Self-Excited, Alternating, High-Voltage Generation Using a Modified Electrostatic Influence Machine

MARKUS ZAHN
RICHARD L. GOSLIN
LARRY F. WICKS
Department of Electrical Engineering
University of Florida
Gainesville, Florida 32601
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The principles of the classic rotary Wimshurst machine, which produces dc high voltage by regenerative electrostatic induction, are used to develop a new alternating high-voltage machine. This device has no electrical inputs but can spontaneously produce either dc or three-phase ac high voltage, depending on external connections. The machine constructed by the authors produces voltages in excess of 18 kV with frequencies of operation up to 2 Hz. Improved design can extend the voltage and frequency operating ranges. Analysis is extended to include multi-phase, multi-frequency operation by modeling the device with a distributed equivalent circuit representation. The natural frequencies of the system are calculated where it is found that many overstable modes are present.

I. INTRODUCTION

The great advances in electrical engineering at the end of the nineteenth century were partly due to the development of reliable electric sources. The primary sources of electricity at this time were electrostatic influence machines which relied on regenerative electrostatic induction.[1] The most popular such generator was the rotating-disc Wimshurst machine which generated high voltage dc.[2] In contrast, these writers have developed an ac version of this classic device, which, to our knowledge, has not been previously known.

The description of "electrostatic" for these devices is a misnomer, as their most interesting aspect is the voltage buildup and discharge, definitely time varying phenomena. As will be shown here, the dc Wimshurst machine has voltage buildup at an exponential rate, with no external electrical excitations. As shown in Fig. 1(a), one version of the Wimshurst machine consists of two rotating circular nonconducting discs, upon which are fixed many metal strips. As these strips are rotated, they come in the vicinity of a conducting plate which is capacitively coupled to the strips. This plate acts as an inducer electrode, for if any charge is on this electrode, opposite charges are induced on the strips as they rotate by. A grounding brush makes contact with the strips when they are at this position to allow a net charge to be deposited on the strips. As the strips leave the grounding brush, they carry this net charge with them until they reach the collecting brush where the charge is given up. The voltage is generated spontaneously because of the reciprocal arrangement whereby the charge inducer near one disc is electrically tied to the collecting brush on the other disc. Any charge unbalance on the inducer, due to random fluctuations (noise) or possibly an initial charge purposely placed on it, induces opposite charges on the adjacent strips. As the charge is collected, it is communicated to the other inducer. Here the process is repeated, such that the net charge on the original inducer has been increased. This positive feedback results in the voltage buildup.

This device is analogous to Lord Kelvin's water dropper whereby two streams of falling water drops are responsible for the generation of high voltages with no external electrical excitations.[3–5] Here, we replace streams of falling water drops by rotating discs with conducting strips. Recent work with Kelvin's dynamo has shown that low-frequency, alternating high voltage can be generated by adding a third stream of falling water.[6–8] The authors have used this concept in

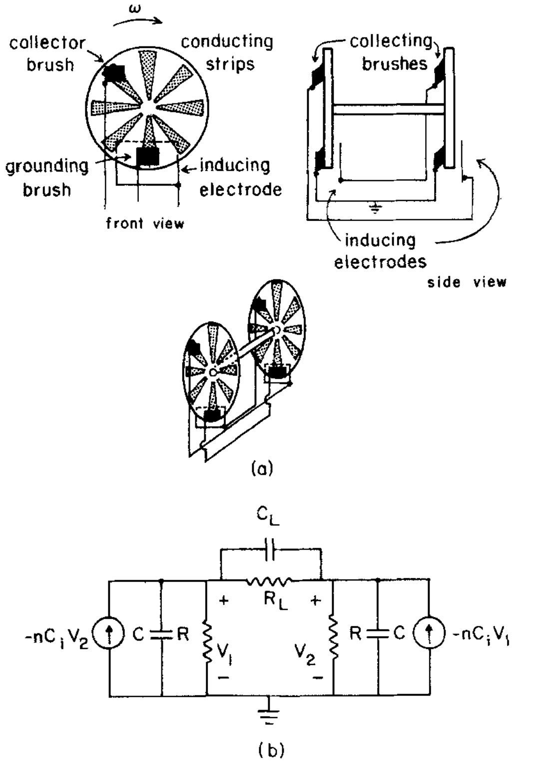

FIG. 1. Basic configuration for self-excited dc voltage buildup: (a) Wimshurst machine, (b) equivalent circuit.

redesigning the classic Wimshurst machine with a third disc to produce three-phase ac high voltage. This design eliminates the nuisance of working with water and improves the reliability and ease of operation. Further advantages of the ac Wimshurst machine include: (1) the elimination of the need to continually supply conducting material for the charge transport although now the shaft must be turned, whereas for the water dropper, the drops fell due to gravity; and (2) the fact that this device is limited in voltage only by the breakdown strength of the surrounding atmosphere, while the water dropper is also limited in voltage by the attraction of the drops to the ring electrodes.

This device has potential applications in such electrostatic processes as printing, painting, and particle precipitation, where an external high-voltage source is usually needed. In configurations similar to those presented here the particles themselves can generate the necessary high voltage, eliminating the need of an external power supply.

The analytical approach for this device as well as for all similar electrostatic influence machines is identical to that presented for the alternating version of Kelvin's water droppers,[8] where an equivalent circuit representation of distributed resistors, capacitors, and dependent sources is developed. Since the differential equations which govern such systems are linear constant coefficient in time, exponential solutions of the form e^{st} can be assumed. To examine for stability, we simply solve for the natural frequencies s. If the real part of s is positive, the system is self excited such that any perturbation will grow at an exponential rate. The imaginary part of s yields the oscillation rate of the resulting overstability.

We will separately discuss the cases of two and three discs to understand the classic Wimshurst machine as well as the new ac modification. Then we will use the results previously developed for N coupled devices to again show the possibilities of multi-frequency operation.[8]

II. MODELING OF THE DC WIMSHURST MACHINE

Figure 1(b) illustrates the equivalent circuit of the two cross-coupled discs. The charge on each strip is proportional to the voltage on the inducer electrode with respect to the grounding brush. If we neglect all other mutual capacitances, the charges on the strips are given by[9]

$$q_1 = -C_i v_2,$$
$$q_2 = -C_i v_1, \qquad (1)$$

where C_i represents the capacitance between the inducer and the strips. The minus signs appear because image charges are induced on the strips. It is assumed that geometrically the two discs are identical. When the discs rotate at angular speed ω, n strips per second pass the collecting brushes; n is proportional to the product of ω and the number of strips per disc. The charge transport is thus modeled as current sources of values

$$nq_1 = -nC_i v_2,$$
$$nq_2 = -nC_i v_1, \qquad (2)$$

where we assume that all the charge is collected.[9]

The collectors as charge storers are represented as capacitors C to ground. The resistance R represents leakage resistance to ground. The capacitance C_L represents the capacitance between the adjacent discs plus the capacitance of a load, such as an electrostatic voltmeter. R_L represents leakage resistance or perhaps a load resistance between the discs. Because the circuit is linear, we may assume exponential solutions of the form

$$v_1 = \hat{V}_1 e^{st},$$
$$v_2 = \hat{V}_2 e^{st}, \tag{3}$$

where the voltages are with respect to ground [see Fig. 2(b)].

Applying Kirchhoff's current law at the nodes results in the relations

$$\begin{bmatrix} \dfrac{R_LC_Ls+1}{R_L}+\dfrac{RCs+1}{R} & \dfrac{-(R_LC_Ls+1)}{R_L}+nC_i \\ \dfrac{-(R_LC_Ls+1)}{R_L}+nC_i & \dfrac{R_LC_Ls+1}{R_L}+\dfrac{RCs+1}{R} \end{bmatrix} \begin{bmatrix} \hat{V}_1 \\ \hat{V}_2 \end{bmatrix} = 0. \tag{4}$$

For non-trivial solution, the determinant of the coefficients must be zero:

$$\left(\frac{R_LC_Ls+1}{R_L}+\frac{RCs+1}{R}\right)^2 = \left[nC_i - \frac{(R_LC_Ls+1)}{R_L}\right]^2. \tag{5}$$

Solving for s yields

$$s_1 = \frac{[nC_i-(2/R_L)-(1/R)]}{(C+2C_L)}, \qquad \hat{V}_1 = -\hat{V}_2;$$
$$s_2 = \frac{[-nC_i-(1/R)]}{C}, \qquad \hat{V}_1 = +\hat{V}_2. \tag{6}$$

For self-excitation, s must have a root which is positive, which yields from s_1 the condition

$$nC_i > (2/R_L)+(1/R). \tag{7}$$

If this condition is met, any slight perturbation will grow at an exponential rate. This growth is usually limited by electrical breakdown. The second root is always negative indicating a decaying solution which becomes negligible after a period of time.

The essential ingredients of self-excitation can be treated by a simpler idealized model with no losses where

$$C_L=0, \qquad R=\infty, \qquad R_L=\infty. \tag{8}$$

The resistances introduce a threshold condition whereby the charge transport must exceed the charge-leakage rate for spontaneous voltage buildup. In the following analysis, we will assume the simplifying conditions of Eqs. (8) are true. However, it must be realized that for finite resistances, the shaft speed must exceed a limit determined by these resistances before self-excitation occurs.

III. ANALYSIS FOR AC OPERATION

Figure 2(a) illustrates the schematic configuration for three coupled discs and Fig. 2(b) shows the lossless, idealized equivalent circuit. The simple circuit relations, with solutions of the exponential form as in Eq. (3) yields

$$\begin{bmatrix} nC_i & Cs & 0 \\ 0 & nC_i & Cs \\ Cs & 0 & nC_i \end{bmatrix} \begin{bmatrix} \hat{V}_1 \\ \hat{V}_2 \\ \hat{V}_3 \end{bmatrix} = 0 \tag{9}$$

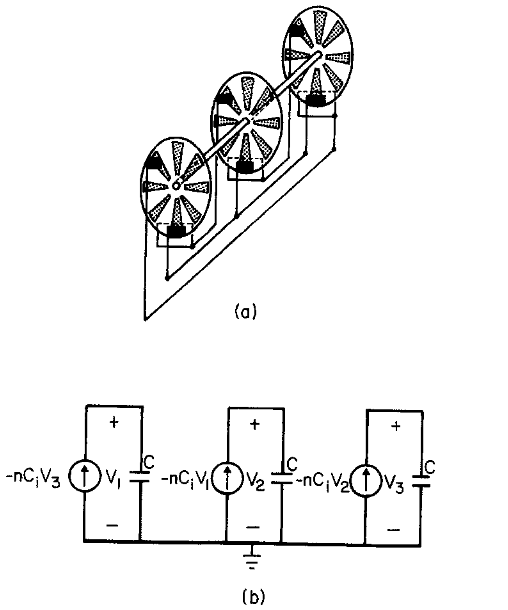

FIG. 2. Basic configuration for self-excited ac voltage buildup: (a) three-disc machine, (b) equivalent circuit.

with solution

$$(Cs)^3 = -(nC_i)^3. \quad (10)$$

Solving for s,

$$s_1 = -nC_i/C, \quad \hat{V}_1 = \hat{V}_2 = \hat{V}_3;$$

$$s_2 = (-nC_i/C)[-\tfrac{1}{2} - (\sqrt{3}/2)j],$$

$$\hat{V}_3/\hat{V}_2 = \hat{V}_2/\hat{V}_1 = \exp(j2\pi/3);$$

$$s_3 = (-nC_i/C)[-\tfrac{1}{2} + (\sqrt{3}/2)j],$$

$$\hat{V}_3/\hat{V}_2 = \hat{V}_2/\hat{V}_1 = \exp(j4\pi/3). \quad (11)$$

The first root strictly decays, while the other two roots are complex conjugates which represent overstability. If leakage became significant, a threshold condition similar to Eq. (7) would be necessary before self-excitation would occur.

We see that the unstable modes have a 120° phase difference between the voltages. Thus, for this configuration three-phase ac high voltage is spontaneously generated. The voltage magnitudes are limited by leakage, saturation or breakdown.

IV. N COUPLED DISCS

With N coupled discs as shown in Fig. 3, the resulting equations are identical to that obtained for N coupled water drop dynamos.[8] In the limits of (8), the equivalent circuit representation shown in Fig. 3 has system equations which can be represented by a linear, constant coefficient difference equation

$$CsV_k + nC_iV_{k-1} = 0. \quad (12)$$

Standard solutions of the form

$$V_k = A\lambda^k \quad (13)$$

substituted into Eq. (12) yield the characteristic equation

$$\lambda = -nC_i/Cs. \quad (14)$$

However, since the last disc is coupled to the first disc

$$V_{N+1} = V_1 \quad (15)$$

so that

$$\lambda^N = 1 \Rightarrow \lambda = -nC_i/Cs = \exp(j2\pi r/N), \quad (16)$$

where $r = 1, 2, \ldots, N$.

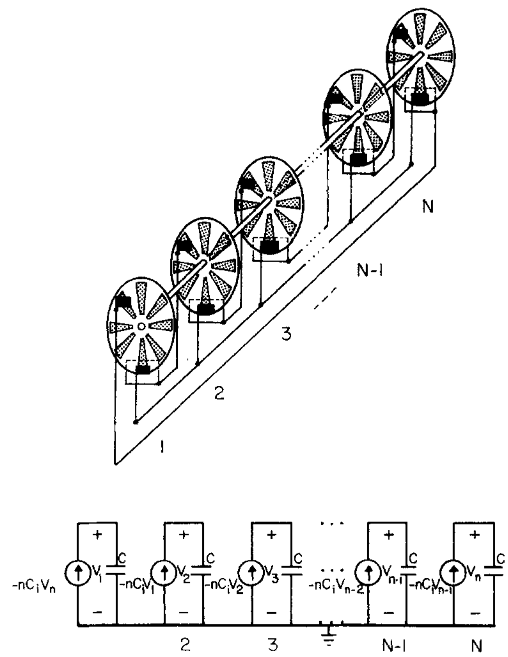

FIG. 3. Basic configuration and equivalent circuit for generalized N-disc machine.

Solving Eq. (16), we have N modes

$$s=(-nC_i/C)\ \exp(-j2\pi r/N),$$

$$V_n/V_{n-1}=\exp(j2\pi r/N), \quad (17)$$

where $r = 1, 2, \ldots, N$.

Equation (17) agrees with the results obtained for $N=2$ and $N=3$, as given by Eqs. (6) and (11) in the limits of Eqs. (8). As these results are identical to the alternating extension of Kelvin's water dropper, the reader is referred to that work[8] for further discussion.

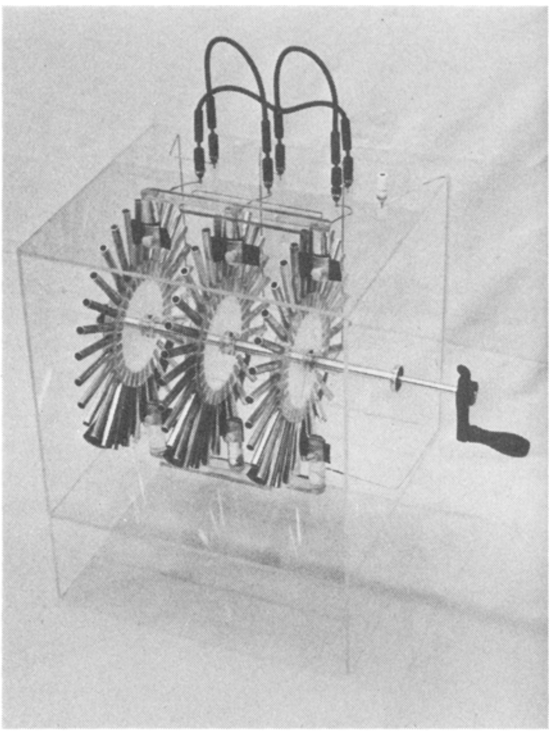

FIG. 4. This device can spontaneously produce high-voltage dc power if only two discs are connected. With all three discs connected, high voltage ac power with frequencies of oscillation up to 2 Hz result. Maximum voltage, plate to plate, is typically 18 kV.

V. EXPERIMENTAL RESULTS

A three-disc version was constructed as shown in Fig. 4. Each disc has a Plexiglas center upon which are mounted aluminum rods which project radially. Semiconducting rubber is used as brushes. Depending on the external connections at the top, it is possible to operate this device either as a two- or three-disc machine, for which dc or three-phase ac can be generated. With more discs, polyphase operation with many natural modes of oscillation can be generated. Losses are very low, so that a hand crank works excellently. Voltage buildup was measured with an electrostatic kilovoltmeter (resistance$\approx 10^{14}$ Ω) and was in excess of 18 kV, limited only by voltage breakdown. To check our theory, a plot of oscillation frequency versus shaft speed was made as

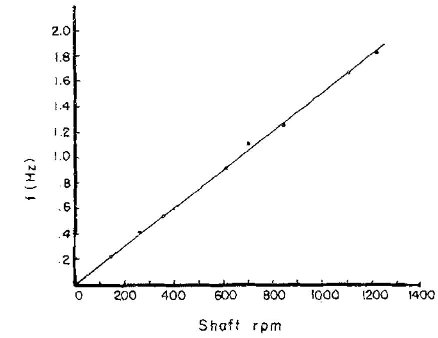

FIG. 5. For the three-disc machine, measured oscillation frequencies as a function of shaft speed agree well with the predicted linear plot.

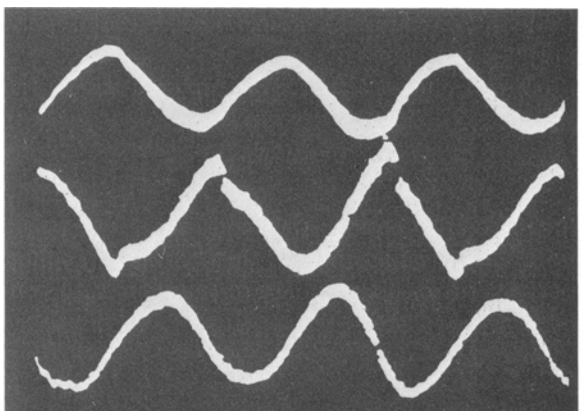

FIG. 6. The three-phase alternating voltages are displayed on a storage oscilloscope through use of a resistor divider. The divider loaded the device, so that the peak voltage is only 2.5 kV with a period of 3 sec.

shown in Fig. 5. The linear plot agrees with Eq. (11). Figure 6 shows the three-phase relationship between the ac voltages. The waveforms are basically sinusoidal with high frequency harmonics and noise due to the discreteness of the rotating rods (similar to slot harmonics in conventional machinery) and to arcing from the electrodes. These high frequency effects can be minimized by using more rods, while the arcing can be decreased by increasing the spacing between the inducer electrodes and the rods and/or replacing the air with an insulant gas such as SF_6.

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