregate probability of equilibrium behavior by timing and role, from the player with the dominant strategy (role 1, who moves last in the sequential timing) to the player who needs to eliminate the highest number of strategies to find the equilibrium (role 3, 4 or 6 depending on the treatment). For treatment **[B]**, we separate between the 4-player games (**[B₄]**) and the 3-player games (**[B₃]**). Keep in mind, however, that **[B₃]** and **[B₄]** correspond to the same participants and sessions. Comparisons between aggregate probabilities of equilibrium play are performed using a two-sided t-test (and, unless otherwise noted, differences are reported as significant when the p-value is below 0.05).

		Prob. of Nash play						observations per role
		Role 1	Role 2	Role 3	Role 4	Role 5	Role 6	
SIM	[B₃]	.99	.87	.80	—	—	—	288
	[B₄]	.99	.89	.70	.62	—	—	216
	[S]	.96	.90	.59	.40	—	—	162
	[E]	.95	.73	.57	.50	.47	.36	108
SEQ	[B₃]	.99	.91	.79	—	—	—	288
	[B₄]	.97	.89	.84	.65	—	—	216
	[S]	.98	.94	.80	.62	—	—	162
	[E]	1.0	.88	.64	.61	.55	.44	108

Table 3: Probability of equilibrium choice by role, timing and treatment.

Table 3 reveals that subjects understand the basics of the game. As in previous experiments (Costa-Gomes et al. (2001); Brocas et al. (2014)), subjects with a dominant strategy (role 1) almost invariably play the equilibrium action. They also play very close to equilibrium in role 2, when it requires eliminating the dominated strategy of only one

other player (between 87% and 94%, with the only exception of treatment **[E]** simultaneous timing in which equilibrium choices are around 73%). Since there is almost no variance in behavior for roles 1 and 2, we will focus on roles 3 and above the rest of the analysis. We make three main observations.

First, Nash compliance decreases with role from role 3 and up. All differences between consecutive roles are statistically significant for treatments **[B₄]** and **[S]**. They are not significant for treatment **[E]** due to the smaller number of observations, but they become significant when we pool observations across orders or pool two roles together. In other words, consistent with theories of limited reasoning as well as with previous experiments on dominance solvable games, Nash behavior is inversely related to the number of strategies that need to be iteratively eliminated in order to find the equilibrium.¹⁵ Large deviations covary with difficulty.

Second, Nash compliance is a function of treatment complexity. In the simultaneous timing, equilibrium play decreases significantly within each role as we move from baseline (**[B₃]** or **[B₄]**) to complex treatments (**[S]** or **[E]**). This is indicative that, just as we expected, removing the presentation cue and adding players makes the game more difficult. By contrast, in the sequential timing, the difference between baseline and complex is significant only for role 3 in **[E]**, which suggests that adding irrelevant players or mixing the presentation is not enough to make the problem more difficult in the sequential timing.

Finally, Nash behavior also depends on timing. Indeed, in the complex treatments, equilibrium play within a role is less frequent under simultaneous than under sequential timing. The difference is highly significant for **[S]** but not for **[E]**, again due to the smaller number of observations (the difference in **[E]** becomes significant once we pool two roles together). This trend is less pronounced in the baseline treatment (it is significant only for role 3 in **[B₄]**), which suggests the existence of an interaction between timing and complexity.

Summary. The analysis in this section answers our first question, namely whether behavior differs across timings. Nash compliance is close to 1 for simple choices (roles 1 and 2) and it decreases significantly with role and treatment complexity. When treatments are sufficiently complex, Nash compliance is further decreased in the simultaneous timing. Therefore, timing has a clear effect on behavior. We would like to note that the results cannot be parsimoniously rationalized with theories based on best response to the empirical

¹⁵In a previous version, we conducted an exhaustive test of limited cognition theories (level k and steps of dominance) and found that level k explains reasonably well the aggregate behavior of our subjects. However, neither level k nor steps of dominance predict the observed differences in choices and lookups across timings (see below). Since differences in lookups between equilibrium and non-equilibrium players and differences in lookups and behavior across timings are the two novel questions addressed in the paper, we decided to remove the level k analysis, but it is available upon request.

distribution or other-regarding preferences. However, for the sake of brevity and clarity, we relegate these discussions to section 7.

Although interesting, these results are expected and do not constitute the main thrust of the paper. Indeed, the novelty of the analysis is to use attentional data to determine why the simultaneous timing is associated with a lower Nash frequency compared to the sequential timing, even though Nash equilibrium and existing theories of limited reasoning do not predict differences in choices. We also want to use lookups as a predictor of choices.

4 The attentional correlates of behavior

One challenge with attentional measures is the large amount of data they provide. For example, subjects in our experiment open as many as 228 payoff cells in one single trial. Finding sensible ways to filter the data is a key step in the analysis of lookups.

4.1 Lookup occurrence as a predictor of equilibrium choice

The simplest measure of attention is *occurrence* of lookups. Occurrence is a binary variable that takes value 1 if a payoff cell has been opened and 0 otherwise.

For each role of each game in each treatment we can determine which cells must *imperatively* be opened in order to find the Nash equilibrium. We call this set of cells the “Minimum Information Necessary” or MIN. For example, role 1 in simultaneous games needs to open all 4 cells of her payoff matrix in order to find her optimal strategy. MIN for other roles can be determined recursively using a simple backward induction algorithm. In Appendix B3, we describe for treatment [B] the cells that belong to MIN by role and order of moves. The logic is similar for treatments [S] and [E].

We want to highlight two simple but crucial points regarding lookup occurrence. First, a subject who looks at MIN may or may not compute the Nash equilibrium. However, a subject who does not look at MIN cannot have performed the traditional game theoretic reasoning needed to play the Nash equilibrium. In other words, opening a cell is a necessary but not sufficient condition for a subject to pay attention and understand the implications of the information. In that respect, MIN is a very conservative measure of attention.

Second, MIN is defined from the perspective of an outside observer (the experimenter) who is aware of the payoffs behind all cells. A subject cannot know *ex ante* what the MIN set is, and therefore she will likely open many more cells than those exact ones. For this reason, we classify an observation as MIN as long as the subject opens all the cells in the MIN set, independently of how many of the other non-essential cells she also opens. An observation is classified as ‘notMIN’ if the subject does not open at least one of the cells in the MIN set, again independently of how many of the other non-essential cells she opens.

Table 4 reports for roles 3 and above, for both timings and for all treatments the percentage of observations where subjects look at the MIN set ($\Pr[\text{MIN}]$). It also shows the probability of equilibrium behavior conditional on MIN and conditional on notMIN ($\Pr[\text{Nash} | \text{MIN}]$ and $\Pr[\text{Nash} | \text{notMIN}]$).

SIMULTANEOUS

	Role 3				Role 4			Role 5	Role 6
	[B ₃]	[B ₄]	[S]	[E]	[B ₄]	[S]	[E]	[E]	[E]
Pr[MIN]	.70	.58	.50	.56	.57	.32	.43	.44	.28
Pr[Nash MIN]	.91	.89	.84	.93	.89	.87	.96	.83	.87
Pr[Nash notMIN]	.56	.44	.33	.13	.26	.17	.16	.20	.17

SEQUENTIAL

	Role 3				Role 4			Role 5	Role 6
	[B ₃]	[B ₄]	[S]	[E]	[B ₄]	[S]	[E]	[E]	[E]
Pr[MIN]	.72	.58	.77	.61	.57	.50	.56	.47	.35
Pr[Nash MIN]	.91	.96	.96	.94	.84	.90	.93	.94	.87
Pr[Nash notMIN]	.49	.67	.29	.17	.39	.33	.21	.19	.20

Table 4: Equilibrium choice based on lookup occurrence (MIN)

Not surprisingly, MIN lookups decrease with role. This is in part explained simply by the fact that the set of MIN cells is larger for higher roles (so statistically less likely).

The most interesting results relate to choices conditional on lookups. In treatment [B], the likelihood of playing Nash after looking at MIN is high, often close to 1, and decreases 29 to 63 percentage points when subjects do not look at all the essential cells. Results are even more dramatic in the more complex treatments. Indeed, in treatments [S] and [E], playing Nash conditional on looking at MIN is still very high (83% and above) whereas playing Nash conditional on not looking at MIN is even lower than before (13% to 33%). The resulting differences in Nash choices between MIN and notMIN are as high as 80 percentage points. All these differences are statistically significant at the 1% level, using a two-sided t-test. Overall, Table 4 suggests that MIN lookup is an excellent predictor of equilibrium choice. It is also a better discriminant of behavior when the decision is more difficult (higher roles, complex treatments). This is natural, since in more difficult settings it is less likely that a subject who does not understand the logic of elimination of strategies opens the “right” cells just by chance.

4.2 Lookup transitions as a predictor of equilibrium choice

To study lookup transitions, we construct the following coarse yet informative measure. For each subject in each trial we determine which role’s payoff matrix a subject opens (independently of the cell(s) within that matrix), and then record the string of transitions between the matrices of the different roles. This means that we ignore the number of clicks in a cell as well as the transitions *within* the matrix of a certain role t . So, for example, a string ‘312’ for a subject in role 3 would capture an individual who first opens one or several cells in her own payoff matrix (role 3), then moves to the payoff matrix of role 1 before finally stopping at the payoff matrix of role 2.¹⁶ For reference, strings in our experiment contain between 0 and 50 digits.

Once these strings are created, we determine for each trial whether the string of a subject in role t contains what we call the “**C**orrect **O**rders of **R**easoning” or COR, which is defined as the sequence of transitions between adjacent matrices from the matrix of the role with a dominant strategy to the subject’s own matrix: 123... t . Intuitively, displaying this sequence should be a strong indicator that the subject follows the logic of elimination of dominated strategies from role 1 to role 2, and so on until role t .¹⁷ As in the case of MIN, we only determine whether the COR sequence is included in the entire string, independently of how many transitions are contained before or after the COR portion of the string. An observation is classified as ‘COR’ if the COR sequence is contained in the string and ‘notCOR’ if it is not contained.

Table 5 reports the average probability that a trial contains the correct sequence (Pr[COR]), by role, timing and treatment. It also presents the probability of Nash choice conditional on performing the correct sequence (Pr[Nash | COR]) and conditional on not performing the correct sequence (Pr[Nash | notCOR]).

We find that COR is also an excellent predictor of equilibrium behavior, possibly better than MIN. The probability that a subject displays the COR sequence in her string of lookups consistently decreases with role complexity.

Equilibrium behavior in treatment [B] given COR is consistently above 90% and drops dramatically under notCOR. Nash choices in that treatment are 43 to 69 percentage points

¹⁶For this particular analysis of lookup transitions, it is key that each matrix contains payoffs of one and only one role, contrary to the traditional normal form representation in game theory. There is obviously a significant loss of information in using this coarse partition. However, it turns out to be highly informative.

¹⁷For roles 4, 5 and 6, we allow also one transition between adjacent matrices in the opposite direction because it seems plausible that a subject may forget some payoff and double check it before restarting the reasoning. So, for example, we allow 1234 but also 123234. However, we never allow transitions between non-adjacent matrices, such as 123134. In principle, we could also allow two or more transitions between adjacent matrices in the opposite direction (for example, 12323234). Adding them would not change significantly the results as these strings are very rare.

SIMULTANEOUS

	Role 3				Role 4			Role 5	Role 6
	[B ₃]	[B ₄]	[S]	[E]	[B ₄]	[S]	[E]	[E]	[E]
Pr [COR]	.67	.57	.54	.55	.54	.33	.40	.35	.24
Pr [Nash COR]	.94	.94	.88	.95	.93	.93	.98	.97	.96
Pr [Nash notCOR]	.51	.39	.24	.12	.24	.13	.18	.20	.17

SEQUENTIAL

	Role 3				Role 4			Role 5	Role 6
	[B ₃]	[B ₄]	[S]	[E]	[B ₄]	[S]	[E]	[E]	[E]
Pr [COR]	.69	.74	.75	.59	.55	.54	.56	.48	.32
Pr [Nash COR]	.93	.96	.98	.97	.91	.93	.93	.92	.91
Pr [Nash notCOR]	.48	.49	.28	.16	.33	.24	.19	.20	.21

Table 5: Equilibrium choice based on lookup transitions (COR)

higher given COR than given notCOR, which is an equal or bigger difference than when we conditioned on MIN. Again, the effect is larger in the more complex treatments [S] and [E], where the differences are as high as 83 percentage points. (All the differences in probability of Nash behavior given COR and notCOR are statistically significant at the 1% level, using a two-sided t-test). Overall, COR is an extremely strong indicator that the subject understands the unraveling logic of the game.¹⁸ Finally, the difference between Pr[Nash | COR] and Pr[Nash | notCOR] is very similar under both timings, which suggests that the difference in Nash behavior across timings for complex treatments are mostly the result of differences in the ability of performing the COR sequence and not in the ability to transform such reasoning into Nash play.

Summary. This section has addressed our second question: Are decision processes different between equilibrium and non-equilibrium players? We have shown that being attentive to all the cells necessary to reach a Nash decision (MIN) and browsing through them in the order predicted by the “elimination of dominated strategies” algorithm (COR) are highly predictive of equilibrium behavior. Those who direct their lookup correctly

¹⁸We also study Nash conditional on performing and not performing the opposite sequence (OPP), from the subject’s own role t to the role of the subject with a dominant strategy ($t...21$). This is a natural way to look at payoffs but one that *does not* help finding the equilibrium (quite the opposite), and therefore *should not* be predictive of equilibrium behavior. We find that, after controlling for COR, OPP has no explanatory power for Nash behavior in any treatment and for any role (data omitted for brevity).

also play Nash very often. By contrast, those who look erratically almost never play Nash. Furthermore, the likelihood of MIN lookups and COR sequences decreases as role complexity and treatment complexity increases. This suggests that directing lookups correctly is more challenging when the game is more difficult to analyze.

5 The path to equilibrium play

The objective of this section is to study how Nash compliers reason and to better assess the effect of timing on that reasoning. To address that question, we provide two complementary approaches. In the first, we focus on trials resulting in equilibrium behavior and we compare lookup transition patterns across timings. In the second, we perform a regression analysis to assess how timing contributes to equilibrium play.

5.1 Lookup transitions among equilibrium players

Denote by tt' the transition from the payoff matrix of role t to the payoff matrix of role t' . Each string constructed in section 4.2 is then made of a sequence of transitions tt' . We can then group all the transitions of a string into three categories. First, “action” transitions. These are the transitions from the matrix of role t to the matrix of the role affected by the action of role t (so $t + 1$ when $t < T$ and 1 when $t = T$, that is, the adjacent transitions from 1 to 2, 2 to 3 and all the way to T , as well as the transition from T to 1 which wraps the argument up). These transitions follow the induction argument which is key to solve the game: “if t chooses action a_t , then $t + 1$ should choose action a_{t+1} , etc.” They closely relate to the COR sequence idea developed in the previous section. Second, “payoff” transitions. These are the transitions from the matrix of role t to the matrix of the role whose action will affect the payoff of role t (so $t - 1$ when $t > 1$ and T when $t = 1$, that is, the adjacent transitions from T to $T - 1$, $T - 1$ to $T - 2$ and all the way to 1, as well as the transition from 1 to T). These are natural transitions to look at, in order to determine potential payoffs associated to a certain strategy. However, they are misleading in that they go against the induction argument and therefore do not help solving the game. All transitions in 3-player games are either action or payoff transitions. The remaining transitions in 4-player and 6-player games are what we call “non-adjacent” transitions, since they skip one or more roles.

Table 6 presents for roles 3 and above and for each treatment, the proportion of action, payoff and non-adjacent transitions. Recall that we are interested in studying differences in cognitive processes between timings in trials that result in equilibrium play, so we focus exclusively on those trials.

SIMULTANEOUS

	Role 3				Role 4			Role 5	Role 6
	[B ₃]	[B ₄]	[S]	[E]	[B ₄]	[S]	[E]	[E]	[E]
action	.58	.57	.56	.53	.57	.55	.53	.53	.50
payoff	.42	.39	.39	.41	.41	.43	.41	.43	.48
non-adjacent	—	.05	.05	.06	.02	.02	.06	.04	.02

SEQUENTIAL

	Role 3				Role 4			Role 5	Role 6
	[B ₃]	[B ₄]	[S]	[E]	[B ₄]	[S]	[E]	[E]	[E]
action	.76	.69	.73	.57	.75	.85	.71	.71	.82
payoff	.24	.26	.20	.35	.22	.13	.20	.23	.16
non-adjacent	—	.06	.07	.08	.03	.02	.09	.06	.02

Table 6: Percentage of action, payoff and non-adjacent transitions for Nash trials

Table 6 shows that the pattern of transitions is very different between the sequential and simultaneous timing. At the same time, differences are stable across roles and treatments. As expected, non-adjacent transitions are always rare.¹⁹ More interestingly, in treatment [B] the overall ratio between action and payoff transitions is around 3 for the sequential timing (75%-25%) and around 1.5 for the simultaneous timing (60%-40%), whereas random lookups would predict a ratio of 1. The difference in ratios between sequential and simultaneous timings is slightly bigger in the complex treatments (except for role 3 in treatment [E]). The result suggests that imposing a sequential timing directs subjects into looking at the matrices in the “right way”, and that this cue provided by sequentiality becomes more helpful when the game is more complex to analyze.

To further investigate the differences in lookup transitions between timings, we construct the same table of transitions, except that we condition on the subject having reached the payoff matrix of role 1. More precisely, we remove from the string all the transitions that occur *before reaching the matrix of role 1 for the first time*. We also remove the observations where the subject never look at role 1’s matrix.²⁰ The reason for such analysis is the conjecture that a main difficulty in finding the equilibrium, specially in the simultaneous case, lies in realizing how the behavior of role 1 is the key to unravel the choices

¹⁹In 3-player games there aren’t any non-adjacent transitions but in 6-player games, 18 out of the 30 possible transitions are non-adjacent so random lookups in that case would result in substantial non-adjacent transitions.

²⁰These are only 13.2% of the observations (recall that we are focusing only on Nash trials).

of roles 2, 3 and up. The results are summarized in Table 7.

SIMULTANEOUS

	Role 3				Role 4			Role 5	Role 6
	[B ₃]	[B ₄]	[S]	[E]	[B ₄]	[S]	[E]	[E]	[E]
action	.85	.82	.81	.73	.89	.93	.87	.85	.83
payoff	.15	.13	.14	.19	.11	.06	.08	.10	.15
non-adjacent	—	.05	.05	.08	.01	.01	.05	.05	.02

SEQUENTIAL

	Role 3				Role 4			Role 5	Role 6
	[B ₃]	[B ₄]	[S]	[E]	[B ₄]	[S]	[E]	[E]	[E]
action	.81	.81	.82	.75	.83	.87	.82	.89	.86
payoff	.19	.14	.13	.17	.15	.11	.11	.08	.12
non-adjacent	—	.05	.05	.08	.02	.02	.07	.03	.02

Table 7: Percentage of action, payoff and non-adjacent transitions for Nash trials, conditional on reaching the payoff matrix of role 1

Once the subject has looked at the payoff matrix of role 1 for the first time and conditional on playing Nash, action transitions become overwhelmingly prevalent (between 81% and 93% with most ratios above 6, except for the outlier role 3 in treatment [E]). Perhaps more surprisingly in light of Table 6, the ratio between action and payoff transitions is now remarkably similar under both timings. Overall, Tables 6 and 7 confirm that the reasoning process is very different under the sequential and simultaneous timings, even for subjects who succeed in playing the equilibrium strategy. It also provides a strong indication of what these differences are. In simultaneous games, it is harder to realize that the choice of role 1 is key to determine the optimal behavior of roles 2, 3 and above. As a result, transitions are more erratic than in sequential games, with considerable back and forth lookups between matrices. However, once the payoff matrix of role 1 is hit under either timing, the dominant strategy is found, and the transition sequence 12...t is triggered fast and efficiently.

5.2 Regression analysis: predicting choice from lookups

The last step in the aggregate analysis consists in using the lookup data to predict choices. We treat each trial as a separate observation and we run Probit regressions to predict whether the subject plays the equilibrium action (= 1) or not (= 0). We run nine regres-

sions to study separately the behavior of subjects by role (3 and above) and treatment, and we pool together observations in the sequential and simultaneous timings (see below).

Since we are interested in the predictive power of attentional data, we include variables related to lookup occurrence and transitions. For occurrence, we introduce a dummy variable that takes value 1 if the subject looked at all the MIN cells, independently of how many other cells he looked at, and 0 otherwise (*min*). For transitions, we introduce two variables: the total number of transitions (*total-t*) and the percentage of transitions that are action transitions (*action-t*). We choose these variables because, according to our previous results, *min* and *action-t* are good candidates to explain equilibrium behavior. Furthermore, *total-t* intuitively captures the total attention and effort put by a subject in thinking about the game. Other interesting lookup variables are highly correlated with the variables in our regression (for example, a natural candidate would be COR, but it is highly correlated with both *min* and *action-t* and worse at discriminating between sequential and simultaneous). Finally but crucially, we also add a timing dummy variable (*seq*) that takes value 1 when the game is sequential and 0 when the game is simultaneous. The goal is to determine if differences in behavior are fully captured by the three lookup variables described above or if we are still missing some lookup aspect that differentiates equilibrium choice between the two timings. Results are presented in Table 8.

	Role 3 [B ₃]	Role 3 [B ₄]	Role 3 [S]	Role 3 [E]	Role 4 [B ₄]	Role 4 [S]	Role 4 [E]	Role 5 [E]	Role 6 [E]
<i>seq</i>	-0.178 (.136)	0.255 (.168)	0.218 (.198)	0.08 (.250)	-0.010 (.145)	0.581** (.189)	-0.081 (.240)	-0.025 (.227)	-0.010 (.220)
<i>min</i>	0.94*** (.178)	1.21*** (.191)	1.71*** (.237)	2.43*** (.299)	1.24*** (.168)	1.96*** (.244)	2.25*** (.312)	1.69*** (.313)	1.67*** (.326)
<i>total-t</i>	-0.006 (.003)	-0.002 (.004)	-0.007 (.006)	0.002 (.006)	0.001 (.002)	-0.012** (.004)	-0.002 (.004)	-0.002 (.004)	-0.000 (.004)
<i>action-t</i>	7.65*** (1.55)	7.22*** (1.34)	7.88*** (1.77)	3.02 (2.31)	6.65*** (1.22)	8.34*** (1.60)	5.50* (2.60)	7.79** (2.81)	7.19*** (2.12)
<i>const.</i>	0.03 (.136)	-0.49** (.173)	-0.96*** (.236)	-1.32*** (.284)	-0.87*** (.164)	-1.29*** (.193)	-1.11*** (.258)	-1.19*** (.264)	-1.36*** (.252)
# obs.	575	430	324	213	431	324	216	215	214
Pseudo R^2	0.233	0.299	0.432	0.554	0.309	0.448	0.516	0.423	0.400

Standard errors in parentheses. * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Table 8: Probit regression of Nash behavior as a function of lookups by role and treatment

The sign and significance of the parameters are remarkably similar across regressions. The coefficient for MIN occurrence (*min*) is always highly significant. The same is true

for action transitions (*action-t*) in all but one regression (role 3 in treatment [E], whose behavior follows different patterns from the rest as we already noted in Table 6). The regression confirms our previous results that these measures of attention are good predictors of equilibrium choice. We also find that the total number of transitions (*total-t*) is either not significant or a (weakly) negative indicator of equilibrium choice. It means that, conditional on looking at MIN and having a high fraction of transitions in the right direction, subjects who spend more time looking at payoffs do not perform better. This captures an interesting kind of misguided search, or wandering, that we will explore in more detail in the next section. Finally but importantly, the sequential order variable (*seq*) is significant in only one regression (role 4 in treatment [S]). It suggests that most differences in choices between the sequential and simultaneous timings can be accounted for by only two simple attentional measures, MIN occurrence and action transitions.

Summary. This section has answered our third question: Are decision processes conditional on equilibrium choices different across timings? We have shown that the timing affects how fast subjects find the role with the dominant strategy (role 1). However, once a subject arrives at the matrix of role 1, the unraveling logic of elimination of dominated strategies is performed equally efficiently under both timings. As such, timing does not explain Nash compliance once we control for lookups variables.

6 Accounting for heterogeneity

Our choice and lookup data have revealed that characteristics that make a game more or less complex affect the likelihood to attend to key features of that game, and hence to play at equilibrium. Still, different subjects are affected differentially by complexity. To describe this heterogeneity, we use the attentional data to group individuals with the objective of finding common patterns of lookups. We follow the clustering methodology developed by Brocas et al. (2014).²¹ An advantage of clustering is that it does not impose any structure of heterogeneity, but rather describes the heterogeneity found in the data as it is.

As highlighted in section 4, there are several attentional variables that contribute to explain choices and these variables are often correlated with each other. In our experiment, the string of lookup transitions is a rich and promising measure that may help differentiate subjects who do and do not follow the logic of elimination of dominated strategies. We therefore focus on this aspect of attention at the expense of lookup occurrence, which may be more noisy and variable. More precisely, we take the string of transitions created

²¹This method has subsequently been used successfully in other eye-tracking studies, such as Devetag et al. (2015) and Polonio et al. (2015).

in section 4.2 and build the following two variables for each subject. First, *%-cor*: the percentage of trials where the individual performs the COR sequence, which we know is highly predictive of equilibrium behavior. Second, *pre-cor*: the average number of matrices opened by the subject before starting the COR sequence. This includes the whole string of transitions when the COR sequence is not performed. It provides a measure of how much the subject looked around before realizing (or not) the COR sequence.²² We informally refer to this second variable as the level of wandering.²³ The choice of these two variables relies heavily on the analysis in sections 4.2 and 5.1, where we reached three conclusions regarding lookup transitions: they are an excellent predictor of equilibrium behavior, they widely differ between sequential and simultaneous timings even among subjects who play the equilibrium strategy, and most of the heterogeneity is concentrated on transitions before reaching the matrix of role 1.

Given the similarities in lookup transitions across treatments and the significant differences across timings, we pool all treatments together but perform a separate cluster analysis for subjects playing under sequential and simultaneous timings respectively. We also concentrate on the behavior of roles 3 and 4, since there is not enough variance for roles 1 and 2 and there is no data for roles 5 and 6 in treatments **[B]** and **[S]**. For each timing, we group the 144 participants in clusters based on the two variables described above: *%-cor* and *pre-cor*. There is a wide array of heuristic clustering methods that are commonly used but they usually require the number of clusters and the clustering criterion to be set ex-ante rather than endogenously optimized. Mixture models, on the other hand, treat each cluster as a component probability distribution. Thus, the choice of number of clusters and model (distribution, volume, shape and orientation of the clusters) can be made using Bayesian statistical methods (Fraley and Raftery, 2002). We implement model-based clustering analysis with the *mclust* package in R (Fraley and Raftery, 2006). Technically, this methodology is the same as Brocas et al. (2014). Conceptually, there are two differences. First, Brocas et al. (2014) cluster individuals based on lookups and choice.²⁴ Clustering only on lookups allows us to study whether a classification made solely on attentional data has a good predictive power of choice. Second, Brocas et al. (2014) introduce six variables. Reducing them to only two makes the comparison across clusters easier. The risk, of course, is to have an insufficient number of variables to adequately

²²We choose average rather than percentage of matrices opened before the COR sequence to distinguish subjects who do not reach the COR sequence after opening few boxes from those who do not reach the COR sequence after opening many boxes. Percentage would be closer to the “density” measure of Costa-Gomes and Crawford (2006).

²³We also explored a third variable: the number of matrices opened after the COR sequence, which we call “post-wandering”. We found little variance across subjects and no systematic patterns for this variable, so we did not include it in the analysis.

²⁴In the appendix, they perform clustering based only on lookups and obtain weaker results.

discriminate behavior.

The `mclust` package requires to set a maximum number of clusters (nine is the default) and models (ten is the default). It then determines the combination of model and number of clusters that yields the maximum Bayesian Information Criterion (BIC). With the default parameters, the models that maximize BIC have seven clusters in the sequential timing and nine clusters in the simultaneous timing. Upon closer inspection, we noticed that the maximum BIC is virtually identical for five clusters and above under each order of moves whereas the interpretation of the data is more contrived the higher the number of clusters. We therefore decided to constrain the maximum number of clusters to five and keep the default maximum number of models. Under this constraint, the models that maximize BIC have five clusters in sequential and three clusters in simultaneous.²⁵

Table 9 shows for each timing the summary statistics of the average value within each cluster of the two variables considered in the analysis (*%-cor* and *pre-cor*) as well as the number of subjects in each cluster (total and by treatment). For reference, it also shows the average performance of subjects within a cluster in terms of percentage of equilibrium choice in the relevant roles 3 and 4 (% Nash) and percentage of MIN lookups also in roles 3 and 4 (% MIN). Recall, however, that these two variables are not used in the clustering. Figure 2 provides a graphical representation of the three clusters in the simultaneous case (left) and the five clusters in the sequential case (right).

Under the sequential timing, about 20% of subjects (cluster A) seem to be lost: they wander for some time, very rarely perform the COR sequence and play the equilibrium strategy less frequently than dictated by chance. Another 20% of subjects (cluster B), try harder on average, perform long strings of transitions but reach the COR sequence only half of the time, which translates into equilibrium choice only slightly more often than random choice would predict. This group has the largest variance in the number of lookups. Finally 60% of subjects (clusters C, D and E) reach reasonably fast the COR sequence and play Nash consistently. Of these, cluster E (almost 20% of individuals) is the absolute portrait of rationality: perfect levels of COR sequence and Nash choices and extremely low wandering.²⁶ Overall, there is a hump-shaped relationship between COR sequence and wandering. For the extreme levels of COR, subjects have low wandering either because they know where to look (clusters C, D and E) or because they are clueless and give

²⁵The BIC of the unconstrained models are -486.2 (sequential, seven clusters) and -516.2 (simultaneous, nine clusters) whereas the BIC of the constrained models are -502.1 (sequential, five clusters) and -521.8 (simultaneous, three clusters), so the difference in performances between constrained and unconstrained are indeed very minor.

²⁶Recall that a subject who first looks at his own payoff matrix and then performs the COR sequence would show a value of 1 in *pre-cor*, which means that many of these subjects go directly to the matrix of role 1. This is the behavior and lookup pattern we would expect of a subject *trained* to perform sequential elimination of strictly dominated strategies.

up (cluster A). In the middle (cluster B), subjects struggle and play Nash sometimes. The representation of subjects from each treatment is remarkably stable with a ratio reasonably close to 2:1:1 in all clusters. Notice also the clear relationship between the average frequency of COR sequence, MIN lookup and Nash choice across clusters.

SEQUENTIAL

Cluster	A	B	C	D	E	all
<i>%-cor</i>	5.4	49.0	81.0	100	100	64.4
<i>pre-cor</i>	3.3	3.9	2.1	2.3	0.6	2.4
# subjects [†]	30 (13, 7, 10)	29 (16, 7, 6)	43 (22, 11, 10)	14 (5, 3, 6)	28 (16, 8, 4)	144 (72, 36, 36)
% Nash	40.0	54.9	81.5	97.2	98.8	72.5
% MIN	10.0	55.7	74.5	94.4	90.6	62.4

[†] Number of subjects from treatments ([**B**], [**S**], [**E**]) in parenthesis.

SIMULTANEOUS

Cluster	A	B	C	all
<i>%-cor</i>	2.6	39.6	86.0	53.7
<i>pre-cor</i>	3.1	3.3	2.7	3.0
# subjects [†]	36 (16, 10, 10)	37 (15, 13, 9)	71 (41, 13, 17)	144 (72, 36, 36)
% Nash	31.1	48.3	85.7	62.7
% MIN	8.1	41.8	84.1	54.6

[†] Number of subjects from treatments ([**B**], [**S**], [**E**]) in parenthesis.

Table 9: Summary statistics by cluster.

The picture looks similar under the simultaneous timing, with some subtle differences however. First, the statistical model groups the most rational players in one (C) rather than three (C, D, E) clusters. Within cluster C, we still observe a number of subjects with lookups and choices very close to pure rational, although fewer than under the sequential timing. Second, the extra difficulty posed by the simultaneous timing in finding the role with the dominant strategy translates into a higher level of wandering on average (3.0 vs. 2.4). Wandering is also less heterogenous across clusters. In other words, since even rational players (especially in treatment [**S**]) need to spend time to find the role with a dominant strategy, we lose the interesting hump-shaped relationship between COR se-

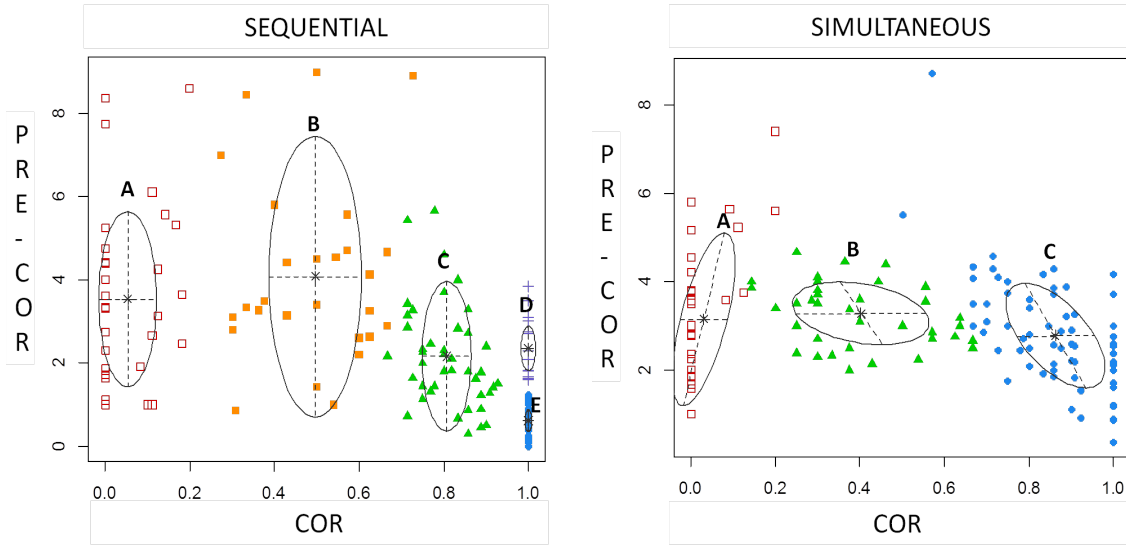


Figure 2 Cluster based on COR sequence ($\% \text{-cor}$) and wandering (pre-cor) for the sequential (left) and simultaneous (right) orders.

quence and wandering. Finally, we also notice that subjects are more likely to be classified as rational in treatment [B] compared to the more complex treatments [S] and [E]. This is again consistent with the fact that Nash play, MIN lookup and COR sequence are less likely when treatment complexity increases under simultaneous timing.

Summary. The cluster analysis shows that the population is composed of different behavioral types using different cognitive processes. Under either timing, some subjects immediately reach the correct lookups and convert them into Nash choices, while others keep wandering, rarely discover the logic of the game or play at equilibrium. It is plausible however that the sequential timing affects the proportions of such subjects. A definite answer to that question would require a within-individual study of timing.

7 Other analyses and extensions

In this section we briefly describe some additional analyses. Details are relegated to Appendix A.

7.1 Empirical best response

One could argue that non compliance with Nash equilibrium may arise because a subset of players do not play at equilibrium, leading others to best respond to non-equilibrium play. To test that theory, we compare the expected payoff of playing each action, for each game, each role, each treatment and each timing, assuming the opponents play according to the empirical probabilities. In our data, for roles 2, 3 and 4, playing Nash is the empirical best-response in all games, all treatments and both timings.²⁷ By contrast, for roles 5 and 6 of treatment [E], playing Nash is the best response only about one-fourth of the time.²⁸ A theory based on empirical best response to non-equilibrium behavior can potentially account for the choices of the remaining three-fourth of the trials. However, this theory predicts in particular that role 4 will play Nash with probability 1 while role 5 will play Nash with probability 0. We do not observe such sharp decline empirically. On the contrary, the probability of equilibrium behavior in the simultaneous version of treatment [E] is 0.50 for role 4 and 0.47 for role 5. As we extensively discussed, our choice and lookup data suggest instead that equilibrium behavior is inversely related to the difficulty of the decision problem.

7.2 Other regarding preferences

Non compliance with Nash equilibrium could result from other regarding concerns. To see that this is likely not the case in our experiment, suppose that players are willing to sacrifice money to reduce inequality, benefit the worst-off player or increase total payoff (as e.g., in Fehr and Schmidt (1999) or Charness and Rabin (2002)). Because studying social preferences is not a main objective of our analysis, the payoffs of the game were chosen in a way that we should not observe significant deviations from equilibrium even with some reasonable degree of social preferences. Stated differently, within the usual range of social preferences parameters, equilibrium behavior is optimal in our games. Furthermore, since the payoff structures are very similar for roles 2 to T , if we were to observe deviations due to social preferences, these should be similar across roles, treatments and timings. This is not what we observe in Table 3. As such, social preference theories do not capture the *differences* in behavior across roles, timings and treatments observed in our game.

²⁷Of the 216 games, there is one exception: game 9, treatment [E], simultaneous order, role 4 where the best response is to not play Nash.

²⁸More precisely, in the sequential order, Nash is best response in all 9 games for role 5 and 1 out of 9 games for role 6. In the simultaneous order, Nash is never a best response for roles 5 or 6.

7.3 Other measures of lookup occurrence

Although MIN lookups are extremely indicative of equilibrium behavior, we can also think of other (complementary) attentional measures that can be suggestive of Nash play for roles 3 and above. First, we determine whether subjects who look relatively more at their own payoffs play the equilibrium strategy less often. This is based on the conjecture that self-centeredness is a sign of insufficient strategic thinking. In our data and consistent with this idea, we find that in trials where the choice is consistent with Nash, subjects spend 10 to 25 percentage points less on their own payoff matrix than in trials where the choice is not consistent with Nash. Second, for each subject we compute the proportion of observations with at least one lookup at the payoff matrix of role 1, the key matrix to initiate the elimination of dominated strategies. Not surprisingly given the results on MIN and COR, we find that in trials where the choice is consistent with Nash, subjects look at role 1's matrix 80 to 94 percent of the time whereas in trials where the choice is not consistent with Nash, subjects look at that matrix only 6 to 47 percent of the time. Overall, the analysis suggests that there are many different measures of attention (often correlated) that can help us understand better the reasoning process of individuals and that creativity in finding the appropriate lookup measures is essential when studying the data. Details of the results can be found in Appendix A2.

7.4 Other cluster analysis

Cluster analysis has the advantage of grouping subjects endogenously rather than imposing which individuals should belong to the same category. However, the method can be sensitive to the variables and the partition of the population. As a robustness check, we performed a cluster analysis with all 288 subjects pooled together. We found the same two qualitative conclusions as in the analysis separated by timing: (i) a hump-shaped relationship between COR sequence and wandering and (ii) highest variance in wandering for subjects who perform the COR sequence with medium and low frequency. The analysis provides also two new insights. First, the most rational cluster (perfect COR, perfect Nash, almost no wandering) is mostly composed of subjects playing the sequential timing, whereas the clusters that never perform COR have a majority of players in the simultaneous timing. For the other clusters, there is a mix of subjects from both timings. Second, the model with all individuals endogenously separates subjects based mostly on the COR variable; it imposes very small variability in COR and allows large variability in wandering. Only for subjects with COR at 100% and COR at 0% it further separates them in two clusters, depending on their level of wandering. That is, among subjects who always perform COR and play Nash, there is a group who immediately understands the

game and another who necessitates to look more carefully. On the other extreme, among subjects who never perform COR and rarely play Nash, there is a group who gives up easily and another who tries hard. Details of the results can be found in Appendix A3.

7.5 Learning

One issue neglected so far is whether subjects learn to play this game. After all, the structure of the game is identical in all trials and subjects receive feedback after each one. Most importantly, subjects change roles so playing in a certain position may help them understand the strategy of their peers and how to best respond to it. The simplest way to study learning is to divide the sample into first- and second-half of trials and conduct the same analysis as we did previously on each subsample. This analysis reveals a substantial amount of learning over the course of the experiment. Nash choices increase significantly between the first and second half of the experiment for all roles and treatments of the simultaneous order and for roles 4 and above in all treatments of the sequential order. Interestingly, this increase is mainly the result of a change in lookup patterns and not so much of a change in how lookups are translated into choices. Indeed, the probability of performing the MIN lookup and the COR sequence increases very significantly over time. By contrast, the likelihood of transforming such lookups into equilibrium choices, $\Pr[\text{Nash} \mid \text{MIN}]$ and $\Pr[\text{Nash} \mid \text{COR}]$, increases much less dramatically and systematically. Finally, the proportion of action transitions among equilibrium players increases over time in the sequential order but not in the simultaneous, suggesting that repeated play facilitates learning about how to perform backward induction efficiently more than about how to eliminate dominated strategies efficiently. The results are confirmed in the regression analysis. Indeed, the inclusion of a dummy variable for late trials in the regressions of section 5.2 has a positive and significant impact on explaining equilibrium behavior in 5 out of 9 regressions. Details of the results can be found in Appendix A4.

7.6 Individual Analysis

Leading theories of limited attention (such as level k , cognitive hierarchy, and steps of dominance) emphasize differences in behavior resulting from different degrees of strategic sophistication. For example, according to level k theory, some subjects use a simple, non-strategic choice rule such as randomization between available actions (level 0). Others are minimally strategic and best respond to randomizers (level 1), others are more strategic and best respond to the best responders of randomizers (level 2), and so on. Following some of the literature on limited cognition (Costa-Gomes et al., 2001), we perform a structural estimation of individual behavior. Briefly, the methodology consists in assuming that each

subject has a type, which remains constant over the entire experiment. We consider 11 possible types that have received support in previous research (such as level 1, level 2, level 3, optimistic, pessimistic, Nash, etc.) and estimate by maximum likelihood the type that best fits the behavior of each individual. We conduct the analysis only for the baseline treatment and separately for the sequential and simultaneous timings.²⁹ The results are somewhat mixed. For the simultaneous order, the Bayesian model can classify 83% of the subjects: 55% as equilibrium players and 28% as level 1 or 2. For the sequential order, the classification is less accurate: 50% are Nash players and 8% are level 2 or 3. There are some important limitations in the individual analysis. First, the number of observations per individual is small and some data (behavior in roles 1 and 2) is not very informative. Second, because of such small number of relevant observations, we are bound to focus on choice data and ignore any attentional measure, even though we know it can be extremely helpful to understand the type of our subjects. Third and as discussed in section 7.5, there is learning in our experiment. It is possible that subjects change types and become more sophisticated over the course of the experiment and the model cannot accommodate such changes. Last, the behavioral theories on which types are based (dominance, level k) predict identical behavior across timings. However, this is empirically not the case. Therefore, even though the analysis is instructive and encouraging, it is not surprising that given the nature of the problem the available data and the behavior of our subjects, the results are not as sharp as one would hope. Details of the analysis and results can be found in Appendix A5.

8 Conclusion

In this paper, we have studied equilibrium behavior in dominance solvable games. We have considered simultaneous and sequential games with identical equilibrium predictions. We found that Nash compliance was slightly higher in sequential than in simultaneous for the baseline treatment and significantly higher for the complex treatments. Indeed, as the difficulty of performing the iterated elimination of dominated strategies increased, the cue provided by the sequential order of moves (which enables the backward induction argument) became more important to find the equilibrium action. Our study also confirms the

²⁹This analysis is possible because our payoff structure allows to disentangle between Nash equilibrium and the different cognitive levels of strategic sophistication. Indeed payoffs are constructed in a way that only role 1 (the one with the dominant strategy) plays the equilibrium action if she best responds to uniform random behavior of her rival (level 1). Role 2 plays the equilibrium if she behaves as level 2 or above. Role 3 plays the equilibrium if she behaves as level 3 or above. We also consider a payoff-variant such that both role 1 and role 2 play the equilibrium action if they best respond to uniform random behavior of other players (level 1). Role 3 plays the equilibrium if she behaves as level 2 or above, and so on. This variant requires a lower degree of sophistication.

value of attentional data in revealing the underlying processes leading to choices and in predicting behavior. A subject who looks at all the payoffs necessary to compute the equilibrium and looks at payoffs in the order predicted by elimination of dominated strategies plays Nash with extremely high probability (around 90% of the time). By contrast, a subject who does not look at all the payoffs necessary to compute the equilibrium and does not look at payoffs in the order predicted by elimination of dominated strategies plays Nash less often than predicted by random choice (between 10% and 50% of the time). We also learned from attentional data that different timings trigger different reasoning algorithms. Among the individuals who do play the equilibrium action, subjects in the sequential order look more efficiently and systematically (less wandering and more correct sequences) than subjects in the simultaneous order. Some subjects know how to solve the game, others need to learn it. By being cued or by trying to rationalize the behavior of past movers, a subject may learn faster or learn something she would not learn in the simultaneous version. Finally, it is interesting to note that our study shows that many of our subjects are not limited in their computational abilities once they understand the logic of the game. Failure to play Nash is often due to an inconclusive information search for the key of the game (the player with a dominant strategy).

In future research, we would like to investigate more deeply the cognitive difficulties that subjects face to find the player with a dominant strategy and how equilibrium choice is tremendously facilitated once this player and strategy are identified. In particular, it would be interesting to know if directing the attention of our subjects to that player can have a long lasting effect on choice. Naturally, the challenge is to devise an ecologically valid mechanism which is powerful yet subtle so that it avoids demand effects from the experimenter. More generally, we believe that choice and non-choice data are strong complementary measures and that experimental research in that direction will improve our understanding of the breadth and the limits of human cognition.

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Appendix A. Supplementary analysis (intended as online supporting material)

A1. Treatment [R]

Recall that the simultaneous order of treatments [R] and [S] are identical: in both cases (and contrary to treatment [B]), the role with a dominant strategy is not necessarily displayed in the rightmost position of the screen. In Table 10, we display the basic statistics for the simultaneous order of treatment [R]

	Role 1	Role 2	Role 3	Role 4	obs. per role
Pr. Nash play	.99	.83	.56	.53	162

	Role 3	Role 4
Pr[MIN]	.62	.56
Pr[Nash MIN]	.81	.76
Pr[Nash notMIN]	.13	.23

	Role 3	Role 4
Pr[COR]	.61	.53
Pr[Nash COR]	.84	.85
Pr[Nash notCOR]	.13	.16

transitions (all)	Role 3	Role 4
action	.53	.56
payoff	.41	.43
non-adjacent	.06	.01

transitions (cond. on role 1)	Role 3	Role 4
action	.76	.91
payoff	.16	.09
non-adjacent	.08	.00

Table 10: Treatment [R] (simultaneous): general statistics

The data in Table 10 is very similar to that in the simultaneous order of treatment [S] presented in Tables 3, 4, 5, 6 and 7. We have a slightly higher level of Nash compliance for role 4 in treatment [R], due to higher rates of MIN lookups and COR sequence, and no noticeable differences for role 3. As in all other treatments, the ratio between action and payoff transitions for Nash players is around 1.5 and dramatically increases when we condition on looking at role 1. Overall, treatment [R] confirms the findings in the simultaneous order of the complex treatments described in the main text.

As discussed in section 2.3 and contrary to all other treatments, the sequential order of treatment [R] is significantly simpler than the simultaneous counterpart, which is why we relegate the analysis of this treatment to the appendix. In Table 11 we present the percentage of equilibrium behavior as a function of the subject's role (role) and the role of

the subject with the dominant strategy (dominant). It also displays the number of steps of dominance required to find the equilibrium (# steps).³⁰

Role	Dominant	# steps	% Equil.
1	1	0	.97
2	1	1	.92
3	1	2	.75
4	1	3	.58
1	2	0	1.0
2	2	0	1.0
3	2	1	.89
4	2	2	.69
1	3	0	1.0
2	3	1	.83
3	3	0	.98
4	3	1	.69
1	4	0	.94
2	4	1	.92
3	4	2	.83
4	4	0	1.0

Table 11: Treatment **[R]** (sequential): equilibrium choices

When role 1 has the dominant strategy (rows 1-4), equilibrium choices are very similar to the previous treatments (close to 1 for roles 1 and 2, lower for role 3 and still lower for role 4). However, when another role has the dominant strategy, the aggregate level of equilibrium compliance is significantly higher than before. Indeed, roles 1 and 2 observe the action of role 4, so they still consistently best respond to that action, independently of which role has the dominant strategy (rows 5, 6, 9, 10, 11 and 12). Role 3 also plays the equilibrium significantly more frequently if roles 2 or 3 have the dominant strategy, since she needs to perform one or no steps of dominance (rows 7 and 11), and the same is true although to a lesser extent for role 4. Overall and just as we expected, there is significantly more equilibrium behavior in this treatment. The comparative statics are also in line with previous findings: equilibrium compliance is inversely related to the number of steps of dominance required to determine the optimal action.

³⁰Equilibrium corresponds to the best response to the *observed* actions of predecessors (whether they played Nash or not).

A2. Other measures of lookup occurrence

Previous research in the alternate bargaining offers game (Camerer et al. (1993); Johnson et al. (2002)) suggests that subjects who play off-equilibrium tend to exhibit a more self-centered behavior, with a majority of lookups in their own payoff-cells. To determine if similar biases occur in our experiment, we look at two other measures of lookup occurrence. First, the proportion of lookups on the subject's own payoff matrix, *own lookups*, with the idea that self-centeredness is an indication of insufficiently strategic thinking. Second, the proportion of observations with at least one lookup in the matrix of role 1, *lookups role 1*. This payoff matrix is often far away from the subject's own payoff matrix and yet it is key to initiate the elimination of dominated strategies. Table 12 summarizes the findings.

SIMULTANEOUS

	Role 3				Role 4			Role 5	Role 6
	[B ₃]	[B ₄]	[S]	[E]	[B ₄]	[S]	[E]	[E]	[E]
<i>own lookups</i>									
Nash	.39	.42	.38	.35	.33	.32	.33	.25	.22
Not Nash	.58	.55	.59	.54	.48	.50	.52	.46	.47
<i>lookups role 1</i>									
Nash	.82	.80	.84	.94	.89	.84	.81	.78	.69
Not Nash	.47	.25	.27	.15	.27	.20	.06	.16	.16

SEQUENTIAL

	Role 3				Role 4			Role 5	Role 6
	[B ₃]	[B ₄]	[S]	[E]	[B ₄]	[S]	[E]	[E]	[E]
<i>own lookups</i>									
Nash	.42	.37	.33	.30	.38	.46	.29	.32	.38
Not Nash	.56	.55	.48	.46	.52	.57	.39	.42	.47
<i>lookups role 1</i>									
Nash	.89	.88	.93	.93	.85	.87	.92	.85	.79
Not Nash	.45	.26	.25	.18	.43	.29	.14	.14	.36

Table 12: Lookup behavior in trials where subjects do and do not play Nash.

Perhaps not surprisingly, the subject's own payoff is a focal point that needs to be overcome when thinking strategically. In trials consistent with Nash play, subjects spend 22 to 46 percent of the time in their own matrix. By contrast, in trials not consistent with Nash play, subjects spend 42 to 59 percent of the time in that same matrix. The difference

is similar across roles and significant for all treatments when we pool roles together.

The difference in the likelihood of looking at role 1’s payoff matrix is more striking. In trials where the equilibrium strategy is reached, subjects fail to look at the crucial payoff matrix of role 1 less than 20% of the time (except for role 6). By contrast, in trials where the equilibrium strategy is not reached, subjects miss that matrix 50% to 90% of the time. Again, the difference is more significant for higher roles and more complex treatments. The result is consistent with Tables 4 and 5, and suggests that many subjects who do not look at all the MIN cells and do not perform the COR sequence are individuals who miss the matrix of role 1 all together.

A3. Other cluster analysis

We present here the results of the cluster analysis performed on all 288 subjects of the sequential and simultaneous order pooled together. Contrary to the analysis in section 6, we do not constrain the maximum number of clusters because the BIC is substantially smaller with 5 clusters or less than with 6 clusters or more. Without such constraint, the model that maximizes BIC has eight clusters.

Table 13 presents the same information as Table 9 using the entire subject pool (in parenthesis, we report for each cluster the number of subjects that play the sequential and the simultaneous timing respectively). Figure 3 provides a graphical representation of the eight clusters.

ALL SUBJECTS

Cluster	A'	B'	C'	D'	E'	F'	G'	H'
%-cor	0.0	0.0	11.7	33.7	65.5	86.1	100	100
pre-cor	1.4	3.6	4.5	3.6	3.3	2.3	2.4	0.7
# subjects [†]	20 (7, 13)	24 (9, 15)	25 (14, 11)	34 (11, 23)	63 (35, 28)	58 (26, 32)	30 (14, 16)	34 (28, 6)
% Nash	27.8	34.9	40.4	44.3	68.5	86.1	97.0	97.7
% MIN	0.0	4.1	21.6	39.2	66.4	83.0	89.3	92.3

[†] Number of subjects from sequential and simultaneous timing in parenthesis.

Table 13: Summary statistics by cluster for the entire pool.

As we can notice from Figure 3, subjects in clusters C', D', E' and F' are grouped mainly by their COR sequence, allowing large differences in wandering. Wandering is slightly lower for those who perform COR (and play Nash) more often. Clusters A' and B' are lost. They never perform the COR sequence (and rarely play Nash) and they either

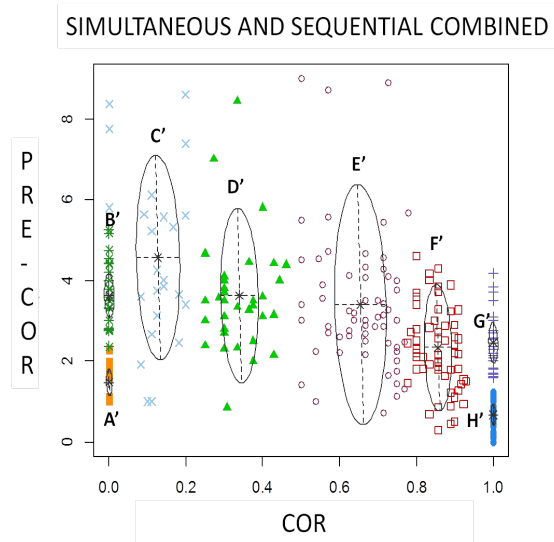


Figure 3 Clusters for the entire sample.

give up fast (A') or try somewhat harder (B'). On the other extreme, clusters G' and H' always look at COR and play Nash. The difference is whether they know right away where to look to perform the elimination of dominated strategies (H') or if it takes them some time to figure it out (G'). As a last note, in the previous version we also performed the cluster analysis with the data from treatment [B] only, and found similar conclusions: four clusters for each order (with close to one, close to zero and two intermediate levels of COR) and a hump-shaped relationship COR vs. wandering under both timings. This suggests that the clustering of individuals is robust across treatments and across orders.

A4. Learning

We present the same information as in the tables of the main text except that we divide the sample into early trials (first half) and late trials (second half). Table 14 presents for roles 3 and above the analogue of Table 3, that is, the proportion of equilibrium choices by treatment, timing and role. We notice the increase in equilibrium behavior which is large in magnitude and highly statistically significant, with the exception of role 3 in sequential order.

Tables 15 and 16 present the analogue of Tables 4 and 5, that is, the information on MIN lookups and COR sequence split between the “early” and “late” trials. There is a significant increase in MIN lookups and COR sequence, except for role 3 in sequential order, where the increase is more moderate. The probability of transforming MIN and

		Prob. of Nash play: early / late			
		Role 3	Role 4	Role 5	Role 6
SIM	[B ₃]	.72 / .88***	—	—	—
	[B ₄]	.59 / .81***	.47 / .76***	—	—
	[S]	.44 / .73***	.27 / .52***	—	—
	[E]	.44 / .70***	.35 / .65***	.37 / .57**	.22 / .50***
SEQ	[B ₃]	.76 / .82	—	—	—
	[B ₄]	.76 / .92***	.56 / .74***	—	—
	[S]	.80 / .80	.49 / .74***	—	—
	[E]	.57 / .70	.46 / .76***	.44 / .65**	.37 / .50

Difference early / late significant at the * 10%, ** 5%, *** 1% level

Table 14: Probability of equilibrium choice by role, order of moves and treatment.

COR into equilibrium choices increases much less over time and not systematically across treatments. Also, the increase is typically more pronounced in simultaneous than in sequential. Part of the reason for the moderate change is that the conditional likelihood of Nash play given MIN and COR is already very high at the beginning of the experiment, severely limiting the amount by which it can increase.

SIMULTANEOUS

	Role 3				Role 4			Role 5	Role 6
	[B ₃]	[B ₄]	[S]	[E]	[B ₄]	[S]	[E]	[E]	[E]
Pr[MIN]	.61/.78	.49/.68	.38/.61	.48/.63	.44/.69	.22/.42	.32/.54	.35/.52	.14/.41
Pr[Nash MIN]	.82/.97	.83/.93	.71/.92	.85/1.0	.77/.96	.78/.91	.88/1.0	.79/.86	.76/.91
Pr[Nash notMIN]	.55/.56	.36/.57	.28/.42	.07/.20	.23/.30	.13/.23	.11/.24	.14/.27	.13/.22

SEQUENTIAL

	Role 3				Role 4			Role 5	Role 6
	[B ₃]	[B ₄]	[S]	[E]	[B ₄]	[S]	[E]	[E]	[E]
Pr[MIN]	.68/.75	.56/.59	.74/.79	.56/.67	.48/.67	.42/.58	.39/.72	.32/.63	.24/.46
Pr[Nash MIN]	.88/.94	.95/.97	.97/.95	.93/.94	.85/.83	.88/.92	.86/.97	.88/.97	.77/.92
Pr[Nash notMIN]	.50/.47	.51/.84	.33/.24	.13/.22	.29/.56	.21/.50	.21/.20	.24/.10	.24/.14

Table 15: Nash choice based on MIN lookup split into early / late trials

Next, we present in Tables 17 and 18 the analogue of Tables 6 and 7 (different types of transitions among equilibrium players) split between the first half and second half of trials. Recall from Table 6 that the main difference across timings is that the proportion of action

SIMULTANEOUS

	Role 3				Role 4			Role 5	Role 6
	[B ₃]	[B ₄]	[S]	[E]	[B ₄]	[S]	[E]	[E]	[E]
Pr[COR]	.53/.80	.44/.69	.42/.67	.46/.63	.40/.69	.21/.46	.26/.54	.26/.44	.11/.37
Pr[Nash COR]	.90/.97	.94/.95	.82/.90	.88/1.0	.86/.97	.88/.95	.93/1.0	1.0/.96	1.0/.95
Pr[Nash notCOR]	.51/.52	.32/.52	.17/.37	.07/.20	.22/.29	.11/.16	.15/.24	.15/.27	.13/.24

SEQUENTIAL

	Role 3				Role 4			Role 5	Role 6
	[B ₃]	[B ₄]	[S]	[E]	[B ₄]	[S]	[E]	[E]	[E]
Pr[COR]	.66/.72	.68/.81	.73/.78	.54/.65	.44/.67	.41/.68	.39/.74	.32/.65	.20/.44
Pr[Nash COR]	.89/.96	.95/.98	.98/.97	.97/.97	.89/.92	.91/.95	.86/.98	.88/.94	.82/.96
Pr[Nash notCOR]	.49/.46	.37/.67	.32/.22	.12/.21	.30/.39	.21/.31	.21/.14	.24/.11	.26/.13

Table 16: Nash choice based on COR transitions split into early / late trials

transitions is substantially higher in sequential than in simultaneous. This difference is exacerbated over the course of the experiment, especially in roles 4, 5 and 6. It suggests that repetition of the game facilitates learning about how to perform backward induction efficiently (sequential) but not so much about how to perform elimination of dominated strategies efficiently (simultaneous). The proportion of action transitions conditional on having reached the matrix of role 1 also increases over the course of the experiment, although not as dramatically and systematically. Indeed, both in the first and second half of the experiment, action transitions become overwhelmingly prevalent once the matrix of role 1 has been looked at.

Finally, we replicate the regression analysis of Table 8 adding a dummy variable that takes value 1 for the observations in the second-half of the trials (*late*). The results are reported in Table 19. For treatments [B] and [S], the *late* variable is positive and statistically significant in 4 out of 5 regressions whereas for treatment [E] it is significant only in 1 out of 4 regressions. These regressions confirm the result that subjects learn how to play the equilibrium over the course of the experiment.

A5. Individual analysis

We perform a structural estimation of individual behavior in treatment [B]. Following Costa-Gomes et al. (2001), we assume that subjects have a *type* that is drawn from a common prior distribution, and that this type remains constant over all trials.³¹ The

³¹Given the documented learning, this is unsatisfactory. Unfortunately, we do not have enough observations to perform an individual estimation if we use only the last 12 trials.

SIMULTANEOUS

	Role 3				Role 4			Role 5	Role 6
	[B ₃]	[B ₄]	[S]	[E]	[B ₄]	[S]	[E]	[E]	[E]
action	.53/.63	.55/.57	.53/.59	.51/.54	.56/.58	.58/.53	.49/.56	.52/.53	.53/.49
payoff	.47/.37	.41/.38	.42/.36	.39/.43	.42/.41	.41/.45	.45/.39	.44/.43	.46/.49
non-adjacent	—	.05/.05	.05/.05	.10/.03	.02/.01	.01/.02	.06/.05	.04/.04	.01/.02

SEQUENTIAL

	Role 3				Role 4			Role 5	Role 6
	[B ₃]	[B ₄]	[S]	[E]	[B ₄]	[S]	[E]	[E]	[E]
action	.72/.82	.62/.78	.72/.74	.49/.68	.67/.81	.77/.91	.63/.76	.66/.75	.70/.89
payoff	.28/.18	.31/.19	.21/.19	.45/.22	.31/.16	.20/.07	.28/.15	.30/.17	.27/.09
non-adjacent	—	.07/.03	.07/.07	.06/.10	.02/.03	.03/.02	.09/.09	.04/.08	.03/.02

Table 17: Transitions for Nash players split into early / late trials

SIMULTANEOUS

	Role 3				Role 4			Role 5	Role 6
	[B ₃]	[B ₄]	[S]	[E]	[B ₄]	[S]	[E]	[E]	[E]
action	.79/.89	.81/.82	.71/.91	.67/.78	.83/.92	.93/.93	.75/.95	.87/.83	.97/.80
payoff	.21/.11	.14/.13	.20/.08	.21/.17	.16/.07	.07/.06	.17/.03	.07/.12	.03/.18
non-adjacent	—	.06/.05	.09/.01	.12/.05	.01/.01	.00/.01	.08/.02	.06/.05	.00/.02

SEQUENTIAL

	Role 3				Role 4			Role 5	Role 6
	[B ₃]	[B ₄]	[S]	[E]	[B ₄]	[S]	[E]	[E]	[E]
action	.77/.85	.77/.85	.79/.85	.70/.79	.80/.85	.81/.92	.77/.85	.84/.92	.80/.89
payoff	.23/.15	.15/.12	.14/.12	.22/.11	.19/.13	.17/.06	.17/.09	.14/.05	.19/.08
non-adjacent	—	.07/.02	.07/.03	.08/.10	.01/.02	.02/.02	.04/.06	.02/.03	.01/.03

Table 18: Transitions for Nash players conditional on reaching the payoff matrix of role 1 split into early / late trials

	Role 3 [B ₃]	Role 3 [B ₄]	Role 3 [S]	Role 3 [E]	Role 4 [B ₄]	Role 4 [S]	Role 4 [E]	Role 5 [E]	Role 6 [E]
<i>seq</i>	-0.188 (.136)	0.290 (.170)	0.219 (.198)	0.10 (.257)	-0.008 (.146)	0.616** (.193)	-0.037 (.243)	-0.015 (.228)	-0.010 (.221)
<i>min</i>	0.92*** (.179)	1.15*** (.194)	1.68*** (.238)	2.43*** (.307)	1.19*** (.171)	1.96*** (.249)	2.17*** (.317)	1.67*** (.315)	1.67*** (.331)
<i>total-t</i>	-0.005 (.003)	-0.001 (.004)	-0.007 (.006)	0.005 (.006)	0.002 (.003)	-0.012** (.004)	-0.000 (.005)	-0.001 (.005)	-0.000 (.005)
<i>action-t</i>	7.49*** (1.55)	7.18*** (1.38)	7.81*** (1.76)	2.73 (2.32)	6.36*** (1.24)	8.32*** (1.64)	5.09* (2.57)	7.74** (2.82)	7.19*** (2.13)
<i>late</i>	0.25 (.140)	0.56*** (.170)	0.20*** (.191)	0.65* (.269)	0.31* (.149)	0.51** (.184)	0.43 (.250)	0.10 (.221)	0.00 (.223)
<i>const.</i>	0.07 (.147)	-0.78*** (.199)	-1.04*** (.253)	-1.75*** (.351)	-0.98*** (.175)	-1.53*** (.219)	-1.34*** (.296)	-1.24*** (.285)	-1.36*** (.271)
# obs.	575	430	324	213	431	324	216	215	214
Pseudo R^2	0.239	0.323	0.435	0.575	0.316	0.465	0.526	0.424	0.400

Standard errors in parentheses. * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Table 19: Probit regression of Nash behavior as a function of lookups by role and treatment

subject’s behavior is determined by her type, possibly with some error. We also assume that subjects treat each trial as strategically independent. In specifying the possible types, we use some of the general behavioral principles that have been emphasized in the literature as being most relevant. We consider the following set of types. Pessimistic [*Pes*] (subjects who maximize the minimum payoff over the rival’s decision), Optimistic [*Opt*] (subjects who maximize the maximum payoff over the rival’s decision), Sophisticated [*Sop*] (subjects who best respond to the aggregate empirical distribution of choices) and Equilibrium [*NE*] (subjects who play Nash). We also include the types corresponding to the steps of dominance and level k theories: $L_1, L_2, L_3, L_4, D_1, D_2, D_3$. This set of 11 types is chosen to be large and diverse enough to accommodate a variety of possible strategies without overly constraining the data analysis, yet small enough to avoid overfitting.³² Each of our types predicts an action for each role in each game.

Econometric model

For the econometric analysis we focus exclusively on decisions. In order to determine

³²Costa-Gomes and Crawford (2006) conducted a second specification test that included “pseudotypes” (types constructed from each of the subjects’ empirical behavior in their experiment) to learn if the behavior of some subjects could be better explained by types omitted in their original specification. Their results suggested no empirically significant omitted types.

how distinctive the behavior of each type is, we first compute the matrix of correlations of choices for the different types.³³ More precisely, for each observation of an individual and given the role and game, we determine whether the action chosen by the subject is consistent with each of the considered types (coded as 1) or not (coded as 0). Naturally, actions will typically be consistent with a subset of types. We then sum up the 24 observations of the individual and calculate the partial correlation matrix for all our types across all individuals. Since D_3 and L_4 subjects play Nash in all games, they are indistinguishable from $[NE]$, so we omit them from the analysis. The results are presented in Table 20 separately for the simultaneous and sequential orders of moves.

SIMULTANEOUS										SEQUENTIAL									
	L_1	L_2	L_3	NE	D_1	D_2	Sop	Pes	Opt		L_1	L_2	L_3	NE	D_1	D_2	Sop	Pes	Opt
L_1	1.0									L_1	1.0								
L_2	.24	1.0								L_2	.33	1.0							
L_3	-.63	.32	1.0							L_3	-.42	.45	1.0						
NE	-.72	.17	.93	1.0						NE	-.64	.23	.90	1.0					
D_1	.78	.67	-.29	-.40	1.0					D_1	.74	.69	.04	-.24	1.0				
D_2	-.43	.56	.91	.79	-.05	1.0				D_2	-.24	.64	.92	.77	.22	1.0			
Sop	-.71	.20	.95	.98	-.36	.83	1.0			Sop	-.64	.23	.90	1.0	-.24	.77	1.0		
Pes	.16	-.17	-.12	-.07	-.02	-.10	-.04	1.0		Pes	.39	.08	-.13	-.14	.15	-.05	-.14	1.0	
Opt	.96	.25	-.52	-.61	.72	-.38	-.60	.11	1.0	Opt	.94	.30	-.35	-.55	.68	-.25	-.55	.33	1.0

Table 20: Matrix of types correlation (shaded cells for correlations > 0.90)

As we already knew from section 7.1, $[Sop]$ play Nash in almost all games and roles, hence the high correlation with $[NE]$. $[Opt]$ are also rarely separated from L_1 and so are D_2 from L_3 . Given these correlations, for the econometric analysis we keep 6 types: $\tau \in \{Pes, D_1, L_1, L_2, L_3, NE\}$.

We conduct a maximum likelihood error-rate analysis of subjects' decisions with the 6 types of players discussed above using the econometric model of Costa-Gomes et al. (2001). A subject of type τ is expected to make a decision consistent with type τ , but in each game makes an error with probability $\varepsilon_\tau \in (0, 1)$. This error rate may be different for different types. Given that our games have only two actions, for a subject of type τ the probability of taking the action consistent with type τ is $(1 - \varepsilon_\tau)$ and the probability of taking the other action is ε_τ . We assume errors are i.i.d. across games and across subjects.

Let $i \in I = \{1, 2, \dots, 72\}$ index the subjects for each order of moves. Denote by N be

³³Ideally, we would like to classify individuals according to choices *and* lookups. In a previous version we performed such exercise. However, the results were not robust mainly because we do not have enough observations to estimate such a large number of parameters.

the total number of trials (24 in our experiment) and by x_τ^i the total number of actions consistent with type τ for subject i . The probability of observing a particular sample with x_τ^i type τ decisions when subject i is type τ can be written as:

$$L_\tau^i(\varepsilon_\tau \mid x_\tau^i) = [1 - \varepsilon_\tau]^{x_\tau^i} [\varepsilon_\tau]^{N-x_\tau^i}$$

Let p_τ denote the subjects' common prior type probabilities, with $\sum_\tau p_\tau = 1$. Weighting the above equation by p_τ , summing over types, taking logarithms, and summing over players yields the log-likelihood function for the entire sample:

$$\ln L(p, \varepsilon \mid x) = \sum_{i \in I} \ln \sum_\tau p_\tau [1 - \varepsilon_\tau]^{x_\tau^i} [\varepsilon_\tau]^{N-x_\tau^i}.$$

With 6 types, we have 11 parameters to estimate: 5 independent probabilities and 6 error rates.

Estimation results

Given the behavioral differences between sequential and simultaneous timings, we compute parameter estimates separately for the two cases. Under our assumptions, maximum likelihood yields consistent parameter estimates (the complexity of the estimation made it impractical to compute standard errors). Table 21 shows the estimated type probabilities and type-dependent error rates.

SIMULTANEOUS			SEQUENTIAL		
Type τ	Prob. p_τ	Error ε_τ	Type τ	Prob. p_τ	Error ε_τ
<i>NE</i>	.60	.06	<i>NE</i>	.59	.04
<i>L₃</i>	.02	.02	<i>L₃</i>	.12	.12
<i>L₂</i>	.22	.11	<i>L₂</i>	.18	.16
<i>L₁</i>	.15	.25	<i>L₁</i>	.07	.26
<i>D₁</i>	.00	.87	<i>D₁</i>	.05	.41
<i>Pes</i>	.01	.04	<i>Pes</i>	.00	.64

Table 21: Estimated type probabilities in simultaneous and sequential timings

The distribution of types is similar under both timings, with more than half of the observations corresponding to [*NE*], and the rest distributed among *L₁*, *L₂* and *L₃*. *D₁* and [*Pes*] are virtually non-existent. Behavior is more sophisticated in the sequential than in the simultaneous timing, with more *L₃* and fewer *L₂* and *L₁* types. Finally, the errors are small for three out of four of the relevant types (*L₂*, *L₃*, *NE*) and somewhat higher for the last one (*L₁*). Overall, the estimation is stable and reasonably accurate.

Given those estimates, we can also characterize the model’s implications for the types of individual subjects. To do this, we calculate the Bayesian posterior conditional on each subject’s decision history. Formally, let x^i be the sequence of actions taken by an individual. By Bayes rule, the probability of this individual being of type τ given x^i is:

$$\Pr(\tau \mid x^i) = \frac{\Pr(x^i \mid \tau) \times p_\tau}{\sum_\tau \Pr(x^i \mid \tau) \times p_\tau}$$

Naturally, the number of subjects that can be classified into a type depends on how harsh is the requirement for a classification. In Table 22 we report the results for the 72 subjects under each timing when a subject is classified into a given type if the posterior estimate of that type, $\Pr(\tau \mid x_i)$, is highest and at least 0.7, 0.8, and 0.9, respectively.

SIMULTANEOUS				SEQUENTIAL			
Type τ	min. criterion $\Pr(\tau \mid x_i)$			Type τ	min. criterion $\Pr(\tau \mid x_i)$		
	0.7	0.8	0.9		0.7	0.8	0.9
<i>NE</i>	42	40	40	<i>NE</i>	40	36	36
<i>L₃</i>	–	–	–	<i>L₃</i>	3	1	1
<i>L₂</i>	13	13	11	<i>L₂</i>	9	8	4
<i>L₁</i>	10	10	9	<i>L₁</i>	4	2	–
<i>D₁</i>	–	–	–	<i>D₁</i>	2	2	2
<i>Pes</i>	1	1	1	<i>Pes</i>	–	–	–
N/C	6	8	11	N/C	14	23	29

Table 22: Individual classification in types (N/C = not classified)

In simultaneous, 55% of subjects are classified as equilibrium players and 28% as L_1 or L_2 , even under the tightest requirement of 0.9 probability of choices fitting a type. The individual classification is much less accurate in sequential (50% classified as equilibrium players and only 7% as some level k) and tilted towards higher sophistication (fewer L_1 and L_2 and more L_3 and unclassified subjects).

Appendix B.

(intended as online supporting material)

B1. Sample of instructions (simultaneous treatment)

Thanks for participating in this experiment on group decision-making. During the experiment we would like to have your undistracted attention. Do not open other applications on your computer, chat with other students, use headphones, read, etc. Make sure to turn your phone to silent mode and not use it during the experiment.

You will be paid for your participation in cash at the end of the experiment. Different participants may earn different amounts. What you earn depends partly on your decisions, partly on the decisions of others, and partly on chance. The entire experiment will take place through computer terminals, and all interaction between participants will take place through the computers. Do not talk or in any way try to communicate with other participants during the experiment.

We will start with a brief instruction period. During the instruction period, you will be given a complete description of the experiment and will be shown how to use the computers. It is very important that you listen carefully and fully understand the instructions since your decisions will affect your earnings. You will be asked some review questions after the instructions, which have to be answered correctly before we can begin the experiment. If you have any questions during the instruction period, raise your hand and your question will be answered so everyone can hear. If any difficulties arise after the experiment has begun, raise your hand, and an experimenter will come and assist you.

At the end of the session, you will be paid the sum of what you have earned in all matches, plus the show-up fee of \$5. Your earnings during the experiment are denominated in tokens. Depending on your decisions, you can earn more or less tokens. At the end of the experiment, we will count the number of tokens you have and you will be paid \$1.00 for every 30 tokens. Everyone will be paid in private and you are under no obligation to tell others how much you earned.

The experiment consists of 24 matches. In each match, you will be grouped with either two or three other participants, which means there will be either 3 or 4 participants in a group. Group size will be different for each match. Since there are 12 participants in today's session, in a match there will be either 3 groups of 4 participants or 4 groups of 3 participants. You are not told the identity of the participants you are grouped with. Your payoff depends only on your decisions, the decisions of the participants you are grouped with and on chance. What happens in the other groups has no effect on your payoff and vice versa. Your decisions are not revealed to participants in the other groups.

We will present the game using screenshots. Your instruction package includes two separate pages, which are screenshots of computer screens. Look at the first page. I will now describe the screenshot in "Display 1". Do you have the Display 1 in front of you? Raise your hand high if you do. If you don't raise your hand we will come around and guide your attention to the separate Display 1 page.

In this match each participant is grouped with two other participants. At the beginning of the match, the computer randomly assigns a role to each of the three members in your group as RED or GREEN or BLUE. In each match, each role is asked to make a choice from two possible actions, X_R or Y_R for the subject in the RED role, X_G or Y_G for the subject in the GREEN role and X_B or Y_B for the subject in the BLUE role. You will choose an action without knowing which actions the other players in your group have chosen.

You will see a screen like the one in Display 1. In this example, you have the GREEN role. The screen says 'GREEN, please choose an action'. Your action can be either X_G or Y_G .

The payoffs you may obtain are the numbers inside the boxes in the left table. In this example, your payoff depends on your action (the rows, X_G or Y_G) and on the action of RED (the columns, X_R or Y_R). For example, if you choose Y_G and RED chooses X_R , then you will earn 80 tokens. If RED chooses Y_R instead, then you will earn 132 tokens.

DISPLAY 1

Subject ID: 3 - Match 1

Payoff of Green

	X_R	Y_R
X_G	66	72
Y_G	80	132

Payoff of Red

	X_B	Y_B
X_R	52	18
Y_R	96	112

Payoff of Blue

	X_G	Y_G
X_B	40	144
Y_B	66	94

Green, please choose an action:

X_G Y_G

Your History

Match	PART	ROLE	ACTIONS	PAYOFFS
1	PART 1			

Total Payoff: 0 (\$0.00)

Switch to Tabbed View

If you are Role RED, you will see a screen similar to Display 1 but it will read, 'RED, please choose an action'. RED must respond by clicking on the X_R or Y_R button. The payoffs RED may obtain are the numbers inside the middle table. In this example, the payoffs RED may obtain depend on his action and the action of BLUE. For example, if RED chooses Y_R and BLUE chooses X_B , RED will earn 96 tokens. Finally, the payoffs BLUE may obtain are the numbers inside the right table. Payoffs that BLUE may obtain depend on his action and the action of GREEN.

Once every member in the group has made a choice, the computer screen will display the actions for all members of your group and your payoff for the match. The payoff is added to your total. This will end the current match.

When a match is finished, we proceed to the next match. For the next match, the computer randomly reassigns all participants to a new group and to a new role. The new assignments do not depend in any way on the past decisions of any participant including you, and are done completely randomly by the computer. The assignments are independent across groups, across participants and across matches. This second match then follows the same rules as the first match with two exceptions. First, the payoffs inside the tables are now different. Second, in the new match you may be grouped with 3 (rather than 2) other participants. If you are grouped with three other participants the roles are "RED", "GREEN", "BLUE" and "ORANGE".

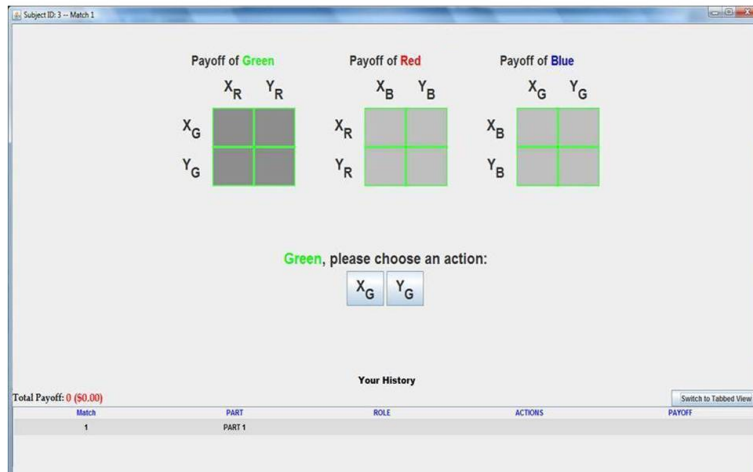
The same procedure continues for 24 matches, after which the experiment ends.

A history screen at the bottom will show a rolling history of your role in that match, the actions of all subjects in your group and your payoff.

Now turn to the Display 2. Do you have the Display 2 page in front of you? Raise your hand high if you do. If you don't raise your hand we will come around and guide you.

This is a similar game but the payoffs are now hidden in boxes. This is the type of screen you will observe during the experiment. In order to find out what your possible payoffs are, or what the other roles' payoffs are, you must move your mouse into the box that shows the payoff from a particular pair of actions in the table, and click-and-hold one of the mouse buttons. If you do not hold down the mouse button the payoff will disappear. When you move the mouse away from the box, the payoff will also disappear. If you move your mouse back into a box, click-and-hold, the exact same payoff will appear again. Clicking does not affect your earnings and you can look at as many of the possible payoffs as you care to, or as few, for as long or as briefly as you like. If you have trouble figuring out how to use the mouse to temporarily

Display 2



reveal the hidden payoffs during the experiment, raise your hand right away and we will come around and help you.

Are there any questions? If there are any problems or questions from this point on, raise your hand and an experimenter will come and assist you.

We will now begin a Practice session and go through one practice match to familiarize you with the computer interface and the procedures. The tokens accumulated in this match do not count towards your final dollar earnings. The practice match is similar to the matches in the experiment. During the practice match, please do not hit any keys until you are asked to, and when you enter information, please do exactly as asked. You are not paid for this practice match. At the end of the practice match you will have to answer some review questions.

[START game]

You now see the first screen of the experiment on your computer. Raise your hand high if you do.

At the top left of the screen, you see your subject ID. Please record that ID in your record sheet. You have been grouped by the computer with two other participants and assigned a role as RED or GREEN or BLUE, which you can see on your screen. The pair assignment and role will remain the same for the entire match. You can also see on the top left of the screen that you are in match 1.

You will see a screen similar to the Display 2 with the payoffs hidden in boxes. Please do not hit any key. Now, use your mouse button to reveal the payoffs in the different boxes. Familiarize yourself with the click-and-hold method. If you have problems revealing the payoffs raise your hand and we will come and assist you.

If you are Role BLUE, please select Y_B . Note that it does not matter which one you choose since you will not be paid for this round. You must wait for other participants in your group to make a choice. If you are Role GREEN, please select Y_G . If you are Role RED, please select X_R .

Once everyone in your group makes a choice, the computer screen will display the actions for all members of your group and your payoff for the match. Please spend some time familiarizing yourself with this screen.

Now click "Continue". The practice match is over. Please complete the review questions before we begin the paid session. Please answer all questions correctly and click to submit. The quiz will disappear from your screen.

Are there any questions before we begin with the paid session?

We will now begin with the 24 paid matches. If there are any problems or questions from this point on, raise your hand and an experimenter will come and assist you.

[START MATCH 1]

[After MATCH 24 read]:

This was the last match of the experiment. Your payoff is displayed on your screen. Your final payoff in the experiment is equal to your stock of tokens in the end converted into dollars plus the show-up fee of \$5. Please record this payoff in your record sheet and remember to CLICK OK after you are done.

We will pay each of you in private in the next room in the order of your Subject ID number. Remember you are under no obligation to reveal your earnings to the other participants. Please put the mouse on the side of the computer and do not use either the mouse or the keyboard. Please remain seated and keep the dividers pulled out until we call you to be paid. Do not converse with the other participants or use your cell phone. Thank you for your cooperation.

QUIZ

1. In this experiment, your payoffs are presented in boxes. Please choose the correct option:
 - i) Payoffs for all cells are always visible.
 - ii) Payoffs for all cells are hidden. They can be viewed by moving your mouse over the cell and clicking-and-holding one of the mouse buttons. There is no cost of opening a cell.
 - iii) Payoffs of some cells are hidden and payoffs of other cells are visible.
 - iv) Payoffs for all cells are hidden. They can be viewed by moving your mouse over the cell and clicking-and-holding one of the mouse buttons. However, tokens are subtracted from your payoff when you view a cell.

2. Look at Display 1. Suppose you are role GREEN. What will be your payoff if you choose X_G and RED chooses Y_R ?
 - i) 72
 - ii) 66
 - iii) It depends on what BLUE chooses

3. What will be the payoff of RED?
 - i) It depends of what BLUE chooses
 - ii) 96
 - iii) 112

4. What will be the payoff of BLUE?
 - i) 66
 - ii) 40 if BLUE chooses X_B and 66 if BLUE chooses Y_B
 - iii) 144 if BLUE chooses X_B and 94 if BLUE chooses Y_B

5. Look at Display 1. If actions chosen by all members of the group are X_G, Y_R, Y_B what will be the earnings of the three roles?
 - i) GREEN: 72 tokens, RED: 96 tokens, BLUE: 94 tokens.
 - ii) GREEN: 72 tokens, RED: 112 tokens, BLUE: 66 tokens.
 - iii) GREEN: 66 tokens, RED: 112 tokens, BLUE: 66 tokens.

6. Look at Display 2. What is your role and which game table hides your own payoffs
 - i) My role is GREEN and my payoffs are hidden in the middle table
 - ii) My role is BLUE and my payoffs are hidden in the right table
 - iii) My role is GREEN and my payoffs are hidden in the left table
 - iv) I cannot know what my role is yet.

B2. Payoff variants used in the experiment

Treatment [B]. There are 24 trials, with 6 different 4-player games and 6 different 3-player games, each played twice. All games are randomly intertwined except that two identical games are never played consecutively. The 12 payoff matrices are presented in Table 23 using the same presentation method as in Table 1. Nash equilibrium cells are highlighted in bold.

Game	Role 4		Role 3		Role 2		Role 1	
1	28	12	4	25	10	20	10	14
	10	24	20	5	25	10	20	26
2	15	25	38	18	14	36	34	26
	30	14	18	32	30	10	20	12
3	15	30	16	28	15	35	18	24
	24	16	30	20	32	10	10	12
4	15	25	8	18	14	6	10	14
	26	10	20	10	6	18	18	24
5	10	22	25	15	28	14	22	32
	18	10	15	30	12	24	14	20
6	32	16	16	34	18	6	10	8
	22	30	30	22	8	22	18	14
7			4	25	10	20	10	14
			20	5	25	10	24	30
8			6	22	12	28	18	12
			28	8	22	10	10	6
9			4	20	8	22	10	8
			15	5	25	10	26	22
10			22	8	10	28	12	10
			10	25	22	12	24	20
11			12	22	10	16	18	14
			28	10	18	10	10	8
12			4	25	10	25	12	20
			20	5	20	12	22	32

Table 23: Payoff variants in treatment [B]

Treatment [S]. There are 18 trials, with 9 different games, each played twice. All games are randomly intertwined except that two identical games are never played consecutively. The 9 payoff matrices are presented in Table 24, and we specify the order of play for each game in the sequential timing. Nash equilibrium cells are highlighted in bold. Only role 1 has a dominant strategy.

Game	Role and payoff matrix							
1	Role 2	Role 4		Role 1		Role 3		
	12 20	28	7	10	14	5	25	
	25	10	6	24	26	20	20	6
2	Role 4	Role 2		Role 1		Role 3		
	12 26	10	36	26	34	38	18	
	30	12	30	8	12	20	16	34
3	Role 4	Role 2		Role 3		Role 1		
	8	28	13	35	16	26	24	18
	24	8	32	14	30	14	10	12
4	Role 3	Role 1		Role 2		Role 4		
	8	28	14	18	12	24	34	10
	24	6	24	30	26	14	12	28
5	Role 1	Role 4		Role 2		Role 3		
	30	38	20	40	12	40	42	20
	16	21	44	22	34	14	22	36
6	Role 1	Role 3		Role 4		Role 2		
	22	28	12	34	8	32	18	42
	14	16	36	14	28	6	40	14
7	Role 3	Role 1		Role 4		Role 2		
	7	23	10	8	26	8	8	18
	20	6	24	18	10	20	24	6
8	Role 2	Role 3		Role 1		Role 4		
	12	34	36	16	32	24	6	24
	30	10	18	30	14	10	28	8
9	Role 2	Role 4		Role 3		Role 1		
	10	34	10	28	8	24	18	16
	30	12	22	12	27	9	8	7

Table 24: Payoff variants in treatment [S]

Treatment [E]. There are 18 trials, with 9 different games, each played twice. All games are randomly intertwined except that two identical games are never played consecutively. The 9 payoff matrices are presented in Table 25. Nash equilibrium cells are highlighted in bold. As in treatment [B], roles are in decreasing order from left (role 6) to right (role 1).

Game	Role 6		Role 5		Role 4		Role 3		Role 2		Role 1	
1	12	30	12	22	28	7	5	25	12	20	10	14
	26	12	28	10	6	24	20	6	25	10	26	20
2	28	7	8	28	12	26	38	18	10	36	26	34
	6	24	24	6	30	12	16	34	30	8	12	20
3	6	28	38	18	8	28	16	26	13	35	24	18
	32	8	16	34	24	8	30	14	32	14	10	12
4	8	28	8	30	34	10	8	28	12	24	14	18
	24	8	36	10	12	28	24	6	26	14	24	30
5	6	24	7	23	20	40	42	20	12	40	30	38
	28	8	20	6	44	22	22	36	34	14	16	21
6	28	12	12	24	8	32	12	34	18	42	22	28
	10	34	26	14	28	6	36	14	40	14	14	16
7	30	18	12	20	26	8	7	23	8	18	10	8
	16	36	25	10	10	20	20	6	24	6	24	18
8	14	36	36	22	6	24	36	16	12	34	32	24
	34	12	20	42	28	8	18	30	30	10	14	10
9	20	10	10	30	10	28	8	24	10	34	18	16
	8	26	34	12	22	12	27	9	30	12	8	7

Table 25: Payoff variants in treatment [E]

Treatment [R]. There are 18 trials, with 9 different games, each played twice. All games are randomly intertwined except that two identical games are never played consecutively. The 9 payoff matrices are presented in Table 26. Nash equilibrium cells are highlighted in bold. As in treatment [B], roles are in decreasing order from left (role 4) to right (role 1). For visual clarity, we add a rightmost column specifying the role with the dominant strategy.

Game	Role 4		Role 3		Role 2		Role 1		Dominant
1	28	7	10	14	10	25	6	20	Role 3
	6	24	26	20	20	12	25	5	
2	12	30	34	16	26	34	8	30	Role 2
	26	12	18	38	12	20	36	10	
3	8	28	16	26	13	35	24	18	Role 1
	24	8	30	14	32	14	10	12	
4	14	18	8	28	12	24	34	10	Role 4
	24	30	24	6	26	14	12	28	
5	22	44	30	38	14	34	42	20	Role 3
	40	20	16	21	40	12	22	36	
6	6	28	14	36	22	28	18	42	Role 2
	32	8	34	12	14	16	40	14	
7	26	8	7	23	8	18	10	8	Role 1
	10	20	20	6	24	6	24	18	
8	32	24	30	18	10	30	8	28	Role 4
	14	10	16	36	34	12	24	6	
9	10	28	18	16	10	34	8	24	Role 3
	22	12	8	7	30	12	27	9	

Table 26: Payoff variants in treatment [R]

B3. Method to determine the MIN set

The MIN set depends on the role, order of play, treatment and the actions consistent with Nash. In sequential, it also depends on the action of the player moving first (role T). We explain MIN in treatment [B₄] with the help of Table 27 (the logic is similar in the other treatments). The values in this table *are not* the payoffs from the game but, instead, code numbers for the cells used here to support the explanation of MIN.

	Role 4		Role 3		Role 2		Role 1		
	X_3	Y_3	X_2	Y_2	X_1	Y_1	X_4	Y_4	
X_4	1	2	X_3	5	6	X_2	9	10	
Y_4	3	4	Y_3	7	8	Y_2	11	12	
							X_1	13	14
							Y_1	15	16

Table 27: Support table to find MIN (values are not payoffs in the game but instead the cell codes given to facilitate the explanation of MIN below)

MIN for simultaneous. Take the convention that the Nash equilibrium is (X_4, X_3, X_2, X_1) . MIN for role 1 are cells 13, 14, 15 and 16, since opening this set enables role 1 to figure out her dominant strategy. MIN for role 2 are cells 13, 14, 15, 16, 9 and 11: opening 13, 14, 15 and 16 enables role 2 to know that X_1 is a dominant strategy for role 1 and then role 2 only needs to open cells 9 and 11, the cells in her payoff matrix corresponding to X_1 . Using the same logic we get that MIN for role 3 are cells 13, 14, 15, 16, 9, 11, 5 and 7 and MIN for role 4 are cells 13, 14, 15, 16, 9, 11, 5, 7, 1 and 3.

MIN for sequential. MIN for role T (role 4 in this treatment) is defined exactly as in the simultaneous order. MIN for the other roles depends on the action taken by role 4. Let us assume that role 4 chooses X_4 . MIN for role 1 are only cells 13 and 15: role 1 observes the action of role 4 so, in order to calculate her Nash strategy, she only needs to compare cells 13 and 15 in her payoff matrix. With an analogous reasoning we get that MIN for role 2 are cells 13, 15, 9 and 11 and MIN for role 3 are cells 13, 15, 9, 11, 5 and 7. Naturally, when we code MIN in each game we need to track which action corresponds to the Nash equilibrium $(X_t$ or $Y_t)$ and which action has been taken by role T .